9. Fiscal Policy in HANK

Adv. Macro: Heterogenous Agent Models

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2024

Introduction

Exam info

- Exam form:
  - Portfolio (the 3 assignments)
  - **36 hours** take home
- Exam office thought it was 48 hours, so some dates were wrong online
  - E.g. info in *Exam schedules* at *https://socialsciences.ku.dk/education/studentservices/exam-schedules* has been **wrong**
- The correct dates are:
  - Start: January 4th morning (9AM)
  - End: January 5th evening (9PM)
- Re-examination: Oral exam

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    - Model with sticky wages
  - **Application**: Fiscal policy in HANK

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    - Model with sticky wages
  - **Application**: Fiscal policy in HANK
- **Literature:** Auclert et. al. (2023), »The Intertemporal Keynesian Cross«
  - Long paper with many (technical) details
  - We will focus on the main results

Sticky Wages

Detour

- Early HANK papers formulated *Heterogeneous Agent New Keynesian* Models by:
  - Take standard NK model from last lecture
  - Replace Representative agent HH block with Heterogeneous Agents HH block
  - See e.g. McKay, Nakamura, and Steinsson (2016), Hagedorn, Manovskii, and Mitman (2019)

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    - Prices are sticky
    - Wages are fully flexible
- Need to make one adjustment to standard NK model before adding HA

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- If HHs have little saving, $c_i \approx w l_i \Rightarrow MPC_i = MPE_i$!

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- Tension between model and data $\Rightarrow$ Standard model with high MPCs imply too large wealth effects on labor supply

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- Will also use this a method of introducing **sticky wages**
  - Empirical evidence typically show that wages adjust more sluggishly than prices w.r.t aggregate shocks

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- where $\frac{\theta^W}{2}\left(W_t^j / W_{t-1}^j - 1\right)^2$ is a quadratic utility cost from changing wages $\Rightarrow$ sticky wages

New Keynesian Wage Phillips curve

- Solving the union's maximization problem in a symmetric equilibrium $L_t^j = L_t,\ W_t^j = W_t$:

$$\pi_t^w \left(1 + \pi_t^w\right) = \kappa^w \left\{ \nu' \left(L_t\right) - \frac{w_t}{\mu^w} \int z_t u' \left(c_{i,t}\right) d\mathcal{D}_{i,t} \right\} L_t + \beta \pi_{t+1}^w \left(1 + \pi_{t+1}^w\right)$$

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  - Increase in labor demand L_t will push up nominal wage inflation
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- Breaks wealth effect on labor supply through two mechanisms:
  - Sticky wages (low κ^w)
  - Wealth effect on agg. L depends on agg. marginal utility $\int z_t u'\left(c_{i,t}\right) d\mathcal{D}_{i,t}$

More tractable version

- Sometimes assume that union maximize the utility of an agent with average consumption $u\left(C_t\right) = u\left(\int c_{it} d\mathcal{D}_{it}\right)$ instead of $\int u\left(c_{i,t}\right) d\mathcal{D}_{it}$

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- $u'\left(C_t\right)$ instead of $\int z_t u'\left(c_{i,t}\right) d\mathcal{D}_{i,t}$
- Slightly easier to implement in code and when working analytically

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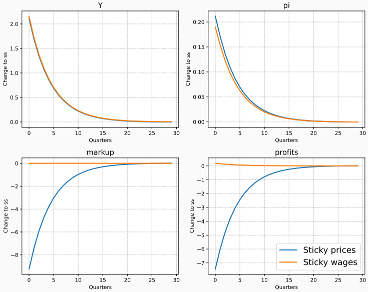
- Sticky wages solve this



HANK

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Households

- **Household problem:**

$$v_t(z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{t+1}, a_t) \right]$$

$$\text{s.t. } a_t + c_t = (1 + r_t^a) a_{t-1} + (1 - \tau_t) \, w_t \ell_t z_t + \chi_t$$

$$\log z_{t+1} = \rho_z \log z_t + \psi_{t+1} \ \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_t] = 1$$

$$a_t \geq 0$$

- **Active decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_t d\mathcal{D}_t$
- **Consumption function:** $C_t^{hh} = C^{hh} \left(\{ r_s^a, (1 - \tau_s) \, w_s \ell_s, \chi_s \}_{s=0}^{\infty} \right)$

Firms

- **Production and profits:**

$$Y_t = \Gamma_t L_t$$
$$\Pi_t = Y_t - \frac{W_t}{p} L_t$$

- **First order condition:**

$$w_t = \Gamma_t$$

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- **FOCs** (no arbitrage conditions):

$$1 + r_t = 1 + r^a_t$$

Union

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} \left(1 - \tau_t\right) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

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$$B_t + T_t + = (1 + r_t)B_{t-1} + G_t + \chi_t$$

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- In which case taxes T_t adjust fully every period to ensure that the budget holds

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- Indeterminacy: Consider limit for nominal rule $i_t = i_{ss} + \phi\pi_{t+1}$ or assume future tightening

Market clearing

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

Fiscal Policy

Simpler consumption function

- **Assumptions:**
  1. One-period real bond
  2. No lump-sum transfers, $\chi_t = 0$
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- **Consumption function in sequence-space:** Simplifies to

$$C_t^{hh} = C^{hh}(\{Y_s - T_s, r_s\}_{s \geq 0}) \Rightarrow \boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r}) = C^{hh}(\boldsymbol{Z}, \boldsymbol{r})$$

Side-note: Two-equation version in $\boldsymbol{Y}$ and $\boldsymbol{r}$

$$
\begin{aligned}
\boldsymbol{Y} &= \boldsymbol{G} + C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T}) \\
\boldsymbol{r} &= \mathcal{R}(\boldsymbol{Y})
\end{aligned}
$$

- **First equation:** Goods market clearing
- **Second equation (**Firms + NKWPC**):**
  1. Given output $\boldsymbol{Y}$, can compute $\boldsymbol{L}$
  2. Firm behavior I: $\boldsymbol{\Gamma}, \boldsymbol{Y} \rightarrow \boldsymbol{L}, \boldsymbol{w}$
  3. NKWC: $\boldsymbol{L}, \boldsymbol{C}, \boldsymbol{w}, \boldsymbol{\tau} \rightarrow \boldsymbol{\pi}^{\boldsymbol{w}}$
  4. Firm behavior II: $\boldsymbol{\pi}^{\boldsymbol{w}}, \boldsymbol{\Gamma} \rightarrow \boldsymbol{\pi}$
  5. Central bank: $\boldsymbol{\pi} \rightarrow \boldsymbol{i}$
  6. Fisher: $\boldsymbol{i}, \boldsymbol{\pi} \rightarrow \boldsymbol{r}$
- Final assumption for today: **Constant $\boldsymbol{r}$**
  - Can replace $\boldsymbol{r} = \mathcal{R}(\boldsymbol{Y})$ with $\boldsymbol{r} = r_{ss}$
  - Entire model boils down to 1 equation

Intertemporal Keynesian Cross

$$Y_t = G_t + C_t^{hh}\left(\{Y_s - T_s\}_{s=0}^{\infty}\right) \qquad \text{Static}$$
$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{Y} - \boldsymbol{T}) \qquad \text{Sequence-space/vector}$$

- **Total differentiation/linearize around ss:**

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} dZ_s = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} \left(dY_s - dT_s\right)$$

- **Intertemporal Keynesian Cross** in vector form

$$d\boldsymbol{Y} = d\boldsymbol{G} + \boldsymbol{M}\left(d\boldsymbol{Y} - d\boldsymbol{T}\right) \Leftrightarrow$$
$$(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}$$

where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes the entire *complexity of HH behavior*

Illustration

- Writing out the IKC:

$$\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} dG_0 \\ dG_1 \\ dG_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \frac{\partial C_0^{hh}}{\partial Z_2} & \cdots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \frac{\partial C_1^{hh}}{\partial Z_2} & \cdots \\ \frac{\partial C_2^{hh}}{\partial Z_0} & \frac{\partial C_2^{hh}}{\partial Z_1} & \frac{\partial C_2^{hh}}{\partial Z_2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \left(\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} - \begin{bmatrix} dT_0 \\ dT_1 \\ dT_2 \\ \vdots \end{bmatrix} \right)$$

iMPC matrix

- $\boldsymbol{M}$ is the Jacobian of aggregate C w.r.t (post-tax) labor income
  - Column s: Response of C at different dates to unit change in Z at date s (IRF)
  - Row s: Change in C at date s to change in income Z at different dates

$$\boldsymbol{M} = \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \cdots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

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- In a quarterly model $\frac{\partial C_0^{hh}}{\partial Z_0}$ is essentially the quarterly MPC
  - Note: Typically define MPCs following change in lump-sum transfer and not labor income, so not quite MPC

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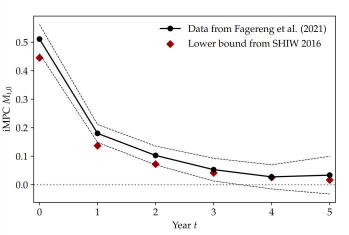
  - Info on first column $\left[\frac{\partial C_0^{hh}}{\partial Z_0}, \frac{\partial C_1^{hh}}{\partial Z_0}, \frac{\partial C_2^{hh}}{\partial Z_0}, \frac{\partial C_3^{hh}}{\partial Z_0}, \ldots\right]'$

Figure 1: iMPCs in the Norwegian and Italian data



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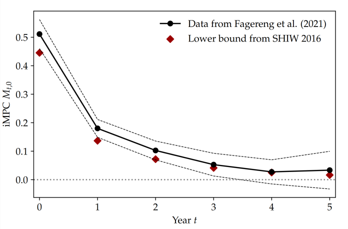

- Very hard to say something about remaining columns (announcement effects)
  - Best we can do: Calibrate model to first column, get rest of $\boldsymbol{M}$ from model

Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depends on current income

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- **Solution**

$$dY_t = dG_t + \frac{\text{mpc}}{1 - \text{mpc}}(dG_t - dT_t)$$

from multiplier-process $\text{mpc} \times \left(1 + \text{mpc} + \text{mpc}^2 \ldots\right) = \frac{\text{mpc}}{1-\text{mpc}}$

NPV-vector

- **NPV-vector:** $\boldsymbol{q} \equiv [1, (1+r_{ss})^{-1}, (1+r_{ss})^{-2}, \dots]'$ - implies $\sum_{t=0}^{\infty}(1+r_{ss})^{-t}x_t = \boldsymbol{q}'\boldsymbol{x}$

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$$\sum_{t=0}^{\infty}(1+r_{ss})^{-t}(dG_t - dT_t) = 0 \Leftrightarrow$$

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$$\sum_{t=0}^{\infty}(1+r_{ss})^{-t}M_{t,s} = \frac{1}{(1+r)^s} \Rightarrow \boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}' \Leftrightarrow \boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M}) = \boldsymbol{0}$$

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- Present value of $\boldsymbol{M}$ columns is 1 - HHs must *eventually* spend income they receive
  - $\sum_{t=0}^{\infty} \frac{MPC_{t,s}}{(1+r_{ss})^{t-s}} = 1$

Form of unique solution

- **Back to IKC**:

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$$\boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M})(\boldsymbol{I} - \boldsymbol{M})^{-1} = \mathbf{0}(\boldsymbol{I} - \boldsymbol{M})^{-1} \Leftrightarrow$$
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- **Result:** If unique solution then on the form

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- **Note:** This is only an issue in infinite horizon
  - When solving numerically we truncate at horizon T, implying that columns in $\boldsymbol{M}$ do not exactly add to 1
  - Can then invert $(\boldsymbol{I}-\boldsymbol{M})$ (but precision becomes worse as horizon T increases)

Response of consumption

$$
\begin{aligned}
d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{G}) \Leftrightarrow \\
(I - \boldsymbol{M})(d\boldsymbol{Y} - d\boldsymbol{G}) &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{C} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})
\end{aligned}
$$

Fiscal multipliers

$$d\boldsymbol{Y} = d\boldsymbol{G} + \underbrace{\mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})}_{d\boldsymbol{C}}$$

- **Balanced budget multiplier:**

$$d\boldsymbol{G} = d\boldsymbol{T} \Rightarrow d\boldsymbol{Y} = d\boldsymbol{G}, d\boldsymbol{C} = 0$$

  Note: Central that income and taxes affect household income proportionally in exactly the same way = no redistribution

- **Deficit multiplier:** $d\boldsymbol{G} \neq d\boldsymbol{T}$
  1. Potentially fiscal multiplier **above** 1
  2. Larger effect of $d\boldsymbol{G}$ than $d\boldsymbol{T}$
  3. *Numerical results needed*

Fiscal multiplier

Impact-multiplier:

$$\frac{\partial Y_0}{\partial G_0}$$

Cumulative-multiplier:

$$\frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dY_t}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dG_t}$$

Comparison with RA model

- **From lecture 1:** $\beta(1+r_{ss})=1$ implies

$$C_t=(1-\beta)\sum_{s=0}^{\infty}\beta^s Y^{hh}_{t+s}+r_{ss}a_{-1}$$

- The **iMPC-matrix** becomes ($\mathbf{1}$ is a square matrix of 1's)

$$\boldsymbol{M}^{RA}=\begin{bmatrix}(1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots\\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots\\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots\\ \vdots & \vdots & \vdots & \ddots\end{bmatrix}=(1-\beta)\mathbf{1}\boldsymbol{q}'$$

- **Consumption response** is zero

$$\begin{aligned}d\boldsymbol{C}^{\boldsymbol{RA}}&=\mathcal{M}\boldsymbol{M}^{\boldsymbol{RA}}(d\boldsymbol{G}-d\boldsymbol{T})\\&=\mathcal{M}(1-\beta)\mathbf{1}\boldsymbol{q}'(d\boldsymbol{G}-d\boldsymbol{T})\\&=\mathbf{0}\Leftrightarrow d\boldsymbol{Y}=d\boldsymbol{G}\end{aligned}$$

- **Fiscal multiplier is 1**

Details on matrix formulation

$$
\begin{aligned}
(1-\beta)\mathbf{1}\boldsymbol{q}' &= \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & (1+r_{ss})^{-1} & (1+r_{ss})^{-2} & \cdots \end{bmatrix} \\
&= \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & \beta & \beta^2 & \cdots \end{bmatrix} \\
&= \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}
\end{aligned}
$$

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- Large litterature on Two Agent New Keyniesan models (TANK) starting with Campbell and Mankiw (1989)
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- Simple to implement+tractable, but some drawbacks
  - No intertemporal MPCs
  - Extremely stylized level of inequality
  - Hard to connect to micro data
  - No precautionary saving

Comparison with TANK model

- **Hand-to-Mouth (HtM) households:** λ share have $C_t = Y_t^{hh}$

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- **Solution:**

$$d\boldsymbol{Y} = d\boldsymbol{G} + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$$

- **Amplification** of fiscal policy with deficit financing $d\boldsymbol{G} > d\boldsymbol{T}$
  - Size of amplification increasing in share of constrained agents λ
  - Solution **very** similar to static, old Keynesian cross (multiplier: $\frac{\text{mpc}}{1-\text{mpc}}$)

TANK Proof

- In TANK we have:

$$\begin{aligned} d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}^{TA}(d\boldsymbol{Y} - d\boldsymbol{T}) \\ (\boldsymbol{I} - \boldsymbol{M}^{TA})d\boldsymbol{Y} &= d\boldsymbol{G} - \boldsymbol{M}^{TA}d\boldsymbol{T} \\ (\boldsymbol{I} - \boldsymbol{M}^{RA})d\boldsymbol{Y} &= \frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right] - \boldsymbol{M}^{RA}d\boldsymbol{T} \\ d\boldsymbol{Y} &= \boldsymbol{M}^{RA}d\boldsymbol{Y} - \boldsymbol{M}^{RA}d\boldsymbol{T} + \frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right] \\ d\boldsymbol{Y} &= d\tilde{\boldsymbol{G}} + \boldsymbol{M}^{RA}(d\boldsymbol{Y} - d\boldsymbol{T}) \end{aligned}$$

- where $d\tilde{\boldsymbol{G}} = \frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right]$
- Recall that in RANK $d\boldsymbol{Y} = d\boldsymbol{G} + \boldsymbol{M}^{RA}(d\boldsymbol{Y} - d\boldsymbol{T})$ with solution $d\boldsymbol{Y} = d\boldsymbol{G}$ thus implying $d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}$ in TANK:

$$\begin{aligned} d\boldsymbol{Y} &= \frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right] \\ d\boldsymbol{Y} &= d\boldsymbol{G} - d\boldsymbol{G} + \frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right] \\ d\boldsymbol{Y} &= d\boldsymbol{G} + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right] \end{aligned}$$

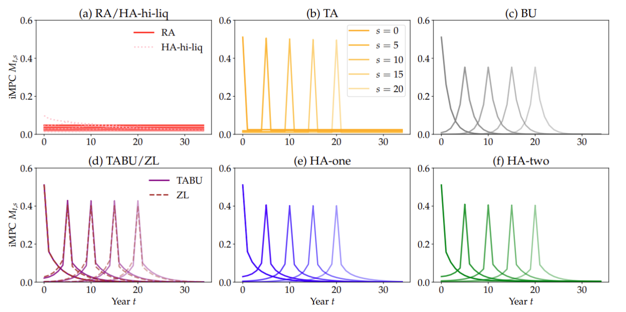
Cumulative multiplier still one

$$\frac{\boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}' d\boldsymbol{G}} = \frac{\boldsymbol{q}' d\boldsymbol{G}_t + \frac{\lambda}{1-\lambda}\boldsymbol{q}' \left[d\boldsymbol{G} - d\boldsymbol{T}\right]}{\boldsymbol{q}' d\boldsymbol{G}}$$

$$= 1$$

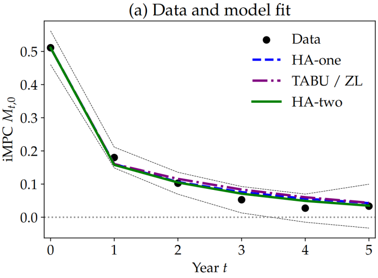
Jacobian columns

- TANK produces positive C response, fiscal multiplier above 1 - do we need HANK?
- Plot columns of $\boldsymbol{M}$ in TANK, HANK + other models
  - Recall columns: dynamic C response to change in Z at various dates



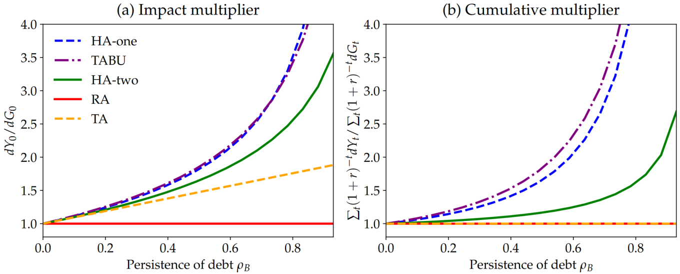
iMPCs in models

Figure 2: iMPCs in the Norwegian data and several models



Multipliers and debt-financing

Figure 5: Multipliers according to the IKC



Note. These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary ρ_B in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

Summary in table

- Summary:

Table 1: Government spending multipliers in the intertemporal Keynesian cross

Fiscal rule	Multiplier	Rep. agent (RA) doesn't match MPC	Two agents (TA) matches MPC	Het. agents (HA) matches iMPCs
balanced budget	impact	1	1	1
	cumulative	1	1	1
deficit financing	impact	1	> 1	> 1
	cumulative	1	1	> 1

Interest rate effects

- We assumed real bonds for tractablity - In reality, bonds are typically **nominal:**
- $$(1 + r_t)\, B_{t-1} \quad \textbf{versus} \quad \frac{1 + i_{t-1}}{1 + \pi_t} B_{t-1}$$

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- Period 0:
$$\frac{1+i_{ss}}{1+\pi_0}B_{ss}$$
- Period 1:
$$\frac{1+i_0}{1+\pi_1}B_0=(1+r)\,B_0$$

Interest rate effects

- We assumed real bonds for tractablity - In reality, bonds are typically **nominal:**
- $$\left(1+r_t\right) B_{t-1} \quad \textbf{versus} \quad \frac{1+i_{t-1}}{1+\pi_t} B_{t-1}$$
- **Fisher**: $\left(1+r_t\right)=\frac{1+i_{t-1}}{1+\pi_t} \approx r_t=i_{t-1}-\pi_t$
- Even if CB keeps ex-ante real rate constant $i_t=i_{ss}+\pi_{t+1}$ real returns on bonds will differ in period 0:
- Period 0:
$$\frac{1+i_{ss}}{1+\pi_0} B_{ss}$$
- Period 1:
$$\frac{1+i_0}{1+\pi_1} B_0=\left(1+r\right) B_0$$
- With nominal bonds **surprise inflation** affects returns - implies capital for households in period 0
    - Positive fiscal shock generates inflation, so **negative** effect on

Generalized IKC

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r^a_{t-1})a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \text{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\text{cap}_0 = 0$
2. Nominal bond:

$$\text{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

Generalized IKC

- **Consumption-function** $\boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T}, \mathbf{cap}_0)$ implies

$$d\boldsymbol{C}^{hh} = \boldsymbol{M}^r d\boldsymbol{r} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + \boldsymbol{m}^{\text{cap}}\text{cap}_0$$

  where

$$\boldsymbol{M}^r_{t,s} = \left[\frac{\partial C^{hh}_t}{\partial r_s}\right], \boldsymbol{m}^{\text{cap}}_t = \left[\frac{\partial C^{hh}_t}{\partial \text{cap}_0}\right]$$

- Capital return effect is negative - can this overturn fiscal multiplier > 1?
  - No - entries in $\boldsymbol{M}$ are large
  - Entries in $\boldsymbol{m}^{\text{cap}}$ are **small**
- MPC out of capital gains approximately 1-4% per year

Fiscal policy in HANK - litterature

- Seminal paper: **Intertemporal Keynesian Cross**
- **Many** other interesting papers:
- McKay and Reis - *The role of automatic stabilizers in the US business cycle* (2016)
  - Analyze the role of automatic stabilizers in a HANK model
- Bayer, Born, Luetticke - *The liquidity channel of fiscal policy* (2023)
  - Role of liquid and illiquid effects of fiscal policy
- Hagedorn, Manovskii, and Mitman - *The fiscal multiplier* (2019)
  - Systematic evaluation of multiplier in HANK + ZLB
- Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm - *Fiscal Multipliers in Small Open Economies With Heterogeneous Households (2024)*
  - Generalize to small open economies

Exercise

Exercise

Consider the standard HANK model outlined in section 2

1. Compute and plot selected columns of the household Jacobian of C w.r.t Z, r, χ
   Hint: use model._compute_jac_hh()to compute the jacobians. You can find the results in model.jac_hh
   1.1 Are the MPCs out of labor income Z and a transfer χ different? why?
2. Compute IRFs to a deficit financed ($\omega = 0.1$) and tax financed (ω large) fiscal spending shock. Compare the responses.
3. Compute the output IRF $d\boldsymbol{Y}$ using the fomula $d\boldsymbol{Y} = d\boldsymbol{G} + \mathcal{M}\boldsymbol{M}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$ where $\mathcal{M} = \left(\boldsymbol{I} - \boldsymbol{M}\right)^{-1}$. Check that you get the same as when using model.find_IRFs()
4. Redo Q2 with active monetary policy, $\phi_\pi = 1.5$. How does the fiscal multiplier change?
5. Redo Q2 with active monetary policy *and* a flatter NKWPC, $\kappa = 0.01$. How does the fiscal multiplier change?

Summary

Summary and next week

- **Today:** Fiscal policy in a HANK model with sticky wages
- **Next week:** Assignment workshop
- **Homework:**
    1. Work on exercise
    2. Work on assignment