# OpenGeoProver Output for conjecture "geothm\_zadatak"

Wu's method used

September 30, 2016

# 1 Invoking the theorem prover

The used proving method is Wu's method. The input system is:

```
= -x_4 + x_1
 p_1
          -x_4 + x_2
           -x_5 + x_1
      = x_5 + x_2 -
           x_3 –
            -x_8 + x_6
      = -x_8 + x_7
      = x_9 + x_6 -
            -x_9 + x_7
      = -x_{13}x_7 + x_{11}
p_{10}
      = x_{13} + x_{12} -
            -x_{14}x_1 + x_{10}
p_{12}
p_{13}
      = -x_{14}x_2 + x_{11}
      = -x_{14}x_3 + x_{12}
p_{14}
           -x_{18}x_7 + x_{16}
p_{15}
      = x_{18} + x_{17} -
      = -x_{19}x_1 + x_{19} + x_{15} -
           -x_{19}x_2 + x_{16}
p_{18}
      = -x_{19}x_3 + x_{17}
p_{19}
      = -x_{23}x_7 + x_{23} + x_{21} -
p_{20}
      = x_{23} + x_{22} -
p_{21}
      = -x_{24}x_1 + x_{20}
p_{22}
      = -x_{24}x_2 + x_{24} + x_{21} -
p_{23}
      = -x_{24}x_3 + x_{22}
     = -x_{28}x_7 + x_{28} + x_{26} -
```

```
\begin{array}{rcl} p_{26} & = & x_{28} + x_{27} - \\ p_{27} & = & -x_{29}x_1 + x_{29} + x_{25} - \\ p_{28} & = & -x_{29}x_2 + x_{29} + x_{26} - \\ p_{29} & = & -x_{29}x_3 + x_{27} \\ p_{30} & = & x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\ p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}
```

#### 1.1 Triangulation, step 1

Choosing variable: Trying the variable with index 33.

Variable  $x_{33}$  selected: The number of polynomials with this variable, with indexes from 1 to 33, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{33}$ . No reduction needed.

The triangular system has not been changed.

#### 1.2 Triangulation, step 2

Choosing variable: Trying the variable with index 32.

Variable  $x_{32}$  selected: The number of polynomials with this variable, with indexes from 1 to 32, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{32}$ . No reduction needed.

The triangular system has not been changed.

## 1.3 Triangulation, step 3

Choosing variable: Trying the variable with index 31.

Variable  $x_{31}$  selected: The number of polynomials with this variable, with indexes from 1 to 31, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{31}$ . No reduction needed.

The triangular system has not been changed.

#### 1.4 Triangulation, step 4

Choosing variable: Trying the variable with index 30.

Variable  $x_{30}$  selected: The number of polynomials with this variable, with indexes from 1 to 30, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{30}$ . No reduction needed.

The triangular system has not been changed.

## 1.5 Triangulation, step 5

Choosing variable: Trying the variable with index 29.

Variable  $x_{29}$  selected: The number of polynomials with this variable, with indexes from 1 to 29, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{29}$  from all other polynomials by reducing them with polynomial  $p_{27}$  from previous step.

```
= -x_4 + x_1
p_1
     = -x_4 + x_2
p_2
 p_3
      = -x_5 + x_1
      = x_5 + x_2 -
 p_4
      = x_3 -
 p_5
      = -x_8 + x_6
 p_6
      = -x_8 + x_7
 p_7
      = x_9 + x_6 -
      = -x_9 + x_7
      = -x_{13}x_7 + x_{11}
p_{10}
      = x_{13} + x_{12} -
      = -x_{14}x_1 + x_{10}
p_{12}
      = -x_{14}x_2 + x_{11}
p_{13}
      = -x_{14}x_3 + x_{12}
      = -x_{18}x_7 + x_{16}
      = x_{18} + x_{17} -
p_{16}
      = -x_{19}x_1 + x_{19} + x_{15} -
      = -x_{19}x_2 + x_{16}
p_{18}
      = -x_{19}x_3 + x_{17}
p_{19}
      = -x_{23}x_7 + x_{23} + x_{21} -
p_{20}
      = x_{23} + x_{22} -
p_{21}
      = -x_{24}x_1 + x_{20}
p_{22}
     = -x_{24}x_2 + x_{24} + x_{21} -
p_{23}
      = -x_{24}x_3 + x_{22}
p_{25}
      = -x_{28}x_7 + x_{28} + x_{26} -
     = x_{28} + x_{27} -
p_{26}
```

```
\begin{array}{rcl} p_{27} & = & -x_{26}x_{1} + x_{26} + x_{25}x_{2} - x_{25} - x_{2} + x_{1} \\ p_{28} & = & -x_{27}x_{1} + x_{27} + x_{25}x_{3} - x_{3} \\ p_{29} & = & -x_{29}x_{1} + x_{29} + x_{25} - \\ p_{30} & = & x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\ p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}
```

## 1.6 Triangulation, step 6

Choosing variable: Trying the variable with index 28.

Variable  $x_{28}$  selected: The number of polynomials with this variable, with indexes from 1 to 28, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{28}$  from all other polynomials by reducing them with polynomial  $p_{25}$  from previous step.

```
= -x_4 + x_1
 p_1
 p_2
     = -x_4 + x_2
     = -x_5 + x_1
 p_3
     = x_5 + x_2 -
 p_4
      = x_3 -
      = -x_8 + x_6
 p_6
     = -x_8 + x_7
 p_7
      = x_9 + x_6 -
      = -x_9 + x_7
 p_9
      = -x_{13}x_7 + x_{11}
p_{10}
      = x_{13} + x_{12} -
p_{11}
     = -x_{14}x_1 + x_{10}
p_{12}
     = -x_{14}x_2 + x_{11}
p_{13}
      = -x_{14}x_3 + x_{12}
p_{14}
      = -x_{18}x_7 + x_{16}
p_{15}
     = x_{18} + x_{17} -
p_{16}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{17}
     = -x_{19}x_2 + x_{16}
p_{18}
     = -x_{19}x_3 + x_{17}
p_{19}
      = -x_{23}x_7 + x_{23} + x_{21} -
     = x_{23} + x_{22} -
p_{21}
     = -x_{24}x_1 + x_{20}
p_{22}
```

```
-x_{24}x_2 + x_{24} + x_{21} -
p_{23}
            -x_{24}x_3 + x_{22}
p_{24}
              -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{25}
        = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{26}
        = -x_{27}x_7 + x_{27} - x_{26} + x_7
p_{27}
              -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
        = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
             x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
       = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
       = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

## 1.7 Triangulation, step 7

Choosing variable: Trying the variable with index 27.

Variable  $x_{27}$  selected: The number of polynomials with this variable, with indexes from 1 to 27, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{27}$  from all other polynomials by reducing them with polynomial  $p_{26}$  from previous step.

```
= -x_4 + x_1
     = -x_4 + x_2
 p_2
     = -x_5 + x_1
     = x_5 + x_2 -
      = x_3 -
 p_5
     = -x_8 + x_6
      = -x_8 + x_7
     = x_9 + x_6 -
 p_8
     = -x_9 + x_7
      = -x_{13}x_7 + x_{11}
p_{10}
     = x_{13} + x_{12} -
p_{11}
     = -x_{14}x_1 + x_{10}
p_{12}
     = -x_{14}x_2 + x_{11}
p_{13}
     = -x_{14}x_3 + x_{12}
p_{14}
     = -x_{18}x_7 + x_{16}
p_{15}
     = x_{18} + x_{17} -
p_{17}
     = -x_{19}x_1 + x_{19} + x_{15} -
p_{18} = -x_{19}x_2 + x_{16}
```

```
p_{19}
            -x_{19}x_3 + x_{17}
            -x_{23}x_7 + x_{23} + x_{21} -
p_{20}
             x_{23} + x_{22} -
p_{21}
              -x_{24}x_1 + x_{20}
             -x_{24}x_2 + x_{24} + x_{21} -
p_{23}
             -x_{24}x_3 + x_{22}
p_{24}
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{25}
       = x_{26}x_1 - x_{26} + x_{25}x_7x_3 - x_{25}x_3 - x_7x_3 - x_7x_1 + x_7 + x_3
p_{26}
             -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
       = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
            x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

## 1.8 Triangulation, step 8

Choosing variable: Trying the variable with index 26.

Variable  $x_{26}$  selected: The number of polynomials with this variable, with indexes from 1 to 26, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{26}$  from all other polynomials by reducing them with polynomial  $p_{25}$  from previous step.

$$\begin{array}{rcl} p_1 & = & -x_4 + x_1 \\ p_2 & = & -x_4 + x_2 \\ p_3 & = & -x_5 + x_1 \\ p_4 & = & x_5 + x_2 - \\ p_5 & = & x_3 - \\ p_6 & = & -x_8 + x_6 \\ p_7 & = & -x_8 + x_7 \\ p_8 & = & x_9 + x_6 - \\ p_9 & = & -x_9 + x_7 \\ p_{10} & = & -x_{13}x_7 + x_{11} \\ p_{11} & = & x_{13} + x_{12} - \\ p_{12} & = & -x_{14}x_1 + x_{10} \\ p_{13} & = & -x_{14}x_2 + x_{11} \\ p_{14} & = & -x_{14}x_3 + x_{12} \end{array}$$

```
p_{15}
       = -x_{18}x_7 + x_{16}
       = x_{18} + x_{17} -
p_{16}
             -x_{19}x_1 + x_{19} + x_{15} -
p_{17}
       = -x_{19}x_2 + x_{16}
       = -x_{19}x_3 + x_{17}
p_{19}
             -x_{23}x_7 + x_{23} + x_{21} -
p_{20}
       = x_{23} + x_{22} -
p_{21}
       = -x_{24}x_1 + x_{20}
p_{22}
             -x_{24}x_2 + x_{24} + x_{21} -
p_{23}
       = -x_{24}x_3 + x_{22}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
             x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
             -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
            x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

## 1.9 Triangulation, step 9

Choosing variable: Trying the variable with index 25.

Variable  $x_{25}$  selected: The number of polynomials with this variable, with indexes from 1 to 25, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{25}$ . No reduction needed.

The triangular system has not been changed.

#### 1.10 Triangulation, step 10

Choosing variable: Trying the variable with index 24.

Variable  $x_{24}$  selected: The number of polynomials with this variable, with indexes from 1 to 24, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{24}$  from all other polynomials by reducing them with polynomial  $p_{22}$  from previous step.

Finished a triangulation step, the current system is:

```
p_1
       = -x_4 + x_1
      = -x_4 + x_2
 p_2
      = -x_5 + x_1
       = x_5 + x_2 -
       = x_3 -
 p_5
      = -x_8 + x_6
 p_6
       = -x_8 + x_7
       = x_9 + x_6 -
       = -x_9 + x_7
 p_9
       = -x_{13}x_7 + x_{11}
p_{10}
      = x_{13} + x_{12} -
p_{11}
      = -x_{14}x_1 + x_{10}
p_{12}
      = -x_{14}x_2 + x_{11}
p_{13}
      = -x_{14}x_3 + x_{12}
p_{14}
      = -x_{18}x_7 + x_{16}
p_{15}
      = x_{18} + x_{17} -
p_{16}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{17}
      = -x_{19}x_2 + x_{16}
p_{18}
      = -x_{19}x_3 + x_{17}
p_{19}
       = -x_{23}x_7 + x_{23} + x_{21} -
p_{20}
      = x_{23} + x_{22} -
p_{21}
p_{22}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
      = -x_{22}x_1 + x_{20}x_3
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
p_{29}
       = -x_{29}x_1 + x_{29} + x_{25} -
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
            x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{33}
      = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

## 1.11 Triangulation, step 11

Choosing variable: Trying the variable with index 23.

Variable  $x_{23}$  selected: The number of polynomials with this variable, with indexes from 1 to 23, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{23}$  from all other polynomials by reducing them with polynomial  $p_{20}$  from previous step.

```
p_1 = -x_4 + x_1
      = -x_4 + x_2
 p_2
      = -x_5 + x_1
 p_3
      = x_5 + x_2 -
 p_4
      = x_3 -
      = -x_8 + x_6
 p_6
      = -x_8 + x_7
 p_7
      = x_9 + x_6 -
      = -x_9 + x_7
p_9
      = -x_{13}x_7 + x_{11}
p_{10}
      = x_{13} + x_{12} -
p_{11}
      = -x_{14}x_1 + x_{10}
p_{12}
      =
          -x_{14}x_2 + x_{11}
p_{13}
      = -x_{14}x_3 + x_{12}
p_{14}
      = -x_{18}x_7 + x_{16}
p_{15}
      = x_{18} + x_{17} -
p_{16}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{17}
      = -x_{19}x_2 + x_{16}
p_{18}
      = -x_{19}x_3 + x_{17}
p_{19}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{20}
      = -x_{22}x_1 + x_{20}x_3
p_{21}
      = -x_{22}x_7 + x_{22} - x_{21} + x_7
p_{22}
      = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
      = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
      = -x_{28}x_7 + x_{28} + x_{26} -
      = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
```

```
\begin{array}{lll} p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}
```

## 1.12 Triangulation, step 12

Choosing variable: Trying the variable with index 22.

Variable  $x_{22}$  selected: The number of polynomials with this variable, with indexes from 1 to 22, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{22}$  from all other polynomials by reducing them with polynomial  $p_{21}$  from previous step.

```
p_1 = -x_4 + x_1
     = -x_4 + x_2
 p_2
      = -x_5 + x_1
     = x_5 + x_2 -
 p_4
     = x_3 -
      = -x_8 + x_6
 p_6
      = -x_8 + x_7
 p_7
     = x_9 + x_6 -
      = -x_9 + x_7
      = -x_{13}x_7 + x_{11}
p_{10}
     = x_{13} + x_{12} -
p_{11}
      = -x_{14}x_1 + x_{10}
     = -x_{14}x_2 + x_{11}
p_{13}
     = -x_{14}x_3 + x_{12}
      = -x_{18}x_7 + x_{16}
     = x_{18} + x_{17} -
p_{16}
     = -x_{19}x_1 + x_{19} + x_{15} -
      = -x_{19}x_2 + x_{16}
      = -x_{19}x_3 + x_{17}
p_{19}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
      = x_{21}x_1 + x_{20}x_7x_3 - x_{20}x_3 - x_7x_1
p_{21}
p_{22}
     = -x_{22}x_1 + x_{20}x_3
     = -x_{23}x_7 + x_{23} + x_{21} - 
p_{23}
      = -x_{24}x_1 + x_{20}
p_{25}
      = \quad -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 + \\
           x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
```

$$\begin{array}{rcl} & -x_3x_1+x_3+x_2x_1-x_2-x_1^2+x_1\\ p_{26} & = & -x_{26}x_1+x_{26}+x_{25}x_2-x_{25}-x_2+x_1\\ p_{27} & = & -x_{27}x_1+x_{27}+x_{25}x_3-x_3\\ p_{28} & = & -x_{28}x_7+x_{28}+x_{26}-\\ p_{29} & = & -x_{29}x_1+x_{29}+x_{25}-\\ p_{30} & = & x_{30}-x_{22}x_{16}+x_{22}x_{11}+x_{21}x_{17}-x_{21}x_{12}-x_{17}x_{11}+x_{16}x_{12}\\ p_{31} & = & x_{31}+x_{22}x_{15}-x_{22}x_{10}-x_{20}x_{17}+x_{20}x_{12}+x_{17}x_{10}-x_{15}x_{12}\\ p_{32} & = & x_{32}-x_{21}x_{15}+x_{21}x_{10}+x_{20}x_{16}-x_{20}x_{11}-x_{16}x_{10}+x_{15}x_{11}\\ p_{33} & = & x_{33}+x_{32}x_{12}+x_{31}x_{11}+x_{30}x_{10} \end{array}$$

## 1.13 Triangulation, step 13

Choosing variable: Trying the variable with index 21.

Variable  $x_{21}$  selected: The number of polynomials with this variable, with indexes from 1 to 21, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{21}$  from all other polynomials by reducing them with polynomial  $p_{20}$  from previous step.

```
p_1
     = -x_4 + x_1
     = -x_4 + x_2
     = -x_5 + x_1
     = x_5 + x_2 -
 p_{4}
     = x_3 -
     = -x_8 + x_6
     = -x_8 + x_7
 p_7
     = x_9 + x_6 -
      = -x_9 + x_7
     = -x_{13}x_7 + x_{11}
p_{10}
     = x_{13} + x_{12} -
p_{11}
      = -x_{14}x_1 + x_{10}
p_{12}
     = -x_{14}x_2 + x_{11}
p_{13}
     = -x_{14}x_3 + x_{12}
p_{14}
      = -x_{18}x_7 + x_{16}
p_{15}
     = x_{18} + x_{17} -
p_{16}
     = -x_{19}x_1 + x_{19} + x_{15} -
     = -x_{19}x_2 + x_{16}
p_{19}
     = -x_{19}x_3 + x_{17}
    = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
```

```
p_{21}
       = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
       = -x_{22}x_1 + x_{20}x_3
p_{22}
             -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
p_{24}
       = -x_{24}x_1 + x_{20}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
             x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
            -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
       =
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
            x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

## 1.14 Triangulation, step 14

Choosing variable: Trying the variable with index 20.

Variable  $x_{20}$  selected: The number of polynomials with this variable, with indexes from 1 to 20, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{20}$ . No reduction needed.

The triangular system has not been changed.

#### 1.15 Triangulation, step 15

Choosing variable: Trying the variable with index 19.

Variable  $x_{19}$  selected: The number of polynomials with this variable, with indexes from 1 to 19, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{19}$  from all other polynomials by reducing them with polynomial  $p_{17}$  from previous step.

$$p_{1} = -x_{4} + x_{1}$$

$$p_{2} = -x_{4} + x_{2}$$

$$p_{3} = -x_{5} + x_{1}$$

$$p_{4} = x_{5} + x_{2} - x_{5}$$

$$p_{5} = x_{3} - x_{5}$$

```
= -x_8 + x_6
       = -x_8 + x_7
       = x_9 + x_6 -
       = -x_9 + x_7
       = -x_{13}x_7 + x_{11}
p_{10}
            x_{13} + x_{12} -
p_{11}
       = -x_{14}x_1 + x_{10}
       = -x_{14}x_2 + x_{11}
p_{13}
       = -x_{14}x_3 + x_{12}
p_{14}
       = -x_{18}x_7 + x_{16}
       = x_{18} + x_{17} -
p_{16}
       = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{17}
       = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
       = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
       = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
       = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
       = -x_{22}x_1 + x_{20}x_3
p_{22}
             -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
       = -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
             x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
            -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
            x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

## 1.16 Triangulation, step 16

Choosing variable: Trying the variable with index 18.

Variable  $x_{18}$  selected: The number of polynomials with this variable, with indexes from 1 to 18, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{18}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

```
= -x_4 + x_1
 p_1
      = -x_4 + x_2
           -x_5 + x_1
 p_3
      = x_5 + x_2 -
 p_4
      = x_3 -
            -x_8 + x_6
 p_6
       = -x_8 + x_7
 p_7
       = x_9 + x_6 -
           -x_9 + x_7
 p_9
       = -x_{13}x_7 + x_{11}
p_{10}
       = x_{13} + x_{12} -
p_{11}
       = -x_{14}x_1 + x_{10}
p_{12}
      = -x_{14}x_2 + x_{11}
p_{13}
      = -x_{14}x_3 + x_{12}
p_{14}
p_{15}
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{16}
      = -x_{17}x_7 - x_{16} + x_7
p_{17}
       = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
       = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p_{22}
      = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
      = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
      = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
     = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
     = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

#### 1.17 Triangulation, step 17

Choosing variable: Trying the variable with index 17.

Variable  $x_{17}$  selected: The number of polynomials with this variable, with indexes from 1 to 17, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{17}$  from all other polynomials by reducing them with polynomial  $p_{16}$  from previous step.

```
p_1 = -x_4 + x_1
     = -x_4 + x_2
 p_2
      = -x_5 + x_1
      = x_5 + x_2 -
 p_4
      = x_3 -
      = -x_8 + x_6
 p_6
 p_7
      = -x_8 + x_7
      = x_9 + x_6 -
      = -x_9 + x_7
p_9
      = -x_{13}x_7 + x_{11}
p_{10}
      = x_{13} + x_{12} -
p_{11}
      = -x_{14}x_1 + x_{10}
p_{12}
      = -x_{14}x_2 + x_{11}
p_{13}
      = -x_{14}x_3 + x_{12}
p_{14}
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{15}
      = x_{16}x_1 - x_{16} + x_{15}x_7x_3 - x_7x_3 - x_7x_1 + x_7
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
      = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p_{22}
      = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
      = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
      = -x_{28}x_7 + x_{28} + x_{26} -
      = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
```

```
\begin{array}{lll} p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}
```

#### 1.18 Triangulation, step 18

Choosing variable: Trying the variable with index 16.

Variable  $x_{16}$  selected: The number of polynomials with this variable, with indexes from 1 to 16, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{16}$  from all other polynomials by reducing them with polynomial  $p_{15}$  from previous step.

```
p_1 = -x_4 + x_1
     = -x_4 + x_2
 p_2
      = -x_5 + x_1
     = x_5 + x_2 -
 p_4
     = x_3 -
 p_5
      = -x_8 + x_6
 p_6
      = -x_8 + x_7
 p_7
     = x_9 + x_6 -
      = -x_9 + x_7
      = -x_{13}x_7 + x_{11}
p_{10}
     = x_{13} + x_{12} -
p_{11}
      = -x_{14}x_1 + x_{10}
     = -x_{14}x_2 + x_{11}
p_{13}
     = -x_{14}x_3 + x_{12}
p_{14}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
           -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
      = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
     = -x_{22}x_1 + x_{20}x_3
p_{22}
     = -x_{23}x_7 + x_{23} + x_{21} -
     = -x_{24}x_1 + x_{20}
p_{24}
     = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
```

$$\begin{array}{rcl} & x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 \\ & -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1 \\ \\ p_{26} & = & -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1 \\ \\ p_{27} & = & -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3 \\ \\ p_{28} & = & -x_{28}x_7 + x_{28} + x_{26} - \\ \\ p_{29} & = & -x_{29}x_1 + x_{29} + x_{25} - \\ \\ p_{30} & = & x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\ \\ p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}$$

#### 1.19 Triangulation, step 19

Choosing variable: Trying the variable with index 15.

**Variable**  $x_{15}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 15, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{15}$ . No reduction needed.

The triangular system has not been changed.

#### 1.20 Triangulation, step 20

Choosing variable: Trying the variable with index 14.

Variable  $x_{14}$  selected: The number of polynomials with this variable, with indexes from 1 to 14, is 3.

Minimal degrees: 3 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{14}$  from all other polynomials by reducing them with polynomial  $p_{12}$  from previous step.

$$p_{1} = -x_{4} + x_{1}$$

$$p_{2} = -x_{4} + x_{2}$$

$$p_{3} = -x_{5} + x_{1}$$

$$p_{4} = x_{5} + x_{2} - x_{5} + x_{2} - x_{5} + x_{6}$$

$$p_{6} = -x_{8} + x_{6}$$

$$p_{7} = -x_{8} + x_{7}$$

$$p_{8} = x_{9} + x_{6} - x_{9} + x_{7}$$

$$p_{9} = -x_{9} + x_{7}$$

$$p_{10} = -x_{13}x_{7} + x_{11}$$

```
p_{11}
       = x_{13} + x_{12} -
       = -x_{11}x_1 + x_{10}x_2
p_{12}
       = -x_{12}x_1 + x_{10}x_3
p_{13}
p_{14}
       = -x_{14}x_1 + x_{10}
       = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
p_{15}
             -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
       = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
       = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
       = -x_{18}x_7 + x_{16}
p_{18}
       = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
       = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
       = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
       = -x_{22}x_1 + x_{20}x_3
p_{22}
       = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
       = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
      = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

#### 1.21 Triangulation, step 21

Choosing variable: Trying the variable with index 13.

Variable  $x_{13}$  selected: The number of polynomials with this variable, with indexes from 1 to 13, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{13}$  from all other polynomials by reducing them with polynomial  $p_{10}$  from previous step.

$$p_1 = -x_4 + x_1$$

$$p_2 = -x_4 + x_2$$

$$p_3 = -x_5 + x_1$$

```
p_4
      = x_5 + x_2 -
       = x_3 -
 p_5
       = -x_8 + x_6
       = -x_8 + x_7
       = x_9 + x_6 -
 p_8
            -x_9 + x_7
 p_9
       = -x_{11}x_1 + x_{10}x_2
p_{10}
       = -x_{12}x_1 + x_{10}x_3
p_{11}
       = -x_{12}x_7 - x_{11} + x_7
p_{12}
       = -x_{13}x_7 + x_{11}
p_{13}
       = -x_{14}x_1 + x_{10}
p_{14}
       = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
p_{15}
             -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
       = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
       = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
       = -x_{18}x_7 + x_{16}
p_{18}
       = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
       = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
       = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
       = -x_{22}x_1 + x_{20}x_3
p_{22}
            -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
       = -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
             x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
       = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

#### 1.22 Triangulation, step 22

Choosing variable: Trying the variable with index 12.

Variable  $x_{12}$  selected: The number of polynomials with this variable, with indexes from 1 to 12, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{12}$  from all other polynomials by reducing them with polynomial  $p_{11}$  from previous step.

```
= -x_4 + x_1
 p_1
      = -x_4 + x_2
 p_2
      = -x_5 + x_1
      = x_5 + x_2 -
 p_4
      = x_3 -
 p_5
      = -x_8 + x_6
      = -x_8 + x_7
      = x_9 + x_6 -
 p_8
      = -x_9 + x_7
      = -x_{11}x_1 + x_{10}x_2
p_{10}
      = x_{11}x_1 + x_{10}x_7x_3 - x_7x_1
p_{11}
     = -x_{12}x_1 + x_{10}x_3
p_{12}
      = -x_{13}x_7 + x_{11}
p_{13}
      = -x_{14}x_1 + x_{10}
p_{14}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
            -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
      = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
       = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p_{22}
      = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
       = -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
     = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

#### 1.23 Triangulation, step 23

Choosing variable: Trying the variable with index 11.

Variable  $x_{11}$  selected: The number of polynomials with this variable, with indexes from 1 to 11, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_{11}$  from all other polynomials by reducing them with polynomial  $p_{10}$  from previous step.

```
p_1 = -x_4 + x_1
     = -x_4 + x_2
     = -x_5 + x_1
      = x_5 + x_2 -
 p_4
      = x_3 -
 p_5
     = -x_8 + x_6
 p_6
      = -x_8 + x_7
      = x_9 + x_6 -
 p_8
      = -x_9 + x_7
      = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
p_{10}
      = -x_{11}x_1 + x_{10}x_2
p_{11}
     = -x_{12}x_1 + x_{10}x_3
p_{12}
      = -x_{13}x_7 + x_{11}
     = -x_{14}x_1 + x_{10}
p_{14}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
           -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
      = -x_{18}x_7 + x_{16}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p22
     = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
           x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
           -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
     = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
     = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
```

```
\begin{array}{rcl} p_{28} & = & -x_{28}x_7 + x_{28} + x_{26} - \\ p_{29} & = & -x_{29}x_1 + x_{29} + x_{25} - \\ p_{30} & = & x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\ p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}
```

## 1.24 Triangulation, step 24

Choosing variable: Trying the variable with index 10.

**Variable**  $x_{10}$  **selected:** The number of polynomials with this variable, with indexes from 1 to 10, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_{10}$ . No reduction needed.

The triangular system has not been changed.

## 1.25 Triangulation, step 25

Choosing variable: Trying the variable with index 9.

Variable  $x_9$  selected: The number of polynomials with this variable, with indexes from 1 to 9, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_9$  from all other polynomials by reducing them with polynomial  $p_8$  from previous step.

```
p_1 = -x_4 + x_1
p_2
     = -x_4 + x_2
     = -x_5 + x_1
p_3
     = x_5 + x_2 -
     = x_3 -
     = -x_8 + x_6
p_6
     = -x_8 + x_7
p_7
     = x_7 + x_6 -
     = x_9 + x_6 -
     = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
     = -x_{11}x_1 + x_{10}x_2
p_{11}
    = -x_{12}x_1 + x_{10}x_3
p_{12}
     = -x_{13}x_7 + x_{11}
     = -x_{14}x_1 + x_{10}
p_{14}
```

$$\begin{array}{rclcrcl} p_{15} & = & -x_{15}x_{7}x_{3}x_{1} + x_{15}x_{7}x_{3} - x_{15}x_{2}x_{1} + x_{15}x_{2} + x_{7}x_{3}x_{1} \\ & -x_{7}x_{3} + x_{7}x_{1}^{2} - 2x_{7}x_{1} + x_{7} + x_{2}x_{1} - x_{2} \\ \end{array}$$
 
$$\begin{array}{rclcrcl} p_{16} & = & -x_{16}x_{1} + x_{16} + x_{15}x_{2} - x_{2} \\ p_{17} & = & -x_{17}x_{1} + x_{17} + x_{15}x_{3} - x_{3} \\ p_{18} & = & -x_{18}x_{7} + x_{16} \\ p_{19} & = & -x_{19}x_{1} + x_{19} + x_{15} - \\ p_{20} & = & -x_{20}x_{7}x_{3}x_{1} + x_{20}x_{3}x_{1} - x_{20}x_{2}x_{1} + x_{20}x_{1} + x_{7}x_{1}^{2} - x_{1}^{2} \\ p_{21} & = & -x_{21}x_{1} + x_{20}x_{2} - x_{20} + x_{1} \\ p_{22} & = & -x_{22}x_{1} + x_{20}x_{3} \\ p_{23} & = & -x_{23}x_{7} + x_{23} + x_{21} - \\ p_{24} & = & -x_{24}x_{1} + x_{20} \\ p_{25} & = & -x_{25}x_{7}x_{3}x_{1} + x_{25}x_{7}x_{3} + x_{25}x_{3}x_{1} - x_{25}x_{3} - x_{25}x_{2}x_{1} + \\ & & & x_{25}x_{2} + x_{25}x_{1} - x_{25} + x_{7}x_{3}x_{1} - x_{7}x_{3} + x_{7}x_{1}^{2} - 2x_{7}x_{1} + x_{7} \\ & & & -x_{26}x_{1} + x_{26} + x_{25}x_{2} - x_{25} - x_{2} + x_{1} \\ p_{26} & = & -x_{26}x_{1} + x_{26} + x_{25}x_{2} - x_{25} - x_{2} + x_{1} \\ p_{27} & = & -x_{27}x_{1} + x_{27} + x_{25}x_{3} - x_{3} \\ p_{28} & = & -x_{28}x_{7} + x_{28} + x_{26} - \\ p_{29} & = & -x_{29}x_{1} + x_{29} + x_{25} - \\ p_{30} & = & x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\ p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}$$

#### 1.26 Triangulation, step 26

Choosing variable: Trying the variable with index 8.

Variable  $x_8$  selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_8$  from all other polynomials by reducing them with polynomial  $p_6$  from previous step.

$$p_{1} = -x_{4} + x_{1}$$

$$p_{2} = -x_{4} + x_{2}$$

$$p_{3} = -x_{5} + x_{1}$$

$$p_{4} = x_{5} + x_{2} - x_{5}$$

$$p_{5} = x_{3} - x_{6} - x_{7} + x_{7} - x_{7} + x_{7} - x_{7} + x_{7} - x$$

```
= -x_8 + x_6
       = x_9 + x_6 -
            -x_{10}x_7x_3x_1-x_{10}x_2x_1+x_7x_1^2
p_{10}
       = -x_{11}x_1 + x_{10}x_2
       = -x_{12}x_1 + x_{10}x_3
p_{12}
            -x_{13}x_7 + x_{11}
p_{13}
       = -x_{14}x_1 + x_{10}
p_{14}
       = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
p_{15}
             -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
       = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
       = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
       = -x_{18}x_7 + x_{16}
p_{18}
       = -x_{19}x_1 + x_{19} + x_{15} -
       = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
            -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
       = -x_{22}x_1 + x_{20}x_3
p_{22}
       = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
            -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
            x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
      = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

#### 1.27 Triangulation, step 27

Choosing variable: Trying the variable with index 7.

Variable  $x_7$  selected: The number of polynomials with this variable, with indexes from 1 to 7, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_7$  from all other polynomials by reducing them with polynomial  $p_6$  from previous step.

```
p_1 = -x_4 + x_1
      = -x_4 + x_2
      = -x_5 + x_1
      = x_5 + x_2 -
 p_5
      = x_3 -
      = 2x_6 -
 p_6
      = x_7 + x_6 -
 p_7
      = -x_8 + x_6
      = x_9 + x_6 -
      = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
      = -x_{11}x_1 + x_{10}x_2
p_{11}
p_{12} = -x_{12}x_1 + x_{10}x_3
      = -x_{13}x_7 + x_{11}
p_{13}
      = -x_{14}x_1 + x_{10}
p_{14}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
p_{15}
            -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
      = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p_{22}
     = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
      = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
      = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
      = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
      = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
     = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
p_{33} = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

## 1.28 Triangulation, step 28

Choosing variable: Trying the variable with index 6.

**Variable**  $x_6$  **selected:** The number of polynomials with this variable, with indexes from 1 to 6, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_6$ . No reduction needed.

The triangular system has not been changed.

#### 1.29 Triangulation, step 29

Choosing variable: Trying the variable with index 5.

**Variable**  $x_5$  **selected:** The number of polynomials with this variable, with indexes from 1 to 5, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_5$  from all other polynomials by reducing them with polynomial  $p_3$  from previous step.

```
= -x_4 + x_1
p_1
 p_2
     = -x_4 + x_2
     = x_3 -
 p_3
     = -x_2 - x_1 + 1
 p_4
      = -x_5 + x_1
      = 2x_6 -
 p_6
     = x_7 + x_6 -
 p_7
      = -x_8 + x_6
      = x_9 + x_6 -
      = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
      = -x_{11}x_1 + x_{10}x_2
     = -x_{12}x_1 + x_{10}x_3
p_{12}
     = -x_{13}x_7 + x_{11}
p_{13}
      = -x_{14}x_1 + x_{10}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
p_{15}
           -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
      = -x_{18}x_7 + x_{16}
      = -x_{19}x_1 + x_{19} + x_{15} -
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
     = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
     = -x_{22}x_1 + x_{20}x_3
```

```
= -x_{23}x_7 + x_{23} + x_{21} -
       = -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
             x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
       = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
       = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
      = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
     = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
p_{33}
```

## 1.30 Triangulation, step 30

Choosing variable: Trying the variable with index 4.

Variable  $x_4$  selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_4$  from all other polynomials by reducing them with polynomial  $p_1$  from previous step.

```
p_1 = x_3 -
 p_2 = -x_2 - x_1 + 1
     = -x_2 + x_1
 p_3
     = -x_4 + x_1
 p_4
     = -x_5 + x_1
 p_5
     = 2x_6 -
 p_6
     = x_7 + x_6 -
 p_7
     = -x_8 + x_6
     = x_9 + x_6 -
 p_9
     = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
p_{10}
     = -x_{11}x_1 + x_{10}x_2
p_{11}
     = -x_{12}x_1 + x_{10}x_3
p_{12}
    = -x_{13}x_7 + x_{11}
p_{13}
     = -x_{14}x_1 + x_{10}
p_{14}
p_{15}
     = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
           -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
```

```
= -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
       = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
             -x_{18}x_7 + x_{16}
p_{18}
            -x_{19}x_1 + x_{19} + x_{15} -
            -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
             -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
       = -x_{22}x_1 + x_{20}x_3
p_{22}
       = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
             -x_{24}x_1 + x_{20}
p_{24}
       = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
             x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
             -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
       = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
       = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
             -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
       = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
       = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
             x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
            x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
      = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

#### 1.31 Triangulation, step 31

Choosing variable: Trying the variable with index 3.

Variable  $x_3$  selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

## 1.32 Triangulation, step 32

Choosing variable: Trying the variable with index 2.

Variable  $x_2$  selected: The number of polynomials with this variable, with indexes from 1 to 2, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

**Polynomial with linear degree:** Removing variable  $x_2$  from all other polynomials by reducing them with polynomial  $p_1$  from previous step.

```
p_1 = -2x_1 + 1
     = -x_2 - x_1 + 1
     = x_3 -
 p_3
      = -x_4 + x_1
      = -x_5 + x_1
 p_5
      = 2x_6 -
 p_6
      = x_7 + x_6 -
 p_7
      = -x_8 + x_6
      = x_9 + x_6 -
      = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
      = -x_{11}x_1 + x_{10}x_2
p_{11}
p_{12} = -x_{12}x_1 + x_{10}x_3
     = -x_{13}x_7 + x_{11}
p_{13}
      = -x_{14}x_1 + x_{10}
p_{14}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
p_{15}
            -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
      = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
p_{20}
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p_{22}
     = -x_{23}x_7 + x_{23} + x_{21} -
p_{23}
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
            x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
      = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
p_{27}
      = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
      = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
      = x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12}
p_{30}
      = x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12}
p_{31}
     = x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11}
p_{32}
p_{33} = x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10}
```

## 1.33 Triangulation, step 33

Choosing variable: Trying the variable with index 1.

Variable  $x_1$  selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is  $p_1$ . No reduction needed.

The triangular system has not been changed.

The triangular system is:

```
p_1 = -2x_1 + 1
     = -x_2 - x_1 + 1
      = x_3 -
      = -x_4 + x_1
 p_4
      = -x_5 + x_1
      = 2x_6 -
      = x_7 + x_6 -
 p_7
      = -x_8 + x_6
      = x_9 + x_6 -
      = -x_{10}x_7x_3x_1 - x_{10}x_2x_1 + x_7x_1^2
p_{10}
      = -x_{11}x_1 + x_{10}x_2
p_{11}
      = -x_{12}x_1 + x_{10}x_3
p_{12}
      = -x_{13}x_7 + x_{11}
p_{13}
      = -x_{14}x_1 + x_{10}
p_{14}
      = -x_{15}x_7x_3x_1 + x_{15}x_7x_3 - x_{15}x_2x_1 + x_{15}x_2 + x_7x_3x_1
            -x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7 + x_2x_1 - x_2
      = -x_{16}x_1 + x_{16} + x_{15}x_2 - x_2
p_{16}
      = -x_{17}x_1 + x_{17} + x_{15}x_3 - x_3
p_{17}
      = -x_{18}x_7 + x_{16}
p_{18}
      = -x_{19}x_1 + x_{19} + x_{15} -
p_{19}
      = -x_{20}x_7x_3x_1 + x_{20}x_3x_1 - x_{20}x_2x_1 + x_{20}x_1 + x_7x_1^2 - x_1^2
      = -x_{21}x_1 + x_{20}x_2 - x_{20} + x_1
p_{21}
      = -x_{22}x_1 + x_{20}x_3
p_{22}
      = -x_{23}x_7 + x_{23} + x_{21} -
      = -x_{24}x_1 + x_{20}
p_{24}
      = -x_{25}x_7x_3x_1 + x_{25}x_7x_3 + x_{25}x_3x_1 - x_{25}x_3 - x_{25}x_2x_1 +
p_{25}
           x_{25}x_2 + x_{25}x_1 - x_{25} + x_7x_3x_1 - x_7x_3 + x_7x_1^2 - 2x_7x_1 + x_7
            -x_3x_1 + x_3 + x_2x_1 - x_2 - x_1^2 + x_1
      = -x_{26}x_1 + x_{26} + x_{25}x_2 - x_{25} - x_2 + x_1
p_{26}
      = -x_{27}x_1 + x_{27} + x_{25}x_3 - x_3
      = -x_{28}x_7 + x_{28} + x_{26} -
p_{28}
      = -x_{29}x_1 + x_{29} + x_{25} -
p_{29}
```

```
\begin{array}{lll} p_{30} & = & x_{30} - x_{22}x_{16} + x_{22}x_{11} + x_{21}x_{17} - x_{21}x_{12} - x_{17}x_{11} + x_{16}x_{12} \\ p_{31} & = & x_{31} + x_{22}x_{15} - x_{22}x_{10} - x_{20}x_{17} + x_{20}x_{12} + x_{17}x_{10} - x_{15}x_{12} \\ p_{32} & = & x_{32} - x_{21}x_{15} + x_{21}x_{10} + x_{20}x_{16} - x_{20}x_{11} - x_{16}x_{10} + x_{15}x_{11} \\ p_{33} & = & x_{33} + x_{32}x_{12} + x_{31}x_{11} + x_{30}x_{10} \end{array}
```

#### 2 Final Remainder

#### 2.1 Final remainder for conjecture geothm\_zadatak

Calculating final remainder of the conclusion:

$$g = x_{33} + x_{32}x_{27} + x_{31}x_{26} + x_{30}x_{25}$$

with respect to the triangular system.

1. Pseudo remainder with  $p_{33}$  over variable  $x_{33}$ :

$$g = x_{32}x_{27} - x_{32}x_{12} + x_{31}x_{26} - x_{31}x_{11} + x_{30}x_{25} - x_{30}x_{10}$$

2. Pseudo remainder with  $p_{32}$  over variable  $x_{32}$ :

$$g = x_{31}x_{26} - x_{31}x_{11} + x_{30}x_{25} - x_{30}x_{10} + x_{27}x_{21}x_{15} - x_{27}x_{21}x_{10} - x_{27}x_{20}x_{16} + x_{27}x_{20}x_{11} + x_{27}x_{16}x_{10} - x_{27}x_{15}x_{11} - x_{21}x_{15}x_{12} + x_{21}x_{12}x_{10} + x_{20}x_{16}x_{12} - x_{20}x_{12}x_{11} - x_{16}x_{12}x_{10} + x_{15}x_{12}x_{11}$$

3. Pseudo remainder with  $p_{31}$  over variable  $x_{31}$ :

$$\begin{array}{lll} g & = & x_{30}x_{25} - x_{30}x_{10} + x_{27}x_{21}x_{15} - x_{27}x_{21}x_{10} - x_{27}x_{20}x_{16} + \\ & & x_{27}x_{20}x_{11} + x_{27}x_{16}x_{10} - x_{27}x_{15}x_{11} - x_{26}x_{22}x_{15} + \\ & & x_{26}x_{22}x_{10} + x_{26}x_{20}x_{17} - x_{26}x_{20}x_{12} - x_{26}x_{17}x_{10} + \\ & & x_{26}x_{15}x_{12} + x_{22}x_{15}x_{11} - x_{22}x_{11}x_{10} - x_{21}x_{15}x_{12} + \\ & & x_{21}x_{12}x_{10} - x_{20}x_{17}x_{11} + x_{20}x_{16}x_{12} + x_{17}x_{11}x_{10} \\ & & -x_{16}x_{12}x_{10} \end{array}$$

4. Pseudo remainder with  $p_{30}$  over variable  $x_{30}$ :

$$\begin{array}{lll} g & = & x_{27}x_{21}x_{15} - x_{27}x_{21}x_{10} - x_{27}x_{20}x_{16} + x_{27}x_{20}x_{11} + \\ & x_{27}x_{16}x_{10} - x_{27}x_{15}x_{11} - x_{26}x_{22}x_{15} + x_{26}x_{22}x_{10} + \\ & x_{26}x_{20}x_{17} - x_{26}x_{20}x_{12} - x_{26}x_{17}x_{10} + x_{26}x_{15}x_{12} + \\ & x_{25}x_{22}x_{16} - x_{25}x_{22}x_{11} - x_{25}x_{21}x_{17} + x_{25}x_{21}x_{12} + \\ & x_{25}x_{17}x_{11} - x_{25}x_{16}x_{12} - x_{22}x_{16}x_{10} + x_{22}x_{15}x_{11} + \\ & x_{21}x_{17}x_{10} - x_{21}x_{15}x_{12} - x_{20}x_{17}x_{11} + x_{20}x_{16}x_{12} \end{array}$$

5. Pseudo remainder with  $p_{29}$  over variable  $x_{29}$ :

```
\begin{array}{lll} g & = & x_{27}x_{21}x_{15} - x_{27}x_{21}x_{10} - x_{27}x_{20}x_{16} + x_{27}x_{20}x_{11} + \\ & x_{27}x_{16}x_{10} - x_{27}x_{15}x_{11} - x_{26}x_{22}x_{15} + x_{26}x_{22}x_{10} + \\ & x_{26}x_{20}x_{17} - x_{26}x_{20}x_{12} - x_{26}x_{17}x_{10} + x_{26}x_{15}x_{12} + \\ & x_{25}x_{22}x_{16} - x_{25}x_{22}x_{11} - x_{25}x_{21}x_{17} + x_{25}x_{21}x_{12} + \\ & x_{25}x_{17}x_{11} - x_{25}x_{16}x_{12} - x_{22}x_{16}x_{10} + x_{22}x_{15}x_{11} + \\ & x_{21}x_{17}x_{10} - x_{21}x_{15}x_{12} - x_{20}x_{17}x_{11} + x_{20}x_{16}x_{12} \end{array}
```

6. Pseudo remainder with  $p_{28}$  over variable  $x_{28}$ :

$$\begin{array}{lll} g & = & x_{27}x_{21}x_{15} - x_{27}x_{21}x_{10} - x_{27}x_{20}x_{16} + x_{27}x_{20}x_{11} + \\ & & x_{27}x_{16}x_{10} - x_{27}x_{15}x_{11} - x_{26}x_{22}x_{15} + x_{26}x_{22}x_{10} + \\ & & x_{26}x_{20}x_{17} - x_{26}x_{20}x_{12} - x_{26}x_{17}x_{10} + x_{26}x_{15}x_{12} + \\ & & x_{25}x_{22}x_{16} - x_{25}x_{22}x_{11} - x_{25}x_{21}x_{17} + x_{25}x_{21}x_{12} + \\ & & x_{25}x_{17}x_{11} - x_{25}x_{16}x_{12} - x_{22}x_{16}x_{10} + x_{22}x_{15}x_{11} + \\ & & x_{21}x_{17}x_{10} - x_{21}x_{15}x_{12} - x_{20}x_{17}x_{11} + x_{20}x_{16}x_{12} \end{array}$$

7. Pseudo remainder with  $p_{27}$  over variable  $x_{27}$ :

$$\begin{array}{lll} g &=& x_{26}x_{22}x_{15}x_1 - x_{26}x_{22}x_{15} - x_{26}x_{22}x_{10}x_1 + x_{26}x_{22}x_{10} \\ &- x_{26}x_{20}x_{17}x_1 + x_{26}x_{20}x_{17} + x_{26}x_{20}x_{12}x_1 - x_{26}x_{20}x_{12} + \\ & x_{26}x_{17}x_{10}x_1 - x_{26}x_{17}x_{10} - x_{26}x_{15}x_{12}x_1 + x_{26}x_{15}x_{12} \\ &- x_{25}x_{22}x_{16}x_1 + x_{25}x_{22}x_{16} + x_{25}x_{22}x_{11}x_1 - x_{25}x_{22}x_{11} + \\ & x_{25}x_{21}x_{17}x_1 - x_{25}x_{21}x_{17} - x_{25}x_{21}x_{15}x_3 - x_{25}x_{21}x_{12}x_1 + \\ & x_{25}x_{21}x_{12} + x_{25}x_{21}x_{10}x_3 + x_{25}x_{20}x_{16}x_3 - x_{25}x_{20}x_{11}x_3 \\ &- x_{25}x_{17}x_{11}x_1 + x_{25}x_{17}x_{11} + x_{25}x_{16}x_{12}x_1 - x_{25}x_{16}x_{12} \\ &- x_{25}x_{16}x_{10}x_3 + x_{25}x_{15}x_{11}x_3 + x_{22}x_{16}x_{10}x_1 - x_{22}x_{16}x_{10} \\ &- x_{22}x_{15}x_{11}x_1 + x_{22}x_{15}x_{11} - x_{21}x_{17}x_{10}x_1 + x_{21}x_{17}x_{10} + \\ &x_{21}x_{15}x_{12}x_1 - x_{21}x_{15}x_{12} + x_{21}x_{15}x_3 - x_{21}x_{10}x_3 + \\ &x_{20}x_{17}x_{11}x_1 - x_{20}x_{17}x_{11} - x_{20}x_{16}x_{12}x_1 + x_{20}x_{16}x_{12} \\ &- x_{20}x_{16}x_3 + x_{20}x_{11}x_3 + x_{16}x_{10}x_3 - x_{15}x_{11}x_3 \end{array}$$

- 8. Pseudo remainder with  $p_{26}$  over variable  $x_{26}$ :
  - Polynomial too big for output (text size is 2657 characters, number of terms is 108)
- 9. Pseudo remainder with  $p_{25}$  over variable  $x_{25}$ :

  Polynomial too big for output (number of terms is 366)

10. Pseudo remainder with  $p_{24}$  over variable  $x_{24}$ :

Polynomial too big for output (number of terms is 366)

11. Pseudo remainder with  $p_{23}$  over variable  $x_{23}$ :

Polynomial too big for output (number of terms is 366)

12. Pseudo remainder with  $p_{22}$  over variable  $x_{22}$ :

Polynomial too big for output (number of terms is 340)

13. Pseudo remainder with  $p_{21}$  over variable  $x_{21}$ :

Polynomial too big for output (number of terms is 404)

14. Pseudo remainder with  $p_{20}$  over variable  $x_{20}$ :

Polynomial too big for output (text size is 7563 characters, number of terms is 232)

15. Pseudo remainder with  $p_{19}$  over variable  $x_{19}$ :

Polynomial too big for output (text size is 7563 characters, number of terms is 232)

16. Pseudo remainder with  $p_{18}$  over variable  $x_{18}$ :

Polynomial too big for output (text size is 7563 characters, number of terms is 232)

17. Pseudo remainder with  $p_{17}$  over variable  $x_{17}$ :

Polynomial too big for output (number of terms is 333)

18. Pseudo remainder with  $p_{16}$  over variable  $x_{16}$ :

Polynomial too big for output (number of terms is 380)

19. Pseudo remainder with  $p_{15}$  over variable  $x_{15}$ :

Polynomial too big for output (text size is 7211 characters, number of terms is 210)

20. Pseudo remainder with  $p_{14}$  over variable  $x_{14}$ :

Polynomial too big for output (text size is 7211 characters, number of terms is 210)

21. Pseudo remainder with  $p_{13}$  over variable  $x_{13}$ :

Polynomial too big for output (text size is 7211 characters, number of terms is 210)

22. Pseudo remainder with  $p_{12}$  over variable  $x_{12}$ :

Polynomial too big for output (text size is 7372 characters, number of terms is 210)

- 23. Pseudo remainder with  $p_{11}$  over variable  $x_{11}$ :

  Polynomial too big for output (text size is 6469 characters, number of terms is 182)
- 24. Pseudo remainder with  $p_{10}$  over variable  $x_{10}$ :

g = 0

25. Pseudo remainder with  $p_9$  over variable  $x_9$ :

g = 0

26. Pseudo remainder with  $p_8$  over variable  $x_8$ :

g = 0

27. Pseudo remainder with  $p_7$  over variable  $x_7$ :

g = 0

28. Pseudo remainder with  $p_6$  over variable  $x_6$ :

g = 0

29. Pseudo remainder with  $p_5$  over variable  $x_5$ :

g = 0

30. Pseudo remainder with  $p_4$  over variable  $x_4$ :

g = 0

31. Pseudo remainder with  $p_3$  over variable  $x_3$ :

g = 0

32. Pseudo remainder with  $p_2$  over variable  $x_2$ :

g = 0

33. Pseudo remainder with  $p_1$  over variable  $x_1$ :

g = 0

## 3 Prover results

Status: Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 404 terms.

Time Complexity: Time spent by the prover is 0.393 seconds.

# 4 NDG Conditions

## NDG Conditions in readable form

• Failed to translate NDG Conditions to readable form