Statistical Modelling of Pakistan's Economy

Introduction

Economic models are fundamental tools used in economics to explain, predict, and analyze various economic phenomena. These models are constructed based on a set of assumptions and mathematical equations that represent the behavior of individuals and groups within an economy. Economic models are essential to understand how the economy works and make informed decisions about economic policies.

Macroeconomic models focus on the behavior of the economy as a whole. These models analyze factors such as gross domestic product (GDP), inflation, unemployment, and interest rates. They also investigate the relationships between these factors and the impact of economic policies on the economy. Phillips' Curve and Okun's Law are one of the two most widely known models used in the economic modelling realm. Phillips Curve demonstrate a relationship between inflation and unemployment rates while Okun's law signifies the relationship between growth rate of real GDP and change in unemployment rate. Thus, in this paper we will be modelling the two aforementioned models with real-world empirical data of Pakistan, a developing country in South Asia.

The data used in this paper is directly retrieved from the International Monetary Fund, a reputable and reliable data provider of the economic indicators, policies and conditions of its member countries. Since, Pakistan is a member country of the International Monetary Fund that receive loans from the IMF regularly, data from the IMF was chosen due to its credibility. There were four main macroeconomic indicators used during the research, inflation rate (annual percentage change) also known as the GDP deflator inflation rate, unemployment rate, real GDP growth rate and real GDP or output in billions of US Dollars of Pakistan (IMF, 2022). The time series data is quantitative in nature and contains information of the indicators from the year *1992 to 2022, a span of 30 years*[1].

Data Processing

The data retrieved was in excel file format and needed manipulation in terms of structure for it to be processed by the Jupyter Notebook software used. Python language was used for the data analysis and a number of python libraries were used to conduct linear regression and visualizations for the economic modelling including pandas, NumPy, SciPy, matplotlib etc.[2]. Since, different indicators contained different years of data, we removed the data points that had no role in our modelling and kept observations for the years 1992 till 2022 of all the required economic indicators[3].

[1] Refer to Appendix Tables for data set used

[2] Refer to Appendix Python Code

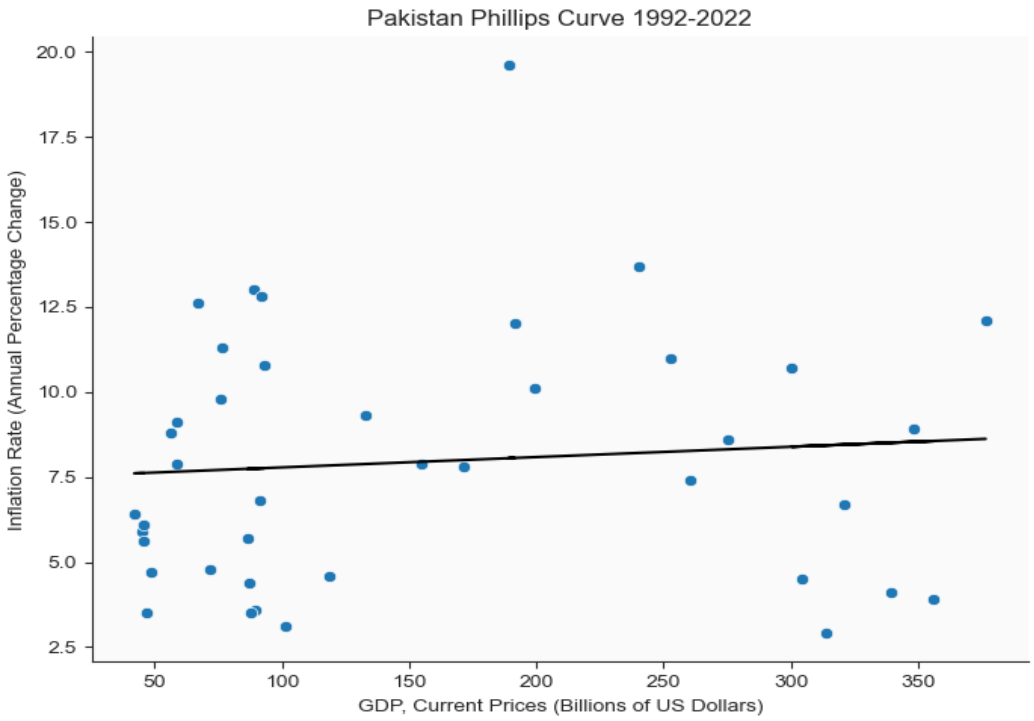
[3] Refer to Appendix Tables

Phillips Curve

In theory, Phillips Curve verifies an inverse relation between unemployment and inflation rates demonstrating a tradeoff between the two. In our model, a slightly different approach is taken with respect to the short-run Phillips curve. The Phillips curve created here shows the positive co-relation between inflation rate and real GDP which is intuitive. The idea being that when there is an economic growth experienced by a country, inflation increases due to an increase in the aggregate demand leading to more jobs that in turn lower the unemployment rate. The equation of the Phillips curve we work with is

$$\pi_t - \pi_{t-1} = a + b\, Y_t$$

The equation above describes a positive corelation between inflation rate and real GDP level and so, the slope of the line would be expected to be positive. After performing a linear regression and calculating the regression line coefficients including the slope and the intercept for the 30 years data from 1992-2022 we have the following regression line and table of coefficients;



Phillips Curve Coefficients	
Regression Coefficient	Value
Slope	0.003034
Intercept	7.477414
R-value	0.089429
P-value	0.578189
Std. Error	0.005412

$$\pi_t - \pi_{t-1} = 7.477414 + 0.003034\, Y_t$$

We take three decimal places for simplicity;

$$\pi_t - \pi_{t-1} = 7.477 + 0.003\, Y_t$$

We now make a crucial assumption to calculate the natural or potential level of GDP for Pakistan which is that inflation rate is equal to the expected inflation rate (inflation expected next year). When the aforementioned assumption holds the GDP level is equal to potential GDP. Suppose,

$$\pi_t = \pi_{t-1}$$

$$7.477 + 0.003\, Y_t = 0$$

$$Y_t = 2492 \cdot 3 = \bar{Y}$$

After rearranging the equation in the form

$$\pi_t - \pi_{t-1} = a\,(Y_t + \bar{Y})$$

we get

$$\pi_t - \pi_{t-1} = 0.003\,(Y_t + 2492.3)$$

This represents that the potential GDP for Pakistan is about $2492 Billion USD when the economy is in equilibrium with no inflationary and recessionary pressures. This indicates that if the right economic policies and institutions are in place, Pakistan has the potential to reach an economy worth over $2 trillion USD in the long-term. This goes hand-in-hand with many renowned projections and reports including by the World Bank for Pakistan to becoming a trillion-dollar economy (Syed, 2019).

The coefficient 0.003 in the Phillips Curve equation represents the sensitivity of the inflation rate to changes in the level of real GDP. In other words, it tells us how much the inflation rate is expected to change in response to a one-unit increase in the level of real GDP. When real GDP increases, inflation tends to increase as well, but the magnitude of this increase is relatively small.

This coefficient is an important parameter in understanding the relationship between the economy's output and inflation. It implies due to its small size that the relationship between real GDP and inflation is not very strong, and that other factors, such as changes in the money supply, interest rates, and expectations, may have a stronger impact on inflation in the short run. Moreover, the R-value that is 0.089 further verifies a weak relationship between real GDP and inflation rate in our model.

The p-value for the Phillips curve regression model is 0.5782 which indicates that there is a high chance that there are more factors apart from the GDP levels that is playing a major role in the inflation rate of Pakistan. The relationship between real GDP and inflation can be affected by a

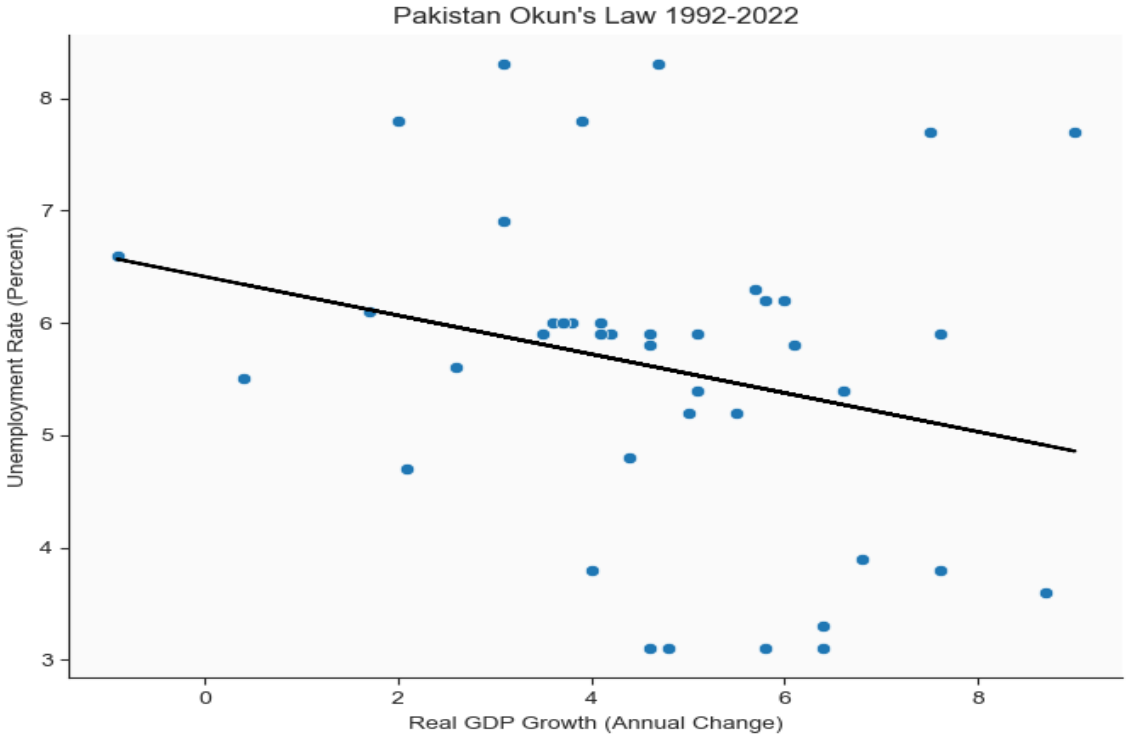
variety of factors, such as changes in productivity, shifts in the composition of output, and changes in the structure of the economy in the long-run. Therefore, it's important to view this coefficient as an estimate rather than a fixed parameter.

Okun's law

In theory, Okun's law is an economic principle that was first empirically observed by the economist Arthur Okun that defines a negative relationship between the unemployment rate and real GDP (Gross Domestic Product) growth rate of an economy. The idea is that there is a trade-off between unemployment and production in the short-run. When unemployment is high, businesses are not producing as much as their output capacity as it is related to the number of employees they hire, and therefore GDP decreases. Okun's law is typically expressed as a simple linear regression model mathematically as follows;

$$u_t - u_{t-1} = -\beta(g_{yt} - \overline{g_y})$$

In this model, the slope of the regression line (beta) represents the magnitude of the relationship between changes in GDP and changes in unemployment. A negative beta value indicates the aforementioned inverse relationship between the two variables. After performing a linear regression and calculating the regression line coefficients including the slope and intercept with for Pakistan's unemployment and GDP growth rates and then a visualzation.



Okun's Law Coefficients	
Regression Coefficient	Value
Slope	-0.17269
Intercept	6.412159
R-value	-0.24579
P-value	0.121382
Std. Error	0.109056

We construct a linear equation with the following numerical values to estimate the relationship between the unemployment rate and GDP growth rate.

$$u_t - u_{t-1} = a + b\ g_{yt}$$

$$u_t - u_{t-1} = 6.412159 - 0.17269\ g_{yt}$$

For simplicity we take three decimal places

$$u_t - u_{t-1} = 6.412 - 0.173\ g_{yt}$$

We now make a pivotal assumption that the unemployment rate is equal to the expected unemployment rate to calculate the normal growth rate of output for Pakistan that would keep the unemployment rate fixed. If the assumption holds, GDP growth rate must be equal to the natural or normal growth rate according to the model. By this assumption we can convert our equation to the standard Okun's law equation.

$$u_t - u_{t-1} = 6.412 - 0.173\ g_{yt}$$

Suppose,

$$u_t = u_{t-1}$$

$$6.412 - 0.173\ g_{yt} = 0$$

$$g_{yt} = 37.064\ =\ \overline{g_y}$$

Thus, Okun's Law equation by substituting our values will be

$$u_t - u_{t-1} = -0.173\left(g_{yt} - 37.064\right)$$

The beta value of -0.173 indicates that there is an inverse relationship between unemployment rate and growth rate as expected. Specifically, a 1% increase in GDP growth would be associated with a 0.173% decrease in the unemployment rate (holding other factors constant). Since our beta value is small, it means that there is a weak relationship between changes in GDP and changes in unemployment. The R-value -0.246 further verifies a weak negative relationship between our variables in the model. Moreover, the p-value for the Okun's law regression model is 0.121 which demonstrates that our data is insignificant to conclude that unemployment rate and GDP growth rate are strongly related. This eventually indicates that there are more factors involved in predicting the unemployment rate.

The magnitude of the beta value in Okun's law may vary depending on various factors such as the structure of the labor market, the degree of economic openness, effectiveness of labor market policies, time and countries, technological changes, policy interventions and demographic trends which are just a few factors amongst many others.

Specifically, the equation implies that for every 1 percentage point deviation of the GDP growth rate from 37.064%, the unemployment rate will change by -0.173 percentage points (i.e., decrease if GDP growth is above 37.064% and increase if GDP growth is below 37.064%). This indicates that every year Pakistan needs about a 37% annual GDP growth rate to keep its unemployment rate fixed i.e., normal growth rate. Any growth rate above the normal growth rate will result in a decrease in unemployment rate and any GDP growth rate would result in an increase in the unemployment rate.

Conclusion

Pakistan is a developing country and there are many factors including political instability, inefficient economic policies and poor vision for a long-term economic growth by the leadership of the country.

The Phillips curve in our model demonstrated a very weak positive co-relation between inflation and GDP levels rather than a strong one due to the aforementioned external factors. Although Pakistan’s economy has a huge potential to become a trillion-dollar economy it is stuck within its constraints of poor infrastructure and technological advancements. Nevertheless, Phillips curve was an insightful tool to represent how inflation and GDP are related.

Moreover, Pakistan’s population has been growing at a rapid speed which is one reason why our Okun’s model demonstrated such a high GDP growth rate to keep the unemployment rate fixed. Okun’s Law is a widely used and useful tool for policymakers and economists to understand the relationship between unemployment and economic growth. Unfortunately, it was unable to capture a strong relationship due to the historic as well as the current economic conditions of Pakistan.

Henceforth, we would need other statistical and econometric tests to evaluate the robustness and significance of the relationship between inflation and output levels as well as between GDP and unemployment. This can be performed by taking more insightful relations like population growth, education indicators and technological advancement and progress etc.

(1785 words excluding graphs and references)

References

IMF. (2022, October 1). *World Economic Outlook (Pakistan)*. Retrieved from IMF Datamapper: https://www.imf.org/external/datamapper/profile/PAK

Syed, R. (2019, March 19). *Pakistan@100: World Bank Suggests How Pakistan Can Become a $2 Trillion Economy.* Retrieved from https://propakistani.pk/2019/03/19/pakistan100-world-bank-suggests-how-pakistan-can-become-a-2-trillion-economy/

Appendix Tables

Year	Unemployment rate (Percent)	Inflation rate, average consumer prices (Annual percent change)
1982	3.8	5.9
1983	3.9	6.4
1984	3.8	6.1
1985	3.6	5.6
1986	3.3	3.5
1987	3.1	4.7
1988	3.1	8.8
1989	3.1	7.9
1990	3.1	9.1
1991	5.9	12.6
1992	5.9	4.8
1993	4.7	9.8
1994	4.8	11.3
1995	5.4	13
1996	5.4	10.8
1997	6.1	12.8
1998	5.9	6.8
1999	5.9	5.7
2000	7.8	3.6
2001	7.8	4.4
2002	8.3	3.5
2003	8.3	3.1
2004	7.7	4.6
2005	7.7	9.3
2006	6.2	7.9
2007	5.2	7.8
2008	5.2	12
2009	5.5	19.6
2010	5.6	10.1
2011	6	13.7
2012	6	11
2013	6	7.4
2014	6	8.6
2015	5.9	4.5
2016	5.9	2.9
2017	5.8	4.1
2018	5.8	3.9
2019	6.9	6.7
2020	6.6	10.7
2021	6.3	8.9
2022	6.2	12.1

Year	Real GDP growth (Annual percent change)	GDP, current prices (Billions of U.S. dollars)
1982	7.6	45.026
1983	6.8	42.038
1984	4	45.639
1985	8.7	45.634
1986	6.4	46.734
1987	5.8	48.848
1988	6.4	56.254
1989	4.8	58.65
1990	4.6	58.862
1991	5.1	66.87
1992	7.6	71.541
1993	2.1	75.718
1994	4.4	76.325
1995	5.1	89.197
1996	6.6	93.145
1997	1.7	91.84
1998	3.5	91.418
1999	4.2	86.573
2000	3.9	89.651
2001	2	87.389
2002	3.1	87.933
2003	4.7	101.103
2004	7.5	118.844
2005	9	132.833
2006	5.8	154.493
2007	5.5	171.512
2008	5	191.425
2009	0.4	188.991
2010	2.6	199.405
2011	3.6	240.385
2012	3.8	252.534
2013	3.7	260.257
2014	4.1	275.063
2015	4.1	304.481
2016	4.6	313.623
2017	4.6	339.229
2018	6.1	356.163
2019	3.1	321.071
2020	-0.9	300.41
2021	5.7	348.227
2022	6	376.493

Python Code

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
import scipy
import seaborn as sns
```

```
unemployment = pd.read_excel("unemployment_rate.xls")
inflation = pd.read_excel("inflation_rate.xls")
gdp_growth = pd.read_excel("real_gdp_growth_rate.xls")
real_output = pd.read_excel("real_output.xls")
```

```
unemployment = unemployment[(unemployment['Year'] >= 1982) &
(unemployment['Year'] <= 2022)]
unemployment = unemployment.reset_index(drop = True)
```

```
inflation = inflation[(inflation['Year'] >= 1982) & (inflation['Year'] <=
2022)]
inflation = inflation.reset_index(drop = True)
```

```
gdp_growth = gdp_growth[(gdp_growth['Year'] >= 1982) & (gdp_growth['Year'] <=
2022)]
gdp_growth = gdp_growth.reset_index(drop = True)
```

```
real_output = real_output[(real_output['Year'] >= 1982) &
(real_output['Year'] <= 2022)]
real_output = real_output.reset_index(drop = True)
```

```
data = pd.merge(unemployment, inflation , on = 'Year')
data = pd.merge(data, gdp_growth, on = 'Year')
data = pd.merge(data, real_output, on = 'Year')
```

```
data.head()
```

	Year	Pakistan Unemployment rate (Percent)	Pakistan Inflation rate, average consumer prices (Annual percent change)	Pakistan Real GDP growth (Annual percent change)	Pakistan GDP, current prices (Billions of U.S. dollars)
0	1982	3.8	5.9	7.6	45.026
1	1983	3.9	6.4	6.8	42.038
2	1984	3.8	6.1	4.0	45.639

	Year	Pakistan Unemployment rate (Percent)	Pakistan Inflation rate, average consumer prices (Annual percent change)	Pakistan Real GDP growth (Annual percent change)	Pakistan GDP, current prices (Billions of U.S. dollars)
3	1985	3.6	5.6	8.7	45.634
4	1986	3.3	3.5	6.4	46.734

```
table = data.to_excel('data_table.xlsx')
#df_cars.to_excel("converted-to-excel.xlsx")
```

```
data.columns
```

```
Index(['Year', 'Pakistan Unemployment rate (Percent) ',
       'Pakistan Inflation rate, average consumer prices (Annual percent
change) ',
       'Pakistan Real GDP growth (Annual percent change)',
       'Pakistan GDP, current prices (Billions of U.S. dollars)'],
      dtype='object')
```

```
model = LinearRegression()
```

```
unemployment_rate = data.iloc[:, 1].to_list()
inflation_rate = data.iloc[:, 2].to_list()
gdp_growth_rate = data.iloc[:, 3].to_list()
real_gdp = data.iloc[:, 4].to_list()
```

```
from scipy.stats import linregress
coeffs_phillips = linregress(real_gdp, inflation_rate)
results_phillips = pd.DataFrame({
    'Variable': ['Slope', 'Intercept', 'R-value', 'P-value', 'Std. Error'],
    'Value': [coeffs_phillips.slope, coeffs_phillips.intercept,
coeffs_phillips.rvalue, coeffs_phillips.pvalue, coeffs_phillips.stderr]
})
```

```
print(results_phillips)
#table = data.to_excel('data_table.xlsx')
results_phillips.to_excel('Phillips_scores.xlsx')
```

```
     Variable     Value
0       Slope  0.003034
1   Intercept  7.477414
2     R-value  0.089429
3     P-value  0.578189
4  Std. Error  0.005412
```

```
from scipy.stats import linregress
```

```
coeffs_okuns = linregress(gdp_growth_rate, unemployment_rate)
results_okuns = pd.DataFrame({
    'Variable': ['Slope', 'Intercept', 'R-value', 'P-value', 'Std. Error'],
    'Value': [coeffs_okuns.slope, coeffs_okuns.intercept,
coeffs_okuns.rvalue, coeffs_okuns.pvalue, coeffs_okuns.stderr]
})

print(results_okuns)
results_okuns.to_excel("Okuns_scores.xlsx")
```

```
     Variable     Value
0       Slope -0.172691
1   Intercept  6.412159
2     R-value -0.245786
3     P-value  0.121382
4  Std. Error  0.109056
```

```
unemployment_rate = data.iloc[:, 1].values.reshape(-1, 1)
inflation_rate = data.iloc[:, 2].values.reshape(-1,1)
gdp_growth_rate = data.iloc[:, 3].values.reshape(-1,1)
real_gdp = data.iloc[:, 4].values.reshape(-1,1)


lr = LinearRegression()
plot_inflation_rate = sns.scatterplot(data = data, x = 'Pakistan GDP, current
prices (Billions of U.S. dollars)',
                                      y = 'Pakistan Inflation rate, average
consumer prices (Annual percent change) ')
plot_inflation_rate.set(title = "Pakistan Phillips Curve 1992-2022", xlabel =
'GDP, Current Prices (Billions of US Dollars)',
                        ylabel = 'Inflation Rate (Annual Percentage Change)')
lr.fit(real_gdp, inflation_rate)
inflation_pred = lr.predict(real_gdp)
plot_inflation_rate.plot(real_gdp, inflation_pred, color = 'black')
sns.despine()
plot_inflation_rate.figure.set_size_inches(8, 6)
sns.set_style(style = 'white', rc = {"axes.facecolor": "#FAFAFA",
'xtick.bottom': True, 'ytick.left': True})



okuns = model.fit(unemployment_rate, gdp_growth_rate)


plot_unemployment_rate = sns.scatterplot(data = data, x = 'Pakistan Real GDP
growth (Annual percent change)',
                                      y = 'Pakistan Unemployment rate
(Percent) ')
plot_unemployment_rate.set(title = "Pakistan Okun's Law 1992-2022", xlabel =
'Real GDP Growth (Annual Change)',
                           ylabel = 'Unemployment Rate (Percent)')
lr.fit(gdp_growth_rate, unemployment_rate)
unemployment_pred = lr.predict(gdp_growth_rate)
```

```
plot_unemployment_rate.plot(gdp_growth_rate, unemployment_pred, color =
'black')
sns.despine()
plot_unemployment_rate.figure.set_size_inches(8, 6)
sns.set_style(style = 'white', rc = {"axes.facecolor": "#FAFAFA",
'xtick.bottom': True, 'ytick.left': True})
```

Figure 1

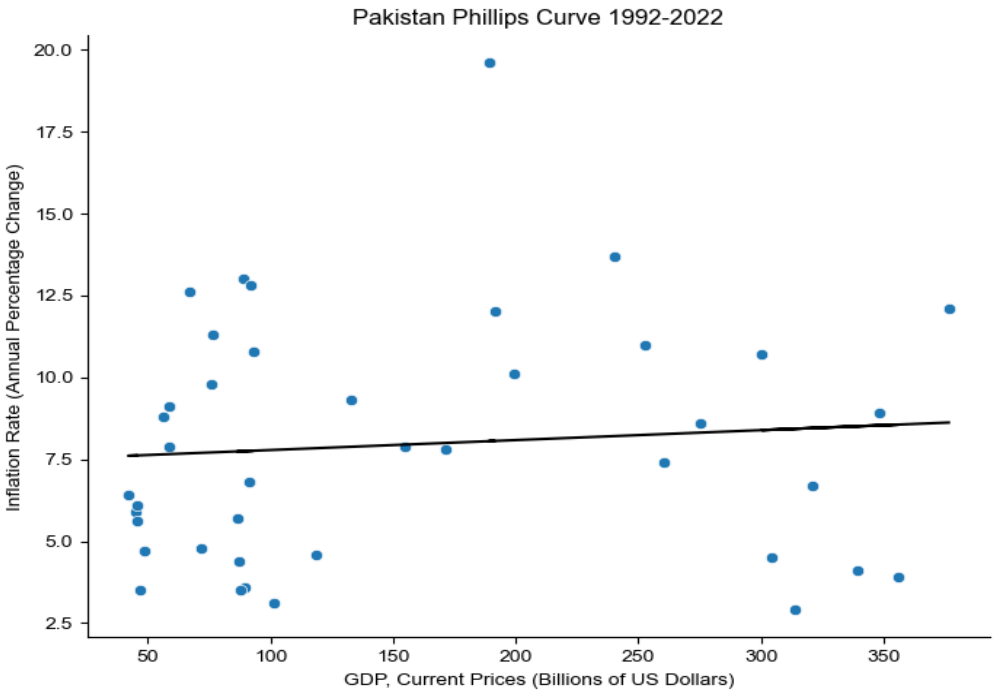


Figure 2