Balkan MO 2017

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1 Find all ordered pairs of positive integers(x, y) such that:

$$x^3 + y^3 = x^2 + 42xy + y^2.$$

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2 Consider an acute-angled triangle ABC with $AB < AC$ and let ω be its circumscribed circle. Let t_B and t_C be the tangents to the circle ω at points B and C, respectively, and let L be their intersection. The straight line passing through the point B and parallel to AC intersects t_C in point D. The straight line passing through the point C and parallel to AB intersects t_B in point E. The circumcircle of the triangle BDC intersects AC in T, where T is located between A and C. The circumcircle of the triangle BEC intersects the line AB (or its extension) in S, where B is located between S and A. Prove that ST, AL, and BC are concurrent.

Vangelis Psychas and Silouanos Brazitikos

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3 Let $\mathbb{N}$ denote the set of positive integers. Find all functions $f : \mathbb{N} \longrightarrow \mathbb{N}$ such that

$$n + f(m) \mid f(n) + nf(m)$$

for all $m, n \in \mathbb{N}$

Proposed by Dorlir Ahmeti, Albania

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4 On a circular table sit $n > 2$ students. First, each student has just one candy. At each step, each student chooses one of the following actions:

(A) Gives a candy to the student sitting on his left or to the student sitting on his right.

(B) Separates all its candies in two, possibly empty, sets and gives one set to the student sitting on his left and the other to the student sitting on his right.

At each step, students perform the actions they have chosen at the same time.

A distribution of candy is called legitimate if it can occur after a finite number of steps.
Find the number of legitimate distributions.

(Two distributions are different if there is a student who has a different number of candy in each of these distributions.)

Forgive my poor English
