Bilateral Trade with Costly Information Acquisition

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June 11, 2024

Introduction

How to design mechanisms if participants can acquire information?

- Which objectives can be implemented?

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Bilateral trade problem with information acquisition.

- Principal proposes a trading mechanism,
- Buyer and Seller privately acquire payoff-relevant information,
- Information acquisition is costly and flexible,
- Information acquisition can be arbitrarily correlated across players.

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- Information acquisition is costly and flexible,
- Information acquisition can be arbitrarily correlated across players.

What do we do?

- Provide implementability conditions,
- Characterize info structures consistent with allocational efficiency.
  - Application: subsidy minimization for efficient trade.

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Endogenous information acquisition can help

- Bikhchandani (2010): FSE $\Rightarrow$ incentives to acquire info about others.
- Bikhchandani and Obara (2017): "inflexible" info $\Rightarrow$ FSE (not always).
  - "inflexible" $=$ finitely many conditionally independent signals.

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    - "inflexible" $=$ finitely many conditionally independent signals.

Flexible endogenous information acquisition addresses the challenge

- Also interesting in its own right.
- Growing literature on flexible info in fixed games.

Preview of results

Tractable characterization of implementabilty

- Finite dimensional system of equations and inequalities.

Information structures consistent with allocational efficiency

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Application: subsidy minimization for efficient trade

- Perfect correlation forces the designer to give up surplus.
  - Compensate for the cost of information acquisition $\Rightarrow$ no *gross* FSE.
  - Prevent further information acquisition $\Rightarrow$ no *net* FSE.

Model: setup

- Principal and two players: Buyer and Seller.
- Buyer and Seller can trade an indivisible good with quality $v \in V$.
  - V is finite,
  - Buyer's valuation is $u^b(v)$, Seller's valuation is $u^s(v)$
    $\Rightarrow$ Interdependent values (nests private and pure common values).
- Principal collects revenue from/subsidizes trade between the players.

Model: information

The true quality $v \in V$ is unknown to anyone at the beginning.

We need a model where players jointly determine info structrure:

$$
\begin{array}{c|cccc}
\text{State } v & s_1^s & s_2^s & \dots & s_J^s \\
\hline
s_1^b & \alpha_{11}(v) & \alpha_{12}(v) & \dots & \alpha_{1J}(v) \\
s_2^b & \alpha_{21}(v) & \alpha_{22}(v) & \dots & \alpha_{2J}(v) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
s_I^b & \alpha_{I1}(v) & \alpha_{I2}(v) & \dots & \alpha_{IJ}(v)
\end{array}
$$

$\Rightarrow$ **player's actions $=$ random variables.**

Model: information acquisition

Commonly known to everyone at the beginning:

- Probability space $(X, \mathcal{F}, \mathbb{P})$, where $X = [0, 1] \ni x$ and $\mathbb{P}$ is uniform.
- A random variable $\mathbf{V} : X \to V$, induces a common prior μ_0 on V.

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Players acquire info about $v \in V$ by choosing other random var's.

- Players have access to a countably infinite set of signal realizations.
- A signal of player $p \in \{b, s\}$ is a pair $\sigma^p = (S^p, \mathbf{S}^p)$, where
  - S^p is a finite non-empty subset of $\mathbb{N}$, $\mathbf{S}^p : X \to S^p$ is a random variable.

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$\left(\mathbf{V}, \sigma^b, \sigma^s\right)$ **induces a joint distribution α over $V \times S^b \times S^s$.**

- Any Bayes-plausible (i.e. $\text{marg}_V \alpha = \mu_0$) α can be induced.

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$\left(\mathbf{V}, \sigma^b, \sigma^s\right)$ induces a joint distribution α over $V \times S^b \times S^s$.

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Signals are costly; $C(\sigma^p)$ is posterior separable.

- **Today:** $C(\sigma^p)$ is proportional to reduction in entropy.

Model: timing

1. Nature draws $x \in X$ uniformly, but nobody observes it.
2. Principal designs a trading mechanism (M, q, t)
   - $M = M^b \times M^s$; M^p is the message space of player p.
   - $q = (q^b, q^s)$; $q^p : M \to [0,1]$ is the allocation function of player p.
   - $t = (t^b, t^s)$; $t^p : M \to \mathbb{R}$ is the payment function of player p.
3. Each player p privately chooses $\sigma^p = (S^p, \mathbf{S}^p)$.
4. Each player p privately observes $s^p = \mathbf{S}^p(x)$ and sends $m^p \in M^p$.
5. Allocations and payments are determined according to (q, t);
   State $v = \mathbf{V}(x)$ is realized.

Roadmap

1. Revelation principle
2. Implementability
3. Information structures consistent with efficient trade
4. Application: subsidy minimization for efficient trade
5. Concluding remarks

Roadmap

Revelation principle

Unlike in standard mechanism design, type space is endogenous.

- Players choose signals in response to principal's mechanism.
- Players' signal realizations become their "types".
- Principal selects equilibrium $\Rightarrow$ correctly anticipates players' choice of signals $\Rightarrow$ can ask about their signal realizations directly.

Revelation principle

Unlike in standard mechanism design, type space is endogenous.

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- Players' signal realizations become their "types".
- Principal selects equilibrium $\Rightarrow$ correctly anticipates players' choice of signals $\Rightarrow$ can ask about their signal realizations directly.

Revelation principle: it is w.l.o.g. to consider direct mechanisms.

- Players could report one of their signal realizations or abstain:

$$M^b = S^b \cup \left\{ m^b_\emptyset \right\}, \qquad M^s = S^s \cup \left\{ m^s_\emptyset \right\},$$

where S^b and S^s are endogenously determined.

Roadmap

Implementability lemma

Lemma (Implementability for the buyer)

(α, q, t) is implementable for the buyer iff there are multipliers $\lambda_j^b(v)$ for all $s_j^s \in S^s$ and $\phi_{ij}^b(v)$ for all $(s_i^b, s_j^s) \in S^b \times S^s$ and all $v \in V$:

$$(\mathrm{ST}^b) \qquad \underbrace{q_{ij}^b u^b(v) - t_{ij}^b}_{\frac{\partial U^b}{\partial \alpha_{ij}(v)}} - \underbrace{\log\left(\mu_i^b(v)\right)}_{\frac{\partial C^b}{\partial \alpha_{ij}(v)}} - \lambda_j^b(v) + \phi_{ij}^b(v) = 0,$$

$$(\mathrm{DF}^b) \qquad \phi_{ij}^b(v) \geq 0,$$

$$(\mathrm{CS}^b) \qquad \alpha_{ij}(v)\phi_{ij}^b(v) = 0,$$

$$(\mathrm{NA}^b) \qquad \sum_{v \in V} \exp\left(-\min_j\{\lambda_j^b(v)\}\right) \leq 1.$$

- Analogous conditions apply to the seller. Seller's implementability

Buyer's problem

Consider a candidate (α, q, t) with α induced by some (σ^b, σ^s).

- Does Buyer have a profitable deviation $\tilde{\sigma}^b$?
- $I + 1$ actions under $\sigma^b \Rightarrow \tilde{\sigma}^b$ with $\leq I + 1$ realizations are w.l.o.g.
- $(\tilde{\sigma}^b, \sigma^s)$ will induce an alternative information structure $\tilde{\alpha}$.
- Can rewrite the best deviation problem in terms of $\tilde{\alpha}$:

$$\mathcal{BD}^b(\alpha, q, t) = \underset{\tilde{\alpha}, \tilde{S}^b}{\operatorname{argmax}} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{v \in V} \tilde{\alpha}_{ij}(v) \big(q_{ij}^b u^b(v) - t_{ij}^b \big) - c^b(\tilde{\alpha}),$$

$$(1) \;\; \tilde{S}^b = S^b \cup \{s_\emptyset^b\}, \;\; \tilde{\alpha} \in \Delta\big(\tilde{S}^b \times S^s \times V\big);$$

$$(2) \;\; \operatorname{marg}_{S^s \times V} \tilde{\alpha} = \operatorname{marg}_{S^s \times V} \alpha.$$

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$$(1) \quad \tilde{S}^b = S^b \cup \{s^b_\emptyset\}, \quad \tilde{\alpha} \in \Delta\big(\tilde{S}^b \times S^s \times V\big);$$

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Implementability condition for the buyer: $(\alpha, S^b) \in \mathcal{BD}^b(\alpha, q, t)$.

Solution to Buyer's problem

We split Buyer's deviations into two classes:

- **Class 1:** induce different $\tilde{\alpha}$'s over the same signal realizations S^b.
- **Class 2:** augment S^b with $s^b_\emptyset$ with positive probability.

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Our approach:

1. Solve **Class 1**-problem, characterize solution in terms of λ, ϕ.
2. Show the following:

Lemma

If α solves **Class 1**-*problem, then (α, S^b) solves* **Class 2**-*problem iff*

$$(\mathrm{NA}^b) \qquad \sum_{v \in V} \exp\big(-\min_j\{\lambda^b_j(v)\}\big) \leq 1.$$

Class 1-problem

$$\alpha \in \underset{\tilde{\alpha}}{\text{argmax}} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{v \in V} \tilde{\alpha}_{ij}(v)\big(q_{ij}^b u^b(v) - t_{ij}^b\big) - c^b(\tilde{\alpha}),$$

$$(1) \quad \tilde{\alpha}_{ij}(v) \geq 0; \qquad \phi_{ij}^b(v)$$

$$(2) \quad \sum_{i=1}^{I} \tilde{\alpha}_{ij}(v) = \sum_{i=1}^{I} \alpha_{ij}(v). \qquad \lambda_j^b(v)$$

Convex optimization problem (concave objective + affine constraints)
$\Rightarrow$ KKT conditions are necessary and sufficient.

Class 2-lemma

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Proof sketch: illustrate the proof using a $2 \times 2 \times 2$ example.

State $\underline{v}$	s_1^s	s_2^s
s_1^b	$\underline{\alpha}_{11} - \epsilon \underline{\beta}_{11}$	$\underline{\alpha}_{12} - \epsilon \underline{\beta}_{12}$
s_2^b	$\underline{\alpha}_{21} - \epsilon \underline{\beta}_{21}$	$\underline{\alpha}_{22} - \epsilon \underline{\beta}_{22}$
$s_\emptyset^b$	$\epsilon \sum_{i=1}^2 \underline{\beta}_{i1}$	$\epsilon \sum_{i=1}^2 \underline{\beta}_{i2}$

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$s_\emptyset^b$	$\epsilon\sum_{i=1}^2 \overline{\beta}_{i1}$	$\epsilon\sum_{i=1}^2 \overline{\beta}_{i2}$

$G_\alpha(\epsilon\beta)$ **is the gain from deviation in direction** $\epsilon\beta$ **from** α**.**

$MG_\alpha(\beta) \equiv \lim_{\epsilon \to 0} \frac{1}{\epsilon} G_\alpha(\epsilon\beta)$ **is the marginal gain.**

Proof of Class 2 lemma

We prove the contrapositive statement:

- Suppose there is a deviation with a positive gain $G_\alpha(\beta) > 0$.
- Convexity of cost function $\Rightarrow \left[G_\alpha(\beta) > 0 \Rightarrow MG_\alpha(\beta) > 0\right]$.
- $MG_\alpha(\beta)$ can be computed in closed form:

$$MG_\alpha(\beta) = -\sum_{i,j,v} \overbrace{\beta_{ij}(v)}^{0 \text{ if } \alpha_{ij}(v)=0} \times \Big[\underbrace{q^b_{ij}u^b(v) - t^b_{ij} - \log\left(\mu^b_i(v)\right)}_{=\lambda^b_j(v) \text{ as long as } \alpha_{ij}(v)>0, \text{ by KKT}} \Big] + F(\beta)$$

$$= -\sum_{i,j,v} \beta_{ij}(v)\lambda^b_j(v) + F(\beta).$$

- Let $\beta^* = \operatorname{argmax}_\beta MG_\alpha(\beta; \lambda^b)$, then $MG_\alpha(\beta^*; \lambda^b) > 0 \Rightarrow \neg(\text{NA}^b)$

Roadmap

Information structures consistent with efficient trade

$V = \{\underline{v}, \overline{v}\}$ **and gains from trade at every quality level:** $u^b(v) > u^s(v)$.

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Order the posteriors wlog: $\overline{\mu}_1^p \geq \overline{\mu}_2^p \geq \ldots$ **for both** $p \in \{b, s\}$

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Order the posteriors wlog: $\overline{\mu}_1^p \geq \overline{\mu}_2^p \geq \dots$ **for both** $p \in \{b, s\}$

Proposition (Efficiency ⇒ Essentially perfect correlation)

If α is consistent with efficient trade, then α has the following form:

$$
\begin{array}{c|ccc|c|ccc}
\text{State } v & s_1^s & \dots & s_k^s & \dots & s_{I-\ell}^s & \dots & s_I^s \\
\hline
s_1^b & \alpha_{11} & \dots & \alpha_{1k} & \dots & 0 & \dots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
s_k^b & \alpha_{k1} & \dots & \alpha_{kk} & \dots & 0 & \dots & 0 \\
\hline
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\hline
s_{I-\ell}^b & 0 & \dots & 0 & \dots & \underline{\alpha}_{I-\ell,I-\ell} & \dots & \underline{\alpha}_{I-\ell,I} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
s_I^b & 0 & \dots & 0 & \dots & \underline{\alpha}_{I,I-\ell} & \dots & \underline{\alpha}_{II}
\end{array}
$$

and the posteriors within each block are equal to each other.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

Lemma (Sets of equal posteriors for the buyer)

For all $j \in S^s$ there exists $\mathcal{I}^*(j) \subseteq S^b$ s.t. $\underline{\alpha}_{ij} = 0$ for all $i > \max \mathcal{I}^*(j) \equiv \bar{i}^*(j)$, and $\overline{\alpha}_{ij} = 0$ for all $i < \min \mathcal{I}^*(j) \equiv \underline{i}^*(j)$. Moreover, for any $i, i' \in \mathcal{I}^*(j)$ we get $\underline{\mu}_i^b = \underline{\mu}_{i'}^b$ and $\overline{\mu}_i^b = \overline{\mu}_{i'}^b$.

$\underline{v}$	s_1^s	$\dots$	s_j^s	$\dots$	s_J^s
s_1^b			$\underline{\alpha}_{1j}$		
$\vdots$			$\vdots$		
$s_{\bar{i}^*(j)-1}^b$			$\underline{\alpha}_{\bar{i}^*(j)-1,j}$		
$s_{\underline{i}^*(j)}^b$					
$\vdots$					
$s_{\bar{i}^*(j)}^b$					
$s_{\bar{i}^*(j)+1}^b$			0		
$\vdots$			$\vdots$		
s_I^b			0		

$\overline{v}$	s_1^s	$\dots$	s_j^s	$\dots$	s_J^s
s_1^b			0		
$\vdots$			$\vdots$		
$s_{\bar{i}^*(j)-1}^b$			0		
$s_{\underline{i}^*(j)}^b$					
$\vdots$					
$s_{\bar{i}^*(j)}^b$					
$s_{\bar{i}^*(j)+1}^b$			$\overline{\alpha}_{\bar{i}^*(j)+1,j}$		
$\vdots$			$\vdots$		
s_I^b			$\overline{\alpha}_{Ij}$		

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

- Define $\bar{i}^*(j) \equiv \max\{i|\underline{\alpha}_{ij} > 0\}$, $\mathcal{I}^*(j) \equiv \{i|\overline{\mu}_i^b = \overline{\mu}_{\bar{i}^*(j)}^b\}$.
- Define $\underline{i}^*(j) \equiv \min \mathcal{I}^*(j)$.

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$\underline{v}$	s_1^s	$\dots$	s_j^s	$\dots$	s_J^s
s_1^b			$\underline{\alpha}_{1j}$		
$\vdots$			$\vdots$		
$s_{\bar{i}^*(j)-1}^b$			$\underline{\alpha}_{\bar{i}^*(j)-1,j}$		
$s_{\underline{i}^*(j)}^b$					
$\vdots$					
$s_{\bar{i}^*(j)}^b$					
$s_{\bar{i}^*(j)+1}^b$			0		
$\vdots$			$\vdots$		
s_I^b			0		

$\overline{v}$	s_1^s	$\dots$	s_j^s	$\dots$	s_J^s
s_1^b			???		
$\vdots$			$\vdots$		
$s_{\bar{i}^*(j)-1}^b$			???		
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$\underline{v}$	s_1^s	$\dots$	s_j^s	$\dots$	s_J^s
s_1^b			$\underline{\alpha}_{1j}$		
$\vdots$			$\vdots$		
$s_{\bar{i}^*(j)-1}^b$			$\underline{\alpha}_{\bar{i}^*(j)-1,j}$		
$s_{\underline{i}^*(j)}^b$					
$\vdots$					
$s_{\bar{i}^*(j)}^b$					
$s_{\bar{i}^*(j)+1}^b$			0		
$\vdots$			$\vdots$		
s_I^b			0		

$\overline{v}$	s_1^s	$\dots$	s_j^s	$\dots$	s_J^s
s_1^b			???		
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$s_{\bar{i}^*(j)+1}^b$			$\overline{\alpha}_{\bar{i}^*(j)+1,j}$		
$\vdots$			$\vdots$		
s_I^b			$\overline{\alpha}_{Ij}$		

- Suppose for a contradiction that $\exists i < \underline{i}^*(j)$ such that $\overline{\alpha}_{ij} > 0$.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

- Define $\bar{i}^*(j) \equiv \max\{i | \underline{\alpha}_{ij} > 0\}$, $\mathcal{I}^*(j) \equiv \{i | \overline{\mu}_i^b = \overline{\mu}^b_{\bar{i}^*(j)}\}$.
- Define $\underline{i}^*(j) \equiv \min \mathcal{I}^*(j)$.
- Suppose for a contradiction that $\exists i < \underline{i}^*(j)$ such that $\overline{\alpha}_{ij} > 0$.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

- Define $\bar{i}^*(j) \equiv \max\{i|\underline{\alpha}_{ij} > 0\}$, $\mathcal{I}^*(j) \equiv \{i|\overline{\mu}_i^b = \overline{\mu}_{\bar{i}^*(j)}^b\}$.
- Define $\underline{i}^*(j) \equiv \min \mathcal{I}^*(j)$.
- Suppose for a contradiction that $\exists i < \underline{i}^*(j)$ such that $\overline{\alpha}_{ij} > 0$.
- $\overline{\mu}_i^b > \overline{\mu}_{\bar{i}^*(j)}^b$ since $i > \min \mathcal{I}^*(j)$ and posteriors are the same in $\mathcal{I}^*(j)$.

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- Define $\bar{i}^*(j) \equiv \max\{i | \underline{\alpha}_{ij} > 0\}$, $\mathcal{I}^*(j) \equiv \{i | \overline{\mu}_i^b = \overline{\mu}_{\bar{i}^*(j)}^b\}$.
- Define $\underline{i}^*(j) \equiv \min \mathcal{I}^*(j)$.
- Suppose for a contradiction that $\exists i < \underline{i}^*(j)$ such that $\overline{\alpha}_{ij} > 0$.
- $\overline{\mu}_i^b > \overline{\mu}_{\bar{i}^*(j)}^b$ since $i > \min \mathcal{I}^*(j)$ and posteriors are the same in $\mathcal{I}^*(j)$.
- Stationarity (combined with CS and DF) is given by:

$$(\mathrm{ST}_{ij}^b) \quad \underline{u}^b - \log\left(\underline{\mu}_i^b\right) - \underline{\lambda}_j^b \le t_{ij}^b, \qquad (\mathrm{ST}_{\bar{i}^*(j),j}^b) \quad \underline{u}^b - \log\left(\underline{\mu}_{\bar{i}^*(j)}^b\right) - \underline{\lambda}_j^b = t_{\bar{i}^*(j),j}^b,$$

$$\overline{u}^b - \log\left(\overline{\mu}_i^b\right) - \overline{\lambda}_j^b = t_{ij}^b. \qquad \overline{u}^b - \log\left(\overline{\mu}_{\bar{i}^*(j)}^b\right) - \overline{\lambda}_j^b \le t_{\bar{i}^*(j),j}^b.$$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

- Define $\bar{i}^*(j) \equiv \max\{i | \underline{\alpha}_{ij} > 0\}$, $\mathcal{I}^*(j) \equiv \{i | \overline{\mu}_i^b = \overline{\mu}_{\bar{i}^*(j)}^b\}$.
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$$
\begin{array}{llll}
(\mathrm{ST}_{ij}^b) & \underline{u}^b - \log\left(\underline{\mu}_i^b\right) - \underline{\lambda}_j^b \leq t_{ij}^b, & (\mathrm{ST}_{\bar{i}^*(j),j}^b) & \underline{u}^b - \log\left(\underline{\mu}_{\bar{i}^*(j)}^b\right) - \underline{\lambda}_j^b = t_{\bar{i}^*(j),j}^b, \\
 & \overline{u}^b - \log\left(\overline{\mu}_i^b\right) - \overline{\lambda}_j^b = t_{ij}^b. & & \overline{u}^b - \log\left(\overline{\mu}_{\bar{i}^*(j)}^b\right) - \overline{\lambda}_j^b \leq t_{\bar{i}^*(j),j}^b.
\end{array}
$$

- $(\mathrm{ST}_{ij}^b) + (\mathrm{ST}_{\bar{i}^*(j),j}^b) \Rightarrow \overline{\mu}_i^b \leq \overline{\mu}_{\bar{i}^*(j)}^b$.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

Lemma (Sets of equal posteriors for the seller)

For all $i \in S^b$ there exists $\mathcal{J}^*(i) \subseteq S^s$ s.t. $\underline{\alpha}_{ij} = 0$ for all $j > \max \mathcal{J}^*(i) \equiv \bar{j}^*(i)$, and $\overline{\alpha}_{ij} = 0$ for all $j < \min \mathcal{J}^*(i) \equiv \underline{j}^*(i)$. Moreover, for any $j, j' \in \mathcal{J}^*(i)$ we get $\underline{\mu}^b_j = \underline{\mu}^b_{j'}$ and $\overline{\mu}^b_j = \overline{\mu}^b_{j'}$.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

Lemma (Sets of equal posteriors for the seller)

For all $i \in S^b$ *there exists* $\mathcal{J}^*(i) \subseteq S^s$ *s.t.* $\underline{\alpha}_{ij} = 0$ *for all* $j > \max \mathcal{J}^*(i) \equiv \bar{j}^*(i)$*, and* $\overline{\alpha}_{ij} = 0$ *for all* $j < \min \mathcal{J}^*(i) \equiv \underline{j}^*(i)$*. Moreover, for any* $j, j' \in \mathcal{J}^*(i)$ *we get* $\underline{\mu}^b_j = \underline{\mu}^b_{j'}$ *and* $\overline{\mu}^b_j = \overline{\mu}^b_{j'}$*.*

- Introduce $\hat{\mathcal{J}}_1 \equiv \{j | \overline{\mu}_j = \overline{\mu}^b_1\}$ and $\tilde{\mathcal{I}}_1 \equiv \{i | \mathcal{J}^*(i) = \hat{\mathcal{J}}_1\}$.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch

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For all $i \in S^b$ *there exists* $\mathcal{J}^*(i) \subseteq S^s$ *s.t.* $\underline{\alpha}_{ij} = 0$ *for all* $j > \max \mathcal{J}^*(i) \equiv \bar{j}^*(i)$*, and* $\overline{\alpha}_{ij} = 0$ *for all* $j < \min \mathcal{J}^*(i) \equiv \underline{j}^*(i)$*. Moreover, for any* $j, j' \in \mathcal{J}^*(i)$ *we get* $\underline{\mu}^b_j = \underline{\mu}^b_{j'}$ *and* $\overline{\mu}^b_j = \overline{\mu}^b_{j'}$*.*

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- Introduce $\hat{\mathcal{J}}_1 \equiv \{j | \overline{\mu}_j = \overline{\mu}^b_1\}$ and $\tilde{\mathcal{I}}_1 \equiv \{i | \mathcal{J}^*(i) = \hat{\mathcal{J}}_1\}$.
- Introduce $\hat{\mathcal{I}}_1 \equiv \{i | \overline{\mu}_i = \overline{\mu}^b_1\}$ and $\tilde{\mathcal{J}}_1 \equiv \{j | \mathcal{I}^*(j) = \hat{\mathcal{I}}_1\}$.

Lemma: $\hat{\mathcal{I}}_1 = \tilde{\mathcal{I}}_1$ **and** $\hat{\mathcal{J}}_1 = \tilde{\mathcal{J}}_1$.

Corollary

1. *For all $i \in \hat{\mathcal{I}}_1$ and $j \notin \hat{\mathcal{J}}_1$ we have $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$.*
2. *For all $i \notin \hat{\mathcal{I}}_1$ and $j \in \hat{\mathcal{J}}_1$ we have $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$.*

Roadmap

Subsidy minimization $\Rightarrow$ perfect correlation is w.l.o.g.

Corollary (Perfect correlation)

If $(\alpha', I', J'; t'; \phi', \lambda')$ is feasible in the subsidy minimization problem, then there is $(\alpha, I, J; t; \phi; \lambda)$, which is also feasible and achieves the same objective value, but $I = J$ and $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$ for $i \neq j$.

State $\underline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\underline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\underline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\underline{\alpha}_I$

State $\overline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\overline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\overline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\overline{\alpha}_I$

Proof: merge signal realizations with equal posteriors.

Subsidy minimization $\Rightarrow$ perfect correlation is w.l.o.g.

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If $(\alpha', I', J'; t'; \phi', \lambda')$ is feasible in the subsidy minimization problem, then there is $(\alpha, I, J; t; \phi; \lambda)$, which is also feasible and achieves the same objective value, but $I = J$ and $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$ for $i \neq j$.

State $\underline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\underline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\underline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\underline{\alpha}_I$

State $\overline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\overline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\overline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\overline{\alpha}_I$

Proof: merge signal realizations with equal posteriors.

Two design concerns for the principal: IC and total cost of info.

- **IC:** More correlated signals $\Rightarrow$ easier to incentivize truthful reporting.
- **Total cost:** Less correlated signals $\Rightarrow$ more info at lower cost.

Subsidy minimization $\Rightarrow$ perfect correlation is w.l.o.g.

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State $\underline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\underline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\underline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\underline{\alpha}_I$

State $\overline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\overline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\overline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\overline{\alpha}_I$

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Two design concerns for the principal: IC and total cost of info.

- **IC:** More correlated signals $\Rightarrow$ easier to incentivize truthful reporting.
- **Total cost:** Less correlated signals $\Rightarrow$ more info at lower cost.

IC overwhelmingly dominates $\Rightarrow$ pay for the same info twice!

Subsidy minimization as Bayesian persuasion

$$\max_{\{\tau,\mu;I;\Lambda\}} \sum_{i=1}^{I} \tau_i T(\underline{\mu}_i, \overline{\mu}_i; \Lambda^b, \Lambda^s)$$

$$\text{(BP)} \qquad \sum_{i=1}^{I} \tau_i \underline{\mu}_i = \underline{\mu}_0, \qquad \sum_{i=1}^{I} \tau_i \overline{\mu}_i = \overline{\mu}_0;$$

$$(\text{NA}^b) \qquad \exp\left(-\underline{\Lambda}^b\right) + \exp\left(-\overline{\Lambda}^b\right) = 1,$$

$$(\text{NA}^s) \qquad \exp\left(-\underline{\Lambda}^s\right) + \exp\left(-\overline{\Lambda}^s\right) = 1.$$

where $\tau_i = \underline{\alpha}_i + \overline{\alpha}_i$, and $\underline{\Lambda}^p = \min_i \left\{\underline{\lambda}_i^p\right\}$ and $\overline{\Lambda}^p = \min_i \left\{\overline{\lambda}_i^p\right\}$.

For a fixed Λ, this is a Bayesian persuasion problem
$\Rightarrow$ look at concave closure of T. Concave closure of T

Example: symmetric problem

Definition (Symmetric subsidy minimization problem)

A subsidy minimization problem is symmetric if the prior is uniform, i.e. $\underline{\mu}_0 = \overline{\mu}_0 = 0.5$, and $\overline{u}^b - \underline{u}^b = \overline{u}^s - \underline{u}^s \equiv \Delta u$.

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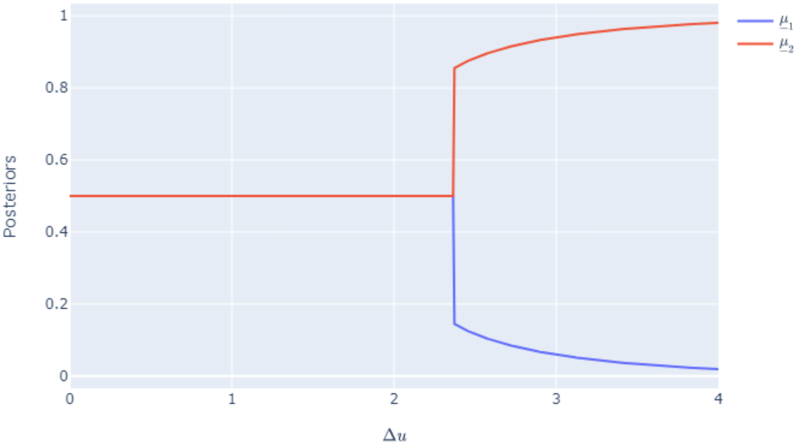
A subsidy minimization problem is symmetric if the prior is uniform, i.e. $\underline{\mu}_0 = \overline{\mu}_0 = 0.5$, and $\overline{u}^b - \underline{u}^b = \overline{u}^s - \underline{u}^s \equiv \Delta u$.

Symmetric solution:

$$1 - \underline{\mu}_1^* = \underline{\mu}_2^* = \begin{cases} \underline{\mu}_0 = 0.5 & \text{for } 0 < \Delta u \leq \Delta u^*, \\ \frac{3}{4} + \frac{1}{4}\sqrt{9 + 8\frac{\exp(\Delta u)}{1-\exp(\Delta u)}} & \text{for } \Delta u > \Delta u^*, \end{cases}$$

where Δu^* is obtained numerically.

Symmetric solution, illustration



Roadmap

Concluding remarks

- Bilateral trade problem with information acquisition.
- Information acquistion is costly and flexible.
- Tractable characterization of implementability.
- Characterization of info structures consistent with efficient trade.
- Solution to subsidy minimization problem.

Appendix

Implementability for the seller

Lemma (Implementability for the seller)

(α, q, t) is globally implementable for the seller iff there are multipliers $\lambda_i^s(v)$ for all $s_i^s \in S^s$ and $\phi_{ij}^s(v)$ for all $(s_i^b, s_j^s) \in S^b \times S^s$ and all $v \in V$:

$$\text{(ST}^s\text{)} \quad \underbrace{t_{ij}^s - q_{ij}^s u^s(v)}_{\frac{\partial U^s}{\partial \alpha_{ij}(v)}} - \underbrace{\log\left(\mu_j^s(v)\right)}_{\frac{\partial C^s}{\partial \alpha_{ij}(v)}} - \lambda_i^s(v) + \phi_{ij}^s(v) = 0,$$

$$\text{(DF)} \quad \phi_{ij}^s(v) \geq 0,$$

$$\text{(CS)} \quad \alpha_{ij}(v)\phi_{ij}^s(v) = 0,$$

$$\text{(NA)} \quad \sum_{v \in V} \exp\left(-\min_i \{\lambda_i^s(v)\}\right) \leq 1.$$

- Analogous conditions apply to the buyer. Buyer's implementability

Appendix

Proof of Class 2 lemma, continued

- $MG_\alpha(\beta)$ can be computed in closed form:

$$MG_\alpha(\beta) = -\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{v\in\{\underline{v},\overline{v}\}} \overbrace{\beta_{ij}(v)}^{0 \text{ if } \alpha_{ij}(v)=0} \times \Big[\underbrace{q_{ij}^{b} u^{b}(v) - t_{ij}^{b} - \log\left(\mu_i^{b}(v)\right)}_{=\textcolor{blue}{\lambda_j^{b}(v)} \text{ as long as } \alpha_{ij}(v)>0, \text{ by KKT}} \Big]$$

$$- \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right],$$

where $\underline{B} \equiv \sum_{i=1}^{2}\sum_{j=1}^{2}\underline{\beta}_{ij}$ and $\overline{B} \equiv \sum_{i=1}^{2}\sum_{j=1}^{2}\overline{\beta}_{ij}$.

Proof of Class 2 lemma, continued

$$MG_\alpha(\beta) = -\sum_{i=1}^{2}\sum_{j=1}^{2}\left[\underline{\beta}_{ij}\underline{\lambda}_j^b + \overline{\beta}_{ij}\overline{\lambda}_j^b\right] - \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right].$$

Proof of Class 2 lemma, continued

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- If $MG_\alpha(\beta) > 0$, then a better direction is also strictly profitable:

$$-\underline{B}\min_j\{\underline{\lambda}_j^b\} - \overline{B}\min_j\{\overline{\lambda}_j^b\} - \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right] > 0.$$

Proof of Class 2 lemma, continued

$$MG_\alpha(\beta) = -\sum_{i=1}^{2}\sum_{j=1}^{2}\left[\underline{\beta}_{ij}\underline{\lambda}_j^b + \overline{\beta}_{ij}\overline{\lambda}_j^b\right] - \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right].$$

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$$-\underline{B}\min_j\{\underline{\lambda}_j^b\} - \overline{B}\min_j\{\overline{\lambda}_j^b\} - \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right] > 0.$$

- Divide through by $\underline{B} + \overline{B}$ and define $P \equiv \frac{\underline{B}}{\underline{B}+\overline{B}}$ to get:

$$-P\min_j\{\underline{\lambda}_j^b\} - (1-P)\min_j\{\overline{\lambda}_j^b\} - P\log(P) - (1-P)\log(1-P) > 0.$$

Proof of Class 2 lemma, continued

$$MG_\alpha(\beta) = -\sum_{i=1}^{2}\sum_{j=1}^{2}\left[\underline{\beta}_{ij}\underline{\lambda}_j^b + \overline{\beta}_{ij}\overline{\lambda}_j^b\right] - \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right].$$

- If $MG_\alpha(\beta) > 0$, then a better direction is also strictly profitable:

$$-\underline{B}\min_j\{\underline{\lambda}_j^b\} - \overline{B}\min_j\{\overline{\lambda}_j^b\} - \left[\underline{B}\log\left(\frac{\underline{B}}{\underline{B}+\underline{B}}\right) + \overline{B}\log\left(\frac{\overline{B}}{\underline{B}+\underline{B}}\right)\right] > 0.$$

- Divide through by $\underline{B} + \overline{B}$ and define $P \equiv \frac{\underline{B}}{\underline{B}+\overline{B}}$ to get:

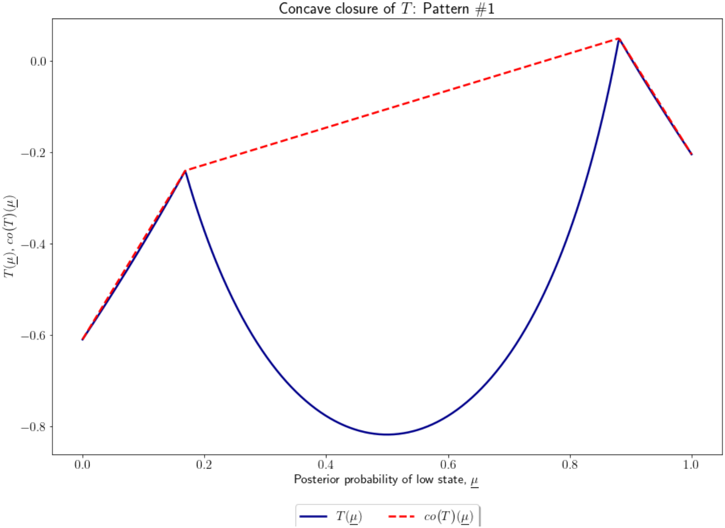
$$-P\min_j\{\underline{\lambda}_j^b\} - (1-P)\min_j\{\overline{\lambda}_j^b\} - P\log(P) - (1-P)\log(1-P) > 0.$$

- Maximizing over P, can find the best direction, profitable if and only if

$$\exp\left(-\min_j\{\underline{\lambda}_j^b\}\right) + \exp\left(-\min_j\{\overline{\lambda}_j^b\}\right) > 1.$$

Appendix

Concave closure of $T(\underline{\mu}, 1-\underline{\mu}; \Lambda^b, \Lambda^s)$



Optimality conditions

Proposition (Optimality conditions)

If the subsidy minimization problem achieves a minimum, then we can set $I = 2$ w.l.o.g., and moreover the optimal posteriors satisfy

$$(\mathrm{Opt}^b) \quad \underline{u}^b - \log(\underline{\mu}_1) - \underline{\Lambda}^b = \overline{u}^b - \log(\overline{\mu}_1) - \overline{\Lambda}^b,$$

$$(\mathrm{Opt}^s) \quad \underline{u}^s + \log(\underline{\mu}_2) + \underline{\Lambda}^s = \overline{u}^s + \log(\overline{\mu}_2) + \overline{\Lambda}^s.$$

Combine (Opt) **with** (NA) **to solve for** Λ **and plug into the objective** $\Rightarrow$ **unconstrained problem for posteriors**.

Go back