# OpenGeoProver Output for conjecture "geothm\_zadatak"

Wu's method used

October 2, 2016

## 1 Invoking the theorem prover

The used proving method is Wu's method. The input system is:

> $p_1$  $= -x_3 + 1$  $= -x_2 + x_1$  $= -x_2 - x_1 + 1$  $= -x_5 + x_4$  $p_4$  $= x_5 + x_4$  $p_5$  $= -x_8x_5 - x_7 + x_5$  $= x_8x_4 + x_6 - x_4$  $p_7$  $= -x_7x_1 + x_6x_2$  $= -x_{11}x_5 - x_{10} + x_5$  $= x_{11}x_4 - x_{11} + x_9 - x_4$  $p_{10}$  $= -x_{10}x_1 + x_{10} + x_9x_2 - x_2$  $p_{11}$  $= -x_{14}x_5 + x_{14} - x_{13} + x_5$  $= x_{14}x_4 + x_{12} - x_4$  $= -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1$  $= -x_{17}x_5 + x_{17} - x_{16} + x_5$  $= x_{17}x_4 - x_{17} + x_{15} - x_4$  $= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1$

#### 1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 17.

Variable  $x_{17}$  selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{17}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{array}{rcl} p_1 & = & -x_3 + 1 \\ p_2 & = & -x_2 + x_1 \\ p_3 & = & -x_2 - x_1 + 1 \\ p_4 & = & -x_5 + x_4 \\ p_5 & = & x_5 + x_4 - \\ p_6 & = & -x_8x_5 - x_7 + x_5 \\ p_7 & = & x_8x_4 + x_6 - x_4 \\ p_8 & = & -x_7x_1 + x_6x_2 \\ p_9 & = & -x_{11}x_5 - x_{10} + x_5 \\ p_{10} & = & x_{11}x_4 - x_{11} + x_9 - x_4 \\ p_{11} & = & -x_{10}x_1 + x_{10} + x_9x_2 - x_2 \\ p_{12} & = & -x_{14}x_5 + x_{14} - x_{13} + x_5 \\ p_{13} & = & x_{14}x_4 + x_{12} - x_4 \\ p_{14} & = & -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1 \\ p_{15} & = & -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1 \\ p_{16} & = & x_{16}x_4 - x_{16} - x_{15}x_5 + x_{15} + x_5 - x_4 \\ p_{17} & = & -x_{17}x_5 + x_{17} - x_{16} + x_5 \end{array}$$

#### 1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 16.

Variable  $x_{16}$  selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{16}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

$$\begin{array}{rcl} p_1 & = & -x_3+1 \\ p_2 & = & -x_2+x_1 \\ p_3 & = & -x_2-x_1+1 \\ p_4 & = & -x_5+x_4 \\ p_5 & = & x_5+x_4- \\ p_6 & = & -x_8x_5-x_7+x_5 \\ p_7 & = & x_8x_4+x_6-x_4 \\ p_8 & = & -x_7x_1+x_6x_2 \\ p_9 & = & -x_{11}x_5-x_{10}+x_5 \end{array}$$

```
\begin{array}{lll} p_{10} & = & x_{11}x_4 - x_{11} + x_9 - x_4 \\ p_{11} & = & -x_{10}x_1 + x_{10} + x_9x_2 - x_2 \\ p_{12} & = & -x_{14}x_5 + x_{14} - x_{13} + x_5 \\ p_{13} & = & x_{14}x_4 + x_{12} - x_4 \\ p_{14} & = & -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1 \\ p_{15} & = & x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_5 + x_4x_2 - x_4 - x_2 + x_1 \\ p_{16} & = & -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1 \\ p_{17} & = & -x_{17}x_5 + x_{17} - x_{16} + x_5 \end{array}
```

#### 1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 15.

Variable  $x_{15}$  selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

#### 1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 14.

Variable  $x_{14}$  selected: The number of polynomials with this variable, with indexes from 1 to 14, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{14}$  from all other polynomials by reducing them with polynomial  $p_{12}$  from previous step.

$$\begin{array}{rcl} p_1 & = & -x_3 + 1 \\ p_2 & = & -x_2 + x_1 \\ p_3 & = & -x_2 - x_1 + 1 \\ p_4 & = & -x_5 + x_4 \\ p_5 & = & x_5 + x_4 - \\ p_6 & = & -x_8x_5 - x_7 + x_5 \\ p_7 & = & x_8x_4 + x_6 - x_4 \\ p_8 & = & -x_7x_1 + x_6x_2 \\ p_9 & = & -x_{11}x_5 - x_{10} + x_5 \\ p_{10} & = & x_{11}x_4 - x_{11} + x_9 - x_4 \\ p_{11} & = & -x_{10}x_1 + x_{10} + x_9x_2 - x_2 \end{array}$$

```
\begin{array}{lll} p_{12} & = & -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1 \\ p_{13} & = & x_{13}x_4 - x_{12}x_5 + x_{12} - x_4 \\ p_{14} & = & -x_{14}x_5 + x_{14} - x_{13} + x_5 \\ p_{15} & = & x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + \\ & & x_5 + x_4x_2 - x_4 - x_2 + x_1 \\ p_{16} & = & -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1 \\ p_{17} & = & -x_{17}x_5 + x_{17} - x_{16} + x_5 \end{array}
```

#### 1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 13.

Variable  $x_{13}$  selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{13}$  from all other polynomials by reducing them with polynomial  $p_{12}$  from previous step.

Finished a triangulation step, the current system is:

```
p_1 = -x_3 + 1
    p_2 = -x_2 + x_1
    p_3 = -x_2 - x_1 + 1
    p_4 = -x_5 + x_4
                     = x_5 + x_4 -
    p_6 = -x_8x_5 - x_7 + x_5
    p_7 = x_8x_4 + x_6 - x_4
                    = -x_7x_1 + x_6x_2
    p_9 = -x_{11}x_5 - x_{10} + x_5
p_{10} = x_{11}x_4 - x_{11} + x_9 - x_4
p_{11} = -x_{10}x_1 + x_{10} + x_9x_2 - x_2
p_{12} = x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1
p_{13} = -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1
p_{14} = -x_{14}x_5 + x_{14} - x_{13} + x_5
p_{15} = x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_{15}x_1 - x_{1
                                            x_5 + x_4x_2 - x_4 - x_2 + x_1
p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1
p_{17} = -x_{17}x_5 + x_{17} - x_{16} + x_5
```

#### 1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 12.

**Variable**  $x_{12}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{12}$ . No reduction needed.

The triangular system has not been changed.

#### 1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 11.

Variable  $x_{11}$  selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{11}$  from all other polynomials by reducing them with polynomial  $p_9$  from previous step.

Finished a triangulation step, the current system is:

```
p_1 = -x_3 + 1
    p_2 = -x_2 + x_1
                     = -x_2 - x_1 + 1
                     = -x_5 + x_4
    p_4
                       = x_5 + x_4 -
                       = -x_8x_5 - x_7 + x_5
    p_6
    p_7 = x_8 x_4 + x_6 - x_4
                        = -x_7x_1 + x_6x_2
                        = -x_{10}x_1 + x_{10} + x_9x_2 - x_2
    p_{\mathbf{q}}
p_{10} = x_{10}x_4 - x_{10} - x_9x_5 + x_5
                          = -x_{11}x_5 - x_{10} + x_5
p_{12} = x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1
p_{13} = -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1
                      = -x_{14}x_5 + x_{14} - x_{13} + x_5
                        = x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_{15}x_1 - x_{15}x_1 - x_{15}x_1 + x_{15}x_2 - x_{15}x_1 - x_{15}x_1 + x_{15}x_2 - x_{15}x_1 - x_{15}x_1 - x_{15}x_1 - x_{15}x_1 + x_{15}x_2 - x_{15}x_1 - x_{15}x_1 - x_{15}x_1 - x_{15}x_1 + x_{15}x_2 - x_{15}x_1 -
p_{15}
                                              x_5 + x_4x_2 - x_4 - x_2 + x_1
                        = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1
                      = -x_{17}x_5 + x_{17} - x_{16} + x_5
```

#### 1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 10.

Variable  $x_{10}$  selected: The number of polynomials with this variable, with indexes from 1 to 10, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{10}$  from all other polynomials by reducing them with polynomial  $p_9$  from previous step.

Finished a triangulation step, the current system is:

```
p_1 = -x_3 + 1
   p_2 = -x_2 + x_1
   p_3 = -x_2 - x_1 + 1
                    = -x_5 + x_4
                    = x_5 + x_4 -
   p_5
                    = -x_8x_5 - x_7 + x_5
   p_6
   p_7 = x_8 x_4 + x_6 - x_4
   p_8
                      = -x_7x_1 + x_6x_2
                     = x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2
                     = -x_{10}x_1 + x_{10} + x_9x_2 - x_2
p_{11} = -x_{11}x_5 - x_{10} + x_5
p_{12} = x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1
p_{13} = -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1
p_{14} = -x_{14}x_5 + x_{14} - x_{13} + x_5
p_{15} = x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_{15}x_1 - x_{1
                                          x_5 + x_4x_2 - x_4 - x_2 + x_1
p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1
p_{17} = -x_{17}x_5 + x_{17} - x_{16} + x_5
```

### 1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 9.

Variable  $x_9$  selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_9$ . No reduction needed.

The triangular system has not been changed.

#### 1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 8.

Variable  $x_8$  selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_8$  from all other polynomials by reducing them with polynomial  $p_6$  from previous step.

Finished a triangulation step, the current system is:

```
p_1 = -x_3 + 1
   p_2 = -x_2 + x_1
   p_3 = -x_2 - x_1 + 1
                      = -x_5 + x_4
                     = x_5 + x_4 -
   p_5
                      = -x_7x_1 + x_6x_2
   p_6
                      = x_7x_4 - x_6x_5
                     = -x_8x_5 - x_7 + x_5
   p_8
                      = x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2
   p_9
                      = -x_{10}x_1 + x_{10} + x_9x_2 - x_2
p_{11} = -x_{11}x_5 - x_{10} + x_5
p_{12} = x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1
p_{13} = -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1
p_{14} = -x_{14}x_5 + x_{14} - x_{13} + x_5
p_{15} = x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_{15}x_1 - x_{1
                                          x_5 + x_4 x_2 - x_4 - x_2 + x_1
p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1
p_{17} = -x_{17}x_5 + x_{17} - x_{16} + x_5
```

#### 1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 7.

Variable  $x_7$  selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_7$  from all other polynomials by reducing them with polynomial  $p_6$  from previous step.

$$\begin{array}{rcl} p_1 & = & -x_3+1 \\ p_2 & = & -x_2+x_1 \\ p_3 & = & -x_2-x_1+1 \\ p_4 & = & -x_5+x_4 \\ p_5 & = & x_5+x_4- \\ p_6 & = & x_6x_5x_1-x_6x_4x_2 \end{array}$$

```
p_7 = -x_7x_1 + x_6x_2
                          = -x_8x_5 - x_7 + x_5
                            = x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2
                            = -x_{10}x_1 + x_{10} + x_9x_2 - x_2
                        = -x_{11}x_5 - x_{10} + x_5
p_{11}
                         = x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1
p_{12}
                        = -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1
p_{13}
p_{14} = -x_{14}x_5 + x_{14} - x_{13} + x_5
                          = x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_{15}x_1 - x_{15}x_1 - x_{15}x_1 + x_{15}x_2 - x_{15}x_1 -
p_{15}
                                                   x_5 + x_4 x_2 - x_4 - x_2 + x_1
p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1
p_{17} = -x_{17}x_5 + x_{17} - x_{16} + x_5
```

#### 1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 6.

Variable  $x_6$  selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

#### 1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 5.

Variable  $x_5$  selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_5$  from all other polynomials by reducing them with polynomial  $p_4$  from previous step.

$$\begin{array}{rcl} p_1 & = & -x_3+1 \\ p_2 & = & -x_2+x_1 \\ p_3 & = & -x_2-x_1+1 \\ p_4 & = & -2x_4+1 \\ p_5 & = & -x_5+x_4 \\ p_6 & = & x_6x_5x_1-x_6x_4x_2 \\ p_7 & = & -x_7x_1+x_6x_2 \\ p_8 & = & -x_8x_5-x_7+x_5 \end{array}$$

```
\begin{array}{rcl} p_9 & = & x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2 \\ p_{10} & = & -x_{10}x_1 + x_{10} + x_9x_2 - x_2 \\ p_{11} & = & -x_{11}x_5 - x_{10} + x_5 \\ p_{12} & = & x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1 \\ p_{13} & = & -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1 \\ p_{14} & = & -x_{14}x_5 + x_{14} - x_{13} + x_5 \\ p_{15} & = & x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_5 + x_4x_2 - x_4 - x_2 + x_1 \\ p_{16} & = & -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1 \\ p_{17} & = & -x_{17}x_5 + x_{17} - x_{16} + x_5 \end{array}
```

#### 1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 4.

Variable  $x_4$  selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_4$ . No reduction needed.

The triangular system has not been changed.

#### 1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 3.

Variable  $x_3$  selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

#### 1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 2.

Variable  $x_2$  selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_2$  from all other polynomials by reducing them with polynomial  $p_1$  from previous step.

```
p_1 = 2x_1 -
                       = -x_2 + x_1
                                             -x_3 + 1
                                      -2x_4 + 1
    p_4
                       = -x_5 + x_4
                         = x_6x_5x_1 - x_6x_4x_2
    p_6
                        = -x_7x_1 + x_6x_2
                       = -x_8x_5 - x_7 + x_5
                         = x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2
                        = -x_{10}x_1 + x_{10} + x_9x_2 - x_2
p_{10}
p_{11} = -x_{11}x_5 - x_{10} + x_5
p_{12} = x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1
p_{13} = -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1
p_{14} = -x_{14}x_5 + x_{14} - x_{13} + x_5
                        = x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + x_{15}x_1 - x_{15}x_1 -
                                            x_5 + x_4x_2 - x_4 - x_2 + x_1
p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1
                                      -x_{17}x_5 + x_{17} - x_{16} + x_5
```

## 1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 1.

Variable  $x_1$  selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{array}{rcl} p_1 & = & 2x_1 - \\ p_2 & = & -x_2 + x_1 \\ p_3 & = & -x_3 + 1 \\ p_4 & = & -2x_4 + 1 \\ p_5 & = & -x_5 + x_4 \\ p_6 & = & x_6x_5x_1 - x_6x_4x_2 \\ p_7 & = & -x_7x_1 + x_6x_2 \\ p_8 & = & -x_8x_5 - x_7 + x_5 \\ p_9 & = & x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2 \end{array}$$

$$\begin{array}{lll} p_{10} & = & -x_{10}x_1 + x_{10} + x_9x_2 - x_2 \\ p_{11} & = & -x_{11}x_5 - x_{10} + x_5 \\ p_{12} & = & x_{12}x_5x_1 - x_{12}x_4x_2 + x_{12}x_4 - x_{12}x_1 \\ p_{13} & = & -x_{13}x_1 + x_{12}x_2 - x_{12} + x_1 \\ p_{14} & = & -x_{14}x_5 + x_{14} - x_{13} + x_5 \\ p_{15} & = & x_{15}x_5x_1 - x_{15}x_5 - x_{15}x_4x_2 + x_{15}x_4 + x_{15}x_2 - x_{15}x_1 - x_5x_1 + \\ & & x_5 + x_4x_2 - x_4 - x_2 + x_1 \\ p_{16} & = & -x_{16}x_1 + x_{16} + x_{15}x_2 - x_{15} - x_2 + x_1 \\ p_{17} & = & -x_{17}x_5 + x_{17} - x_{16} + x_5 \end{array}$$

## 2 Final Remainder

#### 2.1 Final remainder for conjecture geothm\_zadatak

Calculating final remainder of the conclusion:

$$g = -x_{17}x_{13}x_9 + x_{17}x_{13}x_6 + x_{17}x_{12}x_{10} - x_{17}x_{12}x_7 - x_{17}x_{10}x_6 + x_{17}x_{9}x_7 + x_{16}x_{14}x_9 - x_{16}x_{14}x_6 - x_{16}x_{12}x_{11} + x_{16}x_{12}x_8 + x_{16}x_{11}x_6 - x_{16}x_{9}x_8 - x_{15}x_{14}x_{10} + x_{15}x_{14}x_7 + x_{15}x_{13}x_{11} - x_{15}x_{13}x_8 - x_{15}x_{11}x_7 + x_{15}x_{10}x_8 + x_{14}x_{10}x_6 - x_{14}x_{9}x_7 - x_{13}x_{11}x_6 + x_{13}x_{9}x_8 + x_{12}x_{11}x_7 - x_{12}x_{10}x_8$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{17}$  over variable  $x_{17}$ :

$$\begin{array}{lll} g &=& -x_{16}x_{14}x_{9}x_{5} + x_{16}x_{14}x_{9} + x_{16}x_{14}x_{6}x_{5} - x_{16}x_{14}x_{6} \\ & -x_{16}x_{13}x_{9} + x_{16}x_{13}x_{6} + x_{16}x_{12}x_{11}x_{5} - x_{16}x_{12}x_{11} + \\ & x_{16}x_{12}x_{10} - x_{16}x_{12}x_{8}x_{5} + x_{16}x_{12}x_{8} - x_{16}x_{12}x_{7} \\ & -x_{16}x_{11}x_{6}x_{5} + x_{16}x_{11}x_{6} - x_{16}x_{10}x_{6} + x_{16}x_{9}x_{8}x_{5} \\ & -x_{16}x_{9}x_{8} + x_{16}x_{9}x_{7} + x_{15}x_{14}x_{10}x_{5} - x_{15}x_{14}x_{10} \\ & -x_{15}x_{14}x_{7}x_{5} + x_{15}x_{14}x_{7} - x_{15}x_{13}x_{11}x_{5} + x_{15}x_{13}x_{11} + \\ & x_{15}x_{13}x_{8}x_{5} - x_{15}x_{13}x_{8} + x_{15}x_{11}x_{7}x_{5} - x_{15}x_{11}x_{7} \\ & -x_{15}x_{10}x_{8}x_{5} + x_{15}x_{10}x_{8} - x_{14}x_{10}x_{6}x_{5} + x_{14}x_{10}x_{6} + \\ & x_{14}x_{9}x_{7}x_{5} - x_{14}x_{9}x_{7} + x_{13}x_{11}x_{6}x_{5} - x_{13}x_{11}x_{6} \\ & -x_{13}x_{9}x_{8}x_{5} + x_{13}x_{9}x_{8} + x_{13}x_{9}x_{5} - x_{13}x_{6}x_{5} \\ & -x_{12}x_{11}x_{7}x_{5} + x_{12}x_{11}x_{7} + x_{12}x_{10}x_{8}x_{5} - x_{12}x_{10}x_{8} \\ & -x_{12}x_{10}x_{5} + x_{12}x_{7}x_{5} + x_{10}x_{6}x_{5} - x_{9}x_{7}x_{5} \end{array}$$

2. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :

Polynomial too big for output (text size is 3006 characters, number of terms is 132)

- 3. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :

  Polynomial too big for output (number of terms is 480)
- 4. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :

  Polynomial too big for output (number of terms is 646)
- 5. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :

  Polynomial too big for output (number of terms is 638)
- 6. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :

  Polynomial too big for output (number of terms is 414)
- 7. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :

  Polynomial too big for output (number of terms is 357)
- 8. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :

  Polynomial too big for output (number of terms is 546)
- 9. Pseudo remainder with  $p_9$  over variable  $x_9$ :

  Polynomial too big for output (number of terms is 398)
- 10. Pseudo remainder with  $p_8$  over variable  $x_8$ :

  Polynomial too big for output (text size is 7150 characters, number of terms is 199)
- 11. Pseudo remainder with  $p_7$  over variable  $x_7$ :

  Polynomial too big for output (text size is 7694 characters, number of terms is 199)
- 12. Pseudo remainder with  $p_6$  over variable  $x_6$ :

g = 0

13. Pseudo remainder with  $p_5$  over variable  $x_5$ :

g = 0

14. Pseudo remainder with  $p_4$  over variable  $x_4$ :

g = 0

15. Pseudo remainder with  $p_3$  over variable  $x_3$ :

$$g = 0$$

16. Pseudo remainder with  $p_2$  over variable  $x_2$ :

$$g = 0$$

17. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

# 3 Prover results

Status: Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 646 terms.

**Time Complexity:** Time spent by the prover is 0.338 seconds.

# 4 NDG Conditions

#### NDG Conditions in readable form

• Failed to translate NDG Conditions to readable form