

NONHOLONOMIC PATH PLANNING OF SPACE ROBOTS

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ABSTRACT

This paper discusses the nonholonomic mechanical structure of space robots and its path planning. The angular momentum conservation works as a nonholonomic constraint while the linear momentum conservation is a holonomic one. Taking this in to account, a vehicle with a 6 d.o.f. manipulator is described as a 9 variable system with 6 inputs. This fact implies the possibility to control the vehicle orientation as well as the joint variables of the manipulator by actuating the joint variables only if the trajectory is carefully planned, although both of them cannot be controlled independently. Planning a feasible path, a system that consists of a vehicle and a 6 d.o.f. manipulator can be utilized as a 9 d.o.f. kinematically redundant system. In this paper, first, the nonholonomic mechanical structure of space vehicle/manipulator system is shown. Second, a path planning scheme for nonholonomic systems is proposed using Lyapunov functions.

1. INTRODUCTION

The control of space vehicle/manipulator system possesses inherent issues that have not been considered for on-the-earth robot manipulators, such as the micro gravity, momentum conservation, and preciousness of energy. The kinematics and dynamics of space vehicle/manipulator systems have recently been studied by various researchers.

Alexander and Cannon [1] discussed the computation of joint torque for manipulator endpoint control assuming that the thrust force of vehicle was known, and experimentally demonstrated it. Vafa and Dubowsky [2] and Vafa [3] proposed a novel concept to simplify the kinematics and dynamics of space vehicle/manipulator system. A *virtual manipulator* is an imaginary manipulator that has similar kinematic and dynamic structure to the real vehicle/manipulator system but fixed at the total center of mass of the system. By solving the motion of the virtual manipulator for the given endeffector motion, the motion of vehicle/manipulator system is obtained straightforwardly. On the other hand, Umetani and Yoshida [4] reported a method to continuously control the endeffector without actively controlling the vehicle thrust forces. The momentum conservations for linear and angular motion are explicitly represented and used as the constraint equations to eliminate dependent variables and obtain the generalized Jacobian matrix that relates the joint motion and the endeffector motion. Longman, Lindberg, and Zadd [5] also discussed the coupling of manipulator motion and vehicle motion. Miyazaki, Masutani, and Arimoto [6] discussed a sensor feedback scheme using the transposed generalized Jacobian matrix.

Both the linear and angular momentum conservations have been used to eliminate dependent variables [4] [6]. Although both of them are represented by equations of velocities, the linear one can be exhibited by the motion of the center of mass of the total system, which is represented by the equations of positions not of velocities. This implies that the linear momentum conservation is integrable and hence a holonomic

constraint. On the other hand, the angular momentum conservation cannot be represented by an integrated form, which means that it is a nonholonomic constraint. Vafa and Dubowsky [2] proposed cyclic motion of manipulator joints to change the vehicle orientation. This illustrated the possibility of utilizing nonholonomic mechanical structure of space vehicle/manipulator systems. However, this scheme had to assume small cyclic motion to neglect nonlinearity of order greater than two and, therefore, required many cycles to make even small change of vehicle orientation. Furthermore, the scheme has an inherent drawback in the fact that only multiples of vehicle orientation change caused by a single cycle can be attained.

Suppose an n d.o.f. manipulator on a vehicle, the motion of the endeffector is described by $n+6$ variables, n of the manipulator and 6 of the vehicle. By eliminating holonomic constraint of linear momentum conservation, the total system is formulated as a nonholonomic system of $n+3$ variables including 3 dependent variables. Although only n variables out of $n+3$ can be independently controlled, with an appropriate path planning scheme it would be possible to converge all of $n+3$ variables to desired values due to the nonholonomic mechanical structure. A similar situation is experienced in our daily life. Although an automobile has two independent variables to control, that is, wheel rotation and steering, it can be parked at an arbitrary place with an arbitrary orientation in two dimensional space. This can be done because it is a nonholonomic system.

To locate the manipulator endeffector at a desired point with a desired orientation, even a vehicle with a 6 d.o.f. manipulator has redundancy because a variety of vehicle orientation can be chosen at the final time. This kind of nonholonomic redundancy would be utilized (1) when the extended Jacobian control results in an infeasible motion due to the physical joint limitation, (2) when the system requires more degrees of freedom to avoid obstacles at the final location of the endeffector, (3) when the vehicle orientation needs to be changed without using propulsion or a momentum gyro, and so on.

In this paper, we propose a path planning scheme to control both the vehicle orientation and the manipulator joints by actuating manipulator joints only. First, the nonholonomic mechanical structure of space vehicle/manipulator system is shown. Second, a path planning scheme for nonholonomic systems is proposed using Lyapunov functions. This approach deals with the total nonlinearity of the space vehicle/manipulator systems without neglecting nonlinearity of higher order, and enables any allowable change of vehicle orientation. Since the planning scheme is given in a general form, it can be applied to other many nonholonomic planning problems, such as the path planning of 2 d.o.f. vehicles for 3 d.o.f. motion in a plane, planning of contact point motion of multifingered hands with spherical rolling contacts, and so on.

2. ANGULAR MOMENTUM CONSERVATION AS A NONHOLONOMIC CONSTRAINT

2.1 Nomenclature

frame I	Inertia frame.
frame V	Vehicle frame.
frame B	Manipulator base frame
frame E	Manipulator endeffector frame
frame K	k -th body frame. The k -th link frame of manipulator for $k = 1, \dots, n$. The n -th link frame is identical to the manipulator endeffector frame. The vehicle frame for $k = 0$.
m_k	Mass of the k -th body (kg). The 0-th body is the vehicle. The k -th body ($k \geq 1$) is the k -th link of the manipulator.
${}^I \mathbf{r}_k \in R^3$	Position vector from the origin of the inertia frame to the center of mass of the k -th body represented in the inertia frame. (m)
${}^B \mathbf{r}_k \in R^3$	Position vector from the origin of the manipulator base frame to the center of mass of the k -th body represented in the manipulator base frame. (m)
${}^I \boldsymbol{\omega}_k \in R^3$	Angular velocity of the k -th body in the inertia frame. (rad/s)
${}^k \mathbf{I}_k \in R^{3 \times 3}$	Inertia matrix of the k -th body about its center of mass in the k -th body frame. (kgm^2)
${}^I \mathbf{I}_k \in R^{3 \times 3}$	Inertia matrix of the k -th body about its center of mass in the inertia frame. (kgm^2)
$\dot{\boldsymbol{\theta}}_1 \in R^6$	Linear velocity of the center of mass and angular velocity of the vehicle in inertia frame. ($m/s, rad/s$)
$\boldsymbol{\theta}_2 \in R^n$	Joint variables (q_1, \dots, q_n) of the manipulator. (rad)
${}^I \mathbf{A}_B \in R^{3 \times 3}$	Rotation matrix from the inertia frame to the manipulator base frame.
${}^I \mathbf{A}_k \in R^{3 \times 3}$	Rotation matrix from the inertia frame to the k -th body frame (The vehicle frame for $k = 0$, the k -th link frame of the manipulator for $k = 1, \dots, n$).
$\mathbf{J}_2^k \in R^{3 \times n}$	Jacobian matrix of the position of the center of mass of k -th body ($k = 1, \dots, n$) in the manipulator base frame. (m)
$\mathbf{E}_i \in R^{i \times i}$	$i \times i$ identity matrix.
α, β, γ	z-y-x Euler angles.

2.2 Kinematics of Space Vehicle/Manipulator Systems

The basic equations of kinematics of space vehicle/manipulator system is developed in this subsection. Fig. 1 shows a model of space vehicle/manipulator system. Five kinds of frames, the inertia frame, the vehicle frame, the manipulator base frame, the k -th link frames, and the manipulator endeffector frame, are represented by I, V, B, K , and E respectively. The link frames of the manipulator are defined by Denavit-Hartenberg convention [7]. The vehicle frame is assumed to be fixed at the center of mass of the vehicle.

Supposing zero linear and angular momentum at initial time, the linear and angular momentum conservations are represented by

$$\sum_{k=0}^n m_k {}^I \dot{\mathbf{r}}_k = 0, \quad (1)$$

$$\sum_{k=0}^n ({}^I \mathbf{I}_k {}^I \boldsymbol{\omega}_k + m_k {}^I \mathbf{r}_k \times {}^I \dot{\mathbf{r}}_k) = 0, \quad (2)$$

The vehicle and manipulator motions are described by the following $\dot{\boldsymbol{\theta}}_1$ and $\boldsymbol{\theta}_2$.

$$\dot{\boldsymbol{\theta}}_1 = \begin{pmatrix} {}^I \dot{\mathbf{r}}_0 \\ {}^I \boldsymbol{\omega}_0 \end{pmatrix} \quad (3)$$

$$\boldsymbol{\theta}_2 = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix} \quad (4)$$

${}^I \dot{\mathbf{r}}_k$ is computed by

$$\begin{aligned} {}^I \dot{\mathbf{r}}_k &= {}^I \dot{\mathbf{r}}_0 + {}^I \boldsymbol{\omega}_0 \times ({}^I \mathbf{r}_k - {}^I \mathbf{r}_0) + {}^I \mathbf{A}_B \mathbf{J}_2^k \dot{\boldsymbol{\theta}}_2 \\ &= (\mathbf{E}_3 - {}^I \mathbf{R}_{0k}) \dot{\boldsymbol{\theta}}_1 + {}^I \mathbf{A}_B \mathbf{J}_2^k \dot{\boldsymbol{\theta}}_2 \end{aligned} \quad (5)$$

where ${}^I \mathbf{R}_{0k}$ and ${}^I \mathbf{r}_{0k}$ are defined by

$${}^I \mathbf{R}_{0k} = \begin{pmatrix} 0 & -{}^I r_{0kz} & {}^I r_{0ky} \\ {}^I r_{0kz} & 0 & -{}^I r_{0kx} \\ -{}^I r_{0ky} & {}^I r_{0kx} & 0 \end{pmatrix} \quad (6)$$

$${}^I \mathbf{r}_k - {}^I \mathbf{r}_0 = \begin{pmatrix} {}^I r_{0kx} \\ {}^I r_{0ky} \\ {}^I r_{0kz} \end{pmatrix} \quad (7)$$

On the other hand, ${}^I \mathbf{I}_k {}^I \boldsymbol{\omega}_k$ is given by

$${}^I \mathbf{I}_k {}^I \boldsymbol{\omega}_k = {}^I \mathbf{A}_k {}^k \mathbf{I}_k {}^I \mathbf{A}_k^T {}^I \boldsymbol{\omega}_k \quad (8)$$

$${}^I \boldsymbol{\omega}_k = \begin{cases} (0 \quad \mathbf{E}_3) \dot{\boldsymbol{\theta}}_1 & \text{for } k = 0 \\ {}^I \boldsymbol{\omega}_0 + \sum_{j=1}^k {}^I \mathbf{A}_j \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \dot{q}_j & \text{for } k = 1, \dots, n \end{cases} \quad (9)$$

By substituting eqs. (5) and (8) into eqs. (1) and (2) and summarizing them in a matrix form, the linear and angular momentum conservations are represented by the following equation.

$$\mathbf{H}_1 \dot{\boldsymbol{\theta}}_1 + \mathbf{H}_2 \dot{\boldsymbol{\theta}}_2 = 0 \quad (10)$$

$$\mathbf{H}_1 = \begin{pmatrix} \sum_{k=0}^n m_k \mathbf{E}_3 & -\sum_{k=0}^n m_k {}^I \mathbf{R}_{0k} \\ \sum_{k=0}^n m_k {}^I \mathbf{R}_k & \sum_{k=0}^n {}^I \mathbf{A}_k {}^k \mathbf{I}_k {}^I \mathbf{A}_k^T - \sum_{k=0}^n m_k {}^I \mathbf{R}_k {}^I \mathbf{R}_{0k} \end{pmatrix} \quad (11)$$

$$\mathbf{H}_2 = \begin{pmatrix} \sum_{k=0}^n m_k {}^I \mathbf{A}_B \mathbf{J}_2^k \\ \sum_{k=0}^n m_k {}^I \mathbf{R}_k {}^I \mathbf{A}_B \mathbf{J}_2^k + \mathbf{P} \end{pmatrix} \quad (12)$$

where

$${}^I \mathbf{R}_k = \begin{pmatrix} 0 & -{}^I r_{kz} & {}^I r_{ky} \\ {}^I r_{kz} & 0 & -{}^I r_{kx} \\ -{}^I r_{ky} & {}^I r_{kx} & 0 \end{pmatrix} \quad (13)$$

$$\mathbf{P} = (\mathbf{P}_1 \quad \mathbf{P}_2 \quad \dots \quad \mathbf{P}_n) \quad (14)$$

$$\mathbf{P}_i = \left(\sum_{k=i}^n {}^I \mathbf{A}_k {}^i \mathbf{I}_i {}^I \mathbf{A}_i^T \right) {}^I \mathbf{A}_i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

In eq. (13), ${}^I r_{kx}$, ${}^I r_{ky}$ and ${}^I r_{kz}$ are x, y and z components of ${}^I \mathbf{r}_k$ respectively.

The relationship between the endeffector, $\dot{\boldsymbol{\theta}}_1$ and $\dot{\boldsymbol{\theta}}_2$ is described in the following form.

$$\dot{\mathbf{h}} = \mathbf{J}_1 \dot{\boldsymbol{\theta}}_1 + \mathbf{J}_2 \dot{\boldsymbol{\theta}}_2 \quad (15)$$

where

$$\dot{\mathbf{h}} = \begin{pmatrix} {}^I \dot{\mathbf{r}}_E \\ {}^I \boldsymbol{\omega}_E \end{pmatrix}$$

\mathbf{J}_1 and \mathbf{J}_2 are the pure geometrical Jacobian matrices which do not take account of the momentum conservations. In eq. (10), $\mathbf{H}_1 \in R^{6 \times 6}$ is always nonsingular. Therefore, eq. (10) is identical to

$$\dot{\theta}_1 = -H_1^{-1} H_2 \dot{\theta}_2 \quad (16)$$

Substituting eq. (16) into eq. (15) offers

$$\dot{h} = (-J_1 H_1^{-1} H_2 + J_2) \dot{\theta}_2 \quad (17)$$

Umetani and Yoshida [4] named the coefficient matrix of the above equation the *generalized Jacobian matrix*. In this derivation, the momentum conservations of eq. (10) are used as constraints equations and eliminated in the final equation.

2.3 Holonomic and Nonholonomic Constraints

Eq. (1) can be analytically integrated as follows:

$$\int_0^t \sum_{k=0}^n m_k {}^I \dot{\mathbf{r}}_k dt = \sum_{k=0}^n m_k {}^I \mathbf{r}_k(t) - \sum_{k=0}^n m_k {}^I \mathbf{r}_k(0) \quad (18)$$

$$= 0$$

The above equation physically means that the total center of mass of the system does not move. ${}^I \mathbf{r}_k$ is computed by

$${}^I \mathbf{r}_k = {}^I \mathbf{A}_B {}^B \mathbf{r}_k + {}^I \mathbf{r}_0 \quad (19)$$

where ${}^I \mathbf{A}_B$ is a function of the vehicle orientation only. ${}^B \mathbf{r}_k$ is a function of the joint variables of the manipulator only. Knowing the vehicle orientation, the joint variables, and the initial position of the total center of mass, the vehicle position ${}^I \mathbf{r}_k$ can be obtained by substituting eq. (19) into eq. (18). Therefore, the linear momentum conservation is considered a holonomic constraint because it is integrable.

Although eqs. (1) and (2) are both represented by velocities, eq. (2) can not be analytically integrated and, therefore, it is a nonholonomic constraint. The physical characteristic of nonholonomic constraint is exhibited by the fact that even if the manipulator joints return to the initial joint variables after a sequence of motion, the vehicle orientation may not be the same as its initial value. The vehicle orientation can be eliminated as a dependent variable as we did in deriving eq. (17). In next section, we propose to control both the independent and dependent variables by controlling the independent ones only.

The basic system equation is obtained by taking the vehicle orientation and θ_2 as the state variable and the $\dot{\theta}_2$ as the input variable. First, the coefficient matrix of eq. (16) is divided into a top $3 \times n$ matrix and a bottom $3 \times n$ matrix as follows:

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_r \\ \mathbf{H}_w \end{pmatrix} = -\mathbf{H}_1^{-1} \mathbf{H}_2 \quad (20)$$

The state variable \mathbf{x} and the input variable \mathbf{u} are defined by

$$\mathbf{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \theta_2 \end{pmatrix} \in R^{n+3} \quad (21)$$

$$\mathbf{u} = \dot{\theta}_2 \in R^n \quad (22)$$

α, β , and γ are the z-y-x Euler angles of the vehicle with respect to the inertia frame. The relationship between the Euler angles and ${}^I \omega_0$ is given by

$${}^I \omega_0 = \mathbf{N} \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} \quad (23)$$

where

$$\mathbf{N} = \begin{pmatrix} 0 & -\sin \alpha & \cos \alpha \cos \beta \\ 0 & \cos \alpha & \sin \alpha \cos \beta \\ 1 & 0 & -\sin \beta \end{pmatrix}$$

The system equation becomes

$$\dot{\mathbf{x}} = \mathbf{K} \mathbf{u} \quad (24)$$

where

$$\mathbf{K} = \begin{pmatrix} \mathbf{N}^{-1} \mathbf{H}_w \\ \mathbf{E}_n \end{pmatrix} \in R^{(n+3) \times n} \quad (25)$$

2.4 Nonholonomic Redundancy

The system represented by eq. (25) has a unique feature in the fact that the input variable may not be found even if a smooth desired trajectory of \mathbf{x} is provided because it has less number of input variable. It is impossible to plan a feasible trajectory without taking full account of the dynamics of eq. (25). This is a general feature of nonholonomic mechanical systems. An automobile can move around in two dimensional space and orient itself if we drive it properly, although it has only two variables to control, that is, wheel rotation and steering. In this case, the state variables are three and the inputs are two.

By appropriately planning the trajectory, the desired final values of the vehicle orientation and the manipulator joint variables could be reached. To locate the manipulator endeffector at a desired point with a desired orientation, even a vehicle with a 6 d.o.f. manipulator has redundancy because a variety of vehicle orientation can be chosen at the final time. This idea is conceptually illustrated in Fig. 2. The choice of the final vehicle orientation can be done based on the conventional control or planning schemes of kinematically redundant manipulators [8, 9, 10]. It is a problem to find an appropriate configuration among the configurations attained by 3 d.o.f. selfmotion.

The nonholonomic redundancy would be utilized (1) when the extended Jacobian control results in an infeasible motion due to the physical joint limitation, (2) when the system requires more degrees of freedom to avoid obstacles at the final location of the endeffector, (3) when the vehicle orientation needs to be changed without using propulsion or a momentum gyro, and so on.

3. PATH PLANNING USING LYAPUNOV FUNCTIONS

3.1 First Lyapunov function

Vafa and Dubowsky [2] proposed cyclic motion of manipulator joints to change the vehicle orientation. This illustrated the possibility of utilizing nonholonomic mechanical structure of space vehicle/manipulator systems. However, this scheme had to assume small cyclic motion to neglect nonlinearity of order greater than two and, therefore, required many cycles to make even small change of vehicle orientation. Furthermore, the scheme has an inherent drawback in the fact that only multiples of vehicle orientation change caused by a single cycle can be attained.

In this section, the input variable \mathbf{u} is synthesized based on the Lyapunov's direct method [11] so that the vehicle orientation and the joint variables should converge to their desired values. This approach deals with the total nonlinearity of the space vehicle/manipulator systems without neglecting nonlinearity of higher order, and enables any allowable change of vehicle orientation. Since the planning scheme is given in a general form, it can be applied to other many nonholonomic planning problems, such as the path planning of 2 d.o.f. vehicles for 3 d.o.f. motion in a plane, planning of contact point motion of multifingered hands with spherical rolling contacts, and so on.

The following function is chosen as a candidate of the Lyapunov function.

$$v_1 = \frac{1}{2} \Delta \mathbf{x}^T \mathbf{A} \Delta \mathbf{x} \quad (26)$$

$$\Delta \mathbf{x} = \mathbf{x}_d - \mathbf{x} \quad (27)$$

where \mathbf{A} is a positive definite constant matrix. $v_1 = 0$ is attained only when $\mathbf{x}_d = \mathbf{x}$. The time derivative of v_1 is computed as follows:

$$\dot{v}_1 = -\Delta \mathbf{x}^T \mathbf{A} \dot{\mathbf{x}} = -\Delta \mathbf{x}^T \mathbf{A} \mathbf{K} \mathbf{u} \quad (28)$$

where eq. (24) was substituted. Now, choosing the input variable as

$$\mathbf{u}_1 = (\mathbf{A} \mathbf{K})^T \Delta \mathbf{x}, \quad (29)$$

the rate of change of the Lyapunov function becomes

$$\dot{v}_1 = -\mathbf{u}_1^T \mathbf{u}_1 \leq 0 \quad (30)$$

If the equality of eq. (30) holds only when $\mathbf{x}_d = \mathbf{x}$, Lyapunov's theorem [11] can conclude its global stability. However, this condition is not satisfied in our problem. \dot{v}_1 becomes zero when $\Delta \mathbf{x}$ is in the null space of $(\mathbf{A} \mathbf{K})^T$, which is a three dimensional space.

3.2 Avoiding Null Space of $(\mathbf{A} \mathbf{K})^T$

The LaSalle's theorem [12] says that the state variable \mathbf{x} converges to \mathbf{x}_d if $\mathbf{x} = \mathbf{x}_d$ is the unique entry of the maximum invariant set. When $\Delta \mathbf{x}$ is at the null space of $(\mathbf{A} \mathbf{K})^T$ and it stays within the null space thereafter, all the points on this trajectory are the entries of the maximum invariant set. In this subsection, the unit vector is chosen such that $\Delta \mathbf{x}$ should avoid the null space as much as possible and get out of the null space if it is there.

To take account of the null space of $(\mathbf{A} \mathbf{K})^T$ we introduce the second Lyapunov function v_2 such that

$$v_2 = \frac{\Delta \mathbf{x}^T (\mathbf{E}_{n+3} - (\mathbf{A} \mathbf{K}) (\mathbf{A} \mathbf{K})^{\#}) \Delta \mathbf{x}}{\Delta \mathbf{x}^T \Delta \mathbf{x} + \epsilon_1} \quad (31)$$

where ϵ_1 is a positive small constant. v_2 becomes equal to zero when $\Delta \mathbf{x} = 0$

Since

$$\begin{aligned} & \Delta \mathbf{x}^T (\mathbf{E}_{n+3} - (\mathbf{A} \mathbf{K}) (\mathbf{A} \mathbf{K})^{\#}) \Delta \mathbf{x} \\ &= \Delta \mathbf{x}^T (\mathbf{E}_{n+3} - (\mathbf{A} \mathbf{K}) (\mathbf{A} \mathbf{K})^{\#})^T (\mathbf{E}_{n+3} - (\mathbf{A} \mathbf{K}) (\mathbf{A} \mathbf{K})^{\#}) \Delta \mathbf{x} \\ &= \| (\mathbf{E}_{n+3} - (\mathbf{A} \mathbf{K}) (\mathbf{A} \mathbf{K})^{\#}) \Delta \mathbf{x} \|^2, \end{aligned} \quad (32)$$

the numerator of eq. (31) implies the squared Euclidean norm of the orthogonal projection of $\Delta \mathbf{x}$ on the null space of $(\mathbf{A} \mathbf{K})^T$. If we define ϕ such that

$$\cos \phi = \frac{\| (\mathbf{E}_{n+3} - (\mathbf{A} \mathbf{K}) (\mathbf{A} \mathbf{K})^{\#}) \Delta \mathbf{x} \|}{\| \Delta \mathbf{x} \|}, \quad 0 \leq \phi \leq \frac{\pi}{2} \quad (33)$$

ϕ means an angle between $\Delta \mathbf{x}$ and the hyperplane of the null space of $(\mathbf{A} \mathbf{K})^T$, and can be considered as a distance of $\Delta \mathbf{x}$ from the null space as shown in Fig. 3. For $\epsilon_1 = 0$ the second Lyapunov function becomes

$$v_2 = \cos^2 \phi \quad 0 \leq \phi \leq \frac{\pi}{2}. \quad (34)$$

In eq. (31), ϵ_1 allows for v_2 not to take extreme values and to be defined at $\Delta \mathbf{x}$. In eq. (34) v_2 is monotonously reduced as ϕ grows, and takes zero at $\phi = \pi/2$, which means the farthest point from the null space.

Taking the derivative of v_2 with respect to time, we have

$$\dot{v}_2 = \frac{\partial v_2}{\partial \mathbf{x}} \dot{\mathbf{x}} = \frac{\partial v_2}{\partial \mathbf{x}} \mathbf{K} \mathbf{u}. \quad (35)$$

If we choose \mathbf{u}_2 such as

$$\mathbf{u}_2 = -\mathbf{K}^T \left(\frac{\partial v_2}{\partial \mathbf{x}} \right)^T, \quad (36)$$

and use it as \mathbf{u} , then $\dot{v}_2 \leq 0$, and \mathbf{u}_2 works to avoid the null space by driving toward $\phi = \pi/2$.

We integrate \mathbf{u}_1 and \mathbf{u}_2 in a hierarchical manner such that

$$\mathbf{u} = k_1 \mathbf{u}_1 + k_2 (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{u}_2 \quad (37)$$

where $\mathbf{u}_1^{\#}$ is the pseudoinverse of \mathbf{u}_1 , k_1 and k_2 are positive constants. Since $(\mathbf{I} - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{u}_2$ is the orthogonal projection of \mathbf{u}_2 onto the hyperplane that is perpendicular to \mathbf{u}_1 , the first and second terms are mutually perpendicular. This results in following properties of eq. (37).

The second term of eq.(37) has no effect on the convergence speed of v_1 [†]. Indeed substituting eq. (37) into eq. (28), we have

$$\dot{v}_1 = -\mathbf{u}_1^T \{k_1 \mathbf{u}_1 + k_2 (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{u}_2\} = -k_1 \mathbf{u}_1^T \mathbf{u}_1 \quad (38)$$

where $\mathbf{u}_1^T (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) = (\mathbf{u}_1 - \mathbf{u}_1 \mathbf{u}_1^{\#} \mathbf{u}_1)^T = 0$ is used.

Let's consider the effect of the second term of v_2 . Substituting the second term of eq. (37) into eq. (35) along with eq. (36), we obtain

$$\begin{aligned} \dot{v}_2 &= -k_2 \frac{\partial v_2}{\partial \mathbf{x}} \mathbf{K} (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{K}^T \left(\frac{\partial v_2}{\partial \mathbf{x}} \right)^T \\ &= -k_2 \frac{\partial v_2}{\partial \mathbf{x}} \mathbf{K} (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#})^T (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{K}^T \left(\frac{\partial v_2}{\partial \mathbf{x}} \right)^T \leq 0 \end{aligned} \quad (39)$$

\dot{v}_2 becomes zero only when $(\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{K}^T (\partial v_2 / \partial \mathbf{x})^T = 0$. Otherwise \dot{v}_2 is always negative. This means that the second term of eq. (37) tries to reduce v_2 although the total \mathbf{u} of eq. (37) does not guarantee the negativeness of \dot{v}_2 because of the effect of the first term.

The concept of this hierarchical Lyapunov function approach is illustrated in Fig. 4. Since the second term of eq. (37) does not affect the first Lyapunov function, it induces a motion on the equipotential hypersurface of the first Lyapunov function. The equipotential motion is determined by projecting the motion suggested by the second Lyapunov function onto the equipotential hyperplane. Although the second term of eq. (37) does not change the local convergence rate of the first Lyapunov function, it changes the global behavior of motion in the first Lyapunov function field by causing the equipotential motion.

To summarize eqs. (28),(29),(35),and (36), the proposed hierarchical Lyapunov function approach can be represented as follows

$$\mathbf{u} = k_1 \mathbf{u}_1 + k_2 (\mathbf{E}_n - \mathbf{u}_1 \mathbf{u}_1^{\#}) \mathbf{u}_2 \quad (40)$$

$$\mathbf{u}_i = -\mathbf{K}^T \left(\frac{\partial v_i}{\partial \mathbf{x}} \right)^T, \quad \text{for } i = 1, 2 \quad (41)$$

It should be noted that if we consider v_i as the i th manipulation variable, $(\partial v_i / \partial \mathbf{x}) \mathbf{K}$ as its Jacobian matrix with respect to the input variable \mathbf{u} , then eq. (40) is identical to the *task-priority approach* developed for kinematically redundant manipulators[10], having

$$\dot{v}_i = -k_i \frac{\partial v_i}{\partial \mathbf{x}} \mathbf{K} \mathbf{K}^T \left(\frac{\partial v_i}{\partial \mathbf{x}} \right)^T, \quad \text{for } i = 1, 2 \quad (42)$$

as the desired trajectories of the manipulation variables. This approach cannot guarantee that $\mathbf{x} = \mathbf{x}_d$ is the unique entry of the maximum invariant set [12] and, therefore, the trajectory may halt at some point in the null space of $(\mathbf{A} \mathbf{K})^T$. However, if the second Lyapunov function can successfully avoid the null space of $(\mathbf{A} \mathbf{K})^T$, \mathbf{x} converges to \mathbf{x}_d .

4. CONCLUSION

A new insight of the mechanical structure of space vehicle/ manipulator systems was given. By utilizing the nonholonomic structure, not only the manipulator joints, but also vehicle orientation can be

[†] The convergence speed \dot{v}_1 is the same for both \mathbf{u}_1 and \mathbf{u} of eq. (37) only in local sense. Since the global trajectory of \mathbf{x} varies depending on the choice of the input, the global convergence speed would be different.

controlled only by actuating the joint variables, although both of the vehicle motion and the manipulator joints cannot be controlled independently.

It is essential to plan a feasible trajectory. A nonlinear control scheme was synthesized using Lyapunov's direct method. This scheme can be used not only for real-time control, but for planning of a feasible motions of vehicle and manipulator. This approach deals with the total nonlinearity of the space vehicle/manipulator systems without neglecting nonlinearity of higher order, and enables any allowable change of vehicle orientation.

A general graphic simulator that can simulate any open link manipulators on a vehicle has been developed on SUN3 at the Center for Robotic Systems in Microelectronics, University of California, Santa Barbara. A sample of graphic display of motion is shown in Fig. 5. To verify the effectiveness of the proposed approach, numerical simulation is currently being undertaken using this simulator.

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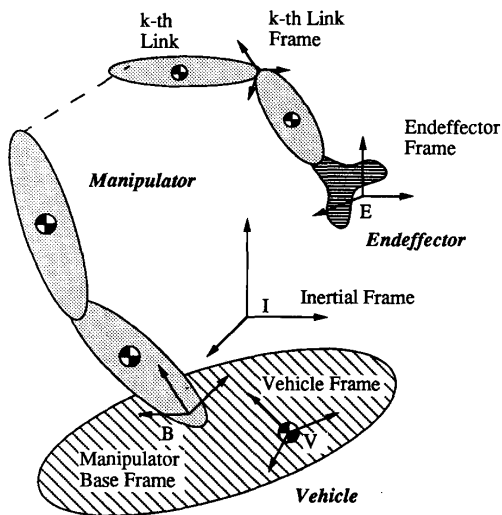


Fig. 1. Five Coordinate Frames for the Space Vehicle / Manipulator System.

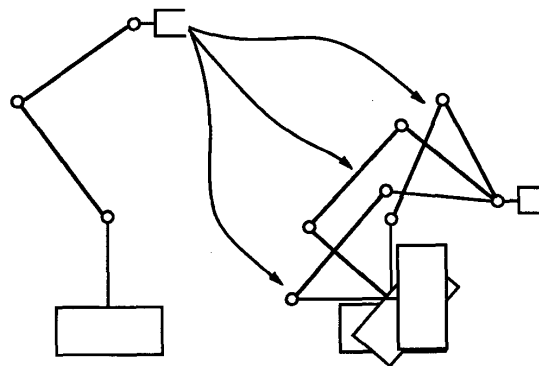
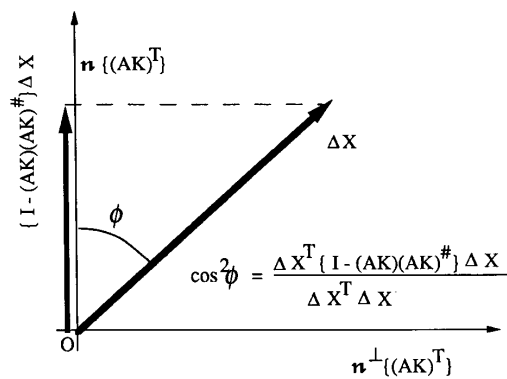


Fig. 2.

Nonholonomic Redundancy:

Although the number of manipulator joints is not redundant, a space vehicle/manipulator system has redundancy in choosing configuration at the final point due to the nonholonomic mechanical structure.



$n \{(AK)^T\}$: the null space of $(AK)^T$
 $n^{\perp} \{(AK)^T\}$: the orthogonal complement space of $n \{(AK)^T\}$

Fig. 3. Physical Meaning of the 2nd Lyapunov Function and Definition of Angle ϕ .

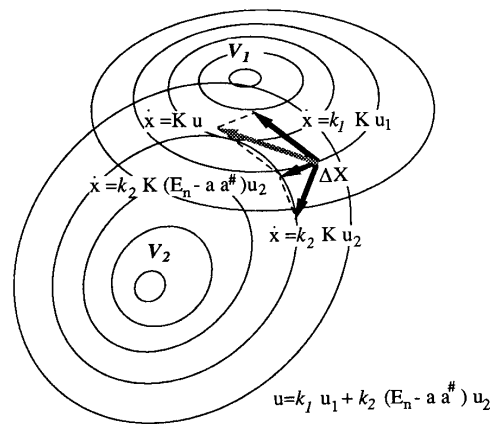


Fig. 4. Physical Meaning of Hierarchical Integration of u_1 and u_2 .

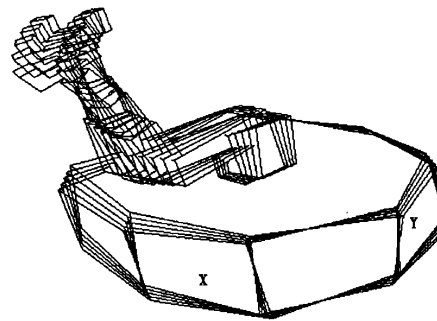
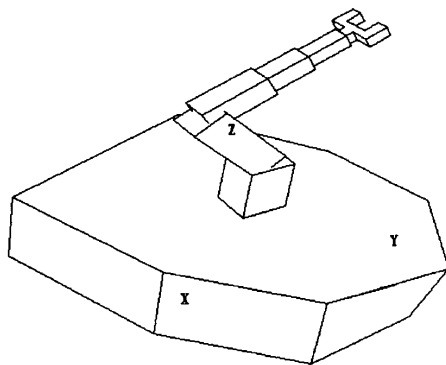


Fig. 5. Graphic Display of the simulated motion of a Space Vehicle/Manipulator System