Forecast reconciliation: Methodological issues and applications

Chapter 6 - Realized volatility forecasting: a reconciliation approach[1]

Online appendix

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Contents

[1]Caporin, M., Di Fonzo, T. and Girolimetto, D. (2023) Exploiting intraday decompositions in Realized Volatility forecasting: A forecast reconciliation approach. arXiv doi:10.48550/arXiv.2306.02952

A Data description

A summary description of the daily RVs for the DJIA index and 26 individual stocks is in Table A.1. Figures A.1, A.2, A.3 show the time series of RV and its SV and $PV(3)$ components for the DJIA index, while the RV of the DJIA index for the 5 time intervals of the day are shown in Figure A.4. The RV graphs for the individual stocks are shown in Figures A.5, A.6, A.7.

Ticker	Company	GICS* Sector	Min	Mean	Median	Max	St.dv.	Skew.	Kurt.
DJIA	*Dow Jones Industrial Average index*		0.020	0.937	0.382	105.107	2.655	17.016	525.675
AAPL	Apple Inc.	Information technology	0.122	3.231	1.899	167.230	5.729	12.440	256.828
AMGN	Amgen	Health Care	0.311	2.408	1.673	104.363	3.708	12.014	222.839
AXP	American Express	Financials	0.131	3.450	1.210	275.174	8.992	10.565	212.313
BA	Boeing	Industrials	0.187	3.147	1.566	336.957	8.591	20.090	598.972
CAT	Caterpillar Inc.	Industrials	0.217	3.017	1.704	167.521	5.463	10.647	212.560
CSCO	Cisco	Information technology	0.237	2.575	1.681	128.759	4.264	11.737	230.317
CVX	Chevron Corporation	Energy	0.219	2.316	1.264	212.251	5.488	16.986	498.805
DIS	Disney	Communication Services	0.168	2.399	1.349	163.579	4.587	13.627	355.131
GS	Goldman Sachs	Financials	0.204	3.622	1.614	468.029	12.412	22.562	709.009
HD	Home Depot (The)	Consumer Discretionary	0.163	2.581	1.344	364.037	7.350	29.328	1287.173
HON	Honeywell	Industrials	0.146	2.365	1.349	166.977	4.922	13.936	334.939
IBM	IBM	Information technology	0.140	1.604	0.916	103.611	3.403	12.103	236.646
INTC	Intel	Information technology	0.337	2.851	1.904	111.192	4.407	11.059	192.990
JNJ	Johnson & Johnson	Health Care	0.118	1.219	0.700	115.272	3.132	19.937	578.804
JPM	JPMorgan Chase	Financials	0.186	3.738	1.410	362.245	11.103	14.267	331.107
KO	The Coca-Cola Company	Consumer Staples	0.188	1.311	0.796	87.093	2.786	14.112	298.911
MCD	McDonald's	Consumer Discretionary	0.125	1.638	0.918	141.630	3.742	17.477	495.913
MMM	3M	Industrials	0.135	1.680	0.979	141.197	3.784	18.862	565.079
MRK	Merck	Health Care	0.242	2.154	1.215	171.104	4.636	16.048	440.157
MSFT	Microsoft	Information technology	0.089	2.162	1.353	80.692	3.495	9.640	143.703
NKE	Nike Inc.	Consumer Discretionary	0.208	2.321	1.249	189.811	4.891	16.803	500.581
PG	Procter & Gamble	Consumer Staples	0.152	1.404	0.733	658.769	10.002	58.625	3811.064
UNH	UnitedHealth Group	Health Care	0.227	3.115	1.592	198.469	6.953	12.739	271.191
VZ	Verizon	Communication Services	0.156	1.864	1.034	232.582	4.999	26.244	1046.776
WBA	Walgreens Boots Alliance	Consumer Staples	0.215	2.581	1.666	119.073	3.959	11.147	219.696
WMT	Walmart	Consumer Staples	0.127	1.464	0.890	122.954	2.974	19.248	635.463

* Global Index Classification Standard.

Table A.1: *Descriptive statistics of the (open-to-close) daily Realized Variances of the DJIA index and 26 individual stocks for the period January 2, 2003 - June 30, 2022 (4,908 daily observations).*

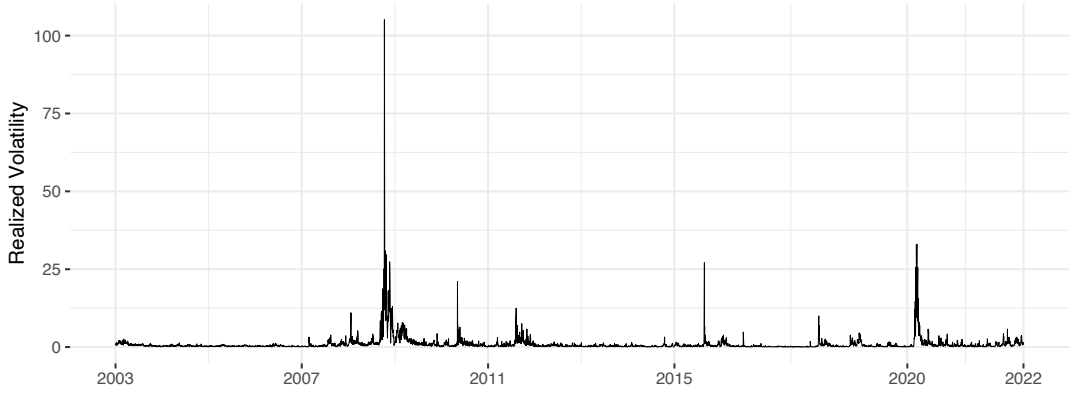

Figure A.1: *DJIA index: daily Realized Variance, January 2, 2003 - June 30, 2022 (4,908 days).*

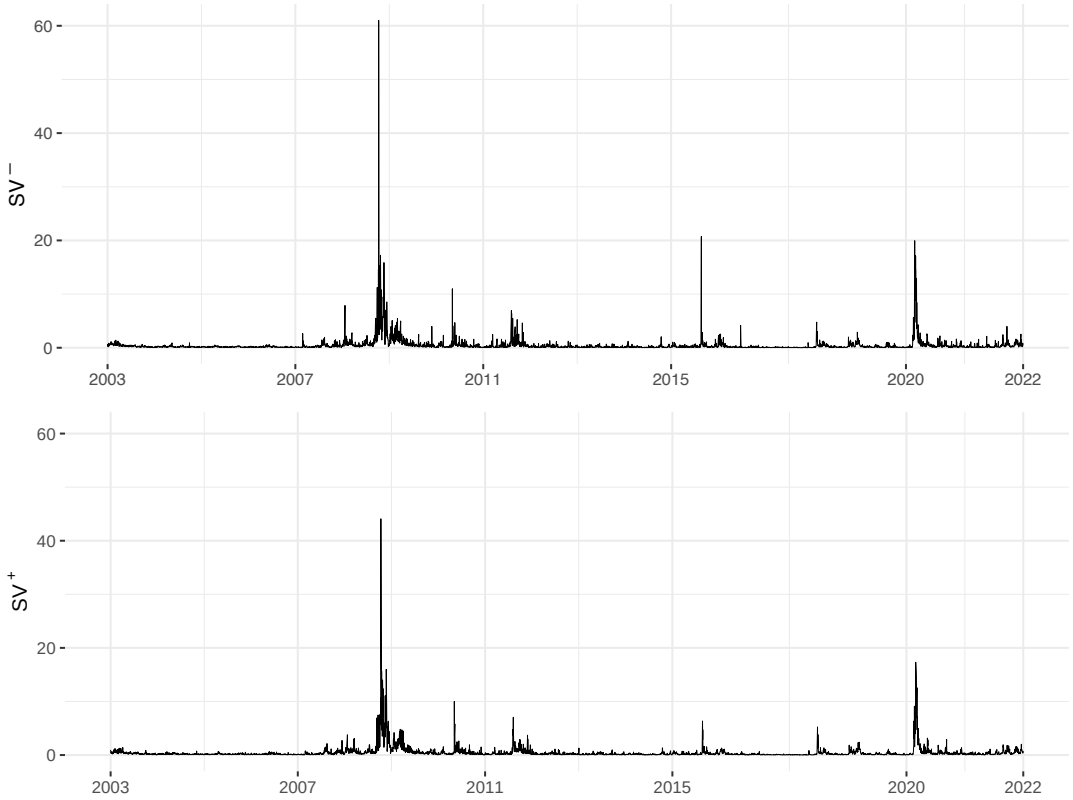

Figure A.2: *DJIA index: daily Realized Semi-Variances, January 2, 2003 - June 30, 2022 (4,908 days).*

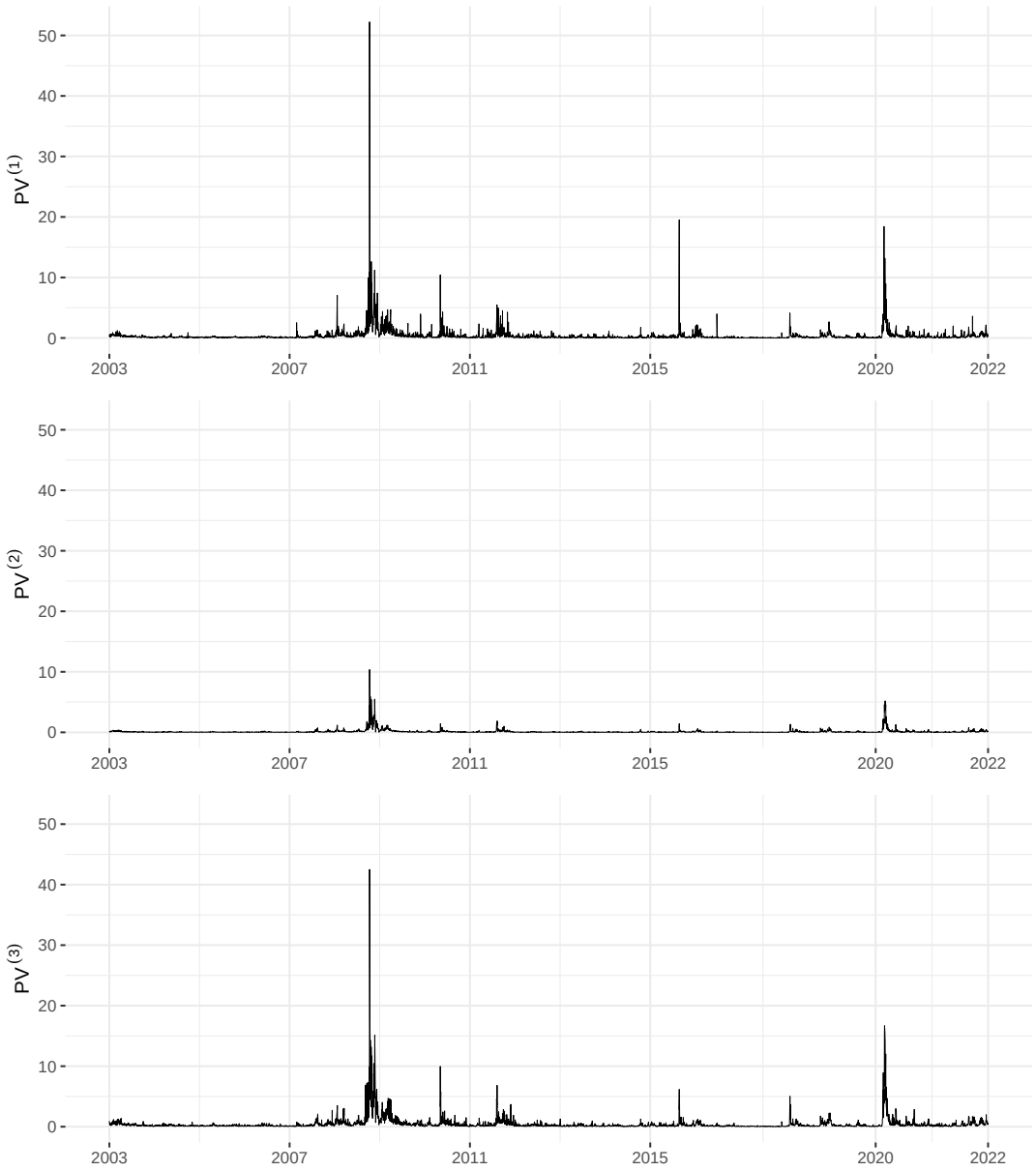

Figure A.3: *DJIA index: daily Realized Power-Variances, January 2, 2003 - June 30, 2022 (4,908 days).*

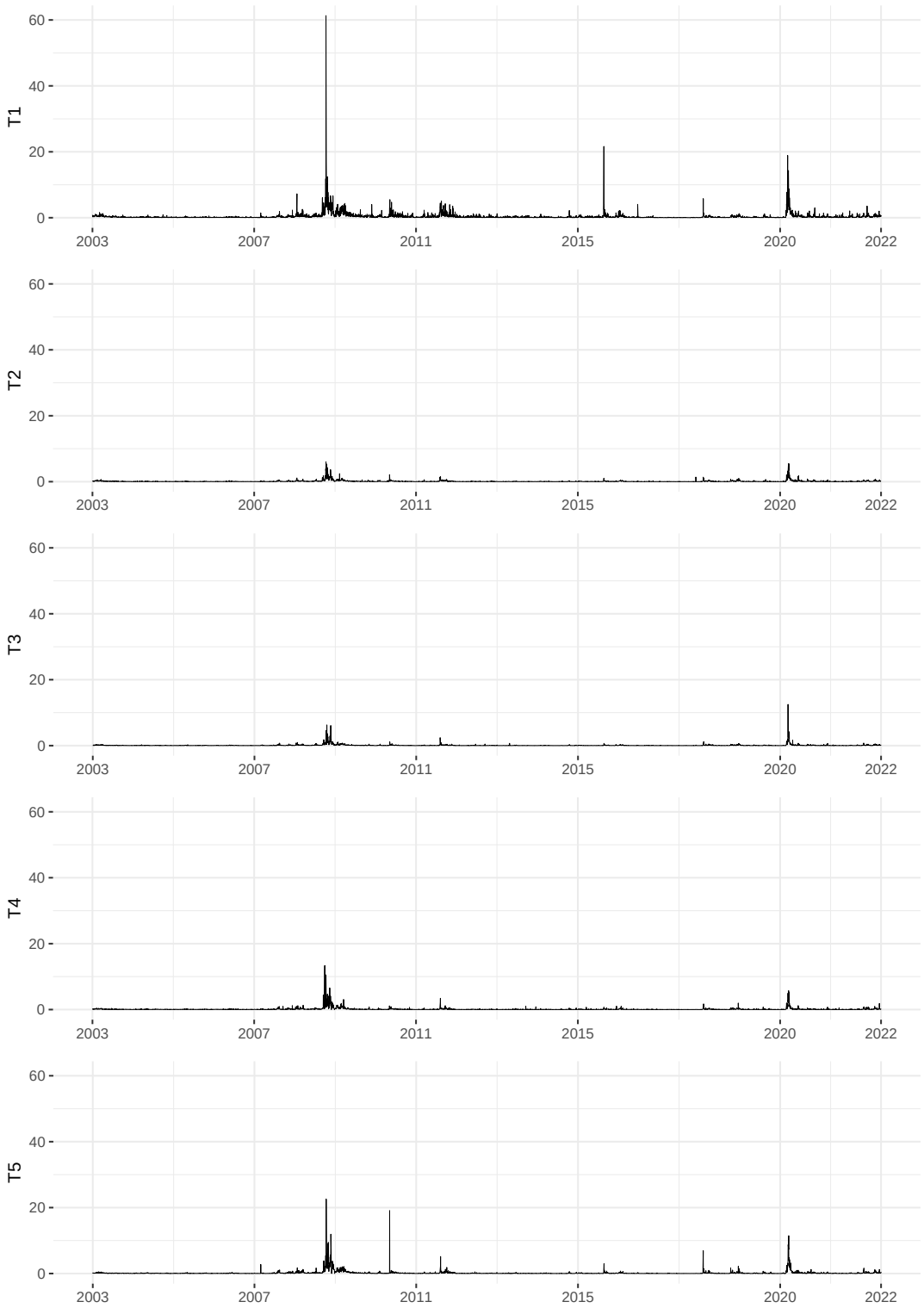

Figure A.4: *DJIA index: Realized Variances in non-overlapping 78-minutes segments of the day, January 2, 2003 - June 30, 2022 (4,908 days).*

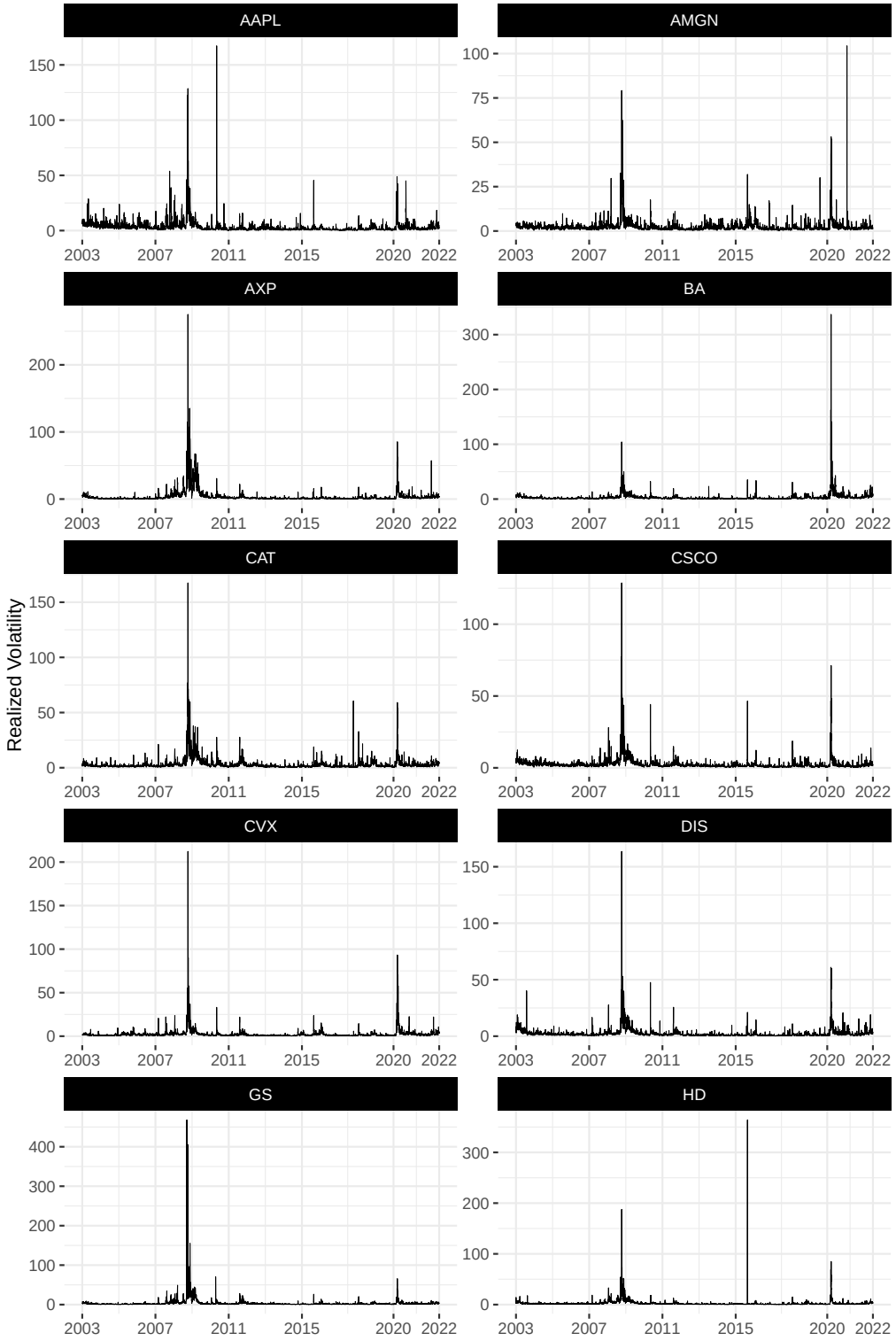

Figure A.5: *Daily Realized Variance of individual stocks, January 2, 2003 - June 30, 2022 (4,908 days). Tickers are described in Table A.1.*

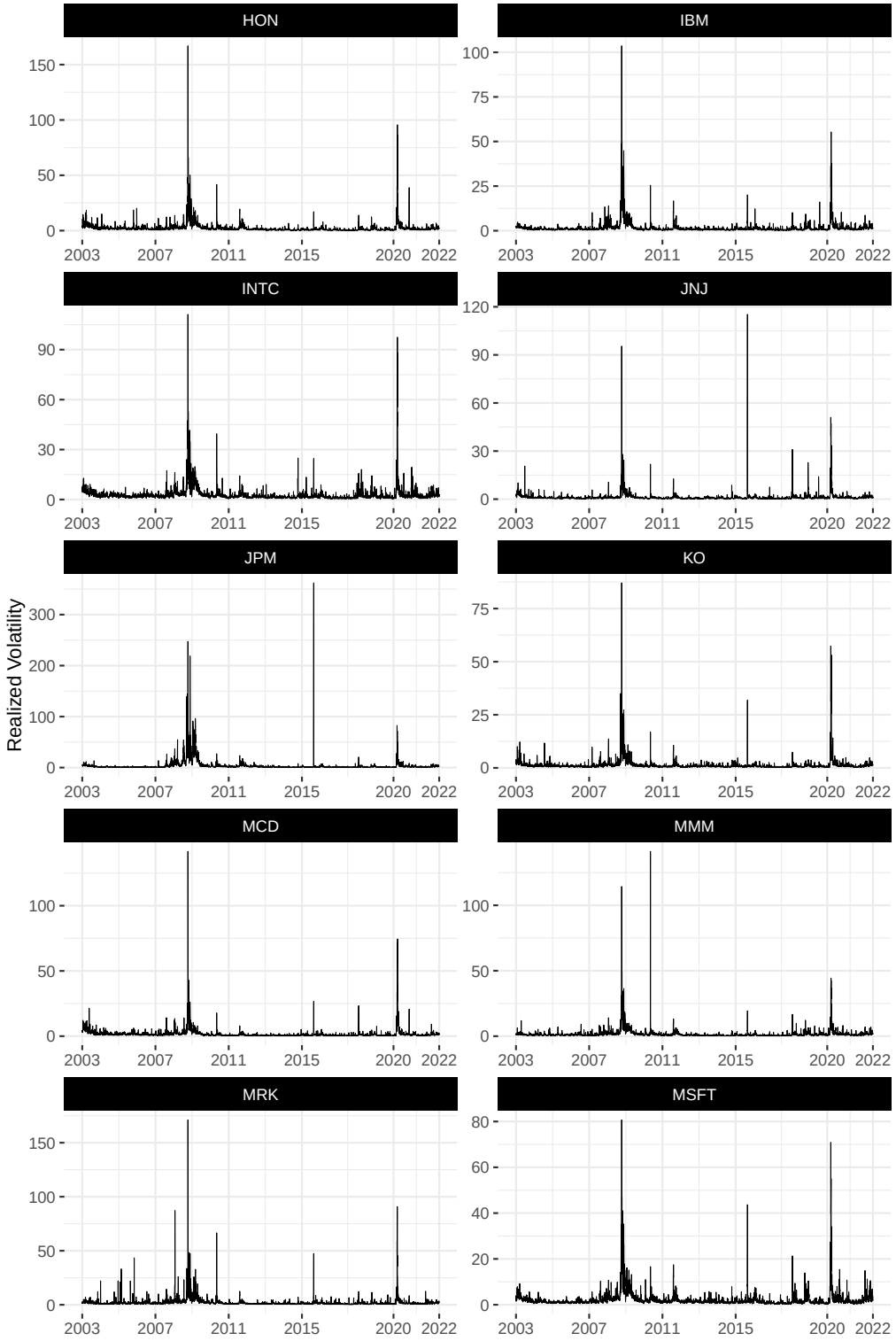

Figure A.6: *Daily Realized Variance of individual stocks, January 2, 2003 - June 30, 2022 (4,908 days). Tickers are described in Table A.1.*

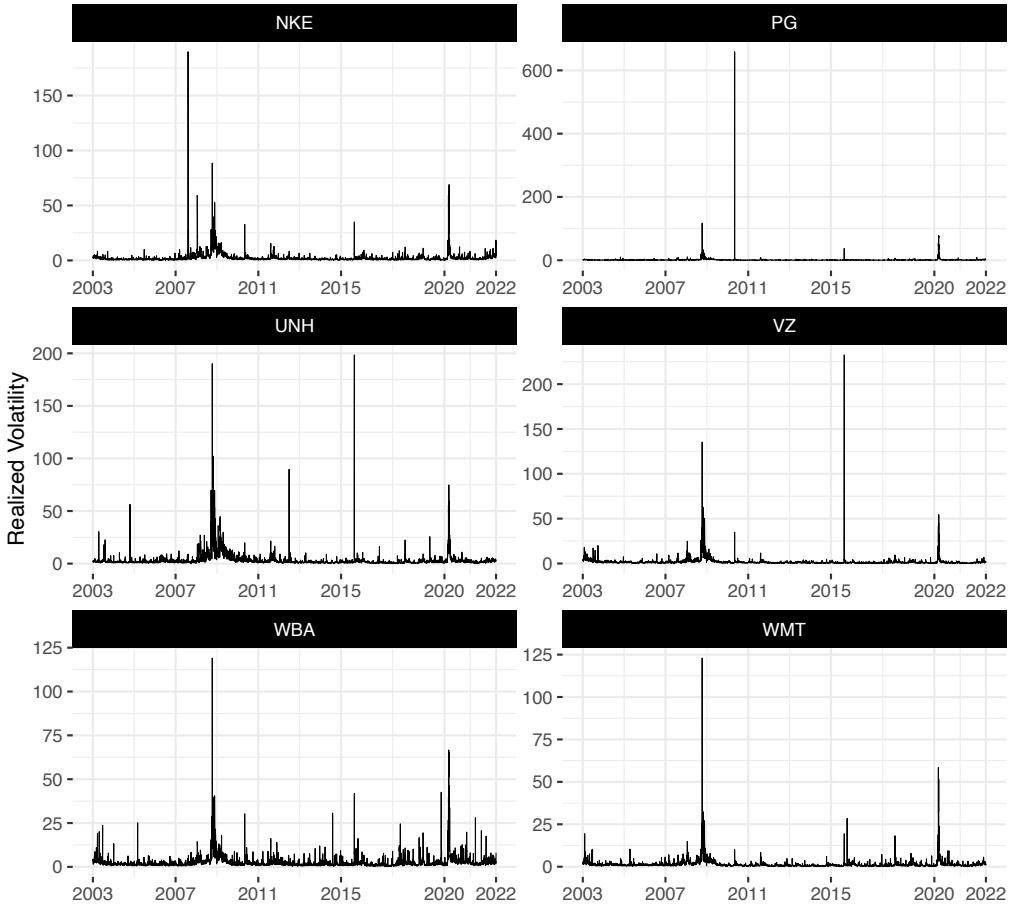

Figure A.7: *Daily Realized Variance of individual stocks, January 2, 2003 - June 30, 2022 (4,908 days). Tickers are described in Table A.1.*

B Alternative hierarchical/grouped representations of different intra-day decompositions of daily RV

In Table B.2 are reported the eleven hierarchies/groupings deriving from different Temporal-and-Volatility based decompositions. A complete hierarchy/grouping involves the bottom level series produced by a decomposition according to time and/or volatility, and the series forming at least one upper level (by time, quantile or both). A simple hierarchy consists of the top-level series (daily RV) and a single level of bottom time series disaggregated according time, quantile, or both. The largest decomposition, CTPV(3), involves 24 series, i.e. daily RV, 3 series distinguished only by quantile thresholds, 5 temporally aggregated series over nonoverlapping, consecutive 78 minutes intervals, 15 series cross-classified by quantile and temporal intervals. This is a grouped series, formed by hierarchies PV(3)-T (see figure B.15) and T-PV(3) (see figure B.16), whose matrix structural representation is given by figure C.27). Instead, the simplest hierarchy, SSV, considers the daily RV as top-level series and two bottom series corresponding to 'Bad' ($r_{i,t} < 0$) and 'Good' ($r_{i,t} > 0$) volatility, respectively.

Table B.2: *Temporal-and-Volatility based intraday RV decompositions*

Name	Intraday decomposition	H/G	n_b	n_a	n	Fig. #	Eq. #
ST	Simple Temporal	H	5	1	6	B.8	C.17
SSV	Simple SV (Good & Bad)	H	2	1	3	B.9	C.18
STSV	Simple Temporal-and-SV	H	10	1	11	B.10	C.19
SV-T	SV with Temporal	H	10	3	13	B.11	C.20
T-SV	Temporal with SV	H	10	6	16	B.12	C.21
CTSV	Complete Temporal-and-SV	G	10	8	18	–	C.22
SPV(3)	Simple PV(3)	H	3	1	4	B.13	C.23
STPV(3)	Simple Temporal-and-PV(3)	H	15	1	16	B.14	C.24
PV(3)-T	PV(3) with Temporal	H	15	4	19	B.15	C.25
T-PV(3)	Temporal with PV(3)	H	15	6	21	B.16	C.26
CTPV(3)	Complete Temporal-and-PV(3)	G	15	9	24	–	C.27

H: hierarchy; G: grouping.

n_b: n. of bottom variables; n_a: n. of upper variables; $n = n_b + n_a$.

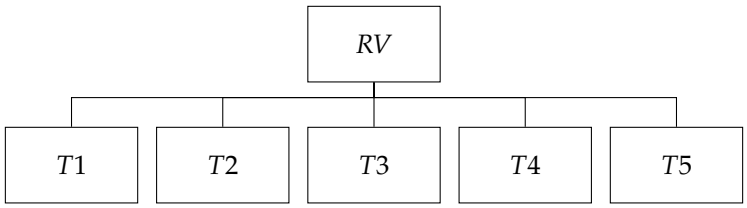

Figure B.8: *Hierarchical representation of the Simple Temporal (ST) decomposition of daily RV using five intraday intervals.* $n_b = 5, n_a = 1, n = 6$. *More details are in Table B.2.*

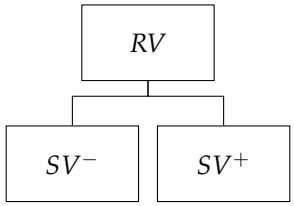

Figure B.9: *Hierarchical representation of the Simple SV (SSV) decomposition of daily RV.* $n_b = 2$, $n_a = 1$, $n = 3$. *More details are in Table B.2.*

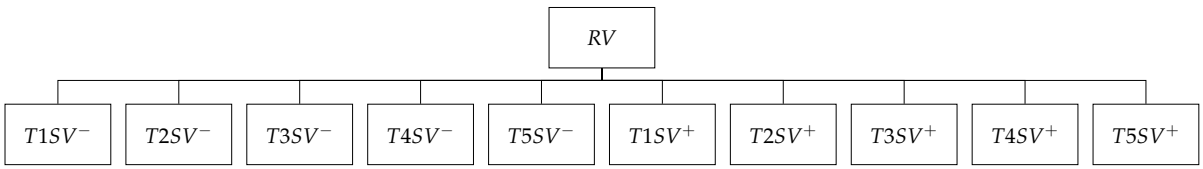

Figure B.10: *Hierarchical representation of the Simple Temporal-and-SV (STSV) decomposition of daily RV.* $n_b = 10$, $n_a = 1$, $n = 11$. *More details are in Table B.2.*

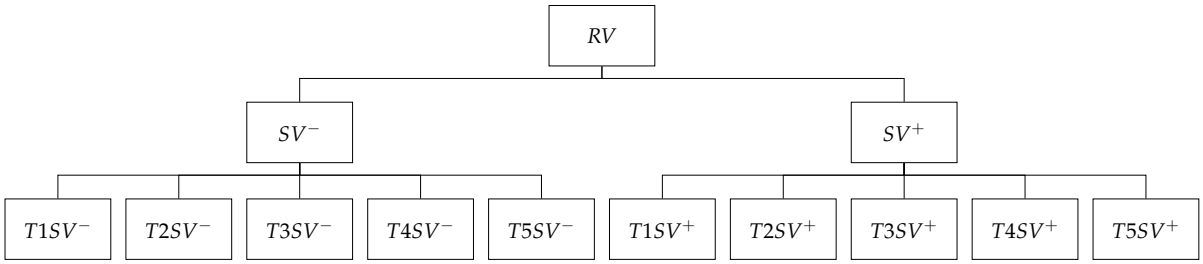

Figure B.11: *Hierarchical representation of the SV with Temporal (SV-T) decomposition of daily RV.* $n_b = 10$, $n_a = 3$, $n = 13$. *More details are in Table B.2.*

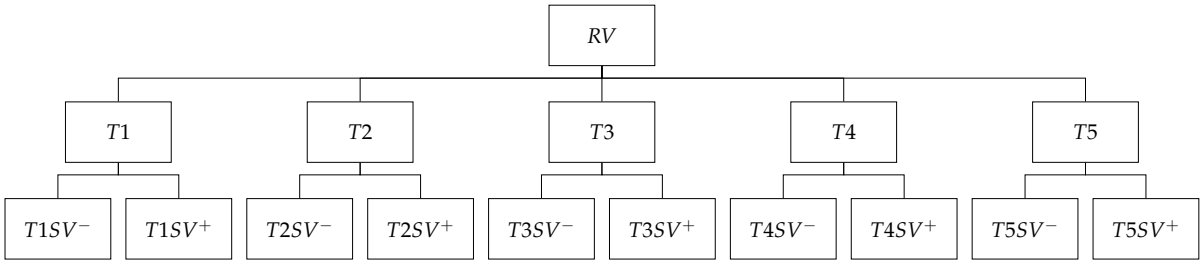

Figure B.12: *Hierarchical representation of the Temporal with SV (T-SV) decomposition of daily RV.* $n_b = 10$, $n_a = 6$, $n = 16$. *More details are in Table B.2.*

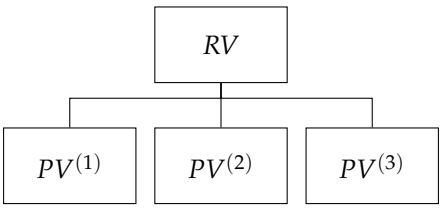

Figure B.13: *Hierarchical representation of the Simple PV(3) (SPV(3)) decomposition of daily RV.* $n_b = 3$, $n_a = 1$, $n = 4$. *More details in Table B.2.*

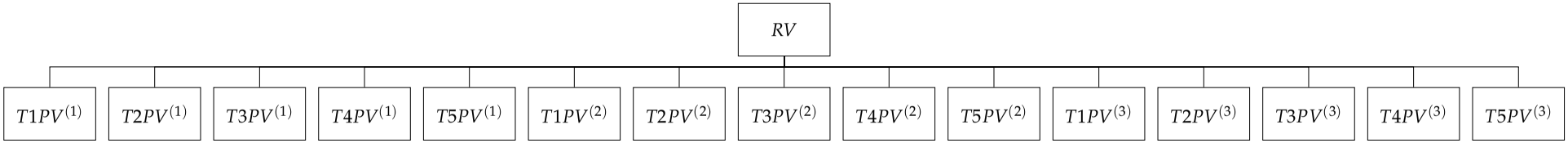

Figure B.14: *Hierarchical representation of the Simple Temporal-and-PV(3) (STPV(3)) decomposition of daily RV.* $n_b = 15$, $n_a = 1$, $n = 16$. *More details in Table B.2.*

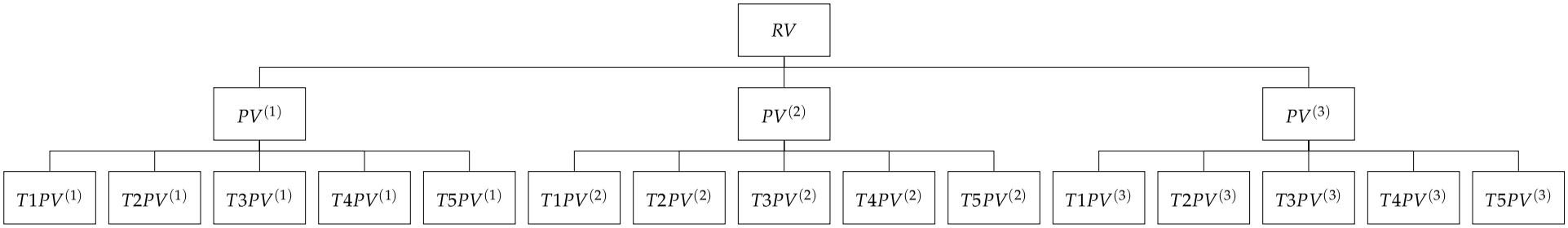

Figure B.15: *Hierarchical representation of the PV(3) with Temporal (PV(3)-T) decomposition of daily RV.* $n_b = 15$, $n_a = 4$, $n = 19$. *More details in Table B.2.*

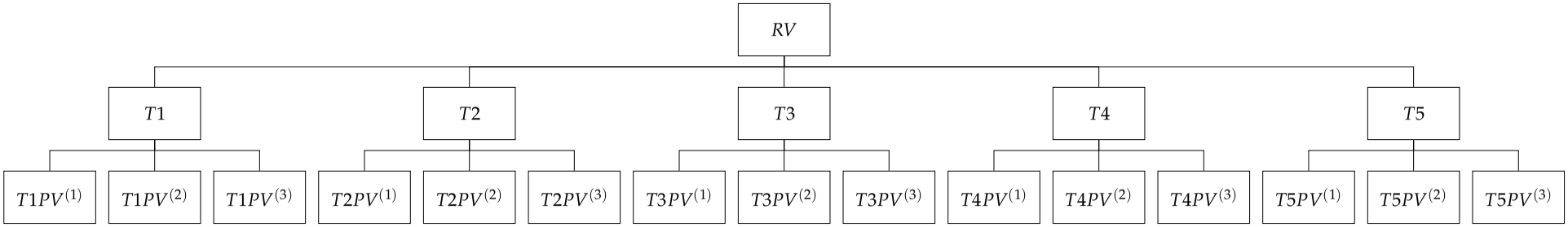

Figure B.16: *Hierarchical representation of the Temporal with PV(3) (T-PV(3)) decomposition of daily RV.* $n_b = 15$, $n_a = 6$, $n = 21$. *More details in Table B.2.*

C Structural representations of hierarchies and groupings for different intraday decompositions of daily RV

$$\begin{bmatrix} RV \\ T1 \\ T2 \\ T3 \\ T4 \\ T5 \end{bmatrix} = \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ \hline & & \mathbf{I}_5 & & \end{array}\right] \begin{bmatrix} T1 \\ T2 \\ T3 \\ T4 \\ T5 \end{bmatrix}$$

Figure C.17: *Structural representation of the Simple Temporal (ST) decomposition of daily RV using five intraday intervals.* $n_b = 5$, $n_a = 1$, $n = 6$.

$$\begin{bmatrix} RV \\ SV^- \\ SV^+ \end{bmatrix} = \left[\begin{array}{cc} 1 & 1 \\ \hline & \mathbf{I}_2 \end{array}\right] \begin{bmatrix} SV^- \\ SV^+ \end{bmatrix}$$

Figure C.18: *Structural representation of the Simple SV (SSV) decomposition of daily RV.* $n_b = 2$, $n_a = 1$, $n = 3$.

$$\begin{bmatrix} RV \\ T1SV^- \\ T2SV^- \\ T3SV^- \\ T4SV^- \\ T5SV^- \\ T1SV^+ \\ T2SV^+ \\ T3SV^+ \\ T4SV^+ \\ T5SV^+ \end{bmatrix} = \left[\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & & & & & \mathbf{I}_{10} & & & & \end{array}\right] \begin{bmatrix} T1SV^- \\ T2SV^- \\ T3SV^- \\ T4SV^- \\ T5SV^- \\ T1SV^+ \\ T2SV^+ \\ T3SV^+ \\ T4SV^+ \\ T5SV^+ \end{bmatrix}$$

Figure C.19: *Structural representation of the Simple Temporal-and-SV (STSV) decomposition of daily RV.* $n_b = 10$, $n_a = 1$, $n = 11$.

$$
\begin{bmatrix} RV \\ SV^{-} \\ SV^{+} \\ T1SV^{-} \\ T2SV^{-} \\ T3SV^{-} \\ T4SV^{-} \\ T5SV^{-} \\ T1SV^{+} \\ T2SV^{+} \\ T3SV^{+} \\ T4SV^{+} \\ T5SV^{+} \end{bmatrix}
=
\left[\begin{array}{cccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
\hline
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & \mathbf{I}_{10} & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & &
\end{array}\right]
\begin{bmatrix} T1SV^{-} \\ T2SV^{-} \\ T3SV^{-} \\ T4SV^{-} \\ T5SV^{-} \\ T1SV^{+} \\ T2SV^{+} \\ T3SV^{+} \\ T4SV^{+} \\ T5SV^{+} \end{bmatrix}
$$

Figure C.20: *Structural representation of the SV with Temporal (SV-T) decomposition of daily RV.* $n_b = 10$, $n_a = 3$, $n = 13$.

$$
\begin{bmatrix} RV \\ T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ T1SV^{-} \\ T2SV^{-} \\ T3SV^{-} \\ T4SV^{-} \\ T5SV^{-} \\ T1SV^{+} \\ T2SV^{+} \\ T3SV^{+} \\ T4SV^{+} \\ T5SV^{+} \end{bmatrix}
=
\left[\begin{array}{cccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\hline
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & \mathbf{I}_{10} & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & &
\end{array}\right]
\begin{bmatrix} T1SV^{-} \\ T2SV^{-} \\ T3SV^{-} \\ T4SV^{-} \\ T5SV^{-} \\ T1SV^{+} \\ T2SV^{+} \\ T3SV^{+} \\ T4SV^{+} \\ T5SV^{+} \end{bmatrix}
$$

Figure C.21: *Structural representation of the Temporal with SV (T-SV) decomposition of daily RV.* $n_b = 10$, $n_a = 6$, $n = 16$.

$$
\begin{bmatrix} RV \\ SV^{-} \\ SV^{+} \\ T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ T1SV^{-} \\ T2SV^{-} \\ T3SV^{-} \\ T4SV^{-} \\ T5SV^{-} \\ T1SV^{+} \\ T2SV^{+} \\ T3SV^{+} \\ T4SV^{+} \\ T5SV^{+} \end{bmatrix}
=
\left[\begin{array}{cccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\hline
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & \mathbf{I}_{10} & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & &
\end{array}\right]
\begin{bmatrix} T1SV^{-} \\ T2SV^{-} \\ T3SV^{-} \\ T4SV^{-} \\ T5SV^{-} \\ T1SV^{+} \\ T2SV^{+} \\ T3SV^{+} \\ T4SV^{+} \\ T5SV^{+} \end{bmatrix}
$$

Figure C.22: *Structural representation of the grouped time series describing the Complete Temporal-and-SV (CTSV) decomposition of daily RV using five intraday intervals and Semi Variances (Bad & Good volatility).* $n_b = 10$, $n_a = 8$, $n = 18$. *The grouped time series is obtained by merging the hierarchies SV-T and T-SV, described in Figures B.11 and B.12, respectively.*

$$
\begin{bmatrix} RV \\ PV^{(1)} \\ PV^{(2)} \\ PV^{(3)} \end{bmatrix}
=
\left[\begin{array}{ccc}
1 & 1 & 1 \\
\hline
 & & \\
 & \mathbf{I}_3 & \\
 & &
\end{array}\right]
\begin{bmatrix} PV^{(1)} \\ PV^{(2)} \\ PV^{(3)} \end{bmatrix}
$$

Figure C.23: *Structural representation of the Simple PV(3) (SPV(3)) decomposition of daily RV.* $n_b = 3$, $n_a = 1$, $n = 4$.

$$
\begin{bmatrix} RV \\ T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix} = \left[\begin{array}{cccccccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \end{array}\right] \begin{bmatrix} T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix}
$$

Figure C.24: *Structural representation of the Simple Temporal-and-PV(3) (STPV(3)) decomposition of daily RV.* $n_b = 15$, $n_a = 1$, $n = 16$.

$$
\begin{bmatrix} RV \\ PV^{(1)} \\ PV^{(2)} \\ PV^{(3)} \\ T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix} = \left[\begin{array}{cccccccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ \hline & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \end{array}\right] \begin{bmatrix} T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix}
$$

Figure C.25: *Structural representation of the PV(3) with Temporal (PV(3)-T) decomposition of daily RV.* $n_b = 15$, $n_a = 4$, $n = 19$.

$$
\begin{bmatrix} RV \\ T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix}
=
\left[\begin{array}{ccccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\hline
\multicolumn{15}{c}{\mathbf{I}_{15}}
\end{array}\right]
\begin{bmatrix} T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix}
$$

Figure C.26: *Structural representation of the Temporal with PV(3) (T-PV(3)) decomposition of daily RV.* $n_b = 15$, $n_a = 6$, $n = 21$.

$$
\begin{bmatrix} RV \\ PV^{(1)} \\ PV^{(2)} \\ PV^{(3)} \\ T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix}
=
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\hline
\multicolumn{15}{c}{\mathbf{I}_{15}}
\end{bmatrix}
\begin{bmatrix} T1PV^{(1)} \\ T2PV^{(1)} \\ T3PV^{(1)} \\ T4PV^{(1)} \\ T5PV^{(1)} \\ T1PV^{(2)} \\ T2PV^{(2)} \\ T3PV^{(2)} \\ T4PV^{(2)} \\ T5PV^{(2)} \\ T1PV^{(3)} \\ T2PV^{(3)} \\ T3PV^{(3)} \\ T4PV^{(3)} \\ T5PV^{(3)} \end{bmatrix}
$$

Figure C.27: *Structural representation of the grouped time series describing the Complete Temporal-and-PV(3) (CTPV(3)) decomposition of daily RV using five intraday intervals and PV(3).* $n_b = 15$, $n_a = 9$, $n = 24$. *The grouped time series is obtained by merging the hierarchies PV(3)-T and T-PV(3), described in Figures B.15 and B.16, respectively.*

D Forecasting accuracy in different test periods

	2007-2022		2007-2010		2011-2014		2015-2019		2020-2022	
	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE
Panel A: DJIA index										
SV	1.024	1.028	0.991	1.069	1.048	0.978	0.973	**0.986**	0.968	0.962
SV_{bu}	0.962	0.960	1.055	0.966	0.923	1.024	1.010	1.069	1.021	0.897
TSV_{bu}	0.801	0.824	1.151	0.914	0.923	1.085	1.085	1.245	1.074	0.906
SV_{shr}	0.976	0.976	1.022	0.997	0.960	0.997	0.983	1.027	0.989	0.896
TSV_{shr}	0.856	0.869	1.066	0.930	0.923	1.020	1.011	1.109	1.011	**0.873**
SV_{hac}	0.976	0.976	1.019	0.991	0.963	0.989	0.969	0.995	0.979	0.893
SV_{ewma}	1.114	1.417	1.018	7.986	1.015	1.022	0.983	1.026	0.985	0.874
$PV(3)$	0.775	0.816	**0.921**	0.997	1.059	2.691	2.272	1.073	**0.873**	6.045
$PV(3)_{bu}$	0.922	0.924	1.036	0.947	**0.911**	1.023	1.001	1.076	1.004	0.874
$TPV(3)_{bu}$	0.799	0.822	1.148	0.911	0.916	1.092	1.090	1.265	1.075	0.893
$PV(3)_{shr}$	0.812	0.833	0.944	0.934	0.947	0.941	0.945	1.047	0.902	0.902
$TPV(3)_{shr}$	0.787	0.808	0.992	**0.900**	0.915	0.967	0.971	1.095	0.944	0.874
$PV(3)_{hac}$	**0.768**	**0.796**	0.938	0.930	0.950	**0.930**	**0.940**	1.043	0.898	0.903
$PV(3)_{ewma}$	0.793	0.852	0.937	1.162	1.193	0.941	1.193	1.049	1.576	0.930
Panel B: Individual stocks										
SV	0.993	1.016	1.022	1.005	1.073	1.124	1.092	1.076	1.080	0.963
SV_{bu}	0.967	0.964	1.000	0.973	0.998	1.000	0.921	1.007	1.006	0.779
TSV_{bu}	1.028	0.991	1.060	0.867	1.023	1.019	0.893	1.119	1.029	0.656
SV_{shr}	0.970	0.980	0.985	0.979	1.029	0.992	0.899	0.988	0.987	0.742
TSV_{shr}	1.009	0.987	1.014	0.907	0.997	0.959	0.850	1.041	0.991	0.632
SV_{hac}	0.968	0.979	0.987	0.979	1.036	0.991	0.898	0.987	0.983	0.749
SV_{ewma}	1.196	1.316	1.002	1.247	1.349	1.124	1.002	0.991	1.033	0.911
$PV(3)$	0.938	0.899	0.973	**0.836**	1.097	1.421	1.364	1.046	1.344	1.035
$PV(3)_{bu}$	0.953	0.943	0.993	0.950	0.994	1.003	0.900	1.010	0.990	0.709
$TPV(3)_{bu}$	1.027	0.982	1.063	0.859	1.011	1.029	0.894	1.134	1.030	0.641
$PV(3)_{shr}$	**0.916**	**0.896**	0.927	0.874	1.030	0.939	0.833	**0.970**	0.898	0.704
$TPV(3)_{shr}$	0.963	0.924	0.963	0.848	**0.991**	**0.923**	**0.805**	1.013	0.918	**0.606**
$PV(3)_{hac}$	0.919	0.899	**0.926**	0.873	1.049	0.941	0.842	0.973	**0.891**	0.745
$PV(3)_{ewma}$	1.023	1.022	0.930	0.919	1.336	1.103	1.018	0.984	1.155	0.872

Table D.3: *Accuracy of the **one-day ahead** forecasts. MSE and QLIKE ratios for the DJIA index (panel A), and geometric means of the MSE and QLIKE ratios for individual stocks (panel B) over the benchmark HAR model. Values larger than one in red. The best index value in each column is highlighted in bold.*

	2007-2022		2007-2010		2011-2014		2015-2019		2020-2022	
	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE
Panel A: DJIA index										
SV	1.012	1.017	1.011	0.999	1.030	0.985	0.997	**0.988**	0.986	1.000
SV_{bu}	0.991	0.972	1.016	0.985	0.921	1.014	0.237	1.049	1.021	0.051
TSV_{bu}	0.774	0.801	1.072	0.977	0.833	1.063	0.249	1.210	1.045	0.051
SV_{shr}	0.998	0.991	1.006	0.983	0.975	0.995	0.233	1.020	0.995	0.053
TSV_{shr}	0.853	0.878	1.031	0.967	0.920	1.009	0.236	1.094	0.999	0.050
SV_{hac^*}	1.058	1.216	1.467	1.050	1.597	1.641	0.735	1.319	0.975	0.619
SV_{hac}	0.989	0.979	1.003	0.981	0.951	0.993	0.231	0.990	0.991	0.051
SV_{ewma^*}	1.129	1.140	1.005	1.456	1.142	1.014	0.236	1.023	1.009	0.053
SV_{ewma}	1.015	1.020	1.005	3.816	**0.760**	1.024	0.232	1.018	1.003	**0.048**
$PV(3)$	1.047	1.077	0.973	1.013	1.163	1.645	0.977	1.021	**0.908**	0.942
$PV(3)_{bu}$	0.981	0.957	1.013	0.979	0.892	1.015	0.235	1.058	1.011	0.049
$TPV(3)_{bu}$	**0.766**	**0.795**	1.078	0.978	0.832	1.067	0.250	1.231	1.049	0.051
$PV(3)_{shr}$	1.010	1.015	**0.963**	0.975	1.033	**0.946**	0.225	1.030	0.938	0.051
$TPV(3)_{shr}$	0.909	0.925	0.984	**0.962**	0.956	0.956	0.227	1.061	0.948	0.050
$PV(3)_{hac^*}$	1.951	1.977	1.417	1.129	2.158	5.247	0.994	1.553	0.962	0.709
$PV(3)_{hac}$	0.982	0.975	0.967	0.969	0.959	0.959	**0.225**	1.018	0.945	0.050
$PV(3)_{ewma^*}$	1.126	1.269	1.001	4.389	1.328	3.382	0.381	1.035	1.201	0.065
$PV(3)_{ewma}$	1.047	1.040	0.968	1.968	0.935	0.964	0.234	1.029	1.013	0.052
Panel B: Individual stocks										
SV	1.014	1.008	0.985	1.001	1.005	1.004	0.998	0.982	0.993	1.025
SV_{bu}	0.991	0.992	1.001	0.991	1.000	1.010	0.977	1.008	0.999	0.946
TSV_{bu}	1.004	1.054	1.048	0.994	1.021	1.076	0.908	1.108	0.994	0.698
SV_{shr}	0.998	0.996	0.987	0.992	1.000	1.001	0.947	0.991	0.990	0.897
TSV_{shr}	0.987	1.007	1.010	0.987	0.988	1.018	0.861	1.043	0.979	0.646
SV_{hac^*}	1.167	1.107	1.295	1.098	1.032	1.913	1.454	1.420	1.114	1.242
SV_{hac}	0.995	0.995	0.989	0.992	1.002	1.000	0.951	0.991	0.990	0.912
SV_{ewma^*}	1.150	1.163	0.991	1.014	1.148	1.084	1.062	0.994	1.011	1.043
SV_{ewma}	1.070	1.045	1.006	1.107	1.000	1.070	1.010	0.994	0.996	0.935
$PV(3)$	1.035	0.976	0.958	**0.943**	0.978	1.139	1.025	**0.955**	1.029	0.865
$PV(3)_{bu}$	0.981	0.982	0.999	0.984	0.990	1.010	0.895	1.013	0.984	0.750
$TPV(3)_{bu}$	0.996	1.039	1.055	0.994	1.000	1.083	0.902	1.123	0.993	0.655
$PV(3)_{shr}$	0.998	**0.964**	0.955	0.949	0.971	**0.956**	0.824	0.965	0.929	0.662
$TPV(3)_{shr}$	**0.973**	0.967	0.972	0.955	**0.965**	0.963	**0.793**	0.990	**0.924**	**0.565**
$PV(3)_{hac^*}$	1.268	1.118	1.183	1.076	1.059	1.775	1.623	1.299	1.212	1.991
$PV(3)_{hac}$	0.976	0.966	**0.952**	0.950	0.989	0.959	0.840	0.970	0.936	0.698
$PV(3)_{ewma^*}$	1.212	1.148	0.978	1.229	1.143	1.322	1.102	0.989	1.038	1.013
$PV(3)_{ewma}$	1.110	1.019	0.963	1.016	1.005	1.097	0.955	0.980	1.073	0.728

Table D.4: *Accuracy of the **five-day ahead** forecasts. MSE and QLIKE ratios for the DJIA index (panel A), and geometric means of the MSE and QLIKE ratios for individual stocks (panel B) over the benchmark HAR model. Values larger than one in red. The best index value in each column is highlighted in bold.*

	2007-2022		2007-2010		2011-2014		2015-2019		2020-2022	
	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE	rMSE	rQLIKE
Panel A: DJIA index										
SV	1.002	1.003	**0.998**	1.003	1.006	1.001	0.928	**0.999**	0.996	0.904
SV_{bu}	1.007	1.005	1.007	0.991	1.003	1.011	0.748	1.014	1.004	0.662
TSV_{bu}	1.121	1.099	1.055	0.991	1.035	1.047	0.385	1.092	1.006	0.170
SV_{shr}	1.002	1.002	1.003	1.000	1.003	1.006	0.728	1.007	0.996	0.637
TSV_{shr}	0.949	0.965	1.044	1.005	1.010	1.014	0.379	1.053	0.997	**0.169**
SV_{hac^*}	1.903	1.840	3.649	1.961	1.459	12.593	4.124	6.538	3.867	3.057
SV_{hac}	1.002	1.002	1.002	0.992	1.003	1.004	0.719	1.001	0.996	0.626
SV_{ewma^*}	0.970	1.011	1.003	1.045	1.153	1.020	0.826	1.008	1.000	0.766
SV_{ewma}	1.016	1.039	1.002	2.499	0.999	1.008	0.713	1.007	1.002	0.617
$PV(3)$	1.030	1.025	1.001	0.988	1.012	**0.995**	0.962	1.011	**0.969**	0.954
$PV(3)_{bu}$	1.010	1.009	1.009	0.988	1.006	1.011	0.596	1.021	1.000	0.460
$TPV(3)_{bu}$	1.125	1.103	1.064	0.991	1.037	1.054	0.386	1.106	1.007	0.169
$PV(3)_{shr}$	1.018	1.015	1.003	0.983	1.005	1.000	0.596	1.015	0.976	0.465
$TPV(3)_{shr}$	**0.914**	**0.939**	1.057	0.989	1.012	0.997	**0.374**	1.032	0.977	0.169
$PV(3)_{hac^*}$	4.920	4.185	3.982	1.844	1.837	23.072	5.711	6.157	2.918	4.181
$PV(3)_{hac}$	1.015	1.012	1.003	**0.981**	1.003	1.005	0.590	1.013	0.981	0.457
$PV(3)_{ewma^*}$	1.003	1.075	1.004	1.221	1.318	1.018	0.819	1.017	0.996	0.757
$PV(3)_{ewma}$	1.031	1.034	1.003	1.715	**0.989**	1.017	0.590	1.015	0.995	0.452
Panel B: Individual stocks										
SV	1.001	1.001	0.990	1.005	1.001	1.000	0.993	0.988	0.997	0.994
SV_{bu}	0.998	0.998	1.001	0.992	1.001	1.008	0.971	1.003	0.995	0.942
TSV_{bu}	1.043	1.019	1.043	0.960	1.006	1.051	0.875	1.055	0.980	0.730
SV_{shr}	0.997	0.998	0.993	0.995	1.000	1.003	0.960	0.995	0.993	0.927
TSV_{shr}	0.994	0.984	1.018	0.971	0.990	1.016	0.853	1.023	0.979	0.706
SV_{hac^*}	2.634	2.356	3.635	3.072	1.539	7.113	4.647	4.455	3.155	3.569
SV_{hac}	0.997	0.998	0.995	0.995	1.001	1.002	0.961	0.997	0.992	0.929
SV_{ewma^*}	1.200	1.134	0.996	1.014	1.052	1.141	1.075	0.998	1.018	1.088
SV_{ewma}	1.022	1.021	1.004	1.106	0.997	1.028	0.982	0.998	0.996	0.942
$PV(3)$	1.049	1.010	**0.990**	**0.941**	0.964	**0.980**	1.055	**0.976**	1.016	1.106
$PV(3)_{bu}$	0.997	0.995	1.004	0.980	0.997	1.009	0.945	1.008	0.982	0.899
$TPV(3)_{bu}$	1.038	1.012	1.053	0.955	1.000	1.058	0.868	1.065	0.978	0.712
$PV(3)_{shr}$	1.017	0.994	0.993	0.949	0.970	0.983	0.916	0.988	0.961	0.865
$TPV(3)_{shr}$	**0.982**	**0.965**	1.013	0.941	0.960	0.989	**0.824**	0.996	**0.956**	**0.672**
$PV(3)_{hac^*}$	4.151	3.219	3.177	1.911	1.521	6.373	4.410	3.842	2.684	4.000
$PV(3)_{hac}$	1.007	0.990	0.993	0.952	0.974	0.984	0.930	0.989	0.962	0.891
$PV(3)_{ewma^*}$	1.935	1.577	0.996	1.014	1.110	1.451	1.475	0.999	1.067	1.823
$PV(3)_{ewma}$	1.068	1.017	0.995	0.991	**0.947**	1.005	0.948	0.994	1.022	0.899

Table D.5: *Accuracy of the **22-day ahead** forecasts. MSE and QLIKE ratios for the DJIA index (panel A), and geometric means of the MSE and QLIKE ratios for individual stocks (panel B) over the benchmark HAR model. Values larger than one in red. The best index value in each column is highlighted in bold.*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	6.352	6.527	5.184	6.099	5.236	5.867	5.219	6.202	5.523	5.293	5.135	6.200	**5.057**
p-value dm_{RV}	–	0.785	0.168	0.142	0.119	0.075	0.121	0.119	0.118	0.066	0.074	0.108	0.073
p-value dm_{SV}	–	–	0.131	0.148	0.112	0.079	0.114	0.115	0.110	**0.044**	0.064	0.111	0.051
p-value dm_{PV}	–	–	–	0.800	0.526	0.759	0.517	0.821	0.659	0.579	0.467	0.820	0.374
p-value MCS	**0.384**	**0.278**	**0.791**	**0.501**	**0.791**	**0.598**	**0.791**	**0.450**	**0.494**	**0.791**	**0.791**	**0.480**	**1.000**
$QLIKE$	0.216	0.210	0.491	0.218	0.235	0.217	0.236	0.212	0.219	0.204	0.210	0.210	**0.203**
p-value dm_{RV}	–	**0.004**	0.908	0.692	1.000	0.528	1.000	0.176	0.698	**0.008**	0.104	0.058	**0.006**
p-value dm_{SV}	–	–	0.911	0.988	1.000	0.956	1.000	0.796	0.997	**0.031**	0.452	0.358	**0.020**
p-value dm_{PV}	–	–	–	0.098	0.112	0.097	0.113	0.093	0.098	0.087	0.092	0.091	0.086
p-value MCS	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**0.232**	**1.000**
Panel B: Individual stocks													
$\overline{MSE}$	39.595	40.993	36.374	37.805	58.265	36.947	55.191	38.970	45.080	**35.553**	37.942	38.765	35.815
p-value dm_{RV}	–	0	0	1	0	2	0	1	2	2	8	1	1
p-value dm_{SV}	–	–	0	0	1	1	1	5	3	2	8	3	2
p-value dm_{PV}	–	–	–	1	0	1	0	2	1	1	2	0	2
p-value MCS	25	24	26	26	26	26	26	26	26	26	26	26	26
$\overline{QLIKE}$	0.185	0.214	0.269	0.170	0.161	0.165	0.161	0.165	0.154	0.151	**0.145**	0.165	0.152
p-value dm_{RV}	–	2	1	2	4	4	4	7	6	19	16	8	15
p-value dm_{SV}	–	–	1	6	5	7	5	10	7	20	17	10	15
p-value dm_{PV}	–	–	–	7	7	8	7	8	8	10	8	8	10
p-value MCS	19	20	17	19	17	20	17	21	20	26	23	21	26

Note: The table reports the **one-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano: p-values < 0.05 are highlighted in bold. MCS: p-values > 0.2 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against HAR, SV, and $PV(3)$ are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. $PV(3)$ denotes the $HAR - PV(3)$ model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.6: *One-day-ahead forecasting performance: 2007-2022 (3,901 days)*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	5.109	5.195	5.503	4.966	4.093	4.888	**4.064**	5.064	4.487	5.185	4.724	4.999	4.982
p-value dm_{RV}	–	0.774	0.833	**0.021**	**0.002**	**0.007**	**0.002**	0.235	**0.000**	0.613	**0.041**	**0.029**	0.227
p-value dm_{SV}	–	–	0.783	0.054	**0.002**	**0.023**	**0.002**	**0.049**	**0.000**	0.485	**0.023**	**0.017**	0.110
p-value dm_{PV}	–	–	–	0.105	**0.010**	0.077	**0.009**	0.139	**0.018**	**0.027**	**0.003**	0.110	**0.026**
p-value MCS	**0.201**	**0.201**	**0.201**	**0.326**	**0.355**	**0.355**	**1.000**	**0.234**	**0.355**	**0.238**	**0.355**	**0.269**	**0.343**
$QLIKE$	0.930	0.927	0.908	0.220	0.231	0.219	0.233	0.217	0.219	0.209	0.211	0.214	**0.209**
p-value dm_{RV}	–	**0.000**	0.441	**0.008**	**0.009**	**0.008**	**0.009**	**0.008**	**0.008**	**0.007**	**0.007**	**0.007**	**0.007**
p-value dm_{SV}	–	–	0.449	**0.008**	**0.009**	**0.008**	**0.009**	**0.008**	**0.008**	**0.007**	**0.007**	**0.008**	**0.007**
p-value dm_{PV}	–	–	–	**0.007**	**0.008**	**0.007**	**0.008**	**0.007**	**0.007**	**0.006**	**0.006**	**0.007**	**0.006**
p-value MCS	**0.478**	**0.480**	0.178	**0.480**	**0.480**	**0.480**	**0.480**	**0.480**	**0.480**	**0.716**	**0.480**	**0.480**	**1.000**
Panel B: Individual stocks													
$\overline{MSE}$	25.109	25.371	22.936	24.804	28.938	24.433	28.072	24.923	25.730	**22.753**	23.185	24.813	23.551
p-value dm_{RV}	–	0	1	2	3	9	4	4	5	2	7	4	5
p-value dm_{SV}	–	–	1	2	4	4	5	4	7	3	5	3	4
p-value dm_{PV}	–	–	–	0	0	0	0	0	0	0	1	0	0
p-value MCS	25	24	26	25	26	25	26	25	26	26	26	25	26
$\overline{QLIKE}$	0.186	0.187	0.201	0.181	0.162	0.161	0.161	0.174	0.153	0.147	**0.141**	0.174	0.149
p-value dm_{RV}	–	6	7	4	5	8	6	9	8	24	22	6	20
p-value dm_{SV}	–	–	7	2	3	4	4	3	5	21	19	1	18
p-value dm_{PV}	–	–	–	2	4	4	4	2	6	8	10	2	7
p-value MCS	15	16	22	14	15	16	15	17	16	25	25	16	25

Note: The table reports the **five-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano: p-values < 0.05 are highlighted in bold. MCS: p-values > 0.2 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against HAR, SV, and $PV(3)$ are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. $PV(3)$ denotes the $HAR - PV(3)$ model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.7: *Five-day-ahead forecasting performance: 2007-2022 (3,897 days)*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	4.424	4.436	4.533	4.448	4.861	4.463	4.879	4.434	4.267	4.488	**4.153**	4.431	4.475
p-value dm_{RV}	–	0.613	0.851	0.631	0.947	0.683	0.956	0.569	0.122	0.780	**0.034**	0.555	0.774
p-value dm_{SV}	–	–	0.843	0.575	0.948	0.646	0.956	0.478	0.079	0.765	**0.016**	0.456	0.750
p-value dm_{PV}	–	–	–	0.209	0.890	0.257	0.904	0.158	**0.030**	0.225	**0.005**	0.148	0.190
p-value MCS	**0.720**	**0.720**	**0.720**	**0.720**	**0.720**	**0.720**	**0.656**	**0.720**	**0.720**	**0.720**	**1.000**	**0.720**	**0.720**
$QLIKE$	0.973	0.902	0.936	0.727	0.375	0.580	0.376	0.708	0.368	0.580	**0.364**	0.699	0.574
p-value dm_{RV}	–	0.160	0.150	**0.027**	**0.000**	**0.002**	**0.000**	**0.016**	**0.000**	**0.002**	**0.000**	**0.014**	**0.002**
p-value dm_{SV}	–	–	0.830	**0.049**	**0.000**	**0.004**	**0.000**	**0.028**	**0.000**	**0.003**	**0.000**	**0.023**	**0.003**
p-value dm_{PV}	–	–	–	**0.031**	**0.000**	**0.002**	**0.000**	**0.017**	**0.000**	**0.002**	**0.000**	**0.014**	**0.002**
p-value MCS	**0.422**	**0.448**	**0.448**	**0.448**	**0.448**	**0.448**	**0.448**	**0.448**	**0.448**	**0.448**	**1.000**	**0.448**	**0.448**
Panel B: Individual stocks													
$\overline{MSE}$	14.243	14.277	14.316	14.205	14.011	14.231	13.905	14.207	13.567	14.122	**13.257**	14.242	14.166
p-value dm_{RV}	–	1	2	2	2	4	3	3	5	3	11	4	6
p-value dm_{SV}	–	–	2	1	2	2	3	2	5	2	11	1	5
p-value dm_{PV}	–	–	–	3	1	2	1	3	1	6	12	3	7
p-value MCS	25	26	25	24	25	25	25	25	26	26	26	25	26
$\overline{QLIKE}$	0.269	0.268	0.292	0.262	0.227	0.255	0.225	0.259	0.222	0.247	**0.214**	0.259	0.250
p-value dm_{RV}	–	5	6	4	12	10	12	6	13	22	22	6	19
p-value dm_{SV}	–	–	5	3	10	6	10	4	13	17	20	5	16
p-value dm_{PV}	–	–	–	8	14	10	14	9	15	14	15	9	13
p-value MCS	18	19	23	17	17	18	17	18	19	25	25	19	23

Note: The table reports the **22-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano: p-values < 0.05 are highlighted in bold. MCS: p-values > 0.2 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against HAR, SV, and $PV(3)$ are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. $PV(3)$ denotes the $HAR - PV(3)$ model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.8: *Twenty-two-day-ahead forecasting performance: 2007-2022 (3,880 days)*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	20.587	21.074	15.957	19.808	16.485	18.972	16.440	20.097	17.614	16.718	16.198	20.089	**15.802**
p-value dm_{RV}	–	0.718	0.162	0.197	0.131	0.107	0.134	0.159	0.136	0.077	0.089	0.145	0.082
p-value dm_{SV}	–	–	0.135	0.211	0.132	0.121	0.134	0.174	0.137	0.060	0.084	0.169	0.065
p-value dm_{PV}	–	–	–	0.820	0.567	0.789	0.560	0.833	0.697	0.639	0.541	0.832	0.459
p-value MCS	**0.534**	**0.454**	**0.740**	**0.454**	**0.740**	**0.492**	**0.740**	**0.517**	**0.534**	**0.740**	**0.740**	**0.566**	**1.000**
$QLIKE$	0.187	0.183	0.505	0.192	0.203	0.192	0.205	0.187	0.191	0.176	0.181	0.185	**0.174**
p-value dm_{RV}	–	**0.002**	0.873	0.999	1.000	0.996	1.000	0.267	0.995	**0.001**	**0.016**	0.103	**0.001**
p-value dm_{SV}	–	–	0.876	1.000	1.000	1.000	1.000	0.999	1.000	**0.024**	0.231	0.952	**0.010**
p-value dm_{PV}	–	–	–	0.131	0.140	0.131	0.141	0.127	0.130	0.119	0.123	0.126	0.118
p-value MCS	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**0.229**	**1.000**
Panel B: Individual stocks													
$\overline{MSE}$	96.815	97.020	95.505	93.645	186.003	92.522	175.154	94.130	130.389	**91.192**	107.195	94.155	93.281
p-value dm_{RV}	–	0	0	0	0	2	0	0	0	3	5	1	3
p-value dm_{SV}	–	–	0	1	1	1	1	2	1	3	4	2	2
p-value dm_{PV}	–	–	–	1	0	1	0	1	0	1	1	0	1
p-value MCS	26	25	26	24	26	25	26	25	26	26	26	25	26
$\overline{QLIKE}$	0.196	0.224	0.287	0.188	0.171	0.191	0.172	0.187	0.163	0.166	**0.156**	0.187	0.166
p-value dm_{RV}	–	4	0	0	1	1	1	3	1	13	9	4	13
p-value dm_{SV}	–	–	1	1	0	0	0	3	0	9	5	3	9
p-value dm_{PV}	–	–	–	4	2	3	2	5	3	6	3	5	7
p-value MCS	23	22	22	23	12	22	11	23	16	26	21	24	26

Note: The table reports **the one-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano: p-values < 0.05 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against HAR, SV, and $PV(3)$ models are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. PV(3) denotes the HAR-PV(3) model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.9: *Forecasting performance: 2007-2010 (1,008 days).*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	0.433	0.429	**0.399**	0.457	0.498	0.449	0.497	0.442	0.462	0.409	0.429	0.441	0.406
p-value dm_{RV}	–	0.339	0.242	0.991	0.997	0.942	0.997	0.995	0.985	0.100	0.389	0.987	0.102
p-value dm_{SV}	–	–	0.266	0.929	0.989	0.849	0.990	0.866	0.951	0.184	0.506	0.850	0.173
p-value dm_{PV}	–	–	–	0.860	0.940	0.834	0.940	0.803	0.867	0.621	0.769	0.797	0.600
p-value MCS	**0.438**	**0.628**	**1.000**	0.117	0.103	0.103	0.102	**0.259**	0.158	**0.643**	**0.214**	**0.311**	**0.700**
$QLIKE$	0.195	**0.192**	0.209	0.209	0.243	0.210	0.247	0.200	0.216	0.204	0.214	0.194	0.204
p-value dm_{RV}	–	**0.003**	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.182	1.000
p-value dm_{SV}	–	–	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.921	1.000
p-value dm_{PV}	–	–	–	0.423	1.000	0.555	1.000	**0.014**	0.948	**0.025**	0.914	**0.000**	**0.007**
p-value MCS	0.131	**1.000**	0.042	0.008	0.000	0.001	0.000	0.042	0.000	0.014	0.000	**0.314**	0.042
Panel B: Individual stocks													
$\overline{MSE}$	1.508	1.568	1.464	1.504	1.564	1.492	1.569	1.494	1.514	1.404	1.445	1.495	**1.402**
p-value dm_{RV}	–	1	4	3	1	5	1	7	2	9	6	4	9
p-value dm_{SV}	–	–	3	1	1	1	1	4	0	10	4	4	9
p-value dm_{PV}	–	–	–	0	0	0	0	0	0	2	1	0	3
p-value MCS	23	19	25	22	10	21	10	22	12	26	19	22	26
$\overline{QLIKE}$	0.123	0.148	0.133	0.124	0.134	0.124	0.136	0.121	0.126	**0.118**	0.123	0.121	0.118
p-value dm_{RV}	–	8	4	1	1	1	1	10	2	14	3	10	13
p-value dm_{SV}	–	–	5	3	1	3	1	5	1	9	2	6	8
p-value dm_{PV}	–	–	–	6	2	6	2	9	5	14	6	9	14
p-value MCS	18	17	17	15	0	11	1	19	1	22	9	19	22

Note: The table reports the **one-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano p-values < 0.05 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against RV-HAR, SV-HAR, and PV(3)-HAR are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. PV(3) denotes the HAR-PV(3) model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.10: *Forecasting performance: 2011-2014 (1,006 days).*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	0.901	0.963	0.899	0.871	0.824	0.854	0.822	0.898	0.839	0.841	**0.811**	0.894	0.838
p-value dm_{RV}	–	0.830	0.485	0.145	0.093	0.113	0.091	0.384	0.062	0.103	0.050	0.267	0.093
p-value dm_{SV}	–	–	0.294	0.155	0.122	0.141	0.121	0.139	0.106	0.124	0.090	0.120	0.118
p-value dm_{PV}	–	–	–	0.324	0.124	0.216	0.114	0.498	0.164	0.056	**0.038**	0.472	**0.044**
p-value MCS	**0.641**	**0.564**	**0.646**	**0.646**	**0.826**	**0.646**	**0.826**	**0.646**	**0.826**	**0.813**	**1.000**	**0.646**	**0.826**
$QLIKE$	0.242	0.234	**0.211**	0.247	0.260	0.243	0.260	0.239	0.245	0.218	0.228	0.237	0.217
p-value dm_{RV}	–	**0.012**	**0.008**	0.866	0.997	0.597	0.999	**0.023**	0.956	**0.001**	**0.010**	0.079	**0.001**
p-value dm_{SV}	–	–	**0.024**	0.951	0.996	0.893	0.998	0.977	0.998	**0.006**	0.112	1.000	**0.005**
p-value dm_{PV}	–	–	–	0.987	0.998	0.984	0.999	0.986	0.996	0.875	0.985	0.986	0.858
p-value MCS	0.021	0.073	**1.000**	0.063	0.009	0.073	0.004	0.044	0.001	**0.259**	0.073	0.063	**0.262**
Panel B: Individual stocks													
$\overline{MSE}$	28.790	28.277	**13.400**	25.753	16.027	24.144	15.524	26.345	19.594	17.863	16.205	26.339	17.856
p-value dm_{RV}	–	2	1	0	1	3	2	4	0	9	12	5	8
p-value dm_{SV}	–	–	3	1	0	1	1	2	1	7	9	2	7
p-value dm_{PV}	–	–	–	0	0	0	0	0	0	0	0	0	1
p-value MCS	21	21	26	16	23	16	23	21	23	26	25	20	26
$\overline{QLIKE}$	0.183	0.239	0.345	0.184	0.185	0.179	0.185	0.179	0.178	0.159	0.163	0.178	**0.157**
p-value dm_{RV}	–	2	5	1	1	7	2	5	7	25	23	7	25
p-value dm_{SV}	–	–	7	2	2	4	2	5	5	24	19	6	22
p-value dm_{PV}	–	–	–	3	3	3	3	4	3	7	6	5	7
p-value MCS	10	12	21	11	12	12	11	12	12	23	17	12	24

Note: The table reports the **one-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano p-values < 0.05 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against RV-HAR, SV-HAR, and PV(3)-HAR are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. PV(3) denotes the HAR-PV(3) model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.11: *Forecasting performance: 2015-2019 (1,258 days).*

	RV	SV	$PV(3)$	SV_{bu}	TSV_{bu}	$PV(3)_{bu}$	$TPV(3)_{bu}$	SV_{shr}	TSV_{shr}	$PV(3)_{shr}$	$TPV(3)_{shr}$	SV_{hac}	$PV(3)_{hac}$
Panel A: DJIA index													
MSE	3.910	4.096	4.141	3.610	3.608	**3.561**	3.583	3.752	3.607	3.702	3.578	3.767	3.716
p-value dm_{RV}	–	0.838	0.854	**0.012**	0.065	**0.011**	0.084	0.076	**0.004**	0.057	**0.005**	0.112	0.084
p-value dm_{SV}	–	–	0.605	**0.016**	**0.038**	**0.013**	**0.044**	**0.015**	**0.015**	**0.019**	**0.012**	**0.013**	**0.016**
p-value dm_{PV}	–	–	–	**0.017**	**0.041**	**0.012**	**0.047**	**0.028**	**0.015**	**0.009**	**0.009**	**0.030**	**0.009**
p-value MCS	**0.500**	**0.500**	**0.217**	**0.723**	**0.723**	**1.000**	**0.920**	**0.441**	**0.723**	**0.500**	**0.920**	**0.500**	**0.500**
$QLIKE$	0.245	0.236	1.482	0.220	0.222	0.214	0.219	0.220	**0.214**	0.221	0.214	0.219	0.221
p-value dm_{RV}	–	0.206	0.847	0.160	0.210	0.128	0.187	0.153	0.126	0.176	0.130	0.146	0.186
p-value dm_{SV}	–	–	0.847	0.133	0.220	0.090	0.184	0.120	0.090	0.158	0.095	0.110	0.175
p-value dm_{PV}	–	–	–	0.153	0.154	0.152	0.153	0.153	0.152	0.153	0.152	0.152	0.153
p-value MCS	**0.783**	**0.714**	**0.392**	**0.783**	**0.783**	**0.981**	**0.867**	**0.783**	**1.000**	**0.783**	**0.981**	**0.867**	**0.783**
Panel B: Individual stocks													
$\overline{MSE}$	30.422	39.697	43.397	30.482	28.721	30.197	**28.037**	35.764	29.017	36.386	28.804	34.463	34.683
p-value dm_{RV}	–	0	0	1	1	3	1	0	2	0	5	0	0
p-value dm_{SV}	–	–	1	3	1	4	1	3	2	2	3	2	2
p-value dm_{PV}	–	–	–	1	1	1	1	2	2	2	2	1	2
p-value MCS	26	26	25	26	26	26	26	26	26	25	26	26	25
$\overline{QLIKE}$	0.269	0.256	0.305	0.186	0.138	0.163	0.135	0.172	0.133	0.162	**0.127**	0.173	0.174
p-value dm_{RV}	–	0	4	2	2	3	2	1	3	10	10	1	4
p-value dm_{SV}	–	–	9	6	4	7	4	6	7	13	13	4	7
p-value dm_{PV}	–	–	–	3	4	5	4	5	5	9	8	5	6
p-value MCS	23	22	24	21	22	24	22	23	24	24	24	22	23

Note: The table reports the **one-step ahead** forecasting performance of the different models. The top panel shows the results for the DJIA index. Diebold-Mariano p-values < 0.05 are highlighted in bold. The bottom panel reports the average loss and 5% rejection frequency of the Diebold-Mariano tests for each individual stocks. The one-sided tests between each forecasting model against RV-HAR, SV-HAR, and PV(3)-HAR are denoted by dm_{HAR}, dm_{SV}, and $dm_{PV(3)}$, respectively. MCS denotes the p-value of that model being in the Model Confidence Set, or the number of times that model is in the 80% Model Confidence Set. PV(3) denotes the HAR-PV(3) model with 3 decompositions defined by two thresholds at 10% and 75%.

Table D.12: *Forecasting performance: 2020-2022 (629 days).*

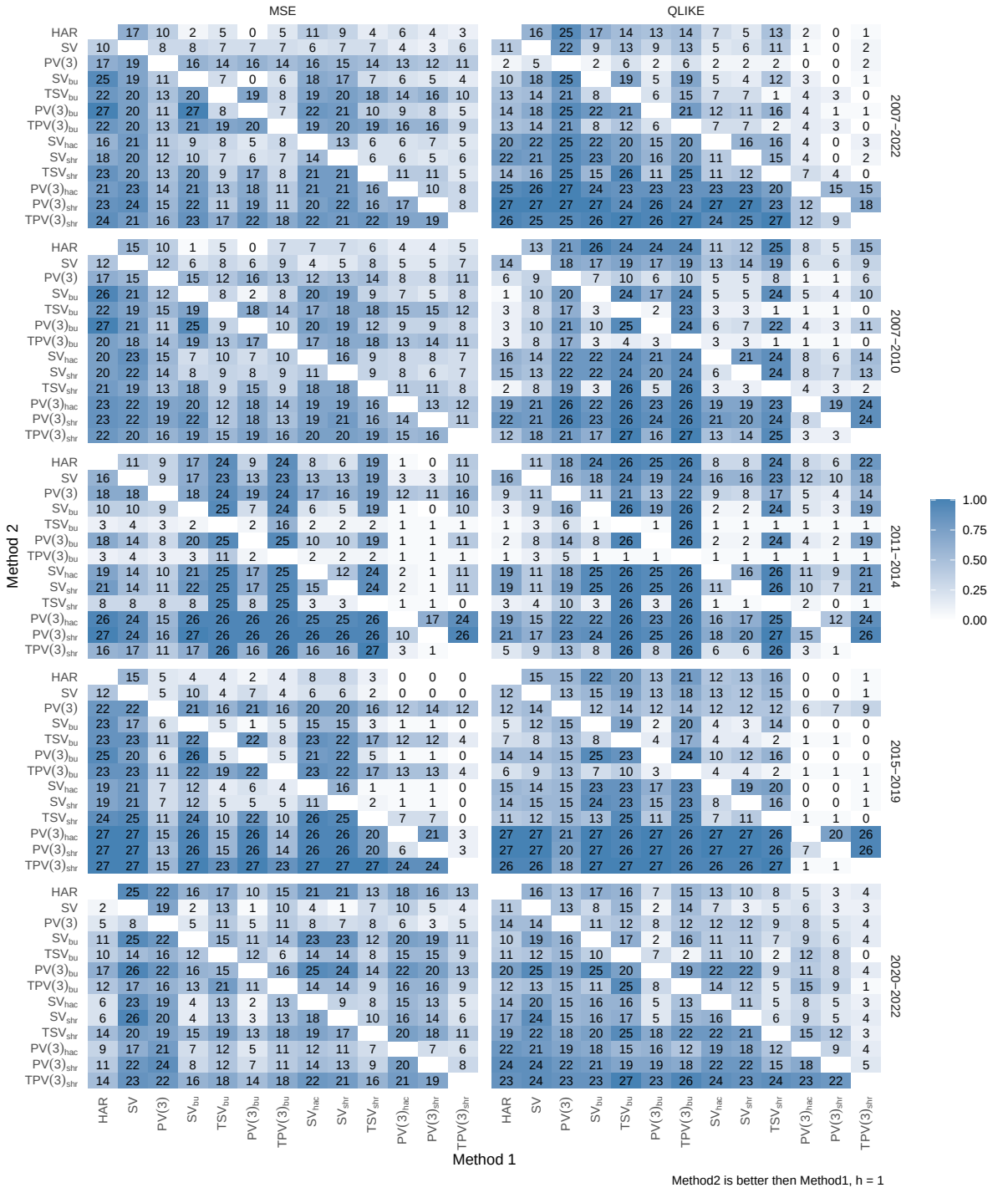

Figure D.28: *Qualitative evaluation of the **one-step ahead** forecasting accuracy. Each cell reports the number of times the forecasting model in the row outperforms the model in the column. Different test periods, from the top: 2007-2010, 2011-2014, 2015-2019, 2020-2022.*

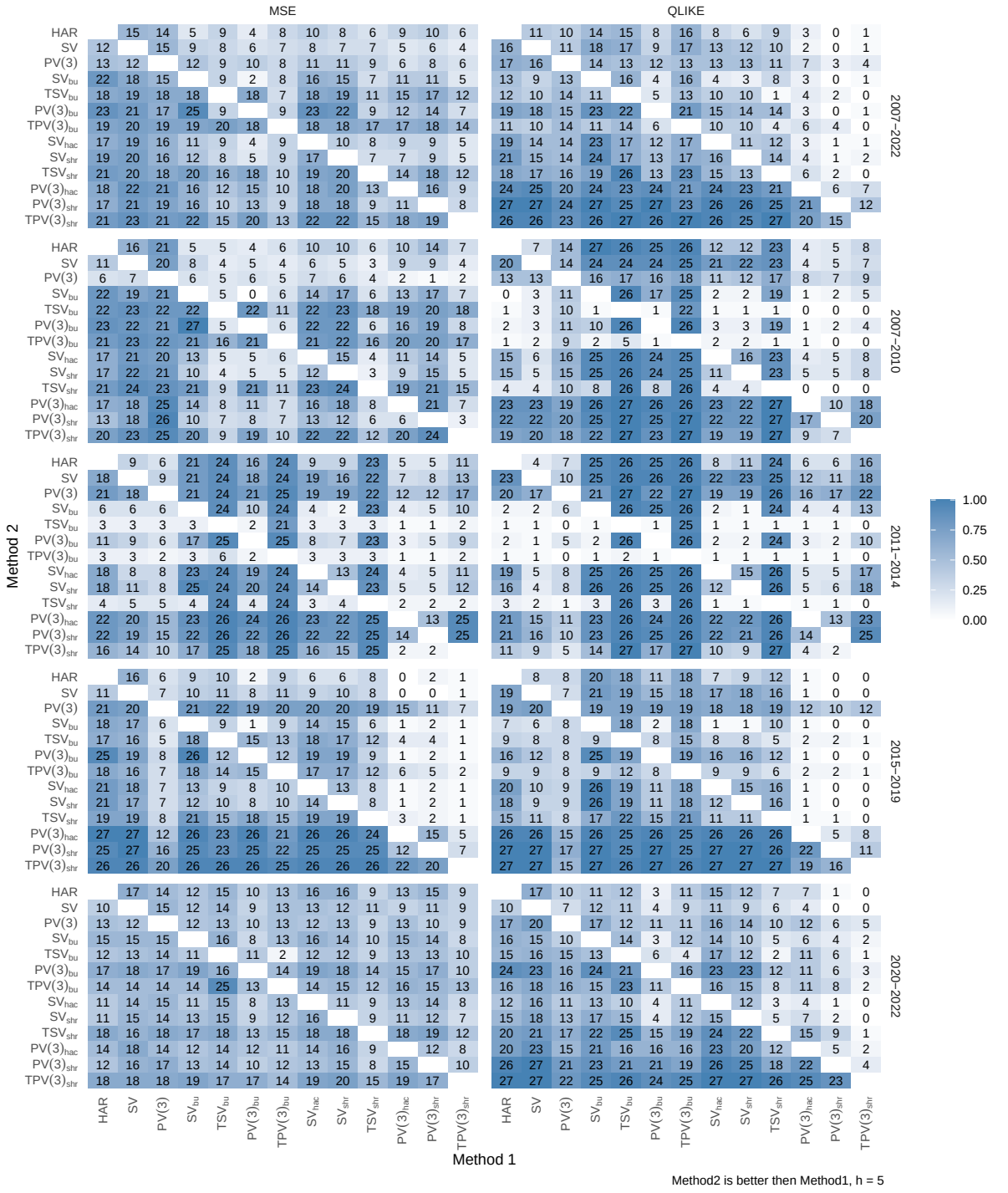

Figure D.29: *Qualitative evaluation of the **five-step ahead** forecasting accuracy. Each cell reports the number of times the forecasting model in the row outperforms the model in the column. Different test periods, from the top: 2007-2010, 2011-2014, 2015-2019, 2020-2022.*

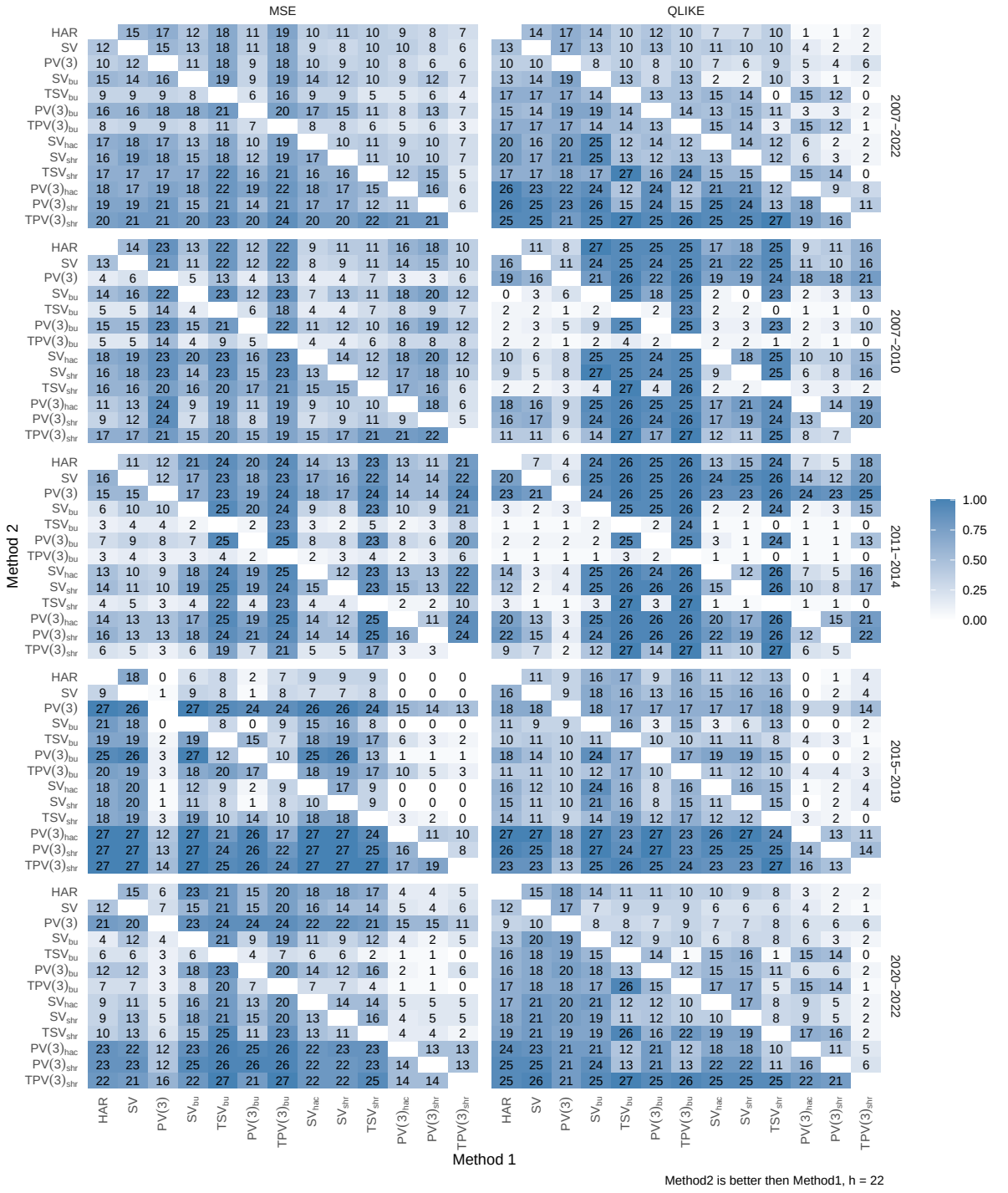

Figure D.30: *Qualitative evaluation of the* ***22-step ahead*** *forecasting accuracy. Each cell reports the number of times the forecasting model in the row outperforms the model in the column. Different test periods, from the top: 2007-2010, 2011-2014, 2015-2019, 2020-2022.*