Balkan MO 1994

– May 10th

1 An acute angle XAY and a point P inside the angle are given. Construct (using a ruler and a compass) a line that passes through P and intersects the rays AX and AY at B and C such that the area of the triangle ABC equals AP^2.

Greece

2 Let n be an integer. Prove that the polynomial $f(x)$ has at most one zero, where

$$f(x) = x^4 - 1994x^3 + (1993 + n)x^2 - 11x + n.$$

Greece

3 Let $a_1, a_2, \dots, a_n$ be a permutation of the numbers $1, 2, \dots, n$, with $n \geq 2$. Determine the largest possible value of the sum

$$S(n) = |a_2 - a_1| + |a_3 - a_2| + \cdots + |a_n - a_{n-1}|.$$

Romania

4 Find the smallest number $n \geq 5$ for which there can exist a set of n people, such that any two people who are acquainted have no common acquaintances, and any two people who are not acquainted have exactly two common acquaintances.

Bulgaria