Image segmentation using SCIP

Advanced practical SS 2017

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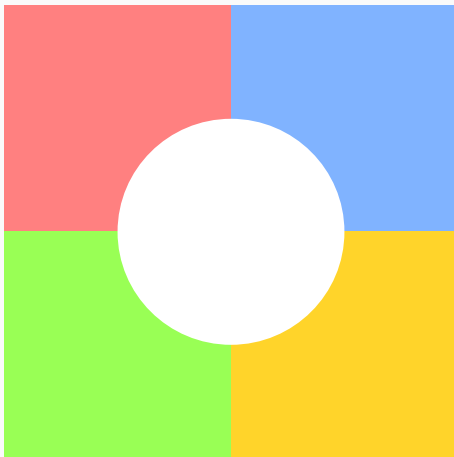August 29, 2017

Content

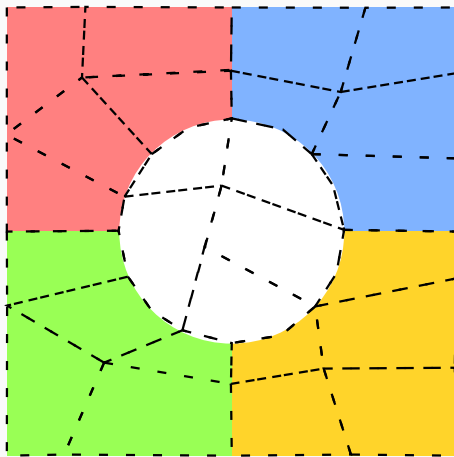# Problem definition

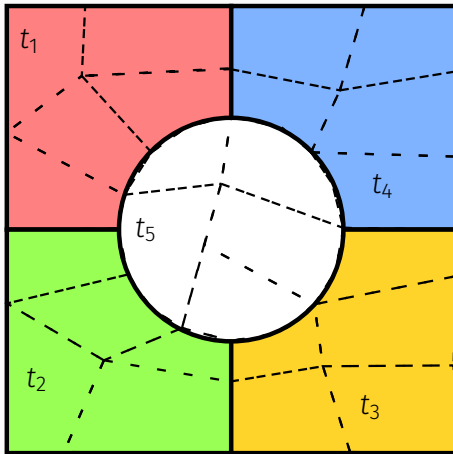**Figure 1:** Input image

Figure 2: Generated superpixels

Figure 3: Master nodes and segments

Master problem

Given data in the theoretical formulation of the master problem:

- $\mathcal{S}$: set of superpixels
- $\mathcal{P} \subset 2^{\mathcal{S}}$: set of segments
- $y_s \geq 0$: color of superpixel $s \in \mathcal{S}$
- $T \subseteq \mathcal{S}$: set of master nodes
- $k = |T|$: number of segments to cover the image with
- $r_P = \sum_{s \in P} |y_t - y_s|$: error of segment $P \in \mathcal{P}$, where $t \in T$ is the single master node for which $t \in P$

Master problem

There is a binary variable x_P for every segment $P \in \mathcal{P}$.

The problem reads as follows:

$$\min \quad \sum_{P \in \mathcal{P}} r_P \cdot x_P \tag{1}$$

$$\text{s.t.} \quad \sum_{\{P \in \mathcal{P} : s \in P\}} x_P = 1 \quad \forall s \in \mathcal{S} \tag{2}$$

$$\sum_{P \in \mathcal{P}} x_P = k \tag{3}$$

$$x_P \in \{0, 1\} \quad \forall P \in \mathcal{P} \tag{4}$$

Dual problem

The dual variables are

- μ_s for all $s \in \mathcal{S}$: corresponding to (2)
- λ: corresponding to (3)

The dual problem reads as follows:

$$\max \quad \sum_{s \in \mathcal{S}} \mu_s + k \cdot \lambda \tag{5}$$

$$\text{s.t.} \quad \sum_{s \in P} \mu_s + \lambda \leq r_P \quad \forall P \in \mathcal{P} \tag{6}$$

$$\mu_s \text{ free} \quad \forall s \in \mathcal{S} \tag{7}$$

$$\lambda \text{ free} \tag{8}$$

Pricing problem

There is a pricing problem for each master node $t \in T$, which generates a segment containing t.

If $s \in \mathcal{S}$ is in the new segment, then $x_s = 1$.

$$\min \quad \underbrace{\sum_{s \in \mathcal{S}} x_s \cdot |y_t - y_s|}_{=r_{\{s \in \mathcal{S} \,:\, x_s = 1\}}} - \sum_{s \in \mathcal{S}} x_s \cdot \mu_s \tag{9}$$

$$\text{s.t.} \quad \text{connectivity constraint} \tag{10}$$

$$x_t = 1 \tag{11}$$

$$x_{t'} = 0 \quad \forall t' \in T \setminus \{t\} \tag{12}$$

$$x_s \in \{0, 1\} \quad \forall s \in \mathcal{S} \tag{13}$$

Constraint (6) of the dual problem is violated by the new segment if and only if

$$\underbrace{\sum_{s \in \mathcal{S}} x_s \cdot |y_t - y_s|}_{=r_{\{s \in \mathcal{S} \,:\, x_s=1\}}} - \sum_{s \in \mathcal{S}} x_s \cdot \mu_s < \lambda.$$

Therefore, a new segment has to satisfy this inequality in order to be added to the master problem.

Cutting planes

We look at the subgraph with vertices $\{s \in \mathcal{S} : x_s = 1\}$. If a component C is not connected to t, we add a cut for each $s \in C$:
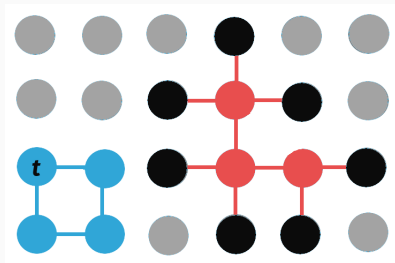
$$\sum_{s' \in \delta(C)} x_{s'} \geq x_s$$

Figure 4: Violated connectivity constraint

About SCIP

From `scip.zib.de`:

> *"SCIP is currently one of the fastest non-commercial solvers for mixed integer programming (MIP) and mixed integer nonlinear programming (MINLP). It is also a framework for constraint integer programming and branch-cut-and-price."*

What is SCIP?

SCIP is developed at the Zuse Institute Berlin (ZIB). It is written in C, but can also be interfaced using

- C++
- Python
- Java

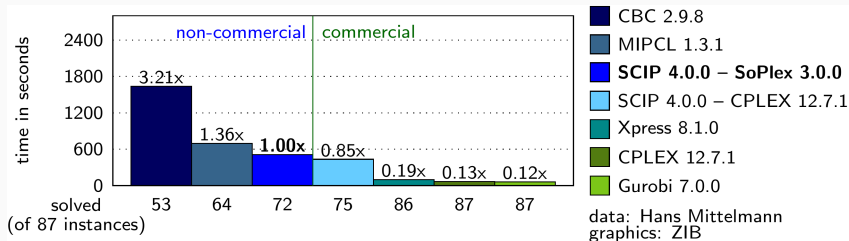SCIP can use different LP solvers, e.g.

- SoPlex
- CPLEX

Figure 5: MIP solver benchmark

Advantages

SCIP is non-commercial and open source.

SCIP supports cutomizing

- Constraint handlers
- Separators
- Variable pricers
- Branching rules

These use callbacks, so there is no need for a custom control loop.

Also, SCIP can automatically do branching on variables declared as integral.

Implementation details

We are using C++. This makes it easy to interface with SCIP and add custom plugins. There are multiple SCIP examples written in C++, notably

- **VRP**, involving a custom pricer
- **TSP**, involving a custom constraint handler

We started off of these and implemented the desired functionality.

The image is first read from a PNG file using libpng and png++.
Then, the image is segmented into superpixels using an
algorithm called SLIC.

```
png::image<png::gray_pixel> pngimage(filename);
float* image = new float[pngimage.get_width() * pngimage.get_height()];
for (png::uint_32 x = 0; x < pngimage.get_width(); ++x)
{
  for (png::uint_32 y = 0; y < pngimage.get_height(); ++y)
  {
    image[x + y * pngimage.get_width()] = pngimage[y][x] / 255.0;
  }
}

segmentation = new vl_uint32[pngimage.get_width() * pngimage.get_height()];
vl_slic_segment(segmentation, image, pngimage.get_width(), pngimage.get_height(), ...);
```

Creating the graph

To represent the graph of superpixels, we use the Boost Graph
Library.

```
Graph g(superpixelcount);
for (auto p = vertices(g); p.first != p.second; ++p.first)
{
  g[*p.first].color = avgcolor[*p.first];
}
// add edges
for (png::uint_32 x = 0; x < width; ++x)
{
  for (png::uint_32 y = 0; y < height; ++y)
  {
    auto current = x + y * width;
    auto right = x + 1 + y * width;
    auto below = x + (y + 1) * width;
    if (x + 1 < width
      && segmentation[current] != segmentation[right])
    {
      // add edge to the superpixel on the right
      auto edge = add_edge(segmentation[current], segmentation[right], g);
      auto weight = boost::get(boost::edge_weight, g, edge.first);
      boost::put(boost::edge_weight, g, edge.first, weight + 1);
    }
    ...
  }
}
```

First, we need to create a SCIP instance.

```
SCIP* scip;
SCIP_CALL(SCIPcreate(&scip));
SCIP_CALL(SCIPcreateProb(scip, "master_problem", ...));
SCIP_CALL(SCIPsetObjsense(scip, SCIP_OBJSENSE_MINIMIZE));
```

We start with initial segments which form a partitioning.

```
std::vector<SCIP_VAR*> vars;
for (auto segment : initial_segments)
{
  SCIP_VAR* var;
  // Set a very high objective value for the initial segments
  // so that they aren't selected in the final solution
  SCIP_CALL(SCIPcreateVar(scip, &var, "x_P", 0.0, 1.0, 10000, SCIP_VARTYPE_BINARY, ...));
  SCIP_CALL(SCIPaddVar(scip, var));
  vars.push_back(var);
}
```

Adding constraints

Finally, we add the partitioning constraints.

```
std::vector<SCIP_CONS*> partitioning_cons;
for (auto p = vertices(g); p.first != p.second; ++p.first)
{
  SCIP_CONS* cons1;
  // 1.0 <= ... <= 1.0
  SCIP_CALL(SCIPcreateConsLinear(scip, &cons1, "first", 0, NULL, NULL, 1.0, 1.0, ...));
  for (size_t i = 0; i != initial_segments.size(); ++i)
  {
    if (initial_segments[i].find(*p.first) != initial_segments[i].end())
    {
      SCIP_CALL(SCIPaddCoefLinear(scip, cons1, vars[i], 1.0));
    }
  }
  SCIP_CALL(SCIPaddCons(scip, cons1));
  partitioning_cons.push_back(cons1);
}

SCIP_CONS* num_segments_cons;
...
```

Pricer callback

```
SCIP_DECL_PRICERREDCOST(scip_redcost)
{
  SCIP_Real lambda = SCIPgetDualsolLinear(scip, num_segments_cons);
  for (size_t i = 0; i < master_nodes.size(); ++i)
  {
    auto p = heuristic(scip, master_nodes[i], lambda); // returns pair<redcost, superpixels>
    if (SCIPisDualfeasNegative(scip, p.first))
      SCIP_CALL(addPartitionVar(scip, master_nodes[i], p.second));
    else
    {
      for (auto s = vertices(g); s.first != s.second; ++s.first)
      {
        SCIP_Real mu_s = SCIPgetDualsolLinear(scip, partitioning_cons[*s.first]);
        SCIP_CALL(SCIPchgVarObj(scip_pricers[i], x[*s.first],
          -mu_s + std::abs(g[master_nodes[i]].color - g[*s.first].color)));
      }
      SCIP_CALL(SCIPsolve(scip_pricers[i]));
      SCIP_SOL* sol = SCIPgetBestSol(scip_pricers[i]);
      if (SCIPisDualfeasNegative(scip, SCIPgetSolOrigObj(scip_pricers[i], sol) - lambda))
        SCIP_CALL(addPartitionVarFromPricer(scip, scip_pricers[i], sol, master_nodes[i]));
    }
  }
  *result = SCIP_SUCCESS; // at least one improving variable was found,
                          // or it is ensured that no such variable exists
  return SCIP_OKAY;
}
```

Pricing heuristic

To make the solving process faster, a heuristic to find segments with negative reduced costs (rc) was added.

Algorithm 1 Simple pricing heuristic

1: $P \leftarrow \{t\}$
2: Let $s \in \delta(P)$ s.t. $rc(P \cup \{s\})$ is minimal.
3: **if** $rc(P) \geq 0$ **then**
4:     $P \leftarrow P \cup \{s\}$
5:     **goto** 2
6: **else if** $rc(P \cup \{s\}) < rc(P)$ **then**
7:     $P \leftarrow P \cup \{s\}$
8:     **goto** 2
9: **else**
10:     **return** P
11: **end if**

First, we create the subgraph of all superpixels in the new component.

```
Graph& subgraph = g.create_subgraph();
std::vector<int> component(num_vertices(g));
for (auto p = vertices(g); p.first != p.second; ++p.first)
{
  if (SCIPisEQ(scip, SCIPgetSolVal(scip, sol, superpixel_vars[*p.first]), 1.0))
  {
    add_vertex(*p.first, subgraph);
  }
}
size_t num_components = connected_components(subgraph, &component[0]);
```

Constraint handler callback

```cpp
for (int i = 0; i < num_components; i++)
{
  if (i == component[master_node])
    continue;

  // all superpixels in the component
  std::vector<Graph::vertex_descriptor> superpixels = ...;
  // all superpixels surrounding the component
  std::vector<Graph::vertex_descriptor> surrounding = ...;

  for (auto s : superpixels)
  {
    // add a constraint/row to the problem
    SCIP_ROW* row;
    SCIP_CALL(SCIPcreateEmptyRowCons(scip, &row, conshdlr, "sepa_con",
      0.0, SCIPinfinity(scip), ...));

    // sum_{all superpixels s surrounding the component} x_s >= ...
    for (auto s_ : surrounding)
      SCIP_CALL(SCIPaddVarToRow(scip, row, superpixel_vars[s_], 1.0));

    // ... >= x_s
    SCIP_CALL(SCIPaddVarToRow(scip, row, superpixel_vars[s], -1.0));

    SCIP_CALL(SCIPflushRowExtensions(scip, row));
    SCIP_CALL(SCIPaddCut(scip, sol, row, ...));
  }
}
```

Solve master problem

```cpp
// include pricer
ObjPricerLinFit* pricer_ptr = new SegmentPricer(scip, g, ...);
SCIP_CALL(SCIPincludeObjPricer(scip, pricer_ptr, true));

// activate pricer
SCIP_CALL(SCIPactivatePricer(scip, SCIPfindPricer(scip, "pricer")));

// solve
SCIP_CALL(SCIPsolve(scip));
SCIP_SOL* sol = SCIPgetBestSol(scip);

// return selected segments
std::vector<std::vector<Graph::vertex_descriptor>> segments;
SCIP_VAR** variables = SCIPgetVars(scip);
for (int i = 0; i < SCIPgetNVars(scip); ++i)
{
  if (SCIPisEQ(scip, SCIPgetSolVal(scip, sol, variables[i]), 1.0))
  {
    auto vardata = (ObjVardataSegment*) SCIPgetObjVardata(scip, variables[i]);
    segments.push_back(vardata->getSuperpixels());
  }
}
```

Possible improvements

Pricer callback

The pricer callback could be improved in several ways:

- Do not generate *k* segments each time
- Begin with a certain master node, based on the last iteration
- Run all *k* heuristics before solving exactly
- Additional heuristic that generates larger segments

Doing this would need a lot of testing to choose the best options.

Ideally, one would develop branching rules specially for the master and pricing problem.

For example, branching on segments with minimal objective values could lead to good results.

This is out of scope for this practical.

Demo

Figure 6: Main Page

Figure 7: Function overview

Documentation

Member Function Documentation

size_t ConnectivityCons::findComponents (SCIP * scip,
SCIP_SOL * sol,
Graph & subgraph,
std::vector< int > & component
) `private`

Finds all connected components in a subgraph.

Returns
    the number of connected components

Parameters

scip	pricer SCIP instance
sol	current primal solution or NULL
subgraph	subgraph with all superpixels for which $x_s = 1$
component	component[s] will be the index of the connected component the superpixel s belongs to

SCIP_RETCODE ConnectivityCons::sepaConnectivity (SCIP * scip,
SCIP_CONSHDLR * conshdlr,
SCIP_SOL * sol,
SCIP_RESULT * result
) `private`

Adds cutting plane, if possible.

If the current solution is infeasible, a cutting plane of the following form is added for every superpixel s in a component C that is not connected to the master node t:

$$\sum_{s' \in \delta(C)} x_{s'} \geq x_s$$

Figure 8: Detailed description

https://daniiki.github.io/image-segmentation-scip

Demo

https://github.com/daniiki/image-segmentation-scip