14. Advanced HANK Topics

Adv. Macro: Heterogenous Agent Models

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2024

Introduction

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- **Today:** How does *policy recommendations* change with the introduction of heterogeneous agents?
  - I.e what is the optimal policy in response to adverse aggregate shocks?
  - Is it affected by heterogeneity?
- This topic is at the **research frontier**
  - At the end of lecture I will briefly mention other topics at the frontier

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  - Optimal steady state: Maximize one function as a function of 1 or 2 variables (ex: labor income tax+capital income tax)
  - Here: Choose entire **path** (length T) of instruments subject to entire dynamic model

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   - Larger role for inequality and redistribution
   - Much more difficult - see end of lecture

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- Around **efficient steady state**, can derive the following approximation of household welfare (see e.g. Woodford book):

$$\mathcal{W} = \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - \nu\left(L_t\right) \right] \approx - \sum_{t=0}^{\infty} \beta^t \left(d\pi_t^2 + \lambda_Y dY_t^2 \right)$$

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- Maximizing welfare same as minizing loss function
  - Planner want to stabilize (varianceof) inflation and output

Solving the planner prob. - efficient steady state

- Sequence-space social planner problem ($\boldsymbol{\beta} = \text{diag}\left(1, \beta^1, \beta^2, ...\right)$):

$$\min \boldsymbol{\pi}' \boldsymbol{\beta} \boldsymbol{\pi} + \lambda_Y d\boldsymbol{Y}' \boldsymbol{\beta} d\boldsymbol{Y} \quad \text{s.t.} \quad H\left(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{\Gamma}\right) = 0$$

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- Stack target variables in vector $\boldsymbol{x} = (\boldsymbol{\pi}, \boldsymbol{Y})'$ and weights $\boldsymbol{\lambda} = (\mathbf{1}, \lambda_Y)$. Then:

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- Choose interest rate to solve this problem. Solution:

$$\boldsymbol{i}^* = -\left[\boldsymbol{J}'_{x,i}\left(\beta \boldsymbol{\lambda}\right) \boldsymbol{J}_{x,i}\right]^{-1} \times \left[\boldsymbol{J}'_{x,i}\left(\beta \boldsymbol{\lambda}\right) \boldsymbol{J}_{x,\Gamma} d\boldsymbol{\Gamma}\right]$$

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- **Easy** to solve for optimal policy given shock $d\boldsymbol{\Gamma}$ - just need jacobians around ss!
  - We need jacobians for π, Y w.r.t to the shock Γ, and the policy instrument i

Interpretation

- How to interpret optimal policy formula:

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- Solution to least-squares problem: $\Rightarrow$ Same form as a OLS least squares estimate (»$cov\left(\frac{\partial x}{\partial i}, \frac{\partial x}{\partial \Gamma} d\boldsymbol{\Gamma}\right)/var\left(\frac{\partial x}{\partial i}\right)$«)
    - If the instrument and the shock co-vary strongly $\Rightarrow$Instrument is good at offsetting shock, should react more
    - If variance of instrument is large (i.e. it has large effects on target variables) $\Rightarrow$Don't need to change instrument much to get effect

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- Choose general social welfare function (i.e. objective of planner):

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- where $\{\phi_j\}$ is a pareto weight. Mckay and Wolf (2022) calibrate these weights such that initial steady state efficiente according to objective $\mathcal{W}$
  - If ss features some HHs with low income/wealth with high marginal utility, then planner will place **low weight** on these

Welfare approximation

- Given $\{\phi_i\}$ we can derive a second-order approximation of social welfare:

$$\begin{aligned}\mathcal{W} &= \sum_{t=0}^{T} \beta^t E_t \int \phi_j \left[u\left(c_{jt}\right) - \nu\left(L_t\right)\right] D_{jt} dj \\ &\approx -\sum_{t=0}^{\infty} \beta^t \left(d\pi_t^2 + \lambda_Y dY_t^2 + \lambda_\omega \int \left(d\omega_{jt}\right)^2 D_j dj\right)\end{aligned}$$

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- Introducing HA give additional motive to social planner to **stabilize consumption shares**
- Implications of inequality for opt. policy depend on distributional incidence of policy
  - Heterogeneity matters for monetary policy if this affect cross-sectional consumption dispersion $\left(\frac{\int \left(d\omega_{jt}\right)^2 D_j dj}{di_t} \neq 0\right)$

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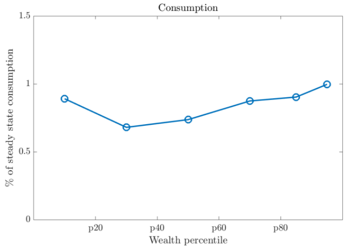

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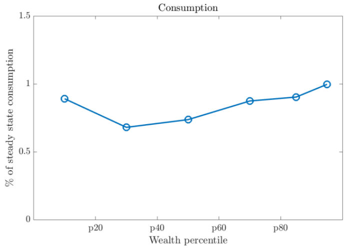

- Relatively flat - mon. pol. affect c of all HHs proportionally $\Rightarrow$ consumption shares relatively stable, $\frac{\int (d\omega_{jt})^2 D_j dj}{di_t} \approx 0$
  - HHs at bottom of dist. respond to changes in labor income
  - HHs at top of dist. respond (less) to large fluctuations in cap income (stock prices + house prices)

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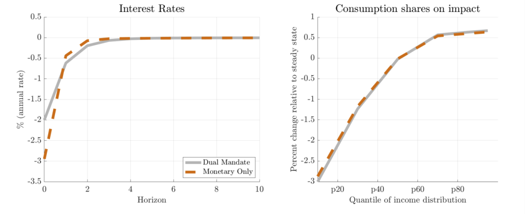
- Inequality term barely affects optimal policy since monetary policy is ill-suited to offset dist. incidence



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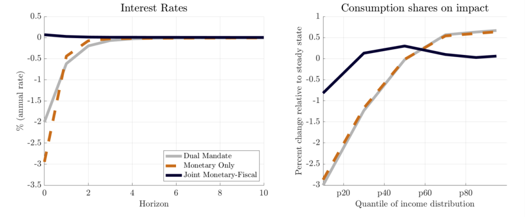

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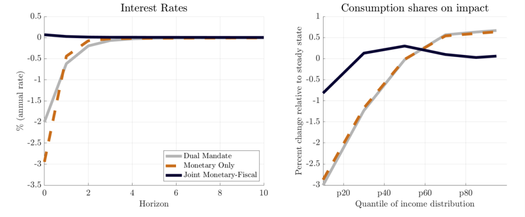

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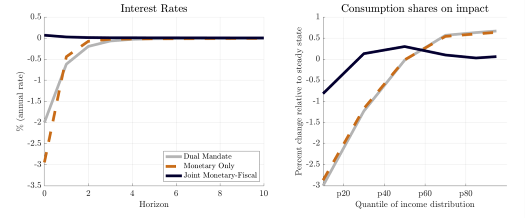

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- Typical sentiment: Fiscal policy should handle redistribution, not monetar policy

Quadratic approx: What do we lose?

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$$\int \phi_j u\left(c_{jt}\right) dj \approx u'\left(C\right) \times dC_t - \int \frac{\phi_j}{c_j^2} dc_{jt}^2 dj$$

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- Weight on variance term dc_{jt}^2 of individual j:
  - Efficient steady state ($\phi_j = u'\left(c_j\right)^{-1} = c_j$): $1/c_j$
  - Infficient steady state and Utilitarian planner ($\phi_j = 1$): $1/c_j^2$

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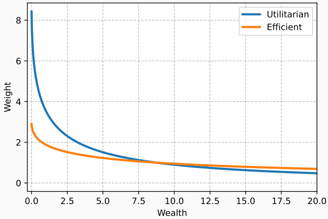
- Weights in standard model



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- If we do not use use planner weights, HA ss is not efficient $\Rightarrow$ Cannot derive quad. loss function
- **How to solve for optimal policy?** Example in simple HANK with optimal monetary policy
- Simple model with HA, linear production and TFP shock - CB controls real rate r (**policy instrument**)

$$\max_{\{r_t, c_{it}, L_t, Y_t, w_t, a_{it}\}_{t=0}^{T}} \mathcal{W} = \sum_{t=0}^{T} \beta^t E_t \int \left[u\left(c_{it}\right) - \nu\left(L_t\right) \right] D_{it} di$$

$$\text{s.t.}$$

$$\begin{aligned} c_{it} &= c_{it}^{*} \\ c_{it} + a_{it} &= w_t e_{it} + \left(1 + r_t\right) a_{it-1} \\ Y_t &= \Gamma_t L_t \\ w_t &= \Gamma_t \\ Y_t &= \int c_{it} D_{it} di \end{aligned}$$

Lagrangian I

- In Lagrangian form (substitute in and use $w_t = \Gamma_t$) and $\boldsymbol{x} = \{L_t \Gamma_t, r_t\}_{t=0}^{T}$

$$\mathcal{L} = \max_{\{r_t, L_t, \lambda_t\}_{t=0}^{T}} \sum_{t=0}^{T} \left\{ \beta^t E_t \int u\left[\left(c_{it}^*\left(\boldsymbol{x}\right)\right) - \nu\left(L_t\right)\right] D_{it}\left(\boldsymbol{x}\right) di \right.$$
$$\left. + \lambda_t \left[\Gamma_t L_t - \int c_{it}^*\left(\boldsymbol{x}\right) D_{it}\left(\boldsymbol{x}\right) di \right] \right\}$$

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- **FOC** (just w.r.t r_s):

$$\frac{\partial}{\partial r_s}\left[\sum_{t=0}^{T} \Big\{ \beta^t E_t \int u\left(c_{it}^{*}(\boldsymbol{x}) - \nu(L_t)\right) D_{it}\, di \right] - \frac{\partial}{\partial r_s} \sum_{t=0}^{T} \lambda_t \int c_{it}^{*}(\boldsymbol{x})\, D_{it}(\boldsymbol{x}) di = 0$$

Lagrangian II

- Denote $\boldsymbol{J}^{\mathcal{U},r}(\boldsymbol{x})$ as jacobian of $\mathcal{U}$ w.r.t r evaluated at $\boldsymbol{x}$

$$\sum_{t=0}^{T} \beta^t \boldsymbol{J}_{t,s}^{\mathcal{U},r}(\boldsymbol{x}) - \sum_{t=0}^{T} \lambda_t \boldsymbol{J}_{t,s}^{C,r}(\boldsymbol{x}) = 0$$

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- Second term is (sum) of jacobian of agg $\boldsymbol{C}$ w.r.t $\boldsymbol{r}$

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- Second term is (sum) of jacobian of agg $\boldsymbol{C}$ w.r.t $\boldsymbol{r}$
- **We know how to compute these using fake-news algorithm**
  - We can **evaluate** the FOCs

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- Continue example from before. Jacobian of residual w.r.t r_k:

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  - Intuition for why this is difficult: the action of every agent c_i, a_i is a choice variable in the planner's problem $\Rightarrow$ **massive** max. problem

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  - Extended to discrete time in Waldstrøm (2024) - use numerical tools from *deep learning* literature to efficiently compute derivatives
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Other HANK Topics

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- **Many** other very interesting topics where household heterogeneity potentially matters
- I will give you some very brief examples
    - *Could serve as inspiration for master thesis*

Overview

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- Calibrate the model to match these channels using micro data
- They find that the contribution of earnings and expenditure heterogeneity channels in the transmission of monetary policy is small

Labor Market Dynamics and Monetary Policy

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- The aggregate MPCs increases by up to 5% in **response** to a monetary policy shock due to this mechanism

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- Consistent with empirical evidence

Conclusion

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- **Today:**
  - A brief look into current research frontier on HANK
  - **Many** other papers which we have not covered
- **Now:** Exam