OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

October 2, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 \\
p_9 &= x_9 + x_4 x_1 - x_4 \\
p_{10} &= x_{10} - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_{11} &= x_{11} - x_4 x_2 - \\
p_{12} &= x_{12} + x_4 x_1 \\
p_{13} &= x_{13} - x_3 x_1 + x_2 x_1 \\
p_{14} &= -x_{18} x_{11} + x_{18} + x_{17} x_{12} - x_{12} \\
p_{15} &= x_{19} x_{11} - x_{19} - x_{17} x_{13} + x_{13} \\
p_{16} &= -x_{18} x_8 + x_{17} x_9 \\
p_{17} &= -x_{15} x_8 + x_{14} x_9 \\
p_{18} &= x_{16} x_5 - x_{16} x_1 - x_{14} x_7 + x_{14} x_4 + x_7 x_1 - x_5 x_4 \\
p_{19} &= -x_{16} x_6 + x_{16} x_3 + x_{15} x_7 - x_{15} x_4 - x_7 x_3 + x_6 x_4
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{18} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 \\
p_9 &= x_9 + x_4 x_1 - x_4 \\
p_{10} &= x_{10} - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_{11} &= x_{11} - x_4 x_2 - \\
p_{12} &= x_{12} + x_4 x_1 \\
p_{13} &= x_{13} - x_3 x_1 + x_2 x_1 \\
p_{14} &= -x_{15} x_8 + x_{14} x_9 \\
p_{15} &= x_{16} x_5 - x_{16} x_1 - x_{14} x_7 + x_{14} x_4 + x_7 x_1 - x_5 x_4 \\
p_{16} &= -x_{16} x_6 + x_{16} x_3 + x_{15} x_7 - x_{15} x_4 - x_7 x_3 + x_6 x_4 \\
p_{17} &= x_{17} x_{12} x_8 - x_{17} x_{11} x_9 + x_{17} x_9 - x_{12} x_8 \\
p_{18} &= -x_{18} x_{11} + x_{18} + x_{17} x_{12} - x_{12} \\
p_{19} &= x_{19} x_{11} - x_{19} - x_{17} x_{13} + x_{13}
\end{aligned}
$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 \\
p_9 &= x_9 + x_4 x_1 - x_4 \\
p_{10} &= x_{10} - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_{11} &= x_{11} - x_4 x_2 - \\
p_{12} &= x_{12} + x_4 x_1 \\
p_{13} &= x_{13} - x_3 x_1 + x_2 x_1 \\
p_{14} &= -x_{15} x_8 + x_{14} x_9 \\
p_{15} &= x_{15} x_7 x_5 - x_{15} x_7 x_1 - x_{15} x_5 x_4 + x_{15} x_4 x_1 - x_{14} x_7 x_6 + \\
&\quad x_{14} x_7 x_3 + x_{14} x_6 x_4 - x_{14} x_4 x_3 + x_7 x_6 x_1 - x_7 x_5 x_3 \\
&\quad -x_6 x_4 x_1 + x_5 x_4 x_3 \\
p_{16} &= x_{16} x_5 - x_{16} x_1 - x_{14} x_7 + x_{14} x_4 + x_7 x_1 - x_5 x_4 \\
p_{17} &= x_{17} x_{12} x_8 - x_{17} x_{11} x_9 + x_{17} x_9 - x_{12} x_8 \\
p_{18} &= -x_{18} x_{11} + x_{18} + x_{17} x_{12} - x_{12} \\
p_{19} &= x_{19} x_{11} - x_{19} - x_{17} x_{13} + x_{13}
\end{aligned}
$$

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{15} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 \\
p_9 &= x_9 + x_4 x_1 - x_4 \\
p_{10} &= x_{10} - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_{11} &= x_{11} - x_4 x_2 - \\
p_{12} &= x_{12} + x_4 x_1 \\
p_{13} &= x_{13} - x_3 x_1 + x_2 x_1 \\
p_{14} &= -x_{14} x_9 x_7 x_5 + x_{14} x_9 x_7 x_1 + x_{14} x_9 x_5 x_4 - x_{14} x_9 x_4 x_1 + \\
&\quad x_{14} x_8 x_7 x_6 - x_{14} x_8 x_7 x_3 - x_{14} x_8 x_6 x_4 + x_{14} x_8 x_4 x_3 \\
&\quad -x_8 x_7 x_6 x_1 + x_8 x_7 x_5 x_3 + x_8 x_6 x_4 x_1 - x_8 x_5 x_4 x_3 \\
p_{15} &= -x_{15} x_8 + x_{14} x_9 \\
p_{16} &= x_{16} x_5 - x_{16} x_1 - x_{14} x_7 + x_{14} x_4 + x_7 x_1 - x_5 x_4 \\
p_{17} &= x_{17} x_{12} x_8 - x_{17} x_{11} x_9 + x_{17} x_9 - x_{12} x_8 \\
p_{18} &= -x_{18} x_{11} + x_{18} + x_{17} x_{12} - x_{12} \\
p_{19} &= x_{19} x_{11} - x_{19} - x_{17} x_{13} + x_{13}
\end{aligned}
$$

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

1.12   Triangulation, step 12

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

1.13   Triangulation, step 13

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

1.14   Triangulation, step 14

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.15   Triangulation, step 15

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

1.16   Triangulation, step 16

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.17   Triangulation, step 17

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.18   Triangulation, step 18

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.19   Triangulation, step 19

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 \\
p_9 &= x_9 + x_4 x_1 - x_4 \\
p_{10} &= x_{10} - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_{11} &= x_{11} - x_4 x_2 - \\
p_{12} &= x_{12} + x_4 x_1 \\
p_{13} &= x_{13} - x_3 x_1 + x_2 x_1 \\
p_{14} &= -x_{14} x_9 x_7 x_5 + x_{14} x_9 x_7 x_1 + x_{14} x_9 x_5 x_4 - x_{14} x_9 x_4 x_1 + \\
&\quad x_{14} x_8 x_7 x_6 - x_{14} x_8 x_7 x_3 - x_{14} x_8 x_6 x_4 + x_{14} x_8 x_4 x_3 \\
&\quad -x_8 x_7 x_6 x_1 + x_8 x_7 x_5 x_3 + x_8 x_6 x_4 x_1 - x_8 x_5 x_4 x_3 \\
p_{15} &= -x_{15} x_8 + x_{14} x_9 \\
p_{16} &= x_{16} x_5 - x_{16} x_1 - x_{14} x_7 + x_{14} x_4 + x_7 x_1 - x_5 x_4 \\
p_{17} &= x_{17} x_{12} x_8 - x_{17} x_{11} x_9 + x_{17} x_9 - x_{12} x_8 \\
p_{18} &= -x_{18} x_{11} + x_{18} + x_{17} x_{12} - x_{12} \\
p_{19} &= x_{19} x_{11} - x_{19} - x_{17} x_{13} + x_{13}
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
g = -x_{19} + x_{16}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{19} over variable x_{19}:

$$
g = -x_{17} x_{13} + x_{16} x_{11} - x_{16} + x_{13}
$$

2. Pseudo remainder with p_{18} over variable x_{18}:

$$
g = -x_{17} x_{13} + x_{16} x_{11} - x_{16} + x_{13}
$$

3. Pseudo remainder with p_{17} over variable x_{17}:

$$
\begin{aligned}
g \quad = \quad & x_{16}x_{12}x_{11}x_8 - x_{16}x_{12}x_8 - x_{16}x_{11}^2 x_9 + 2x_{16}x_{11}x_9 \\
& -x_{16}x_9 - x_{13}x_{11}x_9 + x_{13}x_9
\end{aligned}
$$

4. Pseudo remainder with p_{16} over variable x_{16}:

$$
\begin{aligned}
g \quad = \quad & x_{14}x_{12}x_{11}x_8x_7 - x_{14}x_{12}x_{11}x_8x_4 - x_{14}x_{12}x_8x_7 + \\
& x_{14}x_{12}x_8x_4 - x_{14}x_{11}^2 x_9x_7 + x_{14}x_{11}^2 x_9x_4 + \\
& 2x_{14}x_{11}x_9x_7 - 2x_{14}x_{11}x_9x_4 - x_{14}x_9x_7 + x_{14}x_9x_4 \\
& -x_{13}x_{11}x_9x_5 + x_{13}x_{11}x_9x_1 + x_{13}x_9x_5 - x_{13}x_9x_1 \\
& -x_{12}x_{11}x_8x_7x_1 + x_{12}x_{11}x_8x_5x_4 + x_{12}x_8x_7x_1 \\
& -x_{12}x_8x_5x_4 + x_{11}^2 x_9x_7x_1 - x_{11}^2 x_9x_5x_4 \\
& -2x_{11}x_9x_7x_1 + 2x_{11}x_9x_5x_4 + x_9x_7x_1 - x_9x_5x_4
\end{aligned}
$$

5. Pseudo remainder with p_{15} over variable x_{15}:

$$
\begin{aligned}
g \quad = \quad & x_{14}x_{12}x_{11}x_8x_7 - x_{14}x_{12}x_{11}x_8x_4 - x_{14}x_{12}x_8x_7 + \\
& x_{14}x_{12}x_8x_4 - x_{14}x_{11}^2 x_9x_7 + x_{14}x_{11}^2 x_9x_4 + \\
& 2x_{14}x_{11}x_9x_7 - 2x_{14}x_{11}x_9x_4 - x_{14}x_9x_7 + x_{14}x_9x_4 \\
& -x_{13}x_{11}x_9x_5 + x_{13}x_{11}x_9x_1 + x_{13}x_9x_5 - x_{13}x_9x_1 \\
& -x_{12}x_{11}x_8x_7x_1 + x_{12}x_{11}x_8x_5x_4 + x_{12}x_8x_7x_1 \\
& -x_{12}x_8x_5x_4 + x_{11}^2 x_9x_7x_1 - x_{11}^2 x_9x_5x_4 \\
& -2x_{11}x_9x_7x_1 + 2x_{11}x_9x_5x_4 + x_9x_7x_1 - x_9x_5x_4
\end{aligned}
$$

6. Pseudo remainder with p_{14} over variable x_{14}:
   Polynomial too big for output (text size is 3717 characters, number of terms is 98)

7. Pseudo remainder with p_{13} over variable x_{13}:
   Polynomial too big for output (text size is 4891 characters, number of terms is 126)

8. Pseudo remainder with p_{12} over variable x_{12}:
   Polynomial too big for output (text size is 4922 characters, number of terms is 126)

9. Pseudo remainder with p_{11} over variable x_{11}:
   Polynomial too big for output (text size is 2522 characters, number of terms is 56)

10. Pseudo remainder with p_{10} over variable x_{10}:
   Polynomial too big for output (text size is 2522 characters, number of terms is 56)

11. Pseudo remainder with p_9 over variable x_9:

Polynomial too big for output (text size is 4981 characters, number of terms is 122)

12. Pseudo remainder with p_8 over variable x_8:

Polynomial too big for output (text size is 3397 characters, number of terms is 88)

13. Pseudo remainder with p_7 over variable x_7:

$$
\begin{aligned}
g \;=\; & -x_6x_5x_4^5x_2^4 + x_6x_5x_4^3x_3x_2^3x_1^2 \\
& -x_6x_5x_4^3x_3x_2^3x_1 - x_6x_5x_4^3x_2^4x_1^2 + \\
& x_6x_5x_4^3x_2^4x_1 + x_6x_4^5x_2^4x_1 \\
& -x_6x_4^3x_3x_2^3x_1^3 + x_6x_4^3x_3x_2^3x_1^2 + \\
& x_6x_4^3x_2^4x_1^3 - x_6x_4^3x_2^4x_1^2 \\
& -x_5^2x_4^5x_2^3x_1 + x_5^2x_4^5x_2^3 + \\
& x_5^2x_4^3x_3x_2^2x_1^3 - 2x_5^2x_4^3x_3x_2^2x_1^2 + \\
& x_5^2x_4^3x_3x_2^2x_1 - x_5^2x_4^3x_2^3x_1^3 + \\
& 2x_5^2x_4^3x_2^3x_1^2 - x_5^2x_4^3x_2^3x_1 + \\
& x_5x_4^5x_3x_2^4 + 2x_5x_4^5x_2^3x_1^2 \\
& -2x_5x_4^5x_2^3x_1 + x_5x_4^4x_3x_2^5 + x_5x_4^4x_2^4x_1^2 \\
& -x_5x_4^4x_2^4x_1 - x_5x_4^3x_3^2x_2^3x_1^2 + \\
& x_5x_4^3x_3^2x_2^3x_1 + x_5x_4^3x_3x_2^4x_1^2 \\
& -x_5x_4^3x_3x_2^4x_1 - 2x_5x_4^3x_3x_2^2x_1^4 + \\
& 4x_5x_4^3x_3x_2^2x_1^3 - 2x_5x_4^3x_3x_2^2x_1^2 + \\
& 2x_5x_4^3x_2^3x_1^4 - 4x_5x_4^3x_2^3x_1^3 + \\
& 2x_5x_4^3x_2^3x_1^2 - x_4^5x_3x_2^4x_1 - x_4^5x_2^3x_1^3 + \\
& x_4^5x_2^3x_1^2 - x_4^4x_3x_2^5x_1 - x_4^4x_2^4x_1^3 + \\
& x_4^4x_2^4x_1^2 + x_4^3x_3^2x_2^3x_1^3 \\
& -x_4^3x_3^2x_2^3x_1^2 - x_4^3x_3x_2^4x_1^3 + \\
& x_4^3x_3x_2^4x_1^2 + x_4^3x_3x_2^2x_1^5 \\
& -2x_4^3x_3x_2^2x_1^4 + x_4^3x_3x_2^2x_1^3 \\
& -x_4^3x_2^3x_1^5 + 2x_4^3x_2^3x_1^4 - x_4^3x_2^3x_1^3
\end{aligned}
$$

14. Pseudo remainder with p_6 over variable x_6:

$$
\begin{aligned}
g \;=\; & -x_5^2x_4^5x_2^3x_1 + x_5^2x_4^5x_2^3 + \\
& x_5^2x_4^3x_3x_2^2x_1^3 - 2x_5^2x_4^3x_3x_2^2x_1^2 + \\
& x_5^2x_4^3x_3x_2^2x_1 - x_5^2x_4^3x_2^3x_1^3 + \\
& 2x_5^2x_4^3x_2^3x_1^2 - x_5^2x_4^3x_2^3x_1 + \\
& 2x_5x_4^5x_2^3x_1^2 - 2x_5x_4^5x_2^3x_1 +
\end{aligned}
$$

$$x_5 x_4^4 x_3 x_2^5 + x_5 x_4^4 x_2^4 x_1^2 - x_5 x_4^4 x_2^4 x_1$$
$$-2x_5 x_4^3 x_3 x_2^2 x_1^4 + 4x_5 x_4^3 x_3 x_2^2 x_1^3$$
$$-2x_5 x_4^3 x_3 x_2^2 x_1^2 + 2x_5 x_4^3 x_2^3 x_1^4$$
$$-4x_5 x_4^3 x_2^3 x_1^3 + 2x_5 x_4^3 x_2^3 x_1^2$$
$$-x_4^5 x_2^3 x_1^3 + x_4^5 x_2^3 x_1^2 - x_4^4 x_3 x_2^5 x_1$$
$$-x_4^4 x_2^4 x_1^3 + x_4^4 x_2^4 x_1^2 + x_4^3 x_3 x_2^2 x_1^5$$
$$-2x_4^3 x_3 x_2^2 x_1^4 + x_4^3 x_3 x_2^2 x_1^3$$
$$-x_4^3 x_2^3 x_1^5 + 2x_4^3 x_2^3 x_1^4 - x_4^3 x_2^3 x_1^3$$

15. Pseudo remainder with p_5 over variable x_5:

$$g = 0$$

16. Pseudo remainder with p_4 over variable x_4:

$$g = 0$$

17. Pseudo remainder with p_3 over variable x_3:

$$g = 0$$

18. Pseudo remainder with p_2 over variable x_2:

$$g = 0$$

19. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 126 terms.

Time Complexity: Time spent by the prover is 0.166 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form