k3

1 Class 1

The most important invariant of a k3 surface is intersection form.

There are three classes of manifolds

1. Smooth manifolds

$$\text{smooth manifolds} \xrightarrow{\text{forgetful functor}} \text{PL manifold} \longrightarrow \text{Topological manifolds}$$

Donaldson: contiunally many non-equivalent smooth structures on $\mathbb{R}^4$. K3 surfaces has countably many smooth structures and only one of them is compatible with complex structure.

Definition. Intersection form. Given a quadratic form on a lattice $V_{\mathbb{Z}} = \mathbb{Z}^n$, so

$$q : V_{\mathbb{Z}} \times V_{\mathbb{Z}} \to \mathbb{Z}$$

is ***unimodular*** if

$$V_{\mathbb{Z}} \xrightarrow{q} \mathrm{Hom}(V_{\mathbb{Z}}, \mathbb{Z})$$

is an isomorphism.

Theorem (Universal coefficients formula)**.**

$$H_{n-1}(M, \mathbb{Z}) = \mathbb{Z}^{b_{n-1}(M)} \oplus T_{n-1}(M)$$

$$h^n(M, \mathbb{Z}) = \mathbb{Z}^{b_n(M)} \oplus T_{n-1}(M)$$

Corollary. $H^2(X, \mathbb{Z})$ is torsion free if $\pi_1(X) = 0$ because

Definition. ***Signature*** is $m - n$ if q has signature (m, n).

Theorem (Rokhlm-Wu?)**.** Signature is divisible by 16 for simply-connected (something else).

Remark. The methods used in surgery break down in smooth case because strange topological objects like infinite sums of spheres arise.

Theorem (Freedman, 1982)**.** There are as many 4-manifolds as there are intersection forms. M simply connected 4 manifold homotopy class is uniquely determined by intersection dorm. Moreover, for every unimodular form there exists a unique M with this intersection form.

Theorem (Donaldson, 1986). M smooth compact manifold with positive definite odd intersection form q. Then
$$\begin{pmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Definition. Bilinear symmetric form is ***indefinite*** if it is not positive definite nor negative definite.

Theorem (Classification of unimodular symmetric bilinear forms). Odd are diagonalizable, while even are related to special Lie group E_8.

Definition. A ***K3 surface*** is a Kähler complex surface M with $b_1 = 0$ (simply connected) and $c_1(M, \mathbb{Z}) = 0$.

Kodaira did what André Weil couldn'g classify.

Theorem. K3 surfaces have trivial canonical bundle $K_M = \Lambda^2(\Omega^1 M)$.

2 Class 2

G topological group. ***Principal*** G ***bundle*** is a space with free G-action such that the quotient E/G is Housdorff. There are several conditions that make this work. And then you have $\text{Homotopy}(X, BG)$ =equivalence classes of G-bundles. Vector bundles of a manifold are the same as maps from X to $B\,U(n)$.

Vector bundles up to stable equivalence are classified basically by Chern classes, so by the cohomology in $H^\bullet(B\,U) = Q[c_1, c_2, \ldots, c_n$.

Now look at the loop space of X. Then $H^\bullet(\Omega X)$ is a free graded commutative algebra. Loop space has the interesting property that $\Omega\,U = B\,U$ and $\Omega B\,U = U$.

2.1 Bialgebras

Let A be a superalgebra (graded with antisymmetric product). Then we ask the axiom of coassociativity and that .

Example. G group, and $C(G)$ the ring of k-valued functions $C(G \times G) = C(G) \times C(G)$ so

$$\begin{aligned} G \times G &\longrightarrow G \\ C(G) &\longmapsto C(G) \otimes C(G) \end{aligned}$$

2.2 H-spaces

Definition. H-space is a space M with a map $\mu : M \times M$ to$M$that is homotopy associative,

$$\begin{array}{ccc} M \times M \times M & \xrightarrow{\mu \times \mathrm{id}} & M \times M \\ \downarrow{\scriptstyle \mathrm{id} \times \mu} & & \downarrow{\scriptstyle \mu} \\ M \times M & \xrightarrow{\mu} & M \end{array}$$

which is homotopy commutative. And with homotopy unit.

So it's like a homotopy algebra?

Example. The loop space.

2.3 Bialgebras of finite type

Definition. A bialgebra A is of ***finite type*** if it is the direct sum of $A = \bigoplus_{i \geqslant 0} A^i$ supercommutative and each A^1 is finite dimensional.

Remark. Free commutative algebra is polynomial algebra

Definition. $A = \mathbb{C}[x_1, \ldots, x_n, \ldots] \otimes \Lambda^\bullet(a_1, \ldots, a_n, \ldots)$ is a graded commutative free algebra. In the slides: it is $\mathrm{Sym}_{gr} V^*$ where V^* is a graded vector space.

Theorem (Hopf). A graded commutative bialgebra of finite type over k of 0 characteristic is free graded commutative as a k algebra.

2.4 The cohomology algebra of U(n)

Claim. The cohomology algebra $H^*(U(n), \mathbb{Q})$ is a free graded commutative algebra with generators in degrees $1, 3, 5, \ldots, 2n - 1$.

Demostração. Induction. $U(1)$ is clear because it is a circle. Then do Serre spectral sequence. Differentials vanish on the second page because there's only nonzero groups on even degrees! And we get that $E_2^{p,q} = H^p(S^{2n-1}) \otimes H^q(U(n-1))$. And then the sequence converges to that of the total space which is $U(n)$. □

2.5 Grassman manifolds

Definition. The ***fundamental bundle*** B_{fun} is a rank n vector bundle over $\mathrm{Gr}(n, m)$.

Claim. B, B' vector bundles of rank n, $m - n$, $B \oplus B'$

$$\varphi : X \to \mathrm{Gr}(m, n)$$

$$\varphi(x) = B_x \subset B_x \oplus B'_x = \mathbb{K}^m$$

then $B = \varphi^* B_{\text{fun}}$.

Theorem. If you have B as a bundle on a manifold X then $B \oplus B'$ is trivial for some bundle B'.

Demostração. Embed the total space in a large enough euclidean space. $\square$

Definition. $Gr(n, \infty) = Gr(n)$ is $\bigcup_{m=n_1}^{\infty} Gr(n, m) = Gr(n)$

Corollary. For every bundle B of rank n there is a function $\varphi : X \to Gr(n)$ such that $B = \varphi^* B_{fun}$.

Take a bundle $E \to X$ and G acts freely on E so E principal G bundle. Classifying space BG

Theorem (Atiyah-Bott)**.** Classifying space is unique up to homotopy equivalence.

2.5.1 The fundamental bundle

In class 4 I finally understood that

Definition. The ***fundamental bundle*** on the Grassmanian $Gr(n)$ (the Grassmanian is this space where points are linear spaces) is the vector bundle such that the fiber of one point (which is a vector space) is the vector space that is the point. It's very tautological.

Theorem (Did we prove this?)**.** Let B be a vector bundle of rank n on a cellular space X. Then there exists a continuous map $\varphi : X \to Gr(n)$ such that B is isomorphic to the pullback $\varphi^* B_{fun}$ of the fundamental bundle.

Remark. In fact $Gr(n)$ is the classifying space of vector bundles of rank n, in the sense that isomorphism classes of vector bundles of maps $\varphi : X \to Gr(n)$.

2.6 Stiefel spaces

Definition. $\mathbb{K}^\infty$ is the direct limit of $\mathbb{K}^n$ so its just the direct sum $\bigoplus_{i=n}^{\infty} \mathbb{K}$. Stiefel space is the space of orthonormal n-frames.

If we prove that Stiefel is contractible we obtain our classifying space so let's prove that. We have a fibration

$$U(n) \hookrightarrow St(n, \infty) \to Gr(n, \infty)$$

Theorem. $St(n)$ is contractible.

*Demostração***Step 1** Locally trivial fibration with contractible fiber and base $Y \to X$ then Y is contractible, this is so trivial.

Step 2 Fibration $St(n) \to St(n-1)$ with fiber S^∞

Step 3 Show that S^∞ is contractible.

Step 4 And then some map $\mathbb{R}$ that is not surjective, and construct homotopy of identity to a constant map.

$\square$

Exercise. If $X_\infty = \bigcup X_i$ is the inductive limit of contractible cellular spaces then it is contractible. Use Whitehead theorem.

Theorem (Important)**.** $\mathrm{Gr}(\infty) = \mathrm{B}\,\mathrm{U}$.

2.7 Stable equivalence

Definition. Vector bundles V, W are stable equivalent if $V \oplus A \cong W \cong B$ for trivial vector bundles A and B.

Homotopy classes of equivalent vector bundles are in coorespondance with...

Theorem. $\mathrm{B}\,\mathrm{U}$ is H-space.

Corollary. $H^*(\mathrm{B}\,\mathrm{U}, \mathbb{Q})$ is a free supercommutative algebra.

Claim. $H^*(\mathrm{B}\,\mathrm{U}$ is a free polynomial algebra generated by classes $c_1, c_2, \ldots$ in all even degrees.

3 Class 3

3.1 Reminder

Definition. ***Bialgebra*** is an algebra that is equipped with comultiplication, counit...

Remark. It is when the dual space also has an algebra structure, but we prefer to use the tensor notation.

Let $\sum_{i \geqslant 0} A^i$ with $\dim A^i < \infty$. ***Free commutative algebra*** is a polynomial algebra. ***Free graded commutative algebra*** is

$$\widetilde{\mathrm{Sym}}^\bullet(W^\bullet \oplus V^\bullet) := \mathrm{Sym}^\bullet(W^\bullet) \otimes \Lambda^\bullet(V^\bullet)$$

where

$$W = \bigoplus_i W^{\mathrm{even}} \qquad V = \bigoplus_i V^{\mathrm{odd}}.$$

3.2 Hopf algebra

Definition. A bialgebra is a ***Hopf algebra*** when it is also equipped with an antipode map (S) such that the following diagram commutes

$$\begin{tikzcd}
& H \otimes H \arrow[rr, "S \otimes \mathrm{id}"] & & H \otimes H \arrow[dr, "m"] & \\
H \arrow[ur, "\Delta"] \arrow[rr, "\eta"] \arrow[dr, "\Delta"'] & & \mathbb{C} \arrow[rr, "u"] & & H \\
& H \otimes H \arrow[rr, "\mathrm{id} \otimes S"] & & H \otimes H \arrow[ur] &
\end{tikzcd}$$

[diagram from quantum group minicourse notes]

Example. The cohomology of the loop space, $H^\bullet(\Omega X)$.

3.3 Primitive elements in a bialgebra

Definition. An element of a bialgebra $x \in A$ is ***primitive*** if $\Delta(x) = x \otimes 1 + 1 \otimes x$.

$$\begin{aligned}
\Delta(xy) &= \Delta(x)\Delta(y) \\
&= (1 \otimes x + x \cdot 1)(y \otimes 1 + y \otimes y) \\
&= 1 \otimes xy + xy \otimes 1 + x \otimes y + y \otimes x.
\end{aligned}$$

Remark. We trying to show that Hopf algebras? bialgebras? are generated by primitive elements?

Definition. $A^\bullet$ bialgebra, $\mathscr{P}^\bullet \subset A^\bullet$ space of primitive, and the natural embedding

$$\mathrm{Sym}_{gr}(\mathscr{P}^\bullet) \to A$$

We say that A is ***free up to defree*** k if

$$\bigoplus_{i \leqslant k} \mathrm{Sym}^i_{gr}(P) \xrightarrow{\psi} A$$

is an embedding.

Lemma. Let $A^\bullet$ be a bialgebra which is free up to degree k. Then $A^\bullet$ is free up to degree $k+1$.

Proof.

Step 1 Choose a basis of P, $\{x_i\}$. Chose a polynomial condition $Q(x_1, \ldots, x_n) = 0$ of degree $k+1$. Write this as

$$Q = Q_m x_1^m + Q_{m-1} x_1^{m-1} + \ldots + Q_0.$$

that is

$$Q = \sum_{i=0}^{m} Q_i x_1^i$$

with Q_i invariant somehow. Then we apply comutiplication to obtain

$$\Delta(Q) = Q \otimes 1 + 1 \otimes Q + R$$

where R is some sort of reminder with bounded degree:

$$R \in \mathfrak{U} := \bigoplus_{i \leqslant k} \mathrm{Sym}^i_{gr}(P) \otimes \bigoplus_{i \leqslant k} \mathrm{Sym}^i_{gr}(P)$$

which follows from a similar computation of that which we did after defining primitive elements.

Step 2 Project to drop the terms that have $Q \otimes 1 + 1 \otimes Q$:

$$\Pi : \mathfrak{U} \to x_1 \otimes \bigoplus_{i \leqslant k} \mathrm{Sym}^i_{gr}(P)$$

since the x_i are primitive, i.e. $\Delta(x_i) = x_i \otimes 1 + 1 \otimes x_i$, one has

$$\Delta(x_1^m) = (x_1 \otimes 1 + 1 \otimes x_1)^m$$

we get that

$$\Pi(\Delta(x_1^m)) = m x_1 \otimes x_1^{m-1}$$

while on the board it is written that

$$\Pi(\Delta(x_1^m)) = \Pi((x_1 \otimes 1 + 1 \otimes x_1)^m)$$

Step 3 Let $\Pi(R) := x_1 \otimes R_0$. Since $Q = 0$ in A, its component R_0 is also equal to 0. So $\Pi(\Delta(Q)) = 0$. Then

$$\begin{aligned} 0 &= \Pi\left(\Delta\left(\sum_m x_1^m \cdot Q_m\right)\right) \\ &= \sum_m x_1 \otimes x_1^{m-1} Q_m + \Pi(m x_1 \otimes x_1^{m-1} \cdot \Delta(Q_m)) \\ &= \sum_m x_1 \otimes x_1^{m-1} Q_m \end{aligned}$$

so that

$$\begin{aligned} x_1 \otimes x_1^{m-1} Q_m &= 0 \\ \implies x_1^{m-1} Q_m &= 0 \end{aligned}$$

So we conclude that

$$Q_m = 0$$

$\square$

Remark. We just proved that for any subalgebra generated by finite elements, we didn't use that it is free.

3.4 Algebras with filtration

Definition. A ***filtration on algebra*** is

$$A^\bullet \supset F_1 A^\bullet \supset F_2 A^\bullet \supset \dots$$

such that

$$F_i A^\bullet F_j \subset F_{i+j} A^\bullet$$

Definition. ***Associated graded*** to a filtered algebra is

$$A^\bullet_{gr} = \bigoplus_{i=0}^{\infty} \frac{F^1 A^\bullet}{F^{i+1} A^\bullet}$$

$$F^0 A^\bullet = A^\bullet$$

Definition. $I \subset A$ ideal then I-***adic filtration*** is the filtration by the degrees of the ideal

$$A \supset I \supset I^2 \supset I^3 \dots$$

Lemma. Choose an I-adic filtration. Then A_{gr} is generated by its first and second graded components $A/I \oplus I/I^2$.

Demostração. Indeed, $I^k/^{k+1}$ is generated by products of k elements in (I/I^2). □

Definition. A ***augmentation ideal*** in a bialgebra is the kernel of the counit homomorphism $\varepsilon : A \to k$. We denote it by $Z = \ker A$

Remark.

$$\Delta(x) = 1 \otimes x + x \otimes 1 \mod Z \otimes Z$$

Why? Because

$$x = \varepsilon \otimes \mathrm{id}(\Delta(x)) \qquad \text{up to } Z \otimes A$$
$$\Delta(x) = 1 \otimes x \qquad \text{up to } A \otimes X$$
$$\Delta(x) = x \otimes 1$$

Ok, now we can prove Hopf theorem.

Theorem (Hopf theorem). A finite type bialgebra is generated by primitive elements.

In slides: Let A be a graded bialgebra of finite type over a field k of characteristic 0. Then A is a free graded commutative k-algebra.

Proof.

Step 1 I think this is the computation above.

Step 2 A_{gr} is a bialgebra.

Step 3 A_{gr} is multiplicative generated by Z^1/Z^2. All elements Z^1/Z_2 are primitive, so this algrebra A_{gr} is generated by primitive elements.

Step 4 Let $\{x_i\}$ be a basis of primitive elements of A_{gr}. Then lifts of A have no relations because A_{gr} is already generated by primitive elements. Then there are no relations also for elements in $A^\bullet$ (I think).

$\square$

3.5 Grassmanians (Reminder)

B vector bundle of rank n on X then there exists a map (essentialy unique) $\varphi : X \to \mathrm{Gr}(n)$ such that

$$\varphi^*(B_{\mathrm{fun}} = B$$

which makes the Grassmanian a classifying space, and $\mathrm{Gr}(1) = B\,U(n)$.

The infinite Grassmanian is important.

3.6 BU as an H-space (Reminder)

Bott periodicity identifies the space of loops on U and $B\,U$.

Proposition. Embed $\mathbb{C}^\infty \times \mathbb{C}^\infty$ into $\mathbb{C}^\infty$ taking the basis vectors of the first copy to the even basis vectors and the basis of the second copy to the odd. Then for $L_1 \subset \mathbb{C}^\infty$, $L_2 \subset \mathbb{C}^\infty$, the map

$$L, L' \mapsto S(L, L')$$

defines a structure of H-space on the infinite Grassmanian $B\,U$.

Proof. Just show that H-associatity up to homotopy. $\square$

Corollary. $H^\bullet(B\,U, \mathbb{Q})$ is a free supercommutative algebra.

Proof. Follows from Hopf theorem. $\square$

3.7 Cohomology of BU

Claim. $H^\bullet(B\,U, \mathbb{Q})$ is a free polynomial algebra generated by classes $c_1, c_2, \ldots$ in all even degrees.

Demostração. Leray-Serre spectral sequence. $\square$

3.8 Chern classes: axiomatic definition

Definition. ***Chern classes*** are classes $c_i(B) \in H^{2i}(X)$ for $i = 0, 1, 2, \dots$

Chern classes are $c_i(B) \in H^{2i}(X)$ for a complex vector bundle B over X with axioms

a. $c_0(B) = 1$

b. Functoriality (commutes with bullbacks): for $\varphi : X \to Y$ with B bundle on Y,

$$\varphi^*(c_i(B)) = c_1(\varphi^*(B))$$

c. Define ***total Chern class*** $c_* := \sum_i c_i(B)$ then

$$c_i(B) \cdot c_i(B') = c_*(B \oplus B') \qquad \text{(Whitney)}$$

d. $\mathcal{O}(1)$ on $\mathbb{C}P^n$,

$$c_i(\mathcal{O}(1) = 1 + [H]$$

where $[H]$ is the fundamental class of a hyperplane section.

Suppose we have a class $a \in H^\bullet(BU)$. Then for all B on X

$$\varphi : X \to BU$$

so

$$B \cong \varphi^*(B_{\text{fun}})$$

and so

$$\varphi_B^*(c_*) = c_*(B).$$

4 Class 4

4.1 Reminder

For each rank n bundle B on X there exists $\varphi_B : X \to \text{Gr}(n, \infty) = BU(n)$ such that $\varphi_B^*(B_{\text{fun}} = B$.

The infinite grassmanian is classifying space for (?) stable bundles.

Some more review about H-space structure, primitive elements, a comment on last exercise of homework 2.

Chern classes of $\mathcal{O}(1)$ are hyperplane sections: $c_i(\mathcal{O}(1)) = 1 + [H]$.

4.2 The splitting principle

Exercise. Prove that $BU(1) = \mathbb{C}P^\infty$.

Solution. Hopf fibration on S^∞? It's easier, take $n = 1$, it's just by definition. □

Definition. The ***fundamental bundle*** on $BU(1)^n$ has fiber

$$\ell_1 \oplus \ell_2 \oplus \ldots \ell_n$$

where $\ell_i \in BU(1)$ are product $\ell_1 \times \ell_2 \times \ldots \times \ell_n$.

Remark. Chern classes of B_{fun} are uniquely determined by axioms, because every factor has Chern classes, and fibers are just sums, and pullbacks preserve sums...

$$B_{\text{fun}} = \bigoplus_i \pi_i \mathcal{O}(1)$$

where

$$pi_i : BU(1)^n \to BU(1)$$

is a projection.

Remark. $H^\bullet(BU(1))^n = \mathbb{Z}[z_1, \ldots, zn$ Here at least I remember that the cohomology of $\mathbb{C}P^\infty$ is just polynomials so it looks reasonable that the n-th power is polynomials in more cariables.

Theorem (Splitting principle)**.** Let $\varphi_{\text{fun}} : BU(1)^n \to BU$, the ***fundamental map***, it induces embedding on cohomology up to degree $2n$. For all primer generator $\sigma_i \in H^2(BU)$, $\varphi_{\text{fun}}(\sigma_1) = \lambda \sum_i z_i^k$ with $\lambda \neq 0$.

So

$$\begin{tikzcd}
BU(1)^n \arrow[r] & BU \\
X \arrow[u] \arrow[ur] &
\end{tikzcd}$$

Remark. Wiki Thus, the set of isomorphism classes of circle bundles over a manifold M are in one-to-one correspondence with the homotopy classes of maps from M to $\mathbb{C}P^\infty$

Theorem. Chern classes are unique (uniquely determined by axioms).

Proof.

Step 1 Every bundle is obtained as pullback of the fundamental bundle. So for $A \in H^\bullet(BU)$ and B bundle on X, $A(B) = \varphi_B^*(A) \subset H^\bullet(X)$ so $c_i(B)$ are obtained as pullbacks of c in the fundamental bundle.

Step 2

$$BU(1)^\infty \xrightarrow{\varphi_{\text{fun}}} BU$$

pullback of fundamental bundle is fundamental. (This map is defined from the former by induction).

$$\varphi_{\text{fun}}^*(c_i(B_{\text{fun}}) = c_i(B_{\text{fun}} \text{ on } BU)$$

The Chern classes of the fundamental bundle are already known. Since φ_{fun}^* is injective by the splitting principle we are done.

$\square$

4.3 Primitive generators of $H^*(BU)$

Recall the H-space multiplication:

$$\begin{aligned} BU \times BU &\longrightarrow BU \\ L_1 \times L_2 &\longmapsto L_1 \oplus L_2 \end{aligned}$$

and the comultiplication

$$\Delta : H^\bullet(BU) \to H^\bullet(BU)$$

Generators of $H^\bullet(BU)$ are $c_{h_1}, c_{h_2}, \dots$ with $c_{h_i} \in H^{2i}(BU)$ and we have the comultiplication $\Delta(c_{h_i}) = c_{h_i} \otimes 1 + 1 \otimes c_{h_i}$.

Remark.

$$\varphi = (\varphi_1, \varphi_2) : X \to BU \times BU$$

and we can compose so we have

$$\varphi \circ \mu : X \to BU$$

what does this map do?

$$\begin{aligned} \varphi \circ \mu : X &\longrightarrow BU \\ \varphi^*(B_{\text{fun}} &\longmapsto B_1 \\ (\varphi \circ \mu)^*(B_{\text{fun}}) &= B_1 \oplus B_2 \end{aligned}$$

So then we have

$$\begin{aligned} \varphi^* : H^\bullet(BU) \otimes H^\bullet(BU) &\to H^\bullet(X) \\ \Delta : H^\bullet(BU) &\to H^\bullet(BU) \otimes H^\bullet(BU) \\ \Delta \circ \varphi^* : H^\bullet(BU) &\to H^\bullet(X) \end{aligned}$$

Corollary. For every $x \in H^\bullet(BU)$

$$X(B_1 \oplus B_2) = \Delta(x)(B_1, B_2)$$

Corollary. If $x \in H^*(BU)$ is primitive, then $x(B_1 \oplus B_2) = x(B_1) \oplus X(B_2)$.

Proof. $\Delta(x) = x \otimes 1 + 1 \otimes x$ so $\Delta(x)$ evaluated on (B_1, B_2) ☐

Remark. We will construct the full Chern class $c_*(B)$ as a pullback of a class $C \in H^*(BU)$.

Remark. Then take exponential. Let $\chi_i \in H^{2i}(BU)$ be a primitive generator. Use Hopf theorem to see that it is unique by a constant. Snce $\chi_i(B_1 \oplus B_2) = \chi_i(B_1) + \chi_i(B_2)$, the class $C = e^{\sum_i a_i \chi_i} = 1 + \dots + \frac{\chi_n}{n!} + \dots$ satisfies the Whitney formula.

To construct Chern classes satisfying the axioms it remains to arrange the coefficients a_i in such a way that $C(\mathcal{O}(1)) = 1 + [H]$ I think this means hyperplane section.

Lemma. An embedding

$$BU(1) \overset{\varphi}{\hookrightarrow} BU$$

with $\chi_i \in H^{2i}(BU)$ primitive generator. Then $\varphi^*(\chi_i) \neq 0$

Proof. $H^\bullet(BU) =$ symmetric polynomials in $H^i(BU(1))^n$, $\varphi_{\mathrm{fun}}(x_N) = x \sum_{i=1}^n z_i^k$ so $\varphi(x_k) = \lambda x_1^k$. □

Remark. $\varphi^*(c_i(B_{\mathrm{fun}}) = c_i(\mathcal{O}(1) = 1 + [H]$

Theorem. Choose generators $\chi_i \in H^2(BU)$ primitive. Then $\varphi^*(\sum_i \chi_i = \log(1 + [H])$ where the logarith is a formal power series, namely $\sum_{i=1}^\infty \frac{H^n}{n!}(-1)^n$.

That means $c(B_{\mathrm{fun}}) = \exp\left(\sum_{\chi_i}\right)$.

5 Class 5