

Manipulator Motion Planning for Stabilizing a Mobile-Manipulator

Qiang HUANG and Shigeki SUGANO

Department of Mechanical Engineering

School of Science and Engineering, Waseda University

3-4-1 Okubo, Shinjuku-ku, Tokyo, 169, Japan

e-mail: 63a504@cfi.waseda.ac.jp, sugano@cfi.waseda.ac.jp

Abstract

The stability of a vehicle-mounted mobile manipulator has a close relation with the vehicle's motion, the manipulator's posture and motion, and the end-effector's force. The purpose of this study is to derive the cooperative motions of the vehicle and the manipulator for a stabilization which is compatible with task operation, so that the mobile manipulator can successfully accomplish tasks in environments with various disturbances. The authors have already proposed the stability concepts based on the ZMP criterion to discuss the stabilization and the task operation, and have presented the method of ZMP moved path by a stability potential field to maintain the stability for a mobile manipulator. In this paper, based on the above-mentioned considerations, the manipulator compensatory motion is discussed for stabilizing the mobile manipulator while the vehicle is moving along a given motion.

1: Introduction

Many future applications of robotic systems will require that manipulators operate tasks while being carried by moving vehicles. However, different from a manipulator fixed to the floor tightly, the interaction between the manipulator and the vehicle[1][2], and stability (overturning)[3]-[5] are important problems in controlling such a vehicle-mounted mobile manipulator.

Since the stability of a mobile manipulator has a close relation with the vehicle's motion, the manipulator's posture and motion, and the end-effector's force, it can be said that previous work [3]-[5] on stability control has not sufficiently considered the dynamics of the robot. Furthermore, hardly any research has considered the influence of environmental disturbances. Therefore, the objective of this study is to derive the cooperative motions of the vehicle and the manipulator for a stabilization which is compatible with task operation, so that the mobile manipulator can successfully accomplish tasks in environments with various disturbances.

If we consider the stability of a mobile manipulator executing a target task, we generally have three levels:

- (1) When the vehicle moves along a given motion, deriving the compensatory motion of the manipulator for stabilizing the whole system.
- (2) When the mobile manipulator executes a task in which the motions of the vehicle and the manipulator's endpoint, respectively, are given, deriving the posture changing of the manipulator for stabilizing the whole system.
- (3) When the mobile manipulator executes a task in which only the task information (trajectory, force etc.) of the manipulator's endpoint is given, deriving the cooperative motion of the vehicle and the manipulator.

In level (1), there are no restraints in the endpoint of the manipulator, and a manipulator with two degrees of freedom can stabilize the mobile manipulator. In level (2), it is required that the mobile manipulator move with stability and follow the trajectory of the manipulator's endpoint simultaneously, that is, the stabilization must be compatible with task operation. In such a case, the manipulator must have redundancy. In level (3), it is necessary that the vehicle and the manipulator cooperate to execute the target task by exploiting the vehicle's mobility and the manipulator's manipulation based on level (2).

In order to realize the above motion controls of the mobile manipulator, the authors [6][7] have already proposed stability concepts such as the stability degree and the valid stable region based on the ZMP (Zero Moment Point) criterion. And as a control scheme for maintaining or recovering stability, the planning method of a ZMP moved path by a stability potential field has been presented.

In this paper, the manipulator compensatory motion for stabilizing the mobile manipulator of level (1) is discussed. In section 2, the stability criteria are explained briefly. In section 3, the algorithm of the compensatory motion is formulated: the corrected ZMP trajectory for maintaining the stability of the mobile manipulator is first planned by a potential method, then the compensatory motion of the manipulator is obtained by an iterative algorithm [9] using

the FFT (Finite Fourier Transform [10]) method. Simulation example and discussion are discussed in section 4 and section 5, respectively.

2: Criteria for Stability Evaluation

Some researches have already used the ZMP criterion to distinguish the stability for walking robots. However, different from a walking robot that requires only stability, the mobile manipulator must move with stability and operate tasks simultaneously. To discuss the problem of simultaneous stabilization and task operation, stability concepts such as the stability degree and the valid stable region based on the ZMP criterion have been proposed [6].

ZMP Criterion [8]: The ZMP is defined as the point on the ground about which the sum of all the moments of active force is equal to zero. If ZMP is inside the support polygon (in the following the support polygon is named the stable region), the mobile manipulator is stable.

The Stability Degree is the quantitative measure of a stable extent for a mobile manipulator according to the relationship between the ZMP position and the stable region. The longer the minimal distance from the ZMP to the boundary is, the larger the stability degree is (the maximum of the stability degree is one).

The Valid Stable Region is the area of the ZMP with the safety room. Here, the safety room is the stable room in which the stability degree will not become negative under the disturbance of assumed conditions.

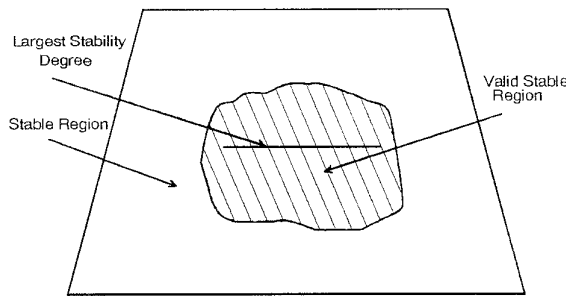


Fig. 1 Valid stable region and stable region

As outlined above, the rule of stabilization and task operation is given as follows (Fig. 1):

In the case of the ZMP being inside the valid stable region, the mobile manipulator is still stable even if it is influenced by disturbances. Thus it is possible for the mobile manipulator to be used for task operation even without stabilization. In the case of the ZMP being inside the stable region but still outside the valid stable region, the mobile manipulator will become unstable if it is influenced by disturbances. Therefore, it is necessary for the mobile manipulator to be controlled for stability and task at the same time. And in the case of the ZMP being outside the stable

region, the mobile manipulator is unstable. In such a case it is necessary that the mobile manipulator be controlled only for stability.

3: Compensatory Motion of the Manipulator

In this section, the compensatory motion of the manipulator for maintaining the stability of the mobile manipulator is discussed when the vehicle moves along a given motion. First, when the vehicle moves along a given motion and the manipulator holds an initial posture without motion, compute the ZMP trajectory (in the following named the *un-corrected ZMP trajectory*). Then, for the part of the un-corrected ZMP trajectory outside the valid stable region, plan the *corrected ZMP trajectory* by the gradient method. Finally, solve the compensatory motion of the manipulator for executing the corrected ZMP trajectory. The algorithm of motion planning is shown in Fig. 2.

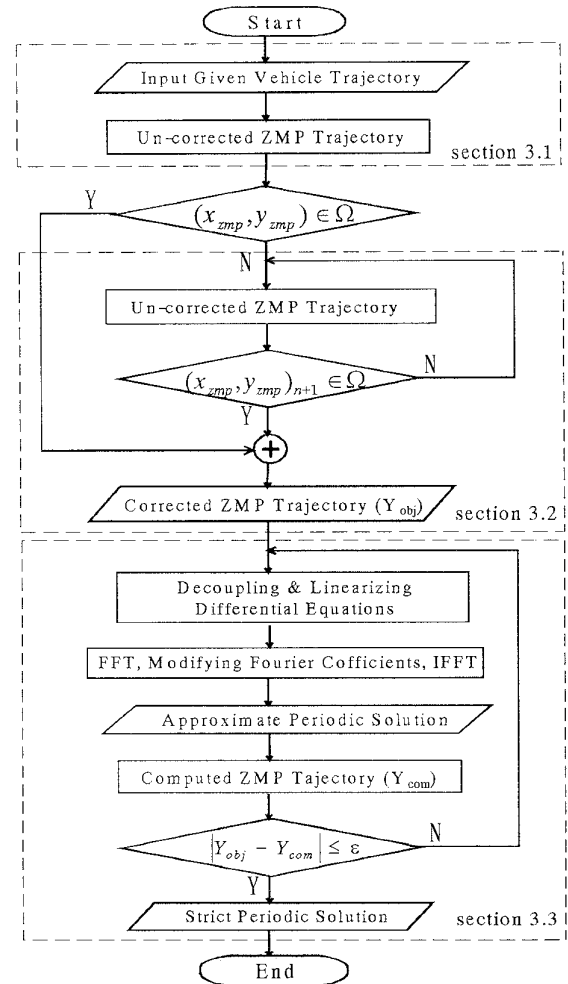


Fig. 2. Algorithm of motion planning for maintaining stability

3.1: Computing the Un-corrected ZMP Trajectory

For convenience, a mobile manipulator including the vehicle, the manipulator and the payload, is considered as a system of particles. In the case of no external force and moment, the ZMP equation is given as follows [9]:

$$x_{zmp} = \frac{\sum_{i=1}^n m_i (\ddot{z}_i + g) x_i - \sum_{i=1}^n m_i \ddot{x}_i z_i}{\sum_{i=1}^n m_i (\ddot{z}_i + g)} \quad (1)$$

$$y_{zmp} = \frac{\sum_{i=1}^n m_i (\ddot{z}_i + g) y_i - \sum_{i=1}^n m_i \ddot{y}_i z_i}{\sum_{i=1}^n m_i (\ddot{z}_i + g)} \quad (2)$$

where m_i is the mass of particle i , $(x_{zmp}, y_{zmp}, 0)$ is the coordinate of ZMP, g is the gravitational acceleration, and (x_i, y_i, z_i) is the coordinate of particle i on the absolute Cartesian coordinate system O-XYZ.

For convenience in computation, the ZMP will be expressed in the vehicle Cartesian system. The relationship between the ZMP in O-XYZ and the one in O'-X'Y'Z' is as follows:

$$\begin{bmatrix} x_{zmp}, y_{zmp}, 0 \end{bmatrix}^T = R \times \begin{bmatrix} \bar{x}_{zmp}, \bar{y}_{zmp}, 0 \end{bmatrix}^T \quad (3)$$

where R is a 3×3 homogeneous transformation matrix about O-XYZ and O'-X'Y'Z', and the superscript T denotes the transpose operation.

Some part of the un-corrected ZMP trajectory might be outside the stable region, or inside the stable region but still outside the valid stable region in the case of the manipulator without motion. To ensure stability, it is necessary that the ZMP be moved into the valid stable region.

3.2: Computing the Corrected ZMP Trajectory by a Potential Method

Since there are multiple moved paths for the ZMP from outside the stable region to inside the stable region, and from inside the stable region to inside the valid stable region, it is necessary to select a suitable ZMP moved path.

Here, the ZMP moved path is planned by a stability potential field. In which the center of the stable region is regarded as the goal state of stability, and the place is regarded as the prohibitive state of stability where the minimal distance from the ZMP outside the stable region to the boundary of the stable region is longer than some distance [7]. By this method, the stability degree of the ZMP moved path near the prohibitive state will increase almost at a maximal rate. And the stability degree of the ZMP moved path near the goal state will not become small, also the ZMP approaches the center of the stable region almost in a straight

line.

For the part of the un-corrected ZMP trajectory outside the valid stable region given in section 3.1, solve for the corrected ZMP trajectory by the gradient method. Specifically, let (x_{zmp}, y_{zmp}) be the ZMP position of a point in the un-corrected trajectory, if (x_{zmp}, y_{zmp}) is outside the valid stable region, then compute the corrected ZMP position as follows:

$$(x_{zmp}, y_{zmp})_{n+1}^T = (x_{zmp}, y_{zmp})_n^T - \Delta S \times \frac{\text{grad}\Phi}{|\text{grad}\Phi|} \quad (4)$$

here, n is the number of times of correcting. $(x_{zmp}, y_{zmp})_n$ is the ZMP position corrected by n times. ΔS is the changed amount of the ZMP position at any one time, and $\text{grad}\Phi$ is the gradient of the potential.

If $(x_{zmp}, y_{zmp})_{n+1}$ does not enter into the valid stable region, then the next ZMP position is computed by equation (4) again. By iterating the above operation, and ZMP position is regarded as the corrected ZMP position when the ZMP enters into the valid stable region.

After correcting all points of the part outside the valid stable region in the un-corrected ZMP trajectory, the corrected ZMP trajectory is obtained by summing the above corrected part and the original part inside the valid stable region.

3.3: Computing the Compensatory Motion of the Manipulator

In this section, the compensatory motion for executing the corrected ZMP trajectory in section 3.2 is discussed. By modifying equations (1) (2), the compensatory terms of the manipulator are regarded as unknown terms on the left-hand side, and the rest are on the right-hand side, indicated by $\psi_1(t)$, $\psi_2(t)$, respectively.

$$\sum_{i=2}^n m_i \ddot{\bar{z}}_i \bar{x}_{zmp} - \sum_{i=2}^n m_i (\ddot{\bar{z}}_i + g) \bar{x}_i + \sum_{i=2}^n m_i (\ddot{x}_i + \ddot{x}_q) (\bar{z}_i + z_q) = \psi_1(t) \quad (5)$$

$$\sum_{i=2}^n m_i \ddot{\bar{z}}_i \bar{y}_{zmp} - \sum_{i=2}^n m_i (\ddot{\bar{z}}_i + g) \bar{y}_i + \sum_{i=2}^n m_i (\ddot{y}_i + \ddot{y}_q) (\bar{z}_i + z_q) = \psi_2(t) \quad (6)$$

where,

$$\psi_1(t) = -(\sum_{i=1}^n m_i g + m_1 \ddot{\bar{z}}_1) \bar{x}_{zmp} + m_1 (\ddot{\bar{z}}_1 + g) \bar{x}_1 - m_1 (\ddot{x}_1 + \ddot{x}_q) (\bar{z}_1 + z_q),$$

$$\psi_2(t) = -(\sum_{i=1}^n m_i g + m_1 \ddot{\bar{z}}_1) \bar{y}_{zmp} + m_1 (\ddot{\bar{z}}_1 + g) \bar{y}_1 - m_1 (\ddot{y}_1 + \ddot{y}_q) (\bar{z}_1 + z_q),$$

are known terms such as the terms of the vehicle and the corrected ZMP trajectory, $(\bar{x}_{zmp}, \bar{y}_{zmp}, 0)$, $(\bar{x}_i, \bar{y}_i, \bar{z}_i)$ are the coordinate of the ZMP and the coordinate of particle i in O'-X'Y'Z', respectively. And (x_q, y_q, z_q) is the coordinate of the origin point O' of O'-X'Y'Z' in O-XYZ.

If there are no restrains in the endpoint of the manipulator, it is possible for a manipulator with two degrees

of freedom to compensate the ZMP of X-Y plane. In the following, we discuss the deriving method of the compensatory motion about the mobile manipulator shown in Fig. 3.

The particle coordinate $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$ of the vehicle is known because its trajectory is given. And the particle coordinate $(\bar{x}_2, \bar{y}_2, \bar{z}_2)$ of link 1 will not change even if link 1 rotates along the Z'-axis in O'-X'Y'Z'. Therefore equations (5) (6) can be modified as follows:

$$\ddot{\bar{z}}_3 \bar{x}_{zmp} - (\ddot{\bar{z}}_3 + g) \bar{x}_3 + \bar{z}_3 (\ddot{\bar{x}}_3 + \ddot{x}_q) = \psi'_1(t) \quad (7)$$

$$\ddot{\bar{z}}_3 \bar{y}_{zmp} - (\ddot{\bar{z}}_3 + g) \bar{y}_3 + \bar{z}_3 (\ddot{\bar{y}}_3 + \ddot{y}_q) = \psi'_2(t) \quad (8)$$

here, $(\bar{x}_3, \bar{y}_3, \bar{z}_3)$ is the particle coordinate of link 2 of the manipulator.

Equations (7) (8) are two-rank non-linear differential equations. Therefore it is difficult to derive analytic solutions from them. The solutions of such equations can be obtained by the iterative algorithm using a FFT method [9], and we briefly outline of this approach.

For decoupling and linearizing equations (7) (8), assume $\ddot{\bar{z}}_3$ is constant, that is $\ddot{\bar{z}}_3 = 0$, then the decoupled linearized differential equations are as follows:

$$\ddot{\bar{x}}_3 - g \bar{x}_3 = \psi''_1(t) \quad (9)$$

$$\ddot{\bar{y}}_3 - g \bar{y}_3 = \psi''_2(t) \quad (10)$$

here, $\psi''_1(t) = \psi'_1(t) - \bar{z}_3 \ddot{x}_q$, $\psi''_2(t) = \psi'_2(t) - \bar{z}_3 \ddot{y}_q$.

Considering the whole motion from beginning motion to ending motion as a periodic motion, the approximate periodic solutions of equations (9) (10) are obtained in the following way:

For equation (9), \bar{x}_3 and $\psi''_1(t)$ are expanded by the Fourier series.

$$\bar{x}_3(t) = \frac{A_0}{2} + \sum_{k=0}^N \{A_k \cos(k\omega t) + B_k \sin(k\omega t)\} \quad (11)$$

$$\psi''_1(t) = \frac{a_0}{2} + \sum_{k=0}^N \{a_k \cos(k\omega t) + b_k \sin(k\omega t)\} \quad (12)$$

Substituting these in equation (9) and comparing the coefficient at each frequency $k\omega t$ ($k = 0, 1, \dots$), we can get:

$$\begin{aligned} k=0 \quad A_0 &= -\frac{a_0}{g} \\ k \neq 0 \quad A_k &= -\frac{a_k}{g + \bar{z}_3(2\pi k f_0)^2} \\ B_k &= -\frac{b_k}{g + \bar{z}_3(2\pi k f_0)^2} \end{aligned} \quad (13)$$

Accordingly, if these are expanded by inverse Fourier series, the solution of \bar{x}_3 is solved. And \bar{y}_3 is also solved by the same method. However the solutions are only approximate.

Then, by substituting the above approximate periodic solutions in the nonlinear equations (1) (2), compute the ZMP trajectory (in the following named *computed ZMP trajectory*), and the error between the corrected ZMP trajectory and the computed ZMP trajectory is obtained. If the error is bigger than the tolerable level, substitute the sum of the computed ZMP trajectory and the error for the ZMP trajectory in equations (9) (10), and compute \bar{x}_3, \bar{y}_3 again. By iterating the above operation, the periodic solutions \bar{x}_3, \bar{y}_3 are regarded as the strict solutions when the error becomes tolerable.

4: Simulation

Consider a vehicle-mounted mobile manipulator with a two-link manipulator (link 1: Z'-axis rotation, link 2: Y'-axis rotation) and a four-wheel vehicle as shown in Fig. 3. The parameters of the mobile manipulator are shown in Table 1 and Table 2.

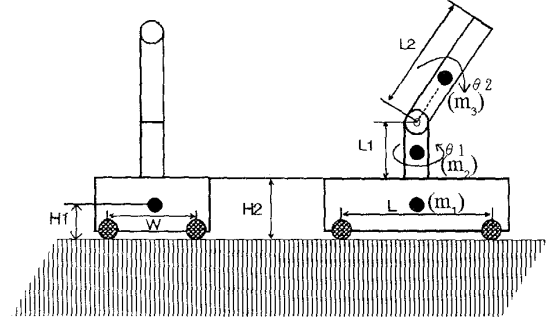


Fig. 3 Outline of the mobile manipulator

Table 1: Parameters of the manipulator

link No.	length L (m)	weight (kg)	angle sphere θ (deg)
1	0.2	30	± 90
2	0.4	50	± 135

Table 2: Parameters of the vehicle

length	width	height		weight
L (m)	W (m)	H1 (m)	H2 (m)	m (kg)
0.6	0.4	0.2	0.3	80

Assume that the vehicle moves at an initial acceleration of 0, and the maximal acceleration of ± 2 [m/s²], and the initial speed of 0, and the maximal speed of 2 [m/s](Fig. 4). In the case of a 60 [kg] payload, when the initial position of the manipulator is (0.28, 0, 0.68) in O'-X'Y'Z' and the vehicle moves from the starting point (0, 0) to the end point (3, 0) in O-XYZ, the simulation is carried out by controlling the manipulator for maintaining stability.

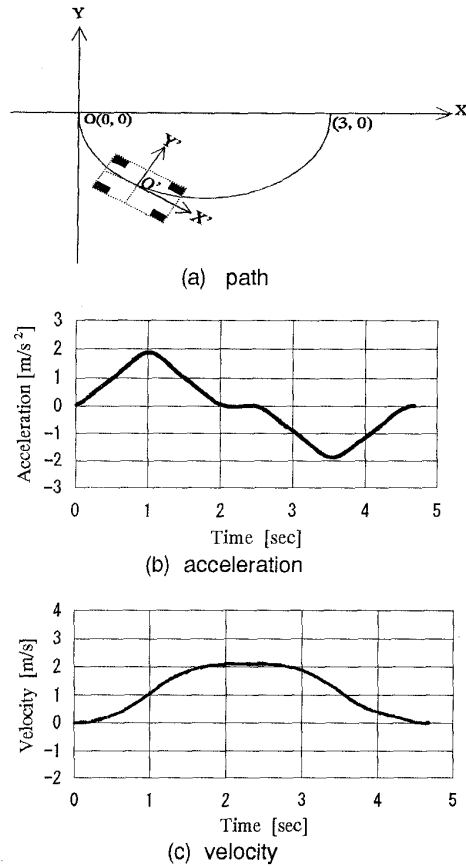


Fig. 4 Given motion of the vehicle

For convenience in simulation, the toleration disturbance is assumed to be a force which can make the mobile manipulator move at an acceleration of $2 \text{ [m/s}^2\text{]}$. In this case, the valid stable region of x_{zmp} is $\pm 0.2 \text{ [m]}$, and y_{zmp} is $\pm 0.1 \text{ [m]}$ in $O'-X'Y'Z'$. Assume that the goal state of the stability potential field is the point $O'(0, 0)$, and the prohibitive state is outside the place $\pm 0.4 \text{ [m]}$ on the X' -axis, $\pm 0.3 \text{ [m]}$ on the Y' -axis in $O'-X'Y'Z'$. As an example, assume the changed amount of the ZMP position in one time $\Delta S = 0.005 \text{ [m]}$.

Some part of the un-corrected ZMP trajectory on the $X'-Y'$ plane is outside the stable region or inside the stable region but outside the valid stable region (Fig. 5). For the part outside the valid stable region, the corrected ZMP trajectory by using the potential method is shown as the broken curve. And the curve from point S to point G is an example of the change process of the ZMP moved path from outside the valid stable region to inside the valid stable region. Specifically, because point S is outside the stable region and the robot is unstable, the stability degree of the ZMP moved path of the starting part from S is increasing almost at a bigger rate.

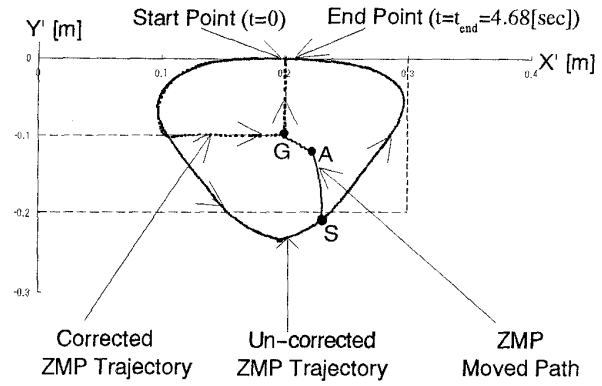


Fig. 5 Simulation for ZMP moved path on $X'-Y'$ plane by the potential method

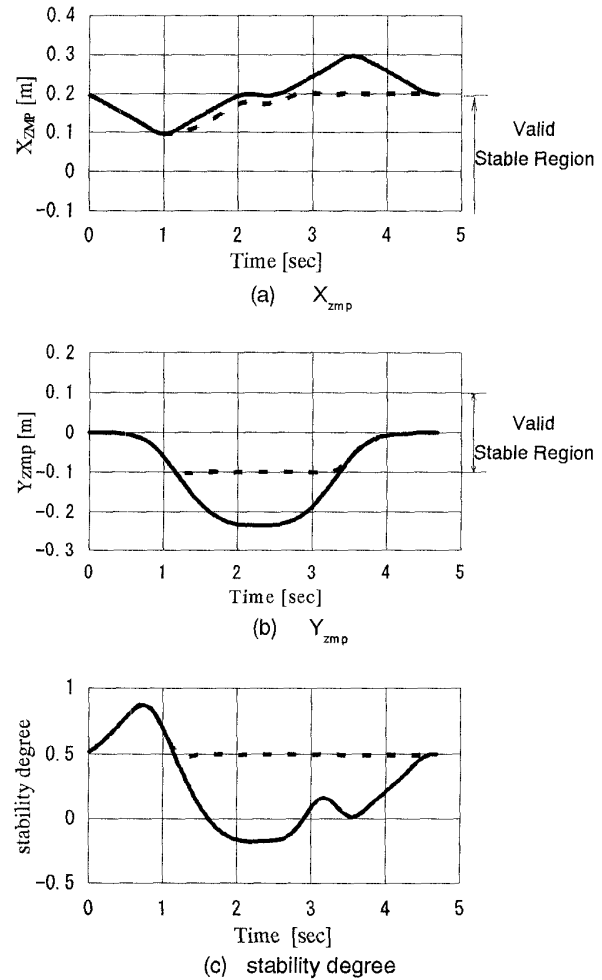


Fig. 6 Simulation for ZMP trajectory

— un-corrected ZMP trajectory
 corrected ZMP trajectory

The ones related time of the un-corrected ZMP trajectory and the corrected ZMP trajectory of X-Y plane are shown in Fig. 6. It is obvious that the ZMP in the corrected ZMP trajectory is always inside the valid stable region, so the mobile manipulator is still stable even if it is influenced by disturbances.

For executing the corrected ZMP trajectory by the potential method, the solved compensatory motion of the manipulator is shown in Fig. 7.

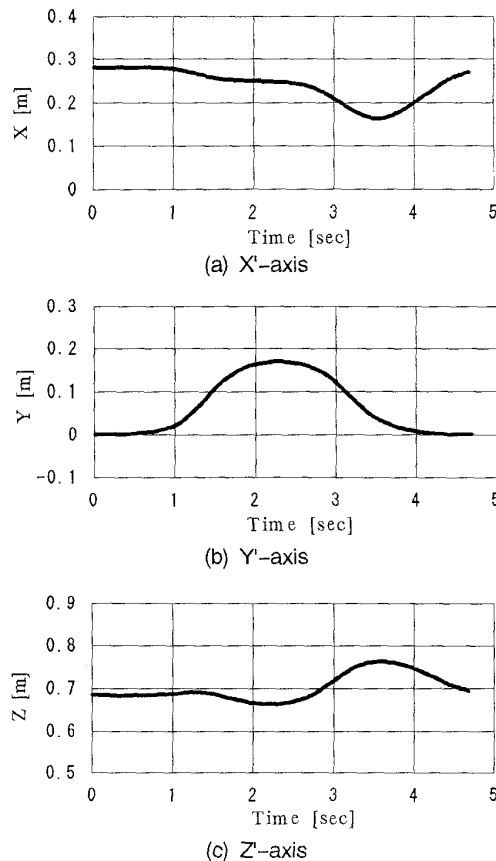


Fig. 7 Position variation of the manipulator's endpoint on the vehicle Cartesian coordinate

5: Conclusion

In the future, robotic systems will be increasingly used in applications in which they are not fixed to factory floors, but are mobile or mounted on vehicles. However, different from a manipulator fixed to the floor tightly, a mobile manipulator might be tip over. In order for the mobile manipulator to successfully accomplish tasks in environments with various disturbances, it is necessary to derive the cooperative motions of the vehicle and the manipulator, in which the stabilization is compatible with task operation. As a step for realizing this motion, in this

paper, the compensatory motion of the manipulator for stabilizing the mobile manipulator has been discussed for case when the vehicle is moving along a given motion: the corrected ZMP trajectory for maintaining stability is first planned by a potential method, then the compensatory motion of the manipulator for executing the corrected ZMP trajectory is solved by an iterative algorithm using the FFT method. Finally, the effectiveness of the method is supported by the simulation.

Future, research will investigate stabilization which is compatible with the task operation of levels (2) and (3) outlined in section 1.

References

- [1] N.Fujiwara, M.Minami, and K.Kanbara, "Non-linear Compensation of Autonomous Mobile Manipulator," in Proc. of 10th Robot Annual Conf. (in Japanese), pp. 525-528 (1992).
- [2] S. Dubowsky and A.B. Tanner, "A Study of the Dynamics and Control of Mobile Manipulators," in Proc. of IV int. Symp. of Robotic Research, pp.111-117 (1987).
- [3] S. Dubowsky and E.E. Vance, "Planning Mobile Manipulator Motions Considering Vehicle Dynamic Stability Constrains," in Proc. of IEEE Int. Conf. on Robotics and Automation, pp.1271-1276 (1989).
- [4] T. Fukuda, and Y. Fujisawa, "Manipulator/Vehicle System for Man-Robot Cooperation," in Proc. of IEEE Int. Conf. on Robotics and Automation, pp.74-79 (1992).
- [5] D.A. Meessuri and C.A. Klein, "Automatic Body Regulation for Maintaining Stability of a Legged Vehicle During Rough-Terrain Locomotion," IEEE J. of Robotics and Automation, Vol. RA-1, No. 3, pp.132-141 (1985)
- [6] S. Sugano, Q. Huang and I. Kato, "Stability Criteria for Mobile robotic Systems," Proc. of IEEE/RSJ Conf. on Intelligent Robots and Systems, Tokyo, Japan, Vol. 2, pp.832-838 (1993)
- [7] Q. Huang, S. Sugano, and I. Kato, "Stability Control for a mobile manipulator using a potential method," Proc. of IEEE/RSJ Conf. on Intelligent Robots and Systems, Munich, Germany, Vol.2, pp.839/846 (1994)
- [8] M. Vukobratovic, A. Frank and D. Juricic, "On the Stability of Biped Locomotion," IEEE Trans. Biomedical Engineering, BME-17, No. 1, pp.25-36 (1970).
- [9] T. Takanishi, M. Ishida, Y. Yamazaki, and I. Kato, "Realization of Dynamic Walking Robot WL-10RD," ICAR, pp.459-466. (1985).
- [10] J.W. Cooley, P.A.W. Lewis, P.D. Welch, "The Finite Fourier Transform," IEEE Trans, Audio and Electroacoustics, vol. AU-17, No. 2, pp 77-85, June 1965.
- [11] A. Madhani, S. Dubowsky, "Motion Planning of Mobile Multi-Limb Robotic Systems Subject to Force and Friction Constraints", Proceedings of the IEEE International Conference on Robot and Automation, Nice, France, pp.233/239 (1992)
- [12] W. Miksch, D. Schroeder, "Performance-Functional Based Controller Design for a Mobile Manipulator", Proceedings of the IEEE International Conference on Robot and Automation, Nice, France, pp.233/239 (1992)