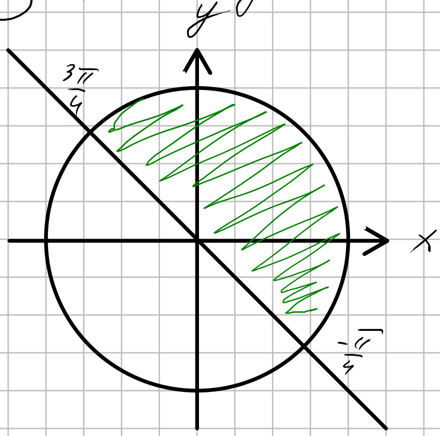
№1.3.

$$\iint\limits_D (2x+y)\,dx\,dy \qquad D=\{x^2+y^2\le R^2,\ y\ge -x\}$$



Обозначим

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \qquad \begin{matrix} r\in[0, R] \\ \varphi\in\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right] \end{matrix}$$

$$x^2+y^2\le R^2$$

$$r^2\cos^2\varphi + r^2\sin^2\varphi \le R^2$$

$$r^2\cancel{\cos^2\varphi} + r^2 - \cancel{r^2\cos^2\varphi} \le R^2$$

$$r^2 \le R^2$$

$$r\le R$$

$$y\ge -x$$

$$r\sin\varphi \ge -r\cos\varphi$$

$$\sin\varphi \ge -\cos\varphi$$

Выполним замену переменной в интеграле.

$$J(r,\varphi) = \frac{D(x,y)}{D(r,\varphi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = (r\cos^2\varphi + r\sin^2\varphi) = r\cdot 1 = r$$

$$\iint\limits_D (2x+y)\,dx\,dy = \iint\limits_D (2r\cos\varphi + r\sin\varphi)\overbrace{r}^{=|J(r,\varphi)|}\,dr\,d\varphi =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_0^R (2r\cos\varphi + r\sin\varphi)\,r\,dr = \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_0^R (2\cos\varphi+\sin\varphi)\,r^2\,dr =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} (2\cos\varphi+\sin\varphi)\,d\varphi \int_0^R r^2\,dr = (1)$$

$$\int_0^R r^2\,dr = \left.\frac{r^3}{3}\right|_0^R = \frac{R^3}{3}$$

$$(1) = \frac{R^3}{3}\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} (2\cos\varphi + \sin\varphi)\,d\varphi = (2)$$

$$\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} (2\cos\varphi + \sin\varphi)\, d\varphi = (\cos\varphi - 2\sin\varphi)\Big|_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} =$$

$$= \left(2\sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}\right) - \left(2\sin\frac{-\pi}{4} - \cos\frac{-\pi}{4}\right) =$$

$$= -\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) + 2\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) + \cos\frac{-\pi}{4} - 2\sin\frac{\pi}{4} =$$

$$= \sin\frac{\pi}{4} + 2\cos\frac{\pi}{4} + \cos\frac{\pi}{4} + 2\sin\frac{\pi}{4} = 3\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right) =$$

$$= 3\sqrt{2}$$

$$(2) = \cancel{3}\sqrt{2} \cdot \frac{R^3}{\cancel{3}} = R^3\sqrt{2}$$