14. Advanced HANK Topics

Adv. Macro: Heterogenous Agent Models

Nicolai Waldstrøm

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Introduction

Introduction

- **So far:**
  - Utilize HANK framework to study **how** stabilization policy (fiscal, monetary) affects the economy
- **Today:** How does *policy recommendations* change with the introduction of heterogeneous agents?
  - I.e what is the optimal policy in response to adverse aggregate shocks?
  - Is it affected by heterogeneity?
- This topic is at the **research frontier**
  - At the end of lecture I will briefly mention other topics at the frontier

Optimal Policy

Optimal Policy

- What is the problem of the social planner?
  - Consider example where the central bank/social planner chooses the path of interest rates
- **Goal**: Maximize aggregate welfare of **all households**
- Subject to:
  - The model (equilibrium conditons, firm + HH behavior, budget constraints etc)
  - Aggregate shocks
- This problem is more complicated than calculating **optimal steady state policy**
  - Optimal steady state: Maximize one function as a function of 1 or 2 variables (ex: labor income tax+capital income tax)
  - Here: Choose entire **path** (length T) of instruments subject to entire dynamic model

Optimal Policy - computation

- Two Approaches:

1. Start from **efficient** steady state
    - Harsh assumption
    - No distortions in steady state
        - No monopolistic competetion, tax distortions
        - No motive for redistribution (i.e. incomplete markets)
    - Can then derive quadratic loss function
    - Easy to minimize $\Rightarrow$ Yields optimal Policy
2. Start from **inefficient** steady state
    - Larger role for inequality and redistribution
    - Much more difficult - see end of lecture

Optimal policy around efficient steady state

- Standard NK model features **inefficient** steady state due to monopolistic competition
- Simple steady state policy can restore efficiency: Give firms a labor subsidiy τ such that they charge lower prices
- Around **efficient steady state**, can derive the following approximation of household welfare (see e.g. Woodford book):

$$\mathcal{W} = \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - \nu\left(L_t\right)\right] \approx -\sum_{t=0}^{\infty} \beta^t \left(d\pi_t^2 + \lambda_Y dY_t^2\right)$$

- Maximizing welfare same as minizing loss function
  - Planner want to stabilize (varianceof) inflation and output

Solving the planner prob. - efficient steady state

- Sequence-space social planner problem ($\boldsymbol{\beta} = \text{diag}\left(1, \beta^1, \beta^2, ...\right)$):

$$\min \boldsymbol{\pi}' \beta \boldsymbol{\pi} + \lambda_Y d\boldsymbol{Y}' \beta d\boldsymbol{Y} \quad \text{s.t.} \quad H\left(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{\Gamma}\right) = 0$$

- Stack target variables in vector $\boldsymbol{x} = \left(\boldsymbol{\pi}, \boldsymbol{Y}\right)'$ and weights $\boldsymbol{\lambda} = \left(\mathbf{1}, \lambda_Y\right)$. Then:

$$\min \boldsymbol{x}' \left(\beta \boldsymbol{\lambda}\right) \boldsymbol{x} \quad \text{s.t.} \quad H\left(\boldsymbol{x}, \boldsymbol{i}, \boldsymbol{\Gamma}\right) = 0$$

- Choose interest rate to solve this problem. Solution:

$$\boldsymbol{i}^* = -\left[\boldsymbol{J}_{x,i}' \left(\beta \boldsymbol{\lambda}\right) \boldsymbol{J}_{x,i}\right]^{-1} \times \left[\boldsymbol{J}_{x,i}' \left(\beta \boldsymbol{\lambda}\right) \boldsymbol{J}_{x,\boldsymbol{\Gamma}} d\boldsymbol{\Gamma}\right]$$

- **Easy** to solve for optimal policy given shock $d\boldsymbol{\Gamma}$ - just need jacobians around ss
  - We need jacobians for π, Y w.r.t to the shock Γ, and the policy instrument i

Interpretation

- How to interpret optimal policy formula:

$$\boldsymbol{i}^{*}=-\left[\boldsymbol{J}_{x,i}'\left(\beta\boldsymbol{\lambda}\right)\boldsymbol{J}_{x,i}\right]^{-1}\times\left[\boldsymbol{J}_{x,i}'\left(\beta\boldsymbol{\lambda}\right)\boldsymbol{J}_{x,\Gamma}d\boldsymbol{\Gamma}\right]$$

- Solution to least-squares problem: $\Rightarrow$ Same form as a OLS least squares estimate (»$cov\left(\frac{\partial x}{\partial i},\frac{\partial x}{\partial\Gamma}d\boldsymbol{\Gamma}\right)/var\left(\frac{\partial x}{\partial i}\right)$«)
  - If the instrument and the shock co-vary strongly $\Rightarrow$Instrument is good at offsetting shock, should react more
  - If variance of instrument is large (i.e. it has large effects on target variables) $\Rightarrow$Don't need to change instrument much to get effect

Optimal policy in HANK - Efficient ss

- How does this work in **HANK**?
  - Main reference: Mckay and Wolf (2022)
- Steady state is **not efficient** due to incomplete markets (i.e. borrowing constraint)
  - But need efficient steady state to get quadratic loss function
- Choose general social welfare function (i.e. objective of planner):

$$\mathcal{W} = \sum_{t=0}^{T} \beta^t E_t \int \phi_j \left[u\left(c_{jt}\right) - \nu\left(L_t\right)\right] D_{jt} dj$$

- where $\{\phi_j\}$ is a pareto weight. Mckay and Wolf (2022) calibrate these weights such that initial steady state efficiente according to objective $\mathcal{W}$
  - If ss features some HHs with low income/wealth with high marginal utility, then planner will place **low weight** on these

Welfare approximation

- Given $\{\phi_i\}$ we can derive a second-order approximation of social welfare:

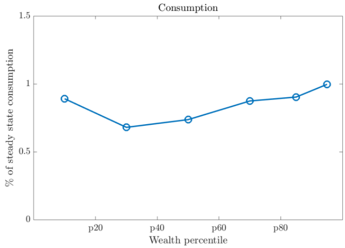
$$\mathcal{W} = \sum_{t=0}^{T} \beta^t E_t \int \phi_j \left[u\left(c_{jt}\right) - \nu\left(L_t\right)\right] D_{jt} dj$$

$$\approx -\sum_{t=0}^{\infty} \beta^t \left(d\pi_t^2 + \lambda_Y dY_t^2 + \lambda_\omega \int \left(d\omega_{jt}\right)^2 D_j dj\right)$$

- where $d\omega_{it} \equiv \frac{c_{it}}{C_t} - \frac{c_{i,ss}}{C_{ss}}$ is the change in individual j's consumption share
- Introducing HA give additional motive to social planner to **stabilize consumption shares**
- Implications of inequality for opt. policy depend on distributional incidence of policy
  - Heterogeneity matters for monetary policy if this affect cross-sectional consumption dispersion $\left(\frac{\int \left(d\omega_{jt}\right)^2 D_j dj}{di_t} \neq 0\right)$

Monetary policy effects

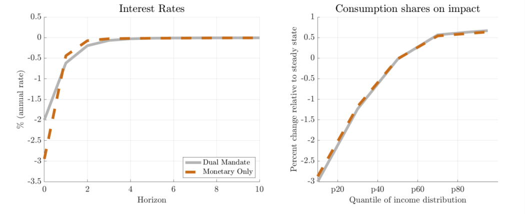
- Match model to empirical effects of monetary policy on labor income + capital gains across wealth distribution:
- Model implied consumption response across wealth distribution:



- Relatively flat - mon. pol. affect c of all HHs proportionally $\Rightarrow$ consumption shares relatively stable, $\frac{\int (d\omega_{jt})^2 D_j dj}{di_t} \approx 0$
  - HHs at bottom of dist. respond to changes in labor income
  - HHs at top of dist. respond (less) to large fluctuations in cap income (stock prices + house prices)

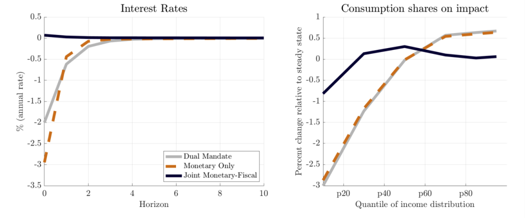
Optimal monetary policy - distributional shock

- Application: Optimal monetary policy response to *distributional* shock (transfer from poor to rich)
  - Dual mandate = No inequality term (loss $= -\sum_{t=0}^{\infty} \beta^t \left(d\pi_t^2 + \lambda_Y dY_t^2\right)$)
  - Monetary only = Dual mandate + inequality term
- Inequality term barely affects optimal policy since monetary policy is ill-suited to offset dist. incidence



Optimal joint monetary-fiscal policy

- If we consider joint monetary **and** fiscal policy:



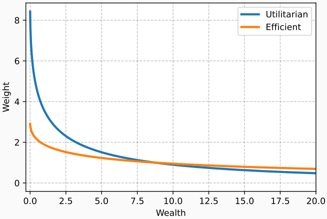
- Fiscal policy good at redistribution - no need for monetary policy here
- Typical sentiment: Fiscal policy should handle redistribution, not monetar policy

Quadratic approx: What do we lose?

- Mckay-Wolf loss function builds on second-order approximation of aggregate utility (assume log utility):

$$\int \phi_j u\left(c_{jt}\right) dj \approx u'\left(C\right) \times dC_t - \int \frac{\phi_j}{c_j^2} dc_{jt}^2 dj$$

- Weight on variance term dc_{jt}^2 of individual j:
  - Efficient steady state ($\phi_j = u'\left(c_j\right)^{-1} = c_j$): $1/c_j$
  - Inficient steady state and Utilitarian planner ($\phi_j = 1$): $1/c_j^2$
- Weights in standard model



Optimal Policy from inefficient steady state (Ramsey)

- If we do not use use planner weights, HA ss is not efficient $\Rightarrow$ Cannot derive quad. loss function
- **How to solve for optimal policy?** Example in simple HANK with optimal monetary policy
- Simple model with HA, linear production and TFP shock - CB controls real rate r (**policy instrument**)

$$\max_{\{r_t, c_{it}, L_t, Y_t, w_t, a_{it}\}_{t=0}^{T}} \mathcal{W} = \sum_{t=0}^{T} \beta^t E_t \int \left[u\left(c_{it}\right) - \nu\left(L_t\right) \right] D_{it} di$$

s.t.

$$c_{it} = c_{it}^{*}$$

$$c_{it} + a_{it} = w_t e_{it} + \left(1 + r_t\right) a_{it-1}$$

$$Y_t = \Gamma_t L_t$$

$$w_t = \Gamma_t$$

$$Y_t = \int c_{it} D_{it} di$$

- In Lagrangian form (substitute in and use $w_t = \Gamma_t$) and $\boldsymbol{x} = \{L_t \Gamma_t, r_t\}_{t=0}^{T}$

$$\mathcal{L} = \max_{\{r_t, L_t, \lambda_t\}_{t=0}^{T}} \sum_{t=0}^{T} \Big\{ \beta^t E_t \int u\left[\left(c_{it}^*\left(\boldsymbol{x}\right)\right) - \nu\left(L_t\right)\right] D_{it}\left(\boldsymbol{x}\right) di \\ + \lambda_t \left[\Gamma_t L_t - \int c_{it}^*\left(\boldsymbol{x}\right) D_{it}\left(\boldsymbol{x}\right) di\right] \Big\}$$

- **FOC** (just w.r.t r_s):

$$\frac{\partial}{\partial r_s}\left[\sum_{t=0}^{T} \Big\{ \beta^t E_t \int u\left(c_{it}^*\left(\boldsymbol{x}\right) - \nu\left(L_t\right)\right) D_{it} di\right] \\ - \frac{\partial}{\partial r_s} \sum_{t=0}^{T} \lambda_t \int c_{it}^*\left(\boldsymbol{x}\right) D_{it}\left(\boldsymbol{x}\right) di = 0$$

Lagrangian II

- Denote $\boldsymbol{J}^{\mathcal{U},r}(\boldsymbol{x})$ as jacobian of $\mathcal{U}$ w.r.t r evaluated at $\boldsymbol{x}$

$$\sum_{t=0}^{T} \beta^t \boldsymbol{J}_{t,s}^{\mathcal{U},r}(\boldsymbol{x}) - \sum_{t=0}^{T} \lambda_t \boldsymbol{J}_{t,s}^{C,r}(\boldsymbol{x}) = 0$$

- First term is (sum) of jacobian of agg. utility $\mathcal{U}$ w.r.t $\boldsymbol{r}$ (gradient)
- Second term is (sum) of jacobian of agg $\boldsymbol{C}$ w.r.t $\boldsymbol{r}$
- **We know how to compute these using fake-news algorithm**
    - We can **evaluate** the FOCs

Lagrangian FOCs I

- Standard techniques (i.e. Fake-news) allow us to evaluate the following system of FOCs:
  - T eqs. for each (instrument, endogenous variable, lagrange multiplier)
- How do we **solve** the system for the optimal policy response?
- Use (Quasi-)Newton/Broyden's method as usual [solve $f(x) = 0$ using $f'(x)$]
- Continue example from before. Jacobian of residual w.r.t r_k:

$$\sum_{t=0}^{T} \beta^t \boldsymbol{J}_{t,s}^{\mathcal{U},r}(\boldsymbol{x}) - \sum_{t=0}^{T} \lambda_t \boldsymbol{J}_{t,s}^{C,r}(\boldsymbol{x}) = 0$$

$$\Rightarrow \sum_{t=0}^{T} \beta^t \frac{\partial \boldsymbol{J}_{t,s}^{\mathcal{U},r}}{\partial r_k}(\boldsymbol{x}) - \sum_{t=0}^{T} \lambda_t \frac{\partial \boldsymbol{J}_{t,s}^{C,r}}{\partial r_k}(\boldsymbol{x}) = 0$$

$$= \sum_{t=0}^{T} \beta^t \frac{\partial^2 \mathcal{U}_t}{\partial r_s \partial r_k}(\boldsymbol{x}) - \sum_{t=0}^{T} \lambda_t \frac{\partial^2 C_t}{\partial r_s \partial r_k}(\boldsymbol{x}) = 0$$

Lagrangian FOCs II

- Last equation repeated:

$$= \sum_{t=0}^{T} \beta^t \frac{\partial^2 \mathcal{U}_t}{\partial r_s \partial r_k} (\boldsymbol{x}) - \sum_{t=0}^{T} \lambda_t \frac{\partial^2 C_t}{\partial r_s \partial r_k} (\boldsymbol{x}) = 0$$

- What is the issue here?
  - We need ***second*** order derivatives to solve this
- Expensive to get - recall:
  - Jacobian of HH problem requires T^2 iterations to get for each input (Fake-news reduce this to T)
  - Hessian (size: $T \times T \times T$) requires T^3 iterations to get
    - Can get down to $\frac{T^2}{2}$ if we use Fake-news + symmetry
    - Still slow if $T = 300 \Rightarrow 45.000$ evaluations of HH problem for each input
    - Getting standard J with Fake-news only requires $T = 300$ evaluations
  - Intuition for why this is difficult: the action of every agent c_i, a_i is a choice variable in the planner's problem $\Rightarrow$ **massive** max. problem

Solutions

- Bhandari, Evans, Golosov, & Sargent (2021)
  - Use small-noise expansions to approximate model w.r.t to both idiosyncratic states + aggregate shocks
  - Can handle aggregate uncertainty, but not occ. binding borrowing constraints
- Le Grand & Ragot (2022)
  - Truncation of state space. Can then derive FOCs by hand
- Dávila & Schaab (2023)
  - Formulate model in continuous time following Nuño & Moll (2018) - can get FOCs of planner problem by hand
  - Extended to discrete time in Waldstrøm (2024) - use numerical tools from *deep learning* literature to efficiently compute derivatives
- Literature not settled - no preferred solution yet

Other HANK Topics

Other HANK topics

- Brief introducion to optimal policy in HANK
- **Many** other very interesting topics where household heterogeneity potentially matters
- I will give you some very brief examples
  - *Could serve as inspiration for master thesis*

Overview

- Schaab and Tan (2023) »Monetary and Fiscal Policy According to HANK-IO«
- Alves & Violante (2023) »Some Like It Hot: Monetary Policy Under Okun's Hypothesis«
- Faccini, Lee, Luetticke, Ravn & Renkin (2024) »Financial Frictions: Macro vs Micro Volatility«
- Ferriere Navarro (2024) »The heterogeneous effects of government spending: It's all about taxes«

HANK-IO

- Schaab and Tan (2023) »Monetary and Fiscal Policy According to HANK-IO«
- A multi-sector HANK model where:
  - Households work in different sectors (earnings heterogeneity)
    - May matter for transmission if high MPC HHs work in sectors that are more cyclical
  - Buy goods from sectors (expenditure heterogeneity)
    - May matter for transmission if higher MPC HHs primarily buy goods from sectors with more flexible prices
- Calibrate the model to match these channels using micro data
- They find that the contribution of earnings and expenditure heterogeneity channels in the transmission of monetary policy is small

Labor Market Dynamics and Monetary Policy

- Alves & Violante (2023) »Some Like It Hot: Monetary Policy Under Okun's Hypothesis«
- Poor households are more exposed to business cycles through LM transition
  - Separation rates + job finding rates are more cyclical
  - Recession have long lasting effects on earnings (recession $\Rightarrow$ leave the LM $\Rightarrow$ hard to get back in)
- **Trade-off** for CB: Running the economy hot generates inflaiton (bad) but keeps poor household in labor market (good)
- Use a HANK model to quantify this trade-off

Financial Frictions

- Faccini, Lee, Luetticke, Ravn & Renkin (2024) »Financial Frictions: Macro vs Micro Volatility«
- Borrowers and savers pay different interest rates
- The spread (interest on loans minus savings) is countercyclical
  - A monetary policy shock that increases the interbank rate causes banks to raise rates on loans more than on savings
  - This reduces the mass of household at $a < 0$ who move to $a = 0$ through deleveraging
- The aggregate MPCs increases by up to 5% in **response** to a monetary policy shock due to this mechanism

Progressive Taxes

- Ferriere & Navarro (2024) »The heterogeneous effects of government spending: It's all about taxes«
  - Uncover *Progressivity-dependent fiscal multipliers*
- Observation: Low income households:
  - 1) Have higher MPCs
  - 2) Have larger labor supply elasticities
- Result: Government spending shocks which are financed by more progressive taxes have higher multipliers
- Consistent with empirical evidence

Conclusion

Conclusion

- **Today:**
  - A brief look into current research frontier on HANK
  - **Many** other papers which we have not covered
- **Now:** Exam