Balkan MO 1989

1 Let n be a positive integer and let $d_1, d_2, , \dots, d_k$ be its divisors, such that $1 = d_1 < d_2 < \dots < d_k = n$. Find all values of n for which $k \geq 4$ and $n = d_1^2 + d_2^2 + d_3^2 + d_4^2$.

2 Let $\overline{a_n a_{n-1} \dots a_1 a_0}$ be the decimal representation of a prime positive integer such that $n > 1$ and $a_n > 1$. Prove that the polynomial $P(x) = a_n x^n + \dots + a_1 x + a_0$ cannot be written as a product of two non-constant integer polynomials.

3 Let G be the centroid of a triangle ABC and let d be a line that intersects AB and AC at B_1 and C_1, respectively, such that the points A and G are not separated by d.
Prove that: $[BB_1GC_1] + [CC_1GB_1] \geq \frac{4}{9}[ABC]$.

4 The elements of the set F are some subsets of $\{1, 2, \dots, n\}$ and satisfy the conditions:
i) if A belongs to F, then A has three elements;
ii)if A and B are distinct elements of F , then A and B have at most one common element.
Let $f(n)$ be the greatest possible number of elements of F. Prove that $\frac{n^2-4n}{6} \leq f(n) \leq \frac{n^2-n}{6}$