Understanding Two-Dimensional Diffusion with Numerical Modelling

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Numerical Methods in Geophysics (OCEA0081-A-a)

Problem Statement

- in a domain $x \in [0,L]$, $y \in [0,L]$; $L=1000$ m, $\kappa=1$ m^2/s

- Initial condition: patch of pollution
  - $C(x, y, t=0) = \begin{cases} 10, & \text{if } (x-L/4)^2 + (y-L/4)^2 < (L/4)^2 \\ 0, & \text{elsewhere} \end{cases}$

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa \frac{\partial C}{\partial y}\right)$$

- Boundary condition in y : $\partial C/\partial y = 0$ in $y = 0, L$

- Boundary condition in x : $\partial C/\partial x = 0$ in $x = 0, L$

What is Diffusion?

- A physical process where substances spread from high to low concentration.
- Governed by the diffusion equation: $\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa \frac{\partial C}{\partial y}\right)$, where κ is a diffusion coefficient
- We used—explicit and implicit schemes—to model the diffusion of a pollutant patch in a closed system.
- Julia programming — efficient simulations & Neumann boundary conditions— mass conservation.
- Model provides insights—pollutant dynamics & numerical scheme trade-offs.

Numerical Techniques Utilized

Numerical method used was a ***Finite Difference Method (FDM)***

- Uses an implicit & explicit scheme.
- Explicit scheme depends on Courier-Friedrich-Lewy condition (CFL) for stability

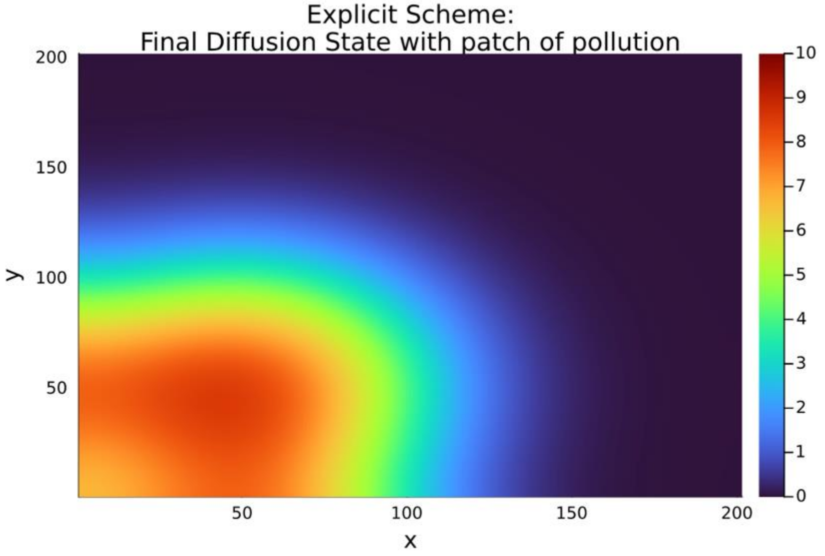
$$\boxed{\Delta t \leq \frac{\min(\Delta x^2, \Delta y^2)}{4D}}$$

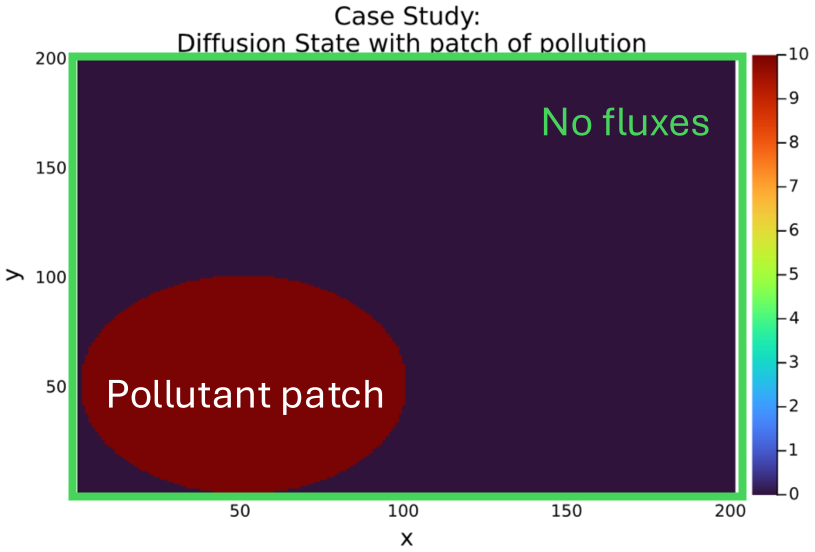
- Implicit scheme is unconditionally stable but computationally extensive.

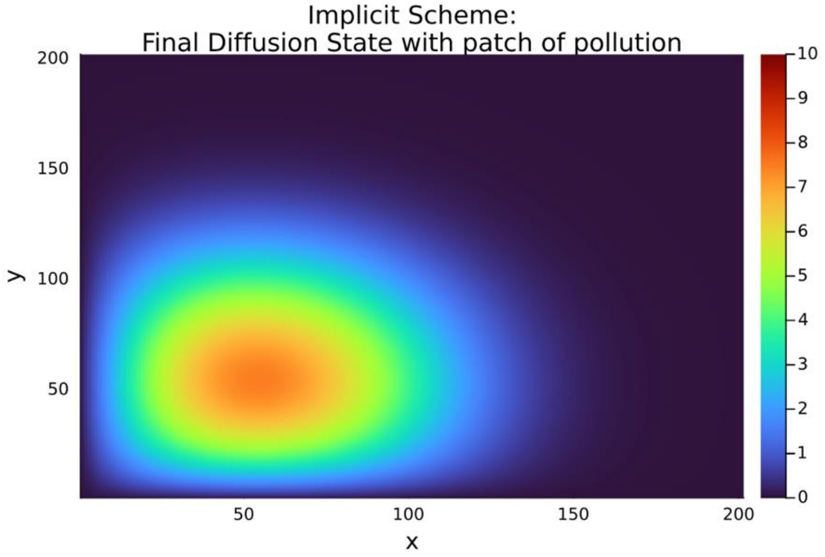
Neumann Boundary condition: Represents a closed system.

- Mass-conserving simulations.

Procedure







<u>'Trial & Error' Method</u>

1- Time and Space discretization

2- Total simulation time

Error: Initial - Final

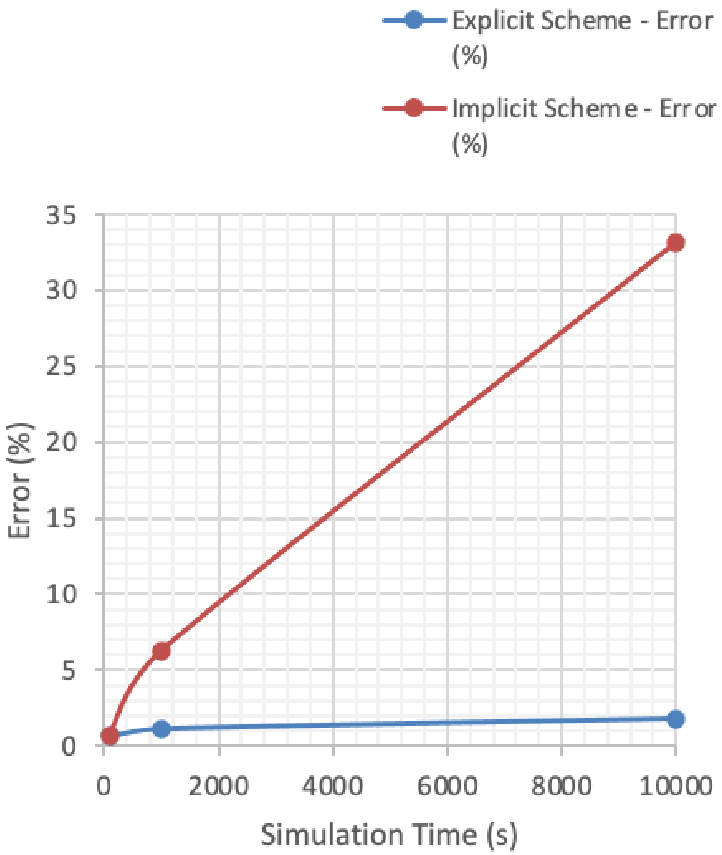
Results

Accuracy **Stability** **Efficiency**

Explicit method is accurate but computationally demanding
Implicit method is stable but less accurate.

Optimal Parameters for both Schemes

	Values	Explicit Scheme - Error (%)	Implicit Scheme - Error (%)
Time Step (dt)	10	1,65E+27	6,26
	5	1,15	6,26
	1	1,23	6,27
	0,1	1,25	6,27
Grid Size (Nx, Ny)	301, 301	1,93E+74	7
	201, 201	1,15	6,26
	101, 101	2,52	5,09
	11, 11	56,19	0,23
Total Simulation Time (s)	10000	1,79	33,18
	1000	1,15	6,26
	100	0,67	0,73



Conclusion

- **Importance of Numerical Modeling:**
  - Model of diffusion processes in 2D geophysical systems (pollutant transport & heat transfer).
  - Insights— Evolution of physical properties over time & space.

- **Comparison of Schemes:**
  - Explicit Scheme — High accuracy, requires small time steps, conditionally stable.
  - Implicit Scheme — Stable for large time steps, less accurate, computationally intensive.

- **Critical Factors:**
  - Neumann boundary conditions — mass conservation in closed systems.
  - Optimal grid resolution and time step balance stability, accuracy, and efficiency.

Importance in Oceanography

- **Dynamic Systems:**
  - Simulates pollutant transport (spread of oil spills, plastic debris), ocean heat uptake, and nutrient dispersal (impact on plankton, fish, etc.).
  - Supports resource management and disaster mitigation.

- **Benefits**:
  - Provides detailed predictions.
  - Tests scenarios under varying conditions.
  - Integrates with climate and hydrodynamic models.

Thank you!

By Danish Ali & Owen Dupuy

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