

DE Homework Daniil Vaino

1. Exact Solution of Differential Equation:

Given IVP:

$$y' = e^{2x} + e^x + y^2 - 2ye^x$$

Solution:



Ricatti:

$$y' + r(x)y + p(x)y^2 = q(x)$$

$$y = y_1 + u$$

y_1 - particular solution, let $y_1 = de^x$

$$y_1 = e^x$$

$$\text{let } z = y - e^x \quad y = z + e^x$$

$$z' = y' - e^x \quad y' = z' + e^x$$

$$y_1' = ae^x$$

$$ae^x = e^{2x} + e^x + a^2e^{2x} - 2e^{2x}$$

$$(ae^x - e^x) + (1 - a^2)e^{2x} = 0$$

$$e^x(a - 1) + (1 - a^2)e^{2x} = 0$$

$$a=1$$



$$z' + e^x = e^x + e^{2x} + e^x + z^2 + 2ze^x + e^{2x} - 2(z + e^x) e^x$$

$$z' = 2e^{2x} + z^2 + 2ze^x - 2ze^x - 2ze^{2x}$$

$$z' = z^2 \quad z = 0 \Rightarrow \text{trivial set}$$

$$\frac{dz}{z^2} = dx \quad \frac{1}{z} = -x \Rightarrow z = \frac{-1}{x} + C$$

$$y = e^x - \frac{1}{x + C} \quad \begin{matrix} y_0 = 0 \\ x_0 = 0 \end{matrix} \Rightarrow C = 1$$

After substitution:

$$y = e^x - \frac{1}{x + 1}$$

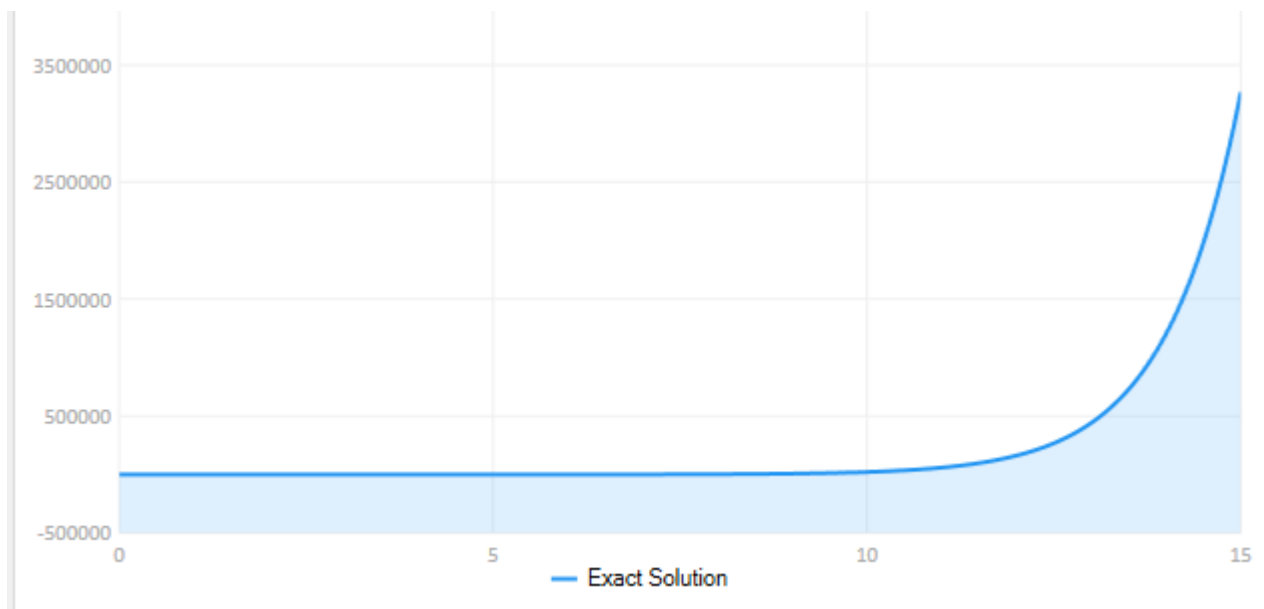
Analysis of discontinuities:

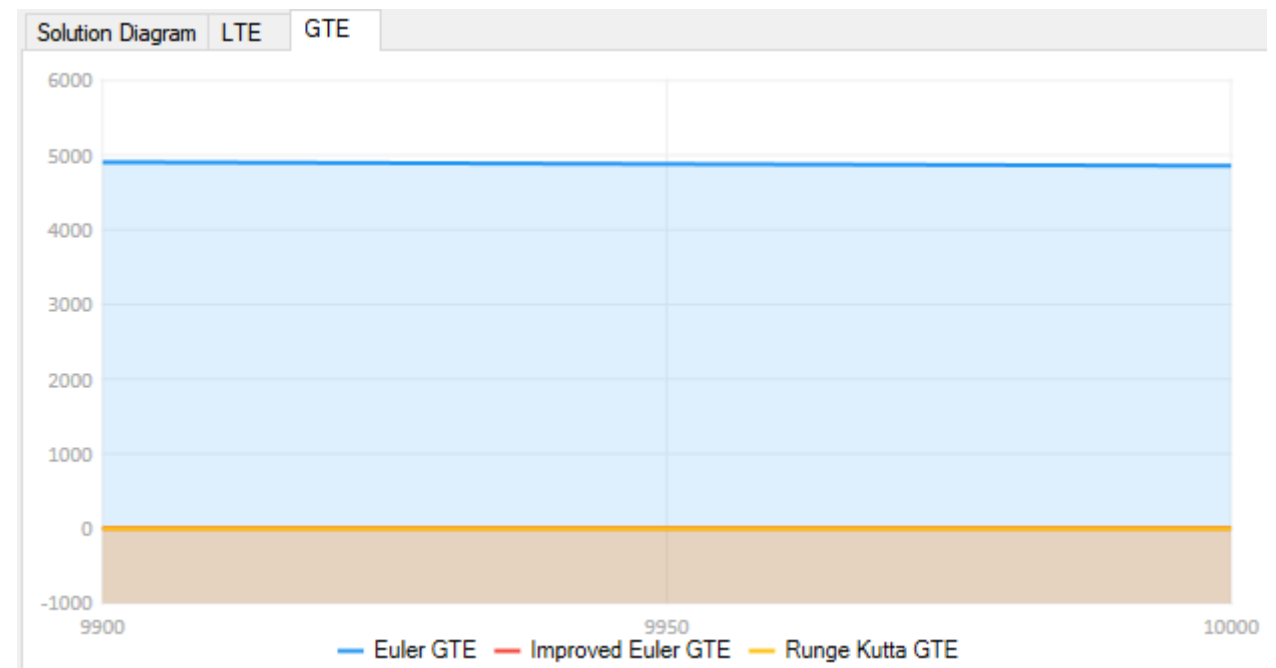
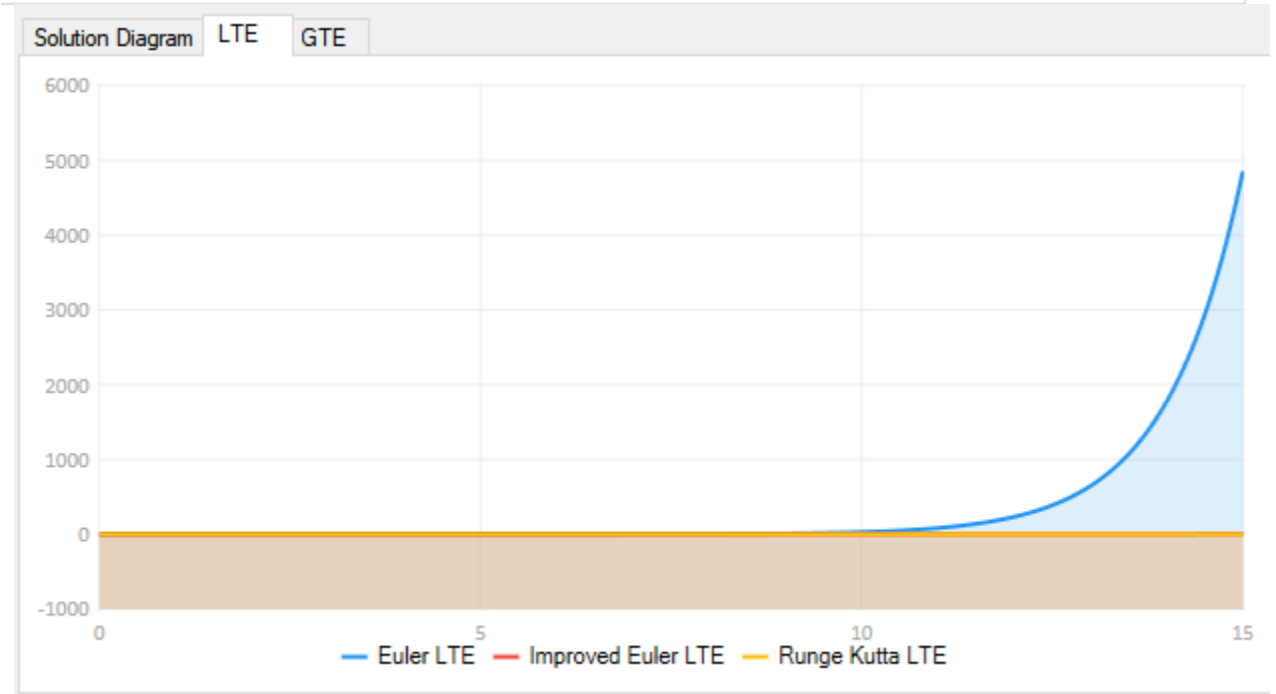
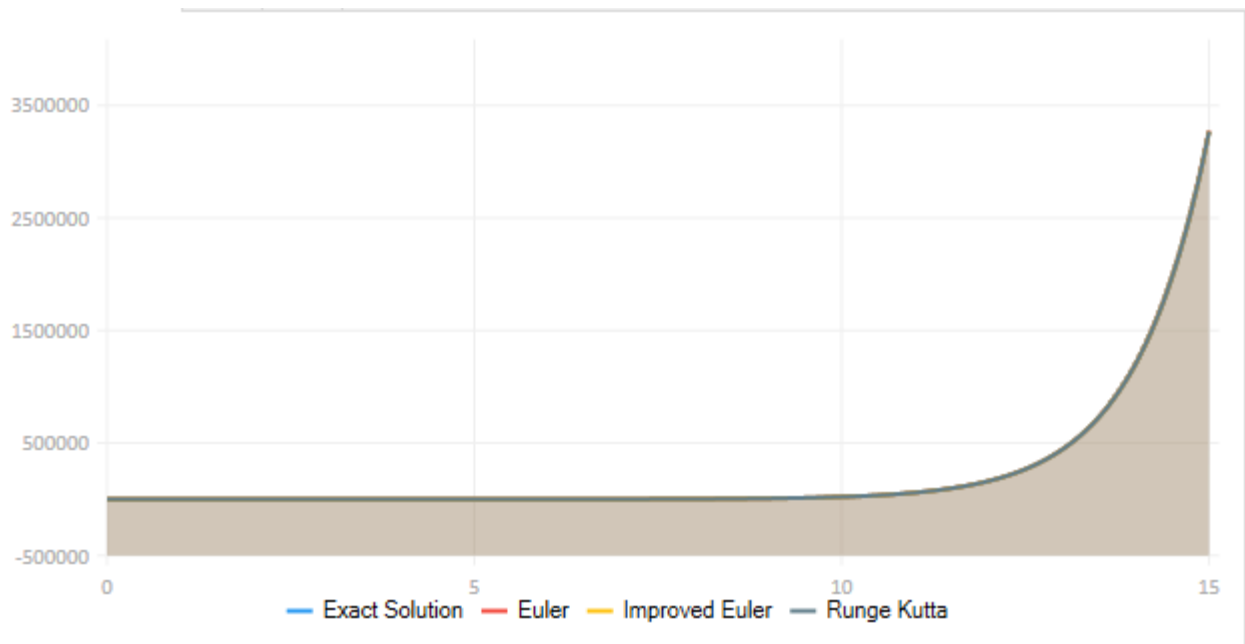
x, x_0 cannot be equal to zero

Solution of the given IVP in point $X = 15$

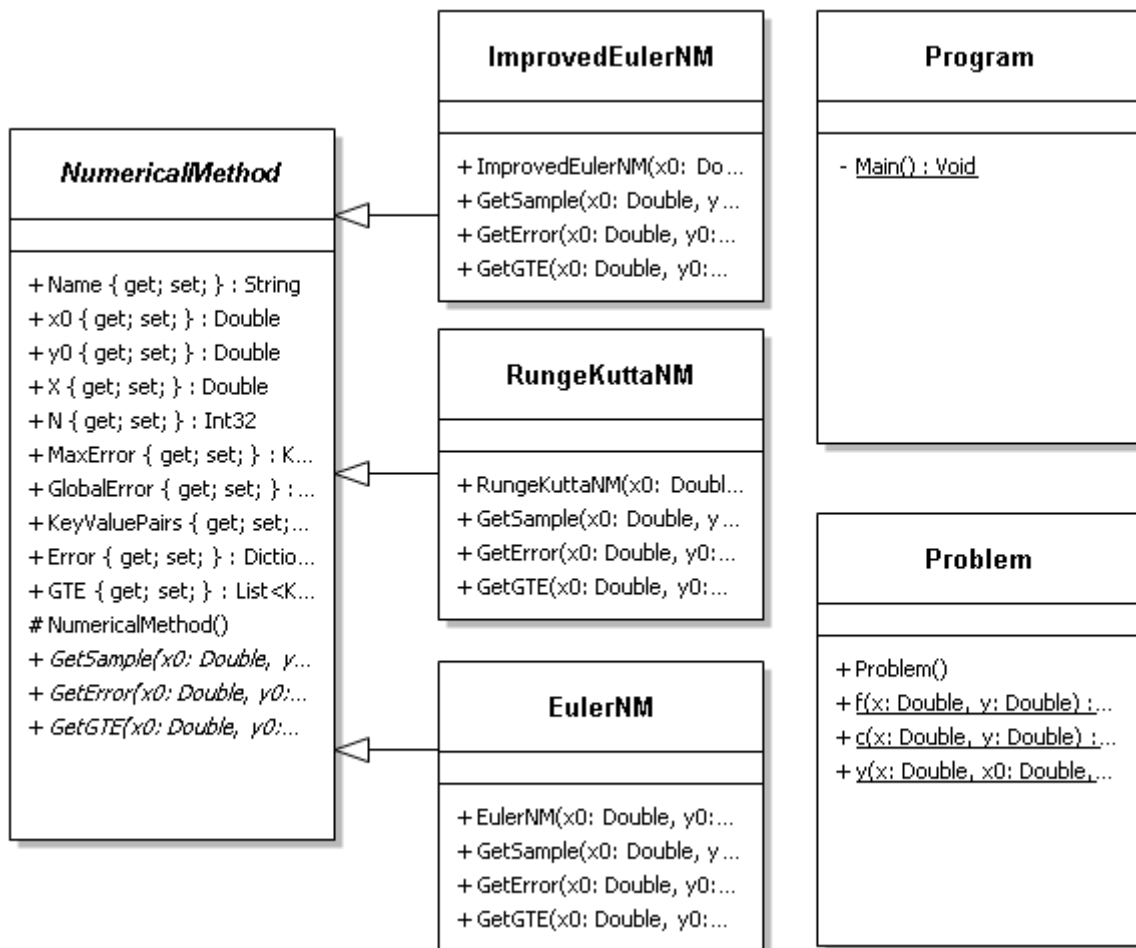
$$y = e^x - \frac{1}{15+1} \approx 3269017.30997$$

2. Visualization of Numerical Methods





3. UML Diagram



Uml diagram

(without automatically generated Windows Forms code)

4. Code comments

- 1) I've implemented **Strategy** pattern for numerical method calculations. Each of specific numerical methods (**EulerNM**, **RungeKuttaNM** and **ImprovedEulerNM**) is a class with methods "**GetSample**", "**GetError**" and "**GetGTE**", derived from abstract class "**NumericalMethod**".
- 2) Class "**Problem**" is used to calculate exact solution, find constant C.
- 3) Class "**Program**" manages the application.
- 4) Class "**CP**" (not show on UML) has automatically generated code that manages Windows Forms UI.