ASSIGNMENT II: THE HANK MODEL

Vision: This project teaches you how to analyze the effects of *deficit financed fiscal stimulus* in a Heterogeneous Agent New Keynesian model.

- **Problem:** The problem consists of
  1. A number of questions (page 2)
  2. A model (page 3 onward)
  3. Some code files you can start from
- **Code:** The problem is designed to be solved with the *GEModelTools* package.
- **Structure:** Your project should consist of
  1. A single self-contained pdf-file with all results
  2. A single Jupyter notebook showing how the results are produced
  3. Well-documented *.py* files
- **Hand-in:** Upload a single zip-file on Absalon (and nothing else)
- **Deadline:** 19th of November 2024
- **Exam:** Your HANK-project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Questions

1. Solve for the non-linear transition path to a shock to government transfers ϵ^T of 1% of steady state output Y_{ss}, which is financed by raising taxes at time H. Analyze the differences between the responses for $H = \{0, 20, 40\}$, and comment on how the *degree of self-financing* (DSF) ν varies with H:

$$\nu = \frac{\sum_{t=0}^{\infty} \left(\frac{1}{1+r_{ss}}\right)^t \tau dY_t}{dB_0}$$

   where $dY_t = Y_t - Y_{ss}$ and $dB_0 = B_0 - B_{ss}$. What is the interpretation of ν?[1]

2. Conduct the same exercise in the *TANK* model. Discuss and interpret the differences to the *HANK* model.
   Note: You will need to write the code in the function TA_HHs() in blocks.py yourself, based on the equations in section 4 below. You will also need to calibrate the steady state.

3. Analyze how your results change in the *HANK* model when the steady state tax rate τ is reduced to $\tau = 0.05$.

4. Analyze how your results change in the *HANK* model when monetary policy is *active*, $\phi_Y = 0.5$. Does the slope of Philips curve κ matter for your results? Would this result change if the government issued nominal bonds instead of real bonds? Explain.

[1] You may find it useful to know that the government's transversality condition imply that $dB_0 = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (d\chi_t + \tau dY_t)$ for a fixed interest rate r.

1. HANK model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_t, and their (end-of-period) savings, a_{t-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, ℓ_t, chosen by a union, and choose consumption, c_t, on their own. Households are not allowed to borrow. The return on savings is r_t, the real wage is w_t, labor income and profits are taxed with the rate $\tau \in [0,1]$, and households pay lumpsum taxes, χ_t.
The household problem is

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}\left[v_{t+1}(z_{t+1}, a_t) \mid z_t, a_t\right] \qquad (1) \\
\text{s.t. } a_t + c_t &= (1+r_t)a_{t-1} + (1-\tau)\left(\Pi_t + w_t \ell_t z_t\right) - \chi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\ \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frish elasticity.
Aggregate quantities are

$$L_t^{hh} = \int \ell_t z_t d\boldsymbol{D}_t \qquad (2)$$

$$C_t^{hh} = \int c_t d\boldsymbol{D}_t \qquad (3)$$

$$A_t^{hh} = \int a_t d\boldsymbol{D}_t \qquad (4)$$

$$MUC_t^{hh} = \int z_t c_t^{-\sigma} d\boldsymbol{D}_t \qquad (5)$$

Firms. A representative firm hires labor, L_t, to produce goods, with the production function

$$Y_t = L_t \qquad (6)$$

Profits are

$$\Pi_t = Y_t - \frac{W_t}{P_t} L_t \qquad (7)$$

where P_t is the price level and W_t is the wage level. Firms choose prices subject to a (virtual) quadratic adjustment cost on prices and monopolistic competition. Optimization implies the following Phillips curve:

$$\pi_t (1+\pi_t) = \kappa \left(w_t - \frac{1}{\mu} \right) + \beta \pi_{t+1} (1+\pi_{t+1}) \tag{8}$$

where inflation is defined as:

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 \tag{9}$$

and $w_t = \frac{W_t}{P_t}$ is the real wage.

Union. A union chooses the labor supply of each household. Each household is chosen to supply the same amount of labor,

$$\ell_t = L_t^{hh} \tag{10}$$

The union maximizes aggregate utility of all households. Optimal labor supply is then given by:

$$\varphi \left(L_t^{hh} \right)^{\nu} = (1-\tau_t)\, w_t MUC_t^{hh} \tag{11}$$

Central bank. The central bank follows a real-rate Taylor rule which reacts to output:

$$r_t = r_{ss} + \phi_Y (Y_t - Y_{ss}) \tag{12}$$

Government. The government lumpsum taxes, χ_t, and the income tax rate, τ. Total tax revenue from income taxes is given by:

$$\mathcal{T}_t \equiv \tau \left(\Pi_t + w_t L_t^{hh} \right) = \tau Y_t \tag{13}$$

The government can finance its expenses with real bonds, B_t which pay out the real rate r_t after one period. The budget constraint for the government is given by:

$$B_t + \mathcal{T}_t + \chi_t = (1+r_t)\, B_{t-1} \tag{14}$$

Lumpsum taxes follows the rule:

$$\chi_t = \begin{cases} \chi_{ss} - \epsilon^T & t \leq H \\ \chi_{ss} - (\mathcal{T}_t - \mathcal{T}_{ss}) + (1 + r_t)(B_t - B_{ss}) & t > H \end{cases} \tag{15}$$

where ϵ^T is shock to government transfers. The No-Ponzi condition for the government is $\lim_{t\to\infty} \prod_{j=0}^{t} \frac{B_t}{1+r_j} = \lim_{t\to\infty} \prod_{j=0}^{t} \frac{B_{ss}}{1+r_j}$.[2]

Market clearing. Market clearing implies

1. Labor market: $L_t = L_t^{hh}$
2. Goods market: $Y_t = C_t^{hh}$
3. Asset market: $B_t = A_t^{hh}$

2. Calibration

The model is calibrated at the quarterly frequency. The parameters and steady state government behavior are as follows:

1. **Preferences and abilities:** $\sigma = 1.0$, $\nu = 1.0$
2. **Income:** $\rho_z = 0.96$, $\sigma_\psi = 0.16$
3. **Production:** $\mu = 1.1, \kappa = 0.03$
4. **Central bank:** $r_{ss} = 0.02$, $\phi^Y = 0$
5. **Government:** $\tau = 0.33$

We let β, φ and the steady state supply of governmet bonds B_{ss} be unspecified and adjust those to obtain the steady state we want.

[2] In principle we do not need to impose this in equilibrium, as it follows from the transversality condition of household combined with market clearing.

3. Finding the steady state

We calibrate the steady state to obtain equilibrium along with an aggregate, annual MPC of 0.5, $MPC = \sum_{h=0}^{3} \int \left(-\frac{\partial c_{t+h}}{\partial \chi_t}\right) dD_{t+h} = 0.5$

1. Guess on β, B_{ss}
2. Set r_{ss} as specified in the calibration
3. Normalize aggregate labor supply to, $L_{ss} = 1$
4. Set steady state inflation to zero, $\pi_{ss} = 0$
5. Calculate the value of all other aggregate steady state variables
6. Solve for and simulate household behavior
7. Calculate $\varphi = \frac{(1-\tau_{ss})w_{ss}MUC_{ss}^{hh}}{\left(L_{ss}^{hh}\right)^{\nu}}$
8. Check 1) the remaining market clearing condition, 2) The aggregate, annual MPC

4. TANK model

In the TANK model we replace the heterogeneous agent household block with a simpler structure that features only two types of households: Unconstrained (R) and constrained (C). Unconstrained agents are on their Euler equation at all times, while constrained agents are up against the borrowing constraint implying they have no savings. The total measure of households is 1, and constrained households make up a share λ of these households. The budget constraints are:

$$c_t^R + \frac{A_t}{1-\lambda} = (1+r_t)\frac{A_{t-1}}{1-\lambda} + (1-\tau)\left(\Pi_t + w_t L_t^{hh}\right) - \chi_t \tag{16}$$

$$c_t^C = (1-\tau)\left(\Pi_t + w_t L_t^{hh}\right) - \chi_t \tag{17}$$

with the Euler equation for unconstrained households being:

$$\left(c_t^R\right)^{-\sigma} = \beta\left(1+r_{t+1}\right)\left(c_{t+1}^R\right)^{-\sigma} \tag{18}$$

Aggregate consumption is given by:

$$C_t^{hh} = (1-\lambda)\,c_t^R + \lambda c_t^c \tag{19}$$

and aggregate savings is simply $A_t^{hh} = A_t$. The aggregate marginal utility of consumption is $MUC_t^{hh} = (1-\lambda)\left(c_t^R\right)^{-\sigma} + \lambda\left(c_t^c\right)^{-\sigma}$.

The model is calibrated to the same aggregate level of bonds and assets as the HANK model, and the share of constrained households λ is set to match an annual MPC of 0.5.

5. Equation system

The model can be summarized by the following equation system

$$\boldsymbol{H}(\boldsymbol{\pi}, \boldsymbol{L}, \boldsymbol{B}, \boldsymbol{w}, \boldsymbol{\epsilon}^T) = \begin{bmatrix} \pi_t\,(1+\pi_t) - \kappa\left(w_t - \frac{1}{\mu}\right) - \beta\pi_{t+1}\,(1+\pi_{t+1}) \\ Y_t - L_t \\ \varphi\left(L_t^{hh}\right)^{\nu} - (1-\tau)\,w_t MUC_t^{hh} \\ r_t - \left(r_{ss} + \phi_Y\left(Y_t - Y_{ss}\right)\right) \\ \mathcal{T}_t - \tau Y_t \\ B_t + \mathcal{T}_t + \chi_t - (1+r_t)\,B_{t-1} \\ \chi_t - \begin{cases} \chi_{ss} - \epsilon^T & t \leq H \\ \chi_{ss} - (\mathcal{T}_t - \mathcal{T}_{ss}) + (1+r_t)\,(B_t - B_{ss}) & t > H \end{cases} \\ L_t - L_t^{hh} \\ B_t - A_t^{hh} \end{bmatrix} = \boldsymbol{0}$$

where household behavior is determined in either the *heterogeneous agent* household block or the *two agent* household block.