

# OpenGeoProver Output for conjecture “geothm\_zadatak”

Wu’s method used

September 30, 2016

## 1 Invoking the theorem prover

The used proving method is Wu’s method.

The input system is:

$$\begin{aligned} p_1 &= -x_4 + x_1 \\ p_2 &= -x_4 + x_2 \\ p_3 &= -x_5 + x_1 \\ p_4 &= x_5 + x_2 - \\ p_5 &= x_3 - \\ p_6 &= -x_8 + x_6 \\ p_7 &= -x_8 + x_7 \\ p_8 &= x_9 + x_6 - \\ p_9 &= -x_9 + x_7 \\ p_{10} &= -x_{13}x_7 + x_{11} \\ p_{11} &= x_{13} + x_{12} - \\ p_{12} &= -x_{14}x_1 + x_{10} \\ p_{13} &= -x_{14}x_2 + x_{11} \\ p_{14} &= -x_{14}x_3 + x_{12} \\ p_{15} &= -x_{18}x_7 + x_{16} \\ p_{16} &= x_{18} + x_{17} - \\ p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\ p_{18} &= -x_{19}x_2 + x_{16} \\ p_{19} &= -x_{19}x_3 + x_{17} \\ p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\ p_{21} &= x_{23} + x_{22} - \\ p_{22} &= -x_{24}x_1 + x_{20} \\ p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\ p_{24} &= -x_{24}x_3 + x_{22} \\ p_{25} &= -x_{28}x_7 + x_{28} + x_{26} - \end{aligned}$$

$$\begin{aligned}
p_{26} &= x_{28} + x_{27} - \\
p_{27} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{28} &= -x_{29}x_2 + x_{29} + x_{26} - \\
p_{29} &= -x_{29}x_3 + x_{27} \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 33.

**Variable  $x_{33}$  selected:** The number of polynomials with this variable, with indexes from 1 to 33, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{33}$ . No reduction needed.

The triangular system has not been changed.

### 1.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 32.

**Variable  $x_{32}$  selected:** The number of polynomials with this variable, with indexes from 1 to 32, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{32}$ . No reduction needed.

The triangular system has not been changed.

### 1.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 31.

**Variable  $x_{31}$  selected:** The number of polynomials with this variable, with indexes from 1 to 31, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{31}$ . No reduction needed.

The triangular system has not been changed.

### 1.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 30.

**Variable  $x_{30}$  selected:** The number of polynomials with this variable, with indexes from 1 to 30, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{30}$ . No reduction needed.

The triangular system has not been changed.

## 1.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 29.

**Variable  $x_{29}$  selected:** The number of polynomials with this variable, with indexes from 1 to 29, is 3.

**Minimal degrees:** 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{29}$  from all other polynomials by reducing them with polynomial  $p_{27}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20} \\
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{26} &= x_{28} + x_{27} -
\end{aligned}$$

$$\begin{aligned}
p_{27} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{28} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 28.

**Variable  $x_{28}$  selected:** The number of polynomials with this variable, with indexes from 1 to 28, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{28}$  from all other polynomials by reducing them with polynomial  $p_{25}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20}
\end{aligned}$$

$$\begin{aligned}
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{26} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{27} &= -x_{27}x_7 + x_{27} - x_{26} + x_7 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 27.

**Variable  $x_{27}$  selected:** The number of polynomials with this variable, with indexes from 1 to 27, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{27}$  from all other polynomials by reducing them with polynomial  $p_{26}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16}
\end{aligned}$$

$$\begin{aligned}
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20} \\
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{26} &= x_{26}x_1 - x_{26} + x_{25}x_7x_3 - x_{25}x_3 - x_7x_3 - x_7x_1 + x_7 + x_3 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 26.

**Variable  $x_{26}$  selected:** The number of polynomials with this variable, with indexes from 1 to 26, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{26}$  from all other polynomials by reducing them with polynomial  $p_{25}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12}
\end{aligned}$$

$$\begin{aligned}
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{24}x_1 + x_{20} \\
p_{23} &= -x_{24}x_2 + x_{24} + x_{21} - \\
p_{24} &= -x_{24}x_3 + x_{22} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 25.

**Variable  $x_{25}$  selected:** The number of polynomials with this variable, with indexes from 1 to 25, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{25}$ . No reduction needed.

The triangular system has not been changed.

## 1.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 24.

**Variable  $x_{24}$  selected:** The number of polynomials with this variable, with indexes from 1 to 24, is 3.

**Minimal degrees:** 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{24}$  from all other polynomials by reducing them with polynomial  $p_{22}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{21} &= x_{23} + x_{22} - \\
p_{22} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{23} &= -x_{22}x_1 + x_{20}x_3 \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 23.



**Variable  $x_{23}$  selected:** The number of polynomials with this variable, with indexes from 1 to 23, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{23}$  from all other polynomials by reducing them with polynomial  $p_{20}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{21} &= -x_{22}x_1 + x_{20}x_3 \\
p_{22} &= -x_{22}x_7 + x_{22} - x_{21} + x_7 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
\end{aligned}$$

$$\begin{aligned}
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.12 Triangulation, step 12

**Choosing variable:** Trying the variable with index 22.

**Variable  $x_{22}$  selected:** The number of polynomials with this variable, with indexes from 1 to 22, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{22}$  from all other polynomials by reducing them with polynomial  $p_{21}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{21} &= x_{21}x_1 + x_{20}x_7x_3 - x_{20}x_3 - x_7x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
\end{aligned}$$

$$\begin{aligned}
& -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.13 Triangulation, step 13

**Choosing variable:** Trying the variable with index 21.

**Variable  $x_{21}$  selected:** The number of polynomials with this variable, with indexes from 1 to 21, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{21}$  from all other polynomials by reducing them with polynomial  $p_{20}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{18} &= -x_{19}x_2 + x_{16} \\
p_{19} &= -x_{19}x_3 + x_{17} \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
\end{aligned}$$

$$\begin{aligned}
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

#### 1.14 Triangulation, step 14

**Choosing variable:** Trying the variable with index 20.

**Variable  $x_{20}$  selected:** The number of polynomials with this variable, with indexes from 1 to 20, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{20}$ . No reduction needed.

The triangular system has not been changed.

#### 1.15 Triangulation, step 15

**Choosing variable:** Trying the variable with index 19.

**Variable  $x_{19}$  selected:** The number of polynomials with this variable, with indexes from 1 to 19, is 3.

**Minimal degrees:** 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{19}$  from all other polynomials by reducing them with polynomial  $p_{17}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 -
\end{aligned}$$

$$\begin{aligned}
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{18}x_7 + x_{16} \\
p_{16} &= x_{18} + x_{17} - \\
p_{17} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{18} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.16 Triangulation, step 16

**Choosing variable:** Trying the variable with index 18.

**Variable  $x_{18}$  selected:** The number of polynomials with this variable, with indexes from 1 to 18, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{18}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{17} &= -x_{17}x_7 - x_{16} + x_7 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.17 Triangulation, step 17

**Choosing variable:** Trying the variable with index 17.

**Variable  $x_{17}$  selected:** The number of polynomials with this variable, with indexes from 1 to 17, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{17}$  from all other polynomials by reducing them with polynomial  $p_{16}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{16} &= x_{16}x_1 - x_{16} + x_{15}x_7x_3 - x_7x_3 - x_7x_1 + x_7 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
\end{aligned}$$

$$\begin{aligned}
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.18 Triangulation, step 18

**Choosing variable:** Trying the variable with index 16.

**Variable  $x_{16}$  selected:** The number of polynomials with this variable, with indexes from 1 to 16, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{16}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11} \\
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{14}x_1 + x_{10} \\
p_{13} &= -x_{14}x_2 + x_{11} \\
p_{14} &= -x_{14}x_3 + x_{12} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
\end{aligned}$$



$$\begin{aligned}
& x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
& - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.19 Triangulation, step 19

**Choosing variable:** Trying the variable with index 15.

**Variable  $x_{15}$  selected:** The number of polynomials with this variable, with indexes from 1 to 15, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

### 1.20 Triangulation, step 20

**Choosing variable:** Trying the variable with index 14.

**Variable  $x_{14}$  selected:** The number of polynomials with this variable, with indexes from 1 to 14, is 3.

**Minimal degrees:** 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{14}$  from all other polynomials by reducing them with polynomial  $p_{12}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{13}x_7 + x_{11}
\end{aligned}$$

$$\begin{aligned}
p_{11} &= x_{13} + x_{12} - \\
p_{12} &= -x_{11}x_1 + x_{10}x_2 \\
p_{13} &= -x_{12}x_1 + x_{10}x_3 \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.21 Triangulation, step 21

**Choosing variable:** Trying the variable with index 13.

**Variable  $x_{13}$  selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{13}$  from all other polynomials by reducing them with polynomial  $p_{10}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1
\end{aligned}$$

$$\begin{aligned}
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= -x_{12}x_1 + x_{10}x_3 \\
p_{12} &= -x_{12}x_7 - x_{11} + x_7 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.22 Triangulation, step 22

**Choosing variable:** Trying the variable with index 12.

**Variable  $x_{12}$  selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{12}$  from all other polynomials by reducing them with polynomial  $p_{11}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_9 + x_6 - \\
p_9 &= -x_9 + x_7 \\
p_{10} &= -x_{11}x_1 + x_{10}x_2 \\
p_{11} &= x_{11}x_1 + x_{10}x_7x_3 - x_7x_1 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.23 Triangulation, step 23

**Choosing variable:** Trying the variable with index 11.

**Variable  $x_{11}$  selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{11}$  from all other polynomials by reducing them with polynomial  $p_{10}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_1 &= -x_4 + x_1 \\ p_2 &= -x_4 + x_2 \\ p_3 &= -x_5 + x_1 \\ p_4 &= x_5 + x_2 - \\ p_5 &= x_3 - \\ p_6 &= -x_8 + x_6 \\ p_7 &= -x_8 + x_7 \\ p_8 &= x_9 + x_6 - \\ p_9 &= -x_9 + x_7 \\ p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\ p_{11} &= -x_{11}x_1 + x_{10}x_2 \\ p_{12} &= -x_{12}x_1 + x_{10}x_3 \\ p_{13} &= -x_{13}x_7 + x_{11} \\ p_{14} &= -x_{14}x_1 + x_{10} \\ p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\ &\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\ p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\ p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\ p_{18} &= -x_{18}x_7 + x_{16} \\ p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\ p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\ p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\ p_{22} &= -x_{22}x_1 + x_{20}x_3 \\ p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\ p_{24} &= -x_{24}x_1 + x_{20} \\ p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\ &\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\ &\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\ p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\ p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \end{aligned}$$

$$\begin{aligned}
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.24 Triangulation, step 24

**Choosing variable:** Trying the variable with index 10.

**Variable  $x_{10}$  selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{10}$ . No reduction needed.

The triangular system has not been changed.

### 1.25 Triangulation, step 25

**Choosing variable:** Trying the variable with index 9.

**Variable  $x_9$  selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_9$  from all other polynomials by reducing them with polynomial  $p_8$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= -x_8 + x_6 \\
p_7 &= -x_8 + x_7 \\
p_8 &= x_7 + x_6 - \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10}
\end{aligned}$$

$$\begin{aligned}
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.26 Triangulation, step 26

**Choosing variable:** Trying the variable with index 8.

**Variable  $x_8$  selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_8$  from all other polynomials by reducing them with polynomial  $p_6$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= x_7 + x_6 - \\
p_7 &= -x_7 + x_6
\end{aligned}$$

$$\begin{aligned}
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.27 Triangulation, step 27

**Choosing variable:** Trying the variable with index 7.

**Variable  $x_7$  selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_7$  from all other polynomials by reducing them with polynomial  $p_6$  from previous step.

Finished a triangulation step, the current system is:



$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= -x_5 + x_1 \\
p_4 &= x_5 + x_2 - \\
p_5 &= x_3 - \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 1.28 Triangulation, step 28

**Choosing variable:** Trying the variable with index 6.

**Variable  $x_6$  selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

## 1.29 Triangulation, step 29

**Choosing variable:** Trying the variable with index 5.

**Variable  $x_5$  selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_5$  from all other polynomials by reducing them with polynomial  $p_3$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -x_4 + x_1 \\
p_2 &= -x_4 + x_2 \\
p_3 &= x_3 - \\
p_4 &= -x_2 - x_1 + 1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3
\end{aligned}$$

$$\begin{aligned}
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.30 Triangulation, step 30

**Choosing variable:** Trying the variable with index 4.

**Variable  $x_4$  selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_4$  from all other polynomials by reducing them with polynomial  $p_1$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= x_3 - \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= -x_2 + x_1 \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
\end{aligned}$$

$$\begin{aligned}
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.31 Triangulation, step 31

**Choosing variable:** Trying the variable with index 3.

**Variable  $x_3$  selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

### 1.32 Triangulation, step 32

**Choosing variable:** Trying the variable with index 2.

**Variable  $x_2$  selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_2$  from all other polynomials by reducing them with polynomial  $p_1$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 - \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} - \\
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

### 1.33 Triangulation, step 33

**Choosing variable:** Trying the variable with index 1.

**Variable  $x_1$  selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_1 &= -2x_1 + 1 \\
p_2 &= -x_2 - x_1 + 1 \\
p_3 &= x_3 - \\
p_4 &= -x_4 + x_1 \\
p_5 &= -x_5 + x_1 \\
p_6 &= 2x_6 - \\
p_7 &= x_7 + x_6 - \\
p_8 &= -x_8 + x_6 \\
p_9 &= x_9 + x_6 - \\
p_{10} &= -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2 \\
p_{11} &= -x_{11}x_1 + x_{10}x_2 \\
p_{12} &= -x_{12}x_1 + x_{10}x_3 \\
p_{13} &= -x_{13}x_7 + x_{11} \\
p_{14} &= -x_{14}x_1 + x_{10} \\
p_{15} &= -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1 \\
&\quad - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2 \\
p_{16} &= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2 \\
p_{17} &= -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3 \\
p_{18} &= -x_{18}x_7 + x_{16} \\
p_{19} &= -x_{19}x_1 + x_{19} + x_{15} - \\
p_{20} &= -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2 \\
p_{21} &= -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1 \\
p_{22} &= -x_{22}x_1 + x_{20}x_3 \\
p_{23} &= -x_{23}x_7 + x_{23} + x_{21} - \\
p_{24} &= -x_{24}x_1 + x_{20} \\
p_{25} &= -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
&\quad x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\
&\quad - x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\
p_{26} &= -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\
p_{27} &= -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\
p_{28} &= -x_{28}x_7 + x_{28} + x_{26} - \\
p_{29} &= -x_{29}x_1 + x_{29} + x_{25} -
\end{aligned}$$

$$\begin{aligned}
p_{30} &= x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\
p_{31} &= x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\
p_{32} &= x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\
p_{33} &= x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
\end{aligned}$$

## 2 Final Remainder

### 2.1 Final remainder for conjecture `geothm_zadatak`

Calculating final remainder of the conclusion:

$$g = -x_{28}x_6 + x_{28} + x_{25} -$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{33}$  over variable  $x_{33}$ :

$$g = -x_{28}x_6 + x_{28} + x_{25} -$$

2. Pseudo remainder with  $p_{32}$  over variable  $x_{32}$ :

$$g = -x_{28}x_6 + x_{28} + x_{25} -$$

3. Pseudo remainder with  $p_{31}$  over variable  $x_{31}$ :

$$g = -x_{28}x_6 + x_{28} + x_{25} -$$

4. Pseudo remainder with  $p_{30}$  over variable  $x_{30}$ :

$$g = -x_{28}x_6 + x_{28} + x_{25} -$$

5. Pseudo remainder with  $p_{29}$  over variable  $x_{29}$ :

$$g = -x_{28}x_6 + x_{28} + x_{25} -$$

6. Pseudo remainder with  $p_{28}$  over variable  $x_{28}$ :

$$g = x_{26}x_6 - x_{26} - x_{25}x_7 + x_{25} + x_7 - x_6$$

7. Pseudo remainder with  $p_{27}$  over variable  $x_{27}$ :

$$g = x_{26}x_6 - x_{26} - x_{25}x_7 + x_{25} + x_7 - x_6$$

8. Pseudo remainder with  $p_{26}$  over variable  $x_{26}$ :

$$g = x_{25}x_7x_1 - x_{25}x_7 - x_{25}x_6x_2 + x_{25}x_6 + x_{25}x_2 - x_{25}x_1 - x_7x_1 + x_7 + x_6x_2 - x_6 - x_2 + x_1$$

9. Pseudo remainder with  $p_{25}$  over variable  $x_{25}$ :

$$\begin{aligned} g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\ & -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\ & -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\ & -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\ & x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1 \end{aligned}$$

10. Pseudo remainder with  $p_{24}$  over variable  $x_{24}$ :

$$\begin{aligned} g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\ & -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\ & -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\ & -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\ & x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1 \end{aligned}$$

11. Pseudo remainder with  $p_{23}$  over variable  $x_{23}$ :

$$\begin{aligned} g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\ & -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\ & -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\ & -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\ & x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1 \end{aligned}$$

12. Pseudo remainder with  $p_{22}$  over variable  $x_{22}$ :

$$\begin{aligned} g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\ & -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\ & -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\ & -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\ & x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1 \end{aligned}$$

13. Pseudo remainder with  $p_{21}$  over variable  $x_{21}$ :



$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

14. Pseudo remainder with  $p_{20}$  over variable  $x_{20}$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

15. Pseudo remainder with  $p_{19}$  over variable  $x_{19}$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

16. Pseudo remainder with  $p_{18}$  over variable  $x_{18}$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

17. Pseudo remainder with  $p_{17}$  over variable  $x_{17}$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

18. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :

$$\begin{aligned}
g = & -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\
& -2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_6 x_1^2 + 2x_7 x_6 x_1 - x_7 x_6 \\
& -x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 + 2x_7 x_1^3 - 5x_7 x_1^2 + 4x_7 x_1 \\
& -x_7 - x_6 x_2 x_1^2 + 2x_6 x_2 x_1 - x_6 x_2 + x_6 x_1^2 - 2x_6 x_1 + x_6 + \\
& x_2 x_1^2 - 2x_2 x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

19. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :

$$\begin{aligned}
g = & -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\
& -2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_6 x_1^2 + 2x_7 x_6 x_1 - x_7 x_6 \\
& -x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 + 2x_7 x_1^3 - 5x_7 x_1^2 + 4x_7 x_1 \\
& -x_7 - x_6 x_2 x_1^2 + 2x_6 x_2 x_1 - x_6 x_2 + x_6 x_1^2 - 2x_6 x_1 + x_6 + \\
& x_2 x_1^2 - 2x_2 x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

20. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :

$$\begin{aligned}
g = & -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\
& -2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_6 x_1^2 + 2x_7 x_6 x_1 - x_7 x_6 \\
& -x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 + 2x_7 x_1^3 - 5x_7 x_1^2 + 4x_7 x_1 \\
& -x_7 - x_6 x_2 x_1^2 + 2x_6 x_2 x_1 - x_6 x_2 + x_6 x_1^2 - 2x_6 x_1 + x_6 + \\
& x_2 x_1^2 - 2x_2 x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

21. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :

$$\begin{aligned}
g = & -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\
& -2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_6 x_1^2 + 2x_7 x_6 x_1 - x_7 x_6 \\
& -x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 + 2x_7 x_1^3 - 5x_7 x_1^2 + 4x_7 x_1 \\
& -x_7 - x_6 x_2 x_1^2 + 2x_6 x_2 x_1 - x_6 x_2 + x_6 x_1^2 - 2x_6 x_1 + x_6 + \\
& x_2 x_1^2 - 2x_2 x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

22. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :

$$\begin{aligned}
g = & -x_7^2 x_1^3 + 3x_7^2 x_1^2 - 3x_7^2 x_1 + x_7^2 + x_7 x_6 x_2 x_1^2 \\
& -2x_7 x_6 x_2 x_1 + x_7 x_6 x_2 - x_7 x_6 x_1^2 + 2x_7 x_6 x_1 - x_7 x_6 \\
& -x_7 x_2 x_1^2 + 2x_7 x_2 x_1 - x_7 x_2 + 2x_7 x_1^3 - 5x_7 x_1^2 + 4x_7 x_1 \\
& -x_7 - x_6 x_2 x_1^2 + 2x_6 x_2 x_1 - x_6 x_2 + x_6 x_1^2 - 2x_6 x_1 + x_6 + \\
& x_2 x_1^2 - 2x_2 x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

23. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

24. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

25. Pseudo remainder with  $p_9$  over variable  $x_9$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

26. Pseudo remainder with  $p_8$  over variable  $x_8$ :

$$\begin{aligned}
g = & -x_7^2x_1^3 + 3x_7^2x_1^2 - 3x_7^2x_1 + x_7^2 + x_7x_6x_2x_1^2 \\
& -2x_7x_6x_2x_1 + x_7x_6x_2 - x_7x_6x_1^2 + 2x_7x_6x_1 - x_7x_6 \\
& -x_7x_2x_1^2 + 2x_7x_2x_1 - x_7x_2 + 2x_7x_1^3 - 5x_7x_1^2 + 4x_7x_1 \\
& -x_7 - x_6x_2x_1^2 + 2x_6x_2x_1 - x_6x_2 + x_6x_1^2 - 2x_6x_1 + x_6 + \\
& x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1
\end{aligned}$$

27. Pseudo remainder with  $p_7$  over variable  $x_7$ :

$$\begin{aligned}
g = & -x_6^2x_2x_1^2 + 2x_6^2x_2x_1 - x_6^2x_2 - x_6^2x_1^3 + \\
& 4x_6^2x_1^2 - 5x_6^2x_1 + 2x_6^2 + x_6x_2x_1^2 - 2x_6x_2x_1 + \\
& x_6x_2 - x_6x_1^2 + 2x_6x_1 - x_6
\end{aligned}$$

28. Pseudo remainder with  $p_6$  over variable  $x_6$ :

$$g = x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1$$

29. Pseudo remainder with  $p_5$  over variable  $x_5$ :

$$g = x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1$$

30. Pseudo remainder with  $p_4$  over variable  $x_4$ :

$$g = x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1$$

31. Pseudo remainder with  $p_3$  over variable  $x_3$ :

$$g = x_2x_1^2 - 2x_2x_1 + x_2 - x_1^3 + 2x_1^2 - x_1$$

32. Pseudo remainder with  $p_2$  over variable  $x_2$ :

$$g = 2x_1^3 - 5x_1^2 + 4x_1 -$$

33. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

### 3 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 29 terms.

**Time Complexity:** Time spent by the prover is 0.203 seconds.

### 4 NDG Conditions

**NDG Conditions in readable form**

- Failed to translate NDG Conditions to readable form