IP-Based Energetic Reasoning for the Resource Constrained Project Scheduling Problem

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Abstract

In this paper, we consider the Resource Constrained Project Scheduling Problem (RCPSP). New feasibility tests for the energetic reasoning are introduced based on new integer programming (IP) formulations. Experimental results are presented based on PSPLIB instances.

Keywords: Resource Constrained Project Scheduling Problem, Lower Bound, Energetic Reasoning.

1   Introduction

The Resource Constrained Project Scheduling Problem (RCPSP) is a challenging combinatorial optimization problem. It can be formulated as follows: given a set $J = \{0, \ldots, n+1\}$ of activities , where activities 0 and $n+1$ represent the start and the end of the project respectively, and a set K of renewable resources. Each activity $j \in J$ has a processing time p_j and a demand b_{jk} on each resource $k \in K$. Each resource k has a capacity R_k that is continuously available from time 0 onwards. Furthermore, precedence relations are considered between activities. The problem consists in scheduling the activities by respecting the precedence and resource constraints in order to minimize the time when the project terminates (or the makespan). Blazewicz et al. [3], have shown that this problem is $\mathcal{NP}$-hard in the strong sense.

The main contribution of this paper is to present new lower bounds for the RCPSP. A recent survey on lower bounds for the RCPSP can be found in [9]. Lower bounds for this problem can be classified into two families: constructive lower bounds and destructive ones. The first family of lower bounds are obtained by relaxing some constraints and solving the resulting relaxed problem. A first bound is the *Capacity* bound which consists in relaxing the precedence constraints. A second simple bound is the *Critical Path* which consists in relaxing the capacity constraints. Stinson et al. [10] derived a bound called *Critical Sequence* bound which improves the Critical Path bound. Another lower bound consists in relaxing the RCPSP to a parallel machine problem with heads and tails and calculate a lower bound to this latter problem [5]. These types of lower bounds are known to be very fast but the gap from optimality may be large. The second family of lower bounds include destructive lower bounds. These bounds are based on the following idea: consider a trivial bound C and try to detect infeasibilities. If it is the case, $C+1$ is a valid lower bound. This procedure is re-iterated until no more infeasibilities are detected. Brucker and Knust [4] and Baptiste and Demassey [1] derived a destructive lower bound using column generation to solve a sequence of feasibility problems. These lower bounds are effective but are very time consuming.

In the sequel, we propose an improvement of the *Classical Energetic Reasoning* introduced by Erschler et al. [6]. In section 2, we recall the principles of the Classical Energetic Reasoning. In section 3, we adapt the framework of

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the *Revisited Energetic Reasoning*, introduced for the parallel machine problem [7], to the RCPSP. In section 4, we introduce a new feasibility test based on a new formulation tacking into account simultaneously all the resources. Experimental results are presented in section 5. A conclusion is presented in section 6.

2 Classical Energetic Reasoning

Following the framework of Erschler et al. [6], we recall that the work of an activity $j \in J$ on a resource $k \in K$ can be written as:

$$W_{jk}(t_1, t_2) = \min \left(W_{jk}^l(t_1, t_2), W_{jk}^r(t_1, t_2) \right)$$

where $W_{jk}^l(t_1, t_2)$ and $W_{jk}^r(t_1, t_2)$ represent the left and right work and are equal respectively to $b_{jk} \min \left(t_2 - t_1, p_j, \max(0, r_j + p_j - t_1) \right)$ and $b_{jk} \min \left(t_2 - t_1, p_j, \max(0, d_j - p_j + t_2) \right)$. r_j and d_j represent the release date and due date of activity j computed according to C and precedence relationship.
The total work over the time-interval $[t_1, t_2]$ for the resource k is denoted $W_k(t_1, t_2)$. It is defined as the sum of the works of all the activities over $[t_1, t_2]$.

The feasibility condition of the Classical Energetic Reasoning is:

Fact 2.1 *if $\exists k \in K$ such that $W_k(t_1, t_2) > R_k(t_2 - t_1)$ then the instance is infeasible.*

If there is no infeasibility, an adjustment procedure may be performed using the concept of slack [2].

3 Revisited Energetic Reasoning

In this section, we present an improvement of the computation of the work $W_k(t_1, t_2)$ over the time-interval $[t_1, t_2]$. The idea of the Revisited Energetic Reasoning was introduced by Hidri et al. [7] for a parallel machine problem. In the following, we adapt this concept to the RCPSP. First, we introduce some notation. Next, we present the mathematical model that allows to compute an enhanced estimate of the total work.

3.1 Notation

In the following, we denote by $\Delta = [t_1, t_2]$ and we define the set J_Δ as:

$$J_\Delta = \{j \in J / r_j + p_j > t_1 \text{ and } d_j - p_j < t_2\}$$

For the sake of clarity, we replace J_Δ, $W_{jk}^l(t_1, t_2)$, $W_{jk}^r(t_1, t_2)$ and $W_k(t_1, t_2)$ by J, W_{jk}^l, W_{jk}^r and W_k.

In order to present the Revisited Energetic Reasoning, the set J is partitioned into 7 subsets:

$$
\begin{aligned}
J_L \;\; &= \{j \in J / r_j + p_j < t_2 \text{ and } d_j - p_j \leq t_1\} \\
J_R \;\; &= \{j \in J / r_j + p_j \geq t_2 \text{ and } d_j - p_j > t_1\} \\
J_I \;\; &= \{j \in J / t_1 < r_j < d_j < t_2\} \\
J_{LI} \;\; &= \{j \in J / r_j \leq t_1, d_j < t_2 \text{ and } d_j - p_j > t_1\} \\
J_{RI} \;\; &= \{j \in J / r_j > t_1, d_j \geq t_2 \text{ and } r_j + p_j < t_2\} \\
J_{LR} \;\; &= \{j \in J / r_j \leq t_1, d_j \geq t_2, r_j + p_j < t_2, d_j - p_j > t_1 \text{ and } p_j \geq t_2 + t_1 - 1\} \\
J_{LIR} &= \{j \in J / r_j \leq t_1, d_j \geq t_2, r_j + p_j < t_2, d_j - p_j > t_1 \text{ and } p_j < t_2 - t_1 - 1\}
\end{aligned}
$$

We also denote by:

$J_{\mathcal{L}} = J_{LI} \cup J_{LR} \cup J_{LIR}$; the set of activities that may be placed at the left position.

$J_{\mathcal{R}} = J_{RI} \cup J_{LR} \cup J_{LIR}$; the set of activities that may be placed at the right position.

$J_{\mathcal{I}} = J_{LI} \cup J_{RI} \cup J_{LIR}$; the set of activities that may be placed inside the interval.

3.2 Mathematical Model

In order to present the mathematical model allowing to calculate the total work W_k over $[t_1, t_2]$, we define for all activity $j \in J$ the following variables:

$$
x_j = \begin{cases} 1 & \text{if } W_{jk} = W_{jk}^l = b_{jk}(r_j + p_j - t_1), \\ 0 & \text{otherwise.} \end{cases}
$$

$$
y_j = \begin{cases} 1 & \text{if } W_{jk} = W_{jk}^r = b_{jk}(t_2 - d_j + p_j), \\ 0 & \text{otherwise.} \end{cases}
$$

$$
z_j = \begin{cases} 1 & \text{if } W_{jk} = W_{jk}^i = b_{jk}p_j, \\ 0 & \text{otherwise.} \end{cases}
$$

Let $m_k^l = \min(\sum_{j \in J_{\mathcal{L}}} b_{jk}, R_k - \sum_{j \in J_L} b_{jk})$ and $m_k^r = \min(\sum_{j \in J_{\mathcal{R}}} b_{jk}, R_k - \sum_{j \in J_R} b_{jk})$

be the capacities available on resource k for activities of the set $J_{\mathcal{L}}$ respectively $J_{\mathcal{R}}$ over the time-interval $[t_1, t_2]$.

The formulation can be written as:

$$E_k = \min \sum_{j \in J_{\mathcal{L}}} W_{jk}^l x_j + \sum_{j \in J_{\mathcal{R}}} W_{jk}^r y_j + \sum_{j \in J_{\mathcal{I}}} W_{jk}^i z_j \tag{1}$$

s.t.

$$\begin{align} x_j + z_j = 1, \quad &\forall j \in J_{LI} \tag{2}\\ y_j + z_j = 1, \quad &\forall j \in J_{RI} \tag{3}\\ x_j + y_j = 1, \quad &\forall j \in J_{LR} \tag{4}\\ x_j + y_j + z_j = 1, \quad &\forall j \in J_{LIR} \tag{5}\\ \sum_{j \in J_{\mathcal{L}}} b_{jk} x_j \le m_k^l \tag{6}\\ \sum_{j \in J_{\mathcal{R}}} b_{jk} y_j \le m_k^r \tag{7}\\ x_j, y_j, z_j \in \{0, 1\}, \quad &\forall j \in J_{LI} \cup J_{RI} \cup J_{LR} \cup J_{LIR} \tag{8} \end{align}$$

$(P1)$

The objectif function (1) allows to minimize the work of activities affected left, right and in the inside. Constraints (2) and (3) state that activities of J_{LI} (J_{RI}) may be placed inside the interval or at the left (right) position. Constraint (4) state that activities of J_{LR} may be placed at the left or right position. Constraint (5) state that activities of J_{LIR} may be placed at the left or at the right or inside the interval. Constraints (6) and (7) impose that demands of activities affected at the left (right) position cannot exceed the available capacity at the left (right).

Let $\overline{W_k}(t_1, t_2) = E_k + \sum_{j \in J_L} W_{jk}^l + \sum_{j \in J_I} W_{jk}^i + \sum_{j \in J_R} W_{jk}^r$ be the *improved* work calculated by the Revisited Energetic Reasoning. We have the following fact:

Fact 3.1 *if* $\exists k \in K$ *such that* $\overline{W_k}(t_1, t_2) > R_k(t_2 - t_1)$ *then the instance is infeasible.*

4 Global Energetic Reasoning

A second improvement of the Classical Energetic Reasoning consists on considering all the resources simultaneously. Indeed, in Formulation (P1), we have considered the resources separately: we have made the feasibility tests on each resource.

Let $\overline{E_k} = R_k(t_2 - t_1) - (\sum_{j \in J_L} W_{jk}^l + \sum_{j \in J_R} W_{jk}^r \sum_{j \in J_I} W_{jk}^i)$ be the work available on resource k for activities of the set $J_{LI} \cup J_{RI} \cup J_{LR} \cup J_{LIR}$.

The *Global Energetic Reasoning* is based on the following mathematical model:

$$\xi = \min \sum_{k \in K} \epsilon_k \qquad (9)$$

s.t.

$$x_j + z_j = 1, \quad \forall j \in J_{LI} \qquad (10)$$
$$y_j + z_j = 1, \quad \forall j \in J_{RI} \qquad (11)$$
$$x_j + y_j = 1, \quad \forall j \in J_{LR} \qquad (12)$$
$$x_j + y_j + z_j = 1, \quad \forall j \in J_{LIR} \qquad (13)$$
$$(P2) \quad \sum_{j \in J_{\mathcal{L}}} b_{jk} x_j \leq m_k^l, \quad \forall k \in K \qquad (14)$$
$$\sum_{j \in J_{\mathcal{R}}} b_{jk} y_j \leq m_k^r, \quad \forall k \in K \qquad (15)$$
$$\sum_{j \in J_{\mathcal{L}}} W_{jk}^l x_j + \sum_{j \in J_{\mathcal{R}}} W_{jk}^r y_j + \sum_{j \in J_{\mathcal{I}}} W_{jk}^i z_j - \epsilon_k \leq E_k, \quad \forall k \in K \qquad (16)$$
$$x_j, y_j, z_j \in \{0, 1\}, \quad \forall j \in J_{LI} \cup J_{RI} \cup J_{LR} \cup J_{LIR} \qquad (17)$$
$$\epsilon_k \geq 0, \quad \forall k \in K \qquad (18)$$

Constraints (10)-(15) have the same signification as in (P1). Constraint (16) imposes that the sum of the works (minus a nonnegative value ϵ_k) of activities placed at the left, at the right and inside the interval must be lower than the total work available on each resource k. The feasibility test becomes:

Fact 4.1 *If $\xi > 0$ then the instance is infeasible.*

5 Computational Results

In this section, we report some computational results for our lower bounds. We tested the lower bounds on the Benchmark **KSD60** of 480 instances which size is 60 activities proposed by Kolish et al. [8]. The calculated bounds are:

- **LB1**: Classical Energetic Reasoning.
- **LB2**: Revisited Energetic Reasoning.
- **LB3**: Global Energetic Reasoning.

All these bounds include time-bound adjustments described in Baptiste et al. [2] i.e. the feasibility tests are repeated until there is no more adjustments or an infeasibility is detected.

The experiments were conducted on a personal computer Core 2 Duo 1,60 Ghz with 1 GB of RAM and running under Windows XP. All the bounds were implemented using C language and CPLEX 8.1.

Table 1 presents the results of the lower bounds. We calculate the mean deviation (**GAP**) from the best known upper bound, the computation time (**TIME**), the number of times the lower bound is equal to the best upper bound (#**LB=UB**), the number of times that the lower bound is the best (#**LB=LBbest**). For the **GAP** and the **TIME**, we calculate the mean (**Avg**) and maximal (**Max**) values.

We also include the results of the lower bound of Brucker and Knust [4] (**LBB**)(this bound is calculated on Sun Ultra 2 workstation 167 MHz).

The set of intervals $[t_1, t_2]$ is $S = \{(t_1, t_2); t_1 \in \Omega_1, t_2 \in \Omega_2\}$ where:

- $\Omega_1 = \{r_j, j = 1, \ldots, n\} \cup \{r_j + p_j, j = 1, \ldots, n\} \cup \{d_j - p_j, j = 1, \ldots, n\}$
- $\Omega_2 = \{d_j, j = 1, \ldots, n\} \cup \{r_j + p_j, j = 1, \ldots, n\} \cup \{d_j - p_j, j = 1, \ldots, n\}$

Table 1
Results of the bounds on KSD60

	GAP(%)		TIME(s)		#LB=UB	#LB=LBbest
	Avg	**Max**	**Avg**	**Max**		
LB1	7.35	32.76	0.06	0.30	82	113
LB2	2.76	22.73	5.21	49.48	**345**	378
LB3	2.76	22.73	2.45	16.42	**345**	379
LBB	1.85	14.74	5.00	62.00	**342**	454

We remark that the new bounds improve the quality of **LB1**. We remark also that the new bounds present the same **GAP**. **LB3** is better than **LB2** only for one instance and is twice faster. We have also tested lower bounds based on linear relaxations of Formulations (P1) and (P2). We note that these lower bounds give the same results that the bounds (**LB2** and **LB3**) based on exact resolutions of (P1) and (P2) in less time. Compared to **LBB**, the new bounds reach the upper bound more frequently.

6   Conclusion

We have introduced two destructive lower bounds based on energetic reasoning. These lower bounds improve the Classical Energetic Reasoning. As a perspective, we plan to develop relaxations to the Revisited Energetic Reasoning in order to accelerate the computation of the lower bounds.

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