

# OpenGeoProver Output for conjecture “geothm\_zadatak”

Wu’s method used

October 2, 2016

## 1 Invoking the theorem prover

The used proving method is Wu’s method.

The input system is:

$$\begin{aligned} p_1 &= 2x_1 - \\ p_2 &= 4x_2^2 - 3 \\ p_3 &= 3x_3 - x_2 \\ p_4 &= 3x_4^2 - 2 \\ p_5 &= x_5 - x_1 \\ p_6 &= x_6 - x_3 \\ p_7 &= x_7 - x_4 - x_2 \\ p_8 &= x_8 - x_4x_2 \\ p_9 &= x_9 + x_4x_1 - x_4 \\ p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\ p_{11} &= x_{11} - x_4x_2 - \\ p_{12} &= x_{12} + x_4x_1 \\ p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\ p_{14} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\ p_{15} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13} \\ p_{16} &= -x_{18}x_8 + x_{17}x_9 \\ p_{17} &= -x_{15}x_8 + x_{14}x_9 \\ p_{18} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\ p_{19} &= -x_{16}x_6 + x_{16}x_3 + x_{15}x_7 - x_{15}x_4 - x_7x_3 + x_6x_4 \end{aligned}$$

### 1.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 19.

**Variable  $x_{19}$  selected:** The number of polynomials with this variable, with indexes from 1 to 19, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

## 1.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 18.

**Variable  $x_{18}$  selected:** The number of polynomials with this variable, with indexes from 1 to 18, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{18}$  from all other polynomials by reducing them with polynomial  $p_{14}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{15}x_8 + x_{14}x_9 \\
p_{15} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{16} &= -x_{16}x_6 + x_{16}x_3 + x_{15}x_7 - x_{15}x_4 - x_7x_3 + x_6x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}$$

## 1.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 17.

**Variable  $x_{17}$  selected:** The number of polynomials with this variable, with indexes from 1 to 17, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{17}$ . No reduction needed.

The triangular system has not been changed.

#### 1.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 16.

**Variable  $x_{16}$  selected:** The number of polynomials with this variable, with indexes from 1 to 16, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{16}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{15}x_8 + x_{14}x_9 \\
p_{15} &= x_{15}x_7x_5 - x_{15}x_7x_1 - x_{15}x_5x_4 + x_{15}x_4x_1 - x_{14}x_7x_6 + \\
&\quad x_{14}x_7x_3 + x_{14}x_6x_4 - x_{14}x_4x_3 + x_7x_6x_1 - x_7x_5x_3 \\
&\quad - x_6x_4x_1 + x_5x_4x_3 \\
p_{16} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}$$

#### 1.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 15.

**Variable  $x_{15}$  selected:** The number of polynomials with this variable, with indexes from 1 to 15, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{15}$  from all other polynomials by reducing them with polynomial  $p_{14}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{14}x_9x_7x_5 + x_{14}x_9x_7x_1 + x_{14}x_9x_5x_4 - x_{14}x_9x_4x_1 + \\
&\quad x_{14}x_8x_7x_6 - x_{14}x_8x_7x_3 - x_{14}x_8x_6x_4 + x_{14}x_8x_4x_3 \\
&\quad - x_8x_7x_6x_1 + x_8x_7x_5x_3 + x_8x_6x_4x_1 - x_8x_5x_4x_3 \\
p_{15} &= -x_{15}x_8 + x_{14}x_9 \\
p_{16} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}$$

## 1.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 14.

**Variable  $x_{14}$  selected:** The number of polynomials with this variable, with indexes from 1 to 14, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{14}$ . No reduction needed.

The triangular system has not been changed.

## 1.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 13.

**Variable  $x_{13}$  selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{13}$ . No reduction needed.

The triangular system has not been changed.

### 1.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 12.

**Variable  $x_{12}$  selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{12}$ . No reduction needed.

The triangular system has not been changed.

### 1.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 11.

**Variable  $x_{11}$  selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{11}$ . No reduction needed.

The triangular system has not been changed.

### 1.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 10.

**Variable  $x_{10}$  selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{10}$ . No reduction needed.

The triangular system has not been changed.

### 1.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 9.

**Variable  $x_9$  selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_9$ . No reduction needed.

The triangular system has not been changed.

### 1.12 Triangulation, step 12

**Choosing variable:** Trying the variable with index 8.

**Variable  $x_8$  selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_8$ . No reduction needed.

The triangular system has not been changed.

### 1.13 Triangulation, step 13

**Choosing variable:** Trying the variable with index 7.

**Variable  $x_7$  selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_7$ . No reduction needed.

The triangular system has not been changed.

### 1.14 Triangulation, step 14

**Choosing variable:** Trying the variable with index 6.

**Variable  $x_6$  selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

### 1.15 Triangulation, step 15

**Choosing variable:** Trying the variable with index 5.

**Variable  $x_5$  selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_5$ . No reduction needed.

The triangular system has not been changed.

### 1.16 Triangulation, step 16

**Choosing variable:** Trying the variable with index 4.

**Variable  $x_4$  selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_4$ . No reduction needed.

The triangular system has not been changed.

### 1.17 Triangulation, step 17

**Choosing variable:** Trying the variable with index 3.

**Variable  $x_3$  selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_3$ . No reduction needed.

The triangular system has not been changed.

### 1.18 Triangulation, step 18

**Choosing variable:** Trying the variable with index 2.

**Variable  $x_2$  selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_2$ . No reduction needed.

The triangular system has not been changed.

### 1.19 Triangulation, step 19

**Choosing variable:** Trying the variable with index 1.

**Variable  $x_1$  selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{14}x_9x_7x_5 + x_{14}x_9x_7x_1 + x_{14}x_9x_5x_4 - x_{14}x_9x_4x_1 + \\
&\quad x_{14}x_8x_7x_6 - x_{14}x_8x_7x_3 - x_{14}x_8x_6x_4 + x_{14}x_8x_4x_3 \\
&\quad - x_8x_7x_6x_1 + x_8x_7x_5x_3 + x_8x_6x_4x_1 - x_8x_5x_4x_3 \\
p_{15} &= -x_{15}x_8 + x_{14}x_9 \\
p_{16} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}$$

## 2 Final Remainder

### 2.1 Final remainder for conjecture geothm\_zadatak

Calculating final remainder of the conclusion:

$$g = x_{19}x_8 - x_{17}x_{10}$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{19}$  over variable  $x_{19}$ :

$$g = x_{17}x_{13}x_8 - x_{17}x_{11}x_{10} + x_{17}x_{10} - x_{13}x_8$$

2. Pseudo remainder with  $p_{18}$  over variable  $x_{18}$ :

$$g = x_{17}x_{13}x_8 - x_{17}x_{11}x_{10} + x_{17}x_{10} - x_{13}x_8$$



3. Pseudo remainder with  $p_{17}$  over variable  $x_{17}$ :

$$g = x_{13}x_{11}x_9x_8 - x_{13}x_9x_8 - x_{12}x_{11}x_{10}x_8 + x_{12}x_{10}x_8$$

4. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :

$$g = x_{13}x_{11}x_9x_8 - x_{13}x_9x_8 - x_{12}x_{11}x_{10}x_8 + x_{12}x_{10}x_8$$

5. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :

$$g = x_{13}x_{11}x_9x_8 - x_{13}x_9x_8 - x_{12}x_{11}x_{10}x_8 + x_{12}x_{10}x_8$$

6. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :

$$g = x_{13}x_{11}x_9x_8 - x_{13}x_9x_8 - x_{12}x_{11}x_{10}x_8 + x_{12}x_{10}x_8$$

7. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :

$$g = -x_{12}x_{11}x_{10}x_8 + x_{12}x_{10}x_8 + x_{11}x_9x_8x_3x_1 \\ - x_{11}x_9x_8x_2x_1 - x_9x_8x_3x_1 + x_9x_8x_2x_1$$

8. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :

$$g = x_{11}x_{10}x_8x_4x_1 + x_{11}x_9x_8x_3x_1 - x_{11}x_9x_8x_2x_1 \\ - x_{10}x_8x_4x_1 - x_9x_8x_3x_1 + x_9x_8x_2x_1$$

9. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :

$$g = x_{10}x_8x_4^2x_2x_1 + x_9x_8x_4x_3x_2x_1 - x_9x_8x_4x_2^2x_1$$

10. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :

$$g = x_9x_8x_4x_3x_2x_1 - x_9x_8x_4x_2^2x_1 + \\ x_8x_4^2x_3x_2x_1^2 - x_8x_4^2x_3x_2x_1 \\ - x_8x_4^2x_2^2x_1^2 + x_8x_4^2x_2^2x_1$$

11. Pseudo remainder with  $p_9$  over variable  $x_9$ :

$$g = 0$$

12. Pseudo remainder with  $p_8$  over variable  $x_8$ :

$$g = 0$$

13. Pseudo remainder with  $p_7$  over variable  $x_7$ :

$$g = 0$$

14. Pseudo remainder with  $p_6$  over variable  $x_6$ :

$$g = 0$$

15. Pseudo remainder with  $p_5$  over variable  $x_5$ :

$$g = 0$$

16. Pseudo remainder with  $p_4$  over variable  $x_4$ :

$$g = 0$$

17. Pseudo remainder with  $p_3$  over variable  $x_3$ :

$$g = 0$$

18. Pseudo remainder with  $p_2$  over variable  $x_2$ :

$$g = 0$$

19. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

### 3 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 12 terms.

**Time Complexity:** Time spent by the prover is 0.093 seconds.

### 4 NDG Conditions

**NDG Conditions in readable form**

- Failed to translate NDG Conditions to readable form