OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

September 30, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17}
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{17} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{18} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{18} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 -
\end{aligned}
$$

$$
\begin{aligned}
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{17} &= -x_{17}x_7 - x_{16} + x_7 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{16} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12}
\end{aligned}
$$

$$
\begin{aligned}
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= x_{16}x_1 - x_{16} + x_{15}x_7x_3 - x_7x_3 - x_7x_1 + x_7 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15}-
\end{aligned}
$$

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
       &\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15}-
\end{aligned}
$$

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{14} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{11}x_1 + x_{10}x_2 \\
p_{13} &= -x_{12}x_1 + x_{10}x_3 \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
       &\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= -x_{12}x_1 + x_{10}x_3 \\
p_{12} &= -x_{12}x_7 - x_{11} + x_7 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= x_{11}x_1 + x_{10}x_7x_3 - x_7x_1 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}$$

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{11} from all other polynomials by reducing them with polynomial p_{10} from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7
\end{aligned}$$

$$
\begin{aligned}
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.10   Triangulation, step 10

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.11   Triangulation, step 11

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_9 from all other polynomials by reducing them with polynomial p_8 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7
\end{aligned}
$$

$$
\begin{aligned}
p_8 &= x_7 + x_6 - \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= x_7 + x_6 - \\
p_7 &= -x_7 + x_6 \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
\end{aligned}
$$

$$p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2$$
$$p_{17} = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3$$
$$p_{18} = -x_{18}x_7 + x_{16}$$
$$p_{19} = -x_{19}x_1 + x_{19} + x_{15}-$$

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_7 from all other polynomials by reducing them with polynomial p_6 from previous step.

Finished a triangulation step, the current system is:

$$p_1 = -x_4 + x_1$$
$$p_2 = -x_4 + x_2$$
$$p_3 = -x_5 + x_1$$
$$p_4 = x_5 + x_2 -$$
$$p_5 = x_3 -$$
$$p_6 = 2x_6 -$$
$$p_7 = x_7 + x_6 -$$
$$p_8 = -x_8 + x_6$$
$$p_9 = x_9 + x_6 -$$
$$p_{10} = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2$$
$$p_{11} = -x_{11}x_1 + x_{10}x_2$$
$$p_{12} = -x_{12}x_1 + x_{10}x_3$$
$$p_{13} = -x_{13}x_7 + x_{11}$$
$$p_{14} = -x_{14}x_1 + x_{10}$$
$$p_{15} = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1$$
$$\qquad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2$$
$$p_{16} = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2$$
$$p_{17} = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3$$
$$p_{18} = -x_{18}x_7 + x_{16}$$
$$p_{19} = -x_{19}x_1 + x_{19} + x_{15}-$$

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_5 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= x_3 - \\
p_4 &= -x_2 - x_1 + 1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
& \quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= x_3 - \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= -x_2 + x_1 \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1 from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 - \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 -
\end{aligned}
$$

$$
\begin{aligned}
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
      &\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} -
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
\begin{aligned}
g &= 4x_{17}^2 - 8x_{17}x_{12} + 4x_{16}^2 - 8x_{16}x_{11} + 4x_{15}^2 - 8x_{15}x_{10} + 4x_{12}^2 + \\
  &\quad 4x_{11}^2 + 4x_{10}^2 -
\end{aligned}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{19} over variable x_{19}:

$$
\begin{aligned}
g &= 4x_{17}^2 - 8x_{17}x_{12} + 4x_{16}^2 - 8x_{16}x_{11} + 4x_{15}^2 - 8x_{15}x_{10} + 4x_{12}^2 + \\
  &\quad 4x_{11}^2 + 4x_{10}^2 -
\end{aligned}
$$

2. Pseudo remainder with p_{18} over variable x_{18}:

$$
\begin{aligned}
g &= 4x_{17}^2 - 8x_{17}x_{12} + 4x_{16}^2 - 8x_{16}x_{11} + 4x_{15}^2 - 8x_{15}x_{10} + 4x_{12}^2 + \\
  &\quad 4x_{11}^2 + 4x_{10}^2 -
\end{aligned}
$$

3. Pseudo remainder with p_{17} over variable x_{17}:

$$
\begin{aligned}
g \;=\; & 4x_{16}^2 x_1^2 - 8x_{16}^2 x_1 + 4x_{16}^2 - 8x_{16}x_{11}x_1^2 + 16x_{16}x_{11}x_1 \\
& -8x_{16}x_{11} + 4x_{15}^2 x_3^2 + 4x_{15}^2 x_1^2 - 8x_{15}^2 x_1 + 4x_{15}^2 \\
& -8x_{15}x_{12}x_3x_1 + 8x_{15}x_{12}x_3 - 8x_{15}x_{10}x_1^2 + 16x_{15}x_{10}x_1 \\
& -8x_{15}x_{10} - 8x_{15}x_3^2 + 4x_{12}^2 x_1^2 - 8x_{12}^2 x_1 + 4x_{12}^2 + \\
& 8x_{12}x_3x_1 - 8x_{12}x_3 + 4x_{11}^2 x_1^2 - 8x_{11}^2 x_1 + 4x_{11}^2 + \\
& 4x_{10}^2 x_1^2 - 8x_{10}^2 x_1 + 4x_{10}^2 + 4x_3^2 - x_1^2 + 2x_1 -
\end{aligned}
$$

4. Pseudo remainder with p_{16} over variable x_{16}:

$$
\begin{aligned}
g \;=\; & 4x_{15}^2 x_3^2 x_1^2 - 8x_{15}^2 x_3^2 x_1 + 4x_{15}^2 x_3^2 + \\
& 4x_{15}^2 x_2^2 x_1^2 - 8x_{15}^2 x_2^2 x_1 + 4x_{15}^2 x_2^2 + \\
& 4x_{15}^2 x_1^4 - 16x_{15}^2 x_1^3 + 24x_{15}^2 x_1^2 - 16x_{15}^2 x_1 + \\
& 4x_{15}^2 - 8x_{15}x_{12}x_3x_1^3 + 24x_{15}x_{12}x_3x_1^2 \\
& -24x_{15}x_{12}x_3x_1 + 8x_{15}x_{12}x_3 - 8x_{15}x_{11}x_2x_1^3 + \\
& 24x_{15}x_{11}x_2x_1^2 - 24x_{15}x_{11}x_2x_1 + 8x_{15}x_{11}x_2 \\
& -8x_{15}x_{10}x_1^4 + 32x_{15}x_{10}x_1^3 - 48x_{15}x_{10}x_1^2 + 32x_{15}x_{10}x_1 \\
& -8x_{15}x_{10} - 8x_{15}x_3^2 x_1^2 + 16x_{15}x_3^2 x_1 - 8x_{15}x_3^2 \\
& -8x_{15}x_2^2 x_1^2 + 16x_{15}x_2^2 x_1 - 8x_{15}x_2^2 + 4x_{12}^2 x_1^4 \\
& -16x_{12}^2 x_1^3 + 24x_{12}^2 x_1^2 - 16x_{12}^2 x_1 + 4x_{12}^2 + \\
& 8x_{12}x_3x_1^3 - 24x_{12}x_3x_1^2 + 24x_{12}x_3x_1 - 8x_{12}x_3 + \\
& 4x_{11}^2 x_1^4 - 16x_{11}^2 x_1^3 + 24x_{11}^2 x_1^2 - 16x_{11}^2 x_1 + \\
& 4x_{11}^2 + 8x_{11}x_2x_1^3 - 24x_{11}x_2x_1^2 + 24x_{11}x_2x_1 - 8x_{11}x_2 + \\
& 4x_{10}^2 x_1^4 - 16x_{10}^2 x_1^3 + 24x_{10}^2 x_1^2 - 16x_{10}^2 x_1 + \\
& 4x_{10}^2 + 4x_3^2 x_1^2 - 8x_3^2 x_1 + 4x_3^2 + 4x_2^2 x_1^2 \\
& -8x_2^2 x_1 + 4x_2^2 - x_1^4 + 4x_1^3 - 6x_1^2 + 4x_1 -
\end{aligned}
$$

5. Pseudo remainder with p_{15} over variable x_{15}:

   Polynomial too big for output (text size is 5720 characters, number of terms is 181)

6. Pseudo remainder with p_{14} over variable x_{14}:

   Polynomial too big for output (text size is 5720 characters, number of terms is 181)

7. Pseudo remainder with p_{13} over variable x_{13}:

   Polynomial too big for output (text size is 5720 characters, number of terms is 181)

8. Pseudo remainder with p_{12} over variable x_{12}:

   Polynomial too big for output (text size is 6130 characters, number of terms is 181)

9. Pseudo remainder with p_{11} over variable x_{11}:

   Polynomial too big for output (text size is 6324 characters, number of terms is 181)

10. Pseudo remainder with p_{10} over variable x_{10}:

    Polynomial too big for output (text size is 2816 characters, number of terms is 84)

11. Pseudo remainder with p_9 over variable x_9:

    Polynomial too big for output (text size is 2816 characters, number of terms is 84)

12. Pseudo remainder with p_8 over variable x_8:

    Polynomial too big for output (text size is 2816 characters, number of terms is 84)

13. Pseudo remainder with p_7 over variable x_7:

    Polynomial too big for output (number of terms is 287)

14. Pseudo remainder with p_6 over variable x_6:

    Polynomial too big for output (text size is 2193 characters, number of terms is 84)

15. Pseudo remainder with p_5 over variable x_5:

    Polynomial too big for output (text size is 2193 characters, number of terms is 84)

16. Pseudo remainder with p_4 over variable x_4:

    Polynomial too big for output (text size is 2193 characters, number of terms is 84)

17. Pseudo remainder with p_3 over variable x_3:

$$
\begin{aligned}
g \;=\; & 48x_2^4x_1^{12} - 288x_2^4x_1^{11} + 720x_2^4x_1^{10} - 960x_2^4x_1^9 + \\
& 720x_2^4x_1^8 - 288x_2^4x_1^7 + 48x_2^4x_1^6 + 32x_2^3x_1^{12} \\
& -192x_2^3x_1^{11} + 480x_2^3x_1^{10} - 640x_2^3x_1^9 + 480x_2^3x_1^8 \\
& -192x_2^3x_1^7 + 32x_2^3x_1^6 - 8x_2^2x_1^{12} + 48x_2^2x_1^{11} \\
& -120x_2^2x_1^{10} + 160x_2^2x_1^9 - 120x_2^2x_1^8 + 48x_2^2x_1^7 \\
& -8x_2^2x_1^6 - 8x_2x_1^{12} + 48x_2x_1^{11} - 120x_2x_1^{10} + 160x_2x_1^9 \\
& -120x_2x_1^8 + 48x_2x_1^7 - 8x_2x_1^6 - x_1^{12} + 6x_1^{11} - 15x_1^{10} + \\
& 20x_1^9 - 15x_1^8 + 6x_1^7 - x_1^6
\end{aligned}
$$

18. Pseudo remainder with p_2 over variable x_2:

$$
\begin{aligned}
g \;=\; & 48x_1^{16} - 512x_1^{15} + 2440x_1^{14} - 6840x_1^{13} + 12487x_1^{12} - 15506x_1^{11} + \\
& 13257x_1^{10} - 7700x_1^9 + 2905x_1^8 - 642x_1^7 + 63x_1^6
\end{aligned}
$$

19. Pseudo remainder with p_1 over variable x_1:

$$g \quad = \quad 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 287 terms.

Time Complexity: Time spent by the prover is 0.226 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form