University of Copenhagen

Assignment 1: *The Aiygari Model*

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1. Define the stationary equilibrium for the model

Below, I define the stationary equilibrium in terms similar to those used in the lecture slides.

In the stationary equilibrium,

1. Quantities K_{ss} and L_{ss},
2. prices r_{ss} and w_{ss},
3. the level of goverment transfers $\mathcal{T}_t$
4. the distribution $\boldsymbol{D}_{\text{ss}}$ over z_{it} and a_{it-1}
5. and the policy functions $a^\star_{\text{ss}}$, $\ell^\star_{\text{ss}}$ and $c^\star_{\text{ss}}$

are such that

1. Households maximize expected utility
2. Firms maximize profits
3. $\boldsymbol{D}_{\text{ss}}$ is the invariant distribution implied by the household problem
4. Mutual fund and government balance sheets are satisfied
5. The markets for capital, labor and goods clear.

In matrix formulation:

$$\begin{bmatrix} r_{\text{ss}} - F_K(\Gamma_{\text{ss}}, K_{\text{ss}}, L_{\text{ss}}) \\ w_{\text{ss}} - F_L(\Gamma_{\text{ss}}, K_{\text{ss}}, L_{\text{ss}}) \\ r_{\text{ss}} - (r^K_{\text{ss}} - \delta) \\ A_{\text{ss}} - K_{\text{ss}} \\ \mathcal{T}_{\text{ss}} - \tau^a_{\text{ss}} r_{\text{ss}} A^{\text{hh}}_{\text{ss}} - \tau^\ell_{\text{ss}} w_{\text{ss}} L^{\text{hh}}_{\text{ss}} \\ A_{\text{ss}} - A^{\text{hh}}_{\text{ss}} \\ L_{\text{ss}} - L^{\text{hh}}_{\text{ss}} \\ A^{\text{hh}}_{\text{ss}} - \int a_{\text{ss}} \, \mathrm{d}\boldsymbol{D}_{\text{ss}} \\ L^{\text{hh}}_{\text{ss}} - \int \ell_{\text{ss}} \, \mathrm{d}\boldsymbol{D}_{\text{ss}} \\ \underline{\boldsymbol{D}}_{\text{ss}} - \Lambda'_{\text{ss}} \Pi'_z \underline{\boldsymbol{D}}_{\text{ss}} \\ a_{\text{ss}} - a^\star_{\text{ss}} \end{bmatrix} = \boldsymbol{0} \tag{1}$$

Due to Walras' law, the remaining market (for goods) clears when the matrix equality above is satisfied. That is, the following equalities are implied:

$$\begin{aligned} I_{\text{ss}} - \delta K_{\text{ss}} &= 0 \\ C^{\text{hh}}_{\text{ss}} - I_{\text{ss}} - Y_{\text{ss}} &= 0 \end{aligned} \tag{2}$$

2. Solve for the stationary equilibrium with positive tax rates

I calibrate the model using the given parameter values and solve for the stationary equilibrium. The resulting aggregate quantities and prices are shown in Listing 1. In Figure 1, I have plotted the Lorenz curve based on the resulting wealth distribution. I compute the Gini-coefficient for wealth at $G = 0.6985$. In comparison, the 2021 wealth Gini of Denmark was $G = 0.739$. In Japan, it was $G = 0.647$.[1]

In our model, wealth inequality is determined by the initial distribution of wealth and by the ideosyncratic productivity shocks, which cause households to have heterogenous earnings, heterogenous expectations and by consequence heterogeneous saving behavior.

```
Gamma          : 1.0000
K              : 5.1537
L              : 1.0000
rK             : 0.1100
w              : 1.1510
Y              : 1.7179
A              : 5.1537
r              : 0.0500
transfer       : 0.3711
tau_l          : 0.3000
tau_a          : 0.1000
A_hh           : 5.1537
C_hh           : 1.4087
ELL_hh         : 1.0570
L_hh           : 1.0000
TAXES_hh       : 0.3711
I              : 0.3092
```

Listing 1: Aggregate quantities and prices.

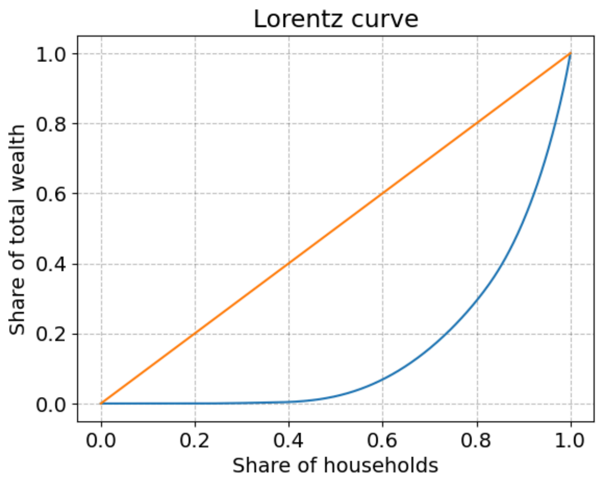


Figure 1: The stationary equilibrium Lorentz curve when $\tau^a = 0.1$ and $\tau^\ell = 0.5$.

[1]Source: https://en.wikipedia.org/wiki/List_of_sovereign_states_by_wealth_inequality

3. Illustrate how changes in the tax rates affect households in partial equilibrium

In Figure 2, I have illustrated the effects of varying $\tau^\ell, \tau^a \in \{0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99\}$ on the aggregates $L_{\text{ss}}^{\text{hh}}, A_{\text{ss}}^{\text{hh}}, C_{\text{ss}}^{\text{hh}}$ in partial equilibrium.

When holding τ^ℓ fixed and increasing τ^a, aggregate assets decreases rapidly as households are discouraged from saving. Consumption decreases slightly and labor supply increases a bit to compensate for the increased tax burden on asset yields.

When fixing τ^a and increasing τ^ℓ, aggregate assets also decreases rapidly as households can afford less savings. Consumption decreases too. Somewhat surprisingly, aggregate labor supply is increasing in τ^ℓ until around $\tau^\ell = 0.9$, indicating that the income effect of labor taxation dominates the substitution effect towards leisure across almost the entire interval of tax rates. Eventually though, the substitution effect starts to dominate and labor supply starts decreasing rapidly above $\tau^\ell = 0.9$.

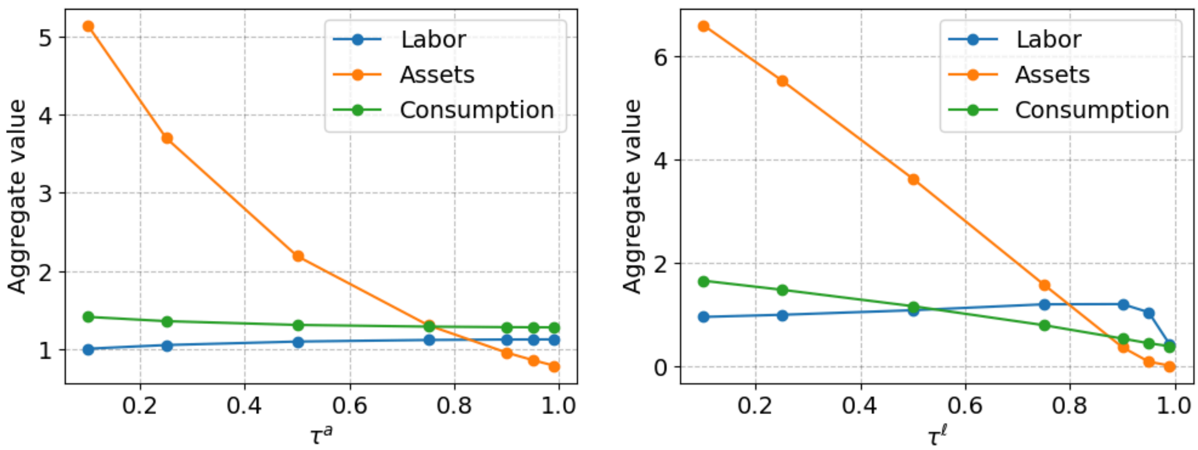


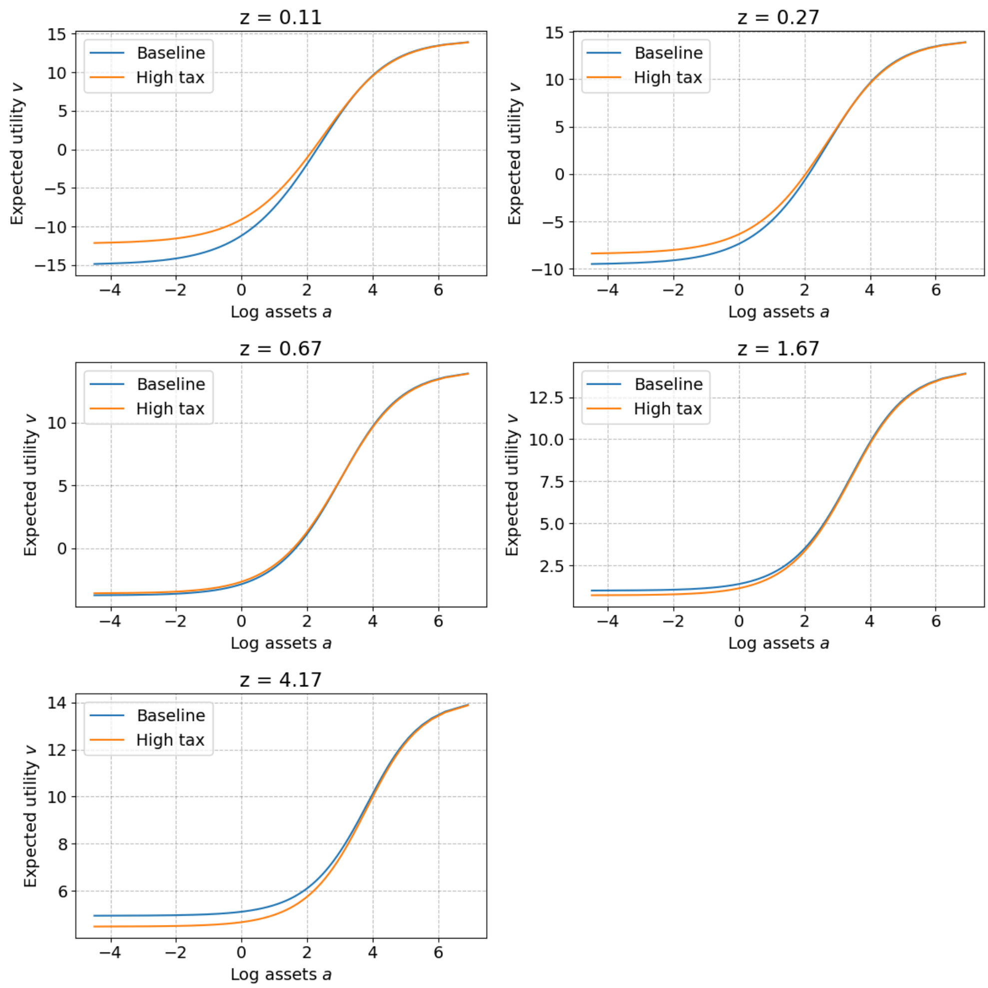
Figure 2: Partial equilibrium effects of tax reforms

4. Implement a tax reform in general equilibrium

I calculate the competetive steady state general equilibrium after the tax reform, where τ^a increases from 10% to 50%. As a result of the reform, aggregate "efficiency" Y_{ss} decreases from 1.7179 to 1.5155 as investment is discouraged by the higher capital tax. Figure 3 shows the impact of the tax reform on expected welfare across the income and wealth distribution. Note that I have taken logs on first axis to make it easier to discern the two curves.

The first panels show the impact for low-earners (low z_t). Here, those in the low end of the wealth distribution get a big boost in expected welfare from the reform, while those in the high end of the asset distribution are indifferent towards the two taxation scenarios. Conversely, in the last panels which depict high-earners (high z_t), those in the low end of the wealth distribution suffer a loss in expected welfare. Those in the high end of the wealth distribution are still indifferent.

The observations might be explained by the fact that while the tax increase finances higher transfers which benefit all households, the expected cost of lower real yields on future investments is bigger for high earners than low earners due to the autoregressive nature of the productivity path (those who earn more today can expect to earn more tomorrow, which means they expect to invest more than low earners).



5. Illustrate welfare effects of transitional dynamics

In the model settings, I add:

- Γ, K, L to the unknowns,
- τ^a, τ^ℓ to the shocks,
- `clearing_A, clearing_L, clearing_Y` to the targets.

Using the high-tax model, I then add a shock in $t = 0$, reducing the capital tax rate from 0.5 to 0.1 to reflect the old equilibrium. In $t = 1$, the capital tax rate immediately jumps back to 0.5 and stays at this level for the remainder of the transition period.

In Figure 4, I plot the transition paths for expected utility across the income and wealth distributions. To illustrate the wealth distribution, I plot curves for the 25th, 50th and 75th quantiles of the asset distribution in the stationary equilibrium when $\tau^a = 0.1$. Note that I have normalized the utility curves by their initial levels to make the comparison clearer.

I'm not sure my approach is correct, as I find it hard to explain the patterns across the transition paths. Maybe I should have modelled the initial state in a different way, using steady state levels from the old equilibrium instead of just adding the τ^a shock.