SF2822 Applied Nonlinear Optimization, 2020/2021
Project assignment 2A
Due Tuesday May 10 2021 23.59

The exercise below is posed with the intent that modeling is made in GAMS. This is highly recommended, but not necessary. If you prefer other tools, e.g., Matlab-routines, you may use them.

Instructions for the project assignments are given in the course PM. Additional clarifications are given below.

- Discussion between the groups is encouraged, but each group must individually solve the assignments. It is *not* allowed to use solutions made by others in any form.
- Instructions for the report:
  - The report should have a leading title page where the project name and the group members' names, personal number and e-mail addresses are clearly stated.
  - The report should be written using a suitable word processor.
  - The contents should be such that another student in the course, who is not familiar with the project, should be able to read the report and easily understand:
    1. What is the problem? What it the problem background? This does *not* mean a copy of the project description, but rather a suitable summary of necessary information needed in order to understand the problem statement.
    2. How has the group chosen to formulate the problem mathematically? What assumptions have been made? If these assumptions affect the solution, this should be noted.
    3. What is the meaning of constraints, variables and objective function in the mathematical formulation?
    4. What is the solution of the formulated optimization problem? If suitable, refer the mathematical solution to the terminology of the (non-mathematical) problem formulation. (There could be more than one optimization problem.)
  - Most project descriptions contain a number of questions to be answered in the report. The report *must* contain the answers to these questions. They should, however, in a natural way be part of the content of the report and not be given in a "list of answers". The purpose of the questions is to suggest suitable issues to consider in the part of the report where the results are interpreted and analyzed. Make use of your knowledge from the course when formulating the problem and analyzing the results. Additional interpretations are encouraged as well as generalizations and other ways of modeling the problem.
  - A suggested outline of the report is as follows:
    1. Possibly a short abstract.
    2. Problem description and background information.
    3. Mathematical formulation.
    4. Results and analysis (interpretation of results).

5. A concluding section with summary and conclusions.

Deviations from the outline can of course be done.

– GAMS code should not be part of the report, and should not be referred to in the report.

- Each group should upload the following documents via the Canvas page of the course no later than by the deadline of the assignment:

  – The report as a pdf file.
  – GAMS files.

  Please upload your documents as individual pdf and gms files, and not as zip files.

- Each student should fill out a paper copy of the self assessment form and hand in at the beginning of the presentation lecture.

In this assignment we will consider traffic modeling. Assume that we have an urban region consisting of a number of areas. One area is the downtown and the remaining areas are the suburbs. Let I denote the number of areas and number these $i = 1, \ldots, I$. The region is modeled as a network, with nodes in the areas (downtown and suburbs), where v_i denotes the node in area i. There are roads between certain regions, given by arcs. A road between areas i and j is given by an arc (i, j). Presumably, arc (j, i) exists if arc (i, j) exists. Note that there normally do not exist direct roads between all areas, this depends on the topology of the region.

We assume that a significant portion of the commuters want to travel by car from suburb to downtown, but also that there are commuters who travel from one suburb to another one. Associated with arc (i, j), we have the time t_{ij}, which is the travel time on arc (i, j) given that there is no congestion. We assume that these numbers are known, given by speed limits.

Basic exercises

1. Construct a region with corresponding network and travel times t_{ij}. The region must be "nontrivial". You may for example base your region on a subset of the Stockholm region. If so, consider adding some new "ring roads" between suburbs.

2. One possible objective function is to minimize the sum of all travel times. If we let all commuters choose their best way from suburb to downtown, without considering congestion, this appears to gives separate shortest-path problems for each commuter. Is this correct? Formulate and solve the optimization problem or optimization problems that arise, preferably by making use of GAMS and associated solvers.

3. If we now want to take congestion into account, one may consider modeling the travel time on arc (i, j) to be modified from t_{ij} to

$$t_{ij} + \frac{a_{ij}x_{ij}}{u_{ij} - x_{ij}},$$

where a_{ij} and u_{ij} are given positive constants, and x_{ij} is the total number of cars on arc (i, j). Here, u_{ij} is an upper limit of the capacity of the road. We assume that there is a large number of cars traveling on the rads, why we may let x_{ij} be a continuous variable.

For your region, assign suitable values on a_{ij} and u_{ij} in addition to the total number of cars. Formulate and solve suitable optimization problem or optimization problems that arise, for example by minimizing the sum of all travel times. Illustrate how the traffic flows change.

4. Can you obtain global optimum to the problems that you have formulated?

Advanced exercises

5. Assume that one considers modifications of certain roads to increase capacity. Give a recommendation as to what roads to modify and what improvement of the throughput that may be expected. Investigate how accurate your prediction is in some selected cases.

6. Discuss potential model improvements. You may consider some other model, for example a model with a different objective function.

Good luck!