Algorithm 1 Online Algorithm for Non-Linear Covering Problems.

Initially, set $A^* \leftarrow \emptyset$ `(where` A^* `is the solution set and` $\forall e \in A^* : x_e = 1$`)`
All primal and dual variables are initially set to 0
During every step, for each feasible solution S, $z_S = \prod_{e \in S} x_e \prod_{e \notin S}(1 - x_e)$ is maintained.
Let τ be the continuous timer during the execution of the algorithm.
for each time t, for the new primal constraint $\sum_e a_e^t x_e \geq 1$ and dual variable $\alpha_{A^*}^t$ **do**
  while $\sum_{e \notin A^*} b_e^t(A^*)\ x_e < 1$ **do** `# Increase primal, dual variables`
    Increase τ with a rate of 1.
    Increase $\alpha_{A^*}^t$ at rate $1\ /\ (\lambda\ \ln(1 + 2d^2/\eta))$
    for $e \notin A^*$ such that $b_e^t(A^*) > 0$ **do**
      if $\beta_e < \frac{1}{\lambda} \nabla_e F(\mathbf{x})$ **then** $\beta_e \leftarrow \frac{1}{\lambda} \nabla_e F(\mathbf{x})$
      Increase x_e at a rate according to the following

$$\frac{\partial x_e}{\partial \tau} \leftarrow \frac{b_e^t(A^*)\ x_e}{\lambda \beta_e} + \frac{\eta}{\lambda \beta_e d} + \frac{(1-\eta) \cdot \mathbb{1}_{\{\texttt{pred}(x_e)=1\}}}{\nabla_e F(\mathbf{x}) \cdot |\{e' : \texttt{pred}(x_{e'}) = 1,\ b_{e'}^t(A^*) > 0\}|}$$

    end for
    if $x_e = 1$ **then** $A^* \leftarrow A^* \cup \{e\}$
    for $e : e \notin A^*$ **do** `# Decrease dual variables`
      while $\sum_{t'=1}^{t} \sum_{A : e \notin A} b_e^{t'}(A)\ \alpha_A^{t'} > \beta_e$ **do**
        for (t_e^*, A) such that $b_e^{t_e^*}(A) = \max\{b_e^{t'}(A) \mid \forall A : e \notin A$ and $\forall t' \leq t$ s.t. $\alpha_A^{t'} > 0\}$ **do**
          Decrease $\alpha_A^{t_e^*}$ continuously with a rate of $\frac{b_e^t(A^*)}{b_e^{t_e^*}(A)} \cdot \frac{1}{\lambda \cdot \ln(1 + 2d^2/\eta)}$
        end for
      end while
    end for
  end while
end for