CAPÍTULO 14

(4) Tome Taylor:

(I) $f(x_0+h) = f(x_0) + hf'(x_0) + \dfrac{h^2 f''(x_0)}{2!} + \dfrac{h^3 f'''(\xi_1)}{3!}$

(II) $f(x_0-h) = f(x_0) - hf'(x_0) + \dfrac{h^2 f''(x_0)}{2!} - \dfrac{h^3 f'''(\xi_2)}{3!}$

(I) + (II)

$$f(x_0+h) + f(x_0-h) = 2f(x_0) + \frac{2h^2 f''(x_0)}{2!} + \underbrace{\left(f'''(\xi_1) - f'''(\xi_2)\right)\left(\frac{h^3}{3!}\right)}_{=K} \quad (*)$$

continuando Taylor

(III) $f(x_0+2h) = f(x_0) + 2hf'(x_0) + \dfrac{4h^2 f''(x_0)}{2!} - \dfrac{8h^3 f'''(\xi_3)}{3!}$

(IV) $f(x_0-2h) = f(x_0) - 2hf'(x_0) + \dfrac{4h^2 f''(x_0)}{2!} - \dfrac{8h^3}{3!} f'''(\xi_4)$

(III) + (IV)

$$f(x_0+2h) + f(x_0-2h) = 2f(x_0) + \frac{8h^2 f''(x_0)}{2!} + \underbrace{\left(f'''(\xi_3) - f'''(\xi_4)\right)\left(\frac{8h^3}{3!}\right)}_{=j} \quad (**)$$

Utilizando as definições j e K, subtraindo $(*)$ de $(**)$

$$g - \hat{g} - Kh + jh = 0 \Rightarrow \frac{g - \hat{g}}{3} = \frac{h}{3}(K - j) = \hat{e} \Rightarrow \hat{e} \text{ é } O(h)$$

Examinando $e - \hat{e}$:

$$e - \hat{e} = -Kh - \left(\frac{g - \hat{g}}{3}\right) = -Kh - \left(\frac{h}{3}(K - j)\right) \Rightarrow e - \hat{e} \text{ é } O(h)$$