CAPÍTULO 14

(8) (a) $2f[x_{-1}x_0x_1] = \dfrac{2f[x_0x_1]-f[x_{-1}x_0]}{x_1-x_{-1}} = 2\dfrac{\left(\dfrac{f(x_1)-f(x_0)}{x_1-x_0} - \dfrac{f(x_0)-f(x_{-1})}{x_0-x_{-1}}\right)}{x_1-x_{-1}}$

Fazendo substituições: $x_0 = x_{-1}+h_0$ $\rightarrow$ $-x_{-1} = -x_0+h_0$

$x_1 = x_0+h_1$

$x_1-x_{-1} = h_0+h_1$

$x_0-x_{-1} = h_0$

$x_1-x_0 = h_1$

$$= \left(\frac{\dfrac{f(x_1)-f(x_0)}{h_1} - \dfrac{f(x_0)-f(x_{-1})}{h_0}}{\dfrac{h_0+h_1}{2}}\right) = \frac{g_{1/2}-g_{-1/2}}{\dfrac{(h_0+h_1)}{2}}$$

(b) $f'(x)-p'(x) = \dfrac{f'''(\xi)}{3!}\left((x-x_1)(x-x_{-1})+(x-x_0)(x-x_1)+(x-x_0)(x-x_{-1})\right)$

$f''(x)-p''(x) = \dfrac{f'''(\xi)}{3!}\left(2(x-x_0)+2(x-x_{-1})+2(x-x_1)\right)$

Em x_0

$f''(x_0)-p''(x_0) = \dfrac{f'''(\xi)}{3!}\left(2(x_0-x_0)+2(x_0-x_{-1})+2(x_0-x_1)\right)$

$= \dfrac{f'''(\xi)}{3!}(2h_0-2h_1) = \dfrac{f'''(\xi)}{3!}2(h_0-h_1)$

Como $h = \max(h_0, h_1)$

$|f''(x_0)-p''(x_0)| \leq \max\limits_{\xi\in(x_{-1},x_1)} \left|\dfrac{f'''(\xi)}{3!}\right| 2h \rightarrow 0$ erro é de ordem $O(h)$