Balkan MO 1992

– May 6th

1 For all positive integers m, n define $f(m, n) = m^{3^{4n}+6} - m^{3^{4n}+4} - m^5 + m^3$. Find all numbers n with the property that $f(m, n)$ is divisible by 1992 for every m.

Bulgaria

2 Prove that for all positive integers n the following inequality takes place

$$(2n^2 + 3n + 1)^n \geq 6^n (n!)^2.$$

Cyprus

3 Let D, E, F be points on the sides BC, CA, AB respectively of a triangle ABC (distinct from the vertices). If the quadrilateral $AFDE$ is cyclic, prove that

$$\frac{4\mathcal{A}[DEF]}{\mathcal{A}[ABC]} \leq \left(\frac{EF}{AD}\right)^2.$$

Greece

4 For each integer $n \geq 3$, find the least natural number $f(n)$ having the property

$\star$ For every $A \subset \{1, 2, \ldots, n\}$ with $f(n)$ elements, there exist elements $x, y, z \in A$ that are pairwise coprime.