Machine Learning – January 16, 2023

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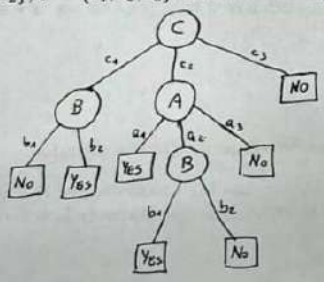
Notes

1. No books, slides, written notes are allowed during the exam.
2. Answers must be explicitly marked with the question they refer to (e.g., **2.1** for question 1 of exercise 2). Cumulative answers which refer to more questions will be evaluated as answering one question only.

Time limit: **2 hours**.

EXERCISE 1

Given a classification problem for the function $f : A \times B \times C \rightarrow \{YES, NO\}$, with $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2\}$, $C = \{c_1, c_2, c_3\}$ and the following decision tree T that is the result of training on some data set:



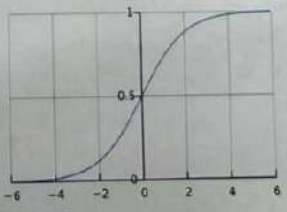
1. Provide a rule based representation of the tree T.
2. Determine if the tree T is consistent with the following set of samples
   $S = \{s_1 = \langle a_1, b_1, c_1, NO\rangle, s_2 = \langle a_2, b_1, c_2, YES\rangle, s_3 = \langle a_1, b_2, c_3, NO\rangle, s_4 = \langle a_3, b_2, c_1, NO\rangle\}$.
   Motivate your answer.
3. Compute the accuracy of T with respect to S.

EXERCISE 2

1. Define mathematically the problem solved by logistic regression
2. Consider the following dataset, two solutions $\vec{w}_1^T$ and $\vec{w}_2^T$ and the sigmoid function plotted below:

x_1	x_2	x_3	t
0	0	1	0
1	2	3	0
4	4	1	1

$\vec{w}_1^T = (2, 0, -2)$
$\vec{w}_2^T = (-2, -2, 4)$



Which one among these solutions fits the data better? Why?
(You do not need to compute explicit values of the model.)

EXERCISE 3

Consider the the k-armed bandit problem (also known as One-state MDP) with stochastic Gaussian behavior and unknown reward functions.

1. **Formally** describe the *model*, provide mathematical definitions and explanations of all the elements of the model.
2. **Formally** describe the *problem* and the *solution concept* (i.e., what is the problem to be solved with the model described in the previous point), provide mathematical definitions and explanations of all the elements of the problem.
3. **Formally** describe the Reinforcement Learning algorithm to compute the optimal policy for this problem, provide explanations of all the terms of the algorithm.

EXERCISE 4

Consider an image classification problem for RGB color images (3 channels) with resolution 128×128 and a LeNet-like CNN model, with a first convolutional layer formed by 5 kernels 7×7 with stride 1 and same padding, followed by max pooling 2×2 with stride 2 and valid padding.

1. Determine the number of trainable parameters of the first convolutional layer.
2. Determine the size of the feature maps after the first convolutional layer.
3. Provide a numerical example (invent some numbers) of the application of a max pooling operation 2×2 with stride 1 and valid padding to a feature map $8 \times 8 \times 1$.

EXERCISE 5

1. Qualitatively explain the maximum margin principle applied to Non-Linear SVM classification.
2. Draw an example of a 2-D binary non-linear classification problem showing qualitatively: i) a generic solution, and ii) a maximum margin solution computed by SVM, describing qualitatively how to obtain such a solution.

EXERCISE 6

1. Describe the concept of bagging in the definition of an ensemble model. Describe precisely the training procedure for such a model and the final formula used for prediction.
2. Discuss the difference between bagging and voting, highlighting in particular the different use of the training data.