Advanced Dynamical Systems (MATH60146/70146)

Coursework 1

Homework issued on: January 17, 2025
Due date: January 24, 2025

Duration: 1 week | Spring 2024-25 | Max mark: 5 Points

Objective: The goal of this homework is to familiarize you with the properties of dynamical systems, numerical methods for solving them, and visualizing their behavior. This assignment will deepen your understanding of flow properties, as well as the comparison of various numerical integration methods.

1. **Flow Candidates and Dynamical Systems Properties.** **(1)**

   Consider the following flow candidates for dynamical systems:

   (a)
   $$\mathbf{x}(t) = \left(c_1 e^t + c_2 e^{-t}, c_1 e^t - c_2 e^{-t}\right)$$

   (b)
   $$\mathbf{x}(t) = \left(\frac{1}{c_1 e^t + c_2 e^{-t}}, \frac{c_1 e^t - c_2 e^{-t}}{c_1 e^t + c_2 e^{-t}}\right)$$

   Based on the analysis of the flow properties, do you consider each flow to represent a valid dynamical system? Justify your answers.

2. **Comparing Numerical Integration Methods for FitzHugh-Nagumo Model** **(4)**

   Consider the following FitzHugh-Nagumo model, which describes the behavior of a simplified neuronal model:

   $$\dot{v} = v - \frac{v^3}{3} - w + I, \quad \dot{w} = \epsilon(v + a - bw)$$

   where v represents the membrane potential, w is the recovery variable, $\epsilon = 0.08$, $a = 0.7$, $b = 0.8$, and $I = 0.5$. Solve this system numerically using four different methods:

   (a) Euler's method (first-order)
   (b) Heun's method (second-order)
   (c) Runge-Kutta 4th order method
   (d) `solve_ivp` from the `scipy` library

   - Use initial conditions of $v(0) = -1.0$ and $w(0) = 0.0$, and integrate the system for $t \in [0, 100]$.
   - Assume that the `solve_ivp` solution provides the best accuracy. Plot the phase portrait of the system only for `solve_ivp` solution by plotting w versus v, and include the vector field on the phase space to visualize the direction of the flow.
   - Compute and plot the error for both the v and w components. For each method, calculate the error between the solution obtained from that method and the `solve_ivp` result. Present the errors over time, and provide a separate plot for each component (v and w).

Submission Requirements:

- A single PDF document containing:
  - Solutions and justifications for Problem 1.
  - Plots and error analysis for Problem 2. Include all code for solving the system with each method and generating the plots.

Best wishes