(RESEARCH ARTICLE)

Vehicle yaw rate simulation and control based on single-track model

Abdussalam Ali Ahmed [1, *], Faraj Ahmed Elzarook Barood [1] and Munir S. Khalifa [2]

[1] Mechanical and Industrial Engineering Department, Bani Waleed University, Bani Waleed/Libya
[2] Computer & IT Department, College of Electronic Technology, Bani Walid, Libya

Abstract

When designing a vehicle, the most important variable that should be taken into account is the vehicle yaw rate, it represents an important indication of the vehicle's stability and control. This paper aims to demonstrate how to simulate and control the yaw rate of a vehicle using two control methods, the first is the Linear Quadratic control method (LQR) and the other one is neural network control. The classical single-track model is prominently used for yaw stability control analysis. One driving conditions performed is the steering input; the steering input in this work is set as step steering angle and a lane change manoeuvre. Simulation results showed that both control methods used produced good and convergent performance results for the vehicle under different driving conditions.

Keywords: Vehicle Stability; Single-Track Model; Yaw Rate; LQR; Neural Network Control.

1. Introduction

As vehicles are becoming more and more performing throughout the years, it turns out that their design process requires better knowledge of their behavior. One of the ways to get to that knowledge is to use mathematical models, which describe the behavior of the vehicle when given relevant parameters. Knowing these parameters is essential to run the models, and get the expected results. This study will focus on a simple vehicle description, known as "single track model" also referred to as "bicycle model". This model contains several parameters influencing its behavior.

In vehicle dynamic studies, the classical single-track model as shown in Figure 1 is prominently used for yaw stability control analysis [1].
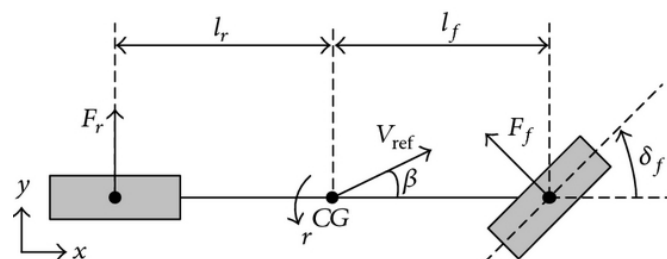

Figure 1 Single track model (bicycle model).

* Corresponding author: Abdussalam Ali Ahmed
Mechanical and Industrial Engineering Department, Bani Waleed University, Bani Waleed/Libya.

In the single-track model the Left and right wheels at the front and rear axle are placed in a single wheel at the centerline of the vehicle.

In order to design an effective control system, it is necessary to determine an appropriate element of yaw stability control system [2]. In this work, the elements of yaw stability control system, that is, vehicle dynamic models, control objectives, active   chassis control, and its control strategies as illustrated in Figure 2.

There are many automatic control methods that are used in the fields of life in general and in the automotive industry in particular [4].

There are several previous studies in which the simple track model of the vehicle was used in different control strategies [5], [6], [7]. In this paper, only two control techniques used which are LQR vehicle lateral stability control and neural network control.

In addition, to examine, and design the controller for yaw stability control system, vehicle dynamics models are essential where the mathematical modeling is obtained.

In general, there are two categories of vehicle dynamic model [2] that is nonlinear vehicle model and linearized vehicle model as painted in Figure 3.
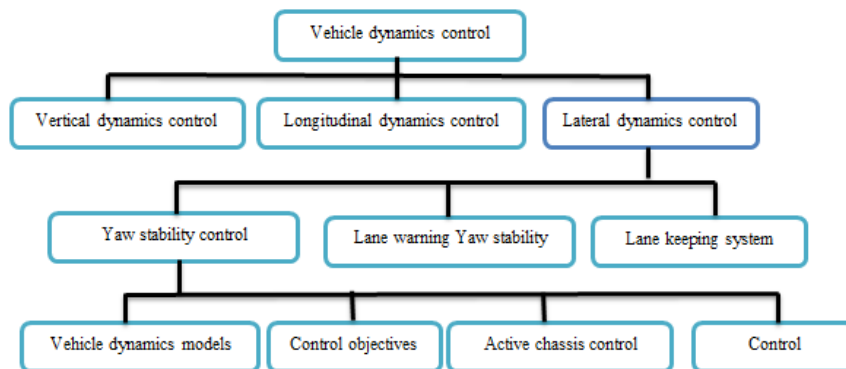

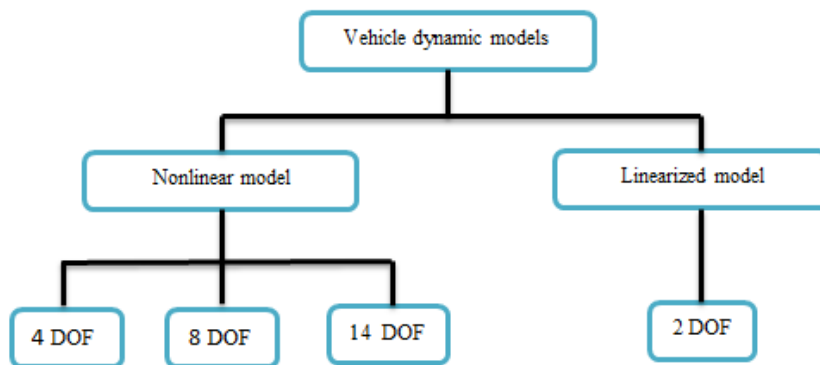Figure 2 **Yaw stability control system for vehicle lateral dynamic.**

Figure 3 Vehicle dynamic model.

2. Single Track Model

The classical single-track model is the simplest model that accurately models the lateral dynamics of a road vehicle. It is valid for non-extreme situations; almost constant longitudinal speed and small side slip angle values. The two tires on the same axle are lumped together, resulting in one front and one rear tire.
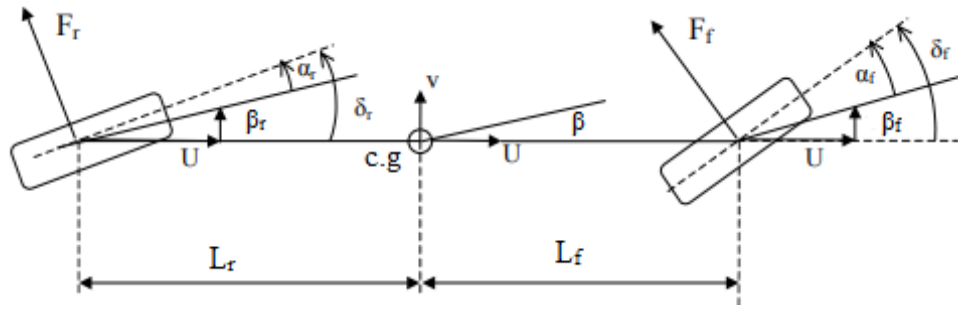
Figure 4 Free body diagram of the single-track model.

Table 1 Nomenclature used.

c.g.	**Center of gravity**	x_o, y_o	**Inertial frame of reference**
V	Vehicle velocity at c.g.	$x - y$	Chassis fixed frame of reference
β	Vehicle side slip angle	V_f	Velocity at center of front tire
l_f	Distance between front axle and c.g	V_r	Velocity at center of rear tire
l_r	Distance between rear axle and c.g	α_f	Front tire side slip angle
F_f	Front tire lateral force	α_r	Rear tire side slip angle
F_r	Rear tire lateral force	β_f	Front tire velocity angle with x axis
δ_f	Front tire steering angle	β_r	Rear tire velocity angle with x axis
δ_r	Rear tire steering angle	ψ	Yaw angle
r	yaw rate		

To complete this work, Matlab Simulink model of single-track model should be completed as shown in figure 5, this model includes Tire Model, Vehicle Dynamics, Steering Angle Projection, Kinematics/Geometry, Simple Driver Model, and Trajectory Calculations.
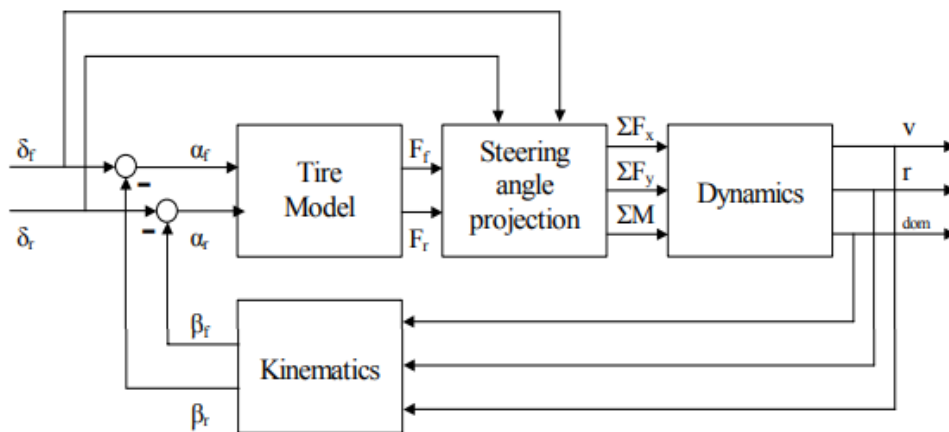

Figure 5 Block diagram of vehicle steering.

2.1. Non-linear track model

Nonlinear Model Equations:

2.1.1. Tire Model

Linear tire model is used in this case:

$$F_f = cfo \times (\delta_f - \beta_f) = cfo \times \alpha_f$$

$$F_r = cfo \times (\delta_r - \beta_r) = cfo \times \alpha_r$$

2.1.2. Vehicle Dynamics

$$mv(\dot{\beta} + \dot{\psi}) = -F_x sin\beta + F_y cos\beta$$

$$m\dot{v} = F_x cos\beta + F_y sin\beta$$

$$J\ddot{\psi} = M_z$$

2.1.3. Steering Angle Projection

$$\sum F_x = -F_f sin\delta_f - F_r sin\delta_r$$

$$\sum F_y = F_f cos\delta_f + F_r cos\delta_r$$

$$\sum M_z = l_f F_f cos\delta_f - l_r F_r cos\delta_r$$

2.1.4. Kinematics/Geometry

$$\beta_f = tan^{-1}\left(tan\,\beta + \frac{\dot{\psi}\,l_f}{vcos\beta}\right)$$

$$\beta_r = tan^{-1}\left(tan\,\beta - \frac{\dot{\psi}\,l_r}{vcos\beta}\right)$$

2.1.5. Simple Driver Model

A simple driver model was added to keep the longitudinal speed constant. The simplest PI cruise controller was used.

$$\sum F_x \rightarrow \sum F_x + G_{PI}(s)(v_{desired} - v_{measured})$$

2.1.6. Trajectory Calculations

Distance traveled in the x_0 and y_0 directions were calculated using following equations;

$$X = \int_0^t vcos(\beta + \psi)dt$$
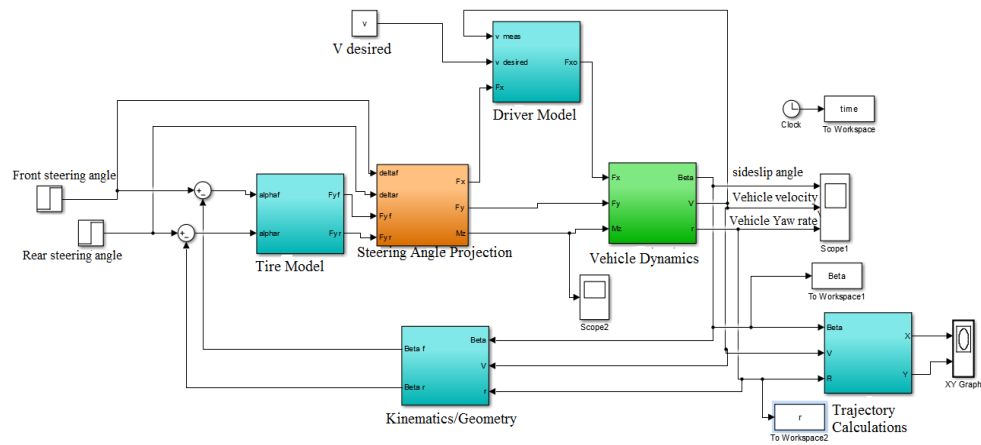
$$Y = \int_0^t vsin(\beta + \psi)dt$$

Figure 6 Simulink model of the nonlinear single-track model.

2.2. Linear track model

In vehicle dynamic studies, the reference vehicle model as shown in Figure 7 is prominently used for yaw stability control analysis and controller design.

The Linear Single Track Model contains the following subsystems:

Tire Model

Vehicle Dynamics

Force/Moment Coordinate Transformation (or Steering Angle Projection)

Kinematics/Geometry

Trajectory Calculations

2.2.1. Tire Model

Linear tire model was used.

$$F_f = cfo \times (\delta_f - \beta_f) = cfo \times \alpha_f$$

$$F_r = cfo \times (\delta_r - \beta_r) = cfo \times \alpha_r$$

2.2.2. Vehicle Dynamics

Original nonlinear vehicle dynamics was

$$mv(\dot{\beta} + \dot{\psi}) = -F_x sin\beta + F_y cos\beta$$

$$m\dot{v} = F_x cos\beta + F_y sin\beta$$

$$J\ddot{\psi} = M_z$$

The force moment coordinate transformation equations will become as follows with small beta and constant velocity assumptions:

$$mv(\dot{\beta} + \dot{\psi}) = F_y$$

$$J\ddot{\psi} = M_z$$

23

2.2.3. Force / Moment Coordinate Transformation

Original nonlinear steering angle projection was

$$\sum F_x = -F_f sin\delta_f - F_r sin\delta_r$$

$$\sum F_y = F_f cos\delta_f + F_r cos\delta_r$$

$$\sum M_z = l_f F_f cos\delta_f - l_r F_r cos\delta_r$$

The force moment coordinate transformation equations will become as follows with small steering angle assumptions:

$$\sum F_y = F_f + F_r$$

$$\sum M_z = l_f F_f - l_r F_r$$

2.2.4. Kinematics Geometry

Original kinematics/geometry relation was

$$\beta_f = tan^{-1}\left(tan\,\beta + \frac{\dot\psi\, l_f}{vcos\beta} \right)$$

$$\beta_r = tan^{-1}\left(tan\,\beta - \frac{\dot\psi\, l_r}{vcos\beta} \right)$$

The kinematics/geometry relation equations will become as follows with small chassis front and rear slip angles assumptions:

$$\beta_f = \left(\beta + \frac{\dot\psi\, l_f}{v} \right)$$

$$\beta_r = \left(\beta - \frac{\dot\psi\, l_r}{v} \right)$$

2.2.5. Trajectory Calculations

In order to investigate about the instantaneous position of the vehicle;

$$X = \int_0^t vcos(\beta + \psi)dt$$

$$Y = \int_0^t vsin(\beta + \psi)dt$$

Figures (5-7) show the Simulink model of the linear single-track model, vehicle yaw rate, and the vehicle body sideslip angle.
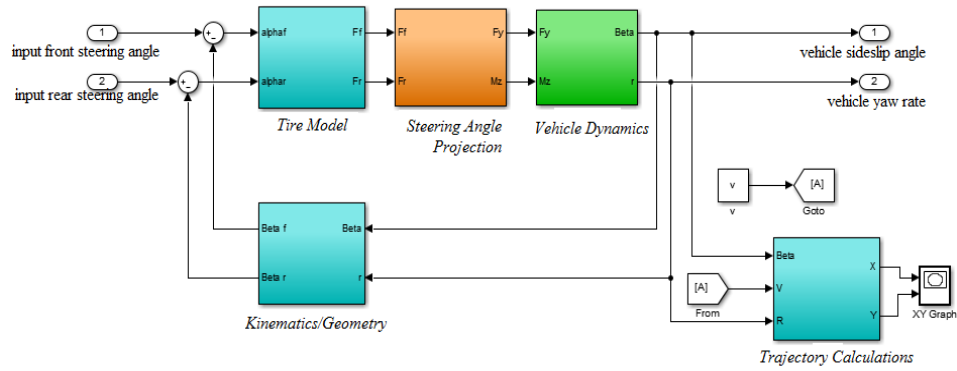
Figure 7 Simulink model of the linear single-track model

In this paper, linear single-track model yaw rate and has been taken as a desired yaw rate at different steering conditions as shown in figures 8 and 9, the steering input is set as step steering angle and a lane change manoeuvre.
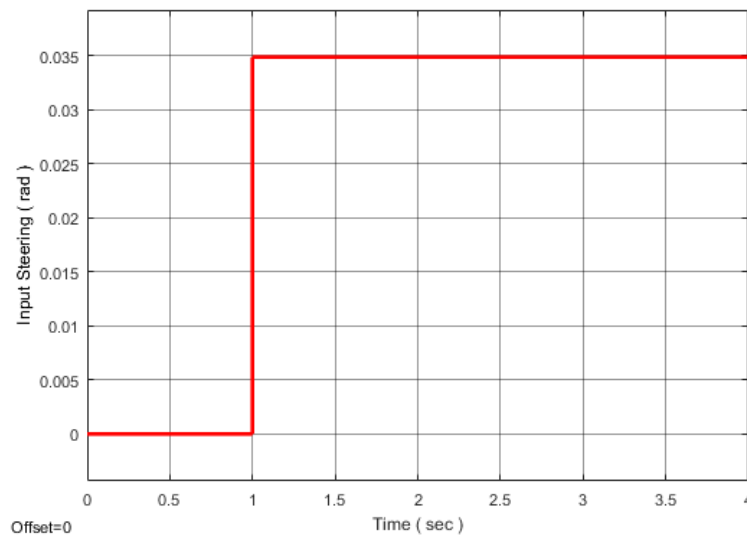

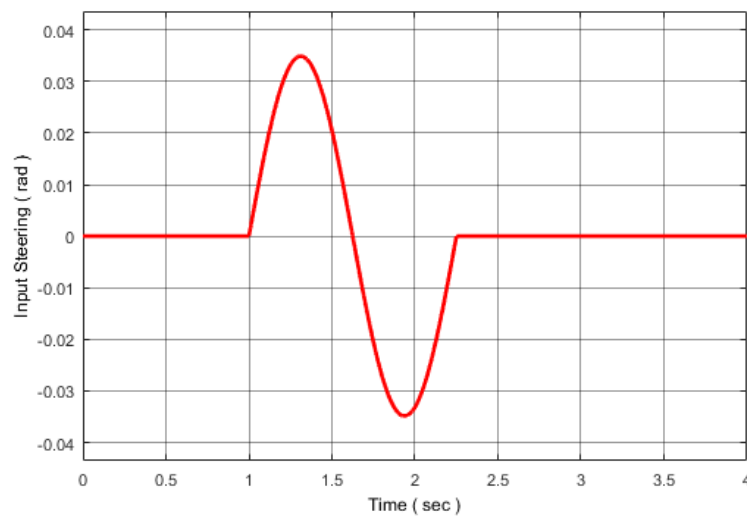**Figure 8** The steering input of vehicle with a step angle.

Figure 9 The steering input of vehicle with a lane-change maneuverer.

3. Control strategy

3.1. Neural Network Controller

Artificial neural networks are forecasting methods that are based on simple mathematical models of the brain. They allow complex nonlinear relationships between the response variable and its predictors. A neural network can be thought of as a network of "neurons" which are organized in layers. The predictors (or inputs) form the bottom layer, and the forecasts (or outputs) form the top layer. There may also be intermediate layers containing "hidden neurons".

In this part, the controller structure uses a neural network with two input signals, which are the plant output and the reference signal, whereas the output signal is delayed control as illustrated in the figure below.

This network works with supervised learning where a data set is provided to train the network system before simulation runs. Figure 10 shows the implemented suspension system includes a neural network controller; this controller block is available in the SIMULINK library.
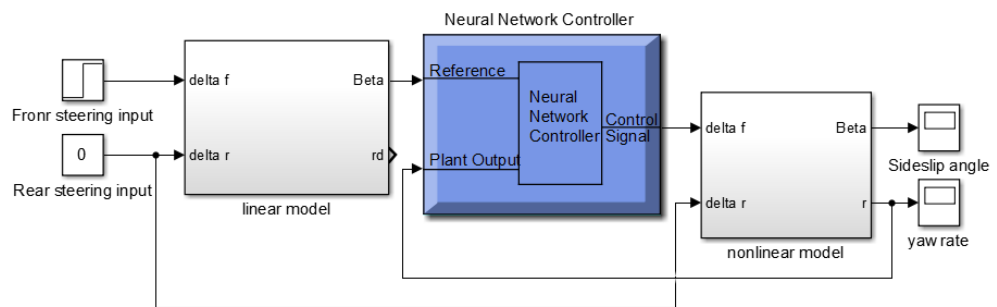

Figure 10 Simulink model for active suspension system with Neural Network Controller.

3.2. LQR Vehicle lateral stability control

The LQR approach to control the vehicle is widely used in the background of many studies in the field of the vehicle dynamics. It was used in a simple quarter car model, a half car model, as well as in a full vehicle model. The strength of the LQR method is that when using the performance factor index that can be weighted according to the engineers' desires and aspirations or other limitations. With this approach type, an optimal result can be achieved when factors of the performance index are taken into account.

The LQR method for the vehicle dynamics has been suggested and proposed to improve passenger comfort and to get better road handling.

In the case of designing an LQR controller (called the gain matrix), one must optimal control vector must choose $u(t)$, so that the quadratic cost function is reduced. The mathematical formula of the quadratic cost function can be written as:

$$J = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t)) dt$$

Where,

x and u are the state vector and the control vector respectively.

The designer must choose the appropriate values for both of R and Q to find the appropriate gain matrix using MATLAB software.

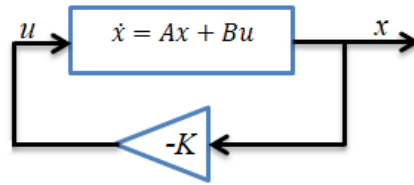The configuration of the state variable feedback is shown in the figure 11.

Figure 11 The configuration of the state variable feedback.

A suitable linear full-state feedback control law used as,

$u(⬚) = -Kx(⬚)$

Where K represents the state feedback gain matrix for the LQR controller, which can be defined by:

$K = R^{-1}B^T P$

The next Algebraic Riccati Equation (ARE) is using to estimate the matrix P.

$$A^T P + AP + PBR^{-1}B^T P + Q = 0$$

The Simulink model for the control system includes the LQR controller is shown below in Figure 12.
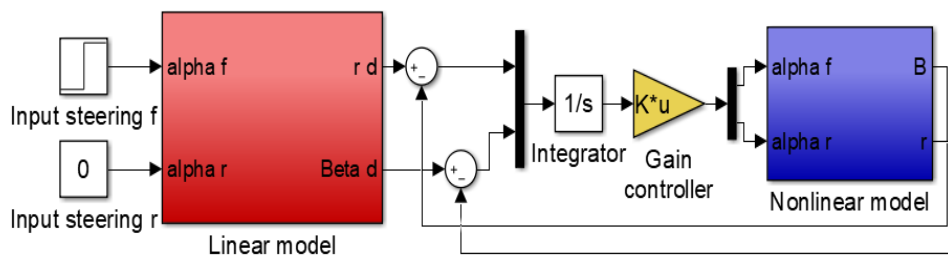

Figure 12 LQR control system of a vehicle.

3.3. Simulations

In order to evaluate the performance of the developed control system, a simulation investigation is performed. The performance and dynamic behaviors of the control system with two control methods using Matlab/Simulink. We assume that the vehicle travels at a constant speed v = 20 m/sec and the friction coefficient of the road is neglected. One driving condition is performed which is the step steering input, the steering input in the case single-track model set as a step signal with value of five degrees and set as lane change manoeuvre.

Modeling and simulations have been performed for vehicle lateral stability and different control techniques used in this work, these control techniques are Linear Quadratic Regulator (LQR) and neural network control.

Figures 13 and 14 show comparison of vehicle yaw rate performance using Linear Quadratic Regulator (LQR) and neural network control.
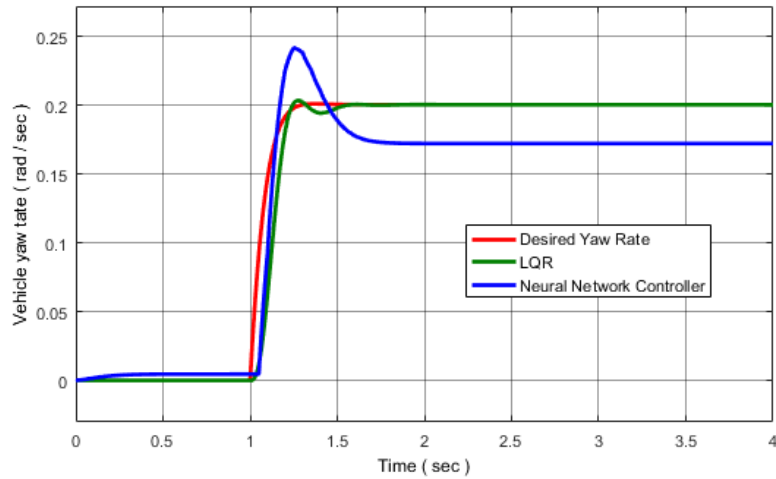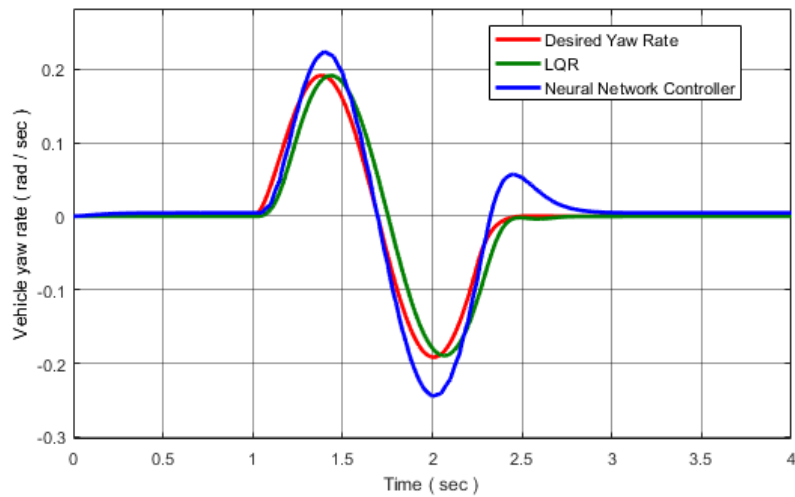
Figure 13 Vehicle yaw rate at step steering.

Figure 14 Vehicle yaw rate at lane change maneuver.

4. Conclusion

The controllers mentioned above are applied to the linear model mentioned at the beginning, we observe acceptable results that yaw rates are close to desired values. It is obvious that when the single-track model is tested, it can be seen that all of these methods have been effected the model to reach the yaw rate desired but the LQR controller is more effective than the neural network controller for yaw rate.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare no competing interests.

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