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Kinematics and Trajectory Planning of a Supporting Medical Manipulator for Vascular Interventional Surgery

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***Abstract* - Conventional vascular interventional surgery (VIS) is performed under fluoroscopic guidance by surgeons. In order to reduce X-ray irradiation to the interventional radiologists, catheter operating systems have been developed to assist surgeons. This paper proposed an active supporting medical manipulator, which can neatly adjust and accurately position the catheter operating system. Forward kinematics and inverse kinematics were solved based on kinematics analysis. Quintic polynomial interpolation was adopted in trajectory planning for smooth motion of manipulator. The kinematics and trajectory planning were validated by simulations. Based on kinematics solutions and trajectory planning, experiment showed that the supporting medical manipulator run smoothly and positioned the catheter operating system accurately, which is essential to the safety of medical robot.**

Index Terms* – *Medical Robot, Vascular Interventional Surgery, Kinematics Analysis, Trajectory Planning

I. INTRODUCTION

Vascular interventional surgery (VIS), which is a minimally invasive therapy, means that surgeons operate catheter to move inside blood vessels under the guidance of digital subtraction angiography (DSA) for treatment of vascular disease. In conventional VIS, Surgeons cut an incision in the groin through which a guide wire and catheter are inserted (Fig. 1 A, B). Then, surgeons control the catheter to the target under fluoroscopic guidance. Finally, therapeutic devices (stent or balloon) are passed or drugs are injected with the conduit of catheter (Fig. 1 C-F) [1-3].

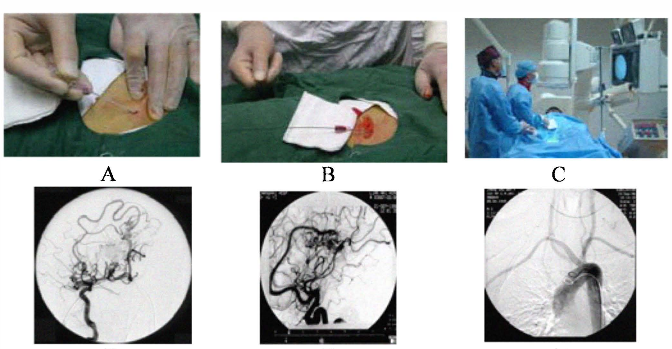

Fig. 1 Procedures of VIS. (A, B) Surgeons insert a guide wire and catheter into blood vessel. (C) Environment of VIS. (D–F) 2D projection images

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In references [1-11], robot-assisted VIS can effectively solve the disadvantages of conventional VIS. However, the existing vascular interventional surgery robots only focus on catheter operating system, which is attached to surgical bed or fixed at the tip of passive manipulator. The catheter operating system can't be neatly and accurately adjusted for catheter operating. Therefore, we developed a novel active supporting medical manipulator to adjust catheter operating system neatly. Kinematics and trajectory planning were analyzed. The procedure and method of kinematics solution avoid a large amount of matrixes inverse multiply operation and the results are simple that it is convenient for trajectory planning. Finally, kinematics and trajectory planning are validated and simulated by using Robot Toolbox in Matlab. Experiment shows that the motion of the supporting medical manipulator was smooth after trajectory planning based on kinematic solutions.

II. THE MEDICAL ROBOT SYSTEM FOR VIS

A. System Overview

As shown in fig. 2, major components of the medical robot system for VIS include the following:

1) A catheter navigator, which is fixed at the tip of the supporting medical manipulator, controls catheter axial (advance and retract) and rotational motions.

2) A supporting medical manipulator is used to accurately position and firmly hold the catheter navigator.

3) A binocular vision is used to guide the supporting medical manipulator.

4) A PC-based interventional workstation is used to provide overall control of the medical robot.

5) A haptic interface device is used to remote control the supporting medical manipulator and the catheter navigator.

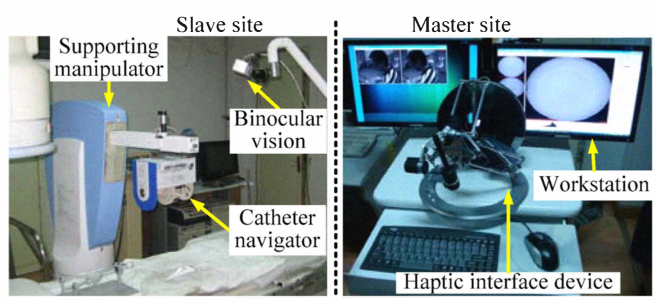
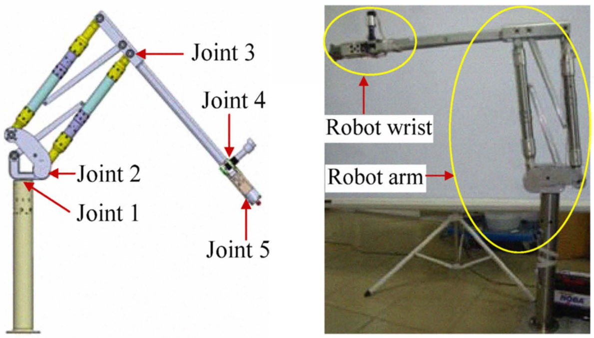


Fig. 2 System overview

The robot-assisted vascular interventional surgery (RVIS) system adopts master-slave structure to protect surgeons from X-ray exposure. The master site is located far away from patient and the slave site is beside operation table.

B. The 5-DOF active supporting medical manipulator

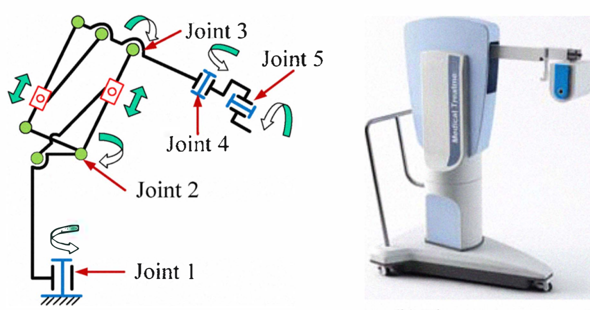
Based on the analysis of position and orientation requirement of the catheter navigator in VIS, the rotation of the catheter navigator around catheter axial (advance and retract) movement is not required. So, we designed a 5-DOF active supporting medical manipulator to adjust the catheter navigator, as shown in fig. 3. The first three DOFs compose robot arm and the remaining two DOFs compose robot wrist.



(a) The CAD model (b) The assembled mechanism

Fig. 3 The 5-DOF active supporting manipulator.

According to the analysis of typical configurations of robot arm patterns, we adopt articulated configuration [12]. In order to diminish the size and improve the stability and positioning accuracy, crank-slider and parallelogram link mechanism are adopted to optimize typical articulated configuration, as shown in fig. 4 (a). Finally, we integrated the mechanic, control unit, cabling, and outward appearance of the supporting medical manipulator, as shown in fig. 4 (b).



(a) The synthesized and optimized structure (b) The prototype

Fig. 4 The structure and prototype of the supporting medical manipulator

The optimized supporting medical manipulator has small-sized structure, large workspace to meet the medical environment, while being able to bear the big load of 4Kg (the weight of the catheter navigator is 3Kg, 25 percent margin of safety). The supporting medical manipulator can adjust the catheter navigator neatly and hold the catheter navigator firmly, so that the catheter, clamped by the catheter navigator, is directed along the vein entry.

III. Kinematics Analysis

A. Kinematics model

We employ Denavit-Hartenberg (D-H) parameters to analysis the kinematics and establish the coordinate system of the supporting medical manipulator. The kinematic chain configuration is shown in fig. 5, and the D-H parameters according to the configuration are presented in table I. Forward kinematics is to calculate the position and orientation of catheter navigator in Cartesian space, given the five joint angles in joint space. Inverse kinematics is to calculate the five joint angles in joint space, given the position and orientation of catheter navigator in Cartesian space. The conversion between Cartesian space and joint space by forward and inverse kinematics is illustrated in fig. 6 [13-15].

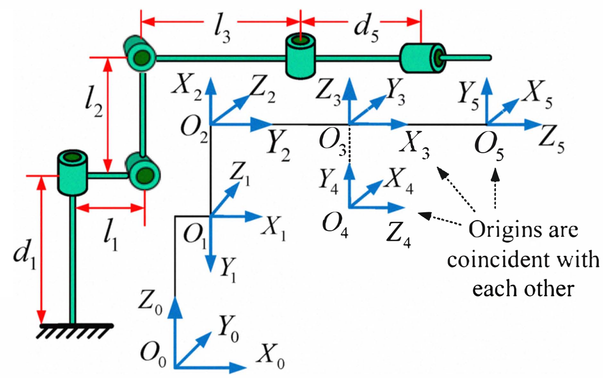


Fig. 5 The kinematic chain configuration

TABLE. I
D-H PARAMETERS

Link	θ_i	d_i	a_i	α_i	Rang of joint (degree)
1	θ_1	d_1	l_1	-90°	-90°~90°
2	θ_2	0	l_2	0°	-125°~-50°
3	θ_3	0	l_3	90°	55°~110°
4	θ_4	0	0	90°	45°~135°
5	θ_5	0	0	0°	-45°~45°

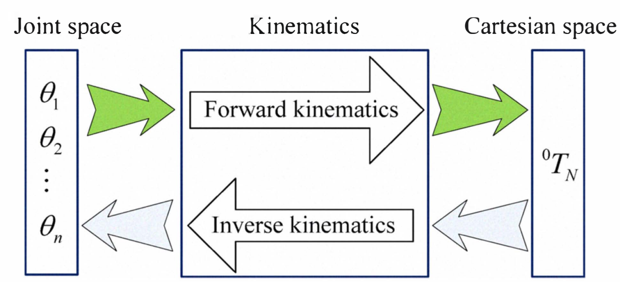


Fig. 6 The relationship of forward and inverse kinematics

B. Forward kinematics

According to the D-H coordinate system and D-H parameters, the homogeneous transformation matrix of link i $(i = 1 \cdots 5)$ is

$$
{}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i \times c\alpha_i & s\theta_i \times s\alpha_i & a_i \times c\theta_i \\ s\theta_i & c\theta_i \times c\alpha_i & -c\theta_i \times s\alpha_i & a_i \times s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

Where, $s\theta_i = \sin\theta_i \quad c\theta_i = \cos\theta_i$

$s\alpha_i = \sin\alpha_i \quad c\alpha_i = \cos\alpha_i$

Starting from the base coordinate frame to the catheter navigator coordinate frame, the transformation matrix between two adjacent coordinate frames can be got by substituting the D-H parameters into formula (1). The transformation matrixes of the supporting medical manipulator are given as follows,

$$
{}^0T_1 = \begin{bmatrix} c_1 & 0 & -s_1 & l_1 \times c_1 \\ s_1 & 0 & c_1 & l_1 \times s_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 \times c_2 \\ s_2 & c_2 & 0 & l_2 \times s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2T_3 = \begin{bmatrix} c_3 & 0 & s_3 & l_3 \times c_3 \\ s_3 & 0 & -c_3 & l_3 \times s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^4T_5 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

Where, $c_i = \cos\theta_i \quad s_i = \sin\theta_i \quad (i = 1 \cdots 5)$

By multiplying rightward these transformation matrix one by one form the first to the fifth, the forward kinematic equations of the supporting manipulator is given as follow,

$$
{}^0T_5 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 \tag{3}
$$

C. Inverse kinematics

Inverse kinematics is to solve the joint variables as the position and orientation of catheter navigator with respect to the base is given by formula (4). The inverse kinematics equation is a nonlinear one due to nonlinear terms in equation [15-17]. The kinematics problem of supporting medical manipulator is represented by formula (5).

$$
{}^{base}T_{end-effector} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

$$
{}^0T_5 = {}^{base}T_{end-effector} \tag{5}
$$

Where, (n_x, n_y, n_z), (o_x, o_y, o_z), and (a_x, a_y, a_z) are the normal vector, approach vector, and orientation vector of the catheter navigator orientation, (p_x, p_y, p_z) is the position vector of the catheter navigator. According to formulas (2) and (3), we can get

$$
{}^1T_5 = {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5
$$

$$
= \begin{bmatrix} c_{23}c_4c_5 + s_{23}s_5 & -c_{23}c_4s_5 + s_{23}c_5 & c_{23}s_4 & c_{23}l_3 + c_2l_2 \\ s_{23}c_4c_5 - c_{23}s_5 & -s_{23}c_4s_5 - c_{23}c_5 & s_{23}s_4 & s_{23}l_3 + s_2l_2 \\ s_4c_5 & -s_4s_5 & -c_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}
$$

Where, $c_{23} = \cos(\theta_2 + \theta_3) \quad s_{23} = \sin(\theta_2 + \theta_3)$

At the same time, according to formulas (2), (4) and (5), we can get

$$
{}^1T_5 = \left({}^0T_1\right)^{-1} {}^0T_5
$$

$$
= \begin{bmatrix} c_1n_x + s_1n_y & c_1s_x + s_1s_y & c_1a_x + s_1a_y & c_1p_x + s_1p_y - l_1 \\ -n_z & -s_z & -a_z & -p_z + d_1 \\ -s_1n_x + c_1n_y & -s_1s_x + c_1s_y & -s_1a_x + c_1a_y & -s_1p_x + c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}
$$

1) The solution of joint angle θ_1

Revolute joint variable θ_1 can be determined by equating (3,4) matrix elements of right side in formula (6) and (7). We can get

$$
-s_1p_x + c_1p_y = 0 \tag{8}
$$

Then, $\theta_1 = \arctan\left(p_y/p_x\right)$ or $\theta_1 = \arctan\left(p_y/p_x\right) + \pi$ (9)

2) The solution of joint angle θ_3

Revolute joint variable θ_3 can be determined by equating (1,4) matrix elements and equating (2,4) matrix elements of right side in formula (6) and (7). We can get

$$
\left(c_{23}l_3 + c_2l_2\right)^2 + \left(s_{23}l_3 + s_2l_2\right)^2 = (c_1p_x + s_1p_y - l_1)^2 + (-p_z + d_1)^2 \tag{10}
$$

Then,

$$
\theta_3 = \pm\arccos\left(\frac{(c_1p_x + s_1p_y - l_1)^2 + (-p_z + d_1)^2 - l_2^2 - l_3^2}{2l_2l_3}\right) \tag{11}
$$

3) The solution of joint angle θ_2

Revolute joint variable θ_2 can be determined by equating (2,4) matrix elements of right side in formula (6) and (7). We can get

$$
s_2\left(c_3l_3 + l_2\right) + c_2s_3l_3 = -p_z + d_1 \tag{12}
$$

Then, $\theta_2 = \pm\arcsin\left(\dfrac{-p_z + d_1}{\sqrt{(c_3l_3 + l_2)^2 + (s_3l)^2}}\right) - \theta_\varphi$ (13)

Where, $\sin(\theta_2 + \theta_\varphi) = \dfrac{-p_z + d_1}{\sqrt{(c_3l_3 + l_2)^2 + (s_3l)^2}}$

4) The solution of joint angle θ_5

Revolute joint variable θ_5 can be determined by equating (1,1) matrix elements and equating (2,1) matrix elements of right side in formula (6) and (7). We can get

$$
\begin{cases} s_{23}c_{23}c_4c_5 + s_{23}s_{23}s_5 = s_{23}\left(c_1n_x + s_1n_y\right) \\ c_{23}s_{23}c_4c_5 - c_{23}c_{23}s_5 = -c_{23}n_z \end{cases} \tag{14}
$$

Then, $$\theta_5 = \pm\arcsin\left(\frac{s_{23}\left(c_1 n_x + s_1 n_y\right) + c_{23} n_z}{s_{23}s_{23} + c_{23}c_{23}}\right) \tag{15}$$

5) The solution of joint angle θ_4

Revolute joint variable θ_4 can be determined by equating (1,1) matrix elements and equating (2,1) matrix elements of right side in formula (6) and (7). We can get

$$\begin{cases} c_{23}c_{23}c_4c_5 + c_{23}s_{23}s_5 = c_{23}\left(c_1 n_x + s_1 n_y\right) \\ s_{23}s_{23}c_4c_5 - s_{23}c_{23}s_5 = -s_{23}n_z \end{cases} \tag{16}$$

Then, $$\theta_4 = \pm\arccos\left(\frac{c_{23}\left(c_1 n_x + s_1 n_y\right) - n_z s_{23}}{c_{23}c_{23}c_5 + s_{23}s_{23}c_5}\right) \tag{17}$$

It is obvious that the inverse kinematics of the supporting medical manipulator has complete analytical solution (closed form solution) in the workspace.

IV. Trajectory Planning

The primary role of the supporting manipulator is to position precisely the catheter navigator without strict path restriction, which is point-to-point (PTP) motion. In order to ensure smooth movement and avoid sudden change of displacement, velocity and acceleration of each joint, we planned trajectory conveniently and reliably with via points by polynomial interpolating algorithm, which mainly includes cubic polynomial function interpolation and quintic polynomial function interpolation. The characteristics of cubic and quintic polynomial function interpolation are validated and contrasted by simulation in Matlab with one via point, as shown from fig. 7 to fig. 9. The joint angle and angular velocity constraints of starting point A, via point B, and desired point C are listed in table. II. The angular velocity of via point is calculated by heuristic algorithm in joint space and the interval of interpolation points is 50ms [15, 18].

TABLE. II
D-H Parameters

Points	A	B	C
Angle values (rad)	$\pi/2$	$3\pi/4$	π
Angular velocity (rad/s)	0		0
Time (s)	0	5	10

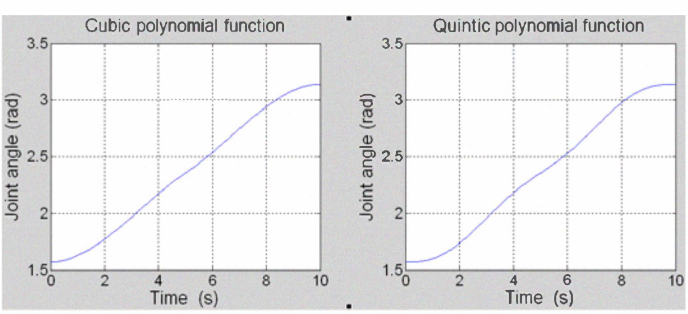


Fig. 7 Trajectory simulation of joint angle by cubic and quintic polynomial function interpolation

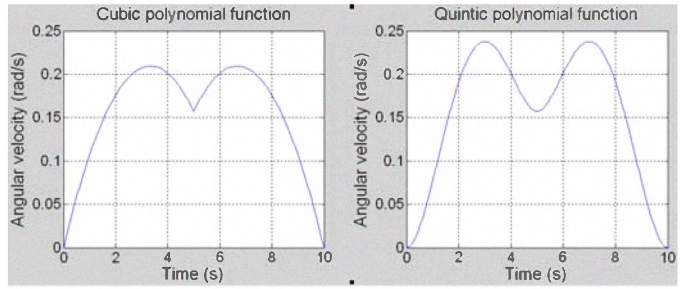


Fig. 8 Trajectory simulation of angular velocity by cubic and quintic polynomial function interpolation

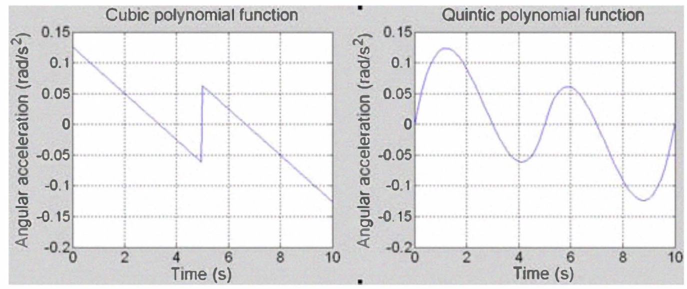


Fig. 9 Trajectory simulation of angular acceleration by cubic and quintic polynomial function interpolation

According to the comparison, the angular acceleration has a sudden change at via point by cubic polynomial function interpolation, whereas quintic polynomial can guarantee the continuity of joint angle, angular velocity and angular acceleration, which is important to smooth motion of the supporting medical manipulator and safety of patients. So, we adopt quintic polynomial function interpolation for trajectory planning.

V. Simulations and Experiments

Firstly, forward kinematics and inverse kinematics were validated by simulation with Matlab. Secondly, trajectory planning was simulated in Matlab to valid smooth motion of the supporting medical manipulator. At last, pre-clinical experiment was conducted with a transparent glass vascular model to assess kinematics and trajectory planning of the supporting medical manipulator in RVIS.

A. Kinematics verification in Matlab

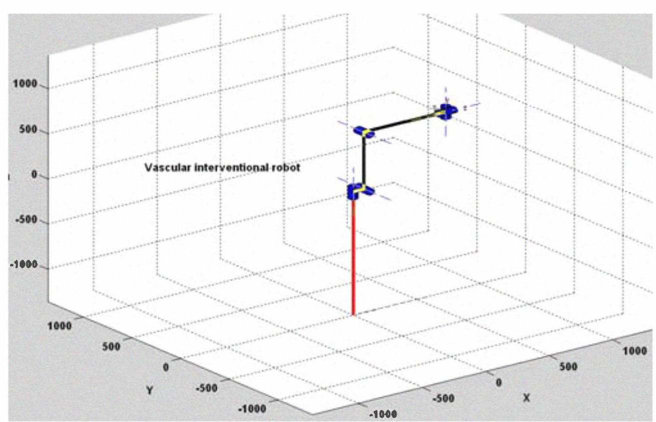


Fig. 10 Robot model in MATLAB

The robot model can be established by robot() function of Robot Toolbox in Matlab, as shown in fig. 10. Let the matrix of five joint angles is $q=[0,-\pi/2,\pi/2,\pi/2,0]$. By using fkine() function of Robot Toolbox in Matlab, the position and orientation matrix of catheter navigator can be get as follows.

```
>>T1 = 1.0e+003 *
        0.0000   -0.0000    0.0010    0.7332
        0.0010    0.0000   -0.0000   -0.0000
        0         0.0010    0.0000    1.3250
        0         0         0         0.0010
```

By using forward kinematics formula (3), the position and orientation matrix of catheter navigator can be get as follows.

$$T2=\begin{bmatrix} 0.0000 & 0.0000 & 1.0000 & 733.2200 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 1325.0000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As T1=T2, the analysis of forward kinematics is correct.

Let the position and orientation matrix of catheter navigator is,

$$T=\begin{bmatrix} 0 & 0 & 1 & 660 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By using ikine() function of Robot Toolbox in Matlab, the matrix of five joint angles can be get as follows.

```
>>theta1 = 0.0000   -1.6680   1.8577   1.5700   -0.0000
```

By using inverse kinematics formulas (9), (11), (13), (15), (17), the matrix of five joint angles can be get as follows.

$$theta2=[0 \quad -1.6678 \quad 1.8578 \quad 1.5708 \quad 0]$$

As theta1=theta2, the analysis of inverse kinematics is correct.

B. Simulation of trajectory planning in Matlab

The catheter navigator moved from starting point D, passed by via point E at 5 second, and then arrived at desired point F at 10 second, as listed in table. III. The interval of interpolation points is 50ms and the results are shown from fig. 11 to fig. 13.

From the results, we can see that trajectories with quintic polynomial function interpolation of joint angle, angular velocity and angular acceleration are continuous and have non sudden change, which guarantee smooth motion of the supporting medical manipulator from starting point to desired point

TABLE. III
D-H PARAMETERS

Points	θ_1 (rad)	θ_2 (rad)	θ_3 (rad)	θ_4 (rad)	θ_5 (rad)
D	0	$-(\pi/2)$	$\pi/2$	$\pi/2$	0
E	$\pi/4$	$-(\pi/3)$	$5\pi/9$	$11\pi/18$	$\pi/6$
F	$\pi/3$	$-(5\pi/18)$	$11\pi/18$	$13\pi/18$	$\pi/4$

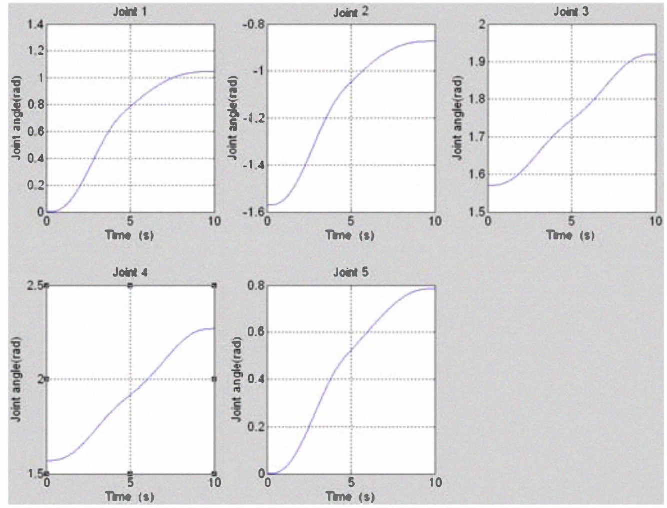


Fig. 11 Trajectory planning of joint angle

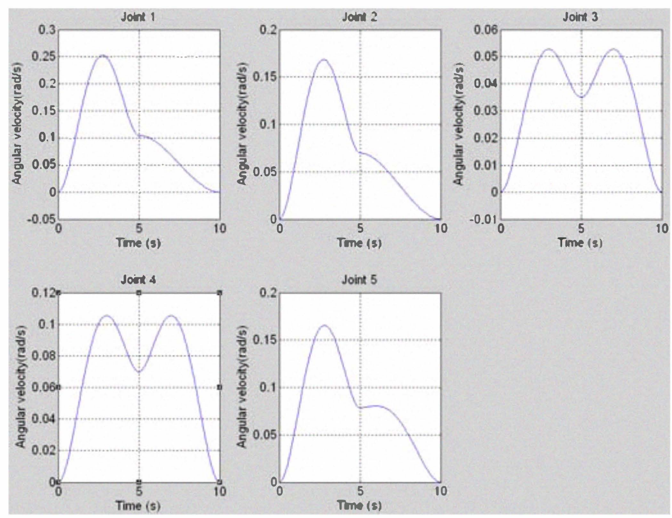


Fig. 12 Trajectory planning of angular velocity

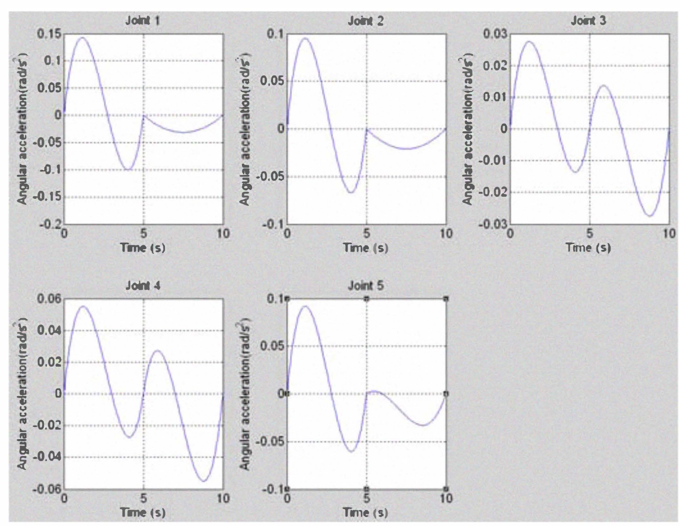


Fig. 13 Trajectory planning of angular acceleration

C. Vascular model experiment

The experiment was conducted with a transparent glass vascular model to assess kinematics and trajectory planning of the supporting medical manipulator in RVIS, as shown in fig. 14.

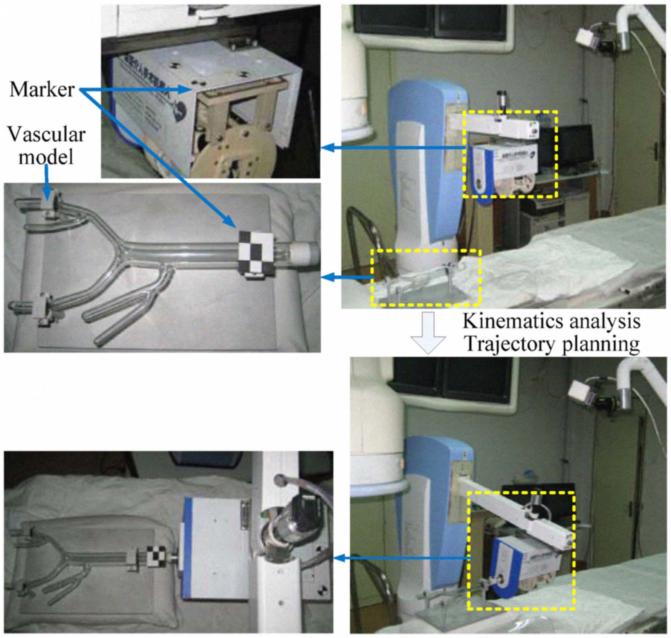


Fig. 14 The transparent glass vascular model experiment

During experiment, the catheter navigator was adjusted from starting point to desired point reposefully by the supporting medical manipulator after kinematics solutions and trajectory planning. The position and orientation were given by binocular vision with markers.

VI. CONCLUSION

A novel active supporting medical manipulator, the end of which is fixed with catheter navigator, was developed to adjust catheter navigator neatly. Inverse kinematics solutions of the supporting medical manipulator were found in closed form, which avoided from computation cost and time consuming. Matrixes inverse multiply was used once and the expressions of solutions are simple, which is convenient for trajectory planning. Simulations in Matlab validated the correctness of kinematic solutions and continuity of trajectory functions in joint space, which is essential to smooth motion of joints. At last, vascular model experiment showed that the supporting medical manipulator moved smoothly by trajectory planning with quintic polynomial function interpolation, positioned the catheter operating system accurately by inverse kinematic solutions, and hold the catheter operating system firmly by optimized structure.

ACKNOWLEDGMENT

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REFERENCES

[1] Liu Da, Dapeng Zhang, Tianmiao Wang. "Overview of the vascular interventional robot," *The International Journal of Medical Robotics and Computer Assisted Surgery*, vol. 4, issue. 4, pp. 289–294, Sep 2008.

[2] Tianmiao Wang, Dapeng Zhang, Liu Da. "Remote-controlled vascular interventional surgery robot," *The International Journal of Medical Robotics and Computer Assisted Surgery*, vol. 6, issue. 2, pp. 194–201, Mar 2010.

[3] LU Wangsheng, LIU Da, TIAN Zengmin, et al. "The analysis of Key Technologies of Intravascular Intervention Surgical Robot," *Journal of Biomedical Engineering Research*, vol. 28, no. 4, pp. 303–306, 2009. (in Chinese)

[4] C. Tercero, S. Ikeda, T. Uchiyama, et al. "Autonomous catheter insertion system using magnetic motion capture sensor for endovascular surgery," *The International Journal of Medical Robotics and Computer Assisted Surgery*, vol. 3, issue. 1, pp. 52–58, Dec 2006.

[5] Govindarajan Srimathveeravalli, Thenkurussi Kesavadas, Xinyan Li. "Design and fabrication of a robotic mechanism for remote steering and positioning of interventional devices," *The International Journal of Medical Robotics and Computer Assisted Surgery*, vol. 6, issue. 2, pp. 160–170, Feb 2010.

[6] Beyar R, Gruberg L, Deleanu D, et al. "Remote-Control Percutaneous Coronary Interventions: Concept, Validation, and First-in-Humans Pilot Clinical Trial," *Journal of the American College of Cardiology*, vol. 47, issue. 2, pp. 296-300, January 2006,

[7] Shuxiang Guo, Yamaji, H, Yousuke Kita, et al. "A novel active catheter system for Ileus treatment," *2008 IEEE International Conference on Automation and Logistics*, pp. 67–72, 2008.

[8] Shuxiang Guo, Jian Guo, Nan Xiao, et al. "Control and experimental results of a catheter operating system," *IEEE International Conference on Robotics and Biomimetics, 2008*, pp. 91–95, 2009.

[9] E Marcelli, L Cercenelli, G Plicchi, et al. "A Novel Telerobotic System to Remotely. Navigate Standard Electrophysiology Catheters," *Computers in Cardiology, 2008*, pp. 137–140, 2008.

[10]Tercero, C., Ikeda, S., Uchiyama, T., et al. "Catheter Insertion Mechanism and Feedback Control using Magnetic Motion Capture Sensor," *SICE-ICASE International Joint Conference 2006*, pp. 1856– 1859, 2006.

[11]Thakur, Y., Bax, J.S., Holdsworth, D.W., et al. "Design and Performance Evaluation of a Remote Catheter Navigation System," *IEEE Transactions on Biomedical Engineering*, vol. 56, issue. 7, pp. 1901–1908, 2009.

[12]LIU Da, WANG Tian-miao, et al. "Structure synthesis of surgical robot orienting to minimally invasive surgery," *ROBOT*, vol. 25, no. 2, pp. 132–135, March 2003. (in Chinese)

[13]Banka N, Lin YJ. "Mechanical design for assembly of a 4-DOF robotic arm utilizing a top-down concept," *Robotica(2003)*, vol. 21, issue. 5, pp. 567-573, 2003.

[14]Xing-guang DUAN, Gui-bin BIAN, Hong-hua ZHAO, Xing-tao WANG, Qiang HUANG. "A medical robot for needle placement therapy in liver cancer," *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, vol. 11, no. 4, pp. 263—269, 2010.

[15]Yujie Cui, Pu Shi, Jianning Hua. "Kinematics Analysis and Simulation of a 6-DOF Humanoid Robot Manipulator," *2010 2nd International Asia Conference on Informatics in Control, Automation and Robotics (CAR)*, pp. 246–249, 2010.

[16]Dequan Guo, Hui Ju, Yuqin Yao, et al. "Efficient Algorithms for the Kinematics and Path Planning of Manipulator," *2009 International Conference on Artificial Intelligence and Computational Intelligence*, pp. 282–287, 2009.

[17]Kucuk, S., Bingul, Z. "The inverse kinematics solutions of industrial robot manipulators," *Proceedings of the IEEE International Conference on Mechatronics, 2004*, pp. 274–279, 2004.

[18]Xing-Guang Duan, Gui-Bin Bian, Yuan-Feng Li, et al. "Trajectory planning and 3-D image reconstruction of an ultrasound guided robot aiming radio frequency ablation surgery," *7th World Congress on Intelligent Control and Automation, 2008*, pp. 8265–8270, 2008.