OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

September 30, 2016

1 Invoking the theorem prover

The used proving method is Wu's method. The input system is:

```
p_1 = 2x_1 -
  p_2 = 4x_2^2 - 3
  p_3 = 3x_3 - x_2
  p_4 = 3x_4^2 - 2
  p_5 = x_5 - x_2
  p_6 = x_6 - x_4 x_2
  p_7 = x_7 + x_4 x_1
  p_8 = x_8 - x_3 x_1 + x_2 x_1
       = x_{10} - x_3
p_{10} = -x_{12}x_5 + x_{11} - x_4
p_{11} = -x_{13}x_6 + x_9 -
p_{12} = -x_{13}x_7 + x_{10}
p_{13} = -x_{13}x_8 + x_{11}
p_{14} = 2x_{14} -
p_{15} = 2x_{15} - x_1 -
p_{16} = 2x_{16} - x_2
p_{17} = 2x_{17} - x_1
p_{18} = 2x_{18} - x_2
p_{19} = x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_{9} + x_{14}x_{9} - x_{11}^{2} - x_{10}^{2} - x_{9}^{2}
p_{20} = x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_{9} + x_{16}x_{10} + x_{15}x_{9} - x_{11}^{2}
p_{21} = x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{22} = x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{23} = x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{24} = x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
```

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{24} . No reduction needed.

The triangular system has not been changed.

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{23} . No reduction needed.

The triangular system has not been changed.

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20} . No reduction needed.

The triangular system has not been changed.

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{21} . No reduction needed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20} . No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{19} . No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18} . No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17} . No reduction needed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16} . No reduction needed.

The triangular system has not been changed.

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15} . No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14} . No reduction needed.

The triangular system has not been changed.

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 13.

Variable x_{13} **selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

```
p_1 = 2x_1 -
 p_2 = 4x_2^2 - 3
 p_3 = 3x_3 - x_2
 p_4 = 3x_4^2 - 2
 p_5 = x_5 - x_2
 p_6 = x_6 - x_4 x_2
 p_7 = x_7 + x_4 x_1
      = x_8 - x_3x_1 + x_2x_1
 p_9 = x_{10} - x_3
p_{10} = -x_{12}x_5 + x_{11} - x_4
p_{11} = -x_{10}x_6 + x_9x_7 - x_7
p_{12} = -x_{11}x_6 + x_9x_8 - x_8
p_{13} = -x_{13}x_6 + x_9 -
p_{14} = 2x_{14} -
p_{15} = 2x_{15} - x_1 -
p_{16} = 2x_{16} - x_2
p_{17} = 2x_{17} - x_1
p_{18} = 2x_{18} - x_2
p_{19} = x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_{9} + x_{14}x_{9} - x_{11}^{2} - x_{10}^{2} - x_{9}^{2}
p_{20} = x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{21} = x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{22} = x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_{9} + x_{16}x_{10} + x_{15}x_{9} - x_{11}^{2}
p_{23} = x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{24} = x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
```

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10} . No reduction needed.

The triangular system has not been changed.

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11} . No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{10} from all other polynomials by reducing them with polynomial p_9 from previous step.

Finished a triangulation step, the current system is:

```
p_1 = 2x_1 -
 p_2 = 4x_2^2 - 3
 p_3 = 3x_3 - x_2
 p_4 = 3x_4^2 - 2
 p_5 = x_5 - x_2
 p_6 = x_6 - x_4 x_2
 p_7 = x_7 + x_4 x_1
 p_8 = x_8 - x_3 x_1 + x_2 x_1
 p_9 = x_9x_7 - x_7 - x_6x_3
p_{10} = x_{10} - x_3
p_{11} = -x_{11}x_6 + x_9x_8 - x_8
p_{12} = -x_{12}x_5 + x_{11} - x_4
p_{13} = -x_{13}x_6 + x_9 -
p_{14} = 2x_{14} -
p_{15} = 2x_{15} - x_1 -
p_{16} = 2x_{16} - x_2
p_{17} = 2x_{17} - x_1
p_{18} = 2x_{18} - x_2
p_{19} = x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_{9} + x_{14}x_{9} - x_{11}^{2} - x_{10}^{2} - x_{9}^{2}
p_{20} = x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_{9}^2
p_{21} = x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_{9}^2
p_{22} = x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_{9} + x_{16}x_{10} + x_{15}x_{9} - x_{11}^{2}
             -x_{10}^2-x_{0}^2
p_{23} = x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
p_{24} = x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
```

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9 . No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8 . No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7 . No reduction needed.

The triangular system has not been changed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6 . No reduction needed.

1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5 . No reduction needed.

The triangular system has not been changed.

1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4 . No reduction needed.

The triangular system has not been changed.

1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3 . No reduction needed.

The triangular system has not been changed.

1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2 . No reduction needed.

1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$g = -x_{24}x_{23}x_{22}^2 + x_{21}x_{20}x_{19}^2$$

with respect to the triangular system.

1. Pseudo remainder with p_{24} over variable x_{24} :

$$g = -x_{23}x_{22}^2x_{16}^2 + 2x_{23}x_{22}^2x_{16}x_{10} - x_{23}x_{22}^2x_{15}^2 + 2x_{23}x_{22}^2x_{15}x_9 - x_{23}x_{22}^2x_{11}^2 - x_{23}x_{22}^2x_{10}^2 - x_{23}x_{22}^2x_{2}^9 + x_{21}x_{20}x_{19}^2$$

2. Pseudo remainder with p_{23} over variable x_{23} :

$$\begin{array}{rcl} g&=&-x_{22}^2x_{16}^2x_{14}^2+2x_{22}^2x_{16}^2x_{14}x_9\\ &-x_{22}^2x_{16}^2x_{11}^2-x_{22}^2x_{16}^2x_{10}^2\\ &-x_{22}^2x_{16}^2x_9^2+2x_{22}^2x_{16}x_{14}^2x_{10}\\ &-4x_{22}^2x_{16}x_{14}x_{10}x_9+2x_{22}^2x_{16}x_{11}^2x_{10}+\\ &2x_{22}^2x_{16}x_{10}^3+2x_{22}^2x_{16}x_{10}x_9^2-x_{22}^2x_{15}^2x_{14}^2+\\ &2x_{22}^2x_{15}^2x_{14}x_9-x_{22}^2x_{15}^2x_{11}^2\\ &-x_{22}^2x_{15}^2x_{10}^2-x_{22}^2x_{15}^2x_{14}^2+\\ &2x_{22}^2x_{15}x_{14}^2x_9-4x_{22}^2x_{15}x_{14}x_9^2+\\ &2x_{22}^2x_{15}x_{11}^2x_9+2x_{22}^2x_{15}x_{10}^2x_9+2x_{22}^2x_{15}x_9^3\\ &-x_{22}^2x_{14}^2x_{11}^2-x_{22}^2x_{14}^2x_{10}^2\\ &-x_{22}^2x_{14}^2x_{10}^2+2x_{22}^2x_{14}x_{11}^2x_9+\\ &2x_{22}^2x_{14}x_{10}^2x_9+2x_{22}^2x_{14}x_{11}^3x_9+2x_{22}^2x_{11}^4\\ &-2x_{22}^2x_{11}^2x_{10}^2-2x_{22}^2x_{11}^2x_{10}^2-2x_{22}^2x_{10}^4\\ &-2x_{22}^2x_{10}^2x_0^2-x_{22}^2x_0^4+x_{21}x_{20}x_{10}^2\end{array}$$

3. Pseudo remainder with p_{22} over variable x_{22} :

Polynomial too big for output (number of terms is 745)

4. Pseudo remainder with p_{21} over variable x_{21} :

Polynomial too big for output (number of terms is 751)

5. Pseudo remainder with p_{20} over variable x_{20} :

Polynomial too big for output (number of terms is 790)

6. Pseudo remainder with p_{19} over variable x_{19} :

Polynomial too big for output (number of terms is 546)

7. Pseudo remainder with p_{18} over variable x_{18} :

Polynomial too big for output (number of terms is 546)

8. Pseudo remainder with p_{17} over variable x_{17} :

Polynomial too big for output (number of terms is 546)

9. Pseudo remainder with p_{16} over variable x_{16} :

Polynomial too big for output (number of terms is 361)

10. Pseudo remainder with p_{15} over variable x_{15} :

Polynomial too big for output (number of terms is 529)

11. Pseudo remainder with p_{14} over variable x_{14} :

Polynomial too big for output (number of terms is 273)

12. Pseudo remainder with p_{13} over variable x_{13} :

Polynomial too big for output (number of terms is 273)

13. Pseudo remainder with p_{12} over variable x_{12} :

Polynomial too big for output (number of terms is 273)

14. Pseudo remainder with p_{11} over variable x_{11} :

Polynomial too big for output (number of terms is 438)

15. Pseudo remainder with p_{10} over variable x_{10} :

Polynomial too big for output (number of terms is 438)

16. Pseudo remainder with p_9 over variable x_9 :

Polynomial too big for output (number of terms is 291)

17. Pseudo remainder with p_8 over variable x_8 :

Polynomial too big for output (number of terms is 471)

18. Pseudo remainder with p_7 over variable x_7 :

Polynomial too big for output (number of terms is 471)

19. Pseudo remainder with p_6 over variable x_6 :

Polynomial too big for output (number of terms is 297)

- 20. Pseudo remainder with p_5 over variable x_5 :

 Polynomial too big for output (number of terms is 297)
- 21. Pseudo remainder with p_4 over variable x_4 :

 Polynomial too big for output (text size is 8177 characters, number of terms is 226)
- 22. Pseudo remainder with p_3 over variable x_3 :

```
g = -56623104x_2^{20}x_1^6 - 254803968x_2^{20}x_1^4 - 382205952x_2^{20}x_1^2
          -191102976x_2^{20} - 56623104x_2^{18}x_1^8 - 835190784x_2^{18}x_1^6 +
          1019215872x_2^{18}x_1^5 - 2123366400x_2^{18}x_1^4 + 3057647616x_2^{18}x_1^3
          -1496973312x_{2}^{18}x_{1}^{2} + 2293235712x_{2}^{18}x_{1} - 1019215872x_{2}^{16}x_{1}^{8} +
          2165833728x_2^{16}x_1^7 - 5995855872x_2^{16}x_1^6 + 13345357824x_2^{16}x_1^5 \\
          -14121713664x_2^{16}x_1^4 + 15144910848x_2^{16}x_1^3
          -11131748352x_2^{16}x_1^2 - 382205952x_2^{14}x_1^{10} + 1146617856x_2^{14}x_1^9
          -6521389056x_2^{14}x_1^8 + 22024617984x_2^{14}x_1^7
          -37169528832x_2^{14}x_1^6 + 52326383616x_2^{14}x_1^5
          -59026931712x_2^{14}x_1^4 + 28092137472x_2^{14}x_1^3
          -3081535488x_2^{12}x_1^{10} + 15479341056x_2^{12}x_1^{9}
          -36882874368x_2^{12}x_1^8 + 74995974144x_2^{12}x_1^7
          -122989694976x_2^{12}x_1^6 + 111616081920x_2^{12}x_1^5
          -39020838912x_2^{12}x_1^4 - 644972544x_2^{10}x_1^{12} +
          3869835264x_2^{10}x_1^{11} - 13114441728x_2^{10}x_1^{10} +
          42890674176x_2^{10}x_1^9 - 104754290688x_2^{10}x_1^8 + \\
          146355019776x_2^{10}x_1^7 - 102980616192x_2^{10}x_1^6 +
          28378791936x_{2}^{10}x_{1}^{5} - 967458816x_{2}^{8}x_{1}^{12} + 8707129344x_{2}^{8}x_{1}^{11}
          -32168005632x_2^8x_1^{10}+62401093632x_2^8x_1^9-66754658304x_2^8x_1^8+\\
          37247164416x_2^8x_1^7 - 8465264640x_2^8x_1^6
```

23. Pseudo remainder with p_2 over variable x_2 :

```
\begin{array}{lll} g&=&-481469424205824x_1^{12}+3851755393646592x_1^{11}-14564450082226176x_1^{10}+\\ &&34425063830716416x_1^9-56121279758991360x_1^8+66081678472249344x_1^7\\ &&-57287338520739840x_1^6+36696997676187648x_1^5-17208770435481600x_1^4+\\ &&5762587170963456x_1^3-1308994997059584x_1^2+180551034077184x_1-11284439629824 \end{array}
```

24. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution

contains 963 terms.

Time Complexity: Time spent by the prover is 0.835 seconds.

4 NDG Conditions

NDG Conditions in readable form

• Failed to translate NDG Conditions to readable form