

OpenGeoProver Output for conjecture “geothm_zadatak”

Wu’s method used

September 30, 2016

1 Invoking the theorem prover

The used proving method is Wu’s method.

The input system is:

$$\begin{aligned} p_1 &= -x_4 + x_1 \\ p_2 &= -x_4 + x_2 \\ p_3 &= -x_5 + x_1 \\ p_4 &= x_5 + x_2 - \\ p_5 &= x_3 - \\ p_6 &= -x_8 + x_6 \\ p_7 &= -x_8 + x_7 \\ p_8 &= x_9 + x_6 - \\ p_9 &= -x_9 + x_7 \\ p_{10} &= -x_{13}x_7 + x_{11} \\ p_{11} &= x_{13} + x_{12} - \\ p_{12} &= -x_{14}x_1 + x_{10} \\ p_{13} &= -x_{14}x_2 + x_{11} \\ p_{14} &= -x_{14}x_3 + x_{12} \\ p_{15} &= -x_{18}x_7 + x_{16} \\ p_{16} &= x_{18} + x_{17} - \\ p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\ p_{18} &= -x_{19}x_2 + x_{16} \\ p_{19} &= -x_{19}x_3 + x_{17} \end{aligned}$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{17} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{18} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{18} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 -
\end{aligned}$$

$$\begin{aligned}
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{17} &= -x_{17}x_7 - x_{16} + x_7 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12}
\end{aligned}$$

$$\begin{aligned}
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= x_{16}x_1 - x_{16} + x_{15}x_7x_3 - x_7x_3 - x_7x_1 + x_7 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15} . No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{14} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{11}x_1 + x_{10}x_2 \\
p_{13} &= -x_{12}x_1 + x_{10}x_3 \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= -x_{12}x_1 + x_{10}x_3 \\
p_{12} &= -x_{12}x_7 - x_{11} + x_7 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= x_{11}x_1 + x_{10}x_7x_3 - x_7x_1 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7
\end{aligned}$$

$$\begin{aligned}
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7
\end{aligned}$$

$$\begin{aligned}
p_8 &= x_7 + x_6 - \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= x_7 + x_6 - \\
p_7 &= -x_7 + x_6 \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
\end{aligned}$$

$$\begin{aligned}
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= x_3 - \\
p_4 &= -x_2 - x_1 + 1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_3 - \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= -x_2 + x_1 \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 - \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 -
\end{aligned}$$

$$\begin{aligned}
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$g = -x_{18}x_6 + x_{18} + x_{15} -$$

with respect to the triangular system.

1. Pseudo remainder with p_{19} over variable x_{19} :

$$g = -x_{18}x_6 + x_{18} + x_{15} -$$

2. Pseudo remainder with p_{18} over variable x_{18} :

$$g = x_{16}x_6 - x_{16} - x_{15}x_7 + x_7$$

3. Pseudo remainder with p_{17} over variable x_{17} :

$$g = x_{16}x_6 - x_{16} - x_{15}x_7 + x_7$$

4. Pseudo remainder with p_{16} over variable x_{16} :

$$g = x_{15}x_7x_1 - x_{15}x_7 - x_{15}x_6x_2 + x_{15}x_2 - x_7x_1 + x_7 + x_6x_2 - x_2$$

5. Pseudo remainder with p_{15} over variable x_{15} :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

6. Pseudo remainder with p_{14} over variable x_{14} :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

7. Pseudo remainder with p_{13} over variable x_{13} :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

8. Pseudo remainder with p_{12} over variable x_{12} :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

9. Pseudo remainder with p_{11} over variable x_{11} :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

10. Pseudo remainder with p_{10} over variable x_{10} :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

11. Pseudo remainder with p_9 over variable x_9 :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

12. Pseudo remainder with p_8 over variable x_8 :

$$\begin{aligned} g &= -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\ &\quad - 2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 \end{aligned}$$

13. Pseudo remainder with p_7 over variable x_7 :

$$\begin{aligned} g = & -x_6^2 x_2 x_1^2 + 2x_6^2 x_2 x_1 - x_6^2 x_2 - x_6^2 x_1^3 + \\ & 3x_6^2 x_1^2 - 3x_6^2 x_1 + x_6^2 + 2x_6 x_2 x_1^2 - 4x_6 x_2 x_1 + \\ & 2x_6 x_2 + 2x_6 x_1^3 - 6x_6 x_1^2 + 6x_6 x_1 - 2x_6 - x_2 x_1^2 + 2x_2 x_1 \\ & - x_2 - x_1^3 + 3x_1^2 - 3x_1 + 1 \end{aligned}$$

14. Pseudo remainder with p_6 over variable x_6 :

$$g = -x_2 x_1^2 + 2x_2 x_1 - x_2 - x_1^3 + 3x_1^2 - 3x_1 + 1$$

15. Pseudo remainder with p_5 over variable x_5 :

$$g = -x_2 x_1^2 + 2x_2 x_1 - x_2 - x_1^3 + 3x_1^2 - 3x_1 + 1$$

16. Pseudo remainder with p_4 over variable x_4 :

$$g = -x_2 x_1^2 + 2x_2 x_1 - x_2 - x_1^3 + 3x_1^2 - 3x_1 + 1$$

17. Pseudo remainder with p_3 over variable x_3 :

$$g = -x_2 x_1^2 + 2x_2 x_1 - x_2 - x_1^3 + 3x_1^2 - 3x_1 + 1$$

18. Pseudo remainder with p_2 over variable x_2 :

$$g = 0$$

19. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 21 terms.

Time Complexity: Time spent by the prover is 0.096 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form