

# Forecast reconciliation: Methodological issues and applications

## Chapter 2 - Cross-temporal reconciliation of solar forecasts<sup>1</sup>

### Online appendix

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#### A Auxiliary tables

**Table A.1:** Number of series with at least one negative reconciled forecast. All temporal aggregation orders, forecast horizon: operating day.

k	method	comb	type	n	min	max	fmin	fmax
1	NA	base	$\hat{y}$	350	4	6	-15.617	0.000
1	ct	ols	$\tilde{y}$	350	109	324	-69.165	0.000
1	ite (cst)	ols-struc	$\tilde{y}$	350	137	324	-20.739	0.000
1	ite (cst)	struc-ols	$\tilde{y}$	350	91	324	-17.961	0.000
1	ct	struc	$\tilde{y}$	350	89	324	-14.292	0.000
1	ite (tcs)	ols2-ols	$\tilde{y}$	350	42	324	-12.993	0.000
1	ite (tcs)	struc2-ols	$\tilde{y}$	350	36	324	-12.994	0.000
1	ite (tcs)	ols2-struc	$\tilde{y}$	350	144	324	-7.526	0.000
1	ite (tcs)	struc2-struc	$\tilde{y}$	350	124	324	-7.642	0.000
2	NA	base	$\hat{y}$	350	171	320	-32.639	0
2	ct	ols	$\tilde{y}$	350	60	324	-32.147	0
2	ite (cst)	ols-struc	$\tilde{y}$	350	56	324	-31.867	0
2	ite (cst)	struc-ols	$\tilde{y}$	350	26	324	-18.728	0
2	ct	struc	$\tilde{y}$	350	33	324	-22.719	0
2	ite (tcs)	ols2-ols	$\tilde{y}$	350	29	324	-27.023	0
2	ite (tcs)	struc2-ols	$\tilde{y}$	350	24	324	-27.025	0
2	ite (tcs)	ols2-struc	$\tilde{y}$	350	82	321	-20.451	0
2	ite (tcs)	struc2-struc	$\tilde{y}$	350	58	322	-17.812	0

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<sup>1</sup>Di Fonzo, T. and Girolimetto, D. (2022a) Enhancements in cross-temporal forecast reconciliation, with an application to solar irradiance forecasts. arXiv doi:10.48550/arXiv.2209.07146

Table A.1 – continued from previous page

k	method	comb	type	n	min	max	fmin	fmax
3	NA	base	$\hat{y}$	350	60	323	-45.120	0
3	ct	ols	$\tilde{y}$	350	6	324	-31.640	0
3	ite (cst)	ols-struc	$\tilde{y}$	350	9	324	-35.059	0
3	ite (cst)	struc-ols	$\tilde{y}$	350	20	324	-28.085	0
3	ct	struc	$\tilde{y}$	350	27	323	-32.377	0
3	ite (tcs)	ols2-ols	$\tilde{y}$	350	3	324	-43.853	0
3	ite (tcs)	struc2-ols	$\tilde{y}$	350	1	324	-43.855	0
3	ite (tcs)	ols2-struc	$\tilde{y}$	350	40	324	-42.255	0
3	ite (tcs)	struc2-struc	$\tilde{y}$	350	18	324	-42.445	0
4	NA	base	$\hat{y}$	350	14	321	-88.776	0
4	ct	ols	$\tilde{y}$	350	11	322	-42.291	0
4	ite (cst)	struc-ols	$\tilde{y}$	350	15	322	-35.129	0
4	ite (cst)	ols-struc	$\tilde{y}$	350	25	318	-44.256	0
4	ct	struc	$\tilde{y}$	350	14	324	-40.562	0
4	ite (tcs)	ols2-ols	$\tilde{y}$	350	2	324	-85.947	0
4	ite (tcs)	struc2-ols	$\tilde{y}$	350	1	324	-85.956	0
4	ite (tcs)	ols2-struc	$\tilde{y}$	350	2	324	-63.026	0
4	ite (tcs)	struc2-struc	$\tilde{y}$	350	1	324	-63.927	0
6	NA	base	$\hat{y}$	350	19	321	-146.702	0
6	ct	ols	$\tilde{y}$	350	3	324	-61.415	0
6	ite (cst)	ols-struc	$\tilde{y}$	350	4	323	-67.327	0
6	ite (cst)	struc-ols	$\tilde{y}$	350	17	323	-53.857	0
6	ct	struc	$\tilde{y}$	350	16	322	-63.281	0
6	ite (tcs)	ols2-ols	$\tilde{y}$	347	0	324	-133.572	0
6	ite (tcs)	struc2-ols	$\tilde{y}$	347	0	324	-133.571	0
6	ite (tcs)	ols2-struc	$\tilde{y}$	347	0	324	-89.955	0
6	ite (tcs)	struc2-struc	$\tilde{y}$	349	0	324	-88.924	0
8	NA	base	$\hat{y}$	212	0	238	-866.818	0.000
8	ct	ols	$\tilde{y}$	300	0	305	-78.762	0.000
8	ct	struc	$\tilde{y}$	276	0	324	-51.806	0.000
8	ite (cst)	ols-struc	$\tilde{y}$	296	0	272	-53.688	0.000
8	ite (cst)	struc-ols	$\tilde{y}$	274	0	319	-42.734	0.000
8	ite (tcs)	ols2-ols	$\tilde{y}$	307	0	320	-680.766	0.000
8	ite (tcs)	struc2-ols	$\tilde{y}$	285	0	324	-680.488	0.000
8	ite (tcs)	ols2-struc	$\tilde{y}$	300	0	241	-56.011	0.000
8	ite (tcs)	struc2-struc	$\tilde{y}$	284	0	268	-52.341	0.000
12	NA	base	$\hat{y}$	46	0	57	-59.405	-0.001
12	ct	ols	$\tilde{y}$	14	0	78	-15.328	-0.001
12	ite (cst)	ols-struc	$\tilde{y}$	13	0	45	-7.949	-0.004
12	ite (cst)	struc-ols	$\tilde{y}$	8	0	31	-6.672	-0.001
12	ct	struc	$\tilde{y}$	6	0	18	-0.537	-0.009
12	ite (tcs)	ols2-ols	$\tilde{y}$	35	0	95	-32.259	0.000
12	ite (tcs)	struc2-ols	$\tilde{y}$	26	0	100	-32.191	0.000
12	ite (tcs)	ols2-struc	$\tilde{y}$	19	0	75	-9.799	0.000
12	ite (tcs)	struc2-struc	$\tilde{y}$	12	0	69	-6.544	-0.002

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k	method	comb	type	n	min	max	fmin	fmax
24	NA	base	$\hat{y}$	11	0	35	-51.205	-0.006
24	ct	ols	$\tilde{y}$	11	0	60	-29.615	-0.001
24	ite (cst)	ols-struc	$\tilde{y}$	11	0	33	-10.363	0.000
24	ite (cst)	struc-ols	$\tilde{y}$	6	0	28	-8.915	-0.008
24	ct	struc	$\tilde{y}$	4	0	11	-1.020	-0.026
24	ite (tcs)	ols2-ols	$\tilde{y}$	10	0	72	-15.037	-0.018
24	ite (tcs)	struc2-ols	$\tilde{y}$	10	0	71	-14.791	-0.005
24	ite (tcs)	ols2-struc	$\tilde{y}$	10	0	43	-12.969	-0.005
24	ite (tcs)	struc2-struc	$\tilde{y}$	5	0	31	-7.042	-0.002

**Table A.2:** Number of replications with at least one negative reconciled forecast. All temporal aggregation orders, forecast horizon: operating day.

k	method	comb	type	#L1	#L2	#L3	#tot	%L1	%L2	%L3	%tot
1	NA	base	$\hat{y}$	1	5	0	6	100.00%	100.00%	0.00%	1.85%
1	ct	ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ite (cst)	ols-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ite (cst)	struc-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ct	struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ite (tcs)	ols2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ite (tcs)	struc2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ite (tcs)	ols2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
1	ite (tcs)	struc2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	NA	base	$\hat{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ct	ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ite (cst)	ols-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ite (cst)	struc-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ct	struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ite (tcs)	ols2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ite (tcs)	ols2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ite (tcs)	struc2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
2	ite (tcs)	struc2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	NA	base	$\hat{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ct	ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ite (cst)	ols-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ite (cst)	struc-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ct	struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ite (tcs)	ols2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ite (tcs)	struc2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ite (tcs)	ols2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
3	ite (tcs)	struc2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%

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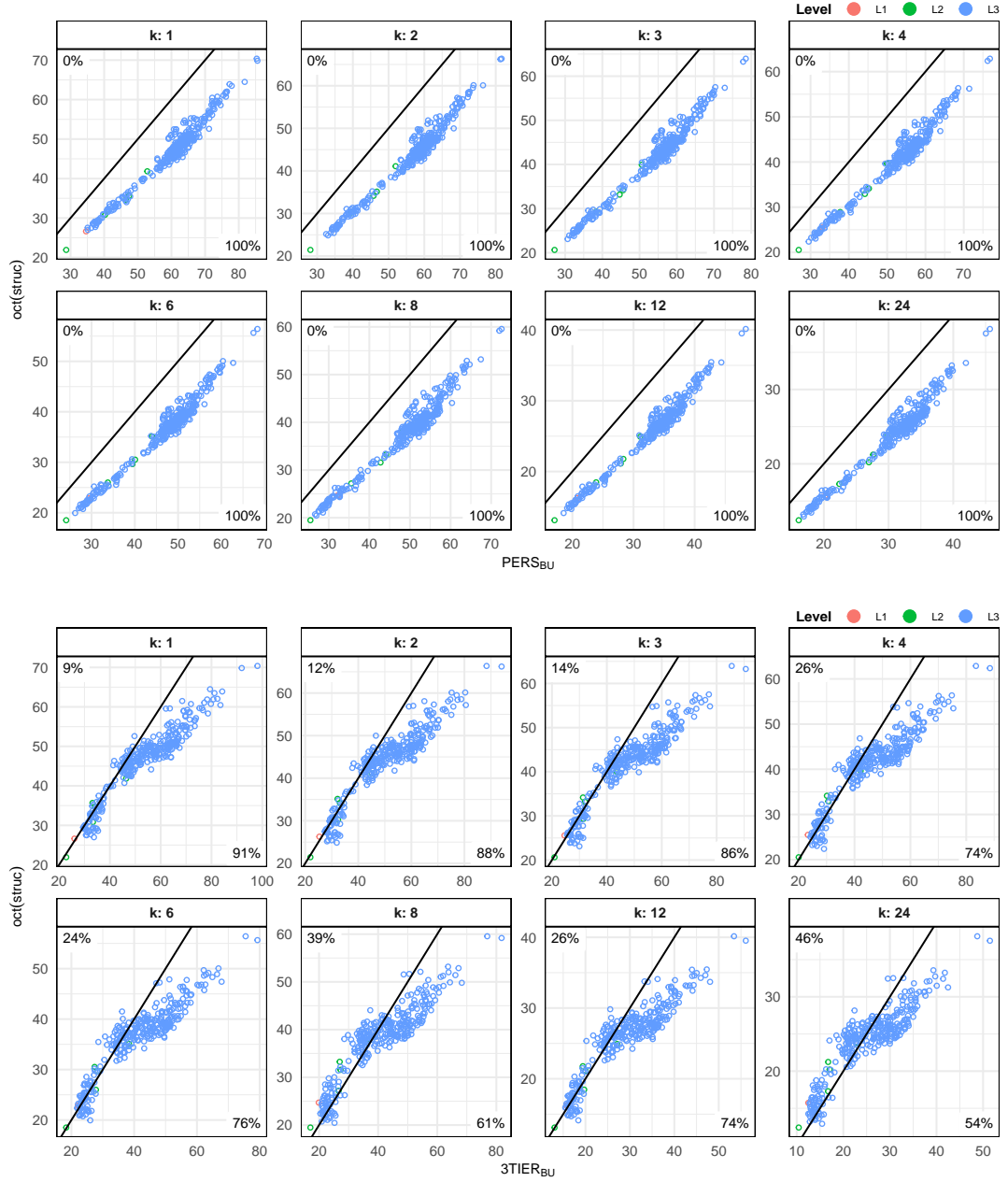
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$k$	method	comb	type	#L1	#L2	#L3	#tot	%L1	%L2	%L3	%tot
4	NA	base	$\hat{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ct	ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ite (cst)	ols-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ite (cst)	struc-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ct	struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ite (tcs)	ols2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ite (tcs)	struc2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ite (tcs)	ols2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
4	ite (tcs)	struc2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	NA	base	$\hat{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ct	ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ite (cst)	ols-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ite (cst)	struc-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ct	struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ite (tcs)	ols2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ite (tcs)	struc2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ite (tcs)	ols2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
6	ite (tcs)	struc2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	NA	base	$\hat{y}$	1	3	318	322	100.00%	60.00%	100.00%	99.38%
8	ct	ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ite (cst)	ols-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ite (cst)	struc-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ct	struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ite (tcs)	ols2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ite (tcs)	struc2-ols	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ite (tcs)	ols2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
8	ite (tcs)	struc2-struc	$\tilde{y}$	1	5	318	324	100.00%	100.00%	100.00%	100.00%
12	NA	base	$\hat{y}$	0	2	135	137	0.00%	40.00%	42.45%	42.28%
12	ct	ols	$\tilde{y}$	0	1	162	163	0.00%	20.00%	50.94%	50.31%
12	ite (cst)	ols-struc	$\tilde{y}$	0	1	99	100	0.00%	20.00%	31.13%	30.86%
12	ite (cst)	struc-ols	$\tilde{y}$	0	1	67	68	0.00%	20.00%	21.07%	20.99%
12	ct	struc	$\tilde{y}$	0	0	28	28	0.00%	0.00%	8.81%	8.64%
12	ite (tcs)	ols2-ols	$\tilde{y}$	0	2	172	174	0.00%	40.00%	54.09%	53.70%
12	ite (tcs)	struc2-ols	$\tilde{y}$	0	2	156	158	0.00%	40.00%	49.06%	48.77%
12	ite (tcs)	ols2-struc	$\tilde{y}$	0	1	126	127	0.00%	20.00%	39.62%	39.20%
12	ite (tcs)	struc2-struc	$\tilde{y}$	0	1	105	106	0.00%	20.00%	33.02%	32.72%

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$k$	method	comb	type	#L1	#L2	#L3	#tot	%L1	%L2	%L3	%tot
24	NA	base	$\hat{\mathbf{y}}$	0	2	58	60	0.00%	40.00%	18.24%	18.52%
24	ct	ols	$\tilde{\mathbf{y}}$	0	1	134	135	0.00%	20.00%	42.14%	41.67%
24	ite (cst)	ols-struc	$\tilde{\mathbf{y}}$	0	1	72	73	0.00%	20.00%	22.64%	22.53%
24	ite (cst)	struc-ols	$\tilde{\mathbf{y}}$	0	1	47	48	0.00%	20.00%	14.78%	14.81%
24	ct	struc	$\tilde{\mathbf{y}}$	0	0	17	17	0.00%	0.00%	5.35%	5.25%
24	ite (tcs)	ols2-ols	$\tilde{\mathbf{y}}$	0	1	121	122	0.00%	20.00%	38.05%	37.65%
24	ite (tcs)	struc2-ols	$\tilde{\mathbf{y}}$	0	1	102	103	0.00%	20.00%	32.08%	31.79%
24	ite (tcs)	ols2-struc	$\tilde{\mathbf{y}}$	0	1	80	81	0.00%	20.00%	25.16%	25.00%
24	ite (tcs)	struc2-struc	$\tilde{\mathbf{y}}$	0	1	41	42	0.00%	20.00%	12.89%	12.96%

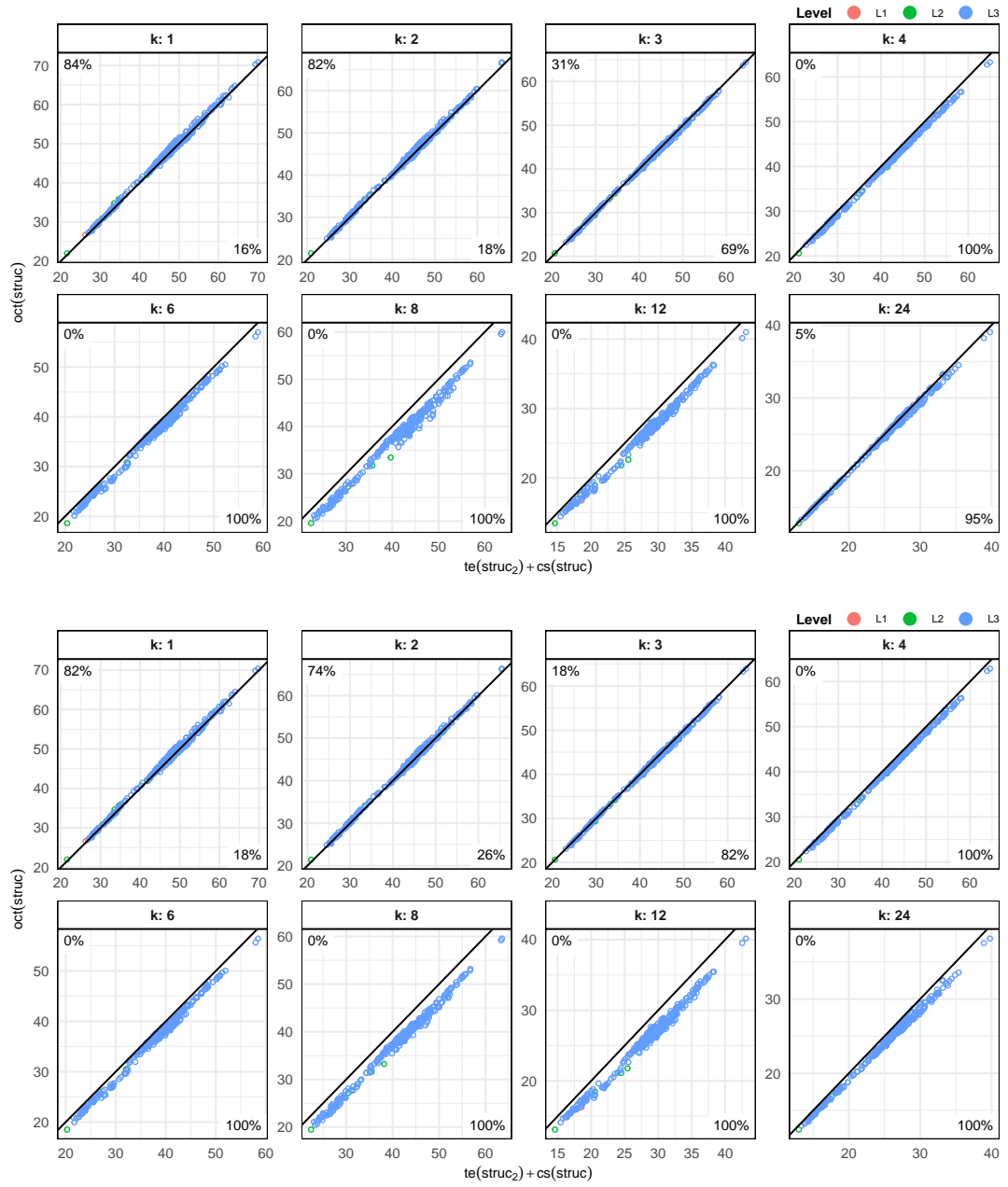


**Figure A.1:** Comparison of  $nRMSE(\%)$  between  $PERS_{BU}$  and not negative oct-struc (top panel), and between  $3TIER_{BU}$  and not negative oct-struc (bottom panel). The black line represents the bisector, where the  $nRMSE$ 's for both approaches are equal. On the top-left (bottom-right) corner of each graph, the percentage of points above (below) the bisector is reported.

A general overview of the accuracy of the considered procedures at any temporal granularity is given by table A.3, while a visual confirmation of the performance of ct-struc is shown by the MCB graphs in figure 5 in the main paper. The ct-struc daily reconciled forecasts show the best performance, significantly different from all the other procedures, while at hourly level it ranks second, the best being t-struc<sub>2</sub> + cs-struc. However, t-struc<sub>2</sub> + cs-struc produces temporally incoherent forecasts, and a punctual assessment of the performance of these two procedures in terms of nRMSE (figure A.2) confirms that their forecasting accuracy is about the same.

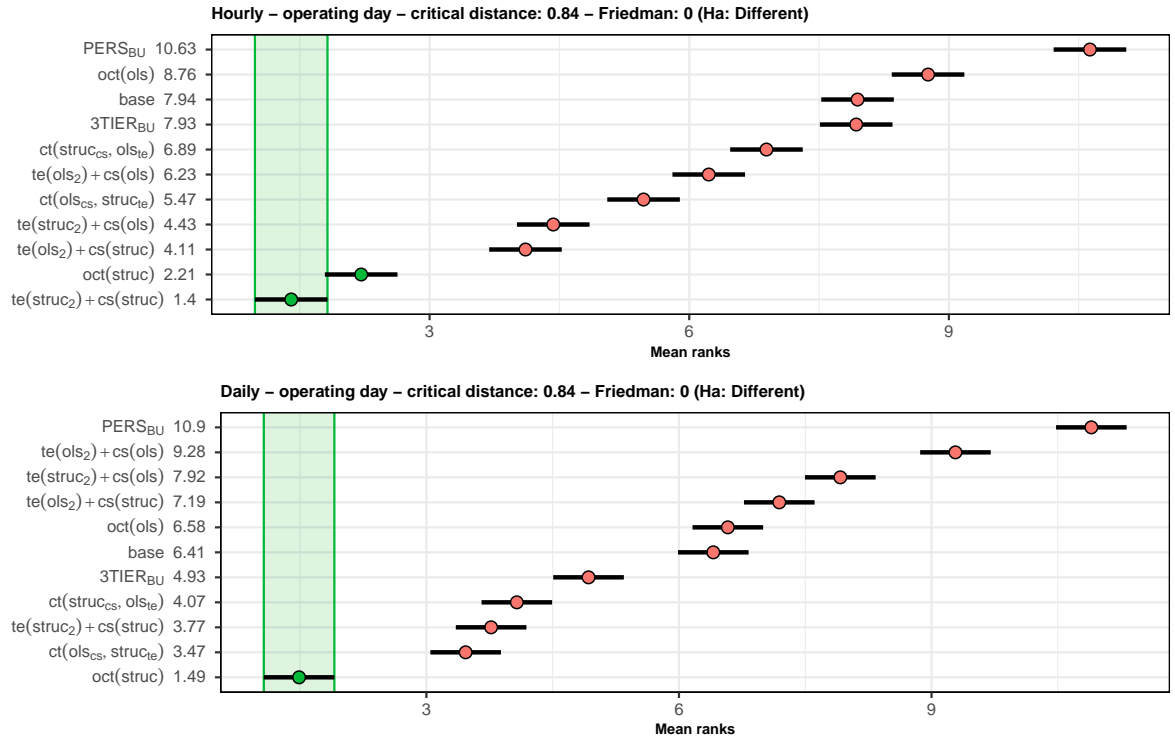
**Table A.3:** Forecast accuracies of sequential reconciliation methods and base forecasts in terms of nRMSE(%). Free reconciliation approaches considered by Yagli et al. (2019), Tables 2, 3, p. 395. All temporal aggregation orders, forecast horizon: operating day.

group	method	comb	type	1	2	3	4	6	8	12	24
L1	bf	pers	$\tilde{y}$	34.62	34.14	33.35	33.11	29.45	32.23	20.83	20.23
L1	ctbu	3TIER	$\tilde{y}$	<b>26.03</b>	<b>25.34</b>	<b>24.56</b>	<b>23.31</b>	<b>22.09</b>	<b>19.89</b>	<b>15.70</b>	<b>12.48</b>
L1	NA	base	$\hat{y}$	27.85	27.55	28.31	29.69	28.01	35.32	21.04	18.17
L1	ct	ols	$\tilde{y}$	30.69	29.96	28.29	28.86	25.26	28.12	18.39	17.91
L1	ite (cst)	ols-struc	$\tilde{y}$	28.74	28.25	27.14	27.38	24.34	26.64	17.82	17.33
L1	ite (cst)	struc-ols	$\tilde{y}$	28.26	27.84	27.09	27.00	24.17	26.24	17.46	16.96
L1	ct	struc	$\tilde{y}$	26.71	26.35	25.67	25.57	23.03	24.80	16.74	16.24
L1	ite (tcs)	ols2-ols	$\tilde{y}$	27.80	27.41	27.91	29.38	27.39	34.11	20.80	17.96
L1	ite (tcs)	struc2-ols	$\tilde{y}$	27.80	27.41	27.90	29.37	27.39	34.10	20.80	17.95
L1	ite (tcs)	ols2-struc	$\tilde{y}$	27.02	26.73	26.73	27.75	25.13	28.93	19.11	17.24
L1	ite (tcs)	struc2-struc	$\tilde{y}$	26.17	25.86	25.83	26.80	24.37	27.95	18.58	16.68
L2	bf	pers	$\tilde{y}$	43.15	42.42	41.27	40.81	36.29	39.19	25.66	24.57
L2	ctbu	3TIER	$\tilde{y}$	33.95	32.94	31.90	<b>30.62</b>	<b>28.02</b>	<b>26.99</b>	<b>19.87</b>	<b>16.75</b>
L2	NA	base	$\hat{y}$	34.24	34.04	33.79	35.01	33.11	40.51	25.16	20.94
L2	ct	ols	$\tilde{y}$	39.17	37.97	35.24	36.12	31.27	34.80	22.58	21.74
L2	ite (cst)	ols-struc	$\tilde{y}$	35.48	34.74	33.24	33.44	29.65	32.16	21.66	20.82
L2	ite (cst)	struc-ols	$\tilde{y}$	35.19	34.57	33.48	33.28	29.74	31.95	21.47	20.61
L2	ct	struc	$\tilde{y}$	33.11	32.54	<b>31.60</b>	31.37	28.19	30.06	20.49	19.64
L2	ite (tcs)	ols2-ols	$\tilde{y}$	34.36	34.52	35.40	36.29	35.64	45.42	26.24	21.80
L2	ite (tcs)	struc2-ols	$\tilde{y}$	34.34	34.50	35.38	36.26	35.62	45.40	26.23	21.79
L2	ite (tcs)	ols2-struc	$\tilde{y}$	33.40	33.07	32.82	33.64	30.70	35.26	23.20	20.62
L2	ite (tcs)	struc2-struc	$\tilde{y}$	<b>32.44</b>	<b>32.07</b>	31.80	32.54	29.81	34.14	22.59	19.96
L3	bf	pers	$\tilde{y}$	59.75	56.81	54.23	52.87	46.82	49.07	33.12	30.65
L3	ctbu	3TIER	$\tilde{y}$	53.46	50.57	48.33	46.19	41.36	40.72	29.28	25.19
L3	NA	base	$\hat{y}$	53.46	47.22	44.87	44.30	39.68	42.66	30.92	25.82
L3	ct	ols	$\tilde{y}$	51.54	48.48	45.06	45.17	39.41	42.39	28.50	26.73
L3	ite (cst)	ols-struc	$\tilde{y}$	49.32	46.47	43.80	43.45	38.38	40.64	27.92	26.14
L3	ite (cst)	struc-ols	$\tilde{y}$	48.20	45.50	43.45	42.51	37.83	39.64	27.28	25.46
L3	ct	struc	$\tilde{y}$	46.74	44.02	<b>42.05</b>	<b>41.07</b>	<b>36.67</b>	<b>38.16</b>	<b>26.56</b>	<b>24.73</b>
L3	ite (tcs)	ols2-ols	$\tilde{y}$	48.52	46.20	45.02	45.58	42.49	50.21	31.50	26.71
L3	ite (tcs)	struc2-ols	$\tilde{y}$	47.77	45.52	44.64	44.96	42.07	49.67	31.19	26.36
L3	ite (tcs)	ols2-struc	$\tilde{y}$	47.76	45.19	43.49	43.72	39.27	43.19	29.26	25.98
L3	ite (tcs)	struc2-struc	$\tilde{y}$	<b>46.25</b>	<b>43.69</b>	42.24	42.12	38.05	41.53	28.38	25.03



**Figure A.2:** Comparison of  $nRMSE(\%)$  between  $t\text{-struc}_2 + cs\text{-struc}$  and  $ct\text{-struc}$  (top panel), and the not negative version (bottom panel). The black line represents the bisector, where the  $nRMSE$ 's for both approaches are equal.





**Figure A.3:** MCB-Nemenyi test on Yang cross-temporal reconciliation approaches with operating day forecast horizon.  $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2$  levels (324 series). Top panel: hourly forecasts; Bottom panel: daily forecasts. The mean rank of each approach is displayed to the right of their names. If the intervals of two forecast reconciliation approaches do not overlap, this indicates a statistically different performance. Thus, approaches that do not overlap with the green interval are considered significantly worse than the best, and vice-versa.