OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

October 1, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_3 - 2 \\
p_4 &= 2x_4 - \\
p_5 &= 2x_5 - \\
p_6 &= 2x_6 - 2 \\
p_7 &= x_7 - x_1 \\
p_8 &= x_8 + x_2 \\
p_9 &= x_9 + x_5 - x_3 \\
p_{10} &= x_{10} + x_6 - x_4 \\
p_{11} &= x_{11} - \\
p_{12} &= x_{12} + x_{11}x_8 \\
p_{13} &= x_{13} - x_{11}x_7 \\
p_{14} &= x_{14} + x_{12}x_1 \\
p_{15} &= x_{15} + x_{11}x_{10} \\
p_{16} &= x_{16} - x_{11}x_9 \\
p_{17} &= x_{17} + x_{16}x_4 + x_{15}x_3 \\
p_{18} &= x_{18} + 1 \\
p_{19} &= x_{19} + 1 \\
p_{20} &= x_{20} \\
p_{21} &= -x_{24}x_{18} + x_{21} \\
p_{22} &= -x_{24}x_{19} + x_{22} \\
p_{23} &= -x_{24}x_{20} + x_{23} - \\
p_{24} &= x_{22}x_{13} + x_{21}x_{12} + x_{14} \\
p_{25} &= -x_{28}x_{18} + x_{25}
\end{aligned}
$$

$$p_{26} = -x_{28}x_{19} + x_{26}$$
$$p_{27} = -x_{28}x_{20} + x_{27} -$$
$$p_{28} = x_{26}x_{16} + x_{25}x_{15} + x_{17}$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 28.

Variable x_{28} selected: The number of polynomials with this variable, with indexes from 1 to 28, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{28} from all other polynomials by reducing them with polynomial p_{25} from previous step.

Finished a triangulation step, the current system is:

$$p_1 = 2x_1 -$$
$$p_2 = 2x_2 -$$
$$p_3 = 2x_3 - 2$$
$$p_4 = 2x_4 -$$
$$p_5 = 2x_5 -$$
$$p_6 = 2x_6 - 2$$
$$p_7 = x_7 - x_1$$
$$p_8 = x_8 + x_2$$
$$p_9 = x_9 + x_5 - x_3$$
$$p_{10} = x_{10} + x_6 - x_4$$
$$p_{11} = x_{11} -$$
$$p_{12} = x_{12} + x_{11}x_8$$
$$p_{13} = x_{13} - x_{11}x_7$$
$$p_{14} = x_{14} + x_{12}x_1$$
$$p_{15} = x_{15} + x_{11}x_{10}$$
$$p_{16} = x_{16} - x_{11}x_9$$
$$p_{17} = x_{17} + x_{16}x_4 + x_{15}x_3$$
$$p_{18} = x_{18} + 1$$
$$p_{19} = x_{19} + 1$$
$$p_{20} = x_{20}$$
$$p_{21} = -x_{24}x_{18} + x_{21}$$
$$p_{22} = -x_{24}x_{19} + x_{22}$$
$$p_{23} = -x_{24}x_{20} + x_{23} -$$
$$p_{24} = x_{22}x_{13} + x_{21}x_{12} + x_{14}$$
$$p_{25} = x_{26}x_{16} + x_{25}x_{15} + x_{17}$$
$$p_{26} = -x_{26}x_{18} + x_{25}x_{19}$$

$$
\begin{aligned}
p_{27} &= -x_{27}x_{18} + x_{25}x_{20} + x_{18} \\
p_{28} &= -x_{28}x_{18} + x_{25}
\end{aligned}
$$

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 27.

Variable x_{27} selected: The number of polynomials with this variable, with indexes from 1 to 27, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{27}. No reduction needed.

The triangular system has not been changed.

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 26.

Variable x_{26} selected: The number of polynomials with this variable, with indexes from 1 to 26, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{26} from all other polynomials by reducing them with polynomial p_{25} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_3 - 2 \\
p_4 &= 2x_4 - \\
p_5 &= 2x_5 - \\
p_6 &= 2x_6 - 2 \\
p_7 &= x_7 - x_1 \\
p_8 &= x_8 + x_2 \\
p_9 &= x_9 + x_5 - x_3 \\
p_{10} &= x_{10} + x_6 - x_4 \\
p_{11} &= x_{11} - \\
p_{12} &= x_{12} + x_{11}x_8 \\
p_{13} &= x_{13} - x_{11}x_7 \\
p_{14} &= x_{14} + x_{12}x_1 \\
p_{15} &= x_{15} + x_{11}x_{10} \\
p_{16} &= x_{16} - x_{11}x_9 \\
p_{17} &= x_{17} + x_{16}x_4 + x_{15}x_3 \\
p_{18} &= x_{18} + 1
\end{aligned}
$$

$$p_{19} = x_{19} + 1$$
$$p_{20} = x_{20}$$
$$p_{21} = -x_{24}x_{18} + x_{21}$$
$$p_{22} = -x_{24}x_{19} + x_{22}$$
$$p_{23} = -x_{24}x_{20} + x_{23} -$$
$$p_{24} = x_{22}x_{13} + x_{21}x_{12} + x_{14}$$
$$p_{25} = x_{25}x_{19}x_{16} + x_{25}x_{18}x_{15} + x_{18}x_{17}$$
$$p_{26} = x_{26}x_{16} + x_{25}x_{15} + x_{17}$$
$$p_{27} = -x_{27}x_{18} + x_{25}x_{20} + x_{18}$$
$$p_{28} = -x_{28}x_{18} + x_{25}$$

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 25.

Variable x_{25} selected: The number of polynomials with this variable, with indexes from 1 to 25, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{25}. No reduction needed.

The triangular system has not been changed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{24} from all other polynomials by reducing them with polynomial p_{21} from previous step.

Finished a triangulation step, the current system is:

$$p_1 = 2x_1 -$$
$$p_2 = 2x_2 -$$
$$p_3 = 2x_3 - 2$$
$$p_4 = 2x_4 -$$
$$p_5 = 2x_5 -$$
$$p_6 = 2x_6 - 2$$
$$p_7 = x_7 - x_1$$
$$p_8 = x_8 + x_2$$
$$p_9 = x_9 + x_5 - x_3$$
$$p_{10} = x_{10} + x_6 - x_4$$

$$
\begin{aligned}
p_{11} &= x_{11} - \\
p_{12} &= x_{12} + x_{11}x_8 \\
p_{13} &= x_{13} - x_{11}x_7 \\
p_{14} &= x_{14} + x_{12}x_1 \\
p_{15} &= x_{15} + x_{11}x_{10} \\
p_{16} &= x_{16} - x_{11}x_9 \\
p_{17} &= x_{17} + x_{16}x_4 + x_{15}x_3 \\
p_{18} &= x_{18} + 1 \\
p_{19} &= x_{19} + 1 \\
p_{20} &= x_{20} \\
p_{21} &= x_{22}x_{13} + x_{21}x_{12} + x_{14} \\
p_{22} &= -x_{22}x_{18} + x_{21}x_{19} \\
p_{23} &= -x_{23}x_{18} + x_{21}x_{20} + x_{18} \\
p_{24} &= -x_{24}x_{18} + x_{21} \\
p_{25} &= x_{25}x_{19}x_{16} + x_{25}x_{18}x_{15} + x_{18}x_{17} \\
p_{26} &= x_{26}x_{16} + x_{25}x_{15} + x_{17} \\
p_{27} &= -x_{27}x_{18} + x_{25}x_{20} + x_{18} \\
p_{28} &= -x_{28}x_{18} + x_{25}
\end{aligned}
$$

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{23}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{22} from all other polynomials by reducing them with polynomial p_{21} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 -
\end{aligned}
$$

$$\begin{aligned}
p_3 &= 2x_3 - 2 \\
p_4 &= 2x_4 - \\
p_5 &= 2x_5 - \\
p_6 &= 2x_6 - 2 \\
p_7 &= x_7 - x_1 \\
p_8 &= x_8 + x_2 \\
p_9 &= x_9 + x_5 - x_3 \\
p_{10} &= x_{10} + x_6 - x_4 \\
p_{11} &= x_{11} - \\
p_{12} &= x_{12} + x_{11}x_8 \\
p_{13} &= x_{13} - x_{11}x_7 \\
p_{14} &= x_{14} + x_{12}x_1 \\
p_{15} &= x_{15} + x_{11}x_{10} \\
p_{16} &= x_{16} - x_{11}x_9 \\
p_{17} &= x_{17} + x_{16}x_4 + x_{15}x_3 \\
p_{18} &= x_{18} + 1 \\
p_{19} &= x_{19} + 1 \\
p_{20} &= x_{20} \\
p_{21} &= x_{21}x_{19}x_{13} + x_{21}x_{18}x_{12} + x_{18}x_{14} \\
p_{22} &= x_{22}x_{13} + x_{21}x_{12} + x_{14} \\
p_{23} &= -x_{23}x_{18} + x_{21}x_{20} + x_{18} \\
p_{24} &= -x_{24}x_{18} + x_{21} \\
p_{25} &= x_{25}x_{19}x_{16} + x_{25}x_{18}x_{15} + x_{18}x_{17} \\
p_{26} &= x_{26}x_{16} + x_{25}x_{15} + x_{17} \\
p_{27} &= -x_{27}x_{18} + x_{25}x_{20} + x_{18} \\
p_{28} &= -x_{28}x_{18} + x_{25}
\end{aligned}$$

1.8   Triangulation, step 8

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{21}. No reduction needed.

The triangular system has not been changed.

1.9   Triangulation, step 9

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20}. No reduction needed.

The triangular system has not been changed.

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{19}. No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18}. No reduction needed.

The triangular system has not been changed.

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16}. No reduction needed.

The triangular system has not been changed.

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

1.25 Triangulation, step 25

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.26   Triangulation, step 26

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.27   Triangulation, step 27

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.28   Triangulation, step 28

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 2x_2 - \\
p_3 &= 2x_3 - 2 \\
p_4 &= 2x_4 - \\
p_5 &= 2x_5 - \\
p_6 &= 2x_6 - 2 \\
p_7 &= x_7 - x_1 \\
p_8 &= x_8 + x_2 \\
p_9 &= x_9 + x_5 - x_3 \\
p_{10} &= x_{10} + x_6 - x_4 \\
p_{11} &= x_{11} - \\
p_{12} &= x_{12} + x_{11}x_8
\end{aligned}
$$

$$
\begin{aligned}
p_{13} &= x_{13} - x_{11}x_7 \\
p_{14} &= x_{14} + x_{12}x_1 \\
p_{15} &= x_{15} + x_{11}x_{10} \\
p_{16} &= x_{16} - x_{11}x_9 \\
p_{17} &= x_{17} + x_{16}x_4 + x_{15}x_3 \\
p_{18} &= x_{18} + 1 \\
p_{19} &= x_{19} + 1 \\
p_{20} &= x_{20} \\
p_{21} &= x_{21}x_{19}x_{13} + x_{21}x_{18}x_{12} + x_{18}x_{14} \\
p_{22} &= x_{22}x_{13} + x_{21}x_{12} + x_{14} \\
p_{23} &= -x_{23}x_{18} + x_{21}x_{20} + x_{18} \\
p_{24} &= -x_{24}x_{18} + x_{21} \\
p_{25} &= x_{25}x_{19}x_{16} + x_{25}x_{18}x_{15} + x_{18}x_{17} \\
p_{26} &= x_{26}x_{16} + x_{25}x_{15} + x_{17} \\
p_{27} &= -x_{27}x_{18} + x_{25}x_{20} + x_{18} \\
p_{28} &= -x_{28}x_{18} + x_{25}
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
\begin{aligned}
g &= x_{27}^2 - 2x_{27}x_{23} + x_{26}^2 - 2x_{26}x_{22} + x_{25}^2 - 2x_{25}x_{21} + x_{23}^2 + \\
  &\quad x_{22}^2 + x_{21}^2 - x_4^2 - x_3^2 + 2x_3x_1 - x_1^2
\end{aligned}
$$

with respect to the triangular system.

1. Pseudo remainder with p_{28} over variable x_{28}:

$$
\begin{aligned}
g &= x_{27}^2 - 2x_{27}x_{23} + x_{26}^2 - 2x_{26}x_{22} + x_{25}^2 - 2x_{25}x_{21} + x_{23}^2 + \\
  &\quad x_{22}^2 + x_{21}^2 - x_4^2 - x_3^2 + 2x_3x_1 - x_1^2
\end{aligned}
$$

2. Pseudo remainder with p_{27} over variable x_{27}:

$$
\begin{aligned}
g &= x_{26}^2 x_{18}^2 - 2x_{26}x_{22}x_{18}^2 + x_{25}^2 x_{20}^2 + x_{25}^2 x_{18}^2 \\
  &\quad -2x_{25}x_{23}x_{20}x_{18} - 2x_{25}x_{21}x_{18}^2 + 2x_{25}x_{20}x_{18} + x_{23}^2 x_{18}^2 \\
  &\quad -2x_{23}x_{18}^2 + x_{22}^2 x_{18}^2 + x_{21}^2 x_{18}^2 - x_{18}^2 x_4^2 \\
  &\quad -x_{18}^2 x_3^2 + 2x_{18}^2 x_3 x_1 - x_{18}^2 x_1^2 + x_{18}^2
\end{aligned}
$$

3. Pseudo remainder with p_{26} over variable x_{26}:

$$
\begin{aligned}
g \;=\; & x_{25}^2 x_{20}^2 x_{16}^2 + x_{25}^2 x_{18}^2 x_{16}^2 + x_{25}^2 x_{18}^2 x_{15}^2 \\
& -2 x_{25} x_{23} x_{20} x_{18} x_{16}^2 + 2 x_{25} x_{22} x_{18}^2 x_{16} x_{15} \\
& -2 x_{25} x_{21} x_{18}^2 x_{16}^2 + 2 x_{25} x_{20} x_{18} x_{16}^2 + \\
& 2 x_{25} x_{18}^2 x_{17} x_{15} + x_{23}^2 x_{18}^2 x_{16}^2 - 2 x_{23} x_{18}^2 x_{16}^2 + \\
& x_{22}^2 x_{18}^2 x_{16}^2 + 2 x_{22} x_{18}^2 x_{17} x_{16} + x_{21}^2 x_{18}^2 x_{16}^2 + \\
& x_{18}^2 x_{17}^2 - x_{18}^2 x_{16}^2 x_4^2 - x_{18}^2 x_{16}^2 x_3^2 + \\
& 2 x_{18}^2 x_{16}^2 x_3 x_1 - x_{18}^2 x_{16}^2 x_1^2 + x_{18}^2 x_{16}^2
\end{aligned}
$$

4. Pseudo remainder with p_{25} over variable x_{25}:

$$
\begin{aligned}
g \;=\; & x_{23}^2 x_{19}^2 x_{18}^2 x_{16}^4 + 2 x_{23}^2 x_{19} x_{18}^3 x_{16}^3 x_{15} + \\
& x_{23}^2 x_{18}^4 x_{16}^2 x_{15}^2 + 2 x_{23} x_{20} x_{19} x_{18}^2 x_{17} x_{16}^3 + \\
& 2 x_{23} x_{20} x_{18}^3 x_{17} x_{16}^2 x_{15} - 2 x_{23} x_{19}^2 x_{18}^2 x_{16}^4 \\
& -4 x_{23} x_{19} x_{18}^3 x_{16}^3 x_{15} - 2 x_{23} x_{18}^4 x_{16}^2 x_{15}^2 + \\
& x_{22}^2 x_{19}^2 x_{18}^2 x_{16}^4 + 2 x_{22}^2 x_{19} x_{18}^3 x_{16}^3 x_{15} + \\
& x_{22}^2 x_{18}^4 x_{16}^2 x_{15}^2 + 2 x_{22} x_{19}^2 x_{18}^2 x_{17} x_{16}^3 + \\
& 2 x_{22} x_{19} x_{18}^3 x_{17} x_{16}^2 x_{15} + x_{21}^2 x_{19}^2 x_{18}^2 x_{16}^4 + \\
& 2 x_{21}^2 x_{19} x_{18}^3 x_{16}^3 x_{15} + x_{21}^2 x_{18}^4 x_{16}^2 x_{15}^2 + \\
& 2 x_{21} x_{19} x_{18}^3 x_{17} x_{16}^3 + 2 x_{21} x_{18}^4 x_{17} x_{16}^2 x_{15} + \\
& x_{20}^2 x_{18}^2 x_{17}^2 x_{16}^2 - 2 x_{20} x_{19} x_{18}^2 x_{17} x_{16}^3 \\
& -2 x_{20} x_{18}^3 x_{17} x_{16}^2 x_{15} + x_{19}^2 x_{18}^2 x_{17}^2 x_{16}^2 \\
& -x_{19}^2 x_{18}^2 x_{16}^4 x_4^2 - x_{19}^2 x_{18}^2 x_{16}^4 x_3^2 + \\
& 2 x_{19}^2 x_{18}^2 x_{16}^4 x_3 x_1 - x_{19}^2 x_{18}^2 x_{16}^4 x_1^2 + \\
& x_{19}^2 x_{18}^2 x_{16}^4 - 2 x_{19} x_{18}^3 x_{16}^3 x_{15} x_4^2 \\
& -2 x_{19} x_{18}^3 x_{16}^3 x_{15} x_3^2 + 4 x_{19} x_{18}^3 x_{16}^3 x_{15} x_3 x_1 \\
& -2 x_{19} x_{18}^3 x_{16}^3 x_{15} x_1^2 + 2 x_{19} x_{18}^3 x_{16}^3 x_{15} + \\
& x_{18}^4 x_{17}^2 x_{16}^2 - x_{18}^4 x_{16}^2 x_{15}^2 x_4^2 \\
& -x_{18}^4 x_{16}^2 x_{15}^2 x_3^2 + 2 x_{18}^4 x_{16}^2 x_{15}^2 x_3 x_1 \\
& -x_{18}^4 x_{16}^2 x_{15}^2 x_1^2 + x_{18}^4 x_{16}^2 x_{15}^2
\end{aligned}
$$

5. Pseudo remainder with p_{24} over variable x_{24}:

$$
\begin{aligned}
g \;=\; & x_{23}^2 x_{19}^2 x_{18}^2 x_{16}^4 + 2 x_{23}^2 x_{19} x_{18}^3 x_{16}^3 x_{15} + \\
& x_{23}^2 x_{18}^4 x_{16}^2 x_{15}^2 + 2 x_{23} x_{20} x_{19} x_{18}^2 x_{17} x_{16}^3 + \\
& 2 x_{23} x_{20} x_{18}^3 x_{17} x_{16}^2 x_{15} - 2 x_{23} x_{19}^2 x_{18}^2 x_{16}^4 \\
& -4 x_{23} x_{19} x_{18}^3 x_{16}^3 x_{15} - 2 x_{23} x_{18}^4 x_{16}^2 x_{15}^2 + \\
& x_{22}^2 x_{19}^2 x_{18}^2 x_{16}^4 + 2 x_{22}^2 x_{19} x_{18}^3 x_{16}^3 x_{15} + \\
& x_{22}^2 x_{18}^4 x_{16}^2 x_{15}^2 + 2 x_{22} x_{19}^2 x_{18}^2 x_{17} x_{16}^3 +
\end{aligned}
$$

$$2x_{22}x_{19}x_{18}^3x_{17}x_{16}^2x_{15} + x_{21}^2x_{19}^2x_{18}^2x_{16}^4 +$$
$$2x_{21}^2x_{19}x_{18}^3x_{16}^3x_{15} + x_{21}^2x_{18}^4x_{16}^2x_{15}^2 +$$
$$2x_{21}x_{19}x_{18}^3x_{17}x_{16}^3 + 2x_{21}x_{18}^4x_{17}x_{16}^2x_{15} +$$
$$x_{20}^2x_{18}^2x_{17}^2x_{16}^2 - 2x_{20}x_{19}x_{18}^2x_{17}x_{16}^3$$
$$-2x_{20}x_{18}^3x_{17}x_{16}^2x_{15} + x_{19}^2x_{18}^2x_{17}^2x_{16}^2$$
$$-x_{19}^2x_{18}^2x_{16}^4x_4^2 - x_{19}^2x_{18}^2x_{16}^4x_3^2 +$$
$$2x_{19}^2x_{18}^2x_{16}^4x_3x_1 - x_{19}^2x_{18}^2x_{16}^4x_1^2 +$$
$$x_{19}^2x_{18}^2x_{16}^4 - 2x_{19}x_{18}^3x_{16}^3x_{15}x_4^2$$
$$-2x_{19}x_{18}^3x_{16}^3x_{15}x_3^2 + 4x_{19}x_{18}^3x_{16}^3x_{15}x_3x_1$$
$$-2x_{19}x_{18}^3x_{16}^3x_{15}x_1^2 + 2x_{19}x_{18}^3x_{16}^3x_{15} +$$
$$x_{18}^4x_{17}^2x_{16}^2 - x_{18}^4x_{16}^2x_{15}^2x_4^2$$
$$-x_{18}^4x_{16}^2x_{15}^2x_3^2 + 2x_{18}^4x_{16}^2x_{15}^2x_3x_1$$
$$-x_{18}^4x_{16}^2x_{15}^2x_1^2 + x_{18}^4x_{16}^2x_{15}^2$$

6. Pseudo remainder with p_{23} over variable x_{23}:

$$
\begin{aligned}
g \quad = \quad & x_{22}^2x_{19}^2x_{18}^4x_{16}^4 + 2x_{22}^2x_{19}x_{18}^5x_{16}^3x_{15} + \\
& x_{22}^2x_{18}^6x_{16}^2x_{15}^2 + 2x_{22}x_{19}^2x_{18}^4x_{17}x_{16}^3 + \\
& 2x_{22}x_{19}x_{18}^5x_{17}x_{16}^2x_{15} + \\
& x_{21}^2x_{20}^2x_{19}^2x_{18}^2x_{16}^4 + \\
& 2x_{21}^2x_{20}^2x_{19}x_{18}^3x_{16}^3x_{15} + \\
& x_{21}^2x_{20}^2x_{18}^4x_{16}^2x_{15}^2 + x_{21}^2x_{19}^2x_{18}^4x_{16}^4 + \\
& 2x_{21}^2x_{19}x_{18}^5x_{16}^3x_{15} + x_{21}^2x_{18}^6x_{16}^2x_{15}^2 + \\
& 2x_{21}x_{20}^2x_{19}x_{18}^3x_{17}x_{16}^3 + \\
& 2x_{21}x_{20}^2x_{18}^4x_{17}x_{16}^2x_{15} + 2x_{21}x_{19}x_{18}^5x_{17}x_{16}^3 + \\
& 2x_{21}x_{18}^6x_{17}x_{16}^2x_{15} + x_{20}^2x_{18}^4x_{17}^2x_{16}^2 + \\
& x_{19}^2x_{18}^4x_{17}^2x_{16}^2 - x_{19}^2x_{18}^4x_{16}^4x_4^2 \\
& -x_{19}^2x_{18}^4x_{16}^4x_3^2 + 2x_{19}^2x_{18}^4x_{16}^4x_3x_1 \\
& -x_{19}^2x_{18}^4x_{16}^4x_1^2 - 2x_{19}x_{18}^5x_{16}^3x_{15}x_4^2 \\
& -2x_{19}x_{18}^5x_{16}^3x_{15}x_3^2 + 4x_{19}x_{18}^5x_{16}^3x_{15}x_3x_1 \\
& -2x_{19}x_{18}^5x_{16}^3x_{15}x_1^2 + x_{18}^6x_{17}^2x_{16}^2 \\
& -x_{18}^6x_{16}^2x_{15}^2x_4^2 - x_{18}^6x_{16}^2x_{15}^2x_3^2 + \\
& 2x_{18}^6x_{16}^2x_{15}^2x_3x_1 - x_{18}^6x_{16}^2x_{15}^2x_1^2
\end{aligned}
$$

7. Pseudo remainder with p_{22} over variable x_{22}:

   Polynomial too big for output (text size is 2089 characters, number of terms is 38)

8. Pseudo remainder with p_{21} over variable x_{21}:

   Polynomial too big for output (text size is 4074 characters, number of terms is 66)

9. Pseudo remainder with p_{20} over variable x_{20}:

   Polynomial too big for output (text size is 3408 characters, number of terms is 56)

10. Pseudo remainder with p_{19} over variable x_{19}:

    Polynomial too big for output (text size is 2970 characters, number of terms is 56)

11. Pseudo remainder with p_{18} over variable x_{18}:

$$
\begin{aligned}
g \;=\; & 2x_{17}^2 x_{16}^2 x_{13}^4 + 4x_{17}^2 x_{16}^2 x_{13}^3 x_{12} + \\
& 2x_{17}^2 x_{16}^2 x_{13}^2 x_{12}^2 - 4x_{17} x_{16}^3 x_{14} x_{13}^3 \\
& -4x_{17} x_{16}^3 x_{14} x_{13}^2 x_{12} - 4x_{17} x_{16}^2 x_{15} x_{14} x_{13}^3 \\
& -4x_{17} x_{16}^2 x_{15} x_{14} x_{13}^2 x_{12} + 2x_{16}^4 x_{14}^2 x_{13}^2 \\
& -x_{16}^4 x_{13}^4 x_4^2 - x_{16}^4 x_{13}^4 x_3^2 + \\
& 2x_{16}^4 x_{13}^4 x_3 x_1 - x_{16}^4 x_{13}^4 x_1^2 \\
& -2x_{16}^4 x_{13}^3 x_{12} x_4^2 - 2x_{16}^4 x_{13}^3 x_{12} x_3^2 + \\
& 4x_{16}^4 x_{13}^3 x_{12} x_3 x_1 - 2x_{16}^4 x_{13}^3 x_{12} x_1^2 \\
& -x_{16}^4 x_{13}^2 x_{12}^2 x_4^2 - x_{16}^4 x_{13}^2 x_{12}^2 x_3^2 + \\
& 2x_{16}^4 x_{13}^2 x_{12}^2 x_3 x_1 - x_{16}^4 x_{13}^2 x_{12}^2 x_1^2 + \\
& 4x_{16}^3 x_{15} x_{14}^2 x_{13}^2 - 2x_{16}^3 x_{15} x_{13}^4 x_4^2 \\
& -2x_{16}^3 x_{15} x_{13}^4 x_3^2 + 4x_{16}^3 x_{15} x_{13}^4 x_3 x_1 \\
& -2x_{16}^3 x_{15} x_{13}^4 x_1^2 - 4x_{16}^3 x_{15} x_{13}^3 x_{12} x_4^2 \\
& -4x_{16}^3 x_{15} x_{13}^3 x_{12} x_3^2 + 8x_{16}^3 x_{15} x_{13}^3 x_{12} x_3 x_1 \\
& -4x_{16}^3 x_{15} x_{13}^3 x_{12} x_1^2 - 2x_{16}^3 x_{15} x_{13}^2 x_{12}^2 x_4^2 \\
& -2x_{16}^3 x_{15} x_{13}^2 x_{12}^2 x_3^2 + \\
& 4x_{16}^3 x_{15} x_{13}^2 x_{12}^2 x_3 x_1 \\
& -2x_{16}^3 x_{15} x_{13}^2 x_{12}^2 x_1^2 + 2x_{16}^2 x_{15}^2 x_{14}^2 x_{13}^2 \\
& -x_{16}^2 x_{15}^2 x_{13}^4 x_4^2 - x_{16}^2 x_{15}^2 x_{13}^4 x_3^2 + \\
& 2x_{16}^2 x_{15}^2 x_{13}^4 x_3 x_1 - x_{16}^2 x_{15}^2 x_{13}^4 x_1^2 \\
& -2x_{16}^2 x_{15}^2 x_{13}^3 x_{12} x_4^2 \\
& -2x_{16}^2 x_{15}^2 x_{13}^3 x_{12} x_3^2 + \\
& 4x_{16}^2 x_{15}^2 x_{13}^3 x_{12} x_3 x_1 \\
& -2x_{16}^2 x_{15}^2 x_{13}^3 x_{12} x_1^2 \\
& -x_{16}^2 x_{15}^2 x_{13}^2 x_{12}^2 x_4^2 \\
& -x_{16}^2 x_{15}^2 x_{13}^2 x_{12}^2 x_3^2 + \\
& 2x_{16}^2 x_{15}^2 x_{13}^2 x_{12}^2 x_3 x_1 \\
& -x_{16}^2 x_{15}^2 x_{13}^2 x_{12}^2 x_1^2
\end{aligned}
$$

12. Pseudo remainder with p_{17} over variable x_{17}:

    Polynomial too big for output (text size is 2186 characters, number of terms is 50)

15

13. Pseudo remainder with p_{16} over variable x_{16}:

Polynomial too big for output (text size is 2666 characters, number of terms is 50)

14. Pseudo remainder with p_{15} over variable x_{15}:

Polynomial too big for output (text size is 2662 characters, number of terms is 50)

15. Pseudo remainder with p_{14} over variable x_{14}:

Polynomial too big for output (text size is 2192 characters, number of terms is 41)

16. Pseudo remainder with p_{13} over variable x_{13}:

Polynomial too big for output (text size is 2143 characters, number of terms is 41)

17. Pseudo remainder with p_{12} over variable x_{12}:

Polynomial too big for output (text size is 2111 characters, number of terms is 41)

18. Pseudo remainder with p_{11} over variable x_{11}:

$$
\begin{aligned}
g = {} & -x_{10}^2 x_9^2 x_8^2 x_7^2 x_4^2 + \\
& x_{10}^2 x_9^2 x_8^2 x_7^2 x_3^2 \\
& -2x_{10}^2 x_9^2 x_8^2 x_7^2 x_3 x_1 + \\
& x_{10}^2 x_9^2 x_8^2 x_7^2 x_1^2 + 2x_{10}^2 x_9^2 x_8 x_7^3 x_4^2 \\
& -2x_{10}^2 x_9^2 x_8 x_7^3 x_3^2 + 2x_{10}^2 x_9^2 x_8 x_7^3 x_1^2 \\
& -x_{10}^2 x_9^2 x_7^4 x_4^2 + x_{10}^2 x_9^2 x_7^4 x_3^2 + \\
& 2x_{10}^2 x_9^2 x_7^4 x_3 x_1 - x_{10}^2 x_9^2 x_7^4 x_1^2 + \\
& 2x_{10} x_9^3 x_8^2 x_7^2 x_4^2 - 4x_{10} x_9^3 x_8^2 x_7^2 x_4 x_3 + \\
& 4x_{10} x_9^3 x_8^2 x_7^2 x_4 x_1 + 2x_{10} x_9^3 x_8^2 x_7^2 x_3^2 \\
& -2x_{10} x_9^3 x_8^2 x_7^2 x_1^2 - 4x_{10} x_9^3 x_8 x_7^3 x_4^2 + \\
& 8x_{10} x_9^3 x_8 x_7^3 x_4 x_3 - 4x_{10} x_9^3 x_8 x_7^3 x_4 x_1 \\
& -4x_{10} x_9^3 x_8 x_7^3 x_3^2 + 4x_{10} x_9^3 x_8 x_7^3 x_3 x_1 \\
& -4x_{10} x_9^3 x_8 x_7^3 x_1^2 + 2x_{10} x_9^3 x_7^4 x_4^2 \\
& -4x_{10} x_9^3 x_7^4 x_4 x_3 + 2x_{10} x_9^3 x_7^4 x_3^2 \\
& -4x_{10} x_9^3 x_7^4 x_3 x_1 + 2x_{10} x_9^3 x_7^4 x_1^2 + \\
& x_9^4 x_8^2 x_7^2 x_4^2 - 4x_9^4 x_8^2 x_7^2 x_4 x_1 \\
& -x_9^4 x_8^2 x_7^2 x_3^2 + 2x_9^4 x_8^2 x_7^2 x_3 x_1 + \\
& x_9^4 x_8^2 x_7^2 x_1^2 - 2x_9^4 x_8 x_7^3 x_4^2 + \\
& 4x_9^4 x_8 x_7^3 x_4 x_1 + 2x_9^4 x_8 x_7^3 x_3^2 \\
& -4x_9^4 x_8 x_7^3 x_3 x_1 + 2x_9^4 x_8 x_7^3 x_1^2 + \\
& x_9^4 x_7^4 x_4^2 - x_9^4 x_7^4 x_3^2 + 2x_9^4 x_7^4 x_3 x_1 \\
& -x_9^4 x_7^4 x_1^2
\end{aligned}
$$

19. Pseudo remainder with p_{10} over variable x_{10}:

   Polynomial too big for output (text size is 3238 characters, number of terms is 79)

20. Pseudo remainder with p_9 over variable x_9:

   Polynomial too big for output (text size is 10316 characters, number of terms is 253)

21. Pseudo remainder with p_8 over variable x_8:

   Polynomial too big for output (text size is 10310 characters, number of terms is 253)

22. Pseudo remainder with p_7 over variable x_7:

   Polynomial too big for output (text size is 9368 characters, number of terms is 253)

23. Pseudo remainder with p_6 over variable x_6:

   Polynomial too big for output (text size is 8779 characters, number of terms is 253)

24. Pseudo remainder with p_5 over variable x_5:

   Polynomial too big for output (text size is 3997 characters, number of terms is 129)

25. Pseudo remainder with p_4 over variable x_4:

$$
\begin{aligned}
g = {} & -1024x_3^6x_2^2x_1^2 - 2048x_3^6x_2x_1^3 - 1024x_3^6x_1^4 + \\
& 2048x_3^5x_2^2x_1^3 + 1024x_3^5x_2^2x_1^2 + 4096x_3^5x_2x_1^4 + \\
& 2048x_3^5x_2x_1^3 + 2048x_3^5x_1^5 + 1024x_3^5x_1^4 + \\
& 1024x_3^4x_2^2x_1^4 - 6144x_3^4x_2^2x_1^3 + \\
& 1536x_3^4x_2^2x_1^2 - 2048x_3^4x_2x_1^5 - 8192x_3^4x_2x_1^4 + \\
& 3072x_3^4x_2x_1^3 - 1024x_3^4x_1^6 - 2048x_3^4x_1^5 + \\
& 1536x_3^4x_1^4 - 1024x_3^3x_2^2x_1^4 + 5632x_3^3x_2^2x_1^3 \\
& -2816x_3^3x_2^2x_1^2 + 2048x_3^3x_2x_1^5 + 6144x_3^3x_2x_1^4 \\
& -5632x_3^3x_2x_1^3 + 1024x_3^3x_1^6 + 512x_3^3x_1^5 \\
& -2816x_3^3x_1^4 + 256x_3^2x_2^2x_1^4 - 2048x_3^2x_2^2x_1^3 + \\
& 1600x_3^2x_2^2x_1^2 - 512x_3^2x_2x_1^5 - 2048x_3^2x_2x_1^4 + \\
& 3200x_3^2x_2x_1^3 - 256x_3^2x_1^6 + 1600x_3^2x_1^4 + \\
& 256x_3x_2^2x_1^3 - 384x_3x_2^2x_1^2 + 256x_3x_2x_1^4 \\
& -768x_3x_2x_1^3 - 384x_3x_1^4 + 32x_2^2x_1^2 + 64x_2x_1^3 + 32x_1^4
\end{aligned}
$$

26. Pseudo remainder with p_3 over variable x_3:

$$
\begin{aligned}
g = {} & 16384x_2^2x_1^4 - 16384x_2^2x_1^3 - 2048x_2^2x_1^2 - 32768x_2x_1^5 + \\
& 16384x_2x_1^4 - 4096x_2x_1^3 - 16384x_1^6 + 32768x_1^5 - 2048x_1^4
\end{aligned}
$$

27. Pseudo remainder with p_2 over variable x_2:

$$g \quad = \quad -65536x_1^6 + 65536x_1^5 + 40960x_1^4 - 24576x_1^3 - 2048x_1^2$$

28. Pseudo remainder with p_1 over variable x_1:

$$g \quad = \quad 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 253 terms.

Time Complexity: Time spent by the prover is 0.297 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form