

# OpenGeoProver Output for conjecture “geothm\_zadatak”

Wu’s method used

September 30, 2016

## 1 Invoking the theorem prover

The used proving method is Wu’s method.

The input system is:

$$\begin{aligned} p_1 &= 2x_1 - \\ p_2 &= 4x_2^2 - 3 \\ p_3 &= 3x_3 - x_2 \\ p_4 &= 3x_4^2 - 2 \\ p_5 &= x_5 - x_2 \\ p_6 &= x_6 - x_4x_2 \\ p_7 &= x_7 + x_4x_1 \\ p_8 &= x_8 - x_3x_1 + x_2x_1 \\ p_9 &= x_{10} - x_3 \\ p_{10} &= -x_{12}x_5 + x_{11} - x_4 \\ p_{11} &= -x_{13}x_6 + x_9 - \\ p_{12} &= -x_{13}x_7 + x_{10} \\ p_{13} &= -x_{13}x_8 + x_{11} \\ p_{14} &= 2x_{14} - \\ p_{15} &= 2x_{15} - x_1 - \\ p_{16} &= 2x_{16} - x_2 \\ p_{17} &= 2x_{17} - x_1 \\ p_{18} &= 2x_{18} - x_2 \\ p_{19} &= x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_9 + x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{20} &= x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_9 + x_{16}x_{10} + x_{15}x_9 - x_{11}^2 \\ &\quad - x_{10}^2 - x_9^2 \\ p_{21} &= x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{22} &= x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \end{aligned}$$

### 1.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 24.

**Variable  $x_{24}$  selected:** The number of polynomials with this variable, with indexes from 1 to 24, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{24}$ . No reduction needed.

The triangular system has not been changed.

### 1.2 Triangulation, step 2

**Choosing variable:** Trying the variable with index 23.

**Variable  $x_{23}$  selected:** The number of polynomials with this variable, with indexes from 1 to 23, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{23}$ . No reduction needed.

The triangular system has not been changed.

### 1.3 Triangulation, step 3

**Choosing variable:** Trying the variable with index 22.

**Variable  $x_{22}$  selected:** The number of polynomials with this variable, with indexes from 1 to 22, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{20}$ . No reduction needed.

The triangular system has not been changed.

### 1.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 21.

**Variable  $x_{21}$  selected:** The number of polynomials with this variable, with indexes from 1 to 21, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{21}$ . No reduction needed.

The triangular system has not been changed.

### 1.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 20.

**Variable  $x_{20}$  selected:** The number of polynomials with this variable, with indexes from 1 to 20, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{20}$ . No reduction needed.

The triangular system has not been changed.

### 1.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 19.

**Variable  $x_{19}$  selected:** The number of polynomials with this variable, with indexes from 1 to 19, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{19}$ . No reduction needed.

The triangular system has not been changed.

### 1.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 18.

**Variable  $x_{18}$  selected:** The number of polynomials with this variable, with indexes from 1 to 18, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{18}$ . No reduction needed.

The triangular system has not been changed.

### 1.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 17.

**Variable  $x_{17}$  selected:** The number of polynomials with this variable, with indexes from 1 to 17, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{17}$ . No reduction needed.

The triangular system has not been changed.

## 1.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 16.

**Variable  $x_{16}$  selected:** The number of polynomials with this variable, with indexes from 1 to 16, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{16}$ . No reduction needed.

The triangular system has not been changed.

## 1.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 15.

**Variable  $x_{15}$  selected:** The number of polynomials with this variable, with indexes from 1 to 15, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

## 1.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 14.

**Variable  $x_{14}$  selected:** The number of polynomials with this variable, with indexes from 1 to 14, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{14}$ . No reduction needed.

The triangular system has not been changed.

## 1.12 Triangulation, step 12

**Choosing variable:** Trying the variable with index 13.

**Variable  $x_{13}$  selected:** The number of polynomials with this variable, with indexes from 1 to 13, is 3.

**Minimal degrees:** 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{13}$  from all other polynomials by reducing them with polynomial  $p_{11}$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 \\
p_8 &= x_8 - x_3x_1 + x_2x_1 \\
p_9 &= x_{10} - x_3 \\
p_{10} &= -x_{12}x_5 + x_{11} - x_4 \\
p_{11} &= -x_{10}x_6 + x_9x_7 - x_7 \\
p_{12} &= -x_{11}x_6 + x_9x_8 - x_8 \\
p_{13} &= -x_{13}x_6 + x_9 - \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_1 - \\
p_{16} &= 2x_{16} - x_2 \\
p_{17} &= 2x_{17} - x_1 \\
p_{18} &= 2x_{18} - x_2 \\
p_{19} &= x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_9 + x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{20} &= x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{21} &= x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_9 + x_{16}x_{10} + x_{15}x_9 - x_{11}^2 \\
&\quad - x_{10}^2 - x_9^2 \\
p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
\end{aligned}$$

### 1.13 Triangulation, step 13

**Choosing variable:** Trying the variable with index 12.

**Variable  $x_{12}$  selected:** The number of polynomials with this variable, with indexes from 1 to 12, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{10}$ . No reduction needed.

The triangular system has not been changed.

### 1.14 Triangulation, step 14

**Choosing variable:** Trying the variable with index 11.

**Variable  $x_{11}$  selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_{11}$ . No reduction needed.

The triangular system has not been changed.

### 1.15 Triangulation, step 15

**Choosing variable:** Trying the variable with index 10.

**Variable  $x_{10}$  selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{10}$  from all other polynomials by reducing them with polynomial  $p_9$  from previous step.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= 4x_2^2 - 3 \\
p_3 &= 3x_3 - x_2 \\
p_4 &= 3x_4^2 - 2 \\
p_5 &= x_5 - x_2 \\
p_6 &= x_6 - x_4x_2 \\
p_7 &= x_7 + x_4x_1 \\
p_8 &= x_8 - x_3x_1 + x_2x_1 \\
p_9 &= x_9x_7 - x_7 - x_6x_3 \\
p_{10} &= x_{10} - x_3 \\
p_{11} &= -x_{11}x_6 + x_9x_8 - x_8 \\
p_{12} &= -x_{12}x_5 + x_{11} - x_4 \\
p_{13} &= -x_{13}x_6 + x_9 - \\
p_{14} &= 2x_{14} - \\
p_{15} &= 2x_{15} - x_1 - \\
p_{16} &= 2x_{16} - x_2 \\
p_{17} &= 2x_{17} - x_1 \\
p_{18} &= 2x_{18} - x_2 \\
p_{19} &= x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_9 + x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{20} &= x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{21} &= x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_9 + x_{16}x_{10} + x_{15}x_9 - x_{11}^2 \\
&\quad - x_{10}^2 - x_9^2 \\
p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\
p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2
\end{aligned}$$

### 1.16 Triangulation, step 16

**Choosing variable:** Trying the variable with index 9.

**Variable  $x_9$  selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_9$ . No reduction needed.

The triangular system has not been changed.

### 1.17 Triangulation, step 17

**Choosing variable:** Trying the variable with index 8.

**Variable  $x_8$  selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_8$ . No reduction needed.

The triangular system has not been changed.

### 1.18 Triangulation, step 18

**Choosing variable:** Trying the variable with index 7.

**Variable  $x_7$  selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_7$ . No reduction needed.

The triangular system has not been changed.

### 1.19 Triangulation, step 19

**Choosing variable:** Trying the variable with index 6.

**Variable  $x_6$  selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

### 1.20 Triangulation, step 20

**Choosing variable:** Trying the variable with index 5.

**Variable  $x_5$  selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_5$ . No reduction needed.

The triangular system has not been changed.

### 1.21 Triangulation, step 21

**Choosing variable:** Trying the variable with index 4.

**Variable  $x_4$  selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_4$ . No reduction needed.

The triangular system has not been changed.

### 1.22 Triangulation, step 22

**Choosing variable:** Trying the variable with index 3.

**Variable  $x_3$  selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_3$ . No reduction needed.

The triangular system has not been changed.

### 1.23 Triangulation, step 23

**Choosing variable:** Trying the variable with index 2.

**Variable  $x_2$  selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_2$ . No reduction needed.

The triangular system has not been changed.



## 1.24 Triangulation, step 24

**Choosing variable:** Trying the variable with index 1.

**Variable  $x_1$  selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned} p_1 &= 2x_1 - \\ p_2 &= 4x_2^2 - 3 \\ p_3 &= 3x_3 - x_2 \\ p_4 &= 3x_4^2 - 2 \\ p_5 &= x_5 - x_2 \\ p_6 &= x_6 - x_4x_2 \\ p_7 &= x_7 + x_4x_1 \\ p_8 &= x_8 - x_3x_1 + x_2x_1 \\ p_9 &= x_9x_7 - x_7 - x_6x_3 \\ p_{10} &= x_{10} - x_3 \\ p_{11} &= -x_{11}x_6 + x_9x_8 - x_8 \\ p_{12} &= -x_{12}x_5 + x_{11} - x_4 \\ p_{13} &= -x_{13}x_6 + x_9 - \\ p_{14} &= 2x_{14} - \\ p_{15} &= 2x_{15} - x_1 - \\ p_{16} &= 2x_{16} - x_2 \\ p_{17} &= 2x_{17} - x_1 \\ p_{18} &= 2x_{18} - x_2 \\ p_{19} &= x_{19} + x_{16}x_{10} - x_{15}x_{14} + x_{15}x_9 + x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{20} &= x_{20} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{21} &= x_{21} - x_{18}^2 + 2x_{18}x_{10} - x_{17}^2 + 2x_{17}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{22} &= x_{22} - x_{18}x_{16} + x_{18}x_{10} - x_{17}x_{15} + x_{17}x_9 + x_{16}x_{10} + x_{15}x_9 - x_{11}^2 \\ &\quad - x_{10}^2 - x_9^2 \\ p_{23} &= x_{23} - x_{14}^2 + 2x_{14}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \\ p_{24} &= x_{24} - x_{16}^2 + 2x_{16}x_{10} - x_{15}^2 + 2x_{15}x_9 - x_{11}^2 - x_{10}^2 - x_9^2 \end{aligned}$$

## 2 Final Remainder

### 2.1 Final remainder for conjecture geothm\_zadatak

Calculating final remainder of the conclusion:

$$g = x_9 - x_1$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{24}$  over variable  $x_{24}$ :

$$g = x_9 - x_1$$

2. Pseudo remainder with  $p_{23}$  over variable  $x_{23}$ :

$$g = x_9 - x_1$$

3. Pseudo remainder with  $p_{22}$  over variable  $x_{22}$ :

$$g = x_9 - x_1$$

4. Pseudo remainder with  $p_{21}$  over variable  $x_{21}$ :

$$g = x_9 - x_1$$

5. Pseudo remainder with  $p_{20}$  over variable  $x_{20}$ :

$$g = x_9 - x_1$$

6. Pseudo remainder with  $p_{19}$  over variable  $x_{19}$ :

$$g = x_9 - x_1$$

7. Pseudo remainder with  $p_{18}$  over variable  $x_{18}$ :

$$g = x_9 - x_1$$

8. Pseudo remainder with  $p_{17}$  over variable  $x_{17}$ :

$$g = x_9 - x_1$$

9. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :

$$g = x_9 - x_1$$

10. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :

$$g = x_9 - x_1$$

11. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :

$$g = x_9 - x_1$$

12. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :

$$g = x_9 - x_1$$

13. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :

$$g = x_9 - x_1$$

14. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :

$$g = x_9 - x_1$$

15. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :

$$g = x_9 - x_1$$

16. Pseudo remainder with  $p_9$  over variable  $x_9$ :

$$g = -x_7x_1 + x_7 + x_6x_3$$

17. Pseudo remainder with  $p_8$  over variable  $x_8$ :

$$g = -x_7x_1 + x_7 + x_6x_3$$

18. Pseudo remainder with  $p_7$  over variable  $x_7$ :

$$g = x_6x_3 + x_4x_1^2 - x_4x_1$$

19. Pseudo remainder with  $p_6$  over variable  $x_6$ :

$$g = x_4x_3x_2 + x_4x_1^2 - x_4x_1$$

20. Pseudo remainder with  $p_5$  over variable  $x_5$ :

$$g = x_4x_3x_2 + x_4x_1^2 - x_4x_1$$

21. Pseudo remainder with  $p_4$  over variable  $x_4$ :

$$g = x_4x_3x_2 + x_4x_1^2 - x_4x_1$$

22. Pseudo remainder with  $p_3$  over variable  $x_3$ :

$$g = x_4x_2^2 + 3x_4x_1^2 - 3x_4x_1$$

23. Pseudo remainder with  $p_2$  over variable  $x_2$ :

$$g = 12x_4x_1^2 - 12x_4x_1 + 3x_4$$

24. Pseudo remainder with  $p_1$  over variable  $x_1$ :

$$g = 0$$

### 3 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.081 seconds.

### 4 NDG Conditions

**NDG Conditions in readable form**

- Failed to translate NDG Conditions to readable form