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Project Assignment 2A: Power Plant Optimization

1 Abstract

This report illustrates a production problem a power company is faced with. It shows how the problem is modelled and optimized. The company needs to fulfill the demand of power production and wants to do so as profitable as possible. This is done by minimizing the company's expected cost. Two scenarios are considered, when the expected demand of power is deterministic and known to the company and when the demand is subject to some fluctuation or randomness. The report shows how the company minimizes its cost and how the uncertainty affects the company's production and cost. The optimal cost for the deterministic problem is 3828.5 kkr per day and the optimal cost for the stochastic problem is 3857.5 kkr per day. There is also a discussion on how the model can be improved.

2 Problem Description

The purpose of this report is to help Varme optimize their costs by modelling their operation as a mixed integer linear problem. Varme is a power production company which needs to satisfy the demand of power. They are faced with the planning of its power production over a 24 hour period. The 24 hours are divided into five time periods with different power demand, as illustrated in Table 1.

Table 1: The demand for power, divided in different period

Time Period	Expected demand (MW)
00-05	50
05-10	60
10-15	80
15-20	70
20-24	60

Varme's machinery consists of three different units with different cost and production capacity, summarized in Table 2. The cost for each unit is made up by its initial starting cost and running cost which depends on how many hours the unit is used for production. The initial cost is in the unit (kkr) and the running cost is in (kkr per MW and hour). Each unit has a lower and an upper bound in its production, i.e. for every time period it can only produce between its minimum and maximum level.

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Table 2: The costs and restriction of each unit

Unit	Inital cost (kkr)	Running cost (kkr/MWh)	Minimum level (MW)	Maximum level (MW)
1	10	2.5	10	50
2	13	2.5	12	45
3	16	2.4	15	55

For every time period, Varme's production has to meet the associated power demand. There is a constraint on all three units such that each unit can be running for at most three time periods in a row. After that, the unit has to be switched off for at least one period. It is considered to be a "warm start" when a unit is switched on after being off for one period and similarly a "cold start" if the unit has been off for two or more periods. The initial cost column in Table 2 represents the initial cost for a warm start. For the cold start, the initial cost is increased by 50% compared to the warm start.

The goal is to help Varme determine the best production plan which minimizes the cost and meets the expected demand and constraints. The production plan to be considered for the 24-hour period should be cyclic, so the objective is can be interpreted as to minimize the cost for an infinite time period.

For a bit more realistic set up, for the advanced part it is assumed that the expected power demand shown in Table 1 is stochastic rather than deterministic. The demand for each period is assumed to be normally distributed with the mean given in the table and standard deviation 5 MW. The demand for each period is known at the beginning of each period and is constant over that certain period. Additional power is available from other suppliers if needed at the cost of 10 kkr/MWh. The problems, both the deterministic and stochastic, are formulated, modelled and then solved using `GAMS`.

3 Mathematical Formulation

3.1 Introduction of Variables

We can categorize each row in Table 1 as one time period. Then there are five periods and 3 units which can be run in each of these periods. Let $t = 1,2,3,4,5$ denote the time period and $i = 1,2,3$ denote the unit. For each unit and time, we also want to know the following information, which are to be the variables to the optimization problem:

- $x_{i,t} \geq 0$ denotes the power of unit i during period t
- $y_{i,t} \in \{0,1\}$ is 1 if unit i is running during period t, and 0 if not
- $v_{i,t} \in \{0,1\}$ is 1 if unit i performs a cold start in period t, and 0 if not
- $w_{i,t} \in \{0,1\}$ is 1 if unit i performs a warm start in period t, and 0 if not

3.2 Dimension Analysis and the Objective Function

The parameters of the optimization problem are given in Watt, which is defined as Joule per second. This can be interpreted as the mean amount of energy produced per time unit. Since the time periods are of different lengths, we need to consider the length of each period when calculating the total cost.

The running cost for each unit is interpreted to be given in kkr per megawatt-hour. If c_i is denoted as the running cost for unit i, then this means that c_i is the cost of running the unit at 1 MJ/s for one hour. Further, $c_i x_{i,t}$ is the cost of running unit i at power $x_{i,t}$ for one hour. We can denote n_t as the number of hours in period t. Then the cost of running unit i for the whole period t is given by $n_t c_i x_{i,t}$.

The objective function to be minimized is defined as the total cost of the operations for one day, where the cost is positive. The total cost for one day is given by the sum of the running costs, cold starting costs, and the warm starting costs, for all units over all periods. Thus, the objective function is

$$\sum_{t=1}^{5}\sum_{i=1}^{3}(n_t c_i x_{i,t} + a_i v_{i,t} + b_i w_{i,t}), \tag{1}$$

where

- n_t is the number of hours in period t
- c_i is the running cost of unit i
- a_i is the cost of a cold start for unit i
- b_i is the cost of a warm start for unit i.

3.3 Introduction of Constraints

There are several constraints to the mathematical problem. The first constraint is that the power produced every period needs to at least satisfy the expected power demand. Power is additive, so the demand for one period should be less than or equal to the sum of the power produced from all three units. If we denote d_t as the demand for period t, then this can be written as

$$\sum_{i=1}^{3} x_{i,t} \geq d_t,\ t = 1,2,3,4,5.$$

Another constraint is that a unit may not be run more than three time periods in a row. This also includes the time periods the day before, so if a unit is run the last three periods one day, it may not be run the first period the next day. Since the optimal solution has to be cyclic, and every 24 hours has to have the same schedule, it is sufficient to restrict the number of periods a unit can be run to be less than three, i.e.

$$\sum_{t=1}^{5} y_{i,t} \leq 3, \ i = 1,2,3.$$

This constraint can be used whenever the schedule is required to be periodic and the number of allowed running periods in a row is less than the total number of periods, in this case three is less than five. This can be argued by that none of the $\binom{5}{3}$ possible combinations of scheduling of a unit violates the requirement that a unit may not be run more than three time periods in a row. However, if the mathematical constraint would be $\sum_{t=1}^{5} y_{i,t} \leq 4$, then it is easy to see that the requirement can be violated. Thus, limiting the number of time periods a unit may be run to three per day is appropriate.

A third constraint to set is so that cold starts are correctly identified. Whether a cold start is initiated at time t depends on if the unit is running at the current time and if it was not running for two consecutive periods before. A cold start can be identified by the inequality

$$y_{i,t} - y_{i,t-1} - y_{i,t-2} \leq v_{i,t}, \ i = 1,2,3, \ t = 1,2,3,4,5.$$

We can see that the left-hand side of the inequality is only 1 if $y_{i,t} = 1$ and $y_{i,t-1} = y_{i,t-2} = 0$, which is when a unit is running and has not been run for the previous two time periods. Otherwise it is 0 or -1. Since $v_{i,t}$, the right-hand side, is associated with a cost, the optimization program will not set $v_{i,t} = 1$ if the constraint would not be violated otherwise. Thus, $v_{i,t} = 1$ if and only if the unit performs a cold start at time t. Since the the schedule is periodic, if $t = 1$ then $t-1 = 5$ and $t-2 = 4$. If $t = 2$ then $t-2 = 5$. In practice, this is handled with the `GAMS` operation `"- -"`.

A forth constraint identifies if a warm start is performed at time t:

$$y_{i,t} - y_{i,t-1} \leq v_{i,t} + w_{i,t}, \ i = 1,2,3, \ t = 1,2,3,4,5.$$

The reasoning for this constraint is similar to the reasoning behind the constraint for cold starts. The left-hand side of the inequality is 1 only if $y_{i,t} = 1$ and $y_{i,t-1} = 0$. Furthermore, we have shown that $v_{i,t} = 1$ if and only if the unit performs a cold start at time t. As in the constraint for cold starts, $w_{i,t}$ is associated with a cost, and the optimization program will not set $w_{i,t} = 1$ if the constraint would not be violated otherwise. Thus, $w_{i,t} = 1$ if and only if a unit is running at time t but not in time $t-1$, and the unit does not perform a cold start at time t. This is the definition of a warm start.

The fifth constraint is that if a unit is used in a period, the power it produces has to be between two levels. Here, we utilize the binary variables so that the limit only applies if the unit is used, and equal to 0 otherwise. If we denote k_i as the maximum and l_i as the minimum power a unit i can be run at if it is used, the mathematical constraints can be written as the set of two inequalities

$$x_{i,t} \leq k_i y_{i,t} \text{ and } x_{i,t} \geq l_i y_{i,t}.$$

for every i and t. We examine the possibilities of these constraints. If $y_{i,t} = 0$, then the two inequalities will together form the equality $x_{i,t} = 0$. In other words, if a unit i is not used at period t, then the power it produces that period is 0. If $y_{i,t} = 1$, then we can write the two inequalities as a single inequality $l_i \leq x_{i,t} \leq k_i$, which means that the power produced must be between the maximum and the minimum level allowed, if the unit is used at period t.

3.4 Formulation of the Deterministic Optimization Problem

To summarize the discussion on variables, parameters, and constraints, the optimization problem can be formulated as the following, where the cost is positive

$$\begin{array}{lll}
\underset{x,y,v,w}{\text{minimize}} & \sum_{t=1}^{5}\sum_{i=1}^{3}(n_t c_i x_{i,t} + a_i v_{i,t} + b_i w_{i,t}) & \\
\text{subject to} & \sum_{i=1}^{3} x_{i,t} \geq d_t, & t = 1,2,3,4,5, \\
& \sum_{t=1}^{5} y_{i,t} \leq 3, & i = 1,2,3, \\
& y_{i,t} - y_{i,t-1} - y_{i,t-2} \leq v_{i,t} & i = 1,2,3,\ t = 1,2,3,4,5, \\
& y_{i,t} - y_{i,t-1} \leq v_{i,t} + w_{i,t} & i = 1,2,3,\ t = 1,2,3,4,5, \\
& x_{i,t} \leq k_i y_{i,t} & i = 1,2,3,\ t = 1,2,3,4,5, \\
& x_{i,t} \geq l_i y_{i,t} & i = 1,2,3,\ t = 1,2,3,4,5, \\
& x_{i,t} \geq 0 & i = 1,2,3,\ t = 1,2,3,4,5, \\
& y_{i,t} \in \{0,1\} & i = 1,2,3,\ t = 1,2,3,4,5, \\
& v_{i,t} \in \{0,1\} & i = 1,2,3,\ t = 1,2,3,4,5, \\
& w_{i,t} \in \{0,1\} & i = 1,2,3,\ t = 1,2,3,4,5.
\end{array} \tag{2}$$

where

- $x_{i,t} \geq 0$ denotes the power of unit i during period t
- $y_{i,t} \in \{0,1\}$ is 1 if unit i is running during period t, and 0 if not
- $v_{i,t} \in \{0,1\}$ is 1 if unit i performs a cold start in period t, and 0 if not

- $w_{i,t} \in \{0,1\}$ is 1 if unit i performs a warm start in period t, and 0 if not
- n_t denotes the number of hours in period t
- c_i denotes the cost of producing 1 MWh of energy from unit i
- a_i denotes the cost of a cold start for unit i
- b_i denotes the cost of a warm start for unit i
- d_t denotes the demand of power for period t
- k_i and l_i are the upper and lower bounds respectively for the amount of power a unit i can produce if it is used.

3.5 Formulation of the Stochastic Optimization Problem

We now consider the case when the demand for each period is not deterministic but normally distributed with a standard deviation of 5 MW. The following interpretations about the conditions of Varme's operations are made:

- At the start of the day, we need to decide which units to run for every period.
- At the start of every period, we can choose and distribute the power production among the units after we know the demand for that period. If needed, we can also buy additional power from other suppliers.
- The demand of power is constant for every period is and does not change within a period.
- The demands of each period are independent from each other, and independent to the demands the day before.

We define $X_t \in N(0,5)$ as the stochastic deviation from the mean demand at time t, and s_t as the outcome of X_t. The stochastic optimization problem is formulated as a two-stage problem, where the first-stage variables are $y_{i,t}, v_{i,t}, w_{i,t}$: Whether a unit is running, if it performs a cold start, or if it performs a warm start at time t. The second-stage variables, which depend on the outcome s_t are the power a unit should produce and the amount of power to be bought from other suppliers. These are denoted $x_{i,t}(s_t)$ and $u_t(s_t)$ respectively.

The cost in the objective function associated with the first-stage variables are formulated the same way as in the deterministic case. For the costs associated with the second stage variables, they are such that we want to minimize the expected value of the objective function. This can be written as

$$\sum_{t=1}^{5}\left(\int_{-\infty}^{\infty} f(s_t) n_t \left(\sum_{i=1}^{3} c_i x_{i,t}(s_t) + 10u_t(s_t)\ ds_t\right) + \sum_{t=1}^{5}\sum_{i=1}^{3}(a_i v_{i,t} + b_i w_{i,t}),\right.$$

where $u_t(s_t)$ is the number of units bought from external suppliers and $f(s_t)$ is the probability density function of X_t. To break it down, the right term is associated with the startup costs and does not depend on the outcome s_t (since we need to have already decided whether to run a unit before we know the demand). The left term is associated with the running costs. If a strategy $x_{i,t}$ is fixed, then it represents the expected value given that strategy.

The number of possible outcomes for a normal distribution is infinite, and the problem becomes even more computational intensive since the theoretical number of possible outcomes is the product of five independent normal distributions. To be able to solve the problem with conventional tools, we can approximate, discreticize, and sample the distributions. The first step is to approximate the integral of the expected value into the sum

$$\sum_{t=1}^{5}\sum_{s_t}(p(s_t)n_t(\sum_{i=1}^{3}c_i x_{i,t}(s_t)+10u_t(s_t))+\sum_{t=1}^{5}\sum_{i=1}^{3}(a_i v_{i,t}+b_i w_{i,t}),$$

where we denote $p(s_t) = P(s_t - \Delta s_t \leq X \leq s_t)$ and $\Delta s_t \to 0$. If Δs_t is small, this problem is still difficult to solve when there are five independent normal distributions. Instead, we can let s^j be a 5×1 vector, where each element is sampled from the normal distribution. The vector is sampled many times. Each outcome of s^j indicates an outcome of the five normal distributions. Let $s^1, s^2 \dots$ be the sample of 5×1 vectors. Technically, this means that the five outcomes in the vector s^j are neither stochastic nor independent, but since they are unknown and simulated from a normal distribution, we can simplify them as stochastic and independent. The probability of each sampled $s^1, s^2 \dots$ is equal. Thus $p(s^k) = 1/N \ \forall k$, where N is the total number of samples of s^j. The objective function can then be written as

$$\sum_{t=1}^{5}\sum_{s^j}(p(s^j)n_t(\sum_{i=1}^{3}c_i x_{i,t}(s^j)+10u_t(s^j))+\sum_{t=1}^{5}\sum_{i=1}^{3}(a_i v_{i,t}+b_i w_{i,t}).$$

As for the constraints, the solution needs to be feasible regardless of the outcome s^j. Therefore, any constraint that depends on the outcome s^j needs to be expanded so that it holds for all s^j. The optimization problem for when the demand is stochastic is given by

$$
\begin{array}{lll}
\underset{x,y,v,w,u}{\text{minimize}} & \sum_{t=1}^{5}\sum_{s^j}(p(s^j)n_t(\sum_{i=1}^{3}c_i x_{i,t}(s^j)+10u_t(s^j))+ & \sum_{t=1}^{5}\sum_{i=1}^{3}(a_i v_{i,t}+b_i w_{i,t}) \\
\text{subject to} & \sum_{i=1}^{3}x_{i,t}+u_t(s^j)\geq d_t+s_t^j & t=1,2,3,4,5,\ \forall s^j, \\
& \sum_{t=1}^{5}y_{i,t}\leq 3 & i=1,2,3, \\
& y_{i,t}-y_{i,t-1}-y_{i,t-2}\leq v_{i,t} & i=1,2,3,\ t=1,2,3,4,5, \\
& y_{i,t} -_{i,t-1}\leq v_{i,t}+w_{i,t}, & i=1,2,3,\ t=1,2,3,4,5, \\
& x_{i,t}(s^j)\leq k_i y_{i,t} & i=1,2,3,\ t=1,2,3,4,5,\forall s^j, \\
& x_{i,t}(s^j)\geq l_i y_{i,t} & i=1,2,3,\ t=1,2,3,4,5,\forall s^j, \\
& x_{i,t}(s^j)\geq 0 & i=1,2,3,\ t=1,2,3,4,5,\ \forall s^j, \\
& u_t(s^j)\geq 0 & t=1,2,3,4,5,\ \forall s^j, \\
& y_{i,t}\in\{0,1\} & i=1,2,3,\ t=1,2,3,4,5, \\
& v_{i,t}\in\{0,1\} & i=1,2,3,\ t=1,2,3,4,5, \\
& w_{i,t}\in\{0,1\} & i=1,2,3,\ t=1,2,3,4,5.
\end{array} \tag{3}
$$

3.6 Formulation of the Wait-and-See Model

It can be interesting to know what the expected cost of the stochastic model would be if we knew the demand for each period precisely at the start of the day. If we knew the outcome s^j at the start of the day, then we could conditionally optimize the stochastic optimization problem. This problem is called the wait-and-see problem. The expected cost of the wait-and-see problem is formulated as

$$
\sum_{s^j}p(s^j)\cdot\ \min\sum_{t=1}^{5}(\sum_{i=1}^{3}(n_t c_i x_{i,t}(s^j)+a_i v_{i,t}+b_i w_{i,t})+10n_t u_t(s^j)). \tag{4}
$$

The set of constraints of this problem is the same as the stochastic problem, but we only need to use one scenario s^j at the time.

3.7 Formulation of the Expected Value Problem

The sections above discuss how it is computationally difficult to solve a problem with many stochastic variables. If one would want to simplify the model, one could optimize the expected value problem instead of the stochastic problem, and evaluate how good it is. This is done by first approximating the stochastic variable s^j as its constant mean and solving the corresponding deterministic problem. Second, the solution for the first-stage variables are saved. Third, the stochastic problem is solved, but instead of treating the first-stage variables as variables, they are instead fixed as the solution from the deterministic problem where s^j was set to its mean.

In the case of our problem, the interpretation of this strategy is to determine which units should be run without taking stochastics into account. We note that since the mean of s^j is 0, the solution to the first step is the same as Problem 2, the deterministic case. Let $\bar{y_{i,t}}, \bar{v_{i,t}}, \bar{w_{i,t}}$ be the fixed solutions to $y_{i,t}, v_{i,t}, w_{i,t}$ in the deterministic problem. Then, the expected cost of the expected value solution is

$$\begin{array}{lll}
\underset{x,u}{\text{minimize}} & \sum_{t=1}^{5}\sum_{s^j}(p(s^j)n_t(\sum_{i=1}^{3}c_i x_{i,t}(s^j)+ & 10u_t(s^j)) + \sum_{t=1}^{5}\sum_{i=1}^{3}(a_i\bar{v_{i,t}} + b_i\bar{w_{i,t}}) \\
\text{subject to} & \sum_{i=1}^{3} x_{i,t} + u_t(s^j) \geq d_t + s_t^j, & t = 1,2,3,4,5,\ \forall s^j, \\
& x_{i,t}(s^j) \leq k_i\bar{y_{i,t}} & i = 1,2,3,\ t = 1,2,3,4,5, \forall s^j, \\
& x_{i,t}(s^j) \geq l_i\bar{y_{i,t}} & i = 1,2,3,\ t = 1,2,3,4,5, \forall s^j, \\
& x_{i,t}(s^j) \geq 0 & i = 1,2,3,\ t = 1,2,3,4,5,\ \forall s^j, \\
& u_t(s^j) \geq 0 & t = 1,2,3,4,5,\ \forall s^j.
\end{array} \tag{5}$$

4 Results

4.1 Results for the Deterministic Model

In the case of the deterministic model, Table 3 shows an optimal solution.

Table 3: The optimal policy for when the demand is deterministic

	Period 1	Period 2	Period 3	Period 4	Period 5
Unit 1	50	10	0	0	48
Unit 2	0	0	25	15	12
Unit 3	0	50	55	55	0

This policy results in the cost 3828.500 kkr per day.

4.2 Results for the Stochastic Model

When the stochastic part is added we get a different result. The expected cost of the optimal policy for the stochastic problem is 3857.456 kkr per day. Table 4 shows the mean amount of power each unit produces from the simulation.

Table 4: The mean values of the simulation for the stochastic model

	Period 1	Period 2	Period 3	Period 4	Period 5
Unit 1	0	10.441	0	37.720	31.610
Unit 2	0	0	25.376	32.396	28.373
Unit 3	49.563	49.634	54.994	0	0

The first thing to observe is that unit 1 now has two warm starts instead of one cold. There have also been some changes compared to the deterministic model between unit 1 and 3 regarding the active periods. Furthermore, the simulations yield a spread on how much each unit produces each period. The minimum values and the maximum values of power each unit generates in the simulation are shown in Table 5 and Table 6 respectively.

Table 5: The minimum values from the simulation for the stochastic model

	Period 1	Period 2	Period 3	Period 4	Period 5
Unit 1	0	10.0	0	11.376	10.0
Unit 2	0	0	12.0	12.0	12.0
Unit 3	35.823	33.772	51.063	0	0

Table 6: The maximum values from the simulation for the stochastic model

	Period 1	Period 2	Period 3	Period 4	Period 5
Unit 1	0	21.827	0	50.0	50.0
Unit 2	0	0	39.822	45.0	45.0
Unit 3	55.0	55.0	55.0	0	0

It is very unlikely that the demand exceeds our capacity level in a period other than period 1. In the simulation, there is no scenario where the demand exceeds the capacity in any other period. Table 7 shows the average amount supplied by our suppliers, the percentage of scenarios where excess power is required, and the largest amount of over-capacity.

Table 7: The mean amount of over-capacity, number of scenarios, and the largest over-capacity

	Mean (MW)	Scenarios above capacity (%)	Largest over-capacity (MW)
Unit 1	0.382	16.5	8.671

4.3 EVPI and VSS

The solution of the wait-and-see problem resulted in an expected a cost of 3844.311 kkr per day. The solution to the expected value problem resulted in an expected cost of 3901.808 kkr per day. Since the solution of expected cost of the optimal solution is 3857.456 kkr per day, we can calculate the expected value of perfect information ($EVPI$) and the value of the stochastic solution (VSS). If we denote the value of the optimal stochastic solution as RP, the optmal value of the wait-and-see problem as WS, and the optimal solution of the expected value problem as EEV, then measures are calculated as

$$EVPI = RP - WS$$

and

$$VSS = EEV - RP$$

In this problem, $EVPI = 13.145$ kkr and $VSS = 44.352$ kkr.

5 Discussion

5.1 Discussion on the Deterministic Model

In the deterministic solution there are two obvious conclusions one can draw. First, it seems that the cold start is preferred over the warm start. This is actually quite logical. To use the warm starts, without breaking the restriction of not exceeding three running periods in a row, one has to use two warm starts for one unit during the cycle. Since the cost of using a cold start is only 1.5 times more expensive than a warm start, but can be used half as many times as a warm start, the cold start is the obvious optimal choice. Therefore, in an optimal policy every unit is running for three straight periods and is then turned off for two straight periods. Second, unit 3 has the lowest running cost of all the units and it is therefore desirable to run it on max capacity as much as possible. However, to be able to use the full capacity, the demand in the period either has to be exactly equal to 55 MW or has to be high enough to exceed the max capacity of unit 3 plus the lowest running capacity of unit 1 or unit 2. Since the max capacity of unit 3 is 55 MW and the lowest running capacities of unit 1 and unit 2 are 10 MW and 12 MW, respectively, the demand has to be at least equal to 65 MW for this scenario to be possible. Since none of the periods has a demand of 55 MW and only period 3 and 4 exceeds 65 MW, it is only possible to run unit 3 on full capacity for those two periods.

5.2 Discussion on the Stochastic Model

In the stochastic model we can see a few changes, as mentioned in section 4.2. Since there are three units and five periods, there are 15 slots to be covered. Those slots are illustrated in the previously shown tables. Since the units can only be active for three periods in one cycle, they can at most cover nine of these time slots. Further, two thirds of 15 is 10, and as mentioned, we can only cover nine. This means that one period has to be covered by only one unit. In the deterministic case, unit 1 covers period 1, which works since the demand does not fluctuate. However, in the stochastic scenario it is reasonable that unit 3, the unit with the highest capacity, solely handles that period. This decreases the number of scenarios where we have to buy excess power. Also, it is logical that period 1 is the period with only one active unit since it is the period with the lowest expected demand.

It is also interesting that we now get a warm start. The explanation behind this is that it is more expensive to buy in extra power than to use an extra start. If we continue from the previous reasoning, i.e. that unit 3 should solely cover period 1 and that the rest of the periods should be covered by two units, there are very few policies that let all three units be active for three consecutive periods. Further, it is favourable to let unit 3 be active during period 1 where the demand is the highest. There is no policy that keeps all of the preferences, i.e.

1. Unit 3 solely active in period 1.
2. Unit 3 active in period 3.
3. Two units active the remaining periods.
4. All units active for three consecutive periods.

This leads to the conclusion that introducing two warm starts instead of one cold start for unit 1, since it has the lowest initial cost, is the cheapest option.

One observation that might be counter-intuitive is that it is only in period 1 that we sometimes have to buy extra power from our suppliers. However, as we can see in Table 6, in both period 4 and period 5, the maximum capacity for both units have been reached in certain scenarios. It is not likely that the demand equals or exceeds $max_{unit1} + max_{unit2}$. The quite simple answer to that somewhat confusing statement is that one megawatt-hour of unit 1 and one megawatt-hour of unit 2 have the same cost. The consequence of this is that in some scenarios unit 1 is maxed out, and in some scenarios unit 2 is fully utilized instead. For example, the scenario with the highest demand in period 4 requires a capacity of 89.883 MW, and the max capacity of unit 1 and unit 2 together is 95 MW.

Finally we can conclude that, just like in the deterministic model, unit 3 is the preferred unit. In a high demand scenario it is running at max capacity in all periods and the average production volume is very close to its maximum capacity. Unit 1 and 2, however, fluctuate a lot more and are used to absorb the shocks in demand. It is easy to see that in periods 2 and 3, where they are combined with unit 3, they sometimes reach their minimum capacity in a low demand scenario. They are however used a lot more in high demand scenarios. Furthermore, in periods 4 and 5 where the utility order does not really matter, both units sometimes reach their maximum and sometimes their minimum, or at least very close to that.

5.3 Potential Improvements

5.3.1 Reducing or Increasing of Randomness in the Model

When solving a stochastic problem the $EVPI$ represents the value of of new information that could increase the knowledge of the deviation in demand. $EVPI$ is the theorical value we would be willing to pay to know the precise scenario of every stochastic outcome. In this problem, the $EVPI$ only has a value of 13.145 kkr, which means that we at most can reduce the cost by that amount if we conduct research to understand the demand better. Therefore, it is probably not worth to attempt improving the model by obtaining more information on the demand. Further, the VSS, which is value of modeling the problem as a stochastic problem instead of a deterministic problem, is 44.352 kkr. If Varme is spending more than 44.352 kkr on say, electricity costs or administrative costs, to solve complex stochastic problems, then a reduction of their costs would actually be to just model the problem as a deterministic one.

5.3.2 Modeling Deviations as Dependent

In the stochastic model we made the assumption that the realized demands of the different periods are independent. However, in a real world application, this might not be completely true. For example, when the society faces a really cold day the power demand will rise in order to keep an acceptable temperature in the homes. This would increase the demand during one whole cycle, not just one period. If this new assumption would appear to be true, we would have to make one minor adjustment to our mathematical model to get that new result. Instead of sampling every element in the 5×1 vector s^j, we sample one value and place that at all indices in that vector (or we could just use the same variable five times). This would instead yield a cost of 3851.9 kkr per cycle, approximately 5500 kr less than with the original assumption. During the space of a whole year this model gives a cost that is 2 million kr less than our original model. Checking the conditions to make the right assumption is therefore essential.

Even if the deviations in demand between the periods are not fully dependent, they may have some correlation. Knowing the demand for one period can be a predictor of how the demand in the next period will be. For example, the deviation in demand could be modeled as a Markov process, where for example a low demand in one period means that it is more likely that the next period will also have a lower demand. By our sampling method of solving the optimization problem, we could easily implement conditional sampling, so that the outcome s_t depends on s_{t-1}. However, this would probably require a simulation of more outcomes for the model to be accurate, which would require more computational power.

5.3.3 Considering Seasonal Fluctuations

One assumption that might have some consequences is that the expected demand is the same through every cycle. A more reasonable assumption would be to assume some kind of seasonality in the demand. Table 8 represents the power usage in Sweden during 2020.

Table 8: Sweden's Power usage 2020. Source: SCB

Month	Power usage (GWh)
Jan	13 477
Feb	13 023
Mar	13 308
Apr	11 020
May	10 638
Jun	8 924
Jul	8 595
Aug	9 277
Sep	9 684
Oct	11 160
Nov	11 402
Dec	13 300

It is obvious that the demand peaks during the winter and reaches the bottom during the summer, with a quite similar "transition period" during fall and spring. To get a more accurate model, these seasonal changes in the demand should be taken into account. In extension to this, there might be some dependencies between the cycles. In our current model, we can in theory reach a "minimum" demand one day, followed by a "maximum" demand the next. This is not really accurate and should be handled. Finally, one can conclude that the periods are pretty long, 4-5 hours, and that the demand probably can be quite volatile during that period. To get a more cost efficient model, the periods might have to be shrunken. However, more data has to be collected before this change could be executed.

5.3.4 Modeling a New Problem Before Every Period

In this report, we have assumed that the plan for production should be cyclic, and that whether a unit should run or not during a period should be decided before the day starts. These decisions are the so-called first-stage decisions. If we were to remove the constraint of periodicity, then we could also not have to decide if a unit should be run or not at the start of the day, but rather right before the period when the demand is known. We could then use dynamic programming approaches, so that the optimal solution prior to every period minimizes the cost for the coming 24 hours, given which units have been running for the previous periods. This would require us to formulate more optimization problems, but it is also possible to automatize the process so that one only needs to click on a button to obtain an optimal plan for the coming period.

6 Conclusion

To conclude, a model is built to find the minimized cost of a cyclic power production company. The company has power demand it has to suffice for five periods in a day and to do so it has three different units it can produce power with. There were constraints on the units that had to be fulfilled, each unit had a lower and an upper bound and each unit could at most be run three periods in a row. The model is built for both when the expected power demand was deterministic and when the demand each period was normally distributed around the deterministic value with standard deviation 5 MW.

The model computes the minimized cost for the deterministic case to be 3828.5 kkr per cycle and the different production for the three different units can be seen in Table 3. The model that takes into account stochasticity of the expected demand returns an optimal value of 3857.5 kkr per day and the production is shown in Table 4. The fact that the cost is higher for the stochastic process than the deterministic is expected due to the uncertainty the stochasticity brings. In some scenarios the expected demand in period 1 exceeds the production capacity and additional power is bought at 10 kkr/MWh. For the other periods the expected demand never exceeded the production capacity so no additional power was needed for those periods. Since the difference in the optimal values of the deterministic and stochastic case is small we conclude that Varme is well equipped to deal with some uncertainty in its expected power demand. The expected value of perfect information was calculated to be 13.1 kkr and the value of the stochastic solution was 44.4 kkr. Lastly, it is discussed that the problem is a simplified model of reality, and can be improved in various ways.