$$
\begin{aligned}
& \min \sum_{i=1}^{n} c_i x_i^T = \min \sum_{i=1}^{n} c_i \sum_{t=1}^{T} \left(x_i^t - x_i^{t-1} \right) && \\
\text{subject to} \quad & \sum_{k=1}^{K} w_k^t = 1 && \forall\, t \\
& x_i^t \geq \sum_{k=1}^{K} w_k^t s_{i,k}^t && \forall\, i, t \\
& x_i^t \geq x_i^{t-1} && \forall\, i, t \\
& w_k^t \geq 0 && \forall\, t, k
\end{aligned}
$$

where $1 \leq t \leq T$ and $1 \leq i \leq n$.

Figure 1: Formulation of the `LIN-COMB` benchmark

Theorem 1 *The algorithm's cost is at most $O(\ln(K\rho))$-competitive in the* `LIN-COMB` *benchmark.*

Proof Lemma 1 proved that our algorithm creates feasible solutions for the dual problem of the `LIN-COMB` benchmark relaxation and for the original covering problem. We show that the algorithm's solution increases the primal objective value of the original covering problem by at most $O(\ln(K\rho))$ times the value of the dual solution, which serves as the lower bound on the `LIN-COMB` benchmark - the best linear combination of the experts' solutions.

$$
\begin{aligned}
\sum_{i=1}^{n} c_i(x_i^t - x_i^{t-1}) &= \frac{1}{\ln(K\rho)} \sum_{i:x_i^t > x_i^{t-1}} c_i(x_i^t - x_i^{t-1}) \\
&\le \sum_{i:x_i^t > x_i^{t-1}} c_i(x_i^t + \delta_i^t) \ln \frac{x_i^{t-1} + \delta_i^t}{x_i^{t-1} + \delta_i^t} && (1)\\
&\le \sum_{i:x_i^t > x_i^{t-1}} c_i(x_i^t + \delta_i^t) \ln \frac{x_i^{t-1} + \delta_i^t}{x_i^{t-1} + \delta_i^{t-1}} && (2)\\
&= \sum_{i:x_i^t > x_i^{t-1}} c_i \left[\left(\sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t + \frac{1}{K} \sum_{k=1}^{K} s_{i,k}^t \right) \ln \left(\frac{\sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t + \delta_i^t}{x_i^{t-1} + \delta_i^{t-1}} \right) \right] && (3)\\
&\le \sum_{i:x_i^t > x_i^{t-1}} c_i \left[\left(\sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t + \frac{1}{K} \sum_{k=1}^{K} s_{i,k}^t \right) \ln \left(\frac{\sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t + \delta_i^t}{\sum_{k=1}^{K} s_{i,k}^{t-1} w_{i,k}^{t-1} + \delta_i^{t-1}} \right) \right] && (4)\\
&= \sum_{i:x_i^t > x_i^{t-1}} \sum_{k=1}^{K} (w_{i,k}^t + 1/K) c_i s_{i,k}^t \ln \left(\frac{\sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t + \delta_i^t}{\sum_{k=1}^{K} s_{i,k}^{t-1} w_{i,k}^{t-1} + \delta_i^{t-1}} \right) \\
&= \sum_{i:x_i^t > x_i^{t-1}} \sum_{k=1}^{K} (w_{i,k}^t + 1/K) \left(a_i^t \hat{s}_{i,k}^t \gamma^t + \lambda_i^t \right) && (5)\\
&\le \sum_{i=1}^{n} \sum_{k=1}^{K} (w_{i,k}^t + 1/K) \left(a_i^t \hat{s}_{i,k}^t \gamma^t + \lambda_i^t \right) \\
&= \sum_{i=1}^{n} a_i^t \left(\sum_{k=1}^{K} w_{i,k}^t \hat{s}_{i,k}^t \right) \gamma^t + \sum_{i=1}^{n} \left(\sum_{k=1}^{K} w_{i,k}^t \right) \lambda_i^t + \frac{1}{K} \sum_{k=1}^{K} \left(\sum_{i=1}^{n} a_i^t \hat{s}_{i,k}^t \right) \gamma^t + \frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{n} \lambda_i^t \\
&= 2\gamma^t + 2 \sum_{i=1}^{n} \lambda_i^t = \ln(K\rho)\alpha^t && (6)
\end{aligned}
$$

The above corresponding transformations hold since:
(2) follows from the inequality $a - b \le a \ln(a/b)$ for all $0 < b \le a$;
(3) holds since $\delta_i^t \ge \delta_i^{t-1}$ (because $s_{i,k}^t \ge s_{i,k}^{t-1}$ for all i, k, t);
(4) is valid because $x_i^t > x_i^{t-1}$, so $x_i^t = \sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t$;
(5) is by the design of the algorithm: $x_i^{t-1} \ge \sum_{k=1}^{K} s_{i,k}^{t-1} w_{i,k}^{t-1}$;
(6) since given that $x_i^t > x_i^{t-1} \ge 0$ (so $\sum_{k=1}^{K} s_{i,k}^t w_{i,k}^t = x_i^t > 0$), the KKT condition applies;
(7) is true due to the complementary slackness conditions and that $\sum_{i=1}^{n} a_i^t \hat{s}_{i,k}^t = 1$.

□

1 Signal Temporal Logic

1.1 Notation

$V = \{x_1, x_2, ..., x_n\}$	Current state of the model as a set of variables
$\mathbb{D} = \mathbb{D}_1 \times \mathbb{D}_2 \times ... \times \mathbb{D}_n$	Domain of V
$\mathbb{T} = \mathbb{R}_{\geq 0}$	Time set
$\mathbb{B} = \{-1, +1\}$	Boolean set
$w : \mathbb{T} \mapsto \mathbb{D}$	A signal w
$M = \{\mu_1, \mu_2, ..., \mu_k\}$	Booleanizers $\mu_j : \mathbb{R}^n \mapsto \mathbb{B}$
φ	STL formula
$(w, t) \models \varphi$	The signal w satisfies the STL formula φ at time t
$\chi(\mu, w, t)$	Characteristic function
$\mathcal{I} \subset \mathbb{T}$	Time interval (open or closed) as (i_1, i_2) and $i_1 < i_2$
$t + \mathcal{I} = \{t + t' \mid t' \in \mathcal{I}\}$	Time interval window from a given time t
$\mathcal{U}_\mathcal{I}$	Until operator with interval $\mathcal{I}$
$\bigcirc_\mathcal{I}$	Next operator with interval $\mathcal{I}$
$\Diamond_\mathcal{I}$	Eventually operator with interval $\mathcal{I}$
$\Box_\mathcal{I}$	Always operator with interval $\mathcal{I}$

1.2 Events

$$\text{NextEvent}(z, r_k) = \begin{cases} \infty \text{ if } (z, \delta z) \text{ is continuous for } r > r_k \\ \min\{r > r_k \mid (z[r^-], \delta z[r^-]) \neq (z[r^+], \delta z[r^+])\} \text{ otherwise} \end{cases} \tag{7}$$