OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

September 30, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20} \\
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
\end{aligned}
$$

$$
\begin{aligned}
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{32}x_{25} + x_{29} - x_1 \\
p_{30} &= -x_{32}x_{26} + x_{30} - x_2 \\
p_{31} &= -x_{32}x_{27} + x_{31} - x_3 \\
p_{32} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28}
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 32.

Variable x_{32} selected: The number of polynomials with this variable, with indexes from 1 to 32, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{32} from all other polynomials by reducing them with polynomial p_{29} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20}
\end{aligned}
$$

$$
\begin{aligned}
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= -x_{31}x_{25} + x_{29}x_{27} - x_{27}x_1 + x_{25}x_3 \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 31.

Variable x_{31} selected: The number of polynomials with this variable, with indexes from 1 to 31, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{31} from all other polynomials by reducing them with polynomial p_{29} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17}
\end{aligned}
$$

$$
\begin{aligned}
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20} \\
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{30} &= x_{30}x_{26}x_{25} + x_{29}x_{27}^2 + x_{29}x_{25}^2 + x_{28}x_{25} - x_{27}^2x_1 + \\
&\quad x_{27}x_{25}x_3 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 30.

Variable x_{30} selected: The number of polynomials with this variable, with indexes from 1 to 30, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{30} from all other polynomials by reducing them with polynomial p_{29} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16}
\end{aligned}
$$

$$
\begin{aligned}
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20} \\
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 29.

Variable x_{29} selected: The number of polynomials with this variable, with indexes from 1 to 29, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{29}. No reduction needed.

The triangular system has not been changed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 28.

Variable x_{28} selected: The number of polynomials with this variable, with indexes from 1 to 28, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{28}. No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 27.

Variable x_{27} selected: The number of polynomials with this variable, with indexes from 1 to 27, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{27}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 26.

Variable x_{26} selected: The number of polynomials with this variable, with indexes from 1 to 26, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{26}. No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 25.

Variable x_{25} selected: The number of polynomials with this variable, with indexes from 1 to 25, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{25}. No reduction needed.

The triangular system has not been changed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{24} from all other polynomials by reducing them with polynomial p_{22} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 -
\end{aligned}
$$

$$
\begin{aligned}
p_{9} &= -x_{9} + x_{7} \\
p_{10} &= -x_{13}x_{7} + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_{1} + x_{10} \\
p_{13} &= -x_{14}x_{2} + x_{11} \\
p_{14} &= -x_{14}x_{3} + x_{12} \\
p_{15} &= -x_{18}x_{7} + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_{1} + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_{2} + x_{16} \\
p_{19} &= -x_{19}x_{3} + x_{17} \\
p_{20} &= -x_{23}x_{7} + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{21}x_{1} + x_{20}x_{2} - x_{20} + x_{1} \\
p_{23} &= -x_{22}x_{1} + x_{20}x_{3} \\
p_{24} &= -x_{24}x_{1} + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^{2}x_{25} - x_{29}x_{26}^{2}x_{25} - x_{29}x_{25}^{3} - x_{28}x_{25}^{2} + \\
&\quad x_{27}^{2}x_{25}x_{1} - x_{27}x_{25}^{2}x_{3} + x_{26}^{2}x_{25}x_{1} - x_{26}x_{25}^{2}x_{2} \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_{1} + x_{25}x_{2} \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_{1}
\end{aligned}
$$

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{23} from all other polynomials by reducing them with polynomial p_{20} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_{1} &= -x_{4} + x_{1} \\
p_{2} &= -x_{4} + x_{2} \\
p_{3} &= -x_{5} + x_{1} \\
p_{4} &= x_{5} + x_{2} -
\end{aligned}
$$

$$
\begin{aligned}
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{21} &= -x_{22}x_1 + x_{20}x_3 \\
p_{22} &= -x_{22}x_7 + x_{22} - x_{21} + x_7 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{22} from all other polynomials by reducing them with polynomial p_{21} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{21} &= x_{21}x_1 + x_{20}x_7x_3 - x_{20}x_3 - x_7x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{21} from all other polynomials by reducing them with polynomial p_{20} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20}. No reduction needed.

The triangular system has not been changed.

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{17} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{18} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
\end{aligned}
$$

$$
\begin{aligned}
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{18} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{17} &= -x_{17}x_7 - x_{16} + x_7
\end{aligned}
$$

$$
\begin{aligned}
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11}
\end{aligned}
$$

$$
\begin{aligned}
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= x_{16}x_1 - x_{16} + x_{15}x_7x_3 - x_7x_3 - x_7x_1 + x_7 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7
\end{aligned}
$$

$$
\begin{aligned}
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{14} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{11}x_1 + x_{10}x_2 \\
p_{13} &= -x_{12}x_1 + x_{10}x_3 \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= -x_{12}x_1 + x_{10}x_3 \\
p_{12} &= -x_{12}x_7 - x_{11} + x_7 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 +
\end{aligned}
$$

$$
\begin{aligned}
& x_{27}^2 x_{25} x_1 - x_{27} x_{25}^2 x_3 + x_{26}^2 x_{25} x_1 - x_{26} x_{25}^2 x_2 \\
p_{30} &= -x_{30} x_{25} + x_{29} x_{26} - x_{26} x_1 + x_{25} x_2 \\
p_{31} &= x_{31} x_{27} + x_{30} x_{26} + x_{29} x_{25} + x_{28} \\
p_{32} &= -x_{32} x_{25} + x_{29} - x_1
\end{aligned}
$$

1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11} x_1 + x_{10} x_2 \\
p_{11} &= x_{11} x_1 + x_{10} x_7 x_3 - x_7 x_1 \\
p_{12} &= -x_{12} x_1 + x_{10} x_3 \\
p_{13} &= -x_{13} x_7 + x_{11} \\
p_{14} &= -x_{14} x_1 + x_{10} \\
p_{15} &= -x_{15} x_7 x_3 x_1 + x_{15} x_7 x_3 - x_{15} x_2 x_1 + x_{15} x_2 + x_7 x_3 x_1 \\
& \quad -x_7 x_3 + x_7 x_1^2 - 2 x_7 x_1 + x_7 + x_2 x_1 - x_2 \\
p_{16} &= -x_{16} x_1 + x_{16} + x_{15} x_2 - x_2 \\
p_{17} &= -x_{17} x_1 + x_{17} + x_{15} x_3 - x_3 \\
p_{18} &= -x_{18} x_7 + x_{16} \\
p_{19} &= -x_{19} x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20} x_7 x_3 x_1 + x_{20} x_3 x_1 - x_{20} x_2 x_1 + x_{20} x_1 + x_7 x_1^2 - x_1^2 \\
p_{21} &= -x_{21} x_1 + x_{20} x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22} x_1 + x_{20} x_3 \\
p_{23} &= -x_{23} x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24} x_1 + x_{20}
\end{aligned}
$$

$$
\begin{aligned}
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

$$
\begin{aligned}
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6
\end{aligned}
$$

$$
\begin{aligned}
p_{7} &= -x_{8} + x_{7} \\
p_{8} &= x_{7} + x_{6} - \\
p_{9} &= x_{9} + x_{6} - \\
p_{10} &= -x_{10}x_{7}x_{3}x_{1} - x_{10}x_{2}x_{1} + x_{7}x_{1}^{2} \\
p_{11} &= -x_{11}x_{1} + x_{10}x_{2} \\
p_{12} &= -x_{12}x_{1} + x_{10}x_{3} \\
p_{13} &= -x_{13}x_{7} + x_{11} \\
p_{14} &= -x_{14}x_{1} + x_{10} \\
p_{15} &= -x_{15}x_{7}x_{3}x_{1} + x_{15}x_{7}x_{3} - x_{15}x_{2}x_{1} + x_{15}x_{2} + x_{7}x_{3}x_{1} \\
&\quad -x_{7}x_{3} + x_{7}x_{1}^{2} - 2x_{7}x_{1} + x_{7} + x_{2}x_{1} - x_{2} \\
p_{16} &= -x_{16}x_{1} + x_{16} + x_{15}x_{2} - x_{2} \\
p_{17} &= -x_{17}x_{1} + x_{17} + x_{15}x_{3} - x_{3} \\
p_{18} &= -x_{18}x_{7} + x_{16} \\
p_{19} &= -x_{19}x_{1} + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_{7}x_{3}x_{1} + x_{20}x_{3}x_{1} - x_{20}x_{2}x_{1} + x_{20}x_{1} + x_{7}x_{1}^{2} - x_{1}^{2} \\
p_{21} &= -x_{21}x_{1} + x_{20}x_{2} - x_{20} + x_{1} \\
p_{22} &= -x_{22}x_{1} + x_{20}x_{3} \\
p_{23} &= -x_{23}x_{7} + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_{1} + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^{2}x_{25} - x_{29}x_{26}^{2}x_{25} - x_{29}x_{25}^{3} - x_{28}x_{25}^{2} + \\
&\quad x_{27}^{2}x_{25}x_{1} - x_{27}x_{25}^{2}x_{3} + x_{26}^{2}x_{25}x_{1} - x_{26}x_{25}^{2}x_{2} \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_{1} + x_{25}x_{2} \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_{1}
\end{aligned}
$$

1.25 Triangulation, step 25

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_{1} &= -x_{4} + x_{1}
\end{aligned}
$$

$$
\begin{aligned}
p_{2} &= -x_{4} + x_{2} \\
p_{3} &= -x_{5} + x_{1} \\
p_{4} &= x_{5} + x_{2} - \\
p_{5} &= x_{3} - \\
p_{6} &= x_{7} + x_{6} - \\
p_{7} &= -x_{7} + x_{6} \\
p_{8} &= -x_{8} + x_{6} \\
p_{9} &= x_{9} + x_{6} - \\
p_{10} &= -x_{10}x_{7}x_{3}x_{1} - x_{10}x_{2}x_{1} + x_{7}x_{1}^{2} \\
p_{11} &= -x_{11}x_{1} + x_{10}x_{2} \\
p_{12} &= -x_{12}x_{1} + x_{10}x_{3} \\
p_{13} &= -x_{13}x_{7} + x_{11} \\
p_{14} &= -x_{14}x_{1} + x_{10} \\
p_{15} &= -x_{15}x_{7}x_{3}x_{1} + x_{15}x_{7}x_{3} - x_{15}x_{2}x_{1} + x_{15}x_{2} + x_{7}x_{3}x_{1} \\
&\quad -x_{7}x_{3} + x_{7}x_{1}^{2} - 2x_{7}x_{1} + x_{7} + x_{2}x_{1} - x_{2} \\
p_{16} &= -x_{16}x_{1} + x_{16} + x_{15}x_{2} - x_{2} \\
p_{17} &= -x_{17}x_{1} + x_{17} + x_{15}x_{3} - x_{3} \\
p_{18} &= -x_{18}x_{7} + x_{16} \\
p_{19} &= -x_{19}x_{1} + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_{7}x_{3}x_{1} + x_{20}x_{3}x_{1} - x_{20}x_{2}x_{1} + x_{20}x_{1} + x_{7}x_{1}^{2} - x_{1}^{2} \\
p_{21} &= -x_{21}x_{1} + x_{20}x_{2} - x_{20} + x_{1} \\
p_{22} &= -x_{22}x_{1} + x_{20}x_{3} \\
p_{23} &= -x_{23}x_{7} + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_{1} + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^{2}x_{25} - x_{29}x_{26}^{2}x_{25} - x_{29}x_{25}^{3} - x_{28}x_{25}^{2} + \\
&\quad x_{27}^{2}x_{25}x_{1} - x_{27}x_{25}^{2}x_{3} + x_{26}^{2}x_{25}x_{1} - x_{26}x_{25}^{2}x_{2} \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_{1} + x_{25}x_{2} \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_{1}
\end{aligned}
$$

1.26 Triangulation, step 26

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.27 Triangulation, step 27

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.28 Triangulation, step 28

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= x_3 - \\
p_4 &= -x_2 - x_1 + 1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
\end{aligned}
$$

$$
\begin{aligned}
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.29 Triangulation, step 29

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_3 - \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= -x_2 + x_1 \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
\end{aligned}
$$

$$
\begin{aligned}
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.30 Triangulation, step 30

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

1.31 Triangulation, step 31

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1
\end{aligned}
$$

$$
\begin{aligned}
p_3 &= x_3 - \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

1.32 Triangulation, step 32

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 - \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= x_{25} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{26} &= x_{26} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{27} &= x_{27} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{28} &= x_{28} + x_{27}x_{12} + x_{26}x_{11} + x_{25}x_{10} \\
p_{29} &= -x_{29}x_{27}^2x_{25} - x_{29}x_{26}^2x_{25} - x_{29}x_{25}^3 - x_{28}x_{25}^2 + \\
&\quad x_{27}^2x_{25}x_1 - x_{27}x_{25}^2x_3 + x_{26}^2x_{25}x_1 - x_{26}x_{25}^2x_2 \\
p_{30} &= -x_{30}x_{25} + x_{29}x_{26} - x_{26}x_1 + x_{25}x_2 \\
p_{31} &= x_{31}x_{27} + x_{30}x_{26} + x_{29}x_{25} + x_{28} \\
p_{32} &= -x_{32}x_{25} + x_{29} - x_1
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$g = -x_{23}x_6 + x_{20}$$

with respect to the triangular system.

1. Pseudo remainder with p_{32} over variable x_{32}:

$$g = -x_{23}x_6 + x_{20}$$

2. Pseudo remainder with p_{31} over variable x_{31}:

$$g = -x_{23}x_6 + x_{20}$$

3. Pseudo remainder with p_{30} over variable x_{30}:

$$g = -x_{23}x_6 + x_{20}$$

4. Pseudo remainder with p_{29} over variable x_{29}:

$$g = -x_{23}x_6 + x_{20}$$

5. Pseudo remainder with p_{28} over variable x_{28}:

$$g = -x_{23}x_6 + x_{20}$$

6. Pseudo remainder with p_{27} over variable x_{27}:

$$g = -x_{23}x_6 + x_{20}$$

7. Pseudo remainder with p_{26} over variable x_{26}:

$$g = -x_{23}x_6 + x_{20}$$

8. Pseudo remainder with p_{25} over variable x_{25}:

$$g = -x_{23}x_6 + x_{20}$$

9. Pseudo remainder with p_{24} over variable x_{24}:

$$g = -x_{23}x_6 + x_{20}$$

10. Pseudo remainder with p_{23} over variable x_{23}:

$$g = x_{21}x_6 - x_{20}x_7 + x_{20} - x_6$$

11. Pseudo remainder with p_{22} over variable x_{22}:

$$g = x_{21}x_6 - x_{20}x_7 + x_{20} - x_6$$

12. Pseudo remainder with p_{21} over variable x_{21}:

$$g = x_{20}x_7x_1 - x_{20}x_6x_2 + x_{20}x_6 - x_{20}x_1$$

13. Pseudo remainder with p_{20} over variable x_{20}:

$$\begin{aligned} g = & -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ & -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

14. Pseudo remainder with p_{19} over variable x_{19}:

$$\begin{aligned} g = & -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ & -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

15. Pseudo remainder with p_{18} over variable x_{18}:

$$\begin{aligned} g = & -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ & -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

16. Pseudo remainder with p_{17} over variable x_{17}:

$$\begin{aligned} g = & -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ & -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

17. Pseudo remainder with p_{16} over variable x_{16}:

$$\begin{aligned} g = & -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ & -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

18. Pseudo remainder with p_{15} over variable x_{15}:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

19. Pseudo remainder with p_{14} over variable x_{14}:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

20. Pseudo remainder with p_{13} over variable x_{13}:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

21. Pseudo remainder with p_{12} over variable x_{12}:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

22. Pseudo remainder with p_{11} over variable x_{11}:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

23. Pseudo remainder with p_{10} over variable x_{10}:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

24. Pseudo remainder with p_9 over variable x_9:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

25. Pseudo remainder with p_8 over variable x_8:

$$\begin{aligned} g &= -x_7^2x_1^3 + x_7x_6x_2x_1^2 - x_7x_6x_1^2 + 2x_7x_1^3 \\ &\quad -x_6x_2x_1^2 + x_6x_1^2 - x_1^3 \end{aligned}$$

26. Pseudo remainder with p_7 over variable x_7:

$$g = -x_6^2 x_2 x_1^2 - x_6^2 x_1^3 + x_6^2 x_1^2$$

27. Pseudo remainder with p_6 over variable x_6:

$$g = -x_2 x_1^2 - x_1^3 + x_1^2$$

28. Pseudo remainder with p_5 over variable x_5:

$$g = -x_2 x_1^2 - x_1^3 + x_1^2$$

29. Pseudo remainder with p_4 over variable x_4:

$$g = -x_2 x_1^2 - x_1^3 + x_1^2$$

30. Pseudo remainder with p_3 over variable x_3:

$$g = -x_2 x_1^2 - x_1^3 + x_1^2$$

31. Pseudo remainder with p_2 over variable x_2:

$$g = 0$$

32. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.187 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form