$\boxed{Q3}$ ① $\psi_m(x) = \phi(x - s^{(m)})$

a) By def. of $y(x) = w_0 + \sum_{m=1}^{M} w_m \phi(x - s^{(m)}) \Rightarrow$

$$y(x^{(n)}) = \begin{bmatrix} w_0 + \sum_{m=1}^{M} w_m \phi(x^{(1)} - s^{(m)}) \\ \vdots \\ w_0 + \sum_{m=1}^{M} w_m \phi(x^{(n)} - s^{(m)}) \end{bmatrix} \overset{\text{by ①}}{=} \begin{bmatrix} w_0 + \sum_{m=1}^{M} w_m \psi_m(x^{(1)}) \\ \vdots \\ w_0 + \sum_{m=1}^{M} w_m \psi_m(x^{(n)}) \end{bmatrix}$$

$$= \begin{bmatrix} w_0 + w_1 \psi_1(x^{(1)}) + \dots w_m \psi_m(x^{(1)}) \\ \vdots \\ w^0 + w_1 \psi_1(x^{(n)}) + \dots + w_m \psi_m(x^{(n)}) \end{bmatrix} = \begin{bmatrix} 1 & \psi_1(x^{(1)}) \dots \psi_m(x^{(1)}) \\ \vdots & \vdots \\ 1 & \psi_1(x^{(n)}) \dots \psi_m(x^{(n)}) \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_m \end{bmatrix}$$

$$= K_{n(m+1)} \vec{w}_{m+1}$$

So Kernel Matrix can be multiplied by a vector $\vec{w}$ to get $y(x^{(n)})$.

b) $\|w\|^2 = \sum_m w_m^2$

$$\frac{\partial \|w\|^2}{\partial \vec{w}} = \frac{\partial (w_1^2 + w_2^2 + \dots + w_m^2)}{\partial \vec{w}}$$

$$\frac{\partial (w_1^2 + w_2^2 + \dots + w_m^2)}{\partial \vec{w}} = \begin{bmatrix} \frac{\partial (w_1^2 + w_2^2 + \dots w_m^2)}{\partial w_1} \\ \frac{\partial (w_1^2 + w_2^2 + \dots + w_m^2)}{\partial w_2} \\ \vdots \\ \frac{\partial (w_1^2 + w_2^2 + \dots + w_m^2)}{\partial w_m} \end{bmatrix} = \begin{bmatrix} 2w_1 \\ 2w_2 \\ \vdots \\ 2w_m \end{bmatrix} =$$

$$= 2\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} = 2\vec{w}$$

So, $\frac{\partial \|w\|^2}{\partial \vec{w}} = 2\vec{w}$ as needed.