Art of Problem Solving
1991 Balkan MO

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1 Let ABC be an acute triangle inscribed in a circle centered at O. Let M be a point on the small arc AB of the triangle's circumcircle. The perpendicular dropped from M on the ray OA intersects the sides AB and AC at the points K and L, respectively. Similarly, the perpendicular dropped from M on the ray OB intersects the sides AB and BC at N and P, respectively. Assume that $KL = MN$. Find the size of the angle $\angle MLP$ in terms of the angles of the triangle ABC.

2 Show that there are infinitely many noncongruent triangles which satisfy the following conditions:
i) the side lengths are relatively prime integers;
ii)the area is an integer number;
iii)the altitudes' lengths are not integer numbers.

3 A regular hexagon of area H is inscribed in a convex polygon of area P. Show that $P \leq \frac{3}{2}H$. When does the equality occur?

4 Prove that there is no bijective function $f : \{1, 2, 3, \ldots\} \to \{0, 1, 2, 3, \ldots\}$ such that $f(mn) = f(m) + f(n) + 3f(m)f(n)$.