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## *Robotics 2*

# Hybrid Force/Motion Control

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# Hybrid force/motion control

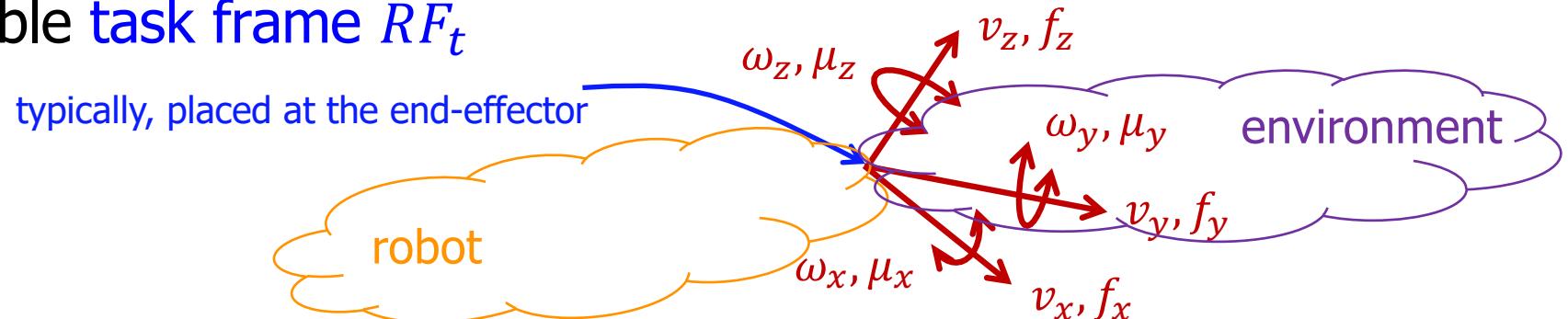
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- we consider **contacts/interactions** between a robot and a stiff environment that **naturally constrains** the end-effector motion
- **compared** to an approach using the constrained/reduced robot dynamics with (bilateral) **geometric constraints**, the **differences** are
  - the hybrid control law is designed in **ideal conditions**, but now unconstrained directions of motion and constrained force directions are defined in a more direct way using a **task frame formalism**
  - all **non-ideal conditions** (compliant surfaces, friction at the contact, errors in contact surface orientation) are handled explicitly in the control scheme by a **geometric filtering of the measured quantities**
    - considering only signal components that should appear in certain directions based on the nominal task model, and treating those that should not be there as **disturbances** to be rejected
- the hybrid control law avoids to introduce conflicting behaviors (force vs. motion control) in any of the task space directions!!



# Natural constraints

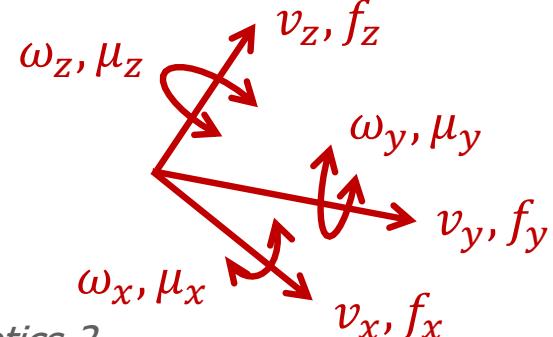
- in **ideal conditions** (robot and environment are perfectly rigid, contact is frictionless), **two sets of generalized directions** can be defined in the **task space**
  - end-effector motion ( $v/\omega$ ) is prohibited along/around  **$6 - k$  directions** (since the environment reacts there with forces/torques)
  - reaction forces/torques ( $f/\mu$ ) are absent along/around  **$k$  directions** (where the environment does not prevent end-effector motions)
- these constraints have been called the **natural constraints** on motion and force associated to the task geometry
- the two sets of directions are characterized through the axes of a suitable **task frame  $RF_t$**





# Artificial constraints

- the way **task execution** should be performed can be expressed in terms of so-called **artificial constraints** that specify the desired values (to be imposed by the control law)
  - for the **end-effector velocities** ( $v/\omega$ ) along/around  $k$  directions where feasible motions can occur
  - for the **contact forces/torques** ( $f/\mu$ ) along/around  $6 - k$  directions where admissible reactions of the environment can occur
- the two sets of directions are **complementary** (they cover the 6D generalized task space) and mutually **orthogonal**, while the **task frame** can be **time-varying** ("moves with task progress")
  - directions are intended as 6D **screws**: twists  $V = (v^T \omega^T)^T$  and wrenches  $F = (f^T \mu^T)^T$

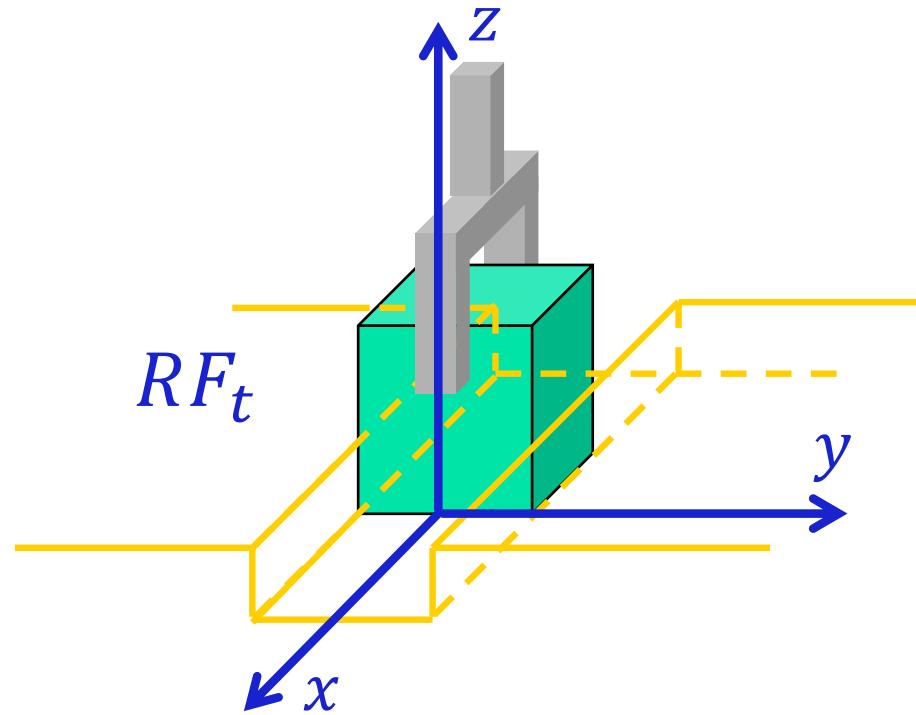


$$F^T V = 0 \Leftrightarrow \text{orthogonality}$$

but **ill-defined** (don't use it!) for  $V_1^T V_2$  or  $F_1^T F_2$



# Task frame and constraints - example 1



$v$  = linear velocity  
 $\omega$  = angular velocity  
 $f$  = force  
 $\mu$  = torque

task: slide the cube along a guide

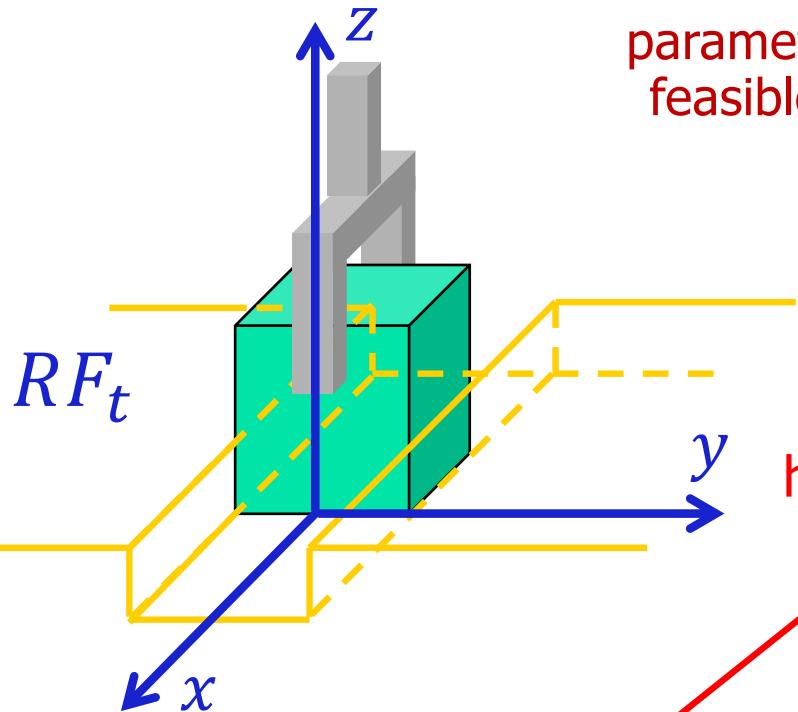
natural (geometric) constraints

$$\left. \begin{array}{l} v_y = v_z = 0 \\ \omega_x = \omega_z = 0 \\ f_x = \mu_y = 0 \end{array} \right\} \begin{array}{l} 6 - k = 4 \\ k = 2 \end{array}$$

$$6 - k = 4 \quad \left\{ \begin{array}{l} f_y = f_{y,des} (= 0) \text{ (to avoid internal stress)} \\ \mu_x = \mu_{x,des} (= 0), \mu_z = \mu_{z,des} (= 0) \\ f_z = f_{z,des} \text{ (to keep contact)} \\ \omega_y = \omega_{y,des} = 0 \text{ (to slide and not to roll !!)} \\ v_x = v_{x,des} \end{array} \right. \quad k = 2$$



# Selection of directions - example 1



parametrization of feasible reactions

$$\begin{pmatrix} f \\ \mu \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_y \\ f_z \\ \mu_x \\ \mu_z \end{pmatrix} = Y \begin{pmatrix} f_y \\ f_z \\ \mu_x \\ \mu_z \end{pmatrix}$$

parametrization of feasible motions

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ \omega_y \end{pmatrix} = T \begin{pmatrix} v_x \\ \omega_y \end{pmatrix}$$

here, constant and unitary  
("selection" of columns from  
the  $6 \times 6$  identity matrix)

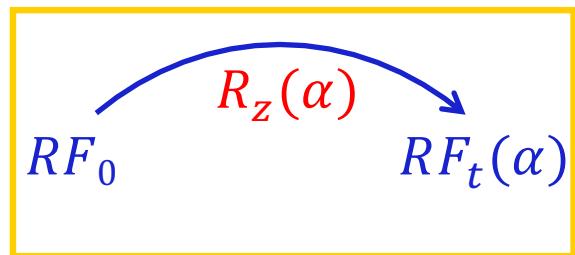
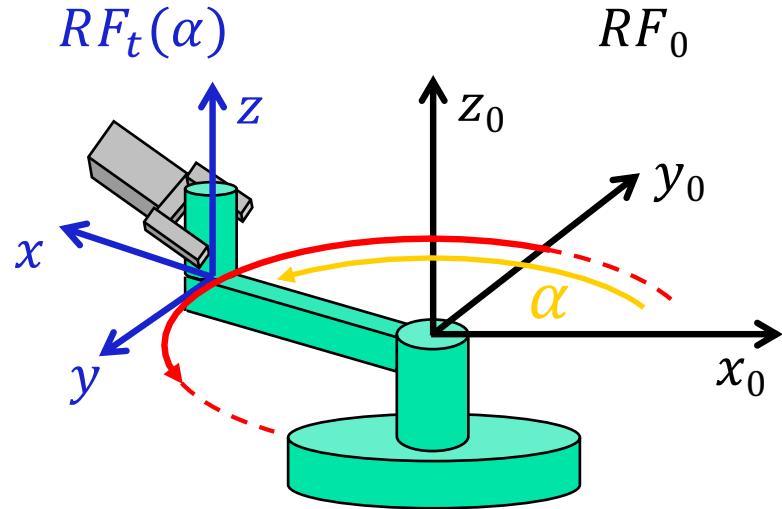
$$T^T Y = 0$$

reaction forces/torques  
do **not** perform work on  
feasible motions

$$(f^T \quad \mu^T) \begin{pmatrix} v \\ \omega \end{pmatrix} = 0$$



# Task frame and constraints - example 2



task: turning a crank  
(free handle)

natural constraints

$$v_x = v_z = 0$$

$$\omega_x = \omega_y = 0$$

$$f_y = \mu_z = 0$$

artificial constraints

$$f_x = f_{x,des} (= 0), f_z = f_{z,des} (= 0)$$

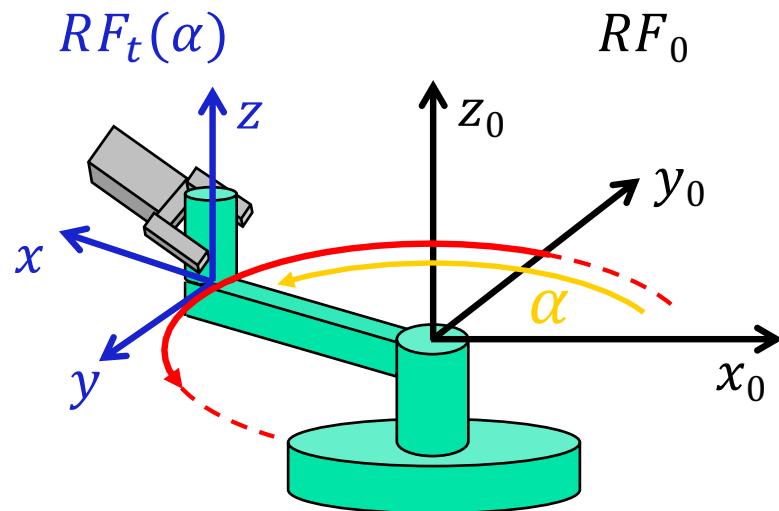
$$\mu_x = \mu_{x,des} (= 0), \mu_y = \mu_{y,des} (= 0)$$

$v_y = v_{y,des}$  (the tangent speed of rotation)

$\omega_z = \omega_{z,des}$  (= 0 if handle should not spin)



# Selection of directions – example 2



parametrization of feasible motions

$$\begin{pmatrix} {}^0v \\ {}^0\omega \end{pmatrix} = \begin{pmatrix} R^T(\alpha) & 0 \\ 0 & R^T(\alpha) \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_y \\ \omega_z \end{pmatrix}$$

$$= T(\alpha) \begin{pmatrix} v_y \\ \omega_z \end{pmatrix}$$

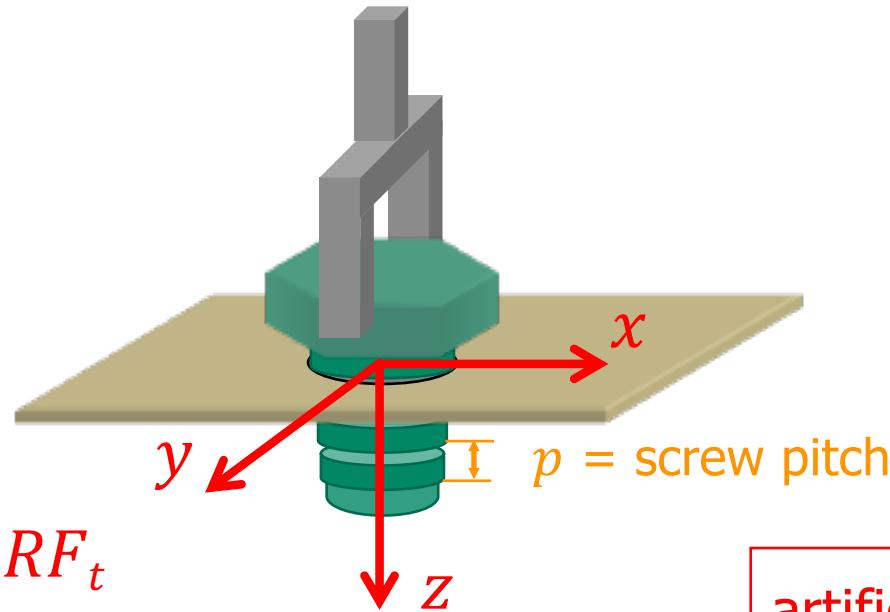
$$T^T(\alpha)Y(\alpha) = 0$$

parametrization of feasible reactions

$$\begin{pmatrix} {}^0f \\ {}^0\mu \end{pmatrix} = \begin{pmatrix} R^T(\alpha) & 0 \\ 0 & R^T(\alpha) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_x \\ f_z \\ \mu_x \\ \mu_y \end{pmatrix} = Y(\alpha) \begin{pmatrix} f_x \\ f_z \\ \mu_x \\ \mu_y \end{pmatrix}$$



# Task frame and constraints - example 3



task: insert a screw  
in a bolt

natural constraints (partial...)

$$v_x = v_y = 0$$

$$\omega_x = \omega_y = 0$$

the screw proceeds **along** and **around** the  **$z$ -axis**, but **not** in an **independent** way! (1 dof)

accordingly,  $f_z$  and  $\mu_z$  **cannot** be **independent**

artificial constraints (abundant...)

$$f_x = f_{x,des} = 0, f_y = f_{y,des} = 0$$

$$\mu_x = \mu_{x,des} = 0, \mu_y = \mu_{y,des} = 0$$

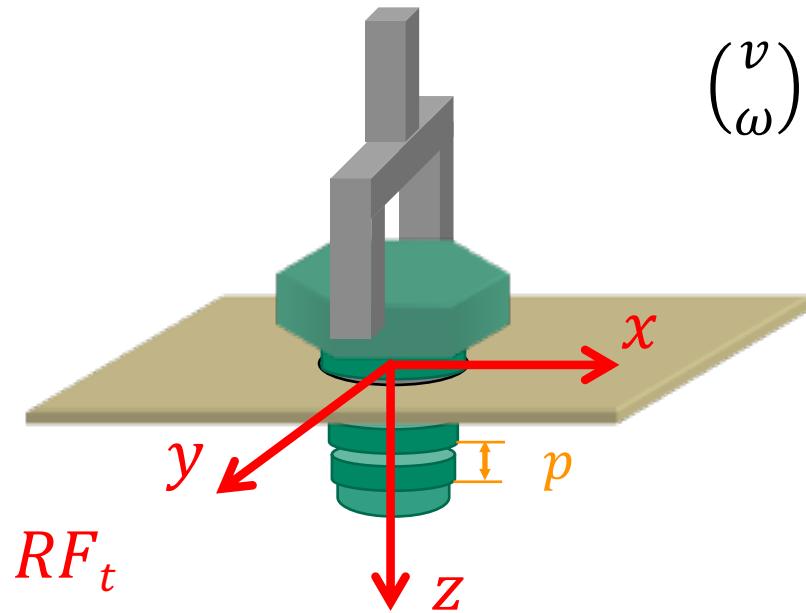
$$v_z = v_{z,des}, \omega_z = \omega_{z,des} = (2\pi/p)v_{z,des}$$

$$f_z = f_{z,des}, \mu_z = \mu_{z,des} \text{ (one function of the other!)}$$

wrench (force/torque) direction should be **orthogonal** to motion twist!



# Selection of directions – example 3

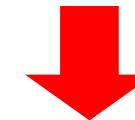


the columns of  $\mathbf{T}$  and  $\mathbf{Y}$   
do not necessarily coincide  
with selected columns  
of the  $6 \times 6$  identity matrix  
 $\Rightarrow$  generalized (screw)  
directions

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & \frac{2\pi}{p} \end{pmatrix}^T v_z = T v_z \quad (k = 1)$$

or  $\omega_z = 2\pi \frac{v_z}{p}$

$\mathbf{Y}$ : such that  $T^T \mathbf{Y} = 0$



$$f_z = -\frac{2\pi}{p} \mu_z$$

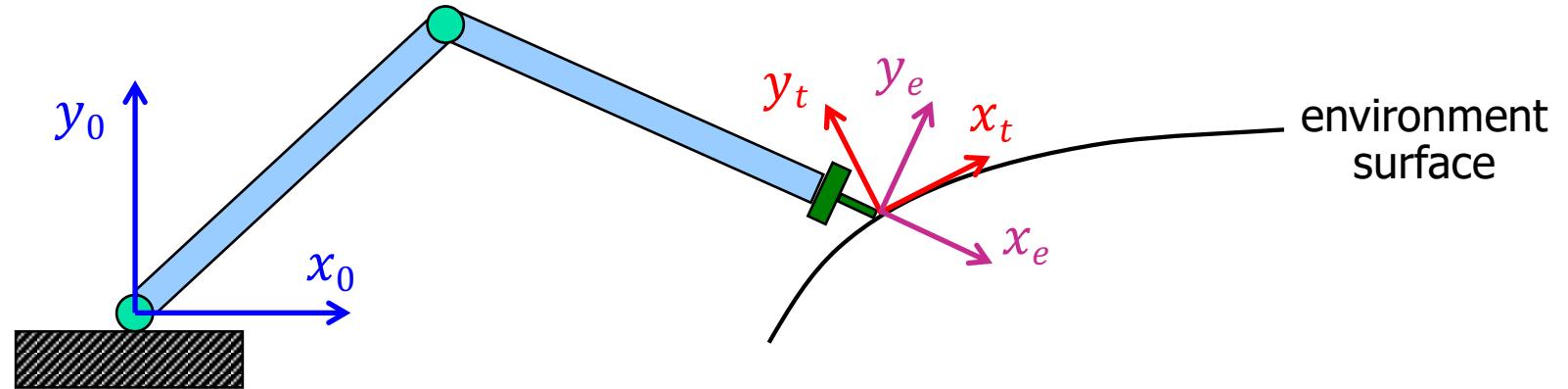
$(6 - k = 5)$

$$\begin{pmatrix} f \\ \mu \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\pi/p \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ \mu_x \\ \mu_y \\ \mu_z \end{pmatrix} = \mathbf{Y} \begin{pmatrix} f_x \\ f_y \\ \mu_x \\ \mu_y \\ \mu_z \end{pmatrix}$$



# Frames of interest – example 4

planar motion of a 2R robot ( $n = 2$ ) in pointwise contact with a surface (task dimension  $m = 2$ )



- **task frame  $RF_t$**  used for an independent definition of the hybrid **reference values** (here:  ${}^t v_{x,des}$  [ $k = 1$ ] and  ${}^t f_{y,des}$  [ $m - k = 1$ ]) and for computing the errors that drive the **feedback control law**
- **sensor frame  $RF_e$**  (here:  $RF_2$ ) where the **force**  ${}^e f = ({}^e f_x, {}^e f_y)$  is measured
- **base frame  $RF_0$**  in which the end-effector **velocity** is expressed (here:  ${}^0 v = ({}^0 v_x, {}^0 v_y)$  of  $O_2$ ), computed using robot Jacobian and joint velocities

all quantities (and errors!) should be expressed ("rotated")  
 in the **same** reference frame  $\Rightarrow$  the **task frame!**



# General parametrization of hybrid tasks

a “description” of robot-environment contact type:  
it implicitly defines the task frame

$$\begin{cases} \begin{pmatrix} v \\ \omega \end{pmatrix} = T(s)\dot{s} & s \in \mathbb{R}^k \\ \begin{pmatrix} f \\ \mu \end{pmatrix} = Y(s)\lambda & \lambda \in \mathbb{R}^{m-k} \end{cases}$$

parametrizes robot E-E free motion  
parametrizes reaction forces/torques

in general, it is  $m = 6$   
(as in most of the previous examples)

+

reaction forces/torques do not perform work on E-E displacements

$$T^T(s)Y(s) = 0$$

axes directions of **task frame** depend in general on  $s$   
(i.e., on robot E-E pose in the environment)

robot dynamics

$$M(q)\ddot{q} + S(q, \dot{q})\dot{q} + g(q) = u + J^T(q) \begin{pmatrix} f \\ \mu \end{pmatrix}$$

robot kinematics

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = J(q)\dot{q}$$



# Hybrid force/velocity control

- **control objective:** to impose desired task evolutions to the parameters  $s$  of **motion** and to the parameters  $\lambda$  of **force**

$$s \rightarrow s_d(t) \quad \lambda \rightarrow \lambda_d(t)$$

- the control law is designed in **two steps**

1. exact **linearization and decoupling** in the **task frame** by feedback

$$\begin{array}{c} \text{closed-loop} \\ \text{model} \end{array} \rightarrow \begin{pmatrix} \ddot{s} \\ \ddot{\lambda} \end{pmatrix} = \begin{pmatrix} a_s \\ a_\lambda \end{pmatrix}$$

2. (**linear**) design of  $a_s$  and  $a_\lambda$  so as to impose the desired dynamic behavior to the errors  $e_s = s_d - s$  and  $e_\lambda = \lambda_d - \lambda$

- **assumptions:**  $n = m$  (= 6 usually),  $J(q)$  out of singularity

**Note:** in “simple” cases,  $\dot{s}$  and  $\dot{\lambda}$  drive single components of  $v$  or  $\omega$  and of  $f$  or  $\mu$ ; accordingly,  $T$  and  $Y$  are just columns of 0/1 selection matrices



# Feedback linearization in task space

$$J(q)\dot{q} = \begin{pmatrix} v \\ \omega \end{pmatrix} = T(s)\dot{s} \rightarrow J\ddot{q} + \dot{J}\dot{q} = T\ddot{s} + \dot{T}\dot{s} \rightarrow \ddot{q} = J^{-1}(T\ddot{s} + \dot{T}\dot{s} - \dot{J}\dot{q})$$

$$M(q)\ddot{q} + S(q, \dot{q})\dot{q} + g(q) = u + J^T(q) \begin{pmatrix} f \\ \mu \end{pmatrix} = u + J^T(q)Y(s)\lambda$$

(under the assumptions made)

$$(M(q)J^{-1}(q)T(s) : -J^T(q)Y(s)) \begin{pmatrix} \ddot{s} \\ \lambda \end{pmatrix} + M(q)J^{-1}(q)(\dot{T}(s)\dot{s} - \dot{J}(q)\dot{q}) + S(q, \dot{q})\dot{q} + g(q) = u$$

$$u = (MJ^{-1}T : -J^TY) \begin{pmatrix} a_s \\ a_\lambda \end{pmatrix} + MJ^{-1}(\dot{T}\dot{s} - \dot{J}\dot{q}) + S\dot{q} + g$$

linearizing and  
decoupling  
control law

$$\rightarrow \begin{pmatrix} \ddot{s} \\ \lambda \end{pmatrix} = \begin{pmatrix} a_s \\ a_\lambda \end{pmatrix} \left. \right\} \begin{matrix} k \\ m - k \end{matrix} \quad \begin{matrix} s \text{ has "relative degree" } = 2 \\ \lambda \text{ has "relative degree" } = 0 \end{matrix}$$



# Stabilization with $a_s$ and $a_\lambda$

as usual, it is sufficient to apply **linear** control techniques for the exponential stabilization of tracking errors (on each single, input-output decoupled channel)

$$a_s = \ddot{s}_d + K_D(\dot{s}_d - \dot{s}) + K_P(s_d - s)$$

$K_P, K_D > 0$   
and diagonal

$$\ddot{e}_s + K_D \dot{e}_s + K_P e_s = 0 \quad \rightarrow \quad e_s = s_d - s \rightarrow 0$$

$K_I \geq 0$   
diagonal

$$a_\lambda = \lambda_d + K_I \int (\lambda_d - \lambda) dt$$

$a_\lambda = \lambda_d$  would be enough,  
but adding an integral  
with the **force error**  
gives more robustness  
to (constant) disturbances

$$e_\lambda + K_I \int e_\lambda dt = 0 \quad \rightarrow \quad e_\lambda = \lambda_d - \lambda \rightarrow 0$$

we need “values” for  $s$ ,  $\dot{s}$  and  $\lambda$  to be  
extracted from actual **measurements** !



# “Filtering” position and force measures

- $s$  and  $\dot{s}$  are obtained from measures of  $q$  and  $\dot{q}$ , equating the descriptions of the end-effector pose and velocity “from the robot side” (direct and differential kinematics) and “from the environment side” (function of  $s, \dot{s}$ )

example

$$\begin{aligned} {}^0r &= {}^0f(q) = \begin{pmatrix} L \cos s \\ L \sin s \\ 0 \end{pmatrix} \rightarrow s = \text{atan2}\{{}^0f_y(q), {}^0f_x(q)\} \\ &\quad \text{Diagram: A 3D coordinate system } (x_0, y_0, z_0) \text{ with a green cylinder at the origin. A blue cylinder is attached to a grey robotic arm. The distance from the origin to the center of the blue cylinder is } L. \text{ The angle between the vertical } z_0 \text{ axis and the line of sight to the center of the blue cylinder is } s = \alpha. \text{ A blue arrow labeled } r \text{ points from the origin to the center of the blue cylinder.} \\ J(q)\dot{q} &= T(s)\dot{s} \rightarrow \dot{s} = T^\#(s)J(q)\dot{q} \end{aligned}$$

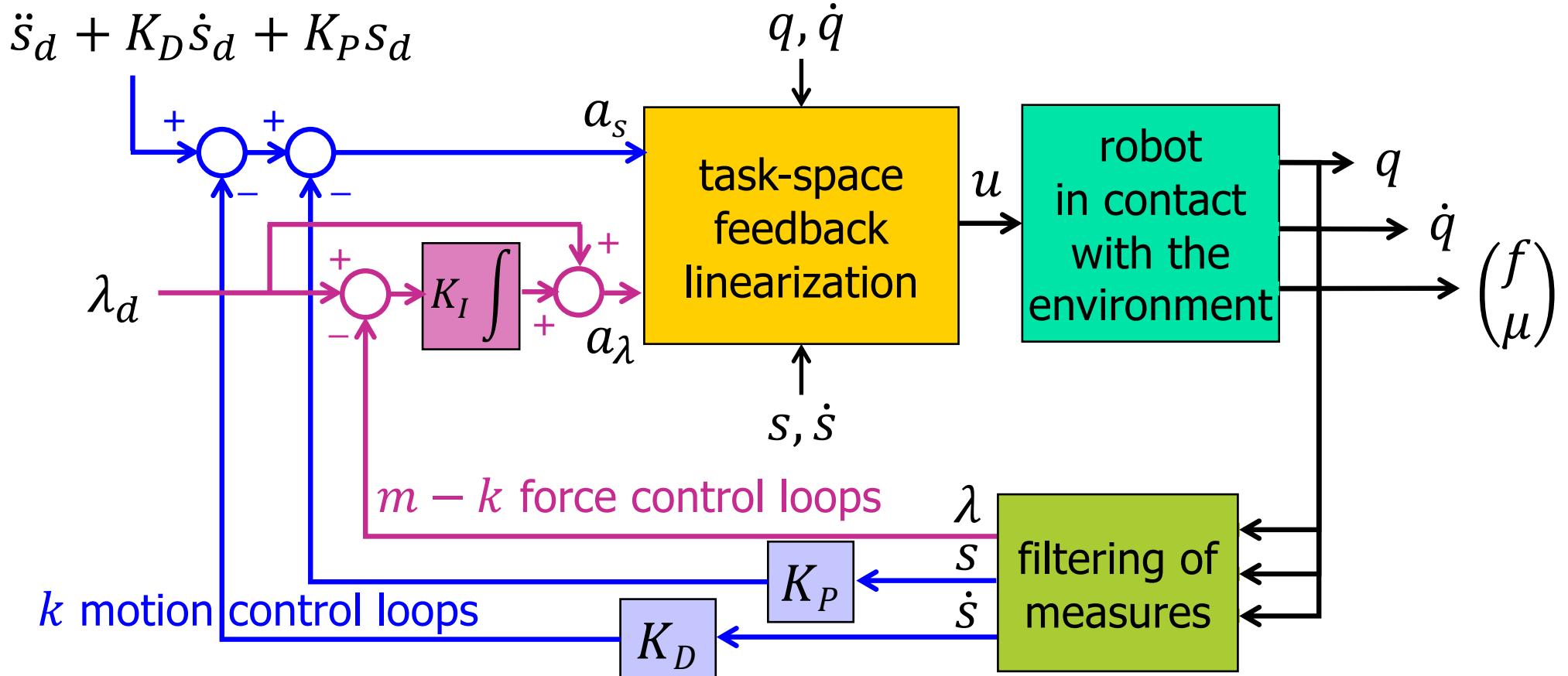
- $\lambda$  is obtained from force/torque measures at end-effector

$$\begin{pmatrix} f \\ m \end{pmatrix} = Y(s)\lambda \rightarrow \lambda = Y^\#(s) \begin{pmatrix} f \\ m \end{pmatrix}$$

pseudoinverses  
of “tall” matrices  
having full  
column rank, e.g.,  
 $T^\# = (T^T T)^{-1} T^T$   
(or weighted)



# Block diagram of hybrid control



usually  $m = 6$  (complete 3D space)

limit cases  $k = m$ : no force control loops, only motion (free motion)

$k = 0$  : no motion control loops, only force ("frozen" robot end-effector)



# Block diagram of hybrid control

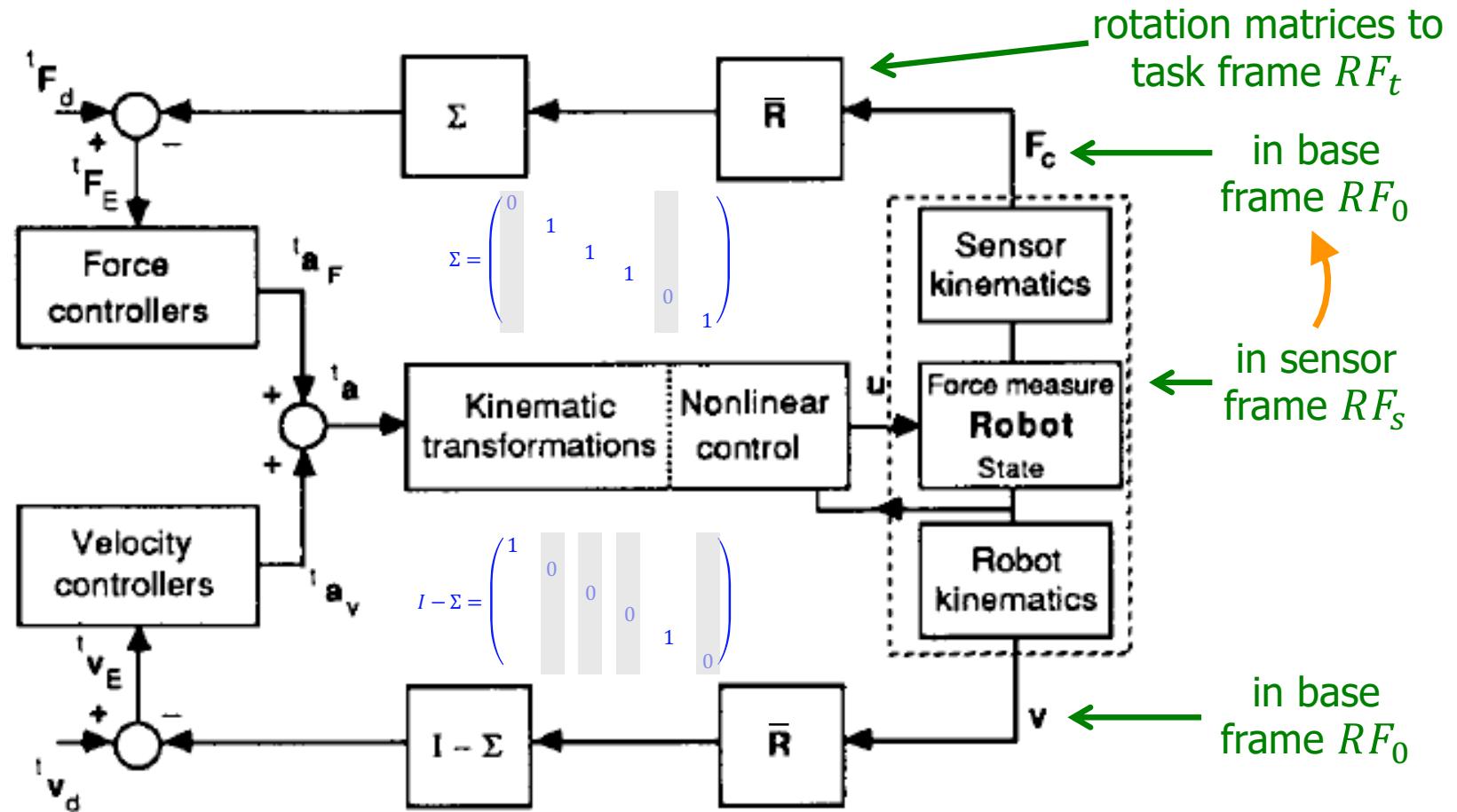
simpler case of 0/1 selection matrices

compact notation  
in this slide

$$F = \begin{pmatrix} f \\ \mu \end{pmatrix}$$

$$V = \begin{pmatrix} v \\ \omega \end{pmatrix}$$

$$\bar{R} = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix}$$



$\lambda$  and  $\dot{s}$  are just single components of  $f$  (or  $\mu$ ) and  $v$  (or  $\omega$ )

$Y$  and  $T$  are replaced by 0/1 selection matrices:  $\Sigma$  and  $I - \Sigma$



# Force control via an impedance model

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- in a force-controlled direction of the hybrid task space, when the **contact stiffness is limited** (i.e., far from infinite, as assumed in the ideal case), one may use **impedance model ideas** to explicitly **control the contact force**
  - let  $x$  be the position of the robot along such a direction,  $x_d$  the (constant) contact point,  $k_s > 0$  the contact (viz., sensor) stiffness, and  $f_d > 0$  the desired contact force
- the impedance model is chosen then as

$$m_m \ddot{x} + d_m \dot{x} + k_s(x - x_d) = f_d$$

where the **force sensor** measures  $f_s = k_s(x - x_d)$ , and only  $m_m > 0$  and  $d_m > 0$  are free model parameters

- after feedback linearization ( $\ddot{x} = a_x$ ), the command  $a_x$  is designed as

$$a_x = (1/m_m)[(f_d - f_s) - d_m \dot{x}]$$

which is a **P-regulator** of the desired force, **with velocity damping**

- the **same** control law works also before the contact ( $f_s = 0$ ), guaranteeing a steady-state speed  $\dot{x}_{ss} = f_d/d_m > 0$  in the **approaching phase**



# First experiments with hybrid control

## First Experiments with Hybrid Force/Velocity Control

Università di Roma "La Sapienza"  
DIS, LabRob  
February 1991



video

## First Experiments with Hybrid Force/Velocity Control

(part II)

Università di Roma "La Sapienza"  
DIS, LabRob  
February 1991



video

MIMO-CRF robot  
(DIS, Laboratorio di Robotica, 1991)

# Sources of inconsistency in force and velocity measurements

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1. presence of **friction** at the contact
    - a reaction force component appears that opposes motion in a “free” motion direction (in case of Coulomb friction, the tangent force intensity depends also on the applied normal force ...)
  2. **compliance** in the robot structure and/or at the contact
    - a (small) displacement may be present also along directions that are nominally “constrained” by the environment
- NOTE:** if the environment geometry at the contact is perfectly known, the task inconsistencies due to 1. and 2. on parameters  $s$  and  $\lambda$  are already **filtered out** by the pseudo-inversion of matrices  $T$  and  $Y$
3. uncertainty on **environment geometry** at the contact
    - can be reduced/eliminated by real-time **estimation processes** driven by external sensors (e.g., vision –but also force!)

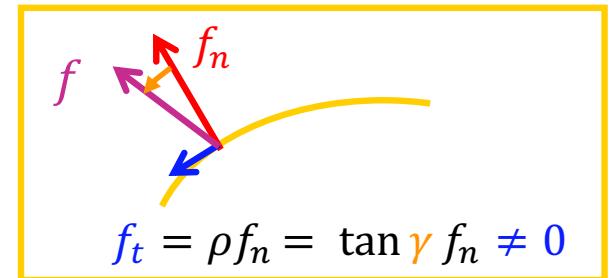


# Estimation of an unknown surface

how difficult is to **estimate** the unknown profile of the environment surface, using information from velocity and force measurements at the contact?

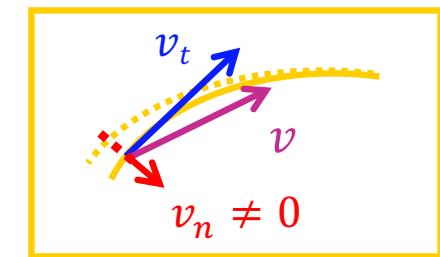
**1. normal** = nominal direction of measured **force**

... in the presence of contact motion with friction, the **measured** force  $f$  is slightly rotated from the actual normal by an (unknown) angle  $\gamma$



**2. tangent** = nominal direction of measured **velocity**

... compliance in the robot structure (joints) and/or at the contact may lead to a **computed** velocity  $v$  having a small component along the actual normal to the surface



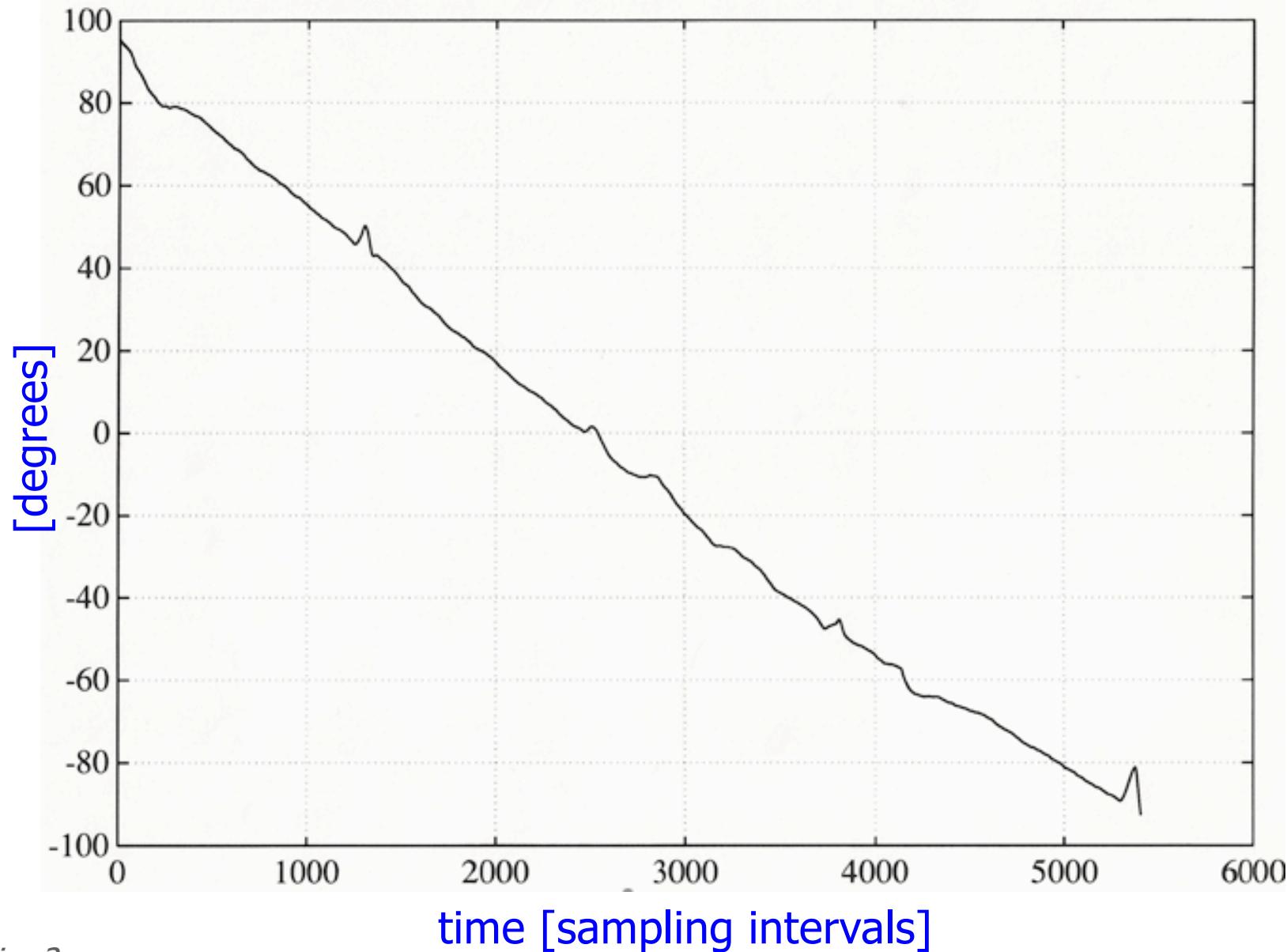
**3. mixed method (**sensor fusion**) with RLS**

- a. tangent direction is estimated by a **recursive least squares** method from position measurements
- b. friction angle is estimated by a **recursive least squares** method, using the current estimate of the tangent direction and from force measurements

to approach an unknown surface or to recover contact (in case of loss), the robot uses simple exploratory moves



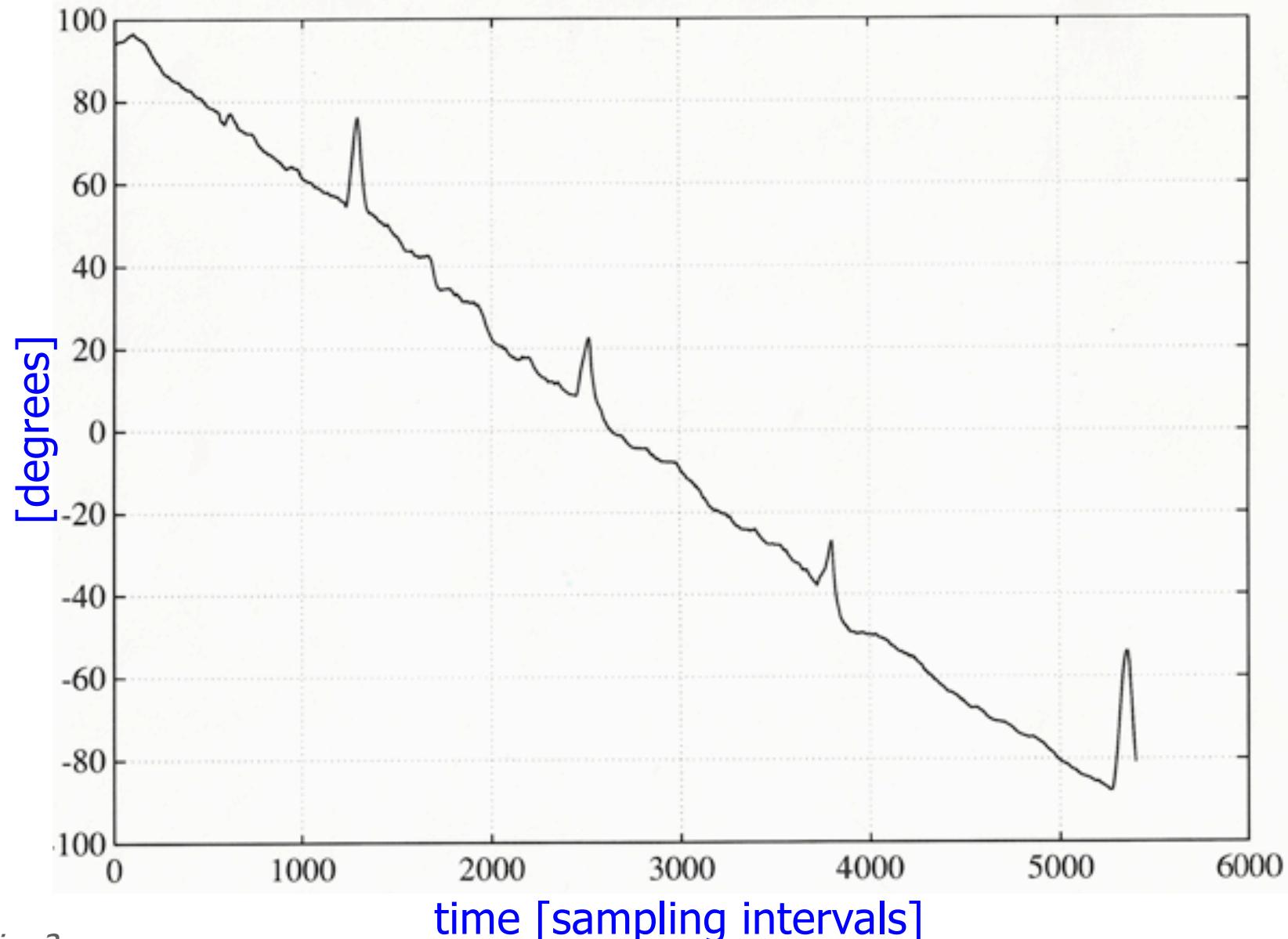
# Position-based estimation of the tangent (for a circular surface traced at constant speed)





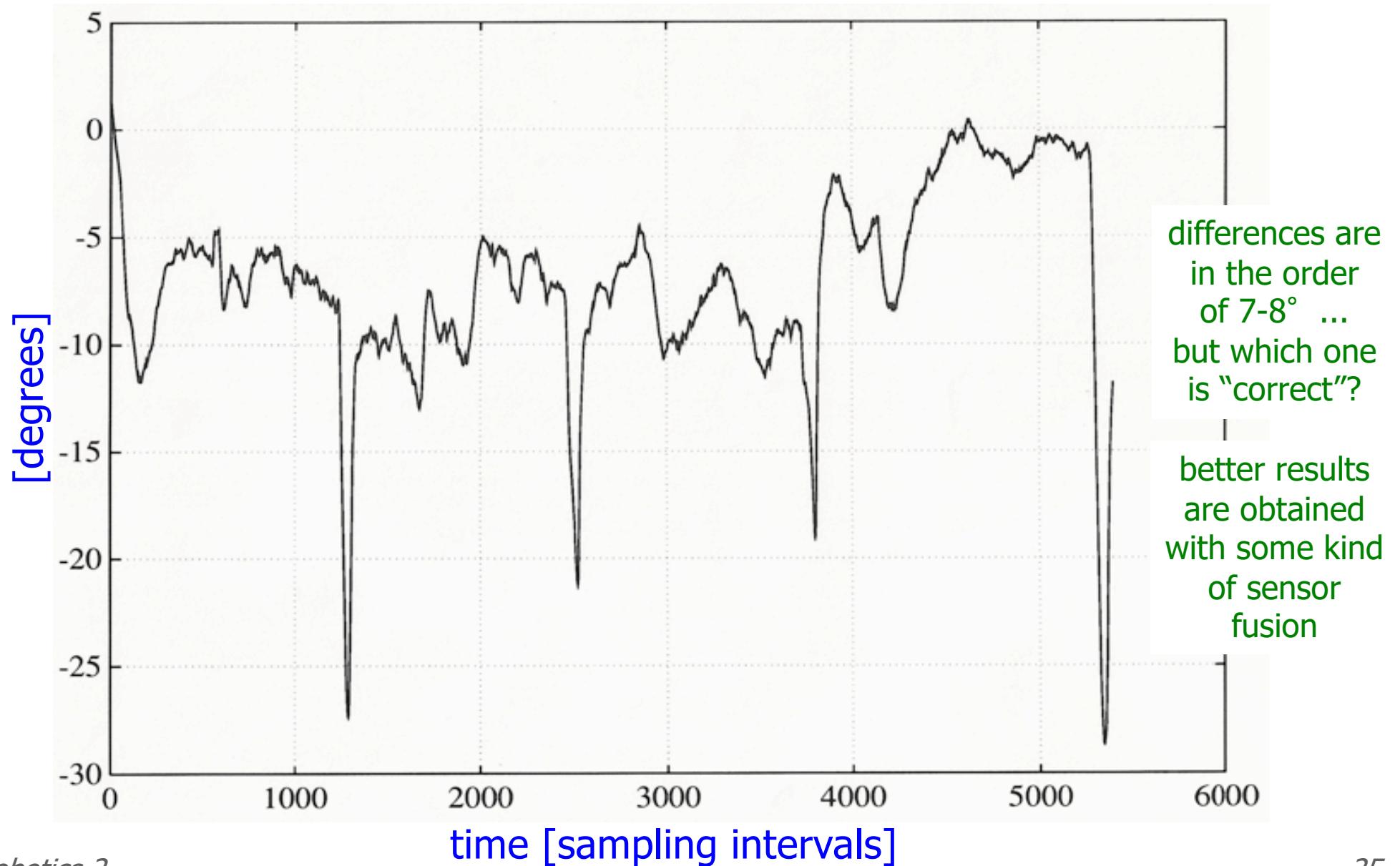
# Force-based estimation of the tangent

(for the same circular surface traced at constant speed)





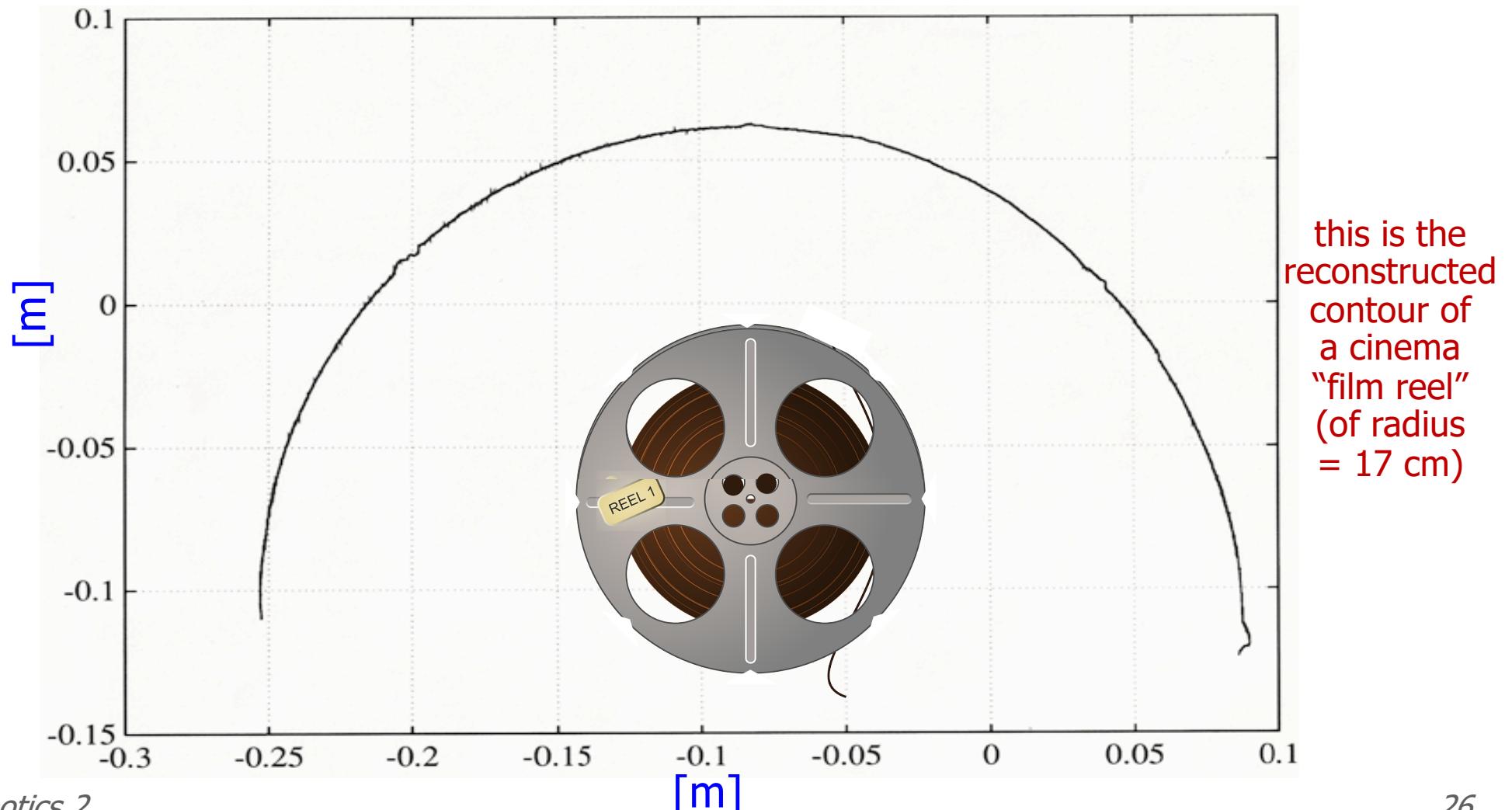
# Difference between estimated tangents





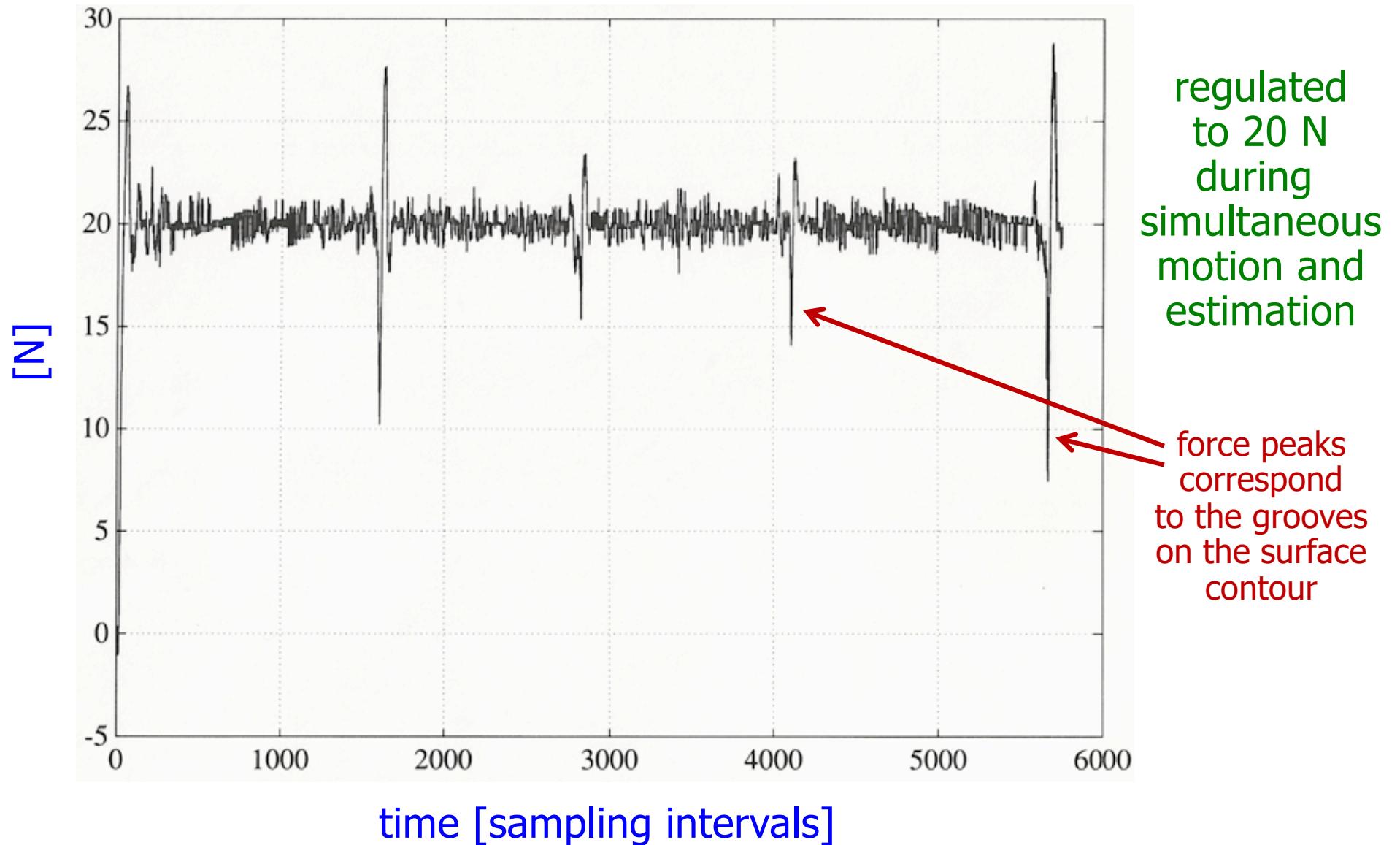
# Reconstructed surface profile

estimation by a RLS (Recursive Least Squares) method: we continuously update the coefficients of two quadratic polynomials that fit locally the unknown contour, using data fusion from both force and position/velocity measurements





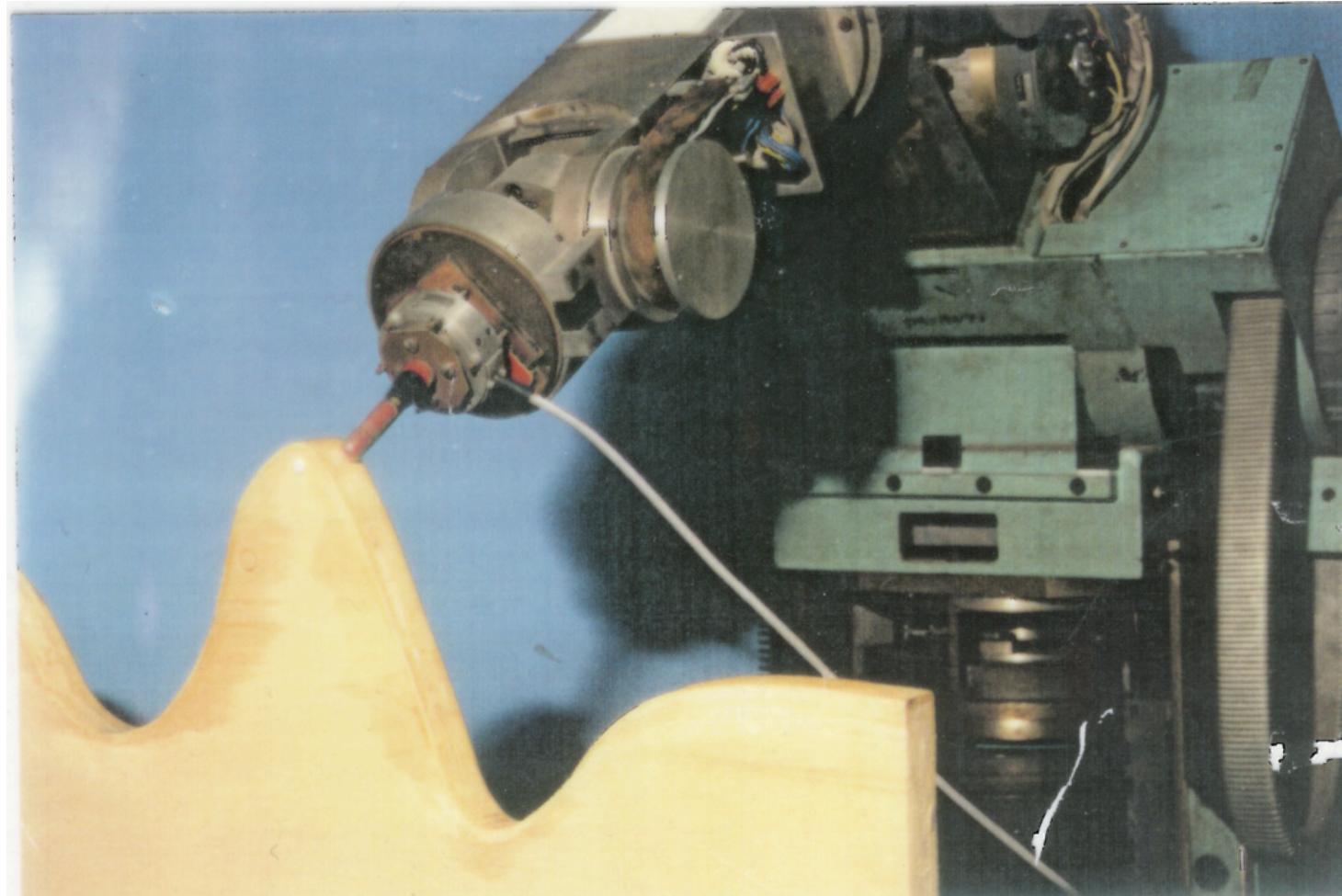
# Normal force





# Contour estimation and hybrid control performed simultaneously

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MIMO-CRF robot (DIS, Laboratorio di Robotica, 1992)



# Contour estimation and hybrid control

## **Hybrid Force/Velocity Control and Identification of Surfaces**

**Università di Roma "La Sapienza"  
DIS, LabRob  
September 1992**



video

# Robotized deburring of car windshields

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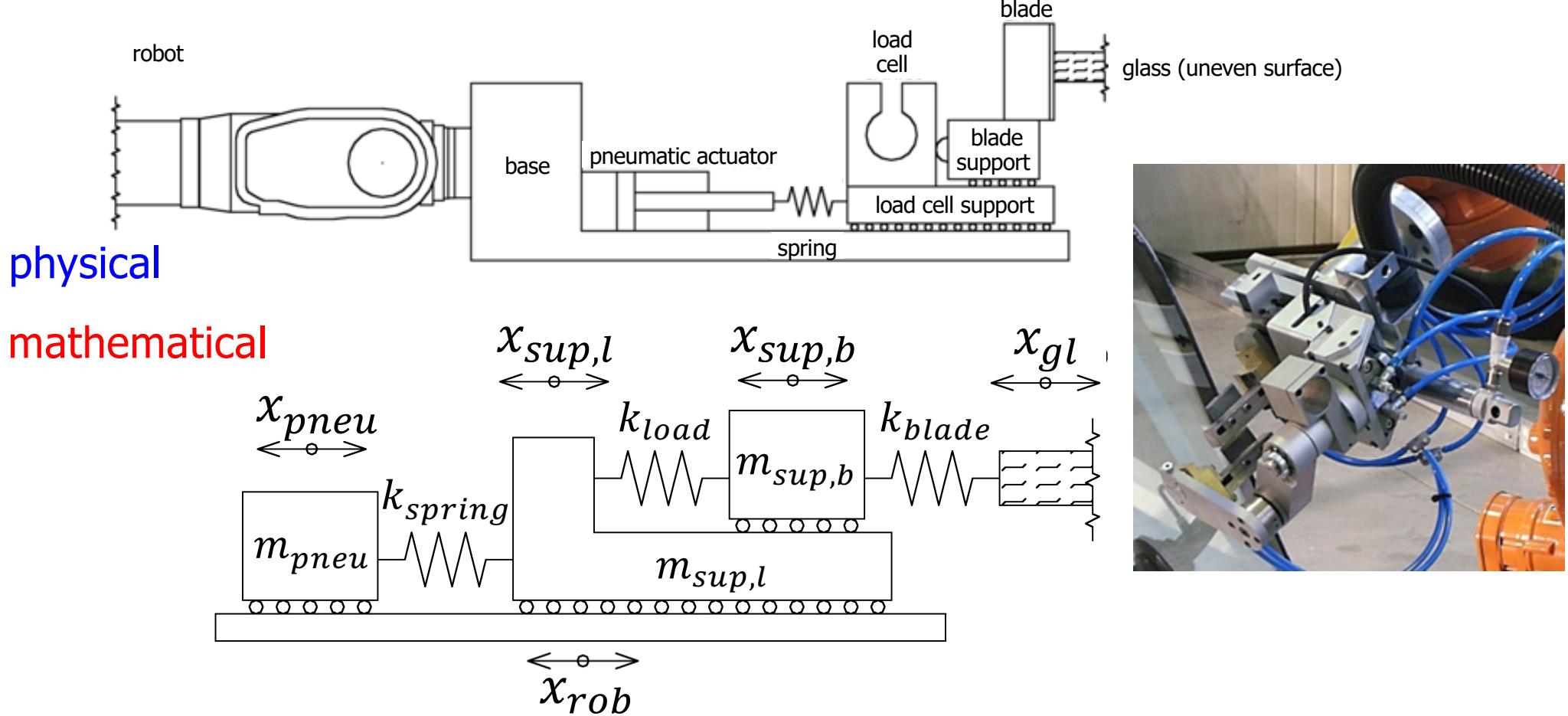
- car windshield with **sharp edges** and fabrication tolerances, with **excess of material** (PVB = Polyvinyl butyral for gluing glass layers) on the contour
- robot end-effector follows the pre-programmed path, despite the small errors w.r.t. the nominal windshield profile, thanks to the **compliance** of the deburring tool
- contact force between tool blades and workpiece can be independently controlled by a **pneumatic actuator** in the tool

the robotic deburring tool contains in particular

- **two blades** for cutting the exceeding plastic material (PVB), the first one actuated, the second passively pushed against the surface by a spring
- a **load cell** for measuring the 1D applied normal force at the contact
- on-board **control system**, exchanging data with the ABB robot controller



# Model of the deburring work tool



for a stability analysis (based on linear models and root locus techniques) of force control in a single direction and in presence of multiple masses/springs, see again Eppinger & Seering, IEEE CSM, 1987 (material in the course web site)



# Summary through video segments



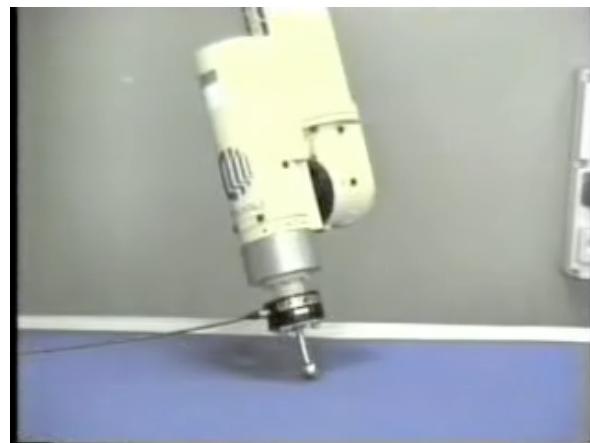
compliance control  
(active Cartesian stiffness  
control **without** F/T sensor)



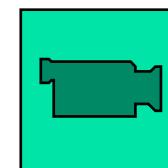
impedance control  
(with F/T sensor)



force control  
(realized as external loop  
providing the reference to  
an internal position loop  
**-see Appendix**)



hybrid force/position control



COMAU Smart robot  
c/o Università di Napoli, 1994  
(full video on course web site)



# Appendix

- force control can also be realized as an external loop providing reference values to an internal motion loop (see video in slide #32)
- inner-outer (or cascaded) control scheme
  - angular position quantities (E-E orientation, errors, commands) can be expressed in different ways (Euler angles  $\phi$ , rotation matrices  $R$ , ...)

