Machine Learning – December 18, 2023

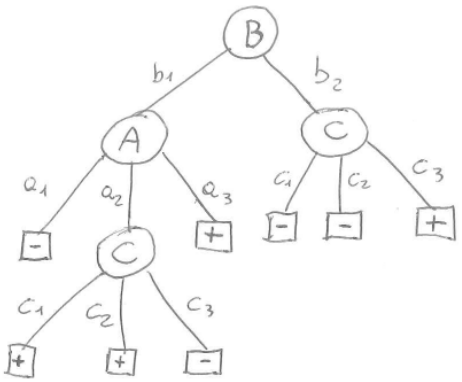
Last Name ..... First Name ..... Matricola .....

1. No books, slides, written notes are allowed during the exam.
2. Answers must be explicitly marked with the question they refer to (e.g., **2.1** for question 1 of exercise 2). Cumulative answers which refer to more questions will be evaluated as answering one question only.

Time limit: **1h 45min**.

EXERCISE 1

Given a classification problem for the function $f : A \times B \times C \to \{+, -\}$, with $A = \{a_1, a_2, a_3\}, B = \{b_1, b_2\}, C = \{c_1, c_2, c_3\}$ and the following decision tree T that is the result of a learning algorithm on a given data set:



1. Provide a rule based representation of the tree T.
2. Provide a **formal definition** of consistency of an hypothesis with respect to a dataset
3. Determine if the tree T is consistent with the following set of samples $S \equiv \{s_1 = \langle a_1, b_1, c_1, - \rangle, s_2 = \langle a_2, b_1, c_2, + \rangle, s_3 = \langle a_1, b_2, c_3, + \rangle, s_4 = \langle a_2, b_2, c_2, + \rangle\}$. Show all the passages needed to get to the answer.

EXERCISE 2

In Bayesian Learning, given a data set D and a hypothesis h, we can express the following relationship between the probability distributions (Bayes theorem):

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

In this context:

1. define *Maximum a posteriori* (MAP) hypotheses and *Maximum likelihood* (ML) hypotheses.
2. formally describe the concept of *Naive Bayes Classifier*
3. describe the assumption made in *Naive Bayes Classifier* and provide a comment about this assumption in terms of practical applicability of the method.

EXERCISE 3

1. Describe the perceptron model for classification and its training rule.
2. Draw a graphical representation of a linearly separable 2D data set for binary classification and provide a qualitative graphical example of a possible evolution of perceptron training (4 images showing a possible temporal evolution of the solution of the algorithm on the sketched data set, with the last image showing a possible final solution).

EXERCISE 4

1. Given a data set D, denoted with its design matrix $\mathbf{X}$ and its output vector $\mathbf{t}$, **formally** describe a kernelized linear model and provide the **formal** definition of the Gram matrix.
2. If the target function is $f : \Re^4 \rightarrow C$, with $|C| = 3$ and D contains 100 samples, provide the dimensions of all the elements of the kernelized linear model.

EXERCISE 5

1. Describe the role of the following algorithms related to parameter estimation of an artificial neural network:
   - Backpropagation
   - Stochastic Gradient Descent
2. Provide the main steps of the Stochastic Gradient Descent algorithm, highlight the hyperparameter(s) of the algorithm and discuss the sensitivity of the solution with respect to the hyperparameters.