Translatorni mehanički sistemi
Modeli fizičkih sistema

Modeliranje i simulacija sistema

Promenljive

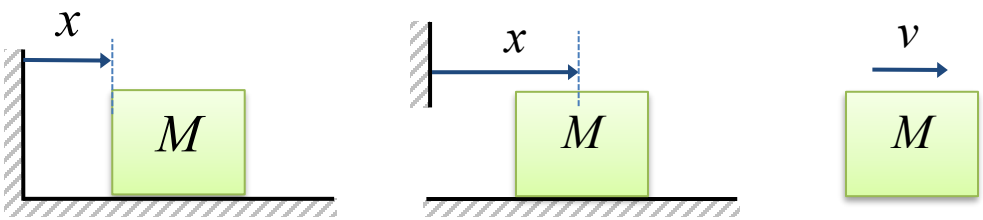
Osnovne promenljive:

- x – rastojanje [m]
- v – brzina [m/s]
- a – ubrzanje [m/s^2]
- f – sila [N]

Sve su funkcije vremena

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



Elementi i njihovi zakoni

Posmatramo elemente i pojave:

- Masa
- Trenje
- Elastičnost

Masa tela

- Masa tela *M* [kg]
- II Njutnov zakon:

$$\frac{d}{dt}(M \cdot v) = f \qquad \text{za } M\text{=const} \qquad M\frac{dv}{dt} = f$$

Trenje

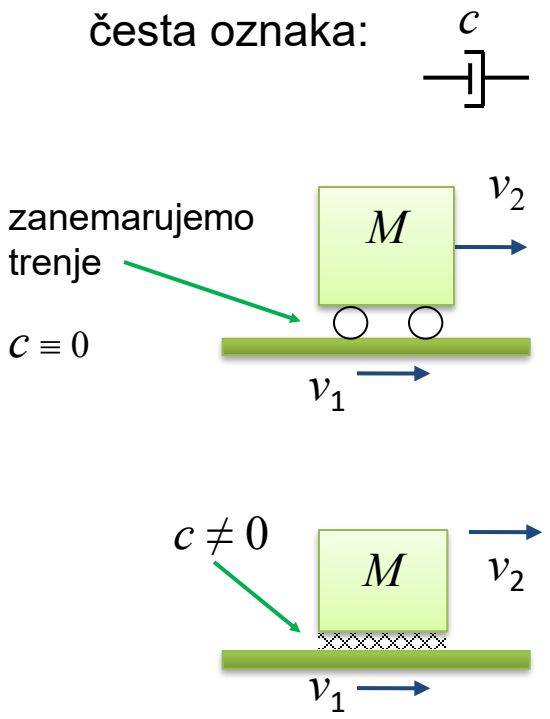
- Sila trenja se javlja kada se dva tela dodiruju i kreću različitim brzinama

$$f = f(\Delta v) \qquad \Delta v = v_2 - v_1$$

- Linearizovana zavisnost: $f = c \cdot \Delta v$

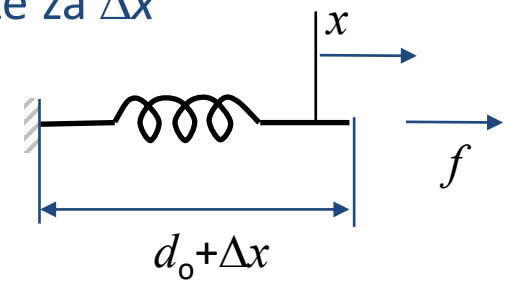
  c – koeficijent trenja (viskoznosti) [Ns/m]

  - direktno je srazmeren površi dodira, a obrnuto srazmeren debljini uljanog filma.



Elastičnost

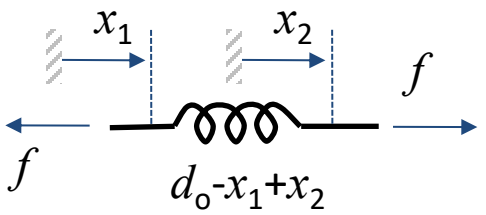
- Opruga
  - Pod dejstvom spoljašnje sile *f* opruga se isteže za Δx
    - d_o - istegnutost opruge bez dejstva sile



- Sila u opruzi: $f = f(\Delta x) \qquad \Delta x = x_2 - x_1$

- Za mala istezanja važi (linearizovano ponašanje) $f = k \cdot \Delta x$

  k - koeficijent elastičnosti [N/m]



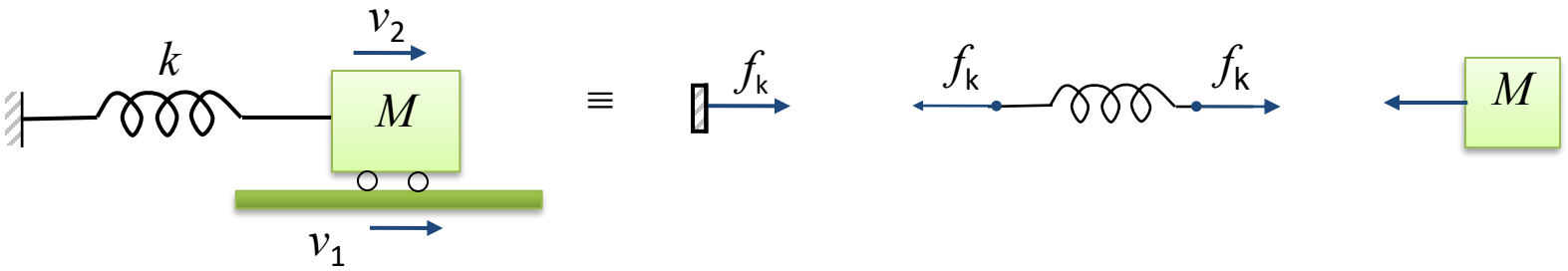
Zakonitosti kod uzajamnog dejstva elemenata

1. Dalamberov zakon (drugačuja formulacija II Njutnovog zakona)

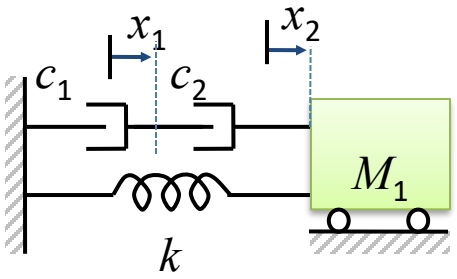
$$\sum_i (f_{ext})_i = M\frac{dv}{dt} \qquad \sum_i (f_{ext})_i - M\frac{dv}{dt} = 0 \qquad \sum_i f_i = 0$$

inercijalna sila
D`Alambert-ova sila

2. Zakon akcije i reakcije (III Njutnov zakon)



3. Zakon pomeraja: suma razlika pomeraja duž zatvorene putanje je 0

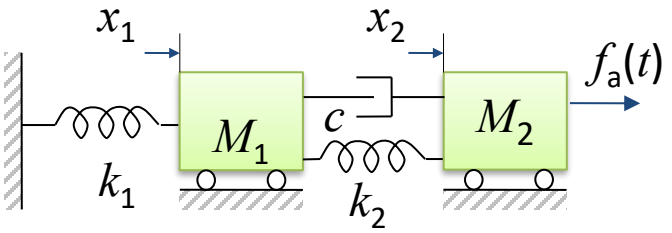


$$\sum_i (\Delta x)_i = 0$$

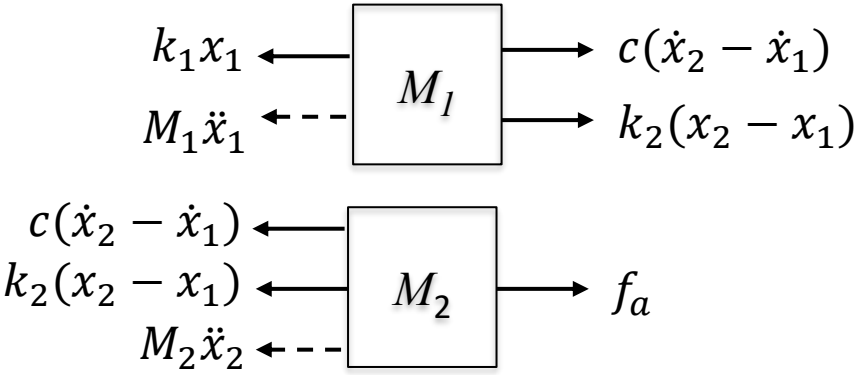
$$\underbrace{x_1}_{\substack{pomeraj \\ B1}} + \underbrace{(x_2 - x_1)}_{\substack{pomeraj \\ B2}} - \underbrace{x_2}_{\substack{pomeraj \\ k}} = 0$$

Dobijanje modela sistema – primer 1

- Kombinuju se zakonitosti elemenata i zakonitosti interakcije (međusobnih veza) elemenata



– Za svako telo posmatramo sile koje na njega deluju

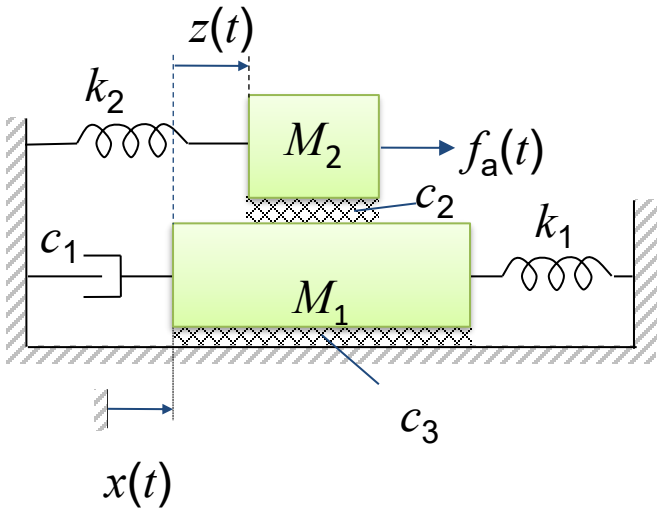


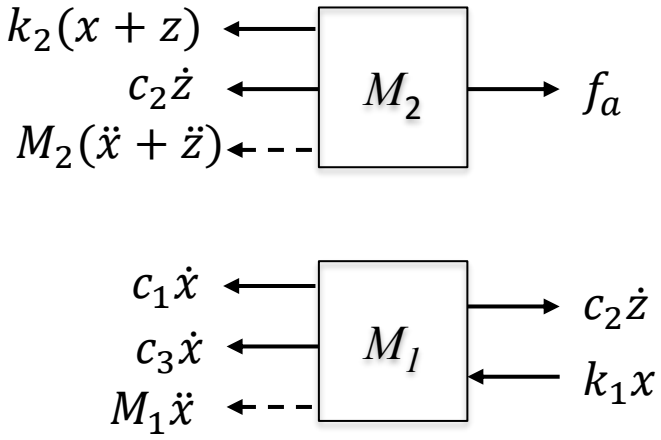
– Na osnovu Dalamberovog zakona pišemo jednačine

$$c(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) - M_1\ddot{x}_1 - k_1 x_1 = 0$$
$$f_a(t) - c(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) - M_2\ddot{x}_2 = 0$$

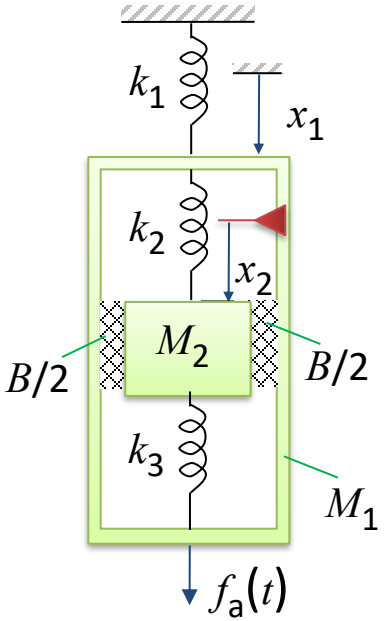
$$M_1\ddot{x}_1 + c\dot{x}_1 + (k_1 - k_2)x_1 - c\dot{x}_2 - k_2 x_2 = 0$$
$$-c\dot{x}_1 - k_2 x_2 + M_2\ddot{x}_2 + c\dot{x}_2 + k_2 x_2 = f_a(t)$$

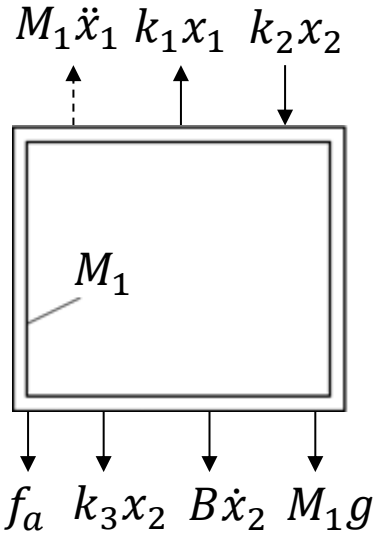
Primer 2

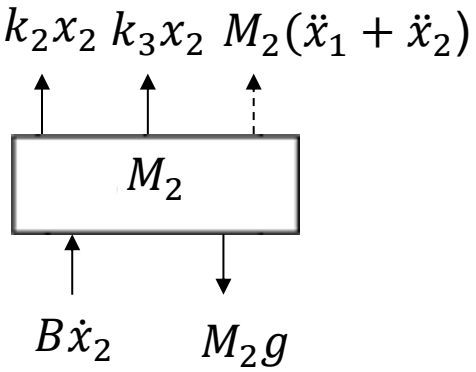

z(t)
k2
M2
fa(t)
c2
k1
c1
M1
c3
x(t)


k2(x + z)
c2ż
M2(ẍ + z̈)
M2
fa
c1ẋ
c3ẋ
M1ẍ
M1
c2ż
k1x

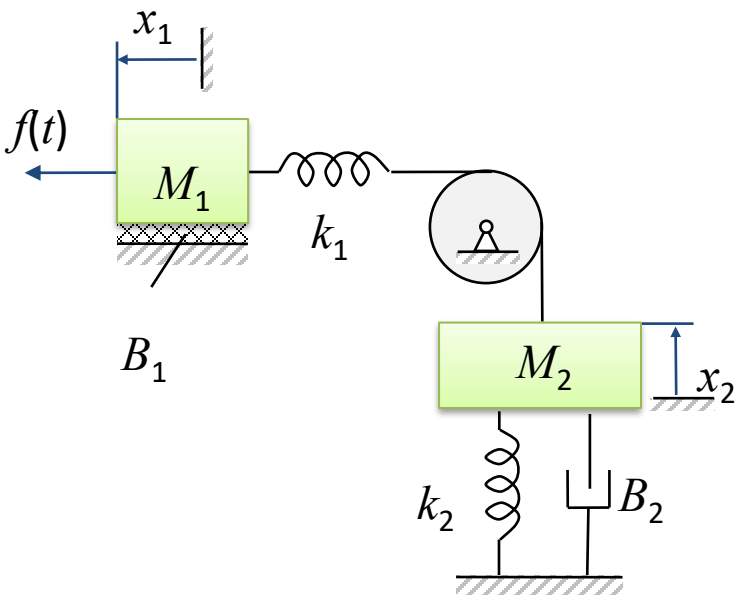
Primer 3

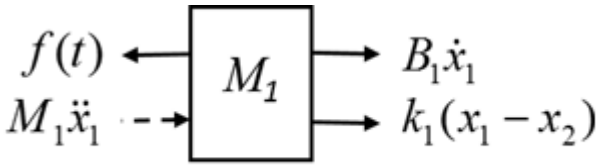

k_1
x_1
k_2
x_2
M_2
$B/2$
$B/2$
k_3
M_1
$f_a(t)$

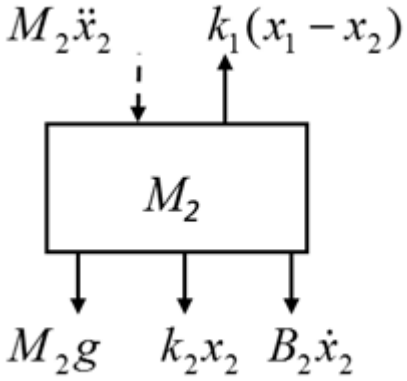

$M_1\ddot{x}_1$ k_1x_1 k_2x_2
M_1
f_a k_3x_2 $B\dot{x}_2$ M_1g


k_2x_2 k_3x_2 $M_2(\ddot{x}_1+\ddot{x}_2)$
M_2
$B\dot{x}_2$ M_2g

Primer 4


x_1
$f(t)$
M_1
k_1
B_1
M_2
x_2
k_2
B_2


$f(t)$
$M_1\ddot{x}_1$
M_1
$B_1\dot{x}_1$
$k_1(x_1 - x_2)$


$M_2\ddot{x}_2$
$k_1(x_1 - x_2)$
M_2
M_2g
k_2x_2
$B_2\dot{x}_2$