

An Alert Regarding a Common Misinterpretation of the Van Genuchten α Parameter

Quirijn de Jong van Lier^{(1)*} and Everton Alves Rodrigues Pinheiro⁽¹⁾

⁽¹⁾ Universidade de São Paulo, Centro de Energia Nuclear na Agricultura, Piracicaba, São Paulo, Brasil.

ABSTRACT: Among the equations available to describe the relation between matric potential and soil water content, the soil water retention function, the most commonly used is the equation proposed by Van Genuchten in his 1980 landmark paper. In soil physics literature, especially in Brazil, several authors relate the inverse of the Van Genuchten parameter α to the air-entry pressure. This study aimed to show this common interpretation to be erroneous, as $1/\alpha$ corresponds to water contents lower than saturation. The deviation depends on the m parameter. In fact, α is merely a scaling parameter relative to the matric potential axis. Recognizing this mathematical fact may improve the interpretation of soil hydraulic properties based on water retention parameters.

Keywords: soil water retention, soil physics, empirical equations.

* Corresponding author:
E-mail: qdjvlier@usp.br

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INTRODUCTION

Several equations are available to describe the relation between matric potential and soil water content, the soil water retention function. The most frequently applied ones are the Brooks and Corey (1964) equation (BC) and the van Genuchten (1980) equation (VG). When expressing effective saturation Θ , a quantity scaling water content from 0 to 1 between residual and saturated values, the BC equation contains two parameters, one of which (h_b) explicitly represents the matric potential corresponding to air-entry. On the other hand, the VG equation, to be applied using the Mualem (1976) or Burdine (1953) parametric restriction, holds also two parameters (α and n), but none of them has a clear physical meaning.

In Brazilian soil physics literature, several authors relate the inverse of parameter α to the air-entry pressure, in analogy to the BC parameter h_b . Some use this kind of identification when describing VG parameters (Souza et al., 2008a; Lima et al., 2014; Oliveira Júnior et al., 2014), others when explicitly interpreting results of α values to the air-entry pressure (Souza et al., 2008b; Silva et al., 2009; Mota et al., 2017). In international literature, a similar description can sometimes be found (Pollaco and Mohanty, 2012; Aschonitis and Antonopoulos, 2013; Aschonitis et al., 2015; Dokooohaki et al., 2017).

Here, we demonstrate that this interpretation of the VG α parameter is incorrect and should therefore be avoided.

DEVELOPMENT

The air-entry pressure, or “bubbling pressure” h_b (m), of a soil or porous material is defined as the matric potential at which the first (largest) pore starts draining its water (Brooks and Corey, 1964). Considering the Young-Laplace capillary equation (Equation 1), it is determined by the radius of the largest pore r_m (m) as:

$$|h_b| = \frac{2\sigma\cos\varphi}{\rho gr_m}, \quad \text{Eq. 1}$$

in which σ (J m⁻²) is the surface tension of water, φ the contact angle between the water surface, the surrounding air, and the pore walls, ρ (kg m⁻³) the density of water, and g (m s⁻²) the gravity. In some water retention models, h_b (m) is an explicit fitting parameter, notably in the Brooks and Corey (1964) model (Equation 2):

$$\Theta = \begin{cases} \left(\frac{h}{h_b}\right)^{\lambda} & \text{for } h < h_b \\ 1 & \text{for } h \geq h_b \end{cases} \quad \text{Eq. 2}$$

$$\Theta = 1 \text{ for } h \geq h_b$$

in which h is the matric potential, $\Theta = (\theta - \theta_r)/(\theta_s - \theta_r)$ is the effective saturation, θ , θ_r , and θ_s are water content, residual water content, and saturated water content, respectively, all on a volume base (m³ m⁻³). The air-entry pressure corresponds to the onset of water content reduction with further decreasing matric potentials. As such, the water content at the air-entry pressure θ_b (m³ m⁻³) equals the saturated water content θ_s and $\Theta = 1$ at $h = h_b$ (Equation 2).

The air-entry pressure is not explicitly present in the frequently used van Genuchten (1980) water retention equation (VG, Equation 3):

$$\Theta = [1 + (\alpha|h|)^n]^{-m} \quad \text{Eq. 3}$$

in which α , n , and m (function of n) are fitting parameters, α having the inverse dimension of h (e.g. m⁻¹). The VG equation is defined together with the theory presented by Mualem (1976) or Burdine (1953), and when applying the respective parametric restrictions (defining m as a function of n), it can be used to estimate the hydraulic conductivity function from retention parameters. The Mualem restriction is as follows (Equation 4):

$$m = 1 - \frac{1}{n} \text{ and } n > 1 \quad \text{Eq. 4}$$

while the Burdine restriction is (Equation 5):

$$m = 1 - \frac{2}{n} \text{ and } n > 2 \quad \text{Eq. 5}$$

Consequently, $0 < m < 1$. As mentioned, many authors assume that α is the inverse of the absolute value of the air-entry pressure h_b , i.e. (Equation 6):

$$|h_b| = \frac{1}{\alpha} \Leftrightarrow \alpha = \frac{1}{|h_b|} \quad \text{Eq. 6}$$

This assumption has its origin in a comparison between equation 2 and 3. If $|h|$ becomes very large, equation 3 reduces to equation 7:

$$\Theta = [(\alpha|h|)^n]^{-m} \quad \text{Eq. 7}$$

Equation 7 is equal to equation 2, with $\lambda = mn$ (or, $l = n - 1$) with the Mualem restriction, equation 4, and $\lambda = n - 2$ with the Burdine restriction, equation 5 and α given by equation 6. However, this does not justify the interpretation of α as the inverse of the bubbling pressure, as equation 7 is only valid for very large values of $|h|$, whereas $|h_b|$ is, in fact, a relatively small value. Fitting soils with several textures from the Hydrus package (Šimůnek et al., 2016), values of $|h_b|$ range between 0.05 and 0.4 m (Figure 1), corresponding to pore diameters of $5.87 \cdot 10^{-4}$ and $7.35 \cdot 10^{-5}$ m, respectively.

Moreover, if the interpretation of α as the inverse of the bubbling pressure were true, then combining equation 6 to equation 3 would result in the following expression (Equation 8) for the effective saturation, corresponding to the bubbling pressure Θ_b :

$$\Theta_b = \frac{1}{2^m} \quad \text{Eq. 8}$$

Equation 8 yields values for Θ_b between 1 (at $m = 0$) and 0.5 (at $m = 1$), as shown in figure 2, and in obvious disagreement with the notion that $\theta_b = \theta_s$; consequently, $\Theta_b = 1$. The value of $m = 0$ implies in $n = 1$ (Mualem restriction, equation 4) or $n = 2$ (Burdine restriction, equation 5). Such values are physically unrealistic, as $m = 0$ results in $\Theta = 1$ for any value of h (Equation 7). Equation 8 implies in the fact that the greater the value

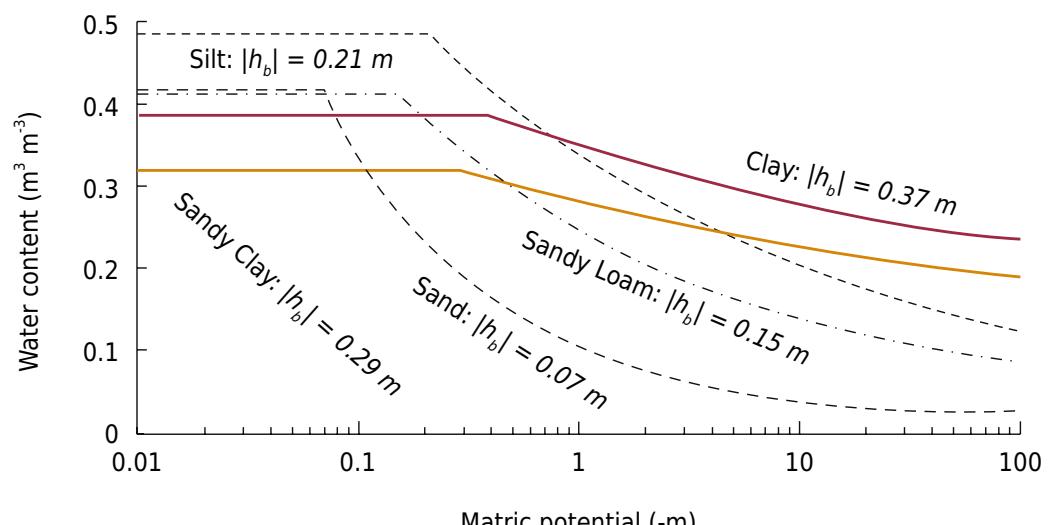


Figure 1. Representative values of $|h_b|$ for soils from the Hydrus package (Šimůnek et al., 2016).

of m (and, from equations 4 and 5, the greater the value of n), the larger the deviation between the inverse of α and the air-entry pressure. This can also be seen in figure 3, showing the retention curves (Θ as a function of the matric potential) for two values of parameter α and $n = 2$ (top) and $n = 5$ (bottom). In this figure, $1/\alpha$ indicates the supposed values of the air-entry pressure according to equation 6, with corresponding Θ_b given by equation 8. Figure 2 also clearly demonstrates the effect of α on the shape of the retention curve, with α being a mere scaling parameter relative to the matric potential axis.

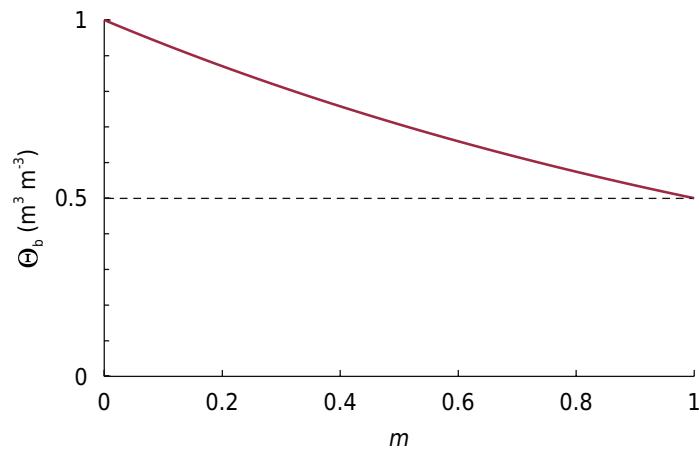


Figure 2. Effective saturation Θ_b as a function of the Van Genuchten parameter m , assuming parameter α to be the inverse of the air-entry pressure.

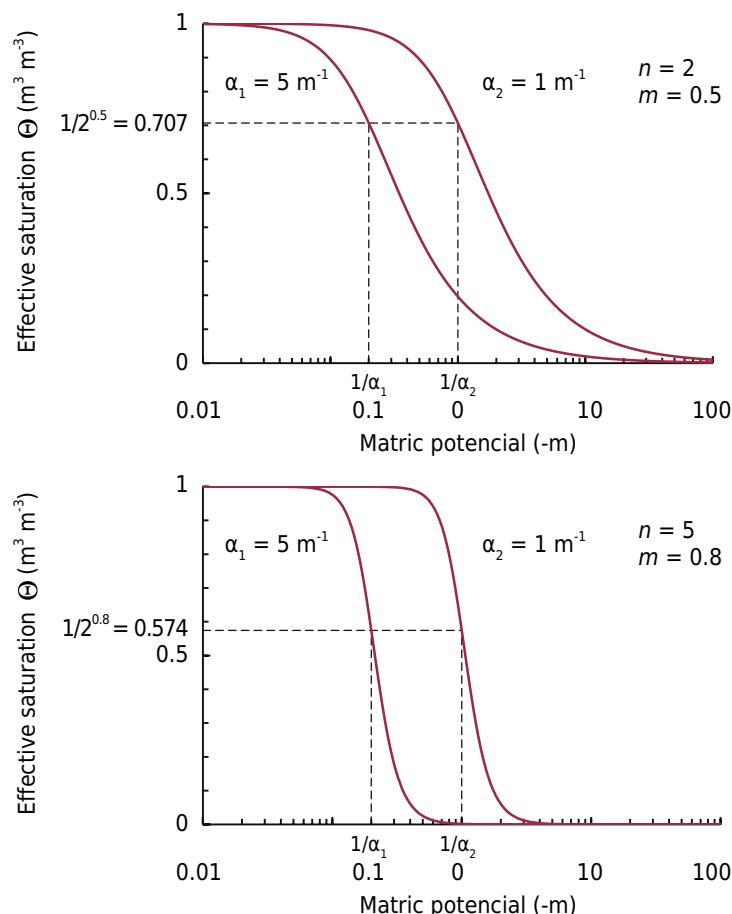


Figure 3. Effective saturation Θ as a function of the matric potential for two values of parameter α and $n = 2$ (top) and $n = 5$ (bottom). The lines $1/\alpha$ indicate the supposed values of the air-entry pressure according to equation 6, with corresponding Θ_b given by equation 8.

CONCLUSION

We showed, mathematically and graphically, that the Van Genuchten retention equation parameter α is not equal to, nor simply correlated to the (inverse of) air-entry matric potential, as frequently alleged. Instead, α is a scaling parameter relative to the matric potential axis. Recognizing this mathematical fact may improve the interpretation of soil hydraulic properties based on water retention parameters and prevent the error of using the relationship shown in equation 6 to correlate parameters from equations 2 and 3.

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