Advanced Dynamical Systems (MATH60146/70146)

Coursework 5

Homework issued on: February 28, 2025
Due date: March 7, 2025

Duration: 1 week | Spring 2024-25 | Max mark: 20 Points

Objective: In this coursework, you will explore **parameter estimation** in chaotic systems using *synchronization-based inference*. Your task is to estimate unknown parameters of a dynamical system by driving a model with real data and identifying the best synchronization.

1. **Parameter Estimation via Synchronization** (20)

   The **Lorenz system** is given by:

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y - x),\\ \frac{dy}{dt} &= x(\rho - z) - y,\\ \frac{dz}{dt} &= xy - \beta z.\end{aligned}$$

   where σ, ρ, β are system parameters.

   Data: You have been provided with a dataset `lorenz_xz_data.txt`, which contains time series of the x and z components from a simulated Lorenz system with unknown parameters.

   (a) Drive the system using the provided $x(t)$ data and find the best synchronization using the $z(t)$ component. You may use any approach to estimate the parameters, but ensure that your method is justified.

   (b) **Parameter Space:** The unknown parameters ρ and β lie within the ranges:

$$\rho \in [26, 29], \quad \beta \in [2.5, 3.2].$$

   The minimum resolution for parameter estimation should be ± 0.05.

   (c) **Visualizing the results:** - Display how the synchronization error E depends on the estimated parameters. - Present your results in a meaningful visualization.

   (d) **Final Discussion:** - Report your estimated parameters. - Discuss potential sources of error in your estimation. - What are the limitations of the method you used?

Hint: Since this is a numerical approximation, the synchronization error E may not be exactly zero. The correct parameters correspond to the **smallest synchronization error**. You may consider grid search, optimization techniques, or other approaches.

Submission Guidelines: Submit a single PDF document containing:

- Your code and explanations.
- Figures and visualizations.
- The estimated parameters and discussion.

Best wishes,