$\boxed{8}$ Como o polinômio é dado por

$$S_i(x) = a_i + b_i(x-t_i) + c_i(x-t_i)^2 + d_i(x-t_i)^3, \quad x \in [t_i, t_{i+1}]$$

Sendo que
$$\begin{cases} S_i(t_i) = f(t_i) \\ S_i(t_{i+1}) = f(t_{i+1}) \\ S_i'(t_i) = f'(t_i) \\ S_i'(t_{i+1}) = f'(t_{i+1}) \end{cases}$$

Substituindo t_i em x:

$$P_i(t_i) = f_i = a_i + b_i(t_i - t_i) + c_i(t_i - t_i)^2 + b_i(t_i - t_i)^3 \to a_i = f_i$$

$$P_i'(t_i) = f'_i = b_i + 2c_i(t_i - t_i) + 3d_i(t_i - t_i)^2 \to b_i = f_i'$$

De modo que $(t_{i+1} - t_i = h_i)$

$$P_i(t_{i+1}) = f_i + f_i' h_i + c_i h_i^2 + d_i h_i^3 \qquad \to \qquad f_{i+1} = f_i + f_i' h_i + c_i h_i^2 + d_i h_i^3$$

$$P_i'(t_{i+1}) = f'_i + 2c_i h_i + 3d_i h_i^2 \qquad\qquad f'_{i+1} = f'_i + 2c_i h_i + 3d_i h_i^2$$

Daí

$$c_i = \frac{f'_{i+1} - f'_i - 3d_i h_i^2}{2h_i} = \frac{f_{i+1} - f_i - f'_i h_i - d_i h_i^3}{h_i^2}$$

Ou seja,

$$\frac{f'_{i+1} - f'_i - 3d_i h_i^2}{2} = \frac{f_{i+1} - f_i - f'_i h_i - d_i h_i^3}{h_i} \Rightarrow d_i = \frac{f'_{i+1} h_i - f'_i h_i - 2f_{i+1} + 2f_i + 2f'_i h_i}{h_i^3}$$

Substituindo em c_i:

$$c_i = \frac{f'_{i+1} - f'_i - 3\frac{(f'_{i+1} + f'_i - 2(f_{i+1} - f_i)}{h_i}}{2h_i} = \frac{-2f'_{i+1} - 4f'_i + 6\frac{(f_{i+1} - f_i)}{h_i}}{2h_i}$$

Como $\tau = \frac{x - t_i}{h_i}$, substituindo em $p(x)$:

$$P_i(x) = f_i + f'_i(x - t_i) + \frac{-2f'_{i+1} - 4f'_i + 6\frac{(f_{i+1} - f_i)}{h_i}}{2h_i}(x - t_i)^2 + \frac{h_i(f'_{i+1} + f'_i) - 2(f_{i+1} - f_i)}{h_i^3}(x - t_i)^3$$

$$= f_i + (h_i f'_i)\tau - h_i f'_{i+1}\tau^2 - \frac{f'_{i+1} + 2f'_i}{h_i}(x - t_i)^2 + \frac{3f_{i+1} - f_i}{h_i^2}(x - t_i)^2 + h_i\frac{f'_{i+1} + f'_i}{h_i^3}(x - t_i)^3$$
$$- \frac{2f_{i+1} - f_i}{h_i^3}(x - t_i)$$

$$= f_i + (h_i f'_i)\tau - h_i(f'_{i+1} + 2f'_i)\tau^2 + 3(f_{i+1} - f_i)\tau^2 + h_i(f'_{i+1} - f'_i)\tau^3 - 2(f_{i+1} - f_i)\tau^3$$

$$= f_i + (h_i f'_i)\tau + (3(f_{i+1} - f_i) - h_i(f'_{i+1} + 2f'_i))\tau^2 + (h_i(f'_{i+1} + f'_i) - 2(f_{i+1} - f_i))\tau^3$$