Advanced Dynamical Systems (MATH60146/70146)

Coursework 2

Homework issued on: January 31, 2025
Due date: February 7, 2025

Duration: 1 week	Spring 2024-25	Max mark: 8 Points

Objective: This coursework focuses on bifurcations and chaotic dynamics in nonlinear systems.

1. **Bifurcation Analysis of a Nonlinear System** (3)

   Consider the nonlinear system given by $\dot{x} = \lambda x + x^2 + x^3$.

   (a) Find the fixed points of this system and determine their stability for different values of the bifurcation parameter λ.

   (b) Compute and plot the bifurcation diagram of the system, showing the locations and stability of fixed points as λ varies.

2. **Chaos in a 4D Nonlinear System** (5)

   Consider the following 4D dynamical system:

   $$\dot{x} = -ax + by - yz; \quad \dot{y} = x + cw^2 + e; \quad \dot{z} = y^2 - z; \quad \dot{w} = dy.$$

   where a, b, c, d, e are system parameters. Use the following values: $a = 2.0, b = 10.0, c = d = 0.1, e = 1$.

   (a) Simulate the system for 100 seconds with initial condition $(x_0, y_0, z_0, w_0) = (1.0, 1.0, 1.0, 1.0)$. Visualize the system by plotting:
      - 3D projections: (x, y, z), (x, z, w).
      - 2D phase portraits: (x, y), (z, w).

   (b) Compute the Lyapunov spectrum to check if the system is in a chaotic regime.

3. **Bonus Question*: Hopf Bifurcation Analysis**

   Consider the Hopf bifurcation, given by:

   $$\dot{x} = \lambda x - y - x(x^2 + y^2),$$
   $$\dot{y} = x + \lambda y - y(x^2 + y^2).$$

   (a) Find the fixed points of the system and determine their stability.

   (b) Transform the system into polar coordinates ($x = r\cos\theta$, $y = r\sin\theta$), and derive the equation governing the evolution of r.

   (c) Construct a bifurcation diagram of r as a function of λ and interpret the meaning of fixed points in terms of oscillatory behavior.

   *Bonus questions are designed as an optional challenge for students who wish to explore the topic further. While they are primarily for enrichment, they can also contribute to the final grade in specific cases. If a student's total score is very close to the next grade boundary (within 1-2 points), completing the bonus question may help bridge the gap. However, bonus questions cannot contribute more than 2 additional points in total, and there are no extra bonuses beyond this limit.

Submission Guidelines:

- Submit a single PDF document containing the codes, solutions, mathematical foundations, justifications and figures.

Best wishes