# OpenGeoProver Output for conjecture "geothm\_zadatak"

Wu's method used

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## 1 Invoking the theorem prover

The used proving method is Wu's method. The input system is:

```
p_1 = 2x_1 -
 p_2 = 2x_2 -
 p_3 = 2x_4 -
 p_4 = 2x_5 - x_1 -
 p_5 = 2x_6 - x_2
 p_6 = 2x_7 - x_3
 p_7 = x_{11}x_2 + x_9 - x_2
     = x_{11}x_3 + x_{10} - x_3
     = -x_{12}x_5 + x_8
p_{10} = -x_{12}x_6 + x_9
p_{11} = -x_{12}x_7 + x_{10}
p_{12} = 2x_{13} - 2
p_{13} = 2x_{14} -
p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{15} = x_{18}x_3 + x_{17} - x_3
p_{16} = -x_{19}x_5 + x_{19} + x_{15} -
p_{17} = -x_{19}x_6 + x_{19} + x_{16} -
p_{18} = -x_{19}x_7 + x_{17}
p_{19} = 2x_{20} -
p_{20} = 2x_{21} - 2
p_{21} = 2x_{22} - x_1 -
p_{22} = 2x_{23} - x_2 -
p_{23} = 2x_{24} - x_3
p_{24} = -x_{28}x_{23} + x_{28} + x_{26} -
p_{25} = -x_{28}x_{24} + x_{27}
```

```
= -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
      = -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
      = x_{29}x_3 + x_{27} - x_3
p_{28}
p_{29}
      = x_{30} -
      = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{30}
      = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{31}
      = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{32}
      = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{33}
p_{34} = x_{35} - x_1
      = x_{36} - x_2
p_{35}
p_{36}
      = -x_{38}x_{30} + x_{37} - x_3
p_{37} = x_{37}x_{30}
      = x_{39} - x_1
p_{38}
      = x_{40} - x_2
p_{40} = -x_{42}x_{30} + x_{41} - x_3
p_{41} = x_{41}x_{33} + x_{40}x_{32} + x_{39}x_{31} + x_{34}
```

## 1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 41.

Variable  $x_{41}$  selected: The number of polynomials with this variable, with indexes from 1 to 41, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{41}$  from all other polynomials by reducing them with polynomial  $p_{40}$  from previous step.

$$\begin{array}{rcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \\ p_{10} & = & -x_{12}x_6 + x_9 \\ p_{11} & = & -x_{12}x_7 + x_{10} \\ p_{12} & = & 2x_{13} - 2 \\ p_{13} & = & 2x_{14} - \end{array}$$

```
p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
     = x_{18}x_3 + x_{17} - x_3
     = -x_{19}x_5 + x_{19} + x_{15} -
p_{16}
      = -x_{19}x_6 + x_{19} + x_{16} -
      = -x_{19}x_7 + x_{17}
p_{19} = 2x_{20} -
     = 2x_{21} - 2
p_{20}
      = 2x_{22} - x_1 -
p_{22} = 2x_{23} - x_2 -
     = 2x_{24} - x_3
p_{23}
      = -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
p_{25} = -x_{28}x_{24} + x_{27}
      = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{26}
      = -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
      = x_{29}x_3 + x_{27} - x_3
p_{28}
      = x_{30} -
p_{29}
p_{30}
      = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
      = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{31}
     = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{32}
      = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{33}
p_{34} = x_{35} - x_1
     = x_{36} - x_2
p_{35}
     = -x_{38}x_{30} + x_{37} - x_3
p_{37} = x_{37}x_{30}
      = x_{39} - x_1
p_{38}
      = x_{40} - x_2
p_{40} = x_{42}x_{33}x_{30} + x_{40}x_{32} + x_{39}x_{31} + x_{34} + x_{33}x_{3}
     = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 40.

**Variable**  $x_{40}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 40, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{40}$  from all other polynomials by reducing them with polynomial  $p_{39}$  from previous step.

$$p_1 = 2x_1 -$$

$$p_2 = 2x_2 -$$

$$p_3 = 2x_4 -$$

$$p_4 = 2x_5 - x_1 -$$

$$p_5 = 2x_6 - x_2$$

$$p_6 = 2x_7 - x_3$$

$$p_7 = x_{11}x_2 + x_9 - x_2$$

$$p_8 = x_{11}x_3 + x_{10} - x_3$$

$$p_9 = -x_{12}x_5 + x_8$$

$$p_{10} = -x_{12}x_6 + x_9$$

$$p_{11} = -x_{12}x_7 + x_{10}$$

$$p_{12} = 2x_{13} - 2$$

$$p_{13} = 2x_{14} -$$

$$p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2$$

$$p_{15} = x_{18}x_3 + x_{17} - x_3$$

$$p_{16} = -x_{19}x_5 + x_{19} + x_{15} -$$

$$p_{17} = -x_{19}x_6 + x_{19} + x_{16} -$$

$$p_{18} = -x_{19}x_7 + x_{17}$$

$$p_{19} = 2x_{20} -$$

$$p_{20} = 2x_{21} - 2$$

$$p_{21} = 2x_{22} - x_1 -$$

$$p_{22} = 2x_{23} - x_2 -$$

$$p_{23} = 2x_{24} - x_3$$

$$p_{24} = -x_{28}x_{23} + x_{28} + x_{26} -$$

$$p_{25} = -x_{28}x_{24} + x_{27}$$

$$p_{26} = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1$$

$$p_{27} = -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2$$

$$p_{28} = x_{29}x_3 + x_{27} - x_3$$

$$p_{29} = x_{30} -$$

$$p_{30} = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}$$

$$p_{31} = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}$$

$$p_{32} = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9$$

$$p_{33} = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8$$

$$p_{34} = x_{35} - x_1$$

$$p_{35} = x_{36} - x_2$$

$$p_{36} = -x_{38}x_{30} + x_{37} - x_3$$

$$p_{37} = x_{37}x_{30}$$

$$p_{38} = x_{39} - x_1$$

$$p_{39} = x_{42}x_{33}x_{30} + x_{39}x_{31} + x_{34} + x_{33}x_3 + x_{32}x_2$$

$$p_{40} = x_{40} - x_2$$

$$p_{41} = -x_{42}x_{30} + x_{41} - x_3$$

## 1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 39.

**Variable**  $x_{39}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 39, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{39}$  from all other polynomials by reducing them with polynomial  $p_{38}$  from previous step.

```
p_1 = 2x_1 -
 p_2 = 2x_2 -
 p_3 = 2x_4 -
 p_4 = 2x_5 - x_1 -
 p_5 = 2x_6 - x_2
 p_6 = 2x_7 - x_3
 p_7 = x_{11}x_2 + x_9 - x_2
     = x_{11}x_3 + x_{10} - x_3
     = -x_{12}x_5 + x_8
     = -x_{12}x_6 + x_9
p_{11} = -x_{12}x_7 + x_{10}
p_{12} = 2x_{13} - 2
p_{13} = 2x_{14} -
p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{15} = x_{18}x_3 + x_{17} - x_3
p_{16} = -x_{19}x_5 + x_{19} + x_{15} -
p_{17} = -x_{19}x_6 + x_{19} + x_{16} -
p_{18} = -x_{19}x_7 + x_{17}
p_{19} = 2x_{20} -
p_{20} = 2x_{21} - 2
p_{21} = 2x_{22} - x_1 -
p_{22} = 2x_{23} - x_2 -
p_{23} = 2x_{24} - x_3
p_{24} = -x_{28}x_{23} + x_{28} + x_{26} -
p_{25} = -x_{28}x_{24} + x_{27}
p_{26} = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{27} = -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
p_{28} = x_{29}x_3 + x_{27} - x_3
p_{29}
     = x_{30} -
p_{30} = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
```

```
= x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{31}
       = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
      = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{33}
p_{34}
       = x_{35} - x_1
       = x_{36} - x_2
p_{35}
       = -x_{38}x_{30} + x_{37} - x_3
p_{36}
       = x_{37}x_{30}
p_{37}
       = x_{42}x_{33}x_{30} + x_{34} + x_{33}x_3 + x_{32}x_2 + x_{31}x_1
p_{38}
      = x_{39} - x_1
p_{39}
      = x_{40} - x_2
p_{40}
      = -x_{42}x_{30} + x_{41} - x_3
```

## 1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 38.

Variable  $x_{38}$  selected: The number of polynomials with this variable, with indexes from 1 to 38, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{36}$ . No reduction needed.

The triangular system has not been changed.

## 1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 37.

Variable  $x_{37}$  selected: The number of polynomials with this variable, with indexes from 1 to 37, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{36}$ . No reduction needed.

The triangular system has not been changed.

#### 1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 36.

Variable  $x_{36}$  selected: The number of polynomials with this variable, with indexes from 1 to 36, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{35}$ . No reduction needed.

The triangular system has not been changed.

## 1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 35.

Variable  $x_{35}$  selected: The number of polynomials with this variable, with indexes from 1 to 35, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{34}$ . No reduction needed.

The triangular system has not been changed.

## 1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 34.

Variable  $x_{34}$  selected: The number of polynomials with this variable, with indexes from 1 to 34, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{34}$  from all other polynomials by reducing them with polynomial  $p_{33}$  from previous step.

```
p_1 = 2x_1 -
     = 2x_2 -
    = 2x_4 -
 p_4 = 2x_5 - x_1 -
     = 2x_6 - x_2
 p_5
     = 2x_7 - x_3
p_6
     = x_{11}x_2 + x_9 - x_2
     = x_{11}x_3 + x_{10} - x_3
p_8
     = -x_{12}x_5 + x_8
p_9
     = -x_{12}x_6 + x_9
     = -x_{12}x_7 + x_{10}
p_{11}
     = 2x_{13} - 2
p_{12}
     = 2x_{14} -
     = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{14}
     = x_{18}x_3 + x_{17} - x_3
p_{15}
     = -x_{19}x_5 + x_{19} + x_{15} -
p_{17} = -x_{19}x_6 + x_{19} + x_{16} -
     = -x_{19}x_7 + x_{17}
p_{18}
     = 2x_{20} -
p_{19}
p_{20}
     = 2x_{21} - 2
p_{21} = 2x_{22} - x_1 -
```

```
= 2x_{23} - x_2 -
p_{22}
              2x_{24} - x_3
p_{23}
              -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
             -x_{28}x_{24} + x_{27}
p_{25}
             -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{26}
              -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
p_{27}
            x_{29}x_3 + x_{27} - x_3
p_{28}
            x_{30} - 
p_{29}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{30}
        = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{31}
            x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{32}
             x_{42}x_{33}x_{30} - x_{33}x_{10} + x_{33}x_3 - x_{32}x_9 + x_{32}x_2 - x_{31}x_8 + x_{31}x_1
p_{33}
            x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
p_{35}
             x_{35} - x_1
p_{36}
             x_{36} - x_2
p_{37}
             x_{37}x_{30}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
             x_{39} - x_1
p_{39}
             x_{40} - x_2
p_{40}
            -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 33.

Variable  $x_{33}$  selected: The number of polynomials with this variable, with indexes from 1 to 33, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{33}$  from all other polynomials by reducing them with polynomial  $p_{32}$  from previous step.

$$\begin{array}{rcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \end{array}$$

```
p_{10} = -x_{12}x_6 + x_9
p_{11} = -x_{12}x_7 + x_{10}
            2x_{13}-2
p_{12}
p_{13} = 2x_{14} -
p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
     = x_{18}x_3 + x_{17} - x_3
p_{15}
      = -x_{19}x_5 + x_{19} + x_{15} -
p_{16}
      = -x_{19}x_6 + x_{19} + x_{16} -
p_{17}
            -x_{19}x_7 + x_{17}
      =
p_{18}
      = 2x_{20} -
p_{19}
      = 2x_{21} - 2
p_{20}
       = 2x_{22} - x_1 -
p_{21}
      = 2x_{23} - x_2 -
p_{22}
      = 2x_{24} - x_3
p_{23}
            -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
      = -x_{28}x_{24} + x_{27}
p_{25}
      = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{26}
            -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
p_{27}
      = x_{29}x_3 + x_{27} - x_3
p_{28}
p_{29}
      = x_{30} -
      = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{30}
      = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{31}
      = x_{42}x_{30}x_{26}x_{15} - x_{42}x_{30}x_{26}x_8 - x_{42}x_{30}x_{25}x_{16} +
p_{32}
             x_{42}x_{30}x_{25}x_9 + x_{42}x_{30}x_{16}x_8 - x_{42}x_{30}x_{15}x_9 - x_{32}x_9 +
             x_{32}x_2 - x_{31}x_8 + x_{31}x_1 - x_{26}x_{15}x_{10} + x_{26}x_{15}x_3 + x_{26}x_{10}x_8
             -x_{26}x_8x_3 + x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_9 + x_{25}x_9x_3
             -x_{16}x_{10}x_8 + x_{16}x_8x_3 + x_{15}x_{10}x_9 - x_{15}x_9x_3
p_{33} = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
     = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
            x_{35} - x_1
p_{35}
            x_{36} - x_2
p_{36}
p_{37}
            x_{37}x_{30}
             -x_{38}x_{30} + x_{37} - x_3
p_{38}
      = x_{39} - x_1
p_{39}
p_{40}
      =
            x_{40} - x_2
            -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 32.

Variable  $x_{32}$  selected: The number of polynomials with this variable, with indexes from 1 to 32, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{32}$  from all other polynomials by reducing them with polynomial  $p_{31}$  from previous step.

```
= 2x_1 -
 p_1
       = 2x_2 -
 p_2
       = 2x_4 -
 p_3
       = 2x_5 - x_1 -
       = 2x_6 - x_2
 p_5
       = 2x_7 - x_3
 p_6
       = x_{11}x_2 + x_9 - x_2
 p_7
       = x_{11}x_3 + x_{10} - x_3
       = -x_{12}x_5 + x_8
 p_9
            -x_{12}x_6 + x_9
p_{10}
           -x_{12}x_7 + x_{10}
p_{11}
       = 2x_{13} - 2
p_{12}
       = 2x_{14} -
p_{13}
       = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{14}
       = x_{18}x_3 + x_{17} - x_3
p_{15}
p_{16}
       = -x_{19}x_5 + x_{19} + x_{15} -
       = -x_{19}x_6 + x_{19} + x_{16} -
p_{17}
       = -x_{19}x_7 + x_{17}
p_{18}
            2x_{20} -
p_{19}
       = 2x_{21} - 2
p_{20}
       = 2x_{22} - x_1 -
p_{21}
       = 2x_{23} - x_2 -
p_{22}
       = 2x_{24} - x_3
p_{23}
            -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
            -x_{28}x_{24} + x_{27}
p_{25}
            -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{26}
            -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
p_{27}
       = x_{29}x_3 + x_{27} - x_3
p_{28}
       = x_{30} -
p_{29}
            x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{30}
            x_{42}x_{30}x_{26}x_{15} - x_{42}x_{30}x_{26}x_8 - x_{42}x_{30}x_{25}x_{16} +
p_{31}
             x_{42}x_{30}x_{25}x_9 + x_{42}x_{30}x_{16}x_8 - x_{42}x_{30}x_{15}x_9 - x_{31}x_8 +
```

$$\begin{array}{rclcrcl} & x_{31}x_{1} + x_{27}x_{15}x_{9} - x_{27}x_{15}x_{2} - x_{27}x_{9}x_{8} + x_{27}x_{8}x_{2} \\ & -x_{26}x_{15}x_{10} + x_{26}x_{15}x_{3} + x_{26}x_{10}x_{8} - x_{26}x_{8}x_{3} - x_{25}x_{17}x_{9} + \\ & x_{25}x_{17}x_{2} + x_{25}x_{16}x_{10} - x_{25}x_{16}x_{3} - x_{25}x_{10}x_{2} + x_{25}x_{9}x_{3} + \\ & x_{17}x_{9}x_{8} - x_{17}x_{8}x_{2} - x_{16}x_{10}x_{8} + x_{16}x_{8}x_{3} + x_{15}x_{10}x_{2} \\ & -x_{15}x_{9}x_{3} \\ & p_{32} & = & x_{32} + x_{27}x_{15} - x_{27}x_{8} - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_{8} - x_{15}x_{10} \\ & p_{33} & = & x_{33} - x_{26}x_{15} + x_{26}x_{8} + x_{25}x_{16} - x_{25}x_{9} - x_{16}x_{8} + x_{15}x_{9} \\ & p_{34} & = & x_{34} + x_{33}x_{10} + x_{32}x_{9} + x_{31}x_{8} \\ & p_{35} & = & x_{35} - x_{1} \\ & p_{36} & = & x_{36} - x_{2} \\ & p_{37} & = & x_{37}x_{30} \\ & p_{38} & = & -x_{38}x_{30} + x_{37} - x_{3} \\ & p_{39} & = & x_{39} - x_{1} \\ & p_{40} & = & x_{40} - x_{2} \\ & p_{41} & = & -x_{42}x_{30} + x_{41} - x_{3} \\ \end{array}$$

## 1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 31.

Variable  $x_{31}$  selected: The number of polynomials with this variable, with indexes from 1 to 31, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{31}$  from all other polynomials by reducing them with polynomial  $p_{30}$  from previous step.

$$\begin{array}{rcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \\ p_{10} & = & -x_{12}x_6 + x_9 \\ p_{11} & = & -x_{12}x_7 + x_{10} \\ p_{12} & = & 2x_{13} - 2 \\ p_{13} & = & 2x_{14} - \\ p_{14} & = & -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2 \\ \end{array}$$

```
p_{15} = x_{18}x_3 + x_{17} - x_3
      = -x_{19}x_5 + x_{19} + x_{15} -
      = -x_{19}x_6 + x_{19} + x_{16} -
p_{17}
       = -x_{19}x_7 + x_{17}
p_{19}
      = 2x_{20} -
      = 2x_{21} - 2
p_{20}
      = 2x_{22} - x_1 -
p_{21}
      = 2x_{23} - x_2 -
p_{23} = 2x_{24} - x_3
      = -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
      = -x_{28}x_{24} + x_{27}
p_{25}
      = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{26}
      = -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
p_{27}
       = x_{29}x_3 + x_{27} - x_3
p_{29}
      = x_{30} -
      = x_{42}x_{30}x_{26}x_{15} - x_{42}x_{30}x_{26}x_8 - x_{42}x_{30}x_{25}x_{16} +
p_{30}
             x_{42}x_{30}x_{25}x_9 + x_{42}x_{30}x_{16}x_8 - x_{42}x_{30}x_{15}x_9 - x_{27}x_{16}x_8 +
             x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2 - x_{27}x_9x_1 + x_{27}x_8x_2 +
             x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} + x_{26}x_{15}x_3 + x_{26}x_{10}x_1
             -x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 + x_{25}x_{16}x_{10} - x_{25}x_{16}x_3
             -x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1 - x_{17}x_8x_2 - x_{16}x_{10}x_1 +
             x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
      = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{32} = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
      = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
      = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
      = x_{35} - x_1
p_{35}
      = x_{36} - x_2
p_{36}
p_{37}
      = x_{37}x_{30}
       = -x_{38}x_{30} + x_{37} - x_3
p_{38}
      = x_{39} - x_1
p_{39}
      = x_{40} - x_2
      = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

#### 1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 30.

Variable  $x_{30}$  selected: The number of polynomials with this variable, with indexes from 1 to 30, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{30}$  from all other polynomials by reducing them with polynomial  $p_{29}$  from previous step.

```
= 2x_1 -
 p_1
       = 2x_2 -
 p_2
      = 2x_4 -
 p_3
       = 2x_5 - x_1 -
 p_4
      = 2x_6 - x_2
 p_5
      = 2x_7 - x_3
 p_6
       = x_{11}x_2 + x_9 - x_2
      = x_{11}x_3 + x_{10} - x_3
 p_8
      = -x_{12}x_5 + x_8
           -x_{12}x_6 + x_9
p_{10}
       = -x_{12}x_7 + x_{10}
p_{11}
      = 2x_{13} - 2
p_{12}
            2x_{14} -
p_{13}
      = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{14}
      = x_{18}x_3 + x_{17} - x_3
           -x_{19}x_5 + x_{19} + x_{15} -
p_{16}
       = -x_{19}x_6 + x_{19} + x_{16} -
p_{17}
      = -x_{19}x_7 + x_{17}
p_{18}
       = 2x_{20} -
p_{19}
       = 2x_{21} - 2
p_{20}
       = 2x_{22} - x_1 -
p_{21}
       = 2x_{23} - x_2 -
p_{22}
       = 2x_{24} - x_3
p_{23}
            -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
       = -x_{28}x_{24} + x_{27}
p_{25}
       = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{26}
            -x_{29}x_{21} + x_{29}x_2 + x_{26} - x_2
p_{27}
       = x_{29}x_3 + x_{27} - x_3
p_{28}
            x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{29}
             -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
             -x_{27}x_9x_1+x_{27}x_8x_2+x_{26}x_{17}x_8-x_{26}x_{17}x_1-x_{26}x_{15}x_{10}+\\
             x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
             x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
             -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
p_{30}
      = x_{30} -
p_{31}
      = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
```

```
= x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
             x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
             x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
p_{35}
             x_{35} - x_1
             x_{36} - x_2
p_{36}
             x_{37}x_{30}
p_{37}
       = -x_{38}x_{30} + x_{37} - x_3
p_{38}
       = x_{39} - x_1
p_{39}
      = x_{40} - x_2
p_{40}
      = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 29.

Variable  $x_{29}$  selected: The number of polynomials with this variable, with indexes from 1 to 29, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{29}$  from all other polynomials by reducing them with polynomial  $p_{26}$  from previous step.

$$\begin{array}{rclcrcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \\ p_{10} & = & -x_{12}x_6 + x_9 \\ p_{11} & = & -x_{12}x_7 + x_{10} \\ p_{12} & = & 2x_{13} - 2 \\ p_{13} & = & 2x_{14} - \\ p_{14} & = & -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2 \\ p_{15} & = & x_{18}x_3 + x_{17} - x_3 \\ p_{16} & = & -x_{19}x_5 + x_{19} + x_{15} - \\ p_{17} & = & -x_{19}x_6 + x_{19} + x_{16} - \\ p_{18} & = & -x_{19}x_7 + x_{17} \\ p_{19} & = & 2x_{20} - \end{array}$$

```
= 2x_{21} - 2
p_{20}
              2x_{22} - x_1 -
p_{21}
              2x_{23} - x_2 -
p22
              2x_{24} - x_3
p_{23}
              -x_{28}x_{23} + x_{28} + x_{26} -
p_{24}
              -x_{28}x_{24} + x_{27}
p_{25}
        = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{26}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 + \\
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
               -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
            -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{27}
            -x_{27}x_{20} + x_{27}x_1 - x_{25}x_3 + x_{20}x_3
p_{28}
              -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
              x_{30} -
p_{30}
              x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
              x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
        =
              x_{35} - x_1
p_{35}
              x_{36} - x_2
        =
p_{36}
p_{37}
              x_{37}x_{30}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
        = x_{39} - x_1
p_{39}
              x_{40} - x_2
p_{40}
p_{41}
              -x_{42}x_{30} + x_{41} - x_3
```

## 1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 28.

**Variable**  $x_{28}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 28, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{28}$  from all other polynomials by reducing them with polynomial  $p_{24}$  from previous step.

$$p_1 = 2x_1 - p_2 = 2x_2 - p_2$$

$$p_3 = 2x_4 -$$

$$p_4 = 2x_5 - x_1 -$$

$$p_5 = 2x_6 - x_2$$

$$p_6 = 2x_7 - x_3$$

$$p_7 = x_{11}x_2 + x_9 - x_2$$

$$p_8 = x_{11}x_3 + x_{10} - x_3$$

$$p_9 = -x_{12}x_5 + x_8$$

$$p_{10} = -x_{12}x_6 + x_9$$

$$p_{11} = -x_{12}x_7 + x_{10}$$

$$p_{12} = 2x_{13} - 2$$

$$p_{13} = 2x_{14} -$$

$$p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2$$

$$p_{15} = x_{18}x_3 + x_{17} - x_3$$

$$p_{16} = -x_{19}x_5 + x_{19} + x_{15} -$$

$$p_{17} = -x_{19}x_6 + x_{19} + x_{16} -$$

$$p_{18} = -x_{19}x_7 + x_{17}$$

$$p_{19} = 2x_{20} -$$

$$p_{20} = 2x_{21} - 2$$

$$p_{21} = 2x_{22} - x_1 -$$

$$p_{22} = 2x_{23} - x_2 -$$

$$p_{23} = 2x_{24} - x_3$$

$$p_{24} = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8 - x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2$$

$$-x_{27}x_{9}x_{1}+x_{27}x_{8}x_{2}+x_{26}x_{17}x_{8}-x_{26}x_{17}x_{1}-x_{26}x_{15}x_{10}+\\x_{26}x_{15}x_{3}+x_{26}x_{10}x_{1}-x_{26}x_{8}x_{3}-x_{25}x_{17}x_{9}+x_{25}x_{17}x_{2}+$$

$$x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1$$

$$-x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3$$

$$p_{25} = -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2$$

$$p_{26} = -x_{27}x_{20} + x_{27}x_1 - x_{25}x_3 + x_{20}x_3$$

$$p_{27} = -x_{27}x_{23} + x_{27} + x_{26}x_{24} - x_{24}$$

$$p_{28} = -x_{28}x_{23} + x_{28} + x_{26} -$$

$$p_{29} = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1$$

$$p_{30} = x_{30} -$$

$$p_{31} = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}$$

$$p_{32} = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}$$

$$p_{33} = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9$$

$$p_{34} = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8$$

$$p_{35} = x_{35} - x_1$$

$$p_{36} = x_{36} - x_2$$

$$p_{37} = x_{37}x_{30}$$

```
p_{38} = -x_{38}x_{30} + x_{37} - x_3
p_{39} = x_{39} - x_1
p_{40} = x_{40} - x_2
p_{41} = -x_{42}x_{30} + x_{41} - x_3
```

## 1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 27.

Variable  $x_{27}$  selected: The number of polynomials with this variable, with indexes from 1 to 27, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{27}$  from all other polynomials by reducing them with polynomial  $p_{24}$  from previous step.

```
2x_1 -
 p_1
           2x_{2} -
      =
      = 2x_4 -
 p_3
      = 2x_5 - x_1 -
      = 2x_6 - x_2
      = 2x_7 - x_3
 p_6
      = x_{11}x_2 + x_9 - x_2
      = x_{11}x_3 + x_{10} - x_3
      = -x_{12}x_5 + x_8
 p_9
      = -x_{12}x_6 + x_9
p_{10}
           -x_{12}x_7 + x_{10}
      = 2x_{13} - 2
p_{12}
      = 2x_{14} -
p_{13}
      = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
      = x_{18}x_3 + x_{17} - x_3
p_{15}
      = -x_{19}x_5 + x_{19} + x_{15} -
p_{16}
           -x_{19}x_6 + x_{19} + x_{16} -
           -x_{19}x_7 + x_{17}
p_{18}
      = 2x_{20} -
p_{19}
      = 2x_{21} - 2
p_{20}
      = 2x_{22} - x_1 -
p_{21}
      = 2x_{23} - x_2 -
p_{22}
      = 2x_{24} - x_3
p_{24}
           -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
      = x_{42}x_{26}x_{20}x_{15} - x_{42}x_{26}x_{20}x_8 - x_{42}x_{26}x_{15}x_1 +
p_{25}
```

```
x_{42}x_{26}x_8x_1 - x_{42}x_{25}x_{20}x_{16} + x_{42}x_{25}x_{20}x_9 +
              x_{42}x_{25}x_{16}x_1 - x_{42}x_{25}x_9x_1 + x_{42}x_{20}x_{16}x_8
              -x_{42}x_{20}x_{15}x_9 - x_{42}x_{16}x_8x_1 + x_{42}x_{15}x_9x_1 +
              x_{26}x_{20}x_{17}x_8 - x_{26}x_{20}x_{17}x_1 - x_{26}x_{20}x_{15}x_{10} +
              x_{26}x_{20}x_{15}x_3 + x_{26}x_{20}x_{10}x_1 - x_{26}x_{20}x_8x_3
              -x_{26}x_{17}x_8x_1 + x_{26}x_{17}x_1^2 + x_{26}x_{15}x_{10}x_1 - x_{26}x_{15}x_3x_1
              -x_{26}x_{10}x_1^2 + x_{26}x_8x_3x_1 - x_{25}x_{20}x_{17}x_9 + x_{25}x_{20}x_{17}x_2 +
              x_{25}x_{20}x_{16}x_{10} - x_{25}x_{20}x_{16}x_3 - x_{25}x_{20}x_{10}x_2 +
              x_{25}x_{20}x_9x_3 + x_{25}x_{17}x_9x_1 - x_{25}x_{17}x_2x_1
              -x_{25}x_{16}x_{10}x_1 + x_{25}x_{16}x_8x_3 - x_{25}x_{15}x_9x_3 +
              x_{25}x_{15}x_3x_2 + x_{25}x_{10}x_2x_1 - x_{25}x_8x_3x_2 + x_{20}x_{17}x_9x_1
              -x_{20}x_{17}x_8x_2 - x_{20}x_{16}x_{10}x_1 + x_{20}x_{16}x_3x_1 +
              x_{20}x_{15}x_{10}x_2 - x_{20}x_{15}x_3x_2 - x_{20}x_9x_3x_1 + x_{20}x_8x_3x_2
              -x_{17}x_9x_1^2 + x_{17}x_8x_2x_1 + x_{16}x_{10}x_1^2 - x_{16}x_8x_3x_1
              -x_{15}x_{10}x_2x_1 + x_{15}x_9x_3x_1
p_{26} = x_{42}x_{26}x_{23}x_{15} - x_{42}x_{26}x_{23}x_8 - x_{42}x_{26}x_{15} + x_{42}x_{26}x_8
              -x_{42}x_{25}x_{23}x_{16} + x_{42}x_{25}x_{23}x_9 + x_{42}x_{25}x_{16} - x_{42}x_{25}x_9 +
              x_{42}x_{23}x_{16}x_8 - x_{42}x_{23}x_{15}x_9 - x_{42}x_{16}x_8 + x_{42}x_{15}x_9
              -x_{26}x_{24}x_{16}x_8 + x_{26}x_{24}x_{16}x_1 + x_{26}x_{24}x_{15}x_9
              -x_{26}x_{24}x_{15}x_2 - x_{26}x_{24}x_9x_1 + x_{26}x_{24}x_8x_2 +
              x_{26}x_{23}x_{17}x_8 - x_{26}x_{23}x_{17}x_1 - x_{26}x_{23}x_{15}x_{10} +
              x_{26}x_{23}x_{15}x_3 + x_{26}x_{23}x_{10}x_1 - x_{26}x_{23}x_8x_3 - x_{26}x_{17}x_8 +
              x_{26}x_{17}x_1 + x_{26}x_{15}x_{10} - x_{26}x_{15}x_3 - x_{26}x_{10}x_1 + x_{26}x_8x_3
              -x_{25}x_{23}x_{17}x_9 + x_{25}x_{23}x_{17}x_2 + x_{25}x_{23}x_{16}x_{10}
              -x_{25}x_{23}x_{16}x_3-x_{25}x_{23}x_{10}x_2+x_{25}x_{23}x_9x_3+x_{25}x_{17}x_9
              -x_{25}x_{17}x_2 - x_{25}x_{16}x_{10} + x_{25}x_{16}x_3 + x_{25}x_{10}x_2 - x_{25}x_9x_3 +
              x_{24}x_{16}x_8 - x_{24}x_{16}x_1 - x_{24}x_{15}x_9 + x_{24}x_{15}x_2 + x_{24}x_9x_1
              -x_{24}x_8x_2 + x_{23}x_{17}x_9x_1 - x_{23}x_{17}x_8x_2 - x_{23}x_{16}x_{10}x_1 +
              x_{23}x_{16}x_{8}x_{3} + x_{23}x_{15}x_{10}x_{2} - x_{23}x_{15}x_{9}x_{3} - x_{17}x_{9}x_{1} +
              x_{17}x_8x_2 + x_{16}x_{10}x_1 - x_{16}x_8x_3 - x_{15}x_{10}x_2 + x_{15}x_9x_3
p_{27} = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
       = -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
        = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
             x_{30} - 
p_{30}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
```

```
= x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
             x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
             x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
p_{35}
             x_{35} - x_1
       = x_{36} - x_2
p_{36}
             x_{37}x_{30}
p_{37}
       = -x_{38}x_{30} + x_{37} - x_3
p_{38}
       = x_{39} - x_1
p_{39}
      = x_{40} - x_2
p_{40}
       = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 26.

Variable  $x_{26}$  selected: The number of polynomials with this variable, with indexes from 1 to 26, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{26}$  from all other polynomials by reducing them with polynomial  $p_{24}$  from previous step.

$$\begin{array}{rclcrcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \\ p_{10} & = & -x_{12}x_6 + x_9 \\ p_{11} & = & -x_{12}x_7 + x_{10} \\ p_{12} & = & 2x_{13} - 2 \\ p_{13} & = & 2x_{14} - \\ p_{14} & = & -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2 \\ p_{15} & = & x_{18}x_3 + x_{17} - x_3 \\ p_{16} & = & -x_{19}x_5 + x_{19} + x_{15} - \\ p_{17} & = & -x_{19}x_6 + x_{19} + x_{16} - \\ p_{18} & = & -x_{19}x_7 + x_{17} \\ p_{19} & = & 2x_{20} - \end{array}$$

```
= 2x_{21} - 2
p_{20}
              2x_{22} - x_1 -
p_{21}
              2x_{23} - x_2 -
p_{22}
              2x_{24} - x_3
p_{23}
p_{24}
p_{25}
              -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
       = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1+x_{27}x_8x_2+x_{26}x_{17}x_8-x_{26}x_{17}x_1-x_{26}x_{15}x_{10}+\\
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
              -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
              -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
              x_{30} -
p_{30}
              x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
              x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
              x_{35} - x_1
        =
p_{35}
              x_{36} - x_2
        =
p_{36}
p_{37}
              x_{37}x_{30}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
        = x_{39} - x_1
p_{39}
              x_{40} - x_2
p_{40}
              -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 25.

Variable  $x_{25}$  selected: The number of polynomials with this variable, with indexes from 1 to 25, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{25}$  from all other polynomials by reducing them with polynomial  $p_{24}$  from previous step.

$$p_1 = 2x_1 - p_2 = 2x_2 - p_2$$

$$p_{34} = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8$$
  
$$p_{35} = x_{35} - x_1$$

$$p_{36} = x_{36} - x_2$$

 $= x_{30} -$ 

 $p_{30}$ 

 $p_{31}$ 

 $p_{32}$ 

 $p_{33}$ 

$$p_{37} = x_{37}x_{30}$$

 $= x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}$ 

 $= x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}$ 

 $= x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9$ 

```
p_{38} = -x_{38}x_{30} + x_{37} - x_3
p_{39} = x_{39} - x_1
p_{40} = x_{40} - x_2
p_{41} = -x_{42}x_{30} + x_{41} - x_3
```

## 1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 24.

Variable  $x_{24}$  selected: The number of polynomials with this variable, with indexes from 1 to 24, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{24}$  from all other polynomials by reducing them with polynomial  $p_{23}$  from previous step.

$$\begin{array}{rclrcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \\ p_{10} & = & -x_{12}x_6 + x_9 \\ p_{11} & = & -x_{12}x_7 + x_{10} \\ p_{12} & = & 2x_{13} - 2 \\ p_{13} & = & 2x_{14} - \\ p_{14} & = & -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2 \\ p_{15} & = & x_{18}x_3 + x_{17} - x_3 \\ p_{16} & = & -x_{19}x_5 + x_{19} + x_{15} - \\ p_{17} & = & -x_{19}x_6 + x_{19} + x_{16} - \\ p_{18} & = & -x_{19}x_7 + x_{17} \\ p_{19} & = & 2x_{20} - \\ p_{20} & = & 2x_{21} - 2 \\ p_{21} & = & 2x_{22} - x_1 - \\ p_{22} & = & 2x_{23} - x_2 - \\ p_{23} & = & \dots \\ p_{24} & = & 2x_{24} - x_3 \\ p_{25} & = & \dots \end{array}$$

```
= -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
       = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
              -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
              -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
p_{30}
             x_{30} - 
              x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
              x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
p_{35}
              x_{35} - x_1
             x_{36} - x_2
p_{36}
              x_{37}x_{30}
p_{37}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
            x_{39} - x_1
p_{39}
             x_{40} - x_2
p_{40}
              -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

#### 1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 23.

Variable  $x_{23}$  selected: The number of polynomials with this variable, with indexes from 1 to 23, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{23}$  from all other polynomials by reducing them with polynomial  $p_{22}$  from previous step.

$$\begin{array}{rclcrcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \end{array}$$

$$p_9 = -x_{12}x_5 + x_8$$

$$p_{10} = -x_{12}x_6 + x_9$$

$$p_{11} = -x_{12}x_7 + x_{10}$$

$$p_{12} = 2x_{13} - 2$$

$$p_{13} = 2x_{14} -$$

$$p_{14} = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2$$

$$p_{15} = x_{18}x_3 + x_{17} - x_3$$

$$p_{16} = -x_{19}x_5 + x_{19} + x_{15} -$$

$$p_{17} = -x_{19}x_6 + x_{19} + x_{16} -$$

$$p_{18} = -x_{19}x_7 + x_{17}$$

$$p_{19} = 2x_{20} -$$

$$p_{20} = 2x_{21} - 2$$

$$p_{21} = 2x_{22} - x_1 -$$

$$p_{22} = \dots$$

$$p_{23} = 2x_{23} - x_2 -$$

$$p_{24} = 2x_{24} - x_3$$

$$p_{25} = \dots$$

$$p_{26} = -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2$$

$$\begin{array}{rcl} p_{27} & = & x_{42}x_{26}x_{15} - x_{42}x_{26}x_{8} - x_{42}x_{25}x_{16} + x_{42}x_{25}x_{9} + x_{42}x_{16}x_{8} \\ & & -x_{42}x_{15}x_{9} - x_{27}x_{16}x_{8} + x_{27}x_{16}x_{1} + x_{27}x_{15}x_{9} - x_{27}x_{15}x_{2} \\ & & -x_{27}x_{9}x_{1} + x_{27}x_{8}x_{2} + x_{26}x_{17}x_{8} - x_{26}x_{17}x_{1} - x_{26}x_{15}x_{10} + \\ & & x_{26}x_{15}x_{3} + x_{26}x_{10}x_{1} - x_{26}x_{8}x_{3} - x_{25}x_{17}x_{9} + x_{25}x_{17}x_{2} + \\ & & x_{25}x_{16}x_{10} - x_{25}x_{16}x_{3} - x_{25}x_{10}x_{2} + x_{25}x_{9}x_{3} + x_{17}x_{9}x_{1} \\ & & -x_{17}x_{8}x_{2} - x_{16}x_{10}x_{1} + x_{16}x_{8}x_{3} + x_{15}x_{10}x_{2} - x_{15}x_{9}x_{3} \end{array}$$

$$p_{28} = -x_{28}x_{23} + x_{28} + x_{26} -$$

$$p_{29} = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1$$

$$p_{30} = x_{30} -$$

$$p_{31} = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}$$

$$p_{32} = x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}$$

$$p_{33} = x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9$$

$$p_{34} = x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8$$

$$p_{35} = x_{35} - x_1$$

$$p_{36} = x_{36} - x_2$$

$$p_{37} = x_{37}x_{30}$$

$$p_{38} = -x_{38}x_{30} + x_{37} - x_3$$

$$p_{39} = x_{39} - x_1$$

$$p_{40} = x_{40} - x_2$$

$$p_{41} = -x_{42}x_{30} + x_{41} - x_3$$

## 1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 22.

Variable  $x_{22}$  selected: The number of polynomials with this variable, with indexes from 1 to 22, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{21}$ . No reduction needed.

The triangular system has not been changed.

## 1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 21.

Variable  $x_{21}$  selected: The number of polynomials with this variable, with indexes from 1 to 21, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{21}$  from all other polynomials by reducing them with polynomial  $p_{20}$  from previous step.

```
p_1 = 2x_1 -
     = 2x_2 -
 p_2
    = 2x_4 -
     = 2x_5 - x_1 -
     = 2x_6 - x_2
 p_5
     = 2x_7 - x_3
p_6
     = x_{11}x_2 + x_9 - x_2
     = x_{11}x_3 + x_{10} - x_3
p_8
     = -x_{12}x_5 + x_8
     = -x_{12}x_6 + x_9
p_{10}
     = -x_{12}x_7 + x_{10}
p_{11}
     = 2x_{13} - 2
p_{12}
     = 2x_{14} -
     = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{14}
     = x_{18}x_3 + x_{17} - x_3
p_{15}
     = -x_{19}x_5 + x_{19} + x_{15} -
p_{16}
     = -x_{19}x_6 + x_{19} + x_{16} -
p_{17}
     = -x_{19}x_7 + x_{17}
p_{18}
      = 2x_{20} -
p_{19}
p_{20}
     = 0
     = 2x_{21} - 2
p_{21}
```

```
= 2x_{22} - x_1 -
p_{22}
        = 2x_{23} - x_2 -
p_{23}
        =
              2x_{24} - x_3
p_{24}
p_{25}
              -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
             x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
             -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
             -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
p_{30}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
             x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
             x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
             x_{35} - x_1
p_{35}
             x_{36} - x_2
p_{36}
             x_{37}x_{30}
       =
p_{37}
             -x_{38}x_{30} + x_{37} - x_3
p_{38}
p_{39}
       = x_{39} - x_1
p_{40}
       = x_{40} - x_2
            -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 20.

Variable  $x_{20}$  selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{19}$ . No reduction needed.

The triangular system has not been changed.

#### 1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 19.

Variable  $x_{19}$  selected: The number of polynomials with this variable, with indexes from 1 to 19, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{19}$  from all other polynomials by reducing them with polynomial  $p_{16}$  from previous step.

```
= 2x_1 -
 p_1
       = 2x_2 -
 p_2
      = 2x_4 -
 p_3
      = 2x_5 - x_1 -
      = 2x_6 - x_2
 p_5
      = 2x_7 - x_3
 p_6
      = x_{11}x_2 + x_9 - x_2
      = x_{11}x_3 + x_{10} - x_3
      = -x_{12}x_5 + x_8
      = -x_{12}x_6 + x_9
p_{10}
      = -x_{12}x_7 + x_{10}
p_{11}
      = 2x_{13} - 2
p_{12}
      = 2x_{14} -
p_{13}
      = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{14}
      = x_{18}x_3 + x_{17} - x_3
p_{15}
p_{16}
      = -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5
p_{17}
      = -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7
p_{18}
       = -x_{19}x_5 + x_{19} + x_{15} -
      = 2x_{20} -
p_{20}
      = 2x_{21} - 2
p_{21}
       = 2x_{22} - x_1 -
p_{22}
      = 2x_{23} - x_2 -
p_{23}
      = 2x_{24} - x_3
p_{24}
p_{25}
            -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
      = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
            -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
            -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
            x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
            x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
            -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
      = -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
p_{30}
       = x_{30} -
      = x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
```

```
= x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
             x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
             x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
p_{35}
             x_{35} - x_1
             x_{36} - x_2
p_{36}
             x_{37}x_{30}
p_{37}
       = -x_{38}x_{30} + x_{37} - x_3
p_{38}
       = x_{39} - x_1
p_{39}
      = x_{40} - x_2
p_{40}
      = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 18.

Variable  $x_{18}$  selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{18}$  from all other polynomials by reducing them with polynomial  $p_{14}$  from previous step.

$$\begin{array}{rclcrcl} p_1 & = & 2x_1 - \\ p_2 & = & 2x_2 - \\ p_3 & = & 2x_4 - \\ p_4 & = & 2x_5 - x_1 - \\ p_5 & = & 2x_6 - x_2 \\ p_6 & = & 2x_7 - x_3 \\ p_7 & = & x_{11}x_2 + x_9 - x_2 \\ p_8 & = & x_{11}x_3 + x_{10} - x_3 \\ p_9 & = & -x_{12}x_5 + x_8 \\ p_{10} & = & -x_{12}x_6 + x_9 \\ p_{11} & = & -x_{12}x_7 + x_{10} \\ p_{12} & = & 2x_{13} - 2 \\ p_{13} & = & 2x_{14} - \\ p_{14} & = & 0 \\ p_{15} & = & -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5 \\ p_{16} & = & -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7 \\ p_{17} & = & -x_{17}x_{14} + x_{17}x_2 - x_{16}x_3 + x_{14}x_3 \\ p_{18} & = & -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2 \\ p_{19} & = & -x_{19}x_5 + x_{19} + x_{15} - \end{array}$$

```
2x_{20} -
p_{20}
              2x_{21}-2
p_{21}
              2x_{22} - x_1 -
p_{22}
              2x_{23} - x_2 -
p_{23}
              2x_{24} - x_3
p_{24}
p_{25}
              -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
       = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1+x_{27}x_8x_2+x_{26}x_{17}x_8-x_{26}x_{17}x_1-x_{26}x_{15}x_{10}+\\
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
              -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
              -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
              x_{30} -
p_{30}
              x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
              x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
              x_{35} - x_1
        =
p_{35}
              x_{36} - x_2
        =
p_{36}
p_{37}
              x_{37}x_{30}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
        = x_{39} - x_1
p_{39}
              x_{40} - x_2
p_{40}
              -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.25 Triangulation, step 25

Choosing variable: Trying the variable with index 17.

Variable  $x_{17}$  selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{17}$  from all other polynomials by reducing them with polynomial  $p_{16}$  from previous step.

$$p_1 = 2x_1 - p_2 = 2x_2 - p_2$$

 $= x_{35} - x_1$ 

 $= x_{36} - x_2$ 

 $p_{35}$ 

 $p_{36}$ 

```
p_{37} = x_{37}x_{30}
p_{38} = -x_{38}x_{30} + x_{37} - x_{3}
p_{39} = x_{39} - x_{1}
p_{40} = x_{40} - x_{2}
p_{41} = -x_{42}x_{30} + x_{41} - x_{3}
```

## 1.26 Triangulation, step 26

Choosing variable: Trying the variable with index 16.

Variable  $x_{16}$  selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{16}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

```
p_1 = 2x_1 -
      = 2x_2 -
 p_2
     = 2x_4 -
 p_3
      = 2x_5 - x_1 -
      = 2x_6 - x_2
 p_5
      = 2x_7 - x_3
p_6
      = x_{11}x_2 + x_9 - x_2
      = x_{11}x_3 + x_{10} - x_3
      = -x_{12}x_5 + x_8
      = -x_{12}x_6 + x_9
p_{10}
      = -x_{12}x_7 + x_{10}
p_{11}
      = 2x_{13} - 2
      = 2x_{14} -
p_{13}
           0
p_{14}
p_{15}
     = -x_{15}x_{14}x_{7}x_{5} + x_{15}x_{14}x_{7} + x_{15}x_{7}x_{5}x_{2} - x_{15}x_{7}x_{2}
            -x_{15}x_6x_5x_3 + x_{15}x_6x_3 + x_{15}x_5x_3 - x_{15}x_3 + x_{14}x_7x_5
            -x_{14}x_7 + x_{14}x_5^2x_3 - 2x_{14}x_5x_3 + x_{14}x_3 - x_7x_5x_2 + x_7x_2 +
           x_6x_5x_3 - x_6x_3 - x_5^2x_3 + x_5x_3
          -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5
      = -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7
      = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
      = -x_{19}x_5 + x_{19} + x_{15} -
p_{20}
      = 2x_{20} -
     = 2x_{21} - 2
p_{21}
```

```
= 2x_{22} - x_1 -
p_{22}
             2x_{23} - x_2 -
p_{23}
              2x_{24} - x_3
p_{24}
p_{25}
              -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
              x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
              -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
              -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
p_{30}
              x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
              x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
              x_{35} - x_1
p_{35}
              x_{36} - x_2
p_{36}
              x_{37}x_{30}
       =
p_{37}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
p_{39}
       = x_{39} - x_1
p_{40}
       = x_{40} - x_2
             -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.27 Triangulation, step 27

Choosing variable: Trying the variable with index 15.

Variable  $x_{15}$  selected: The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

#### 1.28 Triangulation, step 28

Choosing variable: Trying the variable with index 14.

Variable  $x_{14}$  selected: The number of polynomials with this variable, with indexes from 1 to 14, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{13}$ . No reduction needed.

The triangular system has not been changed.

#### 1.29 Triangulation, step 29

Choosing variable: Trying the variable with index 13.

Variable  $x_{13}$  selected: The number of polynomials with this variable, with indexes from 1 to 13, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{12}$ . No reduction needed.

The triangular system has not been changed.

## 1.30 Triangulation, step 30

Choosing variable: Trying the variable with index 12.

Variable  $x_{12}$  selected: The number of polynomials with this variable, with indexes from 1 to 12, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{12}$  from all other polynomials by reducing them with polynomial  $p_9$  from previous step.

```
p_1 = 2x_1 -
p_2 = 2x_2 -
 p_3 = 2x_4 -
     = 2x_5 - x_1 -
     = 2x_6 - x_2
 p_5
     = 2x_7 - x_3
     = x_{11}x_2 + x_9 - x_2
     = x_{11}x_3 + x_{10} - x_3
      = -x_9x_5 + x_8x_6
p_{10}
     = -x_{10}x_5 + x_8x_7
p_{11}
      = -x_{12}x_5 + x_8
     = 2x_{13} - 2
p_{13}
p_{14}
     = 2x_{14} -
p_{15}
     = -x_{15}x_{14}x_7x_5 + x_{15}x_{14}x_7 + x_{15}x_7x_5x_2 - x_{15}x_7x_2
           -x_{15}x_6x_5x_3 + x_{15}x_6x_3 + x_{15}x_5x_3 - x_{15}x_3 + x_{14}x_7x_5
           -x_{14}x_7 + x_{14}x_5^2x_3 - 2x_{14}x_5x_3 + x_{14}x_3 - x_7x_5x_2 + x_7x_2 +
```

```
x_6x_5x_3 - x_6x_3 - x_5^2x_3 + x_5x_3
              -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5
p_{16}
              -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7
p_{17}
              -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
              -x_{19}x_5 + x_{19} + x_{15} -
p_{19}
              2x_{20} -
p_{20}
        = 2x_{21} - 2
p_{21}
        = 2x_{22} - x_1 -
p_{22}
             2x_{23} - x_2 -
p_{23}
             2x_{24} - x_3
p_{24}
p_{25}
              -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
        = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
             -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
             -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
              x_{30} -
        =
p_{30}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
             x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
             x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
              x_{35} - x_1
p_{35}
              x_{36} - x_2
p_{36}
              x_{37}x_{30}
p_{37}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
              x_{39} - x_1
p_{39}
              x_{40} - x_2
p_{40}
              -x_{42}x_{30} + x_{41} - x_3
```

## 1.31 Triangulation, step 31

Choosing variable: Trying the variable with index 11.

Variable  $x_{11}$  selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{11}$  from all other polynomials by reducing them with polynomial  $p_7$  from previous step.

```
= 2x_1 -
 p_1
       = 2x_2 -
 p_2
      = 2x_4 -
 p_3
      = 2x_5 - x_1 -
      = 2x_6 - x_2
      = 2x_7 - x_3
       = -x_9x_5 + x_8x_6
 p_8
      = -x_{10}x_5 + x_8x_7
       = x_{10}x_2 - x_9x_3
p_{10}
      = x_{11}x_2 + x_9 - x_2
p_{11}
      = -x_{12}x_5 + x_8
p_{12}
       = 2x_{13} - 2
p_{13}
      = 2x_{14} -
p_{14}
     = -x_{15}x_{14}x_{7}x_{5} + x_{15}x_{14}x_{7} + x_{15}x_{7}x_{5}x_{2} - x_{15}x_{7}x_{2}
p_{15}
             -x_{15}x_6x_5x_3 + x_{15}x_6x_3 + x_{15}x_5x_3 - x_{15}x_3 + x_{14}x_7x_5
             -x_{14}x_7 + x_{14}x_5^2x_3 - 2x_{14}x_5x_3 + x_{14}x_3 - x_7x_5x_2 + x_7x_2 +
             x_6x_5x_3 - x_6x_3 - x_5^2x_3 + x_5x_3
      = -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5
p_{16}
      = -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7
p_{17}
       = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
       = -x_{19}x_5 + x_{19} + x_{15} -
p_{19}
       = 2x_{20} -
p_{20}
       = 2x_{21} - 2
p_{21}
       = 2x_{22} - x_1 -
       = 2x_{23} - x_2 -
p_{23}
            2x_{24} - x_3
p_{24}
p_{25}
             -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
      = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
             -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
             -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
             x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 + \\
             x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
             -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
      = -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
```

```
-x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
             x_{30} - 
p_{30}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
             x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
              x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
              x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
             x_{35} - x_1
p_{35}
             x_{36} - x_2
p_{36}
              x_{37}x_{30}
p_{37}
       =
        = -x_{38}x_{30} + x_{37} - x_3
p_{38}
       = x_{39} - x_1
p_{39}
       = x_{40} - x_2
p_{40}
       = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

## 1.32 Triangulation, step 32

Choosing variable: Trying the variable with index 10.

**Variable**  $x_{10}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{10}$  from all other polynomials by reducing them with polynomial  $p_9$  from previous step.

```
= 2x_1 -
p_1
      = 2x_2 -
 p_2
     = 2x_4 -
 p_3
     = 2x_5 - x_1 -
      = 2x_6 - x_2
      = 2x_7 - x_3
 p_6
 p_7
      = -x_9x_5 + x_8x_6
      = x_9x_5x_3 - x_8x_7x_2
p_9
      = -x_{10}x_5 + x_8x_7
p_{10}
      = x_{11}x_2 + x_9 - x_2
p_{11}
     = -x_{12}x_5 + x_8
p_{12}
     = 2x_{13} - 2
p_{13}
      = 2x_{14} -
p_{14}
p_{15}
      = -x_{15}x_{14}x_7x_5 + x_{15}x_{14}x_7 + x_{15}x_7x_5x_2 - x_{15}x_7x_2
           -x_{15}x_6x_5x_3 + x_{15}x_6x_3 + x_{15}x_5x_3 - x_{15}x_3 + x_{14}x_7x_5
```

```
-x_{14}x_7 + x_{14}x_5^2x_3 - 2x_{14}x_5x_3 + x_{14}x_3 - x_7x_5x_2 + x_7x_2 +
              x_6x_5x_3 - x_6x_3 - x_5^2x_3 + x_5x_3
       = -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5
p_{16}
            -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7
             -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
p_{18}
              -x_{19}x_5 + x_{19} + x_{15} -
p_{19}
        = 2x_{20} -
p_{20}
        = 2x_{21} - 2
p_{21}
             2x_{22} - x_1 -
p_{22}
        = 2x_{23} - x_2 -
p_{23}
             2x_{24} - x_3
p_{24}
p_{25}
             -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
       = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
              -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
              -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
              x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 +
              x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
              -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
             -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
              -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{29}
             x_{30} -
p_{30}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
             x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
             x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
             x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
             x_{35} - x_1
p_{35}
             x_{36} - x_2
        =
p_{36}
p_{37}
             x_{37}x_{30}
              -x_{38}x_{30} + x_{37} - x_3
p_{38}
            x_{39} - x_1
p_{39}
       =
p_{40}
       =
             x_{40} - x_2
              -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

#### 1.33 Triangulation, step 33

Choosing variable: Trying the variable with index 9.

Variable  $x_9$  selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_9$  from all other polynomials by reducing them with polynomial  $p_8$  from previous step.

```
= 2x_1 -
 p_1
       = 2x_2 -
 p_2
      = 2x_4 -
 p_3
      = 2x_5 - x_1 -
      = 2x_6 - x_2
      = 2x_7 - x_3
 p_6
       = x_8 x_7 x_5 x_2 - x_8 x_6 x_5 x_3
 p_8
      = -x_9x_5 + x_8x_6
       = -x_{10}x_5 + x_8x_7
p_{10}
      = x_{11}x_2 + x_9 - x_2
p_{11}
      = -x_{12}x_5 + x_8
p_{12}
       = 2x_{13} - 2
p_{13}
      = 2x_{14} -
p_{14}
      = -x_{15}x_{14}x_{7}x_{5} + x_{15}x_{14}x_{7} + x_{15}x_{7}x_{5}x_{2} - x_{15}x_{7}x_{2}
p_{15}
             -x_{15}x_6x_5x_3 + x_{15}x_6x_3 + x_{15}x_5x_3 - x_{15}x_3 + x_{14}x_7x_5
             -x_{14}x_7 + x_{14}x_5^2x_3 - 2x_{14}x_5x_3 + x_{14}x_3 - x_7x_5x_2 + x_7x_2 +
             x_6x_5x_3 - x_6x_3 - x_5^2x_3 + x_5x_3
       = -x_{16}x_5 + x_{16} + x_{15}x_6 - x_{15} - x_6 + x_5
p_{16}
       = -x_{17}x_5 + x_{17} + x_{15}x_7 - x_7
p_{17}
       = -x_{18}x_{14} + x_{18}x_2 + x_{16} - x_2
       = -x_{19}x_5 + x_{19} + x_{15} -
p_{19}
       = 2x_{20} -
p_{20}
       = 2x_{21} - 2
p_{21}
       = 2x_{22} - x_1 -
       = 2x_{23} - x_2 -
p_{23}
            2x_{24} - x_3
p_{24}
p_{25}
             -x_{26}x_{20} + x_{26}x_1 + x_{25}x_{21} - x_{25}x_2 - x_{21}x_1 + x_{20}x_2
p_{26}
      = x_{42}x_{26}x_{15} - x_{42}x_{26}x_8 - x_{42}x_{25}x_{16} + x_{42}x_{25}x_9 + x_{42}x_{16}x_8
p_{27}
             -x_{42}x_{15}x_9 - x_{27}x_{16}x_8 + x_{27}x_{16}x_1 + x_{27}x_{15}x_9 - x_{27}x_{15}x_2
             -x_{27}x_9x_1 + x_{27}x_8x_2 + x_{26}x_{17}x_8 - x_{26}x_{17}x_1 - x_{26}x_{15}x_{10} +
             x_{26}x_{15}x_3 + x_{26}x_{10}x_1 - x_{26}x_8x_3 - x_{25}x_{17}x_9 + x_{25}x_{17}x_2 + \\
             x_{25}x_{16}x_{10} - x_{25}x_{16}x_3 - x_{25}x_{10}x_2 + x_{25}x_9x_3 + x_{17}x_9x_1
             -x_{17}x_8x_2 - x_{16}x_{10}x_1 + x_{16}x_8x_3 + x_{15}x_{10}x_2 - x_{15}x_9x_3
      = -x_{28}x_{23} + x_{28} + x_{26} -
p_{28}
```

```
p_{29}
            -x_{29}x_{20} + x_{29}x_1 + x_{25} - x_1
p_{30}
             x_{31} - x_{27}x_{16} + x_{27}x_9 + x_{26}x_{17} - x_{26}x_{10} - x_{17}x_9 + x_{16}x_{10}
p_{31}
             x_{32} + x_{27}x_{15} - x_{27}x_8 - x_{25}x_{17} + x_{25}x_{10} + x_{17}x_8 - x_{15}x_{10}
p_{32}
             x_{33} - x_{26}x_{15} + x_{26}x_8 + x_{25}x_{16} - x_{25}x_9 - x_{16}x_8 + x_{15}x_9
p_{33}
             x_{34} + x_{33}x_{10} + x_{32}x_9 + x_{31}x_8
p_{34}
             x_{35} - x_1
p_{35}
       = x_{36} - x_2
p_{36}
       =
             x_{37}x_{30}
p_{37}
        = -x_{38}x_{30} + x_{37} - x_3
p_{38}
       = x_{39} - x_1
p_{39}
       = x_{40} - x_2
p_{40}
       = -x_{42}x_{30} + x_{41} - x_3
p_{41}
```

#### 1.34 Triangulation, step 34

Choosing variable: Trying the variable with index 8.

Variable  $x_8$  selected: The number of polynomials with this variable, with indexes from 1 to 8, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_8$ . No reduction needed.

The triangular system has not been changed.

#### 1.35 Triangulation, step 35

Choosing variable: Trying the variable with index 7.

Variable  $x_7$  selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

#### 1.36 Triangulation, step 36

Choosing variable: Trying the variable with index 6.

Variable  $x_6$  selected: The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_5$ . No reduction needed.

The triangular system has not been changed.

## 1.37 Triangulation, step 37

Choosing variable: Trying the variable with index 5.

Variable  $x_5$  selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_4$ . No reduction needed.

The triangular system has not been changed.

## 1.38 Triangulation, step 38

Choosing variable: Trying the variable with index 4.

Variable  $x_4$  selected: The number of polynomials with this variable, with indexes from 1 to 4, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_3$ . No reduction needed.

The triangular system has not been changed.

## 1.39 Triangulation, step 39

Choosing variable: Trying the variable with index 3.

Error: Variable with index 3 not found in polynomial system.

## 2 Prover results

Status: Proving failed - general error occurred.