CAPÍTULO 15

① considere um polinômio interpolador de grau $n=2$, usando $x_0 = a$, $x_1 = \frac{a+b}{2}$, $x_2 = b$

Nesse caso $L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \to a_0 = \int_a^b L_0(x)\,dx$

$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \to a_1 = \int_a^b L_1(x)\,dx$

$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \to a_2 = \int_a^b L_2(x)\,dx$

Calculando as integrais:

$$a_1 = \frac{1}{(x_1-x_0)(x_1-x_2)} \int_a^b x^2 - x_2 x - x_0 x + x_0 x_2 \, dx = \frac{1}{(x_1-x_0)(x_1-x_2)} \left[\frac{x^3}{3} - \frac{x_2 x^2}{2} - \frac{x_0 x^2}{2} + x(x_0 x_2) \right]_a^b$$

$$= \frac{\frac{b^3}{3} - \frac{b^2 x_2}{2} - \frac{b^2 x_0}{2} + b(x_0 x_2) - \frac{a^3}{3} + \frac{a^2 x_2}{2} + \frac{a^2 x_0}{2} - a(x_0 x_2)}{\left(\frac{a+b}{2} - a\right)\left(\frac{a+b}{2} - b\right)} = \frac{\frac{(a-b)^3}{6}}{-\frac{(a-b)^2}{4}} = \frac{2(b-a)}{3}$$

$$a_0 = \frac{1}{(x_0-x_1)(x_0-x_2)} \int_a^b x^2 - x_1 x - x_2 x + x_1 x_2 \, dx \cancel{= \frac{1}{(x_0-x_1)(x_0-x_2)} \left[\frac{x^3}{3} - \frac{x_1 x^2}{2} - \frac{x_2 x^2}{2} + x(x_1 x_2) \right]_a^b}$$

$$= \frac{1}{(x_0-x_1)(x_0-x_2)} \int_0^h x^2 - x_1 x - x_2 x + x_1 x_2 \, dn = \int_0^h \frac{(n+a)^2 - (n+a)x_2 - (n+a)x_1 + x_1 x_2 \, dn}{(x_0-x_1)(x_0-x_2)}$$

$\boxed{\begin{array}{l} n = x-a \\ b-a = h \end{array}}$

$$= \left[\frac{n^3}{3} + n^2 a + a^2 n - \frac{n^2}{2} x_2 - an x_2 - \frac{n^2}{2} x_1 - an x_1 + n x_1 x_2 \right]_0^h \frac{1}{(x_0-x_1)(x_0-x_2)}$$

$$= \frac{\frac{h^3}{3} + h^2 a + h a^2 - \frac{h^2}{2} x_2 - a h x_2 - \frac{h^2}{2} x_1 - a h x_1 + h x_1 x_2}{(x_0-x_1)(x_0-x_2)}$$

$$= \frac{\frac{(b-a)^3}{3} + (b-a)^2 a + (b-a)a^2 - \frac{(b-a)^2}{2} \cdot b - a(b-a)b - \frac{(b-a)^2}{2}\left(\frac{a+b}{2}\right) - a(b-a)\left(\frac{a+b}{2}\right) + \frac{(b-a)(a+b)b}{2}}{\left(a - \frac{(a+b)}{2}\right)(a-b)}$$

$$= -\frac{(a-b)^2(a+b)}{12} \cdot \frac{2}{(a+b)(a-b)} = \frac{b-a}{6}$$

$$a_2 = \frac{1}{(x_2-x_0)(x_2-x_1)} \int_a^b x^2 - x_0x - x_1x + x_0x_1 \, dx = \frac{1}{(x_2-x_0)(x_2-x_1)} \int_0^h (h+a)^2 - (h+a)x_0 - (h+a)x_1 + x_0x_1 \, dh$$

$$= \left[\frac{h^3}{3} + h^2 a + a^2 h - \frac{h^2}{2} x_0 - a x_0 h - \frac{h^2}{2} x_1 - a x_1 h + x_0 x_1 h \right]_0^h \frac{1}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{h^3}{3} + h^2 a + a^2 h - \frac{h^2}{2} x_0 - a x_0 h - \frac{h^2}{2} x_1 - a x_1 h + x_0 x_1 h \quad \frac{1}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{\frac{(b-a)^3}{3} + (b-a)^2 a + a^2(b-a) - \frac{(b-a)^2}{2} \cdot a - a^2(b-a) - \frac{(b-a)^2}{2}\left(\frac{a+b}{2}\right) - a\left(\frac{a+b}{2}\right)(b-a) + a\left(\frac{a+b}{2}\right)(b-a)}{(b-a)\left(b - \frac{a+b}{2}\right)}$$

$$= \frac{1}{9(b-a)} \cdot \frac{3(b-a)^2}{2} = \frac{b-a}{6}$$

ou seja

$$\int_a^b f(x)\,dx \simeq \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$