

13. I-HANK

Adv. Macro: Heterogenous Agent Models

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Introduction

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- **Literature:**
 - Auclert, Rognlie, Souchier, & Straub (2024) »Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel«
 - Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm (2024) »The Transmission of Foreign Demand Shocks«
 - Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm (2024) »Fiscal Multipliers in Small Open Economies With Heterogeneous Households«

IHANK Model

Small Open Economy HANK Model

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- IHANK model:
 - Take Gali-Monacelli
 - Add sticky wages
 - Add heterogeneous agents

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- Standard HA: Face idiosyncratic income risk + borrowing constraint
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- **Foreign economy** (mostly exogenous)

- **Household problem:**

$$v_t(z_{it}, a_{it-1}) = \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t [v_{t+1}(z_{it+1}, a_{it})]$$

$$\text{s.t. } a_{it} + c_{it} = (1 + r_t^a) a_{t-1} + Z_t z_{it}$$

$$\log z_{it+1} = \rho_z \log z_{it} + \psi_{it+1}, \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E}[z_{it}] = 1$$

$$a_{it} \geq 0$$

- **Active decisions:** Consumption-saving, c_{it} (and a_{it})
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_{it} d\mathcal{D}_{it}$
- **Consumption function:** $C_t^{hh} = C^{hh}(\{r_s^a, Z_s\}_{s=0}^\infty)$

Consumption basket

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- FOCs:

$$c_{F,it} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} c_{it}, \quad c_{H,it} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} c_{it},$$

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- with $P_t = \text{CPI}$:

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- Aggregating we get:

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- If preferences are non-homothetic HH problem is more complicated

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- where P_{it} is the ideal price index associated with the consumption basket of individual i

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- Note: Usual macro convention, an increase in E_t corresponds to a **depreciation**
 - So $E_t = 7,46$ between DKK and EURO for instance

- Production and profits:

$$Y_t = L_t$$

$$\Pi_t = \frac{P_{H,t}}{P_t} Y_t - \frac{W_t}{P_t} L_t$$

- First order condition:

$$\frac{P_{H,t}}{P_t} \frac{1}{\mu} = w_t$$

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- where $r_t = r_{t+1}^a$ the ex-ante interest rate, r_t^* is the foreign real interest rate, $Q_t = \frac{E_t}{P_t} P_t^*$ is the **real exchange rate**

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi (L_t^{hh})^\nu - \frac{1}{\mu} w_t (C_t^{hh})^{-\sigma} \right) + \beta \pi_{t+1}^w$$

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- with $\phi_E \rightarrow \infty$

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- The current account is:

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- Current account and net foreign asset position are related by [Walras]:

$$NFA_t - NFA_{t-1} = CA_t$$

Market clearing

1. Labor market: $L_t = L_t^{hh}$
2. Goods market (Version 1)

$$Y_t = C_{H,t} + C_{H,t}^*$$

3. Goods market (Version 2)

$$GDP_t = C_t^{hh} + NX_t$$

$$\text{with } NX_t = \frac{P_{H,t}}{P_t} C_{H,t}^* - \frac{P_{F,t}}{P_t} C_{F,t}$$

International Keynesian Cross

Sequence-space - goods market

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$$C_{H,t}^* = \alpha \left(\frac{P_{H,t}^*}{P_{F,t}^*} \right)^{-\eta^*} M_t^* \Rightarrow d\mathbf{C}_H^* = \alpha d\mathbf{M}^* - \eta^* \alpha d \left(\frac{\mathbf{P}_H^*}{\mathbf{P}_F^*} \right)$$

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- For now assume no change in foreign demand, $d\mathbf{M}^* = 0$

$$d\mathbf{Y} = (1 - \alpha) d\mathbf{C}^{hh} - \eta(1 - \alpha) d \left(\frac{P_H}{P} \right) - \eta^* \alpha d \left(\frac{P_H^*}{P_F^*} \right)$$

Sequence-space - trade elasticity

- Linearizing the price index + LOOP gives $dP_{H,t} - dP_t = -\frac{\alpha}{1-\alpha}dQ_t$ and $dP_{H,t}^* = dP_{H,t} - dE_t$ to get:

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- with $\chi = \eta(1 - \alpha) + \eta^*$ being the *trade elasticity*
 - Captures the elasticity of net exports to changes in relative prices (the real EXR Q)
 - Typically called *expenditure switching*
 - If the DKK appreciates ($d\mathbf{Q} \downarrow$) against the USD both DK HHs and US HHs will **substitute toward** US goods ($d\mathbf{Y} \downarrow$)

Sequence-space - HHs

- What can we say about $d\mathbf{C}^{hh}$? Consumption function is $C^{hh}(\{r_s^a, Z_s\}_{s=0}^\infty)$:

$$d\mathbf{C}^{hh} = \mathbf{M}^{r^a} d\mathbf{r}^a + \mathbf{M} d\mathbf{Z}$$

Sequence-space - HHs

- What can we say about $d\mathbf{C}^{hh}$? Consumption function is $C^{hh}(\{r_s^a, Z_s\}_{s=0}^\infty)$:

$$d\mathbf{C}^{hh} = \mathbf{M}^{r^a} dr^a + \mathbf{M} dZ$$

- Use firm FOC + production function

$Z_t = w_t L_t = \frac{P_{H,t}}{P_t} Y_t \Rightarrow dZ_t = -\frac{\alpha}{1-\alpha} dQ_t + dY_t + \text{small valuation effect}$
effect $\mathbf{M}^{r^a} \approx \mathbf{M}^r$:

$$d\mathbf{C}^{hh} = \mathbf{M}^r dr + \mathbf{M} dY - \frac{\alpha}{1-\alpha} \mathbf{M} dQ$$

Sequence-space - Keynesian Cross

- Putting it together:

$$d\mathbf{Y} = \underbrace{(1 - \alpha)\mathbf{M}^r d\mathbf{r}}_{1. \text{ Interest rate}} + \underbrace{(1 - \alpha)\mathbf{M} d\mathbf{Y}}_{2. \text{ Multiplier}} + \underbrace{\chi \frac{\alpha}{1 - \alpha} d\mathbf{Q}}_{3. \text{ Exp. switching}} - \underbrace{\alpha \mathbf{M} d\mathbf{Q}}_{4. \text{ Real income}}$$

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4. **Real income channel of EXR**: Appreciation ($Q \downarrow$) causes foreign goods to be cheaper in home currency \Rightarrow Reduces PF, P , raises real income Z

Monetary Policy

Sequence-space - Keynesian Cross

- Use the Keynesian Cross to analyze monetary policy with heterogeneous agents
 - Reference: Auclert, Rognlie, Souchier, & Straub (2024) »Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel«

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$$1 + r_t = \frac{Q_{t+1}}{Q_t} (1 + r^*)$$

- For given foreign rate r^* an increase in domestic real rate r_t will attract foreign capital flows \Rightarrow Appreciation of Q_t
 - To first-order we have $dQ_t = -\sum_{s \geq t}^{\infty} dr_s$ for a constant r^*

HANK-RANK equivalence

- In closed economy monetary policy was equally effective in HANK/RANK
 - Under some assumption (log utility, no gov bonds ...)

HANK-RANK equivalence

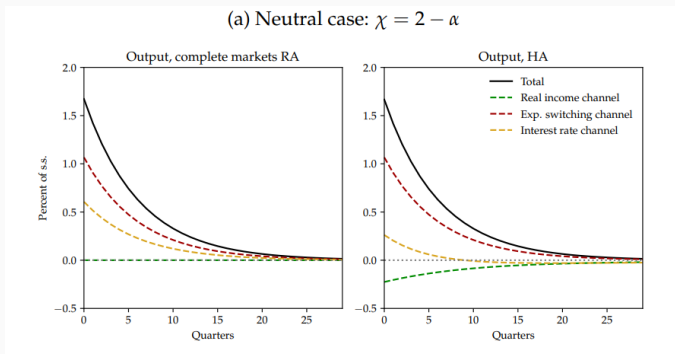
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- Turns out that these effects **balance each other exactly** if trade elasticity $\chi = 2 - \alpha$
 - If $\chi < 2 - \alpha$ then the real income effect dominates and monetary policy *less* effective in HANK
 - If $\chi > 2 - \alpha$ then expenditure switching dominates and monetary policy is *more* effective in HANK

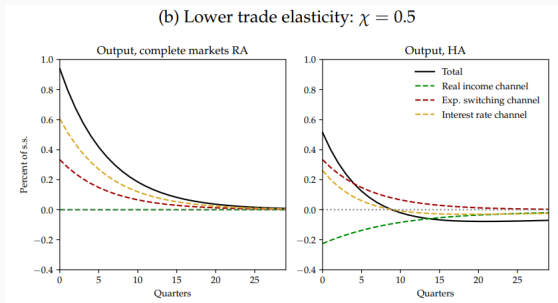
Monetary policy - $\chi = 2 - \alpha$

- Output response in HANK/RANK with neutrality, $\chi = 2 - \alpha$



Monetary policy - $\chi < 2 - \alpha$

- Empirically we expect the trade elasticity to be **low** in the short run
 - Takes time for firms/households to respond to changes in relative prices
 - But *probably* larger in the long run ($\chi > 2 - \alpha$)
- Output response in HANK/RANK with $\chi = 0.5 < 2 - \alpha$
 - Monetary policy **less** effective in HANK



Fiscal Policy

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- Take model from before, add government + Taylor rule

Keynesian cross with G

- Keynesian cross:

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- In fact **isomorphic** to closed economy Cross with $\tilde{M} \equiv (1 - \alpha)M$

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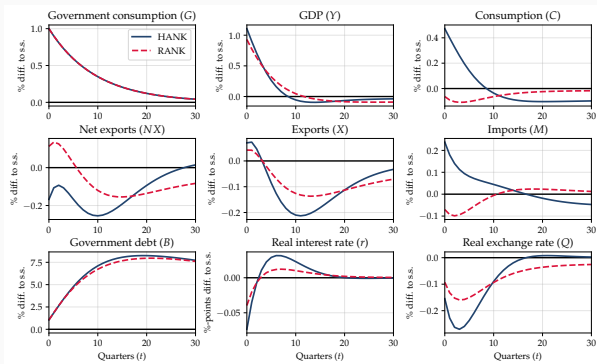
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- ... but some analytical results in paper, for instance:
 - In limit $\alpha \rightarrow 1$ (fully open economy) HANK/RANK equivalence since multiplier effects do not matter

Fiscal spending shocks

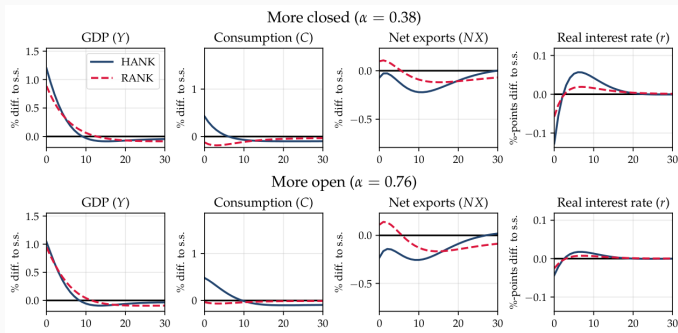
- Main result with deficit financed G shock:



- Relatively similar fiscal multiplier
 - HANK produces **much** larger C response
 - ... But this gets counteracted by larger drop in net exports

Fiscal spending shocks - openness

- How does fiscal multiplier vary with openness α ? (plot IRFs for first and third quartile of $\frac{Imports}{GDP}$ across sample of OECD countries.)



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- Other shocks often studied in open economy context:
 - Foreign monetary policy shocks
 - Capital flow shocks (»sudden stops«)
 - Import price shocks

- Motivation: Go back to international Kenyesian Cross with foreign demand \mathbf{M}^*

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- What is sign of $\text{cov}(d\mathbf{C}, d\mathbf{M}^*)$ empirically?

Empirical estimates of foreign demand shock

- Panel of 38 OECD countries, focus on 31 SOE's

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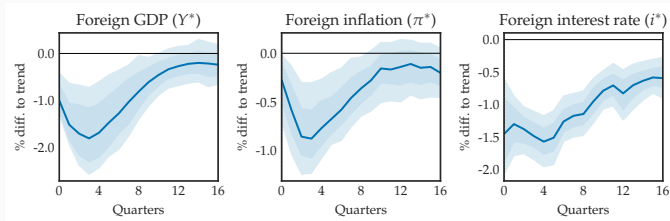
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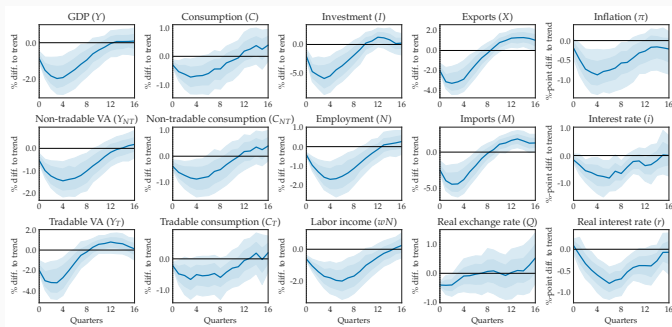
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- Estimated foreign shock



Spillover effects

- Use estimated shock in foreign trading partners to estimate effects on domestic, SOE economy
- Estimate dynamic OLS/LP $y_{c,t+h} = \beta_h i_{c,t}^* + \alpha_h \pi_{c,t}^* + \Theta_h M_{c,t}^*$ where y = domestic outcomes (GDP, C ...)



Why foreign demand shocks?

- Why focus specifically on a **foreign demand shock**?
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- What about domestic demand shock (G)?
 - Identification more difficult
 - Literature ambiguous on whether C increases or decreases

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 - Government
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- Feed in estimated foreign demand shock, compare with empirics

- Household problem:

$$V_t(e_t, a_{t-1}, \beta, s) = \max_{c_t, a_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \nu \frac{L_{s,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} + \beta_t \mathbb{E}_t [V_{t+1}(e_{t+1}, a_t, \beta, s)]$$

s.t.

$$c_t + a_t = (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_{s,t} L_{s,t} e_t + T_t$$

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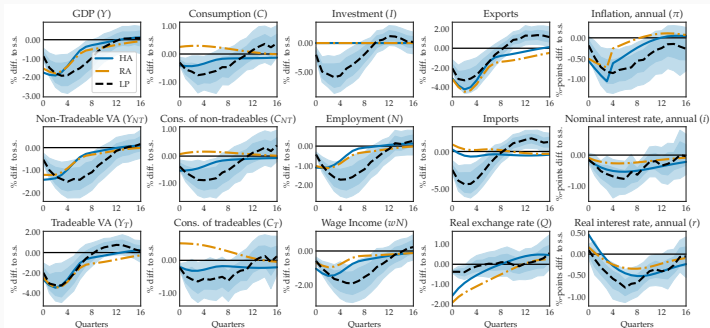
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- Markov matrix for s is $P^s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 - HHs cannot move sectors. Harsh assumption, but consistent with short-run dynamics. Can alleviate by changing P^s
 - Could also have endogenous sector choice at HH level

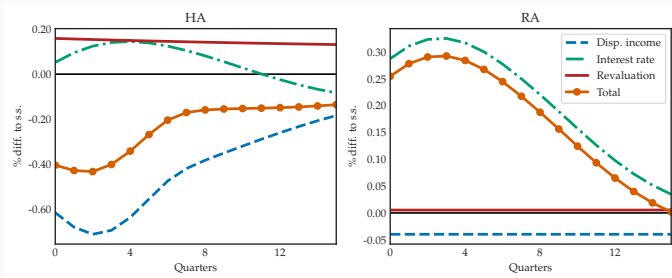
Model fit - floating

- Effects of foreign demand shock with a floating EXR



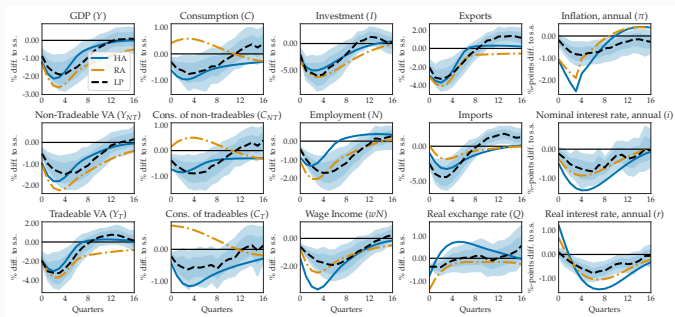
Decomposition

- Decompose dC into effects from interest rate, labor income and capital gain effects



Model fit - floating /w investment

- HANK response amplified by investment
- Note: Getting investment response right requires exogenous shock to investment



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- A foreign demand shock entails a decline in i_t^* (in both model and data).

Fixed exchange rate

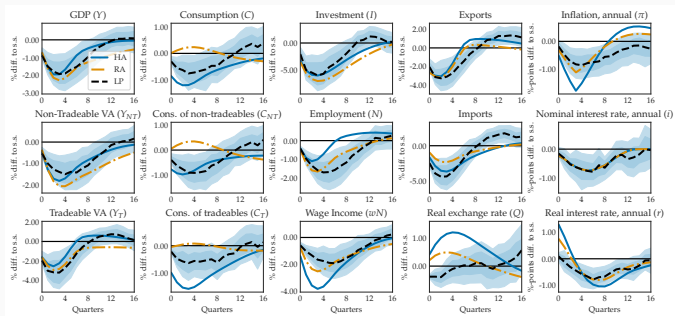
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- Does this mean that our result changes under a fixed exchange rate?
- **No!** UIP condition + Fisher equation:

$$1 + i_t = 1 + i_t^* \Leftrightarrow 1 + r_t = \frac{1 + i_t^*}{1 + \pi_{t+1}}$$

- A foreign demand shock entails a decline in i_t^* (in both model and data).
- UIP forces central bank in SOE to reduce i_t , so $r_t \downarrow$ (unless $\pi_{t+1} \downarrow\downarrow$)

Model fit - fixed

- Similar outcomes with fixed EXR



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Public consumption	1.00	0.08	1.32
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- Fiscal policy loads on NT sector \Rightarrow Very asymmetric effects, barely helps HHs in T sector
 - Issues for countries fixed EXRs or in monetary unions
 - Need targeted transfers

Conclusion

- How does heterogeneity affect transmission of shocks and policies in SOEs?
 - Monetary policy - Likely to be less effective due to real income channel of EXR
 - Fiscal policy - Closer to RANK multipliers due to crowding out of NX
 - Foreign demand shocks - larger transmission to domestic spending

- Covered 3 papers here: Other papers in the litterature on HANK in open economies include:
 1. Guo, X., Ottonello, P., & Perez, D. J. (2023) *Monetary policy and redistribution in open economies*
 - Redistributinal effects of monetary policy in SOEs
 2. Aggarwal, R., Auclert, A., Rognlie, M., & Straub, L. (2023). *Excess savings and twin deficits: The transmission of fiscal stimulus in open economies*
 - Fiscal stimulus in a multi-country model
 3. De Ferra, S., Mitman, K., & Romei, F. (2020). *Household heterogeneity and the transmission of foreign shocks*
 - Effects of exchange rate depreciations when HHs have foreign currency debt
 4. Bayer, C., Kriwoluzky, A., Müller, G. J., & Seyrich, F. (2024). *A HANK² model of monetary unions*. *Journal of Monetary Economics*
 - A 2-country HANK model

Summary

Summary and next week

- **Today:** Small open economy HANK models
- **Next week:**
 - Advanced HANK topics (**research frontier**)
 - Q&A
 - Exam
- **Homework:**
 1. Work on assignment