Art of Problem Solving
1993 Balkan MO

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–	May 5th
1	Let a, b, c, d, e, f be six real numbers with sum 10, such that $$(a-1)^2 + (b-1)^2 + (c-1)^2 + (d-1)^2 + (e-1)^2 + (f-1)^2 = 6.$$ Find the maximum possible value of f. *Cyprus*
2	A positive integer given in decimal representation $\overline{a_n a_{n-1} \ldots a_1 a_0}$ is called *monotone* if $a_n \leq a_{n-1} \leq \cdots \leq a_0$. Determine the number of monotone positive integers with at most 1993 digits.
3	Circles $\mathcal{C}_1$ and $\mathcal{C}_2$ with centers O_1 and O_2, respectively, are externally tangent at point λ. A circle $\mathcal{C}$ with center O touches $\mathcal{C}_1$ at A and $\mathcal{C}_2$ at B so that the centers O_1, O_2 lie inside C. The common tangent to $\mathcal{C}_1$ and $\mathcal{C}_2$ at λ intersects the circle $\mathcal{C}$ at K and L. If D is the midpoint of the segment KL, show that $\angle O_1 O O_2 = \angle ADB$. *Greece*
4	Let p be a prime and $m \geq 2$ be an integer. Prove that the equation $$\frac{x^p + y^p}{2} = \left(\frac{x+y}{2} \right)^m$$ has a positive integer solution $(x, y) \neq (1, 1)$ if and only if $m = p$. *Romania*