DE Homework Daniil Vaino

1. Exact Solution of Differential Equation:

Given IVP:

$$y' = e^{2x} + e^x + y^2 - 2ye^x$$

Solution:

Ricatti:

$$y' + r(x)y + p(x)y^2 = q(x)$$

$$y = y_1 + u$$

y_1 - particular solution, let $y_1 = de^x$

$$y_1 = e^x$$

let $\quad z = y - e^x \qquad y = z + e^x$

$$z' = y' - e^x \qquad y' = z' + e^x$$

$$y_1' = ae^x$$

$$ae^x = e^{2x} + e^x + a^2e^{2x} - 2e^{2x}$$

$$(ae^x - e^x) + (1 - a^2)e^{2x} = 0$$

$$e^x(a - 1) + (1 - a^2)e^{2x} = 0$$

a=1

$$z' + e^x = e^x + e^{2x} + e^x + z^2 + 2ze^x + e^{2x} - 2(z + e^x)\, e^x$$

$$z' = 2e^{2x} + z^2 + 2ze^x - 2ze^x - 2ze^{2x}$$

$$z' = z^2 \qquad z = 0 \Rightarrow trivial\ set$$

$$\frac{dz}{z^2} = dx \qquad \frac{1}{z} = -x \ \Rightarrow z = \frac{-1}{x} + C$$

$$y = e^x - \frac{1}{x + C} \qquad \begin{matrix} y_0 = 0 \\ x_0 = 0 \end{matrix} \ \Rightarrow C = 1$$

After substitution:

$$y = e^x - \frac{1}{x + 1}$$

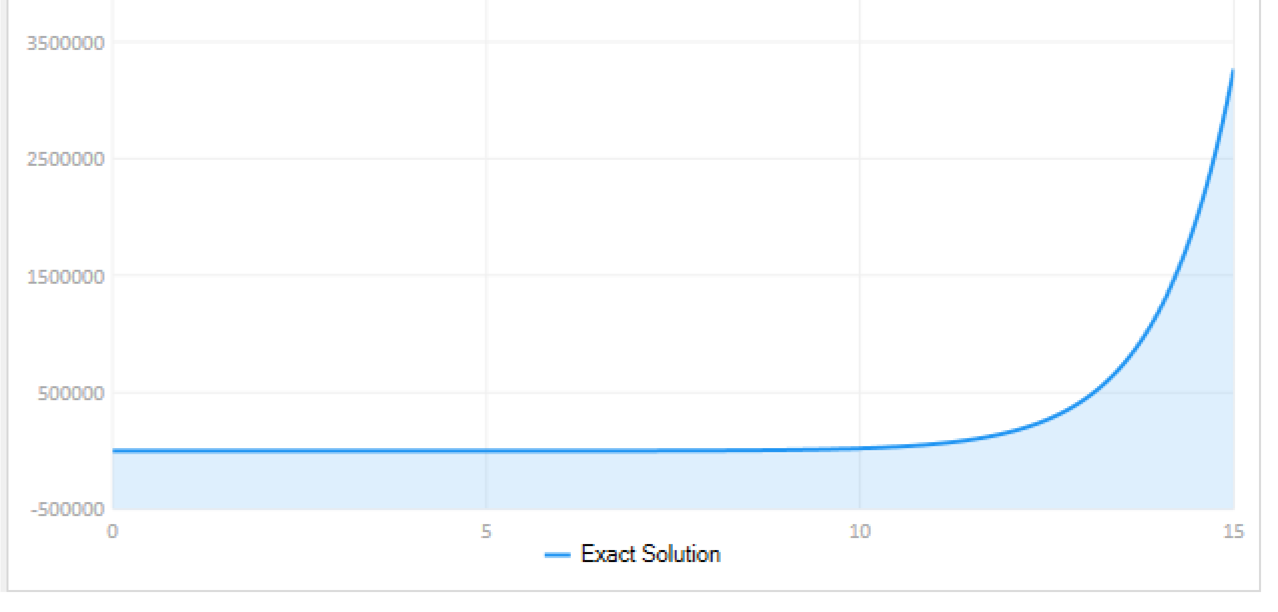
Analysis of discontinuities:

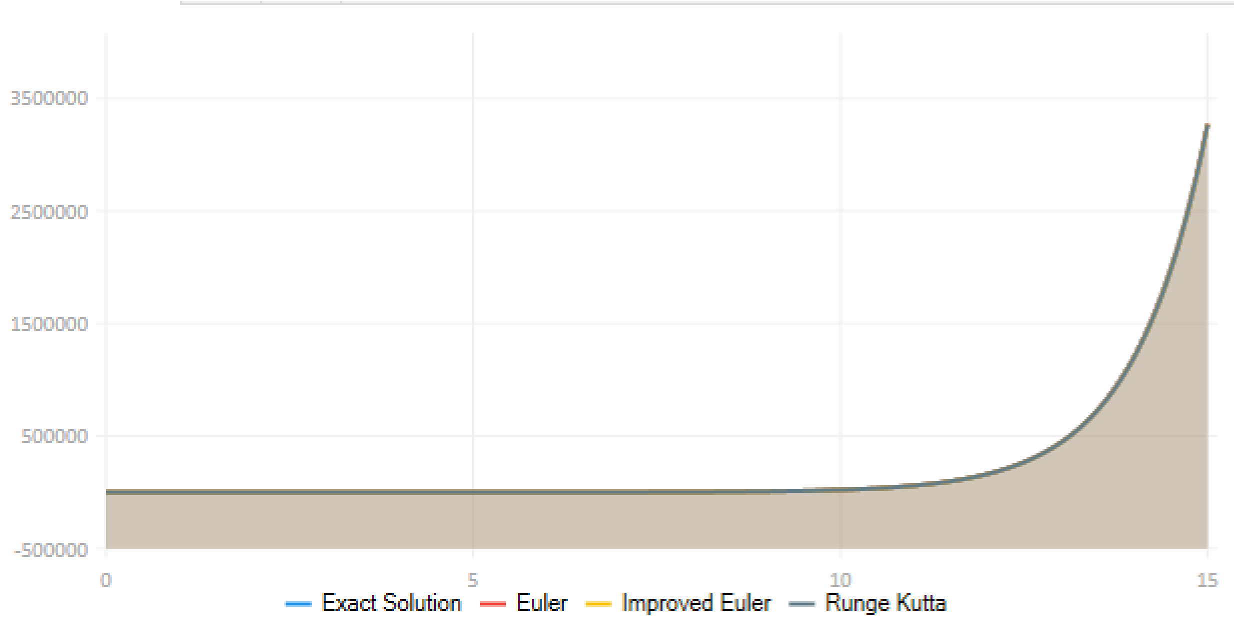
x, x_0 cannot be equal to zero

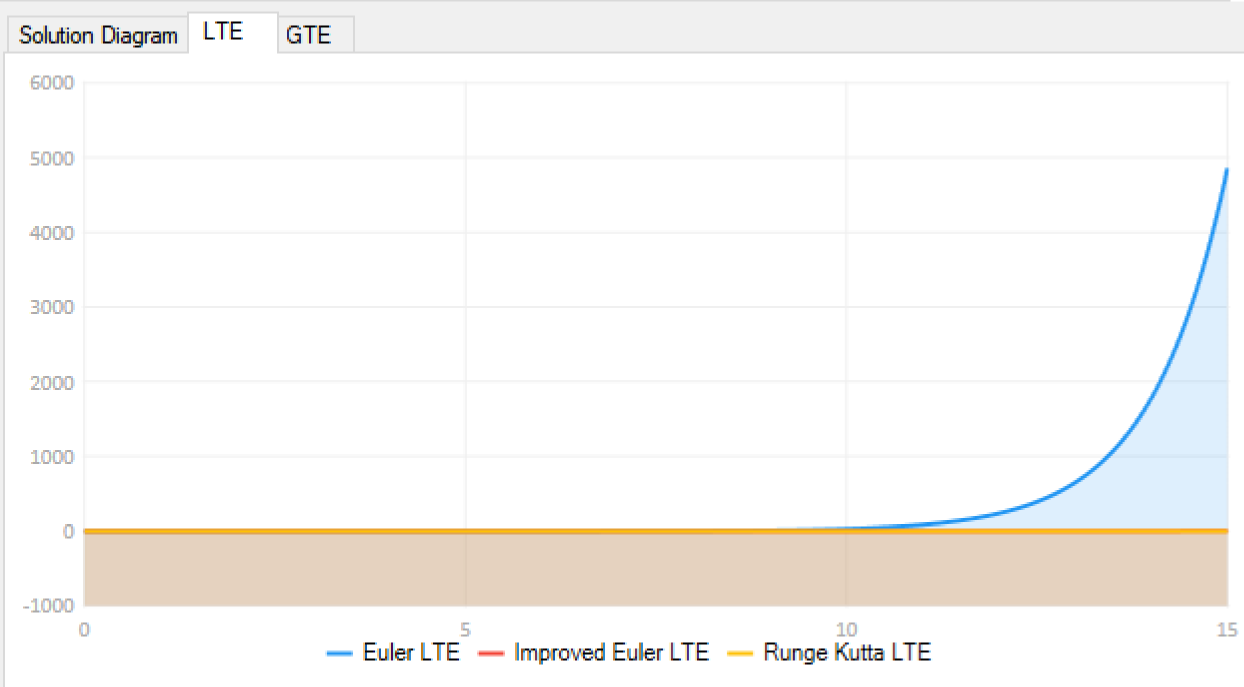
Solution of the given IVP in point X = 15

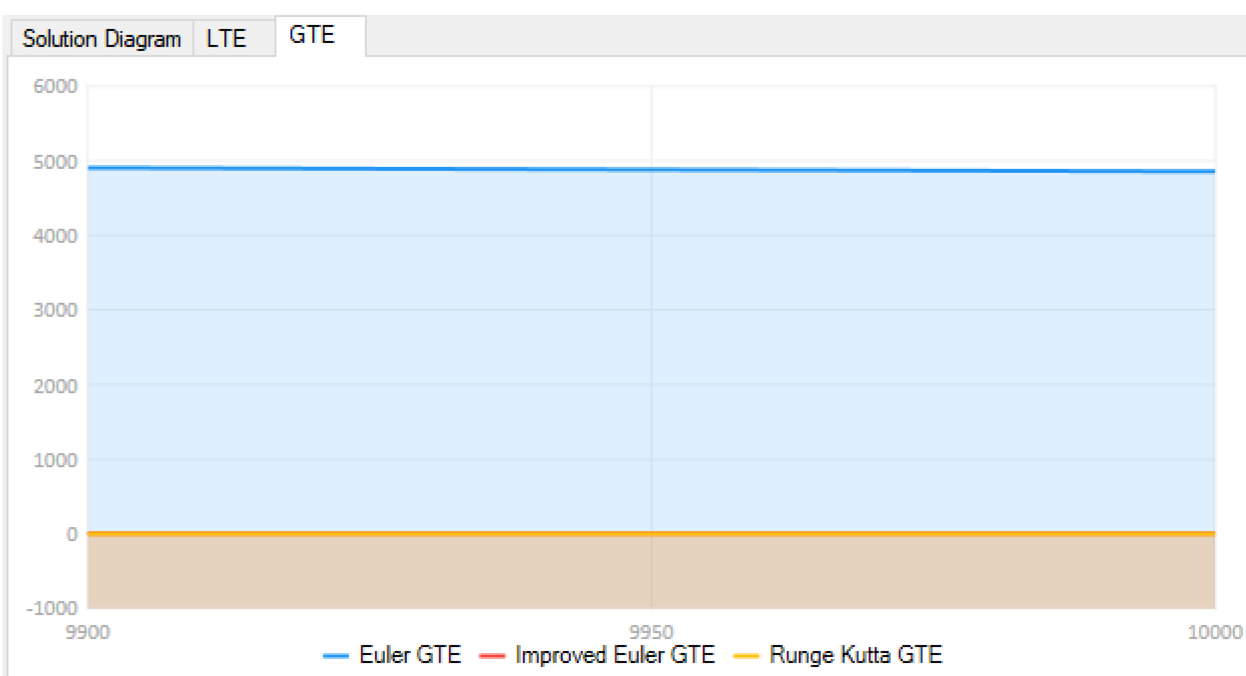
$y = e^x - \frac{1}{15+1}$ ≈ 3269017.30997

2. Visualization of Numerical Methods




3500000
2500000
1500000
500000
-500000
0
5
10
15
Exact Solution
Euler
Improved Euler
Runge Kutta


Solution Diagram
LTE
GTE
6000
5000
4000
3000
2000
1000
0
-1000
0
5
10
15
Euler LTE
Improved Euler LTE
Runge Kutta LTE


Solution Diagram
LTE
GTE
6000
5000
4000
3000
2000
1000
0
-1000
9900
9950
10000
Euler GTE
Improved Euler GTE
Runge Kutta GTE

3. UML Diagram