Exercise 1 (1978 British Math Olympiads (BrMO)) *An altitude of a tetrahedron is a line through a vertex perpendicular to the opposite face. Prove that the four altitudes of a tetrahedron are concurrent if and only if each edge of the tetrahedron is perpendicular to its opposite edge.*

Exercise 2 (1994-95 British Math Olympiads (BrMO) Round 1 P2) ABCDEFGH *is a cube of side* 2 *with* A,B,C,D *above* F,G,H,E*, respectively.* M *is the midpoint of* BC *and* N *is the midpoint of* EF*. Let* P *be the midpoint of* AB*, and* Q *the midpoint of* HE*. Let* AM *meet* CP *at* X*, and* HN *meet* FQ *at* Y*. Find the length of* XY*.*

The solution of this exercise is $|XY| = \frac{2\sqrt{11}}{3}$. We are going to prove that this solution is true.

Exercise 3 (1984 British Math Olympiads2 (BMO2) British FIST p2) ABCD *is a tetrahedron with* DA = DB = DC *and* AB = BC = CA*.* M *and* N *are the midpoints of* AB *and* CD*. A plane π passes through* MN *and cuts* AD *and* BC *at* P *and* Q *respectively. Prove that* AP/AD = BQ/BC*.*

Exercise 4 (1966 International Mathematical Olimpiad Longlist (IMO ILL) Problem 17) *Let $ABCD$ and $A_1B_1C_1D_1$ be two arbitrary parallelograms in the space, and let M, N, P, Q be points dividing the segments AA_1, BB_1, CC_1, DD_1 in equal ratios. Prove that the quadrilateral MNPQ is a parallelogram.*

Exercise 5 (1972 International Mathematical Olimpiad Shortlist (IMO ISL) Problem 5) *Prove the following assertion: The four altitudes of a tetrahedron ABCD intersect in a point if and only if $AB^2+CD^2 = BC^2+AD^2 = CA^2+BD^2$*

Exercise 6 (1981 International Mathematical Olimpiad Shortlist (IMO ISL) Problem 2) *A sphere S is tangent to the edges AB,BC,CD,DA of a tetrahedron ABCD at the points E,F,G,H respectively. The points E,F,G,H are the vertices of a square. Prove that if the sphere is tangent to the edge AC, then it is also tangent to the edge BD.*

Exercise 7 (1991 International Mathematical Olimpiad Shortlist (IMO ISL) Problem 7) *ABCD is a terahedron: AD + BD = AC + BC, BD + CD = BA + CA, CD + AD = CB + AB; M,N,P are the mid–points of BC, CA, AB. OA = OB = OC = OD. Prove that $\angle MOP = \angle NOP = \angle NOM$.*

Exercise 8 (1966 International Mathematical Olimpiad Longlist (IMO ISL) Problem 56) *In a tetrahedron, all three pairs of opposite (skew) edges are mutually perpendicular. Prove that the midpoints of the six edges of the tetrahedron lie on one sphere.*

Exercise 9 1979 (Austrian Polish Mathematical Competition (APMC) Team 2) *Let A, B, C, D be four points in space, and M and N be the midpoints of AC and BD, respectively. Show that $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4MN^2$.*

Exercise 10 1982 (Austrian Polish Mathematical Competition (APMC) Team 2) *Let P be a point inside a regular tetrahedron ABCD with edge length 1. Show that* $d(P, AB)+d(P, AC)+d(P, AD)+d(P, BC)+d(P, BD)+d(P, CD) \geq 3/2\sqrt{2}$ *, with equality only when P is the centroid of ABCD. Here d(P,XY) denotes the distance from point P to line* XY*.*

We are only going to prove equality case.

Exercise 11 (1986 Balkan Mathematical Olympiads (BMO) Problem 2 (BUL)) *Let E, F, G,H , K, L respectively be points on the edges AB, BC, CA, DA, DB, DC of a tetrahedron ABCD. If* $AE \cdot BE = BF \cdot CF = CG \cdot AG = DH \cdot AH = DK \cdot BK = DL \cdot CL$*, prove that the points E, F, G, H, K, L lie on a sphere.*

Exercise 12 (2015 Caucasus Mathematical Olympiads grade XI problem 4) *The midpoint of the edge SA of the triangular pyramid of SABC has equal distances from all the vertices of the pyramid. Let SH be the height of the pyramid. Prove that* $BA^2 + BH^2 = CA^2 + CH^2$*.*

Exercise 13 (2009 International Matehmatics Tournament of Towns (ToT) Fall Senior Problem 2) *A, B, C, D, E and F are points in space such that AB is parallel to DE, BC is parallel to EF, CD is parallel to FA, but* $AB \neq DE$*. Prove that all six points lie in the same plane.*

Exercise 14 (2013 Saint Petersburg Mathematical Olympiads grade XI P3) *Let M and N are midpoint of edges AB and CD of the tetrahedron ABCD, AN = DM and CM = BN. Prove that AC = BD.*