

Balkan MO 1987

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- 1 Let  $a$  be a real number and let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function satisfying:  $f(0) = \frac{1}{2}$  and  $f(x+y) = f(x)f(a-y) + f(y)f(a-x)$ ,  $\forall x, y \in \mathbb{R}$ .  
Prove that  $f$  is constant.
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- 2 Find all real numbers  $x, y$  greater than 1, satisfying the condition that the numbers  $\sqrt{x-1} + \sqrt{y-1}$  and  $\sqrt{x+1} + \sqrt{y+1}$  are nonconsecutive integers.
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- 3 In the triangle  $ABC$  the following equality holds:

$$\sin^{23} \frac{A}{2} \cos^{48} \frac{B}{2} = \sin^{23} \frac{B}{2} \cos^{48} \frac{A}{2}$$

Determine the value of  $\frac{AC}{BC}$ .

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- 4 Two circles  $K_1$  and  $K_2$ , centered at  $O_1$  and  $O_2$  with radii 1 and  $\sqrt{2}$  respectively, intersect at  $A$  and  $B$ . Let  $C$  be a point on  $K_2$  such that the midpoint of  $AC$  lies on  $K_1$ . Find the length of the segment  $AC$  if  $O_1O_2 = 2$
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