Balkan MO 1984

1 Let $n \geq 2$ be a positive integer and $a_1, \dots, a_n$ be positive real numbers such that $a_1 + ... + a_n = 1$. Prove that:

$$\frac{a_1}{1 + a_2 + \cdots + a_n} + \cdots + \frac{a_n}{1 + a_1 + a_2 + \cdots + a_{n-1}} \geq \frac{n}{2n-1}$$

2 Let $ABCD$ be a cyclic quadrilateral and let H_A, H_B, H_C, H_D be the orthocenters of the triangles BCD, CDA, DAB and ABC respectively. Show that the quadrilaterals $ABCD$ and $H_AH_BH_CH_D$ are congruent.

3 Show that for any positive integer m, there exists a positive integer n so that in the decimal representations of the numbers 5^m and 5^n, the representation of 5^n ends in the representation of 5^m.

4 Let a, b, c be positive real numbers. Find all real solutions (x, y, z) of the system:

$$ax + by = (x - y)^2 by + cz = (y - z)^2 cz + ax = (z - x)^2$$