Experimental Investigation of Heuristics for Resource-Constrained Project Scheduling: An Update

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Abstract

This paper considers heuristics for the well–known resource–constrained project scheduling problem (RCPSP). It provides an update of our survey which was published in 2000. We summarize and categorize a large number of heuristics that have recently been proposed in the literature. Most of these heuristics are then evaluated in a computational study and compared on the basis of our standardized experimental design. Based on the computational results we discuss features of good heuristics. The paper closes with some remarks on our test design and a summary of the recent developments in research on heuristics for the RCPSP.

Keywords: Project Scheduling, Resource Constraints, Heuristics, Computational Evaluation.

1 Introduction

The resource–constrained project scheduling problem (RCPSP) can be stated as follows: A single project consists of a number of n activities where each activity has to be processed in order to complete the project. The activities are interrelated by two kinds of constraints. First, precedence constraints force activity j not to be started before all its immediate predecessors have been finished. Second, performing the activities requires resources with limited capacities. Altogether there is a set of $\mathcal{R}$ resources. While being processed, activity j requires $r_{j,k}$ units of resource $k \in \mathcal{R}$ in every time instant of its non–preemptable duration p_j. Resource k has a limited capacity of R_k at any point in time. The parameters p_j, $r_{j,k}$, and R_k are assumed to be nonnegative and deterministic. The objective of the RCPSP is to find precedence and resource feasible completion times for all activities such that the makespan of the project is minimized.

Since its advent (cf. Pritsker et al. [61]) the RCPSP has been a very popular and frequently studied NP–hard optimization problem (cf. Błażewicz et al. [7]). The last 20 years have witnessed a tremendous improvement of both heuristic and exact solution procedures (cf. e.g. the surveys given in Demeulemeester and Herroelen [18], Hartmann and Kolisch [26, 37], Herroelen et al. [27], Kolisch and Padman [38], and Özdamar and Ulusoy [53]). Due to the fact that the RCPSP "is one of the most intractable problems in Operations Research", it has recently "become a popular playground for the the latest optimization techniques, including virtually all local search paradigms" (cf. Möhring et al. [48] p. 330).

The paper at hand is in line with research that has been performed by the authors in [37] and [26]. In [37] we have classified the multitude of heuristic procedures for the RCPSP with respect to their building blocks such as e.g. schedule generation scheme (SGS), metaheuristic strategy, and solution representation. We have also tested the heuristics on three sets of benchmark instances from the PSPLIB library (cf. Kolisch et al. [40], Kolisch and Sprecher [39]). This research has been continued in [26] where we included new methods and performed a rigorous computational study in order to compare the heuristics, to assess the significance of the building blocks, and to evaluate the impact of problem characteristics such as e.g. the scarcity of resources. The new paper has four goals: First, we summarize new heuristics for the RCPSP which have been presented since our last survey. Second, we extend the computational comparison of [26] by including new heuristics and by adding results which have been obtained by applying a larger computational effort. Third, we try to point out promising approaches which promote the progress in the field. Finally, we provide a critical discussion of the test design and its usage by other authors. To restrict the scope of this survey, we consider only heuristics developed for the classical RCPSP. Nevertheless, we have included papers for generalizations of the RCPSP if computational results for standard test instances of the classical RCPSP are given.

2 New Heuristics for the RCPSP

This section summarizes recent heuristics from the literature. Following the categorization of our study [37] the approaches are grouped into priority rule–based X–pass methods (Section 2.1), classical metaheuristics (Section 2.2), non–standard metaheuristics (Section 2.3), and other heuristics (Section 2.4). For a detailed description of basic components of heuristics such as schedule generation schemes, priority rules, and representations, we refer to [37]. Heuristics that have already been summarized in our previous surveys will be briefly mentioned at the beginning of each Section. For a description the reader is referred to [26, 37].

2.1 X-Pass Methods

X-pass methods which have been summarized in our recent study were presented by Alvarez-Valdes and Tamarit [3], Boctor [8], Cooper [13, 14], Davies [15], Davis and Patterson [16], Elsayed [20], Klein [29], Kolisch [33, 34, 35], Kolisch and Drexl [36], Lawrence [41], Li and Willis [44], Özdamar and Ulusoy [52, 54, 55], Patterson [57, 58], Schirmer [63], Schirmer and Riesenberg [64, 65], Thesen [69], Thomas and Salhi [70], Ulusoy and Özdamar [76], Valls et al. [80], and Whitehouse and Brown [82].

Sampling. Coelho and Tavares [12] suggest a so–called global biased random sampling approach which employs the serial SGS. Whereas previous sampling methods compute probabilities for activity selection, this procedure perturbs the priority values by adding a random value $\in [0, 1]$ which is multiplied by a scaling factor. The activites are scheduled in the order prescribed by the modified priority values. An efficient implementation which exploits these "global" perturbations and avoids eligible set computations is also indicated.

2.2 Classical Metaheuristics

In this subsection we present approaches which follow well–known metaheuristic paradigms, namely genetic algorithms, tabu search, simulated annealing, and ant systems. Previously described metaheuristics for the RCPSP include Baar et al. [5], Boctor [9], Bouleimen and Lecocq [10], Cho and Kim [11], Hartmann [24], Kohlmorgen et al. [32], Lee and Kim [42], Leon and Ramamoorthy [43], Naphade et al. [49], Pinson et al. [59] as well as Sampson and Weiss [62].

Genetic algorithms (GAs). Alcaraz and Maroto [1] develop a genetic algorithm based on the activity list representation and the serial SGS. An additional gene decides whether forward or backward scheduling is employed when computing a schedule from an activity list. The crossover operator for activity lists is extended such that a child's activity list can be built up either in forward or in backward direction.

Alcaraz et al. [2] extend the genetic algorithm of Alcaraz and Maroto [1] by adding two features from the literature. First, they take the additional gene that determines the SGS to be used from Hartmann [25] (see below). Second, they employ the forward-backward improvement of Tormos and Lova [73] (see Section 2.4).

Coelho and Tavares [12] present a genetic algorithm which makes use of the activity list representation and the serial SGS. They suggest a new crossover operator for activity lists called late join function crossover. The current version of the working paper gives only the following rough verbal description of the operator: The late join function crossover constructs a new individual by "adopting the father solution and swapping each adjacent pair that is in reverse order in the mother."

Gonçalves and Mendes [23] use a random key representation and a modified parallel SGS. The modified parallel SGS determines all activities to be eligible which can be started up to the schedule time plus a delay time. The random key has twice the length of the number of activities. Each entry is a random number. The first half of the entries biases the activity selection and the second half biases the delay time of the SGS.

Hartmann [25] proposes a so-called self–adapting genetic algorithm. This approach extends the activity list representation by adding a gene that determines whether the serial or the parallel SGS is to be used for transforming an activity list into a schedule. As a prerequisite for the procedure, it is defined how the parallel SGS can be used as decoding procedure for activity lists. The choice of the more successful SGS is left to the inheritance and survival–of–the fittest mechanisms.

Hindi et al. [28] suggest a genetic algorithm based on the activity list representation, the serial SGS, and the related order–preserving crossover strategy (similar to Hartmann [24]). The initial population is produced by a pure random mechanism (whereas LFT–based sampling is used in [24]).

Toklu [72] develops a genetic algorithm which operates directly on schedules (i.e., a vector of start times). Since the genetic operators may produce infeasible offspring schedules, a penalty function is used which evaluates the constraint violations.

Valls et al. [77] extend the activity list–based genetic algorithm with forward–backward improvement of Valls et al. [78] (cf. Section 2.4) to what they call a hybrid genetic algorithm. They develop a peak crossover operator which uses properties of the schedule when recombining activity lists. Generally speaking, this operator aims at inheriting those parts of the parents' activity lists that correspond to peaks in the resource usage. Moreover, a second phase of the evolution is started from neighbors of the best individual found in the first phase. The neighbors are constructed with the approach used in Valls et al. [81] (cf. Section 2.3) which is applied to activity lists here.

Tabu search (TS). Artigues et al. [4] employ their insertion technique (cf. Section 2.4) in order to devise a tabu search procedure. Essentially, the method iteratively selects an activity which is first deleted from the schedule and afterwards re–inserted with a network flow–based insertion algorithm. Chosen activities as well as their resource predecessor and successors are elements of the tabu list.

Klein [30] develops a so–called reactive tabu search method for the RCPSP with time-varying resource constraints. It is based on the activity list representation and the serial

SGS. The neighborhood is given by swap moves which include the shifting of predecessors or successors of the swapped activities if the resulting list would otherwise not be precedence feasible (similar to Baar et al. [5]).

Nonobe and Ibaraki [50] suggest a tabu search approach for a generalized variant of the RCPSP. Considering only the features that are relevant for the standard RCPSP, the heuristic employs the activity list representation, the serial SGS, shift moves, and a specific neighborhood reduction mechanism.

Thomas and Salhi [71] introduce a tabu search method which operates directly on schedules. They define three different moves. Since the resulting neighbor schedules may be infeasible, they employ a repair procedure to turn an infeasible schedule into a feasible one.

Simulated annealing (SA). Valls et al. [78] test a simulated annealing method in a paper that focuses on forward–backward improvement (cf. Section 2.4). The neighborhood definition is taken from Valls et al. [81] (cf. Section 2.3), where a neighbor is constructed by selecting the next activity either in the order of the original solution or by biased random sampling.

Ant systems. Merkle et al. [46] present the first application of ant systems (a metaheuristic strategy developed by Dorigo et al. [19]) to the RCPSP. In their approach, a single ant corresponds to one application of the serial SGS. The eligible activity to be scheduled next is selected using a weighted evaluation of the latest start time (LST) priority rule and so–called pheromones which represent the learning effect of previous ants. A pheromone value τ_{ij} describes how promising it seems to put activity j as the i–th activity into the schedule. Further features of the approach include separate ants for forward and backward scheduling and a 2–opt–based local search phase at the end of the heuristic.

2.3 Non–Standard Metaheuristics

This subsection is devoted to approaches which can be viewed as metaheuristics although they do not follow one of the classic metaheuristic schemes. It summarizes various non–standard local search and population–based methods which have been proposed to solve the RCPSP.

Local search–oriented approaches. Fleszar and Hindi [21] apply a variable neighborhood search (VNS, a metaheuristic strategy introduced by Mladenovic and Hansen [47]) to the RCPSP. They employ the activity list representation, the serial SGS, and an enhanced shift move which allows to shift activities together with their predecessors or successors. During run–time, their approach adds precedence relations on the basis of lower bound calculations.

Palpant et al. [56] embed forward–backward scheduling with the serial SGS and constraint–based optimization of partial schedules in a local search procedure. An initial solution is generated by applying forward–backward scheduling. Afterwards, a so–called block of activities, activities which are processed in parallel or contiguously, is selected randomly and constraint propagation is employed to determine for the selected activities a minimum makespan schedule under the constraints imposed by the non–selected activities. The entire method iterates between the selection of activities, optimization of partial schedules, and forward–backward scheduling until a stopping criterion is met.

Valls et al. [81] propose a two–phase local search method. It is based on the topological order representation (which is a special case of the random key representation) and the serial SGS. Two types of moves (applied either in forward or backward direction) make use of critical activities. A third move employs random sampling within a time window derived from the current solution. The second phase starts from the neighborhood of the best solutions obtained in the first phase. A solution's neighbor is constructed by selecting the next activity either in the order of the original solution or by biased random sampling.

Population–based approaches. Debels et al. [17] apply scatter search (cf. Glover et al. [22]), a population based framework which can be viewed as a variant of genetic algorithms. It makes use of a standardized version of the topological order representation (cf. Valls et al. [81]) and the serial SGS. New solutions are produced by a crossover–like operator that follows a rough analoy to electromagnetism and essentially consists of linear combinations of solutions (it is, in a way, similar to path relinking, cf. Glover et al. [22]). Finally, forward–backward improvement is integrated (cf. Section 2.4).

Kochetov and Stolyar [31] devise an evolutionary algorithm which combines genetic algorithm, path relinking, and tabu search. Solutions are evolved and diversified in a genetic way. Evolution is done by choosing two solutions from the pool and constructing the path of solutions linking the selected solutions (path relinking, cf. Glover et al. [22]). The best solution from the path is chosen and improved via tabu search. The latter employs a neighborhood where the activity list is divided in three parts. For the first and the last part the serial SGS is employed while for the mid part the parallel SGS is used. The best solution from the tabu search is added to the population and the worst solution is removed from the population.

Valls et al. [79] employ the two–phase framework and the topological order representation of Valls et al. [81] which is described above. They introduce an implicit enumeration–based move to increase the resource utilization within a time interval. A binary (and hence crossover–like) operator is defined to produce convex combinations of solutions in the population.

Valls et al. [78] introduce several non–standard population–based schemes in a study which focuses on forward–backward improvement (cf. Section 2.4). Their schemes differ in the way parents are selected for reproduction and in the number of offspring produced for each pair of parent individuals. They use the priority value representation for which four operators are discussed. The change operator (which is considered in our computational study) replaces random positions of the first parent with the corresponding positions of the second parent. This can be viewed as a variant of the uniform crossover.

2.4 Other Methods

This subsection summarizes those heristics that can neither be classified as X–pass construction methods nor as metaheuristics. Previously considered methods of this category are those of Alvarez–Valdes and Tamarit [3], Bell and Han [6], Mausser and Lawrence [45], Oguz and Bala [51], Pollack–Johnson [60], Shaffer et al. [66], and Sprecher [67].

Forward–backward improvement (FBI). Tormos and Lova [73] develop a heuristic which applies forward–backward improvement to schedules computed by sampling. In each iteration, either the serial or the parallel SGS is used to generate a schedule by regret–based sampling with the latest finish time (LFT) priority rule. The resulting schedule is then improved by a backward–forward pass. In the backward pass, the activities are considered from right to left and scheduled at the latest feasible time (i.e., they are shifted to the right). Subsequently, in the forward pass, they are considered from left to right and scheduled at the earliest feasible time (i.e., they are shifted back to the left).

Tormos and Lova [74] enhance this approach. A so–called selective mechanism executes backward–forward improvement passes only if the schedule constructed by sampling is better than the average of the solutions generated by sampling so far.

Tormos and Lova [75] present a few refinements of [74]. In addition to backward–forward improvement passes also forward–backward improvement passes can be executed. The number of passes to be applied to a schedule is selected on the basis of the quality of the schedule.

Valls et al. [78] employ a so–called double justification procedure to improve schedules found by other heuristics. It shifts all activities within a schedule to the right and subsequently to the left in order to obtain a better schedule. In order to demonstrate its power and general applicability, it is tested by adding it to various sampling methods and metaheuristics from the literature as well as to new approaches (cf. Sections 2.2 and 2.3). The best results are reported for the extension of the activity list–based genetic algorithm of Hartmann [24].

Due to the apparently strong similarity of the justification procedure of Valls et al. [78] and the backward–forward pass of Tormos and Lova [73], we refer to both approaches by the notion of forward–backward improvement (FBI).

Further heuristics. Artigues et al [4] have devised an insertion technique based on the parallel SGS and the worst case slack (WCS) priority rule. Other piority rules for the parallel SGS are possible. The method is a multi–pass (MP) approach which solves an RCPSP instance n times where at time j ($j = 1, \dots, n$) activity j is deleted from the instance. Afterwards a network flow–based insertion algorithm is applied in order to perform a makespan–minimal extension of the schedule with activity j. From the at most n different schedules, the schedule with the minimum makespan is selected. The complexity of the entire method is $O(n^3 \cdot m)$ where m denotes the number of renewable resources.

Möhring et al. [48] propose a Lagrange heuristic. The method first generates an upper bound of the project makespan by employing a multi priority rule method. Afterwards, a

Lagrange relaxation (LR) and a list scheduling heuristic are invoked iteratively, in order to generate lower and upper bounds for the RCPSP. The Lagrange relaxation of the RCPSP is solved in polynomial time and the precedence feasible start times are employed as input for a list scheduling procedure. The latter generates a number of different precedence and resource feasible schedules by scheduling the activities in non–increasing order of their start time plus the processing times multiplied by a constant δ ($0 \leq \delta \leq 1$).

Sprecher [68] proposes a network decomposition technique which incorporates exact methodologies into heuristic search. In each iteration, an initial schedule is generated by biased random sampling employing the latest finish time (LFT) priority rule. On the basis of the generated schedule, the problem is divided into subproblems which are solved with the truncated version of the branch–and–bound method of Sprecher [67]. The schedules of the subprojects are concatenated to an improved schedule for the overall project.

3 Computational Comparison

3.1 Test Design

This section presents a computational comparison of many of the heuristics that have been reviewed in Section 2. We use the same test sets and the same stopping criterion as in our previous comparison [26]. We employ the three test sets J30, J60, and J120 that have been constructed by the instance generator ProGen (see Kolisch et al. [40]). The projects in these test sets consist of 30, 60, and 120 activities, respectively. Each set has been generated using a full factorial design of parameters which determine the characteristics of the resource and precedence constraints. In total, we have 480 instances with 30 activities, 480 instances with 60 activities, and 600 instances with 120 activities. The instances have been used by many researchers, and they are available from the project scheduling library PSPLIB in the internet. For more detailed information on the test sets, we refer to Kolisch and Sprecher [39].

As in our last study, we limited the number of generated schedules in the heuristics in order to provide the basis for the comparison. This is based on the assumption that the computational effort for constructing one schedule is similar in most heuristics. This holds in particular for methods which apply the serial or parallel SGS; one pass of an SGS with one start time assignment per activity counts as one schedule. As in our previous comparison, we have selected 1,000 and 5,000 schedules as stopping criteria. Since the speed of computers has increased, larger numbers of schedules can be computed within acceptable run–times. Therefore, we have selected 50,000 schedules as an additional limit.

The advantage of this stopping criterion is that it is independent of the computer platform. Therefore, all heuristics could be tested by their author(s) using the original implementation and the best configuration. Also, future studies can easily make use of the benchmark results presented here by applying the same stopping criterion. Moreover, the tests are independent of compilers and implementational skills, thus we evaluate heuristic concepts rather than program codes. However, the stopping criterion also has a few drawbacks. First, it cannot be applied to all heuristics. For example, it cannot be used if backtracking steps or mixed integer program–based (MIP) methods are included. Nevertheless, the stopping criterion is applicable to most heuristics that have been proposed. Second, different heuristics may require different computation times to compute one schedule. For example, the serial SGS is faster within a metaheuristic based on the activity list representation than within a priority rule–based sampling method (the former simply picks the next activity to be scheduled from the list while the latter has to compute the eligible set, priority values, and selection probabilities). But these differences are rather small—using a time limit with different computers, operating systems, programming languages, and implementational skills would surely lead to much greater inaccuracies. Summing up, we believe that limiting the number of schedules is the best criterion available for such a broad comparison.

After the presentation of the computational results and a summary of the main observations in Subsection 3.2, we close this section with a critical discussion of the stopping criterion in Subsection 3.3.

3.2 Experimental Results

3.2.1 Performance of the Heuristics

The computational results of the tested heuristics can be found in Tables 1, 2, and 3 for the instance sets J30, J60, and J120, respectively. These tables extend those of our previous comparison study [26]. Each heuristic is briefly described by a few keywords, the SGS employed, and the reference. For the J30 set, the results are given in terms of average deviation from the optimal solution. For the other two sets, some of the optimal solutions are unknown. Thus, the average deviation from the well–known critical path–based lower bound is reported.

Each of the three tables is divided into three blocks. The first block considers the test design described in the previous subsection. Here, the results for the three stopping criteria (maximum of 1,000, 5,000, and 50,000 schedules, respectively) are given. Many researchers used our test design in their papers; if results according to our test design were available, we cite them here. Since these papers usually did not cover all test sets and/or stopping criteria, some researchers sent us additional results for this study which are given here as well (note that we accepted only additional results, but no results that improved previously published ones). In order to obtain these additional results, only adjustments of parameter values were allowed but no methodological modifications. These restrictions were necessary to ensure that the results presented here are consistent with the description of the heuristics in the cited papers. In some papers, our stopping criteria were not applied correctly (this was the case for [50], [73], [74], [75]). In these cases, the authors sent us corrected results which are reported here.

Some researchers did not provide results according to our stopping criteria (note that for some methods it is impossible to count the number of schedules in the way required by our criterion, consider in particular the approaches of Möhring et al. [48], Sprecher [68], and Valls et al. [79]). As long as they used the test sets employed here, we have added the results given in their papers in the second or the third block of the tables. The second block of each table contains algorithms where the average deviation together with the average and the maximal number of schedules required is given. Contrary, the third block reports the results of heuristics together with the average and maximal computation time as well as the clockpulse of the computer used. Of course, for the heuristics reported in blocks two and three, the computational effort has to be taken into account when interpreting the results; observe that the reported effort greatly varies between the different methods.

In each block, the heuristics are sorted with respect to increasing deviation. In the first block, the methods are sorted with respect to the results for 50,000 schedules. In case of ties we use the results for 5,000 schedules. Let us also remark that in general only the best performing heuristic of a paper has been considered here. Only if a paper considers substantially different new approaches or if it leads to additional insight, more than one heuristic is considered.

To determine the best heuristics, we use the concept of dominance. A heuristic a is dominated by a heuristic b if a has for at least one combination of instance set and number of generated schedules a higher average deviation than b without having for any of the other combinations a lower average deviation. In our previous study only two heuristics where non–dominated: the procedure of Hartmann [24] and the one of Bouleimen and Lecocq [10]. Using this benchmark for the current study we find six new approaches which dominate each of the two benchmark heuristics. These are (in alphabetical order): Alcaraz et al. [2], Debels et al. [17], Hartmann [25], Kochetov and Stolyar [31], and Valls et al. [77, 78]. Out of these six algorithms, only those of Alcaraz et al. [2], Debels et al. [17], Kochetov and Stolyar [31], and Valls et al. [77] are not dominated by any other heuristic. Hence, these four non–dominated procedures are the best heuristics in our comparison.

Furthermore, several of the new methods of the first block which do not dominate the benchmark outperform the two benchmark heuristics on some instance–schedule–combinations. Finally, some of the heuristics from the second and the third block of the Tables 1 – 3 which were not tested using our test design show a lower average deviation as the benchmark as well.

The overall improvement of heuristic performance can best be seen on the J120 set where the deviations of several new methods are substantially smaller than those reported in our last survey. On the J30 set, several heuristics are now on average so close to the optimum that this test set can hardly be used to evaluate heuristics any longer.

3.2.2 Characteristics of Good Heuristics

In our last study we have resumed that "the best heuristics in our computational analysis are metaheuristics based on the activity list representation and the serial SGS." Furthermore, we have proposed that the use of the parallel scheduling scheme might be beneficial for large problems. Based on the new results in general and the heuristics which dominate the benchmark in particular we can add the following observations.

Algorithm	SGS	Reference	max. #schedules 1,000	5,000	50,000	
GA, TS – path relinking	both	Kochetov, Stolyar [31]	0.10	0.04	0.00	
Scatter Search – FBI	serial	Debels et al. [17]	0.27	0.11	0.01	
GA – hybrid, FBI	serial	Valls et al. [77]	0.27	0.06	0.02	
GA – FBI	serial	Valls et al. [78]	0.34	0.20	0.02	
GA – forw.-backw., FBI	both	Alcaraz et al. [2]	0.25	0.06	0.03	
GA – forw.-backward	serial	Alcaraz, Maroto [1]	0.33	0.12	–	
sampling – LFT, FBI	both	Tormos, Lova [75]	0.25	0.13	0.05	
TS – activity list	serial	Nonobe, Ibaraki [50]	0.46	0.16	0.05	
sampling – LFT, FBI	both	Tormos, Lova [73]	0.30	0.16	0.07	
GA – self-adapting	both	Hartmann [25]	0.38	0.22	0.08	
GA – activity list	serial	Hartmann [24]	0.54	0.25	0.08	
sampling – LFT, FBI	both	Tormos, Lova [74]	0.30	0.17	0.09	
TS – activity list	serial	Klein [30]	0.42	0.17	–	
sampling – random, FBI	serial	Valls et al. [78]	0.46	0.28	0.11	
SA – activity list	serial	Bouleimen, Lecocq [10]	0.38	0.23	–	
GA – late join	serial	Coelho, Tavares [12]	0.74	0.33	0.16	
sampling – adaptive	both	Schirmer [63]	0.65	0.44	–	
TS – schedule scheme	related	Baar et al. [5]	0.86	0.44	–	
sampling – adaptive	both	Kolisch, Drexl [36]	0.74	0.52	–	
GA – random key	serial	Hartmann [24]	1.03	0.56	0.23	
sampling – LFT	serial	Kolisch [35]	0.83	0.53	0.27	
sampling – global	serial	Coelho, Tavares [12]	0.81	0.54	0.28	
sampling – random	serial	Kolisch [33]	1.44	1.00	0.51	
GA – priority rule	serial	Hartmann [24]	1.38	1.12	0.88	
sampling – WCS	parallel	Kolisch [34, 35]	1.40	1.28	–	
sampling – LFT	parallel	Kolisch [35]	1.40	1.29	1.13	
sampling – random	parallel	Kolisch [33]	1.77	1.48	1.22	
GA – problem space	mod. par.	Leon, Ramamoorthy [43]	2.08	1.59	–	
				#schedules		
			result	average	max.	
GA – activity list	serial	Hindi et al. [28]	0.37	1,683	3,068	
MP – network flow	parallel	Artigues et al. [4]	1.74	30	30	
				CPU-time (sec)		
			result	average	max.	computer
decompos. & local opt.	serial	Palpant et al. [56]	0.00	10.26	123.0	2.3 GHz
VNS – activity list	serial	Fleszar, Hindi [21]	0.01	0.64	5.9	1.0 GHz
local search – critical	serial	Valls et al. [81]	0.06	1.61	6.2	400 MHz
population–based	serial	Valls et al. [79]	0.10	1.16	5.5	400 MHz
network decomposition	–	Sprecher [68]	0.12	2.75	39.7	166 MHz

Table 1: Average deviations (%) from optimal makespan — ProGen set $J = 30$

			max. #schedules			
Algorithm	SGS	Reference	1,000	5,000	50,000	
Scatter Search – FBI	serial	Debels et al. [17]	11.73	11.10	10.71	
GA – hybrid, FBI	serial	Valls et al. [77]	11.56	11.10	10.73	
GA, TS – path relinking	both	Kochetov, Stolyar [31]	11.71	11.17	10.74	
GA – FBI	serial	Valls et al. [78]	12.21	11.27	10.74	
GA – forw.-backw., FBI	both	Alcaraz et al. [2]	11.89	11.19	10.84	
GA – self-adapting	both	Hartmann [25]	12.21	11.70	11.21	
GA – activity list	serial	Hartmann [24]	12.68	11.89	11.23	
sampling – LFT, FBI	both	Tormos, Lova [75]	11.88	11.62	11.36	
sampling – LFT, FBI	both	Tormos, Lova [74]	12.14	11.82	11.47	
GA – forw.-backward	serial	Alcaraz, Maroto [1]	12.57	11.86	–	
sampling – LFT, FBI	both	Tormos, Lova [73]	12.18	11.87	11.54	
SA – activity list	serial	Bouleimen, Lecocq [10]	12.75	11.90	–	
TS – activity list	serial	Klein [30]	12.77	12.03	–	
TS – activity list	serial	Nonobe, Ibaraki [50]	12.97	12.18	11.58	
sampling – random, FBI	serial	Valls et al. [78]	12.73	12.35	11.94	
sampling – adaptive	both	Schirmer [63]	12.94	12.58	–	
GA – late join	serial	Coelho, Tavares [12]	13.28	12.63	11.94	
GA – random key	serial	Hartmann [24]	14.68	13.32	12.25	
GA – priority rule	serial	Hartmann [24]	13.30	12.74	12.26	
sampling – adaptive	both	Kolisch, Drexl [36]	13.51	13.06	–	
sampling – WCS	parallel	Kolisch [34, 35]	13.66	13.21	–	
sampling – global	serial	Coelho, Tavares [12]	13.80	13.31	12.83	
sampling – LFT	parallel	Kolisch [35]	13.59	13.23	12.85	
TS – schedule scheme	related	Baar et al. [5]	13.80	13.48	–	
GA – problem space	mod. par.	Leon, Ramamoorthy [43]	14.33	13.49	–	
sampling – LFT	serial	Kolisch [35]	13.96	13.53	12.97	
sampling – random	parallel	Kolisch [33]	14.89	14.30	13.66	
sampling – random	serial	Kolisch [33]	15.94	15.17	14.22	
				#schedules		
			result	average	max.	
VNS – activity list	serial	Fleszar, Hindi [21]	10.94	152,503	1.7 mio	
MP – network flow	parallel	Artigues et al. [4]	14.20	60	60	
				CPU-time (sec)		
			result	average	max.	computer
decompos. & local opt.	serial	Palpant et al. [56]	10.81	38.8	223.0	2.3 GHz
population–based	serial	Valls et al. [79]	10.89	3.7	22.6	400 MHz
local search – critical	serial	Valls et al. [81]	11.45	2.8	14.6	400 MHz
network decomposition	–	Sprecher [68]	11.61	460.2	4311.5	166 MHz
TS – network flow	parallel	Artigues et al. [4]	12.05	3.2	–	450 MHz
LR – activity list	both, mod.par.	Möhring et al. [48]	15.60	6.9	57	200 MHz

Table 2: Average deviations (%) from critical path lower bound — ProGen set $J = 60$

Algorithm	SGS	Reference	max. #schedules 1,000	5,000	50,000	
GA – hybrid, FBI	serial	Valls et al. [77]	34.07	32.54	31.24	
GA – forw.-backw., FBI	both	Alcaraz et al. [2]	36.53	33.91	31.49	
Scatter Search – FBI	serial	Debels et al. [17]	35.22	33.10	31.57	
GA – FBI	serial	Valls et al. [78]	35.39	33.24	31.58	
GA, TS – path relinking	both	Kochetov, Stolyar [31]	34.74	33.36	32.06	
population–based – FBI	serial	Valls et al. [78]	35.18	34.02	32.81	
GA – self-adapting	both	Hartmann [25]	37.19	35.39	33.21	
sampling – LFT, FBI	both	Tormos, Lova [75]	35.01	34.41	33.71	
ant system	serial	Merkle et al. [46]	–	35.43	–	
GA – activity list	serial	Hartmann [24]	39.37	36.74	34.03	
sampling – LFT, FBI	both	Tormos, Lova [74]	36.24	35.56	34.77	
sampling – LFT, FBI	both	Tormos, Lova [73]	36.49	35.81	35.01	
GA – forw.-backward	serial	Alcaraz, Maroto [1]	39.36	36.57	–	
TS – activity list	serial	Nonobe, Ibaraki [50]	40.86	37.88	35.85	
GA – late join	serial	Coelho, Tavares [12]	39.97	38.41	36.44	
sampling – random, FBI	serial	Valls et al. [78]	38.21	37.47	36.46	
SA – activity list	serial	Bouleimen, Lecocq [10]	42.81	37.68	–	
GA – priority rule	serial	Hartmann [24]	39.93	38.49	36.51	
sampling – adaptive	both	Schirmer [63]	39.85	38.70	–	
sampling – LFT	parallel	Kolisch [35]	39.60	38.75	37.74	
sampling – WCS	parallel	Kolisch [34, 35]	39.65	38.77	–	
GA – random key	serial	Hartmann [24]	45.82	42.25	38.83	
sampling – adaptive	both	Kolisch, Drexl [36]	41.37	40.45	–	
sampling – global	serial	Coelho, Tavares [12]	41.36	40.46	39.41	
GA – problem space	mod. par.	Leon, Ramamoorthy [43]	42.91	40.69	–	
sampling – LFT	serial	Kolisch [35]	42.84	41.84	40.63	
sampling – random	parallel	Kolisch [33]	44.46	43.05	41.44	
sampling – random	serial	Kolisch [33]	49.25	47.61	45.60	
			result	#schedules average	max.	
VNS – activity list	serial	Fleszar, Hindi [21]	33.10	1.9 mio	10.8 mio	
MP – network flow	parallel	Artigues et al. [4]	39.34	120	120	
			result	CPU-time (sec) average	max.	computer
population–based	serial	Valls et al. [79]	31.58	59.4	264.0	400 MHz
decompos. & local opt.	serial	Palpant et al. [56]	32.41	207.9	501.0	2.3 GHz
local search – critical	serial	Valls et al. [81]	34.53	17.0	43.9	400 MHz
LR – activity list	both, mod. par.	Möhring et al. [48]	36.00	72.9	654.0	200 MHz
TS – network flow	parallel	Artigues et al. [4]	36.16	67.0	–	450 MHz
network decomposition	–	Sprecher [68]	39.29	458.5	1511.3	166 MHz

Table 3: Average deviations (%) from critical path lower bound — ProGen set $J = 120$

Metaheuristic and Population–based Approaches. Again, the best performing methods are metaheuristics. The six dominating procedures follow a population–based metaheuristic approach. However, for the RCPSP, pure genetic algorithms are hardly developed anymore. Instead, the basic genetic algorithm scheme is modified, or it is extended by integrating further features such as path relinking, forward–backward improvement, self–adapting mechanisms, non–standard crossover techniques, or even other metaheuristics. Sometimes, various modifications and extensions are applied within the same heuristic. While several of these approaches lead to excellent results, it remains unclear in some papers if all modifications and extensions really contribute to the performance.

The best approach in our study that is not a metaheuristic is the sampling method with forward–backward improvement of Tormos and Lova [75]. It is interesting to note that this approach leads to excellent solutions after computing 1,000 schedules but produces only relatively small improvements when computing more schedules. In contrast, the metaheuristics show much larger improvements of the solution quality when computing more schedules. This is due to the fact that they exploit learning effects during run–time.

Schedule Generation Scheme (SGS). In our last study [26] we have stressed the success of the serial SGS. With all of the six dominating methods employing the serial SGS this observation still holds. Three of the six dominating procedures employ the parallel SGS in addition to the serial one. Since neither the serial nor the parallel SGS is consistently superior (which is demonstrated by the results of the sampling methods), it appears to be a good idea to employ both. In fact, comparing the results of Hartmann [25] with those of Hartmann [24], we can state that using the parallel SGS in addition to the serial one improves the results. While most methods that use both SGS construct a single schedule either with the serial or with the parallel one, Kochetov and Stolyar [31] take a different approach. They employ each of the SGS to construct just a part of a schedule. Thus, there are different ways to successfully employ both SGS within one heuristic.

Forward–backward improvement (FBI). A noteworthy trend is the use of forward–backward improvement in a surprisingly large number of recent papers (Alcaraz et al. [2], Debels et al. [17], Tormos and Lova [73, 74, 75], Valls et al. [77, 78]). These heuristics are among the best in our study, and four of the six dominating heuristics include FBI. The power of FBI is also demonstrated by Valls et al. [78] who add it to the simplest project scheduling heuristic, i.e. pure random sampling. Tables 1 – 3 show that adding FBI to random sampling leads to much better results than adding a priority rule. In fact, on the set J120, random sampling with FBI (which is still a remarkably simple procedure) obtains better results than several more complex approaches including that of Bouleimen and Lecocq [10], one of the two non–dominated methods in our last survey. Moreover, as demonstrated by Valls et al. [79], FBI can easily be added to any existing heuristic for the RCPSP because it can be applied to any intermediate schedule. This makes FBI a promising building block of heuristics.

3.3 Critical Remarks on the Use of the Test Design

In the recent years, many researchers have adopted our test design when evaluating their heuristics. This allowed them to compare their approaches with many other heuristics from the literature. However, in some cases our test design has not been used correctly because the number of schedules has not been counted accurately.

In order to avoid misunderstandings or deviating interpretations in the future, we wish to clarify again the assumptions of our stopping criterion. The heuristic to be evaluated is stopped after a certain maximal number of schedules have been constructed (1,000, 5,000, or 50,000 schedules). Generating one schedule corresponds to (at most) one start time assigment per activity, as done by an SGS (regardless of the computational effort, cf. also the discussion in Section 3.1). Each schedule must be counted. This includes, e.g., constructed but then rejected neighbor schedules in local search methods. If FBI is applied, then each single pass has to be counted (e.g., a backward–forward–pass would imply two schedules in addition to the original one). A schedule must be counted as one whole schedule even if it is not constructed completely. In fact, methods in our comparison such as [25] employ approaches to abort the completion of inferior schedules to reduce the computation time, but this does not affect the schedule count.

Researchers who cannot apply the criterion (because their heuristics do not build schedules with an SGS) might still wish to incorporate our comparison into their studies. They may find a suitable way to do so which is, of course, fine.

4 Conclusions

During the years since our last survey [26], research on project scheduling heuristics has led to both a wide range of new methodological ideas and a substantial improvement of computational results with respect to our test design. In our last comparison, the best performing approaches were the genetic algorithm of Hartmann [24] and the simulated annealing procedure of Bouleimen and Lecocq [10]. Now several new methods clearly outperform these former benchmark approaches. These are (in alphabetical order): Alcaraz et al. [2], Debels et al. [17], Hartmann [25], Kochetov and Stolyar [31], and Valls et al. [77, 78].

Starting with the recent work of Tormos and Lova [73], several new papers applied a forward–backward improvement technique (also called justification) to improve schedules constructed by X–pass methods or metaheuristics. This simple procedure—the activities are shifted to the right within the schedule and then to the left—produces excellent results and can be combined with almost any other approach. Four of the six best approaches listed above make use of forward–backward improvement. We expect that forward–backward improvement will become an important component in future heuristics for the RCPSP.

Another main research focus has been on metaheuristics. Genetic algorithms and tabu search have been the most popular strategies. Moreover, the first application of ant systems to the RCPSP as well as various non–standard local search– and population–

based schemes have been proposed. The activity list has been the most widely used representation. It has usually been employed in its classical form, while a few researchers have extended it. Considering the advantages of representations like the activity list, one may in fact wonder why some recent metaheuristics still employ the direct schedule representation with operators that are very likely to produce infeasible solutions (cf. [71], [72]).

Priority rule–based X–pass methods have attracted less attention. As already pointed out in our last survey, they are inferior to metaheuristic approaches which are capable of learning. Finally, several heuristics have been developed which can neither be classified as X–pass methods nor as metaheuristics. Such approaches include a Lagrange method and strategies based on decomposition and optimization. Although these approaches have not yet yielded competitive results, we view them as interesting and promising.

Considering the development during the last years, a general observation is that the recently proposed heuristics (including the six best performing ones listed above) contain more components than earlier procedures. Many methods consider both scheduling directions instead of only forward scheduling, both SGS instead of only one, more than one type of local search operator, or even more than one type of metaheuristic strategy. While recombining merely existing ideas occasionally seems to be less creative than developing new ideas, some of the integration efforts have put well–known techniques into a new and promising context, and the results have often been encouraging.

With the standard test sets which are available on the internet and the computer–independent stopping criterion, the test design and benchmark results of this paper can easily be used by other researchers in future studies. In a few recent papers, however, our comparison has not been used appropriately because the number of schedules (which is the basis for the comparison) was not counted accurately. Therefore, we have emphasized the assumptions of our test design. We believe that the test design and the benchmark results from our previous study have already contributed to significant improvements of heuristic results. On the other hand, such easy comparisons might motivate researchers rather to improve the benchmark results with recombinations or modifications of existing approaches than to develop new and innovative ideas. This is not our intention, and we would like to emphasize the value of new methodologies (even if they are not fully competitive).

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Abbreviations

FBI	Forward–backward improvement
GA	Genetic algorithm
LFT	Latest finish time (priority rule)
LR	Lagrange Relaxation
LST	Latest start time (priority rule)
MIP	Mixed Integer Program
MP	Multi–Pass
SGS	Schedule generation scheme
TS	Tabu search
VNS	Variable neighborhood search
WCS	Worst case slack (priority rule)

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