Discussion of "The Cost of the Cold-Start Problem on Airbnb"

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Comments

1. Welfare losses from the cold-start problem

2. Rational inattention

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A very simple model

Consider the following demand system (for some $\gamma < 1$):

$$q_E = \frac{1}{\omega_E + \omega_I}(\omega_E - p_E + \gamma p_I) \leq 1,$$
$$q_I = \frac{1}{\omega_E + \omega_I}(\omega_I - p_I + \gamma p_E) \leq 1.$$

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- Many disadvantages: corner cases, interpretation as probabilities.
- But perhaps one advantage: simple closed-form solutions.

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The corresponding inverse demand system is

$$p_E = \frac{\omega_E + \omega_I}{1 - \gamma^2}\left(\frac{\omega_E + \gamma\omega_I}{\omega_E + \omega_I} - q_E - \gamma q_I\right) = \frac{\partial U}{\partial q_E},$$
$$p_I = \frac{\omega_I + \omega_E}{1 - \gamma^2}\left(\frac{\omega_I + \gamma\omega_E}{\omega_I + \omega_E} - q_I - \gamma q_E\right) = \frac{\partial U}{\partial q_I}.$$

Period-1 surplus

Cold-start case: Consumers 1 and 2 acting independently.

- Integrating inverse demand, we'll get period 1's total surplus:

$$U^{\text{cold}}(q) = \frac{\omega_I + \omega_E}{1-\gamma^2}\left(\frac{\omega_E + \gamma\omega_I}{\omega_E + \omega_I} q_E + \frac{\omega_I + \gamma\omega_E}{\omega_I + \omega_E} q_I - \frac{1}{2}q_E^2 - \frac{1}{2}q_I^2 - \gamma q_E q_I\right).$$

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"Warm"-start case: single consumer acting over two periods.

- A single consumer would internalize learning about entrant:

$$U^{\text{warm}}(q) = U^{\text{cold}}(q) + q_E u_2^E + q_I u_2^I.$$

Period 1-demand functions (for $\gamma \approx 1$)

Cold-start case: Consumers 1 and 2 acting independently.

$$
\begin{aligned}
q_E &= \frac{1}{\omega_E + \omega_I}(\omega_E - p_E + p_I), \\
q_I &= \frac{1}{\omega_E + \omega_I}(\omega_I - p_I + p_E).
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"Warm"-start case: single consumer acting over two periods.

$$q_E = \frac{1}{\omega_E + \omega_I}(\omega_E + \underbrace{[u_2^E - u_2^I]}_{=\Delta u_2} - p_E + p_I),$$
$$q_I = \frac{1}{\omega_E + \omega_I}(\omega_I - \underbrace{[u_2^E - u_2^I]}_{=\Delta u_2} - p_I + p_E).$$

Period 1-Bertrand equilibria (for $\gamma \approx 1$)

Cold-start case: Consumers 1 and 2 acting independently.

$$p_E^{\text{cold}} = \frac{1}{3}\left(2\big(\omega_E - \underbrace{[\pi_{E2}^{E} - \pi_{E2}^{I}]}_{=\Delta\pi_{E2}}\big) + \big(\omega_I + \underbrace{[\pi_{I2}^{E} - \pi_{I2}^{I}]}_{=\Delta\pi_{I2}}\big)\right),$$

$$p_I^{\text{cold}} = \frac{1}{3}\left(2\big(\omega_I + \underbrace{[\pi_{I2}^{E} - \pi_{I2}^{I}]}_{=\Delta\pi_{I2}}\big) + \big(\omega_E - \underbrace{[\pi_{I2}^{E} - \pi_{I2}^{E}]}_{=\Delta\pi_{E2}}\big)\right).$$

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"Warm"-start case: single consumer acting over two periods.

$$p_E^{\text{warm}} = \frac{1}{3}\left(2\left(\omega_E + \Delta u_2 - \Delta\pi_{E2}\right) + \left(\omega_I - \Delta u_2 + \Delta\pi_{I2}\right)\right) = p_E^{\text{cold}} + \frac{1}{3}\Delta u_2,$$

$$p_I^{\text{warm}} = \frac{1}{3}\left(2\left(\omega_I - \Delta u_2 + \Delta\pi_{I2}\right) + \left(\omega_E + \Delta u_2 - \Delta\pi_{E2}\right)\right) = p_I^{\text{cold}} - \frac{1}{3}\Delta u_2.$$

What happens to welfare?

In terms of welfare, there are three relevant cases:

1. Cold-start Bertrand ($\approx$ Lemma 1): $W(p_E^{cold}, p_I^{cold})$.
2. Warm-start Bertrand: $W(p_E^{warm}, p_I^{warm}) = W\left(p_E^{\text{cold}} + \frac{1}{3}\Delta u_2, p_I^{\text{cold}} - \frac{1}{3}\Delta u_2\right)$.
3. Warm welfare max: ($\approx$ Proposition 1) $W(p_E^{**}, p_I^{**}) = W(0, 0)$.

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3. Warm welfare max: (≈ Proposition 1) $W(p_E^{**}, p_I^{**}) = W(0, 0)$.

Cold-start problem: $W(p_E^{**}, p_I^{**}) > W(p_E^{cold}, p_I^{cold})$.

- I have a couple of questions here.

Questions

Why not $W(p_E^{warm}, p_I^{warm}) > W(p_E^{cold}, p_I^{cold})$**?**

- In words: welfare would $\uparrow$ if we had a single consumer.

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Is $W(p_E^{warm}, p_I^{warm}) > W(p_E^{cold}, p_I^{cold})$ **actually true?**

- There is a strategic response by the firms here.
- Theoretically and/or empirically (feasible in the full model?).

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Where do the welfare gains actually come from?

$$\text{WGain} = \underbrace{W(p_E^{CF}, p_I^{CF}) - W(p_E^{warm}, p_I^{warm})}_{\text{Market power}} + \underbrace{W(p_E^{warm}, p_I^{warm}) - W(p_E^{cold}, p_I^{cold})}_{\text{Lack of social learning}}.$$

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Rational inattention reformulation?

Discrete choice models have an RI foundation.

- Matějka and McKay (2015); Caplin et al. (2019); Fosgerau et al. (2020).

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In a setting with learning, an RI formulation makes a lot of sense.

- Buyers arrive, observe reviews, and engage in costly learning.
- Cost of learning = e.g. expected reduction in entropy.
- Similar logit structure of demand (not harder to estimate?).
- Different notion of welfare (more straightforward comparisons?).
- Other interesting trade-offs: own learning v. relying on reviews, ...
- Highlight the cold-start problem even more?