Brody lemma and Kobayashi metric

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November 7, 2024

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Upshot Brody lemma says Kobayashi hyperbolic is equivalent to contaning $\mathbb{C}$.

$\mathsf{SO}(n)$ acts on $\mathbb{R}^n$ any $\mathsf{SO}(n)$-invariant quadratic form is proportional to $\sum x_i^2$. The orbits fo $\mathsf{SO}(n)$ are spheres...

Definition ***Brody curve*** is a non-constant holomorphic map $f : \mathbb{C} \to M$ such that $|df| \leqslant C$ for some constant C.

Definition (Δ_r, g_r) a disck of raduis r in $\mathbb{C}$ and Poincaré metric rescaled "in a such a way that the unit tangent vector to 0 has length 1". ***Brody map*** is a holomorphic $\Delta_r \to M$ with $|df| \leqslant 1$ and $|df|_\partial = 1$.

Lemma 1 Let $f_r : \Delta_r \to M$ be a sequence of Brody maps, then f_r converges uniformly to a Brody curve.

Proof. Took some time, using Kobayashi metric, theorems of uniform convergence. □

Theorem (Brody lemma) Let Mbe a compact complex manifold that is not Kobayashi hyperbolic. Then M contains a Brody curve.

Proof. Somehow we have a sequence of maps, but they are not Brody. To have Brody curve we need to take the limit of Brody maps. □