

11. Monetary Policy in HANK

Adv. Macro: Heterogenous Agent Models

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Introduction

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- **Today:**
 - Other pillar of stabilization policy: **Monetary policy**
 - Will use as example to study alternatives to **rational expectations** (RE) in HANK
- **Literature:**
 - *Seminal paper:* Kaplan, Moll, Violante (2018) »Monetary policy according to HANK«
 - Auclert Rognlie, Straub (2020) »Micro jumps, macro humps«
 - Alves, Kaplan, Moll, Violante (2020) »A further look at the propagation of monetary policy shocks in HANK«

Monetary Policy in HANK

- Introducing heterogeneous agents into the standard NK model **fundamentally** changes the transmission of Fiscal Policy
 - *Potentially* more effective
 - Important whether policy is deficit financed or tax financed

Monetary Policy

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 - *Potentially* more effective
 - Important whether policy is deficit financed or tax financed
- What about monetary policy?

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 - **Solution:** Firm equity

- **Household problem:**

$$v_t(z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t [v_{t+1}(z_{t+1}, a_t)]$$

$$\text{s.t. } a_t + c_t = (1 + r_t^a) a_{t-1} + Z_t z_t + \chi_t$$

$$\log z_{t+1} = \rho_z \log z_t + \psi_{t+1}, \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E}[z_t] = 1$$

$$a_t \geq 0$$

- with $Z_t = w_t \ell_t$ - real labor income
- **decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_t d\mathcal{D}_t$
- **Consumption function:** $C_t^{hh} = C^{hh}(\{r_s^a, Z_s, \chi_s\}_{s=0}^\infty)$

- **Production and profits:**

$$Y_t = L_t$$

$$\Pi_t = Y_t - w_t L_t$$

- Optimize subject to demand curve (monopolistic competition)

- **First order condition:**

$$w_t = \frac{1}{\mu}$$

- where $\mu > 1$ = markup - firms make positive profits in equilibrium

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- Firms are gonna be symmetric in eq., $p_{j,t}^D = p_t^D$
- Total value of firm equity is then $\int p_t^D v_{j,t} dj = p_t^D$

- Problem:

$$\max_{v_{j,t}} \int (\Pi_{j,t+1} + p_{j,t+1}^D) v_{j,t} - (1 + r_{t+1}^a) A_t$$

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- **Problem:**

$$\max_{v_{j,t}} \int (\Pi_{j,t+1} + p_{j,t+1}^D) v_{j,t} - (1 + r_{t+1}^a) A_t$$

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- Valuation effects: As with nominal gov bonds:

$$1 + r_t^a = \begin{cases} \frac{\Pi_0 + p_0^D}{p_{ss}^D} & t = 0 \\ 1 + r_{t-1} & t > 0 \end{cases}$$

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi (L_t^{hh})^\nu - \frac{1}{\mu} (1 - \tau_t) w_t (C_t^{hh})^{-\sigma} \right) + \beta \pi_{t+1}^w$$

- Two options for monetary policy

Central bank

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- **2. Alternative:** Real rate rule. CB chooses real rate r_t directly

$$r_t = r_{ss} + (\phi - 1) \pi_t$$

Market clearing

1. Asset market: $p_t^D = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh}$

The consumption function

- Model features a consumption function:

$$C_t^{hh} = C_t^{hh}(\{r_s^a, Z_s\}_{s=0}^{\infty}) \Rightarrow \mathbf{C}^{hh} = \mathbf{C}^{hh}(\mathbf{r}^a, \mathbf{Z})$$

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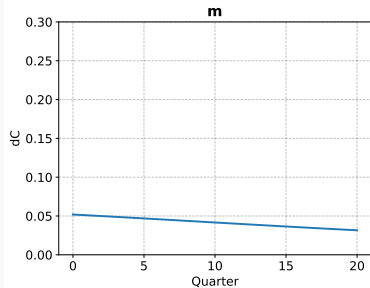
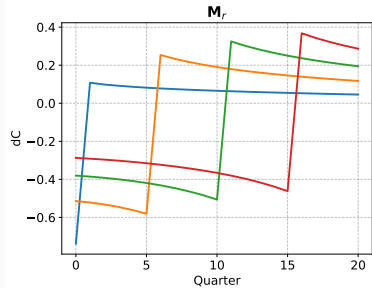
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- Note: \mathbf{m} is a vector not matrix (multiplies onto scalar $d\text{cap}_0$, not vector)

Interest rate Jacobians



Monetary policy in sequence-space

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- Monetary policy operates through:
 - Direct** (partial eq., M_r) effect
 - Indirect** (general eq., M) effect
- Q1:** Sign? Positive/negative?
- Q2:** Do you expect the effects of monetary policy on output to be larger in HANK than RANK?

HANK-RANK equivalence

- Assume logarithmic utility $u(c) = \log(c)$

HANK-RANK equivalence

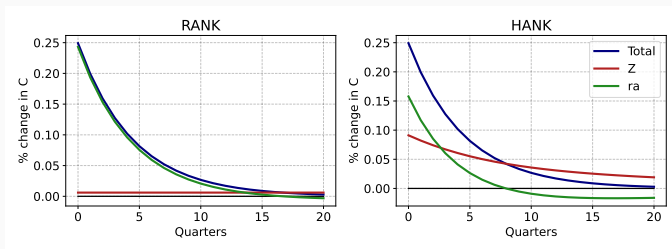
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- Proposition:** Above model features **exact** equivalence between RANK and HANK for the response of aggregates w.r.t a monetary policy shock (*Werning 2015*)
- ... but transmission channel is different
- Decompose $d\mathbf{Y}$ into direct and indirect effect using $d\mathbf{Y}^j = \mathbf{M}_{r^a}^j dr^a + \mathbf{M}^j d\mathbf{Z}$ for $j \in \{HA, RA\}$



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- Exact **equivalence** is the product of a number of simplifying assumptions:
 - Linear production function
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- How does the effectiveness of monetary policy look in more realistic models?
 - Kaplan, Moll, Violante (2018) »Monetary policy according to HANK«
 - Auclert Rognlie, Straub (2020) »Micro jumps, macro humps«

KMV 2018

Monetary policy according to HANK

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- They study the transmission of monetary policy in medium scale HANK model
- Follows Kaplan & Violante (2014) closely
 - See lecture 2
 - Household can hold both liquid and illiquid assets
 - Model features both **poor** and **wealthy** Hand-to-mouth households

Household problem

- Households solve (here converted to discrete time, paper in cont. time):

$$V_t(a_{t-1}, b_{t-1}, z_t) = \max_{c_t, a_t, b_t} u(c_t, \ell_t) + \beta E_t V_{t+1}(a_t, b_t, z_{t+1})$$

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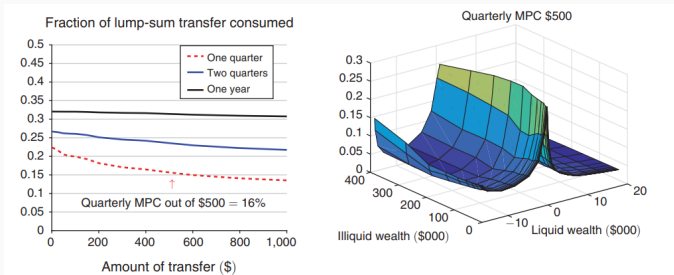
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- with b_t =liquid asset, a_t =illiquid assets, d_t =deposits into illiquid asset
- Return on illiquid asset r_t^a Return on liquid asset r_t^b
 - Household will prefer to hold a_t due to superior return
 - But not good for consumption smoothing as they have to pay adjustment cost to use a_t for smoothing against shocks
 - Some HHs will be wealthy hand-to-mouth

- 1) MPCs for different sizes of stimulus checks, 2) MPCs across the wealth distribution



Direct vs indirect effects

- Amplification in HANK (elasticity of $C^{HANK} = -2.9$ vs $C^{RANK} = -2.07$)

Direct vs indirect effects

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- Baseline HANK: Indirect effects account for majority of transmission ($\approx 80\%$)

TABLE 7—DECOMPOSITION OF THE EFFECT OF MONETARY SHOCK ON AGGREGATE CONSUMPTION

	Baseline (1)	$\omega = 1$ (2)	$\omega = 0.1$ (3)	$\frac{\varepsilon}{\theta} = 0.2$ (4)	$\phi = 2.0$ (5)	$\frac{1}{\nu} = 0.5$ (6)
Change in r^b (pp)	-0.28	-0.34	-0.16	-0.21	-0.14	-0.25
Elasticity of Y	-3.96	-0.13	-24.9	-4.11	-3.94	-4.30
Elasticity of I	-9.43	7.83	-105	-9.47	-9.72	-9.79
Elasticity of C	-2.93	-2.06	-6.50	-2.96	-3.00	-2.87
Partial eq. elasticity of C	-0.55	-0.45	-0.99	-0.57	-0.59	-0.62
<i>Component of percent change in C due to</i>						
Direct effect: r^b	19	22	15	19	20	22
Indirect effect: w	51	56	51	51	51	38
Indirect effect: T	32	38	19	31	31	45
Indirect effect: r^a and q	-2	-16	15	-2	-2	-4

Expectations

Micro Jumps, Macro Humps

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Micro Jumps, Macro Humps

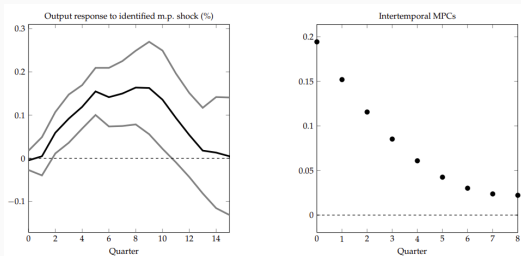
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- Main hurdle: Empirical response of C , Y is hump-shaped to monetary policy shock.

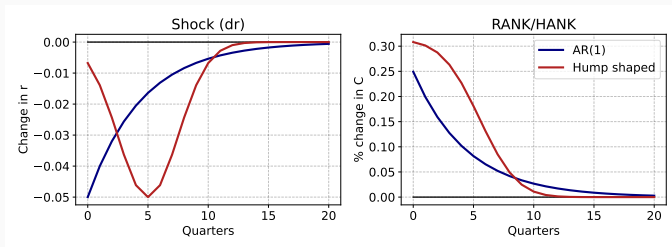
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- Estimate parameters in quantitative HANK model to match estimated effects of causal monetary policy shock
- Main hurdle: Empirical response of C , Y is hump-shaped to monetary policy shock.
- Want a model that simultaneously match hump-shaped agg. response to r and iMPC moment



The problem

- Standard model does **not** give hump shaped for C to standard shock
- Does not matter if **shock** is hump shaped or not



The solution: RANK

- Solution in RANK literature: Habits in utility function:

$$\sum_{t=0}^{\infty} \beta^t u(C_t - \gamma C_{t-1})$$
$$\Rightarrow u'(C_t - \gamma C_{t-1}) = \beta R_{t+1} u'(C_{t+1} - \gamma C_t)$$

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- Generates persistence in C response to shocks because household don't want to deviate too much from last periods consumption level

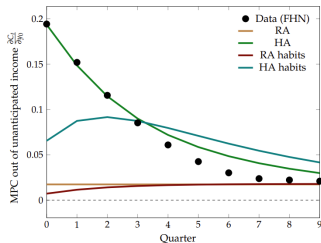
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$$\Rightarrow u'(C_t - \gamma C_{t-1}) = \beta R_{t+1} u'(C_{t+1} - \gamma C_t)$$

- Generates persistence in C response to shocks because household don't want to deviate too much from last periods consumption level
- **However:** Does not work in HANK because it kills iMPCs:



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- Will only implement this to first-order (e.g. linear approximations)
 - Much more difficult if we want full non-linear solution

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- Note that elements above diagonal are affected by expectations (i.e. they concern the **future**)
 - Elements on and below diagonal reflect changes in income **today** or in the **past** (known by HHs)

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- Introduce expectations matrix E :

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- How to get jacobian $\hat{\mathbf{M}}$ associated with \mathbf{E} ?

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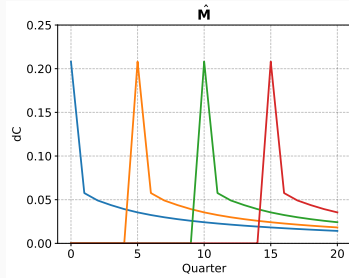
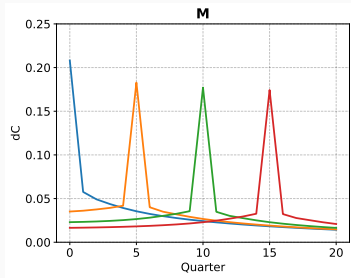
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Jacobians

- Jacobian of C w.r.t Z



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 5. Solve for IRFs:
 - `model.find_IRFs(shocks=[x])`

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 - Response of consumption at 0 to Z_1 is $(1 - \theta) \frac{\partial C_0}{\partial Z_1}$
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- $\theta = 0$ gives us RE, $\theta = 1$ gives us myopic behavior.

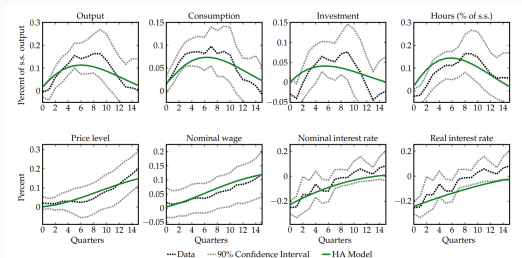
Sticky expectations

- Properties:
 - Response of consumption at 0 to Z_1 is $(1 - \theta) \frac{\partial C_0}{\partial Z_1}$
 - Response of consumption at 1 to Z_1 is $(1 - \theta) \frac{\partial C_1}{\partial Z_1} + \theta \frac{\partial C_0}{\partial Z_0}$ and so forth
- $\theta = 0$ gives us RE, $\theta = 1$ gives us myopic behavior.
- Since households perfectly observe income changes **today and in past** iMPCs are preserved
 - **Unlike** habit formation

- Auclert, Rognlie, Straub (2020) fomulate full HANK model with:
 - Investment
 - Sticky wages + prices
 - Government

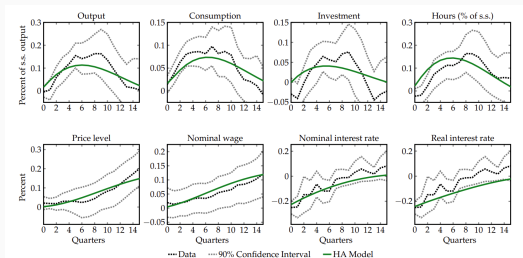
Estimation

- Auclert, Rognlie, Straub (2020) formulate full HANK model with:
 - Investment
 - Sticky wages + prices
 - Government
- Estimate parameters to match empirical evidence on causally identified monetary policy shock in the US (Romer & Romer shock)



Estimation

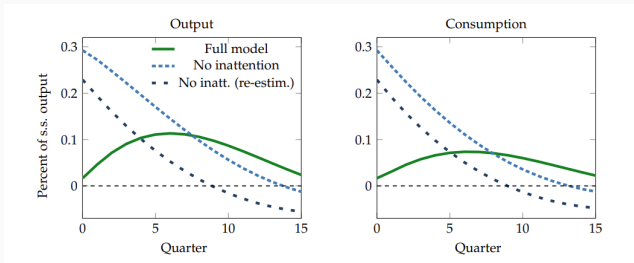
- Auclert, Rognlie, Straub (2020) formulate full HANK model with:
 - Investment
 - Sticky wages + prices
 - Government
- Estimate parameters to match empirical evidence on causally identified monetary policy shock in the US (Romer & Romer shock)



- Estimate $\theta = 0.935 \Rightarrow$ **Large** deviation from RE

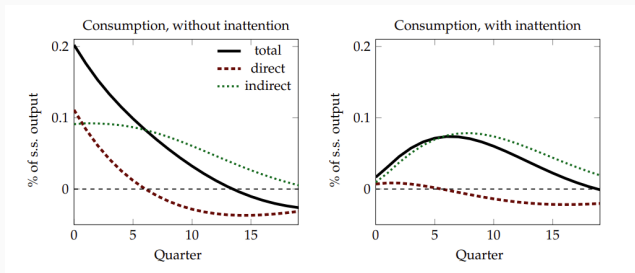
RE vs. Non-RE

- Why we need sticky expectations in order to match empirical response



Direct and indirect effects

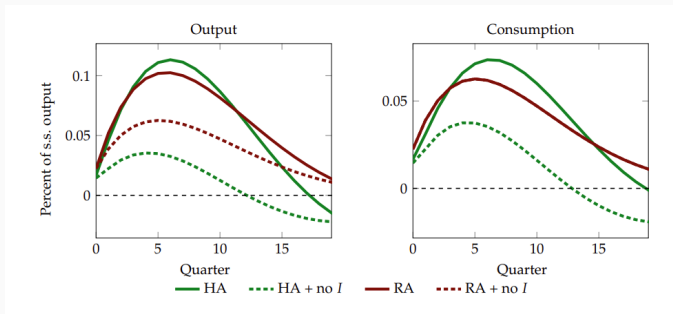
- Can decompose C into direct and indirect as before



- In the estimated model with sticky expectations indirect effect is by far the most important driver of consumption

Importance of Investment

- Importance of indirect effects in HANK partly comes from *investment*



Exercise

Exercise

Consider the HANK model described in section 2

1. Compare a monetary policy shock in HANK and RANK. Decompose the response in HANK into direct and indirect effects using the household Jacobians
2. Solve for a monetary policy shock in HANK and RANK with myopic expectations w.r.t \mathbf{r}, \mathbf{Z} , only \mathbf{r} and only \mathbf{Z}
3. Solve for a monetary policy shock in HANK and RANK with sticky expectations w.r.t \mathbf{r}, \mathbf{Z} , only \mathbf{r} and only \mathbf{Z}
4. Consider a model where household hold nominal government debt instead. Relax the borrowing constraint to -1, $\underline{a} = -1$ and solve for a monetary policy shock (assume rational expectations). Does the presence of household debt amplify or dampen the effects of monetary policy?

Summary

Summary and next week

- **Today:**
 - Monetary policy in HANK
 - Alternatives to rational expectations, and how to implement them using jacobians
- **Next week:** HANK + unemployment risk in GE (**JD**)
- **Homework:**
 1. Work on exercise