1996 Balkan MO

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– April 30th

1 Let O be the circumcenter and G be the centroid of a triangle ABC. If R and r are the circumcenter and incenter of the triangle, respectively,
prove that
$$OG \leq \sqrt{R(R-2r)}.$$

Greece

2 Let p be a prime number with $p > 5$. Consider the set $X = \left\{p - n^2 \mid n \in \mathbb{N},\ n^2 < p\right\}$. Prove that the set X has two distinct elements x and y such that $x \neq 1$ and $x \mid y$.

Albania

3 In a convex pentagon $ABCDE$, the points M, N, P, Q, R are the midpoints of the sides AB, BC, CD, DE, EA, respectively. If the segments AP, BQ, CR and DM pass through a single point, prove that EN contains that point as well.

Yugoslavia

4 Suppse that $X = \{1, 2, \dots, 2^{1996} - 1\}$, prove that there exist a subset A that satisfies these conditions:

a) $1 \in A$ and $2^{1996} - 1 \in A$;

b) Every element of A except 1 is equal to the sum of two (possibly equal) elements from A;

c) The maximum number of elements of A is 2012.

Romania

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