RAMAIAH
Institute of Technology

Department of Mathematics

MA11
Engineering Mathematics–I

- Syllabus
- Lesson plan
- Question Bank
- Model Question Papers

I Semester B.E.

Common to all Branches

Term: 28-12-2020 to 13-03-2021

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(Autonomous Institute, Affiliated to VTU)
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SYLLABUS

ENGINEERING MATHEMATICS – I

Unit I

Differential Calculus-I: Polar curves, angle between the radius vector and the tangent, angle between the curves, length of perpendicular from pole to the tangent, pedal equations.

Partial Differentiation: Partial derivatives, Euler's theorem. Total differential coefficient, differentiation of composite and implicit functions, Jacobians and properties.

Unit II

Integral Calculus-I: Reduction formulae, $\sin^n x$, $\cos^n x$, $\sin^m x \cos^n x$, evaluation of integrals with standard limits, Tracing of curves (both Cartesian and polar).
Application of integration – length of arc of a curve, plane areas, volumes and surface area of revolution.

Unit III

Vector differentiation: Vector functions of a single variable, derivative of a vector function, geometrical interpretation, velocity and acceleration.
Scalar and vector fields, gradient of a scalar field, directional derivative, divergence of a vector field, solenoidal vector, curl of a vector field, irrotational vector, Laplacian operator. Vector identities. (Standard vector identities).

Unit IV

Integral Calculus-II: Multiple integrals- evaluation of double and triple integrals, change of order of integration, change of variables. Applications of double and triple integrals to find areas and volumes.

Unit V

Vector integration: Line integrals, surface integrals and volume integrals. Green's theorem (with proof) and its applications, Stokes' theorem (without proof), and its applications Gauss divergence theorem (without proof) and its applications.

Text Books:

1. G.B. Thomas and R.L. Finney– Calculus and Analytical Geometry, Pearson, 12th edition, 2017.
2. B.S. Grewal – Higher Engineering Mathematics, Khanna Publishers, 44th edition, 2017.

Reference Books:

1. Erwin Kreyszig – Advanced Engineering Mathematics, Wiley publication, 10th edition, 2015.
2. Peter V.O' Neil –Advanced Engineering Mathematics, ThomsonBrooks/Cole, 7th edition,2011.
3. Glyn James–Advanced Modern Engineering Mathematics, Pearson Education, 4th edition, 2010.
4. George B. Thomas, Maurice D. Weir, Joel R. Hass - Thomas' Calculus, Pearson, 13th edition, 2014.

LESSON PLAN

Lesson / Session No	Topics	No. of hours
Unit-I (08 Hours) Differential Calculus-I		
1	Introduction to Polar Coordinates, Polar Curves, Angle between radius vector and tangent.	1 Hr
2	Angle between the polar curves, Problems.	1 Hr
3	Length of the perpendicular from pole to the tangent, Pedal equation and problems.	1 Hr
4	Partial differentiation and problems.	1 Hr
5	Euler's theorem and Problems.	1 Hr
6	Total differential coefficient, Composite & Implicit functions, Problems.	1 Hr
7	Problems on Lesson-5 and 6.	1 Hr
8	Jacobians, Properties of Jacobians and Problems.	1 Hr
Unit-II (09 Hours) Integral Calculus-I		
9	Reduction formula for standard functions.	1 Hr
10	Problems on Lesson-9	1 Hr
11	Tracing of curves – Astroid.	1 Hr
12	Tracing of curves – Strophoid and Cissoid.	1 Hr
13	Tracing of curves – Cycloid and Folium of Descartes.	1 Hr
14	Tracing of curves – Cardioid and three leaved rose.	1 Hr
15	Applications of integration – length of arc of a curve, plane areas.	1 Hr
16	Volume and surface area of revolution.	1 Hr
17	Problems on Lesson–15 &16.	1 Hr
Unit-III (09 Hours) Vector Differentiation		
18	Introduction to derivative of a vector function, Geometrical interpretation.	1 Hr
19	Velocity and Acceleration, Problems.	1 Hr
20	Gradient of a scalar field, Geometrical interpretation, Problems.	1 Hr
21	Angle between two surfaces, Problems.	1 Hr
22	Directional derivatives, Problems.	1 Hr
23	Divergence of a vector field, Solenoidal vectors, Problems.	1 Hr
24	Curl of a vector field, Irrotational vectors, Problems.	1 Hr

Lesson / Session No	Topics	No. of hours
25	Problems continued on Lesson-23 and 24.	1 Hr
26	Vector identities.	1 Hr
	Unit-IV (08 Hours) Integral Calculus-II	
27	Introduction to multiple integrals.	1 Hr
28	Evaluation of double and triple integrals and problems.	1 Hr
29	More problems on Lesson-28.	1 Hr
30	Evaluation of multiple integral over a given region.	1 Hr
31	Evaluation of double integral by changing the order of integration.	1 Hr
32	Change of variables, Problems.	1 Hr
33	Applications of multiple integrals to find areas and volumes.	1 Hr
34	Problems continued on Lesson-33.	1 Hr
	Unit-V (08 Hours) Vector Integration	
35	Introduction to Line integrals, Problems.	1 Hr
36	Green's theorem in a plane, Problems.	1 Hr
37	Problems on verification of Green's theorem.	1 Hr
38	Evaluation of line integrals and computation of area using Green's theorem.	1 Hr
39	Surface integrals, Problems.	1 Hr
40	Stoke's theorem (no proof), Problems.	1 Hr
41	Volume integrals, Problems.	1 Hr
42	Gauss divergence theorem (no proof), Problems	1 Hr

Internal Assessment Details

Test marks = **30 marks** (T_1 and T_2– each carries 30 marks, average of T_1 &T_2)

Quiz = **10 marks**

Assignment = **10 marks**

CIE Total = **50 Marks**

Syllabus for Tests

Test	Unit	Lesson No.
Test - 1	Unit-I, Unit-II and a part of Unit-III	Lesson 1 to Lesson 19
Test - 2	Part of Unit-III, Unit-IV and Unit-V	Lesson 20 to Lesson 37

UNIT–I DIFFERENTIAL CALCULUS-I	
POLAR CURVES (Two & Four Marks Questions)	
1.	Write the relation between cartesian and polar coordinates.
2.	Write the expression for angle between the radius vector and the tangent. Also define the terms involved.
3.	Write the expression for length of the perpendicular from pole to the tangent.
4.	Find the angle between radius vector and the tangent for: (a) $r = a(1+\cos\theta)$ (b) $r^2 cos2\theta = a^2$ (c) $\frac{2a}{r} = 1-\cos\theta$ (d) $r^m = a^m(\cos m\theta + \sin m\theta)$ (e) $\frac{l}{r} = 1 + e\cos\theta$, where l and e are constants (f) $r^2 = a^2\sin 2\theta$
5.	For the following curves find the slope of the tangent at the indicated points: (a) $\frac{2a}{r} = 1-\cos\theta$ at $\theta = \frac{2\pi}{3}$ (b) $2r = a\sin 2\theta$ *at* $\theta = \frac{\pi}{4}$ (c) $r^2\cos 2\theta = a^2$ *at* $\theta = \frac{\pi}{12}$ (d) $r = a\sin 3\theta$ at the pole (e) $r\cos^2\left(\frac{\theta}{2}\right) = a^2$ at $\theta = \frac{2\pi}{3}$ (f) $r\sec^2\left(\frac{\theta}{2}\right) = 4$ at $\theta = \frac{\pi}{2}$
6.	Show that at any point (r,θ), the tangent to the curve $r^n = a^n\sin(n\theta)$ makes an angle $(n+1)\theta$ with the initial line.
7.	Find the slope of the tangent at any point (r,θ) on the curve $r = a(1+\sin\theta)$. Further show that the tangent at the point $\theta = \frac{\pi}{2}$ is parallel to the initial line.
8.	Show that the tangent to the curve $r = a(1+\cos\theta)$ is (a) parallel to the initial line at $\theta = \frac{\pi}{3}$ (b) perpendicular to the initial line at $\theta = \frac{2\pi}{3}$.
9.	Show that the angle between the tangent at any point p and the line joining p to the origin is the same at all points of the curve $\log(x^2+y^2) = k\tan^{-1}\left(\frac{y}{x}\right)$.
10.	Find the pedal equation of the curve $r = a\theta$.
POLAR CURVES (Seven Marks Questions)	
11.	Prove with usual notation $\tan\varphi = r\frac{d\theta}{dr}$.
12.	With usual notation prove that $p = r\sin\varphi$ and hence prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.
13.	Show that the following pairs of curves intersect each other orthogonally: (a) $r = ae^\theta$ and $re^\theta = b$ (b) $r^n = a^n\cos n\theta$ and $r^n = b^n\sin n\theta$ (c) $r = a(1+cos\theta)$ and $r = b(1-cos\theta)$ (d) $r = a\sec^2\left(\frac{\theta}{2}\right)$ and $r = a\,cosec^2\left(\frac{\theta}{2}\right)$
14.	Find the angle of intersection of the following pairs of curves: (a) $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$ (b) $r = a\log\theta$ and $r = \frac{a}{\log\theta}$ (c) $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$ (d) $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$ (e) $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ (f) $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$

15.	Show that the curves $r^2 = a^2\cos 2\theta$ and $r = a(1+\cos\theta)$ intersect at angle $3\sin^{-1}\left(\frac{3}{4}\right)^{1/4}$.
16.	Show that the circle $r = b$ cuts the curve $r^2 = a^2\cos 2\theta + b^2$ at angle $\tan^{-1}\left(\frac{a^2}{b^2}\right)$.
17.	Find the length of perpendicular from pole to the tangent to the following curves: (a) $r = a(1-\cos\theta)$ at $\left(a, \frac{\pi}{2}\right)$ (b) $r^2\cos 2\theta = a^2$ at $\theta = \frac{\pi}{6}$ (c) $r = a\, sec^2\left(\frac{\theta}{2}\right)$ at $\theta = \frac{\pi}{3}$
18.	Find the pedal equation to the following curves: (a) $r^n = a^n\cos n\theta$ (b) $r^2 = a^2\sin 2\theta$ (c) $r = \mathrm{a}\,\mathrm{sech}\, n\theta$ (d) $2r = a\sin 2\theta$ (e) $r = ae^{m\theta}$ (f) $\frac{a(e^2-1)}{r} = 1 + e\cos\theta$ (g) $r = \frac{2a}{1-\cos\theta}$ (h) $r = ae^{\theta\cot\alpha}$, where a and α are constants (i) $\frac{l}{r} = 1 + e\cos\theta$, where l and e are constants
19.	Show that the pedal equation of the curve $r^n = a^n\sin n\theta + b^n\cos n\theta$ is $p^2(a^{2n}+b^{2n}) = r^{2n+2}$.
20.	Prove that the curves $r^n = a^n\sec(n\theta+\alpha)$ and $r^n = b^n\sec(n\theta+\beta)$ intersect at an angle which is independent of a and b.
	PARTIAL DERIVATIVES & EULER'S THEOREM (Two & Four Marks Questions)
21.	Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ in each of the following cases: (a) $u = \sin^{-1}\left(\frac{y}{x}\right)$ (b) $u = x^y + \frac{y}{2x}$ (c) $u = x^3e^{-y^2/4x}$ (d) $u = e^x(x\cos y - y\sin x)$
22.	Find all the first order partial derivatives: (a) $w = \cos(x^2+2y) - e^{4x-z^4y} + y^3$ (b) $f = 8u^2t^3p - \sqrt{v}\,p^2t^{-5} + 2u^2t + 3p^4 - v$
23.	Show that $u = \sin(x-ct)$ satisfies the wave equation $u_{tt} - c^2u_{xx} = 0$.
24.	If $u = xf\left(\frac{y}{x}\right) + \varphi\left(\frac{y}{x}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xf\left(\frac{y}{x}\right)$.
25.	Given $z = 3x^2y - x\sin(xy)$ then verify that $\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial^2 z}{\partial y\partial x}$.
26.	If $u = \frac{y}{z} + \frac{z}{x}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
27.	If $z = e^{ax+by}f(ax-by)$, then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.
28.	If $z = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, then verify that (a) $\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial^2 z}{\partial y\partial x}$ (b) $\frac{\partial^2 z}{\partial x\partial y} = \frac{x^2-y^2}{x^2+y^2}$.
29.	Verify that v satisfies the Laplace's equation $v_{xx} + v_{yy} + v_{zz} = 0$ (a) $v = e^{3x+4y}\cos 5z$ (b) $v = (x^2+y^2+z^2)^{-1/2}$ (c) $v = \cos 3x\cos 4y\sinh 5z$
30.	If $u = e^{a\theta}\cos(a\log r)$, then prove that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} = 0$.
31.	Prove that $u_{xx} + u_{yy} = 0$, if (a) $u = \tan^{-1}\left(\frac{2xy}{x^2-y^2}\right)$ (b) $u = \log(x^2+y^2) + \tan^{-1}\left(\frac{y}{x}\right)$.
32.	The altitude of a right circular cone is 15cm and is increasing at 0.2 cm/s. The radius of the base is 10cm and is decreasing at 0.3cm/s. How fast is the volume changing?
33.	Find the rate at which the area of a rectangle is increasing at a given instant when the sides of the rectangle are 4ft and 3ft and are increasing at the rate of 1.5ft/s and 0.5ft/s respectively.

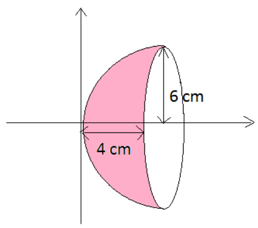
34.	In order that the function $u = 2xy - 3x^2y$ remains constant, what should be the rate of change of y w.r.t. t, given x increases at the rate of $2cm/sec$ at the instant when $x = 3cm$ and $y = 1cm$.
35.	Define homogenous function for two and three variables with an example.
36.	State Euler's theorem on homogenous function for two and three independent variables.
37.	If $f = \frac{1}{x^2} + \frac{1}{xy} + \frac{log\, x - log\, y}{x^2+y^2}$, then prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$.
38.	If $u = tan^{-1}\left\{\frac{x^2+y^2}{\sqrt{x}+\sqrt{y}}\right\}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{4}sin\, 2\,u$.
39.	If $u = \frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
40.	If$u = log\left(\frac{x^4+y^4}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.
41.	If $u = \frac{x^3+y^3}{\sqrt{x+y}}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{5}{2}u$.
	PARTIAL DERIVATIVES & EULER'S THEOREM (Seven Marks Questions)
42.	If $u = log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)u = \frac{3}{x+y+z}$ and hence show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.
43.	Given $u = e^{r\cos\theta}\cos(r\sin\theta)\,, v = e^{r\cos\theta}\sin(r\sin\theta)$, then prove that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$.
44.	Find the value of n so that the equation $v = r^n(3\cos^2\theta - 1)$ satisfies the relation $\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v}{\partial \theta}\right) = 0$.
45.	If $\theta = t^n e^{-r^2/4t}$, what value of n will make $\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right)\right] = \frac{\partial \theta}{\partial t}$.
46.	If $x = r\cos\theta$ and $y = r\sin\theta$then prove that (a) $\frac{\partial^2 r}{\partial x^2}.\frac{\partial^2 r}{\partial y^2} = \left(\frac{\partial^2 r}{\partial x\partial y}\right)^2$ (b) $\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$.
47.	If $u = \frac{1}{r}$ and $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$, where $(x-a),(y-b),(z-c)$ are not simultaneously zero, then show that $u$satisfies the Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$.
48.	State and prove Euler's theorem on homogeneous functions and also prove its extension.
49.	If $u = sin^{-1}\left[\frac{ax+by+cz}{\sqrt{x^n+y^n+z^n}}\right]$, then prove that $xu_x + yu_y + zu_z = \left(1 - \frac{n}{2}\right)tan\, u$.
50.	If $u = \csc^{-1}\left[\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]$, then prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{tan\, u}{6}\left(1 + \frac{sec^2 u}{6}\right)$.
51.	If $u = tan^{-1}\left[\sqrt{x^4+y^4}\right]$, then show that (a) $xu_x + yu_y = sin\, 2\,u$ (b)$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = sin\, 4\,u - sin\, 2\,u$.
52.	Verify Euler's theorem for the following functions (a) $u = \frac{xy}{x+y}$ (b) $u = x^3 + y^3 + z^3 + 3xyz$ (c) $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$
53.	If $w = sin^{-1}u\,, u = \frac{x^2+y^2+z^2}{x+y+z}$ then prove that $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = tan\, w$.
54.	If $u = \sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$, then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left\{\frac{13}{12} + \frac{\tan^2 u}{12}\right\}$.
55.	If $u = tan^{-1}\left(\frac{y^2}{x}\right)$, then prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = -sin\, 2\,u\, sin^2 u$.
56.	If $u = sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, then find $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$.

TOTAL DIFFERENTATION (Two & Four Marks Questions)	
57.	Find $\frac{dy}{dx}$ in each of the following cases using partial derivatives: (a) $e^x + e^y = 2xy$ (b) $y^x = x$ (c) $x \sec y + x^3 y = 1$ (d) $x^3 + y^3 - 3axy = 1$.
58.	Find $\frac{du}{dx}$ in each of the following cases: (a) $u = cos((x^2 - y^3)\,; 2x^2 + 3y^2 = a^2$ (b) $u = tan^{-1}\left(\frac{y}{x}\right); x^2 + y^2 = a^2$ (c) $u = x^2 y; x^2 + xy + y^2 = 1$ (d) $u = ye^{xy}; x^2 + y^2 = a^2$.
59.	If $z = x\log(xy) + y^3$, where $y = \sin(x^2 + 1)$ find $\frac{dz}{dx}$.
60.	If $u = x\, log(\,xy)$, where $x^3 + y^3 + 3xy = 1$ then find $\frac{du}{dx}$.
61.	If $u = tan^{-1}\left(\frac{y}{x}\right)$, where $x = e^t - e^{-t}; y = e^t + e^{-t}$, then find $\frac{du}{dx}$.
62.	Find $\frac{du}{dt}$ for the following functions: (a) $u = x^2 - y^2$, where $x = e^t \cos t\,; y = e^t \sin t$ at $t = 0$. (b) $u = xy + yz + zx$, where $x = \frac{1}{t}, y = e^t$ and $z = e^{-t}$. (c) $u = x^3 y e^z$, where $x = t, y = t^2$ and $z = log\, t$ at $t = 2$.
TOTAL DIFFERENTATION (Seven Marks Questions)	
63.	If $u = e^{xy} \sin(\,yz)$, where $x = t^2, y = t - 1, z = \frac{1}{t}$, then find $\frac{du}{dt}$ at $t = 1$ by partial differentiation.
64.	Find the total derivative of the following functions and also verify the result by direct substitution (a) $u = xy^2 + x^2 y$, where $x = at^2, y = 2at$. (b) $u = sin\left(\frac{x}{y}\right)$, where $x = e^t, y = t^2$. (c) $u = x^2 + y^2 + z^2$, where $x = e^{2t}, y = e^{2t}\cos 3\,t$and $z = e^{2t} \sin 3\,t$. (d) $u = xy + yz + zx$, where$x = t\cos t\,, y = t\sin t\,, z = t$ at $t = \frac{\pi}{4}$.
65.	If $z = f(x,y)$ and $x = u - v, y = uv$ then show that (a) $(u+v)\frac{\partial z}{\partial x} = u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v}$, (b) $(u+v)\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$.
66.	If $z = f(x,y)$ and $x = e^u + e^{-v}; y = e^{-u} - e^v$, then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial v}$.
67.	If $z = f(y - z, z - x, x - y)$ then prove that $u_x + u_y + u_z = 0$.
68.	If $v = f(r, s, t)$, where $r = \frac{x}{y}, s = \frac{y}{z}$ & $t = \frac{z}{x}$, then prove that $xv_x + yv_y + zv_z = 0$.
69.	If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.
70.	If $u = \frac{x}{z}, v = \frac{y}{z}, w = z$ and $f = f(u,v,w)$, then show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = w\frac{\partial f}{\partial w}$.
71.	If $x = u + v + w, y = vw + wu + uv, z = uvw$ and F is a function of x, y, z then show that $u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z}$.
72.	If z is a function of x and y, where $x = e^u \cos v\,, y = e^u \sin v$, then prove that (a) $x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y}$. (b)$\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = \frac{1}{e^{2u}}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right]$.
JACOBIANS (Two & Four Marks Questions)	
73.	Define Jacobian of u, v, w with respect to x, y, z.
74.	State any two properties of Jacobians.
75.	If $u = x + 3y^2 - z^3,\ v = 4x^2 yz,\ w = 2z^2 - xy$ then evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$.

76.	Find $\frac{\partial(u,v)}{\partial(x,y)}$ in each of the following cases: (a) $u = x(1-y), v = xy$ (b) $u = 3x + 5y, v = 4x - 5y$ (c) $u = x^2 - 2y, v = x + y$ (d) $u = e^x \sin y, v = x \log \sin y$.
77.	If $x = a\cosh\theta\cos\varphi$ and $y = a\sinh\theta\sin\varphi$, then find $\frac{\partial(x,y)}{\partial(\theta,\varphi)}$.
78	Find $\frac{\partial(x,y)}{\partial(u,v)}$ in each of the following cases: (a) $x = \frac{u^2}{v}, y = \frac{v^2}{u}$ (b) $x = u(1+v), y = v(1+u)$ (c) $x = u(1-v), y = uv$.
79.	Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ in each of the following cases: (a) $u = x^2 - 2y,\ v = x + y + z,\ w = x - 2y + 3z$ (b) $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$.
	JACOBIANS (Seven Marks Questions)
80.	If $x = r\cos\theta, y = r\sin\theta$ then show that $JJ' = 1$.
81.	If $u = \sqrt{yz}, v = \sqrt{zx}, w = \sqrt{xy}$ and $x = r\cos\varphi\sin\theta, y = r\sin\varphi\sin\theta, z = r\cos\theta$ then find $\frac{\partial(u,v,w)}{\partial(r,\varphi,\theta)}$.
82.	If $x = r\cos\varphi\sin\theta, y = r\sin\varphi\sin\theta, z = r\cos\theta$ then show that $\frac{\partial(x,y,z)}{\partial(r,\varphi,\theta)} = r^2\sin\theta$.
83.	If $x = e^v\sec u$ and $y = e^v\tan u$ then find the Jacobians $\frac{\partial(x,y)}{\partial(u,v)}$ and $\frac{\partial(u,v)}{\partial(x,y)}$.
84.	If $x = a(u+v)$ and $y = b(u-v)$, where $u = r^2\cos 2\theta$ and $v = r^2\sin 2\theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
85.	If $u = x^2 - 2y^2$ and $v = 2x^2 - y^2$, where $x = r\cos\theta, y = r\sin\theta$ then show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3\sin 2\theta$.
86.	If $u = x + y, v = y + z, w = z + x$ then find the inverse Jacobian by first confirming that u, v, w are functionally independent.
87.	Prove that the functions $u = \tan^{-1}x + \tan^{-1}y$ and $v = \frac{x+y}{1-xy}$ are functionally dependent using the concept of Jacobians. Also express u in terms of v.
88.	Show that the functions $u = x + y - z,\ v = x - y + z,\ w = x^2 + y^2 + z^2 - 2yz$ are not independent of one another. Also find the relation between them.
89.	Prove that the functions $u = x + y + z,\ v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are functionally dependent. Also find the relation between them.
90.	Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$ and also find the functional relation between u, v and w.
91.	If $u = x\sqrt{1-y^2} + y\sqrt{1-x^2},\ v = \sin^{-1}x + \sin^{-1}y$, show that u and v are functionally dependent and find the functional relationship.

	UNIT–II **INTEGRAL CALCULUS-I**
	REDUCTION FORMULAE **(Two & Four Marks Questions)**
1.	Write the reduction formula for (a) $\int_0^{\pi/2} \sin^n(x)dx$ (b) $\int \cos^n(x)dx$ (c) $\int_0^{\pi/2} \sin^m(x)\cos^n(x)dx$ where m and n are positive integers.
2.	Evaluate the following integrals by using reduction formula: (a) $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ (b) $\int_0^{\pi/6} \sin^3(6\theta)\cos^4(3\theta)\, d\theta$ (c) $\int_0^{\pi} \frac{\sin^4(x)}{(1+\cos x)^2}dx$ (d) $\int_0^{\pi} x.\sin^4(x)dx$ (e) $\int_0^{\pi} \frac{\sqrt{1-\cos x}}{1+\cos x}\sin^2(x)dx$ (f) $\int_0^{\pi/4} \sin^4(x)\cos^4(x)dx$ (g) $\int_0^{\pi} x\sin^8(x)\cos^6(x)dx$ (h) $\int_0^{\infty} \frac{x^6}{(1+x^2)^4}dx$ (i) $\int_0^1 x^2(1-x^2)^{3/2}dx$ (j) $\int_0^{\pi/4} \cos^6(2t)\, dt$ (k) $\int_0^{\pi/6} \sin^2(6x)\cos^6(3x)\, dx$.
	REDUCTION FORMULAE **(Seven Marks Questions)**
3.	Obtain the reduction formula for $I_n = \int_0^{\pi/2} sin^n(x)\, dx$ and evaluate I_5 and I_6.
4.	Obtain the reduction formula for $I_n = \int_0^{\pi/2} cos^n(x)\, dx$ and evaluate I_5 and I_6.
5.	Prove that $\int_0^{2a} x^n\sqrt{2ax-x^2}\, dx = \pi a^2\left(\frac{a}{2}\right)^n \frac{(2n+1)!}{(n+2)!\, n!}$.
6.	Evaluate $\int_0^{2a} \frac{x^3}{\sqrt{2ax-x^2}}dx$ using reduction formula.
	TRACING OF CURVES **(Two & Four Marks Questions)**
7.	Define a double point and classify it.
8.	Define cusp with an example.
9.	Define node with an example.
10.	Identify the double point at origin of the curve $y^2(1+x) = x^2(1-x)$.
11.	State any two properties while tracing Cartesian curves.
12.	State any two properties while tracing polar curves.
13.	Explain the procedure of finding asymptotes parallel to coordinate axis.
14.	Explain the procedure of finding oblique asymptotes.
15.	Find the equations of the tangents at the origin to the following curves: (a) $x^3 + y^3 = 3axy$ (b) $y^2(2a-x) = x^3, a > 0$ (c) $y^2(a+x) = x^2(a-x)$.
16.	Determine the region of existence of the following curves: (a) $y^2(2a-x) = x^3, a > 0$ (b) $x^{2/3} + y^{2/3} = a^{2/3}$ (c) $r = a(1+cos\theta)$.
17.	Find the oblique asymptotes of the curve $x^3 + y^3 = 3axy$.
	TRACING OF CURVES **(Seven Marks Questions)**
18.	Trace the curve $y^2(a-x) = x^2(a+x)$.
19.	Trace the curve $x^3 + y^3 = 3axy$.
20.	Trace the curve $y^2(2a-x) = x^3, a > 0$.
21.	Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
22.	Trace the curve $r = a(1+cos\theta)$.

23.	Trace the curve $r = a\, sin3\theta,\ a > 0$.
24.	Trace the curve $x = a(\ \theta - sin\theta), y = a(\ 1 - cos\theta), a > 0$.
	APPLICATIONS OF INTEGRATION (Two & Four Marks Questions)
25.	Write the expression to find the surface area of the solid for a cartesian curve when rotated about x and y axis.
26.	Write the expression to find the surface area of the solid for a polar curve when rotated about the initial line and the line $\theta = {}^{\pi}/_{2}$.
27.	Write the expression to find the volume of solid of rotation for a cartesian curve when rotated about x and y axis.
28.	Write the expression to find the volume of solid of rotation for a polar curve when rotated about the initial line and the line $\theta = {}^{\pi}/_{2}$.
29.	Find the total length of the cardioid $r = a(1 + cos\theta), a > 0$.
30.	Find the area of the cardioid $r = a(1 + cos\theta), a > 0$.
31.	Find the total perimeter of the curve asteroid $x^{2/3} + y^{2/3} = a^{2/3}, a > 0$.
32.	Find the area enclosed by the $x^{2/3} + y^{2/3} = a^{2/3}, a > 0$.
33.	Find the length of one arc of the cycloid $x = a(\ \theta - sin\theta), y = a(\ 1 - cos\theta), a > 0,\ 0 \le \theta \le 2\pi$.
34.	Find the area of the arc of the cycloid $x = a(\ \theta - sin\theta), y = a(\ 1 - cos\theta), a > 0, 0 \le \theta \le 2\pi$
35.	Find the length of the curve $3x^2 = y^3$ between $y = 0$ and $y = 1$.
36.	Find the length of the arc of the curve $y = log\left(\frac{e^x - 1}{e^x + 1}\right)$ from $x = 0$ to $x = 2$.
37.	Find the length of the arc of the curve $x = e^{\theta} sin\theta, y = e^{\theta} cos\theta$ from $\theta = 0$to $\theta = {}^{\pi}/_{2}$.
38.	Find the length of the arc of the curve $y = los\ secx$ from $x = 0$ to $x = {}^{\pi}/_{3}$.
	APPLICATIONS OF INTEGRATION (Seven Marks Questions)
39.	Determine the length of the cardioid $r = a(1 - cos\theta), a > 0$ lying outside the circle $r = acos\theta$.
40.	Find the area common to the circles $r = acos\theta$ and $r = asin\theta$.
41.	Prove that the area of loop of the curve $x^3 + y^3 = 3axy$ is $3\frac{a^2}{2}$.
42.	Determine the common area between $r = \frac{3a}{2}$ and $r = a(1 + cos\theta), a > 0$.
43.	Find the surface area of the solid obtained when cardioid $r = a(1 + cos\theta), a > 0$ is rotated about the initial line.
44.	Find the surface area of the solid generated by revolving the curve $x = t^2, y = t - \frac{t^3}{3}$, $0 \le t \le \sqrt{3}$ about x -axis.
45.	Find the length of the parabola $y^2 = 4ax$ cut -off by its latus rectum.
46.	Find the length of the arc of the parabola $y^2 = 4ax$ cut-off by $3y = 8x$.
47.	Find the length of the arc of the curve $y = e^x$ from $(0,1)$ to $(1, e)$.
48.	Find the surface area of the solid obtained when asteroid is rotated about x -axis.
49.	Find the surface area of the solid obtained when cycloid $x = a(\ \theta - sin\theta), y = a(\ 1 - cos\theta), a > 0$, $0 \le \theta \le 2\pi$ is rotated about its base.
50.	Find the surface area of the solid obtained when cardioid $r = a(1 + cos\theta), a > 0$ is rotated about the initial line.

51.	Find the volume of the solid generated by the revolution of the cycloid $x = a(\theta - sin\theta),\ y = a(1 - cos\theta), a > 0,\ \ 0 \le \theta \le 2\pi$ about its base.
52.	Find the volume of revolution of the curve $r = 2a\cos\theta$ about the initial line.
53.	Find the volume of the solid generated by revolution of the cardioid $r = a(1 + cos\theta),\ a > 0$ about the initial line.
54.	Find the cost of plating of the front portion of the parabolic reflector as in figure of an automobile head light of 12cm diameter and 4cm deep if the cost of painting is Rs.10/cm^2. 

UNIT–III **VECTOR DIFFERENTIATION**	
VECTOR FUNCTIONS OF A SINGLE VARIABLE, DERIVATIVE OF A VECTOR FUNCTION, GEOMETRICAL INTERPRETATION. VELOCITY, ACCELERATION, SCALAR AND VECTOR FIELD, GRADIENT OF SCALAR FIELD **(Two And Four Marks Questions)**	
1.	Define vector function of a single variable with an example.
2.	Give the geometrical meaning of the derivative of a vector.
3.	Define velocity and acceleration of vector function of a single variable t.
4.	Define scalar and vector field with an example.
5.	Define gradient of a scalar field.
6.	Show that $\nabla\emptyset$ is a vector perpendicular to the surface $\emptyset(x,y,z)=c$.
7.	If $\vec{r}=sect\,\hat{\imath}+tant\,\hat{\jmath}$ is the position vector of a point P then find the velocity and acceleration of P at $t=\frac{\pi}{6}$.
8.	If $\vec{r}=sin\omega t\,\hat{a}+cos\omega t\,\hat{b}$ where $\bar{a}$, $\bar{b}$ & w are constants, then show that $\frac{d^2r}{dt^2}=-\omega^2\vec{r}$ and $\vec{r}\times\frac{d\vec{r}}{dt}=-\omega(\hat{a}\times\hat{b})$.
9.	At any point on the curve $\vec{r}=3cost\,\hat{\imath}+3sint\,\hat{\jmath}+4t\,\hat{k}$, find unit tangent vector.
10.	If $\emptyset=f(r)$ where $r^2=x^2+y^2+z^2$, then prove that $\nabla\emptyset=\frac{f'(r)}{r}\vec{r}$.
11.	Show that $\nabla r^n=nr^{n-2}\vec{r}$.
12.	Find $\nabla\emptyset$ if a) $\emptyset=\log\lvert\vec{r}\rvert$ b) $\emptyset=\frac{1}{r}$.
13.	Find the unit vector normal to the following surfaces at the indicated points: a)$x^3+y^3+z^3+3xyz=3$ at $(1,2,-1)$ b) $x^2y+2xz=4$ at $(2,-2,3)$.
14.	In a temperature field, heat flows in the direction of maximum decrease of temperature T. Find this direction at P$(2,1)$ when $T=x^3-3xy^2$.
VECTOR FUNCTIONS OF A SINGLE VARIABLE, DERIVATIVE OF A VECTOR FUNCTION, GEOMETRICAL INTERPRETATION. VELOCITY, ACCELERATION, SCALAR AND VECTOR FIELD, GRADIENT OF SCALAR FIELD **(Seven Marks Questions)**	
15.	The position vector of a particle at time t is $\vec{r}=cos(t-1)\,\hat{\imath}+sinh(t-1)\,\hat{\jmath}+at^3\hat{k}$. Find the value of '$a$' such that the acceleration at time $t=1$ is normal to $\vec{r}$.
16.	Find the unit normal to the curve $\vec{r}=4\cos t\,\hat{\imath}+4\sin t\,\hat{\jmath}+3y\hat{k}$.
17.	Find the unit tangent & unit normal vector to the curve $x=2t; y=t^2; z=t^2-1$ at $t=0$.
18.	The position vector of a moving particle at time t is $\vec{r}=t^2\hat{\imath}+t^3\hat{\jmath}-t^4\hat{k}$, find the tangential and normal components of its acceleration at $t=1$.
19.	A particle moves along the curve $x=t^3+1; y=t^2; z=t+5$ where t is time. Find the components of velocity and acceleration at $t=2$ in the direction of $\hat{\imath}+3\hat{\jmath}+2\hat{k}$.
20.	A particle moves along the curve $x=e^{-2t}$, $y=2\cos 5t$, $z=5\sin 2t$, where t is time. Find the magnitude of the velocity and acceleration at $t=0$.
21.	Find the angle between the tangents to the curve $x=t^2+1$, $y=4t-3, z=2t^2-6t$ at $t=1$ and $t=2$.

22.	Find the angle between the tangents to $\vec{r} = t^2\hat{\imath} + 2t\hat{\jmath} - t^3\hat{k}$ at the points $t = \pm 1$.
23.	Find the values of a and b so that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at the point $(1,-1,2)$.
24.	Find the angle between the surfaces $x^2y + z = 3$ and $x\log z - y^2 = 4$ at the point $(-1,2,1)$.
25.	Find the angle between the normal to the surface $x^2yz = 1$ at $(-1,1,1)$ and $(1,-1,-1)$.
26.	Find the angle of intersection of the spheres $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ and $x^2 + y^2 + z^2 = 29$ at the point of intersection being $(4,-3,2)$.
27.	A person going eastwards with a velocity of 4km/hour find that wind appears to blow directly from the north he doubles his speed and the wind seems to comes from the north- east. Find the actual velocity of the wind.
	DIRECTIONAL DERIVATIVES, DIVERGENCE OF A VECTOR FIELD AND SOLENOIDAL VECTOR (Two And Four Marks Questions)
28.	Define directional derivative.
29.	Explain the geometrical meaning of gradient of a scalar field.
30.	Define the divergence of a vector field.
31.	Define solenoidal vector.
32.	Show that the maximum directional derivative takes place in the direction $\nabla\varphi$.
33.	Interpret the symbol $(\vec{A}.\nabla)$. Is it same as $\nabla.\vec{A}$?
34.	Verify $\nabla^2 u = 0$ if $u = x^2 - y^2 + 4z$.
35.	If $\vec{A} = 2yz\,\hat{\imath} - x^2y\,\hat{\jmath} + xz^2\,\hat{k}$ and $\vec{B} = x^2\,\hat{\imath} + yz\,\hat{\jmath} - xy\,\hat{k}$ then find $(\vec{B}.\nabla)\vec{A}$.
36.	If $\varphi = 2x^3y^2z^4$ then find $\nabla.\nabla\varphi$.
37.	Show that $\vec{F} = \frac{x\hat{\imath}+y\hat{\jmath}}{x^2+y^2}$ is a solenoidal vector field.
38.	Find $div\vec{F}$ at the point $(1,2,3)$ given $\vec{F} = (y^2+z^2-x^2)\hat{\imath} + (z^2+x^2-y^2)\hat{\jmath} + (x^2+y^2-z^2)\hat{k}$.
39.	Show that the vector $\vec{F} = 3y^2z^2\hat{\imath} + 4x^3z^2\,\hat{\jmath} + 3x^2y^2\,\hat{k}$ is solenoidal.
40.	If $\vec{F} = (x^2+y^2+z^2)^{-n}$ then find div(grad $\vec{F}$).
	DIRECTIONAL DERIVATIVES, DIVERGENCE OF A VECTOR FIELD AND SOLENOIDAL VECTOR (Seven Marks Questions)
41.	If $\varphi = x^2y^3z^4$ then find the rate of change of φ at $(2,3,-1)$ in the direction making equal angles with the positive $x, y \,\&\, z$ axes.
42.	Find the directional derivative of the function $\varphi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point $(1,1,1)$.
43.	Find the directional derivative of $\varphi = x^2 - 2xy + z^3$ at the point $(1,-2,-1)$ along the vector $2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$.
44.	Find the directional derivative of $x^2 + y^2 + z^2 = 9$ in the direction of the vector $x^2z\,\hat{\imath} + xy^2\,\hat{\jmath} + yz^2\,\hat{k}$ at the point $(2,2,1)$.
45.	Find the directional derivative of $\varphi = y^2x + yz^3$ at the point $(2,-1,1)$ in the direction of the normal to the surface $x\log z - y^2 = -4$ taken at the point $(-1,2,1)$.

46.	If the directional derivative of $\varphi = axy^2 + byz + cz^2x^3$ at (-1, 1, 2) has a maximum magnitude of 32 units in the direction parallel to y-axis then find a, b, c .
47.	In what direction from the point $(2,1,-1)$ is the directional derivative of $\varphi = x^2yz^3$ a maximum? What is its magnitude?
48.	What is the greatest rate of increase of $u = xyz^2$ at the point $(1,0,3)$.
49.	Suppose $\vec{A} = 2yz\hat{\imath} - x^2y\hat{\jmath} + xz^2\hat{k}$ and $\phi = 2x^2yz^3$ then show that $(\vec{A}.\nabla)\phi = \vec{A}.\nabla\phi$.
CURL OF A VECTOR FIELD, IRROTATIONAL VECTOR, LAPLACIAN OPERATOR AND VECTOR IDENTITIES (Two And Four Marks Questions)	
50.	Define the curl of a vector field.
51.	Give the physical meaning of curl of a vector field.
52.	Define irrotational vectors.
53.	Define Laplacian operator.
54.	Write any two vector identities.
55.	If $\vec{a}$ is the constant vector and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then prove that $\nabla \times (\vec{a} \times \vec{r}) = 2\vec{a}$.
56.	Show that the vector field $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$ is irrotational.
57.	Find Curl(Curl$\vec{A}$), where $\vec{A} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ at the point $(1,0,2)$.
58.	If $\vec{A} = 2yz\hat{\imath} - x^2y\hat{\jmath} + xz^2\hat{k}$ and $\phi = 2x^2yz^3$ then find $(\vec{A} \times \nabla\phi)$.
59.	Prove the following (a)$curl(grad\varphi) = \vec{0}$ (or) $\nabla \times (\nabla\varphi) = \vec{0}$. (b) $div(curl\vec{A}) = 0$ (or) $\nabla.(\nabla \times \vec{A}) = 0$.
60.	If $\vec{F} = \frac{x}{r}\hat{\imath} + \frac{y}{r}\hat{\jmath} + \frac{z}{r}\hat{k}$ where $r = \sqrt{x^2 + y^2 + z^2}$ then find $div(\vec{F})$ & $curl(\vec{F})$.
CURL OF A VECTOR FIELD, IRROTATIONAL VECTOR, LAPLACIAN OPERATOR AND VECTOR IDENTITIES (Seven Marks Questions)	
61.	Find the value of the constant 'a' such that $\vec{A} = (axy - z^3)\,\hat{\imath} + (a-2)x^2\,\hat{\jmath} + (1-a)xz^2\,\hat{k}$ is irrotational and hence find a scalar function φ such that $\vec{A} = \nabla\varphi$.
62.	Find the values of the constants *a*, *b*, *c* such that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is conservative. Also find its scalar potential.
63.	Show that $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is irrotational. Also find a scalar function φ such that $\vec{F} = \nabla\varphi$.
64.	Show that $\vec{F} = (\sin z + y)\hat{\imath} + x\hat{\jmath} + x\cos z\,\hat{k}$ is irrotational. Also find a scalar function φ such that $\vec{F} = \nabla\varphi$.
65.	Find the constants *a* and *b* so that $\vec{F} = (axy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (bxz^2 - y)\hat{k}$ is irrotational and find φ such that $\vec{F} = \nabla\varphi$.
66.	If $\vec{A} = xy^2\,\hat{\imath} + 2x^2yz\,\hat{\jmath} - 3y^2z\,\hat{k}$, find $div(\vec{F}), curl(\vec{F}), div(curl\vec{F})$ at (2,1,1).
67.	If $\vec{F} = (x + y + 1)\,\hat{\imath} + \hat{\jmath} - (x + y)\,\hat{k}$, show that $\vec{F}.curl\vec{F} = 0$.
68.	If $\vec{F} = (3x^2y - z)\,\hat{\imath} + (xz^3 + y)\,\hat{\jmath} - 2x^3z^2\,\hat{k}$, find $curl(curl\vec{F})$.

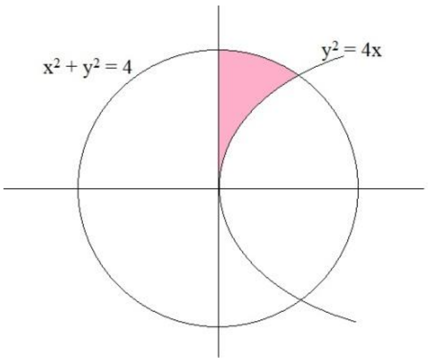
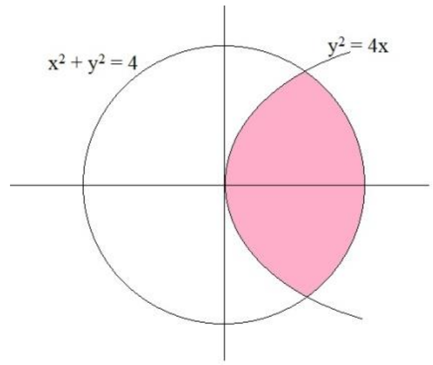
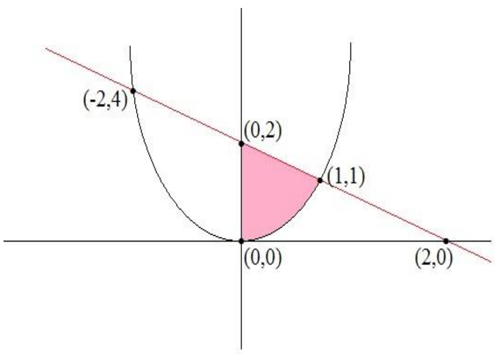
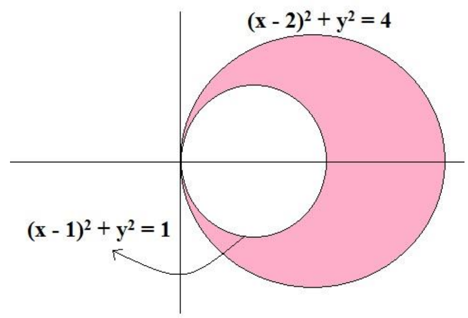
69.	If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx,$ prove that $gradu.[gradv \times gradw] = 0.$
70.	If $\vec{A} = xz^3\,\hat{\imath} - 2x^2yz\,\hat{\jmath} + 2yz^4\,\hat{k}$ find (a) $\nabla.\vec{A}$ (b) $\nabla \times \vec{A}$ (c) $\nabla.(\nabla \times \vec{A})$ at (1,-1,1).
71.	If $\vec{F} = \nabla(xy^3z^2)$ find $div(\vec{F})$ & $curl(\vec{F})$ at the point (1,-1,1).
72.	Show that $\varphi = \frac{1}{r}$ is a solution of Laplace equation $\nabla^2\varphi = 0.$
73.	Find the value of the constant 'a' such that $\vec{A} = y(ax^2 + z)\hat{\imath} + x(y^2 - z^2)\hat{\jmath} + 2xy(z - xy)\hat{k}$ is solenoidal. For this value of 'a' show that $curl\vec{A}$ is also solenoidal.
74.	Find the value of a if the vector $(ax^2y + yz)\hat{\imath} + (xy^2 - xz^2)\hat{\jmath} + (2xyz - 2x^2y^2)\hat{k}$ has zero divergence. Find the curl of the above vector which has zero divergence.
75.	Maxwell's equations of electromagnetic theory are given by $\nabla.\vec{E} = 0, \nabla.\vec{H} = 0, \nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t}$ and $\nabla \times \vec{H} = \frac{\partial \vec{E}}{\partial t}$. Show that $\vec{E}$ and $\vec{H}$ satisfy the wave equation $\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$.
76.	If $\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 + \vec{\omega}_3$ is the constant angular velocity and $\vec{v}$ is the velocity of a particle at a point $p(x, y, z)$ of the moving body having the position vector $\vec{r}$ such that $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{v})$. Interpret the result when $(\nabla \times \vec{v}) = 0$.
77.	If $\vec{r}$ is the position vector of the point (x, y, z) and $r = \sqrt{x^2 + y^2 + z^2}$ then show that $\nabla.\left(\frac{\vec{r}}{r^3}\right) = 0$.
78.	Prove the following: (a) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$ (b) $\nabla.(\phi\vec{A}) = \phi(\nabla.\vec{A}) + (\nabla\phi).\vec{A}$ (c) $\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{A}) + (\nabla\phi \times \vec{A})$ (d) $\nabla.(\vec{A} \times \vec{B}) = \vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$.
79.	Determine the values of m and n such that the vector point function $\vec{F} = (xyz)^m(x^n\hat{\imath} + y^n\hat{\jmath} + z^n\hat{k})$ is irrotational.

UNIT–IV INTEGRAL CALCULUS-II	
DOUBLE & TRIPLE INTEGRALS (Two & Four Marks Questions)	
1.	Define double integral of $f(x,y)$ as a limit of sum.
2.	Define triple integral of $f(x,y,z)$ as a limit of sum.
3.	With the help of a neat diagram mark the region of integration in the following double integrals: a) $\int_0^1\int_x^{\sqrt{x}} f(x,y)\,dy\,dx$ b) $\int_0^1\int_0^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$ c) $\int_0^{\pi}\int_0^{\sin x} f(x,y)\,dy\,dx$ d) $\int_1^4\int_2^{\sqrt{x}} f(x,y)\,dy\,dx$ e) $\int_0^2\int_{\frac{y}{4}}^{3-y} f(x,y)\,dy\,dx$ f) $\int_0^2\int_{2-x}^{\sqrt{4-x^2}} f(x,y)dy\,dx$ g) $\int_{-3}^4\int_0^{\sqrt{25-x^2}} f(x,y)dy\,dx$.
4.	With the help of a neat diagram mark the region of integration in the following double integrals: a) $\int_0^{\pi/2}\int_0^{2cos\theta} f(r,\theta)\,dr\,d\theta$ b) $\int_0^{\pi/2}\int_0^{a} f(r,\theta)\,dr\,d\theta$ c) $\int_0^{2\pi}\int_a^{b} f(r,\theta)\,dr\,d\theta$ d) $\int_0^{\pi/2}\int_0^{\infty} f(r,\theta)\,dr\,d\theta$ e) $\int_0^{\pi}\int_0^{a(1+cos\theta)} f(r,\theta)\,dr\,d\theta$.
5.	Evaluate the following double integrals: a) $\int_0^1\int_0^1 \frac{dy\,dx}{\sqrt{(1-x^2)(1-y^2)}}$ b) $\int_1^2\int_2^3\left(x-\frac{1}{y}\right)^2\,dxdy$ c) $\int_0^1\int_x^{\sqrt{x}} xy\,dydx$ d) $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dydx$ e) $\int_0^1\int_0^{x^2} e^{\frac{y}{x}}\,dydx$ f) $\int_0^4\int_{x^2/4}^{2\sqrt{x}} y\,dydx$ g) $\int_0^{2a}\int_0^{x^2/4} xy\,dydx$ h) $\int_0^1\int_0^{x^2}(x^2+y^2)dydx$.
6.	Evaluate the following double integrals: a) $\int_0^{\pi}\int_0^{acos\theta} rsin\theta\,drd\theta$ b) $\int_0^{\pi}\int_0^{a(1+cos\theta)} r\,drd\theta$ c) $\int_0^{\frac{\pi}{2}}\int_0^{\infty} re^{-r^2}\,drd\theta$ d) $\int_0^{\frac{\pi}{2}}\int_{acos\theta}^{a} sin\theta cos\theta\,drd\theta$ e) $\int_0^{\pi}\int_0^{asin\theta} r\,drd\theta$ f) $\int_0^{\frac{\pi}{4}}\int_0^{asin\theta}\frac{r\,dr\,d\theta}{\sqrt{a^2-r^2}}\,drd\theta$ g) $\int_0^{\frac{\pi}{2}}\int_{a(1-cos\theta)}^{a} r^2\,drd\theta$ h) $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\sin(\theta+\emptyset)\,d\theta\,d\emptyset$.
7.	Evaluate the following triple integrals: a) $\int_0^1\int_0^2\int_1^2 xyz^2\,dxdydz$ b) $\int_0^1\int_0^2\int_1^2 x^2yz\,dxdydz$ c) $\int_0^1\int_1^2\int_2^3(x+y+z)\,dxdydz$ d) $\int_0^1\int_y^1\int_0^{1-x} x\,dz\,dx\,dy$ e) $\int_0^1\int_0^{1-x}\int_0^{1-x-y} dz\,dy\,dx$ f) $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)\,dydzdx$.
8.	Find the limits of integration in the double integral $\iint_R xy\,dx\,dy$ where R is the region bounded by the x – axis; ordinate at $x=2a$ and the curve $x^2=4ay$.
9.	Evaluate $\iint_R \frac{\sin x}{x}dA$ where R is the triangle in the xy –plane bounded by the x – axis, the line $y=x$ and the line $x=1$.
10.	Evaluate $\iint_R \sin(ax+by)\,dxdy$ where R is the triangle region bounded by $x=0$, $y=0$ and $ax+by=1$.

DOUBLE & TRIPLE INTEGRALS (Seven Marks Questions)	
11.	Evaluate the following double integrals: a) $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dy\, dx$ b) $\int_0^{\frac{a}{\sqrt{3}}} \int_0^{\sqrt{x^2+a^2}} \frac{x}{x^2+y^2+a^2}\, dydx$ c) $\int_{-1}^1 \int_0^{1-x} x^{\frac{1}{3}}\, y^{-\frac{1}{2}}(1-x-y)^{\frac{1}{2}}\, dydx$ d) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dydx$ e) $\int_0^1 \int_0^{\sqrt{\frac{1-x^2}{2}}} \frac{1}{\sqrt{1-x^2-y^2}}\, dydx$ f) $\int_0^1 \int_x^{\sqrt{x}} x^2y^2(x+y)dy\, dx$
12.	Evaluate $\iint_R e^{x+y}\, dx\, dy$ where R is the region bounded by the lines $x+y=\pm a$ and $y-x=\pm a$.
13.	Evaluate $\iint_R xy\, dx\, dy$ where R is the region bounded by $\frac{x}{a}+\frac{y}{b}=1; x=0$ and $y=0$.
14.	Evaluate $\iint_R xy(x+y)\, dx\, dy$ where R is region bounded by $y=x^2$ and $y=x$.
15.	Evaluate $\iint_R (x+y)^2\, dx\, dy$ where R is bounded by $y^2=4x$ and $y=x$.
16.	Evaluate $\iint_R xy\, dx\, dy$ where R is the region bounded by the x – axis, ordinate at $x=2a$ and the curve $x^2=4ay$.
17.	Evaluate $\iint_R (x+y)^2\, dx\, dy$ where R is the region bounded by the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
18.	Evaluate $\iint_R xy\, dx\, dy$ where R the region is bounded by the positive quadrant of the circle $x^2+y^2=a^2$.
19.	Evaluate $\iint_R y\, dx\, dy$ where R is the region bounded by the y – axis, the curve $x^2=y$ and the line $x+y=2$ in the first quadrant.
20.	Evaluate $\iint_R x^2\, dx\, dy$ where R is the region bounded by the curves $xy=16$, $x=y$, $y=0$ and $x=8$.
21.	Evaluate $\iint_R r sin\theta\, dr\, d\theta$ where R the region is bounded by the cardioid $r=a(1-cos\theta)$ above the initial line.
22.	Evaluate $\iint_R r^2 sin\theta\, dr\, d\theta$ where R the region is bounded by the semi-circle $r=2acos\theta$ above the initial line.
23.	Evaluate $\iint_R r^3\, dr\, d\theta$ where R the region is included between the circles $r=2sin\theta$ and $r=4sin\theta$.
24.	Evaluate the following triple integrals: a) $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2+y^2+z^2)\, dxdydz$ b) $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dzdxdy$ c) $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)\, dydxdz$ d) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dzdydx$ e) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\;\, dzdydx$ f) $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dzdydx$ g) $\int_0^{\pi/2} \int_0^{asin\theta} \int_0^{\frac{a^2-r^2}{2}} r\;\, dz\, dr\, d\theta$.

25.	Evaluate $\iiint x\, dxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.
26.	Evaluate $\iiint (x+y+z)dx\,dy\,dz$ taken over the region bounded by the $x+y+z=a\ (a>0)$, $x=0$, $y=0$, $z=0$.
	CHANGE THE ORDER OF INTEGRATION (Two & Four Marks Questions)
27.	Write the limits of integration after changing the order of integration with a neat diagram for the following double integrals: a) $\int_0^1\int_x^{\sqrt{x}} f(x,y)\,dydx$ b) $\int_0^1\int_0^{x^2} f(x,y)\,dydx$ c) $\int_0^\infty\int_0^x f(x,y)\,dydx$ d) $\int_0^a\int_0^{\sqrt{a^2-y^2}} f(x,y)\,dxdy$
	CHANGE THE ORDER OF INTEGRATION (Seven Marks Questions)
28.	By changing the order of integration, evaluate the following double integrals: a) $\int_0^1\int_{x^2}^{2-x} xy\,dydx$ b) $\int_0^1\int_{\sqrt{y}}^{2-y} xy\,dxdy$ c) $\int_0^a\int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}dydx$ d) $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$ e) $\int_0^\infty\int_0^x xe^{-x^2/y}\,dydx$ f) $\int_0^1\int_1^{2-x} xy\,dydx$ g) $\int_0^3\int_1^{\sqrt{4-y}} (x+y)\,dxdy$ h) $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ i) $\int_0^a\int_{x/a}^{\sqrt{x/a}} (x^2+y^2)\,dydx$.
29.	Show that $\int_0^6\int_{\frac{y}{2}}^3 \frac{1}{x}e^{y/x}\,dydx = 3(e^2-1)$.
30.	Show that $\int_0^{\frac{\pi}{2}}\int_x^{\frac{\pi}{2}} \frac{\sin y}{y}\,dydx = 1$.
	CHANGE OF VARIABLES (Two & Four Marks Questions)
31.	Write the procedure of evaluating double integral by changing into polar coordinates.
32.	Write the procedure of evaluating triple integral by changing into cylindrical coordinates.
33.	Write the procedure of evaluating triple integral by changing into spherical polar coordinates.
34.	Write the limits of integration with respect to r, θ while evaluating the following integrals: a) $\int_0^a\int_0^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$ b) $\int_0^{4a}\int_{y^2/4a}^{y} f(x,y)\,dx\,dy$ c) $\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} f(x,y)\,dydx$ d) $\int_0^a\int_x^a f(x,y)\,dy\,dx$ e) $\int_0^\infty\int_0^\infty f(x,y)dx\,dy$.
	CHANGE OF VARIABLES (Seven Mark Questions)
35.	By using the transformation $x+y=u;\ y=uv$ show that $\int_0^1\int_0^{1-x} e^{\frac{y}{x+y}}dy\,dx = \frac{1}{2}(e-1)$.
36.	Using the transformation $x+y=u, x-y=v$ evaluate the double integral $\iint_R x^2+y^2\,dxdy$ where R is the region bounded by $u=0, u=2; v=0, v=2$.

37.	If R is the region bounded by $x = 0, y = 0$ and $x + y = 1$ then by using the transformation $x + y = u\,, x - y = v$ then show that $\iint_R sin\left(\frac{x-y}{x+y}\right) dx\, dy = 0$.
38.	Evaluate the following integrals by changing to polar coordinates: a) $\int_0^a \int_x^a \frac{x}{x^2+y^2} dy\, dx$ b) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ c) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} x^2\, dy\, dx$ d) $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{xy}{x^2+y^2} e^{-(x^2+y^2)}\, dy\, dx$ e) $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$ f) $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)\, dy\, dx$.
39.	Transform the integral $\int_0^\pi \int_0^a r^3 sin\theta cos\theta\, dr\, d\theta$ to cartesian form and hence evaluate.
40.	By changing into polar coordinates, evaluate $\iint_R \frac{x^2y^2}{x^2+y^2} dx\, dy$ over the annular region between circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2, (b > a)$.
41.	Using spherical polar coordinates, evaluate $\iiint \frac{dx\, dy\, dz}{x^2+y^2+z^2}$ taken over the volume bounden by the sphere $x^2 + y^2 + z^2 = a^2$.
42.	Evaluate $\iiint_V xyz\,(x^2 + y^2 + z^2)^{\frac{n}{2}}\, dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = b^2$ provided $n + 5 > 0$.
43.	Evaluate $\int_0^\infty \int_0^\infty \int_0^\infty \frac{dx\, dy\, dz}{(1+x^2+y^2+z^2)^2}$ using spherical polar coordinates.
44.	Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{4-x^2-y^2-z^2}}$ by changing into spherical polar coordinates.
45.	Evaluate $\iiint (x^2 + y^2 + z^2)\, dx\, dy\, dz$ taken over the volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$, by transforming into spherical polar coordinates.
46.	By transforming into cylindrical polar coordinates, evaluate $\iiint (x^2 + y^2 + z^2)\, dx\, dy\, dz$ taken over the region $0 \le z \le x^2 + y^2 \le 1$.
	APPLICATIONS OF DOUBLE & TRIPLE INTEGRALS (Seven Marks Questions)
47.	Find the area bounded by ellipse using double integration.
48.	Find the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$.
49.	Find the area bounded by the curves $x^2 + y^2 = 4$ and $y = 2 - x$.
50.	Find the area lying inside the circle $r = asin\theta$ and outside the cardioid $r = a(1 + cos\theta)$.
51.	Find the area lying inside the cardioid $r = a(1 + cos\theta)$ and outside the circle $r = a$.
52.	Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$
53.	Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
54.	Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

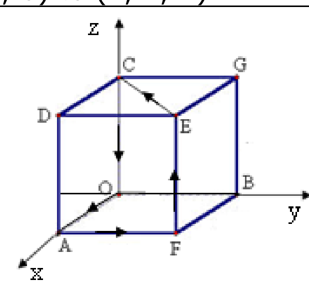
55.	Find the volume bounded by the cylinders $x^2 + y^2 = 4$, $y + z = 4$ and $z = 0$.		
56.	Find the area of the following shaded regions using double integration:		
	a) 		b) 
	c) 		d) 

UNIT–V
VECTOR INTEGRATION

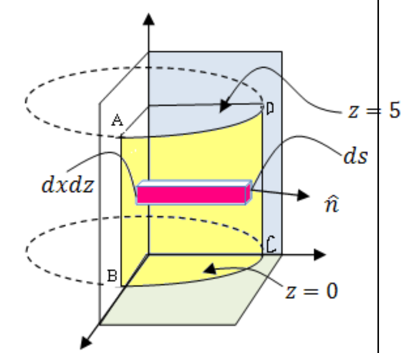
LINE INTEGRALS, SURFACE INTEGRALS AND VOLUME INTEGRALS
(Two and Four marks questions)

1.	Define a simple closed curve with an example.
2.	Define line integral of a vector function.
3.	Give the physical interpretation of $\int_c \vec{F}.d\vec{r}$ if $\vec{F}$ is force on a particle moving along c.
4.	If $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + zx\hat{k}$ evaluate $\int_c \vec{F}.d\vec{r}$ where c is the curve represented by $x = t;\ y = t^2;\ z = t^3. -1 \le t \le 1.$
5.	Evaluate $\int_c (xy\hat{\imath} + (x^2+y^2)\hat{\jmath}).d\vec{r}$ along the straight line joining origin and (1, 2).
6.	Evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = cosy\,\hat{\imath} - xsiny\,\hat{\jmath}$ and c is the curve $y = \sqrt{1-x^2}$ in xy – plane from (1, 0) to (0, 1).
7.	Evaluate $\int_c \vec{F}.d\vec{r}$ along the straight line joining origin and (1, 2), Where a) $\vec{F} = y\hat{\imath} + z\hat{\jmath}$ b) $\vec{F} = 3xy\hat{\imath} - 5xy^2\hat{\jmath}$
8.	Find the circulation of $\vec{F}$ round the curve c where a) $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ and c is the circle $x^2+y^2=1$, $z=0$. b) $\vec{F} = (x-y)\hat{\imath} + (x+y)\hat{\jmath}$, c is the circle $x^2+y^2=4$, $z=0$.
9.	Find the total work done in moving particle in a force field $\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2+1$, $y = 2t^2$ and $z = t^3$ form $t = 1$ to $t = 2$.
10.	If $\vec{F} = 3xy\hat{\imath} - 5y^2\hat{\jmath}$ evaluate $\int_c \vec{F}.d\vec{r}$ where c is the curve $y = 2x^2$ in xy-plane from (0, 0) to (1, 2).
11.	If $\vec{F} = (5xy - 6x^2)\hat{\imath} + (2y-4x)\hat{\jmath}$ evaluate $\int_c \vec{F}.d\vec{r}$ where c is the curve $y = x^3$ from the point (1, 1) to the point (2, 8).
12.	Find the total work done by a force $\vec{F} = 2xy\hat{\imath} - 4z\hat{\jmath} + 5x\hat{k}$ along the curve $x = t^2;\ y = 2t+1;$ $z = t^3$ from $t = 0$ to $t = 1$.

LINE INTEGRALS, SURFACE INTEGRALS AND VOLUME INTEGRALS
(Seven Marks Questions)

13.	If $\vec{F} = (x^2-2y)\hat{\imath} - 6yz\hat{\jmath} + 8xz^2$ evaluate $\int_c \vec{F}.d\vec{r}$ from the point (0, 0, 0) to(1, 1, 1) along the straight lines from (0, 0, 0) to (1, 0, 0), (1, 0, 0) to (1, 1, 0) and(1, 1, 0) to (1, 1, 1).	
14.	Evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = x^2\hat{\imath} - xz\hat{\jmath} + y^2\hat{k}$ along the closed path AFECOA as shown in the figure, which is a unit cube.	
15.	Find the circulation of $\vec{F}$ round the curve c where $\vec{F} = (2x+y^2)\hat{\imath} + (3y-4x)\hat{\jmath}$ and c the curve $x^2 = y$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0).	
16.	If $\vec{F} = (5xy - 6x^2)\hat{\imath} + (2y-4x)\hat{\jmath}$, evaluate $\int_c \vec{F}.d\vec{r}$ where c is the curve $y = x^3$ from the point a) (1, 1) to the point (2, 8) b) (2, 8) to the point (3, 27) c) (1, 1) to the point (3, 27)	

	GREEN'S THEOREM AND SURFACE INTEGRALS (Two and Four marks questions)
17.	State Green's Theorem on a plane.
18.	Explain the method of evaluating the surface integral $\int_s \vec{F}.\hat{n}ds$
19.	Give the physical interpretation of $\int_s \vec{F}.\hat{n}ds$ when $\vec{F}$ represents the velocity of the fluid particle.
20.	Using Green's theorem evaluate $\int(y - sinx)dx + cosx\ dy$ where c is the plane triangle enclosed by the line $y = 0; x = \frac{\pi}{2}; y = \frac{2x}{\pi}$.
21.	By using Green's theorem evaluate $\int(x^2 + xy)dx + (x^2 + y^2)\ dy$ where c is the square formed by the lines $x = \pm 1; y = \pm 1$.
22.	If 'c' is a simple closed curve in the xy-plane by using Green's theorem that the integral $\int \frac{(xdy - y\ dx)}{2}$ represents the area Aenclosed by c.
23.	Evaluate $\int_c (x^2 - \cosh y)dx + (y + sinx)dy$ by Green's theorem where c is the rectangle with vertices$(0,\ 0), (\pi,\ 0),\ (\pi,\ 1)\ and\ (0,\ 1)$.
24.	Prove that if $\int_{P_1}^{P_2} \vec{F}.\overrightarrow{dr}$ is independent of the path joining any two points P_1 and P_2 in a given region, then $\int_c \vec{F}.d\vec{r} = 0$ for all closed paths in the region.
	GREEN'S THEOREM AND SURFACE INTEGRALS (Seven Marks Questions)
25.	State and prove Green's theorem in a plane.
26.	Verify Green's theorem for $\int_c \int (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region enclosed by the lines $x = 0; y = 0; x + y = 1$.
27.	Verify Green's theorem for $\int (xy + y^2)dx + x^2\ dy$ where c is bounded by $y = x$ and $y = x^2$.
28.	Verify Green's theorem for $\int_c (x^2 + y^2)dx - 2xydy$ where c is the rectangle bounded by $y = 0$, $x = 0, y = b$ and $x = a$.
29.	Evaluate by Green's theorem $\int_c e^{-x} siny\ dx + e^{-x}\ cosy\ dy$ where c is the rectangle with vertices$(0,0), (\pi, 0), (\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$.
30.	Apply Green's theorem to evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'c' is the boundary of the area enclosed by the x – axis & upper half of the circle $x^2 + y^2 = a^2$.
31.	Find the area of the ellipse using Green's theorem.
32.	Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem.
33.	Find the area of the asteroid using Green's theorem.
34.	By using Green's theorem evaluate $\int_c (xy)dx + xy^2\ dy$ where 'c' is the square in the xy-plane with vertices (1, 0), (0, 1), (-1, 0) and (0, -1).
35.	Apply Green's theorem to evaluate $\int_c siny\ dx + x(1 + cosy)dy$ where 'c' is the closed path given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
36.	If $\vec{F} = y\hat{\imath} + (x - 2xz)\hat{\jmath} - xy\hat{k}$ then evaluate $\int_s (\nabla \times \vec{F}).\hat{n}ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane.
37.	If $\vec{F} = 4xz\ \hat{\imath} - y^2\hat{\jmath} + yz\ \hat{k}$ then evaluate $\int_s \vec{F}.\hat{n}ds$ directly where s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0\ \&\ z = 1$.
38.	Evaluate $\int_s \vec{F}.\hat{n}ds$ where $\vec{F} = 18z\ \hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and s is the part of the plane $2x + 3y + 6z = 12$ in the first octant.
39.	Evaluate $\int_s \vec{F}.\hat{n}ds$ where $\vec{F} = z\hat{\imath} + x\hat{\jmath} - 3y^2z\hat{k}$ and s is the surface of the cylinder $x^2 + y^2 = 16$ included in the positive octant between $z = 0\ \&\ z = 5$.

40.	Evaluate $\int_s \vec{F}.\hat{n}ds$ where $\vec{F} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ and s is the surface of the cylinder $y^2 + z^2 = 9$ included in the positive octant between $x = 0$ & $x = 5$.
41.	If $\vec{F} = z\,\hat{\imath} + x\hat{\jmath} - 3y^2z\hat{k}$ then evaluate $\int_s \vec{F}.\hat{n}ds$ where s is the curved surface ABCD as shown in the figure which is a part of the cylinder $x^2 + y^2 = 16$. 
	STOKE'S THEOREM & GAUSS DIVERGENCE THEOREM (Two And Four Marks Questions)
42.	State Stoke's theorem.
43.	If $\vec{F}$ is irrotational, then show that $\int_c \vec{F}.d\vec{r} = 0$ for any closed curve c.
44.	State Gauss divergence theorem.
45.	For any closed surface s show that $\int_s (\nabla \times \vec{F}).\hat{n}ds = 0$.
46.	Evaluate by Stoke's theorem $\int_c (e^x dx + 2ydy - dz)$ where c is the curve $x^2 + y^2 = 4,\ z = 2$.
47.	Evaluate by Stoke's theorem $\int_c (ydx + xz^3dy - yz^3dz)$ where c is the curve $x^2 + y^2 = 4,\ z = 1.5$.
48.	Evaluate $\int_s (yz\,\hat{\imath} + zx\hat{\jmath} + xy\,\hat{k}).\hat{n}ds$ where s is the surface of the cube formed by $x = 0,\ x = 0,\ y = 0,$ $y = 1, z = 0, z = 1$ using Gauss divergence theorem.
49.	Evaluate $\int_s (ax\,\hat{\imath} + by\,\hat{\jmath} + cz\,\hat{k}).\hat{n}ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = 1$ using Gauss divergence theorem.
50.	Evaluate $\int_s \vec{r}.\hat{n}ds$ where s is a closed surface, using Gauss divergence theorem.
51.	Show that $\int_s \nabla r^2\,\hat{n}ds = 6V$ where s is a closed surface enclosing a volume V using Gauss divergence theorem.
	STOKE'S THEOREM & GAUSS DIVERGENCE THEOREM (Seven Marks Questions)
52.	Using Stoke's theorem evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$ where c is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6).
53.	Evaluate $\int_c \vec{F}.d\vec{r}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0$ and $y = b$ using Stoke's theorem where $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$.
54.	Evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ where c is the boundary of the upper half of the sphere $x^2 + y^2 + z^2 = 1$ using Stoke's theorem.
55.	Using Stokes theorem, evaluate $\int_c sinzdx - cosxdy + sinydz$, where c is the boundary of rectangle $0 \le x \le \pi,\ 0 \le y \le 1,\ z = 3$.
56.	If $\vec{F} = 2xy\hat{\imath} + yz^2\hat{\jmath} + xz\,\hat{k}$ and s is the rectangular parallelepiped bounded by $x = 0, y = 0,\ z = 0,$ $x = 2, y = 1, z = 3$ then evaluate $\iint_s \vec{F}.\hat{n}\,ds$using Gauss divergence theorem.
57.	Evaluate $\int_s (x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}).\hat{n}ds$where s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$using Gauss divergence theorem
58.	Evaluate $\int_s \vec{F}.\hat{n}ds$ where $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and s is the surface bounding the region$x^2 + y^2 = 4,$ $z = 0, z = 3$ using Gauss divergence theorem.

59.	Evaluate $\int_s \vec{F}.\hat{n}ds$ where $\vec{F} = yz\hat{\imath} + 2y^2\hat{\jmath} + xz^2\hat{k}$ where s is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between $z = 0$ and $z = 2$ using Gauss divergence theorem.
60.	If $\vec{F} = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ evaluate $\int_s \vec{F}.\hat{n}ds$ taken over the surface s of the solid cut off by the plane $x + y + z = a$ in the first octant by using divergence theorem
61.	If $\vec{F} = 2xz\hat{\imath} - x\hat{\jmath} - y^2\hat{k}$, then evaluate $\int_V \vec{F}\, dV$ where V is the region bounded by the surface $x = 0$, $x = 2, y = 0, y = 6, z = x^2$ and $z = 4$
62.	If $\vec{F} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4x\,\hat{k}$, then evaluate $\int_V \nabla.\vec{F}\, dV$ where V is the closed region bounded by the plane $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.
63.	If $\vec{F} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4x\,\hat{k}$, then evaluate $\int_V (\nabla \times \vec{F})\, dV$ where V is the closed region bounded by the plane $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.
64.	If $\vec{F} = 4xz\,\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ then evaluate $\int_s \vec{F}.\hat{n}ds$ using divergence theorem where s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0\ \&\ z = 1$.
65.	Evaluate $\int_s xdydz + ydzdx + zdxdy$ using divergence theorem, where s is the surface of the sphere $(x-2)^2 + (y-3)^2 + (z-4)^2 = 4$.
66.	Evaluate $\int_s x^3dydz + x^2ydzdx + x^2zdxdy$ using divergence theorem, where s is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$and the circular discs $z = 0\ and\ z = b$.

MOBILES ARE BANNED **USN:** 1 M S

DEPARTMENT OF MATHEMATICS

CIE MODEL QUESTION PAPER

Sub Code:	MA11	**Sub:**	Engineering Mathematics-I	**Test:**	01
Semester:	I	**Term:**		**Marks:**	30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q.No.		Questions	Blooms Level	CO's	Marks
1.	(a)	Write the expression for length of the perpendicular from pole to the tangent.	L1	CO1	2
	(b)	If $u = x + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy,$ then evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at$(1,-1,0)$.	L2	CO1	3
	(c)	The altitude of a right circular cone is 15cm and is increasing at 0.2 cm/s. The radius of the base is 10 cm and is decreasing at 0.3 cm/s. How fast is the volume changing?	L3	CO1	5
	(d)	If $u = tan^{-1}\left[\sqrt{x^4 + y^4}\right]$, then show that (a) $xu_x + yu_y = sin\,2u$ (b)$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = sin\,4u - sin\,2u$.	L5	CO1	5
2.	(a)	Write the expression to find the volume of solid of rotation for a polar curve when rotated about the initial line and the line $\theta = {}^{\pi}/_{2}$.	L1	CO2	2
	(b)	Identify the double point at the origin of the curve $y^2(1+x) = x^2(1-x)$.	L2	CO2	3
	(c)	Evaluate$\int_0^{2a} \frac{x^3}{\sqrt{2ax - x^2}}\,dx$ using reduction formula.	L3	CO2	5
	(d)	Find the surface area of the solid obtained when cardioid $r = a(1 + cos\theta), a > 0$ is rotated about the initial line.	L4	CO2	5
3.	(a)	Define gradient of a scalar field.	L1	CO3	2
	(b)	Find the area enclosed by the $x^{2/3} + y^{2/3} = a^{2/3}, a > 0$.	L2	CO2	3
	(c)	The position vector of a moving particle at time t is $\vec{r} = t^2\hat{\imath} + t^3\hat{\jmath} - t^4\hat{k}$, find the tangential and normal components of its acceleration at $t = 1$.	L3	CO3	5
	(d)	If $u = \frac{x}{z}, v = \frac{y}{z}, w = z$ and $f = f(u,v,w)$, then show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = w\frac{\partial f}{\partial w}$.	L4	CO2	5

MOBILES ARE BANNED **USN:**

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DEPARTMENT OF MATHEMATICS

CIE MODEL QUESTION PAPER

Sub Code:	MA11	**Sub:**	Engineering Mathematics-I	**Test:**	02
Semester:	I	**Term:**		**Marks:**	30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q.No.		Questions	Blooms Level	CO's	Marks
1.	(a)	Define Laplacian operator.	L1	CO3	2
	(b)	Show that the maximum directional derivative takes place in the direction $\nabla\varphi$.	L2	CO3	3
	(c)	Find the constants *a* and *b* so that $\vec{F} = (axy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (bxz^2 - y)\hat{k}$ is irrotational and find φ such that $\vec{F} = \nabla\varphi$.	L3	CO3	5
	(d)	Prove that $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$	L4	CO3	5
2.	(a)	Define double integral of $f(x,y)$ as a limit of sum.	L1	CO4	2
	(b)	Write the limits of integration with respect to *r*, *θ* while evaluating the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\, dy\, dx$.	L2	CO4	3
	(c)	Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	L3	CO4	5
	(d)	By changing into polar coordinates, evaluate $\iint_R \frac{x^2y^2}{x^2+y^2} dx\, dy$ over the annular region between circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2, (b > a)$.	L4	CO4	5
3.	(a)	Give the physical meaning of curl of a vector field.	L1	CO3	2
	(b)	Find the total work done by a force $\vec{F} = 2xy\hat{\imath} - 4z\hat{\jmath} + 5x\hat{k}$ along the curve $x = t^2;\ y = 2t + 1;\ z = t^3$ from $t = 0$ to $t = 1$.	L2	CO5	3
	(c)	State Green's Theorem on a plane.	L3	CO5	5
	(d)	Show that $\int_0^6 \int_{\frac{y}{2}}^3 \frac{1}{x} e^{y/x}\, dydx = 3(e^2 - 1)$.	L4	CO4	5

RAMAIAH INSTITUTE OF TECHNOLOGY
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BANGALORE-560054
SEE MODEL QUESTION PAPER-I

Course & Branch	**: B.E. – Common to all Branches**	**Semester:**	**I**
Subject	**: Engineering Mathematics – I**	**Max.Marks:**	**100**
Subject Code	**: MA11**	**Duration:**	**3Hrs**

Instructions to the candidates:
Answer ONE full question from each unit.

		UNIT – I		
1.	a.	Write the expression for length of the perpendicular from pole to the tangent.	CO1	2
	b.	Find the slope of the tangent at any point (r, θ) on the curve $r = a(1 + \sin\theta)$. Further show that the tangent at the point $\theta = \frac{\pi}{2}$ is parallel to the initial line.	CO1	4
	c.	If $u = x^2 - 2y^2$ and $v = 2x^2 - y^2$, where $x = r\cos\theta, y = r\sin\theta$ then show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$.	CO1	7
	d.	If $u = log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) u = \frac{3}{x+y+z}$ and hence show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.	CO1	7
2.	a.	Find $\frac{dy}{dx}$ of $y^x = x$.	CO1	2
	b.	Find the angle between radius vector and the tangent to the curve $\frac{l}{r} = 1 + e\cos\theta$, where l and e are constants.	CO1	4
	c.	If $u = \csc^{-1}\left[\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]$, then prove that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{6}\left(1 + \frac{\sec^2 u}{6}\right)$.	CO1	7
	d.	Show that the pedal equation of the curve $r^n = a^n \sin n\theta + b^n \cos n\theta$ is $p^2(a^{2n} + b^{2n}) = r^{2n+2}$.	CO1	7
		UNIT – II		
3.	a.	Define cusp with an example.	CO2	2
	b.	Find the perimeter of the curve asteroid $x^{2/3} + y^{2/3} = a^{2/3}, a > 0$.	CO2	4
	c.	Find the volume of the solid generated by the revolution of the cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta), a > 0, \quad 0 \le \theta \le 2\pi$ about its base.	CO2	7
	d.	Prove that $\int_0^{2a} x^n \sqrt{2ax - x^2}\, dx = \pi a^2 \left(\frac{a}{2}\right)^n \frac{(2n+1)!}{(n+2)!\,n!}$	CO2	7
4.	a.	Write the expression to find the surface area of the solid for a polar curve when rotated about the initial line and the line $\theta = \pi/2$.	CO2	2
	b.	Evaluate $\int_0^{\pi} \frac{\sqrt{1-\cos x}}{1+\cos x} \sin^2(x) dx$.	CO2	4
	c.	Find the volume of revolution of the curve $r = 2a\cos\theta$ about the initial line.	CO2	7
	d.	Trace the curve $y^2(a - x) = x^2(a + x)$.	CO2	7

UNIT – III				
5.	a.	Give the physical meaning of curl of a vector field.	CO3	2
	b.	Find $div\vec{F}$ at the point$(1,2,3)$ given $\vec{F}=(y^2+z^2-x^2)\hat{\imath}+(z^2+x^2-y^2)\hat{\jmath}+(x^2+y^2-z^2)\hat{k}$.	CO3	4
	c.	Find the directional derivative of $\boldsymbol{\varphi=y^2x+yz^3}$ at the point $(\mathbf{2},-\mathbf{1},\mathbf{1})$ in the direction of the normal to the surface $\boldsymbol{x\log z-y^2=-4}$ taken at the point $(-\mathbf{1},\mathbf{2},\mathbf{1})$.	CO3	7
	d.	The position vector of a moving particle at time t is $\vec{r}=t^2\hat{\imath}+t^3\hat{\jmath}-t^4\hat{k}$, find the tangential and normal components of its acceleration at $t=1$.	CO3	7
6.	a.	Define directional derivative.	CO3	2
	b.	Show that $\nabla r^n=nr^{n-2}\vec{r}$.	CO3	4
	c.	Prove that $\nabla\times(\nabla\times\vec{A})=\nabla(\nabla.\vec{A})-\nabla^2\vec{A}$.	CO3	7
	d.	Find the value of the constant '*a*' such that $\vec{A}=y(ax^2+z)\hat{\imath}+x(y^2-z^2)\hat{\jmath}+2xy(z-xy)\hat{k}$ is solenoidal. For this value of '*a*' show that $curl\vec{A}$ is also solenoidal.	CO3	7
UNIT – IV				
7.	a.	Write the limits of integration after changing the order of integration with a neat diagram for the double integral $\int_0^1\int_x^{\sqrt{x}}f(x,y)\,dydx$.	CO4	2
	b.	Evaluate the triple integral $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dxdydz$.	CO4	4
	c.	Find the volume of the sphere $x^2+y^2+z^2=a^2$	CO4	7
	d.	By changing the order of integration, evaluate the double integral $\int_0^3\int_1^{\sqrt{4-y}}(x+y)$	CO4	7
8.	a.	Write the procedure of evaluating triple integral by changing into spherical polar coordinates.	CO4	2
	b.	Evaluate the double integral $\int_0^{\pi}\int_0^{acos\theta}rsin\theta\,drd\theta$.	CO4	4
	c.	By changing into polar coordinates, evaluate $\iint_R\frac{x^2y^2}{x^2+y^2}dx\,dy$ over the annular region between circles $x^2+y^2=a^2$ and $x^2+y^2=b^2,(b>a)$.	CO4	7
	d.	Evaluate $\iint_R(x+y)^2\,dx\,dy$ where R is bounded by $y^2=4x$ and $y=x$.	CO4	7
UNIT – V				
9.	a.	Define line integral of a vector function.	CO5	2
	b.	Evaluate by Stoke's theorem $\int_c(ydx+xz^3dy-yz^3dz)$ where *c* is the curve $x^2+y^2=4,\ z=1.5$.	CO5	4
	c.	Evaluate $\iint_S(y^2z\hat{\imath}+y^3\hat{\jmath}+xz\hat{k})ds$ where S is the boundary of the cube defined by $-1\le x\le 1,\ -1\le y\le 1$, and $0\le z\le 2$.	CO5	7
	d.	Evaluate $\int_s\vec{F}.\hat{n}ds$ where $\vec{F}=z\hat{\imath}+x\hat{\jmath}-3y^2z\hat{k}$ and s is the surface of the cylinder $x^2+y^2=16$ included in the positive octant between $z=0\,\&\,z=5$.	CO5	7

10	a.	Give the physical interpretation of $\int_s \vec{F}.\hat{n}ds$ when $\vec{F}$ represents the velocity of the fluid particle.	CO5	2
	b.	An object moves in the force field $\vec{F} = \langle \frac{-x}{(x^2+y^2+z^2)^{3/2}}, \frac{-y}{(x^2+y^2+z^2)^{3/2}}, \frac{-z}{(x^2+y^2+z^2)^{3/2}} \rangle$ along the curve $\vec{r} = \langle 1+t,\ t^3,\ tcos(\pi t) \rangle$ as t ranges from 0 to 1. Find the work done by the force on object.	CO5	4
	c.	If $\vec{F} = y\hat{\imath} + (x-2xz)\hat{\jmath} - xy\hat{k}$ then evaluate $\int_s (\nabla \times \vec{F}).\hat{n}ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane.	CO5	7
	d.	Evaluate $\int_c (x^2 - \cosh y)dx + (y + sinx)dy$ by Green's theorem where c is the rectangle with vertices $(0,\ 0), (\pi,\ 0),\ (\pi,\ 1)\ and\ (0,\ 1)$.	CO5	7

Note: <u>students should not be under the impression that questions from model question paper will appear in SEE.</u>

RAMAIAH INSTITUTE OF TECHNOLOGY
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SEE MODEL QUESTION PAPER-II

Course & Branch	**: B.E. – Common to all Branches**	**Semester:**	**I**
Subject	**: Engineering Mathematics – I**	**Max.Marks:**	**100**
Subject Code	**: MA11**	**Duration:**	**3Hrs**

Instructions to the candidates:
Answer ONE full question from each unit.

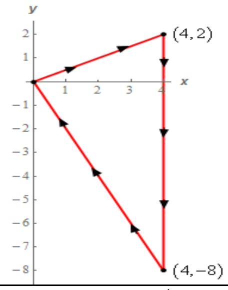
		UNIT – I		
1.	a.	Write the expression for angle between the radius vector and the tangent. Also define the terms involved.	CO1	2
	b.	If $u = cos((x^2 - y^3)\,;2x^2 + 3y^2 = a^2$ then find $\frac{du}{dx}$.	CO1	4
	c.	Find the value of $n$so that the equation $v = r^n(3\,cos^2\theta - 1)$ satisfies the relation $\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(sin\,\theta\frac{\partial v}{\partial\theta}\right) = 0.$	CO1	7
	d.	If $u = x\sqrt{1-y^2} + y\sqrt{1-x^2}, v = sin^{-1}x + sin^{-1}y$, show that u&$v$are functionally dependent and find the functional relationship.	CO1	7
2.	a.	State any two properties of Jacobians.	CO1	2
	b.	Find $\frac{dy}{dx}$for the function $e^x + e^y = 2xy$ using partial derivatives	CO1	4
	c.	Find the pedal equation to the curve $\frac{a(e^2-1)}{r} = 1 + e\,cos\,\theta$	CO1	7
	d.	If z is a function of x and y, where $x = e^u\cos v\,, y = e^u\sin v$, then prove that (a) $x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y}$. (b)$\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = \frac{1}{e^{2u}}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right]$.	CO1	7
		UNIT – II		
3.	a.	Define a double point and classify it.	CO2	2
	b.	Evaluate $\int_0^{\pi}\frac{\sin^4(x)}{(1+\cos x)^2}dx$.	CO2	4
	c.	Trace the curve $x^3 + y^3 = 3axy$.	CO2	7
	d.	Determine the length of the cardioid $r = a(1 - cos\theta), a > 0$ lying outside the circle $r = acos\theta$.	CO2	7
4.	a.	Write the reduction formula for $\int \cos^n(x)dx$.	CO2	2
	b.	Trace the curve $r = a\,sin3\theta,\ a > 0$	CO2	4
	c.	Determine the common area between $r = \frac{3a}{2}$ and $r = a(1 + cos\theta), a > 0$.	CO2	7
	d.	Find the surface area of the solid generated by revolving the curve $x = t^2, y = t - \frac{t^3}{3},\ 0 \le t \le \sqrt{3}$ about x-axis.	CO2	7

		UNIT – III		
5.	a.	Give the geometrical meaning of the derivative of a vector.	CO3	2
	b.	Show that $\nabla\emptyset$ is a vector perpendicular to the surface $\emptyset(x, y, z) = c$	CO3	4
	c.	Find the unit tangent & unit normal vector to the curve $x = 2t; y = t^2; z = t^2 - 1$ *at* $t = 0$.	CO3	7
	d.	Prove that $\nabla.(\vec{A} \times \vec{B}) = \vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$	CO3	7
6.	a.	Define gradient of a scalar field.	CO3	2
	b.	In a temperature field, heat flows in the direction of maximum decrease of temperature *T*. Find this direction at P$(2,1)$ when $T = x^3 - 3xy^2$.	CO3	4
	c.	Find the constants *a* and *b* so that $\vec{F} = (axy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (bxz^2 - y)\hat{k}$ is irrotational and findφ such that $\vec{F} = \nabla\varphi$.	CO3	7
	d.	If $\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 + \vec{\omega}_3$ is the constant angular velocity and $\vec{v}$ is the velocity of a particle at a point $p(x, y, z)$ of the moving body having the position vector $\vec{r}$ such that $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{v})$. Interpret the result when$(\nabla \times \vec{v}) = 0$.	CO3	7
		UNIT – IV		
7.	a.	Define double integral of $f(x, y)$ as a limit of sum.	CO4	2
	b.	Evaluate $\iint_R \sin(ax + by)\, dxdy$ where R is the triangle region bounded by $x = 0$, $y = 0$ and $ax + by = 1$.	CO4	4
	c.	Evaluate $\iint_R (x + y)^2\, dx\, dy$ where R is the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	CO4	7
	d.	By changing the order of integration, evaluate the double integral $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$.	CO4	7
8.	a.	Write the limits of integration after changing the order of integration with a neat diagram for double integral $\int_0^1 \int_x^{\sqrt{x}} f(x, y)\, dydx$.	CO4	2
	b.	Evaluate the Double integral $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} y\, dydx$. .	CO4	4
	c.	Evaluate the triple integral $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{2}} r\ dz\, dr\, d\theta$.	CO4	7
	d.	Find the volume bounded by the cylinders $x^2 + y^2 = 4$, $y + z = 4$ and $z = 0$.	CO4	7
		UNIT – V		
9.	a.	Give the physical interpretation of $\int_c \vec{F}.d\vec{r}$ if $\vec{F}$ is force on a particle moving along c.	CO5	2
	b.	Explain the method of evaluating the surface integral $\int_s \vec{F}.\hat{n}ds$	CO5	4
	c.	If $\vec{F} = (x^2 - 2y)\hat{\imath} - 6yz\,\hat{\jmath} + 8xz^2$ evaluate $\int_c \vec{F}.d\vec{r}$ from the point (0, 0, 0) to (1, 1, 1) along the straight lines from (0, 0, 0) to (1, 0, 0), (1, 0, 0) to (1, 1, 0) and(1, 1, 0) to (1, 1, 1).	CO5	7
	d.	Using Stoke's theorem evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$ where c is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6).	CO5	7

10	a.	If $\vec{F}$ is irrotational, then show that $\int_c \vec{F}\,.\,d\vec{r} = 0$ for any closed curve c.	CO5	2
	b.	Find the total work done by a force $\vec{F} = 2xy\hat{\imath} - 4z\hat{\jmath} + 5x\hat{k}$ along the curve $x = t^2;\ y = 2t+1;\ z = t^3$ from $t = 0$ to $t = 1$.	CO5	4
	c.	If $\vec{F} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4x\,\hat{k}$, then evaluate $\int_V (\nabla \times \vec{F})\, dV$ where V is the closed region bounded by the plane $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.	CO5	7
	d.	Find the area of the ellipse using Green's theorem.	CO5	7

Note: <u>students should not be under the impression that questions from model question paper will appear in SEE.</u>

ASSIGNEMNT QUESTIONS	
1.	Find the angle of intersection of the curves $r = 2 + 2\sin\theta$ and $r = 2 - 2\cos\theta$.
2.	Find the pedal equation of the curve$r = a\sec t\,, \theta = \tan t - t$.
3.	If $u = log(x^3 + y^3 - x^2y - xy^2)$,then prove that $u_{xx} + 2u_{xy} + u_{yy} = \frac{-4}{(x+y)^2}$.
4.	If $u = sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$ then find the value of $xu_x + yu_y + zu_z$.
5.	If $f(x,y) = x^3y^2 + y\sin x$, where $x = \sin 2t$and $y = \log t$, find $\frac{df}{dt}$.
6.	Prove that the function $u = \frac{x-y}{x+y}$ and $v = \frac{xy}{(x+y)^2}$ are functionally dependent. Find the relation between them.
7.	Find the length of the curve $x = e^\theta(sin\,{}^{\theta}/_{2} + 2\,cos\,{}^{\theta}/_{2})$, $y = e^\theta(cos\,{}^{\theta}/_{2} - 2\,sin\,{}^{\theta}/_{2})$ measured from $\theta = 0$ to $\theta = \pi$.
8.	Evaluate $\int_0^4 x^3\sqrt{4x - x^2}\,dx$.
9.	Find the length of the arc of the curve $y = e^x$ from the point $(0,1)$ to $(1, e)$.
10.	Find the area common to the circles $r = a\sqrt{2}$, and $r = 2acos\theta$.
11.	Find the volume of the solid of revolution generated by revolving the curve $x = 2t + 3, y = 4t^2 - 9$ about the axis for $t = -{}^{3}/_{2}$ to $t = {}^{3}/_{2}$.
12.	The curve $r = e^{\theta/2}$ is the revolved about the initial line. Prove that the area of surface of revolution traced out by the part between the points $\theta = 0$ and $\theta = \pi$ is equal to $\frac{\pi}{2}\sqrt{5}\,(e^\pi + 1)$.
13.	Show that the vector field given by $\vec{F} = xyz(yz\,\hat{\imath} + xz\,\hat{\jmath} + xy\,\hat{k})$ irrotational. Also find its scalar potential.
14.	If $f = x^2 + y^2 + z^2$and $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, show that $div(f\,\vec{r}) = 5f$.
15.	A particle moves along the curve $x = t^3 + 1; y = t^2; z = 2t + 5$ where t is time. Find the components of velocity and acceleration at $t = 1$ in the direction of $\hat{\imath} + \hat{\jmath} + 3\hat{k}$.
16.	For the function $= x^2y^3z^4$, calculate the maximum rate of change and the corresponding direction at the point $2\,\hat{\imath} + 3\hat{\jmath} - \hat{k}$.
17.	Prove that $\nabla.\{r\,\nabla\left(\frac{1}{r^3}\right)\} = \frac{3}{r^4}$, where $r^2 = x^2 + y^2 + z^2$.
18.	Find the angle between the curves $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, 1, 2).
19.	Evaluate the double integral $\int_0^4 \int_{\frac{y^2}{4}}^{y} \frac{y}{x^2+y^2}\,dx\,dy$.
20.	Evaluate $\iint(x^2 + y^2)\,dx\,dy$ over the area bounded by the curves $y = 4x$, $x + y = 3$, $y = 0$ and $y = 2$.
21.	Evaluate a) $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^a r^2 sin\theta\ \ dr\,d\theta\,d\emptyset$ b) $\int_0^{2\pi}\int_0^b\int_{-h}^h (z^2 + r^2sin^2\theta)\ \ dz\,dr\,d\theta$.
22.	Evaluate $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ by changing the order of integration.
23.	Find the area of the region bounded by the upper half of the circle $x^2 + y^2 = 25$, the $x - axis$ and the ordinates $x = -3$ and $x = 4$.
24.	Using cylindrical coordinates, evaluate $\iiint(x^2 + y^2)\,dx\,dy\,dz$ taken over the volume bounded by the paraboloid $z = 9 - x^2 - y^2$ and the plane $z = 0$.
25.	Evaluate $\int_0^{4a}\int_{\frac{y^2}{4a}}^{y} \frac{x^2-y^2}{x^2+y^2}\,dx\,dy$ by changing into polar coordinates.
26.	If $f(x,y,z) = z$ and C is the curve in R^3 parametrized by $x = tsint,\ y = tcost$ and $z = t$, $0 \le t \le 8\pi$, evaluate $\int_C f(x,y,z)ds$.
27.	Let S be the elliptic Paraboloid $z = \frac{x^2}{4} + \frac{y^2}{9}$ for $z \le 1$, and C be the boundary curve. Calculate $\int_C \vec{f}.dr$for the vector function $\vec{f} = (9xz + 2y)\hat{\imath} + (2x + y^2)\hat{\jmath} + (2z - 2y^2)\hat{k}$, where C is traversed counter clockwise.

28.	Evaluate $\iint_S (y^2 z\hat{i} + y^3\hat{j} + xz\hat{k})\mathrm{ds}$ where S is the boundary of the cube defined by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, and $0 \leq z \leq 2$.
29.	Use Greens theorem to evaluate $\int_C x^2y^2dx + (yx^3 + y^2)dy$, where C is shown below. 
30.	Evaluate $\int_s \vec{F}.\hat{n}ds$ where $\vec{F} = 2z\,\hat{\imath} + 3\hat{\jmath} + 7y\hat{k}$ and s is the part of the plane $x + 3y + 4z = 12$ in the first octant.