# OpenGeoProver Output for conjecture "geothm_zadatak"

## Wu's method used

### October 2, 2016

## 1 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$
\begin{aligned}
p_1 &= -x_3 + 1 \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_2 - x_1 + 1 \\
p_4 &= -x_5 + x_4 \\
p_5 &= x_5 + x_4 - \\
p_6 &= -x_8 x_5 - x_7 + x_5 \\
p_7 &= x_8 x_4 + x_6 - x_4 \\
p_8 &= -x_7 x_1 + x_6 x_2 \\
p_9 &= -x_{11} x_5 - x_{10} + x_5 \\
p_{10} &= x_{11} x_4 - x_{11} + x_9 - x_4 \\
p_{11} &= -x_{10} x_1 + x_{10} + x_9 x_2 - x_2
\end{aligned}
$$

### 1.1 Triangulation, step 1

**Choosing variable:** Trying the variable with index 11.

**Variable $x_{11}$ selected:** The number of polynomials with this variable, with indexes from 1 to 11, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_{11}$ from all other polynomials by reducing them with polynomial $p_9$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_3 + 1 \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_2 - x_1 + 1
\end{aligned}
$$

$$
\begin{aligned}
p_4 &= -x_5 + x_4 \\
p_5 &= x_5 + x_4 - \\
p_6 &= -x_8 x_5 - x_7 + x_5 \\
p_7 &= x_8 x_4 + x_6 - x_4 \\
p_8 &= -x_7 x_1 + x_6 x_2 \\
p_9 &= -x_{10} x_1 + x_{10} + x_9 x_2 - x_2 \\
p_{10} &= x_{10} x_4 - x_{10} - x_9 x_5 + x_5 \\
p_{11} &= -x_{11} x_5 - x_{10} + x_5
\end{aligned}
$$

## 1.2   Triangulation, step 2

**Choosing variable:** Trying the variable with index 10.

**Variable $x_{10}$ selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_{10}$ from all other polynomials by reducing them with polynomial $p_9$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_3 + 1 \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_2 - x_1 + 1 \\
p_4 &= -x_5 + x_4 \\
p_5 &= x_5 + x_4 - \\
p_6 &= -x_8 x_5 - x_7 + x_5 \\
p_7 &= x_8 x_4 + x_6 - x_4 \\
p_8 &= -x_7 x_1 + x_6 x_2 \\
p_9 &= x_9 x_5 x_1 - x_9 x_5 - x_9 x_4 x_2 + x_9 x_2 - x_5 x_1 + x_5 + x_4 x_2 - x_2 \\
p_{10} &= -x_{10} x_1 + x_{10} + x_9 x_2 - x_2 \\
p_{11} &= -x_{11} x_5 - x_{10} + x_5
\end{aligned}
$$

## 1.3   Triangulation, step 3

**Choosing variable:** Trying the variable with index 9.

**Variable $x_9$ selected:** The number of polynomials with this variable, with indexes from 1 to 9, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_9$. No reduction needed.

The triangular system has not been changed.

## 1.4 Triangulation, step 4

**Choosing variable:** Trying the variable with index 8.

**Variable $x_8$ selected:** The number of polynomials with this variable, with indexes from 1 to 8, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_8$ from all other polynomials by reducing them with polynomial $p_6$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_3 + 1 \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_2 - x_1 + 1 \\
p_4 &= -x_5 + x_4 \\
p_5 &= x_5 + x_4 - \\
p_6 &= -x_7 x_1 + x_6 x_2 \\
p_7 &= x_7 x_4 - x_6 x_5 \\
p_8 &= -x_8 x_5 - x_7 + x_5 \\
p_9 &= x_9 x_5 x_1 - x_9 x_5 - x_9 x_4 x_2 + x_9 x_2 - x_5 x_1 + x_5 + x_4 x_2 - x_2 \\
p_{10} &= -x_{10} x_1 + x_{10} + x_9 x_2 - x_2 \\
p_{11} &= -x_{11} x_5 - x_{10} + x_5
\end{aligned}
$$

## 1.5 Triangulation, step 5

**Choosing variable:** Trying the variable with index 7.

**Variable $x_7$ selected:** The number of polynomials with this variable, with indexes from 1 to 7, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_7$ from all other polynomials by reducing them with polynomial $p_6$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= -x_3 + 1 \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_2 - x_1 + 1 \\
p_4 &= -x_5 + x_4 \\
p_5 &= x_5 + x_4 - \\
p_6 &= x_6 x_5 x_1 - x_6 x_4 x_2 \\
p_7 &= -x_7 x_1 + x_6 x_2
\end{aligned}
$$

3

$$p_8 = -x_8x_5 - x_7 + x_5$$
$$p_9 = x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2$$
$$p_{10} = -x_{10}x_1 + x_{10} + x_9x_2 - x_2$$
$$p_{11} = -x_{11}x_5 - x_{10} + x_5$$

## 1.6 Triangulation, step 6

**Choosing variable:** Trying the variable with index 6.

**Variable $x_6$ selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_6$. No reduction needed.

The triangular system has not been changed.

## 1.7 Triangulation, step 7

**Choosing variable:** Trying the variable with index 5.

**Variable $x_5$ selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_5$ from all other polynomials by reducing them with polynomial $p_4$ from previous step.

Finished a triangulation step, the current system is:

$$p_1 = -x_3 + 1$$
$$p_2 = -x_2 + x_1$$
$$p_3 = -x_2 - x_1 + 1$$
$$p_4 = -2x_4 + 1$$
$$p_5 = -x_5 + x_4$$
$$p_6 = x_6x_5x_1 - x_6x_4x_2$$
$$p_7 = -x_7x_1 + x_6x_2$$
$$p_8 = -x_8x_5 - x_7 + x_5$$
$$p_9 = x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2$$
$$p_{10} = -x_{10}x_1 + x_{10} + x_9x_2 - x_2$$
$$p_{11} = -x_{11}x_5 - x_{10} + x_5$$

## 1.8 Triangulation, step 8

**Choosing variable:** Trying the variable with index 4.

**Variable $x_4$ selected:** The number of polynomials with this variable, with indexes from 1 to 4, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_4$. No reduction needed.

The triangular system has not been changed.

## 1.9 Triangulation, step 9

**Choosing variable:** Trying the variable with index 3.

**Variable $x_3$ selected:** The number of polynomials with this variable, with indexes from 1 to 3, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

## 1.10 Triangulation, step 10

**Choosing variable:** Trying the variable with index 2.

**Variable $x_2$ selected:** The number of polynomials with this variable, with indexes from 1 to 2, is 2.

**Minimal degrees:** 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable $x_2$ from all other polynomials by reducing them with polynomial $p_1$ from previous step.

Finished a triangulation step, the current system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_3 + 1 \\
p_4 &= -2x_4 + 1 \\
p_5 &= -x_5 + x_4 \\
p_6 &= x_6 x_5 x_1 - x_6 x_4 x_2 \\
p_7 &= -x_7 x_1 + x_6 x_2 \\
p_8 &= -x_8 x_5 - x_7 + x_5 \\
p_9 &= x_9 x_5 x_1 - x_9 x_5 - x_9 x_4 x_2 + x_9 x_2 - x_5 x_1 + x_5 + x_4 x_2 - x_2 \\
p_{10} &= -x_{10} x_1 + x_{10} + x_9 x_2 - x_2 \\
p_{11} &= -x_{11} x_5 - x_{10} + x_5
\end{aligned}
$$

## 1.11 Triangulation, step 11

**Choosing variable:** Trying the variable with index 1.

**Variable $x_1$ selected:** The number of polynomials with this variable, with indexes from 1 to 1, is 1.

**Single polynomial with chosen variable:** Chosen polynomial is $p_1$. No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$
\begin{aligned}
p_1 &= 2x_1 - \\
p_2 &= -x_2 + x_1 \\
p_3 &= -x_3 + 1 \\
p_4 &= -2x_4 + 1 \\
p_5 &= -x_5 + x_4 \\
p_6 &= x_6x_5x_1 - x_6x_4x_2 \\
p_7 &= -x_7x_1 + x_6x_2 \\
p_8 &= -x_8x_5 - x_7 + x_5 \\
p_9 &= x_9x_5x_1 - x_9x_5 - x_9x_4x_2 + x_9x_2 - x_5x_1 + x_5 + x_4x_2 - x_2 \\
p_{10} &= -x_{10}x_1 + x_{10} + x_9x_2 - x_2 \\
p_{11} &= -x_{11}x_5 - x_{10} + x_5
\end{aligned}
$$

# 2 Final Remainder

## 2.1 Final remainder for conjecture geothm_zadatak

Calculating final remainder of the conclusion:

$$
\begin{aligned}
g &= 4x_{11}^2 - 8x_{11}x_8 + 4x_{10}^2 - 8x_{10}x_7 + 4x_9^2 - 8x_9x_6 + 4x_8^2 + 4x_7^2 + \\
&\quad 4x_6^2 -
\end{aligned}
$$

with respect to the triangular system.

1. Pseudo remainder with $p_{11}$ over variable $x_{11}$:

$$
\begin{aligned}
g &= 4x_{10}^2x_5^2 + 4x_{10}^2 + 8x_{10}x_8x_5 - 8x_{10}x_7x_5^2 - 8x_{10}x_5 + \\
&\quad 4x_9^2x_5^2 - 8x_9x_6x_5^2 + 4x_8^2x_5^2 - 8x_8x_5^2 + \\
&\quad 4x_7^2x_5^2 + 4x_6^2x_5^2 + 3x_5^2
\end{aligned}
$$

2. Pseudo remainder with $p_{10}$ over variable $x_{10}$:

$$
\begin{aligned}
g &= 4x_9^2x_5^2x_2^2 + 4x_9^2x_5^2x_1^2 - 8x_9^2x_5^2x_1 + \\
&\quad 4x_9^2x_5^2 + 4x_9^2x_2^2 + 8x_9x_8x_5x_2x_1 - 8x_9x_8x_5x_2 \\
&\quad -8x_9x_7x_5^2x_2x_1 + 8x_9x_7x_5^2x_2 - 8x_9x_6x_5^2x_1^2 + \\
&\quad 16x_9x_6x_5^2x_1 - 8x_9x_6x_5^2 - 8x_9x_5^2x_2^2
\end{aligned}
$$

$$-8x_9x_5x_2x_1 + 8x_9x_5x_2 - 8x_9x_2^2 + 4x_8^2x_5^2x_1^2$$
$$-8x_8^2x_5^2x_1 + 4x_8^2x_5^2 - 8x_8x_5^2x_1^2 + 16x_8x_5^2x_1$$
$$-8x_8x_5^2 - 8x_8x_5x_2x_1 + 8x_8x_5x_2 + 4x_7^2x_5^2x_1^2$$
$$-8x_7^2x_5^2x_1 + 4x_7^2x_5^2 + 8x_7x_5^2x_2x_1 - 8x_7x_5^2x_2 +$$
$$4x_6^2x_5^2x_1^2 - 8x_6^2x_5^2x_1 + 4x_6^2x_5^2 +$$
$$4x_5^2x_2^2 + 3x_5^2x_1^2 - 6x_5^2x_1 + 3x_5^2 + 8x_5x_2x_1$$
$$-8x_5x_2 + 4x_2^2$$

3. Pseudo remainder with $p_9$ over variable $x_9$:

   *Polynomial too big for output (text size is 4395 characters, number of terms is 132)*

4. Pseudo remainder with $p_8$ over variable $x_8$:

   *Polynomial too big for output (text size is 3678 characters, number of terms is 110)*

5. Pseudo remainder with $p_7$ over variable $x_7$:

   *Polynomial too big for output (text size is 3956 characters, number of terms is 107)*

6. Pseudo remainder with $p_6$ over variable $x_6$:

$$
\begin{aligned}
g \;=\; & 3x_5^8x_1^8 - 12x_5^8x_1^7 + 18x_5^8x_1^6 - 12x_5^8x_1^5 + \\
& 3x_5^8x_1^4 - 12x_5^7x_4x_2x_1^7 + 42x_5^7x_4x_2x_1^6 \\
& -54x_5^7x_4x_2x_1^5 + 30x_5^7x_4x_2x_1^4 \\
& -6x_5^7x_4x_2x_1^3 + 6x_5^7x_2x_1^7 - 18x_5^7x_2x_1^6 + \\
& 18x_5^7x_2x_1^5 - 6x_5^7x_2x_1^4 + 18x_5^6x_4^2x_2^2x_1^6 \\
& -54x_5^6x_4^2x_2^2x_1^5 + 57x_5^6x_4^2x_2^2x_1^4 \\
& -24x_5^6x_4^2x_2^2x_1^3 + 3x_5^6x_4^2x_2^2x_1^2 \\
& -18x_5^6x_4x_2^2x_1^6 + 48x_5^6x_4x_2^2x_1^5 \\
& -42x_5^6x_4x_2^2x_1^4 + 12x_5^6x_4x_2^2x_1^3 + \\
& 3x_5^6x_2^2x_1^6 - 6x_5^6x_2^2x_1^5 + 3x_5^6x_2^2x_1^4 \\
& -12x_5^5x_4^3x_2^3x_1^5 + 30x_5^5x_4^3x_2^3x_1^4 \\
& -24x_5^5x_4^3x_2^3x_1^3 + 6x_5^5x_4^3x_2^3x_1^2 + \\
& 18x_5^5x_4^2x_2^3x_1^5 - 42x_5^5x_4^2x_2^3x_1^4 + \\
& 30x_5^5x_4^2x_2^3x_1^3 - 6x_5^5x_4^2x_2^3x_1^2 \\
& -6x_5^5x_4x_2^3x_1^5 + 12x_5^5x_4x_2^3x_1^4 \\
& -6x_5^5x_4x_2^3x_1^3 + 3x_5^4x_4^4x_2^4x_1^4 \\
& -6x_5^4x_4^4x_2^4x_1^3 + 3x_5^4x_4^4x_2^4x_1^2 \\
& -6x_5^4x_4^3x_2^4x_1^4 + 12x_5^4x_4^3x_2^4x_1^3 \\
& -6x_5^4x_4^3x_2^4x_1^2 + 3x_5^4x_4^2x_2^4x_1^4 \\
& -6x_5^4x_4^2x_2^4x_1^3 + 3x_5^4x_4^2x_2^4x_1^2
\end{aligned}
$$

7. Pseudo remainder with $p_5$ over variable $x_5$:

$$
\begin{aligned}
g \;=\; & 3x_4^8 x_2^4 x_1^4 - 6x_4^8 x_2^4 x_1^3 + 3x_4^8 x_2^4 x_1^2 \\
& -12x_4^8 x_2^3 x_1^5 + 30x_4^8 x_2^3 x_1^4 - 24x_4^8 x_2^3 x_1^3 + \\
& 6x_4^8 x_2^3 x_1^2 + 18x_4^8 x_2^2 x_1^6 - 54x_4^8 x_2^2 x_1^5 + \\
& 57x_4^8 x_2^2 x_1^4 - 24x_4^8 x_2^2 x_1^3 + 3x_4^8 x_2^2 x_1^2 \\
& -12x_4^8 x_2 x_1^7 + 42x_4^8 x_2 x_1^6 - 54x_4^8 x_2 x_1^5 + \\
& 30x_4^8 x_2 x_1^4 - 6x_4^8 x_2 x_1^3 + 3x_4^8 x_1^8 - 12x_4^8 x_1^7 + \\
& 18x_4^8 x_1^6 - 12x_4^8 x_1^5 + 3x_4^8 x_1^4 - 6x_4^7 x_2^4 x_1^4 + \\
& 12x_4^7 x_2^4 x_1^3 - 6x_4^7 x_2^4 x_1^2 + 18x_4^7 x_2^3 x_1^5 \\
& -42x_4^7 x_2^3 x_1^4 + 30x_4^7 x_2^3 x_1^3 - 6x_4^7 x_2^3 x_1^2 \\
& -18x_4^7 x_2^2 x_1^6 + 48x_4^7 x_2^2 x_1^5 - 42x_4^7 x_2^2 x_1^4 + \\
& 12x_4^7 x_2^2 x_1^3 + 6x_4^7 x_2 x_1^7 - 18x_4^7 x_2 x_1^6 + \\
& 18x_4^7 x_2 x_1^5 - 6x_4^7 x_2 x_1^4 + 3x_4^6 x_2^4 x_1^4 \\
& -6x_4^6 x_2^4 x_1^3 + 3x_4^6 x_2^4 x_1^2 - 6x_4^6 x_2^3 x_1^5 + \\
& 12x_4^6 x_2^3 x_1^4 - 6x_4^6 x_2^3 x_1^3 + 3x_4^6 x_2^2 x_1^6 \\
& -6x_4^6 x_2^2 x_1^5 + 3x_4^6 x_2^2 x_1^4
\end{aligned}
$$

8. Pseudo remainder with $p_4$ over variable $x_4$:

$$
\begin{aligned}
g \;=\; & 3x_2^4 x_1^4 - 6x_2^4 x_1^3 + 3x_2^4 x_1^2 - 6x_2^3 x_1^4 + \\
& 12x_2^3 x_1^3 - 6x_2^3 x_1^2 - 6x_2^2 x_1^6 + 18x_2^2 x_1^5 \\
& -15x_2^2 x_1^4 + 3x_2^2 x_1^2 + 6x_2 x_1^6 - 18x_2 x_1^5 + 18x_2 x_1^4 \\
& -6x_2 x_1^3 + 3x_1^8 - 12x_1^7 + 18x_1^6 - 12x_1^5 + 3x_1^4
\end{aligned}
$$

9. Pseudo remainder with $p_3$ over variable $x_3$:

$$
\begin{aligned}
g \;=\; & 3x_2^4 x_1^4 - 6x_2^4 x_1^3 + 3x_2^4 x_1^2 - 6x_2^3 x_1^4 + \\
& 12x_2^3 x_1^3 - 6x_2^3 x_1^2 - 6x_2^2 x_1^6 + 18x_2^2 x_1^5 \\
& -15x_2^2 x_1^4 + 3x_2^2 x_1^2 + 6x_2 x_1^6 - 18x_2 x_1^5 + 18x_2 x_1^4 \\
& -6x_2 x_1^3 + 3x_1^8 - 12x_1^7 + 18x_1^6 - 12x_1^5 + 3x_1^4
\end{aligned}
$$

10. Pseudo remainder with $p_2$ over variable $x_2$:

$$
g \;=\; 0
$$

11. Pseudo remainder with $p_1$ over variable $x_1$:

$$
g \;=\; 0
$$

# 3 Prover results

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 132 terms.

**Time Complexity:** Time spent by the prover is 0.186 seconds.

# 4 NDG Conditions

## NDG Conditions in readable form

- Failed to translate NDG Conditions to readable form