

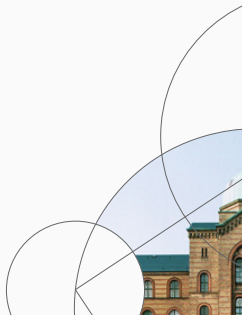


9. Fiscal Policy in HANK

Adv. Macro: Heterogenous Agent Models

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Introduction

Exam info

- Exam form:
 - Portfolio (the 3 assignments)
 - **36 hours** take home
- Exam office thought it was 48 hours, so some dates were wrong online
 - E.g. info in *Exam schedules* at <https://socialsciences.ku.dk/education/student-services/exam-schedules> has been **wrong**
- The correct dates are:
 - Start: January 4th morning (9AM)
 - End: January 5th evening (9PM)
- Re-examination: Oral exam

Introduction

- **Today:**
 - The canonical HANK model
 - Model with sticky wages
 - **Application:** Fiscal policy in HANK
- **Literature:** Auclert et. al. (2023),
»The Intertemporal Keynesian Cross«
 - Long paper with many (technical) details
 - We will focus on the main results

Sticky Wages

- Early HANK papers formulated *Heterogeneous Agent New Keynesian* Models by:
 - Take standard NK model from last lecture
 - Replace Representative agent HH block with Heterogeneous Agents HH block
 - See e.g. McKay, Nakamura, and Steinsson (2016), Hagedorn, Manovskii, and Mitman (2019)
- Turns out that just doing this has undesirable properties when:
 - Prices are sticky
 - Wages are fully flexible
- Need to make one adjustment to standard NK model before adding HA

Sticky wages

- Canonical NK model features **sticky** prices, **flexible** wages
 - Fine for some questions, but can cause issues in HA models (see Auclert, Bardóczy, and Rognlie (2023))
- **EX**: Standard HA model where households have flexible labor supply with FOC (EIS=1, Frisch=1):

$$u'(c_i) w = v'(l_i) \\ \Leftrightarrow c_i^{-1} w = \varphi l_i$$

- What happens if we give household i a lumpsum transfer T ?

$$-c_i^{-2} \frac{\partial c_i}{\partial T} w = \varphi \frac{\partial l_i}{\partial T}$$

- Define $MPC_i = \frac{\partial c_i}{\partial T}$, $MPE_i = -\frac{\partial w l_i}{\partial T}$ as the change in consumption and change in earnings following transfer:

$$MPC_i = \frac{c_i}{w l_i} MPE_i$$

- If HHs have little saving, $c_i \approx w l_i \Rightarrow MPC_i = MPE_i$

MPCs and MPEs

- HA model with flexible labor supply implies tight link btw. MPC_i and MPE_i
 - Plausible?
- **MPCs in the data:**
 - Annual estimates between 0.3 and 0.7
 - Typical value in HA literature: 0.5
- **MPEs in the data:**
 - Range from 0 to 0.04 annually \Rightarrow wealth effects are **small**
 - Increase in (non-wage) income of 1\$ decreases labor earnings by 0-0.04\$
- Tension between model and data \Rightarrow Standard model with high MPCs imply too large wealth effects on labor supply

Union setup

- **Solution:** Take HHs off their individual labor supply curves through **union membership**
 - Litterature: Erceg, Henderson and Levin (2000)
- Households solve the same consumption/saving problem as usual but do not choose labor supply l_{it} themselves
- Instead, they belong to a labor union that determines the level of labor supply
- Will also use this a method of introducing **sticky wages**
 - Empirical evidence typically show that wages adjust more sluggishly than prices w.r.t aggregate shocks

Union problem

- Each household i belong to a union j and face labor demand from firms

$$L_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\epsilon^W} L_t,$$

- Unions maximize aggregate utility of their members, choosing both wages and labor subject to demand:

$$\max_{W_t^j, L_t^j} \sum_{t=0}^{\infty} \beta^t \left(\int \left\{ u(c_{i,t}) - \nu(L_t^j) \right\} dD_{i,t} - \frac{\theta^W}{2} \left(\frac{W_t^j}{W_{t-1}^j} - 1 \right)^2 \right)$$

- where $\frac{\theta^W}{2} \left(W_t^j / W_{t-1}^j - 1 \right)^2$ is a quadratic utility cost from changing wages \Rightarrow sticky wages

New Keynesian Wage Phillips curve

- Solving the union's maximization problem in a symmetric equilibrium $L_t^j = L_t$, $W_t^j = W_t$:

$$\pi_t^w (1 + \pi_t^w) = \kappa^w \left\{ \nu' (L_t) - \frac{w_t}{\mu^w} \int z_t u' (c_{i,t}) d\mathcal{D}_{i,t} \right\} L_t + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w)$$

- with $\kappa^w = \frac{\epsilon^w}{\theta^w}$, $\mu^w = \frac{\epsilon^w}{\epsilon^w - 1}$ and z_{it} is individual productivity ($z_{it} = 1$ in RA)
- **Philips curve** for wages. For given future inflation π_{t+1}^w
 - Increase in labor demand L_t will push up nominal wage inflation
 - Increase in real wage w_t will decrease nominal wage inflation
 - Higher consumption will increase wage inflation (wealth effect)
 - Pass-through depends on slope κ^w
- Breaks wealth effect on labor supply through two mechanisms:
 - Sticky wages (low κ^w)
 - Wealth effect on agg. L depends on agg. marginal utility $\int z_t u' (c_{i,t}) d\mathcal{D}_{i,t}$

More tractable version

- Sometimes assume that union maximize the utility of an agent with average consumption $u(C_t) = u\left(\int c_{it} d\mathcal{D}_{it}\right)$ instead of $\int u(c_{i,t}) d\mathcal{D}_{it}$

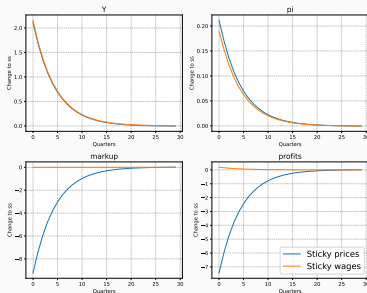
- NKWPC:

$$\pi_t^w (1 + \pi_t^w) = \kappa^w \left\{ \nu'(L_t) - \frac{w_t}{\mu^w} u'(C_t) \right\} L_t + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w)$$

- $u'(C_t)$ instead of $\int z_t u'(c_{i,t}) d\mathcal{D}_{i,t}$
- Slightly easier to implement in code and when working analytically

Profits

- Additional benefit of introducing sticky wages is the **cyclical** of **profits**
- In the data profits are *pro-cyclical* (i.e. go up in boom)
- Not consistent with standard NK model with sticky prices
 - Wages rise more than prices in response to demand shock
- Sticky wages solve this



HANK



Model elements

- The Heterogeneous Agent New Keynesian model
 - Similar to standard NK model but substitute sticky prices with wages + HA instead of RA
- **Households**
 - Standard HA: Face idiosyncratic income risk + borrowing constraint
- **Firms**
 - Produce using labor
- **Unions**
 - Decide on labor supply for HHs subject to wage adjustment cost
- **Mutual fund**
 - Collect household savings and invest in available assets (today: gov' bonds)
- **Central bank**
- **Government**

- **Household problem:**

$$v_t(z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t [v_{t+1}(z_{t+1}, a_t)]$$

$$\text{s.t. } a_t + c_t = (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_t \ell_t z_t + \chi_t$$

$$\log z_{t+1} = \rho_z \log z_t + \psi_{t+1}, \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E}[z_t] = 1$$

$$a_t \geq 0$$

- **Active decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_t d\mathcal{D}_t$
- **Consumption function:** $C_t^{hh} = C^{hh}(\{r_s^a, (1 - \tau_s) w_s \ell_s, \chi_s\}_{s=0}^\infty)$

- Production and profits:

$$Y_t = \Gamma_t L_t$$

$$\Pi_t = Y_t - \frac{W_t}{p} L_t$$

- First order condition:

$$w_t = \Gamma_t$$

Mutual fund and assets

- Mutual fund collect households savings A_t
- Invest in available assets
 - **Real** government bonds which pay real interest rate r
- **Problem:**

$$\max_{B_t} (1 + r_{t+1}) B_t - (1 + r_{t+1}^a) A_t \quad \text{s.t.} \quad B_t = A_t$$

- **FOCs** (no arbitrage conditions):

$$1 + r_t = 1 + r_t^a$$

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi (L_t^{hh})^\nu - \frac{1}{\mu} (1 - \tau_t) w_t (C_t^{hh})^{-\sigma} \right) + \beta \pi_{t+1}^w$$

- **Spending:** G_t

$$B_t + T_t = (1 + r_t)B_{t-1} + G_t + \chi_t$$

- where T_t captures labor income tax revenue:

$$T_t = \tau_t w_t L_t$$

- In response to shocks r_t moves which affect budget
- Need rule to ensure that government debt returns to ss in the long run
- Potential **tax-rule:**

$$\tau_t = \tau_{ss} + \omega \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

- Or simply:

$$B_t = B_{ss}$$

- In which case taxes T_t adjust fully every period to ensure that the budget holds

- Two options for monetary policy
- 1.** Government bonds are nominal, CB chooses nominal interest rate:

$$i_t = i_{ss} + \phi \pi_t$$

- And Fisher equation links nominal rate i to real rate r :

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

- 2. Alternative:** Real rate rule. CB chooses real rate r_t directly

$$r_t = r_{ss} + (\phi - 1) \pi_t$$

- Implicit assumption: CB can control inflation.
 - Today will study special case with »passive« monetary policy, $\phi = 1$
 $\Rightarrow r_t = r_{ss}$
- Indeterminacy: Consider limit for nominal rule $i_t = i_{ss} + \phi \pi_{t+1}$ or assume future tightening

Market clearing

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

Fiscal Policy

Simpler consumption function

- **Assumptions:**

1. One-period real bond
2. No lump-sum transfers, $\chi_t = 0$
3. Fiscal policy in terms of dG_t and dT_t satisfying IBC (e.g. government needs to repay excess debt dB eventually)

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0$$

- **Tax-bill:** $T_t = \tau_t w_t \int \ell_t z_t d\mathbf{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$
- **Household income:** $(1 - \tau_t) w_t \ell_t z_t = \underbrace{(Y_t - T_t)}_{\equiv Z_t} z_t = Z_t z_t$
- **Consumption function in sequence-space:** Simplifies to

$$C_t^{hh} = C^{hh}(\{Y_s - T_s, r_s\}_{s \geq 0}) \Rightarrow \mathbf{C}^{hh} = C^{hh}(\mathbf{Y} - \mathbf{T}, \mathbf{r}) = C^{hh}(\mathbf{Z}, \mathbf{r})$$

Side-note: Two-equation version in Y and r

$$Y = G + C^{hh}(r, Y - T)$$
$$r = \mathcal{R}(Y)$$

- **First equation:** Goods market clearing
- **Second equation (Firms + NKWPC):**
 1. Given output Y , can compute L
 2. Firm behavior I: $\Gamma, Y \rightarrow L, w$
 3. NKWC: $L, C, w, \tau \rightarrow \pi^w$
 4. Firm behavior II: $\pi^w, \Gamma \rightarrow \pi$
 5. Central bank: $\pi \rightarrow i$
 6. Fisher: $i, \pi \rightarrow r$
- Final assumption for today: **Constant r**
 - Can replace $r = \mathcal{R}(Y)$ with $r = r_{ss}$
 - Entire model boils down to 1 equation

Intertemporal Keynesian Cross

$$Y_t = G_t + C_t^{hh}(\{Y_s - T_s\}_{s=0}^{\infty}) \quad \text{Static}$$

$$\mathbf{Y} = \mathbf{G} + C^{hh}(\mathbf{Y} - \mathbf{T}) \quad \text{Sequence-space/vector}$$

- **Total differentiation/linearize around ss:**

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} dZ_s = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} (dY_s - dT_s)$$

- **Intertemporal Keynesian Cross** in vector form

$$\begin{aligned} d\mathbf{Y} &= d\mathbf{G} + \mathbf{M}(d\mathbf{Y} - d\mathbf{T}) \Leftrightarrow \\ (\mathbf{I} - \mathbf{M})d\mathbf{Y} &= d\mathbf{G} - \mathbf{M}d\mathbf{T} \end{aligned}$$

where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes the entire *complexity of HH behavior*

Illustration

- Writing out the IKC:

$$\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} dG_0 \\ dG_1 \\ dG_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \frac{\partial C_0^{hh}}{\partial Z_2} & \cdots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \frac{\partial C_1^{hh}}{\partial Z_2} & \cdots \\ \frac{\partial C_2^{hh}}{\partial Z_0} & \frac{\partial C_2^{hh}}{\partial Z_1} & \frac{\partial C_2^{hh}}{\partial Z_2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \left(\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} - \begin{bmatrix} dT_0 \\ dT_1 \\ dT_2 \\ \vdots \end{bmatrix} \right)$$

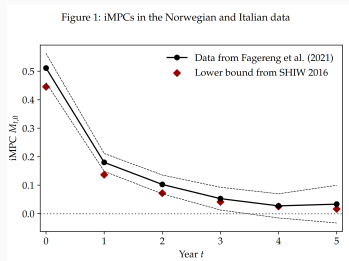
- M is the Jacobian of aggregate C w.r.t (post-tax) labor income
 - Column s : Response of C at different dates to unit change in Z at date s (IRF)
 - Row s : Change in C at date s to change in income Z at different dates

$$M = \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \dots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

- In a quarterly model $\frac{\partial C_0^{hh}}{\partial Z_0}$ is essentially the quarterly MPC
 - Note: Typically define MPCs following change in lump-sum transfer and not labor income, so not quite MPC

iMPCs in the data

- What can we say about the iMPC matrix \mathbf{M} ?
 - Can say a lot about first element $\frac{\partial C_0^{hh}}{\partial Z_0}$ (see lecture 2)
- Fagereng et al (2016) estimate dynamic response of C to income shocks (lottery winnings)
 - Info on first column $\left[\frac{\partial C_0^{hh}}{\partial Z_0}, \frac{\partial C_1^{hh}}{\partial Z_0}, \frac{\partial C_2^{hh}}{\partial Z_0}, \frac{\partial C_3^{hh}}{\partial Z_0}, \dots \right]'$



- Very hard to say something about remaining columns (announcement effects)
 - Best we can do: Calibrate model to first column, get rest of \mathbf{M} from model

Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depends on current income

$$Y_t = G_t + C^{hh}(Y_t - T_t)$$

- **Total differentiate:**

$$\begin{aligned} dY_t &= dG_t + \frac{\partial C_t^{hh}}{\partial Z_t} (dY_t - dT_t) \\ &= dG_t + \text{mpc} \cdot (dY_t - dT_t) \end{aligned}$$

- **Solution**

$$dY_t = dG_t + \frac{\text{mpc}}{1 - \text{mpc}} (dG_t - dT_t)$$

from multiplier-process $\text{mpc} \times (1 + \text{mpc} + \text{mpc}^2 \dots) = \frac{\text{mpc}}{1 - \text{mpc}}$

- **NPV-vector:** $\mathbf{q} \equiv [1, (1 + r_{ss})^{-1}, (1 + r_{ss})^{-2}, \dots]'$ - implies $\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} x_t = \mathbf{q}' \mathbf{x}$

- **Government:** IBC holds

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0 \Leftrightarrow$$

$$\mathbf{q}' (d\mathbf{G} - d\mathbf{T}) = 0$$

- **Households:** IBC holds

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} C_t^{hh} = (1 + r_{ss}) A_{-1} + \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} Z_t \Rightarrow$$

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} M_{t,s} = \frac{1}{(1 + r)^s} \Rightarrow \mathbf{q}' \mathbf{M} = \mathbf{q}' \Leftrightarrow \mathbf{q}' (\mathbf{I} - \mathbf{M}) = \mathbf{0}$$

- Present value of \mathbf{M} columns is 1 - HHs must *eventually* spend income they receive

- $\sum_{t=0}^{\infty} \frac{MPC_{t,s}}{(1 + r_{ss})^{t-s}} = 1$

Form of unique solution

- Back to IKC:

$$(I - M)dY = dG - MdT$$

- **Problem:** $(I - M)^{-1}$ cannot exist because this leads to a contradiction

$$\begin{aligned} q'(I - M)(I - M)^{-1} &= 0(I - M)^{-1} \Leftrightarrow \\ q' &= 0 \end{aligned}$$

- **Result:** If unique solution then on the form

$$\begin{aligned} dY &= \mathcal{M}(dG - MdT) \\ \mathcal{M} &= (K(I - M))^{-1} K \end{aligned}$$

- **Note:** This is only an issue in infinite horizon
 - When solving numerically we truncate at horizon T , implying that columns in M do not exactly add to 1
 - Can then invert $(I - M)$ (but precision becomes worse as horizon T increases)

Response of consumption

$$d\mathbf{Y} = d\mathbf{G} + \mathbf{M}(d\mathbf{Y} - d\mathbf{T}) \Leftrightarrow$$

$$d\mathbf{Y} - d\mathbf{G} = \mathbf{M}(d\mathbf{G} - d\mathbf{T}) + \mathbf{M}(d\mathbf{Y} - d\mathbf{G}) \Leftrightarrow$$

$$(I - \mathbf{M})(d\mathbf{Y} - d\mathbf{G}) = \mathbf{M}(d\mathbf{G} - d\mathbf{T}) \Leftrightarrow$$

$$d\mathbf{Y} - d\mathbf{G} = \mathcal{M}\mathbf{M}(d\mathbf{G} - d\mathbf{T}) \Leftrightarrow$$

$$d\mathbf{C} = \mathcal{M}\mathbf{M}(d\mathbf{G} - d\mathbf{T})$$

$$dY = dG + \underbrace{MM(dG - dT)}_{dC}$$

- **Balanced budget multiplier:**

$$dG = dT \Rightarrow dY = dG, dC = 0$$

Note: Central that income and taxes affect household income proportionally in exactly the same way = no redistribution

- **Deficit multiplier:** $dG \neq dT$
 1. Potentially fiscal multiplier **above** 1
 2. Larger effect of dG than dT
 3. *Numerical results needed*

Fiscal multiplier

Impact-multiplier:

$$\frac{\partial Y_0}{\partial G_0}$$

Cumulative-multiplier:

$$\frac{\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} dY_t}{\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} dG_t}$$

Comparison with RA model

- From lecture 1: $\beta(1 + r_{ss}) = 1$ implies

$$C_t = (1 - \beta) \sum_{s=0}^{\infty} \beta^s Y_{t+s}^{hh} + r_{ss} a_{-1}$$

- The **iMPC-matrix** becomes (**1** is a square matrix of 1's)

$$\mathbf{M}^{RA} = \begin{bmatrix} (1 - \beta) & (1 - \beta)\beta & (1 - \beta)\beta^2 & \dots \\ (1 - \beta) & (1 - \beta)\beta & (1 - \beta)\beta^2 & \dots \\ (1 - \beta) & (1 - \beta)\beta & (1 - \beta)\beta^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = (1 - \beta)\mathbf{1}\mathbf{q}'$$

- Consumption response** is zero

$$\begin{aligned} d\mathbf{C}^{RA} &= \mathcal{M}\mathbf{M}^{RA}(d\mathbf{G} - d\mathbf{T}) \\ &= \mathcal{M}(1 - \beta)\mathbf{1}\mathbf{q}'(d\mathbf{G} - d\mathbf{T}) \\ &= \mathbf{0} \Leftrightarrow d\mathbf{Y} = d\mathbf{G} \end{aligned}$$

- Fiscal multiplier** is 1

- **RANK** cannot produce fiscal multipliers above 1 because it features ricardian equivalence
- Large literature on Two Agent New Keynesian models (TANK) starting with Campbell and Mankiw (1989)
 - Share $1 - \lambda$ is unconstrained; Always on Euler (Ricardian, permanent income HHs, optimizing HHs)
 - Share λ is constrained (no savings) and consume entire income each period, $MPC = 1$ (hand to mouth)

- **M** matrix is:

$$\mathbf{M}^{TA} = (1 - \lambda) \mathbf{M}^{RA} + \lambda \mathbf{I}$$

- Simple to implement+tractable, but some drawbacks
 - No intertemporal MPCs
 - Extremely stylized level of inequality
 - Hard to connect to micro data
 - No precautionary saving

Comparison with TANK model

- **Hand-to-Mouth (HtM) households:** λ share have $C_t = Y_t^{hh}$

$$M^{TA} = (1 - \lambda)M^{RA} + \lambda I$$

- **Intertemporal Keynesian Cross** becomes

$$dY = dG + M^{TA}(dY - dT)$$

$$(I - M^{TA})dY = dG - M^{TA}dT$$

$$(I - M^{RA})dY = \frac{1}{1 - \lambda} [dG - \lambda dT] - M^{RA}dT$$

- **Solution:**

$$dY = dG + \frac{\lambda}{1 - \lambda} [dG - dT]$$

- **Amplification** of fiscal policy with deficit financing $dG > dT$
 - Size of amplification increasing in share of constrained agents λ
 - Solution **very** similar to static, old Keynesian cross (multiplier: $\frac{mpc}{1 - mpc}$)

- In TANK we have:

$$\begin{aligned}d\mathbf{Y} &= d\mathbf{G} + \mathbf{M}^{TA}(d\mathbf{Y} - d\mathbf{T}) \\(I - \mathbf{M}^{TA})d\mathbf{Y} &= d\mathbf{G} - \mathbf{M}^{TA}d\mathbf{T} \\(I - \mathbf{M}^{RA})d\mathbf{Y} &= \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] - \mathbf{M}^{RA}d\mathbf{T} \\d\mathbf{Y} &= \mathbf{M}^{RA}d\mathbf{Y} - \mathbf{M}^{RA}d\mathbf{T} + \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \\d\mathbf{Y} &= d\tilde{\mathbf{G}} + \mathbf{M}^{RA}(d\mathbf{Y} - d\mathbf{T})\end{aligned}$$

- where $d\tilde{\mathbf{G}} = \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}]$
- Recall that in RANK $d\mathbf{Y} = d\mathbf{G} + \mathbf{M}^{RA}(d\mathbf{Y} - d\mathbf{T})$ with solution $d\mathbf{Y} = d\mathbf{G}$ thus implying $d\mathbf{Y} = d\tilde{\mathbf{G}}$ in TANK:

$$\begin{aligned}d\mathbf{Y} &= \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \\d\mathbf{Y} &= d\mathbf{G} - d\mathbf{G} + \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \\d\mathbf{Y} &= d\mathbf{G} + \frac{\lambda}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}]\end{aligned}$$

Cumulative multiplier still one

$$\begin{aligned}\frac{\mathbf{q}' d\mathbf{Y}}{\mathbf{q}' d\mathbf{G}} &= \frac{\mathbf{q}' d\mathbf{G}_t + \frac{\lambda}{1-\lambda} \mathbf{q}' [d\mathbf{G} - d\mathbf{T}]}{\mathbf{q}' d\mathbf{G}} \\ &= 1\end{aligned}$$

Jacobian columns

- TANK produces positive C response, fiscal multiplier above 1 - do we need HANK?
- Plot columns of \mathbf{M} in TANK, HANK + other models
 - Recall columns: dynamic C response to change in Z at various dates

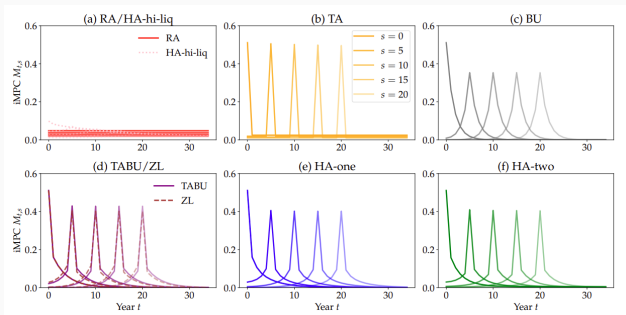
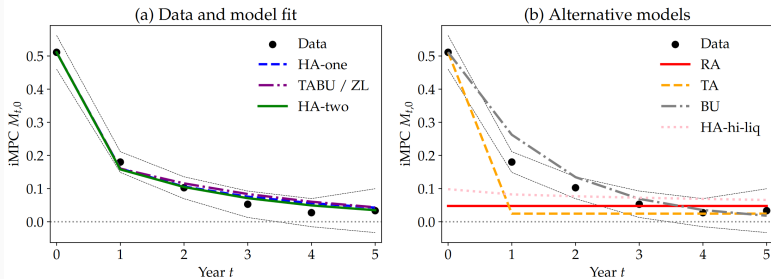
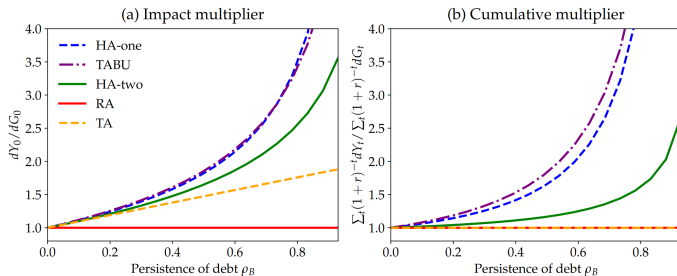


Figure 2: iMPCs in the Norwegian data and several models



Multipliers and debt-financing

Figure 5: Multipliers according to the IKC



Note. These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary ρ_B in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

Summary in table

- Summary:

Table 1: Government spending multipliers in the intertemporal Keynesian cross

Fiscal rule	Multiplier	Rep. agent (RA)	Two agents (TA)	Het. agents (HA)
		doesn't match MPC	matches MPC	matches iMPCs
balanced budget	impact	1	1	1
	cumulative	1	1	1
deficit financing	impact	1	> 1	> 1
	cumulative	1	1	> 1

Interest rate effects

- We assumed real bonds for tractability - In reality, bonds are typically **nominal**:

-

$$(1 + r_t) B_{t-1} \quad \text{versus} \quad \frac{1 + i_{t-1}}{1 + \pi_t} B_{t-1}$$

- **Fisher**: $(1 + r_t) = \frac{1 + i_{t-1}}{1 + \pi_t} \approx r_t = i_{t-1} - \pi_t$
- Even if CB keeps ex-ante real rate constant $i_t = i_{ss} + \pi_{t+1}$ real returns on bonds will differ in period 0:

- Period 0:

$$\frac{1 + i_{ss}}{1 + \pi_0} B_{ss}$$

- Period 1:

$$\frac{1 + i_0}{1 + \pi_1} B_0 = (1 + r) B_0$$

- With nominal bonds **surprise inflation** affects returns - implies capital for households in period 0
 - Positive fiscal shock generates inflation, so **negative** effect on

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r_{t-1}^a)a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \text{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\text{cap}_0 = 0$
2. Nominal bond:

$$\text{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

- Consumption-function $C^{hh} = C^{hh}(r, Y - T, \text{cap}_0)$ implies

$$dC^{hh} = M^r dr + M(dY - dT) + m^{\text{cap}} \text{cap}_0$$

where

$$M_{t,s}^r = \left[\frac{\partial C_t^{hh}}{\partial r_s} \right], m_t^{\text{cap}} = \left[\frac{\partial C_t^{hh}}{\partial \text{cap}_0} \right]$$

- Capital return effect is negative - can this overturn fiscal multiplier > 1 ?
 - No - entries in M are large
 - Entries in m^{cap} are **small**
- MPC out of capital gains approximately 1-4% per year

Fiscal policy in HANK - literature

- Seminal paper: **Intertemporal Keynesian Cross**
- **Many** other interesting papers:
- McKay and Reis - *The role of automatic stabilizers in the US business cycle* (2016)
 - Analyze the role of automatic stabilizers in a HANK model
- Bayer, Born, Luetticke - *The liquidity channel of fiscal policy* (2023)
 - Role of liquid and illiquid effects of fiscal policy
- Hagedorn, Manovskii, and Mitman - *The fiscal multiplier* (2019)
 - Systematic evaluation of multiplier in HANK + ZLB
- Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm - *Fiscal Multipliers in Small Open Economies With Heterogeneous Households* (2024)
 - Generalize to small open economies

Exercise

Exercise

1. Compute and plot selected columns of the household Jacobian of C w.r.t Z, r, χ

Hint: use `model._compute_jac_hh()` to compute the jacobians. You can find the results in `model.jac_hh`

- 1.1 Are the MPCs out of labor income Z and a transfer χ different? why?
2. Compute IRFs to a deficit financed ($\omega = 0.1$) and tax financed (ω large) fiscal spending shock. Compare the responses.
3. Compute the output IRF $d\mathbf{Y}$ using the fomula $d\mathbf{Y} = d\mathbf{G} + \mathcal{M}\mathbf{M}[d\mathbf{G} - d\mathbf{T}]$ where $\mathcal{M} = (\mathbf{I} - \mathbf{M})^{-1}$. Check that you get the same as when using `model.find_IRFs()`
4. Redo Q2 with active monetary policy, $\phi_\pi = 1.5$. How does the fiscal multiplier change?
5. Redo Q2 with active monetary policy *and* a flatter NKWPC, $\kappa = 0.01$. How does the fiscal multiplier change?

Summary

Summary and next week

- **Today:** Fiscal policy in a HANK model with sticky wages
- **Next week:** Assignment workshop
- **Homework:**
 1. Work on exercise
 2. Work on assignment