

### TP 3 Complejos

#### Ejercicio 4. Dados los números complejos:

	Cálculos auxiliares	
$z_1 = \sqrt{2} \left( \cos \frac{7\pi}{4} + \operatorname{sen} \frac{7\pi}{4} \right) = \sqrt{2}_{315^\circ}$		$\frac{7\pi}{4} = \frac{7\pi}{4} \frac{180^\circ}{\pi} = 315^\circ$
$z_2 = 3\sqrt{2}_{135^\circ}$		
$z_3 = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i = 3_{240^\circ}$	$\rho = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} = 3$	$\varphi = \operatorname{arc\,tg} \frac{-\frac{3\sqrt{3}}{2}}{-\frac{3}{2}} = 60^\circ + 180^\circ = 240^\circ$
$z_4 = -4 + 4\sqrt{3}i = 8_{120^\circ}$	$\rho = \sqrt{(-4)^2 + (4\sqrt{3})^2} = 8$	$\varphi = \operatorname{arc\,tg} \frac{4\sqrt{3}}{-4} = -60^\circ + 180^\circ = 120^\circ$
$z_5 = 2 e^{\frac{\pi i}{4}} = 2_{90^\circ}$		
$z_6 = e^{\frac{5\pi i}{4}} = 1_{225^\circ}$		$\frac{5\pi}{4} = \frac{5\pi}{4} \frac{180^\circ}{\pi} = 225^\circ$

ii. Efectuar las operaciones que se indican en notación polar cuando sea posible

$$\begin{aligned}
 \text{d) } (z_3 \cdot z_1)^3 + \frac{z_5^8}{z_6^{-6}} &= (3_{240^\circ} \sqrt{2}_{315^\circ})^3 + (2_{45^\circ})^8 (1_{225^\circ})^6 = (3\sqrt{2}_{555^\circ})^3 + 2^8_{8.45^\circ+6.225^\circ} = 3^3 2\sqrt{2}_{1665^\circ} + 2^8_{1710^\circ} = \\
 &= 3^3 2\sqrt{2}_{225^\circ} + 2^8_{270^\circ} = 27.2\sqrt{2} \left( -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right) + 2^8(-i) = -54\sqrt{2} \frac{\sqrt{2}}{2} (1+i) + 2^8(-i) = -54 - 54i - 256i = \\
 &= \mathbf{-54 - 310i}
 \end{aligned}$$

iii. Efectuar las operaciones que se indican en notación exponencial cuando sea posible

$$\begin{aligned}
 \text{d) } \frac{z_4^{-2}}{z_2^{-4}} &= \\
 \text{d) } \frac{z_4^{-2}}{z_2^{-4}} &= \frac{\left[ \left( 8.e^{\frac{2\pi i}{3}} \right)^{-1} \right]^2}{\left[ \left( 3\sqrt{2}.e^{\frac{3\pi i}{4}} \right)^{-1} \right]^4} = \frac{\left[ \frac{1}{8}e^{\frac{-2\pi i}{3}} \right]^2}{\left[ \frac{1}{3\sqrt{2}}e^{\frac{-3\pi i}{4}} \right]^4} = \frac{\frac{1}{64}e^{\frac{-4\pi i}{3}}}{\frac{1}{81.4}e^{\frac{-12\pi i}{4}}} = \frac{\frac{1}{64}e^{\frac{12\pi i}{4}}}{\frac{1}{324}e^{\frac{4\pi i}{3}}} = \frac{324}{64}e^{\frac{12\pi i}{4} - \frac{4\pi i}{3}} = \frac{81}{16}e^{\frac{5\pi i}{3}} = \frac{\left[ 3\sqrt{2}.e^{\frac{3\pi i}{4}} \right]^4}{\left[ 8.e^{\frac{2\pi i}{3}} \right]^2} = \\
 &= \frac{3^4 4.e^{3\pi i}}{8^2.e^{\frac{4\pi i}{3}}} = \frac{81}{16}e^{\frac{5\pi i}{3}}
 \end{aligned}$$