CHAPTER 4

EQUIVALENT ANNUAL-WORTH COMPARISONS

In its simplest form, justification is provided by the mathematics of answering the question, "Will I make more money by following procedure A, or procedure B?"

Joseph Harrington, Jr.[†]

Dr. Harrington is credited with inventing the phrase *computer-integrated manufacturing (CIM),* whereby integration of all the functions within a manufacturing enterprise leads to a better way of producing a product. These functions include communications, design, production, assembly, marketing, and so on. Harrington did not envisage that all functions within the enterprise would have to be computerized, but the intervening years that have elapsed since the term CIM was coined have seen more and more computerization being implemented in manufacturing. Computer-controlled machining centers, commonly called *direct numerical control (DNC) machines,* may cost hundreds of thousands of dollars. Evaluation of these machines against older, more conventional processes will have to consider quality, productivity, equipment reliability, possible scheduling simplification, work in process reduction, and other potential benefits. Assuming a monetary value can be placed on all the factors that make up the decision process, how might the benefit and cost data be best presented to allow a careful and informed decision to be made? We saw, in Chap. 3, that present-worth evaluations are often preferred because of the relevancy of current timing to the decision maker. In some instances, as discussed also in Chap. 3, evaluation at a time in the future will sometimes be more logical. Many economic decisions can be assisted by determining

[†]Joseph Harrington, Jr., *Computer Integrated Manufacturing,* Krieger Publishing Company, Malabar, FL, reprint edition, 1979.

costs, expenditures, and net worth on the basis of annual or periodic timing. Manufacturing and other engineering alternative evaluations often just make more sense when viewed on an annual basis. The manufacturing manager, e.g., is often required to justify his operation on a monthly or yearly basis. Annual goals are frequently set for marketing and other people in the enterprise. But using a periodic base for economic decisions is not limited to engineering or manufacturing operations, as we shall see in the rest of this chapter.

Not only is it often meaningful to structure an investment solution in terms of annual payments, but also it can be a more convenient way to arrive at a present worth or rate of return (to be covered in Chap. 5). It will be shown that all three methods of analysis will indicate the same preference among engineering alternatives, though one might be more logical to utilize for a specific situation. For example, we will see that present-worth comparison of assets with unequal lives, as shown in Example 3.4, can be much easier to compute by first using an annual-worth comparison.

4.1 UTILIZATION OF EQUIVALENT ANNUAL-WORTH COMPARISON METHOD

With an annual-worth method, all the receipts and disbursements occurring over a period are converted to an equivalent uniform yearly amount.[†] It is a popular method because of the widespread inclination to view a year's gains and losses as a yardstick of progress. Cost accounting procedures, depreciation expenses, tax calculations, and other summary reports are annual. These yearly cost tabulations generally make the annual-worth method easier to apply and comprehend than the other comparison methods.

As mentioned earlier, equivalent annual-worth comparisons produce results compatible with present-worth and rate-of-return comparisons. For a set of common assumptions, a preference for an alternative exhibited by one method will be mirrored by the other two. Annual-worth calculations are frequently part of the computations required to develop present-worth and rate-of-return values, and parallel computations by different methods are useful for complementary comparisons that improve the clarity of an analysis.

The six conditions listed in Chap. 3 for basic present-worth comparisons also apply to basic annual-worth comparisons: cash flows and interest rates are known, cash flows are before taxes and in constant-value dollars, and comparisons include neither intangible considerations nor limits due to availability of financing. These restrictions are relaxed in later chapters.

[†]Equivalent uniform periodic amounts are not restricted to annual periods. The periods may be monthly, quarterly, and so on. A large number of engineering economic decisions are based on annual comparisons, and so the term *equivalent uniform yearly amount* is often used in this text, but the implication will be that the methodologies are just as applicable to other time periods.

4.1.1 Structure of a Capital Recovery Annuity

The cornerstone of annual-worth calculations is the capital recovery factor, which converts a lump sum to an equivalent annuity. This annuity usually represents an investment in an asset that is expected to generate a positive future cash flow, and the duration of the annuity is therefore the life of the asset. Since the cost of an asset is a cash outlay, the resulting annuity is a uniform series of negative payments. This negative cash flow is offset by the positive revenue produced by the asset in establishing the net equivalent annual worth of the investment.

The *capital recovery factor* $(A/P, i, N)$ accounts for both the repayment of invested capital P and the interest earned on the unrecovered portion of the investment. Although payments A are uniform in size, the proportion of capital recovered and interest earned changes each period. The structure of an annuity, in which varying amounts from the equal payments are allocated to capital recovery and interest, is best revealed by examining a sample application.

Assume that an asset is purchased for $40,000. It has an expected life of 4 years and no salvage value at the end of its life. The purchaser intends to recover the $40,000 investment over 4 years *plus* the interest the $40,000 would have earned if it had been invested elsewhere. If an acceptable interest rate is 10 percent, the series of equal payments that will return the capital plus interest is computed as

$$\begin{aligned}\text{Equivalent annual payment } A &= P(A/P, 10, 4)\\ &= \$40{,}000(0.31547) = \$12{,}619\end{aligned}$$

Every year the proportion of a payment allotted to capital recovery and interest changes because interest applies only on the amount of capital not yet recovered, and that amount changes each year. During year 1, before a payment is received, the $40,000 investment earns $40,000 × 0.10 = $4000 interest. The first payment of $12,619 then reduces the unrecovered capital by $12,619 − $4000 = $8619. During year 2, 10 percent interest is paid on $40,000 − $8619 = $31,381, amounting to $3138. Therefore, of the second payment, $3138 is interest, and $12,619 − $3138 = $9481 is allocated to capital recovery. The complete sequence of changing proportions is given in Table 4.1.

Anyone who has had a home mortgage or loan, or who has contemplated one, has probably been told that "You can reduce the total interest paid and the length of time it takes to pay off the mortgage by increasing the annual payments." If the $12,618.83 annual payment shown in Table 4.1 is considered to be a loan payment and is increased by 50 percent to $18,928.25, we get the capital (principal) and interest payments shown in Table 4.2. The final-period payment is less than $18,928.25 since only $9515.74 remains to fully recover the capital.

TABLE 4.1

Pattern of capital recovery and interest charges when capital recovery factor at $i = 10\%$ is applied to purchase of $40,000 asset with life of 4 years and no salvage value

End of period	Capital not recovered by end of period, $	Interest due on unrecovered capital, $	Amount of capital recovered, $	Period capital recovery charge, $
0	40,000.00			
1	31,381.17	4,000.00	8,618.83	12,618.83
2	21,900.45	3,138.12	9,480.72	12,618.83
3	11,471.67	2,190.05	10,428.79	12,618.83
4	0.00	1,147.17	11,471.67	12,618.83
		10,475.33	40,000.00	50,475.33

The total interest paid with the increase in annual payment is reduced from $10,475.33 to $7372.24, and the total amount paid over the four periods is reduced from $50,475.33 to $47,372.24. The time period over which the payments are made is reduced by 25 percent. Whether this type of reduction should be realistically considered is a function of the interest rate, what you can do with the money in the periods saved, and whether you have the money to increase the annual payments.

A more realistic loan case is given in Table 4.3, where computer-generated interest and capital (principal) payments are given for a loan that has a present value of $80,000 which is to be paid off in 48 equal quarterly (3-month) payments, at a quarterly interest rate of 3 percent. The equivalent quarterly equal payments were computed from

$$A = \$80{,}000(A/P, 3, 48)$$

TABLE 4.2

Pattern of capital recovery and interest charges if annual payment given in Table 4.1 is increased by 50%

End of period	Capital not recovered by end of period, $	Interest due on unrecovered capital, $	Amount of capital recovered, $	Actual period payment, $
0	40,000.00			
1	25,071.75	4,000.00	14,928.25	18,928.25
2	8,650.68	2,507.18	16,421.07	18,928.25
3	0.00	865.07	8,650.68	9,515.74
		7,372.24	40,000.00	47,372.24

TABLE 4.3

Loan payment data for $80,000 loan at quarterly 3 percent rate for 48 quarters

End of period	Capital not recovered by end of period, $	Interest due on unrecovered capital, $	Amount of capital recovered, $	Period capital recovery charge, $
0	80,000.00			
1	79,233.78	2,400.00	766.22	3,166.22
2	78,444.57	2,377.01	789.21	3,166.22
3	77,631.69	2,353.34	812.88	3,166.22
4	76,794.41	2,328.95	837.27	3,166.22
5	75,932.02	2,303.83	862.39	3,166.22
6	75,043.77	2,277.96	888.26	3,166.22
7	74,128.86	2,251.31	914.91	3,166.22
8	73,186.50	2,223.87	942.36	3,166.22
9	72,215.88	2,195.59	970.63	3,166.22
10	71,216.13	2,166.48	999.75	3,166.22
11	70,186.40	2,136.48	1,029.74	3,166.22
12	69,125.77	2,105.59	1,060.63	3,166.22
13	68,033.32	2,073.77	1,092.45	3,166.22
14	66,908.10	2,041.00	1,125.22	3,166.22
15	65,749.13	2,007.24	1,158.98	3,166.22
16	64,555.38	1,972.47	1,193.75	3,166.22
17	63,325.81	1,936.66	1,229.56	3,166.22
18	62,059.36	1,899.77	1,266.45	3,166.22
19	60,754.92	1,861.78	1,304.44	3,166.22
20	59,411.35	1,822.65	1,343.57	3,166.22
21	58,027.46	1,782.34	1,383.88	3,166.22
22	56,602.07	1,740.82	1,425.40	3,166.22
23	55,133.91	1,698.06	1,468.16	3,166.22
24	53,621.70	1,654.02	1,512.20	3,166.22
25	52,064.13	1,608.65	1,557.57	3,166.22
26	50,459.84	1,561.92	1,604.30	3,166.22
27	48,807.41	1,513.80	1,652.43	3,166.22
28	47,105.41	1,464.22	1,702.00	3,166.22
29	45,352.35	1,413.16	1,753.06	3,166.22
30	43,546.70	1,360.57	1,805.65	3,166.22
31	41,686.88	1,306.40	1,859.82	3,166.22
32	39,771.26	1,250.61	1,915.62	3,166.22
33	37,798.18	1,193.14	1,973.08	3,166.22
34	35,765.90	1,133.95	2,032.28	3,166.22
35	33,672.65	1,072.98	2,093.25	3,166.22
36	31,516.61	1,010.18	2,156.04	3,166.22
37	29,295.89	945.50	2,220.72	3,166.22
38	27,008.54	878.88	2,287.35	3,166.22
39	24,652.57	810.26	2,355.97	3,166.22
40	22,225.93	739.58	2,426.64	3,166.22

TABLE 4.3 *(Continued)*

End of period	Capital not recovered by end of period, $	Interest due on unrecovered capital, $	Amount of capital recovered, $	Period capital recovery charge, $
41	19,726.49	666.78	2,499.44	3,166.22
42	17,152.06	591.79	2,574.43	3,166.22
43	14,500.40	514.56	2,651.66	3,166.22
44	11,769.19	435.01	2,731.21	3,166.22
45	8,956.04	353.08	2,813.15	3,166.22
46	6,058.50	268.68	2,897.54	3,166.22
47	3,074.03	181.76	2,984.47	3,166.22
48	0.03	92.22	3,074.00	3,166.22
		71,978.70	79,999.98	151,978.58

Table 4.4 shows the effect of doubling the quarterly payment. The total amount of interest paid is reduced by 69 percent ($71,979 to $22,075). The total amount of payments has the same absolute reduction, from $151,978 to $102,075, for a 49 percent reduction. The payout period is reduced from 48 to 17 time periods, a healthy decrease. So, if your interest rates are high and it is not feasible to refinance to a lower rate, the possibility of increasing the period payment might be beneficial.

The computer program CHEER will compute loan analyses, such as we have just seen. Just click Project on the main screen and then click Loan Analysis and insert the required data.

4.1.2 Capital Recovery Calculations

As indicated in Table 4.1, the sum of the four annuity payments is $50,475, of which $10,475 is interest. This tabular format, as was seen, can be used to trace the capital recovery. When only the amount of unrecovered capital at a certain time is sought, it can be determined directly from the present worth of the remaining payments:

$$\text{Unrecovered capital (year 3)} = A(P/A, 10, 1) = \$12{,}618.83(0.90909) = \$11{,}471.65$$

$$\text{Unrecovered capital (year 2)} = A(P/A, 10, 2) = \$12{,}618.83(1.73554) = \$21{,}900.48$$

$$\text{Recovered capital (by year 3)} = \$40{,}000 - \$11{,}471.65 = \$28{,}528.35$$

$$\text{Interest due (year 4)} = \$12{,}618.83 - \$11{,}471.65 = \$1147.18$$

$$\text{Recovered capital (by year 2)} = \$40{,}000 - \$21{,}900.48 = \$18{,}099.52$$

$$\text{Interest due (years 3 and 4)} = 2(\$12{,}618.83) - \$21{,}900.48 = \$3337.18$$

TABLE 4.4

Table 4.3 loan situation when payments are doubled

End of period	Capital not recovered by end of period, $	Interest due on unrecovered capital, $	Amount of capital recovered, $	Actual period payment, $
0	80,000.00			
1	76,067.55	2,400.00	3,932.44	6,332.44
2	72,017.14	2,282.03	4,050.42	6,332.44
3	67,845.21	2,160.51	4,171.93	6,332.44
4	63,548.13	2,035.36	4,297.09	6,332.44
5	59,122.13	1,906.44	4,426.00	6,332.44
6	54,563.34	1,773.66	4,558.78	6,332.44
7	49,867.80	1,636.90	4,695.54	6,332.44
8	45,031.39	1,496.03	4,836.41	6,332.44
9	40,049.89	1,350.94	4,981.50	6,332.44
10	34,918.95	1,201.50	5,130.95	6,332.44
11	29,634.07	1,047.57	5,284.88	6,332.44
12	24,190.65	889.02	5,443.42	6,332.44
13	18,583.92	725.72	5,606.72	6,332.44
14	12,809.00	557.52	5,774.93	6,332.44
15	6,860.82	384.27	5,948.17	6,332.44
16	734.21	205.82	6,126.62	6,332.44
17	0.00	22.03	734.20	756.22
		22,075.32	80,000.00	102,075.34

Review Table 4.1 to verify these results. Minor discrepancies in the results are due to rounding of the $(P/A, 10, N)$ values.

The equivalent annual payment equation can be modified to include a salvage value in two ways. If S = salvage value, we can logically compute

$$\boxed{\text{Equivalent annual cost EAC} = P(A/P, i, N) - S(A/F, i, N)} \tag{4.1}$$

From Chap. 2, we know that

$$(A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1} \tag{4.2}$$

$$(A/F, i, N) = \frac{i}{(1+i)^N - 1} \tag{4.3}$$

Now, subtracting i from Eq. (4.2), we get

$$\frac{i(1+i)^N}{(1+i)^N - 1} - i = \frac{i(1+i)^N - i(1+i)^N + i}{(1+i)^N - 1}$$

which reduces to

$$\frac{i}{(1+i)^N - 1}$$

and which is the right-hand side of Eq. (4.3). Therefore,

$$(A/P, i, N) - i = (A/F, i, N) \tag{4.4}$$

Using Eq. (4.4), we can simplify Eq. (4.1) to get a well-known engineering economy relationship:

$$\boxed{\begin{aligned} \text{EAC} &= P(A/P, i, N) - S[(A/P, i, N) - i] \\ &= (P - S)(A/P, i, N) + Si \end{aligned}} \tag{4.5}$$

Equation (4.1) makes more intuitive sense to the analyst while Eq. (4.5) is generally easier to manipulate. For that reason, Eq. (4.5) will be used most often in this text for computing the EAC.

Consider an asset that costs $60,000 and has a $20,000 salvage value. The net amount that must be recovered from annuity payments is $P - S$ = $60,000 – $20,000 = $40,000; the remaining portion of the purchase price is returned by receipt of the salvage value, $20,000. However, the purchaser is deprived of the use of the $20,000 during the life of the asset, so interest is owed on this amount because it represents unrecovered capital. The Si term in Eq. (4.5) accounts for this interest payment.

If the life of the $60,000 asset with a $20,000 salvage value is 4 years and the interest rate is 10 percent, then

$$\begin{aligned} \text{EAC} &= (\$60{,}000 - \$20{,}000)(A/P, 10, 4) + \$20{,}000(0.10) \\ &= \$40{,}000(0.31547) + \$2000 \\ &= \$12{,}619 + \$2000 = \$14{,}619 \end{aligned}$$

The first term above ($12,619) is the same capital recovery expression depicted in Table 4.1 and accounts for annual decline in the value of the asset. The second term ($2000) is the annual charge for the capital locked in the salvage value S.

The alternative formula for equivalent annual cost, Eq. (4.1), yields the same solution for the given data:

$$\begin{aligned} \text{EAC} &= P(A/P, 10, 4) - S(A/F, 10, 4) \\ &= \$60{,}000(0.31547) - \$20{,}000(0.21547) \\ &= \$18{,}928 - \$4309 = \$14{,}619 \end{aligned}$$

4.2 SITUATIONS FOR EQUIVALENT ANNUAL-WORTH COMPARISONS

The term *annual worth* suggests a positive value, but the calculations can just as well produce a negative value. A negative annual worth indicates that the equivalent value of negative cash flow for disbursements is grea than the corresponding positive flow of receipts. Negative worths usual mean that an alternative is unacceptable. Exceptions occur when proje must be undertaken to satisfy certain requirements such as safety cit tions or building codes, or when a "do nothing" option is not viable. Th the objective is to identify the alternative with the least equivalent annu cost (negative cash flow). We will use the term *equivalent annual co* *(EAC)* to designate comparisons involving only costs, and we will use th term *equivalent annual worth (EAW)* when costs and incomes (benefits) ar both present.

It is often very difficult, and not worth the required study time, to di cover the income derived from only one component in a complex syste having many different components. For instance, the income produced by copying machine is troublesome to derive exactly since its output is utili by many people, often from different departments, working on many pr jects. In this type of situation, alternatives to satisfy the copying needs ar evaluated on the basis of their relative costs, because each alternati capable of meeting the requirements of the system will produce the sam income to the system. When it is apparent that only costs are involved i an evaluation, it is convenient to ignore the minus sign convention and le comparison figures represent the absolute value of costs. Several situatio for applying equivalent annual-worth calculations are described in th examples that follow.

4.2.1 Consolidation of Cash Flows

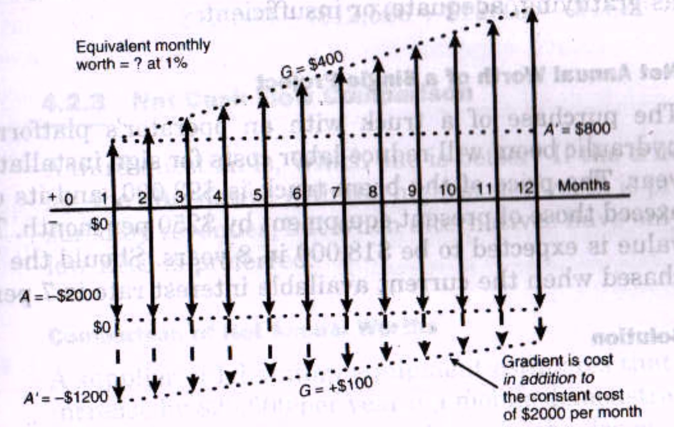
What's it worth? is the decisive query in the appraisal of a proposal. It i difficult to ascertain what to expect from a proposal until the myri receipts and disbursements associated with its conduct are collectivel analyzed. Improvement programs are prime examples. Organizatio regularly engage in programs to improve productivity, reduce accident raise quality, etc. Each is a worthwhile goal, expected to have positiv rewards, but each has costs, too. Consolidating the various costs into a pa tern that can be compared with potential rewards may take the form of net annuity.

EXAMPLE 4.1 **Equivalent Monthly Net Worth of Cash Flows**

A consulting firm proposes to provide "self-inspection" training for clerk who work with insurance claims. The program lasts 1 year, costs $2000 pe month, and professes to improve quality while reducing clerical time. A potential user of the program estimates that savings in the first month should amount to $800 and should increase by $400 per month for the res of the year. However, operational confusion and work interference ar expected to boost clerical costs by $1200 the first month, but this amount

should subsequently decline in equal increments at the rate of $100 per month. If the required return on money is 12 percent compounded monthly and there is a stipulation that the program must pay for itself within 1 year, should the consultants be hired?

Solution



$$i = \frac{r}{m} = \frac{0.12}{12} = 0.01 \text{ per period}$$

$N = 12$ periods

$$\begin{aligned}\text{Equivalent monthly worth of savings} &= \$800 + \$400(A/G, 1, 12)\\ &= \$800 + \$400(5.36815) = \$2947\end{aligned}$$

$$\begin{aligned}\text{Equivalent monthly worth of costs} &= -\$2000 - [\$1200 - \$100(A/G, 1, 12)]\\ &= -\$3200 + \$100(5.36815) = -\$2663\end{aligned}$$

$$\text{Equivalent net monthly cash flow} = \$2947 - \$2663 = \$284$$

The program looks very promising because the equivalent monthly worth is positive during the first year, and savings generated by the training should continue into the future.

Gradient factors were utilized in Example 4.1 to convert uniformly varying cash flows to their equivalent constant worths. Recovery of capital is not an issue, since no property ownership is involved. The comparison is made directly on the basis of expected income versus outgo.

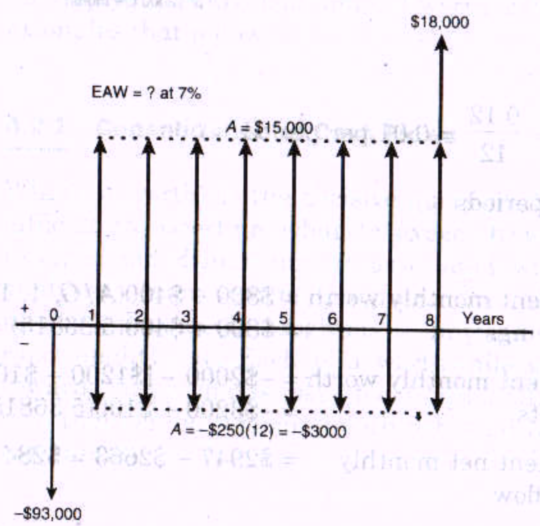
4.2.2 Recovery of Invested Capital

Will it pay off? is the question investors want answered. An adequate payoff recovers the invested capital plus the desired rate of return. Since returns are spread over the life of the investment, it is convenient to convert capital recovery costs to the same annual pattern. The consequential result of combining uniform cost and revenue flows is a *positive, zero,* or *negative* series of payments that, respectively, categorizes the investment as gratifying, adequate, or insufficient.

EXAMPLE 4.2 **Net Annual Worth of a Single Project**

The purchase of a truck with an operator's platform on a telescoping hydraulic boom will reduce labor costs for sign installations by $15,000 per year. The price of the boom truck is $93,000, and its operating costs will exceed those of present equipment by $250 per month. The resale (salvage) value is expected to be $18,000 in 8 years. Should the boom truck be purchased when the current available interest rate is 7 percent?

Solution



$$\begin{aligned} EAW &= -\$93{,}000(A/P, 7, 8) + \$18{,}000(A/F, 7, 8) - \$3000 + \$15{,}000 \\ &= -\$93{,}000(0.16747) + \$18{,}000(0.09747) + \$12{,}000 \\ &= -\$1820 \end{aligned}$$

Equivalent annual-worth calculations indicate that the purchase and use of the boom truck will cause a loss equivalent to $1820 per year for 8 years, compared with other investments that could earn a 7 percent return.

The solution to Example 4.2 was developed from the accompanying cash flow diagram. The capital recovery factor leads to the same solution when capital recovery costs are registered negatively and the net annual savings are positive:

$$\begin{aligned} EAW &= \text{annual savings} - \text{capital recovery costs} \\ &= \$15{,}000 - \$3000 - [(P - S)(A/P, 7, 8) + Si] \\ &= \$12{,}000 - [(\$93{,}000 - \$18{,}000)(0.16747) + \$18{,}000(0.07)] \\ &= \$12{,}000 - (\$12{,}560 + \$1260) = -\$1820 \end{aligned}$$

4.2.3 Net Cash Flow Comparison

A third question is, Which one is better? If the criterion is strictly economics, the alternative with the highest net worth is preferred, if worth is measured by revenues; but when alternatives have only costs and no income, a low EAC is preferred.

EXAMPLE 4.3 **Comparison of Net Annual Worths**

A supplier of laboratory equipment estimates that profit from sales should increase by $20,000 per year if a mobile demonstration unit is built. A large unit with sleeping accommodations for the driver will cost $97,000, while a smaller unit without sleeping quarters will be $63,000. Salvage values for the large and small units after 5 years of use will be, respectively, $9700 and $3500. Lodging costs saved by the larger unit should amount to $11,000 annually, but its yearly transportation costs will exceed those of the smaller unit by $3100. With money at 9 percent, should a mobile demonstration unit be built? And if so, which size is preferable?

Solution

EAW of the large mobile demonstration unit:

Annual increase in profit	$20,000
Savings in lodging costs over smaller unit per year	11,000
Extra transportation costs over smaller unit per year	−3,100
Capital recovery cost: ($97,000 − $9700)(*A/P*, 9, 5) + $9700*i* = $87,300(0.25709) + $9700(0.09)	−23,317
Net AW =	$4,583

EAW of small mobile demonstration unit:

Annual increase in profit	$20,000
Capital recovery cost ($63,000 − $3500)(*A/P*, 9, 5) + $3500*i* = $59,500(0.25709) + $3500(0.09)	−15,612
Net AW =	$4,388

The net annual worths indicate that both alternatives will produce positive cash flows while at the same time they are currently repaying investment costs. The closeness of the two net values indicates that other criteria might need to be considered. For example, will lodging costs *really* be expected to stay about the same over the 5-year plan? If no, the larger unit will benefit, assuming that these costs will increase. *Is it reasonable* to assume that the fuel costs will not be expected to change over the same time period? If this answer is no, the smaller unit should show greater savings in this category. If the original cost estimates do not change, then the less expensive expenditure is probably the safer—invest in the smaller unit. In addition to having a smaller initial expenditure, it has less risk associated with the potential salvage value after 5 years.

Since the profit increase expected from building either of the mobile laboratories is the same, the comparison could have been conducted by first considering only the costs:

	Small unit, $	Large unit, $
Capital recovery cost	15,612	23,317
Net extra cost of smaller unit ($11,000 – $3100)	7,900	
Total annual cost	23,512	23,317

and then evaluating the best resulting alternative in terms of the expected income.

Many economic decisions are based only on costs, thus requiring a net equivalent annual-cost study. Maintaining Environmental Protection Agency (EPA) standards forced the state of Arizona in 1993 to mandate that wood-burning fireplaces could not be operated in the greater Phoenix area on high-pollution days. A homeowner who wanted to have a fire on those days would have three alternatives. The first would be to violate the statute and risk being fined. The second two, which are the viable alternatives, would be to install natural gas artificial logs or a pollutant-screening device in the flue. The latter is very expensive, while the gas logs could have a high operating cost. Possibly, the logical choice would be to not burn wood on high-pollution days, since Phoenix is a warm-weather resort area in the wintertime, and so comply with the statute and save money on the reduced payment for the annual cost of wood. Of course, higher heating bills might accrue.

EXAMPLE 4.4 **Comparison of Net Annual Costs**

A cement plant has been ordered by the state in which it is located to limit particulates being released in the cement-making process, or else the state will order the plant to be closed. Since the plant is located by a quarry with an expected remaining life of 15 years, doing nothing is not a viable alternative. The plant has an agreement with the state that one of three possibilities will satisfy the regulating agency, with the situation to be reviewed again after 5 years:

(*a*) Enclose all existing material conveyors at a first cost of $385,000 with annual maintenance costs of $3500. It is expected that major retrofitting will be required at 5-year intervals: $35,000 after 5 years and $65,000 after 10 years.

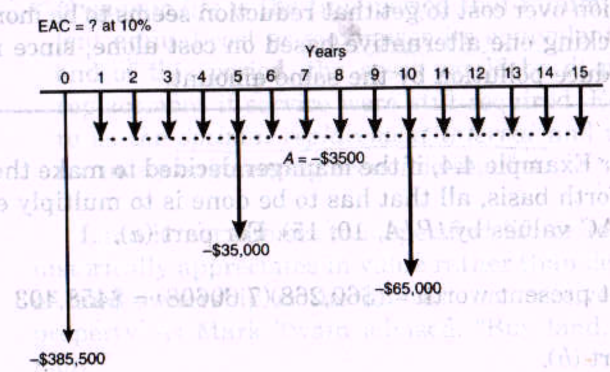
(*b*) Provide filtration equipment on the material silos. This is expected to have a first cost of $271,000 with annual operating costs of $8000.

(*c*) Upgrade the cement kilns. The initial cost of doing this will be $380,000, and lost-time production costs during installation will be $43,000.

From an environmental point of view, the plant should do all three upgrades, but because of the small profit margin on cement this will not be feasible. And so, as mentioned earlier, the state has agreed to just one of the alternatives being implemented. The plant manager decides to go with the alternative that seems to provide the least annual cost over the quarry life of 15 years with an evaluation of other possible changes after 5 years pass. Assuming that productivity does not change with any of the alternatives, the manager has asked you to evaluate the situation, using the prevailing interest rate of 10 percent. Also, even though tax benefits might be possible, these are not known at the moment, since the legislature has deferred until at least next year a decision on such potential benefits.

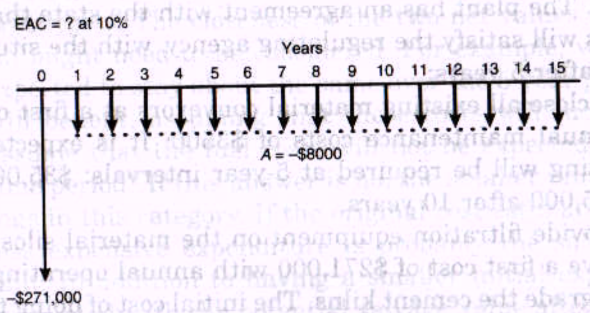
Solution

(*a*) Enclose conveyors:



$$
\begin{aligned}
\text{EAC} &= \$385{,}000(A/P,\ 10,\ 15) + \$35{,}000(P/F,\ 10,\ 5)(A/P,\ 10,\ 15) \\
&\quad + \$65{,}000(P/F,\ 10,\ 10)(A/P,\ 10,\ 15) + \$3500 \\
&= \$385{,}000(0.13147) + \$35{,}000(0.62092)(0.13147) \\
&\quad + \$65{,}000(0.38554)(0.13147) + \$3500 \\
&= \$50{,}616 + \$2857 + \$3295 + \$3500 = \$60{,}268
\end{aligned}
$$

(*b*) Silo filtration:



$$\text{EAC} = \$271{,}000(A/P, 10, 15) + \$8000$$
$$= \$271{,}000(0.13147) + \$8000$$
$$= \$35{,}628 + \$8000 = \$43{,}628$$

(*c*) Upgrade kilns:

$$\text{EAC} = (\$380{,}000 + \$43{,}000)(A/P, 10, 15)$$
$$= \$423{,}000(0.13147) = \$55{,}612$$

In priority order, the equivalent annual costs show that putting fil- tion equipment on the silos is the lowest cost, upgrading the kilns is ne and enclosing the conveyors has the highest cost.

This problem would be ideal for an evaluation by a technique call *benefit/cost (B/C) analysis,* a topic covered in Chap. 8. Benefits will b measure of pollution reduction, and maximizing the ratio of pollu reduction over cost to get that reduction seems to be more appropriate th just picking one alternative based on cost alone, since not all alternati will reduce pollution by the same amount.

For Example 4.4, if the manager decided to make the analysis on a pr sent-worth basis, all that has to be done is to multiply each of the alter tive EAC values by $(P/A, 10, 15)$. For part (*a*),

$$\text{Net present worth} = (\$60{,}268)(7.60608) = \$458{,}403$$

For part (*b*),

$$\text{Net present worth} = \$43{,}628(7.60608) = \$331{,}838$$

For part (*c*),

$$\text{Net present worth} = \$55{,}612(7.60608) = \$422{,}989$$

CONSIDERATION OF ASSET LIFE

Translating cash flows to equivalent annuities is a mechanical process that becomes almost automatic with practice. Understanding the meaning of an economic comparison and being able to explain its significance to others are the critical skills. The discussion of economic asset life, as introduced in Chap. 3, is continued in this section to stress the importance of selecting an appropriate study period in EAW comparisons.

4.3.1 Definitions of Asset Life

In time-value mechanics, N is simply the number of compounding periods appropriate for the analysis of cash flows. And N takes on a special meaning when it represents the life of an asset that loses value as a function of use or time. The more frequently applied terms to describe the life of an asset are listed and defined as follows:

- *Ownership life* or *service life* is the period of time an asset is kept in service by the owner(s). Implied is a period of useful service from the time of purchase until disposal. Actually, under the vague expectation that it might somehow again prove useful, equipment is often retained beyond the point where it is capable of satisfying its intended function. A machine can have a physical life longer than its service life; the machine is still physically sound, but there is no useful function for it to perform.
- *Accounting life* is a life expectancy based primarily on bookkeeping and tax considerations. It may or may not correspond to the period of usefulness and economic desirability.
- *Economic life* is the time period that minimizes the asset's total equivalent annual cost or maximizes its equivalent net annual income. At the end of this period, the asset would be displaced by a more profitable replacement if service were still required. Economic life is also referred to as the *optimal replacement interval* and is the condition appropriate for many engineering economic studies.

Land is not subject to a specified life or to capital recovery, because it historically appreciates in value rather than depreciates with age. The cost of land ownership is the interest not received on funds invested in the property. As Mark Twain advised, "Buy land, they're not making it anymore."

4.3.2 Comparisons of Assets with Equal and Unequal Lives

Examples 4.5 and 4.6 are typical applications of EAC comparisons. The lease-or-buy question posed in Example 4.5 is raised with increasing regularity that corresponds to the rapid growth of leasing companies. It is now possible to lease almost any type of production equipment that is not custom-designed for narrowly specialized service. Important tax and inflation considerations involved in the lease-or-buy choice are discussed in Chap. 10.

EXAMPLE 4.5 **Alternatives with Equal Annual Costs**

A machine needed for 3 years can be purchased for \$77,662 and sold at the end of the period for about \$25,000. A comparable machine can be leased for \$30,000 per year. If a firm expects a return of 20 percent on investments, should it buy or lease the machine when end-of-year payments are expected?

Solution

$$\begin{aligned}\text{Equivalent annual cost to buy} &= (\$77{,}662 - \$25{,}000)(A/P,\ 20,\ 3) \\ &\quad + \$25{,}000(0.20) \\ &= \$52{,}662(0.47473) + \$5000 \\ &= \$30{,}000\end{aligned}$$

$$\text{Annual cost to lease} = \$30{,}000$$

Since the results are the same for both leasing and buying, the decision will likely be affected by the existence or absence of other projects in need of funding. If there are none, then the \$77,662 investment in the machine will return 20 percent, assuming that the salvage value estimates are reasonable.

The two alternatives in Example 4.5 are compared on the basis of their costs because the income resulting from their contribution is not available and it is believed that both are capable of producing that contribution. The question is not whether to get a machine. It is known that the machine is necessary, so the choice is to buy or lease it. In selecting between the alternatives with equal equivalent annual worths, recall that they are equal only after ownership of the machine has earned 20 percent on the capital invested in it.

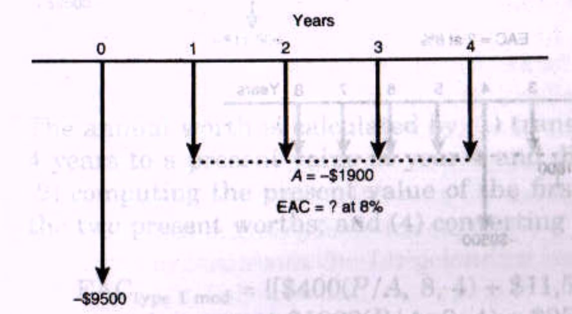
EXAMPLE 4.6 **Comparison of Assets with Unequal Lives**

Two models of small machines perform the same function. Type 1 machine has a low initial cost of \$9500, relatively high operating costs of \$1900 per year more than those of the type 2 machine, and a short life of 4 years. The more expensive, type 2 machine costs \$25,100 and can be kept in service economically for 8 years. The scrap value from either machine at the end

of its life will barely cover its removal cost. Which is preferred when the minimum attractive rate of return is 8 percent?

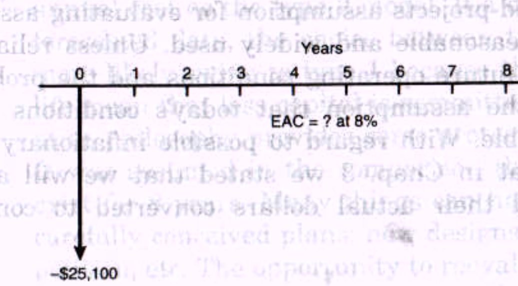
Solution

Type 1 machine:



$$\begin{aligned}\text{EAC}_{\text{type 1}} &= \$1900 + \$9500(A/P,\ 8,\ 4) \\ &= \$1900 + \$9500(0.30192) = \$4768\end{aligned}$$

Type 2 machine:

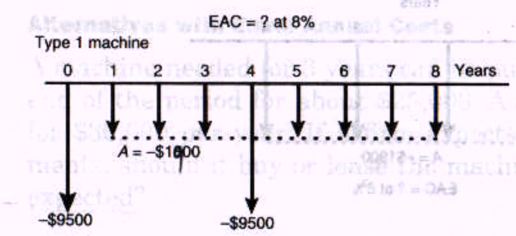


$$\text{EAC}_{\text{type 2}} = \$25{,}100(A/P,\ 8,\ 8) = \$25{,}100(0.17401) = \$4368$$

The type 2 machine has a lower annual cost for service during the next 8 years and is therefore preferred.

The machines described in Example 4.6 exhibit the common feature that more expensive models, designed to serve the same function as less expensive versions, are expected to operate more economically and/or last longer. (If a costlier machine also produces better-quality products, the benefits from improved quality must be included in the analysis to make the outcomes comparable.) The difficulty in comparing alternatives with unequal lives lies in accounting for the service provided during the period in which one outlasts the other.

The implied assumption in the solution to Example 4.6 is that two machines of type 1 will be purchased consecutively to provide the same length of service as one type 2 machine. The equivalent annual cost for 8 years of service from two type 1 machines is, of course, the same as calculated in the solution above:



$$
\begin{aligned}
\text{EAC}_{\text{type 1}} &= \$9500(A/P, 8, 8) + \$9500(P/F, 8, 4)(A/P, 8, 8) + \$1900 \\
&= \$9500(0.17401) + \$9500(0.73503)(0.17401) + \$1900 \\
&= \$1653 + \$1215 + \$1900 = \$4768
\end{aligned}
$$

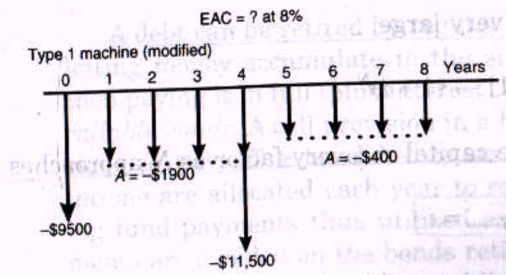
which is the same as calculated earlier.

The repeated-projects assumption for evaluating assets with different lives is fairly reasonable and widely used. Unless reliable forecasts can be made about future operating conditions and the probability of technical advances, the assumption that today's conditions will exist in the future is plausible. With regard to possible inflationary effects on future costs, recall that in Chap. 3 we stated that we will assume that cash flows have had their actual dollars converted to constant dollars by using

$$
\text{Constant dollars} = \text{actual dollars}\left[\frac{1}{(1+f)^k}\right]
$$

where f is the annual inflation rate as a fraction and k is the number of years into the future that the estimated cash transaction takes place, $k = 1, 2, \ldots, N$.

When *future* conditions can be estimated with confidence, excluding inflation, since we are working with constant dollars, these valuations are the data that should be used for equivalent annual-worth calculations. For instance, a confident prediction that current developmental work on the type 1 machine will produce refinements within 4 years to reduce operating costs to $400 above those of the type 2 machine while increasing the purchase price to $11,500 leads to a revised economic analysis:



The annual worth is calculated by (1) translating the cash flow in the last 4 years to a present value at year 4 and discounting this value to year 0; (2) computing the present value of the first 4 years' cash flow; (3) adding the two present worths; and (4) converting the total to an 8-year annuity:

$$
\begin{aligned}
\text{EAC}_{\text{type 1 mod}} &= \{[\$400(P/A, 8, 4) + \$11{,}500]\,(P/F, 8, 4) \\
&\quad + \$1900(P/A, 8, 4) + \$9500\}\,(A/P, 8, 8) \\
&= \{[\$400(3.31213) + \$11{,}500]\,(0.73503) \\
&\quad + \$1900(3.31213) + \$9500\}\,(0.17401) \\
&= (\$9427 + \$15{,}793)(0.17401) = \$4389
\end{aligned}
$$

The modified type 1 machine now has nearly the same equivalent annual cost as the type 2 model. If a decision maker has confidence in the forecasted data, the choice between types can go either way; but it will more likely swing to type 1 because the lower purchase price and shorter life mean that less capital is committed for a shorter period. This conservative philosophy provides some protection from unexpected developments. It was assumed in the comparison that the need for the machine would exist for 8 years. Many things can happen in 8 years to thwart the most carefully conceived plans: new designs, changing markets, successful competition, etc. The opportunity to reevaluate tactics in 4 years, as is possible with the type 1 machine, is a subtle but valued attribute.

4.3.3 Perpetual Life

Occasionally an asset is treated as if it will last forever. The assumption of infinite life in terms of capital recovery is slightly more reasonable than the physical interpretation. Nothing made by humans, even Egyptian pyramids or the Great Wall of China, lasts forever; but the difference between infinity and 100 years in the numerical value of the capital recovery factor is quite small:

$$
(A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}
$$

And as N gets very large,

$$[(1+i)^N - 1] \rightarrow (1+i)^N$$

The limit of the capital recovery factor as N approaches infinity is

$$\boxed{(A/P, i, N)_{N \to \infty} = i}$$

Therefore, in an economic comparison involving an asset with an infinite life, *such as land,* the interest rate replaces the capital recovery factor. The human-made assets most closely approaching perpetual life are dams, tunnels, canals, aqueducts, and monuments. The nature of very-long-lived assets relegates them mostly to public projects, and there the trend has been to set a study period of 50 years or so in recognition of changing public needs and technological advances that generate new ways to fulfill the needs.

The similarity between the perpetual-life assumption for calculating equivalent annual worth and the capitalized cost method associated with present-worth models should be apparent. Both assume infinite life and therefore have very limited application. Note that the difference between capital recovery factors for $N = 50$ and $N = \infty$ is less than 1 percent for i = 10 percent and less than 10 percent for i = 5 percent; computations for "engineering estimates" are still possible.

4.4 USE OF A SINKING FUND

The sinking fund factor $(A/F, i, N)$ was discussed previously as an alternative means of calculating capital recovery costs:

$$(P - S)(A/P, i, N) + Si = P(A/P, i, N) - S(A/F, i, N)$$

As is clear in the equation, the sinking fund factor is applied to compute the annuity required to accumulate a certain future amount. Organizations are sometimes obligated by legislated or contractual agreements to establish a fund, separate from their internal operations, to accumulate a specified amount by a specified time. This accumulation is called a *sinking fund.*

Provision for a sinking fund requires that an organization set aside a portion of the income derived from sales or taxes each year in order to retire a bond issue (or, in some cases, an issue of preferred stock). Failure to meet the sinking fund payments forces the bond issue to be thrown into default, causing serious credit and credibility problems. The payments are a direct cash drain on the organization. That is the purpose of the sinking fund—to protect investors by enforcing an orderly retirement of debt from current income.

A debt can be retired by regular payments from the sinking fund or by letting money accumulate in the sinking fund until the debt is due and then paying it in full (plus interest). Periodic payments are associated with *callable bonds.* A call provision in a bond gives the issuing organization the right to pay off a bond before its maturity date. Funds reserved from income are allocated each year to retire a portion of the bond issue. Sinking fund payments thus utilized earn at the rate at which interest payments are avoided on the bonds retired.

When a sinking fund is established to accumulate sufficient money to meet the bond cost at maturity, annual payments are normally required to be invested in a savings institution. The interest rate for these "savings" typically is lower than the organization earns on its own capital, but the sinking fund is less an earning device than a way to ensure that funds will not be diverted to other ventures.

For example, a firm borrows \$1 million at 9 percent simple interest because it believes that the amount borrowed can be utilized within the firm to earn 18 percent—double the borrowing rate. If \$1 million is acquired by issuing 20-year bonds with the stipulation that a sinking fund is to be set up, the firm has to set aside a payment each year and put it in an external account. This account probably earns less than the 18 percent internally earned on investment funds, but it represents a very secure investment. Assuming that the account pays 6 percent annually compounded interest, we find that the annual payments into the sinking fund would be

$$A = \$1{,}000{,}000(A/F, 6, 20)$$
$$= \$1{,}000{,}000(0.02718) = \$27{,}180$$

to make total annual debt repayment on the principal plus interest of \$27,180 + \$1,000,000(0.09) = \$117,180.

4.5 EQUIVALENT UNIFORM PAYMENTS WHEN INTEREST RATES VARY

Interest rates have fluctuated widely in recent years as a response to different degrees of inflation. Rates also vary according to the type and size of investment; larger investments are often awarded higher interest rates, and riskier investments demand higher returns. It is therefore occasionally appropriate to assign different interest rates to specific periods of cash flow.

Present and future worths of cash flows with changing interest rates are calculated by translating each transaction backward or forward in time according to the prevailing interest rate in each period of the transaction. For example, assume that a deposit of \$5000 was made 4 years ago and a withdrawal of \$2000 was made 2 years ago. The prevailing interest rate during the first year was 6 percent, and the rate was increased by 1 per-

TABLE 4.5

Calculation of future worth with varying interest rates

End of year	Deposit or withdrawal, $	*i* during year, %	Balance in account at end of year, $
0	+5000		
1		6	5000(*F/P*, 6, 1) = 5300
2	−2000	7	5300(*F/P*, 7, 1) − 2000 = 3671
3		8	3671(*F/P*, 8, 1) = 3965
4		9	3965(*F/P*, 9, 1) = 4321.50

cent each year. The amount in the account at the end of each year is shown in Table 4.5.

By the same approach, the present worth (PW) at time 0 of the cash flow is

$$\text{PW} = \$5000 - \$2000(P/F,\ 6,\ 1)(P/F,\ 7,\ 1) = \$3236.63$$

A uniform series of four payments equivalent to the given cash flow with changing interest rates is calculated as

$$\text{PW} = A(P/F,\ 6,\ 1) + A(P/F,\ 6,\ 1)(P/F,\ 7,\ 1) + A(P/F,\ 6,\ 1)(P/F,\ 7,\ 1) \times (P/F,\ 8,\ 1) + A(P/F,\ 6,\ 1)(P/F,\ 7,\ 1)(P/F,\ 8,\ 1)(P/F,\ 9,\ 1)$$

Using the present worth of the cash flow calculated earlier, we find that

$$\begin{aligned}\$3236.63 &= A[0.94340 + 0.94340(0.93458) + 0.94340(0.93458)(0.92593) \\ &\quad + 0.94349(0.93458)(0.92593)(0.91743)] \\ &= A(3.3904)\end{aligned}$$

and

$$A = \frac{\$3236.63}{3.3904} = \$954.65$$

A corresponding calculation of a series of equal payments A from the future worth (FW) is

$$\text{FW} = A + A(F/P,\ 9,\ 1) + A(F/P,\ 8,\ 1)(F/P,\ 9,\ 1) + (F/P,\ 7,\ 1) \times (F/P,\ 8,\ 1)(F/P,\ 9,\ 1)$$

which gives

$$\begin{aligned}A &= \frac{\$4321.50}{1 + 1.09 + 1.08(1.09) + 1.07(1.08)(1.09)} \\ &= \frac{\$4321.50}{4.5268} \\ &= \$954.65\end{aligned}$$

Thus, four year-end payments of \$954.65 yield a future amount of \$4321.50 when interest rates progress from 6 to 7 to 8 to 9 percent during the 4-year span, and the worth of this annuity is equivalent to a cash inflow of \$5000 at time 0 and an outflow of −\$2000 at the end of year 2.

4.6 ANNUITY CONTRACT FOR A GUARANTEED INCOME

To conclude this material on equivalent periodic analysis, we might do well to briefly talk about annuity contracts since all of us, sometime in the future, would probably like to retire with a guaranteed income. Annuities are often associated with retirement plans. In this context, an annuity implies a payment received each year according to a purchased contract. In practice, annuity payments are more likely to be made monthly to the receiver, or *annuitant*.

An annuity contract is essentially the reverse of a life insurance policy. In the latter, an insurance company pays a stipulated sum to heirs based on the amount of payments made during the policyholder's lifetime. In the classical annuity situation, an individual pays a stipulated sum, or number of payments, to a company and then receives regular income payments, starting at a designated date and continuing for life. Annuities of this form were used very early in recorded history; mortality tables for computing their values were compiled by Romans and have been found in Egypt and Babylon.

A vast variety of annuity contracts are available. A straight-life annuity is the classic pattern: Income payments terminate with the death of the annuitant. Since many people are reluctant to purchase such an annuity because they fear that they will die prematurely, causing their investment in the annuity to be wasted, companies offer plans that guarantee a refund to heirs under specified conditions, such as an unusually early death. Annuities may have variable rates that are linked to inflation indexes, gold prices, or foreign currencies. The purpose of indexing is to conserve the buying power of annuity income. As with any investment, the annuity buyer must balance risks against returns.

4.7 REVIEW EXERCISES AND DISCUSSIONS

EXERCISE 1
A hospital purchases a new piece of ultrasound equipment for \$60,000. The hospital expects to use it for 4 years and then sell it to a small clinic for \$20,000. Thus, the salvage value is expected to be \$20,000. If the interest rate is 10 percent, develop a table that shows, by year, capital not recovered, interest due, capital recovered, and the annual capital recovery (ACR) charge. The observant reader will note that these are the same data evaluated in Table 4.1 with the first cost and salvage value both being \$20,000 higher in this exercise; the total capital to be recovered is the same in both cases—\$40,000.

SOLUTION 1
As we found in Sec. 4.1.2,

$$\begin{aligned} EAC &= (P - S)(A/P, i, N) + Si \\ &= (\$60{,}000 - \$20{,}000)(A/P, 10, 4) + \$20{,}000(0.1) \\ &= \$40{,}000(0.31547) + \$2000 = \$14{,}619 \end{aligned}$$

The tabular computations, all in dollars, now mirror those we did to generate Table 4.1:

End of year	Capital not recovered	Interest due on unrecovered capital	Amount of capital recovered	Annual capital recovery charge (ACR)	Interest on unrecovered salvage (Si)	Total equivalent annual cost (Si + ACR)
0	60,000					
1	51,381	6,000	8,619	12,619	2,000	14,619
2	41,900	5,138	9,481	12,619	2,000	14,619
3	31,471	4,190	10,429	12,619	2,000	14,619
4	20,000	3,147	11,471	12,619	2,000	14,619
		18,475	40,000	50,476	8,000	58,476

For convenience, the tabular values of total annual cost given above have been separated into the annual capital recovery charge (which is exactly the same as when we had no salvage value in Table 4.1) and the \$2000 per year that is locked into the expected salvage value. The reader must realize that we have, in effect, a loan of \$60,000; the salvage value is not realized until after all payments have been made. Compared with the situation given in Table 4.1, we have \$2000 additional per payment to cover the annual charge for the capital locked in the salvage value; $Si = \$20{,}000(0.1)$. We therefore have \$8000 in total additional interest payments compared to the case in Table 4.1. Logically, the total annual cost will also increase by the same \$2000.

When the time comes to recover the salvage value, we will probably find that the salvage value is a function of what the "market will bear" or pay. Sometimes, a contractual agreement with a vendor might specify exactly what the salvage value will be, possibly as a down payment on a replacement piece of equipment. Frequently, there is a measure of uncertainty in salvage values.

EXERCISE 2
An asset was purchased 5 years ago for \$52,000. It was expected to have an economic life of 8 years, at which time its salvage value would be \$4000. If the function that the asset was serving is no longer needed, for what price must it be sold now to recover the invested capital when i = 12 percent?

SOLUTION 2
The expected annual cost of the asset over its 8-year life was

$$\begin{aligned} EAC &= (P - S)(A/P, 12, 8) + S(0.12) \\ &= (\$52{,}000 - \$4000)(0.20130) + \$4000(0.12) \\ &= \$48{,}000(0.20130) + \$480 = \$10{,}142.40 \end{aligned}$$

The unrecovered capital at the end of the fifth year is the present worth of the last 3 years of the EAC annuity plus the discounted salvage value:

$$\begin{aligned} \text{Unrecovered capital (year 5)} &= \$10{,}142.40(P/A, 12, 3) + \$4000(P/F, 12, 3) \\ &= \$10{,}142.40(2.40188) + \$4000(0.71178) \\ &= \$24{,}361 + \$2847 = \$27{,}208 \end{aligned}$$

An alternative solution method provides the same result:

$$\begin{aligned} \text{Selling price (year 5)} &= \$48{,}000(A/P, 12, 8)(P/A, 12, 3) + \$4000 \\ &= \$23{,}208 + \$4000 = \$27{,}208 \end{aligned}$$

This sum is a possible price. Realistically, the market value for similar equipment at a similar stage in the asset's life will probably be a more reasonable estimate.

EXERCISE 3
A city maintenance crew has had experience with a conventional backhoe that suggests that its service life is 6 years. A newly designed machine costs 50 percent more than the conventional machine but is quieter in operation, which will make it more adaptable to residential neighborhoods. Both machines will have about the same operating costs, and salvage costs are expected to be negligible. What will the service life of the new backhoe have to be to make its cost comparable to that of the conventional machine at i = 10 percent?

SOLUTION 3
Since the machines will apparently have different lives, it is logical to attack the problem by using an annual-worth evaluation, assuming that replacement costs will be comparable to current costs.

First, we equate the two machines' annual worths:

$$AW_{conv} = AW_{new}$$

$$P_{conv}(A/P, 10, 6) = P_{new}(A/P, 10, N)$$

But we know that $P_{new} = 1.5\ P_{conv}$; therefore,

$$P_{conv}(A/P,\ 10,\ 6) = 1.5\ P_{conv}\ (A/P,\ 10,\ N)$$

Solving for $(A/P, 10, N)$, we get

$$\frac{1}{1.5}\ (A/P,\ 10,\ 6) = (A/P,\ 10,\ N)$$

$$0.667\ (0.22961) = 0.1532 = (A/P,\ 10,\ N)$$

Interpolating in the tables, we find that N lies between 11 and 12 years:

$$N = 11 + \frac{0.15396 - 0.1532}{0.15396 - 0.14676}$$
$$= 11.1 \text{ years}$$

The life would have to be about double that of the conventional machine to justify the additional cost. A life of 12 years might be a lot to expect from such equipment. Possibly, the noise abatement might help justify the new equipment, especially if a worth could be assigned to it in terms of dollars. We discuss such decision theory types of problems in Chap. 15.

EXERCISE 4

In Example 3.4, we looked at a comparison of alternatives with unequal economic lives for a present-worth analysis. Specifically, we had assets A_1 and A_2. Asset A_2 has an initial cost of $3200 and an expected salvage value of $400 at the end of its expected 4-year life. Asset A_1 costs $900 less than A_2, has an economic life of 3 years, and has no salvage value; and its annual operating costs exceed those of A_2 by $250. The required rate of return is 15 percent. Example 3.4 asked for a PW determination of the better alternative by the repeated-projects method. This problem can be solved more efficiently by using an annual-worth comparison.

SOLUTION 4

$$AW_{A_1} = -\$250 - \$2300(A/P,\ 15,\ 3)$$
$$= -\$250 - \$2300(0.43798) = -\$1257$$

$$AW_{A_2} = -\$3200(A/P,\ 15,\ 4) + \$400(A/F,\ 15,\ 4)$$
$$= -\$3200(0.35027) + \$400(0.20027) = -\$1041$$

This says that A_2 has the lower equivalent *annual worth* and so should probably be chosen. Only three factors had to be looked up. The present-worth solution in Example 3.4 required six factors to be applied (one was applied twice), which gave a greater chance for error. If we really wanted a present-worth analysis, then each of the two solutions just found would be multiplied by $(P/A, 15, 12)$, since 12 is the common denominator for the two lives:

$$PW_{A_1} = -\$1257(5.42062) = -\$6816$$
$$PW_{A_2} = -\$1041(5.42062) = -\$5642$$

These are the same results we obtained in Chap. 3.

EXERCISE 5

A short concrete canal can be constructed as part of a flood control project; the placement of a large galvanized culvert will serve the some function. The cost of the canal, which will last indefinitely, is $75,000; and maintenance costs will average $400 per year. A culvert, which will have to be replaced every 30 years, will cost $40,000 and have an annual maintenance cost of $700. Salvage values are negligible for both alternatives, and the government interest rate is 6 percent. Which alternative has the lower equivalent annual cost?

SOLUTION 5

Annual-cost comparison of a canal with perpetual life and a culvert with an economic life of 30 years:

Canal

Annual maintenance	$400
Interest on investment = $75,000(0.06)	4500
Equivalent annual cost	$4900

Culvert

Annual maintenance	$700
Capital recovery = $40,000 (A/P, 6, 30)	
= $40,000(0.07265)	2906
Equivalent annual cost	$3606

The culvert has the advantage of a lower equivalent annual cost.

EXERCISE 6

Exercise 4 in Chap. 3 had data on a possible improvement. The exercise asked that CHEER be used to find the present worth of the alternative action. Use that same CHEER output to determine the EAW of the alternative, and verify that this is correct from the PW result.

SOLUTION 6

The CHEER summary output from the earlier exercise is repeated in Fig. 4.1. In addition to the original PW value, we can read the value of EAW directly: $7058.22. Using the original PW value, we can verify the EAW value:

$$EAW = PW(A/P,\ 10,\ 5)$$
$$= \$26{,}756.22(0.26380) = \$7058$$

which checks with the CHEER result.

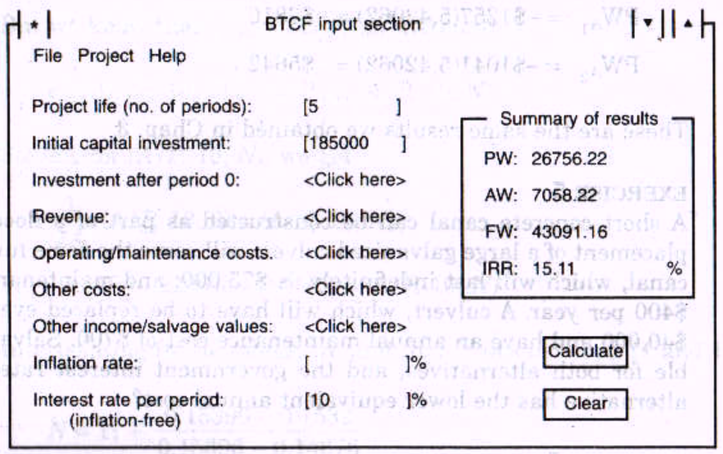


FIGURE 4.1 CHEER screen for finding annual worth.

4.8 PROBLEMS

4.1 You apply to your local bank for a loan of $15,000. The prevailing annual interest rate is 8 percent, and you are to pay off the loan in five equal end-of-year payments. Determine the total interest that you will pay over the 5-year period.

4.2 For the situation given in Prob. 4.1, calculate directly the recovered principal (capital) by the end of year 3.

4.3 A large gasoline station is required by the city to install vapor containment equipment on its gasoline pump nozzles and storage tank vents. The immediate conversion cost will be $180,000 with an estimated $600 per year for maintenance. It will be necessary to update the equipment every 3 years at a cost of $3500. The station pumps an average of 1 million gallons of gasoline per month. On an annual basis, what would be the price increase per gallon necessary to pay for the conversion over a 6-year period? Include the sixth year's update cost in your analysis, and assume an interest rate of 14 percent.

4.4 A grocery chain of four stores is evaluating whether to install video screens on all its grocery carts. These screens will display pricing and "specials" as the cart goes by the pertinent items. Cart location is sensed by ceiling sensors that trigger the appropriate information for the particular screen. The first cost of the equipment is $65,000 per store. Annual programming and screen information would be subcontracted at a total cost of $25,000 per year for the four stores. Because of the novelty, sales are expected to increase by $28,000 per store in the first year and then drop at a rate of $4500 per store per year for each subsequent year, that is, $28,000, $23,500, . . . , $10,000. The technological life of the system is 5 years. If a 12 percent return on investment is required, determine the minimum salvage value of the equipment that would be needed after the 5-year period, basing your analysis on the equivalent annual net worth of the project. Evaluate the result.

4.5 A company owns several gasoline stations in a major city. It is decided that a major television advertising campaign will greatly improve income. Initial development costs for the advertisements will be $120,000. Monthly television airing

costs are quoted at $35,000 for the first month, decreasing by $500 per month thereafter during the period in which the ads will run, which is 18 months. Revenues are expected to increase by $40,000 in the first month and increase by $700 per month thereafter for 11 months more. The last 6 months of the study are expected to see a linear decline of $300 per month from the peak increase. Determine whether the campaign will be economically viable, using an equivalent monthly worth analysis. Assume a nominal annual interest rate of 12 percent with monthly compounding.

4.6 Megabitt Electronics is considering the purchase of a new programmable circuit tester in order to improve its product quality. The equipment has a first cost of $85,000, and the salvage value is predicted to be $6000 after a service life of 5 years. Maintenance and operating costs are expected to be $8000 the first year of operation and to increase by $1500 per year for each additional year of use. Using an interest rate of 10 percent, determine what annual savings must be obtained through the use of this equipment to make it economically justifiable.

4.7 A standby electric power generator was purchased 6 years ago for $8000. At that time it was expected that the equipment would be used for 15 years and would have a salvage value of 10 percent of the first cost. The generator is no longer needed and is to be sold for $2500. Using an interest rate of 15 percent, determine the difference between the anticipated and actual equivalent annual capital costs.

4.8 Granite Rock and Gravel Company is considering the feasibility of purchasing a piece of land for a small quarrying operation. The following cost estimates have been developed for evaluating the venture:

Cost of land	$2,000,000
Site clearing and road preparation	200,000
Annual operating costs:	
First year	400,000
Increase for each additional year of operation	50,000
Site cleanup prior to resale	100,000

The quarry will probably have a useful life of 10 years, and the reclaimed site should have a resale value of $1 million. Using an interest rate of 15 percent, determine the equivalent annual cost of this operation.

4.9 A company can purchase a piece of equipment for $20,000 and sell it for $4000 at the end of a 6-year service life, or it can lease the unit for the same period by making first-of-the-year payments of $3000. Compare the equivalent annual costs of the alternatives, using an interest rate of 15 percent.

4.10 Five years ago, a car owner bought an automobile for $13,000. Today, a trade-in of $1500 was allowed on the purchase of a new car. The old one had been driven 70,000 miles (112,651 kilometers). If the owner's other investments earn 6 percent annually, indicate the old car's cost per mile (kilometer) for capital recovery plus interest during the period of ownership.

4.11 Laser beams are to be used on a major construction project to ensure the exact alignment of components. Two types of laser alignment systems, with the costs shown below, are suitable for the project.

	IC system	UC system
First cost	$5000	$3200
Salvage value	1000	0
Annual operating cost	600	950
Additional taxes and insurance per year	180	0

If both systems have a life of 4 years and the minimum rate of return is 15 percent, which offers the lower equivalent annual cost?

4.12 For the situation given in Prob. 4.11, how much longer would the economic life of the IC system have to be in order to make the equivalent annual costs of the two systems equal?

4.13 Problem 3.26 required the present-worth evaluation of three robot options. Now perform that analysis, using an EAW basis.

4.14 Problem 3.28 asked for a present-worth evaluation of options with different lives. Now perform this analysis with an EAW basis, and comment on whether the annual basis is easier to compute.

4.15 Two methods of supplying water and sewage treatment for a housing development that is outside the districts where water and sewage disposal services are provided by the city are described by the accompanying cash flows for a 40-year study:

Years	Method 1	Method 2
0	−$350,000	−$735,000
1–9	−11,000/year	−8,000/year
10	−36,000	+95,000
11–19	−13,000/year	−13,000/year
20	−163,000	+87,000
21–29	−15,000/year	−15,000/year
30	−40,000	−90,000
31–40	−18,000/year	−15,000/year

Both methods have the same total cash flow without regard for interest (−$1,102,000), and both provide a comparable quality of service. At an interest rate of 8 percent, determine which method has the lower equivalent annual cost.

4.16 A bond issue for $125,000 has been passed by voters to buy six minibuses for a senior citizens' transportation service. It is anticipated that the revenue from the bus service will yield a rate of return of 7 percent on the investment. A provision in the bond issue was that a sinking fund be established through a local bank to accumulate enough money to recover the $125,000 in 6 years. The bond issue is to pay 8 percent simple interest (due in a lump sum at maturity), and the local bank pays annual interest of 6 percent.

(*a*) What annual payment is required for the sinking fund at the bank?

(*b*) What annual return is required to recover the capital invested plus profit?

4.17 A sheltered workshop requires a lift truck to handle pallets for a new contract. A lift truck can be purchased for $27,000. Annual insurance costs are 3 percent of the purchase price, payable on the first of each year. An equivalent truck can be rented for $1500 per month, payable at the end of each month. Operating costs are the same for both alternatives. For what minimum number of months must a purchased truck be used on the contract to make purchasing more attractive than leasing? Interest is 12 percent compounded monthly. (Assume that the purchased truck has no salvage value at any time.)

4.18 An old wooden bridge over a bay is in danger of collapse. The highway department is currently considering two alternatives to alleviate the situation and provide for expected increases in future traffic. One plan is a conventional steel bridge, and the other is a tunnel under the bay. The department is familiar with bridge construction and maintenance but has no experience with maintenance costs for tunnels. The following data have been developed for the bridge:

First cost	$17,000,000
Painting every 6 years	1,000,000
Deck resurfacing every 10 years	3,000,000
Structural overhaul at end of 15 years	4,000,000
Annual maintenance	300,000

The tunnel is expected to cost $24,000,000 and will require repaving every 10 years at a cost of $2,000,000. If both designs are expected to last 30 years with negligible salvage value, determine the maximum equivalent annual amount for maintenance that could be permitted for the tunnel while holding the total EAC to that of the bridge. Let $i = 8$ percent.

4.19 What is the present worth of the following cash flow with nonequal interest rates shown below?

End of year	0		1		2		3		4		5
Interest rate, %		7		7		9		10		8	
Receipts, $	10,000				10,000				10,000		
Payments, $			3000				6000				11,000

4.20 Determine the future worth of the cash flow given in Prob. 4.19.

4.21 Determine the uniform series value A that is equivalent to the cash flow given in Prob. 4.19.

4.22 An electric utility company is looking at two alternatives for tree-trimming equipment. One is to subcontract to an independent maintenance company. The subcontractor's bid calls for $98,000 the first year with additional costs of $8000 per year for subsequent years. The utility company is considering buying equipment with a first cost of $220,000 and annual operating expenses of $65,000 per year. The equipment is expected to have a salvage value of $25,000 at the end of its useful life (to the utility company) of 5 years. Using an interest rate of 12 percent, evaluate the alternatives on an EAC basis.

4.23 The utility company in Prob. 4.22 now has different alternatives. The subcontractor agrees to the same bid but over a 6-year period instead of a 5-year period. The utility company now has a bid on equipment from another company. The equipment will be leased to the utility company for $65,000 per year for the first 3 years and $72,000 per year for the last 3 years, and annual operating expenses are estimated to be $45,000 per year. Evaluate these alternatives on an EAC basis, using the 12 percent interest rate.

4.24 An earth compactor costs $38,000 and has an economic life of 9 years. However, the purchaser needs it for only one project that will be completed in 3 years. At the end of the project, it can be sold for one-half its purchase price. What is the annual cost to the owner, if the required rate of return is 12 percent?

4.25 The athletic department of a university is proposing that a new general-purpose stadium be constructed on campus. A design utilizing a combination earthwork bowl with a steel upper deck and press box is being considered. The following cost estimates have been developed:

First cost of complete construction	$32,000,000
Paint steel structure every 6 years	2,000,000
Replace wooden seats every 10 years	4,000,000
Repave parking facilities and ramps every 12 years	3,000,000
Annual maintenance	1,500,000

Assuming a 60-year life and negligible salvage value, determine the minimum annual revenue that could justify the project, using a tax-free interest rate of 7 percent.

4.26 Two types of power converters, alpha and beta, are under consideration for a specific application. An economic comparison is to be made at an interest rate of 10 percent, and the following cost estimates have been obtained:

	Alpha	Beta
Purchase price	$10,000	$25,000
Estimated service life	5 years	9 years
Salvage value	$0	$5000
Annual operating cost	$2500	$1200

Determine the annual equivalent costs of the alternative systems.

4.27 For the power converters in Prob. 4.26, determine a salvage value for the beta system such that it would have an equivalent annual cost equal to that of the alpha system.

4.28 An asset is expected to depreciate in market value at a constant rate from its purchase price of $20,000 to a zero salvage value during its 8 years of physical life. Operating costs are expected to be $8000 for the first year and to increase at a 10 percent annual rate as the asset gets older. What is the annual cost of ownership, if the asset is replaced every 3 years? The required rate of return is 12 percent.

4.29 You have two alternatives for a loan for a home mortgage. The house you want is for sale at $138,000, but you can afford a down payment of $25,000. Company *A* offers you a loan for the balance at 7.5 percent annual interest to be compounded monthly. The loan is to be paid off in 15 years. Company *B* offers you a rate of 9 percent with the balance to be paid off in 20 years; compounding is also monthly. Evaluate the monthly payments for the two alternatives.

4.30 Suppose that company *A* in Prob. 4.29 will actually give you a loan for the full amount of the house price under the same conditions as given in Prob. 4.29, except that your annual interest rate will increase by a 0.5 percent. You want to use $5000 of the original down payment for draperies for the new house. Would you consider the new financing possibilities if you could get a guaranteed return of 9 percent compounded annually by investing the rest of the down payment for 15 years?

4.31 An airline is evaluating a new reservations mainframe computer that is expected to have a technological life of 3 years. The first cost of the computer is $480,000 with operating costs expected to be $50,000 for the first year and $3000 per year additional for each year afterward. Salvage value is expected to be 10 percent of first cost. The computer supplier suggests that in 3 years a replacement computer will have first costs that are 10 percent less than those of the current machine. Also, operating costs are expected to have the same percentage of first cost and gradient cost reductions. The current computer system is expected to have an EAC over the next 3 years of $285,000. Evaluate the alternatives by the EAC method, using an interest rate of 15 percent. Assume that replacement will be either now or in 3 years.

4.32 A mining and excavating company uses a large number of light pickup trucks for crew transport and general utility duties. The trucks have a first cost of $19,000 and owing to the generally rugged use lose value at a geometric rate of approximately 30 percent per year. Operation and maintenance costs for two-shift use amount to $4000 for the first year and increase about $800 per year for each additional year of service. Current company policy is to keep the vehicles for 5 years before they are sold. The maintenance supervisor has suggested that they be sold 1 year earlier in order to reduce maintenance expenses. Using an interest rate of 12 percent, determine the equivalent annual-cost effect of implementing the supervisor's suggestion.

4.33 A frozen fish company is planning an expansion to a cold storage facility. Four alternative site design proposals are being considered at an interest rate of 15 percent. Plans *A* and *B* require the expenditure of $350,000 for land, and plans *C* and *D* require $425,000 for land. These real estate investments are assumed to be permanent. The buildings are expected to last 30 years, the compressors and related equipment will last 10 years before requiring replacement, and energy costs will increase throughout the building's life. Neither the buildings nor the equipment is expected to have any salvage value. With this information and the data provided on the next page, make an annual-cost comparison to determine which proposal is preferred.

	Proposal			
	A	**B**	**C**	**D**
Building and insulation	$600,000	$700,000	$400,000	$500,000
Compressors	100,000	135,000	85,000	70,000
Expected energy costs:				
First year	65,000	48,000	65,000	54,000
Increase for each additional year	3,000	2,000	3,000	2,000
Annual maintenance expense	20,000	15,000	50,000	40,000

CHAPTER 5

RATE-OF-RETURN CALCULATIONS

No law can reduce the common rate of interest below the lowest ordinary market rate at the time when that law is made.

Adam Smith, *The Wealth of Nations*, 1776

The rate of return is the last of the discounted cash flow comparison methods we need to consider. A *minimum acceptable rate of return (MARR)* is the lowest level at which an independent alternative (or, as we will see, an incremental cash flow between two mutually exclusive alternatives) is still attractive. It varies among and within organizations. Although there are a wide variety of recommendations for determining this lowest level of acceptability, it is generally agreed that the level should be no lower—and most likely considerably higher—than the cost of capital. How much higher depends on the circumstances, objectives, and policies of the organization. The purpose of establishing a minimum acceptable rate of return is to ration capital to the most deserving proposals.

Calculation of an *internal rate of return (IRR)* will allow us to determine, under possible reinvestment constraints, whether an alternative meets the MARR value. The IRR analysis may begin with an equivalent annual-worth (EAW) (covered in Chap. 4), present-worth (PW), or future-worth (FW) (both PW and FW are covered in Chap. 3) formulation. By definition, *the internal rate of return of an investment is the rate of interest earned on the unrecovered balance of an investment where the terminal balance is zero.* Given that the present and future worths of a cash flow are equivalent when either is $0, the IRR computation frequently starts with the PW formulation of the cash flow, which is equal to FW = 0 through PW = $0[1/(1 + i)^N]$.

There is no way to avoid trial-and-error computations in *manually* calculating the IRR for complex formulations, but as we will see, the structure