Stand-Alone Risk Analysis

Risk is inherent in almost every business decision. More so in capital budgetin decisions as they involve costs and benefits extending over a long period of time durin which many things can change in unanticipated ways.

For the sake of expository convenience, we have assumed so far that all investment being considered for inclusion in the capital budget had the same risk as those of th existing investments of the firm. Hence the average cost of capital was used for evaluatin every project. Investment proposals, however, differ in risk. A research and developmer project may be more risky than an expansion project and the latter tends to be more risk than a replacement project. In view of such differences, variations in risk need to b evaluated explicitly in capital investment appraisal.

Risk analysis is one of the most complex and slippery aspects of capital budgeting. Man different techniques have been suggested and no single technique can be deemed as best i all situations. The variety of techniques suggested to handle risk in capital budgeting fa into two broad categories: (i) techniques that consider the stand-alone risk of a projec (ii) techniques that consider the risk of a project in the context of the firm or in the conte of the market. Exhibit 11.1 classifies various techniques into these broad categories.

This chapter discusses different techniques that consider the stand-alone risk of project, examines ways of managing risk, explores various approaches to project selectio under risk, and describes risk analysis in practice. It is divided into eleven sections a follows:

- Sources, measures, and perspectives on risk
- Sensitivity analysis
- Scenario analysis
- Break-even analysis

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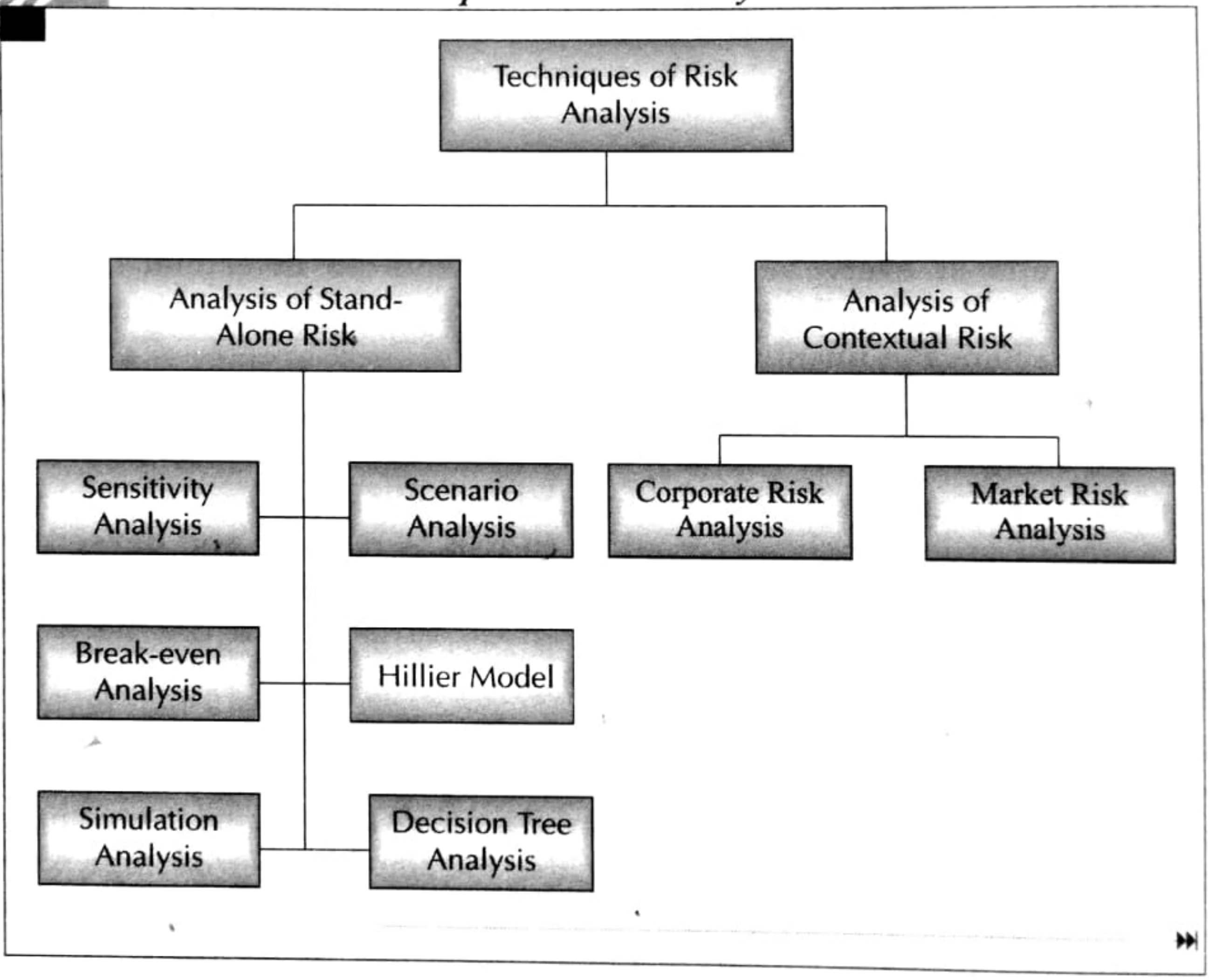
- Hillier model
- Simulation analysis
- Decision tree analysis
- Managing risk
- Project selection under risk
- Risk analysis in practice
- How financial institutions analyse risk

Exhibit 11.1 *Techniques for Risk Analysis*



11.1 SOURCES, MEASURES, AND PERSPECTIVES ON RISK

Sources of Risk

There are several sources of risk in a project. The important ones are project-spec competitive risk, industry-specific risk, market risk, and international risk.

Project-specific risk The earnings and cash flows of the project may be lower than expected because of estimation error or due to some other factors specific to the project like the quality of management.

Competitive risk The earnings and cash flows of the project may be affected by unanticipated actions of the competitors.

Industry-specific risk Unexpected technological developments and regulatory changes, that are specific to the industry to which the project belongs, will have an impact on the earnings and cash flows of the project as well.

Market risk Unanticipated changes in macroeconomic factors like the GDP growth rate, interest rate, and inflation have an impact on all projects, albeit in varying degrees.

International risk In the case of a foreign project, the earnings and cash flows may be different than expected due to the exchange rate risk or political risk.

Measures of Risk

Risk refers to variability. It is a complex and multi-faceted phenomenon. A variety of measures have been used to capture different facets of risk. The more important ones are range, standard deviation, coefficient of variation, and semi-variance.

To illustrate the calculation of these measures, consider a capital investment whose net present value has the following distribution.

NPV	*Probability*
200	0.3
600	0.5
900	0.2

The probability weighted NPV works out to:

$$E(\text{NPV}) = \sum_{i=1}^{3} p_i \, \text{NPV}_i$$
$$= 0.3 \times 200 + 0.5 \times 600 + 0.2 \times 900 = 540$$

Now let us look at the various measures of risk.

Range Obviously the simplest measure of risk, the range of a distribution is the difference between the highest value and the lowest value. The range of the preceding distribution is: 900 – 200 = 700.

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Standard Deviation The standard deviation of a distribution is:

$$\sigma = [\Sigma\, p_i\, (X_i - \overline{X})^2]^{½} \tag{11.1}$$

where σ = standard deviation

p_i = probability associated with the ith value

X_i = i th value

$\overline{X}$ = expected value

The standard deviation of the preceding NPV distribution is:

$$\sigma = [0.3\ (200 - 540)^2 + 0.5\ (600 - 540)^2 + 0.2\ (900 - 540)^2]^{1/2}$$
$$= [62{,}400]^{1/2} = 249.8$$

The square of standard deviation is called variance

$$\text{Variance} = \sigma^2$$

The variance of the above distribution is 62,400

Coefficient of Variation One problem with standard deviation (or variance) is that it is not adjusted for scale. If you go only by standard deviation, an investment with an expected net present value of Rs 10 million and a standard deviation of Rs 100,000 would be considered more risky than an investment with an expected net present value of Rs 1 million and a standard deviation of Rs 90,000.

The coefficient of variation (CV) adjusts standard deviation for scale. It is defined as:

$$CV = \frac{\text{Standard deviation}}{\text{Expected Value}}$$

The coefficient of variation for the investment in our illustration is:

$$CV = \frac{249.8}{540} = 0.46$$

Semi-Variance There is yet another problem with standard deviation (or variance). It considers all deviations, positive as well as negative, from the expected value in the same way. Since investors are concerned about only negative deviations, semi-variance seems to be a more suitable measure of risk.

The semi-variance is computed the way the variance is computed, except that only outcomes below the expected value are taken into account. It is defined as:

$$SV = \Sigma\, p_i\, d_i'^2 \tag{11.2}$$

where d_i' is equal to d_i if $d_i < 0$ and is equal to 0 if $d_i \geq 0$

The semi-variance for the investment in our illustration is:

$$SV = 0.3\ (200 - 540)^2 = 34{,}680$$

The semi-standard deviation is the square root of semi-variance. The semi-standard deviation for the investment in our illustration is:

$$\text{Semi-standard deviation} = (\text{Semi-variance})^{1/2}$$
$$= (34680)^{1/2} = 186.2$$

The Principal Measure Standard deviation is the most commonly used measure of risk in finance. The main reasons for using standard deviation are:

(i) If a variable is normally distributed, its mean and standard deviation contain all the information about its probability distribution.

(ii) If the utility of money is represented by a quadratic function (a function commonly suggested to represent diminishing marginal utility of wealth), then the expected utility is a function of mean and standard deviation.

(iii) Standard deviation is analytically easily tractable.

Use of Subjective Probabilities For measuring the expected value and dispersion of a variable, its probability distribution is required. In some cases the probability distribution can be defined with a fairly high degree of objectivity on the basis of past evidence. A wildcatter, for example, may be able to define with a high degree of objectivity the probabilities associated with certain states of nature if sufficient records for similar ventures are available. Since such a probability distribution is substantially based on objective facts, it may be referred to as 'objective' probability distribution.

However, in most real-life situations, such objective evidence may not be available for defining probability distributions. In such cases, knowledgeable persons may pool their experience and judgment to define the probability distribution. Since there is likely to be a high element of subjectivity in these distributions, such distributions are generally referred to as '*subjective*' probability distributions.

Perspectives on Risk

Regardless of the risk measure employed, there are different perspectives on risk. You can view a project from at least three different perspectives. These are:

Stand-alone risk This represents the risk of a project when it is viewed in isolation

Firm risk Also called *corporate risk*, this reflects the contribution of a project to the risk of the firm.

Systematic risk This represents the risk of a project from the point of view of a diversified investor. It is also called *market risk*.

This chapter focuses on stand-alone risk and the following chapter examines firm risk and market risk. There are several reasons for starting with single investment risk analysis:

- Measuring a project's stand-alone risk is easier than measuring its corporate risk and far easier than measuring its market risk.
- In most of the cases, stand-alone risk, corporate risk, and market risk are highly correlated. If the overall economy does well, the firm too would do well. Further, if the firm does well, most of its projects would do well. Thanks to this high correlation, stand-alone risk may be used as a proxy for corporate risk and market risk.
- The proponent of a capital investment is likely to be judged on the performance of that investment. Hence he will naturally be concerned about its stand alone risk and not about its contribution to the risk of the firm or the risk of a diversified investor.
- In most firms, the capital budgeting committee considers investment proposals one at a time.

11.2 SENSITIVITY ANALYSIS

Since the future is uncertain, you may like to know what will happen to the viability of the project when some variable like sales or investment deviates from its expected value. In other words, you may want to do "what if" analysis or sensitivity analysis.

To understand the nature of sensitivity analysis, let us consider an example. Suppose you are the financial manager of Naveen Flour Mills. Naveen is considering setting up a new flour mill near Bangalore. Based on Naveen's previous experience, the project staff of Naveen has developed the figures shown in Exhibit 11.2 (note that the salvage value has been assumed to be nil).

Exhibit 11.2 ***Cash Flow Forecast for Naveen's Flour Mill Project***

		(Rs in '000)
	Year 0	*Years 1–10*
1. Investment	(20,000)	
2. Sales		18,000
3. Variable costs ($66^2/_3$% of sales)		12,000
4. Fixed costs		1,000
5. Depreciation		2,000
6. Pre-tax profit		3,000
7. Taxes		1,000
8. Profit after taxes		2,000
9. Cash flow from operation		4,000
10. Net cash flow	(20,000)	4,000

Since the cash flow from operations is an annuity, the NPV of the flour mill project is:

$$-20{,}000{,}000 + 4{,}000{,}000 \; PVIFA(r = 12\%, n = 10)$$
$$= -20{,}000{,}000 + 4{,}000{,}000\ (5.650)$$
$$= 2{,}600{,}000$$

The NPV based on the expected values of the underlying variables looks positive. You are, however, aware that the underlying variables can vary widely and hence you would like to explore the effect of such variations on the NPV. So you define the optimistic and pessimistic estimates for the underlying variables. These are shown in the left hand columns of Exhibit 11.3. With this information, you can calculate the *NPV* for the optimistic and pessimistic values of each of the underlying variables.

Exhibit 11.3 ***Sensitivity of NPV to Variations in the Value of Key Variables***

Rs in million

	Range			*NPV*		
Key Variable	*Pessimistic*	*Expected*	*Optimistic*	*Pessimistic*	*Expected*	*Optimistic*
Investment (Rs in million)	24	20	18	−0.65	2.60	4.22
Sales (Rs in million)	15	18	21	−1.17	2.60	6.40
Variable costs as a percent of sales	70	66.66	65	0.34	2.60	3.73
Fixed costs (Rs in million)	1.3	1.0	0.8	1.47	2.60	3.33

To do this, vary one variable at a time. For example, to study the effect of an adverse variation in sales (from the expected Rs 18 million to the pessimistic Rs 15 million), you maintain the values of the other underlying variables at their expected levels. (This means that the investment is held at Rs 20 million, variable costs as a proportion of sales are held at $66^2/_3$ percent, fixed costs are held at Rs 1 million, so on and so forth.)

The NPV when the sales are at their pessimistic level and other variables at their expected level is shown on the right side of Exhibit 11.2. Likewise, you can calculate the effect of variations in the values of the other underlying variables. The NPVs for the pessimistic, expected, and optimistic forecasts are shown on the right side of Exhibit 11.3.

Evaluation

A very popular method for assessing risk, sensitivity analysis has certain *merits*:

- It shows how robust or vulnerable a project is to changes in values of the underlying variables.
- It indicates where further work may be done. If the net present value is highly sensitive to changes in some factor, it may be worthwhile to explore how the variability of that critical factor may be contained.
- It is intuitively a very appealing as it articulates the concerns that project evaluators normally have.

Notwithstanding its appeal and popularity, sensitivity analysis suffers from several *shortcomings*:

- It merely shows what happens to NPV when there is a change in some variable without providing any idea of how likely that change will be.
- Typically, in sensitivity analysis only one variable is changed at a time. In the real world, however, variables tend to move together.
- It is inherently a very subjective analysis. The same sensitivity analysis may lead one decision maker to accept the project while another may reject it.

11.3 SCENARIO ANALYSIS

In sensitivity analysis, typically one variable is varied at a time. If variables are inter related, as they are most likely to be, it will be helpful to look at some plausible scenarios each scenario representing a consistent combination of variables.

Procedure

The steps involved in scenario analysis are as follows:

1. Select the factor around which scenarios will be built. The factor chosen must be the largest source of uncertainty for the success of the project. It may be the state of the economy or interest rate or technological development or response of the market.
2. Estimate the values of each of the variables in investment analysis (investment outlay, revenues, costs, project life, and so on) for each scenario.
3. Calculate the net present value and/or internal rate of return under each scenario.

Illustration

Zen Enterprises is evaluating a project for introducing a new product. Depending on the response of the market—the factor which is the largest source of uncertainty for the success of the project—the management of the firm has identified three scenarios:

Scenario 1: The product will have a moderate appeal to customers across the board at a modest price.

Scenario 2: The product will have a strong appeal to a large segment of the market which is highly price-sensitive.

Scenario 3: The product will appeal to a small segment of the market which will be willing to pay a high price.

Exhibit 11.4 shows the net present value calculation for the project.

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Exhibit 11.4 shows the net present value calculation for the project.

Exhibit 11.4 *Net Present Value Calculation for Three Scenarios*

			(Rs in million)
	Scenario 1	*Scenario 2*	*Scenario 3*
Initial investment	200	200	200
Unit selling price (in rupees)	25	15	40
Demand (in units)	20	40	10
Revenues	500	600	400
Variable costs	240	480	120
Fixed costs	50	50	50
Depreciation	20	20	20
Pre-tax profit	190	50	210
Tax @ 50%	95	25	105
Profit after tax	95	25	105
Annual cash flow	115	45	125
Project life	10 years	10 years	10 years
Salvage value	0	0	0
Net present value (at a discount rate of 15 percent)	377.2	25.9	427.4

Best and Worst Case Analysis

In the preceding illustration, an attempt was made to develop scenarios in which the values of the variables were internally consistent. For example, high selling price and low demand typically go hand in hand. Firms often do another kind of scenario analysis called the best case and worst case analysis. In this kind of analysis the following scenarios are considered:

Best Scenario High demand, high selling price, low variable cost, and so on.

Normal Scenario Average demand, average selling price, average variable cost, and so on.

Worst Scenario Low demand, low selling price, high variable cost, and so on.

The objective of such scenario analysis is to get a feel of what happens under the most favourable or the most adverse configuration of key variables, without bothering much about the internal consistency of such configurations.

Evaluation

Scenario analysis may be regarded as an improvement over sensitivity analysis because it considers variations in several variables together.

However, scenario analysis has its own *limitations*:

- It is based on the assumption that there are few well-delineated scenarios. This may not be true in many cases. For example, the economy does not necessarily lie in three discrete states, viz. recession, stability, and boom. It can in fact be anywhere on the continuum between the extremes. When a continuum is converted into three discrete states some information is lost.

- Scenario analysis expands the concept of estimating the expected values. Thus, in a case where there are 10 inputs, the analyst has to estimate 30 expected values (3x10) to do the scenario analysis.

11.4 BREAK-EVEN ANALYSIS

In sensitivity analysis we ask what will happen to the project if sales decline or costs increase or something else happens. As a financial manager, you will also be interested in knowing how much should be produced and sold at a minimum to ensure that the project does not 'lose money'. Such an exercise is called *break-even analysis* and the minimum quantity at which loss is avoided is called the break-even point. The break-even point may be defined in accounting terms or financial terms.

Accounting Break-even Analysis

Suppose you are the financial manager of Naveen Flour Mills. Naveen is considering setting up a new flour mill near Bangalore. Based on Naveen's previous experience, the project staff of Naveen has developed the figures shown in Exhibit 11.5.

Note that the ratio of variable costs to sales is 0.667 (12/18). This means that every rupee of sales makes a contribution of Rs 0.333. Put differently, the contribution margin ratio is 0.333. Hence the break-even level of sales will be:

$$\frac{\text{Fixed costs} + \text{Depreciation}}{\text{Contribution margin ratio}} = \frac{1+2}{0.333} = \text{Rs 9 million}$$

Exhibit 11.5 ***Cash Flow Forecast for Naveen's Flour Mill Project***

		('000)
	Year 0	*Years 1 – 10*
1. Investment	(20,000)	
2. Sales		18,000
3. Variable costs ($66^2/_3$% of sales)		12,000
4. Fixed costs		1,000
5. Depreciation		2,000
6. Pre-tax profit		3,000
7. Taxes		1,000
8. Profit after taxes		2,000
9. Cash flow from operation		4,000
10. Net cash flow	(20,000)	4,000

By way of confirmation, you can verify that the break-even level of sales is indeed Rs 9 million.

	Rs in million
Sales	9
Variable costs	6
Fixed costs	1
Depreciation	2
Profit before tax	0
Tax	0
Profit after tax	0

A project that breaks even in accounting terms is like a stock that gives you a return of zero percent. In both the cases you get back your original investment but you are not compensated for the time value of money or the risk that you bear. Put differently, you forego the opportunity cost of your capital. Hence a project that merely breaks even in accounting terms will have a negative NPV.

Financial Break-even Analysis

The focus of financial break-even analysis is on NPV and not accounting profit. At what level of sales will the project have a zero NPV?

To illustrate how the financial break-even level of sales is calculated, let us go back to the flour mill project. The annual cash flow of the project depends on sales as follows:

1. Variable costs : 66.67 percent of sales
2. Contribution : 33.33 percent of sales   P.= C × S − F.C
3. Fixed costs : Rs 1 million
4. Depreciation : Rs 2 million
5. Pre-tax profit : (.333 × Sales) – Rs 3 million
6. Tax (at 33.3%) : .333(.333 Sales – Rs 3 million)
7. Profit after tax : .667 (.333 × Sales – Rs 3 million)
8. Cash flow (4+7) : Rs 2 million + .667 (.333 × Sales – Rs 3 million) = 0.222 Sales

Since the cash flow lasts for 10 years, its present value at a discount rate of 12 percent is:

$$\text{PV(cash flows)} = 0.222 \text{ Sales} \times \text{PVIFA (10 years, 12\%)}$$
$$= 0.222 \text{ Sales} \times 5.650$$
$$= 1.254 \text{ Sales}$$

The project breaks even in NPV terms when the present value of these cash flows equals the initial investment of Rs 20 million. Hence, the financial break-even occurs when

PV (cash flows) = Investment

1.254 Sales = Rs 20 million

Sales = Rs 15.95 million

Thus, the sales for the flour mill must be Rs 15.94 million per year for the investment to have a zero NPV. Note that this is significantly higher than Rs 9 million which represents the accounting break-even sales.

11.5 HILLIER MODEL

Under certain circumstances, the expected net present value and the standard deviation of net present value may be obtained through analytical derivation as suggested by F.S. Hillier. Two cases of such analysis are discussed here: (i) no correlation among cash flows and (ii) perfect correlation among cash flows.

Uncorrelated Cash Flows

When the cash flows of different years are uncorrelated, the cash flow for year t is independent of the cash flow for year $t - r$. Put differently, there is no relationship between cash flows from one period to another. In this case the expected net present value and the standard deviation of net present value are defined as follows:

$$\overline{\text{NPV}} = \sum_{t=1}^{n} \frac{\bar{A}_t}{(1+i)^t} - I \tag{11.3}$$

$$\sigma(\text{NPV}) = \left[\sum_{t=1}^{n} \frac{\sigma_t^2}{(1+t)^{2t}} \right]^{1/2} \qquad (11.4)[1]$$

where $\overline{\text{NPV}}$ = expected net present value

$\bar{A}_t$ = expected cash flow for year t

i = risk-free interest rate

I = initial outlay

$\sigma(\text{NPV})$ = standard deviation of net present value

σ_t = standard deviation of the cash flow for year t

Note that in the above formulae the discount rate is the risk-free interest rate because we try to separate the time value of money and the risk factor. The risk of the project, reflected in $\sigma(\text{NPV})$, is considered in conjunction with $\overline{\text{NPV}}$ computed with the risk-free

[1] In Eqs (11.3) and (11.4) we have assumed that the initial investment is known with certainty. The formula can, however, be easily modified to consider the variability of I.

discount rate. If $\overline{\text{NPV}}$ is computed using a risk-adjusted discount rate and then if this is viewed along with σ(NPV), the risk factor would be counted twice.

Example A project involving an outlay of Rs 10,000 has the following benefits associated with it:

Year 1		Year 2		Year 3	
Net cash flow	*Probability*	*Net cash flow*	*Probability*	*Net cash flow*	*Probability*
Rs 3,000	0.3	Rs 2,000	0.2	Rs 3,000	0.3
5,000	0.4	4,000	0.6	5,000	0.4
7,000	0.3	6,000	0.2	7,000	0.3

The cash flows of different years are uncorrelated. Calculate $\overline{\text{NPV}}$ and σ(NPV), assuming that i = 6 percent

$$\overline{\text{NPV}} = \sum_{t=1}^{3} \frac{\overline{A}_t}{(1+i)^t} - I$$

$$= \frac{5{,}000}{1.06} + \frac{4{,}000}{(1.06)^2} + \frac{5{,}000}{(1.06)^3} - 10{,}000 = \text{Rs } 2{,}475$$

$$\sigma(\text{NPV}) = \left[\Sigma \frac{\sigma_t^2}{(1+i)^{2t}}\right]^{1/2}$$

$$= \left[\frac{2{,}400{,}000}{(1.06)^2} + \frac{1{,}600{,}000}{(1.06)^4} + \frac{2{,}400{,}000}{(1.06)^6}\right]^{1/2} = \text{Rs } 2{,}258$$

Perfectly Correlated Cash Flows

If cash flows are perfectly correlated, the behaviour of cash flows in all periods is alike. This means that if the actual cash flow in one year is α standard deviations to the left of its expected value, cash flows in other years will also be α standard deviations to the left of their respective expected values. Put in other words, cash flows of all years are linearly related to one another. The expected value and the standard deviation of net present value, when cash flows are perfectly correlated, are as follows:

$$\overline{\text{NPV}} = \sum_{t=1}^{n} \frac{\overline{A}_t}{(1+i)^t} - I \tag{11.5}$$

$$\sigma(\text{NPV}) = \sum_{t=1}^{n} \frac{\sigma_t}{(1+i)^t} \tag{11.6}$$

Example An investment project involves a current outlay of Rs 10,000. The mean and standard deviation of cash flows, which are perfectly correlated, are as follows:

Year	$\bar{A}_t$	σ_t
1	Rs 5,000	1,500
2	3,000	1,000
3	4,000	2,000
4	3,000	1,200

Calculate $\overline{\text{NPV}}$ and σ (NPV), assuming a risk-free interest rate of 6 percent.

$$\overline{\text{NPV}} = \sum_{t=1}^{3} \frac{\bar{A}_t}{(1+i)^t} - I$$

$$= \frac{5,000}{1.06} + \frac{3,000}{(1.06)^2} + \frac{4,000}{(1.06)^3} + \frac{3,000}{(1.06)^4} - 10,000$$

$$= \text{Rs } 3,121$$

$$\sigma(\text{NPV}) = \sum_{t=1}^{4} \frac{\sigma_t}{(1+i)^t}$$

$$= \frac{1,500}{(1.06)} + \frac{1,000}{(1.06)^2} + \frac{2,000}{(1.06)^3} + \frac{1,200}{(1.06)^4} = \text{Rs } 4,935$$

11.6 SIMULATION ANALYSIS

Sensitivity analysis indicates the sensitivity of the criterion of merit (NPV, IRR, or any other) to variations in basic factors and provides information of the following type: If the quantity produced and sold decreases by 1 percent, other things being equal, the NPV falls by 6 percent. Such information, though useful, may not be adequate for decision making. The decision maker would also like to know the likelihood of such occurrences. This information can be generated by simulation analysis which may be used for developing the probability profile of a criterion of merit by randomly combining values of variables which have a bearing on the chosen criterion.

Procedure

The steps involved in simulation analysis are as follows:

1. Model the project. The model of the project shows how the net present value is related to the parameters and the exogenous variables. (Parameters are input variables specified by the decision maker and held constant over all simulation runs. Exogenous variables are input variables which are stochastic in nature and outside the control of the decision maker.)

2. Specify the values of parameters and the probability distributions of the exogenous variables.
3. Select a value, at random, from the probability distributions of each of the exogenous variables.
4. Determine the net present value corresponding to the randomly generated values of exogenous variables and pre-specified parameter values.
5. Repeat steps (3) and (4) a number of times to get a large number of simulated net present values.
6. Plot the frequency distribution of the net present value.

Illustration

In real life situations, simulation is done only on the computer because of the computational tedium involved. However, to give you a flavour of what goes on in simulation, we will work with a simple example where simulation has been done manually.

Zenith Chemicals is evaluating an investment project whose net present value has been modelled as follows:

$$\text{NPV} = \sum_{t=1}^{n} \frac{\text{Annual Cash Flow}}{(1 + \text{Risk-free Rate})} - \text{Initial Investment} \qquad (11.7)$$

In the NPV model embodied in Eq. (11.7), the risk-free rate and the initial investment are parameters with the following values: risk-free rate = 10 percent and initial investment = Rs 13,000. The annual cash flow and the project life (n) are stochastic exogenous variables with the following distributions:

Annual Cash Flow

Value	*Probability*
Rs 1,000	0.02
1,500	0.03
2,000	0.15
2,500	0.15
3,000	0.30
3,500	0.20
4,000	0.15

Project Life

Value	*Probability*
3 years	0.05
4	0.10
5	0.30
6	0.25
7	0.15
8	0.10
9	0.03
10	0.02

The firm wants to perform 10 manual simulation runs for this project. To perform the simulation runs, we have to generate values, at random, for the two exogenous variables: annual cash flow and project life. For this purpose, we have to (i) set up the correspondence between the values of exogenous variables and random numbers, and (ii) choose some random number generating device. Exhibit 11.6 shows the correspondence between various variables and two digit random numbers. Exhibit 11.7 presents a table of random digits that will be used for obtaining two digit random numbers.[2]

Now we are ready for simulation. In order to obtain random numbers from Exhibit 11.9 we may begin anywhere at random in the table and read any pair of adjacent columns (since we are interested in a two-digit random number) and read column-wise or row-wise.

For our example, let us use the first two columns of Exhibit 11.7. Starting from the top, we will read down the column. For the first simulation run we need two two-digit random numbers, one for the annual cash flow and the other for the project life. These numbers are 53 and 97 and the corresponding values for annual cash flow and project life are Rs 3,000 and 9 years respectively. We go further in this manner. Exhibit 11.8 shows the random numbers so obtained and the results of simulation.

Exhibit 11.6 *Correspondence between Values of Exogenous Variables and Two Digit Random Numbers*

Annual Cash Flow				*Project Life*			
Value	*Probability*	*Cumulative probability*	*Two digit random numbers*	*Value*	*Probability*	*Cumulative probability*	*Two digit random numbers*
Rs				*Years*			
1,000	0.02	0.02	00 to 01	3	0.05	0.05	00 to 04
1,500	0.03	0.05	02 to 04	4	0.10	0.15	05 to 14
2,000	0.15	0.20	05 to 19	5	0.30	0.45	15 to 44
2,500	0.15	0.35	20 to 34	6	0.25	0.70	45 to 69
3,000	0.30	0.65	35 to 64	7	0.15	0.85	70 to 84
3,500	0.20	0.85	65 to 84	8	0.10	0.95	85 to 94
4,000	0.15	1.00	86 to 99	9	0.03	0.98	95 to 97
				10	0.02	1.00	98 to 99

Exhibit 11.7 *Random Numbers*[2]

53479	81115	98036	12217	59526
97344	70328	58116	91964	26240
66023	38277	74523	71118	84892
99776	75723	03172	43112	83086
30176	48979	92153	38416	42436
81874	83339	14988	99937	13213
19839	90630	71863	95053	55532
09337	33435	53869	52769	18801
31151	58925	40823	41330	21093
67619	52515	03037	81699	17106

[2] Extracted from Rand Corporation, *A Million Random Digits with 100,000 Normal Deviates*, Glencoe, Illinois: The Free Press.

Exhibit 11.8 *Simulation Results*

Run	Annual Cash Flow: Random number	Annual Cash Flow: Corresponding value of annual cash flow	Project Life: Random number	Project Life: Corresponding value of project life	Net present value
1	53	3,000	97	9	4277
2	66	3,500	99	10	8506
3	30	2,500	81	7	(829)
4	19	2,000	09	4	(7660)
5	31	2,500	67	6	(2112)
6	81	3,500	70	7	4039
7	38	3,000	75	7	1605
8	48	3,000	83	7	1605
9	90	4,000	33	5	2163
10	58	3,000	52	6	66

Obtaining Probability Distributions of Basic Variables

Defining the probability distributions of basic variables is an important step in simulation. In defining these distributions it must be borne in mind that often it is impossible to find the true distributions. The distributions that are defined in practice are based on the judgment of experts. Great care should be exercised in translating the judgment of experts into probability distributions.

Two approaches may be used for obtaining probability distributions. These may be called the *'portrait'* approach and the *'building block'* approach.

The portrait approach is similar to the portrait method used for identifying suspects. According to this approach a standard probability distribution (normal, beta, chi-square, poisson, uniform, exponential, or any other) is drawn up, usually by a statistician, on the basis of the judgment expressed by the expert (informant). This is shown to the expert for his comments. The expert may suggest changes if the distribution does not conform with his judgment. For example, he may suggest that the probabilities at the tails should be greater or the probability of the modal value should be higher. The statistician modifies the earlier distribution to incorporate the changes suggested by the expert is satisfied that the probability distribution represents his judgment well.

This method suffers from a major limitation. The expert may accept a smooth distribution because he may be beguiled by the appearance of smooth curves and charmed by complicated formulae.

In the second approach, the 'building block' approach the probability distribution is defined by the expert. He attempts to quantity his judgment by a procedure which is as follows: (i) he chooses the range encompassing possible values; (ii) he divides the range into intervals which he thinks have different probabilities associated with them; (iii) he assigns probabilities to these intervals such that $\sum p_i = 1$; (iv) he may divide intervals into sub-intervals if he feels that the probabilities within an interval are different; and (v) he continues this process till he arrives at a distribution which represents his judgment well.

This process often leads to a step rectangular distribution and has the following advantages (i) the expert has complete freedom in expressing his judgment; and (ii) it squares well with the principle of using all available information, no more no less.

Some Commonly Used Distributions

Defining probability distributions foı the variables which reflect expert judgment is a very critical element of risk analysis. It is also regarded as one of the most troublesome aspects of risk analysis. In this context we may ask, how useful are certain distributions in reflecting expert judgments? To answer this question, we will look at the following distributions and consider their utility:

- Uniform distribution
- Trapezoidal distribution
- Step rectangular distribution
- Normal distribution

These distributions are shown graphically in Exhibit 11.9.

Uniform Distribution This distribution may be employed where the expert, because of vagueness in his judgment, is unable to differentiate between values within the range of the variable. Such a distribution, however, makes very little sense. For example, it may be absurd to claim that the project has an equal probability of costing anywhere between Rs 20 million and Rs 30 million, but will not under any circumstance cost Rs 19.99 million or Rs 30.01 million. Hence, the uniform distribution should be used only for variables which are not critical or when the appraiser wants to play safe by over-estimating the probability of extreme values in the range of the variables.

Trapezoidal Distribution Under normal circumstances, a variable lies in a small range around its best estimate. Under unusual circumstances, of course, it may fall outside this range. However, the likelihood of a value tends to diminish the farther the value is from the range in which it would normally fall. The trapezoidal distribution represents these possibilities rather well. Hence, it appears to reflect a large class of subjective judgments quite satisfactorily.

Step Rectangular Distribution A commonly used distribution in practice, the step rectangular distribution may be viewed as a refinement over the trapezoidal distribution. It enables the expert to divide the range of possible values into a few intervals and assign different probabilities to these intervals.

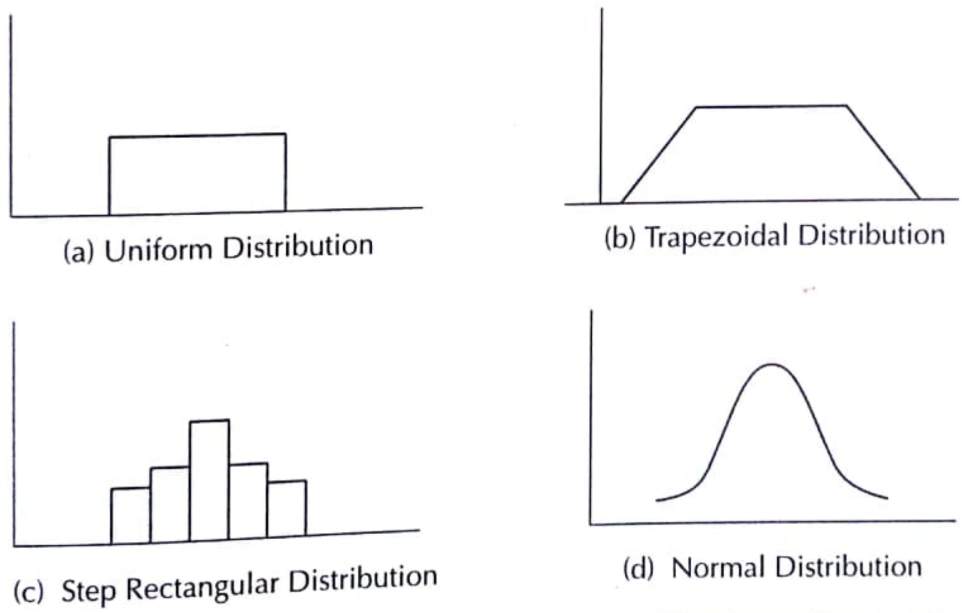
Normal Distribution This is a very important theoretical distribution. However, it is of limited use in risk analysis wherein the variations that are sought to be captured are caused neither by statistical errors nor random disturbances.

Problem of Correlation

We considered an example in which it was assumed that the probability distributions of various factors affecting the NPV were independent. In practice, correlations may exist among the distribution of several factors. For example, the number of units sold may be correlated with the price per unit.

When such a dependency exists the factors which are correlated should be considered together. For this purpose, the joint probability distribution of correlated factors has to be developed. This adds immensely to the problem of estimation.

Exhibit 11.9 ***Some Probability Distributions***



(a) Uniform Distribution

(b) Trapezoidal Distribution

(c) Step Rectangular Distribution

(d) Normal Distribution

In this context we must consider the choice relating to the level of disaggregation. The nature of this choice may be illustrated with an example. In an investment project the cost of production may be considered at different levels of details as shown in Exhibit 11.10. Now the problem is, to which level of detail should we go? Should we define the probability distribution of cost of production without explicitly considering the probability distribution of various elements like cost of raw materials, cost of fuels and utilities, cost of manpower? Or, should we consider explicitly the distributions of these elements? Should we go further and consider explicitly the distributions of sub-elements like cost of imported raw materials, so on and so forth?

Ideally, the greater the degree of disaggregation, the better it is because it contributes to clarity of judgement. However, disaggregated analysis calls for considering correlations explicitly, which is often a difficult task. By limiting the degree of disaggregation we consider correlations implicitly. If we choose to define the probability distribution of cost of production, without doing disaggregated analysis, we do not have to consider explicitly the correlations between, say, cost of raw material and cost of fuel. The distribution of cost of production would implicitly consider this.

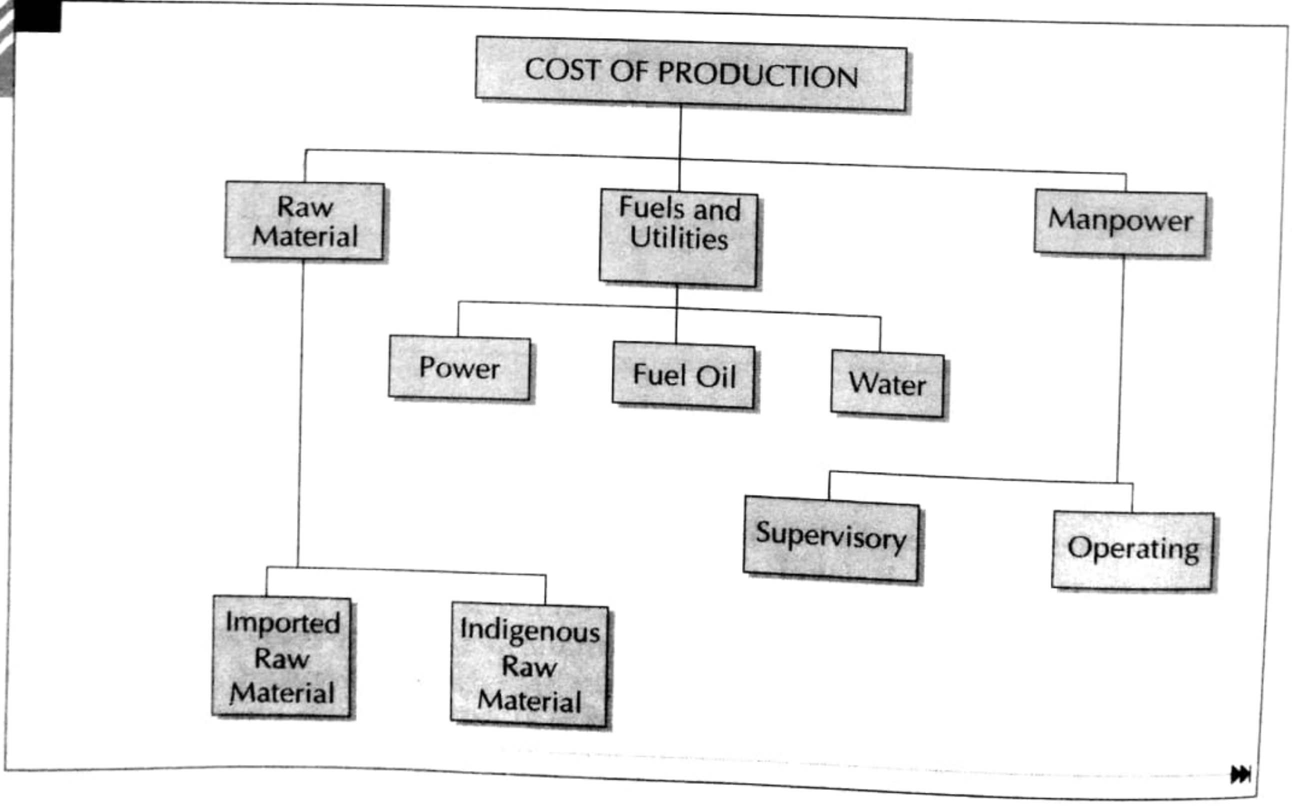
The choice of the level of aggregation or disaggregation would be finally based on the trade-off between the advantages of clarity of judgement and the complexities of disaggregated analysis. Since the influence of correlations is more significant than that of the shape of any particular distribution, it may be preferable to limit disaggregation.

Issues in Applying Simulation

Some of the important issues in the application of simulation are:

- What should the output be?
- Is project variability enough?
- How should the extreme values be used?
- How should the results of simulation be used?

Exhibit 11.10 ***Levels of Detail***



What Should the Output Be? Typically the output of simulation is the probability distribution of internal rate of return or net present value. The problem with the probability distribution of internal rate of return is that it only shows what the risk will be in the long run. It does not reflect the risk borne by investors in the capital market, which is captured by the distributions of successive one-period rates of return investors will earn, when investment is made in the project.

Is the probability distribution of net present value a better alternative? Here there is a problem about the discount rate to be used in the net present value calculation. If the cost of capital (which reflects the risk factor) is used as the discount rate, any further risk adjustment would mean double counting. If the risk-free rate of interest is used as the discount rate, then the underlying assumption is that the uncertainty about the project's cash flows would be resolved almost immediately. Since uncertainty is not resolved in this manner, the meaning of the simulated probability distribution is somewhat unclear.

In view of the problems associated with the distributions of internal rate of return and net present value, it perhaps makes sense to look at the distributions of cash flows (or earnings). If a project has an expected economic life of eight years, the project evaluator may examine eight separate cash flow distributions. These distributions may be scaled by their respective expected values for comparisons to be made between the projects.

Is Project Variability Enough? According to the Capital Asset Pricing Model (this model is discussed in following chapter), what matters is the systematic (non-diversifiable) risk of an investment and not its unsystematic (diversifiable) risk. Simulation can provide information on the total risk (which consists of systematic risk plus unsystematic risk), but it cannot separately show the systematic risk. Hence, a user of simulation must regard total risk as a proxy for systematic risk. Even though this is a strong assumption, it may not be unreasonable because empirical studies show a fairly reliable association between systematic risk and various measures of earnings volatility. Further, systematic risk tends to be proportional to total risk for firms operating in a single homogeneous industry.

How Should the Extreme Values Be Interpreted? The proponents of simulation argue that, *inter alia,* it helps in answering questions like "What is the probability that NPV would be less than zero?" or "What is the probability that IRR would be less than *x* per cent?" These questions can be answered satisfactorily when the "tails" of the simulated distribution are reliable. However, the tails tend to be the least reliable part of the simulated distribution.

There is yet another, and perhaps more fundamental, problem in interpreting the tails of the simulated distribution. When a simulation model is constructed, it is assumed that the management will follow a strategy based on general business conditions it expects to encounter. Hence, the model cranks out numbers on the basis of "business-as-usual" strategy even when it runs into surprises. In real life situations, however, managements modify their strategies to cope with surprises. When adverse developments occur, new actions are likely to be initiated. In extreme cases, the project may be abandoned to save future losses. Thus, the tails of the simulated distribution, which are important elements in risk assessment, are likely to reflect unrealistic management strategies.

How Should the Results of Simulation Be Used? Hertz[3] advocated simulation on the grounds that a probability distribution of the criterion of merit (NPV or IRR) leads to better decisions. This implies that a decision maker is able to reach a decision on the basis of the probability distribution. Not many executives, however, may be capable of reviewing the probability distribution of project's NPV or IRR and deciding confidently whether the project is worthwhile or not. (Of course, there will be some projects which will be sure winners and some projects which will be losers—in these cases, however, simulation itself may be redundant.) A better approach to use the results of simulation may call for a two-stage decision making process:

Stage 1 Review the probability distributions of the project (i.e. probability distributions of cash flows, IRR, etc.) along with other factors and make an assessment.

Stage 2 Define the appropriate discount rate to be employed on the basis of the business risk assessment in Stage 2.

World Bank's Experience

It may be instructive here to review the experience of World Bank. The following are its summary remarks on simulation.

1. Simulation is a powerful technique which permits use of a great deal of information which would otherwise be lost.
2. It is a highly efficient medium of communication.
3. It is not a technique which replaces skilled judgment. On the contrary, it often requires the use of far more judgment than the traditional analysis.
4. Despite the method's value, the treatment of correlations between variables remains a major problem. It is clear that results can be completely misleading if correlations are not handled properly.

Evaluation

An important tool of risk analysis, simulation offers certain advantages:

- Its principal strength lies in its versatility. It can handle problems characterised by (a) numerous exogenous variables following any kind of distribution, and (b) complex

3. David B. Hertz, "Risk Analysis in Capital Investment", *Harvard Business Review* (Jan–Feb 1964, pp 95–106).

interrelationships among parameters, exogenous variables, and endogenous variables. Such problems often defy the capabilities of analytical methods.

- It compels the decision maker to explicitly consider the interdependencies and uncertainties characterising the project.

Simulation, however, is a controversial tool which suffers from several shortcomings.

- It is difficult to model the project and specify the probability distributions of exogenous variables.
- Simulation is inherently imprecise. It provides a rough approximation of the probability distribution of net present value (or any other criterion of merit). Due to its imprecision, the simulated probability distribution may be misleading when a tail of the distribution is critical.
- A realistic simulation model, likely to be complex, would most probably be constructed by a management scientist, not the decision maker. The decision maker, lacking understanding of the model, may not use it.
- To determine the net present value in a simulation run the risk-free discount rate is used. This is done to avoid prejudging risk which is supposed to be reflected in the dispersion of the distribution of net present value. Thus the measure of net present value takes a meaning, very different from its usual one, that is difficult to interpret.

11.7 DECISION TREE ANALYSIS

The scientists at Pharmalab have come up with a new molecule. The firm is ready for pilot production which is estimated to cost Rs 8 million and take one year. If the results of pilot production are encouraging the next step would be to test market the product. This will cost Rs 3 million and take two months. Based on the outcome of the test marketing, a manufacturing decision may be taken. The firm may, however, skip the test marketing phase and take a decision whether it should manufacture the product or not. If the firm decides to manufacture the product commercially it is confronted with two options: a small plant or a large plant. This decision hinges mainly on the size of the market. While the level of demand in the short run may be gauged by the results of the test market, the demand in the long run would depend on how satisfied the initial users are.

If the firm builds a large plant initially it can cater to the needs of the market when demand growth is favourable. However, if the demand turns out to be weak, the plant would operate at a low level of capacity utilisation. If the firm builds a small plant, to begin with, it need not worry about a weak market and the consequent low level capacity utilisation. However, if the market turns out to be strong it will have to build another plant soon (and thereby incur a higher total outlay) in order to save itself from competitive encroachment.

To analyse situations of this kind where sequential decision making in the face of risk is involved, decision tree analysis is a useful tool. This section discusses the technique of decision tree analysis.

Steps in Decision Tree Analysis

The key steps in decision tree analysis are:

1. Identifying the problem and alternatives
2. Delineating the decision tree
3. Specifying probabilities and monetary outcomes
4. Evaluating various decision alternatives

Identifying the Problem and Alternatives To understand the problem and develop alternatives, information from different sources—marketing research, engineering studies, economic forecasting, financial analysis, etc.—has to be tapped. Imaginative effort must be made to identify the nature of alternatives that may arise as the decision situation unfolds itself and assess the kinds of uncertainties that lie ahead with respect to market size, market share, prices, cost structure, availability of raw material and power, technological changes, competitive action, and governmental regulation.

Recognising that risk and uncertainty are inherent characteristics of investment projects, persons involved in analysing the situation must be encouraged to express freely their doubts, uncertainties, and reservation and motivated to suggest contingency plans and identify promising opportunities in the emerging environment.

Delineating the Decision Tree The decision tree, exhibiting the anatomy of the decision situation, shows:

- The decision points (also called decision forks) and the alternative options available for experimentation and action at these decision points.
- The chance points (also called chance forks) where outcomes are dependent on a chance process and the likely outcomes at these points.

The decision tree reflects in a diagrammatic form the nature of the decision situation in terms of alternative courses of action and chance outcomes which have been identified in the first step of the analysis.

A decision tree can easily become very complex and cumbersome if an attempt is made to consider the myriad possible future events and decisions. Such a decision tree, however, is not likely to be a very useful tool of analysis. Over-elaborate, it may obfuscate the critical issues. Hence an effort should be made to keep the decision tree somewhat simple so that the decision makers can focus their attention on major future alternatives without being drowned in a mass of trivia. One must remember the advice of Brealey and Myers: "Decision trees are like grapevines; they are productive only if vigorously pruned."[4]

Specifying Probabilities and Monetary Values for Outcomes Once the decision tree is delineated, the following data have to be gathered:

- Probabilities associated with each of the possible outcomes at various chance forks, and
- Monetary value of each combination of decision alternative and chance outcome.

The probabilities of various outcomes may sometimes be defined objectively. For example, the probability of a good monsoon may be based on objective, historical data. More often, however, the possible outcomes encountered in real life are such that objective probabilities for them cannot be obtained. How can you, for example, define objectively the probability that a new product like an electric moped will be successful in the market? In such cases, probabilities have to be necessarily defined subjectively. This does not, however, mean that they are drawn from a hat. To be useful they have to be based on the experience, judgment, intuition, and understanding of informed and knowledgeable executives. Assessing the cash flows associated with various possible outcomes, too, is a difficult task. Again, the judgment of experts play an important role.

Evaluating the Alternatives Once the decision tree is delineated and data about probabilities and monetary values gathered, decision alternatives may be evaluated as follows:

1. Start at the right-hand end of the tree and calculate the expected monetary value at various chance points that come first as we proceed leftward.
2. Given the expected monetary values of chance points in step 1, evaluate the alternatives at the final stage decision points in terms of their expected monetary values.
3. At each of the final stage decision points, select the alternative which has the highest expected monetary value and truncate the other alternatives. Each decision point is assigned a value equal to the expected monetary value of the alternative selected at that decision point.
4. Proceed backward (leftward) in the same manner, calculating the expected monetary value at chance points, selecting the decision alternative which has the highest expected monetary value at various decision points, truncating inferior decision alternatives, and assigning values to decision points, till the first decision point is reached.

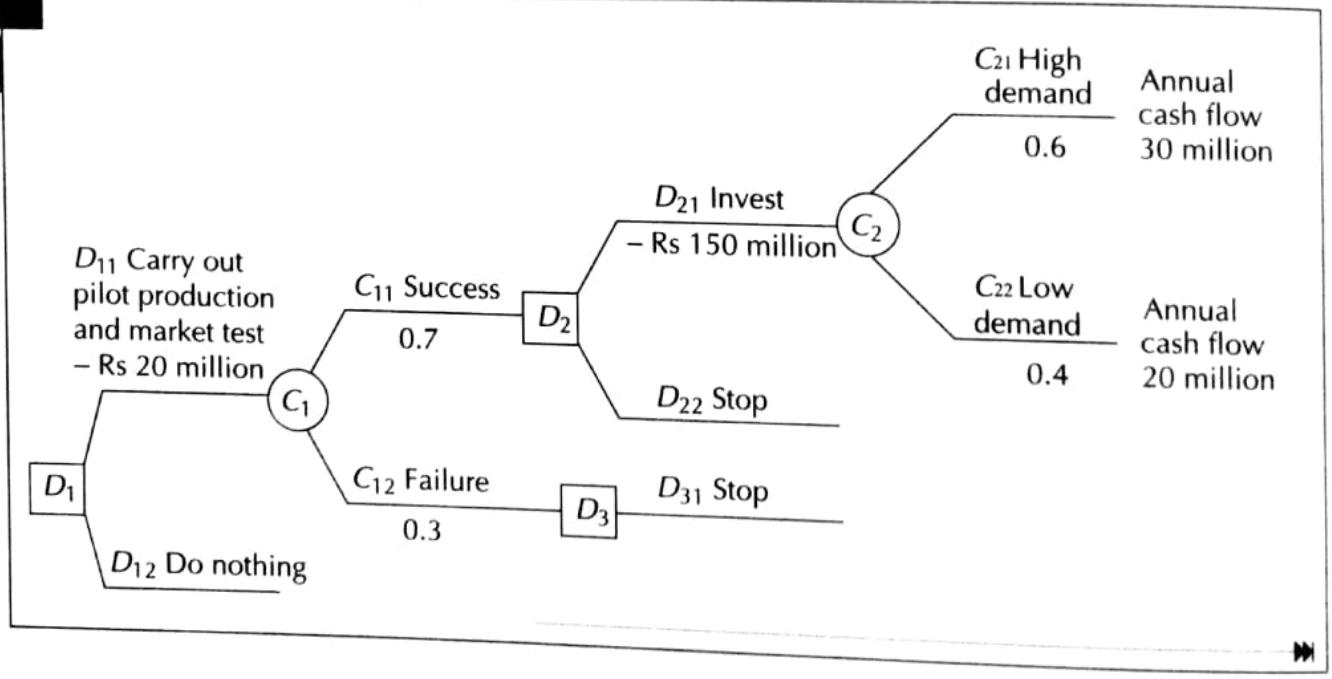
[4] R. Brealey and S. Myers, *Principles of Corporate Finance*, New York, McGraw-Hill Company, 1981, p. 212.

A Simple Example

The scientist at Spectrum have come up with an electric moped. The firm is ready for pilot production and test marketing. This will cost Rs 20 million and take six months. Management believes that there is 70 per cent chance that the pilot production and test marketing will be successful. In case of success, Spectrum can build a plant costing Rs 150 million. The plant will generate an annual cash inflow of Rs 30 million for 20 years if the demand is high or an annual cash inflow of Rs 250 million if the demand is low. High demand has a probability of 0.6; low demand has a probability of 0.4. What is the optimal course of action using decision tree analysis?

The decision tree for the electric moped project of Spectrum is shown in Exhibit 11.11.

Exhibit 11.11 ***Decision Tree***



The optimal course of action is determined as follows:

1. Start at the right-hand end of the tree and calculate the expected monetary value (EMV) at chance point C_2 that comes first as we proceed leftward.

$$\text{EMV } (C_2) = 0.6\ [30 \times \text{PVIFA } (20, 12\%)] + 0.4\ [20 \times \text{PVIFA } (20, 12\%)]$$
$$= \text{Rs } 194.2 \text{ million}$$

2. Evaluate the EMV of the decision alternatives at D_2 the last stage decision point.

Alternative	*EMV*
D_{21} (Invest Rs 150 million)	Rs 44.2 million
D_{22} (Stop)	0

3. Select D_{21} and truncate D_{22} as EMV (D_{21}) > EMV (D_{22})
4. Calculate the EMV at chance point C_1 that comes next as we roll backwards.

$$\text{EMV } (C_1) = 0.7\ [44.2] + 0.3\ [0]$$
$$= \text{Rs } 30.9 \text{ million}$$

5. Evaluate the EMV of the decision alternatives at D_1 the first stage decision point

Alternative	*EMV*
D_{11} (Carry out pilot production and market test at a cost of Rs 20 million)	Rs 10.9 million
D_{12} (Do nothing)	0

Based on the preceding evaluation, we find that the optimal decision strategy is as ollows: Choose D_{11} (carry out pilot production and market test) at the decision point D_1 and wait for the outcome at the chance point C_1. If the outcome at C_1 is C_{11} (success), nvest Rs 150 million, if the outcome at C_1 is C_{12} (failure) stop.

A Tougher Example[1]

Airways Limited has been set up to run an air taxi service in western India. The company s debating whether it should buy a turboprop aircraft or a piston engine aircraft. The turboprop aircraft costs 3500 and has a larger capacity. It will serve if the demand turns out to be high. The piston engine aircraft costs 1800 and has a smaller capacity. It will serve if the demand is low, but it will not suffice if the demand is high.

The company believes that the chances of demand being high and low in year 1 are 0.6 and 0.4. If the demand is high in year 1, there is an 80 percent chance that it will be high in subsequent years (year 2 onward) and a 20 percent chance that it will be low in subsequent years.

The technical director of Airways Limited thinks that if the company buys a piston engine aircraft now and the demand turns out to be high the company can buy a second-hand piston engine aircraft for 1400 at the end of year 1. This would double its capacity and enable it to cope reasonably well with high demand from year 2 onwards.

The payoffs associated with high and low demand for various decision alternatives are shown in Exhibit 11.12.The payoffs shown for year 1 are the payoffs occurring at the end of year 1 and the payoffs shown for year 2 are the payoffs for year 2 and the subsequent years, evaluated as of year 2, using a discount rate of 12 percent which is the weighted average cost of capital for Airways Limited.

[1] Adapted from Brealey and Myers op. cit.

operation and disposing off the aircraft at the end of year 1, ... so. Suppose after 1 year of use the turboprop aircraft can be sold for Rs. 3,600 and th piston-engine aircraft for 1400.

If the demand in year 1 turns out to be low, the payoffs for 'continuation' an 'abandonment' as of year 1 are as follows.

	Turboprop Aircraft		Piston Engine Aircraft
Continuation:	0.4 (7000) + 0.6(600)	Continuation:	0.2(2500) + 0.8 (800)
	= 3160/(1.12) = 2821		= 1140/(1.12) = 1018
Abandonment:	3600	Abandonment:	1400

Thus in both the cases it makes sense to sell off the aircraft after year 1, if the demand i year 1 turns out to be low.

The revised decision tree, taking into account the abandonment options, is shown i Exhibit 11.13.

Given the decision tree with abandonment possibilities, let us calculate the NPV of th turboprop aircraft and the piston engine aircraft.

$$\text{NPV (Turboprop)} = -4000 + \frac{0.6[1000 + \{0.8(7000) + 0.2(1000)\} / (1.12)] + 0.4(200 + 3600)}{(1.12)}$$

$$= 667$$

$$\text{NPV (Piston engine)} = -1800 + \frac{0.6(500 + 2993) + 0.4(300 + 1400)}{(1.12)}$$

$$= 641$$

Note that the possibility of abandonment increases the NPV of the Turboprop aircraf from 389 to 667. This means that the value of the option to abandon is:

= NPV with abandonment - NPV without abandonment

Value of abandonment option = 667 – 389 = 278

For the piston engine aircraft the possibility of abandonment increases the NPV fror 505 to 641. Hence the value of the *abandonment option* is 136.

Evaluation

Decision trees are useful for analysing a project that has the following characteristics:

- Decisions on continuing the project are made in well-defined stages.

- The outcomes at each stage fall into few broad classes.
- The probabilities and the cash flows associated with various outcomes can be specified at the beginning of the project. This means that the firm has experience of doing similar projects in the past.

Obviously, decision tree analysis requires enormous information before it can be applied. The oil drilling project is one case where the required information may be available. However, it may be much more difficult to apply decision tree analysis to a project where the product or service is new and the firm has very little information on how the market will respond to it. Decision trees are also not easy to apply when investments are gradually made over a period of time rather than in a few well-defined stages.

11.8 MANAGING RISK

Managers are not merely content with measuring risk. They want to explore ways and means of mitigating risk. Some of the ways of doing this are discussed below. These risk reduction strategies have a cost associated with them, and whether they are profitable in a given situation will depend on circumstances.

Fixed and Variable Costs A common way to modify the risk of an investment is to change the proportion of fixed and variable costs. For example, in the early 1980s Ford Motor Company restructured its operations. Essentially it decided to buy most of its components from outside suppliers instead of manufacturing them in-house. This decreased its fixed costs and increased its variable costs. The net effect was that its break-even level declined.

Pricing Strategy Pricing strategy is used by many firms to manage risk. A lower price increases potential demand, but also raises the break-even level. This is the reason why publishers first bring out a hard-cover edition at a higher price and then introduce a soft-cover edition at a lower price.

Sequential Investment If you are not sure about the market response to your product or service, you may start small and later expand as the market grows. This strategy may entail higher capital cost per unit because capacity is created in stages. However, it reduces risk exposure. You can employ decision tree analysis to hammer out the optimal sequence of investment in face of risk.

Improving Information An African proverb says *don't test the depth of a river with both feet*. You may like to gather more information about the market and technology before taking the plunge. Additional study often improves the quality of forecasts but involves direct costs (the cost of the study) as well as opportunity costs of delayed action. You have to weigh the costs and benefits of further study and decide how much of additional information should be gathered.

Financial Leverage We discussed how reducing the proportion of fixed operating costs lowers risk. Likewise reducing the dependence on debt lowers risk. Remember that debt entails a definite contractual commitment whereas equity carries no fixed burden. Hence if the operating risk of the project is high, it makes sense to go for a low level of financial leverage.

Insurance You can get an insurance cover against a variety of risks like physical damage, theft, loss of key person, and so on. Insurance is a pure antidote for such risks. Of course to protect yourself against such risks you have to pay insurance premium.

Long-term Arrangements One way to mitigate risk is to enter into long-term arrangements with suppliers, employees, lenders, and customers. A long-term contract with suppliers ensures availability of inputs at a predictable price; a long-term wage contract with employees removes uncertainty about employee cost; a long term debt contract reduces risk about interest rate; finally, a long-term sales contract with customers eliminates revenue risk.

Often long-term contracts are indexed. This means that the prices are periodically adjusted in line with the movement of some index which essentially reflects inflation. For example, a supply contract may have an escalator clause that links the supply price to some price index like the Wholesale Price Index. Price indexing protects both the buyer and the seller against inflation risk because indexing ensures that the real price (price in terms of purchasing power) is constant.

Strategic Alliance When the resources required for a project or the risks inherent in a project are beyond the capacity of a single company, strategic alliance may be the way out. A strategic alliance, also referred to as a joint venture, represents a partnership between two or more independent companies which join hands to achieve a common purpose. It is usually organised as a newly created company, though the partners may choose any other form of organisation. Typically, the partners partake in the equity of the common enterprise, contribute resources (technology, facilities, distribution networks, brands, key manpower, and so on), and share management and control. The massive resource requirements and huge risks in modern enterprises have compelled many traditional rivals to work together. Competitors are beginning to cooperate leading to a phenomenon called as 'cooptition'.

Derivatives Derivative instruments like options and futures can be used for managing risk. An option gives its owner the right to buy or sell an underlying asset on or before a given date at a predetermined price. An option to buy is a call option. Options gives flexibility which is very valuable in volatile markets. For example, a call option embedded in a debt instrument gives the issuing firm the right to prematurely redeem (buy back) the debt instrument at a certain price. Such an option is very valuable when the interest rate falls.

A futures contract is an agreement between two parties to exchange an asset for cash at a predetermined future date for a price that is specified today. Futures contracts eliminate price risk. For example, a refinery may buy an oil futures contract for its oil requirement. Doing so entitles the refinery to get delivery of oil at a specified future date at a price that is fixed today.

11.9 PROJECT SELECTION UNDER RISK

Once information about expected return (measured as net present value or internal rate of return or some other criterion of merit) and variability of return (measured in terms of range or standard deviation or some other risk index) has been gathered, the next question is, should the project be accepted or rejected. There are several ways of incorporating risk in the decision process: judgmental evaluation, payback period requirement, risk profile method, certainty equivalent method, and risk adjusted discount rate method.

Judgmental Evaluation

Often managers look at risk and return characterises of a project and decide judgmentally whether the project should be accepted or rejected, without using any formal method for incorporating risk in the decision making process. The decision may be based on the collective view of some group like the capital budgeting committee, or the executive committee, or the board of directors. If judgmental decision making appears highly subjective or haphazard, consider how most of us make important decisions in our personal life. We rarely use formal selection methods or quantitative techniques for choosing a career or a spouse or an employer. Instead, we rely on our judgment.

Payback Period Requirement

In many situations companies use NPV or IRR as the principal selection criterion, but apply a payback period requirement to control for risk. Typically, if an investment is considered more risky, a shorter payback period is required even if the NPV is positive or IRR exceeds the hurdle rate. This approach assumes that risk is a function of time.

Ordinarily it is true that the farther a benefit lies in future the more uncertain it is likely to be because economic and competitive conditions tend to change overt time. However, risk is influenced by things other than the mere passage of time. Hence the payback period requirement may not be an adequate method for risk adjustment or control.

Risk Adjusted Discount Rate Method

The risk adjusted discount rate method calls for adjusting the discount rate to reflect project risk. If the risk of the project is equal to the risk of the existing investment of the firm, the discount rate used is the average cost of capital of the firm; if the risk of the project is greater than the risk of the existing investments of the firm, the discount rate used is higher than the average cost of capital of the firm; if the risk of the project is less than the risk of the existing investments of the firm the discount rate used is less than the average cost of capital of the firm. The risk adjusted discount rate is:

(11.8)

where r_k = risk-adjusted discount rate for project k
i = risk-free rate of interest
n = adjustment for the firm's normal risk
d_k = adjustment for the differential risk of project k

It may be noted that $(i+n)$ measures the firm's cost of capital, and that d_k may be positive or negative depending on how the risk of the project under consideration compares with the existing risk of the firm.

The adjustment for the differential risk of project k, quite understandably, depends on management's perception of the project risk and management's attitude towards risk (risk-return preference). A large pharmaceutical concern, for example, uses the following risk-adjusted discount rates for various types of investments.

Investment Category	*Risk-adjusted Discount Rate*
Replacement investments	Cost of capital
Expansion investments	Cost of capital + 3%
Investments in related lines	Cost of capital + 6%
Investments in new lines	Cost of capital + 10%

Once the project's risk-adjusted discount rate (r_k) is specified, the project is accepted if its net present value, calculated as follows, is positive.

$$\text{NPV} = \sum_{t=1}^{n} \frac{\bar{A}_t}{(1+r_k)^t} - I \tag{11.9}$$

where NPV = net present value of project k
$\bar{A}_t$ = expected cash flow for year t
r_k = risk adjusted discount rate for project k

Example The expected cash flows of a project, which involves an investment outlay of Rs 1,000,000, are as follows:

Year	*Cash flow (Rs)*
1	200,000
2	300,000
3	400,000
4	300,000
5	200,000

The risk-adjusted discount rate for this project is 18 percent. Is the project worthwhile? The net present value of the project, using the risk-adjusted discount rate is:

11.10 RISK ANALYSIS IN PRACTICE[5]

Several methods to incorporate the risk factor into capital expenditure analysis are used in practice. The most common ones are discussed here.

Conservative Estimation of Revenues In many cases the revenues expected from a project are conservatively estimated to ensure that the viability of the projects is not easily threatened by unfavourable circumstances. The capital budgeting systems often have built-in devices for conservative estimation. This is indicated by the following remarks made by two executives:

"We ask the project sponsor to estimate revenues conservatively. This checks the optimism common among project sponsors."

"The capital budgeting committee requires justification for revenue figures given by those who propose capital expenditures. This has a sobering effect on them."

Safety Margin in Cost Figures A margin of safety is generally included in estimating cost figures. This varies between 10 percent and 30 percent of what is deemed as normal cost. The size of the margin depends on what management feels about the likely variation in cost. The following observation suggests this:

"In estimating the cost of raw material we add about 20 to 25 percent to the current prices as the raw material price is not stable and often we pay a high price to get it. For labour cost we add about 10 to 12 percent as this is the annual increase."

Flexible Investment Yardsticks The cut-off point an investment varies according to the judgment of management about the riskiness of the project. In one company replacement investments are okayed if the expected post-tax return exceeds 15 percent but new investments are undertaken only if the expected post-tax return is greater than 20 percent. Another company employs a short payback period of three year for new investments. Its decision rule was stated by its financial controller as follows:

"Our policy is to accept a new project only if it has a payback period of three years. We have never, as far as I know, deviated from this. The use of a short payback period automatically weeds out risky projects."

Acceptable Overall Certainty Index Some companies calculate what may be called the overall certainty index, based on a few crucial factors affecting the success of the project.

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[5]The discussion in this section is based on a survey conducted by the author which covered 20 firms. The findings of this survey are reported in "Risk Analysis in Capital Expenditures", *Indian Management*, October 1975.

The calculation of an overall certainty index may be illustrated by an example drawn m a large engineering concern. The overall index for a capital expenditure considered this firm was calculated as follows:

	Certainty index %
Raw material availability	70
Power availability	60
Freedom from competition	80
Overall certainty $= \dfrac{70+60+80}{3} =$	70

This company accepted the project because it regarded a 70 percent level of overall rtainty as satisfactory.

ıdgment on Three Point Estimates In some companies three estimates are developed for ıe or more aspects of the proposed investments. The top management or the board of irectors decides on the basis of such information. Two examples of this are given here:

In a pharmaceutical company sponsors are required to give three estimates of rate of eturn: most pessimistic, most likely, and most optimistic.

In a shipping company three estimates, labelled high, medium, and low respectively, are eveloped for proposed investments.

valuation The methods of conservative estimation of revenues, safety margin in cost igures, and flexible investment yardstick, in common practice, do not generally employ xplicitly defined probability distributions, which seem alien to current practices. They are ased on subjective, implicit evaluations. The three-point estimates method is based on hree points in the range of variation and provides some idea of probability distribution. The overall certainty index method is based on two-level discrete probability distributions of important factors affecting the outcome of investments. However, the manner in which these probabilities are used for developing the overall certainty index of a project whose success is dependent on the attainment of three states A, B and C, is $p(\text{A}) \times p(B/A) \times p(\text{C}/\text{A} \,\&\, \text{B})$ and not $[p(\text{A}) + p(\text{B}) + p(\text{C})]/3$ as defined by the overall certainty index method used in practice.

Room for Improvement A significant improvement in the analysis of the risk factor in practice can be brought about if probability distributions of the key factors underlying investments are developed and information is communicated in this form. This means the use of (i) subjectively understood phrases like the 'most likely return', 'low chances of failure' and (ii) single point estimates will have to be replaced by probability distributions. This has certain advantages.

The calculation of an overall certainty index may be illustrated by an example drawn from a large engineering concern. The overall index for a capital expenditure considered by this firm was calculated as follows:

	Certainty index %
Raw material availability	70
Power availability	60
Freedom from competition	80

$$\text{Overall certainty} = \frac{70 + 60 + 80}{3} = 70$$

This company accepted the project because it regarded a 70 percent level of overall certainty as satisfactory.

Judgment on Three Point Estimates In some companies three estimates are developed for one or more aspects of the proposed investments. The top management or the board of directors decides on the basis of such information. Two examples of this are given here:

In a pharmaceutical company sponsors are required to give three estimates of rate of return: most pessimistic, most likely, and most optimistic.

In a shipping company three estimates, labelled high, medium, and low respectively, are developed for proposed investments.

Evaluation The methods of conservative estimation of revenues, safety margin in cost figures, and flexible investment yardstick, in common practice, do not generally employ explicitly defined probability distributions, which seem alien to current practices. They are based on subjective, implicit evaluations. The three-point estimates method is based on three points in the range of variation and provides some idea of probability distribution. The overall certainty index method is based on two-level discrete probability distributions of important factors affecting the outcome of investments. However, the manner in which these probabilities are used for developing the overall certainty index of a project whose success is dependent on the attainment of three states A, B and C, is $p(A) \times p(B/A) \times p(C/A \& B)$ and not $[p(A) + p(B) + p(C)]/3$ as defined by the overall certainty index method used in practice.

Room for Improvement A significant improvement in the analysis of the risk factor in practice can be brought about if probability distributions of the key factors underlying investments are developed and information is communicated in this form. This means the use of (i) subjectively understood phrases like the 'most likely return', 'low chances of failure' and (ii) single point estimates will have to be replaced by probability distributions. This has certain advantages.

1. *It would alleviate ambiguity in communication.* When judgments about uncertainty are transmitted in qualitative phrases they are likely to be interpreted in a sense different from what the communicator has in mind because of the vagueness of communication. Such vagueness is reduced if judgments are quantified in terms of probability distribution.
2. *It would reduce bias in reporting.* When managers and other informants are asked to provide single-point estimates (a common practice) they often tend to underestimate revenues and overestimate costs when performance evaluation emphasises negative variations. If there is no performance evaluation, revenues may be overestimated and costs underestimated for projecting a favourable picture of a project. Such tendencies may be checked to some extent if managers and other informants are asked to provide probability distributions instead of single point estimates.
3. *It utilises more information.* When an expert, say, a market analyst, is asked to give an estimate of market demand in terms of a single number, he has to necessarily discard a good deal of his knowledge about the market which otherwise can be communicated in the form of a probability distribution.
4. *It is convenient.* With experience it is easy to express judgments in probabilistic terms than in terms of a single-point estimate. Often experts find it convenient to communicate their judgment in terms of probability distribution rather than in the form of a single-point estimate.