Some problems in Hamiltonian geometry of PDEs

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Abstract The Hamiltonian formulation of Partial Differential Equations is one of the cornerstones of the theory of Integrable Systems. Being carried out by analogy with finite-dimensional Hamiltonian systems, it has an intrinsic geometric nature. In this talk we will review differential-geometric aspects of the Hamiltonian theory of PDEs as well as new projective-geometric properties of known Integrable Systems that are emerging in recent years.

Contents

- Introduction to the Hamiltonian formalism of PDEs
- Geometry of the Hamiltonian formalism.
- Classification of bi-Hamiltonian ?

Definition A is a ***Hamiltonian operator*** if and only if

$$[F, G] = \int \frac{\delta G}{\delta u^i} A^{ij} \frac{\delta}{\delta ?}$$

Example KdV equation leads to a bi-Hamiltonian system.

- Motivation for Hamiltonian PDEs: A Hamiltonian maps conservation laws to symmetries.
- Classification of (?bi-)Hamiltonians leads to classification of PDEs.

Remark There is some sort of projective invariance of solutions of WDVV equation. Does that lead to some interesting result about solutions?

Projective geometry of homogeneous second order Hamiltonian operators Classification of linear operators making them correspond to algebraic varieties. See Vergallo-Vitolo, 2023, *Projective geometry of homogeneous second order Hamiltonian operators* for a Fano variety found this way.

3 order Hamilt op	quadratic line complex
2 order	system of line compl
R_2 first order	quadratic line complex