CSC384 Assignment 2

Student Name: Shibo Zhang Student Number: 998542475 CDF account: c1zhangu

Question 4

1.

The following sudoku boards are tested in both model 1 and model 2, the result is written below:

```
 \begin{array}{l} \mathbf{c1} = [[0,\ 0,\ 0,\ 0,\ 3,\ 0,\ 4,\ 0,\ 0], \\ [0,\ 0,\ 1,\ 9,\ 0,\ 0,\ 0,\ 5,\ 8], \\ [0,\ 0,\ 5,\ 7,\ 1,\ 0,\ 2,\ 6,\ 9], \\ [0,\ 0,\ 5,\ 7,\ 1,\ 0,\ 2,\ 6,\ 9], \\ [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\ [0,\ 0,\ 0], \\
```

	Model 1		Model 2	
	Find Solution	Time (s)	Find Solution	Time (s)
C1	Yes	46.49	Yes	0.84
C2	No	39.13	Yes	1.31
C3	No	41.46	Yes	2.12
C4	No	40.5	No	1.45

As the above data and the test sample provided by the prof shown, Model 2 has both better ability on finding the solutions and shorter time consumption. Therefore it proves Model 2 is always more powerful than the other.

2.

The way I take is to calculate their upper bound to compare the space consumption for two models, assume the input board is a very hard sudoku problem, which has almost all of its cells blank (has value 0). For the convienience in calculation, consider all cells are blank (which theoretically can not be true, otherwise the board is unsolvable), and each element in a tuple takes one unit of space.

In this case, approximately the space required for model 1 is:

Number of constraints * Number of tuples in each constraints * Size of the tuple

$$= 972 * (9 * 8) * 2 = 139,968$$

The space required for model 2 is:

Number of constraints * Number of tuples in each constraints * Size of the tuple

$$=27 * 9 ! * 9 = 88, 179, 840$$

As shown, model 2 has much larger uppper bound than model 1, and because their actual space consumption is potisively porportion to the difficulty of the board, thus can conclude model 2 requires much more space than model 1

The time for both models to run the sample tests are shown below:

Test	Model 1 (s)	Model 2 (s)
b1	50.77	1.64
b2	51.17	0.71
b3	40.76	13.47
b4	41.19	0.95
b5	43.98	1.40
b6	42.49	0.94
b7	40.77	3.87

Also with the data provided in part 1, I can conclude that the time it takes for model 2 to complete the job is around 0.7s to 2s in most cases, in some cases it will take upto about 15s. But for model 1, it usually take over 40s to find the solution of a sudoku board.

The CPU time that this algorithm takes primarily depends on the size of the conQueue from the gac_enforce() function. Because the total number of constraints is 972 in model 1 after the set up, and 27 in model 2 after the set up, therefore, although model 2 has the disadvantage of having a more complicate add_tuple function, its advantage in size of conQueue outweights its disadvantage, and thus model 2 consumes should be significantly shorter amount of time.