# algebraic geometry

**exercise** (2.14)**.** Let $\psi : \mathbb{P}^r \times \mathbb{P}^s \to \mathbb{P}^N$ be the map defined by sending the order pair $(a_0, \dots, a_r) \times (b_0, \dots, b_s)$ to $(\dots, a_i b_j, \dots)$ in lexicographic order, where $N = rs + r + s$. Note that $\psi$ is well-defined and injective. It is called the ***Segre embedding***. Show that the image of $\psi$ is a subvariety of $\mathbb{P}^N$. [*Hint*: Let the homogeneous coordinates of $\mathbb{P}^N$ be $\{z_{ij} : i = 0, \dots, r, j = 0, \dots, s\}$ and let $\mathfrak{a}$ be the kernel of the homomorphism $k[\{z_{ij}\}] \to k[x_0, \dots, x_r, y_0, \dots, y_s]$ which sends $z_{ij}$ to $x_i y_j$. Then show that $\operatorname{img} \psi = Z(\mathfrak{a})$.

*Solution.* content... $\square$

**exercise** (H-3.16, Products of Quasi-Projective Varieties)**.** Use the Segre embedding (Ex. 2.14) to identify $\mathbb{P}^n \times \mathbb{P}^m$ with its image and hence give it a structure of projective variety. Now for any two quasi-projective varieties $X \subseteq \mathbb{P}^n$ and $Y \subseteq \mathbb{P}^m$ consider $X \times Y \subseteq \mathbb{P}^n \times \mathbb{P}^m$.

(a) Show that $X \times Y$ is a quasi-projective variety.

(b) If $X, Y$ are both projective, show that $X \times Y$ is projective.

(c) Show that $X \times Y$ is a product in the category of varieties.

*Solution.* content... $\square$