CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

12. HANK-SAM

Adv. Macro: Heterogenous Agent Models

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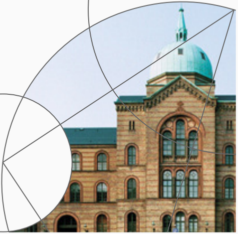

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Introduction

- **From RANK to HANK:**
  1. Income effects more important relative to substitution effects
  2. Cash-flows more important relative to relative prices
- **Central:** High MPCs

I. Idiosyncratic risk + incomplete markets $\rightarrow$

II. Precautionary saving and liquidity constraint $\rightarrow$

III. Concave consumption function $\rightarrow$ high MPCs

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  New: *Endogenous fluctuations in idiosyncratic risk*

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- **SAM:** Search-And-Matching labor market

  New: *Endogenous fluctuations in idiosyncratic risk*
- **Today:**

  GEModelTools: Model description of HANK-SAM

  Broer, Druedahl, Harmenberg and Öberg:

  2024: »Stimulus effects of common fiscal policies«
  2023: »The Unemployment-Risk Channel in Business-Cycle Fluctuations«

HANK-SAM

Overview

- Intermediate **producers:**
  1. Hire and fire in search-and-matching labor market
  2. Sell homogeneous good at price p_t^X.
- Wholesale **price-setters:**
  1. Set prices in monopolistic competition subject to adjustment costs
  2. Pay out dividends
- Final producers: Aggregate to final good
- **Government:**
  1. Pay transfers and unemployment insurance
  2. Collect taxes and issues debt
- **Central bank:** Sets nominal interest rate
- **Households:** Consume and save

Equilibrium dynamics

1. **Incomplete markets:** Unemployment risk $\rightarrow$ demand

   Complete markets / representative agent:
   Only total income matters

2. **Sticky prices:** Demand $\rightarrow$ profitability

3. **Frictional labor market:** Profitability $\rightarrow$ unemployment risk

Household problem

$$v_t(\beta_i, u_{it}, a_{it-1}) = \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[v_{t+1}\left(\beta_i, u_{it+1}, a_{it}\right)\right]$$

$$\text{s.t. } a_{it} + c_{it} = (1+r_t)a_{it-1} + (1-\tau_t)y_t(u_{it}) + \text{div}_t + \text{transfer}_t$$

$$a_{it} \geq 0$$

1. **Dividends and government transfers:** div_t and transfer_t
2. **Real wage**: w_{ss}
3. **Income tax:** τ_t
4. **Separation rate** for employed: δ_{ss}
5. **Job-finding rate** for unemployed: $\lambda_t^{u,s} s(u_{it-1})$
   (where $s(u_{it-1})$ is exogenous search effectiveness)
6. US-style duration-dependent **UI system:**
   a) High replacement rate $\overline{\phi}$, first $\overline{u}$ months
   b) Low replacement rate $\underline{\phi}$, after $\overline{u}$ months

Income process

- **Income** is

$$y_{it}(u_{it}) = w_{ss} \cdot \begin{cases} 1 & \text{if } u_{it} = 0 \\ \overline{\phi}\mathrm{UI}_{it} + (1 - \mathrm{UI}_{it})\underline{\phi} & \text{else} \end{cases}$$

where the share of the month with UI is

$$\mathrm{UI}_{it} = \begin{cases} 0 & \text{if } u_{it} = 0 \\ 1 & \text{else if } u_{it} < \overline{u} \\ 0 & \text{else if } u_{it} > \overline{u} + 1 \\ \overline{u} - (u_{it} - 1) & \text{else} \end{cases}$$

- **Note:** *Hereby $\overline{u}$ becomes a continuous variable.*

Transition probabilities

- **Beginning-of-period value function:**

$$\underline{v}_t\left(\beta_i, u_{it-1}, a_{it-1}\right) = \mathbb{E}\left[v_t(\beta_i, u_{it}, a_{it-1}) \,\middle|\, u_{it-1}, a_{it-1}\right]$$

- **Grid:** $u_{it} \in \{0, 1, \ldots, \#_u - 1\}$
- **Employed** with $u_{it-1} = 0$: $u_{it} = \begin{cases} 0 & \text{with prob. } 1 - \delta_{ss} \\ 1 & \text{with prob. } \delta_{ss} \end{cases}$
- **Unemployed** with $u_{it-1} = 1$:

$$u_{it} = \begin{cases} 0 & \text{with prob. } \lambda_t^{u,s} s(u_{it-1}) \\ u_{it-1} + 1 & \text{with prob. } 1 - \lambda_t^{u,s} s(u_{it-1}) \end{cases}$$

Trick: $u_{it} = \min\left\{u_{it-1} + 1, \#_u - 1\right\}$

- **All unemployed search:** $s(u_{it-1}) = \begin{cases} 0 & \text{if } u_{it-1} = 0 \\ 1 & \text{else} \end{cases}$

Aggregation

- **Distributions:**
  1. Beginning-of-period: $\underline{\boldsymbol{D}}_t$ over β_i, u_{it-1} and a_{it-1}
  2. At decision: $\boldsymbol{D}_t$ over β_i, u_{it} and a_{it-1}
- **Stochastic (time-varying) transition matrix:** $\Pi_{t,z} = \Pi_z(\lambda_t^u)$
- **Deterministic savings policy matrix:** Λ_t'
- **Transition steps:**

$$\boldsymbol{D}_t = \Pi_{t,z}' \underline{\boldsymbol{D}}_t$$

$$\underline{\boldsymbol{D}}_{t+1} = \Lambda_t' \boldsymbol{D}_t$$

- **Searchers:** $S_t = \int s(\beta_i, u_{it-1}, a_{it-1}) d\underline{\boldsymbol{D}}_t$
- **Savings:** $A_t^{hh} = \int a_t^*(\beta_i, u_{it}, a_{it-1}) d\boldsymbol{D}_t$
- **Consumption:** $C_t^{hh} = \int c_t^*(\beta_i, u_{it}, a_{it-1}) d\boldsymbol{D}_t$

EGM

- **Beginning-of-period value function:**

$$\underline{v}_{a,t}(\beta_i, u_{it-1}, a_{it-1}) = \mathbb{E}_t\left[v_{a,t}(\beta_i, u_{it}, a_{it-1})\right] = \mathbb{E}_t\left[(1+r_t)c_{it}^{-\sigma}\right]$$

- **Endogenous grid method:** Vary u_{it} and a_{it} to find

$$c_{it} = \left(\beta \underline{v}_{a,t+1}(\beta_i, u_{it}, a_{it})\right)^{-\frac{1}{\sigma}}$$
$$m_{it} = c_{it} + a_{it}$$

- **Consumption and labor supply:** Use linear interpolation to find

$$c_t^*(\beta_i, u_{it}, a_{it-1}) \text{ with } m_{it} = (1+r_t)a_{it-1}$$

- **Savings:** $a^*(u_{it}, a_{it-1}) = (1+r_t)a_{it-1} - c_t^*(\beta_i, u_{it}, a_{it-1})$

Producers: Hiring and firing

- **Job value:**

$$V_t^j = p_t^X Z_t - w_{ss} + \beta^{\text{firm}} \mathbb{E}_t \left[(1 - \delta_{ss}) V_{t+1}^j \right]$$

- **Vacancy value:**

$$V_t^v = -\kappa + \lambda_t^v V_t^j + (1 - \lambda_t^v)(1 - \delta_{ss}) \beta^{\text{firm}} \mathbb{E}_t \left[V_{t+1}^v \right]$$

- **Free entry** implies

$$V_t^v = 0$$

Labor market dynamics

- **Labor market tightness** is given by

$$\theta_t = \frac{\text{vacancies}_t}{\text{searchers}_t} = \frac{v_t}{S_t}$$

- **Cobb-Douglas matching function**

$$\text{matches}_t = AS_t^{\alpha} v_t^{1-\alpha}, \;\; \alpha \in (0,1)$$

implies the job-filling and job-finding rates:

$$\lambda_t^v = \frac{\text{matches}_t}{v_t} = A\theta_t^{-\alpha}$$
$$\lambda_t^{u,s} = \frac{\text{matches}_t}{S_t} = A\theta_t^{1-\alpha}$$

- **Law of motion for unemployment:**

$$u_t = u_{t-1} + \delta_t(1 - u_{t-1}) - \lambda_t^{u,s} S_t$$

Price setters

- **Intermediate goods price:** p_t^X
- Dixit-Stiglitz **demand curve** $\Rightarrow$ **Phillips curve** relating marginal cost, $MC_t = p_t^x$, and **final goods price inflation**, $\Pi_t = P_t/P_{t-1}$,

$$1 - \epsilon + \epsilon p_t^x = \phi \pi_t (1 + \pi_t) - \phi \beta^{\text{firm}} \mathbb{E}_t \left[\pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right]$$

  with output $Y_t = Z_t(1 - u_t)$
- **Flexible price limit:** $\phi \to 0$
- **Dividends:**

$$\text{div}_t = Y_t - w_t(1 - u_t)$$

Central bank

- **Taylor rule:**

$$1 + i_t = (1 + i_{ss}) \left(\frac{1 + \pi_t}{1 + \pi_{ss}} \right)^{\delta_\pi}$$

Government

- **Unemployment insurance:** $\Phi_t = w_{ss}\left(\overline{\phi}\mathrm{UI}_t^{hh} + \underline{\phi}\left(u_t - \mathrm{UI}_t^{hh}\right)\right)$
- **Total expenses:** $X_t = \Phi_t + G_t + \mathrm{transfer}_t$
- **Total taxes:** $\mathrm{taxes}_t = \tau_t\left(\Phi_t + w_{ss}(1-u_t)\right)$
- **Government budget** is

$$q_t B_t = (1 + q_t\delta_q)B_{t-1} + X_t - \mathrm{taxes}_t$$

  Long-term debt: Real payment stream is $1, \delta, \delta^2, \dots$.
  The real bond price is q_t.

- **Tax rule:**

$$\tilde{\tau}_t = \frac{\left(1 + q_t\delta_q\right)B_{t-1} + X_t - q_{ss}B_{ss}}{\Phi_t + w_{ss}(1-u_t)}$$

$$\tau_t = \omega\tilde{\tau}_t + (1-\omega)\tau_{ss}$$

- **Transfers:** $\mathrm{transfer}_t = -\mathrm{div}_{ss}$

Financial markets: No arbitrage

1. **Pricing of government debt:**

$$\frac{1+\delta_q q_{t+1}}{q_t} = \frac{1+i_t}{1+\pi_{t+1}} = 1 + r_{t+1}$$

2. **Ex post real return:**

$$1 + r_t = \begin{cases} \frac{(1+\delta_q q_0)B_{-1}}{A_{-1}^{hh}} & \text{if } t = 0 \\ \frac{1+i_{t-1}}{1+\pi_t} & \text{else} \end{cases}$$

Market clearing

1. Asset market: $A_t^{hh} = q_t B_t$
2. Goods market: $Y_t = C_t^{hh} + G_t$

Tip: *You should be able to verify Walras' law.*

Market clearing

1. **Shocks:** G_t
2. **Unknowns:** p_t^X, V_t^j, v_t, u_t, S_t, π_t, $\mathrm{UI}_t^{\mathrm{guess}}$
3. **Targets:**
   - 3.1 Error in Job Value
   - 3.2 Error in Vacancy Value
   - 3.3 Error in Law-of-Motion for u_t
   - 3.4 Error in Philips Curve
   - 3.5 Error in Asset Market Clearing
   - 3.6 $u_t = U_t^{hh} = \int 1\{u_{it} > 0\} d\boldsymbol{D}_t$
   - 3.7 $\mathrm{UI}_t^{\mathrm{guess}} = \mathrm{UI}_t^{hh} = \int \mathrm{UI}_{it} d\boldsymbol{D}_t$

Steady State

1. **Zero inflation:** $\pi_t = 0$
2. **SAM:** Choose A and κ to ensure $\delta_{ss} = 0.02$ and $\lambda_{ss}^{u,s} = 0.30$
3. **HANK:** Enforce *asset market clearing*
   3.1 Set r_{ss}
   3.2 Calculate implied A_{ss}^{hh}
   3.3 Adjust G_{ss} so $q_{ss} B_{ss} = A_{ss}^{hh}$

Calibration

1. **Real interest rate:** $1 + r_t = 1.02^{\frac{1}{12}}$
2. **Households:** $\sigma = 2.0$
   30%: $\beta_i = \beta^{\text{HtM}} = 0$
   60%: $\beta_i = \beta^{\text{BS}} = 0.94^{\frac{1}{12}}$
   10%: $\beta_i = \beta^{\text{PIH}} = 0.975^{\frac{1}{12}}$
3. **Matching and bargaining:** $\alpha = 0.60$**,** $\theta = 0.60$, $w_{ss} = 0.90$
4. **Producers:** $\beta^{\text{firm}} = 0.975^{\frac{1}{12}}$
5. **Price-setters:** $\epsilon = 6$ and $\phi = 600$
6. **Monetary policy:** $\phi = 1.5$
7. **Government:**
   Tax: $\tau = 0.30$
   Debt: $\delta_q = 1 - \frac{1}{36}$ and $\omega = 0.05$
   UI: $\overline{\phi} = 0.70$, $\underline{\phi} = 0.40$, and $\overline{u} = 6$

Steady state analysis

In **steady state:**

1. Look at the consumption functions
2. Look at the distribution of savings
3. Look at how consumption evolves in unemployment

Policy analysis

Shock: Consider a 1% shock to government consumption

$$G_t - G_{ss} = 0.80^t \cdot 0.01 \cdot G_{ss}$$

Look at **impulse responses** for**:**

1. Output
2. Unemployment (risk)
3. Tax rate

What drives the consumption response?

1. Interest rate
2. Tax rate
3. Job-finding rate
4. Dividends

Is the effect from the job-finding rate larger than an equivalent change in income causes by wages? Why?

Stimulus Effects of Common Fiscal Policies

Introduction

- **Many types of fiscal policy:**
  1. Government consumption, G_t
  2. Universal transfer, $T_t = \text{transfer}_t$
  3. Higher unemployment benefits, $\overline{\phi}_t$
  4. Longer unemployment benefit duration, $\overline{u}_t$
  5. Hiring subsidies, hs_t
  6. Retention subsidies, rs_t
- **Extended model:**
  1. Endogenous separations + sluggish entry
  2. Dividends distributed equally
  3. Decreasing search intensity/efficiency while unemployed
  4. Risk of no unemployment benefits
  5. More detailed calibration
- **Previous paper:** Broer et. al. (2023) in zero liquidity

Model summary

- **Notation:** $\boldsymbol{x} = [X_0 - X_{ss}, X_1 - X_{ss}, \ldots]$
- **Household policies:**

$$\boldsymbol{h} = \left[\boldsymbol{g}, \boldsymbol{t}, \overline{\boldsymbol{\phi}}, \overline{\boldsymbol{u}}\right]'$$

- **Firm policies:**

$$\boldsymbol{f} = [\boldsymbol{hs}, \boldsymbol{rs}]'$$

- **Income process:**

$$\boldsymbol{inc} = [\boldsymbol{\delta}, \boldsymbol{\lambda^u}, \boldsymbol{div}]'$$

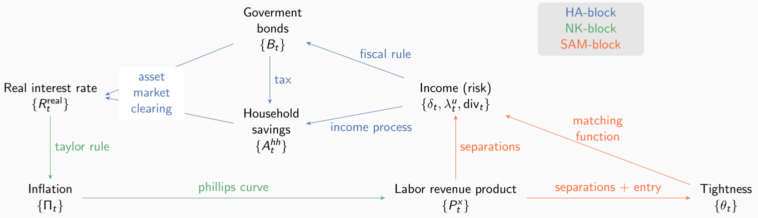
- **Model summary:**

$$\boldsymbol{r^{real}} = M_{HA}\boldsymbol{inc} + M_{h,r}\boldsymbol{h} + M_{f,r}\boldsymbol{f}, \tag{1}$$

$$\boldsymbol{p^x} = M_{NK}\boldsymbol{r^{real}}, \tag{2}$$

$$\boldsymbol{inc} = M_{SAM}\boldsymbol{p^x} + M_{s,inc}\boldsymbol{f}. \tag{3}$$

Directed Cycle Graph



Directed Cycle Process

Let $||\cdot||$ denote the operator norm. If $||M_{SAM}M_{NK}M_{HA}|| < 1$, there is a unique solution to the system (1)-(3) given by

$$\boldsymbol{inc} = \underbrace{\mathcal{G}}_{\text{GE}} \times \left(\underbrace{M_{\text{SAM}}M_{\text{NK}}\underbrace{M_{h,r}\boldsymbol{h}}_{\text{direct}}}_{\text{first round, household transfer policy}} + \underbrace{M_{SAM}M_{NK}\underbrace{M_{f,r}\boldsymbol{f}}_{\text{direct}} + \underbrace{M_{f,\text{inc}}\boldsymbol{f}}_{\text{direct}}}_{\text{first round, firm transfer policy}} \right),$$

where $\mathcal{G}$ is defined by

$$\mathcal{G} = (I - M_{SAM}M_{NK}M_{HA})^{-1}.$$

Fiscal multipliers

- **Fiscal multiplier:**

$$\mathcal{M} = \text{cumulative fiscal multiplier} = \frac{\mathbf{1}'\boldsymbol{y}}{\mathbf{1}'\boldsymbol{taxes}}.$$

$$\boldsymbol{taxes} = M_{\text{inc,taxes}}\boldsymbol{inc} + M_{h,taxes}\boldsymbol{h}$$

- **Household policies 0 and 1:** If same direct PE real interest rate

$$M_{h,r}\boldsymbol{h}^0 = M_{h,r}\boldsymbol{h}^1$$

then output and income are the same $\boldsymbol{y}^0 = \boldsymbol{y}^1$ and $\boldsymbol{inc}^0 = \boldsymbol{inc}^1$. Differences in taxes are due to direct fiscal costs

$$\mathbf{1}'\boldsymbol{taxes}^0 - \mathbf{1}'\boldsymbol{taxes}^1 = \mathbf{1}'M_{h,\text{taxes}}\boldsymbol{h}^0 - \mathbf{1}'M_{h,\text{taxes}}\boldsymbol{h}^1,$$

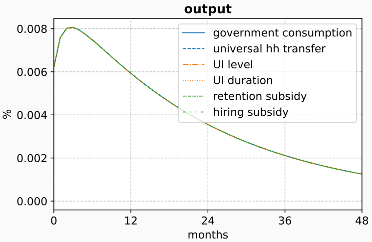
Fiscal multipliers are ordered by direct fiscal costs:

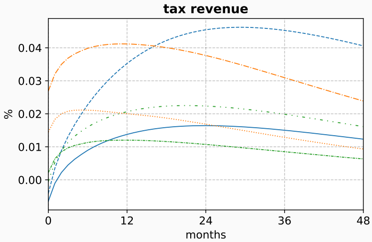
$$\mathcal{M}_{h^0} \gtreqless \mathcal{M}_{h^1} \iff \mathbf{1}'M_{h,\text{taxes}}\boldsymbol{h}^0 \lesseqgtr \mathbf{1}'M_{h,\text{taxes}}\boldsymbol{h}^1.$$

- **Firm policies:** Same result, but only with representative agent

Policy experiment

- **Experiment:** Same output path for different policies.





Different fiscal multipliers

	G [level]	Household transfers: Transfer	Household transfers: Level	Household transfers: Duration	Firm transfers: Retention	Firm transfers: Hiring
1. Relative fiscal multiplier	1.0 [0.99]	0.28	0.44	1.03	1.64	0.72
2. Relative tax response	1.00	3.64	2.29	0.97	0.61	1.39
3. PE relative tax response	1.47	4.11	2.77	1.45	0.57	1.56
4. GE relative tax response	-0.47	-0.47	-0.47	-0.47	0.04	-0.17

- **Relative fiscal multiplier:** $\frac{\mathcal{M}_{h^j}}{\mathcal{M}_{h^G}}$
- **Relative tax respones:** $\frac{\mathbf{1}'\boldsymbol{taxes}^j}{\mathbf{1}'\boldsymbol{taxes}^G}$
  Decomposition for household transfers:

$$\boldsymbol{taxes}^j = M_{\text{inc,taxes}}\boldsymbol{inc}^j + M_{h,taxes}\boldsymbol{h}^j$$
$$\boldsymbol{taxes}^{j,\text{PE}} = M_{h,\text{taxes}}\boldsymbol{h}^j$$
$$\boldsymbol{taxes}^{j,\text{GE}} = M_{\text{inc,taxes}}\boldsymbol{inc}^j$$

Determinants of fiscal multipliers

	G [level]	Household transfers: Transfer	Household transfers: Level	Household transfers: Duration	Firm transfers: Retention	Firm transfers: Hiring
1. Baseline	1.0 [0.99]	0.28	0.44	1.03	1.64	0.72
2. Less sticky prices ($\phi = 178$)	1.0 [0.61]	0.30	0.47	1.03	3.43	1.15
3. More reactive mp ($\delta_\pi = 2$)	1.0 [0.64]	0.30	0.47	1.03	3.33	1.13
4. Representative agent	1.0 [0.54]	0.00	0.00	0.00	1.92	0.57
5. Fewer HtM (17.4%)	1.0 [0.80]	0.19	0.41	1.11	1.80	0.69
6. More tax financing ($\omega = 0.10$)	1.0 [0.84]	0.19	0.40	1.10	1.70	0.67
7. Exo. separations ($\psi = 0$)	1.0 [0.13]	0.35	0.52	1.02	1.39	3.38
8. Free entry ($\xi = \infty$)	1.0 [0.54]	0.31	0.47	1.03	1.50	1.21
9. Wage rule ($\eta_e = 0.50$)	1.0 [0.73]	0.29	0.46	1.03	1.55	0.74
10. 95% of div. to PIH	1.0 [0.82]	0.28	0.43	0.99	0.72	0.16

Endogenous search

Endogenous search

- **Search decision:**
  1. Discrete search choice: $s_{it} \in \{0, 1\}$
  2. Search cost: λ if $s_{it} = 1$
  3. Taste shocks: $\varepsilon(s_{it}) \sim$ Extreme value (Iskhakov et. al., 2017)
- **See also:** Bardóczy (2021)
- **Note:** Drop β_i for notational simplicity
- **Warning:** *This is advanced! Only for the interested if time permits.*

Discrete search decision

- **Search intensity matter for transition:**

$$\underline{v}_t\left(\beta_i, u_{it-1}, a_{it-1} \,|\, s_{it}\right) = \mathbb{E}\left[v_t(\beta_i, u_{it}, a_{it-1}) \,|\, u_{it-1}, a_{it-1}, s_{it}\right]$$

- **Standard logit formula:**

$$\begin{aligned}\underline{v}_t(u_{it-1}, a_{it-1}) &= \max_{s_{it}\in\{0,1\}} \left\{\underline{v}_t(u_{it-1}, a_{it-1} \,|\, s_{it}) - \lambda \mathbf{1}_{s_{it}=1} + \sigma_\varepsilon \varepsilon\left(s_{it}\right)\right\} \\ &= \sigma_\varepsilon \log\left(\exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \,|\, 0)}{\sigma_\varepsilon} + \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \,|\, 1)}{\sigma_\varepsilon}\right)\end{aligned}$$

Envelope condition

- **Choice probabilities:**

$$P_t(s \mid u_{it-1}, a_{it-1}) = \frac{\exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid s)}{\sigma_\xi}}{\sum_{s' \in \{0,1\}} \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid s')}{\sigma_\xi}}$$

- **Envelope condition:**

$$\begin{aligned} \underline{v}_{a,t}(u_{t-1}, a_{t-1}) &= \sum_{s \in \{0,1\}} P_t(s \mid u_{it-1}, a_{it-1}) \pi_t(u_{it} \mid u_{it-1}, s) v_{a,t}(u_{it}, a_{it-1}) \\ &= \sum_{s \in \{0,1\}} P_t(s \mid u_{it-1}, a_{it-1}) \pi_t(u_{it} \mid u_{it-1}, s) c_t^*(u_{it}, a_{it-1})^{-\sigma} \end{aligned}$$

- **Break of *monotonicity*** $\Rightarrow$ FOC still *necessary*, but not *sufficient*
  1. **Normally:** Savings $\uparrow \Rightarrow$ future consumption $\uparrow \Rightarrow$ marginal utility $\downarrow$
  2. **Now also:** Future search jump $\downarrow \Rightarrow$ future income $\downarrow$
     $\Rightarrow$ future consumption $\downarrow \Rightarrow$ marginal utility $\uparrow$

Upper envelope for given z^{i_z}

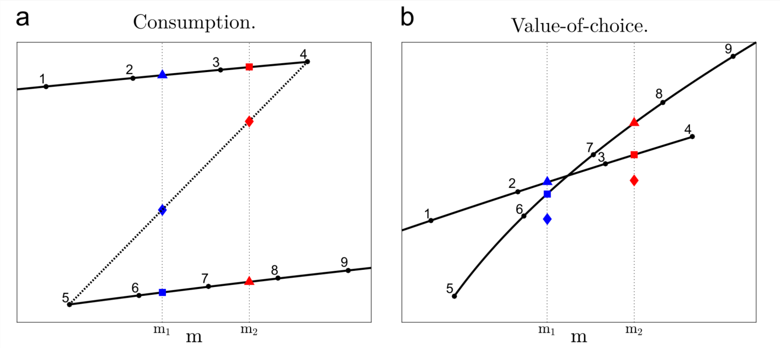
1. **Generate candidate points:** $\forall i_a \in \{0, 1, \dots, \#_a - 1\}$

$$
\begin{aligned}
w^{i_a} &= \beta \underline{v}_{t+1}(z^{i_z}, a^{i_a}) \\
c^{i_a} &= u'^{-1}\left(\beta \underline{v}_{a,t+1}(z^{i_z}, a^{i_a})\right) \\
m^{i_a} &= a^{i_a} + c^{i_a} \\
v^{i_a} &= u\left(c^{i_a}\right) + w^{i_a}
\end{aligned}
$$

2. **Apply upper-envelope:** $\forall i_{a-} \in \{0, 1, \dots, \#_a - 1\}$

$$
\begin{aligned}
c^*(a^{i_{a-}}) &= \max_{j \in \{0,1,\dots \#_a - 2\}} u\left(c^{i_{a-}}\right) + w^{i_{a-}} \text{ s.t.} \\
m^{i_{a-}} &= (1 + r_t) a^{i_{a-}} + w_t z^{i_z} \in \left[m^j, m^{j+1}\right] \\
c^{i_{a-}} &= \min\left\{\text{interp } \left\{m^{i_a}\right\} \to \left\{c^{i_a}\right\} \text{ at } m^{i_{a-}}, m^{i_{a-}}\right\} \\
a^{i_{a-}} &= m^{i_{a-}} - c^{i_{a-}} \\
w^{i_{a-}} &= \text{interp } \left\{a^{i_a}\right\} \to \left\{w^{i_a}\right\} \text{ at } a^{i_{a-}}
\end{aligned}
$$

Illustration



1. **Numbering:** Different levels of end-of-period assets, a^{i_a}
2. **Problem:** Find the consumption function at m_1 and m_2
3. **Largest value-of-choice:** Denoted by the *triangles*

Source: Druedahl and Jørgensen (2017), G^2EGM

Example

- **Beg.-of-period value function:**

$$\underline{v}_{t+1}(a_t) = \sqrt{m_{t+1}} + \eta \max\left\{m_{t+1} - \underline{m}, 0\right\}$$
$$\text{where } m_{t+1} = (1+r)a_t + 1$$

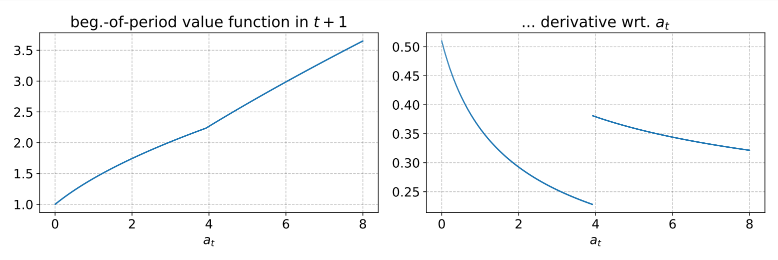
- **Derivative:**

$$\underline{v}_{a,t+1}(a_t) = \frac{1}{2}(1+r)m_{t+1}^{-\frac{1}{2}} + (1+r)\eta\mathbf{1}\left\{m_{t+1} > \underline{m}\right\}$$

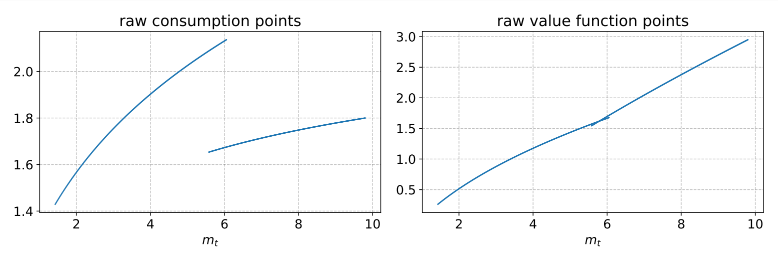
- **Budget constraint:**

$$a_t + c_t = (1+r)a_{t-1} + 1$$

Next-period values

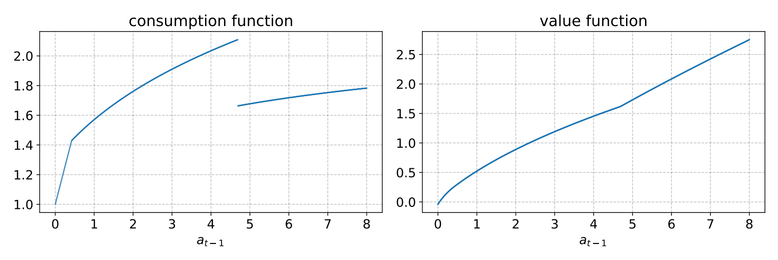


Raw values of c^{i_a} and v^{i_a}



Problem: Overlaps $\Rightarrow$ not a function m_t!

Result after upper envelope



General problem structure

- **General problem structure with *nesting*:**

$$\overline{v}_t\left(\overline{x}_t, d_t, e_t, m_t\right) = \max_{c_t \in [0, m_t]} u(c_t, d_t, e_t) + \beta \underline{v}_{t+1}\left(\underline{\Gamma}_t\left(\overline{x}_t, d_t, e_t, a_t\right)\right)$$

$$\text{with } a_t = m_t - c_t$$

$$v(x_t) = \max_{d_t \in \Omega^d(x_t)} \overline{v}_t\left(\overline{\Gamma}_t\left(x_t, d_t\right)\right)$$

$$\underline{v}_t\left(\underline{x}_t\right) = \max_{e_t \in \Omega^e(\underline{x}_t)} \mathbb{E}\left[v\left(\Gamma\left(\underline{x}_t, e_t\right)\right) \mid \underline{x}_t, e_t\right]$$

- **Finding c_t:** *EGM with upper envelope can (typically) still be used*
- **Finding d_t and e_t:**
  1. Combination of discrete and continuous choices
  2. Typically requires use of numerical optimizer or root-finder
- **Druedahl (2021),** »A Guide on Solving Non-Convex Consumption-Saving Models« (costly with extra states in $\overline{v}$)

Summary

Summary

- **HANK-SAM:**
  1. More realistic labor market and income process
  2. Allow for fluctuations in idiosyncratic risk
  3. Laboratory for studying e.g. fiscal policy
- **Solution methods:**
  1. Time-varying transition matrix is straightforward
  2. Non-sufficient Euler-equation create serious problems

Adds

Økonomisk Eksploratorium

#40 Nobelpris: Velstand handler om gode institutioner

Nobelprisen i økonomi i 2024 gik til Daron Acemoglu, Simon Johnson og James Robinson for at have demonstreret at sociale institutioner er afgørende for et lands velstand. Samfund med et svagt retssystem og institutioner, som udbytter befolkningen, genererer ikke vækst eller ændringer til det bedre.

Jeanet Bentzen (lektor på Økonomisk Institut) giver et overblik over den prisvindende forskning, deriblandt hendes egen forskning med en af prisvinderne.
Bagefter vil der være tid til spørgsmål og diskussioner.

Detaljer

Tid: 28. nov. 2024, kl. 16.00

Sted: Lokale 26.3.21 (Bygning 26, 3. sal), Øster Farimagsgade 5, 1353 København K

Arrangør: Økonomisk Institut

PhD (4+4 or 5+3)

PhD scholarship at the Department of Economics, University of Copenhagen (UCPH)	Det Samfundsvidenskabelige Fakultet	Department of Economics	15-01-2025
PhD scholarships at Center for Economic Behaviour and Inequality (CEBI) at the Department of Economics, University of Copenhagen (UCPH)	Det Samfundsvidenskabelige Fakultet	Department of Economics	15-01-2025
PhD scholarships as part of the research project, "Behavioral Barriers to the Green Transition", CEBI, University of Copenhagen (UCPH)	Det Samfundsvidenskabelige Fakultet	Department of Economics	15-01-2025
PhD scholarship as part of the research project "The Role of Preferences and Beliefs in Shaping Physician Careers", CEBI, University of Copenhagen (UCPH)	Det Samfundsvidenskabelige Fakultet	Department of Economics	15-01-2025

Interested? Come talk to me

Additional positions:

1. **Nationalbanken:** Link
2. **EUI** (in Florence)**:** Virtual Open Day Dec 2 at 13 (link)