29 октября.

$f(x) = x^2$ $\qquad f(-x) = f^2(x)$

а) $x \in (-\pi; \pi)$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2}{\pi}\frac{x^3}{3}\Big|_0^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx \, d x = \frac{2}{\pi}\left(\frac{x^2 \sin x}{n} - \frac{-2nx\cos nx + 2\sin nx}{n^3}\right)\Big|_0^{\pi}$$

$$= \frac{2}{\pi} \cdot \sin(-1)^n +$$

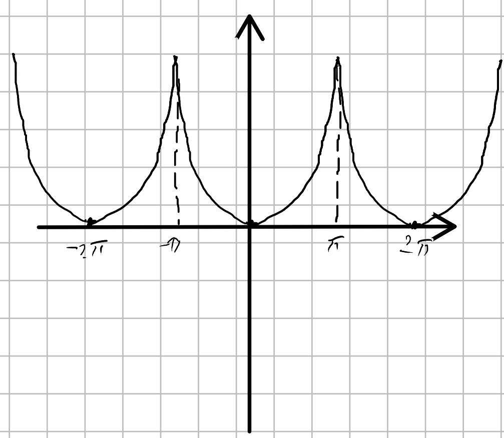
$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \underbrace{\cos nx \, d x}_{d\frac{\sin nx}{n}} = \frac{2}{\pi}\left(x^2 \sin x\Big|_0^{\pi} - \int_0^{\pi} 2x \overbrace{\sin nx \, dx}^{-\frac{1}{n} d \cos nx}\right) =$$

$$= \frac{4}{\pi n^2}\left(x\cos nx\Big|_0^{\pi} - \int_0^{\pi}\cos nx \, dx\right) = \frac{4}{\pi n^2}\left((-1)^n \pi - \frac{\sin x}{n}\Big|_0^{\pi}\right) = \frac{4(-1)^n}{n^2}$$

Тригонометрическая система: $\frac{1}{2}$, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, ...

Ряд Фурье: $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$

$\exists f'(x) \Rightarrow f(x) = \lim_{n\to\infty} S_n(x)$



$$\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$x = \pi$

$$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cdot(-1)^n =$$

$$= \frac{\pi^2}{3} + 4\sum^{\infty}\frac{1}{n^2} \Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{4}\left(\pi^2 - \frac{\pi^2}{3}\right) = \frac{\pi^2}{6}$$

№ 247. Разложить в ряд Фурье периодическую функцию

$f(x) = \operatorname{sign}(\cos x)$

Кусочно-заданная функция

$f(x) = \operatorname{sign}(\cos(x))$

$f(-x) = f(x)$

$a_0 = \frac{2}{\pi}\int_0^{\pi}\operatorname{sign}(\cos x)\,dx = \frac{2}{\pi}\left(\int_0^{\frac{\pi}{2}}dx - \int_{\frac{\pi}{2}}^{\pi}dx\right) = \frac{2}{\pi}\left(x\Big|_0^{\frac{\pi}{2}} - x\Big|_{\frac{\pi}{2}}^{\pi}\right) = 0 =$

$= \frac{2}{\pi}\left(\frac{\sin nx}{n}\Big|_0^{\frac{\pi}{2}} - \frac{\sin nx}{n}\Big|_{\frac{\pi}{2}}^{\pi}\right) = \frac{2}{\pi n}\left(\sin\frac{\pi n}{2} - 0 + \sin\frac{\pi n}{2}\right) = \frac{4}{\pi n}\sin\frac{\pi n}{2}$

$n = 2k \quad a_{2k} = 0$

$n = 2k+1 \quad a_{2k+1} = \frac{4}{\pi(2k+1)}\sin\frac{\pi(2k+1)}{2}$

Порядок проведения коллока.

На лекцию приходить ко второй половине первой пары (к 9:20). Наш коллок