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Project Assignment 1C: Blending Problem

1 Abstract

This report assists the oil-refining company Oljeblandarna on how to maximize their profits for a time span of three weeks. Their refinery can produce three products called A, B and C. A is of higher quality and can be sold for 1000 kr per unit while B is of lower quality and is sold for 740 kr per unit. Product C cannot be sold as it is and needs to be further processed to reconvert it to product A or B. There is also an option to reconvert B to produce more of Product A. However, reconverting adds an additional costs. Oljeblandarna have the choice of buying two different types of crude oils that convert into the products. Additional parameters are that product B can be stored for the next week, the amount that could be sold varied every week and that excess amounts of oil produced may be discarded. The result of this problem is to buy 300 units of crude oil 1 every week and buy 0, 60 and 80 units of crude oil 2 for week 1, 2 and 3 respectively. Furthermore, since the buying limit of crude oil 1 is always reached, Oljeblandarna are offered by their suppliers to buy more of crude oil 1 for a price of 700 kr per unit. This is found to be a severely overpriced offer. Lastly, this report investigates how Oljeblandarnas profit is affected by additional disturbances, for example if the cost of storage were to be increased, if there would be a limit of storage, or if the supply of crude oil 1 is stochastic. The results show that the profit decreases linearly with both increased storage cost and decreased storage limit. The results further show that a stochastic supply of crude oil 1 does not affect the profitability significantly, and that the operation is very profitable with low risks.

2 Problem Description

The purpose of this paper is to analyze Oljeblandarna's distillation of crude oil under a three week period and optimize their profit with the help of the coding language GAMS. The production plan is based on two different types of crude oil (type 1 and 2) which both produce three types of products (A, B, and C). The production plan is described in further detail in Figure 1.

More specifically, we start by formulating a specific objective function and constraints, connected to Oljeblandarna's production plan with the hopes of finding an optimal solution by using the simplex method. We also hope to gather enough insight so we can help Oljeblandarna decide whether an extra in-purchase of crude oil 1 for a more expensive price of 700 kr, would be beneficial, if the option exists.

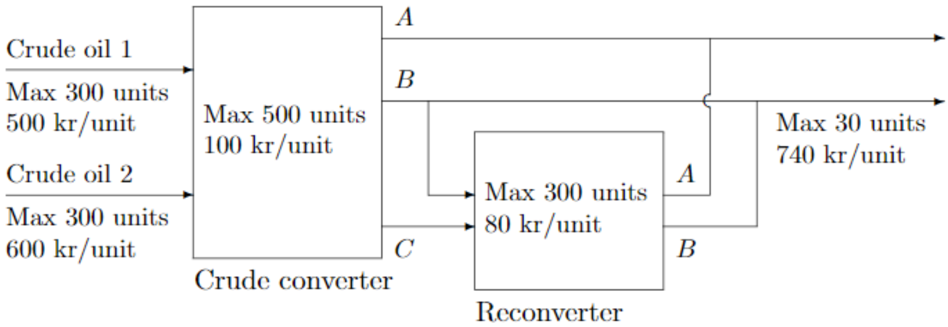

Figure 1: Oil Refining system

Oljeblandarna have also raised concerns about the sensitivity of their production plan. To be precise, they want to understand how a change in storage management and an uncertainty in the supply of crude oil 1 may effect the optimal plan. For now, storage of one unit has a cost of 20 kr with no capacity limit, but the questions remains on how an increase in cost and a decrease of capacity would effect our model.

As a further analysis, the distribution of the supply of crude oil 1 can be approximated to have a normal distribution with expected value 300 and standard deviation 20. A problem arises, since Oljeblandarna needs to decide their production plan before the supply is known. If the supply is less than the planned purchase for a week, then they must purchase crude oil 1 to a more expensive price of 700 kr per unit. In this problem, Oljeblandarna want to maximize their expected profit.

2.1 Description of Oil-Refining System

The oil refining system has two inputs, crude oil 1 and crude oil 2, both with a in-purchase limit of 300 units. The base price for each oil type is 500 kr and 600 kr per unit respectively. When converting the oil, 50 percent of crude oil 1 and 70 percent of crude oil 2 transforms to something we call Product A, which sells at a market price of 1000 kr per unit. At the same time, 30 percent of crude oil 1 and 20 percent of crude oil 2 transform into Product B. Product B entails three options, either sell it to a unit price of 740 kr, store it for a unit cost of 20 kr per week, or reconvert it for 80 kr per unit where 90 percent of the reconverted oil from product B transforms into product A and 10 percent will be left unchanged. The final product that is created through the converting process is Product C. 20 percent of crude oil 1 and 20 percent of crude oil 2 converts to product C which only option is to reconvert with 40 percent will become product A and 60 percent becomes product B. The exchange (in percent) for the crude converter and the reconverter are shown in Tables 1 and 2.

Other aspects of the oil refining system that are important for the model formulation are the capacity restrictions. For the crude converter and reconverter there exists a limit of 500 units and 300 units respectively. There are also restrictions regarding the amount of crude oil that

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Table 1: Crude converter exchange

	Prod. A	Prod. B	Prod.C
Crude oil 1	50	30	20
Crude oil 2	70	20	10

Table 2: Reconverter exchange

	Prod. A	Prod. B
Prod. B	90	10
Prod. C	40	60

can be bought. The maximum units per week for both crude oil 1 and crude oil 2 is 300 units. At the same time, there exists time dependent restrictions which limit the total number of units Oljeblandarna can sell for each week. The maximum amount of Product B which may be sold each week are are 30, 130, and 130 for week 1, 2, and 3. The maximum amount of Product A to be sold is 250 for each week.

3 Mathematical Formulation

3.1 Model Explanation

The objective of this project is to maximize the profit, which is defined as the sales revenue minus the purchase cost and the running costs. Hence the objective function, which is to be maximized, is the sum of these revenues and costs. The objective function should maximize the total profit over three weeks.

The problem is modeled by assigning a variable to each of the "flows". A flow is defined as a passage between two "nodes" where oil is transported in the system. A node, in turn, is defined as an intersection between two or more flows. Furthermore, the volume of oil in the crude converter and the reconverter are also assigned a variable. The setup is illustrated in Figure 2. This model uses up to 16 variables for each week. It is possible to reduce the problem to just six variables, since the only real decisions to be made are:

- How much of crude oil 1 to be purchased.
- How much of crude oil 2 to be purchased.
- How much of Product B to be sent to the reconverter.
- How much of Product B to be stored.
- How much of Product A to be dumped from the system.

- How much of Product A to be dumped from the system.

However, modeling the problem as 16 variables makes it easier to set and understand the constraints of the problem and analyzing specific flows in the system. We define the variables for week 1, 2 and 3 as w, x and y respectively. The setup for week 2 is shown in Figure 2.

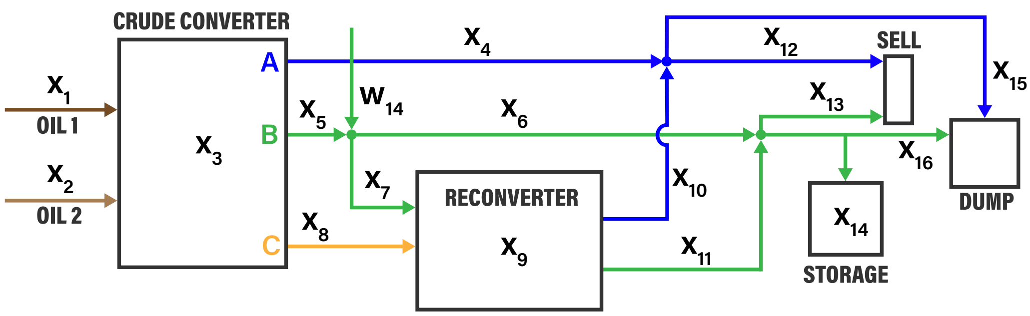

Figure 2: Schematic of the setup for week 2

The other two weeks are similar to week 2 however, there are some slight differences in how the system works and what decisions are possible for each of the three weeks. Firstly, for week 1 and 2, there is a possibility to store Product B for future weeks, while there is no point to store Product B after the last week has passed. Second, if Product B was stored in the previous week, it is then used in week 2 and 3, while we assume there is no oil stored in the beginning of week 1. Third, the number of products which can be sold differs depending on the week. This means that, in general, we cannot optimize the total profit for all three weeks by only considering a part of the problem. Instead, we consider the three weeks as one large system. The corresponding optimization problem then has 47 variables. Note that w and x are vectors of length 16 while y is a vector of length 15, since there is no option to store Product B in week 3.

The optimization problem has several constraints which need to be considered in the solution. The constraints can be divided into three categories:

- Input-output constraints. Each node has to be balanced, i.e. the amount of input must be equal to the amount of output. This is because no oil can be lost in the system. If each node is balanced, then the whole system is also balanced, and the volume of the crude oil bought is equal to the volume of the oil products sold. An example of such a constraint is $x_1 + x_2 = x_3$, since the amount of oil in the crude converter is equal to the sum of the input of crude oil 1 and crude oil 2.

- Converter contraints. The crude converter and the reconverter produces a fixed distribution of products based on the input. The distributions of products that these converters produce are considered in the converter constraints. An example of such constraint is

$0.5x_1 + 0.7x_2 = x_4$, since the amount of Product A produced from the crude converter is equal to 50 percent of the crude oil 1 plus 70 percent of the crude oil 2.

- Capacity restrictions. These constraints describe restrictions of capacity that exist. Examples include the restrictions on how much that can be bought and sold each week, the capacity of the converters and the storage limit, if applicable. An example is the constraint $x_2 \leq 300$, which states that we may not buy more than 300 units of crude oil 2 for the week.

Lastly, we assume that the oil and products can only transport in one direction through the flows, so that none of the variables can be negative.

3.2 Mathematical Model Definition

3.2.1 Base Problem

As mentioned before, the objective function for the mathematical optimization problem is the sum of sales revenue minus the purchase and running costs. More specifically, the sales revenue consists of the amount of sold units of Product A and Product B, the purchase costs consists of purchases of crude oil 1 and crude oil 2, and the running costs consists of the cost of running the crude converter and the reconverter, as well as storing units for the coming week. As Figure 2 shows, for each week, variables with specific indices i correspond to specific volumes:

- $i = 1$ represents the amount of crude oil 1 purchased.
- $i = 2$ represents the amount of crude oil 2 purchased.
- $i = 3$ represents the amount put in the crude converter.
- $i = 9$ represents the amount put in the reconverter.
- $i = 12$ represents the amount of Product A sold.
- $i = 13$ represents the amount of Product B sold.
- $i = 14$ represents the amount of Product B stored to next week.

If we let w, x, y denote the decisions made in week 1, 2, and 3 respectively, and

$$v = \begin{bmatrix} w \\ x \\ y \end{bmatrix}.$$

we can then write the problem as a linear optimization problem, where the cost is negative and the revenue is positive.

$$
\begin{aligned}
\underset{w,x,y}{\text{maximize}} \quad & -500w_1 - 600w_2 - 100w_3 - 80w_9 + 1000w_{12} + 740w_{13} - 20w_{14} \\
& -500x_1 - 600x_2 - 100x_3 - 80x_9 + 1000x_{12} + 740x_{13} - 20x_{14} \\
& -500y_1 - 600y_2 - 100y_3 - 80y_9 + 1000y_{12} + 740y_{13} \\
\text{subject to} \quad & A_1 v = 0, \\
& A_2 v = 0, \\
& A_3 v \leq b_3, \\
& v \geq 0.
\end{aligned}
\tag{1}
$$

Here, $A_1 v = 0$, corresponds to the input-output constraints, $A_2 v = 0$, corresponds to the converter contraints and $A_3 \leq b_3$, corresponds to the capacity restrictions.

3.2.2 Optimization with the Option to Purchase Crude Oil At Higher Price

If Oljeblandarna has the option to purchase crude oil 1 to a more expensive price of 700 kr per unit in addition to the crude oil 1 with price 500 kr per unit, then the total crude oil 1 put in the system for a week is the sum of the volume of the cheap and the expensive option, for example for week 2,

$$x_1 = x_{cheap} + x_{exp}$$

where, x_{cheap} is the volume of crude oil 1 bought at an cheap price and x_{exp} is the volume of crude oil 1 at a expensive price. As before, x_1 denotes the total input of crude oil 1 into a system for the week. Naturally, one only wants to purchase the expensive alternative if it is no longer possible to buy more of the cheap option. Nevertheless, it is sufficient that the only constraint on x_{exp} is that it is larger than or equal to 0, since we only are interested in optimal solutions and the program will not increase x_{exp} if x_{cheap} is not on its boundary. This can be argued by that if we buy the expensive alternative without having maxed out the cheap alternative, then we can buy less of the expensive alternative and more of the cheap alternative. This way, we reduce the purchase costs without reducing the revenue. Therefore, any solution that buys the expensive alternative without having maxed out the cheap alternative cannot be an optimal solution. Mathematically, the corresponding problem can be written with just some minor modifications to Equation 1:

$$
\begin{aligned}
\underset{w,x,y}{\text{maximize}} \quad & -500w_{cheap} - 700w_{exp} - 600w_2 - 100w_3 - 80w_9 + 1000w_{12} + 740w_{13} - 20w_{14} \\
& -500x_{cheap} - 700x_{exp} - 600x_2 - 100x_3 - 80x_9 + 1000x_{12} + 740x_{13} - 20x_{14} \\
& -500y_{cheap} - 700y_{exp} - 600y_2 - 100y_3 - 80y_9 + 1000y_{12} + 740y_{13} \\
\text{subject to} \quad & A_1 v = 0, \\
& A_2 v = 0, \\
& A_4 v \leq b_4, \\
& v \geq 0, \\
& w_1 - w_{cheap} - w_{exp} = 0, \\
& x_1 - x_{cheap} - x_{exp} = 0, \\
& y_1 - y_{cheap} - y_{exp} = 0, \\
& w_{cheap} \leq 300, \\
& x_{cheap} \leq 300, \\
& y_{cheap} \leq 300, \\
& w_{cheap},\ w_{exp},\ x_{cheap},\ x_{exp},\ y_{cheap},\ y_{exp} \geq 0.
\end{aligned} \tag{2}
$$

The definitions of the notations are the same as in Equation 1. The only difference between A_4, b_4 and A_3, b_3 used in the original problem is that the total amount of crude oil 1 bought each week is unlimited (but is still required to be positive) in the modified problem. The restrictions are instead put on w_{cheap}, x_{cheap}, and y_{cheap} .

3.3 Effects of Storage Constraints

To better understand the effects of changes to storage management, a simulation is used. We start by implementing the correct changes to our previous model which included replacing the constant cost of storing with a variable $q_c \in [20, \infty]$ and adding an upper bound constraint, $q_s \in [0, \infty]$, to the storage variable for each week. Important to note is that these changes are done separate to each other in order to better understand the consequence of each condition. The new model is presented as

$$
\begin{aligned}
\underset{w,x,y}{\text{maximize}} \quad & -500w_1 - 600w_2 - 100w_3 - 80w_9 + 1000w_{12} + 740w_{13} - q_c w_{14} \\
& -500x_1 - 600x_2 - 100x_3 - 80x_9 + 1000x_{12} + 740x_{13} - q_c x_{14} \\
& -500y_1 - 600y_2 - 100y_3 - 80y_9 + 1000y_{12} + 740y_{13} \\
\text{subject to} \quad & A_1 v = 0, \\
& A_2 v = 0, \\
& A_3 v \leq b_3, \\
& v \geq 0, \\
& w_{14} \leq q_s \\
& x_{14} \leq q_s
\end{aligned}
\tag{3}
$$

The second part of this problem is to formulate the actual iteration of the simulation. For this step we simply change the value for the constraints q_c and q_s respectively and noted its effect on the profit.

3.4 Optimal Strategy when Supply is Stochastic

Suppose that the supply of crude oil 1 is not deterministic but normally distributed with mean 300 and standard deviation 20. Nevertheless, Oljeblandarna needs to determine their production plan before they know the actual supply. If there is a shortage of crude oil 1, Oljeblandarna must buy it for a more expensive price of 700 kr per unit. Oljeblandarna wants to determine the strategy that maximizes their expected return.

To determine the optimal solution to a stochastic linear programming problem, we use the method introduced in Birge and Louveaux (1997). The problem is a two-stage decision problem, where some decisions need to be made before the stochastic outcome is known (first-stage decisions) and some decision may be made after the outcome is known (second-stage decision). In our case, the stochastic outcome is the available supply of crude oil 1. The second-stage decisions are the amount of the cheap and the expensive oil to be bought, and the first-stage decisions are every other decision in the production plan (including the total amount of crude oil 1 to be bought). Let X denote the stochastic supply of crude oil 1, which is normally distributed with mean 300 and standard deviation 20. The expected profit is written as

$$
\begin{aligned}
z = & -600w_2 - 100w_3 - 80w_9 + 1000w_{12} + 740w_{13} - 20w_{14} \\
& -600x_2 - 100x_3 - 80x_9 + 1000x_{12} + 740x_{13} - 20x_{14} \\
& -600y_2 - 100y_3 - 80y_9 + 1000y_{12} + 740y_{13} \\
& -\int_{-\infty}^{\infty} f(t)(500w_{cheap}(t) + 700w_{exp}(t) + 500x_{cheap}(t) + 700x_{exp}(t) + 500y_{cheap}(t) + 700y_{exp}(t))dt
\end{aligned}
$$

Here $f(t)$ denotes the probability density function of X. We cannot solve this with linear programming. Instead, we can approximate the integral as a sum of probabilities:

$$
\begin{aligned}
z = & -600w_2 - 100w_3 - 80w_9 + 1000w_{12} + 740w_{13} - 20w_{14} \\
& -600x_2 - 100x_3 - 80x_9 + 1000x_{12} + 740x_{13} - 20x_{14} \\
& -600y_2 - 100y_3 - 80y_9 + 1000y_{12} + 740y_{13} \\
& -\sum p(s)(500w_{cheap}(s) + 700w_{exp}(s) + 500x_{cheap}(s) + 700x_{exp}(s) + 500y_{cheap}(s) + 700y_{exp}(s))
\end{aligned}
$$

Here, we approximate X to have discrete outcomes. We denote $p(s) = P(s - \Delta s \leq X \leq s)$ and $\Delta s \to 0$ (In theory, in practice we use $\Delta s = 10$ for computational purposes). This problem can be solved as a linear programming problem. We write it in the so-called extensive form:

$$
\begin{aligned}
\underset{w,x,y}{\text{maximize}} \quad & -600w_2 - 100w_3 - 80w_9 + 1000w_{12} + 740w_{13} - 20w_{14} \\
& -600x_2 - 100x_3 - 80x_9 + 1000x_{12} + 740x_{13} - 20x_{14} \\
& -600y_2 - 100y_3 - 80y_9 + 1000y_{12} + 740y_{13} \\
& -\sum p(s)(500w_{cheap}(s) + 700w_{exp}(s) + 500x_{cheap}(s) + 700x_{exp}(s) + 500y_{cheap}(s) + 700y_{exp}(s)) \\
\text{subject to} \quad & A_1 v = 0, \\
& A_2 v = 0, \\
& A_4 v \leq b_4, \\
& v \geq 0, \\
& w_1 - w_{cheap}(s) - w_{exp}(s) = 0 \; \forall s, \\
& x_1 - x_{cheap}(s) - x_{exp}(s) = 0, \; \forall s \\
& y_1 - y_{cheap}(s) - y_{exp}(s) = 0, \; \forall s \\
& w_{cheap}(s) \leq s, \; \forall s, \\
& x_{cheap}(s) \leq s, \; \forall s, \\
& y_{cheap}(s) \leq s, \; \forall s, \\
& w_{cheap}(s),\; w_{exp}(s),\; x_{cheap}(s),\; x_{exp}(s),\; y_{cheap}(s),\; y_{exp}(s) \geq 0, \; \forall s.
\end{aligned} \tag{4}
$$

Theoretically, for every possible value of X, we add an additional constraint specifying that the amount of cheap crude oil 1 bought that week cannot exceed s. This is done by defining a new variable, for example $w_{cheap}(s)$ for week 1 represents the amount of cheap crude oil 1 to be bought if $X = s$. Another constraint is that regardless of the outcome of X, the total crude oil 1 bought is the sum of the cheap and the expensive alternative. For each possible outcome s of X, we define six new variables: $w_{cheap}(s)$, $w_{exp}(s)$, $x_{cheap}(s)$, $x_{exp}(s)$, $y_{cheap}(s)$, and $y_{exp}(s)$. The number of variables for this stochastic problem increases with the number of

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different partitions made when we discretize the normal distribution as a sum of probabilities. For example, if we divide the normal distribution into 100 different outcomes, then the linear program will have 600 additional variables.

4 Results

4.1 Results when Supply is Deterministic

Table 3: Results for the optimal strategy to maximize profits

Week	1	2	3
Crude oil 1 to buy	300	300	300
Crude oil 2 to buy	0	60	80
Product A to sell	250	250	250
Product B to sell	30	130	130
Product B to store	20	0	0
Product A to dump	0	0	0
Product B to dump	0	0	0

The optimal strategy when the supply of crude oil 1 is deterministic is to maximize sales, selling a total of 750 units of Product A and a total of 290 units of Product B. A more detailed overview of the results can be seen in Table 3. Table 3 also shows that the sum of amount of bought oil is equal to the sum of the sold oil, meaning that nothing is lost in the system. The expected total profit for this strategy is approximately 299 600 kr. The solutions in a schematic figure for week 1, 2 and 3 are presented below in Figure 3-5 respectively for a clearer overview of the flows of the oils.

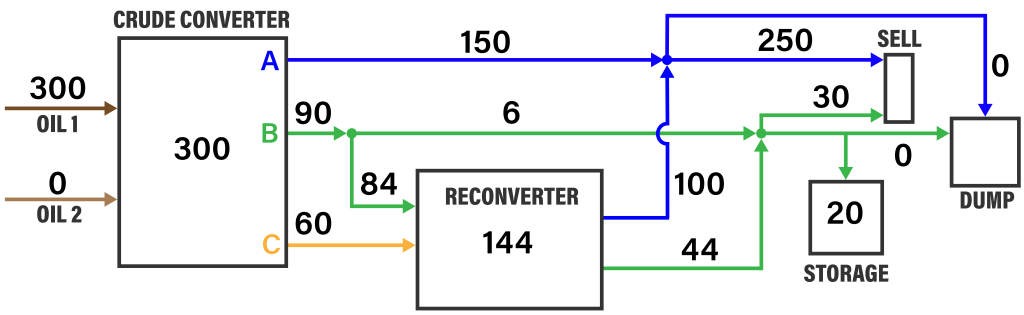

Figure 3: Optimal flow for week 1

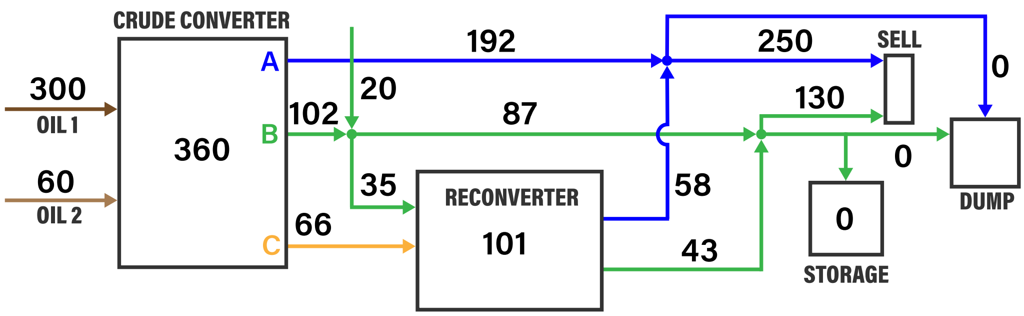

Figure 4: Optimal flow for week 2

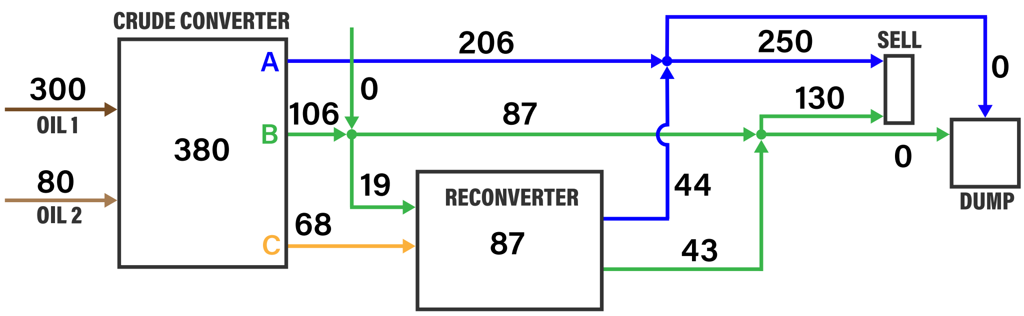

Figure 5: Optimal flow for week 3

The optimal solution can be found in Table 4 for when Oljeblandarna has the opportunity to buy more of crude oil 1 for a higher price if the limit of buying 300 units is reached. The result is that the more expensive variant is not worth the additional price increase of 200 kr per unit. Therefore, none of it was bought. However, through iteration, if the price of the expensive variant of crude oil 1 cost 577 kr per unit or less, the algorithm buys only crude oil 1 and none of crude oil 2.

Table 4: Results for the optimal strategy to maximize profits introducing more crude oil 1 for a price of 700 kr per unit

Week	1	2	3
Crude oil 1 (cheap) to buy	300	300	300
Crude oil 1 (expensive) to buy	0	0	0
Crude oil 2 to buy	0	60	80
Product A to sell	250	250	250
Product B to sell	30	130	130
Product B to store	20	0	0
Product A to dump	0	0	0
Product B to dump	0	0	0

4.2 The Effect of Storage Sensitivity

When analyzing the effect of storage costs and storage limits on the total profit, we consider the original problem, without the option to buy crude oil 1 to a higher price and where the supply is deterministic. The first result given by the simulation is that regardless of the storage cost, or the storage limit, it is never recommended to store anything at the end of week 2. Thus, in the following results, we only present the amount of stored units at the end of week 1.

When increasing the the cost of storing product B, we see a clear negative linear relation to Oljeblandarnas profitability. At a closer look, it seems that there exists a breaking point between two lines at 78 kr per unit and 643, see Figure 6. By looking at Figure 7, which shows plots the number of stored units in the optimal solution versus the storage cost, we can see that there are two breaking points where the decision changes. If the storage cost is less than 78 kr per unit, then it is worth to deliberately store Product B to use or sell in the next week. If the storage cost is between 78 and 643 kr per unit, then it is recommended to only store the 14 units of Product B that are left after everything has been sold or converted to Product A. If the storage cost is greater than 643, then it is cheaper to throw out the remainder of Product B than to store it. We can interpret this as that the most expensive unit of Product B produced in week 2 is 643 kr. The linear relationship in Figure 6 is simply because the profit has a piecewise linear relationship with the number of units stored.

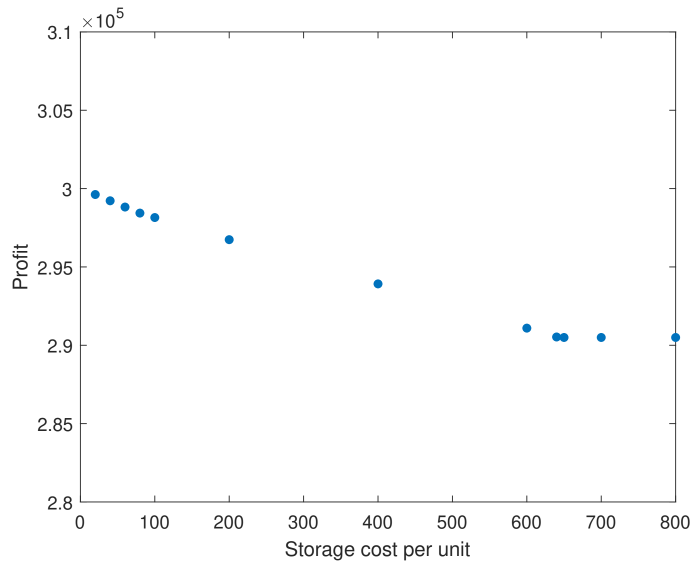

Figure 6: The optimal profit for some storage costs

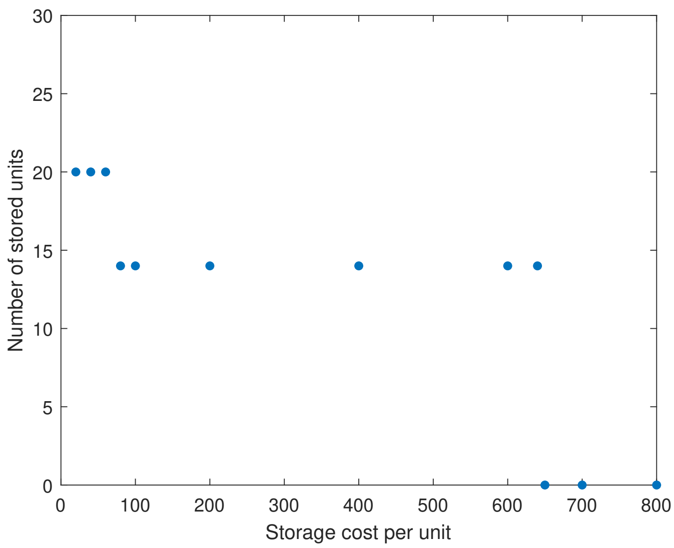

Figure 7: The optimal number of stored units for some storage costs

The same can be said when we analyze how storage space impacts profitability with a breaking point at both 14 stored units and 299 282 kr profit and another breaking point at 20 stored units with a profit of 299 622 kr, see Figure 8. This, again linear relationship, can be explained by Figure 9. Not surprisingly, it is always optimal to fully utilize the storage, as long as the limit is under 20 units per week. However if the limit is under 14, it seems to be cheaper to simply throw away the remainder of the product instead of changing the sales strategy. Hence, the linear relation is different between the storage limit lower than 14 and the storage limit higher than it.

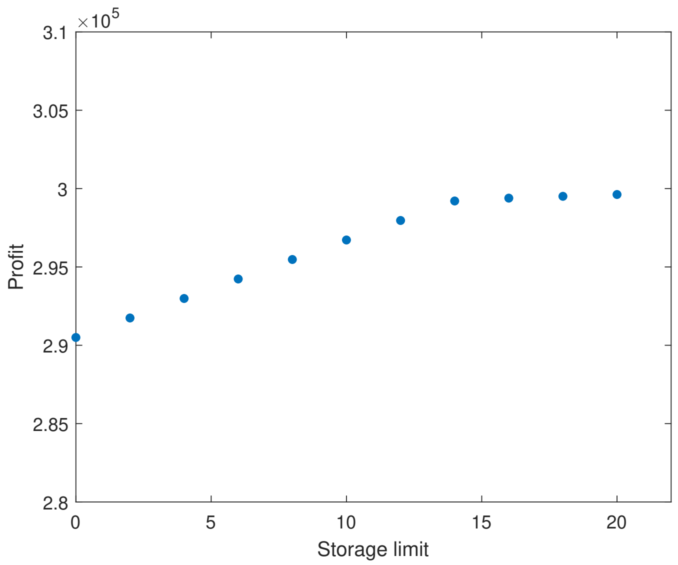

Figure 8: The optimal profit for some storage limits

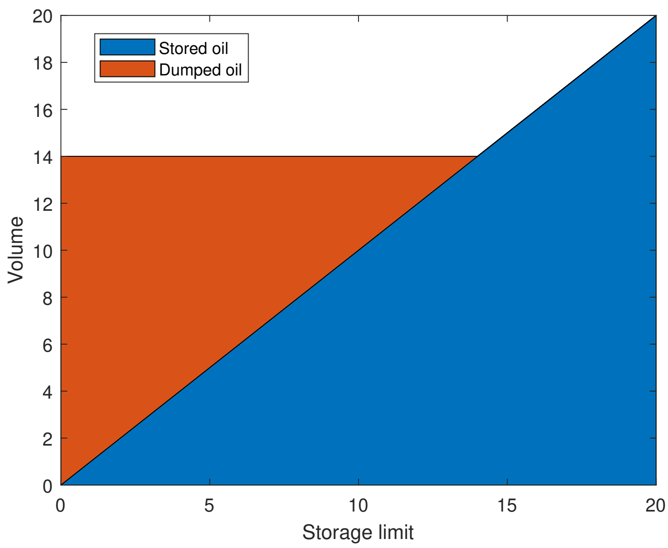

Figure 9: The optimal number of stored and dumped units for different storage limits

4.3 Results when Suppply is Stochastic

The optimal solution when Oljeblandarna considers the supply of crude oil 1 as an approximately normally distributed stochastic variable with mean 300 and standard deviation 20 is shown in Table 5. We can see that the only difference between this result and the result where the supply is deterministic is that we store a bit less of Product B at then end of week 1 and instead purchase more of crude oil 2 at the beginning of week 2. The expected profit is 296 000 kr.

Table 5: Results for the optimal strategy to maximize profits when the supply of crude oil 1 is approximately normally distributed

Week	1	2	3
Crude oil 1 to buy	294	300	300
Crude oil 2 to buy	0	66	80
Product A to sell	250	250	250
Product B to sell	30	130	130
Product B to store	14	0	0
Product A to dump	0	0	0
Product B to dump	0	0	0

5 Discussion

An interesting results from Table 3 is that the amount of crude oil 1 to buy is always maximized and crude oil 2 is not nearly as much bought. From this result one might get the impression that oil 1 is far more cost efficient than oil 2. This of course raised the question, how much more cost efficient is it? Table 4 shows that buying 700 kr per unit, a 200 kr per unit increase, is not more efficient than buying crude oil 2 for 600 kr per unit. The results shows that for it to be more worth to buy more of oil 1, after the limit is reached, rather than oil 2, the price of crude oil 1 could be increased with maximum 77 kr per unit. If the price would increase more than that, it would be more cost efficient to buy oil 2 to maximize profit. This is much less than the 200 kr per unit increase the suppliers want. Thus, we suggest that Oljeblandarna should not accept the given price of 700 kr per unit and negotiate a more reasonable price with their suppliers since they are getting swindled.

When examining the effects of storage limit and costs we seem to have found two different optimal strategy's depending on the value of the two variables. What is worth noting is that because there are no fixed costs and there exists an optimal solution with no excess products for the constraints of week 2, it will never be optimal to store that week. The first strategy, which applies when the cost is between 0 to 78 kr or a storage space higher than 14, suggests the same optimal strategy as in the initial problem. In this situation the profit is relatively stable. Important to note is that an increase of storage space or decrease of cost will also not have a significant effect on the profit either. The second strategy, applied when the cost is between 78 and 643 suggests buying 294 units of product A, reconverting 88 units, and storing as much of the leftover product as possible. This strategy is similarly sensitive to changes in the cost as the previous strategy but drastically more sensitive to changes in storage space. This is important for Oljeblanderna to know, that if changes to the storage system has to be made, they should prioritize increasing the cost rather than the capacity if the second strategy is being used. Thus, it may be worth for them to pay for extra storage space.

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The profit is also not very sensitive to fluctuations in the supply of crude oil 1. For example, if the available supply would be 280 for each of the three weeks, which is in the worst 0.5 percent of cases, the optimal strategy would still yield 285 000 kr in profit, only around 11 000 kr less than the expected profit and only around 15 000 kr less than if the supply were deterministic. To put this number into perspective, the revenue for the three-week period is 964 600 kr. Thus, we can conclude that the operations are very profitable with low risks.

6 Conclusion

In this report we have found optimal solutions to four different problems:

1. When the supply is deterministic with no option to buy additional crude oil 1 for a higher price
2. When the supply is deterministic with option to buy additional crude oil 1 for a higher price
3. When the supply is deterministic with no option to buy additional crude oil 1 for a higher price and the storage cost is increased or the storage space is limited
4. When the supply is stochastic and the production strategy needs to be decided before the supply is known.

For all four problems, it is optimal to maximize sales.

1. When the supply is deterministic with no option to buy additional crude oil 1 for a higher price, it is optimal to buy 900 units of crude oil 1 and 140 units of crude oil 2.
2. If the option to buy more of crude oil 1 for a price exceeding 577 kr Oljeblandarna should not take it. Thus, if they are offered additional crude oil 1 to a price of 700 kr per unit, the optimal strategy should be the same as when this option is not available.
3. Changes in storage costs and storage limits have a piecewise linear relation to the profit. When storage management is changed, it is more profitable to increase the storage cost than to decrease the storage capacity, since capacity has a stronger negative correlation.
4. When the supply is stochastic, the optimal solution and the expected profit is only marginally worse than then when the supply is deterministic. Furthermore, the risks are small and the profitability is good even in the worst case scenarios.