OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

September 30, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 - x_4 \\
p_8 &= x_8 - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4x_2 \\
p_{11} &= x_{11} + x_4x_1 \\
p_{12} &= x_{12} - x_3x_1 + x_2x_1 \\
p_{13} &= x_{14} - x_3 \\
p_{14} &= -x_{16}x_5 + x_{15} - x_4 \\
p_{15} &= -x_{17}x_6 + x_{13} \\
p_{16} &= -x_{17}x_7 + x_{14} \\
p_{17} &= -x_{17}x_8 + x_{15} \\
p_{18} &= -x_{21}x_7 + x_{19} \\
p_{19} &= -x_{21}x_8 + x_{20} \\
p_{20} &= -x_{22}x_{10} + x_{18} - \\
p_{21} &= -x_{22}x_{11} + x_{19} \\
p_{22} &= -x_{22}x_{12} + x_{20}
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{22} from all other polynomials by reducing them with polynomial p_{20} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 - x_4 \\
p_8 &= x_8 - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4x_2 \\
p_{11} &= x_{11} + x_4x_1 \\
p_{12} &= x_{12} - x_3x_1 + x_2x_1 \\
p_{13} &= x_{14} - x_3 \\
p_{14} &= -x_{16}x_5 + x_{15} - x_4 \\
p_{15} &= -x_{17}x_6 + x_{13} \\
p_{16} &= -x_{17}x_7 + x_{14} \\
p_{17} &= -x_{17}x_8 + x_{15} \\
p_{18} &= -x_{21}x_7 + x_{19} \\
p_{19} &= -x_{21}x_8 + x_{20} \\
p_{20} &= -x_{19}x_{10} + x_{18}x_{11} - x_{11} \\
p_{21} &= -x_{20}x_{10} + x_{18}x_{12} - x_{12} \\
p_{22} &= -x_{22}x_{10} + x_{18}-
\end{aligned}
$$

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{21} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 - x_4 \\
p_8 &= x_8 - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4x_2 \\
p_{11} &= x_{11} + x_4x_1 \\
p_{12} &= x_{12} - x_3x_1 + x_2x_1 \\
p_{13} &= x_{14} - x_3 \\
p_{14} &= -x_{16}x_5 + x_{15} - x_4 \\
p_{15} &= -x_{17}x_6 + x_{13} \\
p_{16} &= -x_{17}x_7 + x_{14} \\
p_{17} &= -x_{17}x_8 + x_{15} \\
p_{18} &= -x_{19}x_{10} + x_{18}x_{11} - x_{11} \\
p_{19} &= -x_{20}x_{10} + x_{18}x_{12} - x_{12} \\
p_{20} &= -x_{20}x_7 + x_{19}x_8 \\
p_{21} &= -x_{21}x_7 + x_{19} \\
p_{22} &= -x_{22}x_{10} + x_{18}-
\end{aligned}
$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{20} from all other polynomials by reducing them with polynomial p_{19} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2
\end{aligned}
$$

$$
\begin{aligned}
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4 x_2 \\
p_7 &= x_7 + x_4 x_1 - x_4 \\
p_8 &= x_8 - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4 x_2 \\
p_{11} &= x_{11} + x_4 x_1 \\
p_{12} &= x_{12} - x_3 x_1 + x_2 x_1 \\
p_{13} &= x_{14} - x_3 \\
p_{14} &= -x_{16} x_5 + x_{15} - x_4 \\
p_{15} &= -x_{17} x_6 + x_{13} \\
p_{16} &= -x_{17} x_7 + x_{14} \\
p_{17} &= -x_{17} x_8 + x_{15} \\
p_{18} &= -x_{19} x_{10} + x_{18} x_{11} - x_{11} \\
p_{19} &= -x_{19} x_{10} x_8 + x_{18} x_{12} x_7 - x_{12} x_7 \\
p_{20} &= -x_{20} x_{10} + x_{18} x_{12} - x_{12} \\
p_{21} &= -x_{21} x_7 + x_{19} \\
p_{22} &= -x_{22} x_{10} + x_{18} -
\end{aligned}
$$

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{19} from all other polynomials by reducing them with polynomial p_{18} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4 x_2 \\
p_7 &= x_7 + x_4 x_1 - x_4 \\
p_8 &= x_8 - x_3 x_1 + x_3 + x_2 x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4 x_2 \\
p_{11} &= x_{11} + x_4 x_1
\end{aligned}
$$

$$
\begin{aligned}
p_{12} &= x_{12} - x_3x_1 + x_2x_1 \\
p_{13} &= x_{14} - x_3 \\
p_{14} &= -x_{16}x_5 + x_{15} - x_4 \\
p_{15} &= -x_{17}x_6 + x_{13} \\
p_{16} &= -x_{17}x_7 + x_{14} \\
p_{17} &= -x_{17}x_8 + x_{15} \\
p_{18} &= -x_{18}x_{12}x_{10}x_7 + x_{18}x_{11}x_{10}x_8 + x_{12}x_{10}x_7 - x_{11}x_{10}x_8 \\
p_{19} &= -x_{19}x_{10} + x_{18}x_{11} - x_{11} \\
p_{20} &= -x_{20}x_{10} + x_{18}x_{12} - x_{12} \\
p_{21} &= -x_{21}x_7 + x_{19} \\
p_{22} &= -x_{22}x_{10} + x_{18}-
\end{aligned}
$$

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18}. No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{17} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 - x_4 \\
p_8 &= x_8 - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_9 &= x_9 + x_6
\end{aligned}
$$

$$
\begin{aligned}
p_{10} &= x_{10} - x_4 x_2 \\
p_{11} &= x_{11} + x_4 x_1 \\
p_{12} &= x_{12} - x_3 x_1 + x_2 x_1 \\
p_{13} &= x_{14} - x_3 \\
p_{14} &= -x_{16} x_5 + x_{15} - x_4 \\
p_{15} &= -x_{14} x_6 + x_{13} x_7 \\
p_{16} &= -x_{15} x_6 + x_{13} x_8 \\
p_{17} &= -x_{17} x_6 + x_{13} \\
p_{18} &= -x_{18} x_{12} x_{10} x_7 + x_{18} x_{11} x_{10} x_8 + x_{12} x_{10} x_7 - x_{11} x_{10} x_8 \\
p_{19} &= -x_{19} x_{10} + x_{18} x_{11} - x_{11} \\
p_{20} &= -x_{20} x_{10} + x_{18} x_{12} - x_{12} \\
p_{21} &= -x_{21} x_7 + x_{19} \\
p_{22} &= -x_{22} x_{10} + x_{18} -
\end{aligned}
$$

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{14} from all other polynomials by reducing them with polynomial p_{13} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 - x_4 \\
p_8 &= x_8 - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4x_2 \\
p_{11} &= x_{11} + x_4x_1 \\
p_{12} &= x_{12} - x_3x_1 + x_2x_1 \\
p_{13} &= x_{13}x_7 - x_6x_3 \\
p_{14} &= x_{14} - x_3 \\
p_{15} &= -x_{15}x_6 + x_{13}x_8 \\
p_{16} &= -x_{16}x_5 + x_{15} - x_4 \\
p_{17} &= -x_{17}x_6 + x_{13} \\
p_{18} &= -x_{18}x_{12}x_{10}x_7 + x_{18}x_{11}x_{10}x_8 + x_{12}x_{10}x_7 - x_{11}x_{10}x_8 \\
p_{19} &= -x_{19}x_{10} + x_{18}x_{11} - x_{11} \\
p_{20} &= -x_{20}x_{10} + x_{18}x_{12} - x_{12} \\
p_{21} &= -x_{21}x_7 + x_{19} \\
p_{22} &= -x_{22}x_{10} + x_{18} -
\end{aligned}
$$

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 - x_4 \\
p_8 &= x_8 - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_9 &= x_9 + x_6 \\
p_{10} &= x_{10} - x_4x_2 \\
p_{11} &= x_{11} + x_4x_1 \\
p_{12} &= x_{12} - x_3x_1 + x_2x_1
\end{aligned}
$$

$$
\begin{aligned}
p_{13} &= x_{13}x_7 - x_6x_3 \\
p_{14} &= x_{14} - x_3 \\
p_{15} &= -x_{15}x_6 + x_{13}x_8 \\
p_{16} &= -x_{16}x_5 + x_{15} - x_4 \\
p_{17} &= -x_{17}x_6 + x_{13} \\
p_{18} &= -x_{18}x_{12}x_{10}x_7 + x_{18}x_{11}x_{10}x_8 + x_{12}x_{10}x_7 - x_{11}x_{10}x_8 \\
p_{19} &= -x_{19}x_{10} + x_{18}x_{11} - x_{11} \\
p_{20} &= -x_{20}x_{10} + x_{18}x_{12} - x_{12} \\
p_{21} &= -x_{21}x_7 + x_{19} \\
p_{22} &= -x_{22}x_{10} + x_{18}-
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$g = -x_{21}x_6 + x_{18}$$

with respect to the triangular system.

1. Pseudo remainder with p_{22} over variable x_{22}:

$$g = -x_{21}x_6 + x_{18}$$

2. Pseudo remainder with p_{21} over variable x_{21}:

$$g = x_{19}x_6 - x_{18}x_7$$

3. Pseudo remainder with p_{20} over variable x_{20}:

$$g = x_{19}x_6 - x_{18}x_7$$

4. Pseudo remainder with p_{19} over variable x_{19}:

$$g = -x_{18}x_{11}x_6 + x_{18}x_{10}x_7 + x_{11}x_6$$

5. Pseudo remainder with p_{18} over variable x_{18}:

$$g = -x_{12}x_{10}^2x_7^2 + x_{11}x_{10}^2x_8x_7$$

6. Pseudo remainder with p_{17} over variable x_{17}:

$$g = -x_{12}x_{10}^2x_7^2 + x_{11}x_{10}^2x_8x_7$$

7. Pseudo remainder with p_{16} over variable x_{16}:

$$g = -x_{12}x_{10}^2x_7^2 + x_{11}x_{10}^2x_8x_7$$

8. Pseudo remainder with p_{15} over variable x_{15}:

$$g = -x_{12}x_{10}^2x_7^2 + x_{11}x_{10}^2x_8x_7$$

9. Pseudo remainder with p_{14} over variable x_{14}:

$$g = -x_{12}x_{10}^2x_7^2 + x_{11}x_{10}^2x_8x_7$$

10. Pseudo remainder with p_{13} over variable x_{13}:

$$g = -x_{12}x_{10}^2x_7^2 + x_{11}x_{10}^2x_8x_7$$

11. Pseudo remainder with p_{12} over variable x_{12}:

$$g = x_{11}x_{10}^2x_8x_7 - x_{10}^2x_7^2x_3x_1 + x_{10}^2x_7^2x_2x_1$$

12. Pseudo remainder with p_{11} over variable x_{11}:

$$g = -x_{10}^2x_8x_7x_4x_1 - x_{10}^2x_7^2x_3x_1 + x_{10}^2x_7^2x_2x_1$$

13. Pseudo remainder with p_{10} over variable x_{10}:

$$\begin{aligned} g = & -x_8x_7x_4^3x_2^2x_1 - x_7^2x_4^2x_3x_2^2x_1 + \\ & x_7^2x_4^2x_2^3x_1 \end{aligned}$$

14. Pseudo remainder with p_9 over variable x_9:

$$\begin{aligned} g = & -x_8x_7x_4^3x_2^2x_1 - x_7^2x_4^2x_3x_2^2x_1 + \\ & x_7^2x_4^2x_2^3x_1 \end{aligned}$$

15. Pseudo remainder with p_8 over variable x_8:

$$\begin{aligned} g &= -x_7^2x_4^2x_3x_2^2x_1 + x_7^2x_4^2x_2^3x_1 \\ &\quad -x_7x_4^3x_3x_2^2x_1^2 + x_7x_4^3x_3x_2^2x_1 + \\ &\quad x_7x_4^3x_2^3x_1^2 - x_7x_4^3x_2^3x_1 \end{aligned}$$

16. Pseudo remainder with p_7 over variable x_7:

$$g = 0$$

17. Pseudo remainder with p_6 over variable x_6:

$$g = 0$$

18. Pseudo remainder with p_5 over variable x_5:

$$g = 0$$

19. Pseudo remainder with p_4 over variable x_4:

$$g = 0$$

20. Pseudo remainder with p_3 over variable x_3:

$$g = 0$$

21. Pseudo remainder with p_2 over variable x_2:

$$g = 0$$

22. Pseudo remainder with p_1 over variable x_1:

$$g = 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.101 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form