ImagesToLARModel, a tool for creation of three-dimensional models from a stack of images

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Abstract

Here we will present a software for creating a three-dimensional model from a stack of images. This can be useful because of the simplicity of these type of representations. In particular a scope of use can be offered by medicine, where there is an enormous number of images but with very complex two-dimensional representations.

This work will use the LAR representation ([CL13]) with the Julia language, because of its simplicity, showing how it can be used for quickly process image data.

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1 Introduction

This work has the aim to transform a two-dimensional representation of a model (based on a stack of images) into a three-dimensional representation based on the LAR schema. In particular, it will produce a single obj model which can be viewed with standard graphics softwares.

In the past were developed other softwares using same principles (see [PDFJ15]). However, they were optimized for speed and cannot be able to accept huge amounts of data. With the rise of the big data era, we now have more and more data available for research purposes, so softwares must be able to deal with them. A typical hardware environment is based on a cluster of computers where computation can be distributed among a lot of different processes. However, as stated by Amdahl's law, the speedup of a program using multiple processors is limited by the time needed for the sequential fraction of the program. So use of parallel techniques for dealing with big data is not important for time performance gain but for memory space gain. In fact, our biggest problem is lack of memory, due to model sizes. As a consequence, every parts of this software is written with the clear objective of minimizing memory usage at the cost of losing something in terms of time performance. So, for example, images will be converted in blocks determined by a grid size (see section 5) among different processes and different machines of the cluster



Figure 1: Amdahl's law

1.1 Why Julia

Ricordare che precedenti versioni erano in python Semplicita Efficienza Capacita di realizzare programmi paralleli con poco sforzo

2 Software structure

2.1 Julia packages

This software will be distributed as a Julia Package. For the actual release (Julia 0.4) a package is a simple git project with the structure showed in figure 2



Figure 2: Julia module structure

Source code must be in folder src, while in test folder there are module tests with a runtests.jl for executing them and with a REQUIRE file for specifying tests dependencies. For listing dependencies for the entire project, there is another REQUIRE file in main folder. As an example in figure 3 there is the REQUIRE file for ImagesToLARModel.jl.

After creating this structure for a project it can be pushed on a git repository and installed on Julia systems. The usual installation procedure use this syntax:

Pkg.add("Package-name")

This will check for that package in METADATA.jl repository on github where there are all official Julia package. However it is also possible to install an unofficial package (on a public git repository) using this sintax:

julia 0.3 JSON Logging PyCall Images Colors Clustering

Figure 3: REQUIRE contents for ImagesToLARModel.jl

Pkg.clone("git://repository-address.git")

This will install the package on your system with all the dependencies listed in RE-QUIRE file.

2.2 Architecture of ImagesToLARModel

In previous section we have seen how to create a Julia package for distribute our application. Now we focus on the structure of our application. In **src** folder we can find the following modules:

ImagesToLARModel.jl: main module for the software, it takes input parameters and start images conversion

ImagesConversion.jl: it is called by ImagesToLARModel.jl module and controls the entire conversion process calling all other modules

GenerateBorderMatrix.jl: it generates the boundary operator for grid specified in input, saving it in a JSON file

PngStack2Array3dJulia.jl: it is responsible of images loading and conversion into computable data

Lar2Julia.jl: it contains a small subset of LAR functions written in Julia language

LARUtils.jl: it contains utility functions for manipulation of LAR models

Smoother.jl: it contains function for smoothing of LAR models

Model2Obj.jl: it contains function that manipulates obj files

larcc.py: python larcc module for boundary computation. In next releases of the software it will be rewritten in Julia language



Figure 4: Schema of module dependencies of ImagesToLARModel

In figure 4 there is a simple schema of dependencies between modules.

Next sections of this document will explain in details all these modules showing also the code involved in conversion

3 Images To LAR Model

This is the main module for the application; it takes the input data and start conversion calling ImagesConversion.jl.

3.1 Calling modules

As we have already said, this first module has the responsibility of starting the conversion calling all other modules in the package. In Julia calling modules requires that they are in a path specified by LOAD_PATH array. So at the beginning of this module we need to add this line:

Pkg.dir() function gives us the path of the Julia installation, so Pkg.dir("ImagesToLARModel/src") returns " $\langle Julia - path \rangle / ImagesToLARModel/src$ "

After this line we can now import all modules defined here and export public functions:

```
⟨ modules import ImagesToLARModel 7 ⟩ ≡
   import JSON
   import ImagesConversion
   using Logging
   export convertImagesToLARModel
   ◊
```

Fragment referenced in 71.

Fragment referenced in 71.

3.2 Input loading

Images conversion takes several parameters:

- inputDirectory: The path of the directory containing the stack of images
- outputDirectory: The path of the directory containing the output
- bestImage: Image chosen for centroid computation (see section 4)
- nx, ny, nz: Sizes of the grid chosen for image segmentation (see section 4)
- DEBUG_LEVEL: Debug level for Julia logger
- parallelMerge (experimental): Choose between sequential or parallel merge of files (see section 10)

Because of their number it has been realized a function for simply loading them from a JSON configuration file; this is the code:

```
\langle load \ JSON \ configuration \ 8 \rangle \equiv
     function loadConfiguration(configurationFile)
       load parameters from JSON file
       configurationFile: Path of the configuration file
       configuration = JSON.parse(configurationFile)
       DEBUG_LEVELS = [DEBUG, INFO, WARNING, ERROR, CRITICAL]
       parallelMerge = false
       try
         if configuration["parallelMerge"] == "true"
           parallelMerge = true
           parallelMerge = false
         end
       catch
       end
       return configuration["inputDirectory"], configuration["outputDirectory"],
              configuration["bestImage"],
              configuration["nx"], configuration["ny"], configuration["nz"],
              DEBUG_LEVELS[configuration["DEBUG_LEVEL"]],
             parallelMerge
     end
Fragment referenced in 71.
A valid JSON file has the following structure:
  "inputDirectory": "Path of the input directory",
  "outputDirectory": "Path of the output directory",
  "bestImage": "Name of the best image (with extension)",
  "nx": x grid size,
  "ny": y grid size,
  "nz": border z,
  "DEBUG_LEVEL": julia Logging level (can be a number from 1 to 5)
```

```
"parallelMerge": "true" or "false"

For example, we can write:

{
    "inputDirectory": "/home/juser/IMAGES/"
    "outputDirectory": "/home/juser/OUTPUT/",
    "bestImage": "0009.tiff",
    "nx": 2,
    "ny": 2,
    "nz": 2,
    "DEBUG_LEVEL": 2
}
```

As we can see, in a valid JSON configuration file DEBUG_LEVEL can be a number from 1 to 5. Instead, when we explicitly define parameters, DEBUG_LEVEL can only be one of the following Julia constants:

- DEBUG
- INFO
- WARNING
- ERROR
- CRITICAL

3.3 Starting conversion

As we have already said, this module has the only responsibility to collect data input and starts other modules. These are the functions that start the process and the only exposed to the application users:

```
\langle \, Start \,\, conversion \,\, from \,\, JSON \,\, file \,\, 9 \,\rangle \equiv \\  \,\, function \,\, convertImagesToLARModel(configurationFile) \\  \,\, """ \\  \,\, Start \,\, conversion \,\, of \,\, a \,\, stack \,\, of \,\, images \,\, into \,\, a \,\, 3D \,\, model \\  \,\, loading \,\, parameters \,\, from \,\, a \,\, JSON \,\, configuration \,\, file \\  \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, Path \,\, of \,\, the \,\, configuration \,\, file \\ \,\, configurationFile: \,\, ConfigurationFile: \,\, Configuration \,\, file \\ \,\, configurationFile: \,\, Configuration
```

```
inputDirectory, outputDirectory, bestImage, nx, ny, nz,
           DEBUG_LEVEL, parallelMerge = loadConfiguration(open(configurationFile))
       convertImagesToLARModel(inputDirectory, outputDirectory, bestImage,
                              nx, ny, nz, DEBUG_LEVEL, parallelMerge)
     end
     \Diamond
Fragment referenced in 71.
\langle Start \ manual \ conversion \ 10 \rangle \equiv
     function convertImagesToLARModel(inputDirectory, outputDirectory, bestImage,
                                        nx, ny, nz, DEBUG_LEVEL = INFO, parallelMerge = false)
       Start conversion of a stack of images into a 3D model
       inputDirectory: Directory containing the stack of images
       outputDirectory: Directory containing the output
       bestImage: Image chosen for centroids computation
       nx, ny, nz: Border dimensions (Possibly the biggest power of two of images dimensions)
       DEBUG_LEVEL: Debug level for Julia logger. It can be one of the following:
         - DEBUG
         - INFO
         - WARNING
         - ERROR
         - CRITICAL
       # Create output directory
         mkpath(outputDirectory)
       catch
       end
       Logging.configure(level=DEBUG_LEVEL)
       ImagesConversion.images2LARModel(nx, ny, nz, bestImage,
                inputDirectory, outputDirectory, parallelMerge)
     end
     \Diamond
```

4 PngStack2Array3dJulia

This module has the responsibility of convert a png image into an array of values that will be passed to other modules

4.1 Module imports

These are modules needed for this part of the package and the public functions exported

```
⟨ modules import PngStack2Array3dJulia 11⟩ ≡
    using Images # For loading png images
    using Colors # For grayscale images
    using PyCall
    using Clustering
    using Logging
    @pyimport scipy.ndimage as ndimage

NOISE_SHAPE_DETECT=10

export calculateClusterCentroids, pngstack2array3d, getImageData, convertImages
    ⋄
```

Fragment referenced in 81.

We need Images and Colors packages for manipulating png images and PyCall for using Python functions for clustering and filtering images. As a consequence, we need a python environment with scipy to be able to run the package

4.2 Convert input to png

First thing to do in our program is getting our input folder and convert the stack of images into png format. This process lets us to avoid managing an enormous variety of formats during computation, simplifying code used for transformation.

Conversion needs the following parameters:

- inputPath: path of the folder containing the original images
- outputPath: path where we will save png images
- bestImage: name of the image chosen for centroids computing (see section 4.4)

After conversion *outputPath* will contain our png images and the function will return the new name chosen for the best image.

Now we can examine single parts of conversion process. First of all we need to specify a new name for images, keeping the right order between them; so we need to define a prefix based on number of images:

```
⟨ Define string prefix 12a⟩ ≡
   imageFiles = readdir(inputPath)
   numberOfImages = length(imageFiles)
   outputPrefix = ""
   for i in 1: length(string(numberOfImages)) - 1
      outputPrefix = string(outputPrefix, "0")
   end ◊
```

Fragment referenced in 14b.

Next we need to open the single image doing the following operations:

- 1. Open images using Images library (which relies on ImageMagick) and save them in greyscale png format
- 2. if one or both dimensions of the image are odd we need to remove one row (or column) of pixels to make it even. This will be more clear when we will introduce the grid for parallel computation (see section 5)

```
⟨ Greyscale conversion 12b⟩ ≡
    rgb_img = convert(Image{ColorTypes.RGB}, img)
    gray_img = convert(Image{ColorTypes.Gray}, rgb_img) ◊
```

Fragment referenced in 14b.

As we can see, we first need to convert image to RGB and then reconverting to greyscale. Without the RGB conversion these rows will return a stackoverflow error due to the presence of alpha channel

```
\langle Image\ resizing\ 12c \rangle \equiv
```

```
# resizing images if they do not have even dimensions
     dim = size(img)
     if(dim[1] \% 2 != 0)
       debug("Image has odd x; resizing")
       xrange = 1: dim[1] - 1
     else
       xrange = 1: dim[1]
     end
     if(dim[2] % 2 != 0)
       debug("Image has odd y; resizing")
       yrange = 1: dim[2] - 1
     else
       yrange = 1: dim[2]
     end
     img = subim(gray_img, xrange, yrange) 
Fragment referenced in 14b.
Next we just have to search for the best image and add one image if they are odd (for same
reasons we need even image dimensions)
\langle Search for best image 13a \rangle \equiv
     # Searching the best image
     if(imageFile == bestImage)
       newBestImage = string(outputPrefix[length(string(imageNumber)):end],
                                    imageNumber,".png")
     end
     imageNumber += 1 \diamond
Fragment referenced in 14b.
\langle Add \ one \ image \ 13b \rangle \equiv
     # Adding another image if they are odd
     if(numberOfImages % 2 != 0)
       debug("Odd images, adding one")
       imageWidth, imageHeight = getImageData(string(outputPath, "/", newBestImage))
```

if(imageWidth % 2 != 0)
 imageWidth -= 1

Fragment referenced in 14b.

Fianlly we have to reduce noise on the image. The better choice is using a *median filter* from package scipy.ndimage because it preserves better the edges of the image:

```
⟨ Reduce noise 14a⟩ ≡
    # Denoising
    imArray = raw(img)
    imArray = ndimage.median_filter(imArray, NOISE_SHAPE_DETECT) ◊
Fragment referenced in 14b.
```

Where imArray is an array containing all raw data from images Finally this is the code for the entire function:

```
(Convert to png 14b) =
  function convertImages(inputPath, outputPath, bestImage)
    """
  Get all images contained in inputPath directory
  saving them in outputPath directory in png format.
  If images have one of two odd dimensions, they will be resized
  and if folder contains an odd number of images another one will be
  added

inputPath: Directory containing input images
  outputPath: Temporary directory containing png images
  bestImage: Image chosen for centroids computation

Returns the new name for the best image
```

```
⟨ Define string prefix 12a⟩
  newBestImage = ""
  imageNumber = 0
  for imageFile in imageFiles
    img = imread(string(inputPath, imageFile))
    ⟨ Greyscale conversion 12b⟩
    \langle Image\ resizing\ 12c \rangle
    outputFilename = string(outputPath, outputPrefix[length(string(imageNumber)):end],
                                   imageNumber,".png")
    imwrite(img, outputFilename)
    ⟨ Search for best image 13a ⟩
    ⟨ Reduce noise 14a ⟩
    img = grayim(imArray)
    imwrite(img, outputFilename)
  end
  \langle Add \ one \ image \ 13b \rangle
  return newBestImage
end
```

Fragment referenced in 81.

11 11 11

4.3 Getting data from a png

Now we need to load information data from png images. In particular we are interested in getting width and height of an image. As stated in [W3C] document, a standard PNG file contains a *signature* followed by a sequence of *chunks* (each one with a specific type).

The signature always contain the following values:

```
137 80 78 71 13 10 26 10
```

This signature indicates that the remainder of the datastream contains a single PNG image, consisting of a series of chunks beginning with an IHDR chunk and ending with an IEND chunk. Every chunk is preceded by four bytes indicating its length.

As we are interested in width and height we need to parse the IHDR chunk. It is the first chunk in PNG datastream and its type field contains the decimal values:

```
73 72 68 82
```

The header also contains:

4 bytes
4 bytes
1 bytes
1 byte
1 byte
1 byte
1 byte

So for reading width and height we need first 24 bytes; the first eight contain the signature, then we have four bytes for length, four bytes for the type field and eight bytes for information we are interested in. This is the code:

```
{ Get image data 16 \) =
    function getImageData(imageFile)
    """
    Get width and height from a png image
    """
    input = open(imageFile, "r")
    data = readbytes(input, 24)

    if (convert(Array{Int},data[1:8]) != reshape([137 80 78 71 13 10 26 10],8))
        error("This is not a valid png image")
    end

    w = data[17:20]
    h = data[21:24]

    width = reinterpret(Int32, reverse(w))[1]
    height = reinterpret(Int32, reverse(h))[1]

    close(input)

    return width, height
    end

    **Tender**
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```

Fragment referenced in 81.

4.4 Centroids computation

As we have seen above, this package uses greyscale images for conversion into threedimensional models and for next steps we need binary images so we can distinguish between the background and the model we want to represent. We can use clustering techniques for obtaining this result. First step is centroids calculation from a chosen image (this choice must be made from the user, because we cannot knowing in advance what is the best image for finding clusters). Moreover we compute these centroids only for an image and then reuse them when we want to cluster all other images, saving processing time.

Actually we need only two centroids, because next steps should only recognize between background and foreground pixels. This is the code used for centroid computation:

```
\langle Centroid\ computation\ 17 \rangle \equiv
     function calculateClusterCentroids(path, image, numberOfClusters = 2)
       Loads an image and calculate cluster centroids for segmentation
       path: Path of the image folder
       image: name of the image
       numberOfClusters: number of desidered clusters
       imageFilename = string(path, image)
       img = imread(imageFilename) # Open png image with Julia Package
       imArray = raw(img)
       imageWidth = size(imArray)[1]
       imageHeight = size(imArray)[2]
       # Getting pixel values and saving them with another shape
       image3d = Array(Array{Uint8,2}, 0)
       # Inserting page on another list and reshaping
       push!(image3d, imArray)
       pixel = reshape(image3d[1], (imageWidth * imageHeight), 1)
       centroids = kmeans(convert(Array{Float64}, transpose(pixel)), 2).centers
       return convert(Array{Uint8}, trunc(centroids))
     end
     \Diamond
```

Fragment referenced in 81.

4.5 Transform pixels to three-dimensional array

Now we can study the most important part of this module, where images are converted into data usable by other modules for the creation of the three-dimensional model. The basic concept consists in transforming every single pixel in an integer value representing color, and then clustering them all using centroids computed earlier. So, we can obtain a matrix containing only two values (the two centroids) representing background and foreground of the image.

Now we will follow the code. This function uses four parameters

• path: Path of the images directory

• minSlice: First image to read

• maxSlice: Last image to read

• centroids: Array containing centroids for clustering

For every image we want to transform in the interval [minSlice, maxSlice) we have to read it from disk and save pixel informations into a multidimensional Array:

```
⟨ Read raw data 18a ⟩ ≡
  img = imread(imageFilename) # Open png image with Julia Package
  imArray = raw(img) # Putting pixel values into RAW 3d array ◊
Fragment referenced in 19.
```

The Images.jl raw function, get all pixel values saving them in an Array. In Figure 7 we can see how the array will be like for a sample greyscale image.

Finally we have to compute clusters obtaining images with only two values:

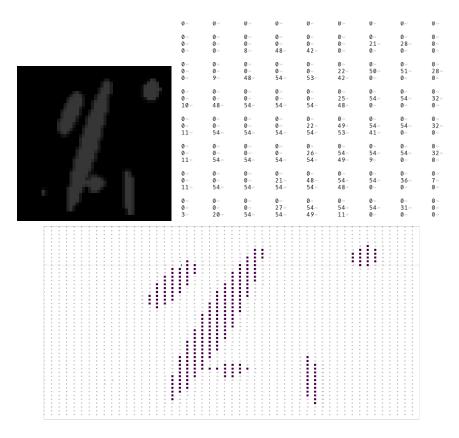


Figure 5: Reading raw data from image. (a) Original greyscale image (b) A view of raw data array (c) The entire raw data array with main color highlighted

This is the complete code:

 $\langle Pixel \ transformation \ 19 \rangle \equiv$



Figure 6: Image transformation. (a) Original greyscale image (b) Denoised image (c) Two-colors image

```
function pngstack2array3d(path, minSlice, maxSlice, centroids)
 Import a stack of PNG images into a 3d array
 path: path of images directory
 minSlice and maxSlice: number of first and last slice
 centroids: centroids for image segmentation
 # image3d contains all images values
 image3d = Array(Array{Uint8,2}, 0)
 debug("maxSlice = ", maxSlice, " minSlice = ", minSlice)
 files = readdir(path)
 for slice in minSlice : (maxSlice - 1)
   debug("slice = ", slice)
   imageFilename = string(path, files[slice + 1])
   debug("image name: ", imageFilename)
    ⟨ Read raw data 18a ⟩
   debug("imArray size: ", size(imArray))
   # Inserting page on another list and reshaping
   push!(image3d, imArray)
 end
 # Quantization
 for page in 1:length(image3d)
```

```
\langle \ Clustering \ images \ 18b \ \rangle end return image3d end \diamond
```

Fragment referenced in 81.

5 ImagesConversion

Now we will study the most important module for this package: ImagesConversion. It has the responsibility of doing the entire conversion process delegating tasks to the other modules.

5.1 General algorithm

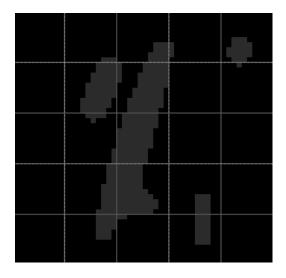
Now we will examine, in a general way, the algorithm used for conversion from a twodimensional to a three-dimensional representation of our biomedical models.

We have already seen in section 4 how to get information from a png image, obtaining arrays with only two values; one for the **background** color and one for **foreground** color. This is only the first step of the complete conversion process.

Now we focus only on a single image of the stack. Our two-dimensional representation, consists of pixels of two different colors (where only the one associated with foreground is significant); so we can obtain a three-dimensional representation simply replacing every foreground pixel with a small cube. Focusing on the entire stack of images, the full three-dimensional representation can be obtained simply overlapping all the image representations

This algorithm is very simple, however we does not considered problems concerning lack of memory. In fact, we could have images so big that we cannot build these models entirely in memory; moreover they would require a lot of CPU time for computation. A good solution to these problems consists in taking our representation based on images and divide according to a **grid**.

So, instead of converting the entire model with a unique process, we can subdivide the input among a lot of processes, where every process will execute the conversion process on



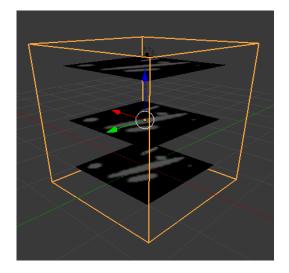


Figure 7: The grid used for parallel computation (a) 2D grid on a single image (b) 3D grid for the stack of images

a small number of **blocks** according to the grid subdivision.

Summing up we can define the following terms, which will be used in next parts of this documentation:

• **Grid:** It is the subdivision of the entire stack of images, with sizes defined by the user. They should be powers of two (for increasing performance during border matrix computation which we will see in section 6)

• Block: It is a single cell of the grid

• xBlock: It is the x-coordinate of a block

• yBlock: It is the y-coordinate of a block

• **zBlock:** It is the z-coordinate of a block

xBlock and yBlock are defined on a single image, while zBlock is defined on different images; in the code it will often be replaced by terms **StartImage** and **EndImage**, which indicate the first image and the last image of that block respectively.

In next subsections we will examine the conversion algorithm in detail, showing what happens for every block of the grid.

5.2 Module imports

These are modules needed for this part of the package and the public functions exported.

```
⟨ modules import ImagesConversion 23a⟩ ≡
  import GenerateBorderMatrix
  import PngStack2Array3dJulia
  import Lar2Julia
  import Model2Obj
  import LARUtils
  import Smoother

using Logging
  export images2LARModel
  ◊
```

Fragment referenced in 72a.

5.3 Data preparation

As a first thing, we will see how to prepare our data for conversion process. Firstly we need to convert input images to greyscale png; so we need to create a temporary directory for saving them.

Later, we need to compute the LAR boundary operator for finding boundaries of our cells (for the generation see section 6) getting width and height from our images.

Finally we can start conversion with all these parameters calling startImageConversion function, which will be explained in next subsection.

```
mkdir(string(outputDirectory, "TEMP"))
  catch
  end
 tempDirectory = string(outputDirectory, "TEMP/")
 newBestImage = PngStack2Array3dJulia.convertImages(inputDirectory, tempDirectory,
                                                        bestImage)
  imageWidth, imageHeight = PngStack2Array3dJulia.getImageData(
                                      string(tempDirectory,newBestImage))
  imageDepth = length(readdir(tempDirectory))
  # Computing border matrix
  info("Computing border matrix")
   mkdir(string(outputDirectory, "BORDERS"))
  catch
  end
 borderFilename = GenerateBorderMatrix.getOriented3BorderPath(
                                        string(outputDirectory, "BORDERS"), nx, ny, nz)
 # Starting images conversion and border computation
 info("Starting images conversion")
  startImageConversion(tempDirectory, newBestImage, outputDirectory, borderFilename,
                       imageHeight, imageWidth, imageDepth,
                       nx, ny, nz,
                       numberOfClusters, parallelMerge)
end
```

Fragment referenced in 72a.

5.4 Conversion pipeline

Now we can see how conversion of images works. In section 5.1 we have seen how to execute the single conversion of a pixel into a voxel using our grid for parallel computation. However, with that algorithm, we obtain models with internal boundaries between blocks and with squared edge. So we need to create a **conversion pipeline** which will progressively refine our models. In Figure 8 there are the steps used for our conversion

Every single step of the pipeline, is executed in parallel for every block of the grid; so we need a general purpose function for blocks iteration which will take as a parameter a



Figure 8: Images conversion pipeline

function that will execute it. So we can define the iterateOnBlocks function which takes the following parameters:

- inputDirectory: Directory which contains input files for the process function
- imageHeight, imageWidth, imageDepth: Sizes of the stack of images
- imageDx, imageDy, imageDz: Sizes of the grid
- **processFunction**: Function that contains instructions for execution of a single step of the pipeline for a single block
- outputDirectory: Directory which will contains the output
- centroids: Centroids from the best image
- boundaryMat: Boundary operator for the chosen grid

This function will iterate on all blocks of the image grid executing the process function, which will be different for every pipeline step. This is the code used:

```
\langle parallel block iteration 25 \rangle \equiv
     function iterateOnBlocks(inputDirectory,
                               imageHeight, imageWidth, imageDepth,
                               imageDx, imageDz,
                               processFunction, outputDirectory,
                               centroidsCalc, boundaryMat)
       11 11 11
       Simple function that iterates on blocks for executing
       a task described by a processFunction
       inputDirectory: Directory which contains input files for the process function
       imageHeight, imageWidth, imageDepth: Images sizes
       imageDx, imageDy, imageDz: Sizes of cells grid
       processFunction: Function that will be executed on a separate task on
       the entire z-Block
       outputDirectory: Directory which will contains the output
       centroidsCalc: Centroids from the best image
       boundaryMat: Boundary operator for the chosen grid
```

```
beginImageStack = 0
  endImage = beginImageStack
 tasks = Array(RemoteRef, 0)
 for zBlock in 0:(imageDepth / imageDz - 1)
   startImage = endImage
   endImage = startImage + imageDz
   task = @spawn processFunction(inputDirectory,
                                   startImage, endImage,
                                   imageDx, imageDy,
                                   imageWidth, imageHeight,
                                   outputDirectory,
                                   centroidsCalc, boundaryMat)
   push!(tasks, task)
  end
 # Waiting for tasks
  for task in tasks
   wait(task)
  end
end <
```

First of all we need to iterate on the grid finding the zBlock coordinate; we saw earlier that the imageDz parameter must be a divisor of the image depth, so we will have exactly imageDepth/imageDz blocks on the z coordinate. Moreover, at every zBlock correspond a startImage and an endImage where endImage - startImage = imageDz.

Now we can simply parallelize the conversion process spawning a new process for every zBlock, so we open at most imageDz images for process. Finally, we have to wait for tasks completion.

Now we can see the entire pipeline for images conversion.

First of all we need to compute the centroids from the best image using module PngStack2Array3dJulia and get the previously computed border matrix in csc sparse array format

```
\langle \ compute \ centroids \ and \ get \ border \ matrix \ 26 \ \rangle \equiv # Create clusters for image segmentation info("Computing image centroids")
```

Fragment referenced in 72a.

Fragment referenced in 28a.

Fragment referenced in 28a.

Now we can start the pipeline:

```
⟨ pipeline conversion 27 ⟩ ≡

# Starting pipeline conversion
info("Starting images conversion")
⟨ pixels to voxels conversion step 33a ⟩

info("Merging boundaries")
⟨ boundaries merge step 36a ⟩

info("Merging blocks")
⟨ block merge step 38a ⟩

info("Smoothing models")
⟨ smoothing step 42a ⟩

info("Merging obj models")
⟨ final file merge 42b ⟩
end ⋄
```

As we can see, last pipeline step does not require iteration on all grid blocks. This is the code for the function that starts the pipeline, with the parts explained earlier:

Fragment referenced in 72a.

5.4.1 Images conversion step

Now we will focus on the first step of our pipeline conversion: *images conversion*. First thing to do is read an image calling the PngStack2Array3dJulia, after that is necessary to sort the centroid array for choosing correct background and foreground pixels.

Fragment referenced in 32.

Now we can start iterating on other blocks of the grid getting the corresponding slice of the image:

Fragment referenced in 32.

Here xStart and yStart are the absolute coordinates of the model and are calculated from the block coordinates. At the end of this process, we have an array called image with size (imageDz, imageDx, imageDy).

Now we can get the value of the single pixel into this array and, if it represents a foreground point, put it into an array called chain3D. This structure contains indexes of the linearized array created from the matrix. In Figure 9 there is a sample conversion from the matrix to the array

$$\begin{pmatrix} 0^0 & 0^2 \\ 0^1 & 0^3 \end{pmatrix} \begin{pmatrix} 46^4 & 0^6 \\ 46^5 & 46^7 \end{pmatrix} \rightarrow 0^0 \quad 0^1 \quad 0^2 \quad 0^3 \quad 46^4 \quad 46^5 \quad 0^6 \quad 46^7$$

$$\begin{pmatrix} 0^0 & 0^2 \\ 0^1 & 0^3 \end{pmatrix} \begin{pmatrix} 0^4 & 46^6 \\ 46^5 & 46^7 \end{pmatrix} \rightarrow 0^0 \quad 0^1 \quad 0^2 \quad 0^3 \quad 0^4 \quad 46^5 \quad 46^6 \quad 46^7$$

Figure 9: Transformation of a matrix resulting from a 2x2x2 grid into a linearized array (with cells indexes) (a) First example (b) Second example

As we can see from that figure, from a 2x2x2 grid we can obtain eight values for the single block (or **cell**), where the indexes for the foreground pixels represent indexes of non-empty cells in a 2x2x2 cuboidal geometry

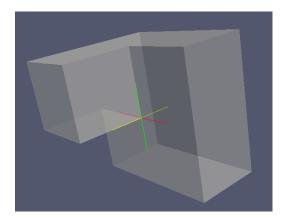
This is the code for getting foreground pixels:

```
\langle get foreground pixels 29b \rangle \infty
nz, nx, ny = size(image)
chains3D = Array(Uint8, 0)
zStart = startImage
```

```
for y in 0:(nx - 1)
  for x in 0:(ny - 1)
    for z in 0:(nz - 1)
    if(image[z + 1, x + 1, y + 1] == foreground)
       push!(chains3D, y + ny * (x + nx * z))
    end
  end
end
end
```

Fragment referenced in 32.

Now that we have full cells for the geometry, we can convert them into a LAR model. In particular, we are interested in cell boundaries for every block (as we want to obtain only the boundaries for the final model) so we can call function larBoundaryChain from Lar2Julia module (which will be explained in section 7). In Figure 10 there are some examples of models extracted from a single $2 \times 2 \times 2$ block.



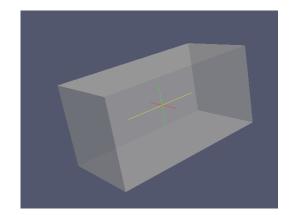


Figure 10: Sample models of 2x2x2 blocks

After model computation, next step is getting vertices and faces from model cells writing results to file. However, as we have already said, we are only interested in boundaries of the final model while now we have only boundaries of a single block. Consequently, we have to separate boundaries from the inner faces of the block on different files (boundaries separation will be explained in section 8). As we can see later, we will merge boundaries together deleting common faces on both block borders, obtaining a model without internal faces. These are pieces of code for getting the inner block model with the boundaries and for file writing:

```
# IMPORTANT: inverting xStart and yStart for obtaining correct rotation of the model
     models = LARUtils.computeModelAndBoundaries(nx, ny, nz,
                                                  yStart, xStart, zStart, objectBoundaryChain)
     V, FV = models[1][1] # inside model
     V_left, FV_left = models[2][1]
     V_right, FV_right = models[3][1] # right boundary
     V_top, FV_top = models[4][1] # top boundary
     V_bottom, FV_bottom = models[5][1] # bottom boundary
     V_front, FV_front = models[6][1] # front boundary
     V_back, FV_back = models[7][1] # back boundary
Fragment referenced in 32.
\langle write \ block \ models \ to \ file \ 31b \rangle \equiv
     # Writing all models on disk
     model_outputFilename = string(outputDirectory, "MODELS/model_output_", xBlock,
                                      "-", yBlock, "_", startImage, "_", endImage)
     Model2Obj.writeToObj(V, FV, model_outputFilename)
     left_outputFilename = string(outputDirectory, "MODELS/left_output_", xBlock,
                                      "-", yBlock, "_", startImage, "_", endImage)
     Model20bj.writeToObj(V_left, FV_left, left_outputFilename)
     right_outputFilename = string(outputDirectory, "MODELS/right_output_", xBlock,
                                      "-", yBlock, "_", startImage, "_", endImage)
     Model2Obj.writeToObj(V_right, FV_right, right_outputFilename)
     top_outputFilename = string(outputDirectory, "MODELS/top_output_", xBlock,
                                      "-", yBlock, "_", startImage, "_", endImage)
     Model20bj.writeToObj(V_top, FV_top, top_outputFilename)
     bottom_outputFilename = string(outputDirectory, "MODELS/bottom_output_", xBlock,
                                      "-", yBlock, "_", startImage, "_", endImage)
     Model2Obj.writeToObj(V_bottom, FV_bottom, bottom_outputFilename)
     front_outputFilename = string(outputDirectory, "MODELS/front_output_", xBlock,
                                      "-", yBlock, "_", startImage, "_", endImage)
     Model2Obj.writeToObj(V_front, FV_front, front_outputFilename)
     back_outputFilename = string(outputDirectory, "MODELS/back_output_", xBlock,
```

```
"-", yBlock, "_", startImage, "_", endImage)
     Model2Obj.writeToObj(V_back, FV_back, back_outputFilename)
     \Diamond
Fragment referenced in 32.
This is the processFunction for this pipeline step
\langle image\ conversion\ process\ 32 \rangle \equiv
     function imageConversionProcess(sliceDirectory,
                                      startImage, endImage,
                                      imageDx, imageDy,
                                      imageWidth, imageHeight,
                                      outputDirectory,
                                      centroids, boundaryMat)
       11 11 11
       Support function for converting a stack of image on a single
       independent process
       11 11 11
       ⟨ image read and centroids sort 28b⟩
       for xBlock in 0:(imageHeight / imageDx - 1)
         for yBlock in 0:(imageWidth / imageDy - 1)
           yStart = xBlock * imageDx
           xStart = yBlock * imageDy
           xEnd = xStart + imageDx
           yEnd = yStart + imageDy
           debug("********")
           debug(string("xStart = ", xStart, " xEnd = ", xEnd))
           debug(string("yStart = ", yStart, " yEnd = ", yEnd))
            debug("theImage dimensions: ", size(theImage)[1], " ",
                             size(theImage[1])[1], " ", size(theImage[1])[2])
            ⟨ get image slice 29a ⟩
            ⟨ get foreground pixels 29b⟩
            if(length(chains3D) != 0)
              # Computing boundary chain
              debug("chains3d = ", chains3D)
              debug("Computing boundary chain")
              objectBoundaryChain = Lar2Julia.larBoundaryChain(boundaryMat, chains3D)
```

```
debug("Converting models into obj")
    try
        mkdir(string(outputDirectory, "MODELS"))
    catch
    end
        ⟨ get inner model and boundaries 31a⟩

        ⟨ write block models to file 31b⟩
    else
        debug("Model is empty")
    end
    end
    end
    end
end
end
```

Tragment referenced in 12a.

This is the code for starting this pipeline step:

Fragment referenced in 27.

5.4.2 Boundaries merge step

Next step of our pipeline consists in *boundaries merge*. In fact, we have already seen that for every non-empty cell we create files for the inner parts and for the boundaries of the block. So if we want a final model without boundaries between internal blocks, we need to merge them removing duplicated faces on both sides (see Section 8.6 for a better explanation of this step). The following is the processFunction:

```
outputDirectory = None,
                                  centroidsCalc = None, boundaryMat = None)
  Helper function for mergeBoundaries.
  It is executed on different processes
  modelDirectory: Directory containing model files
  startImage: Block start image
  endImage: Block end image
  imageDx, imageDy: x and y sizes of the grid
  imageWidth, imageHeight: Width and Height of the image
  for xBlock in 0:(imageHeight / imageDx - 1)
    for yBlock in 0:(imageWidth / imageDy - 1)
      # Merging right Boundary
      firstPath = string(modelDirectory, "/right_output_", xBlock, "-", yBlock,
                        "_", startImage, "_", endImage)
      secondPath = string(modelDirectory, "/left_output_", xBlock, "-", yBlock + 1,
                        "_", startImage, "_", endImage)
      mergeBoundariesAndRemoveDuplicates(firstPath, secondPath)
      # Merging top boundary
      firstPath = string(modelDirectory, "/top_output_", xBlock, "-", yBlock,
                         "_", startImage, "_", endImage)
      secondPath = string(modelDirectory, "/bottom_output_", xBlock, "-", yBlock,
                         "_", endImage, "_", endImage + (endImage - startImage))
      {\tt mergeBoundariesAndRemoveDuplicates(firstPath, secondPath)}
      # Merging front boundary
      firstPath = string(modelDirectory, "/front_output_", xBlock, "-", yBlock,
                        "_", startImage, "_", endImage)
      secondPath = string(modelDirectory, "/back_output_", xBlock + 1, "-", yBlock,
                        "_", startImage, "_", endImage)
      mergeBoundariesAndRemoveDuplicates(firstPath, secondPath)
    end
  end
end <
```

Fragment referenced in 72a.

For every block we do the following merges:

• right boundary with the left boundary of the next block on the right

- top boundary with the bottom boundary of the next block on the top
- front boundary with the back boundary of the next block on the front

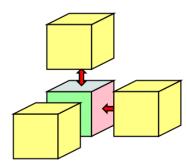


Figure 11: Merging of boundary faces. For a single block we need adjacent blocks on the right, top and front

all merges are executed by the function mergeBoundariesAndRemoveDuplicates which does the work calling the Model2Obj and LARUtils libraries for loading and cleaning of the boundaries models.

```
\langle merge \ boundaries \ utility \ function \ 35 \rangle \equiv
     function mergeBoundariesAndRemoveDuplicates(firstPath, secondPath)
       Merge two boundary files removing common faces between
       them
       firstPath, secondPath: Prefix of paths to merge
       firstPathV = string(firstPath, "_vtx.stl")
       firstPathFV = string(firstPath, "_faces.stl")
       secondPathV = string(secondPath, "_vtx.stl")
       secondPathFV = string(secondPath, "_faces.stl")
       if(isfile(firstPathV) && isfile(secondPathV))
         V, FV = Model2Obj.getModelsFromFiles([firstPathV, secondPathV],
                                                 [firstPathFV, secondPathFV])
         # Writing model to file
         rm(firstPathV)
         rm(firstPathFV)
         rm(secondPathV)
```

```
rm(secondPathFV)
    Model2Obj.writeToObj(V, FV, firstPath)
    end
end
```

Fragment referenced in 72a.

This is the code used to start this pipeline step:

Fragment referenced in 27.

5.4.3 Block merge step

At this step of the computation, we have files with the inner parts of a single block model and the remaining boundaries. Now we need to merge the blocks removing double vertices and faces, so we can save space and prepare our model to the *smoothing step*. This is the code of the processFunction:

```
for xBlock in 0:(imageHeight / imageDx - 1)
         for yBlock in 0:(imageWidth / imageDy - 1)
           blockCoordsV = string(xBlock, "-", yBlock, "_", startImage,
                                 "_", endImage, "_vtx.stl")
           blockCoordsFV = string(xBlock, "-", yBlock, "_", startImage,
                                 "_", endImage, "_faces.stl")
           arrayV = [string(modelDirectory, "/left_output_", blockCoordsV),
                     string(modelDirectory, "/right_output_", blockCoordsV),
                     string(modelDirectory, "/top_output_", blockCoordsV),
                     string(modelDirectory, "/bottom_output_", blockCoordsV),
                     string(modelDirectory, "/front_output_", blockCoordsV),
                     string(modelDirectory, "/back_output_", blockCoordsV),
                     string(modelDirectory, "/model_output_", blockCoordsV)]
           arrayFV = [string(modelDirectory, "/left_output_", blockCoordsFV),
                      string(modelDirectory, "/right_output_", blockCoordsFV),
                      string(modelDirectory, "/top_output_", blockCoordsFV),
                      string(modelDirectory, "/bottom_output_", blockCoordsFV),
                      string(modelDirectory, "/front_output_", blockCoordsFV),
                      string(modelDirectory, "/back_output_", blockCoordsFV),
                      string(modelDirectory, "/model_output_", blockCoordsFV)]
           V, FV = Model2Obj.getModelsFromFiles(arrayV, arrayFV)
           for i in 1:length(arrayV)
             if(isfile(arrayV[i]))
               rm(arrayV[i])
               rm(arrayFV[i])
             end
           end
           Model2Obj.writeToObj(V, FV, string(modelDirectory, "/model_output_",
                                    xBlock, "-", yBlock, "_", startImage, "_", endImage))
         end
       end
     end <
Fragment referenced in 72a.
```

For a better explanation of the LARUtils function that remove duplicated vertices, you can see Section 8.4

This is the code for block merge starting

Fragment referenced in 27.

5.4.4 Smoothing step

Now we have obtained models without internal boundaries between blocks and without double vertices and faces in a single block. However this partial model has squared edges, so we need to smooth them. The processFunction for this step, is the following:

```
\langle Smooth \ block \ process \ function \ 38b \rangle \equiv
     function smoothBlocksProcess(modelDirectory,
                                    startImage, endImage,
                                    imageDx, imageDy,
                                    imageWidth, imageHeight,
                                    outputDirectory = None,
                                    centroidsCalc = None, boundaryMat = None)
       Smoothes a block in a single process
       modelDirectory: Path of the directory containing all blocks
                        that will be smoothed
       startImage, endImage: start and end image for this block
       imageDx, imageDy: sizes of the grid
       imageWidth, imageHeight: sizes of the images
       for xBlock in 0:(imageHeight / imageDx - 1)
         for yBlock in 0:(imageWidth / imageDy - 1)
           # Loading the current block model
           blockFileV = string(modelDirectory, "/model_output_", xBlock, "-", yBlock,
                                "_", startImage, "_", endImage, "_vtx.stl")
           blockFileFV = string(modelDirectory, "/model_output_", xBlock, "-", yBlock,
                                 "_", startImage, "_", endImage, "_faces.stl")
           if isfile(blockFileV)
             # Loading only model of the current block
```

```
blockModelV, blockModelFV = Model2Obj.getModelsFromFiles([blockFileV], [blockFileFV])
             # Loading a unique model from this block and its adjacents
             modelsFiles = Array(String, 0)
             for x in xBlock - 1:xBlock + 1
               for y in yBlock - 1:yBlock + 1
                 for z in range(startImage - (endImage - startImage),(endImage - startImage), 3)
                   push!(modelsFiles, string(modelDirectory, "/model_output_",
                                             x, "-", y, "_", z, "_", z + (endImage - startImage)))
                 end
               end
             end
             modelsFilesV = map((s) -> string(s, "_vtx.stl"), modelsFiles)
             modelsFilesFV = map((s) -> string(s, "_faces.stl"), modelsFiles)
             modelV, modelFV = Model2Obj.getModelsFromFiles(modelsFilesV, modelsFilesFV)
             # Now I have to save indices of vertices of the current block model
             blockVerticesIndices = Array(Int, 0)
             for i in 1:length(blockModelV)
               for j in 1:length(modelV)
                 if blockModelV[i] == modelV[j]
                   push!(blockVerticesIndices, j)
                 end
               end
               # Now I can apply smoothing on this model
               V_sm, FV_sm = Smoother.smoothModel(modelV, modelFV)
               # Now I have to get only block vertices and save them on the new model
               V_final = Array(Array{Float64}, 0)
               for i in blockVerticesIndices
                 push!(V_final, V_sm[i])
               end
               outputFilename = string(modelDirectory, "/smoothed_output_", xBlock, "-",
                                       yBlock, "_", startImage, "_", endImage)
               Model20bj.writeToObj(V_final, blockModelFV, outputFilename)
             end
           end
         end
       end
     end <
Fragment referenced in 72a.
```

An explanation of the smoothing algorithm used there, can be found in Section 9.2. What we need to remember here, is the importance of having the adjacent vertices for every vertex of our block. In fact, according to the chosen smoothing algorithm, every vertex is replaced with a new one with coordinates computed from the mean positions of its adjacent. However, loading of the entire model into memory cannot be done because of its sizes; so we created a simple algorithm which loads only near blocks to the current one. In fact, for every block we want to smooth, we load the twenty six adjacent blocks on all directions and the chosen one. We create a unique model with it (removing double vertices and faces) and then smoothing it with the algorithm in Smoother module. Finally we save only smoothed vertices for the chosen block and continue with the other blocks. In Figure 12 there is a graphical explanation for the algorithm.

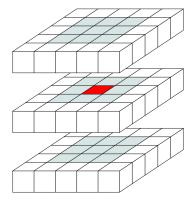


Figure 12: Smoothing of a single block. The red block at the center of the figure is the current one, while the other twenty six colored ones are the blocks that will be part of the model which will be smoothed for this iteration

Moreover, this **processFunction** can only execute a single iteration of the smoothing algorithm, so we need a function that can be able to execute more times the algorithm:

Fragment referenced in 72a.

We can see that after every smoothing iteration on the complete model, we need to rename the output files for the next iterations. In fact, this parallel algorithm works because for every block we do not need the current smoothed vertices for the adjacent blocks but only the old ones. However after first iteration we will have a lot of files with both the new smoothed model and the previous version; as a consequence we need to remove the old model and prepare the smoothed data for the next smoothing iteration. This is the code for the file renaming:

```
if(isfile(f_V))
              if VERSION >= v"0.4"
                mv(f_V, replace(f_V, "smoothed", "model"), remove_destination = true)
                mv(f_FV, replace(f_FV, "smoothed", "model"), remove_destination = true)
              else
                mv(f_V, replace(f_V, "smoothed", "model"))
                mv(f_FV, replace(f_FV, "smoothed", "model"))
              end
            end
          end
        end
     end <
Fragment referenced in 40.
This is the code for starting this step:
\langle smoothing step 42a \rangle \equiv
     @time smoothBlocks(string(outputDirectory, "MODELS"),
                    imageHeight, imageWidth, imageDepth,
                     imageDx, imageDy, imageDz) 
Fragment referenced in 27.
```

5.4.5 Model creation step

At this point of the pipeline, we have a lot of files containing models for a single block; now we can merge them in a unique obj file. As we will see in Section 10, there are two different algorithms for file merging. The first one use a serial merging and it is better for traditional filesystems. The other one use a parallel algorithm which is better on a distributed filesystem. This is the code for invocation of the step:

```
⟨final file merge 42b⟩ ≡
    if parallelMerge
        @time Model2Obj.mergeObjParallel(string(outputDirectory, "MODELS"))
    else
        @time Model2Obj.mergeObj(string(outputDirectory, "MODELS")) ◊
Fragment referenced in 27.
```

6 GenerateBorderMatrix

This module has the responsibility for the generation of the border matrix operator for models boundary computation.

6.1 Module imports

These are modules needed for this part of the package and the public functions exported

Fragment referenced in 72b.

We can notice some lines for importing larcc python library, which will be used in subsection 6.5

6.2 Get border matrix from file

As we have already seen in previous sections, we need to compute boundaries for every block of the model grid. This can be done using the topological boundary operator from LAR package. However, the resulting matrix depends only on grid sizes; so it could be reused for other models. Consequently first time we need a border operator we compute it and then save it on disk for next conversions. This function does that work searching for a file containing the border and, if it does not exist, calculate and save it:

```
⟨ get Border matrix 43b⟩ ≡
function getOriented3BorderPath(borderPath, nx, ny, nz)
"""
Try reading 3-border matrix from file. If it fails matrix
is computed and saved on disk in JSON format
```

```
borderPath: path of border directory
nx, ny, nz: image dimensions
"""

filename = string(borderPath, "/border_", nx, "-", ny, "-", nz, ".json")
if !isfile(filename)
  border = computeOriented3Border(nx, ny, nz)
  writeBorder(border, filename)
end
return filename
```

Fragment referenced in 72b.

6.3 Write border matrix on file

We have already seen that for performance reasons border operator matrix is saved on file; here we will see code used for this scope. Firstly, we have defined a function writeBorder, which takes as parameters a PyObject containing a matrix (computed in subsection 6.4) and the output file path. When porting of larce library will be completed, code for conversion of python csr matrix into csc julia matrix will not be necessary.

```
    function writeBorder(boundaryMatrix, outputFile)
        """
        Write 3-border matrix on json file

        boundaryMatrix: matrix to write on file
        outputFile: path of the outputFile
        """

        fullBorder = pycall(boundaryMatrix["toarray"], PyAny)
        cscBorder = sparse(fullBorder)
        row = findn(cscBorder)[1]
        col = findn(cscBorder)[2]
        data = nonzeros(cscBorder)

        matrixObj = MatrixObject(0, 0, row, col, data)

        outfile = open(string(outputFile), "w")
        JSON.print(outfile, matrixObj)
```

```
close(outfile)
end <>
Fragment referenced in 72b.
```

We can see that, in final JSON file, we write an object called MatrixObject which has the following definition:

```
⟨ Matrix object for JSON file 45a⟩ ≡
type MatrixObject
ROWCOUNT
COLCOUNT
ROW
COL
DATA
end ⋄
```

Fragment referenced in 72b.

The most important fields of this object are the last three ones; the first two contain all coordinates of the non-zero elements, the last contains all non-zero elements of the sparse matrix. So considering the full matrix V we will have that S[ROW[k], COL[k]] = V[k].

6.4 Compute border matrix

Here we can see code used for computation of the border operator. As we can see, we call the python larcc module, from the LAR module, which returns a PyObject containing a sparse csr matrix. In next versions this function will be probably changed and the code for boundary computation will be moved in LAR2Julia module (also transforming all csr matrix in csc matrix) avoiding python calls.

```
⟨ compute border matrix 45b⟩ ≡
    # Compute the 3-border operator
    function computeOriented3Border(nx, ny, nz)
        """
        Compute the 3-border matrix using a modified
        version of larcc
        """
        V, bases = LARUtils.getBases(nx, ny, nz)
        boundaryMat = larcc.signedCellularBoundary(V, bases)
        return boundaryMat
    end ◊
```

6.5 Transform border matrix

Last function we will see, extracts the MatrixObject in Section 6.3 converting it into a common Julia csc sparse matrix

```
\langle transform\ border\ matrix\ in\ csc\ format\ 46 \rangle \equiv
     function getBorderMatrix(borderFilename)
       Get the border matrix from json file and convert it in
       CSC format
       11 11 11
       # Loading borderMatrix from json file
       borderData = JSON.parsefile(borderFilename)
       # Converting Any arrays into Int arrays
       row = Array(Int64, length(borderData["ROW"]))
       col = Array(Int64, length(borderData["COL"]))
       data = Array(Int64, length(borderData["DATA"]))
       for i in 1: length(borderData["ROW"])
         row[i] = borderData["ROW"][i]
       for i in 1: length(borderData["COL"])
         col[i] = borderData["COL"][i]
       end
       for i in 1: length(borderData["DATA"])
         data[i] = borderData["DATA"][i]
       end
       return sparse(row, col, data)
     end <
```

7 Lar2Julia

Fragment referenced in 72b.

This module contains functions used in LAR library which are converted using Julia syntax. Next versions of the software will contain more and more functions from the original LAR

library (which is written in python)

7.1 Module imports

These are modules used for Lar2Julia and the public functions

```
⟨ modules import Lar2Julia 47a⟩ ≡
   import JSON
   using Logging
   export larBoundaryChain, cscChainToCellList ◊
Fragment referenced in 73a.
```

7.2 Get boundary chain from a model

Now we will observe how to compute the boundary chain of a LAR model given the list of non-empty cells and the boundary operator stored as a csc sparse matrix. This algorithm is very simply: firstly we need to convert the list of cells into a sparse array containing the LAR model. So, the resulting array (which will be called cscChain) will contain a one for every cscChain[i][1] $\forall i \in \text{brcCellList}$. Next, we just have to compute the product between the two sparse matrices and convert all values of the result into one of these: $\{-1; +1; 0\}$ using function cscBinFilter.

```
{ get boundary chain 47b \) \equiv function larBoundaryChain(cscBoundaryMat, brcCellList)
    """

    Compute boundary chains
    """

# Computing boundary chains
    n = size(cscBoundaryMat)[1]
    m = size(cscBoundaryMat)[2]

debug("Boundary matrix size: ", n, "\t", m)

data = ones(Int64, length(brcCellList))

i = Array(Int64, length(brcCellList))

for k in 1:length(brcCellList)
    i[k] = brcCellList[k] + 1
    end

end

for k in definition of the content o
```

```
j = ones(Int64, length(brcCellList))
 debug("cscChain rows length: ", length(i))
 debug("cscChain columns length: ", length(j))
 debug("cscChain data length: ", length(brcCellList))
 debug("rows ", i)
 debug("columns ", j)
 debug("data ", data)
 cscChain = sparse(i, j, data, m, 1)
 cscmat = cscBoundaryMat * cscChain
 out = cscBinFilter(cscmat)
 return out
end
function cscBinFilter(CSCm)
 k = 1
 data = nonzeros(CSCm)
 sgArray = copysign(1, data)
 while k <= nnz(CSCm)
   if data[k] % 2 == 1 || data[k] % 2 == -1
      data[k] = 1 * sgArray[k]
   else
      data[k] = 0
   end
   k += 1
 end
 return CSCm
end
```

Fragment referenced in 73a.

7.3 Get oriented cells from a chain

Another operation that could be useful (even if it is not actually used in the package) consists in getting of "+1" oriented cells from a chain. For obtaining this result, it is necessary to get all non-zeros element from the sparse Julia array (remembering that if the user manually write a zero into the array it will be returned from nonzeros function anyway) and then returning only indices of cells that have a "+1" in nonzero element array.

```
\langle get \ oriented \ cells \ from \ a \ chain \ 49a \rangle \equiv
     function cscChainToCellList(CSCm)
       Get a csc containing a chain and returns
       the cell list of the "+1" oriented faces
       data = nonzeros(CSCm)
       # Now I need to remove zero element (problem with Julia nonzeros)
       nonzeroData = Array(Int64, 0)
       for n in data
         if n != 0
            push!(nonzeroData, n)
         end
       end
       cellList = Array(Int64,0)
       for (k, theRow) in enumerate(findn(CSCm)[1])
         if nonzeroData[k] == 1
            push!(cellList, theRow)
         end
       end
       return cellList
     end <
```

Fragment referenced in 73a.

7.4 Transform relationships from arrays of arrays to a sparse matrix

```
⟨ transform relationships to csc 49b⟩ ≡

function relationshipListToCSC(larRelation)

"""

Get a LAR relationship
   and convert it into a CSC matrix

"""

# Build I and J arrays for creation of
   # sparse matrix
   data = Array(Int, 0)
   I = Array(Int, 0)
   J = Array(Int, 0)
   for (k,row) in enumerate(larRelation)
      for col in row
      push!(I, k)
      push!(J, col)
```

```
push!(data, 1)
  end
end

return sparse(I, J, data)
end <>/pre>
Fragment referenced in 73a.
```

8 LARUtils

This module contains functions used for manipulation of LAR models

8.1 Module imports

These are modules used in LARUtils and the functions exported

```
\langle \ modules \ import \ LARUtils \ 50a \rangle \equiv \\ using \ Logging \\ export \ ind, \ invertIndex, \ getBases, \ removeDoubleVerticesAndFaces, \\ computeModel, \ computeModelAndBoundaries \\ \diamond
```

Fragment referenced in 73b.

8.2 Transformation from matrix to array

First utility functions we will see, transform a matrix into an array and vice versa. We have already seen in section 5.4.1 uses of this linearized matrices; now we can focus on code for transformation.

```
⟨ conversion from matrix to array 50b⟩ ≡
function ind(x, y, z, nx, ny)
"""

Transform coordinates into linearized matrix indexes
"""
return x + (nx + 1) * (y + (ny + 1) * (z))
end ◊
```

Fragment referenced in 73b.

Here we have defined also the inverse transformation from the array to the matrix, which is useful for obtaining vertices coordinates from a cell

```
⟨ conversion from array to matrix 51a⟩ ≡

function invertIndex(nx,ny,nz)

"""

Invert indexes

"""

nx, ny, nz = nx + 1, ny + 1, nz + 1

function invertIndex0(offset)

a0, b0 = trunc(offset / nx), offset % nx
a1, b1 = trunc(a0 / ny), a0 % ny
a2, b2 = trunc(a1 / nz), a1 % nz

return b0, b1, b2

end

return invertIndex0
end ◊
```

Fragment referenced in 73b.

8.3 Get bases of a LAR model

For generation of LAR models from an array of non-empty cells, we need to define a function for obtaining a base for every model, which will contain all LAR relationships:

- V: the array of vertices of a LAR model
- VV: the relationship between a vertex and itself
- EV: the relationship between an edge and its vertices
- FV: the relationship between a face and its vertices
- CV: the relationship between a cell and its vertices

From a geometrical point of view these bases create a chain composed from $nx \times ny \times nz$ square cells (where nx ny and nz are the grid size).

Now we will see in details how to obtain all LAR relationships. First of all we need to compute vertices for the geometry:

```
\langle compute \ vertices 51b \rangle \equiv
```

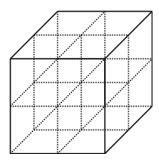


Figure 13: LAR bases geometry for a $2 \times 2 \times 2$ grid

```
# Calculating vertex coordinates (nx * ny * nz)
V = Array{Int64}[]
for z in 0:nz
  for y in 0:ny
   for x in 0:nx
     push!(V,[x,y,z])
  end
  end
end
```

Fragment referenced in 54c.

So we assume that our cube geometry has only integers coordinates that can vary from (0,0,0) to (nx,ny,nz)

Next we have to compute the CV relationship:

```
⟨ compute CV 52⟩ ≡

# Building CV relationship
CV = Array{Int64}[]
for z in 0:nz-1
    for y in 0:ny-1
        for x in 0:nx-1
            push!(CV,the3Dcell([x,y,z]))
        end
    end
end ⇔
```

Fragment referenced in 54c.

For every coordinate in the space delimited by the grid size, it is called function the 3Dcell, which get the coordinate values returning a cell in the three-dimensional space:

Fragment referenced in 54c.

Now we have to compute the FV relationship, which will be widely used in this package:

```
\langle compute FV 53b \rangle \equiv
     # Building FV relationship
     FV = Array{Int64}[]
     v2coords = invertIndex(nx,ny,nz)
     for h in 0:(length(V)-1)
       x,y,z = v2coords(h)
       if (x < nx) && (y < ny)
         push! (FV, [h, ind(x+1, y, z, nx, ny), ind(x, y+1, z, nx, ny), ind(x+1, y+1, z, nx, ny)])
       end
       if (x < nx) && (z < nz)
         push!(FV, [h,ind(x+1,y,z,nx,ny),ind(x,y,z+1,nx,ny),ind(x+1,y,z+1,nx,ny)])
       end
       if (y < ny) && (z < nz)
         push!(FV,[h,ind(x,y+1,z,nx,ny),ind(x,y,z+1,nx,ny),ind(x,y+1,z+1,nx,ny)])
       end
     end <
```

Fragment referenced in 54c.

Finally we have the VV relationship (which is trivial)

```
\langle compute VV 54a \rangle \equiv
      # Building VV relationship
      VV = map((x) \rightarrow [x], 0:length(V) - 1) \diamond
Fragment referenced in 54c.
and the EV relationship
\langle compute EV 54b \rangle \equiv
      # Building EV relationship
      EV = Array{Int64}[]
      for h in 0:length(V)-1
         x,y,z = v2coords(h)
         if (x < nx)
           push!(EV, [h,ind(x+1,y,z,nx,ny)])
         if (y < ny)
           push!(EV, [h,ind(x,y+1,z,nx,ny)])
         end
         if (z < nz)
           push!(EV, [h,ind(x,y,z+1,nx,ny)])
         end
      end \diamond
Fragment referenced in 54c.
This is the complete code for the function getBases
\langle get \ LAR \ bases \ 54c \rangle \equiv
      function getBases(nx, ny, nz)
         Compute all LAR relations
         \langle compute \ three-dimensional \ cells \ 53a \rangle
         ⟨ compute vertices 51b ⟩
         \langle compute CV 52 \rangle
```

 $\langle compute FV 53b \rangle$

```
\langle \ compute \ VV \ 54a \rangle
\langle \ compute \ EV \ 54b \rangle
# return all basis return V, (VV, EV, FV, CV) end \diamond
```

Fragment referenced in 73b.

8.4 Double vertices and faces removal

Another useful function for our models is *removal of double vertices and faces*. In fact, when we produce a LAR model getting only full cell from the geometry in Figure 13 we could obtain double vertices (and consequently double faces). Figure 14 shows an example of a model with these vertices:

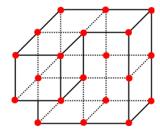


Figure 14: A sample model taken from a $2 \times 2 \times 2$ grid with double vertices between faces in red (remember that we have only the boundaries faces for the model as we have seen in section 5.4.1)

As we can see, for every model there are a lot of double vertices, so we need to remove them for obtaining a compact representation and for next smoothing of the objects. First of all we have to identify double vertices, so it can be useful to define an order between them. Unfortunately Julia does not define a function for order array containing coordinates (which is format used in V array); so we have to define first a custom ordering function:

```
\langle vertices\ comparator\ function\ 55 \rangle \equiv
function lessThanVertices(v1, v2)
"""
Utility function for comparing vertices coordinates
"""

if v1[1] == v2[1]
```

```
if v1[2] == v2[2]
    return v1[3] < v2[3]
  end
  return v1[2] < v2[2]
  end
  return v1[1] < v2[1]
end </pre>
```

Fragment referenced in 58.

Now we can remove double vertices from the V array simply ordering them and removing all consecutive equal vertices. This procedure is more complex than a simple call to Julia **unique** function for removal of double elements because we need the new vertices indices for renaming faces (as we can see later)

```
\langle removal \ of \ double \ vertices \ 56 \rangle \equiv
     function removeDoubleVertices(V)
       Remove double vertices from a LAR model
       V: Array containing all vertices of the model
       11 11 11
       # Sort the vertices list and returns the ordered indices
       orderedIndices = sortperm(V, lt = lessThanVertices, alg=MergeSort)
       orderedVerticesAndIndices = collect(zip(sort(V, lt = lessThanVertices),
                                                  orderedIndices))
       newVertices = Array(Array{Float64}, 0)
       indices = zeros(Int, length(V))
       prevv = Nothing
       i = 1
       for (v, ind) in orderedVerticesAndIndices
         if v == prevv
           indices[ind] = i - 1
           push!(newVertices, v)
           indices[ind] = i
           i += 1
           prevv = v
         end
       end
       return newVertices, indices
     end <
```

Fragment referenced in 58.

As we can see the algorithm does the following steps:

- 1. Sort of vertices list
- 2. Set the current vertex index counter to 1
- 3. For every couple (vertex, index into V array) do:
 - (a) If the current *vertex* is equal to the previous one put into the indices array at position *index* the value for the current vertex index count
 - (b) If the current *vertex* is not equal to the previous one save it into a new V array, insert the indices array at position *index* the current index count and increment it by one

So at the end of this function the array new Vertices will contain all unique vertices, while the *indices* array will contain the correct index for every vertex into new Vertices and the index corresponding to the saved vertex for every deleted vertex.

Now we can use these informations for renaming all faces.

```
⟨ renaming of faces 57 ⟩ ≡
    function reindexVerticesInFaces(FV, indices, offset)
    """
    Reindex vertices indices in faces array

FV: Faces array of the LAR model
    indices: new Indices for faces
    offset: offset for faces indices
    """

for f in FV
    for i in 1: length(f)
        f[i] = indices[f[i] - offset] + offset
        end
    end
    return FV
end ◊
```

Fragment referenced in 58.

Here we can observe a *offset* parameter, which is necessary only if we are renaming faces whose indices doesn't start from zero; actually in ImagesToLARModel it is always equal to zero.

Finally for removing double faces, we only have to call unique function on renamed faces. This is the final code

```
⟨ removal of double vertices and faces 58⟩ ≡
    ⟨ vertices comparator function 55⟩

function removeDoubleVerticesAndFaces(V, FV, facesOffset)
    """
    Removes double vertices and faces from a LAR model

    V: Array containing all vertices
    FV: Array containing all faces
    facesOffset: offset for faces indices
    """

    newV, indices = removeDoubleVertices(V)
    reindexedFaces = reindexVerticesInFaces(FV, indices, facesOffset)
    newFV = unique(FV)

    return newV, newFV

end
    ⟨ removal of double vertices 56⟩
    ⟨ renaming of faces 57⟩ ⋄
Fragment referenced in 73b.
```

8.5 Creation of a LAR model

Now we can see code used for creation of a LAR model given the sparse array containing full cells of our block (**objectBoundaryChain** as we had seen in Section 7.2). We also need the following parameters:

- imageDx, imageDy, imageDz: The grid size
- xStart, yStart, zStart: The coordinate offsets for the current block vertices
- facesOffset: The offset for faces of this block

First thing to do is define models that will be returned from the function:

```
\langle models \ definition \ 59 \rangle \equiv
     V_model = Array(Array{Int}, 0)
     FV_model = Array(Array{Int}, 0)
     V_left = Array(Array{Int},0)
     FV_left = Array(Array{Int},0)
     V_right = Array(Array{Int},0)
     FV_right = Array(Array{Int},0)
     V_top = Array(Array{Int},0)
     FV_top = Array(Array{Int},0)
     V_bottom = Array(Array{Int},0)
     FV_bottom = Array(Array{Int},0)
     V_front = Array(Array{Int},0)
     FV_front = Array(Array{Int},0)
     V_back = Array(Array{Int},0)
     FV_back = Array(Array{Int},0) 
Fragment referenced in 63.
```

We can see from Figure 15 that our grid is divided into seven parts.

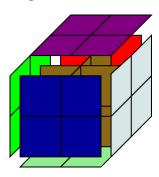


Figure 15: Decomposition of a LAR model into seven parts: the inside model (brown), the left boundary (green), the right boundary (light blue), the top boundary (purple), the bottom boundary (light green), the front boundary(blue), the back boundary (red)

We need this decomposition because we are interested in boundaries of the entire model, while we currently have boundaries only for blocks. So we need to split the inner parts

of a single block model, as we need to freely merge boundaries between adjacent blocks removing the common faces. Function for boundaries merging are shown in subsection 8.6.

After model definition we have to get the cells indices from the block boundary chain and for every non-empty cell we have found, choose the correct model for it. We can observe that every boundary face has a fixed coordinate; for example all faces on the top boundary have the maximum z-coordinate, or faces on right boundary have the maximum y-coordinate (as shown in Figure 16)

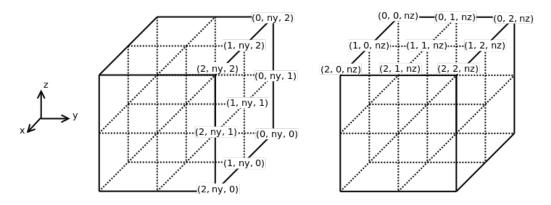


Figure 16: Boundaries coordinates for top and right boundaries of a $2 \times 2 \times 2$ grid. We can observe that every boundary has a fixed coordinate

So we can define a series of functions for checking the membership of a given face to a boundary exploiting these fixed coordinates:

```
⟨ check membership of a face to a boundary 60⟩ ≡
function isOnLeft(face, V, nx, ny, nz)

"""

Check if face is on left boundary

"""

for(vtx in face)
    if(V[vtx + 1][2] != 0)
        return false
    end
    end
    return true

end

function isOnRight(face, V, nx, ny, nz)

"""
```

```
Check if face is on right boundary
  for(vtx in face)
    if(V[vtx + 1][2] != ny)
      return false
    end
  end
 return true
end
function isOnTop(face, V, nx, ny, nz)
  Check if face is on top boundary
  for(vtx in face)
    if(V[vtx + 1][3] != nz)
      return false
    end
  end
  return true
end
function isOnBottom(face, V, nx, ny, nz)
  Check if face is on bottom boundary
  for(vtx in face)
    if(V[vtx + 1][3] != 0)
      return false
    end
  end
  return true
end
function isOnFront(face, V, nx, ny, nz)
  Check if face is on front boundary
  for(vtx in face)
    if(V[vtx + 1][1] != nx)
      return false
```

```
end
end
return true
end

function isOnBack(face, V, nx, ny, nz)
"""
  Check if face is on back boundary
"""

for(vtx in face)
  if(V[vtx + 1][1] != 0)
    return false
  end
end
return true
end
```

Fragment referenced in 63.

After choosing of the right model, we have to insert our face into it. We can do it with the following function, which takes vertices and faces of the base and the model, the face, and the offset of the current face for the model chosen:

```
\langle add \ a \ face \ to \ a \ model \ 62 \rangle \equiv
     function addFaceToModel(V_base, FV_base, V, FV, face, vertex_count)
       Insert a face into a LAR model
       V_base, FV_base: LAR model of the base
       V, FV: LAR model
       face: Face that will be added to the model
       vertex_count: Indices for faces vertices
       new_vertex_count = vertex_count
       for vtx in FV_base[face]
         push!(V, [convert(Int, V_base[vtx + 1][1] + xStart),
                           convert(Int, V_base[vtx + 1][2] + yStart),
                          convert(Int, V_base[vtx + 1][3] + zStart)])
         new_vertex_count += 1
       end
       push!(FV, [vertex_count, vertex_count + 1, vertex_count + 3])
       push!(FV, [vertex_count, vertex_count + 3, vertex_count + 2])
```

```
\begin{array}{c} \texttt{return new\_vertex\_count} \\ \texttt{end} \ \diamondsuit \end{array}
```

Fragment referenced in 63.

As we can see, for every face we put into the model FV array two faces, in fact our final representation is not based on square faces but on triangular faces.

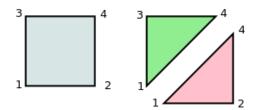


Figure 17: Triangulation of a single face

This is the complete code for creation of a model

```
\langle \, LAR \,\, model \,\, creation \,\, 63 \, \rangle \equiv
      ⟨ check membership of a face to a boundary 60 ⟩
      function computeModelAndBoundaries(imageDx, imageDy, imageDz,
                                xStart, yStart, zStart,
                                objectBoundaryChain)
        .....
        Takes the boundary chain of a part of the entire model
        and returns a LAR model splitting the boundaries
        imageDx, imageDy, imageDz: Boundary dimensions
        xStart, yStart, zStart: Offset of this part of the model
        objectBoundaryChain: Sparse csc matrix containing the cells
        11 11 11
        \langle add \ a \ face \ to \ a \ model \ 62 \rangle
        \langle models \ definition \ 59 \rangle
        V, bases = getBases(imageDx, imageDy, imageDz)
        FV = bases[3]
        vertex_count_model = 1
        vertex_count_left = 1
```

```
vertex_count_right = 1
vertex_count_top = 1
vertex_count_bottom = 1
vertex_count_front = 1
vertex_count_back = 1
# Get all cells (independently from orientation)
b2cells = findn(objectBoundaryChain)[1]
debug("b2cells = ", b2cells)
for f in b2cells
  old_vertex_count_model = vertex_count_model
  old_vertex_count_left = vertex_count_left
  old_vertex_count_right = vertex_count_right
  old_vertex_count_top = vertex_count_top
  old_vertex_count_bottom = vertex_count_bottom
  old_vertex_count_front = vertex_count_front
  old_vertex_count_back = vertex_count_back
  # Choosing the right model for vertex
  if(isOnLeft(FV[f], V, imageDx, imageDy, imageDz))
    vertex_count_left = addFaceToModel(V, FV, V_left, FV_left,
                                f, old_vertex_count_left)
  elseif(isOnRight(FV[f], V, imageDx, imageDy, imageDz))
    vertex_count_right = addFaceToModel(V, FV, V_right, FV_right,
                                f, old_vertex_count_right)
  elseif(isOnTop(FV[f], V, imageDx, imageDy, imageDz))
    vertex_count_top = addFaceToModel(V, FV, V_top, FV_top,
                                f, old_vertex_count_top)
  elseif(isOnBottom(FV[f], V, imageDx, imageDy, imageDz))
    vertex_count_bottom = addFaceToModel(V, FV, V_bottom, FV_bottom,
                                f, old_vertex_count_bottom)
  elseif(isOnFront(FV[f], V, imageDx, imageDy, imageDz))
    vertex_count_front = addFaceToModel(V, FV, V_front, FV_front,
                                f, old_vertex_count_front)
  elseif(isOnBack(FV[f], V, imageDx, imageDy, imageDz))
    vertex_count_back = addFaceToModel(V, FV, V_back, FV_back,
                                f, old_vertex_count_back)
  else
    vertex_count_model = addFaceToModel(V, FV, V_model, FV_model,
                                f, old_vertex_count_model)
  end
end
```

```
# Removing double vertices
return [removeDoubleVerticesAndFaces(V_model, FV_model, 0)],
[removeDoubleVerticesAndFaces(V_left, FV_left, 0)],
[removeDoubleVerticesAndFaces(V_right, FV_right, 0)],
[removeDoubleVerticesAndFaces(V_top, FV_top, 0)],
[removeDoubleVerticesAndFaces(V_bottom, FV_bottom, 0)],
[removeDoubleVerticesAndFaces(V_front, FV_front, 0)],
[removeDoubleVerticesAndFaces(V_back, FV_back, 0)]
end
```

Fragment referenced in 73b.

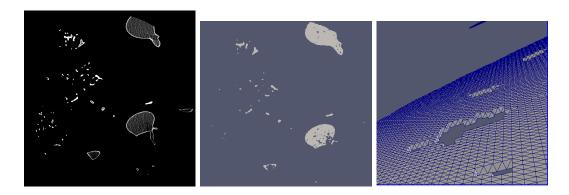


Figure 18: Creation of a sample model. (a) The original image (b) The three-dimensional model (c) The three-dimensional model (detail with triangular faces)

8.6 Removing double faces and vertices from boundaries

In previous section, we have seen how to create a LAR model from the chain list. However this model contains all borders between blocks, while we are only interested in borders for the entire image. So, we will see functions for boundaries merging with removal of double faces and vertices from both sides.

The algorithm for the removal is very simply. First of all we need to remove double vertices from models in the usual way using removeDoubleVertices function and re-indexing all faces. Next we count all elements in re-indexed faces array removing elements with more than one occurrence and create an array of faces with an explicit representation of vertices ($FV_{-}vertices$). Now we can safely remove double vertices on the other side of the boundary without losing the correct indexing in the faces. Finally we can create the final faces array with only remaining vertices comparing coordinates in $FV_{-}vertices$ with the

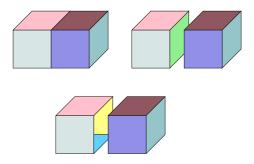


Figure 19: Removal of double faces from boundaries. (a) Two adjacent blocks (b) The same blocks exploded on x axis (c) Result of the removal on the exploded blocks

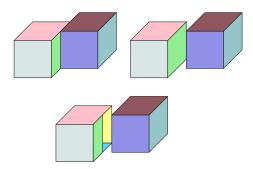


Figure 20: Same as Figure 19 with another model

ones in the last vertices array. Code for this function is the following:

```
⟨Removal of double vertices and faces from boundaries 66⟩ ≡
function removeVerticesAndFacesFromBoundaries(V, FV)

"""

Remove vertices and faces duplicates on
boundaries models

V,FV: lar model of two merged boundaries

"""

newV, indices = removeDoubleVertices(V)
uniqueIndices = unique(indices)

# Removing double faces on both boundaries
FV_reindexed = reindexVerticesInFaces(FV, indices, 0)
FV_unique = unique(FV_reindexed)

FV_cleaned = Array(Array{Int}, 0)
```

```
for f in FV_unique
    if(count((x) \rightarrow x == f, FV\_reindexed) == 1)
      push!(FV_cleaned, f)
    \quad \text{end} \quad
  end
  # Creating an array of faces with explicit vertices
  FV_vertices = Array(Array{Array{Float64}}, 0)
  for i in 1 : length(FV_cleaned)
    push!(FV_vertices, Array(Array{Float64}, 0))
    for vtx in FV_cleaned[i]
      push!(FV_vertices[i], newV[vtx])
    end
  end
  V_final = Array(Array{Float64}, 0)
  FV_final = Array(Array{Int}, 0)
  # Saving only used vertices
  for face in FV_vertices
    for vtx in face
      push!(V_final, vtx)
    end
  end
  V_final = unique(V_final)
  # Renumbering FV
  for face in FV_vertices
    tmp = Array(Int, 0)
    for vtx in face
      ind = findfirst(V_final, vtx)
      push!(tmp, ind)
    push!(FV_final, tmp)
  end
  return V_final, FV_final
end \diamond
```

9 Smoother

This module contains functions used for smoothing LAR models

9.1 Get adjacent vertices

As we will see in next subsection, for executing a smoothing algorithm we need to know adjacent vertices to a given one. So we need a VV relationship, where for every vertex index i, we have a list of adjacent vertices.

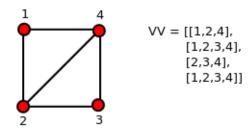


Figure 21: VV relationship for a simple model

Algorithm is very simple and exploit the following property: for triangular faces all vertices are linked together. So we just need to search for every vertex i all faces that contain it and add all their vertices to a list. VV will contain a concatenation of all these lists

```
\langle get \ adjacent \ vertices \ 68 \rangle \equiv
     function adjVerts(V, FV)
        Compute the adjacency graph of vertices
        of a LAR model
        V, FV: LAR model
        Returns the list of indices of vertices adjacent
        to a vertex
        11 11 11
        VV = Array{Int}[]
        for i in 1:length(V)
          row = Array(Int, 0)
          for face in FV
            if i in face
               for v in face
                 push!(row, v)
               end
            end
```

```
end
  if length(row) == 0
    push!(row, i)
  end
  push!(VV, collect(unique(row)))
  end
  return VV
end
```

Fragment referenced in 82a.

9.2 Laplacian smoothing

There are many different algorithms for mesh smoothing. The simpler and the one we used in this library is **laplacian smoothing**. For each vertex in a mesh, a new position is chosen according to local information (such as the coordinates of neighbors) and the vertex is moved there. If that mesh is topologically a rectangular grid (so each internal vertex is connected to four neighbors) then this operation produces the *Laplacian* of the mesh.

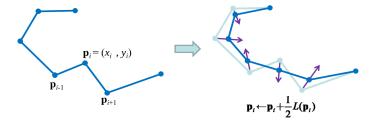


Figure 22: Laplacian smoothing (picture taken from the *Geometry Processing Algorithms* course at Stanford University)

As we can see from Figure 22, with substitution of every vertex position with the mean of the neighbors positions, we can obtain a curve with smoothed edges. This procedure can be repeated many times, so we can obtain a smoother model. For example, in Figure 24, we can see this algorithm applied on a sample mesh with three iterations.

This is the code for the smoothing function; it takes a single LAR model and returns the smoothed model.

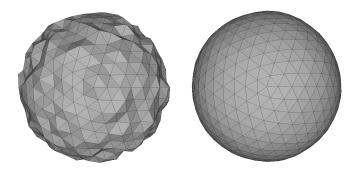


Figure 23: Laplacian smoothing for a sample mesh. (a) Original mesh (b) Mesh after three iterations of the smoothing algorithm (picture taken from a *Digital Geometry Processing* course at IMPA)

```
V, FV: LAR model
 VV = adjVerts(V, FV)
 newV = Array(Array{Float64},0)
 V_temp = Array(Array{Float64},0)
 for i in 1:length(VV)
    adjs = VV[i]
    # Get all coordinates for adjacent vertices
    coords = Array(Array{Float64}, 0)
    for v in adjs
     push!(coords, V[v])
    end
    # Computing sum of all vectors
    sum = [0.0, 0.0, 0.0]
    for v in coords
     sum += v
    end
    # Computing convex combination of vertices
   push!(newV, sum/length(adjs))
 end
 return newV, FV
end \diamond
```

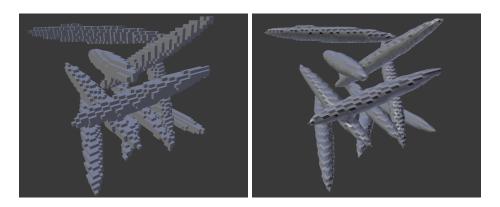


Figure 24: Smoothing of a sample model made with ImagesToLARModel

10 Model2Obj

11 Exporting the library

${\bf Images To LAR Model}$

```
"src/ImagesToLARModel.jl" 71≡
module ImagesToLARModel

⟨update load path 6⟩

⟨modules import ImagesToLARModel 7⟩
⟨load JSON configuration 8⟩
⟨Start conversion from JSON file 9⟩
⟨Start manual conversion 10⟩
end

⋄
```

ImagesConversion

```
"src/ImagesConversion.jl" 72a\(\) module ImagesConversion
\(\) \( modules import ImagesConversion 23a \)
\(\) \( main function for ImagesConversion 23b \)
\(\) \( parallel block iteration 25 \)
\(\) \( start conversion of images 28a \)
\(\) \( image conversion process 32 \)
\(\) \( boundary merge process function 33b \)
\(\) \( merge boundaries utility function 35 \)
\(\) \( Block merge process function 36b \)
\(\) \( Smooth block process function 38b \)
\(\) \( execute smoothing function 40 \)
end
\(\)
```

GenerateBorderMatrix

```
"src/GenerateBorderMatrix.jl" 72b\(\)\text{module GenerateBorderMatrix}\\ \langle Matrix object for JSON file 45a \rangle \\ \langle modules import GenerateBorderMatrix 43a \rangle \langle compute border matrix 45b \rangle \langle write Border matrix 44 \rangle \langle get Border matrix 43b \rangle
```

```
\langle \ transform \ border \ matrix \ in \ csc \ format \ 46 \, \rangle end \diamond
```

Lar2Julia

```
"src/Lar2Julia.jl" 73a≡
module Lar2Julia

⟨modules import Lar2Julia 47a⟩

⟨get boundary chain 47b⟩

⟨get oriented cells from a chain 49a⟩

⟨transform relationships to csc 49b⟩
end

⋄
```

LARUtils

```
"src/LARUtils.jl" 73b≡
module LARUtils

⟨modules import LARUtils 50a⟩

⟨conversion from matrix to array 50b⟩

⟨conversion from array to matrix 51a⟩

⟨get LAR bases 54c⟩

⟨removal of double vertices and faces 58⟩

⟨Removal of double vertices and faces from boundaries 66⟩

⟨LAR model creation 63⟩
end

⋄
```

Model2Obj

```
"src/Model20bj.jl" 74\equiv
     module Model20bj
     import LARUtils
     using Logging
     export writeToObj, mergeObj, mergeObjParallel
     function writeToObj(V, FV, outputFilename)
       Take a LAR model and write it on obj file
       V: array containing vertices coordinates
       FV: array containing faces
       outputFilename: prefix for the output files
       if (length(V) != 0)
         outputVtx = string(outputFilename, "_vtx.stl")
         outputFaces = string(outputFilename, "_faces.stl")
         fileVertex = open(outputVtx, "w")
         fileFaces = open(outputFaces, "w")
         for v in V
           write(fileVertex, "v ")
           write(fileVertex, string(v[1], " "))
           write(fileVertex, string(v[2], " "))
           write(fileVertex, string(v[3], "\n"))
         end
         for f in FV
           write(fileFaces, "f ")
           write(fileFaces, string(f[1], " "))
           write(fileFaces, string(f[2], " "))
           write(fileFaces, string(f[3], "\n"))
         end
         close(fileVertex)
         close(fileFaces)
       end
```

```
end
```

```
function mergeObj(modelDirectory)
 Merge stl files in a single obj file
 modelDirectory: directory containing models
 files = readdir(modelDirectory)
 vertices_files = files[find(s -> contains(s, string("_vtx.stl")), files)]
 faces_files = files[find(s -> contains(s, string("_faces.stl")), files)]
 obj_file = open(string(modelDirectory, "/", "model.obj"), "w") # Output file
 vertices_counts = Array(Int64, length(vertices_files))
 number_of_vertices = 0
 for i in 1:length(vertices_files)
   vtx_file = vertices_files[i]
   f = open(string(modelDirectory, "/", vtx_file))
   # Writing vertices on the obj file
   for ln in eachline(f)
     write(obj_file, ln)
     number_of_vertices += 1
   # Saving number of vertices
   vertices_counts[i] = number_of_vertices
   close(f)
  end
 for i in 1 : length(faces_files)
   faces_file = faces_files[i]
   f = open(string(modelDirectory, "/", faces_file))
   for ln in eachline(f)
     splitted = split(ln)
     write(obj_file, "f ")
     if i > 1
        write(obj_file, string(parse(splitted[2]) + vertices_counts[i - 1], " "))
        write(obj_file, string(parse(splitted[3]) + vertices_counts[i - 1], " "))
        write(obj_file, string(parse(splitted[4]) + vertices_counts[i - 1]))
        write(obj_file, string(splitted[2], " "))
        write(obj_file, string(splitted[3], " "))
        write(obj_file, splitted[4])
     write(obj_file, "\n")
```

```
end
   close(f)
 close(obj_file)
 # Removing all tmp files
 for vtx_file in vertices_files
   rm(string(modelDirectory, "/", vtx_file))
 end
 for fcs_file in faces_files
   rm(string(modelDirectory, "/", fcs_file))
 end
end
function assignTasks(startInd, endInd, taskArray)
 This function choose the first files to merge
 creating a tree where number of processes is maximized
 startInd: starting index for array subdivision
 endInd: end index for array subdivision
 taskArray: array containing indices of files to merge for first
  if (endInd - startInd == 2)
   push!(taskArray, startInd)
 elseif (endInd - startInd < 2)</pre>
   if (endInd % 4 != 0 && startInd != endInd)
      # Stop recursion on this branch
     push!(taskArray, startInd)
   end
   # Stop recursion doing nothing
 else
   assignTasks(startInd, startInd + trunc((endInd - startInd) / 2), taskArray)
   assignTasks(startInd + trunc((endInd - startInd) / 2) + 1, endInd, taskArray)
 end
end
function mergeVerticesFiles(file1, file2, startOffset)
 Support function for merging two vertices files.
 Returns the number of vertices of the merged file
 file1: path of the first file
 file2: path of the second file
```

```
startOffset: starting face offset for second file
 f1 = open(file1, "a")
 f2 = open(file2)
 debug("Merging ", file2)
 number_of_vertices = startOffset
 for ln in eachline(f2)
   write(f1, ln)
   number_of_vertices += 1
 end
 close(f2)
 close(f1)
 return number_of_vertices
end
function mergeFacesFiles(file1, file2, facesOffset)
 Support function for merging two faces files
 file1: path of the first file
 file2: path of the second file
 facesOffset: offset for faces
 f1 = open(file1, "a")
 f2 = open(file2)
 for ln in eachline(f2)
   splitted = split(ln)
   write(f1, "f ")
   write(f1, string(parse(splitted[2]) + facesOffset, " "))
   write(f1, string(parse(splitted[3]) + facesOffset, " "))
   write(f1, string(parse(splitted[4]) + facesOffset, "\n"))
 end
 close(f2)
 close(f1)
end
function mergeObjProcesses(fileArray, facesOffset = Nothing)
```

```
Merge files on a single process
 fileArray: Array containing files that will be merged
 facesOffset (optional): if merging faces files, this array contains
   offsets for every file
  if(contains(fileArray[1], string("_vtx.stl")))
   # Merging vertices files
   offsets = Array(Int, 0)
   push!(offsets, countlines(fileArray[1]))
   vertices_count = mergeVerticesFiles(fileArray[1], fileArray[2], countlines(fileArray[1]))
   rm(fileArray[2]) # Removing merged file
   push!(offsets, vertices_count)
   for i in 3: length(fileArray)
      vertices_count = mergeVerticesFiles(fileArray[1], fileArray[i], vertices_count)
     rm(fileArray[i]) # Removing merged file
     push!(offsets, vertices_count)
   \quad \text{end} \quad
   return offsets
  else
   # Merging faces files
   mergeFacesFiles(fileArray[1], fileArray[2], facesOffset[1])
   rm(fileArray[2]) # Removing merged file
   for i in 3 : length(fileArray)
     mergeFacesFiles(fileArray[1], fileArray[i], facesOffset[i - 1])
     rm(fileArray[i]) # Removing merged file
   end
  end
end
function mergeObjHelper(vertices_files, faces_files)
 Support function for mergeObj. It takes vertices and faces files
 and execute a single merging step
 vertices_files: Array containing vertices files
 faces_files: Array containing faces files
 numberOfImages = length(vertices_files)
 taskArray = Array(Int, 0)
 assignTasks(1, numberOfImages, taskArray)
 # Now taskArray contains first files to merge
 numberOfVertices = Array(Int, 0)
  tasks = Array(RemoteRef, 0)
```

```
for i in 1 : length(taskArray) - 1
   task = @spawn mergeObjProcesses(vertices_files[taskArray[i] : (taskArray[i + 1] - 1)])
   push!(tasks, task)
   #append!(numberOfVertices, mergeObjProcesses(vertices_files[taskArray[i] : (taskArray[i +
  end
 # Merging last vertices files
 task = @spawn mergeObjProcesses(vertices_files[taskArray[length(taskArray)] : end])
 push!(tasks, task)
  #append!(numberOfVertices, mergeObjProcesses(vertices_files[taskArray[length(taskArray)] : e.
 for task in tasks
   append!(numberOfVertices, fetch(task))
  end
 debug("NumberOfVertices = ", numberOfVertices)
  # Merging faces files
 tasks = Array(RemoteRef, 0)
 for i in 1 : length(taskArray) - 1
   task = @spawn mergeObjProcesses(faces_files[taskArray[i] : (taskArray[i + 1] - 1)],
                                    numberOfVertices[taskArray[i] : (taskArray[i + 1] - 1)])
   push!(tasks, task)
   #mergeObjProcesses(faces_files[taskArray[i] : (taskArray[i + 1] - 1)],
                       numberOfVertices[taskArray[i] : (taskArray[i + 1] - 1)])
  end
  #Merging last faces files
 task = @spawn mergeObjProcesses(faces_files[taskArray[length(taskArray)] : end],
                                  numberOfVertices[taskArray[length(taskArray)] : end])
 push!(tasks, task)
 #mergeObjProcesses(faces_files[taskArray[length(taskArray)] : end],
                       numberOfVertices[taskArray[length(taskArray)] : end])
 for task in tasks
   wait(task)
 end
end
function mergeObjParallel(modelDirectory)
```

```
Merge stl files in a single obj file using a parallel
 approach. Files will be recursively merged two by two
 generating a tree where number of processes for every
 step is maximized
 Actually use of this function is discouraged. In fact
 speedup is influenced by disk speed. It could work on
 particular systems with parallel accesses on disks
 modelDirectory: directory containing models
 files = readdir(modelDirectory)
  # Appending directory path to every file
 files = map((s) -> string(modelDirectory, "/", s), files)
 # While we have more than one vtx file and one faces file
 while(length(files) != 2)
   vertices_files = files[find(s -> contains(s,string("_vtx.stl")), files)]
   faces_files = files[find(s -> contains(s,string("_faces.stl")), files)]
   # Merging files
   mergeObjHelper(vertices_files, faces_files)
   files = readdir(modelDirectory)
   files = map((s) -> string(modelDirectory, "/", s), files)
 mergeVerticesFiles(files[2], files[1], 0)
 mv(files[2], string(modelDirectory, "/model.obj"))
 rm(files[1])
end
function getModelsFromFiles(arrayV, arrayFV)
 Get a LAR models for two arrays of vertices
 and faces files
 arrayV: Array containing all vertices files
 arrayFV: Array containing all faces files
 V = Array(Array{Float64}, 0)
 FV = Array(Array(Int), 0)
 offset = 0
```

```
for i in 1:length(arrayV)
    if isfile(arrayFV[i])
      f_FV = open(arrayFV[i])
      for ln in eachline(f_FV)
        splitted = split(ln)
        push!(FV, [parse(splitted[2]) + offset, parse(splitted[3]) + offset, parse(splitted[4]
      end
      close(f_FV)
      f_V = open(arrayV[i])
      for ln in eachline(f_V)
        splitted = split(ln)
        push!(V, [parse(splitted[2]), parse(splitted[3]), parse(splitted[4])])
      \quad \text{end} \quad
      close(f_V)
    end
  end
  return LARUtils.removeVerticesAndFacesFromBoundaries(V, FV)
end
end
```

PngStack2Array3dJulia

```
"src/PngStack2Array3dJulia.jl" 81 \equiv module PngStack2Array3dJulia  \langle \ modules \ import \ PngStack2Array3dJulia \ 11 \rangle   \langle \ Convert \ to \ png \ 14b \rangle   \langle \ Get \ image \ data \ 16 \rangle   \langle \ Centroid \ computation \ 17 \rangle   \langle \ Pixel \ transformation \ 19 \rangle  end
```

Smoother

```
"src/Smoother.jl" 82a\equiv module Smoother \langle \ get \ adjacent \ vertices \ 68 \ \rangle \langle \ laplacian \ smoothing \ 69 \ \rangle end \diamond
```

11.1 Installing the library

- 12 Conclusions
- 12.1 Results
- 12.2 Further improvements

References

- [CL13] CVD-Lab, *Linear Algebraic Representation*, Tech. Report 13-00, Roma Tre University, October 2013.
- [PDFJ15] Alberto Paoluzzi, Antonio DiCarlo, Francesco Furiani, and Miroslav Jirik, CAD models from medical images using LAR, Computer-Aided Design and Applications 13 (2015), To appear.
- [W3C] W3C, Portable Network Graphics (PNG) Specification (Second Edition), Tech. report.

A Utility functions

B Tests

Generation of the border matrix

[&]quot;test/generateBorderMatrix.jl" 82b=

```
push!(LOAD_PATH, "../../")
import GenerateBorderMatrix
import JSON
using Base.Test
function testComputeOriented3Border()
 Test function for computeOriented3Border
 boundaryMatrix = GenerateBorderMatrix.computeOriented3Border(2,2,2)
 rowcount = boundaryMatrix[:shape][1]
 @test rowcount == 36
 colcount = boundaryMatrix[:shape][2]
 @test colcount == 8
 row = boundaryMatrix[:indptr]
 col = boundaryMatrix[:indices]
 data = boundaryMatrix[:data]
 end
function testWriteBorder()
 Test for writeBorder
 boundaryMatrix = GenerateBorderMatrix.computeOriented3Border(2,2,2)
 filename = "borderFile"
 GenerateBorderMatrix.writeBorder(boundaryMatrix, filename)
 @test isfile(filename)
 # Loading borderMatrix from json file
 borderData = JSON.parsefile(filename)
 row = Array(Int64, length(borderData["ROW"]))
 col = Array(Int64, length(borderData["COL"]))
 data = Array(Int64, length(borderData["DATA"]))
 @test borderData["ROW"] == [0,1,2,3,4,5,7,8,9,11,12,13,15,17,18,19,20,22,23,24,26,27,29,30,3
 @test borderData["COL"] == [0,0,0,1,1,0,1,1,2,0,2,2,3,1,3,2,3,3,2,3,0,4,4,4,1,5,5,4,5,5,2,6,
 rm(filename)
```

```
end
function executeAllTests()
  @time testComputeOriented3Border()
  @time testWriteBorder()
  println("Tests completed.")
end
executeAllTests()
```

Conversion of a png stack to a 3D array

```
"test/pngStack2Array3dJulia.jl" 84\equiv
     push!(LOAD_PATH, "../../")
     import PngStack2Array3dJulia
     using Base.Test
     function testGetImageData()
       Test function for getImageData
       11 11 11
       width, height = PngStack2Array3dJulia.getImageData("images/0.png")
       @test width == 50
       @test height == 50
     end
     function testCalculateClusterCentroids()
       {\tt Test \ function \ for \ calculateClusterCentroids}
       path = "images/"
       image = 0
       centroids = PngStack2Array3dJulia.calculateClusterCentroids(path, image, 2)
       expected = [0, 253]
       centroids = vec(reshape(centroids, 1, 2))
       @test sort(centroids) == expected
```

```
end
```

```
function testPngstack2array3d()
  Test function for pngstack2array3d
  path = "images/"
  minSlice = 0
  maxSlice = 4
  centroids = PngStack2Array3dJulia.calculateClusterCentroids(path, 0, 2)
  image3d = PngStack2Array3dJulia.pngstack2array3d(path, minSlice, maxSlice, centroids)
  @test size(image3d)[1] == 5
  @test size(image3d[1])[1] == 50
  @test size(image3d[1])[2] == 200
end
function executeAllTests()
  @time testCalculateClusterCentroids()
  @time testPngstack2array3d()
  @time testGetImageData()
  println("Tests completed.")
end
executeAllTests()
```

Test for LAR utilities

```
"test/LARUtils.jl" 85
    push!(LOAD_PATH, "../../")
    import LARUtils
    using Base.Test

function testInd()
    """
    Test function for ind
    """
    nx = 2
    ny = 2
```

```
@test LARUtils.ind(0, 0, 0, nx, ny) == 0
@test LARUtils.ind(1, 1, 1, nx, ny) == 13
@test LARUtils.ind(2, 5, 4, nx, ny) == 53
@test LARUtils.ind(1, 1, 1, nx, ny) == 13
@test LARUtils.ind(2, 7, 1, nx, ny) == 32
@test LARUtils.ind(1, 0, 3, nx, ny) == 28
end

function executeAllTests()
@time testInd()
println("Tests completed.")
end

executeAllTests()
```