Art of Problem Solving
1998 Balkan MO

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– May 5th

1 Consider the finite sequence $\left\lfloor \frac{k^2}{1998} \right\rfloor$, for $k = 1, 2, \ldots, 1997$. How many distinct terms are there in this sequence?

Greece

2 Let $n \geq 2$ be an integer, and let $0 < a_1 < a_2 < \cdots < a_{2n+1}$ be real numbers. Prove the inequality

$$\sqrt[n]{a_1} - \sqrt[n]{a_2} + \sqrt[n]{a_3} - \cdots + \sqrt[n]{a_{2n+1}} < \sqrt[n]{a_1 - a_2 + a_3 - \cdots + a_{2n+1}}.$$

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3 Let $\mathcal{S}$ denote the set of points inside or on the border of a triangle ABC, without a fixed point T inside the triangle. Show that $\mathcal{S}$ can be partitioned into disjoint closed segemnts.

Yugoslavia

4 Prove that the following equation has no solution in integer numbers:

$$x^2 + 4 = y^5.$$

Bulgaria