



ENGR 101 – Lecture 5

Logical Operations and Indexing

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Lecture Goals

- **Previous lecture: Functions and Vectorization**
 - Reducing code duplication
 - Functions
 - Vectorization
 - Project 1 Overview
 - **Suggested readings, Attaway, Chap 3.7 and 6.1-6.2**
- **Today's lecture: Logic Operations and Logical Indexing**
 - Relational operators (gt, lt, not eq, ...)
 - Logical operators (and, or, not)
 - Final MATLAB precedence table
 - Indexing matrices with logical arrays
 - **Suggested readings, Attaway, Chap 1.6 and 2.5**

Recall: Binary Arithmetic Operations

➤ Essentially, doing math with two operands.

	Operator	Function	Example	Result
Addition	+	plus	2 + 3	5
Subtraction	-	minus	5 - 3	2
Multiplication	.*	times	5 .* 3	15
Exponentiation	.^	power	2 .^ 3	8
Division	./	rdivide	11 ./ 4	2.75
Modulo		mod	mod(11,4)	3

Asking Questions

- Consider the following expression:

$$x < 5$$

- ~~In math, this would mean that x must be less than 5.~~
- In programming, this is a **question**.
 - "Check the current value of x... Is it less than 5?"
 - The result is called a **truth value**, and is either false or true.
 - False and true are encoded as 0 and 1, eg. `disp(3 < 5) = 1`

false	true
0	1

Relational Operations

➤ These operations check for **equality** or perform **comparisons**.

	Operator	Function	Example	Result
Equality	<code>==</code>	<code>eq</code>	<code>2 == 3</code>	<code>0</code>
Inequality	<code>~=</code>	<code>ne</code>	<code>2 ~= 3</code>	<code>1</code>
Less Than	<code><</code>	<code>lt</code>	<code>5 < 5</code>	<code>0</code>
Less Than or Equal	<code><=</code>	<code>le</code>	<code>5 <= 5</code>	<code>1</code>
Greater Than	<code>></code>	<code>gt</code>	<code>gt(3, 4)</code>	<code>0</code>
Greater Than or Equal	<code>>=</code>	<code>ge</code>	<code>ge(4, 4)</code>	<code>1</code>

Relational Operations on Matrices

In MATLAB, anything that you can do with scalars, you can do with arrays (i.e. vectors and matrices).

The image shows a MATLAB interface with several variables and their values:

- ans**: A 2x3 matrix with values $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
- a**: A 2x3 matrix with values $\begin{bmatrix} 3 & 4 & 2 \\ 6 & 5 & 1 \end{bmatrix}$.
- b**: A 2x3 matrix with values $\begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 7 \end{bmatrix}$.
- c**: A 1x1 matrix with value 1.
- d**: A 2x2 matrix with values $\begin{bmatrix} 1 & 7 \\ 2 & 5 \end{bmatrix}$.

The command window shows the operation `a < b` and its result:

`a < b`

$\begin{bmatrix} 3 & 4 & 2 \\ 6 & 5 & 1 \end{bmatrix} < \begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 7 \end{bmatrix}$

ans = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

A box contains the following MATLAB expressions:

```
b == c
b = c

~d
~(~d)
```

Source: <https://goo.gl/FLGU2D> Web Viewer Terms | Privacy & Cookies Edit

Logical Operations

- We can use **logical operators** to combine two truth values according to the rules of **formal logic**.
- For example, the & operator implements **logical and**.
 - If BOTH operands are true, the whole result is true. Otherwise false.

<https://goo.gl/syLgsb>

(5<6)&(4==3)
(5<6)&(4==4)

Inclusive vs. Exclusive Or

➤ There are two kinds of "or"...

➤ Inclusive Or

- One or the other, or both.
- "Were you in sports or music in high school?"

or

➤ Exclusive Or

- One or the other, but NOT both.
- "Would you like the soup or the salad?"

xor

Logical Operations

- Essentially, combining two **truth values** in a particular way.
- Most operations have both a symbolic operator and a function.

	Operator	Function	Example	Result
Logical And	&	and	2 < 3 & 5 > 6	0
Logical Or		or	2 < 3 5 > 6	1
Exclusive Or		xor	xor(0,1)	1
Not	~	not	~(1 == 2)	1

The precedence of ~ is higher than ==, so we need parentheses here.

- Order of precedence
 1. (highest) parentheses
 2. exponentiation
 3. **negation**
 4. multiplication, division
 5. addition, subtraction
 6. colon operator (:))
 7. **Relational operators**
 8. **Logical AND**
 9. **Logical OR**
 10. (lowest) assignment (=)

*if two or more operations have the same precedence,
the expression is executed from left to right*



4
min

Your turn: Logical Operator Truth Table

- The formal way to define the logical operators is with a truth table that shows the result for all combinations of operands.
- Fill in the missing pieces of the table...

A	B	$\sim A$	$\sim B$	$A \& B$	$A B$	$\text{xor}(A, B)$
0	0			0	0	
0	1	1				
1	0					
1	1		0			0

Solution: Logical Operator Truth Table

- A formal way to define the logical operators is with a truth table that shows the result for all combinations of operands.
- Fill in the missing pieces of the table...

A	B	$\sim A$	$\sim B$	A & B	A B	xor(A, B)
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	1	0

Logical Operations with Matrices

- Of course, we can perform these same operations with matrices as well as scalars.

<https://goo.gl/SgvMKg>

```
a == b + 1  
a(:,2) == b(:,3)
```

Careful!

10

x

- Let's say we wanted to check whether a number is in a particular interval. It's tempting to try something like this:

$$1 < x < 5$$

MATLAB claims for
x=10 this is true.

- MATLAB sees this as:

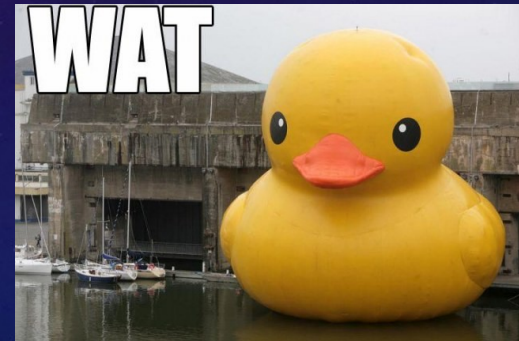
$$(1 < x) < 5$$

$$(1 < 10) < 5$$

1 < 10 is true, so
it evaluates to 1.

$$(1) < 5$$

1



- Instead, use & to specify the lower and upper bounds separately:

$$1 < x \& x < 5$$



5
min

Your turn: Logical Operations

goo.gl/5AQbNV

➤ Given these variables:

1	2	4
3	3	2

a

0	2	9
3	3	6

b

2

c

➤ Find the result of the following five expressions:

$$a \sim= b$$

$$b - a == c + 3$$

$$a > c$$

$$a(2,:) == 3 .* [1,1,1]$$

$$a + 1 > 3 \& b > 2$$

Solution: Logical Operations

➤ Given these variables:

1	2	4
3	3	2

a

0	2	9
3	3	6

b

2

c

➤ Find the result of the following expressions:

$a \sim= b$

1	0	1
0	0	1

$b - a == c + 3$

0	0	1
0	0	0

$a > c$

0	0	1
1	1	0

$a(2,:) == 3 .* [1,1,1]$

1	1	0
---	---	---

$a + 1 > 3 \& b > 2$

0	0	1
1	1	0

Logical Arrays

- The result of a relational or logical operation is a **logical array**.
- The 1s and 0s in a logical array are a completely different type of data than regular numbers.
- In many contexts, an expression will behave differently with logical vs. regular numbers.
 - Indexing is an important example.

```
>> x = [1,2;3,4]
```

```
x =
```

```
    1    2  
    3    4
```

```
>> which = x > 2
```

```
which =
```

```
    0    0  
    1    1
```

```
>> whos which
```

```
2x2 logical array
```


Recall: Matrix Indexing

- Each element has a sequential index.
- When we put a **[regular array]** between the `()` in an indexing expression, we select those indices.

1 2	3 8	5 2
2 0	4 4	6 3

x

ans

2	0	4
---	---	---

x

2	8	2
0	4	3

x([1,2,4])

1 2	3 8	5 2
2 0	4 4	6 3

x

ans =

2	0	4
---	---	---

Logical Indexing

- We can also use a **logical array** to index.
- In this case, we select all the elements from the source matrix that correspond to positions with a 1 in the logical matrix.

1 2	3 8	5 2
2 0	4 4	6 3

X

x 2	8	2
0	4	3

```
a = x
which = x < 3
x = x(which)
x = a
x = x(x < 3)
```

A few notes

- If you want to hardcode a logical matrix, you need to wrap it up in the **logical** function (otherwise it's just regular numeric 1s and 0s).

`[1,0;0,1]`

1	0
0	1

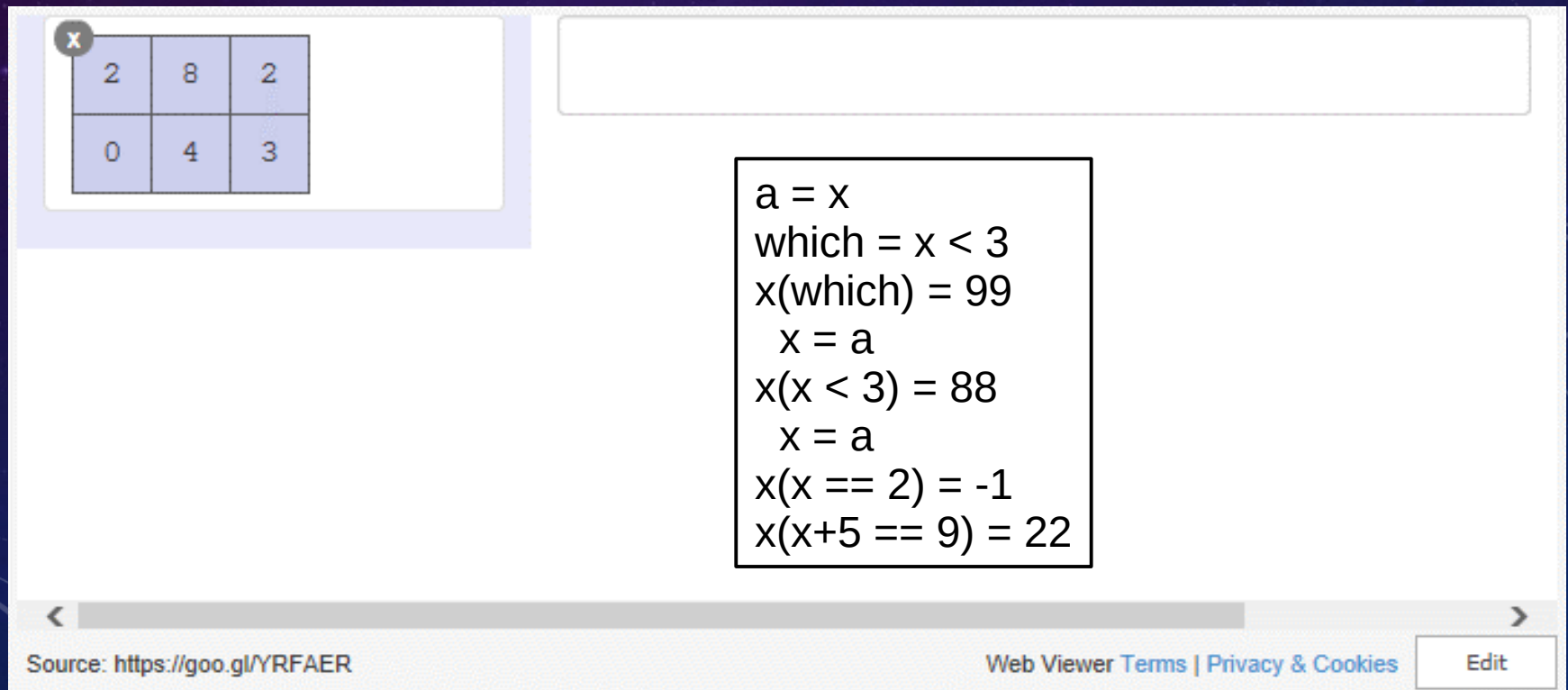
`logical([1,0;0,1])`

1	0
0	1

- You can use “whos” in the command window to check the data types

Writing Into a Logically Indexed Matrix

- One can assign elements in a logically indexed expression.
- The RHS must either be a scalar or have the same number of elements as there are 1s in the logical index matrix.



The image shows a MATLAB interface. On the left, a variable **x** is displayed as a 2x3 matrix:

2	8	2
0	4	3

On the right, a script editor contains the following code:

```
a = x
which = x < 3
x(which) = 99
x = a
x(x < 3) = 88
x = a
x(x == 2) = -1
x(x+5 == 9) = 22
```

At the bottom, the source is listed as <https://goo.gl/YRFAER>, and there are links for [Web Viewer Terms](#), [Privacy & Cookies](#), and an [Edit](#) button.

Logical Indexing with Parallel Matrices

- You can use a logical matrix derived from a different matrix than the source into which you are indexing.
- The two "parallel" matrices should be the same size.

a

1	2	10
3	3	10

b

0	2	9
3	3	6

c

2

$a(b > 5) = 10$

0	1	0	2	1	4
0	3	0	3	1	2

=

10

a

a is now

1	2	10
3	3	10

$a(b > 5) = 10$
 $a(a \sim= b) = c$

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your turn: Logical Indexing

5
min

➤ Write code for each of the following:

goo.gl/VEfrMK

1. Double all elements in `a` that are between 5 and 10 (inclusive).
2. Set all matrix elements in `b` to their absolute value.

<https://goo.gl/VEfrMK>

Solution: Logical Indexing

➤ Write code for each of the following:

Double all elements in `a` that are between 5 and 10 (inclusive).

```
a(5 <= a & a <= 10) = 2 .* a(5 <= a & a <= 10)
```

Set all matrix elements in `b` to their absolute value.

```
b(b < 0) = -1 .* b(b < 0)
```



I will stay for about one hour
to answer questions (Project 1). Will be
behind Stamps Auditorium

See you next time !!!