OpenGeoProver Output for conjecture “geothm_zadatak”

Wu’s method used

September 30, 2016

1 Invoking the theorem prover

The used proving method is Wu’s method.
The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 2x_9 - x_3 \\
p_9 &= 2x_{10} - x_1 - \\
p_{10} &= 2x_{11} - x_2 - \\
p_{11} &= 2x_{12} - x_3 \\
p_{12} &= 2x_{13} - x_1 \\
p_{13} &= 2x_{14} - x_2 - \\
p_{14} &= 2x_{15} - x_3 \\
p_{15} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{16} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{17} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5 \\
p_{18} &= x_{19} + x_{18}x_6 + x_{17}x_5 + x_{16}x_4
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{18} from all other polynomials by reducing them with polynomial p_{17} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 2x_9 - x_3 \\
p_9 &= 2x_{10} - x_1 - \\
p_{10} &= 2x_{11} - x_2 - \\
p_{11} &= 2x_{12} - x_3 \\
p_{12} &= 2x_{13} - x_1 \\
p_{13} &= 2x_{14} - x_2 - \\
p_{14} &= 2x_{15} - x_3 \\
p_{15} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{16} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{17} &= x_{19} + x_{17}x_5 + x_{16}x_4 + x_{11}x_7x_6 - x_{11}x_6x_4 - x_{10}x_8x_6 + \\
&\quad x_{10}x_6x_5 + x_8x_6x_4 - x_7x_6x_5 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3
\end{aligned}
$$

$$
\begin{aligned}
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 2x_9 - x_3 \\
p_9 &= 2x_{10} - x_1 - \\
p_{10} &= 2x_{11} - x_2 - \\
p_{11} &= 2x_{12} - x_3 \\
p_{12} &= 2x_{13} - x_1 \\
p_{13} &= 2x_{14} - x_2 - \\
p_{14} &= 2x_{15} - x_3 \\
p_{15} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{16} &= x_{19} + x_{16}x_4 - x_{12}x_7x_5 + x_{12}x_5x_4 + x_{11}x_7x_6 - x_{11}x_6x_4 + \\
&\quad x_{10}x_9x_5 - x_{10}x_8x_6 - x_9x_5x_4 + x_8x_6x_4 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 2x_9 - x_3 \\
p_9 &= 2x_{10} - x_1 - \\
p_{10} &= 2x_{11} - x_2 - \\
p_{11} &= 2x_{12} - x_3 \\
p_{12} &= 2x_{13} - x_1 \\
p_{13} &= 2x_{14} - x_2 - \\
p_{14} &= 2x_{15} - x_3 \\
p_{15} &= x_{19} + x_{12}x_8x_4 - x_{12}x_7x_5 - x_{11}x_9x_4 + x_{11}x_7x_6 + x_{10}x_9x_5
\end{aligned}
$$

$$
\begin{array}{rcl}
 & & -x_{10}x_8x_6 \\
p_{16} & = & x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} & = & x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} & = & x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{array}
$$

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 2x_9 - x_3 \\
p_9 &= 2x_{10} - x_1 - \\
p_{10} &= 2x_{11} - x_2 - \\
p_{11} &= 2x_{19} - 2x_{11}x_9x_4 + 2x_{11}x_7x_6 + 2x_{10}x_9x_5 - 2x_{10}x_8x_6 + \\
&\quad x_8x_4x_3 - x_7x_5x_3 \\
p_{12} &= 2x_{12} - x_3 \\
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}$$

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2
\end{aligned}$$

$$
\begin{aligned}
p_8 &= 2x_9 - x_3 \\
p_9 &= 2x_{10} - x_1 - \\
p_{10} &= 4x_{19} + 4x_{10}x_9x_5 - 4x_{10}x_8x_6 - 2x_9x_4x_2 - 2x_9x_4 + 2x_8x_4x_3 + \\
&\quad 2x_7x_6x_2 + 2x_7x_6 - 2x_7x_5x_3 \\
p_{11} &= 2x_{11} - x_2 - \\
p_{12} &= 2x_{12} - x_3 \\
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{10} from all other polynomials by reducing them with polynomial p_9 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 2x_9 - x_3 \\
p_9 &= 8x_{19} + 4x_9x_5x_1 + 4x_9x_5 - 4x_9x_4x_2 - 4x_9x_4 - 4x_8x_6x_1 \\
&\quad -4x_8x_6 + 4x_8x_4x_3 + 4x_7x_6x_2 + 4x_7x_6 - 4x_7x_5x_3 \\
p_{10} &= 2x_{10} - x_1 - \\
p_{11} &= 2x_{11} - x_2 - \\
p_{12} &= 2x_{12} - x_3 \\
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6
\end{aligned}
$$

$$
\begin{aligned}
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 2x_8 - x_2 \\
p_8 &= 16x_{19} - 8x_8x_6x_1 - 8x_8x_6 + 8x_8x_4x_3 + 8x_7x_6x_2 + 8x_7x_6 \\
&\quad -8x_7x_5x_3 + 4x_5x_3x_1 + 4x_5x_3 - 4x_4x_3x_2 - 4x_4x_3 \\
p_9 &= 2x_9 - x_3 \\
p_{10} &= 2x_{10} - x_1 - \\
p_{11} &= 2x_{11} - x_2 - \\
p_{12} &= 2x_{12} - x_3 \\
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_7 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3 \\
p_6 &= 2x_7 - x_1 - \\
p_7 &= 32x_{19} + 16x_7x_6x_2 + 16x_7x_6 - 16x_7x_5x_3 - 8x_6x_2x_1 - 8x_6x_2 + \\
&\quad 8x_5x_3x_1 + 8x_5x_3 - 8x_4x_3 \\
p_8 &= 2x_8 - x_2 \\
p_9 &= 2x_9 - x_3 \\
p_{10} &= 2x_{10} - x_1 - \\
p_{11} &= 2x_{11} - x_2 - \\
p_{12} &= 2x_{12} - x_3 \\
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}$$

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_4 - x_1 \\
p_4 &= 2x_5 - x_2 \\
p_5 &= 2x_6 - x_3
\end{aligned}$$

$$p_6 = 64x_{19} + 16x_6x_1 + 16x_6 - 16x_4x_3$$
$$p_7 = 2x_7 - x_1 -$$
$$p_8 = 2x_8 - x_2$$
$$p_9 = 2x_9 - x_3$$
$$p_{10} = 2x_{10} - x_1 -$$
$$p_{11} = 2x_{11} - x_2 -$$
$$p_{12} = 2x_{12} - x_3$$
$$p_{13} = 2x_{13} - x_1$$
$$p_{14} = 2x_{14} - x_2 -$$
$$p_{15} = 2x_{15} - x_3$$
$$p_{16} = x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6$$
$$p_{17} = x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6$$
$$p_{18} = x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5$$

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_5 from previous step.

Finished a triangulation step, the current system is:

$$p_1 = 2x_1 -$$
$$p_2 = 2x_2 -$$
$$p_3 = 2x_4 - x_1$$
$$p_4 = 2x_5 - x_2$$
$$p_5 = 128x_{19} - 32x_4x_3 + 16x_3x_1 + 16x_3$$
$$p_6 = 2x_6 - x_3$$
$$p_7 = 2x_7 - x_1 -$$
$$p_8 = 2x_8 - x_2$$
$$p_9 = 2x_9 - x_3$$
$$p_{10} = 2x_{10} - x_1 -$$
$$p_{11} = 2x_{11} - x_2 -$$
$$p_{12} = 2x_{12} - x_3$$
$$p_{13} = 2x_{13} - x_1$$
$$p_{14} = 2x_{14} - x_2 -$$
$$p_{15} = 2x_{15} - x_3$$
$$p_{16} = x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6$$

$$
\begin{aligned}
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 256x_{19} + 32x_3 \\
p_4 &= 2x_4 - x_1 \\
p_5 &= 2x_5 - x_2 \\
p_6 &= 2x_6 - x_3 \\
p_7 &= 2x_7 - x_1 - \\
p_8 &= 2x_8 - x_2 \\
p_9 &= 2x_9 - x_3 \\
p_{10} &= 2x_{10} - x_1 - \\
p_{11} &= 2x_{11} - x_2 - \\
p_{12} &= 2x_{12} - x_3 \\
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 256x_{19} + 32x_3 \\
p_4 &= 2x_4 - x_1 \\
p_5 &= 2x_5 - x_2 \\
p_6 &= 2x_6 - x_3 \\
p_7 &= 2x_7 - x_1 - \\
p_8 &= 2x_8 - x_2 \\
p_9 &= 2x_9 - x_3 \\
p_{10} &= 2x_{10} - x_1 - \\
p_{11} &= 2x_{11} - x_2 - \\
p_{12} &= 2x_{12} - x_3
\end{aligned}
$$

$$
\begin{aligned}
p_{13} &= 2x_{13} - x_1 \\
p_{14} &= 2x_{14} - x_2 - \\
p_{15} &= 2x_{15} - x_3 \\
p_{16} &= x_{16} - x_{12}x_8 + x_{12}x_5 + x_{11}x_9 - x_{11}x_6 - x_9x_5 + x_8x_6 \\
p_{17} &= x_{17} + x_{12}x_7 - x_{12}x_4 - x_{10}x_9 + x_{10}x_6 + x_9x_4 - x_7x_6 \\
p_{18} &= x_{18} - x_{11}x_7 + x_{11}x_4 + x_{10}x_8 - x_{10}x_5 - x_8x_4 + x_7x_5
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
g = x_{19} + x_{18}x_{15} + x_{17}x_{14} + x_{16}x_{13}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{18} over variable x_{18}:

$$
\begin{aligned}
g = \; & x_{19} + x_{17}x_{14} + x_{16}x_{13} + x_{15}x_{11}x_7 - x_{15}x_{11}x_4 - x_{15}x_{10}x_8 + \\
& x_{15}x_{10}x_5 + x_{15}x_8x_4 - x_{15}x_7x_5
\end{aligned}
$$

2. Pseudo remainder with p_{17} over variable x_{17}:

$$
\begin{aligned}
g = \; & x_{19} + x_{16}x_{13} + x_{15}x_{11}x_7 - x_{15}x_{11}x_4 - x_{15}x_{10}x_8 + x_{15}x_{10}x_5 + \\
& x_{15}x_8x_4 - x_{15}x_7x_5 - x_{14}x_{12}x_7 + x_{14}x_{12}x_4 + x_{14}x_{10}x_9 \\
& -x_{14}x_{10}x_6 - x_{14}x_9x_4 + x_{14}x_7x_6
\end{aligned}
$$

3. Pseudo remainder with p_{16} over variable x_{16}:

$$
\begin{aligned}
g = \; & x_{19} + x_{15}x_{11}x_7 - x_{15}x_{11}x_4 - x_{15}x_{10}x_8 + x_{15}x_{10}x_5 + \\
& x_{15}x_8x_4 - x_{15}x_7x_5 - x_{14}x_{12}x_7 + x_{14}x_{12}x_4 + x_{14}x_{10}x_9 \\
& -x_{14}x_{10}x_6 - x_{14}x_9x_4 + x_{14}x_7x_6 + x_{13}x_{12}x_8 - x_{13}x_{12}x_5 \\
& -x_{13}x_{11}x_9 + x_{13}x_{11}x_6 + x_{13}x_9x_5 - x_{13}x_8x_6
\end{aligned}
$$

4. Pseudo remainder with p_{15} over variable x_{15}:

$$
\begin{aligned}
g = \; & 2x_{19} - 2x_{14}x_{12}x_7 + 2x_{14}x_{12}x_4 + 2x_{14}x_{10}x_9 - 2x_{14}x_{10}x_6 \\
& -2x_{14}x_9x_4 + 2x_{14}x_7x_6 + 2x_{13}x_{12}x_8 - 2x_{13}x_{12}x_5 \\
& -2x_{13}x_{11}x_9 + 2x_{13}x_{11}x_6 + 2x_{13}x_9x_5 - 2x_{13}x_8x_6 + x_{11}x_7x_3 \\
& -x_{11}x_4x_3 - x_{10}x_8x_3 + x_{10}x_5x_3 + x_8x_4x_3 - x_7x_5x_3
\end{aligned}
$$

5. Pseudo remainder with p_{14} over variable x_{14}:

$$\begin{aligned} g \quad = \quad & 4x_{19} + 4x_{13}x_{12}x_8 - 4x_{13}x_{12}x_5 - 4x_{13}x_{11}x_9 + 4x_{13}x_{11}x_6 + \\ & 4x_{13}x_9x_5 - 4x_{13}x_8x_6 - 2x_{12}x_7x_2 - 2x_{12}x_7 + 2x_{12}x_4x_2 + \\ & 2x_{12}x_4 + 2x_{11}x_7x_3 - 2x_{11}x_4x_3 + 2x_{10}x_9x_2 + 2x_{10}x_9 \\ & -2x_{10}x_8x_3 - 2x_{10}x_6x_2 - 2x_{10}x_6 + 2x_{10}x_5x_3 - 2x_9x_4x_2 \\ & -2x_9x_4 + 2x_8x_4x_3 + 2x_7x_6x_2 + 2x_7x_6 - 2x_7x_5x_3 \end{aligned}$$

6. Pseudo remainder with p_{13} over variable x_{13}:

$$\begin{aligned} g \quad = \quad & 8x_{19} + 4x_{12}x_8x_1 - 4x_{12}x_7x_2 - 4x_{12}x_7 - 4x_{12}x_5x_1 + 4x_{12}x_4x_2 + \\ & 4x_{12}x_4 - 4x_{11}x_9x_1 + 4x_{11}x_7x_3 + 4x_{11}x_6x_1 - 4x_{11}x_4x_3 + \\ & 4x_{10}x_9x_2 + 4x_{10}x_9 - 4x_{10}x_8x_3 - 4x_{10}x_6x_2 - 4x_{10}x_6 + \\ & 4x_{10}x_5x_3 + 4x_9x_5x_1 - 4x_9x_4x_2 - 4x_9x_4 - 4x_8x_6x_1 + \\ & 4x_8x_4x_3 + 4x_7x_6x_2 + 4x_7x_6 - 4x_7x_5x_3 \end{aligned}$$

7. Pseudo remainder with p_{12} over variable x_{12}:

$$\begin{aligned} g \quad = \quad & 16x_{19} - 8x_{11}x_9x_1 + 8x_{11}x_7x_3 + 8x_{11}x_6x_1 - 8x_{11}x_4x_3 + \\ & 8x_{10}x_9x_2 + 8x_{10}x_9 - 8x_{10}x_8x_3 - 8x_{10}x_6x_2 - 8x_{10}x_6 + \\ & 8x_{10}x_5x_3 + 8x_9x_5x_1 - 8x_9x_4x_2 - 8x_9x_4 - 8x_8x_6x_1 + \\ & 8x_8x_4x_3 + 4x_8x_3x_1 + 8x_7x_6x_2 + 8x_7x_6 - 8x_7x_5x_3 \\ & -4x_7x_3x_2 - 4x_7x_3 - 4x_5x_3x_1 + 4x_4x_3x_2 + 4x_4x_3 \end{aligned}$$

8. Pseudo remainder with p_{11} over variable x_{11}:

$$\begin{aligned} g \quad = \quad & 32x_{19} + 16x_{10}x_9x_2 + 16x_{10}x_9 - 16x_{10}x_8x_3 - 16x_{10}x_6x_2 \\ & -16x_{10}x_6 + 16x_{10}x_5x_3 + 16x_9x_5x_1 - 16x_9x_4x_2 - 16x_9x_4 \\ & -8x_9x_2x_1 - 8x_9x_1 - 16x_8x_6x_1 + 16x_8x_4x_3 + 8x_8x_3x_1 + \\ & 16x_7x_6x_2 + 16x_7x_6 - 16x_7x_5x_3 + 8x_6x_2x_1 + 8x_6x_1 \\ & -8x_5x_3x_1 \end{aligned}$$

9. Pseudo remainder with p_{10} over variable x_{10}:

$$\begin{aligned} g \quad = \quad & 64x_{19} + 32x_9x_5x_1 - 32x_9x_4x_2 - 32x_9x_4 + 16x_9x_2 + 16x_9 \\ & -32x_8x_6x_1 + 32x_8x_4x_3 - 16x_8x_3 + 32x_7x_6x_2 + 32x_7x_6 \\ & -32x_7x_5x_3 - 16x_6x_2 - 16x_6 + 16x_5x_3 \end{aligned}$$

10. Pseudo remainder with p_9 over variable x_9:

$$\begin{aligned} g &= 128x_{19} - 64x_8x_6x_1 + 64x_8x_4x_3 - 32x_8x_3 + 64x_7x_6x_2 + 64x_7x_6 \\ &\quad -64x_7x_5x_3 - 32x_6x_2 - 32x_6 + 32x_5x_3x_1 + 32x_5x_3 - 32x_4x_3x_2 \\ &\quad -32x_4x_3 + 16x_3x_2 + 16x_3 \end{aligned}$$

11. Pseudo remainder with p_8 over variable x_8:

$$\begin{aligned} g &= 256x_{19} + 128x_7x_6x_2 + 128x_7x_6 - 128x_7x_5x_3 - 64x_6x_2x_1 - 64x_6x_2 \\ &\quad -64x_6 + 64x_5x_3x_1 + 64x_5x_3 - 64x_4x_3 + 32x_3 \end{aligned}$$

12. Pseudo remainder with p_7 over variable x_7:

$$g = 512x_{19} + 128x_6x_1 - 128x_4x_3 + 64x_3$$

13. Pseudo remainder with p_6 over variable x_6:

$$g = 1024x_{19} - 256x_4x_3 + 128x_3x_1 + 128x_3$$

14. Pseudo remainder with p_5 over variable x_5:

$$g = 1024x_{19} - 256x_4x_3 + 128x_3x_1 + 128x_3$$

15. Pseudo remainder with p_4 over variable x_4:

$$g = 2048x_{19} + 256x_3$$

16. Pseudo remainder with p_3 over variable x_3:

$$g = 0$$

17. Pseudo remainder with p_2 over variable x_2:

$$g = 0$$

18. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 25 terms.

Time Complexity: Time spent by the prover is 0.108 seconds.

4 NDG Conditions

NDG Conditions in readable form

- There are no NDG conditions for this theorem