Reptile Search Algorithm (RSA): A novel nature-inspired meta-heuristic optimizer

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Abstract
This paper proposes a novel nature-inspired meta-heuristic optimizer, called Reptile Search Algorithm (RSA), motivated by the hunting behaviour of Crocodiles. Two main steps of Crocodile behaviour are implemented, such as encircling, which is performed by high walking or belly walking, and hunting, which is performed by hunting coordination or hunting cooperation. The mentioned search methods of the proposed RSA are unique compared to other existing algorithms. The performance of the proposed RSA is evaluated using twenty-three classical test functions, thirty CEC2017 test functions, ten CEC2019 test functions, and seven real-world engineering problems. The obtained results of the proposed RSA are compared to various existing optimization algorithms in the literature. The results of the tested three benchmark functions revealed that the proposed RSA achieved better results than the other competitive optimization algorithms. The results of the Friedman ranking test proved that the RSA is a significantly superior method than other comparative methods. Finally, the results of the examined engineering problems showed that the RSA obtained better results compared to other various methods.

Keyword: Reptile Search Algorithm (RSA); Optimization Algorithms; Meta-heuristics; Real-word Problems; Optimization Problems.

1. Introduction

There is a well-known theory said, “survival of the fittest”. Hence, predators have to choose a robust approach to maximize the hunting the prey in natural life (Bartumeus et al., 2005). Generally, the foraging activity of various animals in nature is a dramatically random walk approach; a stochastic manner in which the next position is subject to the current situation/position and a change likelihood to the next position which can be mathematically represented as an optimization technique (Shcherbacheva, 2019; Viswanathan et al., 1999). These approaches have been developed by the environment and typically chosen by predators to survive.

There are two main classes for optimization techniques: (1) deterministic methods, which are divided into linear and non-linear approaches (Horst & Tuy, 2013). These approaches are the most used deterministic methods, designated by utilizing the gradient erudition of the problem to explore the search space and find the optimal solution (Abualigah & Diabat, 2020; Abualigah, 2020a). In spite of these approaches are useful for linear search problems (Unimodal), they are susceptible to local optima trap when implementing to non-linear search problems (Multimodal), including real-world optimization problems. To overcome this problem, several methods can be used such as a different initial population strategy, hybridize, or modify the algorithm (Luenberger et al., 1984). (2) Stochastic methods, which are another alternative that generates and utilizes random variables such as the meta-heuristic optimization algorithms. These optimization algorithms are utilized to globally search in the available search space of the problem to obtain the near-optimal solution (Gardiner et al., 1985). The advantages of these algorithms are simplicity, flexibility, independency to the problem, easy to use, and gradient-free nature (Abualigah, 2020b; Abualigah et al., 2020b).

Recently, meta-heuristic algorithms have been successfully employed to address various complicated optimization problems (Gandomi & Alavi, 2012; Gandomi, 2014). These algorithms are exposed to be more robust than the other traditional methods that are based on conventional logic or mathematical programming. Exploration (diversification) and exploitation (Intensification) search strategies are two chief merits of the meta-heuristic algorithms. The exploration search strategy works to ensure that the algorithm explores the given search space widely and efficiently. While the exploitation search strategy searches around the obtained-optimal solution to find the best candidate solution. The main objectives of introducing advanced meta-heuristic algorithms are to solve various complicated optimization problems faster and to get more robust optimization methods (Erol & Eksin, 2006; Kaveh & Farhoudi, 2013; Geem et al., 2001).

According to the nature of inspiration, meta-heuristic algorithms are classified into main four classes (Abualigah & Diabat, 2021): (1) evolutionary-based algorithms (EA) (Fonseca & Fleming, 1995), (2) swarm-based intelligence (SI) (Parpinelli & Lopes, 2011), (3) physics-based methods (PM) (Biswas et al., 2013), and (4) human-based methods (HM) (Kosorukoff, 2001). EAs mimic the behaviour of natural evolution. These algorithms utilize operators (i.e., crossover and mutation) inspired by biologies. The most commonly utilized EA is GA. GA uses these operators to generate improved solutions. Other examples of the EA class are evolutionary programming (Fogel et al., 1966), differential evolution (Storn & Price, 1997), and evolution strategy (Hansen et al., 2003). SI simulates the social behaviour of animals in swarms (i.e., herds, flocks, or schools). Mainly, the characteristic of this class is the sharing of joint information of all animals through the optimization process. The most utilized algorithm in this class is particle swarm optimization (PSO), developed by Kennedy and Eberhart (Eberhart & Kennedy, 1995). The Fox Red Optimization Algorithm is suggested based on a mathematical representation of red fox rules and hunting for food (Połap & Woźniak, 2021). Black Widow Optimization Algorithm is proposed in (Hayyolalam & Kazem, 2020) based on the inspiration of mating behavior of black widow spiders. Other examples of SI class are Salp Swarm Algorithm (Abualigah et al., 2019), Ant Colony Optimization (Dorigo et al., 2006), and Dolphin Echolocation (Kaveh & Farhoudi, 2013). PMs are inspired by physical laws in life, and mainly defines the communication of the candidate solutions based on controlling rules of the physical methods. One of the most commonly utilized algorithms in PM class is Simulated Annealing, which utilizes thermodynamics laws to heating and the next handler cooling of an element to grow the volume of its crystals. Other examples of PM class are Gravitational Search Algorithm (Rashedi et al., 2009), Henry Gas Solubility Optimization (Hashim et al., 2019), and Charged System Search (Kaveh & Talatahari, 2010b). Finally, HMs are motivated by human communications and behaviour in communities. This class is consequently emphasized to generate better solutions until met the termination criteria. Examples of HMs are the Imperialist Competitive Algorithm (ICA) (Atashpaz-Gargari & Lucas, 2007), and Teaching-Learning-Based Optimization (TLBO) (Rao et al., 2011). Moreover, these various methods have been widely used to solve different problems such as image segmentation (Abuowaida et al., 2021), microscopy image analysis (Połap, 2020), (Altabeeb et al., 2021), task scheduling (Abd Elaziz et al., 2021), economic emission dispatch (Hassan et al., 2021), optimal allocation of power resources (Eid et al., 2021), feature selection (Jiang et al., 2021), vulnerability detection (Şahin & Abualigah, 2021), images classification (Yousri et al., 2021), intrusion detection system (Safaldin et al., 2021), identifying photovoltaic models (Yousri et al., 2020), and others.

Genetic Algorithms (GA) (Holland et al., 1992), Harmony Search (HS) Algorithm (Geem et al., 2001), Cuckoo Search Optimization (CS) (Yang & Deb, 2009), Krill Herd Algorithm (KHA) (Gandomi & Alavi, 2012), Gray Wolf Optimizer (GWO) (Mirjalili et al., 2014), Artificial Bee Colony (ABC) (Karaboga & Akay, 2009), Aquila Optimizer (AO) (Abualigah et al., 2021b), and Arithmetic Optimization Algorithm (AOA) (Abualigah et al., 2021a) are some of the common traditional meta-heuristics optimization algorithms. Despite the success of traditional and recent optimization algorithms, no algorithm can guarantee to achieve the best global optimum solutions for various optimization problems. This has been proven by the theorem of the No-Free-Lunch in search and optimization (Abualigah et al., 2020a; Wolpert & Macready, 1997). This theory motivated us to introduce a new optimization algorithm and solve various optimization problems more efficiently.

We intend to propose a more dynamic and effective algorithm; this paper introduces a new natural-inspired based meta-heuristic optimizer, called Reptile Search Algorithm (RSA). This algorithm is stimulated by the encircling and hunting behaviours of Crocodiles in the real-life. The main difference between the proposed RSA and others is that RSA has a unique procedure modeled to update the solutions' positions using four novel mechanisms. For example, encircling is performed by high walking or belly walking, and hunting is conducted by hunting coordination or hunting cooperation. The main motivation behind RSA is to find powerful search methods that can produce better quality solutions for the complicated problems and get new best results that can help solve complex real-world problems. A set of twenty-three classical test functions, thirty CEC2017 test functions, and ten CEC2019 test functions is used to verify the robustness and effectiveness of the proposed RSA rigorously. Moreover, eight real-world engineering problems are used to investigate the effectiveness of the proposed RSA further.

The remainder of the paper is organized as: Section 2 describes the Reptile Search Algorithm developed in this paper. Section 3 presents the results, discussion, and evaluation of RSA on various optimization problems. Section 4 presents the conclusion of the current work and recommends future directions. Thus, two mathematical models were introduced to update the positions of candidate solutions; one for diverse search and another toward the optimal search region.

2. The Reptile Search Algorithm (RSA)

In this section, the exploration (global search) and exploitation (local search) phases of the proposed Reptile Search Algorithm (RSA) are presented, which is inspired by the encircling mechanisms, hunting mechanisms, and the social behaviour of Crocodiles in nature. Crocodiles behaviours consist of encircling and hunting the prey. These mechanisms are mathematically modeled to present the proposed RSA and perform the optimization processes. RSA is a population-based and gradient-free method, so it can be used to address complicated or straightforward optimization problems subject to specific constraints. Cohesive groups are helpful for active co-operation between Crocodiles and also maximize their robustness.

2.1. Biology and behaviour of Crocodiles

Crocodiles (subfamily includes the true crocodiles) are colossal semi-aquatic semiaquatic creepers that live everywhere in tropics such as Australia, Africa, Asia, and the Americas. The word crocodile refers to only the kinds within the subfamily of "Crocodylinae". Generally, a crocodile's physical characteristics support them to be a strong predator (Dinets et al., 2015; Kushlan & Mazzotti, 1989). Their external shape is a sign of its water and predatory lifestyle. Crocodiles have very little resistance to a flow of air and water (streamlined body); this shape increases its speed. It makes their movement easier, which enables them to move quickly. As well, Crocodiles also raise their feet to the side during walking, which runs faster. Crocodiles' webbed feet allow them to walk turns and sudden movements quickly in the swimming. These feet are a distinctive feature, where the animals usually move from one place to another by walking (Dinets, 2015; Platt et al., 2006). The main characterizations of the Crocodile behaviour are given as follows.

Vision: Crocodiles have perfect night eyesight and are mainly nightly hunters. They utilize the weakness (i.e., miserable night eyesight) of prey animals to their support.

Hunting and diet: Crocodiles are snare predators, looking for nearby fish or land animals, and then running out to attack. Crocodiles hunt fish, reptiles, crustaceans, amphibians, mollusks, mammals, and birds, and sometimes they eat smaller crocodiles. Crocodiles often hunt small fish and invertebrates, gradually moving on to larger prey. Crocodiles are predators (in cold-blooded); they have a prolonged metabolism so that they can survive for a long time without food. Notwithstanding their appearance of moving slow, crocodiles have a swift beating. They are top and powerful predators in their habitat, and several classes have been recognized, attacking and killing other predators such as sharks and deer. As well, when given a chance, they would prey on young or dying elephants and other different animals. Evidence implies that crocodiles also feed on several kinds of fruits.

Locomotion: Crocodiles can run very fast over small distances, even out of water. When a crocodile moves quickly, it keeps its legs in a straighter and more straightforward situation under its body (called the high walk). This kind of walk allows crocodiles a high speed.

Cognition: Crocodiles own some advanced cognitive skills. They can recognize and utilize patterns of prey behaviour, such as while prey closes to the river to drink frequently.

Hunting: Crocodiles are advanced hunters: cooperate as a team to hunt their prey. Hunting is carried out by crocodiles cooperatively based on coordination and collaboration. Coordinated hunting is a sophisticated form of collaborative hunting, in which particular predators associate in each other's movements adjusted on the target-prey during organized hunting. Coordinated hunting is a sophisticated form of cooperative hunting, in which specific predators associate in each other's actions, which is considered to be uncommon for the animal lacking a backbone. In some instances, individual Crocodile runs on the same role (being the ambusher or the driver) during various hunts, as seen in lions.

Coordination and collaboration of crocodiles: Crocodiles hunt in a team, as the modern study has shown. This distinguished them, members, as one of the best sophisticated and intelligent teams that can make cooperation between different individuals with different roles. Seeing crocodiles hunting operation is very complicated. They hunt by ambush; they occasionally eat due to their metabolism is slow, and almost all hunting happens at night and maybe in shallow waters. For example, the crocodiles moved together to force a group of fishes to gather in a small group (dense group). They then took turns hunting fish from this group (bait ball). Next, the crocodiles will take turns cutting over the circle centroid, attack the fishes. Often, Crocodiles of various sizes follow various roles. More giant Crocodiles lead fishes from the deeper area of a lagoon into the shallows, where smaller, more intelligent Crocodiles prevents its escape. Other cases included a crocodile scared a pig or a zebra (wildebeests), making it move quickly into a lagoon where other crocs were disappearing and waiting for the attack. Hunting behaviour of Crocodiles is relatively constant; when a Crocodile caught a prey, it would move out of the hunting space and join the Crocodiles again after eating its victim. It seems that the animals after hunting take some resting and wait their turn to join the active hunting team.

A conclusion is that crocodiles are one of the most intelligent and expert hunters and possibly next behind humans. We modeled Crocodile behaviours as a mathematical optimization, and it is determining the best solution subjected to specific constraints. Optimization problems occur in various quantitative disciplines, from engineering, economics, and computer sciences to operations research and industry, and improvements in searching techniques have been attracting interest in several domains of sciences. The main inspiration of the proposed algorithm (RSA) derived from the encircling and hunting the prey. In the following subsections, the descriptions of these processes in the RSA are discussed. RSA is then proposed based on the mathematical model.

2.2. Initialization phase

In RSA, the optimization process starts with a set of candidate solutions (X) as shown in Equation (1), which is generated stochastically, and the best-obtained solution is considered as the nearly the optimum in each iteration.

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & x_{1,n-1} & x_{1,n} \\ x_{2,1} & \cdots & x_{2,j} & \cdots & x_{2,n} \\ \cdots & \cdots & x_{i,j} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{N-1,1} & \cdots & x_{N-1,j} & \cdots & x_{N-1,n} \\ x_{N,1} & \cdots & x_{N,j} & x_{N,n-1} & x_{N,n} \end{bmatrix} \tag{1}$$

where X is a set of the candidate solutions that are generated randomly by using Equation (2), $x_{i,j}$ denotes to the j_{th} position of the i_{th} solution, N is the number of candidate solutions, and n denotes to the dimension size of the given problem,.

$$x_{ij} = rand \times (UB - LB) + LB, \; j = 1, 2, ..., n \tag{2}$$

where $rand$ is a random value, LB and UB denote to the lower and upper bound of the given problem, respectively.

2.3. Encircling phase (Exploration)

In this section, the exploratory behaviour (encircling) of RSA is introduced. According to the encircling behaviour, Crocodiles have two movements during the encircling are high walking and belly walk. These movements refer to different reigns, which commitment to the exploration search (globally). Crocodile movements (high and belly walking) cannot allow them to approach the target prey due to their disturbance easily, unlike another search phase (hunting phase). Hence, the exploration search discovers a wide search space; it can find the density area maybe after several endeavors. In addition, the exploration mechanisms (high and belly walking) are operated at this stage of optimization to support the other phase (hunting/exploration) in the search process through extensive and spread research.

The RSA can transfer between encircling (exploration) and hunting (exploitation) search phases, this change between various behaviours is done based on four conditions; divide the total number of iterations into four parts. The exploration mechanisms of RSA explore the search regions and approach to find a better solution based on two main search strategies (high walking strategy and belly walking strategy).

This phase of searching is conditioned on two conditions. The high walking movement strategy is conditioned by $t \leq \frac{T}{4}$, and the belly walking movement strategy is conditioned by $t \leq 2\frac{T}{4}$ and $t > \frac{T}{4}$. This means that this condition will be satisfied for almost the half number of exploration iterations (High walking) and another half for the Belly walking. These are two exploration search methods. Note, a stochastic scaling coefficient is examined for the element to generate more diverse-solutions and explore diverse-regions. We employed the most straightforward rule, which can mimic the encircling behaviour of Crocodiles. In this paper, the position updating equations are proposed for the exploration phase as in Equation (3).

$$x_{(i,j)}(t+1) = \begin{cases} Best_j(t) \times -\eta_{(i,j)}(t) \times \beta - R_{(i,j)}(t) \times rand, & t \leq \frac{T}{4} \\ Best_j(t) \times x_{(r_1,j)} \times ES(t) \times rand, & t \leq 2\frac{T}{4} \; and \; t > \frac{T}{4} \end{cases} \tag{3}$$

where $Best_j(t)$ is the j_{th} position in the best-obtained solution so far, $rand$ denotes to a random number between 0 and 1, t is the number of the current iteration, and T is the maximum number of iterations. $\eta_{(i,j)}$ denotes to the hunting operator for the j_{th} position in the i_{th} solution, which is calculated using Equation (4). β is a sensitive parameter, controls the exploration accuracy (i.e., High walking) for encircling phase over the course of iterations, which is fixed equal to 0.1. Reduce function ($R_{(i,j)}$) is a value used to reduce the search area, which is calculated using Equation (5). r_1 is a random number between [1 N] and $x_{(r_1,j)}$ denotes to a random position of the i_{th} solution. N is the number of the candidate solutions. Evolutionary Sense ($ES(t)$) is a probability ratio takes randomly decreasing values between 2 and -2 throughout the number of iterations, which is calculated using Equation (6).

$$\eta_{(i,j)} = Best_j(t) \times P_{(i,j)}, \tag{4}$$

$$R_{(i,j)} = \frac{Best_j(t) - x_{(r_2,j)}}{Best_j(t) + \epsilon}, \tag{5}$$

$$ES(t) = 2 \times r_3 \times \left(1 - \frac{1}{T}\right), \tag{6}$$

where, ϵ a small value and r_2 is a random number between [1 N]. In Equation (6), 2 is used as a correlation value to give values between 2 and 0, r_3 denotes to a random integer number between -1 and 1. $P_{(i,j)}$ is the percentage difference between the j_{th} position of the best-obtained solution and the j_{th} position of the current solution, which is calculated using Equation (7).

$$P_{(i,j)} = \alpha + \frac{x_{(i,j)} - M(x_i)}{Best_j(t) \times (UB_{(j)} - LB_{(j)}) + \epsilon}, \tag{7}$$

where $M(x_i)$, as in Equation (7), is the average positions of the i_{th} solution, which is calculated using Equation (8). $UB_{(j)}$ and $LB_{(j)}$ are the upper and lower boundaries of the j_{th} position, respectively. α is a sensitive parameter, controls also the exploration accuracy (the difference between candidate solutions) for the hunting cooperation over the course of iterations, which is fixed equal to 0.1 in this paper.

$$M(x_i) = \frac{1}{n} \sum_{j=1}^{n} x_{(i,j)}, \tag{8}$$

2.4. Hunting phase (Exploitation)

In this section, the exploitative behaviour (hunting) of RSA is introduced. According to the hunting behaviour, Crocodiles have two strategies during the hunting are hunting coordination and cooperation. These strategies refer to different intensify techniques, which commitment to the exploitation search (locally).

Crocodile strategies (hunting coordination and cooperation) allow them to approach the target prey easily due to their intensification, unlike encircling mechanisms. Hence, the exploitation search discovers the near-optimal solution, maybe after several endeavors. Besides, the exploitation mechanisms are operated at this stage of optimization to conduct an intensification search near the optimal solution and emphasized communication between them.

The exploitation mechanisms of RSA exploit the search space and approach to find the optimal solution based on using two main search strategies (i.e., (1) hunting coordination and (2) hunting cooperation), which is modelled as in Equation (9). The searching in this phase is conditioned as the hunting coordination strategy is conditioned by $t \leq 3\frac{T}{4}$ and $t > 2\frac{T}{4}$, otherwise, the hunting cooperation strategy is performed, when $t \leq T$ and $t > 3\frac{T}{4}$. Note, stochastic coefficients are considered to generate more dense-solutions and exploit the promising regions (locally). We employed the most straightforward rule, which can mimic the hunting behaviour of Crocodiles. In this paper, the following position updating equations are proposed for the exploitation phase (Equation (9)):

$$x_{(i,j)}(t+1) = \begin{cases} Best_j(t) \times P_{(i,j)}(t) \times rand, & t \leq 3\frac{T}{4} \ and \ t > 2\frac{T}{4} \\ Best_j(t) - \eta_{(i,j)}(t) \times \epsilon - R_{(i,j)}(t) \times rand, & t \leq T \ and \ t > 3\frac{T}{4} \end{cases} \tag{9}$$

where $Best_j(t)$ is the j_{th} position in the best-obtained solution so far, $\eta_{(i,j)}$ denotes to the hunting operator for the j_{th} position in the i_{th} solution, which is calculated using Equation (4). $P_{(i,j)}$ is the percentage difference between the j_{th} position of the best-obtained solution and the j_{th} position of the current solution, which is calculated using Equation (7). $\eta_{(i,j)}$ denotes to the hunting operator for the j_{th} position in the i_{th} solution, which is calculated using Equation (4). ϵ a small value. $R_{(i,j)}$ is a value used to reduce the search area, which is calculated using Equation (5).

In this respect, Figures 1 and 2 show that when $t \leq \frac{T}{2}$, the encircling phase (exploration) happens, otherwise; when $t > \frac{T}{2}$, the hunting phase (exploitation) occurs to be close enough to prey when attacking.

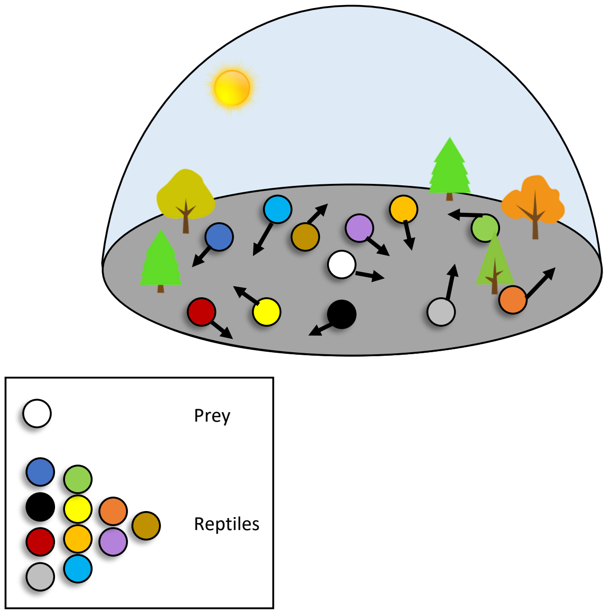


Figure 1: Encircling the prey, when ($t \leq \frac{T}{2}$).

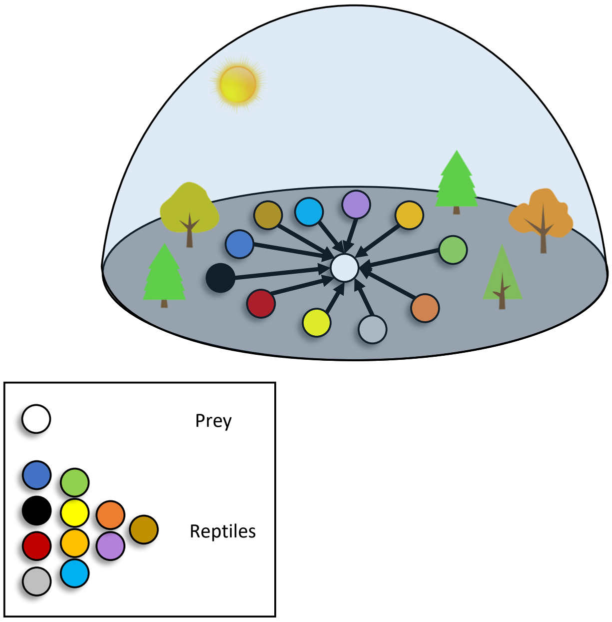


Figure 2: Attacking the prey, when ($t > \frac{T}{2}$).

Exploitation search mechanisms (hunting coordination and cooperation) are attempting to evade getting trapped in the local optima. These procedures assist the exploration search in determining the optimal solution and maintain the diversity over the candidate solutions. We carefully designed two parameters (i.e., β and α) to produce a stochastic value at each iteration, continue exploration not only during the first iterations but also last iterations. This part of searching is beneficial in the situation of local optima stagnation, particularly in the last iterations.

2.5. Pseudo-code of the Reptile Search Algorithm (RSA)

To recap, in RSA, the optimization process begins with generating a random set of candidate solutions (population). During the trajectory of repetition, the search mechanisms of the RSA explore the possible positions of the near-optimal solution. Each solution replaces its positions from the best-obtained solution according to the processes of the proposed RSA.

To emphasize exploration and exploitation, the search processes are divided into two main methods (exploration and exploitation) with four strategies. Exploration: high walking strategy and belly walking strategy. Exploitation: hunting coordination and cooperation. Candidate solutions attempt to expand the search area when $t \leq \frac{T}{2}$ and attempt to converge towards the near-optimal solution when $t > \frac{T}{2}$. In the exploration phase, the high walking movement strategy is performed when $t \leq \frac{T}{4}$, and the belly walking movement strategy is performed when $t \leq 2\frac{T}{4}$ and $t > \frac{T}{4}$. In the exploitation phase, the hunting coordination strategy is performed when $t \leq 3\frac{T}{4}$ and $t > 2\frac{T}{4}$, otherwise, the hunting cooperation strategy is performed, when $t \leq T$ and $t > 3\frac{T}{4}$. Finally, the RSA is stopped when it met the end criterion. The Pseudo-code of the proposed RSA is detailed in Algorithm 1. The intuitive and detailed process of RSA is shown in Figure 3.

Algorithm 1 Pseudo-code of the Reptile Search Algorithm (RSA)

Initialization phase
Initialize RSA parameters α, β, etc.
Initialize the solutions' positions randomly. X: $i = 1, ..., N$.
while ($t < T$) **do**
  Calculate the Fitness Function for the candidate solutions (X).
  Find the Best solution so far.
  Update the ES using Equations (6).
  The beginning of the RSA
  for (i=1 to N) **do**
    for (j=1 to n) **do**
      Update the η, R, P and values using Equations (4), (5) and (7), respectively.
      if ($t \leq \frac{T}{4}$) **then**
        $x_{(i,j)}(t+1) = Best_j(t) \times$ - $\eta_{(i,j)}(t) \times \beta$ - $R_{(i,j)}(t) \times rand$, $\triangleright$ {**High walking**}
      else if ($t \leq 2\frac{T}{4}$ and $t > \frac{T}{4}$) **then**
        $x_{(i,j)}(t+1) = Best_j(t) \times x_{(r_1,j)} \times ES(t) \times rand$, $\triangleright$ {**Belly walking**}
      else if ($t \leq 3\frac{T}{4}$ and $t > 2\frac{T}{4}$) **then**
        $x_{(i,j)}(t+1) = Best_j(t) \times P_{(i,j)}(t) \times rand$, $\triangleright$ {**Hunting coordination**}
      else
        $x_{(i,j)}(t+1) = Best_j(t)$ - $\eta_{(i,j)}(t) \times \epsilon$ - $R_{(i,j)}(t) \times rand$, $\triangleright$ {**Hunting cooperation**}
      end if
    end for
  end for
  t=t+1
end while
Return the best solution ($Best(X)$).

2.6. Time complexity

The complexity of the developed RSA depends on the complexity of three main parameters (i.e., initialization processes, number of fitness evaluation, and updating of solutions) and it is given as follows.

Note that the computational complexity of the initialization process, with N solutions, is $O(N)$. The computational complexity of the updating processes is $O(T \times N) + O(T \times N \times D)$. Consequently, the computational complexity of the proposed RSA is as follows.

$$O(RSA) = O(N \times (T \times D + 1)) \tag{10}$$

where, T number of iterations, N presents the number of used solutions, and D presents the solution size.

3. Experimental results and discussions

In this study, the new developed meta-heuristic method RSA is applied to solve different global optimization problems, including twenty-three classical, thirty CEC2017, and ten CEC2019 benchmark test functions. In addition, it is used to find the best solution for seven engineering problems and several feature selection problems. During the evaluation of RSA over the tested problems, its results are compared with grasshopper optimisation algorithm (GOA) (Saremi et al., 2017), salp swarm algorithm (SSA) (Mirjalili et al., 2017), whale optimization algorithm (WOA) (Mirjalili & Lewis, 2016), sine cosine algorithm (SCA) (Mirjalili, 2016b), dragonfly algorithm (DA) (Mirjalili, 2016a), grey wolf optimizer (GWO) (Mirjalili et al., 2014), PSO (Eberhart & Kennedy, 1995), ant lion optimizer (ALO) (Mirjalili, 2015a), marine predators

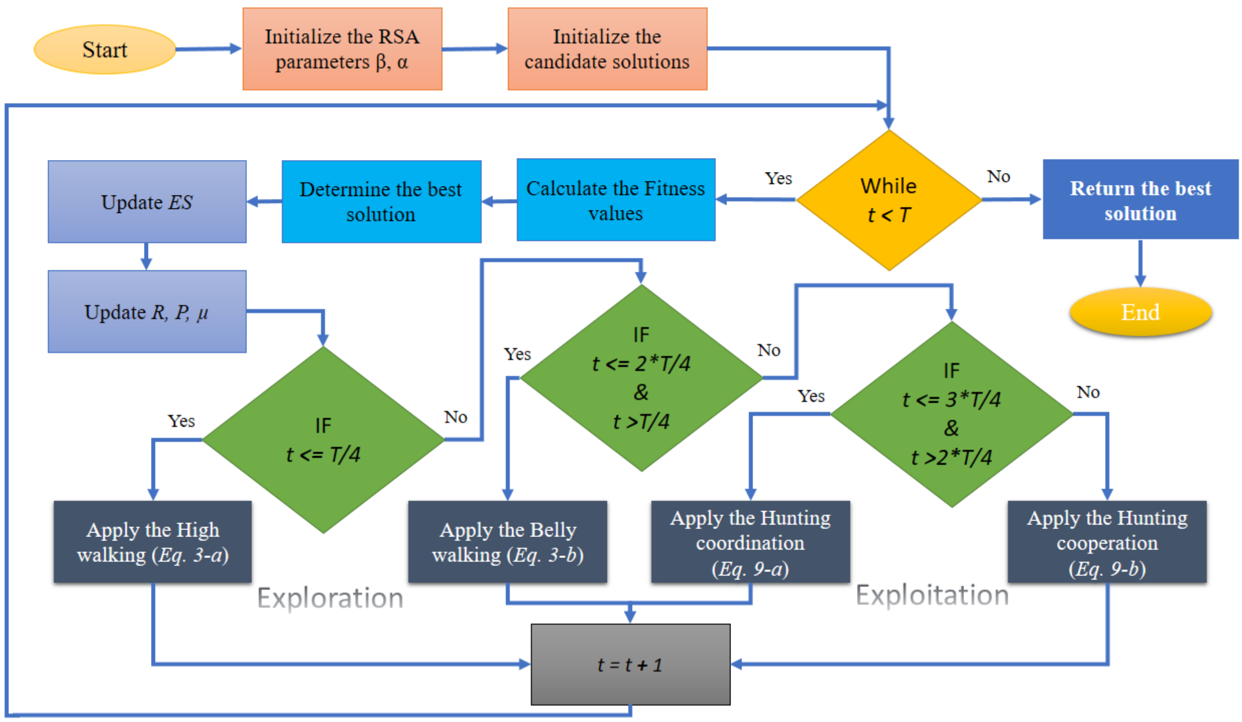


Figure 3: Flowchart of the proposed Reptile Search Algorithm (RSA).

Algorithm (MPA) (Faramarzi et al., 2020a), equilibrium optimizer (EO) (Faramarzi et al., 2020b), and covariance matrix adaptation evolution strategy (CMAES) (Hansen & Ostermeier, 2001).

The common parameters in the current study, such as the total number of iterations is set to 500, and the number of Crocodiles is 30. In addition, the statistical measures are computed for each algorithm after performing it 25 runs. So, the number of function evaluations is 15000. Also, the parameter setting of each algorithm is defined according to its implementation, whereas, the parameters of the RSA are given as α is fixed equal to 0.1 and β is fixed equal to 0.1 by experiments. The experiments are conducted using a computer Core i7-8565U with 16GB RAM and 64-bit for Microsoft Windows 10. The source code is implemented using MATLAB (R2015a).

To assess the quality of the obtained results by RSA and other Comparative algorithms, a set of performance measures is used. Such as the worst, best, average, and standard deviation (STD) of the fitness value overall the independents 25 times for each tested problem. Moreover, a non-parametric statistical tests called Friedman ranking test is applied for a fair comparison. In general, Friedman ranking test produces a mean rank for each algorithm and ranking them (Martin et al., 1993).

3.1. Definition of twenty-three classical test functions

The process of evaluating the quality of RSA starting by using a set of twenty-three 2005 classical functions (Suganthan et al., May 2005). This set of functions is classified into three categories; the first category is called unimodal functions, and the main characteristics that discriminant this type is that they have a single extreme point in the search domain. The functions F1-F7 are examples of this type, and their definition are given in Table 1. The second and third type is named multimodal and multimodal with fixed dimension. These functions have more than an extreme solution. Functions F8-F13 are multimodal functions (see Table 2) and functions F14-F23 are examples from multimodal with fixed dimension, and their definitions are given in Table 3.

Table 1: Unimodal benchmark functions.

Function	Description	Dimensions	Range	f_{min}
F1	$f(x)=\sum_{i=1}^{n} x_i^2$	30, 100, 500, 1000	[-100,100]	0
F2	$f(x)=\sum_{i=0}^{n}\lvert x_i\rvert+\prod_{i=0}^{n}\lvert x_i\rvert$	30, 100, 500, 1000	[-10,10]	0
F3	$f(x)=\sum_{i=1}^{d}(\sum_{j=1}^{i} x_j)^2$	30, 100, 500, 1000	[-100,100]	0
F4	$f(x)=max_i\{\lvert x_i\rvert, 1\le i\le n\}$	30, 100, 500, 1000	[-100,100]	0
F5	$f(x)=\sum_{i=1}^{n-1}[100(x_i^2-x_{i+1})^2+(1-x_i)^2]$	30, 100, 500, 1000	[-30,30]	0
F6	$f(x)=\sum_{i=1}^{n}([x_i+0.5])^2$	30, 100, 500, 1000	[-100,100]	0
F7	$f(x)=\sum_{i=0}^{n} ix_i^4+\text{random}[0,1)$	30, 100, 500, 1000	[-128,128]	0

Table 2: Multimodal benchmark functions.

Function	Description	Dimensions	Range	f_{min}
F8	$f(x)=\sum_{i=1}^{n}(-x_i sin(\sqrt{\lvert x_i\rvert}))$	30, 100, 500, 1000	[-500,500]	-418.9829× n
F9	$f(x)=\sum_{i=1}^{n}[x_i^2-10cos(2\pi x_i)+10]$	30, 100, 500, 1000	[-5.12,5.12]	0
F10	$f(x) = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2}) - exp(\frac{1}{n}\sum_{i=1}^n cos(2\pi x_i))+20+e$	30, 100, 500, 1000	[-32,32]	0
F11	$f(x)=1+\frac{1}{4000}\sum_{i=1}^n x_i^2-\prod_{i=1}^n cos(\frac{x_i}{\sqrt{i}})$	30, 100, 500, 1000	[-600,600]	0
F12	$f(x)=\frac{\pi}{n}\{10sin(\pi y_1)\}+\sum_{i=1}^{n-1}(y_i-1)^2[1+10sin^2(\pi y_{i+1})+\sum_{i=1}^n u(x_i,10,100,4)]$, where $y_i = 1+\frac{x_i+1}{4}, u(x_i,a,k,m)\begin{cases}K(x_i-a)^m & \text{if } x_i>a\\ 0 & -a\le x_i\ge a\\ K(-x_i-a)^m & -a\le x_i\end{cases}$	30, 100, 500, 1000	[-50,50]	0
F13	$f(x)=0.1(sin^2(3\pi x_1)+\sum_{i=1}^n(x_i-1)^2[1+sin^2(3\pi x_i+1)]+(x_n-1)^2 1+sin^2(2\pi x_n))+\sum_{i=1}^n u(x_i,5,100,4)$	30, 100, 500, 1000	[-50,50]	0

Table 3: Fixed-dimension multimodal benchmark functions.

Function	Description	Dimensions	Range	f_{min}
F14	$f(x)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}}(x_i-a_{ij})\right)^{-1}$	2	[-65,65]	1
F15	$f(x)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	4	[-5,5]	0.00030
F16	$f(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	[-5,5]	-1.0316
F17	f(x)=$\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10(1-\frac{1}{8\pi})cos\ x_1+10$	2	[-5,5]	0.398
F18	$f(x)=\left[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\right]\times\left[30+(2x_1-3x_2)^2\times(18-32x_i+12x_1^2+48x_2-36x_1x_2+27x_2^2)\right]$	2	[-2,2]	3
F19	$f(x)=-\sum_{i=1}^4 c_i exp\left(-\sum_{i=1}^3 a_{ij}(x_j-p_{ij})^2\right)$	3	[-1,2]	-3.86
F20	$f(x)=-\sum_{i=1}^4 c_i exp\left(-\sum_{i=1}^6 a_{ij}(x_j-p_{ij})^2\right)$	6	[0,1]	-.32
F21	$f(x)=-\sum_{i=1}^5\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,1]	-10.1532
F22	$f(x)=-\sum_{i=1}^7\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,1]	-10.4028
F23	$f(x)=-\sum_{i=1}^{10}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,1]	-10.5363

3.1.1. Parameters sensitivity of the RSA

In this section, the effect of adjusting the parameters' values of RSA is tested. Two main sensitive parameters (i.e., $\alpha = 0.1$ and $\beta = 0.1$) in RSA are studied, so we test different scenarios according to the value of these parameters. These parameters are evaluated at one value from 0.1, 0.5, and 0.9; therefore, we have nine scenarios (as in Table 4). We conducted extensive experiments before adding the results in this paper, in which we tested the parameter values, and we recognized that the parameter ranges are between 0.1 and 1. Thus, we presented coverage samples (0.1, 0.5, and 0.9) to show the effects of these values on the performance of the proposed optimizer. Note that a set of values were taken in the range of [0.1 0.9] to study their effect on the proposed RSA's performance. These numbers are chosen to cover the specified range with several equal periods. More extensive values can be taken to determine the best deals for these variables.

Table 5 depicts the statistical results achieved in each scenario among the used thirteen functions. From these results, it can be seen that the first scenario (i.e., $\alpha = 0.1$ and $\beta = 0.1$) has better results among all the tested functions in all scenarios. Followed by sixth and fifth scenarios that earmarked the second and third rank, respectively. Nevertheless, it can be seen that the performance of the proposed RSA at all of these scenarios is similar at F8, F10, and F11. Moreover, by comparing the third scenario (first rank) with the ninth scenario (final rank), it can be remarked that the RSA at the third scenario wins eight times while at another scenario wins three times. Additionally, in the third scenario, RSA lost only five times but RSA using the ninth scenario lost ten times.

Table 4: Scenarios of the tuning perimeters.

Scenario No.	α value	β value
1	0.1	0.1
2	0.1	0.5
3	0.1	0.9
4	0.5	0.1
5	0.5	0.5
6	0.5	0.9
7	0.9	0.1
8	0.9	0.5
9	0.9	0.9

3.1.2. Qualitative analysis for the convergence of RSA

In this section, we qualitatively analyze the convergence of the RSA using trajectories and convergence curves. Figure 4 shows the different qualitative metrics for convergence of RSA. For example, the first column in Figure 4 illustrates the shape of the tested functions in 2D to focus on the topology of the domain.

Moreover, the second column in Figure 4 indicates the search history that illustrates the behaviour of collective and interactional between crocodiles and prey. This provides RSA with a suitable tool to detect the modality of the collective search of Crocodiles. This modality illustrates the collection of Crocodiles around the optimal point in unimodal functions, whereas the scattering of Crocodiles when RSA is used to solve the multimodal functions. All of these properties of modality helps in exploring and exploiting the results when RSA is applied to multimodal and unimodal functions, respectively.

Furthermore, the average fitness value of agents during the iterations is given in the third column in Figure 4. It can be seen from these curves that the agents have variant values at the initial the iterations and in most cases this variation still to the end iterations. This verified the RSA has a high ability to maintain the diversity over the iterations.

In addition, the topological variations of a Crocodiles during the process of optimization is measures using the trajectory metric as in the fourth column. It can be noticed from the curve of trajectory that the first solution in the first dimension has high frequency and magnitude, specifically at the early generations, which can be vanished in the last generations. This refers to that RSA has a high exploration ability at the initial process of optimization, then switching to the exploitation phase in the later period of generation. According to this behaviour, the RSA can converge to the global solution (Van den Bergh & Engelbrecht, 2006).

Also, one can be seen that the frequency and magnitude of topological variations for multimodal and composition functions are occurred several times than those for the unimodal functions and especially last longer. This indicates the high robustness and adaptability of the RSA when used to solve different optimization functions since it has a high ability to balance between exploration and exploitation.

From the last column in Figure 4 it can be seen the convergence curve that represents the best agent determined so far. According to the type of each function, the convergence curve has a specific pattern. For

Table 5: The influence of the RSA parameters (i.e., α and β) tested on various test functions.

Fun	Measure	**Scenario No.**								
		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
F1	Worst	2.9726E-139	4.5873E-146	1.6338E-157	5.1699E-171	1.7344E-165	4.5736E-148	1.6476E-154	1.4740E-153	9.62E-168
	Average	5.9452E-140	9.1747E-147	3.2679E-158	1.1228E-171	4.7511E-166	9.1473E-149	3.3267E-155	2.9480E-154	1.93E-168
	Best	4.3644E-159	6.6540E-166	3.5363E-174	6.5145E-178	4.1126E-174	5.9392E-185	9.8708E-171	9.5832E-179	2.14E-174
	STD	1.3294E-139	2.0515E-146	7.3062E-158	0.0000E+00	0.0000E+00	2.0454E-148	7.3511E-155	6.5920E-154	0.00E+00
	Rank	9	8	4	1	3	7	5	6	2
F2	Worst	1.0336E-64	7.8114E-83	1.9162E-87	8.8143E-89	1.3987E-86	1.6705E-91	1.4983E-81	6.1081E-86	3.27E-84
	Average	2.5840E-65	1.9536E-83	4.8503E-88	4.2369E-89	3.4969E-87	7.8710E-92	3.7456E-82	1.5816E-86	8.17E-85
	Best	1.3589E-81	1.6671E-93	1.8202E-95	3.1742E-93	5.5908E-94	6.4421E-94	4.5301E-93	2.6738E-94	9.94E-94
	STD	5.1680E-65	3.9052E-83	9.5415E-88	4.8728E-89	6.9935E-87	7.9993E-92	7.4913E-82	3.0184E-86	1.63E-84
	Rank	9	7	3	2	4	1	8	5	6
F3	Worst	1.9258E-96	1.0538E-103	6.7198E-126	4.0726E-111	5.9064E-130	3.1028E-112	4.0648E-119	4.9366E-123	4.25E-122
	Average	4.8144E-97	2.7844E-104	2.3623E-126	1.0182E-111	1.4766E-130	7.7570E-113	1.0406E-119	1.2342E-123	1.06E-122
	Best	3.0968E-112	1.2938E-120	2.4357E-141	2.3414E-147	2.7999E-151	4.7313E-130	1.6856E-124	2.2717E-153	1.89E-142
	STD	9.6289E-97	5.1743E-104	3.1772E-126	2.0363E-111	2.9532E-130	1.5514E-112	2.0164E-119	2.4683E-123	2.12E-122
	Rank	9	8	2	7	1	6	5	3	4
F4	Worst	7.1543E-63	4.7120E-67	5.3059E-79	5.5196E-78	7.5783E-79	2.9953E-76	1.7624E-75	4.1342E-75	9.55E-78
	Average	1.7887E-63	1.3062E-67	1.3267E-79	1.3799E-78	1.8963E-79	1.2007E-76	4.7395E-76	1.1448E-75	2.39E-78
	Best	5.2599E-69	1.4221E-78	1.3705E-88	1.0499E-90	1.5501E-83	5.0723E-79	1.8144E-85	2.1469E-87	1.25E-84
	STD	3.5771E-63	2.2834E-67	2.6529E-79	2.7598E-78	3.7880E-79	1.4285E-76	8.5960E-76	2.0039E-75	4.77E-78
	Rank	9	8	1	3	2	5	6	7	4
F5	Worst	2.8981E+01	2.8983E+01	2.8995E+01	2.8992E+01	2.8990E+01	2.8980E+01	2.8987E+01	2.8984E+01	2.90E+01
	Average	2.8931E+01	2.8976E+01	2.8991E+01	2.8984E+01	2.8980E+01	2.8975E+01	2.8979E+01	2.8980E+01	2.90E+01
	Best	2.8812E+01	2.8969E+01	2.8984E+01	2.8974E+01	2.8970E+01	2.8960E+01	2.8971E+01	2.8972E+01	2.89E+01
	STD	8.0079E-02	6.8193E-03	5.2820E-03	7.5520E-03	8.0779E-03	9.7193E-03	7.0393E-03	5.6050E-03	2.22E-02
	Rank	1	4	9	8	6	3	5	7	2
F6	Worst	6.6513E+00	7.0289E+00	7.4195E+00	7.2288E+00	7.1179E+00	7.2659E+00	7.1355E+00	7.1569E+00	7.28E+00
	Average	6.4199E+00	6.7310E+00	7.3179E+00	7.0070E+00	7.0125E+00	7.0855E+00	7.0092E+00	7.0307E+00	7.14E+00
	Best	6.2061E+00	6.5659E+00	7.2507E+00	6.7356E+00	6.8213E+00	6.8716E+00	6.8980E+00	6.9506E+00	6.80E+00
	STD	2.2444E-01	2.0413E-01	7.1784E-02	2.1039E-01	1.3266E-01	1.7479E-01	9.7678E-02	8.9708E-02	2.28E-01
	Rank	1	2	9	3	5	7	4	6	8
F7	Worst	1.8584E-04	4.6636E-04	2.5236E-03	1.0740E-03	9.8080E-04	4.0462E-04	1.1080E-03	2.2067E-03	3.41E-03
	Average	8.5611E-05	2.0017E-04	1.3702E-03	3.7573E-04	5.9518E-04	2.9627E-04	5.9084E-04	8.5127E-04	1.33E-03
	Best	7.6870E-06	2.9867E-05	2.4748E-05	9.4919E-05	1.1975E-04	1.2066E-04	1.7343E-04	2.2633E-05	3.34E-05
	STD	7.9871E-05	1.9039E-04	1.2968E-03	4.6681E-04	3.7166E-04	1.2552E-04	3.9577E-04	9.9148E-04	1.52E-03
	Rank	1	2	9	4	6	3	5	7	8
F8	Worst	-4.8545E+03	-2.8849E+03	-1.2858E+03	-1.9232E+03	-2.0809E+03	-4.8269E+03	-2.6107E+03	-4.1314E+03	-1.95E+03
	Average	-5.2996E+03	-4.1857E+03	-3.5857E+03	-3.4489E+03	-3.8105E+03	-5.0747E+03	-4.2818E+03	-4.8366E+03	-2.87E+03
	Best	-5.6561E+03	-5.1984E+03	-5.5435E+03	-5.3504E+03	-5.6182E+03	-5.4177E+03	-5.3888E+03	-5.4841E+03	-4.23E+03
	STD	3.3664E+02	1.0474E+03	1.8107E+03	1.6898E+03	1.8798E+03	2.5223E+02	1.1872E+03	6.8133E+02	1.03E+03
	Rank	1	5	7	8	6	2	4	3	9
F9	Worst	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Average	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Best	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Rank	1	1	1	1	1	1	1	1	1
F10	Worst	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.88E-16
	Average	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.88E-16
	Best	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.8818E-16	8.88E-16
	STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Rank	1	1	1	1	1	1	1	1	1
F11	Worst	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Average	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Best	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00E+00
	Rank	1	1	1	1	1	1	1	1	1
F12	Worst	1.4808E+00	1.4296E+00	1.5564E+00	1.6158E+00	1.5458E+00	1.5738E+00	1.5213E+00	1.5712E+00	1.63E+00
	Average	1.1817E+00	1.2638E+00	1.3766E+00	1.5422E+00	1.3887E+00	1.4567E+00	1.4875E+00	1.4791E+00	1.55E+00
	Best	7.3178E-01	1.0229E+00	9.0488E-01	1.4412E+00	1.0528E+00	1.3599E+00	1.4639E+00	1.4430E+00	1.47E+00
	STD	3.3233E-01	2.0011E-01	3.1487E-01	7.7043E-02	2.3072E-01	1.0209E-01	2.4895E-02	6.1508E-02	7.15E-02
	Rank	1	2	3	8	4	5	7	6	9
F13	Worst	2.9970E+00	2.9947E+00	2.9997E+00	3.1230E+00	2.9987E+00	2.9977E+00	2.9977E+00	2.9992E+00	3.00E+00
	Average	2.9723E+00	2.9671E+00	2.9896E+00	3.0300E+00	2.9976E+00	2.9909E+00	2.9969E+00	2.9848E+00	3.00E+00
	Best	2.9445E+00	2.9399E+00	2.9611E+00	2.9990E+00	2.9963E+00	2.9732E+00	2.9964E+00	2.9433E+00	3.00E+00
	STD	2.8529E-02	2.8321E-02	1.9043E-02	6.1946E-02	1.0335E-03	1.1851E-02	5.9045E-04	2.7716E-02	1.49E-03
	Rank	2	1	4	9	8	5	6	3	7
Mean	rank	3.54	3.85	4.15	4.31	3.69	3.62	4.46	4.31	4.77
Final	ranking	1	4	5	6	3	2	8	6	9

example, the convergence curve of the unimodal functions is smooth and it's enhanced with a small number of iterations. Whereas, the convergence curve of the multimodal functions is improved in stepwise behaviour since those functions are more complex than unimodal functions. Moreover, it can be seen that the agents of RSA at the initial iterations are encircling the optimum agent and they aim to improve the position of agents with increasing the iteration. Meanwhile, the agents explore the search space from the first iteration till the last iteration with trying to determine the better position of agents.

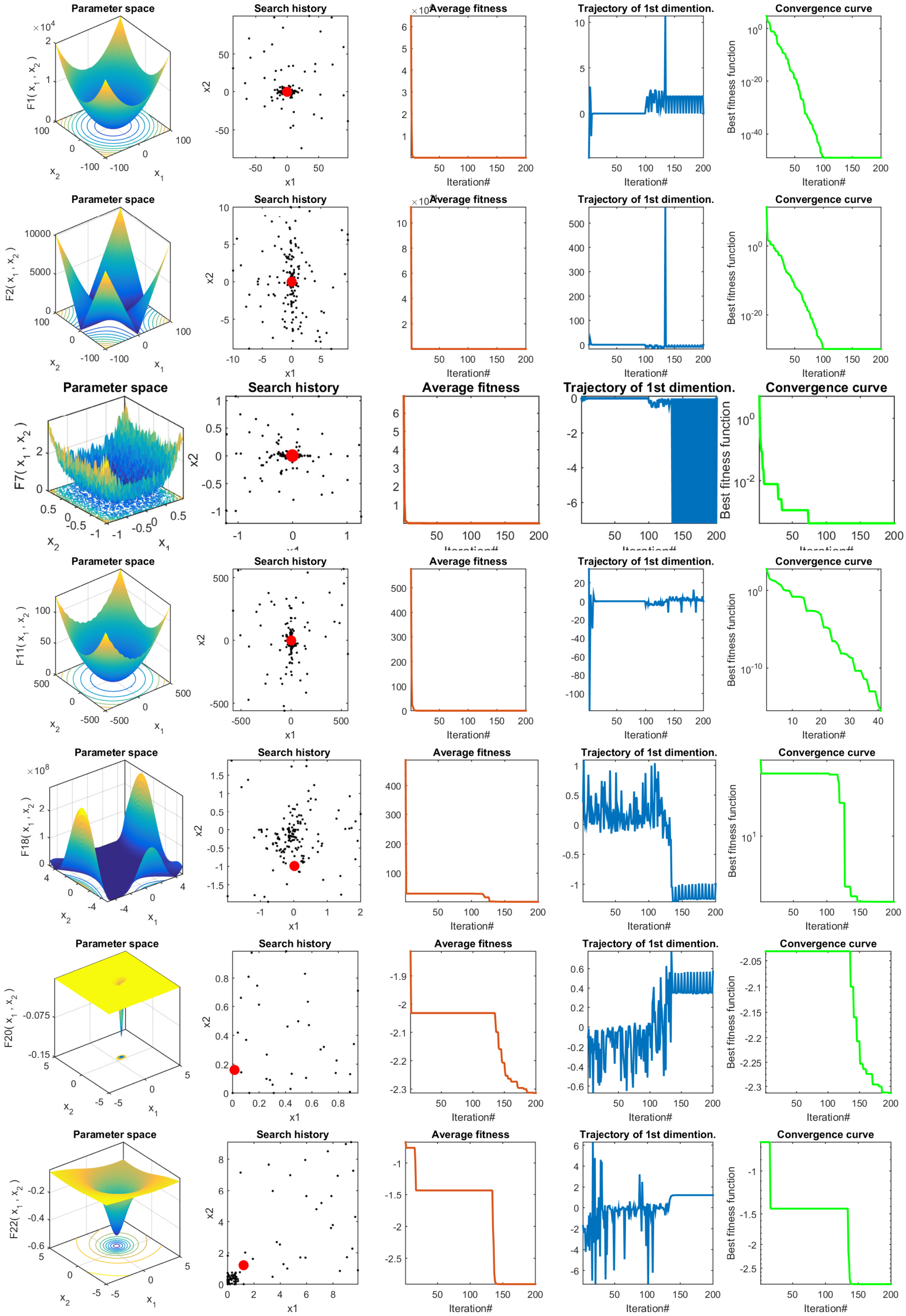


Figure 4: Qualitative results for the studied problems

3.1.3. Comparison with other MH methods using classical functions

In this section, the performance of the RSA to find the optimal solution for unimodal and multimodal functions is evaluated. Tables 6-9 show the comparison results between the RSA and other meta-heuristic techniques, including GOA, SSA, WOA, SCA, DA, GWO, PSO, ALO, MPA, EO, and CMA-ES. Those MH algorithms are used since they established their abilities to solve this kind of global optimization problems.

From Table 6 it can be seen that the RSA achieves the first rank overall other algorithms followed by CMA-ES, MPA, and EO, which allocate second, third, and fourth rank, respectively. Moreover, from thirteen functions, the RSA provides a high ability to find the smallest average of fitness value at eight functions which represent nearly 62%. While, CMA-ES has a better average of fitness value at six functions (i.e., 46%) followed by PSO, which has the best value at function F6.

In addition, the performance of RSA and other MH techniques is assessed at variant dimensions 50, 100, and 5000 as in Tables 7-9. From these tables, it can be noticed that the RSA provides a better average of fitness value at these dimensions, which has the first rank. However, MPA at dimension 100 has the same rank (first rank) similar to RSA. However, by analyzing the behaviour of the two algorithms at this dimension, it can be observed that RSA has the first rank at eight functions. At the same time, MPA achieves better performance only at function F9. The main reason for both of them (i.e., MPA and RSA) have the same performance is that the RSA allocates later rank at functions F5, F6, F8, and F12 however, its performance at those functions are competitive.

Furthermore, it can be observed that the performance of RSA provides better results than other methods at high dimension (i.e., 500). As well as, there is no large difference between the performance of the RSA at these dimensions which indicates RSA is more applicable than other methods for applying it in different real-world applications that depend on high dimensions.

Table 6: Results of the RSA using unimodal and multimodal test functions (F1-F13), with 10 dimensions

Fun	Measure	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
F1	Worst	6.37E-171	3.70E-04	1.49E-09	3.00E-69	8.10E-10	2.36E+01	1.46E-48	6.23E-19	1.43E-07	1.09E-29	7.75E-55	7.52E-46
	Average	1.27E-171	9.23E-05	8.89E-10	1.02E-69	2.04E-10	1.31E+01	7.03E-49	1.68E-19	4.38E-08	5.16E-30	1.94E-55	3.85E-46
	Best	2.85E-181	4.53E-06	4.53E-10	1.30E-74	2.30E-15	9.41E-01	3.40E-51	2.89E-21	1.39E-08	4.83E-31	4.33E-58	9.01E-47
	STD	0.00E+00	1.57E-04	4.06E-10	1.43E-69	3.49E-10	8.56E+00	5.58E-49	2.59E-19	5.62E-08	3.90E-30	3.32E-55	2.46E-46
	Rank	1	11	9	2	8	12	4	7	10	6	3	5
F2	Worst	6.89E-86	8.07E+00	1.60E-01	7.05E-49	1.93E-08	5.00E+00	5.05E-27	5.68E-09	1.82E+00	2.66E-16	1.74E-32	1.30E-22
	Average	1.38E-86	1.99E+00	3.30E-02	1.41E-49	7.16E-09	2.68E+00	1.83E-27	1.82E-09	6.57E-01	1.11E-16	4.75E-33	7.53E-23
	Best	4.05E-94	4.28E-02	7.86E-06	2.33E-54	2.27E-11	1.12E+00	5.23E-29	1.85E-10	2.06E-05	1.56E-17	4.46E-34	4.53E-23
	STD	3.08E-86	3.46E+00	7.11E-02	3.15E-49	9.79E-09	1.49E+00	2.27E-27	2.32E-09	9.09E-01	1.30E-16	7.11E-33	3.52E-23
	Rank	1	11	9	2	8	12	4	7	10	6	3	5
F3	Worst	2.67E-142	7.92E+00	1.61E-02	9.11E+02	1.58E+00	2.30E+02	5.93E-19	8.94E-05	3.57E+01	8.90E-13	1.37E-26	2.10E-37
	Average	6.35E-143	2.37E+00	3.25E-03	4.20E+02	3.19E-01	1.51E+02	1.44E-19	2.44E-05	9.77E+00	2.95E-13	2.80E-27	7.68E-38
	Best	1.20E-149	7.67E-01	5.50E-06	1.80E+01	8.79E-06	4.94E+01	5.84E-22	3.02E-07	7.71E-01	4.33E-15	3.28E-30	8.51E-39
	STD	1.15E-142	3.12E+00	7.21E-03	3.74E+02	7.07E-01	6.70E+01	2.52E-19	3.72E-05	1.49E+01	4.10E-13	6.08E-27	8.74E-38
	Rank	1	9	7	12	8	11	4	6	10	5	3	2
F4	Worst	1.04E-81	3.68E-01	6.63E-04	1.02E+01	4.25E-03	5.49E+00	5.96E-15	9.07E-04	1.44E-01	8.96E-13	4.50E-18	9.80E-21
	Average	2.70E-82	1.60E-01	1.73E-04	3.16E+00	1.33E-03	3.12E+00	2.14E-15	2.49E-04	3.42E-02	4.83E-13	9.10E-19	4.95E-21
	Best	2.17E-89	8.75E-02	1.62E-05	2.10E-03	4.38E-06	1.59E+00	2.36E-16	2.09E-05	1.05E-03	2.31E-13	7.25E-22	2.73E-21
	STD	4.44E-82	1.18E-01	2.79E-04	4.05E+00	1.88E-03	1.62E+00	2.55E-15	3.73E-04	6.17E-02	2.71E-13	2.01E-18	2.88E-21
	Rank	1	10	6	12	8	11	4	7	9	5	3	2
F5	Worst	3.98E+00	4.73E+02	6.43E+02	8.93E+00	8.23E+00	1.21E+04	8.06E+00	7.65E+00	2.13E+02	8.82E+00	5.82E+00	2.36E-02
	Average	3.17E+00	1.32E+02	1.87E+02	7.65E+00	7.68E+00	2.71E+03	6.99E+00	4.92E+00	1.16E+02	8.78E+00	5.53E+00	1.87E-02
	Best	2.38E+00	3.59E+00	7.89E+00	6.82E+00	7.28E+00	1.35E+02	6.24E+00	1.02E-01	6.00E+00	8.76E+00	5.30E+00	1.46E-02
	STD	6.10E-01	1.98E+02	2.60E+02	8.26E-01	4.45E-01	5.28E+03	7.64E-01	2.84E+00	1.01E+02	2.38E-02	2.44E-01	3.31E-03
	Rank	2	10	11	6	7	12	5	3	9	8	4	1
F6	Worst	1.73E-09	7.23E-04	9.36E-01	2.34E-01	5.16E-01	4.58E+01	5.33E-06	2.03E-16	1.18E-07	8.61E-11	8.55E-14	4.18E-04
	Average	1.31E-09	2.23E-04	6.15E-01	5.02E-02	4.11E-01	1.69E+01	3.59E-06	4.36E-17	3.29E-08	2.88E-11	4.11E-14	3.29E-04
	Best	1.12E-09	3.70E-07	3.13E-01	1.20E-03	2.91E-01	1.03E+00	2.53E-06	2.89E-21	7.90E-09	3.76E-12	4.84E-16	7.90E-04
	STD	2.45E-10	2.95E-04	2.36E-01	1.03E-01	9.67E-02	1.86E+01	1.11E-06	8.96E-17	4.77E-08	3.28E-11	3.29E-14	4.77E-04
	Rank	4	7	11	9	10	12	6	1	5	3	2	8
F7	Worst	9.04E-05	1.52E-01	3.64E-02	8.91E-03	3.84E-03	4.22E-02	1.34E-03	1.61E-02	6.18E-02	2.17E-03	1.87E-03	1.94E-03
	Average	6.76E-05	1.08E-01	2.18E-02	5.21E-03	2.42E-03	2.40E-02	6.44E-04	9.41E-03	3.49E-02	1.08E-03	1.31E-03	1.09E-03
	Best	2.02E-05	5.45E-02	1.40E-02	1.99E-03	1.48E-03	7.39E-03	3.95E-04	3.96E-03	1.64E-02	3.95E-04	7.18E-04	5.06E-04
	STD	2.85E-05	4.36E-02	8.74E-03	3.33E-03	9.66E-04	1.64E-02	3.90E-04	4.37E-03	1.91E-02	7.20E-04	4.89E-04	5.82E-04
	Rank	1	12	9	7	6	10	2	8	11	3	5	4
F8	Worst	-2.88E+03	-2.41E+03	-2.49E+03	-2.60E+03	-1.90E+03	-2.58E+03	-2.29E+03	-1.52E+03	-2.03E+03	-3.24E+03	-2.64E+03	6.55E+04
	Average	-2.04E+03	-2.75E+03	-2.98E+03	-3.09E+03	-2.00E+03	-2.75E+03	-2.71E+03	-2.11E+03	-2.46E+03	-3.53E+03	-3.20E+03	6.55E+04
	Best	-3.59E+03	-3.32E+03	-3.26E+03	-3.82E+03	-2.10E+03	-3.00E+03	-3.14E+03	-2.59E+03	-3.22E+03	-3.95E+03	-3.59E+03	6.55E+04
	STD	5.62E+02	3.47E+02	3.01E+02	4.47E+02	7.80E+01	1.65E+02	3.48E+02	4.43E+02	4.97E+02	2.72E+02	3.91E+02	6.55E+04
	Rank	3	7	9	10	2	8	6	4	5	12	11	1
F9	Worst	0.00E+00	6.27E+01	2.59E+01	1.42E-14	6.37E+00	4.73E+01	1.51E+01	1.29E+01	4.28E+01	5.03E-07	0.00E+00	1.61E+01
	Average	0.00E+00	4.34E+01	1.43E+01	2.84E-15	1.27E+00	3.13E+01	4.31E+00	5.61E+00	3.14E+01	1.01E-07	0.00E+00	6.33E+00
	Best	0.00E+00	2.89E+01	5.97E+00	0.00E+00	2.71E-12	1.06E+01	0.00E+00	1.99E+00	1.59E+01	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	1.41E+01	8.75E+00	6.36E-15	2.85E+00	1.51E+01	6.66E+00	4.31E+00	1.14E+01	2.25E-07	0.00E+00	7.50E+00
	Rank	1	12	9	3	5	10	6	7	11	4	1	8
F10	Worst	8.88E-16	3.57E+00	2.81E+00	7.99E-15	6.91E-06	1.27E+01	1.51E-14	1.20E-09	2.01E+00	7.99E-15	4.44E-15	8.88E-16
	Average	8.88E-16	1.68E+00	1.49E+00	4.44E-15	1.72E-06	4.98E+00	1.15E-14	7.06E-10	4.03E-01	5.15E-15	4.44E-15	8.88E-16
	Best	8.88E-16	1.04E-03	9.23E-06	8.88E-16	5.66E-08	1.43E+00	7.99E-15	3.21E-10	8.18E-05	4.44E-15	4.44E-15	8.88E-16
	STD	0.00E+00	1.30E+00	1.10E+00	3.55E-15	2.95E-06	4.44E+00	3.55E-15	3.73E-10	9.00E-01	1.59E-15	0.00E+00	0.00E+00
	Rank	1	11	10	3	8	12	6	7	9	5	3	1
F11	Worst	0.00E+00	2.95E-01	2.65E-01	2.46E-01	4.73E-02	1.05E+00	5.92E-02	5.34E-01	3.76E-01	0.00E+00	2.95E-02	0.00E+00
	Average	0.00E+00	2.58E-01	1.94E-01	9.75E-02	1.22E-02	5.89E-01	1.39E-02	2.45E-01	2.35E-01	0.00E+00	5.90E-03	0.00E+00
	Best	0.00E+00	1.96E-01	1.33E-01	0.00E+00	6.63E-12	6.52E-02	0.00E+00	1.01E-01	1.21E-01	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	4.19E-02	5.83E-02	1.33E-01	2.04E-02	3.75E-01	2.57E-02	1.79E-01	1.13E-01	0.00E+00	1.32E-02	0.00E+00
	Rank	1	11	8	7	5	12	6	10	9	1	4	1
F12	Worst	9.17E-02	3.53E+00	5.18E-01	2.32E-02	2.19E-01	4.64E+00	2.03E-02	2.81E-19	5.63E+00	4.35E-11	6.85E-14	4.71E-32
	Average	6.18E-03	1.27E+00	1.93E-01	1.15E-02	1.34E-01	1.63E+00	1.02E-02	5.82E-20	2.90E+00	2.54E-11	1.78E-14	4.71E-32
	Best	3.55E-04	8.35E-04	5.15E-11	1.48E-03	6.60E-02	7.58E-02	2.74E-06	2.04E-21	2.63E-01	1.17E-11	5.98E-17	4.71E-32
	STD	2.81E-02	1.43E+00	2.14E-01	9.02E-03	6.02E-02	1.80E+00	9.98E-03	1.24E-19	1.95E+00	1.32E-11	2.88E-14	6.12E-48
	Rank	5	10	9	7	8	11	6	2	12	4	3	1
F13	Worst	3.94E-01	5.56E-02	1.10E-02	1.57E-01	4.75E-01	1.59E+00	2.00E-01	1.10E-02	1.10E-02	9.50E-11	4.39E-02	1.35E-32
	Average	1.40E-04	2.23E-02	2.20E-03	7.27E-02	3.93E-01	9.39E-01	4.00E-02	2.20E-03	4.40E-03	5.67E-11	8.79E-03	1.35E-32
	Best	3.76E-09	8.25E-04	5.78E-10	2.50E-03	3.35E-01	2.89E-01	6.55E-06	8.61E-22	2.25E-07	3.59E-11	4.03E-16	1.35E-32
	STD	1.58E-01	2.29E-02	4.91E-03	6.47E-02	5.09E-02	4.69E-01	8.94E-02	4.91E-03	6.02E-03	2.38E-11	1.97E-02	0.00E+00
	Rank	3	8	5	10	11	12	9	4	6	2	7	1
Mean	rank	1.9231	9.0000	8.4000	7.2000	7.6000	9.9333	5.6000	5.6667	8.2667	4.9333	5.0667	4.1333
Final	Ranking	1	11	10	7	8	12	5	6	9	3	4	2

Table 7: Results of the RSA using unimodal and multimodal test functions (F1-F13), with 50 dimensions

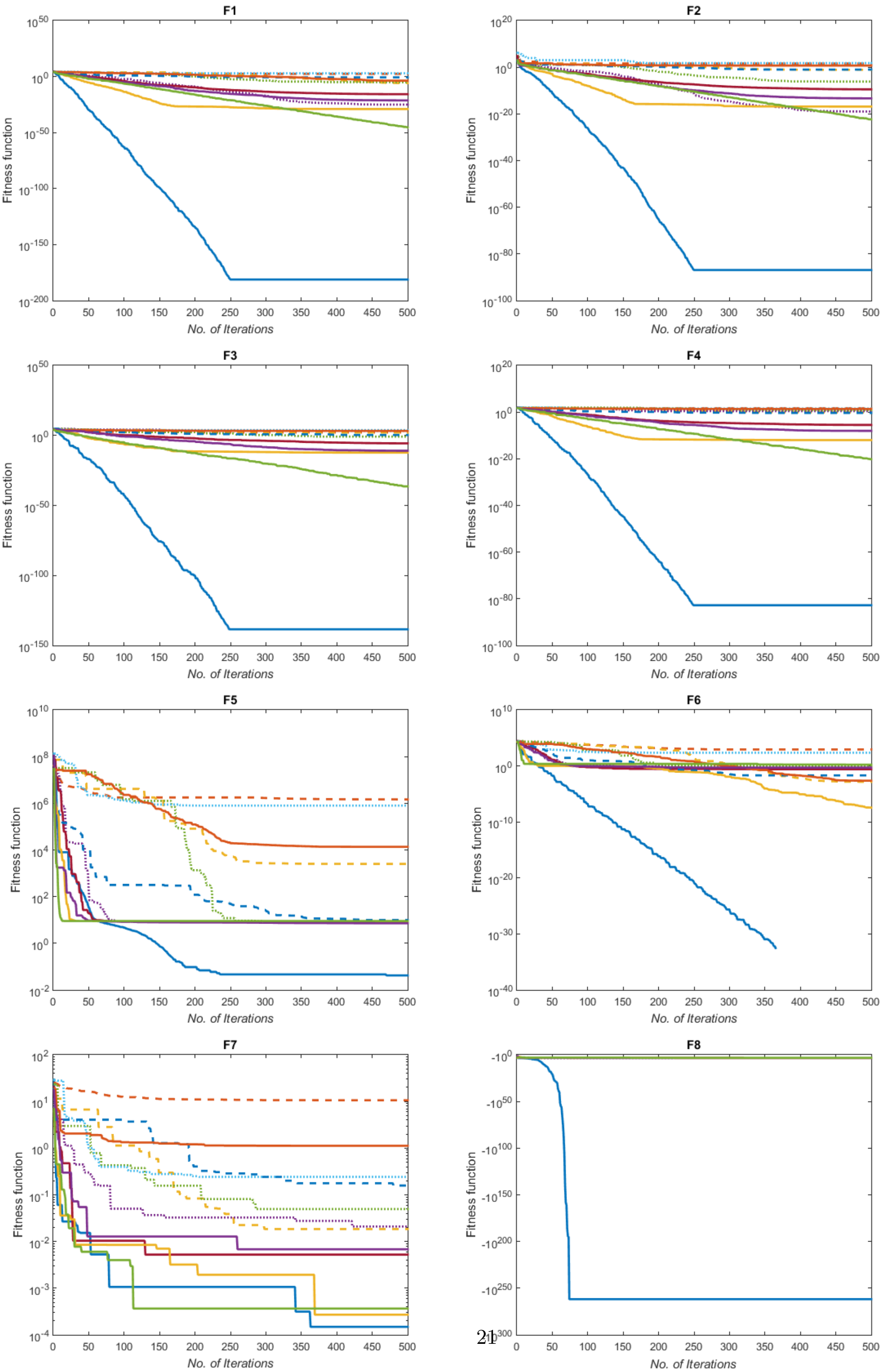
Fun	Measure	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
F1	Worst	1.20E-159	7.80E+03	1.02E+03	5.58E-42	7.02E+03	3.64E+03	3.70E-11	6.97E+00	1.11E+04	1.55E-21	4.07E-21	2.91E-06
	Average	2.99E-160	6.72E+03	6.14E+02	1.40E-42	3.28E+03	3.02E+03	1.21E-11	3.99E+00	6.73E+03	4.25E-22	1.08E-21	2.11E-06
	Best	8.94E-169	4.78E+03	2.80E+02	4.56E-58	5.74E+02	2.48E+03	3.01E-12	2.36E+00	3.74E+03	3.66E-24	3.20E-23	1.03E-06
	STD	5.98E-160	1.36E+03	3.05E+02	2.79E-42	2.70E+03	5.52E+02	1.66E-11	2.03E+00	3.11E+03	7.48E-22	1.99E-21	8.47E-07
	Rank	1	11	8	2	10	9	5	7	12	3	4	6
F2	Worst	5.79E-81	4.13E+24	1.76E+06	9.37E-36	1.58E+01	1.23E+02	7.17E-05	1.09E+02	3.83E+02	7.92E-12	1.29E-11	2.49E-01
	Average	1.45E-81	1.03E+24	4.45E+05	3.66E-36	7.97E+00	8.00E+01	6.72E-05	9.39E+01	2.16E+02	3.44E-12	9.36E-12	2.13E-01
	Best	3.55E-86	5.80E+02	2.31E+02	2.97E-43	1.43E+00	5.45E+01	6.35E-05	7.56E+01	1.44E+02	5.30E-13	5.36E-12	1.89E-01
	STD	2.89E-81	2.07E+24	8.75E+05	4.07E-36	7.01E+00	3.03E+01	3.52E-06	1.39E+01	1.12E+02	3.55E-12	3.37E-12	2.70E-02
	Rank	1	12	11	2	7	8	5	9	10	3	4	6
F3	Worst	2.24E-104	8.16E+04	6.14E+04	5.53E+05	1.52E+05	1.64E+05	3.37E+03	1.65E+04	1.67E+05	1.22E+01	2.16E+02	6.54E+00
	Average	5.60E-105	7.19E+04	5.64E+04	3.89E+05	8.90E+04	1.28E+05	1.62E+03	1.39E+04	1.20E+05	3.28E+00	6.02E+01	3.25E+00
	Best	5.26E-128	5.74E+04	5.21E+04	3.11E+05	3.35E+04	8.91E+04	2.77E+02	9.04E+03	8.76E+04	8.41E-02	1.83E+00	1.71E+00
	STD	1.12E-104	1.03E+04	4.66E+03	1.11E+05	5.65E+04	3.89E+04	1.58E+03	3.44E+03	3.72E+04	5.94E+00	1.04E+02	2.27E+00
	Rank	1	8	7	12	9	11	5	6	10	3	4	2
F4	Worst	7.22E+08	7.04E+08	8.00E+08	7.98E+08	7.30E+08	6.80E+08	8.53E+08	7.56E+08	8.89E+08	5.93E+08	6.87E+08	9.03E+08
	Average	5.52E+08	6.51E+08	6.77E+08	6.98E+08	6.31E+08	5.59E+08	6.45E+08	6.51E+08	7.61E+08	5.37E+08	6.18E+08	7.87E+08
	Best	4.80E+08	5.58E+08	4.58E+08	6.49E+08	4.50E+08	4.47E+08	4.83E+08	5.53E+08	6.73E+08	4.91E+08	4.40E+08	5.91E+08
	STD	6.25E+07	6.40E+07	1.51E+08	6.78E+07	1.24E+08	9.54E+07	1.56E+08	8.32E+07	9.23E+07	4.23E+07	1.19E+08	1.39E+08
	Rank	2	7	9	10	5	3	6	7	11	1	4	12
F5	Worst	4.90E+01	7.21E+07	3.21E+07	4.88E+01	5.68E+07	9.10E+07	4.88E+01	1.45E+05	2.54E+07	4.87E+01	4.86E+01	1.46E+02
	Average	4.60E+01	5.04E+07	2.40E+07	4.88E+01	2.52E+07	4.29E+07	4.87E+01	6.08E+04	1.65E+07	4.87E+01	4.79E+01	6.90E+01
	Best	4.89E+01	3.80E+07	1.78E+07	4.88E+01	1.97E+06	7.77E+06	4.86E+01	1.68E+04	5.52E+06	4.85E+01	4.72E+01	4.29E+01
	STD	1.80E-02	1.50E+07	6.57E+06	2.87E-02	2.76E+07	3.70E+07	1.06E-01	6.03E+04	8.22E+06	1.09E-01	7.60E-01	5.13E+01
	Rank	1	12	9	5	10	11	3	7	8	3	2	6
F6	Worst	1.22E+01	2.86E+04	1.66E+04	8.39E+00	8.98E+03	3.52E+04	7.91E+00	8.91E+01	2.90E+04	5.26E+00	6.77E+00	1.79E-06
	Average	1.18E+01	2.03E+04	1.07E+04	6.55E+00	4.02E+03	2.37E+04	7.11E+00	6.90E+01	2.41E+04	4.33E+00	6.37E+00	1.45E-06
	Best	1.14E+01	1.42E+04	5.78E+03	5.44E+00	4.04E+02	1.36E+04	6.23E+00	4.91E+01	2.17E+04	3.69E+00	5.67E+00	8.61E-07
	STD	3.54E-01	6.06E+03	4.48E+03	1.35E+00	3.76E+03	9.34E+03	6.91E-01	1.79E+01	3.34E+03	7.14E-01	4.79E-01	4.29E-07
	Rank	6	10	9	4	8	11	5	7	12	2	3	1
F7	Worst	3.02E-04	4.04E+02	1.22E+01	3.08E-02	2.02E+01	3.05E+01	5.04E-02	3.20E+02	3.15E+01	8.14E-03	1.70E-02	6.32E-03
	Average	1.75E-04	3.00E+02	1.05E+01	1.62E-02	1.52E+01	1.52E+01	2.69E-02	2.01E+02	1.88E+01	4.80E-03	6.78E-03	5.87E-03
	Best	7.86E-07	2.48E+02	8.51E+00	7.29E-03	4.56E+00	1.90E+00	1.21E-02	1.22E+02	9.03E+00	2.47E-03	1.08E-03	5.21E-03
	STD	1.49E-04	7.02E+01	1.69E+00	1.09E-02	7.22E+00	1.18E+01	1.74E-02	9.00E+01	1.03E+01	2.49E-03	7.12E-03	4.96E-04
	Rank	1	12	7	5	8	8	6	11	10	2	4	3
F8	Worst	-6.65E+03	-8.25E+03	-8.64E+03	-8.60E+03	-3.67E+03	-4.20E+03	-6.84E+03	-2.82E+03	-9.03E+03	-1.04E+04	-7.91E+03	-5.77E+12
	Average	-7.50E+03	-9.30E+03	-9.08E+03	-1.24E+04	-4.02E+03	-5.67E+03	-7.70E+03	-3.90E+03	-9.03E+03	-1.14E+04	-1.07E+04	-2.53E+17
	Best	-8.80E+03	-9.85E+03	-9.61E+03	-1.50E+04	-4.25E+03	-6.61E+03	-8.88E+03	-5.27E+03	-9.03E+03	-1.26E+04	-1.28E+04	-9.98E+17
	STD	9.24E+02	7.23E+02	4.37E+02	2.99E+03	2.48E+02	1.12E+03	8.91E+02	1.02E+03	0.00E+00	9.64E+02	2.08E+03	4.97E+17
	Rank	9	5	6	2	11	10	8	12	7	3	4	1
F9	Worst	0.00E+00	6.92E+02	4.36E+02	0.00E+00	3.56E+02	5.32E+02	3.06E+01	4.74E+02	4.37E+02	0.00E+00	9.98E-01	3.34E+02
	Average	0.00E+00	6.02E+02	3.83E+02	0.00E+00	2.03E+02	4.51E+02	2.08E+01	3.59E+02	3.80E+02	0.00E+00	4.99E-01	3.23E+02
	Best	0.00E+00	5.22E+02	3.11E+02	0.00E+00	1.07E+02	3.09E+02	8.94E+00	3.02E+02	2.95E+02	0.00E+00	1.14E-13	3.08E+02
	STD	0.00E+00	7.00E+01	5.84E+01	0.00E+00	1.10E+02	9.89E+01	1.08E+01	8.07E+01	6.74E+01	0.00E+00	5.76E-01	1.13E+01
	Rank	1	12	10	1	6	11	5	8	9	1	4	7
F10	Worst	4.44E-15	2.06E+01	1.90E+01	1.47E-13	2.06E+01	1.89E+01	2.18E-04	6.38E+00	1.79E+01	2.31E-12	3.52E-08	4.15E-04
	Average	1.78E-15	2.00E+01	1.83E+01	5.06E-14	1.95E+01	1.55E+01	1.44E-04	4.97E+00	1.27E+01	1.60E-12	1.76E-08	2.98E-04
	Best	8.88E-16	1.93E+01	1.72E+01	4.44E-15	1.62E+01	1.27E+01	9.61E-05	4.37E+00	1.73E+00	2.92E-13	8.23E-09	2.50E-04
	STD	1.78E-15	5.43E-01	7.76E-01	6.58E-14	2.19E+00	2.56E+00	5.22E-05	9.42E-01	7.42E+00	8.95E-13	1.20E-08	7.83E-05
	Rank	1	12	10	2	11	9	5	7	8	3	4	6
F11	Worst	0.00E+00	2.17E+02	1.63E+02	0.00E+00	2.21E+01	1.28E+02	7.68E-02	1.33E+02	2.57E+02	1.05E-02	7.99E-15	1.68E-05
	Average	0.00E+00	1.53E+02	1.03E+02	0.00E+00	1.46E+01	9.05E+01	4.71E-02	1.17E+02	2.26E+02	2.63E-03	2.47E-15	1.60E-05
	Best	0.00E+00	1.13E+02	7.03E+01	0.00E+00	2.58E+00	4.30E+01	5.22E-06	8.97E+01	2.05E+02	0.00E+00	0.00E+00	1.47E-05
	STD	0.00E+00	4.45E+01	4.16E+01	0.00E+00	8.81E+00	3.79E+01	3.30E-02	1.94E+01	2.47E+01	5.26E-03	3.74E-15	9.94E-07
	Rank	1	11	9	1	7	8	6	10	12	5	3	4
F12	Worst	1.37E+00	7.29E+07	1.94E+07	7.04E-01	9.21E+07	7.48E+05	7.40E-01	2.29E+01	4.71E+07	2.01E-01	3.57E-01	5.41E-08
	Average	1.26E+00	3.61E+07	1.01E+07	4.79E-01	6.45E+07	4.76E+05	6.13E-01	1.44E+01	1.57E+07	1.37E-01	3.01E-01	4.06E-08
	Best	9.54E-01	1.69E+07	2.10E+06	2.25E-01	1.33E+07	1.68E+01	4.20E-01	9.81E+00	2.12E+06	7.66E-02	2.58E-01	2.57E-08
	STD	2.03E-01	2.57E+07	7.46E+06	2.38E-01	3.50E+07	3.38E+05	1.53E-01	5.81E+00	2.13E+07	5.52E-02	4.65E-02	1.16E-08
	Rank	6	11	9	4	12	8	5	7	10	2	3	1
F13	Worst	5.00E+00	1.77E+08	4.00E+07	4.45E+00	2.12E+08	5.52E+07	4.62E+00	1.24E+03	1.78E+08	4.70E+00	4.34E+00	1.33E-06
	Average	5.00E+00	1.29E+08	2.55E+07	3.75E+00	9.36E+07	1.91E+07	4.05E+00	3.38E+02	9.67E+07	4.22E+00	4.11E+00	1.08E-06
	Best	5.00E+00	7.40E+07	7.59E+06	3.21E+00	1.91E+07	4.13E+06	3.59E+00	3.22E+01	2.65E+07	3.71E+00	3.82E+00	8.29E-07
	STD	1.30E-03	4.41E+07	1.34E+07	5.45E-01	8.26E+07	2.43E+07	4.91E-01	5.99E+02	7.71E+07	4.08E-01	2.20E-01	2.09E-07
	Rank	6	12	9	2	10	8	3	7	11	5	4	1
Mean	rank	2.8462	10.2000	8.2000	4.2667	7.8667	9.1333	5.8000	8.0667	10.2667	3.3333	3.5333	4.2667
Final	Ranking	1	11	9	4	7	10	6	8	12	2	3	4

Table 8: Results of the RSA using unimodal and multimodal test functions (F1-F13), with 100 dimensions

Fun	Measure	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
F1	Worst	2.58E-157	3.97E+04	2.04E+04	9.72E-41	1.57E+04	1.72E+04	1.04E-06	1.39E+02	5.77E+04	2.59E-21	2.30E-17	1.28E-02
	Average	6.48E-158	3.51E+04	1.89E+04	2.43E-41	9.44E+03	1.18E+04	5.19E-07	1.03E+02	3.48E+04	1.56E-21	6.21E-18	9.47E-03
	Best	9.29E-165	3.12E+04	1.75E+04	2.74E-47	4.82E+03	7.07E+03	2.13E-07	7.61E+01	1.79E+04	3.72E-22	2.09E-19	5.88E-03
	STD	1.29E-157	3.65E+03	1.19E+03	4.86E-41	4.97E+03	5.29E+03	3.64E-07	2.91E+01	2.01E+04	1.03E-21	1.12E-17	3.17E-03
	Rank	1	12	10	2	8	9	5	7	11	3	4	6
F2	Worst	2.66E-83	1.15E+21	2.96E+06	1.01E-20	3.74E+00	2.01E+02	1.63E-04	7.77E+01	2.12E+02	4.21E-13	5.29E-09	5.14E-03
	Average	7.23E-84	2.86E+20	7.41E+05	2.53E-21	2.14E+00	1.47E+02	9.64E-05	6.05E+01	2.01E+02	1.56E-13	3.35E-09	3.32E-03
	Best	4.15E-89	2.28E+07	1.43E+02	6.11E-30	6.79E-01	7.48E+01	4.54E-05	4.19E+01	1.82E+02	5.07E-14	1.09E-09	2.30E-03
	STD	1.30E-83	5.73E+20	1.48E+06	5.06E-21	1.61E+00	5.81E+01	5.32E-05	1.86E+01	1.33E+01	1.77E-13	1.85E-09	1.26E-03
	Rank	1	12	11	2	7	9	5	8	10	3	4	6
F3	Worst	1.28E-91	8.72E+04	6.59E+04	4.79E+05	9.56E+04	1.98E+05	6.84E+02	1.57E+04	1.78E+05	1.84E+00	5.99E+01	6.75E+00
	Average	3.21E-92	6.71E+04	5.60E+04	3.82E+05	6.78E+04	1.28E+05	4.61E+02	1.24E+04	1.30E+05	4.94E-01	1.79E+01	3.87E+00
	Best	3.70E-116	5.42E+04	4.63E+04	3.04E+05	4.03E+04	5.59E+04	1.40E+02	1.07E+04	9.11E+04	5.12E-07	4.40E-02	4.07E-01
	STD	6.42E-92	1.45E+04	8.63E+03	7.67E+04	2.81E+04	5.79E+04	2.29E+02	2.34E+03	3.90E+04	8.99E-01	2.85E+01	2.79E+00
	Rank	1	8	7	12	9	10	5	6	11	2	4	3
F4	Worst	4.42E-71	6.60E+01	6.96E+01	9.82E+01	8.97E+01	5.53E+01	1.48E+00	2.84E+01	7.09E+01	1.09E-08	7.86E-03	2.07E-02
	Average	1.11E-71	6.27E+01	6.51E+01	8.54E+01	7.95E+01	4.39E+01	1.17E+00	2.48E+01	5.54E+01	5.97E-09	4.20E-03	1.62E-02
	Best	1.23E-81	5.54E+01	5.27E+01	5.46E+01	6.90E+01	3.29E+01	8.81E-01	2.19E+01	3.51E+01	2.89E-09	4.23E-04	1.27E-02
	STD	2.21E-71	4.95E+00	8.28E+00	2.06E+01	8.65E+00	9.37E+00	3.12E-01	3.29E+00	1.49E+01	3.71E-09	3.19E-03	3.36E-03
	Rank	1	9	10	12	11	7	5	6	8	2	3	4
F5	Worst	4.90E+01	1.38E+08	3.33E+07	4.89E+01	3.12E+07	1.69E+07	4.88E+01	3.66E+04	3.12E+07	4.87E+01	4.80E+01	4.41E+01
	Average	4.90E+01	9.11E+07	1.23E+07	4.88E+01	1.66E+07	9.53E+06	4.87E+01	2.80E+04	1.94E+07	4.84E+01	4.79E+01	4.34E+01
	Best	4.90E+01	4.41E+07	3.81E+06	4.87E+01	7.65E+05	2.21E+05	4.86E+01	1.92E+04	8.88E+06	4.79E+01	4.75E+01	4.27E+01
	STD	8.25E-03	4.45E+07	1.41E+07	6.84E-02	1.67E+07	8.68E+06	1.12E-01	9.62E+03	9.27E+06	3.74E-01	2.60E-01	5.93E-01
	Rank	6	12	9	5	10	8	4	7	11	3	2	1
F6	Worst	1.21E+01	2.57E+04	1.41E+04	7.31E+00	5.27E+03	3.35E+04	7.36E+00	1.33E+02	4.25E+04	5.18E+00	6.22E+00	2.91E-06
	Average	1.20E+01	1.96E+04	1.08E+04	6.91E+00	3.02E+03	1.62E+04	6.99E+00	6.39E+01	2.78E+04	4.05E+00	5.90E+00	2.11E-06
	Best	1.18E+01	1.29E+04	7.11E+03	6.33E+00	4.93E+02	5.27E+03	5.96E+00	3.08E+01	1.99E+04	2.79E+00	5.48E+00	1.23E-06
	STD	1.62E-01	5.28E+03	3.39E+03	4.14E-01	1.98E+03	1.35E+04	6.84E-01	4.76E+01	1.04E+04	1.11E+00	3.51E-01	7.53E-07
	Rank	6	11	9	4	8	10	5	7	12	2	3	1
F7	Worst	1.69E-03	4.84E+02	1.64E+01	1.79E-02	2.24E+01	4.36E+01	2.76E-02	2.61E+02	3.09E+01	4.56E-03	9.19E-03	1.07E-02
	Average	5.63E-04	3.67E+02	1.11E+01	9.30E-03	1.40E+01	1.92E+01	1.97E-02	9.58E+01	1.79E+01	2.12E-03	6.43E-03	8.12E-03
	Best	1.38E-04	2.41E+02	8.74E+00	4.10E-03	5.78E+00	8.19E+00	1.28E-02	3.17E+01	7.18E+00	3.02E-04	5.11E-03	5.90E-03
	STD	7.53E-04	1.18E+02	3.55E+00	6.26E-03	7.49E+00	1.64E+01	7.09E-03	1.11E+02	1.08E+01	1.78E-03	1.87E-03	2.00E-03
	Rank	1	12	7	5	8	10	6	11	9	2	3	4
F8	Worst	-9.52E+03	-8.98E+03	-8.15E+03	-7.83E+03	-3.46E+03	-3.62E+03	-6.68E+03	-2.77E+03	-9.03E+03	-1.03E+04	-8.43E+03	-1.30E+14
	Average	-1.03E+04	-9.87E+03	-8.58E+03	-8.87E+03	-3.89E+03	-5.10E+03	-7.85E+03	-3.92E+03	-9.05E+03	-1.14E+04	-1.01E+04	-4.15E+17
	Best	-1.22E+04	-1.05E+04	-9.06E+03	-9.99E+03	-4.08E+03	-6.36E+03	-8.86E+03	-5.85E+03	-9.13E+03	-1.31E+04	-1.08E+04	-1.13E+18
	STD	1.24E+03	6.55E+02	4.09E+02	5.11E+02	2.88E+02	1.28E+03	1.16E+03	1.34E+03	4.82E+01	1.26E+03	1.15E+03	5.38E+17
	Rank	3	5	8	7	12	10	9	11	6	2	4	1
F9	Worst	0.00E+00	7.07E+02	4.03E+02	0.00E+00	1.71E+02	5.68E+02	1.79E+01	3.39E+02	3.74E+02	0.00E+00	2.09E+00	3.33E+02
	Average	0.00E+00	6.63E+02	3.51E+02	0.00E+00	7.28E+01	4.51E+02	1.40E+01	2.94E+02	3.37E+02	0.00E+00	5.23E-01	3.17E+02
	Best	0.00E+00	6.30E+02	3.27E+02	0.00E+00	2.65E+01	3.84E+02	8.78E+00	2.47E+02	2.94E+02	0.00E+00	5.68E-14	2.94E+02
	STD	0.00E+00	3.19E+01	3.61E+01	0.00E+00	6.61E+01	8.20E+01	4.17E+00	3.78E+01	4.02E+01	0.00E+00	1.05E+00	1.67E+01
	Rank	1	12	10	1	6	11	5	7	9	1	4	8
F10	Worst	8.88E-16	1.99E+01	3.40E+00	2.93E-14	4.34E-01	8.46E+00	7.85E-10	1.93E-02	1.63E+01	2.58E-14	1.64E-13	8.88E-16
	Average	8.88E-16	1.94E+01	1.82E+00	1.15E-14	1.10E-01	7.77E+00	4.15E-10	9.45E-03	9.81E+00	9.77E-15	1.03E-13	8.88E-16
	Best	8.88E-16	1.90E+01	1.16E+00	8.88E-16	6.61E-04	7.09E+00	6.48E-11	3.38E-04	2.32E+00	4.44E-15	4.35E-14	8.88E-16
	STD	0.00E+00	4.00E-01	1.07E+00	1.26E-14	2.16E-01	6.80E-01	3.22E-10	8.02E-03	5.75E+00	1.07E-14	5.39E-14	0.00E+00
	Rank	1	12	9	4	8	10	6	7	11	3	5	1
F11	Worst	0.00E+00	5.88E-01	3.29E-01	3.01E-01	6.75E-01	1.50E+01	5.94E-02	3.10E+01	4.60E-01	3.95E-02	2.02E-02	1.14E-02
	Average	0.00E+00	5.41E-01	1.99E-01	1.32E-01	3.80E-01	5.95E+00	2.75E-02	9.08E+00	2.30E-01	1.67E-02	5.06E-03	2.84E-03
	Best	0.00E+00	4.50E-01	7.15E-02	0.00E+00	4.41E-04	2.64E+00	1.11E-16	5.46E-01	1.13E-01	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	6.17E-02	1.05E-01	1.56E-01	3.31E-01	6.06E+00	3.20E-02	1.47E+01	1.56E-01	1.99E-02	1.01E-02	5.69E-03
	Rank	1	10	7	6	9	11	5	12	8	4	3	2
F12	Worst	1.08E+00	5.31E+04	9.94E+00	3.54E-01	3.24E-01	4.49E+06	6.43E-02	4.23E-04	4.32E+01	2.52E-03	1.99E-02	4.71E-32
	Average	3.83E-01	1.34E+04	4.59E+00	2.00E-01	2.30E-01	1.12E+06	5.63E-02	1.34E-04	2.44E+01	7.86E-04	7.27E-03	4.71E-32
	Best	1.62E-01	2.22E+01	1.23E+00	1.38E-01	1.50E-01	1.02E+01	4.13E-02	3.60E-06	1.36E+01	3.89E-10	4.81E-05	4.71E-32
	STD	5.98E-01	2.65E+04	3.80E+00	1.03E-01	7.67E-02	2.24E+06	1.04E-02	1.96E-04	1.35E+01	1.19E-03	9.43E-03	0.00E+00
	Rank	8	11	9	6	7	12	5	2	10	3	4	1
F13	Worst	9.69E-01	3.70E+04	3.06E-01	7.31E-01	6.83E-01	8.41E+05	6.01E-01	2.13E-02	3.17E+01	1.18E-01	7.71E-01	1.35E-32
	Average	8.12E-02	1.49E+04	1.65E-01	4.49E-01	5.46E-01	2.40E+05	3.80E-01	5.47E-03	1.31E+01	3.73E-02	2.89E-01	1.35E-32
	Best	6.10E-02	5.80E+01	1.70E-02	1.54E-01	4.60E-01	2.50E+03	2.15E-01	1.71E-05	1.99E+00	2.73E-04	9.93E-02	1.35E-32
	STD	1.49E-01	1.82E+04	1.19E-01	3.22E-01	9.61E-02	4.03E+05	1.65E-01	1.06E-02	1.29E+01	5.44E-02	3.23E-01	0.00E+00
	Rank	4	11	5	8	9	12	7	2	10	3	6	1
Mean	rank	2.692	10.0666	8.0666	6.040	7.8666	10.2	6.1333	7.400	9.8666	3.000	3.5333	3.1333
Final	Ranking	1	11	9	5	8	12	6	7	10	2	4	3

Table 9: Results of the RSA using the unimodal and multimodal functions (F1-F13), with 500 dimensions

Fun	Measure	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
F1	Worst	1.91E-150	9.19E+04	9.45E+04	9.49E-24	2.11E+04	4.81E+04	3.03E-03	2.28E+03	1.25E+05	2.62E-22	4.23E-12	1.82E-02
	Average	4.78E-151	8.89E+04	7.61E+04	2.67E-24	1.55E+04	3.74E+04	1.68E-03	1.67E+03	9.06E+04	1.34E-22	1.53E-12	1.24E-02
	Best	2.69E-162	8.19E+04	5.89E+04	1.48E-37	1.09E+04	2.74E+04	7.69E-04	1.23E+03	5.92E+04	4.72E-24	1.37E-13	9.97E-03
	STD	9.55E-151	4.70E+03	1.46E+04	4.58E-24	5.19E+03	8.70E+03	9.85E-04	4.54E+02	3.07E+04	1.06E-22	1.91E-12	3.86E-03
	Rank	1	11	10	2	8	9	5	7	12	3	4	6
F2	Worst	6.28E-77	1.72E+137	1.33E+171	3.93E-21	2.31E+02	1.24E+03	2.95E+00	1.62E+161	2.69E+247	1.94E-10	5.08E-06	9.24E+01
	Average	1.60E-77	4.31E+136	3.32E+170	1.18E-21	1.78E+02	1.04E+03	1.82E+00	4.05E+160	6.72E+246	1.06E-10	3.80E-06	8.48E+01
	Best	1.12E-83	2.25E+103	1.69E+132	6.41E-25	1.35E+02	7.16E+02	3.96E-01	1.04E+67	2.06E+03	1.38E-11	2.73E-06	7.94E+01
	STD	3.12E-77	8.61E+136	6.55E+04	1.87E-21	4.39E+01	2.25E+02	1.14E+00	6.55E+04	6.55E+04	9.52E-11	1.12E-06	5.91E+00
	Rank	1	9	11	2	7	8	5	10	12	3	4	6
F3	Worst	1.57E-73	7.64E+06	8.51E+06	1.44E+08	1.23E+07	1.70E+07	1.05E+06	2.71E+06	1.05E+07	8.77E+04	2.01E+05	9.85E+06
	Average	3.91E-74	6.05E+06	5.69E+06	8.56E+07	8.56E+06	1.15E+07	8.89E+05	1.55E+06	8.13E+06	4.89E+04	1.18E+05	7.58E+06
	Best	1.84E-95	3.39E+06	2.79E+06	3.38E+07	6.29E+06	5.97E+06	7.38E+05	7.85E+05	4.81E+06	1.51E+04	3.61E+04	5.89E+06
	STD	7.83E-74	1.87E+06	2.35E+06	5.71E+07	2.60E+06	4.69E+06	1.47E+05	8.78E+05	2.39E+06	3.92E+04	9.32E+04	1.68E+06
	Rank	1	7	6	12	10	11	4	5	9	2	3	8
F4	Worst	1.87E-67	9.55E+01	9.76E+01	9.96E+01	9.97E+01	9.95E+01	9.09E+01	6.33E+01	8.83E+01	1.21E-05	9.82E+01	3.58E+01
	Average	4.92E-68	8.97E+01	9.71E+01	9.24E+01	9.96E+01	9.86E+01	8.06E+01	5.83E+01	8.41E+01	3.57E-06	7.55E+01	3.32E+01
	Best	4.26E-71	8.70E+01	9.64E+01	7.99E+01	9.95E+01	9.68E+01	7.12E+01	5.53E+01	7.93E+01	2.96E-07	5.25E+01	3.14E+01
	STD	9.19E-68	3.91E+00	5.55E-01	8.76E+00	1.15E-01	1.23E+00	1.01E+01	3.51E+00	4.41E+00	5.70E-06	1.94E+01	1.94E+00
	Rank	1	8	10	9	12	11	6	4	7	2	5	3
F5	Worst	4.99E+02	2.59E+09	4.92E+09	4.98E+02	3.21E+09	1.81E+09	1.07E+06	5.87E+08	3.37E+09	4.99E+02	4.99E+02	1.17E+05
	Average	4.99E+02	2.41E+09	4.38E+09	4.98E+02	2.54E+09	1.16E+09	6.54E+05	5.51E+08	2.20E+09	4.98E+02	4.99E+02	7.99E+04
	Best	4.99E+02	2.29E+09	3.97E+09	4.97E+02	1.77E+09	6.90E+08	1.30E+05	4.51E+08	1.77E+09	4.98E+02	4.99E+02	6.18E+04
	STD	5.09E-03	1.27E+08	4.03E+08	4.21E-01	6.13E+08	5.33E+08	3.97E+05	6.67E+07	7.81E+08	8.98E-02	2.51E-02	2.54E+04
	Rank	3	10	12	1	11	8	6	7	9	1	3	5
F6	Worst	1.25E+02	8.08E+05	1.05E+06	1.04E+02	2.69E+05	3.58E+05	1.63E+02	2.30E+05	1.09E+06	1.08E+02	1.19E+02	5.00E+02
	Average	1.24E+02	7.61E+05	9.53E+05	1.01E+02	2.52E+05	3.15E+05	1.45E+02	2.27E+05	1.00E+06	1.07E+02	1.18E+02	4.89E+02
	Best	1.24E+02	7.33E+05	8.59E+05	9.86E+01	2.37E+05	2.31E+05	1.20E+02	2.22E+05	8.73E+05	1.06E+02	1.17E+02	4.81E+02
	STD	1.97E-01	4.13E+04	9.81E+04	2.62E+00	1.63E+04	7.24E+04	2.24E+01	4.29E+03	1.14E+05	1.44E+00	9.18E-01	9.39E+00
	Rank	4	10	11	1	8	9	5	7	12	2	3	6
F7	Worst	3.60E-04	5.11E+04	1.87E+04	4.92E-02	2.07E+04	7.92E+03	9.85E-01	6.18E+04	1.39E+04	8.09E-03	3.05E-02	5.01E-01
	Average	1.79E-04	4.73E+04	1.17E+04	3.31E-02	1.76E+04	4.06E+03	7.11E-01	5.84E+04	1.02E+04	4.48E-03	1.82E-02	4.23E-01
	Best	3.85E-05	4.25E+04	8.65E+03	1.30E-02	1.52E+04	1.21E+03	5.28E-01	5.23E+04	5.95E+03	2.46E-03	9.02E-03	3.61E-01
	STD	1.51E-04	3.57E+03	4.70E+03	1.55E-02	2.74E+03	2.82E+03	2.18E-01	4.30E+03	4.13E+03	2.49E-03	1.01E-02	5.90E-02
	Rank	1	11	9	4	10	7	6	12	8	2	3	5
F8	Worst	-7.46E+04	-3.32E+04	-2.67E+04	-1.26E+05	-1.21E+04	-1.50E+04	-1.02E+04	-9.56E+03	-9.03E+04	-5.10E+04	-3.85E+04	-6.06E+04
	Average	-7.86E+04	-3.70E+04	-3.25E+04	-1.39E+05	-1.33E+04	-1.70E+04	-2.91E+04	-1.19E+04	-9.03E+04	-5.53E+04	-4.26E+04	-6.64E+04
	Best	-8.10E+04	-3.95E+04	-3.73E+04	-1.50E+05	-1.45E+04	-1.90E+04	-3.63E+04	-1.56E+04	-9.03E+04	-5.96E+04	-4.58E+04	-7.17E+04
	STD	2.78E+03	2.69E+03	4.71E+03	9.97E+03	1.04E+03	1.64E+03	1.26E+04	2.85E+03	0.00E+00	4.72E+03	3.26E+03	5.66E+03
	Rank	3	7	8	1	11	10	9	12	2	5	6	4
F9	Worst	0.00E+00	8.48E+03	6.00E+03	0.00E+00	1.48E+03	6.65E+03	3.58E+02	8.35E+03	6.18E+03	0.00E+00	1.02E+00	4.87E+03
	Average	0.00E+00	8.16E+03	5.95E+03	0.00E+00	1.27E+03	5.97E+03	2.77E+02	7.82E+03	5.79E+03	0.00E+00	2.55E-01	4.78E+03
	Best	0.00E+00	8.04E+03	5.87E+03	0.00E+00	1.14E+03	5.27E+03	1.21E+02	7.26E+03	5.29E+03	0.00E+00	5.06E-10	4.74E+03
	STD	0.00E+00	2.11E+02	5.46E+01	0.00E+00	1.44E+02	5.81E+02	1.06E+02	5.34E+02	3.79E+02	0.00E+00	5.09E-01	5.85E+01
	Rank	1	12	9	1	6	10	5	11	8	1	4	7
F10	Worst	8.88E-16	2.04E+01	2.08E+01	3.92E-09	2.09E+01	1.91E+01	2.52E+00	1.89E+01	1.98E+01	3.77E-09	1.58E-03	4.38E+00
	Average	8.88E-16	2.01E+01	2.05E+01	1.21E-09	2.09E+01	1.85E+01	2.14E+00	1.86E+01	1.35E+01	2.42E-09	7.09E-04	4.18E+00
	Best	8.88E-16	1.98E+01	2.02E+01	1.11E-10	2.09E+01	1.73E+01	1.48E+00	1.83E+01	8.88E-16	1.41E-09	1.91E-04	3.94E+00
	STD	0.00E+00	2.61E-01	2.60E-01	1.82E-09	2.17E-02	8.65E-01	4.57E-01	2.52E-01	9.08E+00	1.01E-09	6.27E-04	1.79E-01
	Rank	1	10	11	2	12	8	5	9	7	3	4	6
F11	Worst	0.00E+00	6.37E+03	6.65E+03	0.00E+00	2.24E+03	2.80E+03	1.34E+00	1.97E+03	8.28E+03	0.00E+00	1.46E-09	5.00E+00
	Average	0.00E+00	5.72E+03	6.51E+03	0.00E+00	1.65E+03	2.05E+03	1.14E+00	1.89E+03	6.46E+03	0.00E+00	7.45E-10	4.89E+00
	Best	0.00E+00	5.29E+03	6.15E+03	0.00E+00	7.95E+02	1.44E+03	9.19E-01	1.81E+03	5.57E+03	0.00E+00	2.62E-10	4.81E+00
	STD	0.00E+00	4.84E+02	2.41E+02	0.00E+00	6.26E+02	5.64E+02	1.71E-01	6.67E+01	1.23E+03	0.00E+00	5.19E-10	8.53E-02
	Rank	1	10	12	1	7	9	5	8	11	1	4	6
F12	Worst	0.00E+00	6.35E+03	6.16E+03	1.11E-16	3.36E+03	3.24E+03	1.28E+00	2.06E+03	6.24E+03	0.00E+00	7.08E-09	5.35E+00
	Average	0.00E+00	5.87E+03	5.65E+03	2.78E-17	2.23E+03	2.13E+03	1.01E+00	1.93E+03	5.49E+03	0.00E+00	3.66E-09	5.05E+00
	Best	0.00E+00	5.44E+03	5.25E+03	0.00E+00	1.30E+03	1.23E+03	3.67E-01	1.83E+03	4.29E+03	0.00E+00	5.50E-10	4.61E+00
	STD	0.00E+00	3.75E+02	3.83E+02	5.55E-17	8.86E+02	8.65E+02	4.31E-01	9.73E+01	8.61E+02	0.00E+00	2.80E-09	3.61E-01
	Rank	1	12	11	3	9	8	5	7	10	1	4	6
F13	Worst	5.00E+01	7.80E+09	8.62E+09	4.78E+01	1.06E+10	2.76E+09	3.06E+02	2.83E+08	7.83E+09	4.98E+01	4.98E+01	2.15E+02
	Average	5.00E+01	7.20E+09	7.83E+09	3.90E+01	8.82E+09	1.59E+09	2.04E+02	2.64E+08	7.23E+09	4.98E+01	4.96E+01	1.97E+02
	Best	5.00E+01	6.57E+09	7.27E+09	3.01E+01	6.56E+09	1.12E+09	1.05E+02	2.45E+08	6.63E+09	4.98E+01	4.92E+01	1.82E+02
	STD	1.19E+03	6.05E+08	6.34E+08	8.05E+00	1.72E+09	7.84E+08	8.38E+01	1.55E+07	5.79E+08	3.71E-02	2.93E-01	1.39E+01
	Rank	4	9	11	1	12	8	6	7	10	3	2	5
Mean	rank	1.7692	8.8000	9.6666	3.73333	8.3333	9.2000	6.1333	8.2666	9.4000	2.2000	3.9333	5.6667
Final	Ranking	1	9	12	3	8	10	6	7	11	2	4	5


F1
F2
F3
F4
F5
F6
F7
F8
Fitness function
No. of Iterations

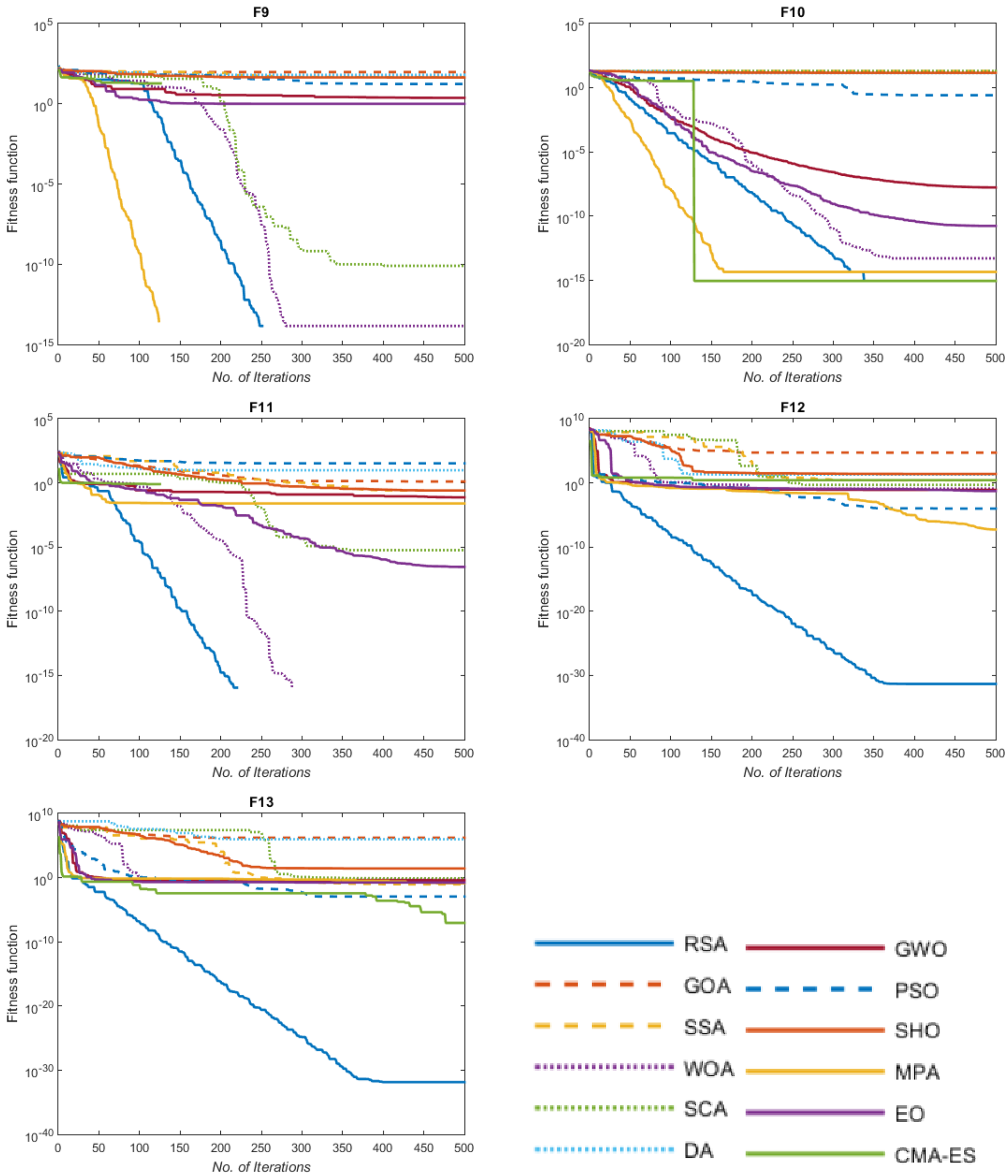


Figure 5: Convergence behaviour results for the studied problems from F1-F13

Figure 5 depicts the convergence curve of the RSA and other MH method. It can be seen from these figures that the RSA has a convergence rate faster than other algorithms among the tested functions except at F6, F8, and F13. However, its convergence still competitive with other MH methods. In addition, it can be observed that the RSA need less than 350 iterations to reached to optimal value in most tested functions.

In Table 10, the comparison between the algorithms to find the solution for the fixed-dimension multi-modal. From these results, it can be noticed that RSA and MPA are the best algorithms that ranked first. Where RSA allocates the best value overall functions except at F15, and F21-F23, while, MPA does not achieve the best value at F15 and F20. The PSO and EO allocate the second rank which provide better results than rest algorithms.

Table 10: Results of the RSA using the fixed-dimension multimodal benchmark functions (F14-F23)

Fun	Measure	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
F14	Worst	9.98E-01	1.46E+01	2.98E+00	1.08E+01	2.98E+00	1.99E+00	1.08E+01	3.97E+00	5.93E+00	9.98E-01	9.98E-01	7.98E+00
	Average	9.98E-01	1.23E+01	1.59E+00	4.14E+00	1.49E+00	1.40E+00	6.29E+00	2.78E+00	3.57E+00	9.98E-01	9.98E-01	6.92E+00
	Best	9.98E-01	9.80E+00	9.98E-01	9.98E-01	9.98E-01	9.98E-01	2.98E+00	1.99E+00	9.98E-01	9.98E-01	9.98E-01	4.00E+00
	STD	0.00E+00	1.86E+00	8.88E-01	4.23E+00	8.59E-01	5.44E-01	4.10E+00	1.08E+00	2.27E+00	0.00E+00	1.57E-16	1.66E+00
	Rank	1	12	6	9	5	4	10	7	8	1	1	11
F15	Worst	8.39E-04	2.18E-02	2.08E-02	1.52E-03	1.91E-03	2.04E-02	2.04E-02	9.93E-04	2.04E-02	3.07E-04	3.53E-04	0.00E+00
	Average	5.76E-04	9.79E-03	4.80E-03	7.74E-04	1.28E-03	5.72E-03	8.45E-03	9.20E-04	4.95E-03	3.07E-04	3.21E-04	0.00E+00
	Best	3.64E-04	7.85E-04	7.43E-04	3.13E-04	6.92E-04	1.50E-03	3.08E-04	8.66E-04	7.78E-04	3.07E-04	3.08E-04	0.00E+00
	STD	2.24E-04	1.05E-02	8.95E-03	4.56E-04	4.90E-04	8.19E-03	1.09E-02	6.64E-05	8.65E-03	2.90E-15	1.98E-05	0.00E+00
	Rank	4	12	8	5	7	10	11	6	9	2	3	1
F16	Worst	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	0.00E+00
	Average	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	0.00E+00
	Best	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	0.00E+00
	STD	1.19E-03	2.14E-13	1.52E-14	1.18E-09	2.17E-05	4.23E-07	2.80E-08	1.92E-16	2.93E-13	0.00E+00	1.92E-16	0.00E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	12
F17	Worst	3.98E-01	0.00E+00	0.00E+00	3.98E-01	4.02E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	0.00E+00
	Average	3.98E-01	0.00E+00	0.00E+00	3.98E-01	4.00E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	0.00E+00
	Best	3.98E-01	0.00E+00	0.00E+00	3.98E-01	3.99E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	0.00E+00
	STD	1.57E-02	0.00E+00	0.00E+00	1.98E-04	1.37E-03	1.19E-06	1.05E-05	0.00E+00	1.80E-13	4.37E-14	0.00E+00	0.00E+00
	Rank	1	10	10	1	9	1	1	1	1	1	1	10
F18	Worst	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	8.40E+01	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Average	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	1.92E+01	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Best	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	STD	1.75E-12	1.75E-12	5.95E-13	1.55E-04	1.13E-03	4.63E-13	3.62E+01	1.26E-15	1.54E-12	1.13E-15	1.13E-15	0.00E+00
	Rank	1	1	1	1	1	1	12	1	1	1	1	1
F19	Worst	-3.86E+00	-3.08E+00	-3.86E+00	-3.85E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Average	-3.86E+00	-3.54E+00	-3.86E+00	-3.85E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Best	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	STD	3.37E-01	3.29E-01	2.81E-10	4.46E-03	1.32E-03	1.48E-03	1.82E-03	3.14E-16	4.65E-11	3.14E-16	6.28E-16	0.00E+00
	Rank	1	12	1	10	10	1	1	1	1	1	1	1
F20	Worst	-1.03E+00	-3.20E+00	-3.16E+00	-3.09E+00	-2.60E+00	-3.18E+00	-3.08E+00	-3.20E+00	-3.19E+00	-3.20E+00	-3.20E+00	-3.20E+00
	Average	-2.05E+00	-3.30E+00	-3.24E+00	-3.26E+00	-2.93E+00	-3.21E+00	-3.20E+00	-3.25E+00	-3.27E+00	-3.30E+00	-3.30E+00	-3.23E+00
	Best	-2.63E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.07E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00
	STD	6.34E-01	5.56E-02	7.70E-02	9.92E-02	1.89E-01	6.05E-02	8.59E-02	6.51E-02	6.90E-02	5.32E-02	5.32E-02	5.32E-02
	Rank	1	10	6	8	2	4	3	7	9	10	10	5
F21	Worst	-5.06E+00	-2.68E+00	-5.06E+00	-5.05E+00	-4.96E-01	-2.63E+00	-1.01E+01	-2.63E+00	-2.68E+00	-1.02E+01	-2.94E-01	-1.02E+01
	Average	-9.13E+00	-6.15E+00	-6.08E+00	-8.07E+00	-8.94E-01	-6.63E+00	-1.01E+01	-7.15E+00	-4.61E+00	-1.02E+01	-4.66E-01	-1.02E+01
	Best	-1.02E+01	-1.02E+01	-1.02E+01	-1.01E+01	-2.10E+00	-1.02E+01	-1.02E+01	-1.02E+01	-5.10E+00	-1.02E+01	-7.11E-01	-1.02E+01
	STD	2.28E+00	3.78E+00	2.27E+00	2.75E+00	6.93E-01	3.37E+00	2.99E-03	4.11E+00	1.08E+00	1.78E-11	1.59E-01	0.00E+00
	Rank	4	8	9	5	11	7	3	6	10	1	12	1
F22	Worst	-5.09E+00	-2.75E+00	-3.72E+00	-2.94E-01	-9.06E-01	-2.75E+00	-1.04E+01	-2.75E+00	-3.72E+00	-1.04E+01	-5.09E+00	-1.04E+01
	Average	-9.25E+00	-7.34E+00	-8.01E+00	-5.56E-01	-1.90E+00	-6.38E+00	-1.04E+01	-6.47E+00	-7.73E+00	-1.04E+01	-9.34E+00	-1.04E+01
	Best	-1.04E+01	-1.04E+01	-1.04E+01	-8.58E-01	-4.02E+00	-1.03E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01
	STD	2.33E+00	4.19E+00	3.31E+00	2.72E-01	1.43E+00	3.54E+00	1.34E-03	3.68E+00	3.66E+00	1.04E-11	2.38E+00	0.00E+00
	Rank	5	8	6	12	11	10	1	9	7	1	4	1
F23	Worst	-2.87E+00	-1.68E+00	-1.05E+01	-5.10E+00	-2.46E+00	-1.68E+00	-1.05E+01	-1.05E+01	-3.84E+00	-7.89E-01	-3.84E+00	-1.05E+01
	Average	-6.59E+00	-4.26E+00	-1.05E+01	-9.42E+00	-4.14E+00	-6.52E+00	-1.05E+01	-1.05E+01	-9.20E+00	-2.09E+00	-9.20E+00	-1.05E+01
	Best	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-7.27E+00	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-5.13E+00	-1.05E+01	-1.05E+01
	STD	3.69E+00	3.60E+00	2.10E-11	2.42E+00	1.90E+00	3.74E+00	1.98E-03	1.54E-15	3.00E+00	1.90E+00	3.00E+00	0.00E+00
	Rank	8	10	1	5	11	9	1	1	6	8	6	1
Mean	rank	2.7	8.4	4.9	5.7	6.8	4.8	4.4	4	5.3	2.7	4	4.4
Final	Ranking	1	12	8	10	11	7	5	3	9	1	3	5

In order to further investigate the effectiveness of RSA, the running time used by algorithms is recorded to discover the solutions for F1–F13 functions with 10 dimensions, and the results are presented in Table 11. As the results given in Table 11, we found that the RSA takes a fairly fast and competitive performance in determining the best solutions related to other used algorithms. According to the average running time on thirteen functions, the RSA works faster than GOA, SSA, WOA, DA, ALO, MPA, EO, and CMA-ES algorithms. These observations are further in conformity with the time complexity of RSA.

Table 11: Comparison results of average running time (seconds) over 10 independent runs for test problems (F1-F13) with 10 dimensions.

Function	Metric	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
F1	Average	3.2172E-02	2.7875E+01	1.4338E-01	3.4594E-01	9.4782E-02	4.4401E+00	1.1545E-01	4.9913E-01	5.0246E+00	3.2567E-01	2.4921E-01	2.4116E+00
	STD	5.4976E-02	4.4465E-01	5.3080E-03	6.3660E-03	2.4486E-03	6.2985E-02	1.0307E-02	1.3794E-03	4.7268E-02	1.1314E-02	1.2481E-02	1.5425E-02
	Rank	1	12	4	7	2	10	3	8	11	6	5	9
F2	Average	5.8828E-01	9.8145E+01	5.6230E-01	1.1134E+00	4.5597E-01	1.6442E+01	4.8294E-01	2.3644E-01	1.8685E+01	1.1860E+00	8.8009E-01	8.4677E+00
	STD	1.7538E+00	1.7741E+01	1.2131E-01	1.8789E-01	6.6271E-02	2.3875E+00	7.9893E-02	3.9256E-02	2.4342E+00	3.2151E-01	2.2735E-01	1.3641E+00
	Rank	5	12	4	7	2	10	3	1	11	8	6	9
F3	Average	8.6391E-01	9.9638E+01	1.3412E+00	1.8652E+00	1.2049E+00	1.7218E+01	1.2642E+00	1.0838E+00	1.8767E+01	2.8935E+00	1.7321E+00	1.2643E+01
	STD	1.9116E+00	1.3803E+01	1.5503E-01	2.6547E-01	1.4965E-01	2.2633E+00	1.6665E-01	1.1012E-01	2.3936E+00	3.3900E-01	1.2822E-01	1.2818E+00
	Rank	1	12	5	7	3	10	4	2	11	8	6	9
F4	Average	6.3668E-01	1.0128E+02	5.8734E-01	1.0231E+00	4.3502E-01	1.5402E+01	5.5706E-01	2.9102E-01	1.9093E+01	1.5229E+00	1.1256E+00	9.6949E+00
	STD	1.7131E+00	1.3956E+01	1.4401E-01	2.9237E-01	1.1216E-01	1.9936E+00	5.4488E-02	3.6138E-02	1.8516E+00	1.6390E-01	7.7539E-02	7.3131E-01
	Rank	5	12	4	6	2	10	3	1	11	8	7	9
F5	Average	6.0534E-01	1.0129E+02	8.7885E-01	1.4350E+00	7.1702E-01	1.6520E+01	7.7544E-01	4.3968E-01	1.8404E+01	1.8817E+00	1.3109E+00	1.0017E+01
	STD	1.8343E+00	1.1361E+01	1.5260E-01	1.8468E-01	1.1536E-01	1.7670E+00	6.5519E-02	3.3253E-02	2.2054E+00	1.3334E-01	6.9053E-02	8.3396E-01
	Rank	2	12	5	7	3	10	4	1	11	8	6	9
F6	Average	6.8281E-02	1.0177E+02	4.9876E-01	1.0577E+00	3.9925E-01	1.6225E+01	4.1733E-01	2.1842E-01	1.8591E+01	1.3243E+00	1.0024E+00	9.3970E+00
	STD	5.4480E-01	1.1022E+01	4.2569E-02	1.1180E-01	4.9893E-02	9.6447E-01	3.6281E-02	3.0515E-02	2.1904E+00	1.3282E-01	1.7435E-01	1.3470E+00
	Rank	1	12	5	7	3	10	4	1	11	8	6	9
F7	Average	6.5693E+00	1.0093E+02	6.2153E-01	1.2433E+00	5.0066E-01	1.6072E+01	5.5113E-01	3.0141E-01	1.8811E+01	1.5786E+00	1.1968E+00	9.9114E+00
	STD	7.0767E-01	1.0025E+01	6.1293E-02	1.3001E-01	6.9859E-02	1.8000E+00	2.8436E-02	2.0819E-02	2.2745E+00	6.0705E-02	7.6440E-02	7.7761E-01
	Rank	8	12	4	6	2	10	3	1	11	7	5	9
F8	Average	7.2978E-01	1.0122E+02	5.6452E-01	1.1411E+00	4.5463E-01	1.5469E+01	4.5497E-01	2.1675E-01	1.9257E+01	1.4413E+00	1.0674E+00	9.1825E+00
	STD	1.1514E+00	1.1564E+01	4.2612E-02	7.0411E-02	6.4605E-02	1.9428E+00	5.0117E-02	2.1778E-02	1.7361E+00	1.0574E-01	4.5980E-02	1.6726E+00
	Rank	5	12	4	7	2	10	3	1	11	8	6	9
F9	Average	6.7546E-01	8.4904E+01	6.5542E-01	1.2058E+00	4.8481E-01	1.4253E+01	5.2338E-01	2.8175E-01	1.5867E+01	1.2326E+00	9.7515E-01	8.3052E+00
	STD	7.8670E-01	8.7717E+00	2.7999E-02	9.6884E-02	3.2282E-02	3.4804E-01	9.6827E-02	2.8887E-02	6.0103E-01	9.9609E-02	4.8100E-03	3.6052E-01
	Rank	5	12	4	7	2	10	3	1	11	8	6	9
F10	Average	1.5403E-01	8.6224E+01	6.3696E-01	1.0763E+00	5.2417E-01	1.3460E+01	5.5973E-01	3.0979E-01	1.6048E+01	1.2763E+00	9.6439E-01	7.5298E+00
	STD	1.1758E+00	7.5414E+00	8.3612E-02	1.2452E-01	4.3289E-02	5.3142E-01	8.1630E-02	1.0022E-01	3.2669E-01	2.4381E-01	1.2767E-01	9.2171E-01
	Rank	1	12	5	7	3	10	4	2	11	8	6	9
F11	Average	7.1132E-01	8.7313E+01	6.1277E-01	1.0923E+00	4.4249E-01	1.3900E+01	4.9247E-01	2.9836E-01	1.4868E+01	1.2838E+00	9.3286E-01	7.1048E+00
	STD	7.8975E-01	4.7479E+00	1.2756E-01	1.1171E-01	6.4018E-02	6.7009E-01	2.8571E-02	2.2482E-02	1.8047E+00	1.4572E-01	1.7244E-01	2.3576E+00
	Rank	5	12	4	7	2	10	3	1	11	8	6	9
F12	Average	9.2742E-01	8.6970E+01	9.2729E-01	1.3327E+00	7.9022E-01	1.2516E+01	8.2789E-01	6.2175E-01	1.5072E+01	1.9835E+00	1.1935E+00	9.8499E+00
	STD	8.2052E-01	3.6139E+00	2.0303E-01	2.9486E-01	1.2054E-01	2.7038E+00	1.0553E-01	9.0698E-02	2.5856E+00	4.4094E-01	3.7534E-01	1.2597E+00
	Rank	5	12	4	7	2	10	3	1	11	8	6	9
F13	Average	9.4400E-01	8.5774E+01	9.5983E-01	1.4259E+00	8.3512E-01	1.3270E+01	8.8748E-01	6.4852E-01	1.5467E+01	1.9280E+00	1.2378E+00	9.2636E+00
	STD	9.1199E-01	4.4410E+00	1.9925E-01	1.2138E-01	1.3035E-01	2.4908E+00	1.5567E-01	1.9785E-01	2.7953E+00	6.2193E-01	2.6149E-01	2.2980E+00
	Rank	4	12	5	7	2	10	3	1	11	8	6	9
Mean	rank	3.69	12.00	4.38	6.85	2.31	10.00	3.31	1.69	11.00	7.77	5.92	9.00
Final	Ranking	4	12	5	7	2	10	3	1	11	8	6	9

3.2. Results comparisons using CEC2017 test functions

In this experiment, we evaluate the ability of RSA to find the solution for CEC2017 functions (Awad et al., 2016) that defined in Table 12. These functions are more complex than those functions illustrated in the previous experiment which have at least half of them are the challenging composition and hybrid functions. The algorithms are compared using the same parameter setting except for the dimension of CEC2017 functions is set to 10.

Table 12: Review of CEC2017 benchmark function problems.

Type	No.	Description	Fi*
Unimodal functions	1	Shifted and Rotated Bent Cigar Function	100
	3	Shifted and Rotated Zakharov Function	300
Simple Multimodal Functions	4	Shifted and Rotated Rosenbrock's Function	400
	5	Shifted and Rotated Rastrigin's Function	500
	6	Shifted and Rotated Expanded Scaffer's F6 Function	600
	7	Shifted and Rotated Lunacek Bi-Rastrigin Function	700
	8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
	9	Shifted and Rotated Levy Function	900
	10	Shifted and Rotated Schwefel's Function	1000
Hybrid functions	11	Hybrid Function 1 (N=3)	1100
	12	Hybrid Function 2 (N=3)	1200
	13	Hybrid Function 3 (N=3)	1300
	14	Hybrid Function 4 (N=4)	1400
	15	Hybrid Function 5 (N=4)	1500
	16	Hybrid Function 6 (N=4)	1600
	17	Hybrid Function 6 (N=5)	1700
	18	Hybrid Function 6 (N=5)	1800
	19	Hybrid Function 6 (N=5)	1900
	20	Hybrid Function 6 (N=6)	2000
Composition Functions	21	Composition Function 1 (N=3)	2100
	22	Composition Function 2 (N=3)	2200
	23	Composition Function 3 (N=4)	2300
	24	Composition Function 4 (N=4)	2400
	25	Composition Function 5 (N=5)	2500
	26	Composition Function 6 (N=5)	2600
	27	Composition Function 7 (N=6)	2700
	28	Composition Function 8 (N=6)	2800
	29	Composition Function 9 (N=3)	2900
	30	Composition Function 10 (N=3)	3000

Table 13 shows the average and STD of the fitness value, as well as, the mean rank of the Friedman test (i.e., Rank). It is observed that the RSA allocates the first rank according to the Friedman test in the last row. Followed by the EO and PSO that allocate the second and third rank, respectively. For more details, the RSA provides a better average of fitness value at twelve functions which represent nearly 42% from the total test CEC2017 functions (i.e., F6-F8, F10, F11, F14, F17, F22-F25, and F30). In addition, it is noticed from these results that the mean ranks of RSA, EO, and PSO are far from the other MH techniques and this reflects the high performance of RSA as a new optimizer. In addition, RSA has more stability than other comparative algorithms as observed from the results of STD.

Table 13: Results of the RSA using the CEC2017 test functions

Fun	Measure	RSA	GA	PSO	EO	GWO	SSA	CS	GSA	CMA-ES
CEC-2017-F1	Average	2.47E+03	9.80E+03	3.96E+03	2.47E+03	3.25E+05	3.40E+03	2.96E+02	2.96E+02	1.00E+02
	STD	2.65E+02	5.94E+03	4.46E+03	2.21E+03	1.07E+05	3.67E+03	2.75E+02	2.75E+02	0.00E+00
	Rank	4	8	7	4	9	6	2	2	1
CEC-2017-F3	Average	1.51E+03	8.72E+03	3.00E+02	3.00E+02	1.54E+03	3.00E+02	1.08E+04	1.08E+04	3.00E+02
	STD	2.54E+01	5.90E+03	1.90E-10	2.40E-08	1.89E+03	0.00E+00	1.60E+03	1.62E+03	0.00E+00
	Rank	5	7	1	1	6	1	8	8	1
CEC-2017-F4	Average	4.04E+02	4.11E+02	4.06E+02	4.04E+02	4.10E+02	4.06E+02	4.07E+02	4.07E+02	4.00E+02
	STD	8.17E+00	1.85E+01	3.28E+00	7.91E-01	7.55E+00	1.01E+01	2.92E+00	2.92E+00	0.00E+00
	Rank	2	9	4	2	8	4	6	6	1
CEC-2017-F5	Average	5.13E+02	5.16E+02	5.13E+02	5.11E+02	5.14E+02	5.22E+02	5.57E+02	5.57E+02	5.30E+02
	STD	2.43E+01	6.93E+00	6.54E+00	3.67E+00	6.10E+00	1.05E+01	8.41E+00	8.40E+00	5.83E+01
	Rank	2	5	2	1	4	6	8	8	7
CEC-2017-F6	Average	6.00E+02	6.00E+02	6.00E+02	6.00E+02	6.01E+02	6.10E+02	6.22E+02	6.22E+02	6.82E+02
	STD	1.40E+00	6.68E-02	9.80E-01	1.50E-04	8.80E-01	8.26E+00	9.02E+00	9.02E+00	3.54E+01
	Rank	1	1	1	1	5	6	7	7	9
CEC-2017-F7	Average	7.13E+02	7.28E+02	7.19E+02	7.21E+02	7.30E+02	7.41E+02	7.15E+02	7.15E+02	7.13E+02
	STD	4.30E+00	7.29E+00	5.10E+00	5.74E+00	8.60E+00	1.66E+01	1.56E+00	1.55E+00	1.63E+00
	Rank	1	7	5	6	8	9	3	3	1
CEC-2017-F8	Average	8.09E+02	8.21E+02	8.11E+02	8.10E+02	8.14E+02	8.23E+02	8.21E+02	8.21E+02	8.29E+02
	STD	8.01E+00	8.96E+00	5.47E+00	2.92E+00	8.26E+00	9.95E+00	4.69E+00	4.69E+00	5.30E+01
	Rank	1	5	3	2	4	8	5	5	9
CEC-2017-F9	Average	9.10E+02	9.10E+02	9.00E+02	9.00E+02	9.11E+02	9.44E+02	9.00E+02	9.00E+02	4.67E+03
	STD	2.00E+01	1.52E+01	5.90E-14	2.27E-02	1.95E+01	1.05E+02	0.00E+00	5.90E-15	2.06E+03
	Rank	5	5	1	1	7	8	1	1	9
CEC-2017-F10	Average	1.41E+03	1.72E+03	1.47E+03	1.42E+03	1.53E+03	1.86E+03	2.69E+03	2.69E+03	2.59E+03
	STD	3.50E+01	2.52E+02	2.15E+02	2.62E+02	2.87E+02	2.95E+02	2.98E+02	2.98E+02	4.14E+02
	Rank	1	5	3	2	4	6	8	8	7
CEC-2017-F11	Average	1.11E+03	1.13E+03	1.11E+03	1.11E+03	1.14E+03	1.18E+03	1.13E+03	1.13E+03	1.11E+03
	STD	1.12E+01	2.38E+01	6.28E+00	5.02E+00	5.41E+01	5.98E+01	1.05E+01	1.05E+01	2.54E+01
	Rank	1	5	1	1	8	9	5	5	1
CEC-2017-F12	Average	1.52E+04	3.73E+04	1.45E+04	1.03E+04	6.25E+05	1.98E+06	7.10E+05	7.03E+05	1.63E+03
	STD	2.68E+03	3.48E+04	1.13E+04	9.79E+03	1.13E+06	1.91E+06	4.20E+05	4.21E+04	1.98E+02
	Rank	4	5	3	2	6	9	8	7	1
CEC-2017-F13	Average	6.82E+03	1.08E+04	8.60E+03	8.02E+03	9.84E+03	1.61E+04	1.11E+04	1.11E+04	1.32E+03
	STD	4.26E+03	8.93E+03	5.12E+03	6.72E+03	5.63E+03	1.05E+04	2.11E+03	2.11E+03	7.83E+01
	Rank	2	6	4	3	5	9	7	7	1
CEC-2017-F14	Average	1.45E+03	7.05E+03	1.48E+03	1.46E+03	3.40E+03	1.51E+03	7.15E+03	7.15E+03	1.45E+03
	STD	2.24E+01	8.16E+03	4.25E+01	3.25E+01	1.95E+03	5.11E+01	1.49E+03	1.49E+03	5.60E+01
	Rank	1	7	4	3	6	5	8	8	1
CEC-2017-F15	Average	1.58E+03	9.30E+03	1.71E+03	1.59E+03	3.81E+03	2.24E+03	1.80E+04	1.80E+04	1.51E+03
	STD	1.28E+02	8.98E+03	2.83E+02	4.80E+01	3.86E+03	5.71E+02	5.50E+03	5.50E+03	1.64E+01
	Rank	2	7	4	3	6	5	8	8	1
CEC-2017-F16	Average	1.73E+03	1.79E+03	1.86E+03	1.65E+03	1.73E+03	1.73E+03	2.15E+03	2.15E+03	1.82E+03
	STD	1.20E+02	1.29E+02	1.28E+02	5.09E+01	1.24E+02	1.27E+02	1.06E+02	1.06E+02	2.30E+02
	Rank	2	5	7	1	2	2	8	8	6
CEC-2017-F17	Average	1.73E+03	1.75E+03	1.76E+03	1.73E+03	1.76E+03	1.77E+03	1.86E+03	1.86E+03	1.83E+03
	STD	3.45E+01	3.98E+01	4.75E+01	1.81E+01	3.13E+01	3.42E+01	1.08E+02	1.08E+02	1.76E+02
	Rank	1	3	4	1	4	6	8	8	7
CEC-2017-F18	Average	7.44E+03	1.57E+04	1.46E+04	1.25E+04	2.58E+04	2.34E+04	8.72E+03	8.72E+03	1.83E+03
	STD	4.52E+03	1.28E+04	1.19E+04	1.14E+04	1.58E+04	1.40E+04	5.06E+03	5.06E+03	1.35E+01
	Rank	2	7	6	5	9	8	3	3	1
CEC-2017-F19	Average	1.95E+03	9.69E+03	2.60E+03	1.95E+03	9.87E+03	2.92E+03	4.50E+04	1.37E+04	1.92E+03
	STD	5.53E+01	6.77E+03	2.19E+03	4.71E+01	6.37E+03	1.87E+03	1.90E+04	1.92E+04	2.87E+01
	Rank	2	6	4	2	7	5	9	8	1
CEC-2017-F20	Average	2.02E+03	2.06E+03	2.09E+03	2.02E+03	2.08E+03	2.09E+03	2.27E+03	2.27E+03	2.49E+03
	STD	2.53E+01	6.00E+01	6.23E+01	2.23E+01	5.20E+01	4.93E+01	8.17E+01	8.17E+01	2.43E+02
	Rank 1	3	5	1	4	5	7	7	9	
CEC-2017-F21	Average	2.23E+03	2.30E+03	2.28E+03	2.31E+03	2.32E+03	2.25E+03	2.36E+03	2.36E+03	2.32E+03
	STD	4.35E+01	4.38E+01	5.40E+01	2.10E+01	7.00E+00	6.04E+01	2.82E+01	2.82E+01	6.78E+01
	Rank 1	4	3	5	6	2	8	8	6	
CEC-2017-F22	Average	2.28E+03	2.30E+03	2.31E+03	2.30E+03	2.31E+03	2.30E+03	2.30E+03	2.30E+03	3.53E+03
	STD	1.33E+01	2.38E+00	6.61E+01	1.84E+01	1.68E+01	1.18E+01	7.00E-02	7.20E-02	8.48E+02
	Rank	1	2	7	2	7	2	2	2	9
CEC-2017-F23	Average	2.61E+03	2.63E+03	2.62E+03	2.62E+03	2.62E+03	2.62E+03	2.74E+03	2.74E+03	2.73E+03
	STD	4.12E+00	1.34E+01	9.23E+00	5.53E+00	8.47E+00	8.69E+00	3.91E+01	3.91E+01	2.43E+02
	Rank	1	6	2	2	2	2	8	8	7
CEC-2017-F24	Average	2.62E+03	2.76E+03	2.69E+03	2.74E+03	2.74E+03	2.73E+03	2.74E+03	2.74E+03	2.70E+03
	STD	7.96E+01	1.49E+01	1.08E+02	6.90E+00	8.73E+00	6.44E+01	5.55E+00	5.52E+00	7.34E+01
	Rank	1	9	2	5	5	4	5	5	3
CEC-2017-F25	Average	2.92E+03	2.95E+03	2.92E+03	2.93E+03	2.94E+03	2.92E+03	2.94E+03	2.94E+03	2.93E+03
	STD	1.26E+01	1.93E+01	2.50E+01	1.98E+01	2.36E+01	2.39E+01	1.53E+01	1.54E+01	2.09E+01
	Rank	1	9	1	4	6	1	6	6	4
CEC-2017-F26	Average	3.11E+03	3.11E+03	2.95E+03	2.97E+03	3.22E+03	2.90E+03	3.44E+03	3.44E+04	3.46E+03
	STD	2.89E+02	3.35E+02	2.50E+02	1.65E+02	4.27E+02	3.66E+01	6.29E+02	6.29E+02	5.99E+02
	Rank	4	4	2	3	6	1	7	9	8
CEC-2017-F27	Average	3.11E+03	3.12E+03	3.12E+03	3.09E+03	3.10E+03	3.09E+03	3.26E+03	3.26E+03	3.14E+03
	STD	2.09E+01	1.92E+01	2.50E+01	2.24E+00	2.18E+01	2.78E+00	4.17E+01	4.17E+01	2.14E+01
	Rank	4	5	5	1	3	1	8	8	7
CEC-2017-F28	Average	2.30E+03	3.32E+03	3.32E+03	3.30E+03	3.39E+03	3.21E+03	3.46E+03	3.46E+03	3.40E+03
	STD	1.24E+02	1.26E+02	1.22E+02	1.34E+02	1.02E+02	1.13E+02	3.38E+01	3.38E+01	1.31E+02
	Rank	1	4	4	3	6	2	8	8	7
CEC-2017-F29	Average	3.21E+03	3.25E+03	3.20E+03	3.17E+03	3.19E+03	3.21E+03	3.45E+03	3.45E+03	3.21E+03
	STD	5.66E+01	8.20E+01	5.23E+01	2.47E+01	4.29E+01	5.17E+01	1.71E+02	1.71E+02	1.10E+02
	Rank	4	7	3	1	2	4	1	1	4
CEC-2017-F30	Average	2.96E+05	5.37E+05	3.51E+05	2.97E+05	2.98E+05	4.21E+05	9.40E+05	1.30E+06	3.05E+05
	STD	2.14E+04	6.37E+05	5.05E+05	4.59E+05	5.28E+05	5.68E+05	3.60E+05	3.64E+05	4.45E+05
	Rank	1	7	5	2	3	6	8	9	4
Mean	rank	2.034	5.6206	3.5517	2.413	5.4482	4.896	6.4482	6.4827	4.586
Final	Ranking	1	6	3	2	7	5	8	9	4

3.3. Results comparisons using CEC2019 test functions

This experiment assesses the performance of RSA to solve another set of complex functions called CEC2019 (Price et al., 2018). In CEC2019, there are ten functions and each of them has its own dimension and search range as described in Table 14. In contrast to other CEC benchmarks used in this study (i.e., classical functions and CEC2017), the complexity of the functions of CEC2019 is significantly increased. In addition, the process of finding an accurate solution has more attention to evaluate the algorithm. The same algorithms used in the previous experiments are used with the same parameters.

Table 14: Review of CEC2019 benchmark function problems.

No.	Functions	$F_i^* = F_i(x^*)$	Dim	Search range
1	Storn's Chebyshev Polynomial Fitting Problem	1	9	[-8192, 8192]
2	Inverse Hilbert Matrix Problem	1	16	[-16384, 16384]
3	Lennard-Jones Minimum Energy Cluster	1	18	[-4,4]
4	Rastrigin's Function	1	10	[-100,100]
5	Griewangk's Function	1	10	[-100,100]
6	Weierstrass Function	1	10	[-100,100]
7	Modified Schwefel's Function	1	10	[-100,100]
8	Expanded Schaffer's F6 Function	1	10	[-100,100]
9	Happy Cat Function	1	10	[-100,100]
10	Ackley Function	1	10	[-100,100]

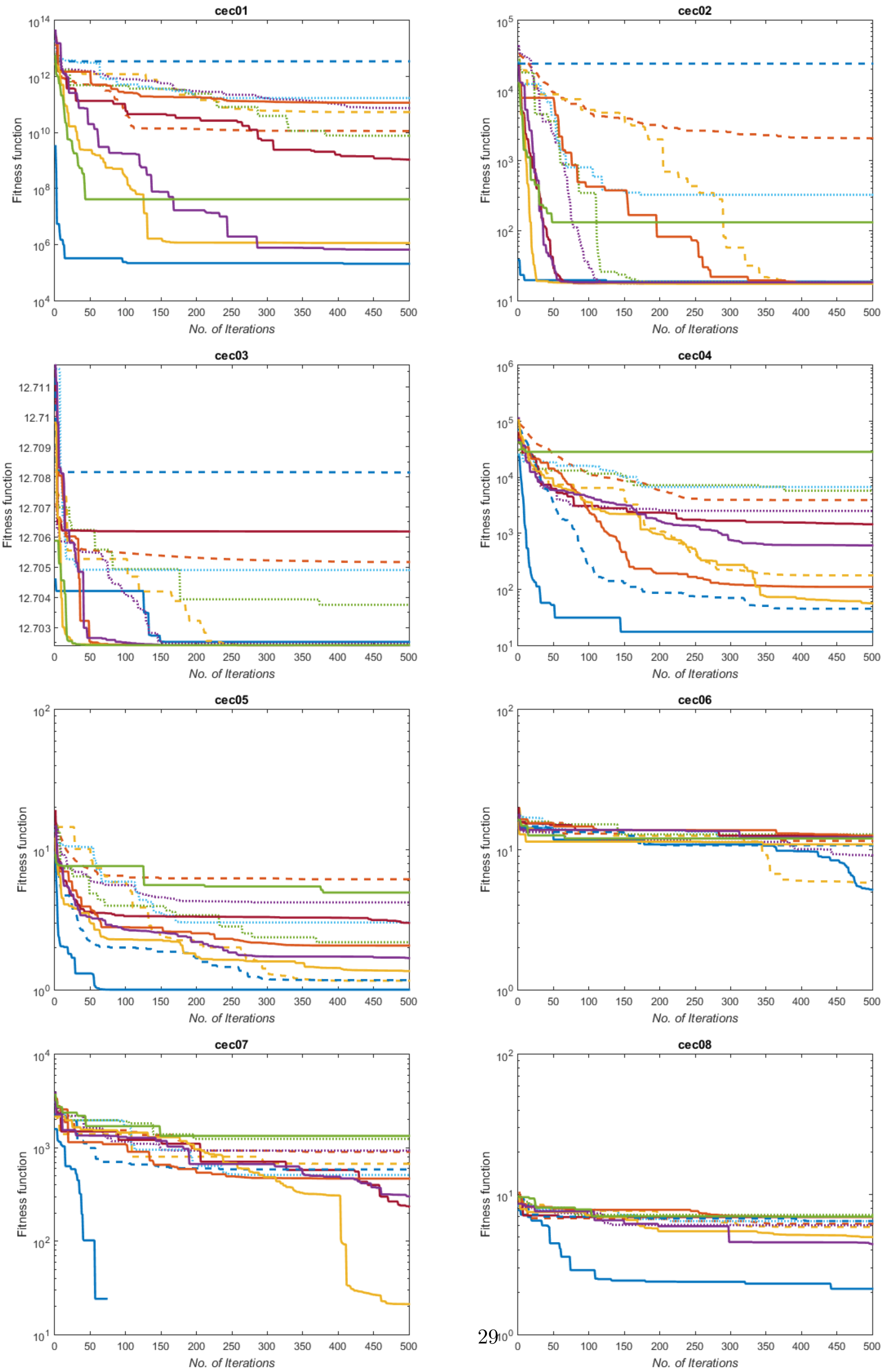
The comparison results between the competitive algorithms in terms of performance metrics are given in Table 15. From those results, one can be seen that the mean rank of RSA overall the ten tested functions are better than other methods which has the first rank. Followed by the MPA which has mean rank 3.9, then the EO allocates the 3^{rd} rank with mean 4.5. In addition, RSA has a better average in the most tested ten functions which indicates its ability to solve this kind of function CEC2019.

Table 15: Results of the RSA using the CEC2019 test functions

Fun	Measure	RSA	GOA	SSA	WOA	SCA	DA	GWO	PSO	ALO	MPA	EO	CMA-ES
CEC-2019-F1	Worst	6.30E+04	4.01E+10	1.13E+10	1.24E+11	1.20E+10	3.62E+10	3.28E+08	2.15E+13	5.31E+10	4.41E+04	2.85E+09	1.02E+09
	Average	5.62E+04	1.32E+10	7.32E+09	5.74E+10	4.61E+09	2.11E+10	1.07E+08	8.04E+12	2.44E+10	3.75E+04	9.31E+08	5.16E+08
	Best	5.01E+04	2.98E+09	2.46E+09	1.53E+09	1.31E+09	1.29E+10	5.17E+05	8.20E+11	4.52E+09	3.24E+04	3.97E+04	2.88E+08
	STD	4.84E+03	1.54E+10	3.48E+09	4.70E+10	4.22E+09	9.29E+09	1.51E+08	8.64E+12	2.20E+10	4.79E+03	1.33E+09	2.92E+08
	Rank	2	8	7	11	6	9	3	12	10	1	5	4
CEC-2019-F2	Worst	1.73E+01	1.75E+01	1.74E+01	1.74E+01	1.75E+01	6.04E+02	1.73E+01	1.58E+04	1.73E+01	1.73E+01	1.73E+01	1.28E+02
	Average	1.73E+01	1.74E+01	1.74E+01	1.74E+01	1.75E+01	1.35E+02	1.73E+01	1.23E+04	1.73E+01	1.73E+01	1.73E+01	8.54E+01
	Best	1.73E+01	1.74E+01	1.73E+01	1.73E+01	1.74E+01	1.73E+01	1.73E+01	8.28E+03	1.73E+01	1.73E+01	1.73E+01	4.38E+01
	STD	1.26E-07	4.08E-02	2.08E-02	6.19E-03	4.61E-02	2.62E+02	1.40E-04	3.02E+03	2.21E-03	1.26E-07	5.02E-15	3.18E+01
	Rank	2	8	7	6	9	11	4	12	5	2	1	10
CEC-2019-F3	Worst	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01
	Average	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01
	Best	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01
	STD	5.43E-05	8.19E-12	1.78E-15	7.43E-07	8.14E-05	1.12E-03	1.79E-06	0.00E+00	8.88E-16	0.00E+00	1.54E-15	1.16E-05
	Rank	1	7	5	8	11	12	9	2	2	2	5	10
CEC-2019-F4	Worst	2.75E+01	5.47E+01	4.88E+01	8.66E+02	3.62E+03	3.22E+03	1.68E+02	2.39E+01	1.13E+02	1.30E+01	2.41E+01	2.75E+01
	Average	2.45E+01	3.29E+01	3.42E+01	5.89E+02	1.98E+03	1.28E+03	7.59E+01	1.73E+01	5.61E+01	8.99E+00	1.79E+01	2.45E+01
	Best	2.09E+01	8.96E+00	2.19E+01	2.27E+02	1.10E+03	3.43E+01	4.91E+01	4.98E+00	1.09E+01	5.00E+00	7.00E+00	2.09E+01
	STD	2.42E+00	1.97E+01	1.08E+01	2.83E+02	9.63E+02	1.47E+03	5.16E+01	7.38E+00	3.71E+01	3.38E+00	6.83E+00	2.42E+00
	Rank	4	6	7	10	12	11	9	2	8	1	3	4
CEC-2019-F5	Worst	1.09E+00	1.39E+00	1.36E+00	1.53E+00	2.65E+00	1.29E+00	2.66E+00	1.63E+00	1.09E+00	1.93E+00	2.86E+00	1.73E+00
	Average	1.05E+00	1.21E+00	1.18E+00	1.25E+00	2.34E+00	1.21E+00	2.39E+00	1.48E+00	1.05E+00	1.05E+00	2.14E+00	1.56E+00
	Best	1.00E+00	1.05E+00	1.09E+00	1.11E+00	2.14E+00	1.17E+00	2.15E+00	1.21E+00	1.00E+00	1.27E+00	1.72E+00	1.33E+00
	STD	4.23E-02	1.38E-01	1.20E-01	1.63E-01	1.87E-01	5.05E-02	1.97E-01	1.59E-01	4.23E-02	5.13E-01	5.30E-01	1.72E-01
	Rank	2	5	4	7	11	6	12	8	2	1	10	9
CEC-2019-F6	Worst	8.58E+00	9.78E+00	6.28E+00	1.13E+01	1.18E+01	1.08E+01	1.21E+01	1.13E+01	1.22E+01	1.40E+01	1.14E+01	1.13E+01
	Average	5.29E+00	8.48E+00	5.49E+00	9.64E+00	1.11E+01	9.63E+00	1.16E+01	9.98E+00	1.15E+01	1.23E+01	1.08E+01	1.04E+01
	Best	2.98E+00	7.66E+00	4.51E+00	8.28E+00	9.87E+00	8.52E+00	1.15E+01	8.90E+00	1.09E+01	1.12E+01	9.89E+00	9.13E+00
	STD	2.36E+00	8.76E-01	8.53E-01	1.15E+00	7.51E-01	8.85E-01	2.40E-01	9.32E-01	4.74E-01	1.13E+01	6.60E-01	7.99E-01
	Rank	1	3	2	5	9	4	11	6	10	12	8	7
CEC-2019-F7	Worst	6.65E+02	7.68E+02	5.64E+02	1.13E+03	1.06E+03	1.26E+03	6.59E+02	4.15E+02	1.40E+03	1.46E+02	5.42E+02	6.08E+02
	Average	4.43E+02	5.01E+02	4.61E+02	8.06E+02	8.50E+02	9.48E+02	3.53E+02	2.01E+02	6.56E+02	4.73E+01	2.04E+02	2.60E+02
	Best	2.83E+02	2.50E+02	2.89E+02	4.62E+02	4.78E+02	4.93E+02	1.24E+02	-3.41E+00	1.87E+02	-8.88E+01	-2.23E+02	-2.92E+01
	STD	1.57E+02	2.19E+02	1.18E+02	2.53E+02	2.24E+02	3.07E+02	2.29E+02	1.80E+02	4.47E+02	9.78E+01	2.86E+02	2.91E+02
	Rank	6	8	7	10	11	12	5	2	9	1	3	4
CEC-2019-F8	Worst	6.58E+00	6.20E+00	6.71E+00	6.64E+00	6.69E+00	6.47E+00	6.81E+00	5.63E+00	6.10E+00	4.90E+00	5.60E+00	6.62E+00
	Average	5.60E+00	5.59E+00	5.48E+00	6.10E+00	5.90E+00	5.95E+00	5.48E+00	4.95E+00	5.70E+00	4.09E+00	3.91E+00	5.85E+00
	Best	4.67E+00	5.16E+00	4.86E+00	5.67E+00	5.38E+00	5.20E+00	2.90E+00	4.35E+00	5.12E+00	2.56E+00	2.42E+00	4.58E+00
	STD	7.59E-01	4.30E-01	7.11E-01	4.24E-01	5.74E-01	4.93E-01	1.69E+00	6.17E-01	3.92E-01	9.14E-01	1.16E+00	8.08E-01
	Rank	7	6	4	12	10	11	5	3	8	2	1	9
CEC-2019-F9	Worst	2.44E+00	2.98E+00	2.93E+00	6.40E+00	4.59E+02	1.96E+02	7.14E+00	2.44E+00	2.83E+00	2.50E+00	2.48E+00	2.41E+00
	Average	2.40E+00	2.66E+00	2.73E+00	5.40E+00	2.03E+02	5.64E+01	4.93E+00	2.41E+00	2.60E+00	2.42E+00	2.40E+00	2.39E+00
	Best	2.37E+00	2.52E+00	2.38E+00	3.95E+00	4.52E+01	2.74E+00	3.27E+00	2.39E+00	2.41E+00	2.35E+00	2.36E+00	2.38E+00
	STD	3.20E-02	1.83E-01	2.19E-01	9.34E-01	1.58E+02	8.01E+01	1.53E+00	3.38E-02	1.74E-01	5.96E-02	4.82E-02	1.12E-02
	Rank	2	7	8	10	12	11	9	4	6	5	3	1
CEC-2019-F10	Worst	2.05E+01	2.00E+01	2.00E+01	2.05E+01	2.06E+01	2.06E+01	2.06E+01	2.07E+01	2.01E+01	2.10E+01	2.05E+01	2.05E+01
	Average	2.03E+01	2.00E+01	2.00E+01	2.03E+01	2.05E+01	2.04E+01	2.05E+01	2.04E+01	2.00E+01	2.10E+01	2.04E+01	2.04E+01
	Best	2.02E+01	2.00E+01	2.00E+01	2.02E+01	2.04E+01	2.03E+01	2.05E+01	2.01E+01	2.00E+01	2.10E+01	2.03E+01	2.04E+01
	STD	1.22E-01	3.72E-04	1.88E-03	1.02E-01	5.89E-02	1.40E-01	4.52E-02	2.02E-01	3.93E-02	3.56E-02	5.61E-02	6.71E-02
	Rank	4	1	2	5	10	9	11	8	3	12	6	7
Mean	rank	3.1	5.9	5.3	8.4	10.1	9.6	7.8	5.9	6.3	3.9	4.5	6.5
Final	Ranking	1	5	4	10	12	11	9	5	7	2	3	8

To further analysis, the performance of RSA to find the optimal solution for CEC2019 functions, Figure 6 shows the convergence curves of RSA and other methods. It can be seen that the RSA has a fast convergence rate than other methods at some functions such as F1, F4, and F5. As well as, at the most functions its convergence rate is competitive with other methods especially CMA-ES, SHO, and MPA. Moreover, the accounts for the satisfactory achievement of the proposed RSA in sustaining the stability of exploitation and exploration capabilities can be probably attributed to the following features:

- The adaptive ES value allows the RSA to keep a certain disturbance rate while supporting fast convergence, so evading optimal local trapping through high-speed convergence.
- η parameter supports the individual position of RSA to engage in a specific way to generate better solution positions, thus guaranteeing the effectiveness of the initial exploration and the efficiency of the next exploitation.
- The satisfactory utilization of the best fitness values allows RSA to obtain better decisions based on historical learning.


cec01
cec02
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cec04
cec05
cec06
cec07
cec08
Fitness function
No. of Iterations

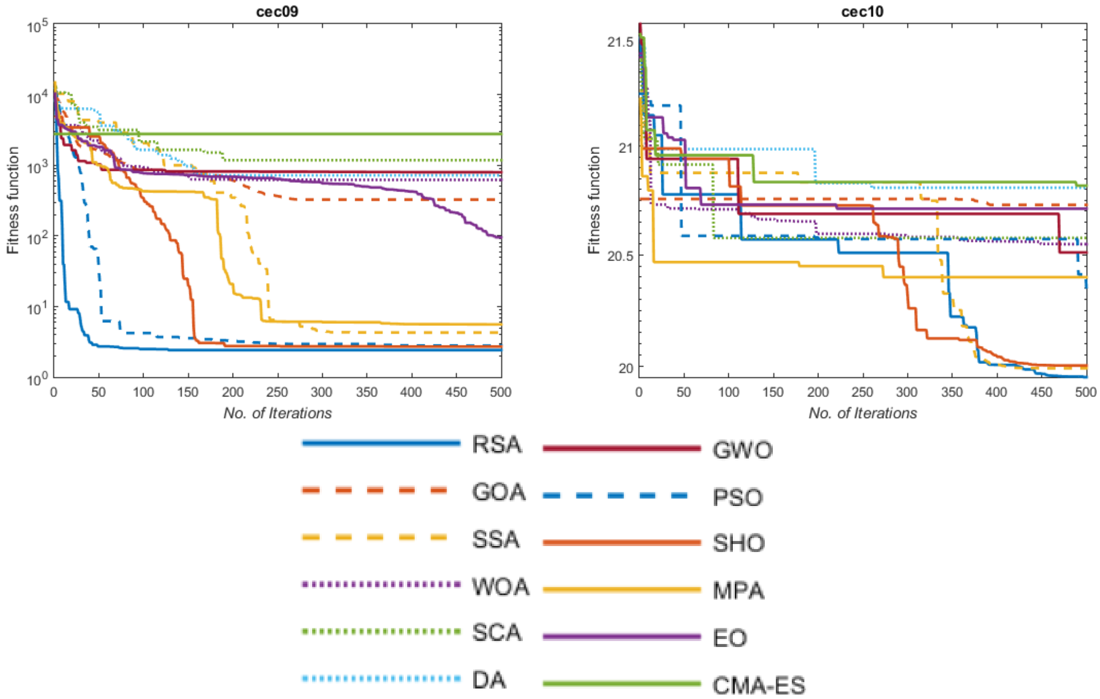


Figure 6: Convergence behaviour results for the CEC2019 test problems

3.4. Real-world applications

In this section, we applied the RSA to solve real-world engineering problems with constraints. These problems including pressure vessel design, welded beam design, tension/compression spring design, 3-bar truss design problem, Speed reducer problem, Cantilever beam design problem, and Multiple disc clutch brake problem.

The parameters setting for RSA is similar to the above experiments. This means the number of iterations is 500 and the population size is 30. The mathematical definition of these problems can be found in (Chen et al., 2019; Cheng & Prayogo, 2017; Samma et al., 2019; Gandomi & Yang, 2011).

3.4.1. Welded beam design problem

The performance of RSA to solve this engineering problem (i.e., welded beam design) is discussed in this section. Figure 7 shows the shape of the welded beam as can be seen in this figure that there are four parameters deflection (h), shear stress (l), buckling load (t) and bending stress (p). The main aim of any optimization technique used to solve this problem is to minimize the total fabrication cost. In this experiment, the results of the RSA are compared with other literature including OBSCA (Elaziz et al., 2017), GSA (Saremi et al., 2017), RSA (Kaveh & Khayatazad, 2012) , DAVID (Ragsdell & Phillips, 1976), MVO (Mirjalili et al., 2016), WOA (Mirjalili & Lewis, 2016), CPSO (He & Wang, 2007a), HS (Lee & Geem, 2005), APPROX (Ragsdell & Phillips, 1976), SIMPLEX (Ragsdell & Phillips, 1976), CSCA (Huang et al., 2007), and GA (Deb, 1991). The mathematical representation of this problem can be found below.

Consider $\overrightarrow{x}=[x_1\ x_2\ x_3\ x_4] = [\ h\ l\ t\ p]$,
Minimize $f(\overrightarrow{x}) = 1.10471 x_1^2 x_1 + 0.04811 x_3 x_4 (14.0 + x_2)$
Subject to $g_1(\overrightarrow{x}) = \tau\ (\overrightarrow{x})$ - $\tau_{max} \leq 0$,
$g_2(\overrightarrow{x}) = \sigma\ (\overrightarrow{x})$ - $\sigma_{max} \leq 0$,
$g_3(\overrightarrow{x}) = \delta\ (\overrightarrow{x})$ - $\delta_{max} \leq 0$,
$g_4(\overrightarrow{x}) = x_1$ - $x_4 \leq 0$,
$g_5(\overrightarrow{x}) = p$ - $p_c(\overrightarrow{x}) \leq 0$,

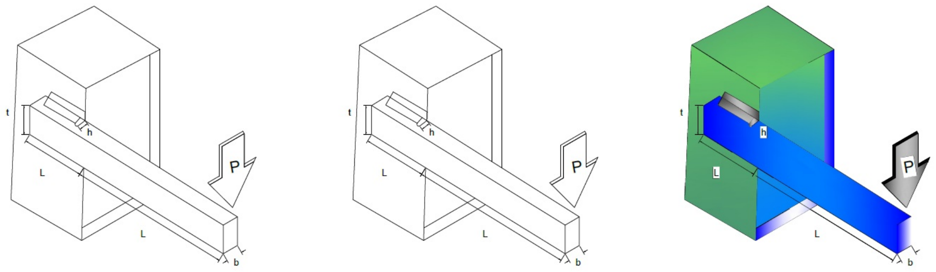


Figure 7: Welded beam design problem.

$g_6(\overrightarrow{x})$=0.125 - $x_1 \leq 0$,
$g_7(\overrightarrow{x})$=1.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 0.5 ≤ 0,

where,
$\tau(\overrightarrow{x})=\sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')}$, $\tau^{'}=\frac{P}{\sqrt{2x_1x_2}}$,

$\tau^{''}$ = $\frac{MR}{J}$, M=P(L+$\frac{x_2}{2}$), R= $\sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2}$,

$P_c(\overrightarrow{x})=\frac{4.013E\sqrt{\frac{x_3^2x_4^6}{0}}}{L^2}$, (1 - $\frac{x_3}{2L}$ $\sqrt{\frac{E}{4G}}$), $\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$,

J=2 $\left\{\sqrt{2x_1x_2}\left[\frac{x_x^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\}$, σ $(\overrightarrow{x})$= $\frac{6PL}{x_4x_3^2}$, δ $(\overrightarrow{x})$=$\frac{6PL^3}{Ex_4x_3^2}$,

Note that, P=6000 lb, L=14 in, δ_{max}=0.25 in, E=30 $\times$ 1^6 psi, G=12 $\times$ 10^6 psi, τ_{max}=13600 psi, and σ_{max}= 30000 psi.

Table 16 shows the performance of the proposed method compared to the other methods. One can be seen that the smallest cost function is that obtained by the RSA, followed by the OBSCA algorithm. In addition, Figure 8 depicts the behaviour of RSA to solve WBD during the iterations. It can be seen objective values for each parameter among the iterations as in the second column. While the trajectory of the first solution is represented in the third column which indicates the RSA starting exploration small then pass exploitation followed by exploration again and finally back again to the exploitation. This refers to the ability of RSA to search the domain and avoiding attractive at the local point. Also, from the fourth column that represents the convergence curve of RSA, it can be seen its fitness value is decreased in step-wise form nearly after 100 iterations.

Table 16: Results of the comparative algorithms for solving the welded beam design problem.

Algorithm	**Optimal values for variables**				Optimal
	h	l	t	b	cost
OBSCA (Elaziz et al., 2017)	0.230824	3.069152	8.988479	0.208795	1.722315
GSA (Saremi et al., 2017)	0.182129	3.856979	10.000	0.202376	1.87995
RSA (Kaveh & Khayatazad, 2012)	0.203687	3.528467	9.004233	0.207241	1.735344
CSCA (Huang et al., 2007)	0.203137	3.542998	9.033498	0.206179	1.733461
GA (Deb, 1991)	0.2489	6.1730	8.1789	0.2533	2.4300
DAVID (Ragsdell & Phillips, 1976)	0.2434	6.2552	8.2915	0.2444	2.3841
SIMPLEX (Ragsdell & Phillips, 1976)	0.2792	5.6256	7.7512	0.2796	2.5307
APPROX (Ragsdell & Phillips, 1976)	0.2444	6.2189	8.2915	0.2444	2.3815
HS (Lee & Geem, 2005)	0.2442	6.2231	8.2915	0.2400	2.3807
CPSO (He & Wang, 2007a)	0.202369	3.544214	9.04821	0.205723	1.72802
WOA (Mirjalili & Lewis, 2016)	0.205396	3.484293	9.037426	0.206276	1.730499
MVO (Mirjalili et al., 2016)	0.205463	3.473193	9.044502	0.205695	1.72645
MPA (Faramarzi et al., 2020a)	0.205728	3.470509	9.036624	0.205730	1.724853
RSA	0.14468	3.514	8.9251	0.21162	1.6726

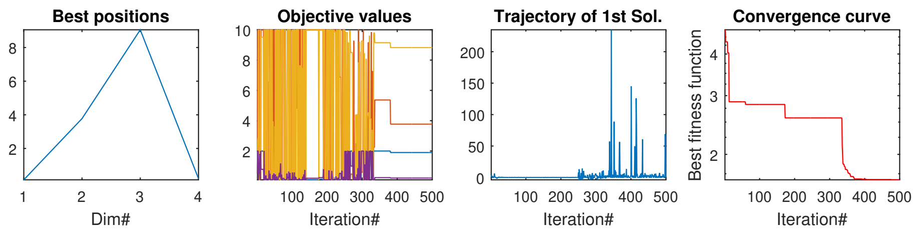


Figure 8: Qualitative results for the welded beam design problem.

3.4.2. Tension/compression spring design problem

The main target of this engineering problem is to determine the value of three parameters to minimize the weight of a tension/compression spring (see Figure 9). The three parameters are the wire diameter(d), number of active coils (N). mean coil diameter (D). The mathematical representation of this problem can be found below.

Consider $\overrightarrow{x}$=[x_1 x_2 x_3] = [d D N],

Minimize $f(\overrightarrow{x})$=$(x_3+2)x_2x_1^2$,

Subject to $g_1(\overrightarrow{x})$=1-$\frac{x_3x_2^3}{71785x_1^4}$ ≤ 0,

g_2=$(\overrightarrow{x})$=$\frac{4x_2^2-x_1x_1}{12566(x_2x_1^3-x_1^4)}+\frac{1}{5108x_1^2}$ ≤ 0,

g_3=$(\overrightarrow{x})$=1-$\frac{140.54x_1}{x_2^2x_3}$ ≤ 0,

g_4=$(\overrightarrow{x})$=$\frac{x_1x_2}{1.5}$-1 ≤ 0,

Variables range $(0.05 \leq x_1 \leq 2)$, $(0.25 \leq x_2 \leq 1.30)$, $(2.00 \leq x_3 \leq 15)$,

This problem has been solved by using several techniques we compared the results of RSA with some of them as in Table 17. It is observed that most of the algorithms nearly have the same optimal weight except the RSA which has the smallest weight overall of them. Moreover, Figure 10 shows the qualitative results of RSA during solving this problem. It can be observed that the convergence curve of the RSA is reached to the optimal weight nearly in the second period of iterations (i.e., after 450 iterations) which stay fixed till the end of iterations. The same observation can be noticed from the trajectory of the first solution as in the third column in the figure.

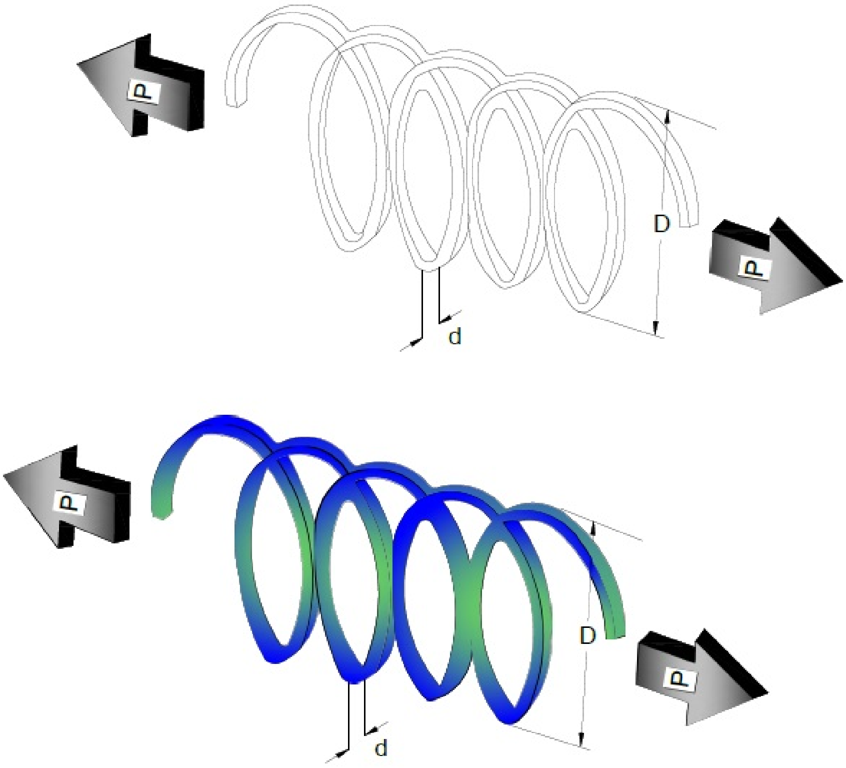


Figure 9: Tension/compression spring design problem.

Table 17: Results of the comparative algorithms for solving the tension/compression spring design problem.

Algorithm	**Optimal values for variables**			Optimal
	d	D	N	weight
OBSCA (Elaziz et al., 2017)	0.05230	0.31728	12.54854	0.012625
GSA (Saremi et al., 2017)	0.050276	0.323680	13.525410	0.0127022
RSA (Kaveh & Khayatazad, 2012)	0.051370	0.349096	11.76279	0.0126788
CPSO (He & Wang, 2007a)	0.051728	0.357644	11.244543	0.0126747
CSCA (Huang et al., 2007)	0.051609	0.354714	11.410831	0.0126702
GA (Coello, 2000)	0.051480	0.351661	11.632201	0.01270478
CC (Arora, 2004)	70.050000	0.315900	14.250000	0.0128334
HS (Mahdavi et al., 2007)	0.051154	0.349871	12.076432	0.0126706
PSO (He & Wang, 2007a)	0.051728	0.357644	11.244543	0.0126747
ES (Mezura-Montes & Coello, 2008)	0.051643	0.355360	11.397926	0.012698
WOA (Mirjalili & Lewis, 2016)	0.051207	0.345215	12.004032	0.0126763
MVO (Mirjalili et al., 2016)	0.05251	0.37602	10.33513	0.012790
RSA	0.057814	0.58478	4.0167	0.01176

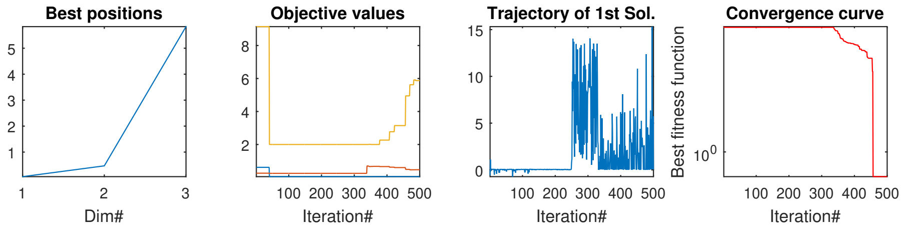


Figure 10: Qualitative results for the tension/compression spring design problem.

3.4.3. Pressure vessel design problem

The pressure vessel design (PVD) problem is one of the most popular design problems of mixed-integer type. Figure 11 depicts the PVD which aims to minimize the cost of material, welding, and forming of the pressure vessel. In the PVD problem, there are four variables that control on the optimization process including the inner radius (R), the thickness of the head (Th), length of the cylindrical section of the vessel (L), and the thickness of the shell (T_s). According to the requirement of the ASME boiler, TS and Th are integer times 0.0625 inch. The mathematical representation of this problem can be found below.

Consider $\overrightarrow{x}$=[x_1 x_2 x_3 x_4] = [T_s T_h R L],
Minimize $f(\overrightarrow{x})$=0.6224$x_1x_3x_4$ + 1.7781$x_2x_3^2$ + 3.1661$x_1^2x_4$ + 19.84$x_1^2x_3$,
Subject to $g_1(\overrightarrow{x})$=-x_1 + 0.0193$x_3 \leq 0$,
$g_2(\overrightarrow{x})$=-x_3 + 0.000954$x_3 \leq 0$,
$g_3(\overrightarrow{x})$=-π x_3^2 x_4 - $\frac{4}{3}$ π x_3^3 +1296000 ≤ 0,
$g_4(\overrightarrow{x})$=x_4 - 240 ≤ 0,
Variables range ($0 \leq x_1$, $x_2 \leq 99$), ($10 \leq x_3$, $x_4 \leq 200$),

The results of RSA are given in Figure 12 and Table 18. One can be observed from the results in Table 18 that the RSA has the smallest cost with nearly 24.955 difference between it and HPSO that followed it. In addition, from Figure 12 it can be seen the convergence curve of RSA reached the optimal cost nearly after 400 iterations similar to the previous problem. The main change is the behaviour of RSA occurs when moving from exploration to exploitation phase as reflected from the trajectory of the first solution.

Table 18: Results of the comparative algorithms for solving the pressure vessel design problem.

Algorithm	**Optimal values for variables**				Optimal
	T_s	T_h	R	L	cost
Branch-bound (Sandgren, 1990)	1.125	0.625	48.97	106.72	7982.5
GWO (Mirjalili et al., 2014)	0.8125	0.4345	42.0892	176.7587	6051.5639
MVO (Mirjalili et al., 2016)	0.8125	0.4375	42.090738	176.73869	6060.8066
WOA (Mirjalili & Lewis, 2016)	0.812500	0.437500	42.0982699	176.638998	6059.7410
ES (Mezura-Montes & Coello, 2008)	0.8125	0.4375	42.098087	176.640518	6059.74560
CPSO (He & Wang, 2007a)	0.8125	0.4375	42.091266	176.7465	6061.0777
CSCA (Huang et al., 2007)	0.8125	0.4375	42.098411	176.63769	6059.7340
HS (Mahdavi et al., 2007)	1.125000	0.625000	58.29015	43.69268	7197.730
GA (Coello, 2000)	0.81250	0.43750	42.097398	176.65405	6059.94634
PSO-SCA (Liu et al., 2010)	0.8125	0.4375	42.098446	176.6366	6059.71433
HPSO (He & Wang, 2007b)	0.8125	0.4375	42.0984	176.6366	6059.7143
ACO (Kaveh & Talatahari, 2010a)	0.812500	0.437500	42.098353	176.637751	6059.7258
GSA (Rashedi et al., 2009)	1.125	0.625	55.9886598	84.4542025	8538.8359
RSA	0.8400693	0.4189594	43.38117	161.5556	6034.7591

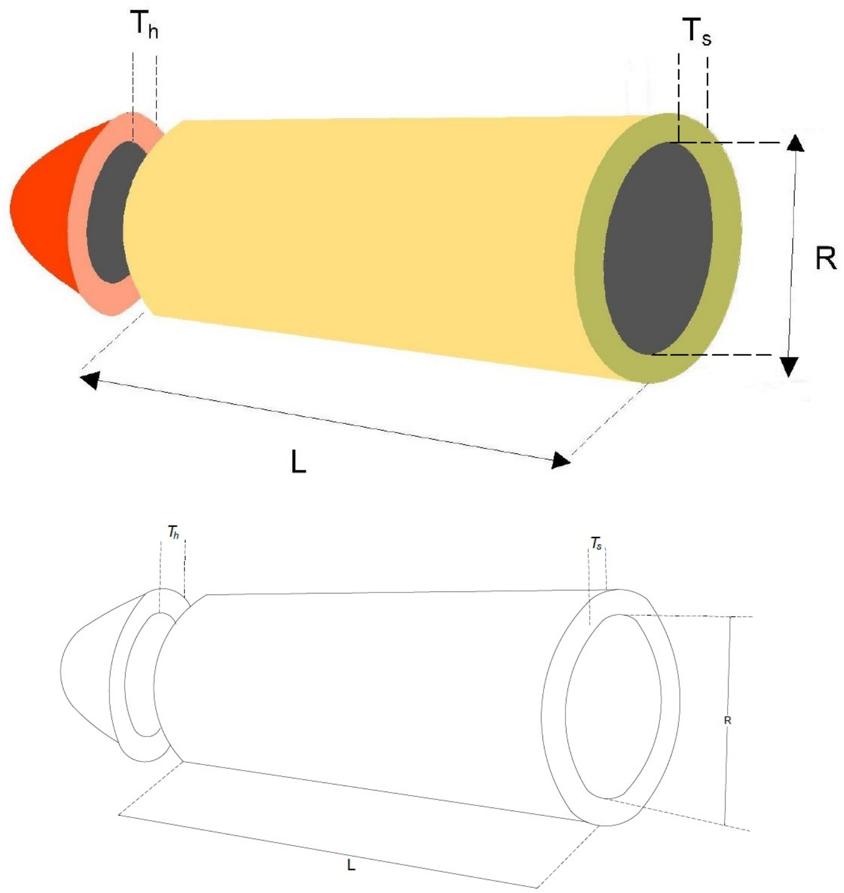


Figure 11: Pressure vessel design problem

3.4.4. 3-bar truss design problem

The main objective of study the three-bar truss design problem is to minimize the structure weight subject to supporting a total load P acting vertically downward (Ray & Liew, 2003; Mirjalili, 2015b). The geometry structure of this problem in given Figure 13 where the cross-sectional areas represent the variables of design. Due to the symmetric of the system, so the cross-sections with $A_1(= x_1)$ and $A_2(= x_2)$ are need to be determined (Ray & Liew, 2003). The mathematical representation of this problem can be found below.

Minimize $f(x)$=$(2\sqrt{2x_1}$ + $x_2)\times$ l

Subject to:

g_1x=$\frac{\sqrt{x1}x_1+x_2}{\sqrt{2x_1^2+2x_1x_2}}$ P - σ ≤ 0

g_2x=$\frac{x_2}{\sqrt{2x_1^2+2x_1x_2}}$ P-σ ≤ 0

g_3x=$\frac{1}{\sqrt{2x_2+x_1}}$ P-σ ≤ 0

l=100cm, P=2 kN/cm^2, σ=2 kN/cm^2,

Variables range ($0 \leq x_i \leq 1$, i=1,2)

The RSA is applied to determine the design variables and it is compared with some optimization techniques that have been applied to solve the problem as given in Table 19. From the results in this table, it is observed that the RSA provides the best solution since it obtained the smallest weight. Moreover, the

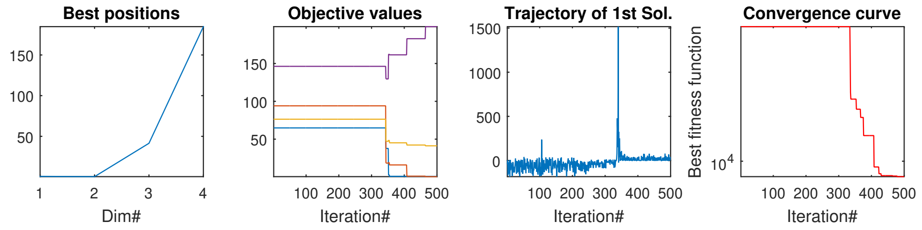


Figure 12: Qualitative results for the pressure vessel design problem

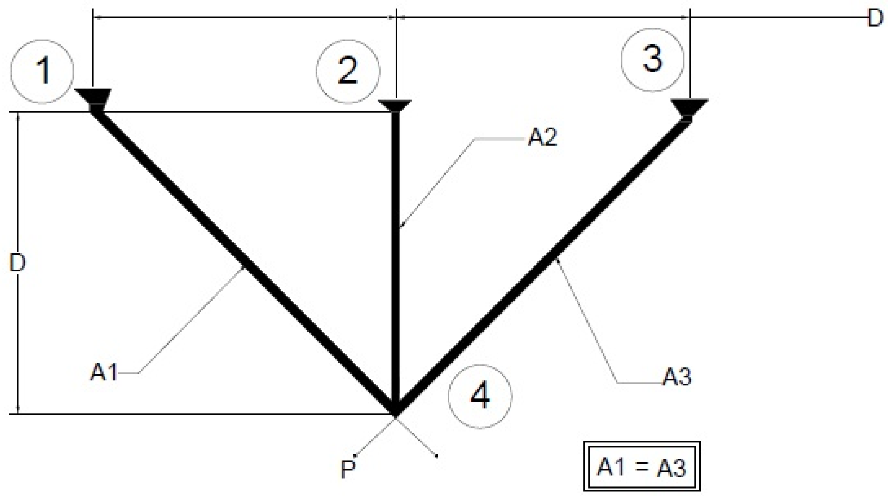


Figure 13: 3-bar truss design problem.

Qualitative results of RSA to solve the 3-bar truss design problem are given in Figure 14. From this figure, it can be noticed the best position over the dimensions and the objective value of each dimension overall the iterations. In addition, the ability of RSA to searching the space using the exploration and exploitation phases is represented by the trajectory of the first solution. Also, from the last column in this figure, it can be seen the convergence of the RSA that similar to previous experiments and this indicates the stable in the behaviour of RSA overall the tested problems.

Table 19: Results of the comparative algorithms for solving the 3-bar truss design problem.

Algorithm	**Optimal values for variables**		Optimal
	x_1	x_2	weight
DEDS (Zhang et al., 2008)	0.78867513	0.40824828	263.89584
SSA (Mirjalili et al., 2017)	0.78866541	0.408275784	263.89584
MBA (Sadollah et al., 2013)	0.7885650	0.4085597	263.89585
PSO-DE (Liu et al., 2010)	0.7886751	0.4082482	263.89584
Ray and Sain (Ray & Saini, 2001)	0.795	0.395	264.3
CS (Gandomi et al., 2013)	0.78867	0.40902	263.9716
AAA (YILDIRIM & KARCI, 2018)	0.7887354	0.408078	263.895880
GOA (Saremi et al., 2017)	0.78889755557	0.40761957011	263.89588149
RSA	0.78873	0.40805	263.8928

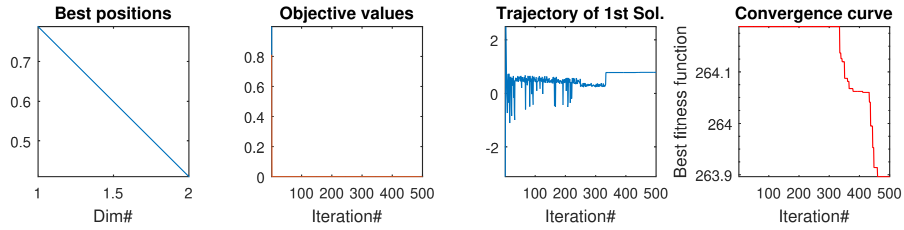


Figure 14: Qualitative results for the 3-bar truss design problem.

3.4.5. Speed reducer problem

The Speed reducer problem is a gearbox between propeller and engine of airplane to allow each of them to rotate with efficient speed (as in Figure 15). The objective of any optimization technique used to solve the problem of speed reducer (Siddall, 1972) is to minimize the weight of the design subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shafts. To solve this problem, the optimal value of the seven design variables are required to be determined. Those variables including face width (x_1), the module of teeth (x_2), number of teeth on pinion (x_3), length of the first shaft between bearings (x_4), length of the second shaft between bearings (x_5), the diameter of the first shaft (x_6), and diameter of second shaft (x_7). According to the definition of the Speed reducer problem, it is high complexity than other problems since it has a higher number of constraints. The mathematical representation of this problem can be found below.

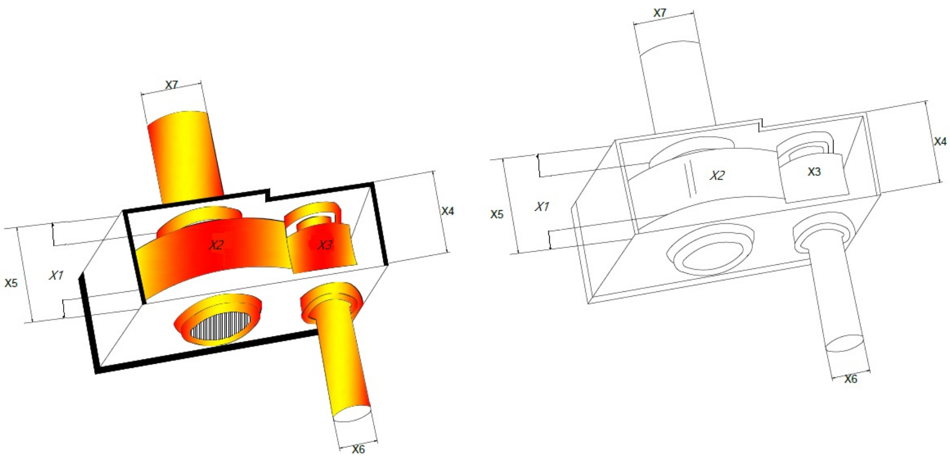


Figure 15: Speed reducer problem.

Consider $\overrightarrow{x}$=$0.7854x_1x_2^2(3.3333x_3^2+14.9334x_3-43.0934)-1.508x_1(x_6^2+x_7^2)+7.4777x_6^3+x_7^3+0.7854x_4x_6^2+x_5x_7^2$

Subject to:

$g(1)=\frac{27}{x_1x_2^2x_3}-1 \leq 0,$

$g(2)=\frac{397.5}{x_1x_2^2x_3^2}-1 \leq 0,$

$g(3)=\frac{1.93x_4^3}{x_2x_3x_6^4}-1 \leq 0,$

$g(4)=\frac{1.93x_5^3}{x_2x_3x_7^4}-1 \leq 0,$

$g(5)=\frac{1}{110x_6^3}\sqrt{(\frac{745x_4}{x_2x_3})^2+16.9\times10^6}-1 \leq 0,$

$g(6)=\frac{1}{85x_7^3}\sqrt{(\frac{745x_5}{x_2x_3})^2+157.5\times10^6}$ -1 $\leq 0,$

$g(7) = \frac{x_2 x_3}{40} - 1 \leq 0,$
$g(8) = \frac{5x_2}{x_1} - 1 \leq 0,$
$g(9) = (x_1/12x_2) - 1 \leq 0,$
$g(10) = \frac{1.5x_6+1.9}{x_4} - 1 \leq 0,$
$g(11) = \frac{1.1x_7+1.9}{x_5} - 1 \leq 0$

Variables range $(2.6 \leq x_1 \leq 3.6)$, $(0.7 \leq x_2 \leq 0.8)$, $(17 \leq x_3 \leq 28)$, $(7.3 \leq x_4 \leq 8.3)$, $(7.3 \leq x_5 \leq 8.3)$, $(2.9 \leq x_6 \leq 3.9)$, $(5 \leq x_7 \leq 5.5)$

The qualitative results of the speed reducer problem over 500 iterations are given in Figure 16. It depicts the convergence curve has step-wise behaviour. Also, it is clear that the trajectory begins the exploration search than changing the state of the solution to the exploitation stage to obtain the solution in the feasible region.

The comparison results of RSA and other MH techniques published in the literature are given in Table 20. It can be noticed that the RSA outperforms most of the compared techniques and, in general, it allocates the $3^r d$ rank. Since, CS (Gandomi et al., 2013)and SBSM (Akhtar et al., 2002) which allocate the first and second rank, respectively. However, the no significant difference between the three algorithms since all of them nearly have the same optimal weight.

Table 20: Results of the comparative algorithms for solving the speed reducer design problem.

Algorithm	Optimal values for variables							Optimal
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	weight
GA (SARUHAN & UYGUR, 2003)	3.510253	0.7	17	8.35	7.8	3.362201	5.287723	3067.561
GSA (Rashedi et al., 2009)	3.600000	0.7	17	8.3	7.8	3.369658	5.289224	3051.120
HS (Geem et al., 2001)	3.520124	0.7	17	8.37	7.8	3.366970	5.288719	3029.002
SES (Mezura-Montes et al., 2003)	3.506163	0.700831	17	7.460181	7.962143	3.362900	5.308949	3025.005127
MDA (Lu & Kim, 2010)	3.5	0.7	17	7.3	7.670396	3.542421	5.245814	3019.583365
SBSM (Akhtar et al., 2002)	3.506122	0.700006	17	7.549126	7.859330	3.365576	5.289773	3008.08
SCA (Mirjalili, 2016b)	3.508755	0.7	17	7.3	7.8	3.461020	5.289213	3030.563
CS (Gandomi et al., 2013)	3.5015	0.7000	17	7.6050	7.8181	3.3520	5.2875	3000.9810
PSO (Stephen et al., 2018)	3.5001	0.7000	17.0002	7.5177	7.7832	3.3508	5.2867	3145.922
FA (Baykasoğlu & Ozsoydan, 2015)	3.507495	0.7001	17	7.719674	8.080854	3.351512	5.287051	3010.137492
hHHO-SCA (Kamboj et al., 2020)	3.506119	0.7	17	7.3	7.99141	3.452569	5.286749	3029.873076
RSA	3.50279	0.7	17	7.30812	7.74715	3.35067	5.28675	2996.5157

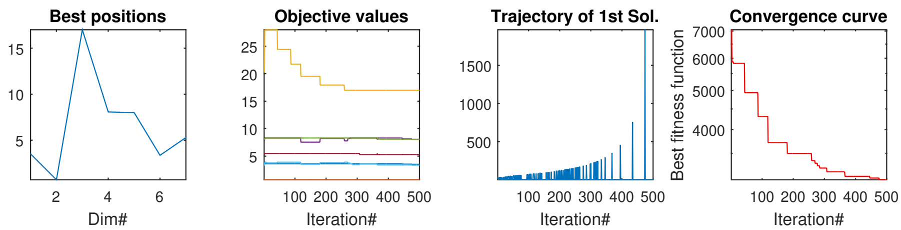


Figure 16: Qualitative results for the speed reducer design problem.

3.4.6. Cantilever beam design problem

In this part, the proposed RSA is applied to solve another design problem called Cantilever beam (Chickermane & Gea, 1996). This problem has five design variables that should be determined during the optimization process as depicted in Figure 17. The mathematical representation of this problem can be found below.

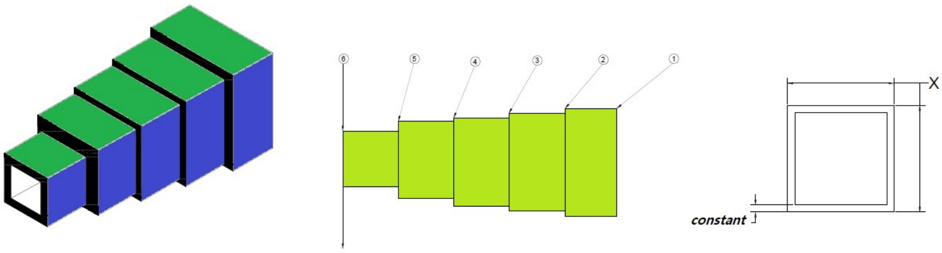


Figure 17: Cantilever beam design problem.

Consider $x = [x_1\ x_2\ x_3\ x_4\ x_5]$
Minimise $f(\vec{x}) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5)$,
subject to $\quad g(\vec{x}) = \frac{60}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0$
Variable range $0.01 \leq x_1, x_2, x_3, x_4, x_5 \leq 100$

Table 21 shows the values of the five design variables and the optimal weight. From this table, it can be observed that the RSA provides better weight than other methods. Followed by CS (Gandomi et al., 2013) which allocates the second rank.

Figure 18 depicts the Qualitative results of RSA over 500 iterations which can be noticed the convergence curve has step-wise behaviour. Also, the trajectory starts by exploration the search space followed by changing the status of the solution to the exploitation phase to find the solution in the feasible region.

Table 21: Results of the comparative algorithms for solving the cantilever beam design problem.

Algorithm	**Optimal values for variables**					Optimal
	x_1	x_2	x_3	x_4	x_5	weight
GCA_I (Chickermane & Gea, 1996)	6.0100	5.30400	4.4900	3.4980	2.1500	1.3400
GCA_II (Chickermane & Gea, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
ALO (Mirjalili, 2015a)	6.01812	5.31142	4.48836	3.49751	2.158329	1.33995
CS (Gandomi et al., 2013)	6.0089	5.3049	4.5023	3.5077	2.1504	1.3399
SOS (Cheng & Prayogo, 2014)	6.01878	5.30344	4.49587	3.49896	2.15564	1.33996
MMA (Chickermane & Gea, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
RSA	6.0231	5.4457	4.277	3.5853	2.1767	1.3386

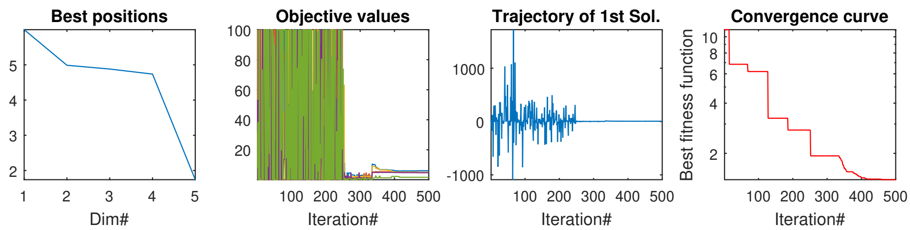


Figure 18: Qualitative results for the cantilever beam design problem.

3.4.7. Multiple disc clutch brake problem

The main objective of study the Multiple disc clutch brake (MDCB) problem is to find the value of five design variables to minimize the mass of the MDCB (Sadollah et al., 2013). The design variables are inner radius x_1, outer radius x_2, thickness of the disc x_3, actuating force x_4, and number of friction surfaces x_5

as depicted in Figure 19. The mathematical representation of this problem can be found below.

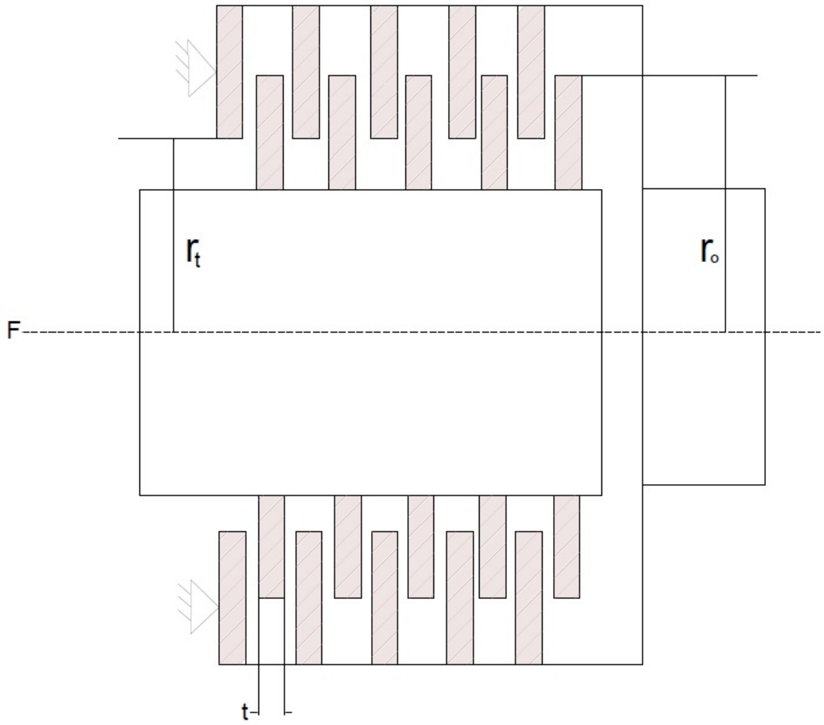


Figure 19: Multiple disc clutch brake problem.

$f(x) = \Pi(r_o^2 - r_i^2)t(Z+1)\rho$
$subject \quad to \quad g_1(x) = r_o - r_i - \Delta r \geq 0$
$g_2(x) = l_{max} - (Z+1)(t+\delta) \geq 0$
$g_3(x) = Pmax - Prz \geqslant 0$
$g_4(x) = P_{max}\ \nu_{sr\ max} - P_{rz}\ \nu_{sr} \geq 0$
$g_5(x) = \nu_{sr\ max} - \nu_{sr} \geq 0$
$g_6 = T_{max} - T \geq 0$
$g_7(x) = M_h - sM_s \geq 0$
$g_8(x) = T \geq 0$
$where$
$M_h = \frac{2}{3}\mu FZ\frac{r_o^3 - r_i^2}{r_o^2 - r_i^3}, P_{rz} = \frac{F}{\Pi(r_o^2 - r_i^2)},$
$\nu_{rz} = \frac{2\Pi(r_o^3 - r_i^3)}{90(r_o^2 - r_i^2)}, T = \frac{I_z\ \Pi\ n}{30(M_h + M_f)}$
$\Delta r = 20\ mm,\ I_z = 55\ kgmm^2,\ P_{max} = 1\ MPa,\ F_{max} = 1000\ N,$
$T_{max} = 15\ s,\ \mu = 0.5,\ s = 1.5,\ M_s = 40Nm,$
$M_f = 3Nm,\ n = 250\ rpm,$
$\nu_{sr\ max} = 10\ m/s,\ l_{max} = 30\ mm,\ r_{i\ min} = 60,$
$r_{i\ max} = 80, r_{o\ min} = 90,$
$r_{o\ max} = 110,\ t_{min} = 1.5, t_{max} = 3,\ F_{min} = 600,$
$F_{max} = 1000,\ Z_{min} = 2,\ Z_{max} = 9$

The results obtained by RSA and other MH techniques collected from literatures are given in Table 22. From this table, one can be seen that the optimal cost obtained by RSA (i.e., 0.31176) followed by WCA, MVO, CMVO, MFO which have the same performance (i.e., 0.313656).

The qualitative results of the MDCB problem are given in Figure 20. It can be noticed that the behaviour of RSA during the optimization process is similar to the previous problems except it takes more iterations to solve the MDCB problem which reached the optimal nearly after 450 iterations. Also, the trajectory of the fist solution shows the ability of RSA to balance between the exploration and exploitation of the optimization.

Table 22: Results of the comparative algorithms for solving the multiple disc clutch brake problem.

Algorithm	**Optimal values for variables**					Optimal weight
	x_1	x_2	x_3	x_4	x_5	
TLBO (Rao et al., 2011)	70	90	1	810	3	0.313656611
NSGA-II (Deb & Srinivasan, 2008)	70	90	1.5	1000	3	0.470400
WCA (Eskandar et al., 2012)	70	90	1	910	3	0.313656
MVO (Sayed et al., 2018)	70	90	1	910	3	0.313656
CMVO (Sayed et al., 2018)	70	90	1	910	3	0.313656
MFO (Bhesdadiya et al., 2018)	70	90	1	910	3	0.313656
RSA	70.0347	90.0349	1.0000	801.7285	2.9740	0.31176

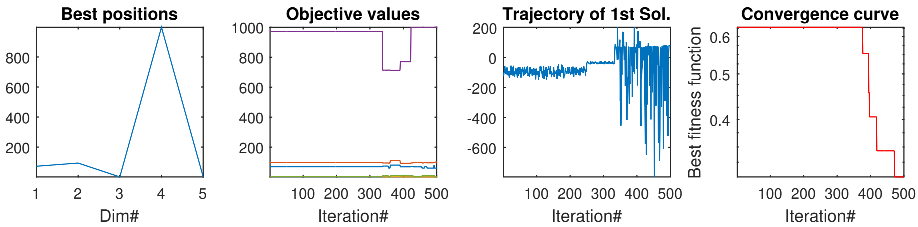


Figure 20: Qualitative results for the multiple disc clutch brake problem.

3.4.8. Rolling element bearing design (REBD) problem

In this section, we discuss the RSA method's performance to solve the rolling element bearing design optimization problem (as in Figure 21). This problem is considered as one of the most complex engineering problems, which have ten optimization variables. This problem's main target is to determine the value of these variables that will maximize the dynamic load-carrying capacity subject to nine constraints on assembly and geometric-based restrictions.

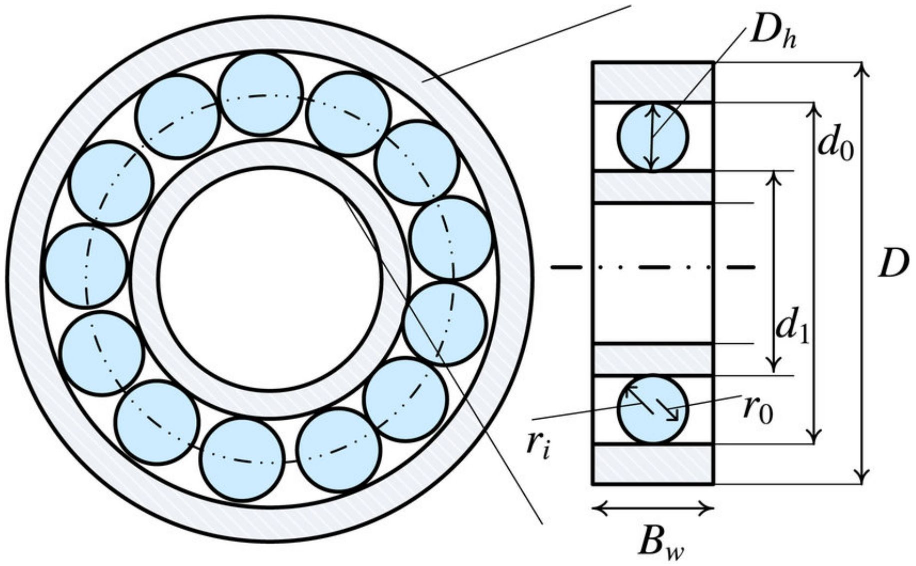


Figure 21: Rolling element bearing design.

The mathematics formulation of REBD problem can be given as:

$$
\begin{aligned}
&Maximize \quad && C_d = f_c U^{2/3} D_b^{1.8},\ if\, D \leq 25.4mm,\\
& && C_d = 3.647 f_c U^{2/3} D_b^{1.4},\ if\, D \leq 25.4mm,\\
&Subject\ to && g_1(\vec{c}) = \frac{\phi_0}{2sin^{-1}(D_b/D_m)} \leqslant 0,\\
& && g_2(\vec{u}) = 2D_b - K_{Dmin}(D-d) > 0,\\
& && g_3(\vec{u}) = K_{Dmax}(D-d) - 2D_b > 0,\\
& && g_4(\vec{u}) = \xi B_w - D_b \leqslant 0,\\
& && g_5(\vec{u}) = D_m - 0.5(D-d) \geq 0,\\
& && g_6(\vec{u}) = (0.5+e)(D+d) - D_m \geq 0,\\
& && g_7(\vec{u}) = 0.5(D - D_m - D_b) - \epsilon D_b \geq 0,\\
& && g_8(\vec{u}) = f_i \geq 0.515,\\
& && g_9(\vec{u}) = f_0 \geq 0.515,\\
&where && f_c = 37.91\left[1+\left\{1.64(\frac{1-\gamma}{1-\gamma})^{1.72}\left(\frac{f_i(2f_0-1)}{f_0(2f_i-1)}^{0.41}\right)\right\}^{10/3}\right]^{-0.3} \times \left[\frac{\gamma^{0.3}(1-\gamma)^{0.39}}{(1-\gamma)^{1/3}}\right]\left[\frac{2f_i}{2f_i-1}\right]^{0.41},\\
& && x = [\{(D-d)/2 - 3(T/4)\}^2 + \{D/2 - T/4 - D_b\}^2 - \{d/2 + T/4\}^2],\\
& && y = 2\{(D-d)/2 - 3(T/4)\}\{D/2 - T/4 - D_b\},\\
& && \phi_0 = 2\pi - cos^{-1}(\frac{u}{y}),\\
& && \gamma = \frac{D_b}{D_m}, f_i = \frac{r_i}{D_b}, f_0 = \frac{r_0}{D_b}, T = D - d - 2D_b, D = 160, d = 90,\\
& && B = 30, r_i = r_0 = 11.033, 0.5(D+d) \leqslant D_m \leqslant 0.6(D+d),\\
& && 0.15(D-d) \leqslant D_b \leqslant 0.45(D-d), 4 \leqslant Z \leqslant 50, 0.515 \leqslant f_i and f_o \leqslant 0.6,\\
& && 0.4 \leqslant K_{Dmin} \leqslant 0.5,\\
& && 0.6 \leqslant K_{Dmax} \leqslant 0.7, 0.3 \leqslant e \leqslant 0.4, 0.02 \leqslant e \leqslant 0.1, 0.6 \leqslant \xi \leqslant 0.85
\end{aligned}
\tag{11}
$$

The results of RSA are compared with other MH techniques including TLBO (Rao et al., 2011), CMVO-HHO (Ewees & Elaziz, 2020), MVO, PVs (Savsani & Savsani, 2016), HHO, LSHSPCM, LSHcEpS, and SaDE. Table 23 shows the comparison between the RSA and other methods. It can be observed that RSA has the largest cost value than other methods. It is followed by HHO, which allocates the second rank. The same observation can be noticed from Figure 22.

Table 23: Comparison results between RSA and other MH techniques to solve REBD problem.

Variable	HHO	TLBO	PVS	LSHSPCM	LSHcEpS	SaDE	RSA
D_m	125	125.7191	125.7191	125.4293	125.7227	125.7227	125.1722
D_b	21	21.42559	21.42559	20.7121	21.4233	21.4233	21.29734
Z	11.09207	11	11	9.8719	10.14	10.14	10.88521
f_i	0.515	0.515	0.515	0.5226	0.515	0.515	0.515253
f_o	0.515	0.515	0.515	0.5173	0.515	0.515	0.517764
K_dmin	0.4	0.424266	0.40043	0.4776	0.4565	0.4264	0.41245
K_dmax	0.6	0.633948	0.68016	0.6217	0.6494	0.6313	0.632338
ϵ	0.3	0.3	0.3	0.3041	0.3	0.3	0.301911
e	0.050474	0.068858	0.07999	0.0285	0.0246	0.0237	0.024395
ξ	0.6	0.799498	0.7	0.6041	0.6079	0.6432	0.6024
capacity	83011.88	81859.74	81859.74	63734.24	81014.88	81014.88	83486.64

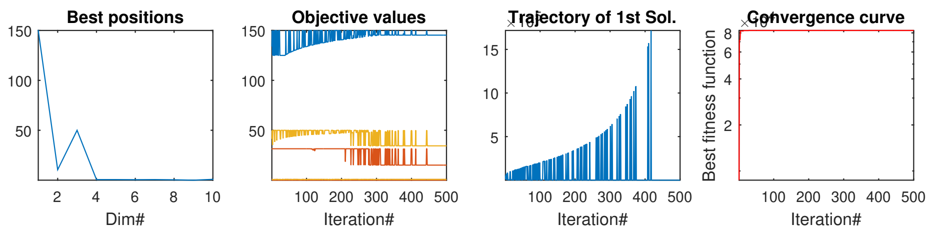


Figure 22: Qualitative results for the rolling element bearing design.

3.5. Feature selection application

Recently, feature selection becomes one of the most prepossessing steps in data analysis since it aims to reduce the high dimensional of the data by removing the irrelevant and redundancy features. Therefore, they have been applied in several fields including detection of cancer (Halvorsen et al., 2017), medical diagnosis (Tourassi et al., 2001), genomics (Allen et al., 2003), combinatorial chemistry (Osborne & Ellington, 1997), and computer vision (Zhang et al., 2014),

To tackling this FS problem, the main target of the FS optimization technique is to determine the relevant features with minimization of the classification error. The mathematical formulation of FS can be represented as a minimize optimization problem:

$$Fit_i = \lambda \times \gamma_i + (1 - \lambda) \times (\frac{|BX_i|}{Dim}) \tag{12}$$

In Equation (12), the classification error is represented by γ_i, and it is computed using the KNN classifier based on selected features from the training set. Dim denotes the total number of features, and $|BX_i|$ refers to the total number of features chosen which corresponding to ones in the binary solution BX_i and it is obtained using the following formula:

$$BX_{ij} = \begin{cases} 1 & if\ X_{ij} > 0.5 \\ 0 & otherwise \end{cases} \tag{13}$$

In addition, $\lambda \in [0, 1]$, in Equation (12), represents the weight parameter applied to balance between the two terms of Equation (12).

Table 24: Datasets description

Datasets	Number of features	Number of instances	Number of classes	Data category
Breastcancer	9	699	2	Biology
BreastEW	30	569	2	Biology
CongressEW	16	435	2	Politics
Exactly	13	1000	2	Biology
Exactly2	13	1000	2	Biology
HeartEW	13	270	2	Biology
IonosphereEW	34	351	2	Electromagnetic
KrvskpEW	36	3196	2	Game
Lymphography	18	148	2	Biology
M-of-n	13	1000	2	Biology
PenglungEW	325	73	2	Biology
SonarEW	60	208	2	Biology
SpectEW	22	267	2	Biology
Tic-tac-toc	9	958	2	Game
Vote	16	300	2	Politics
WaveformEW	40	5000	3	Physics
WineEW	13	178	3	Chemistry

3.5.1. Datasets descriptions

In this study, a set of sixteen UCI datasets is used to evaluate the applicability of the proposed method to tackling real-world applications. The properties of these datasets are given in Table 24, and they contain data collected from variant fields.

3.5.2. Results and Discussion

The comparison results of the RSA and other MH techniques are given in Tables 25-31 and Figure 23. The average and standard deviation of the classification accuracy are given in Table 25 and Table 26, respectively. It can be seen from these tables that the RSA has better accuracy at seven datasets, followed by MPA, which has high accuracy at four datasets. In addition, by analyzing the stability of the algorithms as given in Table 26, one can be observed that the RSA is more stable than others which has the smallest standard deviation at five datasets. In contrast, the MRFO has the smallest standard deviation at four datasets, followed by AEO and GA, which have the best stability at three datasets.

Moreover, Tables 27-30 show the performance of the algorithms in terms of fitness value measures (i.e., average, standard deviation, best and worst). From Table 27 it can be seen that the RSA can reach the smallest fitness value at five datasets, while both MPA and HHO have the smallest average fitness value at three datasets. In terms of standard deviation, one can be observed that most of the competitive algorithms nearly have the same stability in fitness value. For example, RSA, TLBO, and MPA have the best standard deviation at three datasets, followed by MRFO and GA since each of them has the best standard deviation at only one dataset. In addition, from Table 29, which shows the best fitness value obtained by each algorithm at each dataset, it can be concluded that the algorithm nearly has the same fitness value at some datasets such as Exactly and M-of-n, as well as the RSA and MPA, nearly have the same performance in terms of best fitness value. In terms of the worst fitness value obtained by each algorithm, as in Table 30, it can be observed that the worst fitness value obtained by RSA is better than other methods in six datasets, followed by MPA, which provides results better than other methods at three datasets.

Table 31 depicts the average number of selected features obtained by each algorithm over the tested datasets. From these results, it can be noticed that the RSA has the smallest number of features chosen at six datasets. In comparison, the WOA and TLBO have the best-selected features in the three datasets. In addition, the average of the selected features overall in the tested datasets is better than other methods. The same observation can be noticed from Figure 23 which depicts the average of each measure. These results indicate the behavior of RSA to balance between exploration and exploitation, and this leads to improving the convergence rate and the quality of output.

Table 25: Average of accuracy for each algorithm

Mean	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	**0.9843**	0.9557	0.9819	0.9619	0.9643	0.9752	0.9476	0.9567	0.9462	0.9790	0.9729
BreastEW	0.9737	**0.9807**	0.9749	0.9760	0.9731	0.9333	0.9433	0.9415	0.9351	0.9719	0.9573
CongressEW	0.9885	**0.9923**	0.9655	0.9716	0.9640	0.9854	0.9448	0.9157	0.9609	0.9594	0.9655
Exactly	**1.0000**	0.9990	**1.0000**	0.9993	0.9720	0.9743	0.8960	0.8987	0.8667	0.9587	**1.0000**
Exactly2	0.7750	0.7437	0.7357	0.7650	0.7450	0.7260	0.7703	**0.7900**	0.7130	0.7653	0.7507
HeartEW	0.8704	0.9049	0.8568	0.8654	**0.9062**	0.8914	0.7988	0.8272	0.8753	0.8975	0.8877
IonosphereEW	**0.9872**	0.9408	0.9624	0.9681	0.9268	0.9117	0.9174	0.9380	0.9577	0.9700	0.9859
KrvskpEW	**0.9844**	0.9720	0.9741	0.9779	0.9725	0.9495	0.9507	0.9577	0.9616	0.9655	0.9547
Lymphography	0.9533	0.9542	0.9660	0.9289	0.9133	0.9319	0.8821	0.8756	0.8844	**0.9685**	0.9337
M-of-n	**1.0000**	**1.0000**	0.9993	0.9990	0.9947	0.9853	0.9497	0.9627	0.9477	0.9637	0.9990
PenglungEW	0.9857	0.9378	**1.0000**	**1.0000**	0.9556	**1.0000**	0.9686	0.9822	0.8667	**1.0000**	0.9511
SonarEW	0.9524	0.9683	0.9476	0.9571	0.9571	0.9556	0.9825	0.9381	**0.9937**	0.9190	0.9794
SpectEW	**0.9333**	0.8790	0.8469	0.8506	0.9309	0.9148	0.7593	0.7716	0.8580	0.8099	0.8531
Tic-tac-toe	0.8229	**0.8556**	0.8375	0.8226	0.8177	0.8101	0.7809	0.7819	0.8250	0.7990	0.8354
Vote	**0.9833**	0.9622	0.9656	0.9778	0.9667	0.9444	0.9622	0.9700	0.9600	0.9567	0.9711
WaveformEW	0.7590	0.7589	**0.7764**	0.7665	0.7287	0.7283	0.7283	0.7186	0.7533	0.7381	0.7530

Table 26: Standard deviation of accuracy.

STD	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	0.0130	0.0055	0.0053	**0.0052**	0.0108	0.0076	0.0110	0.0184	0.0085	0.0083	0.0090
BreastEW	**0.0057**	0.0068	0.0114	0.0062	0.0062	0.0104	0.0178	0.0147	0.0065	0.0089	0.0144
CongressEW	**0.0000**	0.0071	0.0123	0.0150	0.0105	0.0053	0.0164	0.0207	0.0105	0.0122	0.0000
Exactly	0.0487	0.0387	**0.0000**	0.0018	0.0794	0.0314	0.1008	0.1064	0.0820	0.0406	**0.0000**
Exactly2	0.0018	0.0117	0.0080	**0.0000**	0.0000	0.0173	0.0181	0.0000	0.0151	0.0193	0.0026
HeartEW	0.0321	0.0196	0.0191	0.0204	0.0226	0.0327	0.0231	0.0249	**0.0085**	0.0251	0.0275
IonosphereEW	0.0263	0.0095	0.0182	0.0195	0.0170	0.0164	0.0219	0.0117	**0.0092**	0.0139	0.0106
KrvskpEW	0.0065	0.0050	0.0053	**0.0046**	0.0092	0.0123	0.0164	0.0116	0.0099	0.0115	0.0128
Lymphography	**0.0020**	0.0204	0.0181	0.0117	0.0246	0.0159	0.0520	0.0317	0.0278	0.0087	0.0253
M-of-n	0.0120	**0.0000**	0.0026	0.0028	0.0093	0.0194	0.0365	0.0408	0.0395	0.0350	0.0028
PenglungEW	0.0267	0.0172	**0.0000**	**0.0000**	0.0411	**0.0000**	0.0348	0.0305	0.0000	**0.0000**	0.0396
SonarEW	0.0110	0.0147	0.0184	0.0184	0.0161	0.0199	0.0168	0.0296	**0.0109**	0.0197	0.0218
SpectEW	**0.0110**	0.0138	0.0130	0.0434	0.0226	0.0195	0.0313	0.0218	0.0134	0.0191	0.0191
Tic-tac-toe	0.0044	**0.0037**	0.0129	0.0054	0.0136	0.0048	0.0200	0.0319	0.0267	0.0167	0.0038
Vote	**0.0051**	0.0076	0.0204	0.0059	0.0126	0.0133	0.0076	0.0157	0.0207	0.0123	0.0160
WaveformEW	0.0109	0.0094	**0.0067**	0.0131	0.0126	0.0151	0.0121	0.0153	0.0070	0.0069	0.0124

Table 27: Average of fitness value obtained by each algorithm

Mean	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	0.0555	0.0724	0.0578	0.0684	**0.0521**	0.0601	0.0738	0.0679	0.1018	0.0552	0.0824
BreastEW	0.0638	**0.0400**	0.0709	0.0456	0.0529	0.0911	0.0699	0.0809	0.1280	0.0777	0.0752
CongressEW	0.0373	0.0377	0.0681	0.0476	0.0495	0.0302	0.0738	0.1075	0.1018	0.0895	**0.0191**
Exactly	**0.0467**	0.0501	0.0492	0.0539	0.0770	0.0852	0.1598	0.1415	0.1923	0.1008	0.1103
Exactly2	0.2357	0.2615	0.2805	0.2192	0.2372	0.2902	0.2170	**0.1998**	0.3306	0.2743	0.2868
HeartEW	0.1457	0.1297	0.1915	0.1647	**0.1137**	0.1301	0.2160	0.2038	0.1958	0.1450	0.1733
IonosphereEW	**0.0352**	0.0809	0.0913	0.0525	0.0938	0.1057	0.0993	0.0817	0.1206	0.0831	0.0887
KrvskpEW	0.0784	**0.0656**	0.0856	0.0745	0.0777	0.0949	0.0971	0.0955	0.1148	0.0947	0.0953
Lymphography	0.0938	0.0919	0.0866	0.1318	0.1258	0.1009	0.1287	0.1564	0.1818	**0.0835**	0.1424
M-of-n	**0.0491**	0.0497	0.0514	0.0512	0.0622	0.0768	0.1176	0.0998	0.1179	0.0999	0.1181
PenglungEW	0.0546	0.0739	0.0394	**0.0150**	0.0514	0.0239	0.0408	0.0489	0.2022	0.0570	0.1308
SonarEW	**0.0571**	0.0777	0.1058	0.0835	0.0799	0.0832	0.0673	0.0965	0.0890	0.1317	0.1213
SpectEW	0.1568	0.1456	0.1926	0.1557	**0.1001**	0.1115	0.2336	0.2353	0.2047	0.2284	0.2058
Tic-tac-toe	0.2148	**0.1981**	0.2107	0.2315	0.2300	0.2228	0.2572	0.2548	0.2279	0.2461	0.2276
Vote	0.0568	0.0636	0.0689	0.0378	0.0604	0.0465	0.0457	0.0533	0.1052	0.0823	**0.0353**
WaveformEW	**0.2568**	0.2639	0.2766	0.2750	0.2905	0.2942	0.2996	0.3026	0.3075	0.3027	0.2969

Table 28: STD of fitness value obtained by each algorithm

	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	0.0073	**0.0013**	0.0054	0.0066	0.0034	0.0018	0.0113	0.0061	0.0120	0.0053	0.0045
BreastEW	0.0122	0.0068	0.0115	0.0068	**0.0053**	0.0058	0.0115	0.0114	0.0067	0.0081	0.0087
CongressEW	0.0056	0.0062	0.0140	0.0087	0.0051	0.0019	0.0153	0.0196	0.0119	0.0087	**0.0000**
Exactly	0.0521	0.0077	0.0049	0.0065	0.0647	0.0330	0.0963	0.0810	0.0823	0.0418	**0.0020**
Exactly2	0.0073	0.0236	0.0223	**0.0000**	0.0000	0.0417	0.0262	0.0049	0.0164	0.0164	0.0116
HeartEW	0.0143	0.0152	0.0188	0.0277	0.0378	0.0182	0.0225	0.0319	**0.0105**	0.0240	0.0202
IonosphereEW	0.0140	0.0110	0.0239	0.0165	0.0174	0.0165	0.0177	0.0094	0.0091	0.0174	**0.0079**
KrvskpEW	0.0119	0.0080	0.0079	**0.0050**	0.0107	0.0079	0.0150	0.0141	0.0106	0.0092	0.0077
Lymphography	**0.0070**	0.0216	0.0187	0.0075	0.0278	0.0134	0.0597	0.0255	0.0271	0.0132	0.0297
M-of-n	0.0565	**0.0040**	0.0066	0.0069	0.0141	0.0236	0.0410	0.0416	0.0425	0.0384	0.0069
PenglungEW	0.0399	0.0148	0.0146	0.0075	0.0405	0.0116	0.0314	0.0266	**0.0023**	0.0025	0.0357
SonarEW	**0.0097**	0.0164	0.0251	0.0139	0.0142	0.0170	0.0128	0.0202	0.0106	0.0168	0.0184
SpectEW	0.0155	0.0097	0.0108	0.0223	0.0181	0.0152	**0.0072**	0.0177	0.0119	0.0158	0.0163
Tic-tac-toe	0.0084	**0.0006**	0.0070	0.0009	0.0109	0.0093	0.0180	0.0177	0.0231	0.0188	0.0035
Vote	**0.0014**	0.0070	0.0259	0.0073	0.0087	0.0053	0.0168	0.0201	0.0232	0.0173	0.0147
WaveformEW	0.0080	0.0135	**0.0067**	0.0111	0.0096	0.0103	0.0174	0.0115	0.0085	0.0102	0.0121

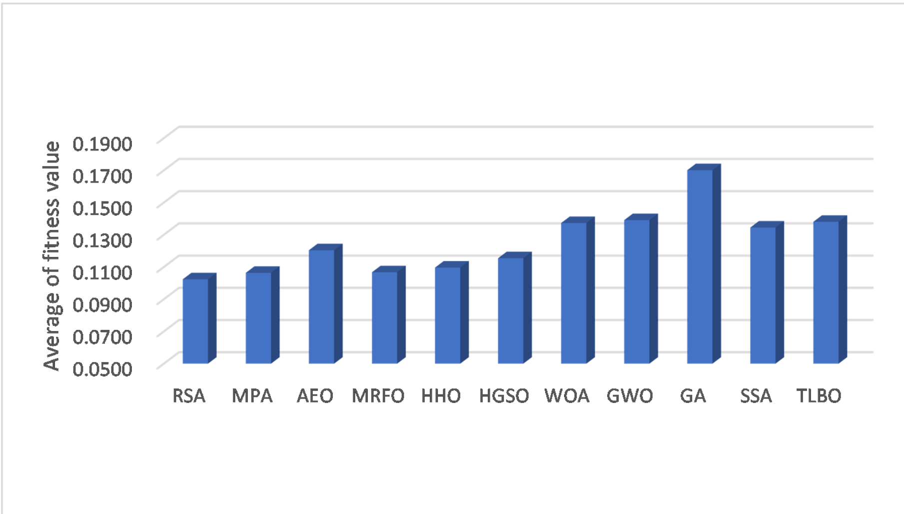
Table 29: Best of fitness value obtained by each algorithm

	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	0.0526	0.0719	0.0526	0.0655	0.0497	0.0590	0.0590	0.0573	0.0830	**0.0462**	0.0766
BreastEW	0.0458	**0.0291**	0.0512	0.0325	0.0437	0.0786	0.0491	0.0607	0.1107	0.0646	0.0595
CongressEW	0.0373	0.0313	0.0394	0.0394	0.0435	0.0291	0.0560	0.0664	0.0769	0.0769	**0.0166**
Exactly	**0.0462**	**0.0462**	**0.0462**	**0.0462**	**0.0462**	0.0538	**0.0462**	**0.0462**	0.0615	0.0538	0.0538
Exactly2	0.2327	0.2372	0.2507	0.2192	0.2372	0.2417	0.2102	**0.1967**	0.3032	0.2505	0.2794
HeartEW	0.1205	0.1128	0.1628	0.1231	**0.0744**	0.1064	0.1731	0.1628	0.1692	0.1128	0.1551
IonosphereEW	**0.0176**	0.0616	0.0518	0.0274	0.0595	0.0654	0.0742	0.0674	0.1048	0.0441	0.0751
KrvskpEW	0.0645	**0.0503**	0.0641	0.0642	0.0572	0.0809	0.0660	0.0683	0.1003	0.0822	0.0758
Lymphography	0.0556	0.0699	0.0500	0.1156	0.0989	0.0624	0.0471	0.1065	0.1322	**0.0444**	0.1322
M-of-n	**0.0462**	**0.0462**	**0.0462**	**0.0462**	**0.0462**	0.0538	0.0615	0.0538	0.0615	0.0538	0.0538
PenglungEW	0.0071	0.0222	0.0108	0.0052	0.0037	0.0108	**0.0031**	0.0203	0.1982	0.0532	0.0760
SonarEW	**0.0300**	0.0467	0.0631	0.0598	0.0614	0.0564	0.0481	0.0648	0.0750	0.1045	0.1057
SpectEW	0.1394	0.1242	0.1712	0.0985	**0.0652**	0.0894	0.2182	0.1939	0.1773	0.2045	0.1818
Tic-tac-toe	0.2120	**0.1979**	0.2073	0.2307	0.2214	0.2179	0.2354	0.2307	0.2120	0.2290	0.2196
Vote	0.0275	0.0550	0.0338	**0.0250**	0.0488	0.0375	0.0363	0.0338	0.0625	0.0525	0.0338
WaveformEW	**0.2354**	0.2380	0.2660	0.2482	0.2779	0.2758	0.2730	0.2847	0.2951	0.2843	0.2877

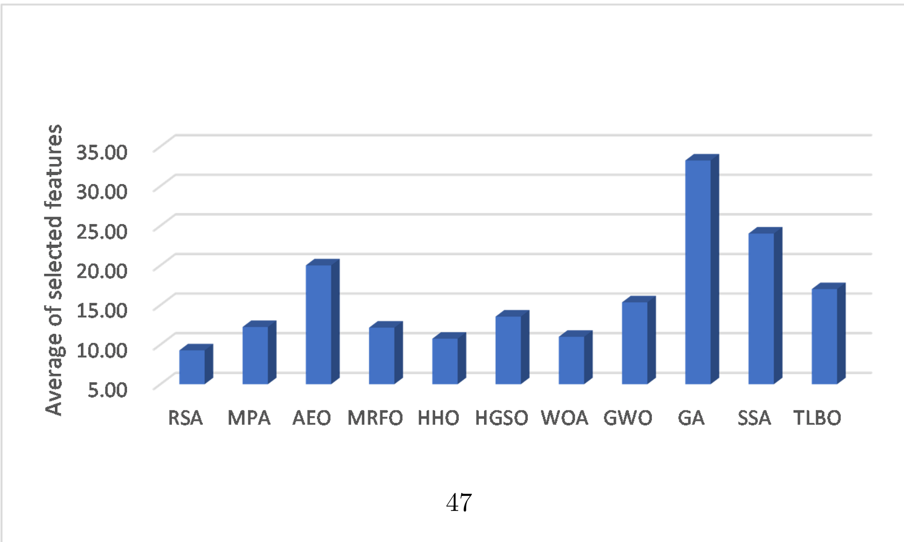
Table 30: Worst of fitness value obtained by each algorithm

	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	0.0655	0.0766	0.0702	0.0894	**0.0590**	0.0637	0.0976	0.0818	0.1228	0.0637	0.0941
BreastEW	0.0819	**0.0537**	0.0995	0.0604	0.0616	0.1023	0.0856	0.1011	0.1365	0.0882	0.0907
CongressEW	0.0373	0.0478	0.0873	0.0769	0.0623	0.0332	0.1037	0.1431	0.1330	0.1080	**0.0291**
Exactly	**0.0538**	0.0750	0.0615	0.0660	0.3021	0.1547	0.2822	0.2372	0.3096	0.1727	0.1894
Exactly2	0.2775	0.2950	0.3032	0.2192	0.2372	0.3334	0.3117	**0.2121**	0.3559	0.3000	0.2961
HeartEW	0.1731	**0.1551**	0.2269	0.2115	0.1628	0.1577	0.2513	0.2692	0.2090	0.2026	0.1910
IonosphereEW	**0.0489**	0.1075	0.1428	0.0851	0.1270	0.1299	0.1279	0.0996	0.1389	0.1057	0.1093
KrvskpEW	0.0894	**0.0755**	0.0975	0.0821	0.1033	0.1051	0.1160	0.1215	0.1340	0.1089	0.1049
Lymphography	0.1254	0.1289	0.1232	0.1456	0.2089	0.1198	0.2467	0.2052	0.2278	**0.1022**	0.1489
M-of-n	**0.0538**	0.0705	0.0705	0.0705	0.0917	0.1219	0.1836	0.1785	0.2106	0.1868	0.1804
PenglungEW	0.1348	0.0825	0.0594	**0.0338**	0.1375	0.0542	0.0852	0.0905	0.2065	0.0618	0.1843
SonarEW	**0.0862**	0.1029	0.1376	0.1076	0.1126	0.1110	0.0867	0.1243	0.1081	0.1621	0.1295
SpectEW	0.1939	0.1652	0.2091	0.1712	**0.1288**	0.1439	0.2379	0.2652	0.2227	0.2591	0.2212
Tic-tac-toe	**0.1997**	0.2214	0.2243	0.2325	0.2635	0.2418	0.2917	0.2924	0.2793	0.2964	0.2418
Vote	0.0738	0.0763	0.1250	0.0500	0.0800	0.0550	0.0950	0.0975	0.1438	0.1100	**0.0363**
WaveformEW	**0.2741**	0.2882	0.2858	0.2942	0.3114	0.3152	0.3229	0.3215	0.3215	0.3157	0.3062

(a) Accuracy



(b) Fitness



(c) Selected features

Figure 23: Comparisons among the results by the RSA and other in terms of average of (a) Accuracy, (b) Fitness value, and (c) Selected features.

Table 31: Average of selected features by each algorithm

	RSA	MPA	AEO	MRFO	HHO	HGSO	WOA	GWO	GA	SSA	TLBO
Breastcancer	3.60	2.93	3.73	3.07	2.80	3.40	**2.40**	2.60	4.80	3.27	3.00
BreastEW	11.20	6.80	14.47	7.20	8.60	9.33	**5.67**	8.47	20.87	15.73	10.27
CongressEW	**1.40**	4.93	5.93	3.53	2.73	2.73	3.87	5.07	10.67	8.47	5.93
Exactly	8.60	6.40	6.40	6.93	6.73	8.07	8.60	6.53	9.40	8.27	**6.20**
Exactly2	9.20	4.00	5.53	**3.00**	**3.00**	5.67	3.33	3.40	9.40	8.20	**3.00**
HeartEW	5.20	5.73	8.13	5.67	**3.80**	4.20	4.53	6.27	10.87	6.87	5.47
IonosphereEW	8.60	9.40	19.53	**8.07**	9.47	8.93	8.47	8.80	28.07	19.07	11.00
KrvskpEW	**10.80**	14.53	22.40	19.67	19.07	17.80	19.00	20.67	28.87	22.93	16.20
Lymphography	11.60	9.13	10.07	12.20	8.60	7.13	**4.07**	8.00	14.00	9.93	7.40
M-of-n	8.80	6.47	6.60	6.53	7.47	8.27	9.40	8.60	9.20	8.73	**6.13**
PenglungEW	**19.60**	58.13	128.00	48.67	37.20	77.73	40.53	106.80	267.27	185.33	134.60
SonarEW	**18.80**	29.47	35.20	26.93	24.80	25.93	30.93	24.47	50.00	35.33	28.67
SpectEW	6.40	8.07	12.07	4.67	8.33	7.67	**3.73**	6.53	16.93	12.60	7.87
Tic-tac-toe	5.80	6.13	5.80	6.47	5.93	**4.67**	5.40	5.27	6.33	5.87	5.00
Vote	**3.80**	4.73	6.07	5.73	4.87	5.20	3.87	4.20	11.07	6.93	4.07
WaveformEW	**15.00**	18.73	30.13	25.93	18.53	19.87	22.00	19.73	34.20	26.80	17.33

3.6. The pros and cons of the proposed RSA

From the above analysis, it is observed the high capability of RSA to handle variant problems of engineering design with different constraints, and it can obtain the optimal solution for those problems. So, it can be concluded that the RSA is a powerful, effective, and attractive meta-heuristic algorithm to solve a global optimization problems with/without constraints. The main advantages of the proposed RSA are ease of implementation, finding new best solutions for the tested problems, fast response, smooth converge curves, better-targeted delivery of new solutions, a few control parameters, etc. Also, some minor weaknesses can be handled, such as the miss self-learning mechanism, the effect of the fitness values on the updating solutions' mechanisms, and a new transition method between the search methods.

4. Conclusion and potential future works

This paper presented a novel natural-inspired meta-heuristic called Reptile Search Algorithm (RSA), which is motivated by the Crocodile hunting behaviour. Two main steps of Crocodile behaviour are implemented, i.e., encircling, which is performed by high walking or belly walking, and hunting, which is performed by hunting coordination or hunting cooperation. Thus, two mathematical models were introduced to update the positions of candidate solutions; one for diverse search and another toward the optimal search region.

In this paper, four metrics (i.e., search history, average fitness function, the trajectory of the first dimension, and convergence curve) are implemented to investigate the proposed RSA qualitatively. Next, the RSA is proposed for solving a set of various optimization problems belonging to classical, CEC-2017, and CEC-2019 benchmark test functions to investigate the exploration, exploitation, local optima escape, and convergence performance. The results demonstrated the effectiveness of RSA towards achieving optimal global solutions having more reliable convergence compared to other well-known optimization algorithms published in the literature. Freidman ranking test is applied to evaluate the efficacy of the proposed RSA scientifically. The statistical results demonstrated that the RSA can guarantee the effectiveness of explorations while producing excellent exploitation, hence maintaining an equilibrium between exploitation and explorations strategies, which reveals the superior performance of the RSA in a statistical sense against other comparative algorithms. Moreover, seven real-world engineering problems are used to investigate the effectiveness of the proposed RSA further. The results of the engineering design problems proved that the RSA achieved extremely better results against the other well-known optimization algorithms, and it can handle various constraints problems.

GOA tackled single-objective optimization problems with contentious variables. Several research directions can be recommended for future works. Binary, modify, and multi-objective variants of the RSA may be extended to tackle various discrete, multi-objective, and many-objective real-life optimization problems. As well, tackling problems in different fields (i.e., neural network, image processing, feature selection, scheduling, text and data mining, big data, signal denoising, smart home, networking, industry, etc.) and tuning the controlling parameters of RSA could be a valuable contribution and beneficial.

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