Logo

Uncertainty propagation in neural networks for sparse coding

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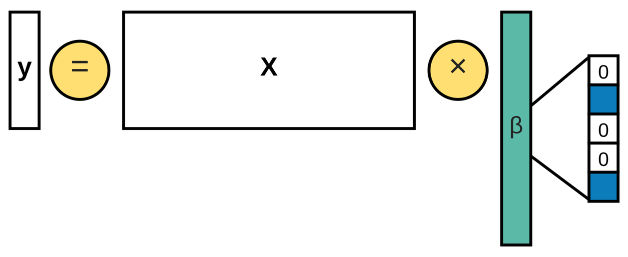
1-LISTA

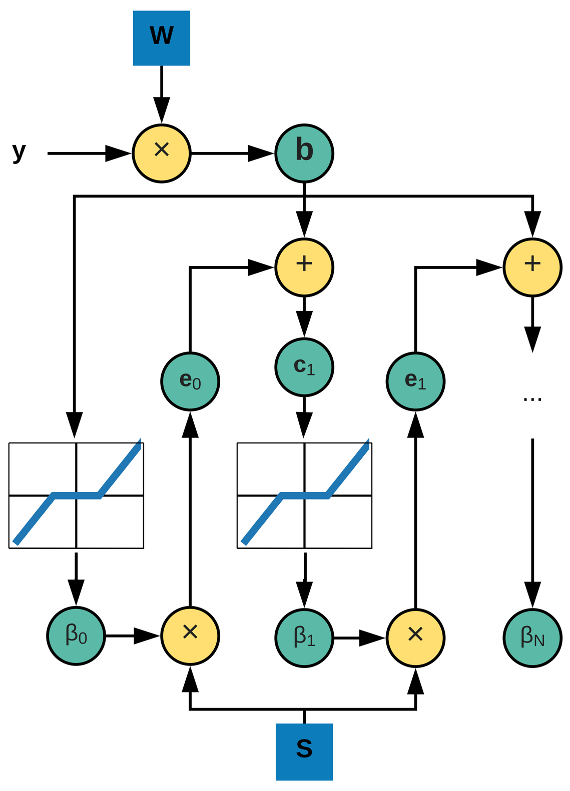
Estimate $\boldsymbol{\beta}$ from observations $\mathbf{y}$ collected as $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, s.t. elements $\boldsymbol{\beta}$ contain zeros.



LISTA

- Represent iterative soft-thresholding algorithm as RNN with shared weights
- Learn weights with BPTT

Init. Dense $\mathbf{b} \leftarrow \mathbf{W}\mathbf{y}$
Init. Soft-thresholding $\widehat{\boldsymbol{\beta}}_0 \leftarrow h_\lambda(\mathbf{b})$
for $l = 1$ **to** L **do**
  Dense $\mathbf{c}_l \leftarrow \mathbf{b} + \mathbf{S}\widehat{\boldsymbol{\beta}}_{l-1}$
  Soft-thresholding $\widehat{\boldsymbol{\beta}}_l \leftarrow h_\lambda(\mathbf{c}_l)$
end for
return $\widehat{\boldsymbol{\beta}} \leftarrow \widehat{\boldsymbol{\beta}}_L$



Overfitting

No uncertainty estimation

2-BayesLISTA

Add priors for NN weights

$$p(\mathbf{W}) = \prod_{d=1}^{D}\prod_{k=1}^{K}\mathcal{N}(w_{ij}; 0, \eta^{-1}), \quad p(\mathbf{S}) = \prod_{d'=1}^{D}\prod_{d''=1}^{D}\mathcal{N}(s_{d'd''}; 0, \eta^{-1}), \tag{1}$$

Propagate distribution for $\widehat{\boldsymbol{\beta}}$ through layers

Compute prediction as noisy NN output

$$p(\boldsymbol{\beta}|\mathbf{y}, \mathbf{W}, \mathbf{S}, \gamma, \lambda) = \prod_{d=1}^{D}\mathcal{N}\left(\beta_d; [f(\mathbf{y}; \mathbf{S}, \mathbf{W}, \lambda)]_d, \gamma^{-1}\right) \tag{2}$$

Update weights with PBP

3-Uncertainty propagation

At every step the output of soft-thresholding can be closely approximated with spike and slab distribution

1. $\mathbf{b} = \mathbf{W}\mathbf{y}$ is Gaussian-distributed
2. $\widehat{\boldsymbol{\beta}}_0 = h_\lambda(\mathbf{b})$ is approximated with the spike and slab distribution
3. $\mathbf{e}_l = \mathbf{S}\widehat{\boldsymbol{\beta}}_{l-1}$ is approximated with the Gaussian distribution
4. $\mathbf{c}_l = \mathbf{b} + \mathbf{e}_l$ is Gaussian-distributed
5. $\widehat{\boldsymbol{\beta}}_l = h_\lambda(\mathbf{c}_l)$ is approximated with the spike and slab distribution

- All latent variables are modelled with parametrised distributions
- We can apply approximate Bayesian inference methods

4-BackProp-PBP

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5-Results

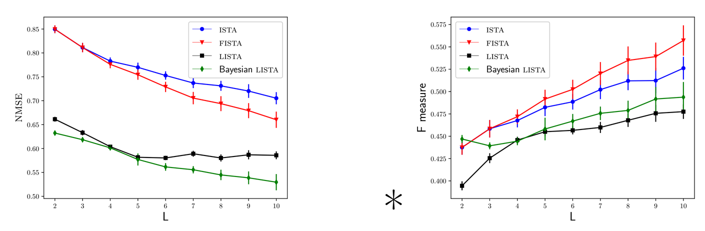

Fig. 3: Different depth performance

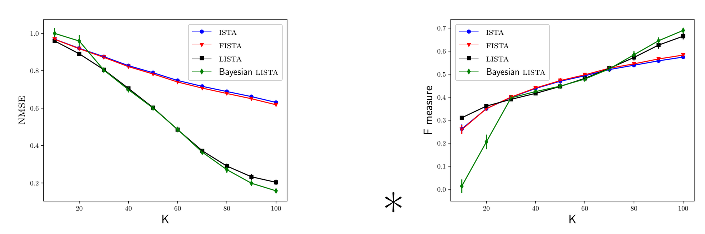

Fig. 4: Different observation size performance

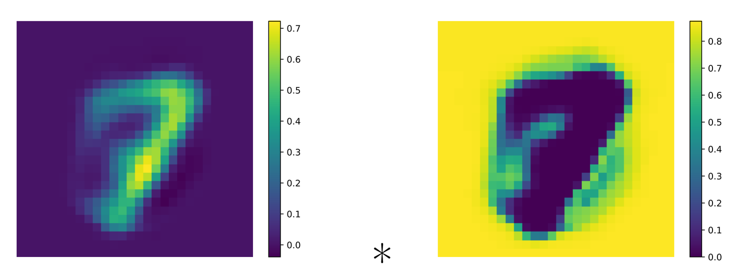

Fig. 5: Posterior parameters for an image of digit 7

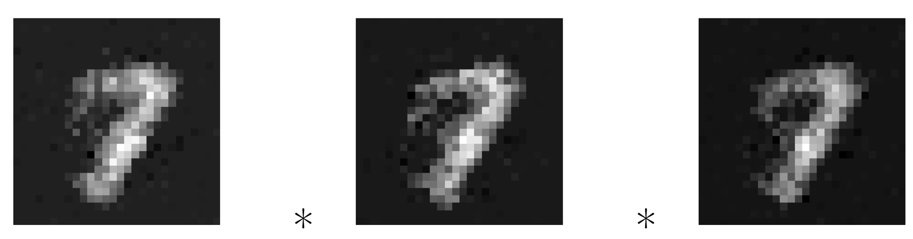

Fig. 6: Samples from the posterior for an image of digit 7

Use the estimated uncertainty to choose next training data with largest variance

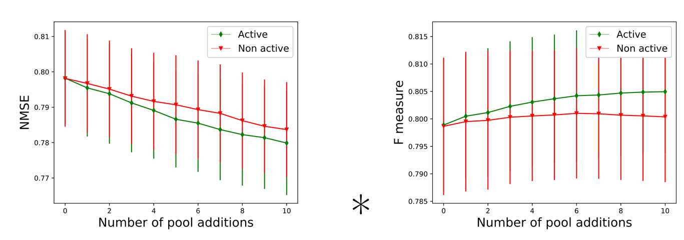

Fig. 7: Sequential pool additions

6-Summary

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