Advanced Macroeconomics: Heterogenous Agent Models
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Assignment 3: *Inheritance taxation in HANC with bequests and endogenous labor supply*

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1. Introduction

Inheritances play a key role in explaining the long term persistence of wealth inequality in society. For this reason, a much-debated political question is how much we should tax inheritances. Higher inheritance taxes could potentially increase social mobility by redistributing wealth from rich dynasties to poor households. However, taxing inheritances could also influence labor supply, as parental savings and labor decisions could be shaped by their expectations about the ability to transfer wealth to their descendants, while the labor supply decisions of children may adjust based on their anticipated inheritance.

This assignment explores these mechanisms within a heterogeneous-agent (HA) framework, using an extended Aiyagari HANC-model with endogenous labor supply and bequest motives. The aim is to study how inheritance taxation impact household behavior and aggregate outcomes.

2. Model

Below, I will formally state my model. Parts of the model description are replicated directly from the Ayagari-model with endogenous labor supply from Assignment 1, and from the "HANC" and "HANC-gov" notebooks from the GEModelTools repository. I have also found inspiration in the lecture slides on topic 6: Wealth Inequality, and in my own submission of Assignment 1.

2.1. Households

Households are homogeneous *ex ante*, but heterogeneous *ex post* with respect to their productivity z_t, their (end-of-period) assets a_t and whether they are still alive s_t. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Each period households choose their labor supply ℓ_t and consumption c_t subject to a no-borrowing constraint, $a_t \geq 0$. The real interest rate is r_t, the real wage is w_t, and real profits are Π_t. Households receive real lumpsum transfers $\mathcal{T}_t$ from the government. Interest-rate income is taxed with the rate $\tau_t^\alpha \in [0, 1]$. Labor income is taxed with the rate τ_t^ℓ.

Households live a finite number of periods. s_t indicates whether the household is still alive in period t, and s_t^p indicates whether the parent of the household is still alive. When a household dies, they gain utility through the bequest motif $\phi(a_{t-1})$. Child households receive an inheritance b_t when their parent dies, corresponding to the terminal wealth of the parent household a_{t-1}^p. Inheritances are taxed with the rate $\tau_t^b \in [0, 1]$.

The household problem is

$$
\begin{aligned}
v_t(s_t, z_t, a_{t-1}) = \max_{c_t, \ell_t, a_t} \; & s_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\left(\frac{1}{\nu}\right)}}{1+\left(\frac{1}{\nu}\right)} \right] + (1 - s_t)\phi(\tilde{a}_{t-1}) \\
& + \beta \mathbb{E}\left[v_{t+1}\left(s_{t+1}, z_{t+1}, a_t\right) \mid s_t, z_t, a_t \right] \\
\text{s.t.} \quad c_t + a_t &= (1 + \tilde{r}_t) a_{t-1} + \tilde{w}_t \ell_t z_t + \Pi_t + \mathcal{T}_t + \tilde{b}_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1}, \psi \sim \mathcal{N}\left(\mu_\psi, \sigma_\psi\right), \mathbb{E}[z_t] = 1 \\
b_t &= (1 - s_t^p) a_{t-1}^p \\
\phi(a_t) &= \gamma \frac{a_t^{1-\eta}}{1-\eta} \\
a_t &\geq 0
\end{aligned}
\tag{1}
$$

where $\tilde{r}_t = (1 - \tau_t^a) r_t$, $\tilde{w}_t = \left(1 - \tau_t^b\right) w_t$, $\tilde{a}_t = \left(1 - \tau_t^b\right) a_t$, and $\tilde{b}_t = \left(1 - \tau_t^b\right) b_t$. The binary random variable s_t is Bernoulli distributed and equal to 1 with a probability p_t which decreases over time.

Likewise, s_t^p is equal to 1 with probability p_t^p. New households enter the economy when old ones die such that a stationary equilibrium can be obtained. Households observe their parents' wealth a_{t-1}^p and have perfect foresight with regards to wages w_t, interest rates r_t, taxes $\tau_t^\ell, \tau_t^r, \tau_t^b$, profits Π_t, transfers $\mathcal{T}_t$ and survival probabilities p_t, p_t^p, such that implicitly

$$v_t(s_t, z_t, a_{t-1}) = v_t\Big(s_t, z_t, a_{t-1}, a_{t-1}^p, \{w_t, r_t, \tau_t^\ell, \tau_t^r, \tau_t^b, \Pi_t, \mathcal{T}_t, p_t, p_t^p\}_{t=0}^{\infty}\Big) \tag{2}$$

Aggregate quantities are

$$\begin{aligned} L_t^{\text{hh}} &= \int \ell_t z_t \,\mathrm{d}\boldsymbol{D}_t \\ C_t^{\text{hh}} &= \int c_t \,\mathrm{d}\boldsymbol{D}_t \\ A_t^{\text{hh}} &= \int a_t \,\mathrm{d}\boldsymbol{D}_t \end{aligned} \tag{3}$$

2.2. Firms

A representative firm rents capital, K_{t-1}, and hires labor, L_t, to produce goods, with the production function

$$Y_t = F(\Gamma_t, K_{t-1}, L_t) = \Gamma_t K_{t-1}^\alpha L_t^{1-\alpha} \tag{4}$$

Capital depreciates with the rate $\delta \in (0, 1)$. The real rental price of capital is r_t^K and the real wage is w_t. Profits are

$$\Pi_t = Y_t - w_t L_t - r_t^K K_{t-1} \tag{5}$$

The law of motion for capital is

$$K_t = (1 - \delta)K_{t-1} + I_t \tag{6}$$

Profit maximization by

$$\max_{K_{t-1}, L_t} Y_t - w_t L_t - r_t^k K_{t-1} \tag{7}$$

implies the standard pricing equations

$$\begin{aligned} r_t^k &= \alpha \Gamma_t (K_{t-1}/L_t)^{\alpha-1} \\ w_t &= (1-\alpha)\Gamma_t (K_{t-1}/L_t)^\alpha \end{aligned} \tag{8}$$

where r_t^k is the rental rate of capital and w_t is the wage rate.

The households own the representative firm in equal shares.

2.3. Government

The government raises taxes from the capital income tax, the labor income tax, and the inheritance tax, and transfers this back to the households as a lump sum transfer. The budget constraint for the government is

$$\mathcal{T}_t = \int \left[\tau_t^a r_t a_{t-1} + \tau_t^\ell w_t \ell_t z_t + (1 - s_t)\tau^b a_{t-1}\right] \mathrm{d}\boldsymbol{D}_t \tag{9}$$

2.4. Market clearing

Market clearing implies

1. Labor market: $L_t = L_t^{\mathrm{hh}}$
2. Goods market: $Y_t = C_t^{\mathrm{hh}} + I_t$
3. Asset market: $K_t = A_t^{\mathrm{hh}}$

2.5. Solving the model

Using the notation

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \nu(\ell_t) = \varphi \frac{\ell^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{10}$$

the Lagrangian for the household problem is:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t s_t \big(u\big(1 + \tilde{r} a_{t-1} + \tilde{w}_t \ell_t z_t + \Pi_t + \mathcal{T}_t + \tilde{b}_t - a_t\big) - \nu(\ell_t) \big) \\ +(1-s_t)\phi\big(\big(1-\tau^b\big)a_{t-1}\big) + \beta^t \lambda_t (a_t - 0) \end{aligned} \tag{11}$$

where λ_t is the Lagrangian multiplier on the borrowing constraint. The first order conditions with regards to savings and labor supply are:

$$\begin{aligned} s_t u'(c_t) &= \beta \mathbb{E}_t \big[s_{t+1} (1 + \tilde{r}_{t+1}) u'(c_{t+1}) + (1 - s_{t+1}) \big(1 + \tau_t^b\big) \phi'(\tilde{a}_t) \big] + \lambda_t \\ &= p_{t+1} \beta (1 + \tilde{r}_{t+1}) \mathbb{E}_t [u'(c_{t+1})] + (1 - p_{t+1}) \beta \big(1 + \tau_t^b\big) \phi'(\tilde{a}_t) + \lambda_t \\ \nu'(\ell_t) &= s_t \tilde{w}_t z_t u'(c_t) \end{aligned} \tag{12}$$

The envelope condition is:

$$\underline{v}_{a,t}(s_t, a_t, z_t) \equiv \frac{\partial \underline{v}_t(s_t, a_t, z_t)}{\partial a_t} = \mathbb{E}_t[(1 + \tilde{r}_t) u'(c_t)] \tag{13}$$

In the case where the household is unconstrained ($\lambda_t = 0$) and still alive ($s_t = 1$), the Euler equation is derived using the first order condition and the envelope condition:

$$u'(c_t) = p_{t+1} \beta \underline{v}_{a,t+1}(s_t, a_t, z_t) + (1 - p_{t+1}) \beta \big(1 + \tau_t^b\big) \phi'(\tilde{a}_t) \tag{14}$$

which can be inverted to obtain

$$c_t = \big[p_{t+1} \beta \underline{v}_{a,t+1}(s_t, a_t, z_t) + (1 - p_{t+1}) \beta \big(1 + \tau_t^b\big) \phi'(\tilde{a}_t) \big]^{\frac{1}{-\sigma}} \tag{15}$$

In the case where the household is constrained ($\lambda_t \neq 0$), we set $a_t = 0$ and solve for c_t, ℓ_t using the budget constraint and the labor supply FOC which still holds with equality. In the final case where the household is no longer alive ($s_t = 0$) in period t, their wealth is transferred to the child household through inheritance, and the parent household is removed from the economy.

2.6. Stationary equilibrium

In the stationary equilibrium,

1. Quantities K_{ss} and L_{ss},
2. prices r_{ss} and w_{ss},
3. the level of government transfers $\mathcal{T}_{\text{ss}}$
4. the distribution $\boldsymbol{D}_{\text{ss}}$ over s_t, z_t, a_{t-1}
5. and the policy functions a^*_{ss}, ℓ^*_{ss} and c^*_{ss}

are such that

1. Households maximize expected utility
2. Firms maximize profits
3. $\boldsymbol{D}_{\text{ss}}$ is the invariant distribution implied by the household problem
4. Government balance sheet is satisfied
5. The markets for capital, labor and goods clear.

3. Implementation and analysis

In the model specification stated above, the decisions of the child household depends on state variables in the parent household. In particular, at the time of the parent's death (when $s^p_t = 0$), the terminal wealth of the parents a^p_{t-1} enters the budget constraint of the child household. This implies that implementing the model requires a full Monte Carlo-simulation, where individual child households are linked to parent households. Since such a simulation is computationally expensive, it is relevant to investigate whether we can simplify the model such that household decisions do not depend on state variables of other households.

3.1. Simplifying restrictions

If we introduce two additional restrictions, we can remove the child household's dependency on the parent household's state variables:

$$\begin{aligned} a^p_{t-1} &= a_0 \\ p^p_t &= p_{t+k} \end{aligned} \tag{16}$$

The first restriction replaces the endogenous inheritance size with a fixed amount, which we equate with the child household's initial wealth. This is a simplistic way to remove the interdependency while still capturing some degree of social heritage, such that households with low initial wealth can expect low inheritance and vice versa. If the economy starts in a stationary equilibrium, the initial wealth distribution will reflect the equilibrium.

In the second restriction, we set the "age" of the parents to be some fixed amount of periods older than the child household. In a yearly model, we could e.g. set $k = 30$. Thus, instead of endogenously keeping track of the parent household's survival chances, we can instead derive it from the survival chances of the child household.

3.2. Deterministic vs. stochastic length of life

Overall, we have options for implementing the survival chances p_t. The first option is using a deterministic length of life, for example by setting

$$p_t = s_t = \begin{cases} 0 & t = J \\ 1 & \text{o/w} \end{cases} \tag{17}$$

Alternatively, the chance of still being alive each period p_t could be calibrated using empirical mortality rates.[1]

3.3. Analyzing inheritance taxation

To analyze how the size of inheritance taxation affects household behavior and aggregate outcomes, the model can initially be solved with $\tau^b = 0$. From this stationary equilibrium, we can compute the transition path to a new equilibrium with a positive τ^b, e.g. $\tau^b = 0.15$ to match the current Danish inheritance tax rate. Then, we can observe how Y_t, L_t and K_t changes over the transition path and in the stationary equilibrium.

In particular it is interesting to observe the effects on L_t, where we should expect inheritance taxation to negatively affect the labor supply of parent households, as their expected bequest utility diminishes with the tax. However, child households could be expected to increase their labor supply when we introduce the inheritance tax, since they will have to work more to meet their asset buffer target if they expect less net inheritance – especially when they reach an age where their parents' survival chances start diminishing.

Another object of interest is wealth inequality, which we can compute at each stage of the transition path using e.g. the Gini coefficient or a similar inequality measure. The inheritance tax is expected to lower inequality, as the government can afford higher transfers and wealthy households are less inclined to save.

In further analysis, interactions between inheritance tax and the labor and capital taxes could be investigated. For instance, we could allow the labor tax to adjust while keeping transfers fixed, and thus perhaps mitigate some of the negative effects on labor supply from the inheritance tax. It is also possible that the behavioral responses to an inheritance tax is stronger if the capital gains tax is low, since the incentives to save could amplify one another.

Under all circumstances, it is obvious that the behavioral response to the inheritance tax depends on the strength of the bequest motif $\phi(a_{t-1})$, which is captured in the parameter in my model specification. γ could perhaps be calibrated to match the empirical size of the ratio between inheritances and consumption levels, although this is likely to differ greatly for different wealth groups, which would require a more elaborate model to capture.

[1]For instance, using data from the 2024 Human Mortality Database:
https://ourworldindata.org/grapher/annual-death-rate-by-age-group?country=~DNK