OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

October 2, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.
The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 - \\
p_9 &= x_9 + x_4 x_1 \\
p_{10} &= x_{10} - x_3 x_1 + x_2 x_1 \\
p_{11} &= 2x_{14} - \\
p_{12} &= 2x_{15} - x_1 - \\
p_{13} &= 2x_{16} - x_2 \\
p_{14} &= 2x_{17} - x_1 \\
p_{15} &= 2x_{18} - x_2 \\
p_{16} &= x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{17} &= x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
&\quad - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{18} &= x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{19} &= x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{20} &= x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{21} &= x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{22} &= -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9 \\
p_{23} &= x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7 x_1 - x_5 x_4 \\
p_{24} &= -x_{13}x_6 + x_{13}x_3 + x_{12}x_7 - x_{12}x_4 - x_7 x_3 + x_6 x_4
\end{aligned}
$$

1.1   Triangulation, step 1

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{21}. No reduction needed.

The triangular system has not been changed.

1.2   Triangulation, step 2

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20}. No reduction needed.

The triangular system has not been changed.

1.3   Triangulation, step 3

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.4   Triangulation, step 4

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18}. No reduction needed.

The triangular system has not been changed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.9   Triangulation, step 9

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.10   Triangulation, step 10

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.11   Triangulation, step 11

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.12   Triangulation, step 12

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 - \\
p_9 &= x_9 + x_4 x_1 \\
p_{10} &= x_{10} - x_3 x_1 + x_2 x_1 \\
p_{11} &= -x_{12} x_8 + x_{12} + x_{11} x_9 - x_9 \\
p_{12} &= x_{12} x_7 x_5 - x_{12} x_7 x_1 - x_{12} x_5 x_4 + x_{12} x_4 x_1 - x_{11} x_7 x_6 + \\
&\quad x_{11} x_7 x_3 + x_{11} x_6 x_4 - x_{11} x_4 x_3 + x_7 x_6 x_1 - x_7 x_5 x_3 \\
&\quad -x_6 x_4 x_1 + x_5 x_4 x_3 \\
p_{13} &= x_{13} x_5 - x_{13} x_1 - x_{11} x_7 + x_{11} x_4 + x_7 x_1 - x_5 x_4 \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_1 - \\
p_{16} &= 2x_{16} - x_2 \\
p_{17} &= 2x_{17} - x_1 \\
p_{18} &= 2x_{18} - x_2 \\
p_{19} &= x_{19} + x_{16} x_{12} - x_{15} x_{14} + x_{15} x_{11} + x_{14} x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{20} &= x_{20} - x_{16}^2 + 2x_{16} x_{12} - x_{15}^2 + 2x_{15} x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{21} &= x_{21} - x_{18}^2 + 2x_{18} x_{12} - x_{17}^2 + 2x_{17} x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{22} &= x_{22} - x_{18} x_{16} + x_{18} x_{12} - x_{17} x_{15} + x_{17} x_{11} + x_{16} x_{12} + x_{15} x_{11} \\
&\quad -x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{23} &= x_{23} - x_{14}^2 + 2x_{14} x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{24} &= x_{24} - x_{16}^2 + 2x_{16} x_{12} - x_{15}^2 + 2x_{15} x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
\end{aligned}$$

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$p_1 = 2x_1 -$$
$$p_2 = 4x_2^2 - 3$$
$$p_3 = 3x_3 - x_2$$
$$p_4 = 3x_4^2 - 2$$
$$p_5 = x_5 - x_1$$
$$p_6 = x_6 - x_3$$
$$p_7 = x_7 - x_4 - x_2$$
$$p_8 = x_8 - x_4 x_2 -$$
$$p_9 = x_9 + x_4 x_1$$
$$p_{10} = x_{10} - x_3 x_1 + x_2 x_1$$
$$
\begin{aligned}
p_{11} = {} & -x_{11}x_9x_7x_5 + x_{11}x_9x_7x_1 + x_{11}x_9x_5x_4 - x_{11}x_9x_4x_1 + \\
& x_{11}x_8x_7x_6 - x_{11}x_8x_7x_3 - x_{11}x_8x_6x_4 + x_{11}x_8x_4x_3 \\
& -x_{11}x_7x_6 + x_{11}x_7x_3 + x_{11}x_6x_4 - x_{11}x_4x_3 + x_9x_7x_5 \\
& -x_9x_7x_1 - x_9x_5x_4 + x_9x_4x_1 - x_8x_7x_6x_1 + x_8x_7x_5x_3 + \\
& x_8x_6x_4x_1 - x_8x_5x_4x_3 + x_7x_6x_1 - x_7x_5x_3 - x_6x_4x_1 + \\
& x_5x_4x_3
\end{aligned}
$$
$$p_{12} = -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9$$
$$p_{13} = x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4$$
$$p_{14} = 2x_{14} -$$
$$p_{15} = 2x_{15} - x_1 -$$
$$p_{16} = 2x_{16} - x_2$$
$$p_{17} = 2x_{17} - x_1$$
$$p_{18} = 2x_{18} - x_2$$
$$p_{19} = x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2$$
$$p_{20} = x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2$$
$$p_{21} = x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2$$
$$
\begin{aligned}
p_{22} = {} & x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
& -x_{13}^2 - x_{12}^2 - x_{11}^2
\end{aligned}
$$
$$p_{23} = x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2$$
$$p_{24} = x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2$$

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

1.19   Triangulation, step 19

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.20   Triangulation, step 20

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

1.21   Triangulation, step 21

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.22   Triangulation, step 22

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4 x_2 - \\
p_9 &= x_9 + x_4 x_1 \\
p_{10} &= x_{10} - x_3 x_1 + x_2 x_1 \\
p_{11} &= -x_{11}x_9 x_7 x_5 + x_{11}x_9 x_7 x_1 + x_{11}x_9 x_5 x_4 - x_{11}x_9 x_4 x_1 + \\
&\quad x_{11}x_8 x_7 x_6 - x_{11}x_8 x_7 x_3 - x_{11}x_8 x_6 x_4 + x_{11}x_8 x_4 x_3 \\
&\quad -x_{11}x_7 x_6 + x_{11}x_7 x_3 + x_{11}x_6 x_4 - x_{11}x_4 x_3 + x_9 x_7 x_5 \\
&\quad -x_9 x_7 x_1 - x_9 x_5 x_4 + x_9 x_4 x_1 - x_8 x_7 x_6 x_1 + x_8 x_7 x_5 x_3 + \\
&\quad x_8 x_6 x_4 x_1 - x_8 x_5 x_4 x_3 + x_7 x_6 x_1 - x_7 x_5 x_3 - x_6 x_4 x_1 + \\
&\quad x_5 x_4 x_3 \\
p_{12} &= -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9 \\
p_{13} &= x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7 x_1 - x_5 x_4 \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_1 - \\
p_{16} &= 2x_{16} - x_2
\end{aligned}
$$

$$
\begin{aligned}
p_{17} &= 2x_{17} - x_1 \\
p_{18} &= 2x_{18} - x_2 \\
p_{19} &= x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{20} &= x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{21} &= x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
&\quad - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{24} over variable x_{24}:

$$
g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}
$$

2. Pseudo remainder with p_{23} over variable x_{23}:

$$
g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}
$$

3. Pseudo remainder with p_{22} over variable x_{22}:

$$
g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}
$$

4. Pseudo remainder with p_{21} over variable x_{21}:

$$
g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}
$$

5. Pseudo remainder with p_{20} over variable x_{20}:

$$
g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}
$$

6. Pseudo remainder with p_{19} over variable x_{19}:

$$g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}$$

7. Pseudo remainder with p_{18} over variable x_{18}:

$$g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}$$

8. Pseudo remainder with p_{17} over variable x_{17}:

$$g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}$$

9. Pseudo remainder with p_{16} over variable x_{16}:

$$g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}$$

10. Pseudo remainder with p_{15} over variable x_{15}:

$$g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}$$

11. Pseudo remainder with p_{14} over variable x_{14}:

$$g = x_{13}x_8 - x_{13} - x_{11}x_{10} + x_{10}$$

12. Pseudo remainder with p_{13} over variable x_{13}:

$$g = -x_{11}x_{10}x_5 + x_{11}x_{10}x_1 + x_{11}x_8x_7 - x_{11}x_8x_4 - x_{11}x_7 + x_{11}x_4 + x_{10}x_5 - x_{10}x_1 - x_8x_7x_1 + x_8x_5x_4 + x_7x_1 - x_5x_4$$

13. Pseudo remainder with p_{12} over variable x_{12}:

$$g = -x_{11}x_{10}x_5 + x_{11}x_{10}x_1 + x_{11}x_8x_7 - x_{11}x_8x_4 - x_{11}x_7 + x_{11}x_4 + x_{10}x_5 - x_{10}x_1 - x_8x_7x_1 + x_8x_5x_4 + x_7x_1 - x_5x_4$$

14. Pseudo remainder with p_{11} over variable x_{11}:
    Polynomial too big for output (text size is 2202 *characters, number of terms is* 80*)*

15. Pseudo remainder with p_{10} over variable x_{10}:
    Polynomial too big for output (text size is 3283 *characters, number of terms is* 112*)*

11

16. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (text size is 3328 characters, number of terms is 112)

17. Pseudo remainder with p_8 over variable x_8:

$$
\begin{aligned}
g \quad = \quad & -x_7^2 x_5 x_4^2 x_3 x_2^2 - x_7^2 x_5 x_4^2 x_2 x_1^2 + \\
& x_7^2 x_5 x_4^2 x_2 x_1 + x_7^2 x_4^2 x_3 x_2^2 x_1 + \\
& x_7^2 x_4^2 x_2 x_1^3 - x_7^2 x_4^2 x_2 x_1^2 + \\
& x_7 x_6 x_5 x_4^3 x_2^2 - x_7 x_6 x_5 x_4 x_3 x_2 x_1^2 + \\
& x_7 x_6 x_5 x_4 x_3 x_2 x_1 + x_7 x_6 x_5 x_4 x_2^2 x_1^2 \\
& -x_7 x_6 x_5 x_4 x_2^2 x_1 - x_7 x_6 x_4^3 x_2^2 x_1 + \\
& x_7 x_6 x_4 x_3 x_2 x_1^3 - x_7 x_6 x_4 x_3 x_2 x_1^2 \\
& -x_7 x_6 x_4 x_2^2 x_1^3 + x_7 x_6 x_4 x_2^2 x_1^2 + \\
& x_7 x_5^2 x_4^3 x_2 x_1 + x_7 x_5^2 x_4 x_3^2 x_2 x_1 \\
& -x_7 x_5^2 x_4 x_3 x_2^2 x_1 + x_7 x_5 x_4^3 x_3 x_2^2 \\
& -2x_7 x_5 x_4^3 x_2 x_1 - x_7 x_5 x_4 x_3^2 x_2 x_1^2 \\
& -x_7 x_5 x_4 x_3^2 x_2 x_1 + x_7 x_5 x_4 x_3 x_2^2 x_1^2 + \\
& x_7 x_5 x_4 x_3 x_2^2 x_1 - x_7 x_4^3 x_3 x_2^2 x_1 \\
& -x_7 x_4^3 x_2 x_1^3 + 2x_7 x_4^3 x_2 x_1^2 + \\
& x_7 x_4 x_3^2 x_2 x_1^2 - x_7 x_4 x_3 x_2^2 x_1^2 \\
& -x_6 x_5 x_4^4 x_2^2 + x_6 x_5 x_4^2 x_3 x_2 x_1^2 \\
& -x_6 x_5 x_4^2 x_3 x_2 x_1 - x_6 x_5 x_4^2 x_2^2 x_1^2 + \\
& x_6 x_5 x_4^2 x_2^2 x_1 + x_6 x_4^4 x_2^2 x_1 \\
& -x_6 x_4^2 x_3 x_2 x_1^3 + x_6 x_4^2 x_3 x_2 x_1^2 + \\
& x_6 x_4^2 x_2^2 x_1^3 - x_6 x_4^2 x_2^2 x_1^2 \\
& -x_5^2 x_4^4 x_2 x_1 - x_5^2 x_4^2 x_3^2 x_2 x_1 + \\
& x_5^2 x_4^2 x_3 x_2^2 x_1 + x_5 x_4^4 x_2 x_1^2 + x_5 x_4^4 x_2 x_1 + \\
& x_5 x_4^2 x_3^2 x_2 x_1^2 + x_5 x_4^2 x_3^2 x_2 x_1 \\
& -x_5 x_4^2 x_3 x_2^2 x_1^2 - x_5 x_4^2 x_3 x_2^2 x_1 \\
& -x_4^4 x_2 x_1^2 - x_4^2 x_3^2 x_2 x_1^2 + x_4^2 x_3 x_2^2 x_1^2
\end{aligned}
$$

18. Pseudo remainder with p_7 over variable x_7:

$$
\begin{aligned}
g \quad = \quad & x_6 x_5 x_4^3 x_2^3 - x_6 x_5 x_4 x_3 x_2^2 x_1^2 + \\
& x_6 x_5 x_4 x_3 x_2^2 x_1 + x_6 x_5 x_4 x_2^3 x_1^2 \\
& -x_6 x_5 x_4 x_2^3 x_1 - x_6 x_4^3 x_2^3 x_1 + \\
& x_6 x_4 x_3 x_2^2 x_1^3 - x_6 x_4 x_3 x_2^2 x_1^2 \\
& -x_6 x_4 x_2^3 x_1^3 + x_6 x_4 x_2^3 x_1^2 + x_5^2 x_4^3 x_2^2 x_1 + \\
& x_5^2 x_4 x_3^2 x_2^2 x_1 - x_5^2 x_4 x_3 x_2^3 x_1
\end{aligned}
$$

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$$-x_5 x_4^3 x_3 x_2^3 - 2 x_5 x_4^3 x_2^2 x_1^2$$
$$-x_5 x_4^2 x_3 x_2^4 - x_5 x_4^2 x_2^3 x_1^2 + x_5 x_4^2 x_2^3 x_1$$
$$-x_5 x_4 x_3^2 x_2^2 x_1^2 - x_5 x_4 x_3^2 x_2^2 x_1 +$$
$$x_5 x_4 x_3 x_2^3 x_1^2 + x_5 x_4 x_3 x_2^3 x_1 + x_4^3 x_3 x_2^3 x_1 +$$
$$x_4^3 x_2^2 x_1^3 + x_4^2 x_3 x_2^4 x_1 + x_4^2 x_2^3 x_1^3$$
$$-x_4^2 x_2^3 x_1^2 + x_4 x_3^2 x_2^2 x_1^2 - x_4 x_3 x_2^3 x_1^2$$

19. Pseudo remainder with p_6 over variable x_6:

$$
\begin{aligned}
g \;=\; & x_5^2 x_4^3 x_2^2 x_1 + x_5^2 x_4 x_3^2 x_2^2 x_1 \\
& -x_5^2 x_4 x_3 x_2^3 x_1 - 2 x_5 x_4^3 x_2^2 x_1^2 \\
& -x_5 x_4^2 x_3 x_2^4 - x_5 x_4^2 x_2^3 x_1^2 + x_5 x_4^2 x_2^3 x_1 \\
& -2 x_5 x_4 x_3^2 x_2^2 x_1^2 + 2 x_5 x_4 x_3 x_2^3 x_1^2 + \\
& x_4^3 x_2^2 x_1^3 + x_4^2 x_3 x_2^4 x_1 + x_4^2 x_2^3 x_1^3 \\
& -x_4^2 x_2^3 x_1^2 + x_4 x_3^2 x_2^2 x_1^3 - x_4 x_3 x_2^3 x_1^3
\end{aligned}
$$

20. Pseudo remainder with p_5 over variable x_5:

$$g \;=\; 0$$

21. Pseudo remainder with p_4 over variable x_4:

$$g \;=\; 0$$

22. Pseudo remainder with p_3 over variable x_3:

$$g \;=\; 0$$

23. Pseudo remainder with p_2 over variable x_2:

$$g \;=\; 0$$

24. Pseudo remainder with p_1 over variable x_1:

$$g \;=\; 0$$

3   Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 112 terms.

Time Complexity: Time spent by the prover is 0.19 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form