4a

i	z_i	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
0	5	-1	—	—	—
1	5	-1	8	—	—
2	6	4	5	-3	—
3	4	2	1	4	-7

$\frac{4-y_1}{6-5} = 5 \rightarrow y_1 = -1$ $\qquad$ $\frac{2-4}{4-6} = f[z_2, z_3] = 1$ $\qquad$ $z_0 = 5$, $f(5) = -1 \Rightarrow f(z_1) = -1$

$\frac{5 - f[z_0, z_1]}{6-5} = -3 \rightarrow f[z_0, z_1] = 8$ $\qquad$ $f[z_1 z_2 z_3] = \frac{1-5}{4-5} = 4$ $\qquad$ $f[z_0 z_1 z_2 z_3] = \frac{4+3}{4-5} = -7$

4b Temos $P_2(x) = ax^2 + bx + c$ e $P_2'(x) = 2ax + b$

Substituindo os pontos $(5, f(5))$; $(5, f'(5))$ e $(6, f(6))$:

$$\begin{cases} P_2(z_1) = f(z_1) \rightarrow P_2(5) = -1 \\ P_2(z_2) = f(z_2) \rightarrow P_2(6) = 4 \\ P_2'(z_1) = f'(z_1) \rightarrow P_2'(5) = 8, \text{ pois } f[z_0 z_1] = \frac{f^{(1-0)}(z_0)}{(1-0)!} \rightarrow f'(z_0) = 8 \end{cases}$$

$$\begin{cases} -1 = a(5)^2 + b(5) + c \\ 4 = a(6)^2 + b(6) + c \\ 8 = 2a(5) + b \end{cases} \rightarrow \begin{cases} 10a + b = 8 \\ 36a + b + c = 4 \\ 25a + 5b + c = -1 \end{cases} \rightarrow \begin{cases} 10a + b = 8 \\ 24a - c = 44 \\ 25a - c = 41 \end{cases} \rightarrow \begin{array}{l} a = -3 \\ b = 38 \\ c = -116 \end{array}$$

O polinômio osculante é $P(x) = -3x^2 + 38x - 116$