

13. Sea la matriz $N = \begin{pmatrix} 5-k & 0 & -2 \\ 4 & -k-1 & 3 \\ 2 & 0 & 1-k \end{pmatrix}$

b) Determinar los valores de k para que N resulte una matriz singular

$$|N|=0 \rightarrow \text{Matriz Singular}$$

$$\begin{vmatrix} 5-k & 0 & -2 \\ 4 & -k-1 & 3 \\ 2 & 0 & 1-k \end{vmatrix} = (5-k) \cdot \begin{vmatrix} -k-1 & 3 \\ 0 & 1-k \end{vmatrix} - 2 \cdot \begin{vmatrix} 4 & -k-1 \\ 2 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 5-k & 0 & -2 \\ 4 & -k-1 & 3 \\ 2 & 0 & 1-k \end{vmatrix} = (5-k) \cdot [(-1-k)(1-k)] - 2 \cdot (-2k-2) = (5-k) \cdot (k^2-1) - 4k-4 = 0$$

$$5k^2 - 5 - k^3 + k - 4k - 4 = -k^3 + 5k^2 - 3k - 9 = 0$$

$$\text{Si } k_1 = -1 \rightarrow -k^3 + 5k^2 - 3k - 9 = 0 ; \text{ Por Ruffini } \rightarrow -k^2 + 6k - 9 = 0$$

$$k^2 - 6k + 9 = 0 \rightarrow K_{23} = \frac{6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 9}}{2} = 3$$

$$\text{Si } k_1 = -1 \text{ o } k_2 = 3 \text{ o } k_3 = 3 \rightarrow N \text{ es singular}$$

c) Calcular la inversa de N para $k=2$

$$N = \begin{pmatrix} 3 & 0 & -2 \\ 4 & -3 & 3 \\ 2 & 0 & -1 \end{pmatrix}$$

$$N = \begin{pmatrix} 3 & 0 & -2 \\ 4 & -3 & 3 \\ 2 & 0 & -1 \end{pmatrix} ; \quad AdjN = (Cof N)^t = \begin{bmatrix} 3 & 10 & 6 \\ 0 & 1 & 0 \\ 6 & -17 & -9 \end{bmatrix}^t = \begin{vmatrix} 3 & 0 & 6 \\ 10 & 1 & -17 \\ 6 & 0 & -9 \end{vmatrix}$$

$$|N| = (9-12) = -3$$

$$N^{-1} = \frac{AdjN}{|N|} = \frac{\begin{vmatrix} 3 & 0 & 6 \\ 10 & 1 & -17 \\ 6 & 0 & -9 \end{vmatrix}}{-3} = \begin{bmatrix} -1 & 0 & -2 \\ -10/3 & -1/3 & 17/3 \\ -2 & 0 & 3 \end{bmatrix}$$

