OpenGeoProver Output for conjecture “geothm_zadatak”

Wu’s method used

October 2, 2016

1 Invoking the theorem prover

The used proving method is Wu’s method.
The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{15} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13} \\
p_{16} &= -x_{18}x_8 + x_{17}x_9 \\
p_{17} &= -x_{15}x_8 + x_{14}x_9 \\
p_{18} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{19} &= -x_{16}x_6 + x_{16}x_3 + x_{15}x_7 - x_{15}x_4 - x_7x_3 + x_6x_4
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{18} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{15}x_8 + x_{14}x_9 \\
p_{15} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{16} &= -x_{16}x_6 + x_{16}x_3 + x_{15}x_7 - x_{15}x_4 - x_7x_3 + x_6x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}
$$

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{16} from all other polynomials by reducing them with polynomial p_{15} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{15}x_8 + x_{14}x_9 \\
p_{15} &= x_{15}x_7x_5 - x_{15}x_7x_1 - x_{15}x_5x_4 + x_{15}x_4x_1 - x_{14}x_7x_6 + \\
&\quad x_{14}x_7x_3 + x_{14}x_6x_4 - x_{14}x_4x_3 + x_7x_6x_1 - x_7x_5x_3 \\
&\quad -x_6x_4x_1 + x_5x_4x_3 \\
p_{16} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}
$$

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{15} from all other polynomials by reducing them with polynomial p_{14} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{14}x_9x_7x_5 + x_{14}x_9x_7x_1 + x_{14}x_9x_5x_4 - x_{14}x_9x_4x_1 + \\
&\quad x_{14}x_8x_7x_6 - x_{14}x_8x_7x_3 - x_{14}x_8x_6x_4 + x_{14}x_8x_4x_3 \\
&\quad -x_8x_7x_6x_1 + x_8x_7x_5x_3 + x_8x_6x_4x_1 - x_8x_5x_4x_3 \\
p_{15} &= -x_{15}x_8 + x_{14}x_9 \\
p_{16} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}
$$

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 \\
p_9 &= x_9 + x_4x_1 - x_4 \\
p_{10} &= x_{10} - x_3x_1 + x_3 + x_2x_1 - x_2 \\
p_{11} &= x_{11} - x_4x_2 - \\
p_{12} &= x_{12} + x_4x_1 \\
p_{13} &= x_{13} - x_3x_1 + x_2x_1 \\
p_{14} &= -x_{14}x_9x_7x_5 + x_{14}x_9x_7x_1 + x_{14}x_9x_5x_4 - x_{14}x_9x_4x_1 + \\
&\quad x_{14}x_8x_7x_6 - x_{14}x_8x_7x_3 - x_{14}x_8x_6x_4 + x_{14}x_8x_4x_3 \\
&\quad -x_8x_7x_6x_1 + x_8x_7x_5x_3 + x_8x_6x_4x_1 - x_8x_5x_4x_3 \\
p_{15} &= -x_{15}x_8 + x_{14}x_9 \\
p_{16} &= x_{16}x_5 - x_{16}x_1 - x_{14}x_7 + x_{14}x_4 + x_7x_1 - x_5x_4 \\
p_{17} &= x_{17}x_{12}x_8 - x_{17}x_{11}x_9 + x_{17}x_9 - x_{12}x_8 \\
p_{18} &= -x_{18}x_{11} + x_{18} + x_{17}x_{12} - x_{12} \\
p_{19} &= x_{19}x_{11} - x_{19} - x_{17}x_{13} + x_{13}
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$g = x_{16}x_8 - x_{14}x_{10}$$

with respect to the triangular system.

1. Pseudo remainder with p_{19} over variable x_{19}:

$$g = x_{16}x_8 - x_{14}x_{10}$$

2. Pseudo remainder with p_{18} over variable x_{18}:

$$g = x_{16}x_8 - x_{14}x_{10}$$

3. Pseudo remainder with p_{17} over variable x_{17}:

$$g = x_{16}x_8 - x_{14}x_{10}$$

4. Pseudo remainder with p_{16} over variable x_{16}:

$$\begin{aligned} g = {} & -x_{14}x_{10}x_5 + x_{14}x_{10}x_1 + x_{14}x_8x_7 - x_{14}x_8x_4 - x_8x_7x_1 + \\ & x_8x_5x_4 \end{aligned}$$

5. Pseudo remainder with p_{15} over variable x_{15}:

$$\begin{aligned} g = {} & -x_{14}x_{10}x_5 + x_{14}x_{10}x_1 + x_{14}x_8x_7 - x_{14}x_8x_4 - x_8x_7x_1 + \\ & x_8x_5x_4 \end{aligned}$$

6. Pseudo remainder with p_{14} over variable x_{14}:

$$\begin{aligned} g = {} & -x_{10}x_8x_7x_6x_5x_1 + x_{10}x_8x_7x_6x_1^2 + \\ & x_{10}x_8x_7x_5^2x_3 - x_{10}x_8x_7x_5x_3x_1 + \\ & x_{10}x_8x_6x_5x_4x_1 - x_{10}x_8x_6x_4x_1^2 \\ & -x_{10}x_8x_5^2x_4x_3 + x_{10}x_8x_5x_4x_3x_1 + \\ & x_9x_8x_7^2x_5x_1 - x_9x_8x_7^2x_1^2 - x_9x_8x_7x_5^2x_4 + \\ & x_9x_8x_7x_4x_1^2 + x_9x_8x_5^2x_4^2 - x_9x_8x_5x_4^2x_1 \\ & -x_8^2x_7^2x_5x_3 + x_8^2x_7^2x_3x_1 + x_8^2x_7x_6x_5x_4 \\ & -x_8^2x_7x_6x_4x_1 + x_8^2x_7x_5x_4x_3 - x_8^2x_7x_4x_3x_1 \\ & -x_8^2x_6x_5x_4^2 + x_8^2x_6x_4^2x_1 \end{aligned}$$

7. Pseudo remainder with p_{13} over variable x_{13}:

$$\begin{aligned} g = {} & -x_{10}x_8x_7x_6x_5x_1 + x_{10}x_8x_7x_6x_1^2 + \\ & x_{10}x_8x_7x_5^2x_3 - x_{10}x_8x_7x_5x_3x_1 + \\ & x_{10}x_8x_6x_5x_4x_1 - x_{10}x_8x_6x_4x_1^2 \\ & -x_{10}x_8x_5^2x_4x_3 + x_{10}x_8x_5x_4x_3x_1 + \\ & x_9x_8x_7^2x_5x_1 - x_9x_8x_7^2x_1^2 - x_9x_8x_7x_5^2x_4 + \\ & x_9x_8x_7x_4x_1^2 + x_9x_8x_5^2x_4^2 - x_9x_8x_5x_4^2x_1 \\ & -x_8^2x_7^2x_5x_3 + x_8^2x_7^2x_3x_1 + x_8^2x_7x_6x_5x_4 \\ & -x_8^2x_7x_6x_4x_1 + x_8^2x_7x_5x_4x_3 - x_8^2x_7x_4x_3x_1 \\ & -x_8^2x_6x_5x_4^2 + x_8^2x_6x_4^2x_1 \end{aligned}$$

8. Pseudo remainder with p_{12} over variable x_{12}:

$$\begin{aligned} g \;=\;& -x_{10}x_8x_7x_6x_5x_1 + x_{10}x_8x_7x_6x_1^2 + \\ & x_{10}x_8x_7x_5^2x_3 - x_{10}x_8x_7x_5x_3x_1 + \\ & x_{10}x_8x_6x_5x_4x_1 - x_{10}x_8x_6x_4x_1^2 \\ & -x_{10}x_8x_5^2x_4x_3 + x_{10}x_8x_5x_4x_3x_1 + \\ & x_9x_8x_7^2x_5x_1 - x_9x_8x_7^2x_1^2 - x_9x_8x_7x_5^2x_4 + \\ & x_9x_8x_7x_4x_1^2 + x_9x_8x_5^2x_4^2 - x_9x_8x_5x_4^2x_1 \\ & -x_8^2x_7^2x_5x_3 + x_8^2x_7^2x_3x_1 + x_8^2x_7x_6x_5x_4 \\ & -x_8^2x_7x_6x_4x_1 + x_8^2x_7x_5x_4x_3 - x_8^2x_7x_4x_3x_1 \\ & -x_8^2x_6x_5x_4^2 + x_8^2x_6x_4^2x_1 \end{aligned}$$

9. Pseudo remainder with p_{11} over variable x_{11}:

$$\begin{aligned} g \;=\;& -x_{10}x_8x_7x_6x_5x_1 + x_{10}x_8x_7x_6x_1^2 + \\ & x_{10}x_8x_7x_5^2x_3 - x_{10}x_8x_7x_5x_3x_1 + \\ & x_{10}x_8x_6x_5x_4x_1 - x_{10}x_8x_6x_4x_1^2 \\ & -x_{10}x_8x_5^2x_4x_3 + x_{10}x_8x_5x_4x_3x_1 + \\ & x_9x_8x_7^2x_5x_1 - x_9x_8x_7^2x_1^2 - x_9x_8x_7x_5^2x_4 + \\ & x_9x_8x_7x_4x_1^2 + x_9x_8x_5^2x_4^2 - x_9x_8x_5x_4^2x_1 \\ & -x_8^2x_7^2x_5x_3 + x_8^2x_7^2x_3x_1 + x_8^2x_7x_6x_5x_4 \\ & -x_8^2x_7x_6x_4x_1 + x_8^2x_7x_5x_4x_3 - x_8^2x_7x_4x_3x_1 \\ & -x_8^2x_6x_5x_4^2 + x_8^2x_6x_4^2x_1 \end{aligned}$$

10. Pseudo remainder with p_{10} over variable x_{10}:

$$\begin{aligned} g \;=\;& x_9x_8x_7^2x_5x_1 - x_9x_8x_7^2x_1^2 - x_9x_8x_7x_5^2x_4 + \\ & x_9x_8x_7x_4x_1^2 + x_9x_8x_5^2x_4^2 - x_9x_8x_5x_4^2x_1 \\ & -x_8^2x_7^2x_5x_3 + x_8^2x_7^2x_3x_1 + x_8^2x_7x_6x_5x_4 \\ & -x_8^2x_7x_6x_4x_1 + x_8^2x_7x_5x_4x_3 - x_8^2x_7x_4x_3x_1 \\ & -x_8^2x_6x_5x_4^2 + x_8^2x_6x_4^2x_1 \\ & -x_8x_7x_6x_5x_3x_1^2 + x_8x_7x_6x_5x_3x_1 + \\ & x_8x_7x_6x_5x_2x_1^2 - x_8x_7x_6x_5x_2x_1 + \\ & x_8x_7x_6x_3x_1^3 - x_8x_7x_6x_3x_1^2 - x_8x_7x_6x_2x_1^3 + \\ & x_8x_7x_6x_2x_1^2 + x_8x_7x_5^2x_3^2x_1 \\ & -x_8x_7x_5^2x_3^2 - x_8x_7x_5^2x_3x_2x_1 + \\ & x_8x_7x_5^2x_3x_2 - x_8x_7x_5x_3^2x_1^2 + \\ & x_8x_7x_5x_3^2x_1 + x_8x_7x_5x_3x_2x_1^2 \\ & -x_8x_7x_5x_3x_2x_1 + x_8x_6x_5x_4x_3x_1^2 \end{aligned}$$

$$-x_8x_6x_5x_4x_3x_1 - x_8x_6x_5x_4x_2x_1^2 +$$
$$x_8x_6x_5x_4x_2x_1 - x_8x_6x_4x_3x_1^3 + x_8x_6x_4x_3x_1^2 +$$
$$x_8x_6x_4x_2x_1^3 - x_8x_6x_4x_2x_1^2$$
$$-x_8x_5^2x_4x_3^2x_1 + x_8x_5^2x_4x_3^2 +$$
$$x_8x_5^2x_4x_3x_2x_1 - x_8x_5^2x_4x_3x_2 +$$
$$x_8x_5x_4x_3^2x_1^2 - x_8x_5x_4x_3^2x_1$$
$$-x_8x_5x_4x_3x_2x_1^2 + x_8x_5x_4x_3x_2x_1$$

11. Pseudo remainder with p_9 over variable x_9:

$$\begin{aligned}
g \quad = \quad & -x_8^2x_7^2x_5x_3 + x_8^2x_7^2x_3x_1 + x_8^2x_7x_6x_5x_4 \\
& -x_8^2x_7x_6x_4x_1 + x_8^2x_7x_5x_4x_3 - x_8^2x_7x_4x_3x_1 \\
& -x_8^2x_6x_5x_4^2 + x_8^2x_6x_4^2x_1 \\
& -x_8x_7^2x_5x_4x_1^2 + x_8x_7^2x_5x_4x_1 + \\
& x_8x_7^2x_4x_1^3 - x_8x_7^2x_4x_1^2 \\
& -x_8x_7x_6x_5x_3x_1^2 + x_8x_7x_6x_5x_3x_1 + \\
& x_8x_7x_6x_5x_2x_1^2 - x_8x_7x_6x_5x_2x_1 + \\
& x_8x_7x_6x_3x_1^3 - x_8x_7x_6x_3x_1^2 - x_8x_7x_6x_2x_1^3 + \\
& x_8x_7x_6x_2x_1^2 + x_8x_7x_5^2x_4^2x_1 \\
& -x_8x_7x_5^2x_4^2 + x_8x_7x_5^2x_3^2x_1 \\
& -x_8x_7x_5^2x_3^2 - x_8x_7x_5^2x_3x_2x_1 + \\
& x_8x_7x_5^2x_3x_2 - x_8x_7x_5x_3^2x_1^2 + \\
& x_8x_7x_5x_3^2x_1 + x_8x_7x_5x_3x_2x_1^2 \\
& -x_8x_7x_5x_3x_2x_1 - x_8x_7x_4^2x_1^3 + x_8x_7x_4^2x_1^2 + \\
& x_8x_6x_5x_4x_3x_1^2 - x_8x_6x_5x_4x_3x_1 \\
& -x_8x_6x_5x_4x_2x_1^2 + x_8x_6x_5x_4x_2x_1 \\
& -x_8x_6x_4x_3x_1^3 + x_8x_6x_4x_3x_1^2 + x_8x_6x_4x_2x_1^3 \\
& -x_8x_6x_4x_2x_1^2 - x_8x_5^2x_4^3x_1 + x_8x_5^2x_4^3 \\
& -x_8x_5^2x_4x_3^2x_1 + x_8x_5^2x_4x_3^2 + \\
& x_8x_5^2x_4x_3x_2x_1 - x_8x_5^2x_4x_3x_2 + \\
& x_8x_5x_4^3x_1^2 - x_8x_5x_4^3x_1 + x_8x_5x_4x_3^2x_1^2 \\
& -x_8x_5x_4x_3^2x_1 - x_8x_5x_4x_3x_2x_1^2 + \\
& x_8x_5x_4x_3x_2x_1
\end{aligned}$$

12. Pseudo remainder with p_8 over variable x_8:

$$\begin{aligned}
g \quad = \quad & -x_7^2x_5x_4^2x_3x_2^2 - x_7^2x_5x_4^2x_2x_1^2 + \\
& x_7^2x_5x_4^2x_2x_1 + x_7^2x_4^2x_3x_2^2x_1 + \\
& x_7^2x_4^2x_2x_1^3 - x_7^2x_4^2x_2x_1^2 + \\
& x_7x_6x_5x_4^3x_2^2 - x_7x_6x_5x_4x_3x_2x_1^2 +
\end{aligned}$$

$$
\begin{aligned}
& x_7x_6x_5x_4x_3x_2x_1 + x_7x_6x_5x_4x_2^2x_1^2 \\
& -x_7x_6x_5x_4x_2^2x_1 - x_7x_6x_4^3x_2^2x_1 + \\
& x_7x_6x_4x_3x_2x_1^3 - x_7x_6x_4x_3x_2x_1^2 \\
& -x_7x_6x_4x_2^2x_1^3 + x_7x_6x_4x_2^2x_1^2 + \\
& x_7x_5^2x_4^3x_2x_1 - x_7x_5^2x_4^3x_2 + \\
& x_7x_5^2x_4x_3^2x_2x_1 - x_7x_5^2x_4x_3^2x_2 \\
& -x_7x_5^2x_4x_3x_2^2x_1 + x_7x_5^2x_4x_3x_2^2 + \\
& x_7x_5x_4^3x_3x_2^2 - x_7x_5x_4x_3^2x_2x_1^2 + \\
& x_7x_5x_4x_3^2x_2x_1 + x_7x_5x_4x_3x_2^2x_1^2 \\
& -x_7x_5x_4x_3x_2^2x_1 - x_7x_4^3x_3x_2^2x_1 \\
& -x_7x_4^3x_2x_1^3 + x_7x_4^3x_2x_1^2 - x_6x_5x_4^4x_2^2 + \\
& x_6x_5x_4^2x_3x_2x_1^2 - x_6x_5x_4^2x_3x_2x_1 \\
& -x_6x_5x_4^2x_2^2x_1^2 + x_6x_5x_4^2x_2^2x_1 + \\
& x_6x_4^4x_2^2x_1 - x_6x_4^2x_3x_2x_1^3 + \\
& x_6x_4^2x_3x_2x_1^2 + x_6x_4^2x_2^2x_1^3 \\
& -x_6x_4^2x_2^2x_1^2 - x_5^2x_4^4x_2x_1 + x_5^2x_4^4x_2 \\
& -x_5^2x_4^2x_3^2x_2x_1 + x_5^2x_4^2x_3^2x_2 + \\
& x_5^2x_4^2x_3x_2^2x_1 - x_5^2x_4^2x_3x_2^2 + \\
& x_5x_4^4x_2x_1^2 - x_5x_4^4x_2x_1 + x_5x_4^2x_3^2x_2x_1^2 \\
& -x_5x_4^2x_3^2x_2x_1 - x_5x_4^2x_3x_2^2x_1^2 + \\
& x_5x_4^2x_3x_2^2x_1
\end{aligned}
$$

13. Pseudo remainder with p_7 over variable x_7:

$$
\begin{aligned}
g \quad = \quad & x_6x_5x_4^3x_2^3 - x_6x_5x_4x_3x_2^2x_1^2 + \\
& x_6x_5x_4x_3x_2^2x_1 + x_6x_5x_4x_2^3x_1^2 \\
& -x_6x_5x_4x_2^3x_1 - x_6x_4^3x_2^3x_1 + \\
& x_6x_4x_3x_2^2x_1^3 - x_6x_4x_3x_2^2x_1^2 \\
& -x_6x_4x_2^3x_1^3 + x_6x_4x_2^3x_1^2 + x_5^2x_4^3x_2^2x_1 \\
& -x_5^2x_4^3x_2^2 + x_5^2x_4x_3^2x_2^2x_1 \\
& -x_5^2x_4x_3^2x_2^2 - x_5^2x_4x_3x_2^3x_1 + \\
& x_5^2x_4x_3x_2^3 - x_5x_4^3x_3x_2^3 \\
& -2x_5x_4^3x_2^2x_1^2 + 2x_5x_4^3x_2^2x_1 \\
& -x_5x_4^2x_3x_2^4 - x_5x_4^2x_2^3x_1^2 + x_5x_4^2x_2^3x_1 \\
& -x_5x_4x_3^2x_2^2x_1^2 + x_5x_4x_3^2x_2^2x_1 + \\
& x_5x_4x_3x_2^3x_1^2 - x_5x_4x_3x_2^3x_1 + \\
& x_4^3x_3x_2^3x_1 + x_4^3x_2^2x_1^3 - x_4^3x_2^2x_1^2 + \\
& x_4^2x_3x_2^4x_1 + x_4^2x_2^3x_1^3 - x_4^2x_2^3x_1^2
\end{aligned}
$$

14. Pseudo remainder with p_6 over variable x_6:

$$
\begin{aligned}
g \quad = \quad & x_5^2x_4^3x_2^2x_1 - x_5^2x_4^3x_2^2 + \\
& x_5^2x_4x_3^2x_2^2x_1 - x_5^2x_4x_3^2x_2^2 \\
& -x_5^2x_4x_3x_2^3x_1 + x_5^2x_4x_3x_2^3 \\
& -2x_5x_4^3x_2^2x_1^2 + 2x_5x_4^3x_2^2x_1 \\
& -x_5x_4^2x_3x_2^4 - x_5x_4^2x_2^3x_1^2 + x_5x_4^2x_2^3x_1 \\
& -2x_5x_4x_3^2x_2^2x_1^2 + 2x_5x_4x_3^2x_2^2x_1 + \\
& 2x_5x_4x_3x_2^3x_1^2 - 2x_5x_4x_3x_2^3x_1 + \\
& x_4^3x_2^2x_1^3 - x_4^3x_2^2x_1^2 + x_4^2x_3x_2^4x_1 + \\
& x_4^2x_2^3x_1^3 - x_4^2x_2^3x_1^2 + x_4x_3^2x_2^2x_1^3 \\
& -x_4x_3^2x_2^2x_1^2 - x_4x_3x_2^3x_1^3 + x_4x_3x_2^3x_1^2
\end{aligned}
$$

15. Pseudo remainder with p_5 over variable x_5:

$$g \quad = \quad 0$$

16. Pseudo remainder with p_4 over variable x_4:

$$g \quad = \quad 0$$

17. Pseudo remainder with p_3 over variable x_3:

$$g \quad = \quad 0$$

18. Pseudo remainder with p_2 over variable x_2:

$$g \quad = \quad 0$$

19. Pseudo remainder with p_1 over variable x_1:

$$g \quad = \quad 0$$

3 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 52 terms.

Time Complexity: Time spent by the prover is 0.145 seconds.

4 NDG Conditions

NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form