CAPÍTULO 15

(4) (a) $f(x) - P_3(x) = f[x_0 x_1 x_2 x_3 x](x-a)^2(x-b)^2$

$$E = \int_a^b f[x_0 x_1 x_2 x_3 x](x-a)^2(x-b)^2\,dx \overset{(*)}{=} \overbrace{f[x_0 x_1 x_2\ \xi]}^{K} \int_a^b (x-a)^2(x-b)^2\,dx$$

$$= K\int_0^h \underbrace{r^2(r-h)^2}\,dr$$

$r^2(r^2-2rh+h^2)$

$r^4 - 2r^3 h + r^2 h^2$

pois
$x = a + r \Rightarrow r = x - a$
$a + r - a = r$
$a + r - b = r + a - b = r - h$
$b - a = h$

$$= K\left[\frac{r^5}{5} - 2h\frac{r^4}{4} + h^2\frac{r^3}{3}\right]_0^h = K\left(\frac{h^5}{5} - \frac{2h^5}{4} + \frac{h^5}{3}\right) = K\left(\frac{12h^5 - 30h^5 + 20h^5}{60}\right) = K\left(\frac{1}{30}h^5\right) = \left(\frac{(b-a)^5}{30}\right)K$$

$$= f[x_0 x_1 x_2 x_3\ \xi]\,\frac{(b-a)^5}{30} \overset{(**)}{=} \frac{f^{(4)}(\eta)}{4!}\cdot\frac{(b-a)^5}{30} = \frac{f^{(4)}(\eta)}{720}(b-a)^5$$

(*) Funciona pois $\int_a^b g(x)\,\Psi(x)\,dx = g(x)\int_a^b \Psi(x)\,dx$

(**) Funciona pelo teorema da página 314