OpenGeoProver Output for conjecture "geothm_zadatak"

Wu's method used

October 2, 2016

1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 - \\
p_9 &= x_9 + x_4x_1 \\
p_{10} &= x_{10} - x_3x_1 + x_2x_1 \\
p_{11} &= 2x_{14} - \\
p_{12} &= 2x_{15} - x_1 - \\
p_{13} &= 2x_{16} - x_2 \\
p_{14} &= 2x_{17} - x_1 \\
p_{15} &= 2x_{18} - x_2 \\
p_{16} &= x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{17} &= x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
&\quad -x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{18} &= x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{19} &= x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{20} &= x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{21} &= x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{22} &= -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9 \\
p_{23} &= x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4 \\
p_{24} &= -x_{13}x_6 + x_{13}x_3 + x_{12}x_7 - x_{12}x_4 - x_7x_3 + x_6x_4
\end{aligned}
$$

1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 24.

Variable x_{24} selected: The number of polynomials with this variable, with indexes from 1 to 24, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{21}. No reduction needed.

The triangular system has not been changed.

1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 23.

Variable x_{23} selected: The number of polynomials with this variable, with indexes from 1 to 23, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{20}. No reduction needed.

The triangular system has not been changed.

1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 22.

Variable x_{22} selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 21.

Variable x_{21} selected: The number of polynomials with this variable, with indexes from 1 to 21, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{18}. No reduction needed.

The triangular system has not been changed.

1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 20.

Variable x_{20} selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{17}. No reduction needed.

The triangular system has not been changed.

1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 19.

Variable x_{19} selected: The number of polynomials with this variable, with indexes from 1 to 19, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{16}. No reduction needed.

The triangular system has not been changed.

1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 18.

Variable x_{18} selected: The number of polynomials with this variable, with indexes from 1 to 18, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{15}. No reduction needed.

The triangular system has not been changed.

1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 17.

Variable x_{17} selected: The number of polynomials with this variable, with indexes from 1 to 17, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{14}. No reduction needed.

The triangular system has not been changed.

1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 16.

Variable x_{16} selected: The number of polynomials with this variable, with indexes from 1 to 16, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{13}. No reduction needed.

The triangular system has not been changed.

1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 15.

Variable x_{15} selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{12}. No reduction needed.

The triangular system has not been changed.

1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 14.

Variable x_{14} selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 13.

Variable x_{13} selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{13} from all other polynomials by reducing them with polynomial p_{12} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 - \\
p_9 &= x_9 + x_4x_1 \\
p_{10} &= x_{10} - x_3x_1 + x_2x_1 \\
p_{11} &= -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9 \\
p_{12} &= x_{12}x_7x_5 - x_{12}x_7x_1 - x_{12}x_5x_4 + x_{12}x_4x_1 - x_{11}x_7x_6 + \\
&\quad x_{11}x_7x_3 + x_{11}x_6x_4 - x_{11}x_4x_3 + x_7x_6x_1 - x_7x_5x_3 \\
&\quad -x_6x_4x_1 + x_5x_4x_3 \\
p_{13} &= x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4 \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_1 - \\
p_{16} &= 2x_{16} - x_2 \\
p_{17} &= 2x_{17} - x_1 \\
p_{18} &= 2x_{18} - x_2 \\
p_{19} &= x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{20} &= x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{21} &= x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
&\quad -x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
\end{aligned}
$$

1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 12.

Variable x_{12} selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_{12} from all other polynomials by reducing them with polynomial p_{11} from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_{1} &= 2x_{1} - \\
p_{2} &= 4x_{2}^{2} - 3 \\
p_{3} &= 3x_{3} - x_{2} \\
p_{4} &= 3x_{4}^{2} - 2 \\
p_{5} &= x_{5} - x_{1} \\
p_{6} &= x_{6} - x_{3} \\
p_{7} &= x_{7} - x_{4} - x_{2} \\
p_{8} &= x_{8} - x_{4}x_{2} - \\
p_{9} &= x_{9} + x_{4}x_{1} \\
p_{10} &= x_{10} - x_{3}x_{1} + x_{2}x_{1} \\
p_{11} &= -x_{11}x_{9}x_{7}x_{5} + x_{11}x_{9}x_{7}x_{1} + x_{11}x_{9}x_{5}x_{4} - x_{11}x_{9}x_{4}x_{1} + \\
&\quad x_{11}x_{8}x_{7}x_{6} - x_{11}x_{8}x_{7}x_{3} - x_{11}x_{8}x_{6}x_{4} + x_{11}x_{8}x_{4}x_{3} \\
&\quad -x_{11}x_{7}x_{6} + x_{11}x_{7}x_{3} + x_{11}x_{6}x_{4} - x_{11}x_{4}x_{3} + x_{9}x_{7}x_{5} \\
&\quad -x_{9}x_{7}x_{1} - x_{9}x_{5}x_{4} + x_{9}x_{4}x_{1} - x_{8}x_{7}x_{6}x_{1} + x_{8}x_{7}x_{5}x_{3} + \\
&\quad x_{8}x_{6}x_{4}x_{1} - x_{8}x_{5}x_{4}x_{3} + x_{7}x_{6}x_{1} - x_{7}x_{5}x_{3} - x_{6}x_{4}x_{1} + \\
&\quad x_{5}x_{4}x_{3} \\
p_{12} &= -x_{12}x_{8} + x_{12} + x_{11}x_{9} - x_{9} \\
p_{13} &= x_{13}x_{5} - x_{13}x_{1} - x_{11}x_{7} + x_{11}x_{4} + x_{7}x_{1} - x_{5}x_{4} \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_{1} - \\
p_{16} &= 2x_{16} - x_{2} \\
p_{17} &= 2x_{17} - x_{1} \\
p_{18} &= 2x_{18} - x_{2} \\
p_{19} &= x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^{2} - x_{12}^{2} - x_{11}^{2} \\
p_{20} &= x_{20} - x_{16}^{2} + 2x_{16}x_{12} - x_{15}^{2} + 2x_{15}x_{11} - x_{13}^{2} - x_{12}^{2} - x_{11}^{2} \\
p_{21} &= x_{21} - x_{18}^{2} + 2x_{18}x_{12} - x_{17}^{2} + 2x_{17}x_{11} - x_{13}^{2} - x_{12}^{2} - x_{11}^{2} \\
p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
&\quad -x_{13}^{2} - x_{12}^{2} - x_{11}^{2} \\
p_{23} &= x_{23} - x_{14}^{2} + 2x_{14}x_{11} - x_{13}^{2} - x_{12}^{2} - x_{11}^{2} \\
p_{24} &= x_{24} - x_{16}^{2} + 2x_{16}x_{12} - x_{15}^{2} + 2x_{15}x_{11} - x_{13}^{2} - x_{12}^{2} - x_{11}^{2}
\end{aligned}
$$

1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 11.

Variable x_{11} selected: The number of polynomials with this variable, with indexes from 1 to 11, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{11}. No reduction needed.

The triangular system has not been changed.

1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 10.

Variable x_{10} selected: The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_{10}. No reduction needed.

The triangular system has not been changed.

1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 9.

Variable x_9 selected: The number of polynomials with this variable, with indexes from 1 to 9, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_9. No reduction needed.

The triangular system has not been changed.

1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_8. No reduction needed.

The triangular system has not been changed.

1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7. No reduction needed.

The triangular system has not been changed.

1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_6. No reduction needed.

The triangular system has not been changed.

1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5. No reduction needed.

The triangular system has not been changed.

1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_4. No reduction needed.

The triangular system has not been changed.

1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3. No reduction needed.

The triangular system has not been changed.

1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2. No reduction needed.

The triangular system has not been changed.

1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_1 \\
p_6 &= x_6 - x_3 \\
p_7 &= x_7 - x_4 - x_2 \\
p_8 &= x_8 - x_4x_2 - \\
p_9 &= x_9 + x_4x_1 \\
p_{10} &= x_{10} - x_3x_1 + x_2x_1 \\
p_{11} &= -x_{11}x_9x_7x_5 + x_{11}x_9x_7x_1 + x_{11}x_9x_5x_4 - x_{11}x_9x_4x_1 + \\
&\quad x_{11}x_8x_7x_6 - x_{11}x_8x_7x_3 - x_{11}x_8x_6x_4 + x_{11}x_8x_4x_3 \\
&\quad -x_{11}x_7x_6 + x_{11}x_7x_3 + x_{11}x_6x_4 - x_{11}x_4x_3 + x_9x_7x_5 \\
&\quad -x_9x_7x_1 - x_9x_5x_4 + x_9x_4x_1 - x_8x_7x_6x_1 + x_8x_7x_5x_3 + \\
&\quad x_8x_6x_4x_1 - x_8x_5x_4x_3 + x_7x_6x_1 - x_7x_5x_3 - x_6x_4x_1 + \\
&\quad x_5x_4x_3 \\
p_{12} &= -x_{12}x_8 + x_{12} + x_{11}x_9 - x_9 \\
p_{13} &= x_{13}x_5 - x_{13}x_1 - x_{11}x_7 + x_{11}x_4 + x_7x_1 - x_5x_4 \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_1 - \\
p_{16} &= 2x_{16} - x_2
\end{aligned}
$$

$$
\begin{aligned}
p_{17} &= 2x_{17} - x_1 \\
p_{18} &= 2x_{18} - x_2 \\
p_{19} &= x_{19} + x_{16}x_{12} - x_{15}x_{14} + x_{15}x_{11} + x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{20} &= x_{20} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{21} &= x_{21} - x_{18}^2 + 2x_{18}x_{12} - x_{17}^2 + 2x_{17}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{12} - x_{17}x_{15} + x_{17}x_{11} + x_{16}x_{12} + x_{15}x_{11} \\
&\quad -x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2 \\
p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{12} - x_{15}^2 + 2x_{15}x_{11} - x_{13}^2 - x_{12}^2 - x_{11}^2
\end{aligned}
$$

2 Final Remainder

2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
g = -x_{24}x_{23}x_{22}^2 + x_{21}x_{20}x_{19}^2
$$

with respect to the triangular system.

1. Pseudo remainder with p_{24} over variable x_{24}:

$$
\begin{aligned}
g = {} & -x_{23}x_{22}^2x_{16}^2 + 2x_{23}x_{22}^2x_{16}x_{12} - x_{23}x_{22}^2x_{15}^2 + \\
& 2x_{23}x_{22}^2x_{15}x_{11} - x_{23}x_{22}^2x_{13}^2 - x_{23}x_{22}^2x_{12}^2 \\
& -x_{23}x_{22}^2x_{11}^2 + x_{21}x_{20}x_{19}^2
\end{aligned}
$$

2. Pseudo remainder with p_{23} over variable x_{23}:

$$
\begin{aligned}
g = {} & -x_{22}^2x_{16}^2x_{14}^2 + 2x_{22}^2x_{16}^2x_{14}x_{11} \\
& -x_{22}^2x_{16}^2x_{13}^2 - x_{22}^2x_{16}^2x_{12}^2 \\
& -x_{22}^2x_{16}^2x_{11}^2 + 2x_{22}^2x_{16}x_{14}^2x_{12} \\
& -4x_{22}^2x_{16}x_{14}x_{12}x_{11} + 2x_{22}^2x_{16}x_{13}^2x_{12} + \\
& 2x_{22}^2x_{16}x_{12}^3 + 2x_{22}^2x_{16}x_{12}x_{11}^2 \\
& -x_{22}^2x_{15}^2x_{14}^2 + 2x_{22}^2x_{15}^2x_{14}x_{11} \\
& -x_{22}^2x_{15}^2x_{13}^2 - x_{22}^2x_{15}^2x_{12}^2 \\
& -x_{22}^2x_{15}^2x_{11}^2 + 2x_{22}^2x_{15}x_{14}^2x_{11} \\
& -4x_{22}^2x_{15}x_{14}x_{11}^2 + 2x_{22}^2x_{15}x_{13}^2x_{11} + \\
& 2x_{22}^2x_{15}x_{12}^2x_{11} + 2x_{22}^2x_{15}x_{11}^3 \\
& -x_{22}^2x_{14}^2x_{13}^2 - x_{22}^2x_{14}^2x_{12}^2 \\
& -x_{22}^2x_{14}^2x_{11}^2 + 2x_{22}^2x_{14}x_{13}^2x_{11} + \\
& 2x_{22}^2x_{14}x_{12}^2x_{11} + 2x_{22}^2x_{14}x_{11}^3 - x_{22}^2x_{13}^4 \\
& -2x_{22}^2x_{13}^2x_{12}^2 - 2x_{22}^2x_{13}^2x_{11}^2 - x_{22}^2x_{12}^4 \\
& -2x_{22}^2x_{12}^2x_{11}^2 - x_{22}^2x_{11}^4 + x_{21}x_{20}x_{19}^2
\end{aligned}
$$

3. Pseudo remainder with p_{22} over variable x_{22}:

   Polynomial too big for output (number of terms is 745)

4. Pseudo remainder with p_{21} over variable x_{21}:

   Polynomial too big for output (number of terms is 751)

5. Pseudo remainder with p_{20} over variable x_{20}:

   Polynomial too big for output (number of terms is 790)

6. Pseudo remainder with p_{19} over variable x_{19}:

   Polynomial too big for output (number of terms is 546)

7. Pseudo remainder with p_{18} over variable x_{18}:

   Polynomial too big for output (number of terms is 546)

8. Pseudo remainder with p_{17} over variable x_{17}:

   Polynomial too big for output (number of terms is 546)

9. Pseudo remainder with p_{16} over variable x_{16}:

   Polynomial too big for output (number of terms is 361)

10. Pseudo remainder with p_{15} over variable x_{15}:

    Polynomial too big for output (number of terms is 529)

11. Pseudo remainder with p_{14} over variable x_{14}:

    Polynomial too big for output (number of terms is 273)

12. Pseudo remainder with p_{13} over variable x_{13}:

3 Prover results

Status: Proving failed - Space limit has been reached.

Space Complexity: The biggest polynomial obtained during prover execution contains 3339 terms.

Time Complexity: Time spent by the prover is 0.481 seconds.