OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

October 2, 2016

1 Invoking the theorem prover

The used proving method is Wu's method. The input system is:

```
p_1 = 2x_1 -
p_2 = 4x_2^2 - 3
 p_3 = 3x_3 - x_2
 p_4 = 3x_4^2 - 2
p_5 = x_5 - x_1
p_6 = x_6 - x_3
p_7 = x_7 - x_4 - x_2
      = x_8 - x_4 x_2 -
      = x_9 + x_4 x_1
     = x_{10} - x_3x_1 + x_2x_1
p_{10}
      = 2x_{14} -
p_{11}
p_{12}
     = 2x_{15} - x_1 -
p_{13}
     = 2x_{16} - x_2
p_{14} = 2x_{17} - x_1
p_{15} = 2x_{18} - x_2
p_{16} = x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
     = x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11}
             -x_{13}^2 - x_{12}^2 - x_{11}^2
     = x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{18}
     = x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
     = x_{23} - x_{14}^{2} + 2x_{14}x_{11} - x_{13}^{2} - x_{12}^{2} - x_{11}^{2}
     = x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{22} = -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9
     = x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4
p_{24} = -x_{13}x_6 + x_{13}x_3 + x_{12}x_7 - x_{12}x_4 - x_7x_3 + x_6x_4
```

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{21} . No reduction needed.

The triangular system has not been changed.

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20} . No reduction needed.

The triangular system has not been changed.

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17} . No reduction needed.

The triangular system has not been changed.

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18} . No reduction needed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17} . No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16} . No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15} . No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13} . No reduction needed.

The triangular system has not been changed.

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12} . No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11} . No reduction needed.

The triangular system has not been changed.

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

```
p_1 = 2x_1 -
 p_2 = 4x_2^2 - 3
 p_3 = 3x_3 - x_2
 p_4 = 3x_4^2 - 2
 p_5 = x_5 - x_1
      = x_6 - x_3
 p_6
      = x_7 - x_4 - x_2
 p_7
       = x_8 - x_4 x_2 -
       = x_9 + x_4 x_1
 p_9
       = x_{10} - x_3 x_1 + x_2 x_1
p_{10}
      = -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9
p_{12} = x_{12}x_7x_5 - x_{12}x_7x_1 - x_{12}x_5x_4 + x_{12}x_4x_1 - x_{11}x_7x_6 +
             x_{11}x_7x_3 + x_{11}x_6x_4 - x_{11}x_4x_3 + x_7x_6x_1 - x_7x_5x_3
              -x_6x_4x_1 + x_5x_4x_3
       = x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4
p_{13}
      = 2x_{14} -
p_{14}
      = 2x_{15} - x_1 -
p_{15}
       = 2x_{16} - x_2
p_{16}
      = 2x_{17} - x_1
p_{17}
      = 2x_{18} - x_2
p_{18}
      = x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{20} = x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{21} = x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
      = x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11}
             -x_{13}^2 - x_{12}^2 - x_{11}^2
p_{23} = x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{24} = x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
```

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

```
p_1 = 2x_1 -
 p_2 = 4x_2^2 - 3
 p_3 = 3x_3 - x_2
 p_4 = 3x_4^2 - 2
      = x_5 - x_1
       = x_6 - x_3
 p_6
      = x_7 - x_4 - x_2
       = x_8 - x_4x_2 -
       = x_9 + x_4 x_1
 p_9
      = x_{10} - x_3 x_1 + x_2 x_1
p_{10}
      = -x_{11}x_9x_7x_5 + x_{11}x_9x_7x_1 + x_{11}x_9x_5x_4 - x_{11}x_9x_4x_1 +
p_{11}
              x_{11}x_8x_7x_6 - x_{11}x_8x_7x_3 - x_{11}x_8x_6x_4 + x_{11}x_8x_4x_3
              -x_{11}x_7x_6 + x_{11}x_7x_3 + x_{11}x_6x_4 - x_{11}x_4x_3 + x_9x_7x_5
              -x_9x_7x_1 - x_9x_5x_4 + x_9x_4x_1 - x_8x_7x_6x_1 + x_8x_7x_5x_3 +
              x_8x_6x_4x_1 - x_8x_5x_4x_3 + x_7x_6x_1 - x_7x_5x_3 - x_6x_4x_1 +
             x_5 x_4 x_3
      = -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9
p_{12}
             x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4
p_{13}
       = 2x_{14} -
p_{14}
       = 2x_{15} - x_1 -
p_{15}
       = 2x_{16} - x_2
p_{16}
       = 2x_{17} - x_1
p_{17}
      = 2x_{18} - x_2
p_{18}
      = x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
     = x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{21} = x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
       = x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11}
             -x_{13}^2 - x_{12}^2 - x_{11}^2
p_{23} = x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
p_{24} = x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
```

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11} . No reduction needed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9 . No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8 . No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 7.

Variable x_7 **selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7 . No reduction needed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

The triangular system has not been changed.

1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5 . No reduction needed.

The triangular system has not been changed.

1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4 . No reduction needed.

The triangular system has not been changed.

1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3 . No reduction needed.

1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2 . No reduction needed.

The triangular system has not been changed.

1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

```
p_1 = 2x_1 -
 p_2 = 4x_2^2 - 3
 p_3 = 3x_3 - x_2
 p_4 = 3x_4^2 - 2
     = x_5 - x_1
 p_5
     = x_6 - x_3
 p_6
 p_7 = x_7 - x_4 - x_2
     = x_8 - x_4 x_2 -
 p_8
     = x_9 + x_4 x_1
p_9
     = x_{10} - x_3x_1 + x_2x_1
     = -x_{11}x_9x_7x_5 + x_{11}x_9x_7x_1 + x_{11}x_9x_5x_4 - x_{11}x_9x_4x_1 +
p_{11}
          x_{11}x_8x_7x_6 - x_{11}x_8x_7x_3 - x_{11}x_8x_6x_4 + x_{11}x_8x_4x_3
           -x_{11}x_7x_6 + x_{11}x_7x_3 + x_{11}x_6x_4 - x_{11}x_4x_3 + x_9x_7x_5
           -x_9x_7x_1-x_9x_5x_4+x_9x_4x_1-x_8x_7x_6x_1+x_8x_7x_5x_3+
           x_8x_6x_4x_1 - x_8x_5x_4x_3 + x_7x_6x_1 - x_7x_5x_3 - x_6x_4x_1 +
           x_5 x_4 x_3
p_{12} = -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9
p_{13} = x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4
     = 2x_{14} -
p_{15}
     = 2x_{15} - x_1 -
p_{16} = 2x_{16} - x_2
```

$$\begin{array}{rcl} p_{17} & = & 2x_{17} - x_1 \\ p_{18} & = & 2x_{18} - x_2 \\ p_{19} & = & x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\ p_{20} & = & x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\ p_{21} & = & x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\ p_{22} & = & x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\ - x_{13}^2 - x_{12}^2 - x_{11}^2 \\ p_{23} & = & x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\ p_{24} & = & x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \end{array}$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$g = -x_{13}x_9 + x_{12}x_{10}$$

with respect to the triangular system.

1. Pseudo remainder with p_{24} over variable x_{24} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

2. Pseudo remainder with p_{23} over variable x_{23} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

3. Pseudo remainder with p_{22} over variable x_{22} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

4. Pseudo remainder with p_{21} over variable x_{21} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

5. Pseudo remainder with p_{20} over variable x_{20} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

6. Pseudo remainder with p_{19} over variable x_{19} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

7. Pseudo remainder with p_{18} over variable x_{18} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

8. Pseudo remainder with p_{17} over variable x_{17} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

9. Pseudo remainder with p_{16} over variable x_{16} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

10. Pseudo remainder with p_{15} over variable x_{15} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

11. Pseudo remainder with p_{14} over variable x_{14} :

$$g = -x_{13}x_9 + x_{12}x_{10}$$

12. Pseudo remainder with p_{13} over variable x_{13} :

$$g = x_{12}x_{10}x_5 - x_{12}x_{10}x_1 - x_{11}x_9x_7 + x_{11}x_9x_4 + x_9x_7x_1 - x_9x_5x_4$$

13. Pseudo remainder with p_{12} over variable x_{12} :

$$\begin{array}{lll} g & = & -x_{11}x_{10}x_{9}x_{5} + x_{11}x_{10}x_{9}x_{1} + x_{11}x_{9}x_{8}x_{7} - x_{11}x_{9}x_{8}x_{4} \\ & & -x_{11}x_{9}x_{7} + x_{11}x_{9}x_{4} + x_{10}x_{9}x_{5} - x_{10}x_{9}x_{1} - x_{9}x_{8}x_{7}x_{1} + \\ & & x_{9}x_{8}x_{5}x_{4} + x_{9}x_{7}x_{1} - x_{9}x_{5}x_{4} \end{array}$$

14. Pseudo remainder with p_{11} over variable x_{11} :

Polynomial too big for output (text size is 2592 characters, number of terms is 80)

- 15. Pseudo remainder with p_{10} over variable x_{10} :

 Polynomial too big for output (text size is 3837 characters, number of terms is 112)
- 16. Pseudo remainder with p_9 over variable x_9 :

 Polynomial too big for output (text size is 3991 characters, number of terms is 112)
- 17. Pseudo remainder with p_8 over variable x_8 :

 Polynomial too big for output (text size is 2054 characters, number of terms is 52)
- 18. Pseudo remainder with p_7 over variable x_7 :

$$g = -x_6x_5x_4^4x_2^3x_1 + x_6x_5x_4^2x_3x_2^2x_1^3$$

$$-x_6x_5x_4^2x_3x_2^2x_1^2 - x_6x_5x_4^2x_2^3x_1^3 +$$

$$x_6x_5x_4^2x_2^3x_1^2 + x_6x_4^4x_2^3x_1^2$$

$$-x_6x_4^2x_3x_2^2x_1^4 + x_6x_4^2x_3x_2^2x_1^3 +$$

$$x_6x_4^2x_2^3x_1^4 - x_6x_4^2x_2^3x_1^3$$

$$-x_5^2x_4^4x_2^2x_1^2 - x_5^2x_4^2x_2^3x_2^2x_1^2 +$$

$$x_5^2x_4^2x_3x_2^3x_1^2 + x_5x_4^4x_3x_2^3x_1 +$$

$$2x_5x_4^4x_2^2x_1^3 + x_5x_4^3x_3x_2^4x_1 +$$

$$x_5x_4^3x_2^3x_1^3 - x_5x_4^3x_2^3x_1^2 +$$

$$x_5x_4^2x_3^2x_2^2x_1^3 + x_5x_4^2x_3^2x_2^2x_1^2 +$$

$$-x_5x_4^2x_3^2x_2^3x_1^3 - x_5x_4^2x_3x_2^2x_1^2 -$$

$$-x_5x_4^2x_3x_2^3x_1^3 - x_5x_4^2x_3x_2^3x_1^2 -$$

$$-x_4^4x_3x_2^3x_1^2 - x_4^4x_2^2x_1^4 -$$

$$-x_4^4x_3x_2^4x_1^2 - x_4^3x_2^3x_1^4 + x_4^3x_2^3x_1^3 -$$

$$-x_4^2x_3^2x_2^2x_1^3 + x_4^2x_3x_2^3x_1^3 +$$

$$-x_4^2x_3^2x_2^2x_1^3 + x_4^2x_3x_2^3x_1^3 +$$

19. Pseudo remainder with p_6 over variable x_6 :

$$\begin{array}{ll} g &=& -x_5^2x_4^4x_2^2x_1^2 - x_5^2x_4^2x_3^2x_2^2x_1^2 + \\ & x_5^2x_4^2x_3x_2^3x_1^2 + 2x_5x_4^4x_2^2x_1^3 + \\ & x_5x_4^3x_3x_2^4x_1 + x_5x_4^3x_2^3x_1^3 \\ & -x_5x_4^3x_2^3x_1^2 + 2x_5x_4^2x_3^2x_2^2x_1^3 \\ & -2x_5x_4^2x_3x_2^3x_1^3 - x_4^4x_2^2x_1^4 \\ & -x_4^3x_3x_2^4x_1^2 - x_4^3x_2^3x_1^4 + x_4^3x_2^3x_1^3 \\ & -x_4^2x_3^2x_2^2x_1^4 + x_4^2x_3x_2^3x_1^4 \end{array}$$

20. Pseudo remainder with p_5 over variable x_5 :

$$g = 0$$

21. Pseudo remainder with p_4 over variable x_4 :

$$g = 0$$

22. Pseudo remainder with p_3 over variable x_3 :

$$g = 0$$

23. Pseudo remainder with p_2 over variable x_2 :

$$g = 0$$

24. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 112 terms.

Time Complexity: Time spent by the prover is 0.154 seconds.

4 NDG Conditions

NDG Conditions in readable form

• Failed to translate NDG Conditions to readable form