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# **BMS INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

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## **DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING**

### **COURSE MATERIAL**

## **AUTOMATA THEORY AND COMPUTABILITY (17CS54 / 18CS54 )**

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## INTRODUCTION TO FINITE AUTOMATA

FA are useful model for many imp kinds of hardware & software. Some of them are  
is s/w for designing & checking the behavior of digital circuits.

- as The "lexical analyzer" of a typical compiler, i.e. the computer component that breaks the input text into logical units, such as identifiers, keywords and punctuation.
  - 2) Uses for scanning large bodies of text, such as collections of web pages, to find occurrences of words, phrases or other patterns.
  - 3) Uses for verifying systems of all types that have a finite no. of distinct states, like communication protocols or protocols for secure exchange of information.

## The central concepts of Automata Theory

## Alphabets:

An alphabet is a finite, non-empty set of symbols. It is conventionally denoted by the symbol  $\Sigma$ . Common alphabets are

- 1)  $\Sigma = \{0, 1\}$ , the binary alphabet.
  - 2)  $\Sigma = \{a, b, \dots, z\}$ , the set of all lower-case letters
  - 3) The set of all ASCII characters, or the set of all printable ASCII characters.

## Sutengs:

A string (or word) is a finite sequence of symbols chosen from some alphabet.

For eg 011, 111 are strings from binary alphabet  $\Sigma = \{0, 1\}$ .

The empty string as a string with zero occurrences of symbols. It is denoted by  $\epsilon$ , which may be chosen from any alphabet.

Length of a string as the number of symbols in that string. The standard notation for length of string  $w$  is  $|w|$ .

$$\text{e.g. } |011|=3, |\epsilon|=0$$

### Power of an alphabet

If  $\Sigma$  is any alphabet, we can express the set of all strings of a certain length from that alphabet by using an exponential notation.

$\Sigma^k$  is defined to be the set of strings of length  $k$ , each of whose symbols is in  $\Sigma$ .

$$\text{Note: } \Sigma^0 = \{\epsilon\}$$

i.e.  $\epsilon$  is the only string of length 0.

$$\text{If } \Sigma = \{a, b, c\}$$

$$\Sigma^1 = \{a, b, c\}$$

$$\Sigma^2 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$$

$$\Sigma^3 = \{aaa, aab, aac, aba, abb, abc, aca, acb, ace, baa, bab, bac, bba, bbb, bbc, bca, bcb, bcc, caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc\} \text{ etc.}$$

Note the confusion between  $\Sigma$  and  $\Sigma^1$ .

$\Sigma$  is an alphabet with members  $a, b$  and  $c$  as symbols.

$\Sigma^1$  is a set of strings with members as the strings  $a, b$  and  $c$  of length 1.

The set of all strings over an alphabet  $\Sigma$  is denoted  $\Sigma^*$ . For instance,

$$\{0, 1\}^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, \dots\}$$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$$

The set of non-empty strings from alphabet  $\Sigma$  is denoted  $\Sigma^+$ . Hence

- $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$
- $\Sigma^* = \Sigma^+ \cup \{\epsilon\}$

### Concatenation of strings

Let  $x$  and  $y$  be strings. Then  $xy$  denotes the concatenation of  $x$  and  $y$ , i.e. the string formed by making a copy of  $x$  and following it by a copy of  $y$ .

If  $x$  is the string composed of  $i$  symbols,  $x = a_1 a_2 \dots a_i$  and  $y$  is the string composed of  $j$  symbols  $y = b_1 b_2 \dots b_j$ , then  $xy$  is the string of length  $i+j$ .

$$xy = a_1 a_2 \dots a_i b_1 b_2 \dots b_j$$

$$\text{Ex: Let } x = 110 \quad y = 001$$

$$\text{Then } xy = 110001, \quad yx = 001110$$

For any string  $w$ , the eqns  
 $ew = we = w$  hold.

i.e.  $\epsilon$  is the identity for concatenation.

### Languages

A set of strings all of which are chosen from some  $\Sigma^*$ , where  $\Sigma$  is a particular alphabet, is called a language.

If  $\Sigma$  is an alphabet and  $L \subseteq \Sigma^*$  then  $L$  is a language over  $\Sigma$ .

② A language over  $\Sigma$  need not include strings with all the symbols of  $\Sigma$ .

Grammar:

A grammar  $G$  is a quadruple or 4-tuple

$$G = (V, T, P, S)$$
 where

$V \rightarrow$  set of variables or non-terminals

$T \rightarrow$  set of terminals

$P \rightarrow$  set of productions

$S \rightarrow$  start symbol.

Each prodn is of the form  $\alpha \rightarrow \beta$  where

$\alpha \rightarrow$  non-empty string of terminals and/or non-terminals

$\beta$  is string of terminals and/or non-terminals

including the null string

i.e.  $\alpha$  is a string in  $(VUT)^+$

$\beta$  is a string in  $(VUT)^*$ .

The gr. is also called phrase structure grammar.

Problems:

- 1) Let  $\Sigma = \{a, b\}$ . Obtain a grammar  $G$  generating set of all palindromes over  $\Sigma$ .

Soln

$$V = \{S\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow a|b|e \}$$

$$S \rightarrow aa|bb|ab|ba|$$

{

$S$  is the start symbol.

- 2) Obtain a grammar to generate a language of all non-palindromes over  $\{a, b\}$ .

$$V = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow aa|bb|A \}$$

$$A \rightarrow aBb|bBa$$

$$B \rightarrow AB|BA|e$$

$S$  is the start symbol.

$$P = \{ S \rightarrow asb|bsa|abs|bas|e \}$$

(or)

Obtain a grammar to generate the language

$$L = \{ w | n_a(w) = n_b(w) \}$$

abbq

$$V = \{S\}$$

ab

$$T = \{a, b\}$$

bq

$$P = \{ S \rightarrow asb|bsa|ss|e \}$$

ss

$S$  is the start symbol.

- 4) Obtain a gr. to generate the language

$$L = \{ 0^n 1^{n+1} | n \geq 0 \}$$

Soln

$$V = \{S, A\}$$

$$T = \{0, 1\}$$

$$P = \{ S \rightarrow A \}$$

$$A \rightarrow 0A1|e \}$$

$0^n 1^n \Rightarrow 0A1/e$

$\Rightarrow 0A1|1e$

$\Rightarrow 00A11$

$$P = \{ S \rightarrow 0S1|1S \}$$

$S$  is the start symbol.

- 5) What is the language generated by the gr.

$$S \rightarrow 0A|e$$

$$A \rightarrow 1S$$

$$S \Rightarrow e \text{ (null string)}$$

$$S \Rightarrow 0A$$

$$\Rightarrow 01S \Rightarrow 010A \Rightarrow 0101S \Rightarrow 01010A$$

$$\Rightarrow 010101S \Rightarrow 010101$$

i.e. The lang. generated by the gr. is

$$L = \{ w | w \in (01)^* \}$$
 or

$$L = \{ (01)^n | n \geq 0 \}$$

Note:  $S \rightarrow 01S|e$  } These two produce the  
and  $S \rightarrow 0S1|e$  } same language.

6) obtain a CFG to generate a string of balanced parentheses.

Soln: The grammar G to generate a string of balanced parentheses is given by  $G = (V, T, P, S)$  where

$$V = \{S\}$$

$$T = \{(), [], {}, 2, 3\}$$

$$P = \{ S \rightarrow (S) | [S] | \{S\} | SS \\ S \rightarrow \epsilon \}$$

S is the start symbol

7) Obtain a gr. to generate the language

$$L = \{w w^R \mid w \in \{a, b\}^*\}$$

where  $w^R$  is reverse of w.

Soln: The gr. to find the reverse is given by

$$G = (V, T, P, S)$$
 where

$$V = \{S\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow aSa | bSb | \epsilon \}$$

S is the start symbol

Note: For the language  $w w^R$

$$P \Rightarrow = \{ S \rightarrow aSa | bSb | c \}$$

8) Obtain the gr. to generate the language

$$L = \{0^n 1^m 2^n \mid m \geq 1 \text{ and } n \geq 0\}$$

Soln: If  $n=0$ , there will be m no of 0's & 1's

$$A \rightarrow 01 | 0A1$$

Otherwise when  $n > 0$ ,

$$S \rightarrow A S_2$$

Thus the gr. can be written as

$$G = (V, T, P, S)$$
 where

$$V = \{S, A\}$$

$$T = \{0, 1, 2\}$$

$$P = \{ S \rightarrow A | S_2 \\ A \rightarrow 01 | 0A1 \}$$

S is the start symbol.

a) Obtain a gr. to generate the lang

$$L = \{0^n 1^{2n} \mid n \geq 0\}$$

Soln:

$$V = \{S\}$$

$$T = \{0, 1\}$$

$$P = \{ S \rightarrow 0S11 | \epsilon \}$$

S is the start symbol.

10) Obtain a gr. to generate the language

$$L = \{a^{n+2} b^m \mid n \geq 0 \text{ and } m > n\}$$

Soln: The strings can be represented by the language

$$L = \{aabb^*, aaabbb^*, aaaabbbb^*, \dots\}$$

i.e min no of b's is 1 less than no of a's.

In each string,

- No of b's can be one less than no of a's.
- No of b's may be equal to total no of a's.
- No of b's may be greater than total no of a's.

$$V = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow aAB \\ A \rightarrow aAb | ab \\ B \rightarrow bB | \epsilon \}$$

S is the start symbol.  
Derivation

$$S \Rightarrow aAB$$

$$S \Rightarrow aAB$$

$$S \Rightarrow aAB$$

$$\Rightarrow a aAbB$$

$$\Rightarrow aabB$$

$$\Rightarrow aabB$$

$$\Rightarrow a aabbB$$

$$\Rightarrow aabB$$

$$\Rightarrow a aabbB$$

$$\Rightarrow aaabbB$$

$$\Rightarrow aabB$$

$$\Rightarrow a aabbB$$

$$\Rightarrow aaabbB$$

$$\Rightarrow aabB$$

$$\Rightarrow a aabbB$$

$$\Rightarrow aaabbB$$

$$\Rightarrow aabB$$

$$\Rightarrow a aabbB$$

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12) Obtain a gr. to generate the language

$$L = \{a^n b^m \mid n \geq 0, m \geq n\}$$

Soln: The set of strings that can be generated by this language

$$L = \{bb^*, abbb^*, aabbba^*, \dots\}$$

To generate equal no. of a's &amp; b's

$$A \rightarrow aAb \mid a\epsilon$$

To generate one or more b's

$$B \rightarrow bB \mid b$$

Always a's precede b's. So

$$S \rightarrow AB$$

So, the gr. to generate the given lang. is

$$V = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow AB\}$$

$$A \rightarrow aAb \mid a\epsilon$$

$$B \rightarrow bB \mid b$$

{}

S is the start symbol.

13) Obtain a gr. to generate the language

$$L = \{a^n b^{n-3} \mid n \geq 3\}$$

Soln:  $L = \{aaa, aaaab, aaaaabb, \dots\}$

The gr.  $G = (V, T, P, S)$  is

$$V = \{S, A\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow aaaA\}$$

$$A \rightarrow aAb \mid a\epsilon$$

{}

S is the start symbol.

13) Obtain the gr. to generate the language

$$L = L_1 L_2 \text{ where}$$

$$L_1 = \{a^n b^m \mid n \geq 0, m \geq n\}$$

$$L_2 = \{0^n 1^m \mid n \geq 0, m \geq 0\}$$

The gr.  $L_1$  is

$$V = \{S_1, A, B\}$$

$$T = \{a, b\}$$

$$P = \{S_1 \rightarrow AB\}$$

$$A \rightarrow aAb \mid a\epsilon$$

$$B \rightarrow bB \mid b$$

{}

S<sub>1</sub> is the start symbol.The gr.  $L_2$  is given by

$$V = \{S_2\}$$

$$T = \{0, 1\}$$

$$P = \{S_2 \rightarrow 0S_2 \mid 1S_2 \mid \epsilon\}$$

S<sub>2</sub> is the start symbol.The resulting gr.  $L = L_1 L_2$  can be obtained by concatenating S<sub>1</sub> with S<sub>2</sub>. i.e

$$S \rightarrow S_1 S_2$$

Where S is the start symbol of resultant gr.

The final gr. which accepts  $L = L_1 L_2$  is

$$V = \{S, S_1, S_2, A, B\}$$

$$T = \{0, 1, a, b\}$$

$$P = \{$$

$$S \rightarrow S_1 S_2$$

$$S_1 \rightarrow AB$$

~~$$S_2 \rightarrow 0S_2 \mid 1S_2 \mid \epsilon$$~~

$$A \rightarrow aAb \mid a\epsilon$$

$$B \rightarrow bB \mid b$$

{}

S is the start symbol.

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- 14) Obtain a gr. to generate the language  
 $L = L_1 \cup L_2$  where

$$L_1 = \{a^n b^m \mid n \geq 0, m > n\}$$

$$L_2 = \{a^m \mid n \geq 0\}$$

Soln The start symbol is either  $S_1$  or  $S_2$ .

$$S \rightarrow S_1 \mid S_2$$

The final gr. is

$$V = \{S, S_1, S_2, A, B\}$$

$$T = \{a, b, 0, 1\}$$

$$P = \{ S \rightarrow S_1 \mid S_2 \}$$

$$S_1 \rightarrow AB$$

$$S_2 \rightarrow 0S_2 \mid 1$$

$$A \rightarrow aAb \mid abA$$

$$B \rightarrow bB \mid b$$

{

$S$  is the start symbol.

- 15) Obtain a gr. to generate the language

$$L = \{w : |w| \bmod 3 = 0\} \text{ on } \Sigma = \{a\}$$

Soln The lang. accepted by the gr. can be written as

$$L = \{e, aaa, aaaaa, aaaaaaaaa, \dots\}$$

∴ The gr. is  $G$  is

$$V = \{S\}$$

$$T = \{a\}$$

$$P = \{ S \rightarrow aaa \mid e \}$$

$S$  is the start symbol.

- 16) Obtain a gr. to generate the language

$$L = \{w : |w| \bmod 3 = 0\} \text{ on } \Sigma = \{a, b\}$$

Soln The gr. is  $G = (V, T, P, S)$  where

$$V = \{S, AS\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow AAA \mid e \}$$

$$A \rightarrow a \mid b$$

{}

$S$  is the start symbol.

- 17) Obtain a gr. to generate the language

$$L = \{w : |w| \bmod 3 > 0\} \text{ on } \Sigma = \{a\}$$

Soln The gr. is

$$V = \{S, A, B\}$$

$$T = \{a\}$$

$$P = \{ S \rightarrow AA \}$$

$A \rightarrow aAb \mid abA$  (Accepts  $w$  of  $|w| = 1, 4, 7, \dots$ )

$B \rightarrow aas \mid e$  (Accepts  $w$  of  $|w| = 2, 5, 8, \dots$ )

{}

$S$  is the start symbol.

$$L = \{a, aa, aaaa, aaaaaa, \dots\}$$

$$\begin{array}{llll} S \Rightarrow OA & S \Rightarrow QA & S \Rightarrow GA & S \Rightarrow AA \\ \Rightarrow aAB & \Rightarrow aAB & \Rightarrow a & \Rightarrow aab \\ \Rightarrow aaaS & \Rightarrow a & & \Rightarrow aaaa \\ \Rightarrow aaaaA & & & \Rightarrow aaaaa \\ \Rightarrow aaaa & & & \end{array}$$

- 18) Obtain a gr. to generate the language

$$L = \{w : |w| \bmod 3 \neq |w| \bmod 2\} \text{ on } \Sigma = \{a\}$$

Soln The above lang. can be written as  $|w| \bmod 3 \neq |w| \bmod 2$

$$L = \{aa, aaaa, aaaa, aaaaaaaaa, \dots\}$$

or

$$L = \{w : w=a^k \text{ and } |w|=8i \text{ or } 3i \text{ or } 4i\}$$

or  $s_i$  for  $i =$

$$1, 4, 7, 10, 13, \dots$$

	$1$
	$3$
	$5$
	$7$
	$9$
	$11$
	$13$
	$15$
	$17$

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$$V = \{S, A, B, C, D\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow aaA, \dots \}$$

$$A \rightarrow AB/\epsilon \quad (\text{Accepts w.g } P) = \{1, 8, \dots\}$$

$$B \rightarrow acle \quad (\text{---} \rightarrow = \{3, 9, \dots\})$$

$$C \rightarrow adle \quad (\text{---} \rightarrow = \{4, 10, \dots\})$$

$$D \rightarrow as/\epsilon \quad (\text{---} \rightarrow = \{5, 11, \dots\})$$

{}

S is the start symbol.

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Q1) Obtain a gr. to generate the set of all strings with exactly one a when  $\Sigma = \{a, b\}$

$$\text{Soln: } V = \{S, AS\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow AA | bs \}$$

$$A \rightarrow ba | \epsilon \quad \{\}$$

S is the start symbol.

$$\begin{aligned} S &\rightarrow AA \\ &\Rightarrow aba \\ &\Rightarrow abba \\ &\Rightarrow abb \end{aligned}$$

$$\begin{aligned} S &\rightarrow bs \\ &\Rightarrow baa \\ &\Rightarrow babaa \\ &\Rightarrow bab \end{aligned}$$

Q2) Obtain a gr. to generate the set of all strings with atleast one a when  $\Sigma = \{a, b\}$

$$\text{Soln: } V = \{S, AS\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow bs | AA \}$$

$$A \rightarrow aa | ba | \epsilon \quad \{\}$$

S is the start symbol.

Q3) Obtain a gr. to generate the set of all strings with no more than three a's when  $\Sigma = \{a, b\}$

$$\text{Soln: } V = \{S, A, B, C\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow bs | aaA | \epsilon \}$$

$$A \rightarrow ba | ab | \epsilon \quad \{\}$$

$$B \rightarrow bb | ac | \epsilon \quad \{\}$$

$$C \rightarrow bc | \epsilon \quad \{\}$$

S is the start symbol.

Q3) Obtain the gr. to generate the language.

$$L = \{w \mid n_a(w) = n_b(w) + 1\}$$

Soln One extra 'a' either in the beginning or at end can be got by  $S \rightarrow Aaa$ .

'A' generates equal no. of a's & b's using the prod.,

$$A \rightarrow aAb$$

$$A \rightarrow bAa$$

$$A \rightarrow AA$$

$$A \rightarrow \epsilon$$

The final gr. to generate the given language

$$G = (V, T, P, S)$$

$$V = \{S, A\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow Aaa$$

$$A \rightarrow aAb \mid bAa \mid AA \mid \epsilon\}$$

}

S is the start symbol.

Q4) Obtain the gr. to generate the language,

$$L = \{w \mid n_a(w) > n_b(w)\}$$

Soln More no. of a's can appear either in the beginning or at the middle or at the end. So,

$$S \rightarrow AB \mid BA \mid ABA.$$

$$B \rightarrow aBa$$

Final gr. is

$$V = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow AB \mid BA \mid ABA$$

$$A \rightarrow aAb \mid bAa \mid AA \mid \epsilon\}$$

$$B \rightarrow aBa\}$$

S is the start symbol.

## DETERMINISTIC FINITE AUTOMATION (DFA)

### Definition

The term "deterministic" refers to the fact that on each input there is one and only one state to which the automation can transition from its current state.

A deterministic finite automation consists of:

- i) A finite set of states, often denoted  $Q$ .
- ii) A finite set of input symbols, often denoted  $\Sigma$ .
- iii) A transition function that takes as arguments a state and an input symbol and returns a state. The transition function will commonly be denoted  $\delta$ .  
 Eg: If  $q$  is a state, and  $a$  is an input symbol then  $\delta(q, a)$  is that state  $p$  such that there is an arc labeled  $a$  from  $q$  to  $p$ .
- iv) A start state, one of the states in  $Q$ .
- v) A set of final states or accepting states  $F$ . The set  $F$  is a subset of  $Q$ .

A DFA is a "five-tuple" or quintuple

$$A = (Q, \Sigma, \delta, q_0, F)$$

where  $A$  is the name of DFA,

$Q$  is the set of states

$\Sigma$  its input symbols

$\delta$  its transition function

$q_0$  its start state

$F$  its set of accepting states.

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### How a DFA processes strings

The "language" of the DFA is the set of all strings that the DFA accepts.

Suppose  $a_1 a_2 \dots a_n$  is a sequence of input symbols. We start out with the DFA in its start state,  $q_0$ . We consult the transition function  $\delta$ , say  $\delta(q_0, a_1) = q_1$ , to find the state that the DFA  $A$  enters after processing the first input symbol  $a_1$ . We process the next input symbol,  $a_2$ , by evaluating  $\delta(q_1, a_2)$ ; let us suppose this state is  $q_2$ . We continue in this manner finding states  $q_3, q_4, \dots, q_n$  such that  $\delta(q_{i-1}, a_i) = q_i$  for each  $i$ .

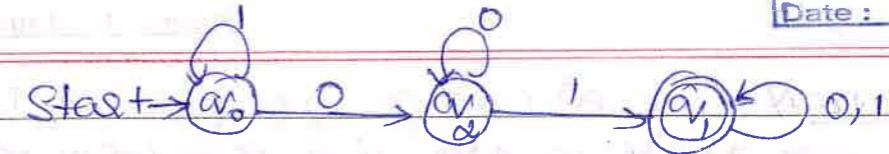
If  $q_n$  is a member of  $F$ , then the input  $a_1 a_2 \dots a_n$  is accepted. And if not then it is "Rejected".

### Transition Diagrams

- A " " for a DFA  $A = (Q, \Sigma, \delta, q_0, F)$  as a graph defined as follows:
- For each state in  $Q$  there is a node.
  - For each state  $q$  in  $Q$  and each i/p symbol  $a$  in  $\Sigma$ , let  $\delta(q, a) = p$ . Then the tr. diagram has an arc from node  $q$  to node  $p$ , labeled  $a$ . If there are several i/p symbols that cause transitions from  $q$  to  $p$ , then the tr. dia can have one arc labeled by the rest of these symbols.

- There is an arrow into the start state  $q_0$ , labeled start. This arrow does not originate at any node.

- Nodes corresponding to accepting states (that in  $F$ ) are marked by a double circle. States not in  $F$



### Transition Table (TT):

A TT is a conventional, tabular representation of a function like  $\delta$  that takes two arguments and returns a value. The rows of the table correspond to the states, and the columns correspond to the inputs.

The entry for the row corresponding to state  $q_1$  and the column corresponding to input  $a$  is the state  $\delta(q_1, a)$ .

Ex	0	1
$\rightarrow q_0$	$q_2$	$q_0$
$\times q_1$	$q_1$	$q_1$
$q_2$	$q_2$	$q_1$

### Extending the Transition Function to strings

If  $\delta$  is the to, for, then the extended to for constructed from  $\delta$  will be called  $\tilde{\delta}$ .

The ext to for is a to that takes a state  $q$  and a string  $w$  and returns a state  $p$  - the state that the automaton reaches when starting in state  $q$  and processing the sequence of i/p's  $w$ . We define  $\tilde{\delta}$  by induction on the length of the i/p string, as follows:

Basis:  $\tilde{\delta}(q, \epsilon) = q$ . That is, if we are in state  $q$  and read no i/p, then we are still in state  $q$ .

INDUCTION:

Suppose  $w$  is a string of the form  $xa$ , i.e.,  $a$  is the last symbol of  $w$ , and  $x$  is the string consisting of all but the last symbol.

For ex.,  $w = 1101$  is broken into  $w = 110$  &  $a = 1$ . Then

$$\hat{\delta}(a, w) = \delta(\hat{\delta}(a, x), a) \quad \text{--- (1)}$$

To compute  $\hat{\delta}(a, w)$  first compute  $\hat{\delta}(a, x)$ , the state that the automaton is in after processing all but the last symbol of  $w$ .

Suppose this state is  $p$ , i.e.  $\hat{\delta}(a, x) = p$ . Then

$\hat{\delta}(a, w)$  is what we get by making a transition from state  $p$  on input  $a$ , the last symbol of  $w$ . i.e.  $\hat{\delta}(a, w) = \delta(p, a)$ .

Properties of Transition Function

$$\begin{aligned} \delta(a, \epsilon) &= \delta^*(a, \epsilon) = a \\ \delta(a, wa) &= \delta(\delta^*(a, w), a) \\ \delta(a, aw) &= \delta^*(\delta(a, a), w) \end{aligned}$$

Ex.  $w = 01101$  (TT - prev page) - (Villman)

$$\delta(q_0, \epsilon) = q_0$$

$$\delta(q_0, 0) = \delta(\hat{\delta}(q_0, \epsilon), 0) = \delta(q_0, 0) = q_1$$

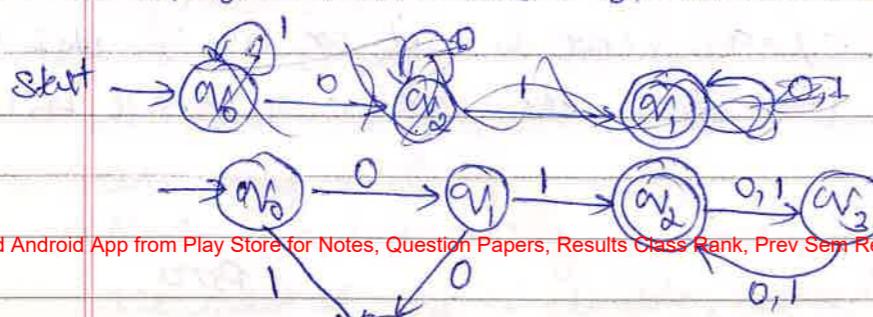
$$\delta(q_0, 01) = \delta(\hat{\delta}(q_0, 0), 1) = \delta(q_1, 1) = q_2$$

$$\delta(q_0, 011) = \delta(\hat{\delta}(q_0, 01), 1) = \delta(q_2, 1) = q_3$$

$$\delta(q_0, 0111) = \delta(\hat{\delta}(q_0, 011), 1) = \delta(q_3, 1) = q_4$$

$$\delta(q_0, 01110) = \delta(\hat{\delta}(q_0, 0111), 0) = \delta(q_4, 0) = q_3$$

$$\delta(q_0, 011101) = \delta(\hat{\delta}(q_0, 01110), 1) = \delta(q_3, 1) = q_4$$

The language of a DFA

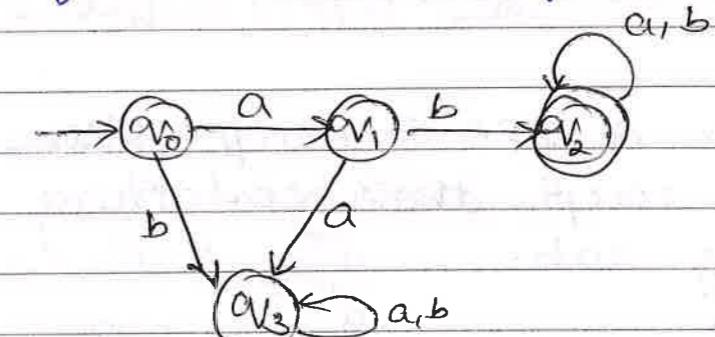
The lang. of a DFA  $A = (Q, \Sigma, \delta, q_0, F)$  denoted as  $L(A)$  is defined by

$$L(A) = \{w \mid \delta(q_0, w) \text{ is in } F\} \\ \text{i.e.}$$

The lang. of  $A$  is the set of strings  $w$  that take the start state  $q_0$  to one of the accepting states. If  $L$  is  $L(A)$  for some DFA  $A$ , then we say  $L$  is a regular language.

Problems

- v) Obtain a DFA to accept strings of ab and ba starting with the string ab.



The DFA to accept strings starting with ab is

$$A = (Q, \Sigma, \delta, q_0, F) \text{ where } \Sigma = \{a, b\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

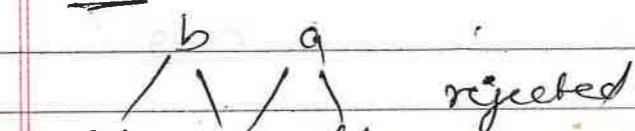
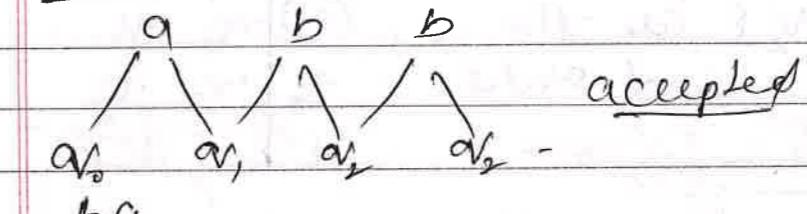
$$\Sigma = \{a, b\}$$

$q_0$  is the start state

$q_3$  is the final state

$\delta$	a	b
$q_0$	$q_1$	$q_2$
$q_1$	$q_3$	$q_2$
$q_2$	$q_2$	$q_3$
$q_3$	$q_3$	$q_3$

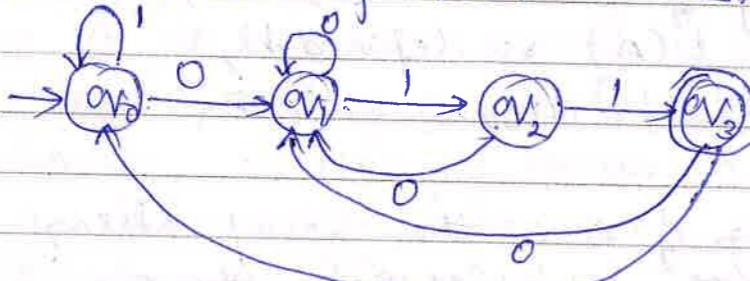
abb



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- 2) Design a DFA to accept strings of  $a^8$  and  $b^8$  ending with the string  $011$ .



$$\begin{array}{l} 011 \\ 10011 \\ 0110^x \\ 011011 \\ 01101^x \\ 111011 \end{array}$$

$$L = \{ (0+1)^* 011 \}$$

The DFA  $A = (Q, \Sigma, \delta, q_0, F)$  is

$$Q = \{q_0, q_1, q_2, q_3\}$$

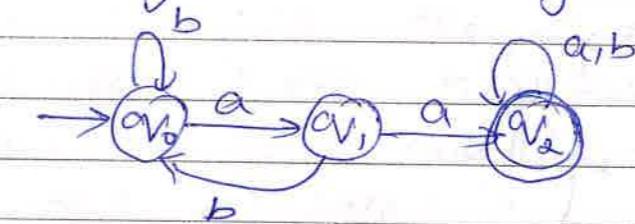
$$\Sigma = \{0, 1\}$$

$q_0$  is the start state

$q_3$  is the final state

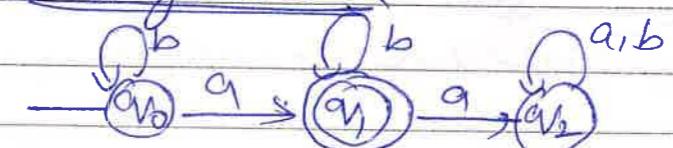
	0	1
$\delta(q_0, 0)$	$q_1, q_3$	
$\delta(q_1, 0)$	$q_1, q_2$	
$\delta(q_2, 0)$	$q_1, q_3$	
$\delta(q_3, 0)$	$q_1, q_3$	

- 4) Obtain a DFA to accept strings of  $a^8$  &  $b^8$  having a substring  $aa$ .

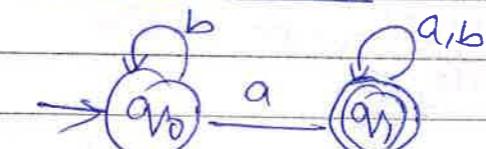


- 5) Obtain DFA's to accept strings of  $a^8$  &  $b^8$  having
- i) exactly one  $a$
  - ii) atleast one  $a$
  - iii) not more than three  $a^8$

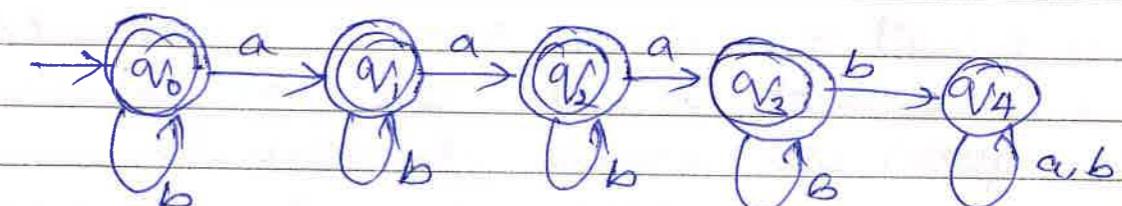
- i) exactly one  $a$



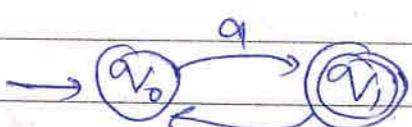
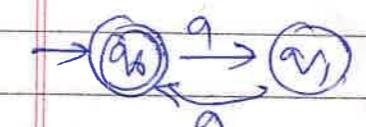
- ii) atleast one  $a$



- iii) not more than three  $a^8$



- 6) Obtain a DFA to accept even no of  $a^8$  and odd no of  $a^8$ .



even

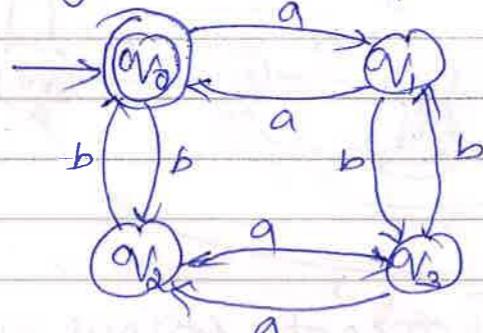
odd

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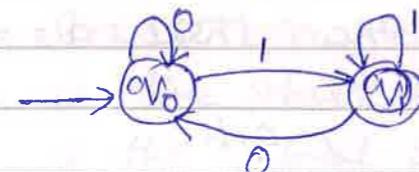
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## NONDETERMINISTIC FINITE AUTOMATA (NFA)

- 7) Obtain a DFA to accept strings of  $a^k b^l$  having even no of  $a$ s and  $b$ s.

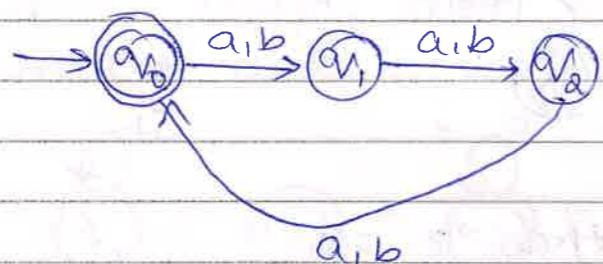


- 8) Obtain a DFA to accept binary odd nos



001  
011  
101

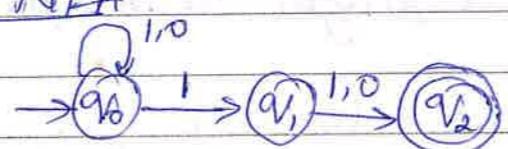
- 9) Obtain a DFA to accept the language  $L = \{w : |w| \bmod 3 = 0\}$  on  $\Sigma = \{a, b\}$



An NFA has the power to be in several states at once. This ability is often expressed as an ability to "guess" something about its input.

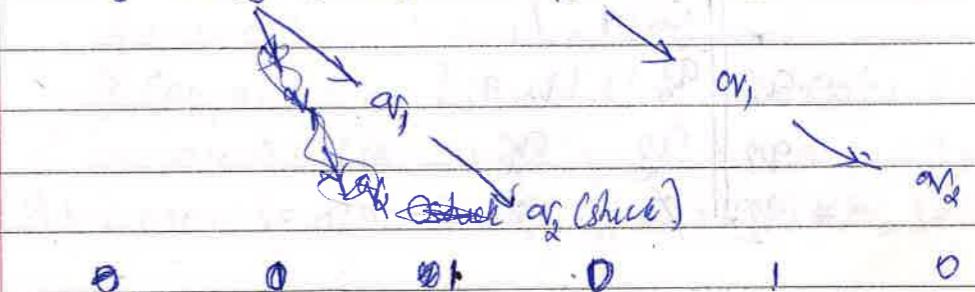
For instance, when the automaton is used to search for certain sequences of characters (eg: keywords) in a long text string, it is helpful to "guess" that we are at the beginning of one of those strings and use a sequence of states to do nothing but check that the string appears, character by character.

An NFA



Note that now there are 2 possible transitions labeled 1 or 0 as & hence NFA has 2 options - it can move to either  $q_0$  or  $q_1$ . In fact at each such point, it starts a new thread corresponding to each of the new transitions. Also note that there are no transitions out of  $q_2$ . Whenever this happens, the corresponding thread just ceases to exist.

Ex:  $q_0 \rightarrow q_1 \rightarrow q_0 \rightarrow q_0 \rightarrow q_1 \rightarrow q_2$



Definition of NFA

An NFA is represented like a DFA:

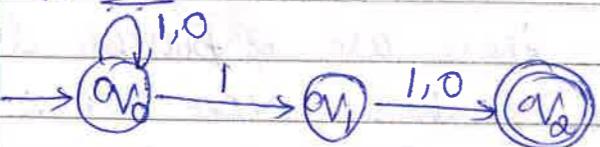
$$A = (Q, \Sigma, \delta, q_0, F)$$

Where

- 1)  $Q$  is a finite set of states
- 2)  $\Sigma$  is a finite set of input symbols
- 3)  $q_0$ , a member of  $Q$ , is the start state
- 4)  $F$ , a subset of  $Q$ , is the set of final (or accepting) states.
- 5)  $\delta$ , the transition function is a function that takes a state in  $Q$  and an input symbol in  $\Sigma$  as arguments and returns a subset of  $Q$ .

Note:  $\delta$  returns a set of states in the case of an NFA and a single state in the case of a DFA.

For ex



$$A = (Q, \Sigma, \delta, q_0, F) \text{ where}$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

$q_0$  is the start state

$q_2$  is the final state

	0	1
$\rightarrow q_0$	$q_0$	$\{q_0, q_1\}$
$q_1$	$q_2$	$q_2$
$* q_2$	$\emptyset$	$\emptyset$

The Extended Transition Function

We extend the to to of an NFA to a function  $\tilde{\delta}$  that takes a state  $q$  and a string of input symbols  $w$ , and returns the set of states that the NFA is in if it starts in state  $q$  and processes the string  $w$ .

Formally, we define  $\tilde{\delta}$  for an NFA's to. for  $\delta$  by:

Basis:

$\tilde{\delta}(q, \epsilon) = \{q\}$ . That is, without reading any input symbols, we are only in the state we began in.

INDUCTION:

Suppose  $w$  is of the form  $w = xa$ , where  $a$  is the final symbol of  $w$  and  $x$  is the rest of  $w$ . Also suppose that  $\tilde{\delta}(q, x) = \{p_1, p_2, \dots, p_k\}$ .

Let

$$\bigcup_{i=1}^k \delta(p_i, a) = \{r_1, r_2, \dots, r_m\}$$

$$\text{Then } \tilde{\delta}(q, w) = \{r_1, r_2, \dots, r_m\}$$

Let formally, we compute  $\tilde{\delta}(q, w)$  by first computing  $\tilde{\delta}(q, x)$ , and then following any transition from any of these states that is labeled  $a$ .

Ex: Process the string 01010 using  $\tilde{\delta}$  by NFA.

$$\tilde{\delta}(q_0, \epsilon) = \{q_0\}$$

$$\tilde{\delta}(q_0, 0) = \delta(q_0, 0) = \{q_0\}$$

$$\tilde{\delta}(q_0, 01) = \delta(q_0, 1) = \{q_0, q_1\}$$

$$\tilde{\delta}(q_0, 010) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$$

$$\tilde{\delta}(q_0, 0101) = \delta(q_0, 1) \cup \delta(q_2, 1) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$$

$$\tilde{\delta}(q_0, 01010) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$$

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### The Language of an NFA

If  $A = (Q, \Sigma, \delta, q_0, F)$  is an NFA, then,  
 $L(A) = \{w \mid \delta(q_0, w) \cap F \neq \emptyset\}$

That is  $L(A)$  is the set of strings  $w$  in  $\Sigma^*$  such that  $\delta(q_0, w)$  contains at least one accepting state.

### EQUIVALENCE OF DFA & NFA OR CONVERSION FROM NFA TO DFA

#### SUBSET CONSTRUCTION METHOD

The subset construction starts from an NFA  $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ . Its goal is the description of a DFA  $D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$  such that  $L(D) = L(N)$ . The i/p alphabets of the two automata are the same, and the start state of  $D$  is the set containing only the start state of  $N$ . The other components of  $D$  are constructed as follows.

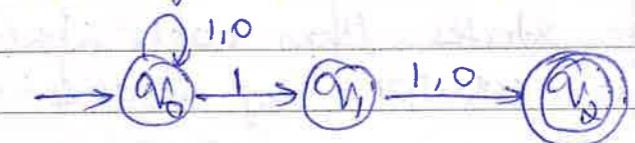
- $Q_D$  is the set of subsets of  $Q_N$ , i.e.  $Q_D$  is the powerset of  $Q_N$ . If  $Q_N$  has  $n$  states, then  $Q_D$  will have  $2^n$  states. Often, not all these states are accessible from the start state of  $Q_D$ . Inaccessible states can be "thrown away", so effectively, the no. of states of  $D$  may be much smaller than  $2^n$ .

- $F_D$  is the set of subsets  $S$  of  $Q_N$  such that  $S \cap F_N \neq \emptyset$ . That is  $F_D$  is all sets of  $N$ 's states that include at least one accepting state

- For each set  $S \subseteq Q_N$  and for each input symbol  $a$  in  $\Sigma$ .

$$\delta_D(S, a) = \bigcup_{p \in S} \delta_N(p, a)$$

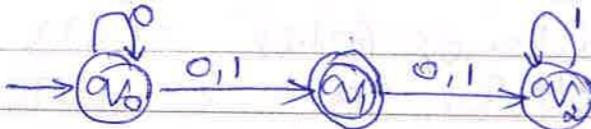
That is, to compute  $\delta_D(S, a)$ , we look at all the states  $p$  in  $S$ , see what states  $N$  goes to from  $p$  on input  $a$ , and take the union of all those states.



$\delta$	0	1
$\emptyset$	$\emptyset$	$\emptyset$
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_1\}$	$\{q_1\}$	$\{q_1, q_2\}$
$\{q_2\}$	$\emptyset$	$\emptyset$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$

## Problems :

- Q) Convert the following NFA into an equivalent DFA.



Step 1:  $q_0$  is the start of DFA.

$$\text{So } Q_D = \{[q_0]\} - - - (1)$$

Step 2 Find the new states from each state in  $Q_D$  and obtain the corresponding transitions.

Consider the state  $[q_0]$ :

When  $a=0$ ,

$$\delta_D([q_0], 0) = \delta_N([q_0], 0) \\ = [q_0, q_1] - - - (2)$$

When  $a=1$ ,

$$\delta_D([q_0], 1) = \delta_N([q_0], 1) \\ = [q_1] - - - (3)$$

Since the states obtained in (2) & (3) are not in  $Q_D$  (1), add these two states to  $Q_D$

so that

$$Q_D = \{[q_0], [q_0, q_1], [q_1]\}$$

Consider the state  $[q_0, q_1]$ :

When  $a=0$ ,

$$\delta_D([q_0, q_1], 0) = \delta_N([q_0, q_1], 0) \\ = \delta_N(q_0, 0) \cup \delta_N(q_1, 0) \\ = \{q_0, q_1\} \cup \{q_2\} \\ = [q_0, q_1, q_2] - - - (4)$$

When  $a=1$

$$\delta_D([q_0, q_1], 1) = \delta_N([q_0, q_1], 1) \\ = \delta_N(q_0, 1) \cup \delta_N(q_1, 1) \\ = \{q_1, q_2\} - - - (5)$$

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Since these 2 states are not in  $Q_D$  add them to  $Q_D$  so that

$$Q_D = \{[q_0], [q_0, q_1], [q_1], [q_0, q_1, q_2], [q_1, q_2]\}$$

The transitions are

		$\delta$	0	1
	$\uparrow$	$[q_0]$	$[q_0, q_1]$	$[q_1]$
$Q$	$\downarrow$	$[q_1]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
		$[q_0, q_1, q_2]$		
		$[q_1, q_2]$		

Consider the state  $[q_1]$ :

When  $a=0$

$$\delta_D([q_1], 0) = \delta_N([q_1], 0) \\ = [q_2] - - - (6)$$

When  $a=1$

$$\delta_D([q_1], 1) = \delta_N([q_1], 1) \\ = [q_2] - - - (7)$$

Since  $[q_2]$  is not present in  $Q_D$  add it to it

$$Q_D = \{[q_0], [q_0, q_1], [q_1], [q_0, q_1, q_2], [q_1, q_2], [q_2]\}$$

Consider the state  $[q_0, q_1, q_2]$ :

When  $a=0$ ,

$$\delta_D([q_0, q_1, q_2], 0) = \delta_N([q_0, q_1, q_2], 0) \\ = \delta_N(q_0, 0) \cup \delta_N(q_1, 0) \cup \delta_N(q_2, 0) \\ = \{q_0, q_1, q_2\} \cup \{q_2\} \cup \emptyset \\ = [q_0, q_1, q_2]$$

When  $a=1$ ,

$$\delta_D([q_0, q_1, q_2], 1) = \delta_N([q_0, q_1, q_2], 1) \\ = \delta_N(q_0, 1) \cup \delta_N(q_1, 1) \cup \delta_N(q_2, 1) \\ = \{q_1, q_2\} \cup \{q_2\} \cup \{q_1\} \\ = \{q_1, q_2\}$$

Since these two states are already in  $Q_D$  do not add them to  $Q_D$ . But add the corresponding Tr. to Tr. table.

Consider the state  $[q_1, q_2]$

When  $a=0$ ,

$$\begin{aligned} d_D([q_1, q_2], 0) &= d_N([q_1, q_2], 0) \\ &= d_N(q_1, 0), d_N(q_2, 0) \\ &= \{q_1\} \cup \emptyset \\ &= [q_1] \end{aligned}$$

When  $a=1$

$$\begin{aligned} d_D([q_1, q_2], 1) &= d_N([q_1, q_2], 1) \\ &= d_N(q_1, 1) \cup d_N(q_2, 1) \\ &= \{q_1\} \cup \{q_2\} \\ &= [q_1, q_2] \end{aligned}$$

Since  $[q_2]$  is already in  $Q_D$ , just add the tr. to  $\pi$ .

Consider the state  $[q_2]$ :

When  $a=0$ ,

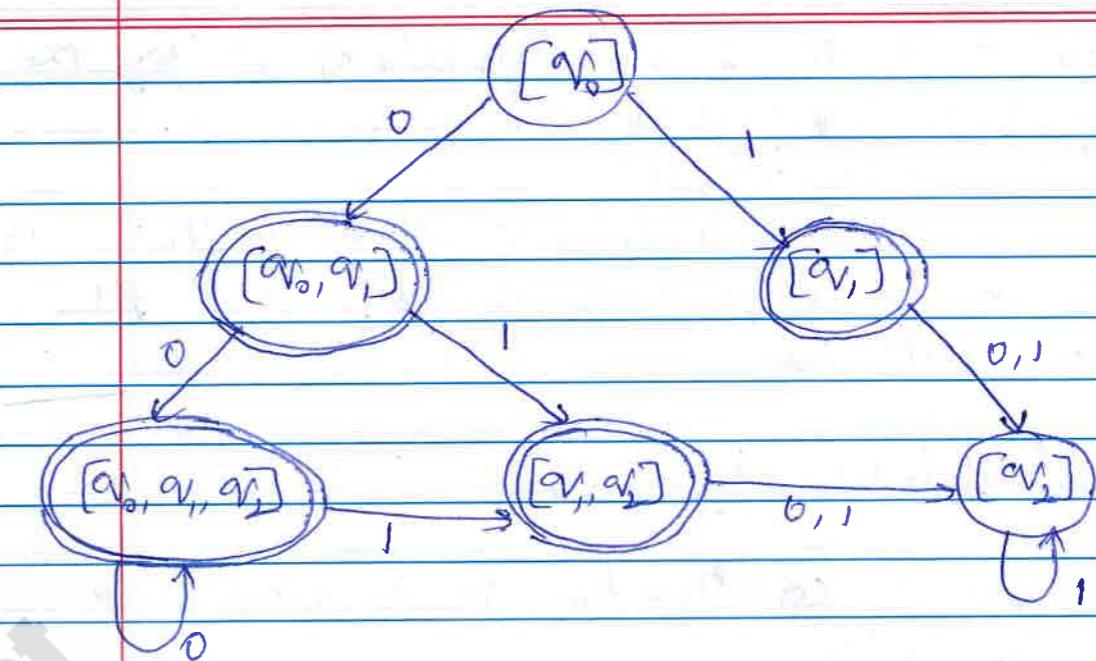
$$\begin{aligned} d_D([q_2], 0) &= d_N([q_2], 0) \\ &= \emptyset \end{aligned}$$

When  $a=1$

$$\begin{aligned} d_D([q_2], 1) &= d_N([q_2], 1) \\ &= [q_2] \end{aligned}$$

The final  $\pi$  is  $\leftarrow \Sigma \rightarrow$

$\delta$	0	1
$[q_0]$	$[q_0, q_1]$	$[q_1]$
$(q_0, q_1)$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
$[q_1]$	$[q_2]$	$[q_1]$
$(q_2, q_1, q_2)$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
$(q_1, q_2)$	$[q_2]$	$[q_2]$
$[q_2]$	$\emptyset$	$[q_2]$



### Theorem - 1

If  $D = \{Q_D, \Sigma, d_D, \{q_0\}, F_D\}$  is the DFA constructed from NFA  $N = (Q_N, \Sigma, d_N, q_0, F_N)$  by the subset construction- then  $L(D) = L(N)$ .

Play: What we actually prove first, by induction on  $|\omega|$ , is that

$$d_D(\{q_0\}, \omega) = d_N(q_0, \omega)$$

Notice that each  $\delta$  in the  $\delta$  for selecting a set of states from  $Q_N$ , but  $\delta_D$  interprets this set as one of the states of  $Q_D$  (whereas  $\delta$  is the power set of  $Q_N$ ), while  $\delta_N$  interprets this set as a subset of  $Q_N$ .

BY BASIC: Let  $|\omega| = 0$ ; that is  $\omega = \epsilon$ . By the basic definition of  $\delta$  for DFA's and NFA's both  $d_D(\{q_0\}, \epsilon)$  and  $d_N(q_0, \epsilon)$  are  $\{q_0\}$ .

### INDUCTION:

Let  $\omega$  be of length  $n+1$ , and assume the hint for length  $n$ . Break  $\omega$  up as  $\omega_1, \omega_2, \dots, \omega_n, \omega_{n+1}$ .

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where  $a$  is the final symbol of  $w$ . By the inductive hypothesis,

$$\delta_D^*(\{q_0\}, x) = \delta_N^*(q_0, x).$$

Let both these sets of NFA states be  $\{P_1, P_2, \dots, P_k\}$ .

The inductive part of the defn of  $\delta$  for NFA's tells us

$$\delta_N^*(q_0, w) = \bigcup_{i=1}^k \delta_N^*(P_i, a) \quad \dots (1)$$

The subset construction, on the other hand, tells us that

$$\delta_D^*(\{P_1, P_2, \dots, P_k\}, a) = \bigcup_{i=1}^k \delta_D^*(P_i, a) \quad \dots (2)$$

Now, let us use (2) and the fact that  $\delta_D^*(\{q_0\}, x) = \{P_1, P_2, \dots, P_k\}$  in the inductive part of the defn. of  $\delta$  for DFA's:

$$\begin{aligned} \delta_D^*(\{q_0\}, w) &= \delta_D^*(\delta_D^*(\{q_0\}, x), a) \\ &= \delta_D^*(\{P_1, P_2, \dots, P_k\}, a) \end{aligned}$$

$$= \bigcup_{i=1}^k \delta_N^*(P_i, a) \quad \dots (3)$$

Thus eqns (1) & (3) demonstrate that

$\delta_D^*(\{q_0\}, w) = \delta_N^*(q_0, w)$ , when we observe that  $D$  and  $N$  both accept  $w$  if and only if  $\delta_D^*(\{q_0\}, w)$  or  $\delta_N^*(q_0, w)$  respectively, contain a state in  $F_N$ , we have a complete proof that  $L(D) = L(N)$ .

### Problem - 2.

A lang.  $L$  is accepted by some DFA iff  $L$  is accepted by some NFA.

Proof: (if) the "if" part is the subset construction

8. Thm-G1

(Only if). In this part, we have only to convert a DFA into an identical NFA.

If we have the tr. diagram for a DFA, we can also interpret it as the tr. diag of an NFA, which happens to have exactly one choice of  $\alpha$  in any situation.

More formally, let  $D = (Q, \Sigma, \delta_D, q_0, F)$  be a DFA. Define  $N = (Q, \Sigma, \delta_N, q_0, F)$  to be the equivalent NFA, where  $\delta_N$  is defined by the rule:

- If  $\delta_D(q, a) = p$ , then  $\delta_N(q, a) = \{p\}$ .

It is then easy to show by induction on  $|w|$ , that if  $\delta_D^*(q_0, w) = p$  then

$$\delta_N^*(q_0, w) = \{p\}.$$

$w$  is accepted by  $D$  if and only if it is accepted by  $N$  i.e.  $L(D) = L(N)$ .

### Finite Automata with Epsilon-Transitions

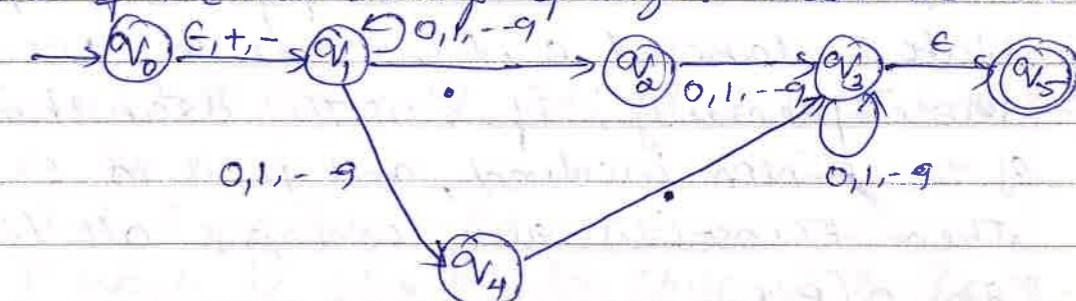
Construct an E-NFA that accepts decimal numbers consisting of

- i) an optional + or - sign

ii) A string of digits

iii) A decimal point .

iv) Another string of digits. Either the string of digits or the string (ii) can be empty, but at least one of the two strings of digits must be nonempty.



The Formal Notation for an E-NFA:-

We represent an E-NFA  $A$  by  $A = (Q, \Sigma, \delta, q_0, F)$  where all components have the same interpretation as for an NFA, except that  $\delta$  is now a function that takes as arguments:

- i. A state in  $Q$  and
- ii. A member of  $\Sigma \cup \{\epsilon\}$ , that is either an input symbol, or the symbol  $\epsilon$ . We require that  $\epsilon$ , the symbol for the empty string, cannot be a member of the alphabet  $\Sigma$ .

Epsilon-closures :-

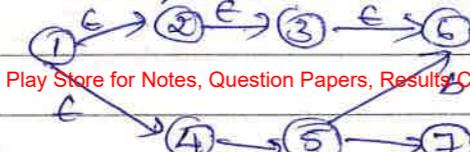
We  $\epsilon$ -close a state  $q$  by following all transitions out of  $q$  that are labeled  $\epsilon$ . However, when we get to other states by following  $\epsilon$ , we follow the  $\epsilon$ -transitions out of those states & so on, eventually finding every state that can be reached from  $q$  along any path whose arcs are all labeled  $\epsilon$ .

Recursive Defn :-

Basis: State  $q$  is in  $\text{ECLOSE}(q)$ .

Induction: If state  $p$  is in  $\text{ECLOSE}(q)$ , and there is a transition from state  $p$  to state  $r$  labeled  $\epsilon$ , then  $r$  is in  $\text{ECLOSE}(q)$ .

More precisely, if  $\delta$  is the transition function of the E-NFA involved, and  $p$  is in  $\text{ECLOSE}(q)$ , then  $\text{ECLOSE}(q)$  also contains all the states in  $\delta(p, \epsilon)$ .

Ex

$$\text{Eclose}(1) = \{1, 2, 3, 4, 6, 7\}$$

Eliminating E-Transitions

Let the E-NFA  $E = (Q_E, \Sigma, \delta_E, q_{0E}, F_E)$ :

Then the equivalent DFA

$$D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$$

is defined as follows:

i)  $Q_D$  is the set of subsets of  $Q_E$ . More precisely, we shall find that the only accessible states of  $D$  are the  $\epsilon$ -closed subsets of  $Q_E$ , i.e. those sets  $S \subseteq Q_E$  such that  $S = \text{ECLOSE}(S)$ .

Put another way, the  $\epsilon$ -closed sets of states  $S$  are those such that any  $\epsilon$ -transition out of one of the states in  $S$  lead to a state that is also in  $S$ . Note that  $\emptyset$  is an  $\epsilon$ -closed set.

ii)  $q_{0D} = \text{ECLOSE}(q_{0E})$ ; i.e. we get the start state of  $D$  by closing the set consisting of only the start state of  $E$ . Note that this rule differs from the original subset construction, where the start state of the constructed automata was just the set containing the start state of the given NFA.

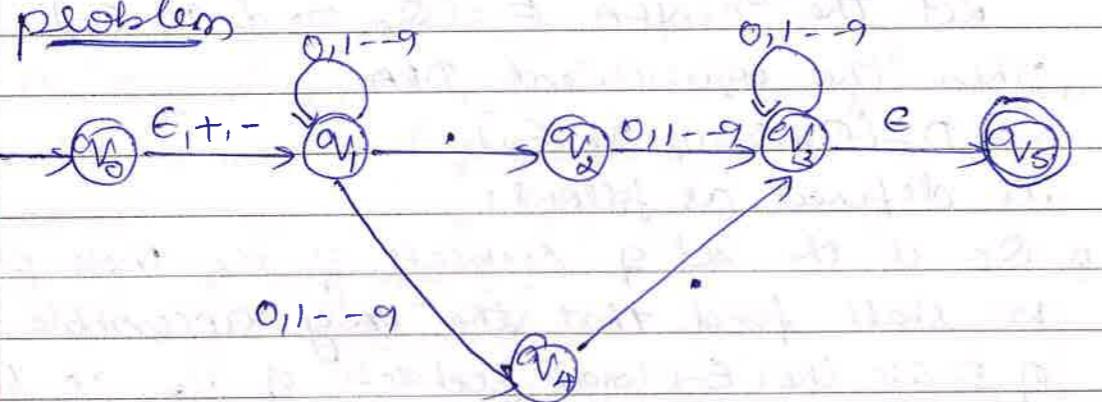
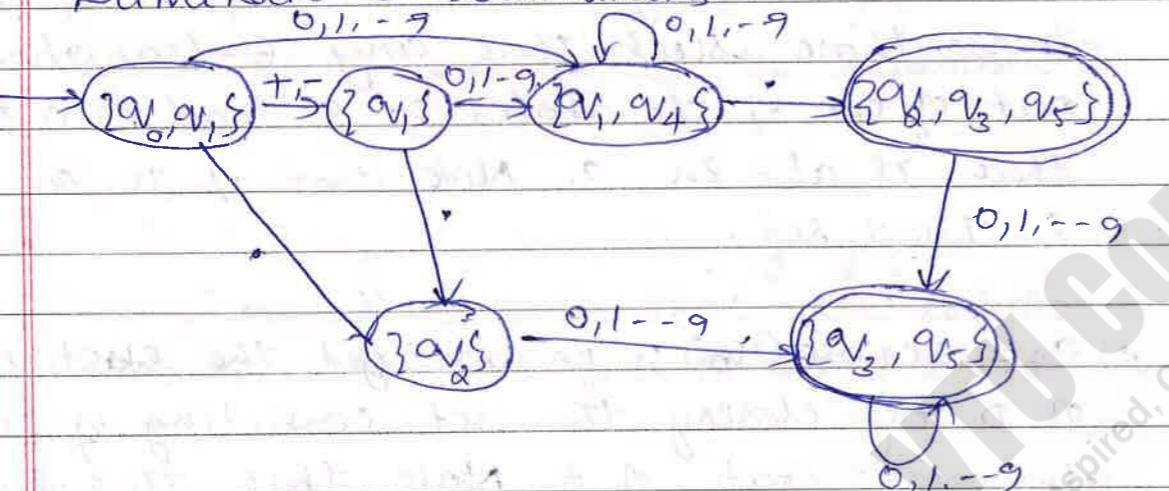
iii)  $F_D$  is those sets of states that contain at least one accepting state of  $E$ . That is  $F_D = \{S | S \text{ is in } Q_D \text{ and } S \cap F_E \neq \emptyset\}$

iv)  $\delta_D(S, a)$  is computed, for all  $a$  in  $\Sigma$  and sets  $S$  in  $Q_D$  by:

(a) Let  $S = \{p_1, p_2, \dots, p_k\}$

(b) Compute  $\bigcup_{i=1}^k \delta(p_i, a)$ ; let this set be  $\{r_1, r_2, \dots, r_m\}$

(c) Then  $\delta_D(S, a) = \bigcup_{j=1}^m \text{ECLOSE}(r_j)$ .

REGULAR EXPRESSIONSproblemEliminate e-transitions

$$d_p(q_0, q_1, S, +) = \{q_1\}$$

$$d_p(\{q_0, q_1\}, -) = \{q_1\}$$

$$d_p(q_0, q_1, S, \cdot) = \{q_2\}$$

$$d_p(q_0, q_1, S, 0) = \{q_1, q_4\}$$

$$d_p(\{q_0, q_1\}, 1) = \{q_1, q_4\}$$

$$d_p(q_2, 0) = \{q_3, q_4\}$$

$$d_p(q_2, 1) = \{q_2, q_3, q_4\}$$

$$d_p(q_3, 0) = \{q_3\}$$

$$d_p(q_3, 1) = \{q_3, q_4\}$$

$$d_p(q_4, \cdot) = \{q_2\}$$

$$d_p(q_4, 0) = \{q_1\}$$

$$d_p(q_4, 1) = \{q_1\}$$

$$d_p(q_1, q_4, S, +) = \{q_4\}$$

$$d_p(q_1, q_4, S, -) = d_p(q_1, \cdot) \cup d_p(q_1, 0)$$

$$= \{q_2\} \cup \{q_3, q_4\}$$

$$d_p(q_1, q_4, S, \cdot) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, S, 0) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, 1) = \emptyset \cup \{q_2\}$$

$$d_p(q_1, q_4, 0) = \emptyset$$

$$d_p(q_1, q_4, \cdot) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, 1) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, 0) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, \cdot) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, 1) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, \cdot, 0) = \{q_1, q_3, q_4\}$$

$$d_p(q_1, q_4, \cdot, 1) = \{q_1, q_3, q_4\}$$

REGULAR EXPRESSIONS

Regular exps define the regular language that various forms of automata describe.

R.E. offers a declarative way to express the strings we want to accept.

The Operators of R.Es :-

$\cup$  The union of two languages L and M denoted  $L \cup M$ , is the set of strings that are in either L or M or both.

$\cdot$  or  $\circ$  if  $L = \{001, 10, 111\}$  and  $M = \{\epsilon, 001\}$  then  $L \cup M = \{\epsilon, 10, 001, 111\}$ .

$\cdot^*$  The concatenation of languages L and M is the set of strings that can be formed by taking any string in L and concatenating it with any string in M denoted with a dot or with no operator.

$\cdot^*$  if  $L = \{001, 10, 111\}$  and  $M = \{\epsilon, 001\}$  then  $L \cdot M$  or  $M \cdot L = \{001, 10, 111, 001001, 10001, 111001\}$

${}^*$  The closure (or star or Kleene closure) of a lang. L is denoted  $L^*$  and represents the set of those strings that can be formed by taking any no. of strings from L, possibly with repetitions and concatenating all of them.

Ex: Let  $L = \{0, 1\}$

$$L^0 = \{\epsilon\}$$

$L^1 = L$ , represents choice of one string from L.

$L^2 = \{00, 01, 10, 11\}$ , pick two strings from L with repetitions allowed.

$L^3 = \{000, 001, 010, 110, 0111, 11011, 11110, 11111\}$

To compute  $L^*$ , compute  $L^i$  for each i & take the union of all these languages.  $L^i$  has  $2^i$  members. L is finite here.

2) Let  $L$  be the set of all strings of 0's.  
 $L$  is infinite.

$$L^0 = \{ \epsilon \}$$

$$L^1 = L, L^2 = L \cdot L, L^3 = L \cdot \dots$$

$\therefore$  Every string of 0's can be written as the concatenation of two strings of 0's.

$\therefore L^* = L^0 \cup L^1 \cup L^2 \cup \dots$  is  $L$  in this case.

3)  $\emptyset^* = \{ \epsilon \}$ .

$$\emptyset^0 = \{ \emptyset \}$$

$\emptyset^i$  for any  $i \geq 1$  is empty.

$\emptyset$  is one of only two langs whose closure is finite.

### Building Reg. Exprns

The algebra of R.E.s uses constants & variables and operators for 3 operations viz union, dot and star.

### Recursive defn. of R.E.'s

Basis: The basis consists of 3 parts:

i) The constants  $\epsilon$  and  $\emptyset$  are R.E.s denoting the langs  $\{ \epsilon \}$  and  $\emptyset$  resp., that is  $L(\epsilon) = \{ \epsilon \}$  and  $L(\emptyset) = \emptyset$ .

ii) If  $a$  is any symbol, then  $a$  is a R.E. This exprs denotes the lang.  $\{ a \}$ . i.e.  $L(a) = \{ a \}$

iii) A var, usually capitalized & italic such as  $L$  is a var, representing any language.

### INDUCTION:

i) If  $E$  &  $F$  are R.E.s, then  $E+F$  is a RE denoting the union of  $L(E)$  and  $L(F)$ , i.e.

$$L(E+F) = L(E) \cup L(F).$$

ii) If  $E$  and  $F$  are REs then  $EF$  is a RE denoting the concatenation of  $L(E)$  and  $L(F)$ . i.e.  $L(ER) = L(E)L(F)$ .

iii) If  $E$  is a RE, then  $E^*$  is a RE, denoting the closure of  $L(E)$ , i.e.  $L(E^*) = (L(E))^*$

iv) If  $E$  is a RE, then  $(E)$ , a parenthesized  $E$ , is also a RE denoting the same language. Formally,  $L((E)) = L(E)$ .

### Precedence of Reg. Exprn Operators

- i) The star operator is of highest precedence.
- ii) Concatenation or dot operator.
- iii) All unions (+ operators) with their operands.

### Finite Automata & Regular Expressions:-

W.K.t DFA & the two kinds of NFA (with & without  $\epsilon$ -transitions) accept the same class of languages. T.S.T RE define the same class, we must show that

- i) Every lang. defined by one of these automata is also defined by a RE.
- ii) Every lang. defined by a RE is defined by one of these automata. For this we show that there is an NFA with  $\epsilon$ -transitions accepting the same language.

pbns) Obtain a RE to accept a lang. consisting of strings of a's and b's of even length.

soln  $(aa+ab+ba+bb)^*$  or

$$L(R) = \{aa+ab+ba+bb\}^n \mid n \geq 0\}$$

2) Obtain a RE to accept a lang. consisting of strings of a's and b's of odd length.

$$(a+b)(aa+ab+ba+bb)^*$$

3) Obtain a RE to accept strings of 0's & 1's ending with 'b' and has no substring aa.

$$(b+ab)^*$$

using set notation,

$$L(R) = \{(b+ab)^n \mid n \geq 1\}$$

4) Obtain a RE to accept strings of 0's & 1's having no two consecutive zeros.

$$(1+01)^*(01+1)$$

5) Obtain a RE such that  $L(R) = \{w \mid w \in \{0,1\}^*\}$  with atleast three consecutive zeros.

soln  $(0+1)^* 000(0+1)^*$  or

$$L(R) = \{ (0+1)^m 000(0+1)^n \mid m \geq 0 \text{ and } n \geq 0 \}$$

6) Obtain a RE to accept strings of a's and b's of length  $\leq 10$ .

$$(a+b)^{10}$$

7) Obtain a RE to accept strings of a's and b's starting with 'a' and ending with 'b'.

$$a(a+b)^*$$

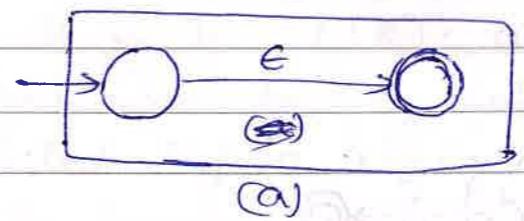
## CONVERTING REG. EXPNS TO AUTOMATA

### Theorem

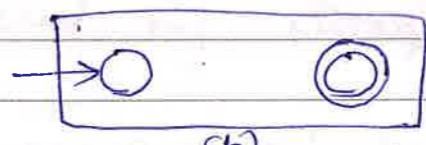
Every language defined by a RE is also defined by a finite automaton.

Proof: Suppose  $L = L(R)$  for a RE  $R$ , we show that  $L = L(E)$  for some E-NFA  $E$  with:

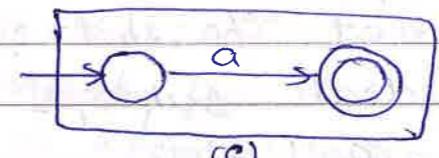
- i) Exactly one accepting state,
- ii) No arcs into the initial state
- iii) No arcs out of the accepting state.



(a)



(b)



(c)

Basis: There are three parts to the basis as shown in above fig.

In part (a) we see how to handle the expr  $\epsilon$ .

The lang. of the automation is easily seen to be  $\{\epsilon\}$ . Since the only path from the start state to an accepting state is labeled  $\epsilon$ .

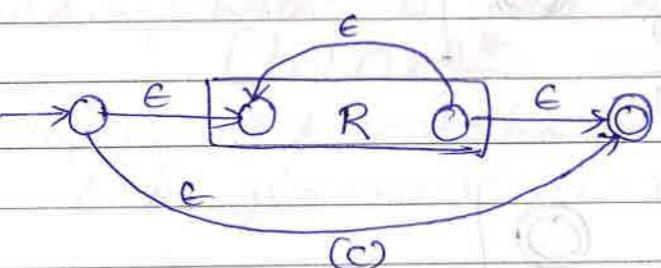
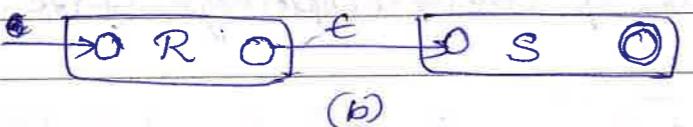
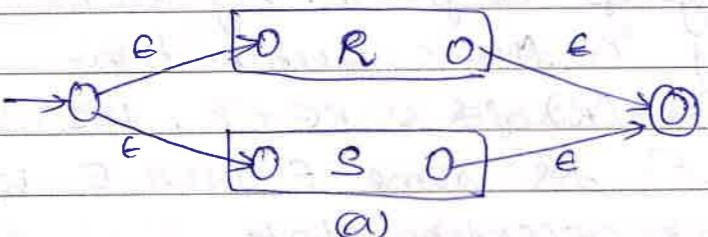
Part (b) shows the construction for  $\emptyset$ . Since there are no paths from start state to accepting state, so  $\emptyset$  is the lang. of this automation.

Part (c) gives the automation for a RE  $a$ .

The lang. of this automaton is  $L(a)$ , consist of one string  $a$ .

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INTRODUCTION:-

The three parts of concatenation are shown above. We assume that the start of the two is true for the immediate subexpressions of a given  $R \circ S$ . The four cases are:

- i) The expr. is  $R+S$  for some smaller exprs  $R$  and  $S$ . Then automation (a) serves. i.e. Starting at the new start state, we can go to the start state of either the automation for  $R$  or the automation for  $S$ . We then reach the accepting state of one of these automata, following a path labeled by some string in  $L(R)$  or  $L(S)$ , respectively. From there, we follow one of the  $\epsilon$ -arcs to the accepting state of the new automation. Thus, the lang. of the automation is  $L(R) \cup L(S)$ .

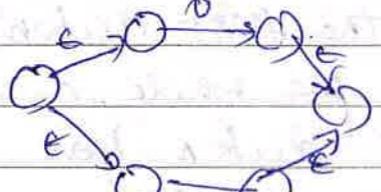
ii) The expr. is  $RS$  for some smaller exprs  $R$  and  $S$ . The automation is shown in (b). The start state of the first automation becomes the start state of the whole, and the accepting state of the second automation becomes the accepting state of the whole. The only paths from start to accepting state go first through the automation for  $R$ , where it must follow a path labeled by a string in  $L(R)$ , and then through the automation for  $S$ , where it follows a path labeled by a string in  $L(S)$ . Thus the paths in the automation of fig (b) are those labeled by strings in  $L(R)L(S)$ .

- iii) The expr. is  $R^*$  for some smaller expr  $R$ . We use automation (c). That allows us to go either:
  - a) Directly from the start state to the accepting state along a path labeled  $\epsilon$ . That path lets us accept  $\epsilon$ , which is in  $L(R^*)$  no matter what expr.  $R$  is.
  - b) To the start state of the automation for  $R$ , through that automation one or more times and then to the accepting state. This set of paths allows us to accept strings in  $L(R)$ ,  $L(R)L(R)$ , and so on thus covering all strings in  $L(R^*)$  except  $\epsilon$ .
- iv) The expr. is  $(R)$  for some smaller expr  $R$ . The automation for  $R$  also serves as the automaton for  $(R)$  since the parentheses do not change

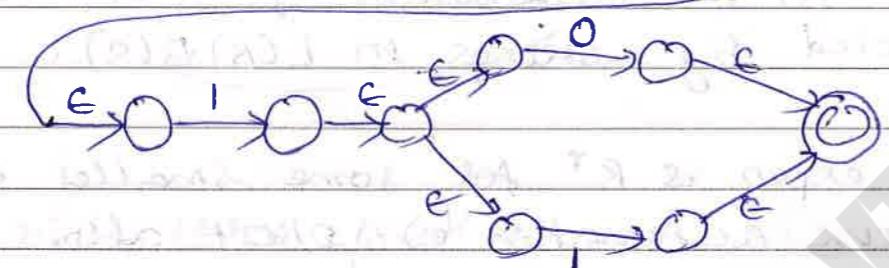
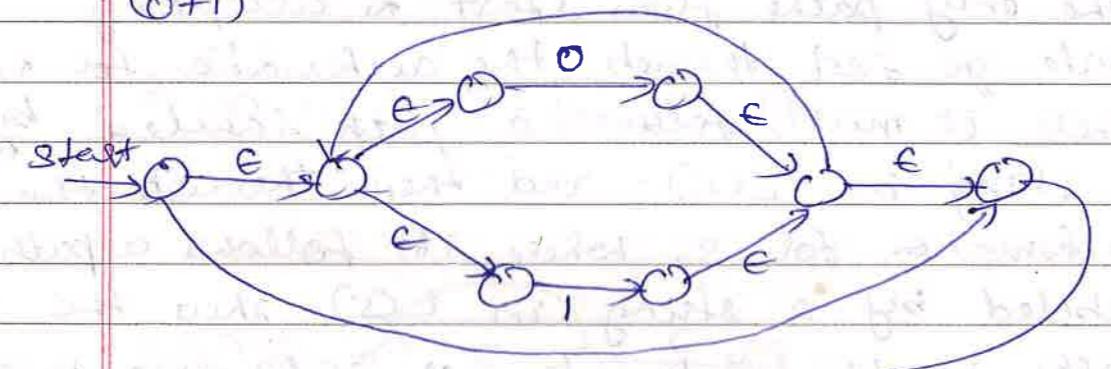
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- 4) Convert the RE  $(0+1)^* 1 (0+1)$  to an E-NFA.
- Soln Consider  $(0+1)$

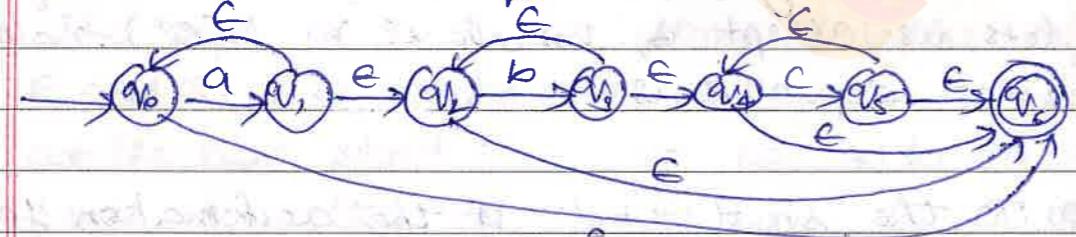


$(0+1)^*$

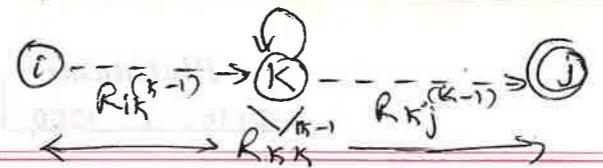
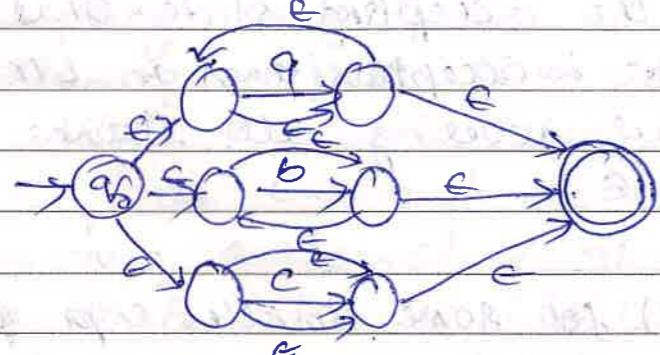


- 2) Obtain an NFA for the RE  $(a+b)^* aa(a+b)^*$

- 3) Obtain an NFA for the RE  $a^* b^* c^*$



- 4) Obtain an NFA for the RE  $a^* + b^* + c^*$



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Theorem: - Kleene's Theorem :-

If  $L = L(A)$  for some DFA A, then there is a RE R such that  $L = L(R)$ .

Proof: Let us suppose that A's states are  $\{1, 2, \dots, n\}$  for some integer n. We have to construct a collection of REs that describe progressively broader sets of paths in the DFA A.

Let us use  $R_{ij}^{(k)}$  as the name of a RE whose lang, is the set of strings w such that w is the label of a path from state i to state j in A and that path has no intermediate node whose number is  $\geq k$ .

Note: The beginning & end pts of the path are not "intermediate," so there is no constraint that i and/or j be less than or equal to k.

To construct the express  $R_{ij}^{(k)}$ , we use the following inductive defn starting at  $k=0$  and finally reaching  $k=n$ . Notice that when  $k=n$ , there is no restriction at all on the paths represented, since there are no states greater than n.

Basis: The basis is  $k=0$ . Since all states are numbered 1 or above, the restriction on paths is that the path must have no intermediate states at all. There are only two kinds of paths that meet such a condition:

- An arc from node (state) i to node j.
- A path of length 0 that consists of only some node i.

If  $i=j$  then only case (i) is possible. We must examine the DFA  $A$  and find those input symbols  $a$  such that there is a transition from state  $i$  to state  $j$  on symbol  $a$ .

- (a) If there is no such symbol  $a$ , then

$$R_{ij}^{(0)} = \emptyset$$

- (b) If there is exactly one such symbol  $a$ , then

$$R_{ij}^{(0)} = a$$

- (c) If there are symbols  $a_1, a_2, \dots, a_k$  that label arcs from state  $i$  to state  $j$ , then

$$R_{ij}^{(0)} = a_1 + a_2 + \dots + a_k.$$

However, if  $i=j$  then the legal paths are the paths of length 0 and all loops from  $i$  to itself. The path of length 0 is represented by the RE  $\epsilon$  since that path has no symbols along it. Thus we add  $\epsilon$  to the various exprs devised in (a) thru (c) above.

### INDUCTION:-

Suppose there is a path from state  $i$  to state  $j$  that goes through no state higher than  $k$ . There are two possible cases to consider:

- (i) The path does not go through state  $k$  at all. In this case, the label of the path is in the lang. of  $R_{ij}^{(k-1)}$ .

- (ii) The path goes thro' state  $k$  atleast once. Then we can break the path into several pieces. The first goes from state  $i$  to state  $k$  without passing through  $k$ , the last piece goes from  $k$  to  $j$  without passing thro'  $k$ , and all the

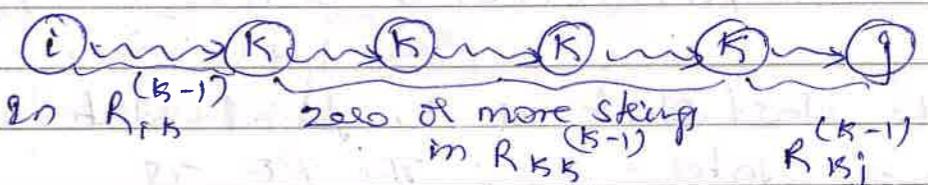
pieces in the middle go from  $k$  to itself without passing through  $k$ .

Note that if the path goes thro' state  $k$  only once, then there are no "middle" pieces, just a path from  $i$  to  $k$ , and a path from  $k$  to  $j$ . The set of labels for all paths of the type  $z_k$  is represented by a RE  $R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$ .

When we combine the expns for the paths of two types above, we have,

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ij}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$

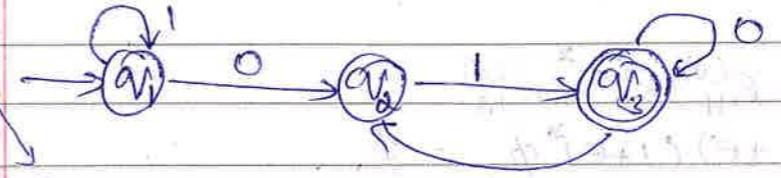
for the labels of all paths from state  $i$  to state  $j$  that go thro' no state higher than  $k$ .



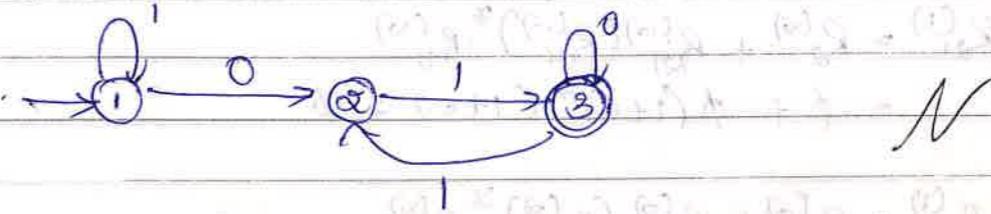
if convert the DFA to a RE.

then the DFA accepts all strings corresponding to RE  $1^* 01 (0+1)^*$ .

However, lets use the fm to identify the expr



Rewritten as:



Bases		$\kappa=1$		Platinum	
		Induction	Induction	Date: 11/12/200	
$R_{11}^{(0)}$	$1+\epsilon$	$R_{11}^{(1)}$	$1^*$	$R_{11}^{(2)}$	$1^*$
$R_{12}^{(0)}$	0	$R_{12}^{(1)}$	$1^*0$	$R_{12}^{(2)}$	$1^*0$
$R_{13}^{(0)}$	$\phi$	$R_{13}^{(1)}$	$\phi$	$R_{13}^{(2)}$	$1^*01$
$R_{21}^{(0)}$	$\phi$	$R_{21}^{(1)}$	$\phi$	$R_{21}^{(2)}$	$\phi$
$R_{22}^{(0)}$	$\epsilon$	$R_{22}^{(1)}$	$\epsilon$	$R_{22}^{(2)}$	$\epsilon$
$R_{23}^{(0)}$	1	$R_{23}^{(1)}$	1	$R_{23}^{(2)}$	1
$R_{31}^{(0)}$	$\phi$	$R_{31}^{(1)}$	$\phi$	$R_{31}^{(2)}$	$\phi$
$R_{32}^{(0)}$	1	$R_{32}^{(1)}$	1	$R_{32}^{(2)}$	1
$R_{33}^{(0)}$	$0+\epsilon$	$R_{33}^{(1)}$	$0+\epsilon$	$R_{33}^{(2)}$	$0+\epsilon+11$

(a)

(b)

(c)

RE's for paths that can go through  
 (a) no state (b) state 1 only (c) states 1 & 2 only.

Inductive part

$$R_{ij}^{(1)} = R_{ij}^{(0)} + R_{ii}^{(0)}(R_{ij}^{(0)})^* R_{ij}^{(0)}$$

$$\therefore R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\ = 0 + (1+\epsilon)(1+\epsilon)^* 0 = 1^*0$$

$$\because (1+\epsilon)^* = 1^*$$

So start state is 1, final state is 3,

no. of states = 3. i.e. The RE is

$$R_{13}^{(2)} = R_{13}^{(1)} + R_{13}^{(1)}(R_{23}^{(1)})^* R_{23}^{(1)}$$

$$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ = (1+\epsilon) + (1+\epsilon)(1+\epsilon)^* (1+\epsilon) \\ = 1^*$$

$$R_{13}^{(1)} = R_{13}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\ = \phi + (1+\epsilon)(1+\epsilon)^* \phi = \phi$$

$$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ = \phi + \phi(1+\epsilon)^* (1+\epsilon) = \phi$$

$$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ = \epsilon + \phi(1+\epsilon)^* (0) = \epsilon$$

$$R_{23}^{(1)} = R_{23}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\ = 1 + \phi(1+\epsilon)^* \phi = 1$$

$$R_{31}^{(1)} = R_{31}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ = \phi + \phi(1+\epsilon)^* (1+\epsilon) = \phi$$

$$R_{32}^{(1)} = R_{32}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\ = 1 + \phi(1+\epsilon)^* 0 = 1$$

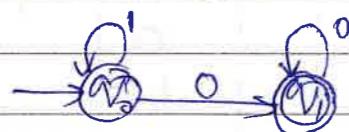
$$R_{33}^{(1)} = R_{33}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\ = (0+\epsilon) + \phi(0+\epsilon)^* \phi = 0+\epsilon \quad \text{Continue for other RE's}$$

$$R_{13}^{(2)} = R_{13}^{(1)} + R_{12}^{(1)}(R_{23}^{(1)})^* R_{23}^{(1)} \\ = \phi + 1^*0(\epsilon)^* 1 = 1^*01$$

$$R_{33}^{(2)} = R_{33}^{(1)} + R_{32}^{(1)}(R_{23}^{(1)})^* R_{23}^{(1)} \\ = (0+\epsilon) + 1(\epsilon)^* 1 \\ = 0+\epsilon+11$$

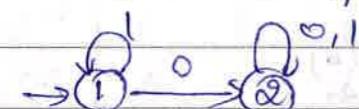
$$\text{Now, } R_{13}^{(3)} = R_{13}^{(2)} + R_{13}^{(2)}(R_{33}^{(2)})^* R_{33}^{(2)} \\ = 1^*01 + 1^*01(0+\epsilon+11)^*(0+\epsilon+11) \\ = \underline{1^*01(0+11)^*}$$

Q1) Obtain a RE for the FA shown below:



What is the lang. corresponding to the RE?

Soln Let  $a_0 = 1, a_1 = 2$



By Kleene's thm,

Basis:  $k=0$

$$R_{11}^{(0)} = \epsilon + 1 \quad R_{12}^{(0)} = 0 \quad R_{21}^{(0)} = \emptyset \quad R_{22}^{(0)} = \epsilon + 0 + 1$$

Induction:

The RE corresponding to the paths from state  $i$  to state  $j$  thro' a state which is not higher than  $k$  is given by:

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kj}^{(k+1)})^* R_{kj}^{(k-1)}$$

When  $k=1$ ,

$$\begin{aligned} R_{11}^{(1)} &= R_{11}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)} \\ &= (\epsilon + 1) + (\epsilon + 1) (\epsilon + 1)^* (\epsilon + 1) \\ &= (\epsilon + 1) + (\epsilon + 1) 1^* (\epsilon + 1) \\ &= (\epsilon + 1) + 1^* \\ &= 1^* \end{aligned}$$

$$\begin{aligned} R_{12}^{(1)} &= R_{12}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} \\ &= 0 + (\epsilon + 1) (\epsilon + 1)^* 0 \\ &= 0 + 1^* 0 = 1^* 0 \end{aligned}$$

$$\begin{aligned} R_{21}^{(1)} &= R_{21}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)} \\ &= \emptyset + \emptyset (\epsilon + 1)^* (\epsilon + 1) = \emptyset \end{aligned}$$

$$\begin{aligned} R_{22}^{(1)} &= R_{22}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} \\ &= (\epsilon + 0 + 1) + \emptyset (\epsilon + 1) 0 \end{aligned}$$

When  $k=2$

$$\begin{aligned} R_{11}^{(2)} &= R_{11}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \\ &= 1^* + 1^* 0 (\epsilon + 0 + 1)^* \emptyset \\ &= 1^* \end{aligned}$$

$$\begin{aligned} R_{12}^{(2)} &= R_{12}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \\ &= 1^* 0 + 1^* 0 (\epsilon + 0 + 1)^* (\epsilon + 0 + 1) \\ &= 1^* 0 + 1^* 0 (0+1)^* (\epsilon + 0 + 1) \\ &= 1^* 0 (0+1)^* \\ &= 1^* 0 (0+1)^* \end{aligned}$$

$$\begin{aligned} R_{21}^{(2)} &= R_{21}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \\ &= \emptyset + (\epsilon + 0 + 1) (\epsilon + 0 + 1)^* \emptyset \\ &= \emptyset \end{aligned}$$

$$\begin{aligned} R_{22}^{(2)} &= R_{22}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \\ &= (\epsilon + 0 + 1) + (\epsilon + 0 + 1) (\epsilon + 0 + 1)^* (\epsilon + 0 + 1) \\ &= (\epsilon + 0 + 1) + (0+1)^* \\ &= (0+1)^* \end{aligned}$$

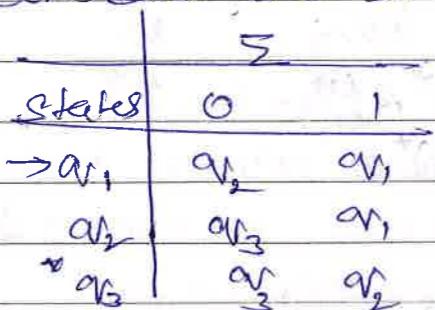
Since the total no of states is 2, the max val of  $k=2$ .

Since the start state is 1 & final state is 2, the RE is given by

$$R_{12}^{(2)} = 1^* 0 (0+1)^*$$

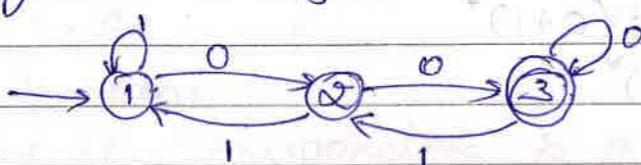
i.e. The RE for DPA is  $1^* 0 (0+1)^*$  which is the lang. consisting of any no of 1's followed by a 0 followed by strings of 0's & 1's.

Consider the DFA shown below:



Observe the RE  $R_{ij}^{(0)}$ ,  $R_{ij}^{(1)}$  and simplify the RE as much as possible.

From By renaming the states of constructing RA,



By applying Kleene's thm:

Base: When  $k=0$ ,

$$\begin{aligned} R_{11}^{(0)} &= 1 + \epsilon & R_{21}^{(0)} &= 1 & R_{31}^{(0)} &= \emptyset \\ R_{12}^{(0)} &= 0 & R_{22}^{(0)} &= \epsilon & R_{32}^{(0)} &= 1 \\ R_{13}^{(0)} &= \emptyset & R_{23}^{(0)} &= 0 & R_{33}^{(0)} &= 0 + \epsilon \end{aligned}$$

Induction

When  $k=1$ ,

$$\begin{aligned} R_{11}^{(1)} &= R_{11}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ &= (1 + \epsilon) + (1 + \epsilon)(1 + \epsilon)^* (1 + \epsilon) \\ &= (1 + \epsilon) + 1^* = 1^* \end{aligned}$$

$$\begin{aligned} R_{12}^{(1)} &= R_{12}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\ &= 0 + (1 + \epsilon)(1 + \epsilon)^* 0 \\ &= 0 + 1^* 0 = 1^* 0 \end{aligned}$$

$$\begin{aligned} R_{13}^{(1)} &= R_{13}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\ &= \emptyset + (1 + \epsilon)(1 + \epsilon)^* \emptyset = \emptyset \end{aligned}$$

$$\begin{aligned} R_{21}^{(1)} &= R_{21}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ &= 1 + 1(1 + \epsilon)^* (1 + \epsilon) \\ &= 1 + 1 \cdot 1^* = 11^* \end{aligned}$$

$$\begin{aligned} R_{22}^{(1)} &= R_{22}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\ &= \emptyset + 1(1 + \epsilon)^* 0 \\ &= \emptyset + 11^* 0 \end{aligned}$$

$$\begin{aligned} R_{23}^{(1)} &= R_{23}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\ &= 0 + 1(1 + \epsilon)^* \emptyset \\ &= 0 \end{aligned}$$

$$\begin{aligned} R_{31}^{(1)} &= R_{31}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\ &= \emptyset + \emptyset (1 + \epsilon)^* \emptyset = \emptyset \end{aligned}$$

$$\begin{aligned} R_{32}^{(1)} &= R_{32}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\ &= 1 + \emptyset (1 + \epsilon)^* = 1 \end{aligned}$$

$$\begin{aligned} R_{33}^{(1)} &= R_{33}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\ &= (0 + \epsilon) + \emptyset (1 + \epsilon)^* \\ &= (0 + \epsilon) \end{aligned}$$

When  $k=2$

$$\begin{aligned} R_{11}^{(2)} &= R_{11}^{(1)} + R_{11}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\ &= 1^* + 1^* 0 \cdot (\emptyset + 11^* 0)^* 11^* \\ &= 1^* + 1^* 0 (11^* 0)^* 11^* \end{aligned}$$

$$\begin{aligned} R_{12}^{(2)} &= R_{12}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\ &= 1^* 0 + 1^* 0 (\emptyset + 11^* 0)^* (\emptyset + 11^* 0) \\ &= 1^* 0 + 1^* 0 (11^* 0)^* \\ &= 1^* 0 (11^* 0)^* \end{aligned}$$

$$\begin{aligned} R_{13}^{(2)} &= R_{13}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\ &= \emptyset + 1^* 0 (\emptyset + 11^* 0)^* 0 \\ &= 1^* 0 (11^* 0)^* 0 \end{aligned}$$

$$\begin{aligned} R_{21}^{(2)} &= R_{21}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)} \\ &= 11^* + (\epsilon + 11^* 0)(\epsilon + 11^* 0)^* 11^* \\ &= N^* F N^* 0^* N^* \\ &= 11^* + (\epsilon + 11^* 0)(11^* 0)^* 11^* \end{aligned}$$

$$\begin{aligned} R_{22}^{(2)} &= R_{22}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\ &= (\epsilon + 11^* 0) + (\epsilon + 11^* 0)(\epsilon + 11^* 0)^* (\epsilon + 11^* 0) \\ &= (\epsilon + 11^* 0) + (\epsilon + 11^* 0)(11^* 0)^* (\epsilon + 11^* 0) \end{aligned}$$

$$\begin{aligned} R_{23}^{(2)} &= R_{23}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\ &= 0 + (\epsilon + 11^* 0)(\epsilon + 11^* 0)^* 0 \\ &= 0 + (\epsilon + 11^* 0)(11^* 0)^* 0 \end{aligned}$$

$$\begin{aligned} R_{31}^{(2)} &= R_{31}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)} \\ &= \emptyset + 1 \cdot (\epsilon + 11^* 0)^* 11^* \\ &= 1(11^* 0)^* 11^* \end{aligned}$$

$$\begin{aligned} R_{32}^{(2)} &= R_{32}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\ &= 1 + 1(\epsilon + 11^* 0)^* (\epsilon + 11^* 0) \\ &= 1 + 1(11^* 0)^* (\epsilon + 11^* 0) \end{aligned}$$

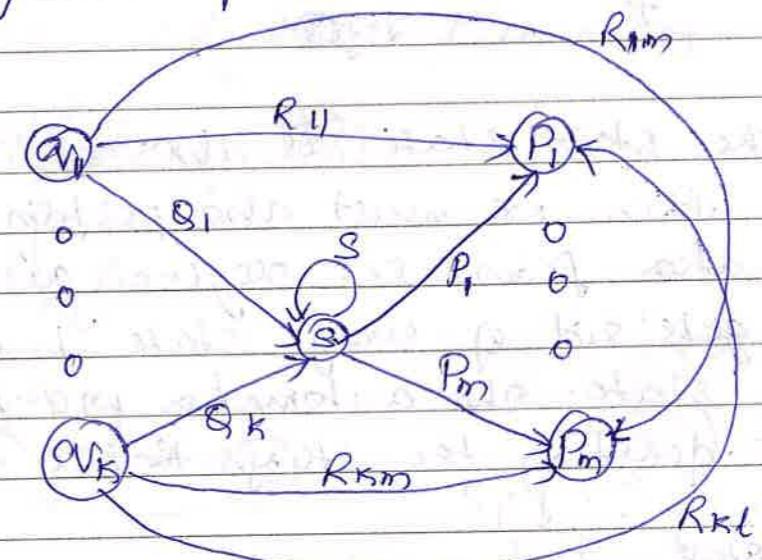
$$\begin{aligned} R_{33}^{(2)} &= R_{33}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\ &= (0 + \epsilon) + 1(\epsilon + 11^* 0)^* 0 \\ &= (0 + \epsilon) + 1(11^* 0)^* 0 \end{aligned}$$

The RE as given by  $R_{13}^{(2)}$  is calculated as

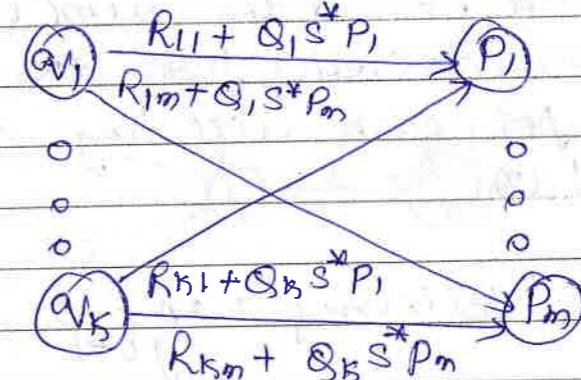
$$\begin{aligned} R_{13}^{(2)} &= R_{13}^{(2)} + R_{13}^{(2)}(R_{33}^{(2)})^* R_{33}^{(2)} \\ &= 1^* 0(11^* 0)^* 0 + 1^* 0(11^* 0)^* 0[(0 + \epsilon) + 1(11^* 0)^* 0] \\ &\quad [(0 + \epsilon) + 1(11^* 0)^* 0] \end{aligned}$$

Converting DFA's to Reg. express by Eliminating States :-

The lang. of the automaton is the union over all paths from the start state to an accepting state of the lang. formed by concatenating the languages of the RE's along that path.



fig① A state  $S$  about to be eliminated



fig② Result of eliminating state  $S$  from fig ①

The strategy for constructing a RE from RA is as follows:

- 1) For each accepting state  $q_f$ , apply the above reduction process to produce an equivalent automaton with RE labels on the arcs. Eliminate all states except  $q_f$  and start state  $q_0$ .

- 2) If  $a_1 \neq a_2$ , then we shall be left with a two-state automaton that looks like fig ③. The RE for the accepted strings can be described in various ways. One is  $(R + S U^* T)^* S U^*$ .



fig ③

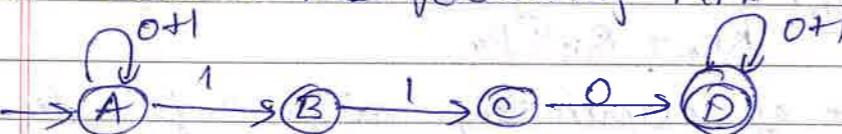
- 3) If the start state is also an accepting state, then we must also perform a state-elimination from the original automaton that gets rid of every state but the start state. The automaton will be like fig ④. The RE denoting the strings that it accepts is  $R^*$ .



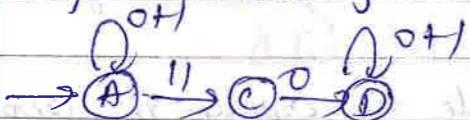
fig ④

- 4) The desired RE is the sum(union) of all the expressions derived from the reduced automata for each accepting state by Rule (2) and (3).

Pbm) Reduce the following NFA.



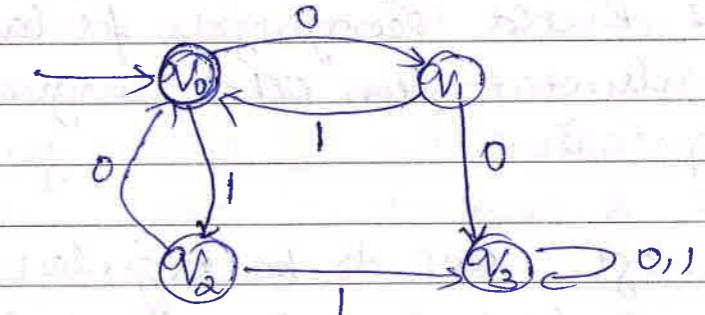
Soln) By eliminating state B,



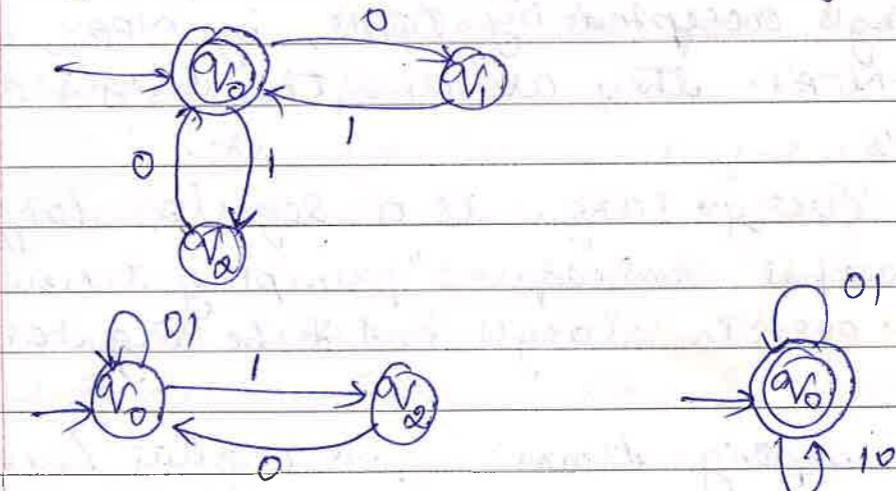
By eliminating C,



- 2) Obtain a RE for the FA shown below:

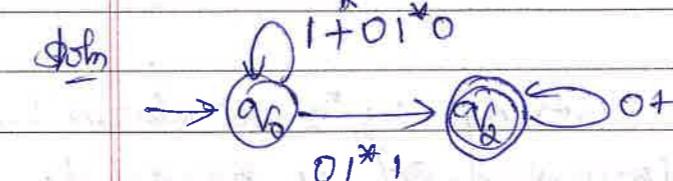
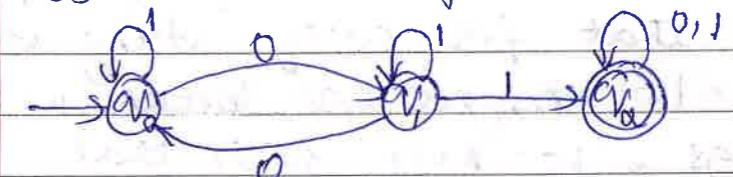


Soln)  $a_3$  is dead state. Eliminating  $a_3$ ,



$\therefore$  The RE is  $(01 + 10)^*$

- 3) Obtain a RE for the FA shown below.



$$\text{By comparing it with fig ③} \\ R = 1 + 01^* 0 \quad T = \emptyset \quad (0+1)^* [ ]^* \cdot 01^* 1 (0+1)^*$$

$$S = 01^* 1 \quad U = 0+1 \quad = (1+01^* 0) + (\emptyset)^* \\ = (1+01^* 0) + 01^* 1 (0+1)^*$$

$$(R + S U^* T)^* S U^* \\ (1+01^* 0) +$$

## PROPERTIES OF REGULAR LANGUAGES

Regular languages exhibit closure property, which let us build recognizers for languages that are constructed from other languages by certain operations.

### Proving Languages not to be Regular

The class of languages known as Reg. lang's has atleast four different descriptions. They are lang's accepted by DFA's, by NFA's, and by E-NFA's. They are also the lang's defined by RE's.

Not every lang. is a regular language. A powerful technique "pumping lemma" shows certain lang's not to be regular.

### The pumping lemma for Regular Languages

#### Theorem:

The pumping principle

Let  $L$  be a regular language. Then there exists a constant  $n$  (which depends on  $L$ ) such that for every string  $w \in L$  such that  $|w| \geq n$ , we can break  $w$  into three strings,  $w = xyz$  such that

- 1)  $y \neq \epsilon$
  - 2)  $|ny| \leq n$
  - 3) For all  $k \geq 0$ , the string  $xy^kz$  is also in  $L$ .
- That is, we can always find a nonempty string  $y$  not too far from the beginning of  $w$  that can be "pumped"; i.e. repeating  $y$  any no. of times or deleting it (the case  $k=0$ ), keeps the resulting string in the lang.  $L$ .

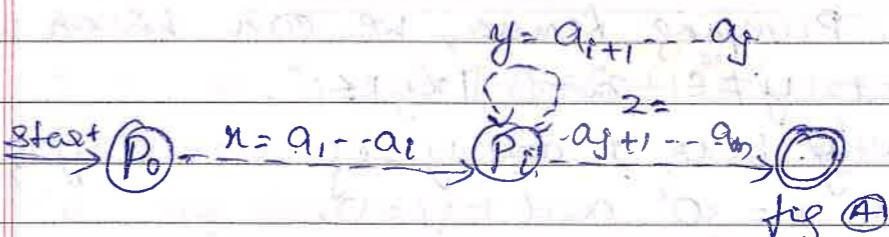
Proof: Suppose  $L$  is regular. Then  $L=L(A)$  for some DFA  $A$ . Suppose  $A$  has  $n$  states. Now, consider any string  $w$  of length  $n$  or more, say  $w=a_1a_2\dots a_m$ , where  $m \geq n$  and each  $a_i$  is an input symbol. For  $i=0, 1, \dots, n$  define state  $P_i$  to be  $\delta(q_0, a_1, a_2, \dots, a_i)$  where  $\delta$  is the ts fn of  $A$ , and  $q_0$  as the start state of  $A$ . That is  $p_i$  is the state  $A$  is in after reading the first  $i$  symbols of  $w$ . Note that  $p_0 = q_0$ .

By the pigeonhole principle, it is not possible for the  $n+1$  different  $p_i$ 's for  $i=0, 1, \dots, n$  to be distinct, since there are only  $n$  different states. Thus, we can find two diff integers  $i$  and  $j$ , with  $0 \leq i < j \leq n$ , such that  $p_i = p_j$ . Now, we can break  $w=xyz$  as follows:

$$1) \quad w = a_1a_2\dots a_n$$

$$2) \quad y = a_{i+1}a_{i+2}\dots a_j$$

$$3) \quad z = a_{j+1}a_{j+2}\dots a_m$$



That is,  $w$  takes us to  $p_i$  once;  $y$  takes us from  $p_i$  back to  $p_0$  (since  $p_i$  is also  $p_j$ ) and  $z$  is the balance of  $w$ . The relationships among the strings & states are suggested by fig A. Note that  $x$  may be empty, in case that  $i=0$ . Also,  $z$  may be empty if  $j=n=m$ . However,  $y$  cannot be empty, since  $i$  is strictly less than  $j$ .

Now, consider that automaton A receives the input  $xy^kz$  for any  $k \geq 0$ . If  $k=0$  then the automaton goes from the start state  $q_0$  (which is also  $p_0$ ) to  $p_1$  on input  $x$ . Since  $p_1$  is also  $p_0$ , it must be that A goes from  $p_1$  to the accepting state shown in fig. on  $\epsilon/p_2$ . Thus A accepts  $xz$ .

If  $k > 0$ , then A goes from  $q_0$  to  $p_1$  on  $\epsilon/p_2$ ,  $x$ , cycles from  $p_1$  to  $p_1$   $k$  times on  $\epsilon/p_2$ , and then goes to the accepting state on  $\epsilon/p_2$ . Thus for any  $k \geq 0$ ,  $xy^kz$  is also accepted by A; ie  $xy^kz$  is in L.

### Applications of pumping lemma:

Sol S.T. the lang. L consisting of all palindromes over  $(0+1)^*$  is not regular.

Soln Suppose L was regular. Then there exists a constant  $n$  satisfying the conditions of pumping lemma.

$$\text{Let } w = 0^n 1 0^n$$

Using Pumping Lemma, we can break  $w = xyz$  so that  $y \neq \epsilon$  and  $|xy| \leq n$ .

Obviously both x and y consist only of 0's.

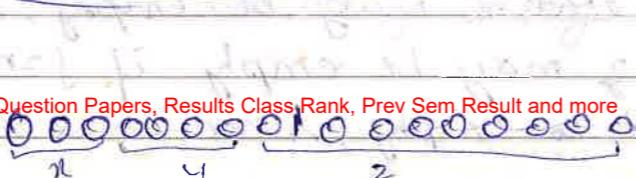
$$\text{Suppose } x = 0^i \text{ and } y = 0^j.$$

Then w.s.t by pumping lemma that  $xy^0z$  is in L.

$$\text{Thus } 0^n 1 0^n \in L.$$

This is not possible.

Hence the lang. is not regular.



3) S.T.  $L = \{a^n b^n \mid n \geq 0\}$  is not regular.

Let L is regular if n be the no of states in A. Consider the string  $w = a^n b^n$

$$|w| = 2n \text{ ie } |w| \geq 0$$

So, we can split w into xyz such that  $|xy| \leq n$  &  $|y| \geq 1$  as shown

$$w = \underbrace{aaaaaa}_{x} \underbrace{aaaaaa}_{y} \underbrace{bbbbbb}_{z}$$

where

$$|x| = n-1 \quad |xy| = |x| + |y|$$

$$|y| = 1 \quad = n-1+1 = n$$

According to pumping lemma,

$$ay^kz \in L \text{ for } k = 0, 1, 2, \dots$$

If  $k=0$ , y does not appear.

$\therefore$  No of a's < no of b's.

III<sup>rd</sup> if  $k=2, 3, \dots$

no. of a's > no. of b's

But accg to PL,  $n^{th}$  of a's should be followed by  $n^{th}$  of b's which is a contradiction to the assumption that the string is regular.  
 $\therefore$  the lang  $L = \{a^n b^n \mid n \geq 0\}$  is not regular

3) S.T.  $L = \{ww^k \mid w \in (0+1)^*\}$  is not regular.

Let L is regular if n is no of states in A. Consider the string  $w = \underbrace{0^n 1 0^n}_{w} \underbrace{0^n 1 0^n}_{w^k}$

$$\underbrace{0^n 1 0^n 0^n 1 0^n}_{w^k}$$

Since  $|w| \geq n$ , string w can be split into xyz such that  $|xy| \leq n$  &  $|y| \geq 1$  as shown

$$\underbrace{0^n 1 0^n}_{x} \underbrace{0^n 1 0^n}_{y} \underbrace{0^n 1 0^n}_{z}$$

where  $|x| = n-1$   $|uv| = n-1+1 = n$  is true.  
 $|y| = 1$

Accg to P.  $xy^kz \in L$  for  $k=0,1,2\dots$

If  $k=0$ ,  $y$  doesn't appear.

So, no of 0's in left of  $w$  will be less than no of 0's on right of  $w$ .

So the string is not of the form  $ww^k$ .

So  $xy^kz \notin L$  when  $k=0$ .

This is a contradiction to the assumption that the lang is regular.

So the given lang. is not regular.

4) S.T  $L = \{a^n \mid n \geq 0\}$  is not regular.

Let  $L$  is regular &  $n$  be no of states in FA.

Let  $w = a^n$ . It is clear that  $|w| \geq n$ .

So, split  $w$  into  $xyz$  such that

$$|xy| \leq n \quad |y| \geq 1$$

$$w = a^n =$$

$$\overbrace{a^i}^u \underbrace{a^k}_{v} \overbrace{a^{n-j-k}}_w \quad \text{where } |u|=j \\ |v|=k$$

$$|uv| = j+k \leq n$$

Accg to P.  $xy^kz \in L$  for  $k=0,1,2\dots$

i.e.  $a^i(a^k)^p a^{n-j-k} \in L$

If we choose  $i=0$ , it means that

$a^s a^{n-j-k} \in L$  i.e.  $xz \in L$

$\Rightarrow a^{n-k} \in L$

$$n! > n! - k$$

$$\text{When } k=1, \quad n! > n! - 1$$

But accg to pm  $\log n! = n! - 1$  which is false

& is a contradiction, so,  $L$  cannot be regular.

So the lang  $L = \{a^n \mid n \geq 0\}$  is not regular.

Equivalence & Minimization of Automata

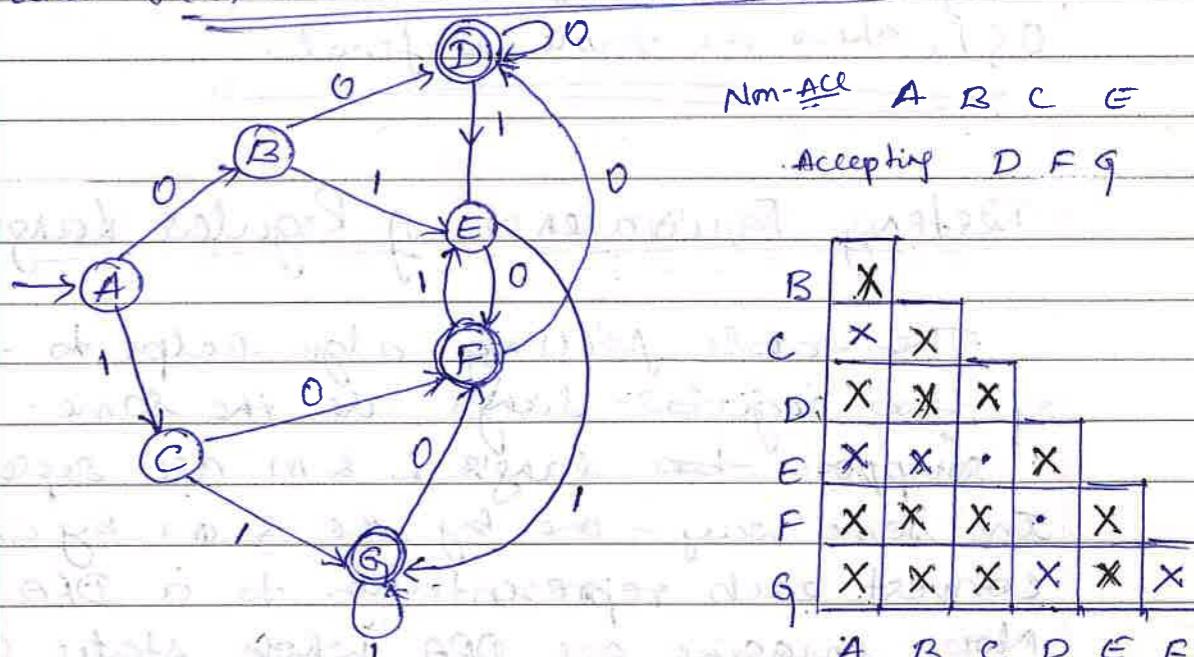
Testing Equivalence of states :-

Basis :-

If  $p$  is an accepting state &  $q$  is non accepting, Then the pair  $\{p, q\}$  is distinguishable.

Induction :-

Let  $p \neq q$  be states such that for some input symbol  $a$ ,  $r = \delta(p, a)$  &  $s = \delta(q, a)$  are a pair of states known to be distinguishable. Then  $\{p, q\}$  is a pair of distinguishable states. The reason this rule makes sense is that there must be some string  $w$  that distinguishes  $r$  from  $s$ ; i.e. exactly one of  $\delta(r, w)$  and  $\delta(s, w)$  is accepting. Then string  $aw$  must distinguish  $p$  from  $q$ , since  $\delta(p, aw)$  and  $\delta(q, aw)$  is the same pair of states as  $\delta(r, w)$  and  $\delta(s, w)$ .



for ①

Table of state equivalence

An Automaton with  
Equivalent states  $\{B, E\}$

Ref pg 93  
600

Initially the entire table is blank. Now, since states D, F and G are the only accepting states, they cannot be equivalent to A, B, C and E. Accordingly, we fill the corresponding squares with X. Now, we can use these distinguishable pairs to find others. For instance, since {B, E} is distinguishable & states B and E go to E and G on input 1, we find that {B, E} is also distinguishable. In a similar manner each of the blank squares is examined and distinguishable pairs identified whenever we find that the transition from a pair of states either on 0 or 1 goes to distinguishable states.

After one pass we find there are no more distinguishable pairs. The two remaining pairs {C, E} and {D, F} are equivalent since they have identical transitions on both 0 & 1. Thus the table is final.

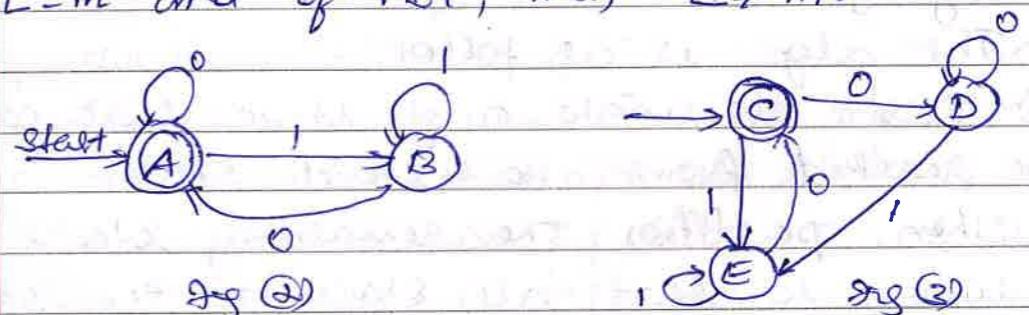
### Testing Equivalence of Regular Languages

The table filling algo. helps to test if two regular langs are the same.

Suppose two langs L & M are represented in some way - one by R.E & one by an NFA. Convert each representation to a DFA.

Now, imagine one DFA whose states are the union of the states of the DFAs for L and M. Technically, this DFA has two start states, but it is irrelevant as far as

testing state equivalence is concerned, so make any state the lone start state. Now, test if the start states of the two original DFAs are equivalent, using the table filling algo. If they are equivalent, then  $L = M$  and if not, then  $L \neq M$ .



Two equivalent DFA's

The lang. of RE is  $L + (0+1)^* 0$   
For fig (1) the partition of states into blocks is {A}, {B}, {C, E}, {D}, {F}, {G}

### Minimization of DFA's

It means for each DFA, we can find an equivalent DFA that has as few states as any DFA accepting the same language. The minimum-state DFA is unique for the language.

The algo is as follows:

- 1) First, eliminate any state that cannot be reached from the start state.
- 2) Then, partition the remaining states into blocks, so that all states in the same block are equivalent, and no pair of states from diff. blocks are equivalent.

Thm

The equivalence of states is transitive. i.e if in some DFA A, states  $p \& q$  are equivalent & also  $q$  and  $r$  are egr, then it must be that  $p$  and  $r$  are equivalent.

Thm 4.24

If we create for each state  $q$  of a DFA a block consisting of  $q$  and all the states equivalent to  $q$ , then the different blocks of states form a partition of the set of states. i.e each state is in exactly one block.

All members of a block are equivalent and no pair of states chosen from diff. blocks are equivalent.

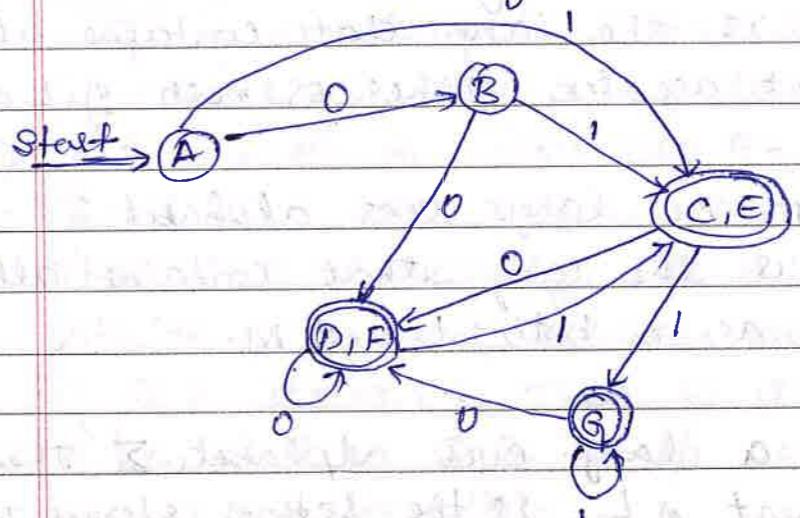
We state the algo. for minimizing a DFA  $A = (Q, \Sigma, \delta, q_0, F)$

1) Use the Table-filling algo. to find all the pairs of equivalent states.

2) Partition the set of states  $Q$  into blocks of mutually equivalent states, by the method described above.

3) Construct the minimum-state equivalent DFA  $B$  by using the blocks as its states. Let  $\delta'$  be the tr. fn. of  $B$ . Suppose  $S$  is a set of equivalent states of  $A$ , and  $a$  is an ip symbol. Then there must exist one block  $T$  of states such that for all states  $q_i$  in  $S$ ,  $\delta(q_i, a)$  is a member of block  $T$ . For if not, then ip symbol  $a$  takes two states  $P$  and  $q_i$  of  $S$  to states in diff. blocks, and those states are distinguishable by Thm 4.24. That fact lets us conclude that  $P \& q_i$  are not equivalent and they did not both belong in  $S$ . As a consequence, we can let  $\delta'(S, a) = T$ . In addition,

- (a) The start state of  $B$  is the block containing the start state of  $A$ .
- (b) The set of accepting states of  $B$  is the set of blocks containing accepting states of  $A$ . Note that if one state of a block is accepting, then all the states of that block must be accepting.



Closure Properties of Regular Languages

Closure properties express the idea that when one (or several) lang's are regular, then certain related lang's are also regular.

Summary of the principal closure properties for regular languages:

- 1) The union of two regular languages is regular
- 2) The intersection
- 3) The complement of a
- 4) The difference of two
- 5) The reversal of a
- 6) The closure (star) of a
- 7) The concatenation of
- 8) A homomorphism (substitution of strings for symbols) of a reg. lang. is regular.
- 9) The inverse homomorphism of a reg. lang. is regular.

Closure of Regular Lang's under Boolean Operations:

The 3 boolean op are union, intersection & complementation.

- 1) Let L and M be languages over alphabet  $\Sigma$ . Then  $L \cup M$  is the lang. that contains all strings that are in either or both of L and M.
- 2) Let L and M be lang's over alphabet  $\Sigma$ . Then  $L \cap M$  is the lang. that contains all strings that are in both L and M.
- 3) Let L be a lang. over alphabet  $\Sigma$ . Then  $\bar{L}$ , the complement of L, is the set of strings in  $\Sigma^*$  that are not in L.

The reg. lang's are closed under all three of the boolean operations.

Closure under Union:-

Ques:

If L and M are reg. lang's then so is  $L \cup M$ .

Proof:

Since L and M are regular, they have reg. exps, say  $L = L(R)$ , and  $M = L(S)$ . Then  $L \cup M = L(R+S)$  by the defn. of the + operator for reg. exps.

Closure under Complementation

Starting with a RE, we could find a RE for its complement as follows:

- 1) Convert the RE to an E-NFA.
- 2) Convert that E-NFA to a DFA by the subset construction.
- 3) Complement the accepting states of that DFA.
- 4) Given the complement DFA back into a RE using the construction technique.

Ques:

If L is a reg. lang. over alphabet  $\Sigma$ , then  $\bar{L} = \Sigma^* - L$  is also a reg. lang.

- Proof: Let  $L = L(A)$  for some DFA  $A = (\mathcal{Q}, \Sigma, \delta, q_0, F)$ . Then  $\bar{L} = L(B)$ , where B is the DFA  $(\mathcal{Q}, \Sigma, \delta, q_0, Q-F)$ . That is, B is exactly like A, but the accepting states of A have become non-accepting states of B & vice versa. Then w is in  $L(B)$  if  $(q_0, w) \in Q-F$ , which occurs if & only if w is not in  $L(A)$ .

Closure under Intersection :-

The intersection of lang L & m is given by,  
 $L \cap m = \overline{L \cup \overline{m}}$  - (De-morgan's)

In general, the intersection of two sets is the set of elements that are not in the complement of either set.

The other De-Morgan's Law,

$$L \cup m = \overline{L \cap \overline{m}}$$

Refer Thm:

Closure under Difference :-

In terms of languages,  $L - m$ , the difference of L and m is the set of strings that are in lang. L but not in m.

Thm: If L & m are reg. langs then so is  $L - m$ .

Proof: Observe that  $L - m = L \cap \overline{m}$ . By thm 4.5,

$\overline{m}$  is regular & by thm 4.8  $L \cap \overline{m}$  is reg.

Therefore  $L - m$  is regular.

Reversal

The reversal of a string  $a_1 a_2 \dots a_n$  is the string written backwards i.e.  $a_n a_{n-1} \dots a_1$ . We use  $w^R$  for the reversal of string w.

$$e^R = e$$

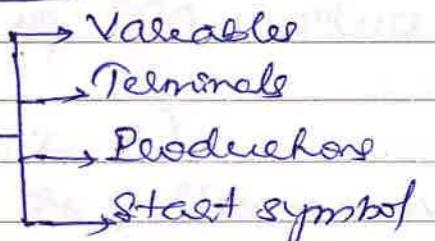
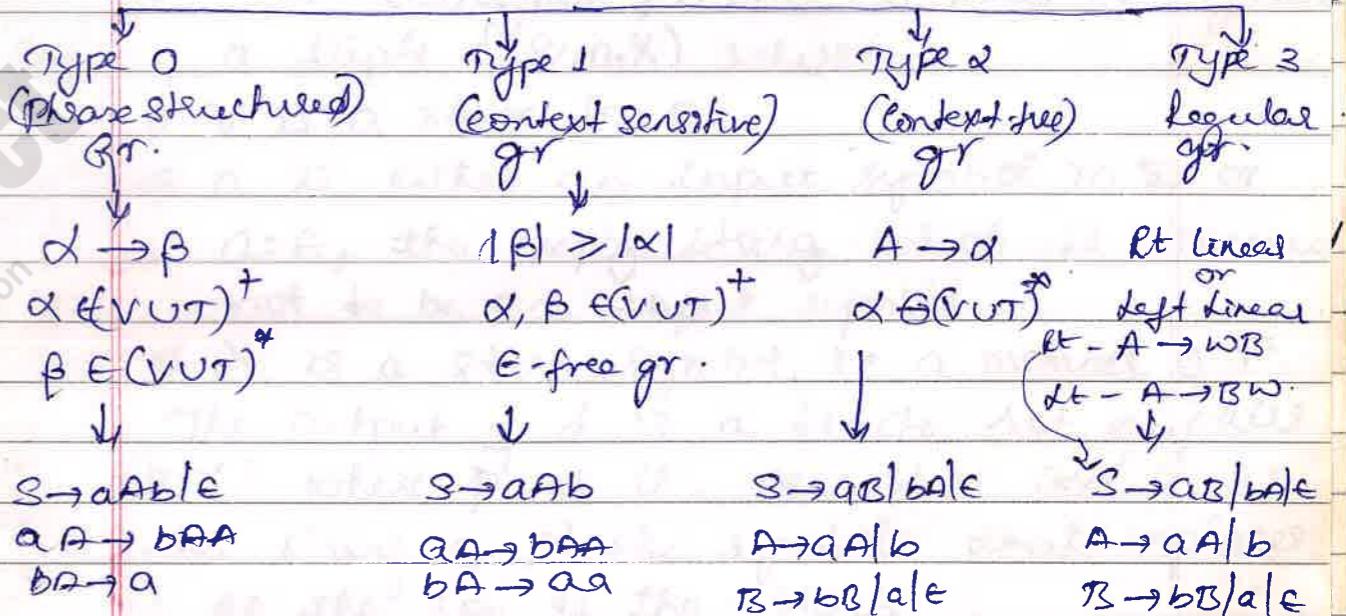
The reversal of a lang L, written  $L^R$  is the lang. consisting of the reversals of all its strings.

Ex: If  $L = \{001, 10, 111\}$  then

$$L^R = \{100, 01, 111\}$$

CONTEXT-FREE GRAMMARS

Components of Grammars

Chomsky Hierarchy :-Types of grammar

$$G = (V, T, P, S)$$

The 4 imp components of grammar are :

1. Non-terminals or syntactic categories : A finite set of variables. Each var represents a language.
2. Terminals or Terminal symbols : A finite set of symbols that form the strings of the language being defined.
3. Start symbol : Represents the language being defined.
4. Productions : A finite set of production or rules represent the recursive definition of a language. Each production consists of :

## PUSHDOWN AUTOMATION

The formal defn. of PDA involves seven components.

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

The components have the following meanings:

$Q$ : A finite set of states.

$\Sigma$ : A finite set of input symbols.

$\Gamma$ : A finite stack alphabet. It is the set of symbols allowed to push on to the stack.

$\delta$ : The transition function.  $\delta$  takes as argument a triple  $\delta(q, a, X)$  where:

q)  $q$  is a state in  $Q$ .

as  $a$  is either an input symbol in  $\Sigma$  or  $a = \epsilon$ , the empty string which is assumed not to be an input symbol.

3)  $X$  is a stack symbol, i.e. a member of  $\Gamma$ .

The output of  $\delta$  is a finite set of pairs  $(p, r)$  where  $p$  is the new state and  $r$  is the string of stack symbols that replaces  $X$  at the top of the stack.

For instance, if  $\delta = \epsilon$  then the stack is popped.

If  $\delta = X$  then stack is intercal unchanged.

if  $\delta = YZ$ , then  $X$  is replaced by  $Z$  and  $Y$  is pushed on to the stack.

$q_0$ : The start state. The PDA is in this state before making any transition.

$z_0$ : The stack start symbol. Initially, the PDA's stack consists of one instance of the symbol and nothing else.

$F$ : The set of accepting states or final states.

Q Design a PDA to accept the language  
 $L = \{a^i b^k c^j \mid i+j=k\}$

$$\delta(\alpha_0, a, z_0) = (\alpha_0, a z_0)$$

$$\delta(\alpha_0, a, a) = (\alpha_1, aa)$$

$$\delta(\alpha_0, b, a) = (\alpha_1, \epsilon)$$

$$\delta(\alpha_1, b, a) = (\alpha_1, \epsilon)$$

$$\delta(\alpha_1, b, b) = (\alpha_1, bb)$$

$$\delta(\alpha_1, b, z_0) = (\alpha_1, bz_0)$$

$$\delta(\alpha_1, c, b) = (\alpha_1, \epsilon)$$

$$\delta(\alpha_1, \epsilon, z_0) = (\alpha_2, z_0)$$

## TURING MACHINES

Design a TM to accept the language consisting of all palindromes of 0's and 1's.

BabaB

B X b a B

State	a	b	x	y	B
$\alpha_0$	( $\alpha_1, X, R$ )	( $\alpha_1, Y, R$ )	( $\alpha_6, X, R$ )	( $\alpha_6, Y, R$ )	( $\alpha_6, B, R$ )
$\alpha_1$	( $\alpha_1, a, R$ )	( $\alpha_1, b, R$ )	( $\alpha_2, X, L$ )	( $\alpha_2, Y, L$ )	( $\alpha_2, B, L$ )
$\alpha_2$	( $\alpha_2, a, R$ )	( $\alpha_2, b, R$ )	( $\alpha_4, X, L$ )	( $\alpha_4, Y, L$ )	( $\alpha_4, B, L$ )
$\alpha_3$	( $\alpha_5, X, L$ )	-	( $\alpha_6, X, R$ )	( $\alpha_6, Y, R$ )	-
$\alpha_4$	-	( $\alpha_5, Y, L$ )	( $\alpha_6, X, R$ )	( $\alpha_6, Y, R$ )	-
$\alpha_5$	( $\alpha_5, a, L$ )	( $\alpha_5, b, L$ )	( $\alpha_0, X, R$ )	( $\alpha_0, Y, R$ )	-

PDA

- \* Define PDA and construct a PDA that accepts the following language.  
 $L = \{ w : w \in (a+b)^* \text{ and } n_a(w) = n_b(w) \}$ . Write the instantaneous description for the string  $aabbab$ . (Jan-09) 12 Marks

The PDA is

$$Q = \{ q_0, q_f \}$$

$$\Sigma = \{ a, b \}$$

$$\Gamma = \{ a, b, z_0 \}$$

$\delta$ :

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, b, z_0) = (q_0, bz_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, b) = (q_0, bb)$$

$$\delta(q_0, a, b) = (q_f, \epsilon)$$

$$\delta(q_0, b, a) = (q_f, \epsilon)$$

$$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$$

$aabbab$



- \* Discuss the languages accepted by a PDA. Design a PDA for the language that accepts the strings with  $n_a(w) < n_b(w)$  where  $w \in (a+b)^*$  & show the ID8 of the PDA on input  $aabbab$ . (AP) marks: July-09

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, b, z_0) = (q_0, bz_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, b) = (q_0, bb)$$

$$\delta(q_0, a, b) = (q_0, \epsilon)$$

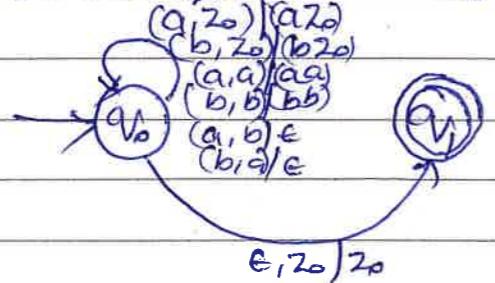
$$\delta(q_0, b, a) = (q_0, \epsilon)$$

$$\delta(q_0, \epsilon, b) = (q_f, \epsilon) - \text{final state}$$

$q_0 \in Q$  is the start state

$z_0 \in \Gamma$  - initial symbol on stack

$F = \{ q_f \}$  is the final state.



I.D

$(q_0, aabbab, z_0) \xrightarrow{\quad} (q_0, ababb, az_0)$   
 $\xrightarrow{\quad} (q_0, babb, aaaz_0)$   
 $\xrightarrow{\quad} (q_0, abb, aaz_0)$   
 $\xrightarrow{\quad} (q_0, bb, aaaz_0)$   
 $\xrightarrow{\quad} (q_0, b, aaz_0)$   
 $\xrightarrow{\quad} (q_0, \epsilon, z_0)$   
 $\xrightarrow{\quad} (q_f, \epsilon, z_0)$

- ④ Obtain a PDA to accept the language  
 $L = \{a^n b^n \mid n > 0\}$  by a final state.

Give the graphical representation for PDA obtained. Show the moves made by the PDA for the string aaabbb. (10) M J T Reo 06

Soln The PDA is

$$Q = \{S_0\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{Z_0\}$$

$$\delta: \begin{aligned} \delta(S_0, \epsilon, Z_0) &= (S_0, Z_0) \\ \delta(S_0, a, Z_0) &= (S_1, aZ_0) \\ \delta(S_1, a, a) &= (S_1, aa) \\ \delta(S_1, a, b) &= (S_2, \epsilon) \end{aligned}$$

$$\delta(S_2, b, a) = (S_2, \epsilon)$$

$\delta(S_2, \epsilon, Z_0) = (S_2, Z_0)$  - Accepted by final state

$\delta(S_2, \epsilon, \epsilon) = (S_2, \epsilon)$  - Accepted by empty stack.

$$(S_0, \epsilon) \xrightarrow{\delta} (S_0, Z_0)$$

$$(S_0, a) \xrightarrow{\delta} (S_1, aZ_0)$$

$$(S_1, a) \xrightarrow{\delta} (S_1, aa)$$

$$(S_1, a) \xrightarrow{\delta} (S_2, \epsilon)$$



## Equivalence of PDA's and CFG's From Grammars to PDA

Let  $G = (V, T, Q, S)$  be a CFG. Construct the PDA  $P$  that accepts  $L(G)$  by empty stack as follows:

$$P = (\{S\}, T, V \cup T, \delta, S)$$

where transition function  $\delta$  is defined by:

i) For each variable  $A$ ,

$$\delta(A, \epsilon, A) = \{(\alpha, \beta) \mid A \Rightarrow \beta \text{ is a prodn of } P\}$$

as for each terminal  $a$ ,

$$\delta(A, a, a) = \{(\alpha, \beta)\}$$

Ex Convert the given grammar to PDA

$$I \rightarrow a/b/ia \mid a/b/2a/2b$$

$$E \rightarrow 2/E+E \mid E+E \mid (E)$$

The set of terminals for the PDA are

$$\{a, b, 2, +, *, (\, , )\}$$

The transition function for the PDA is

a)  $\delta(V, \epsilon, I) = \{(V, a), (V, b), (V, ia), (V, ib), (V, 2a), (V, 2b)\}$

b)  $\delta(V, \epsilon, E) = \{(V, 2), (V, E+E), (V, E+E), (V, (E))\}$

c)  $\delta(V, a, a) = \{(V, a)\}$        $\delta(V, (, )) = \{(V, \epsilon)\}$

$\delta(V, b, b) = \{(V, \epsilon)\}$        $\delta(V, +, +) = \{(V, \epsilon)\}$

$\delta(V, 2, 2) = \{(V, \epsilon)\}$        $\delta(V, +, +) = \{(V, \epsilon)\}$

$\delta(V, *, *) = \{(V, \epsilon)\}$        $\delta(V, (, )) = \{(V, \epsilon)\}$

Note (a) & (c) come from rule(1) & g-transitions come from rule(2). Also,  $\delta$  is empty except as defined by (a) through (c).

- (\*) Design a TM to accept all set of palindromes over  $\{0, 1\}^*$ . Also write its transition diagram and ID on the string 10101. (14) Marks

	0	1	X	Y	Z
$q_0$	$(q_1, X, R)$	$(q_2, Y, R)$	$(q_7, X, R)$	$(q_7, Y, R)$	
$q_1$	$(q_1, 0, R)$	$(q_1, 1, R)$	$(q_2, X, L)$	$(q_3, Y, L)$	$(q_3, B, L)$
$q_2$	$(q_2, 0, R)$	$(q_2, 1, R)$	$(q_4, X, L)$	$(q_4, Y, L)$	$(q_4, B, L)$
$q_3$	$(q_5, X, L)$	-	$(q_7, X, R)$	-	
$q_4$	-	$(q_6, Y, L)$	-	$(q_7, X, R)$	
$q_5$	$(q_5, 0, L)$	$(q_5, 1, L)$	$(q_6, X, L)$	$(q_6, Y, L)$	
$q_6$	$(q_6, 0, L)$	$(q_6, 1, L)$	$(q_6, X, R)$	$(q_6, Y, R)$	
$q_7$			$(q_7, X, R)$	$(q_7, Y, R)$	$(q_8, B, R)$
$q_8$					

- (\*) Obtain a TM to accept the language  $L = \{wwR \mid w \in (a+b)^*\}$

$\Rightarrow$  If it is a palindrome of even length.

f	a	b	R
$q_0$	-	-	$q_1, B, R$
$q_1$	$q_2, B, R$	$q_5, D, R$	$q_7, B, R$
$q_2$	$q_1, a, R$	$q_4, b, R$	$q_3, B, L$
$q_3$	$q_4, B, L$	-	$q_7, B, R$
$q_4$	$q_4, a, L$	$q_5, b, L$	$q_1, B, R$
$q_5$	$q_5, a, R$	$q_5, b, R$	$q_6, B, L$
$q_6$	-	$q_4, B, L$	$q_5, B, R$
$q_7$	-	-	-

palindrome  
extra  
entries

- (\*) Design a TM to accept the language  $L = \{a^n b^n c^n \mid n \geq 1\}$ , Give the graphical representation for the TM obtained.

	a	b	c	x	y	z	B
$q_0$	$q_1, x, R$						
$q_1$							
$q_2$							
$q_3$							
$q_4$							
$q_5$							
$q_6$							

Design a TM for the language to accept the set of strings with equal no of 0's and 1's and also give the ID for the input 110100.

Minimize the following DFA using Table-filling algorithm where A is the start state. The states C, F and I are final states.

	0	1	A	B	C	D	E	F	G	H
A	B	E		B						
B	C	F		X						
C*	D	H			D					
D	E	H			E					
E	F	I			*F					
F*	G	B			G					
G	H	B			H					
H	I	C			I					
I*	A	E			A	B	C	D	E	F

N.A: A B D E G  
A: C F I

Initial Horizontal marking:-

C, F, I are final states, The pairs (C, A), (F, A), (F, B), (E, D), etc have one final state & one non-final state. So mark them with X.

Initial vertical marking:

The pairs (C, E), (C, G)(C H) etc are distinguishable so mark them with X in the table.

B	X									
C*	X	X								
D		X	X							
E	X		X	X						
F*	X	X		X	X					
G		X	X		X	X				
H	X		X	X		X	X			
I*	X	X		X	X		X	X		
A										
B										
C*										
D										
E										
F										
G										
H										

Got from next slide in next page.

$\delta$	$\alpha$	$\beta$	
(A, B)	(B, C)	(E, F)	(B, C) is marked - so mark (A, B)
(A, D)	(B, E)	(E, H)	
(A, E)	(B, F)	(E, I)	(D, F) is .. .. (A, E)
(A, G)	(B, H)	(E, B)	
(A, H)	(B, I)	(E, C)	(B, I) - (A, H)
(B, D)	(C, E)	(F, H)	(C, E) - (B, D)
(B, E)	(C, F)	(F, I)	
(B, G)	(C, H)	(F, B)	(B, F) - (B, G)
(B, H)	(C, I)	(F, C)	
(C, F)	(D, G)	(H, B)	
(C, I)	(D, A)	(H, E)	
(D, E)	(E, F)	(H, I)	(E, F) - (D, E)
(D, G)	(E, H)	(H, B)	
(D, H)	(E, I)	(H, C)	(D, H) - (C, H)
(E, G)	(F, H)	(I, B)	(B, I) - (E, G)
(E, H)	(F, I)	(I, C)	
(F, I)	(G, A)	(B, E)	
(G, H)	(H, I)	(B, C)	(G, H) - (B, C) - (B, C)

$\delta$	$\alpha$	$\beta$	
(A, D)	(B, E)	(E, H)	
(A, G)	(B, H)	(E, B)	
(B, E)	(C, F)	(F, I)	
(B, H)	(C, I)	(F, C)	
(C, F)	(D, G)	(H, B)	No markings
(C, I)	(D, A)	(H, E)	(A, D) is marked - so mark (C, I)
(D, G)	(E, H)	(H, B)	
(E, H)	(F, I)	(I, C)	
(F, I)	(G, A)	(B, E)	

Table

4

(A, D) is marked - so mark (C, I)

## Minimizing DFA:

1) Find the distinguishable &amp; indistinguishable pairs.

~~(A, B), (A, G), (B, E), (B, H), (F, G), (E, H), (F, I)~~

indistinguishable:

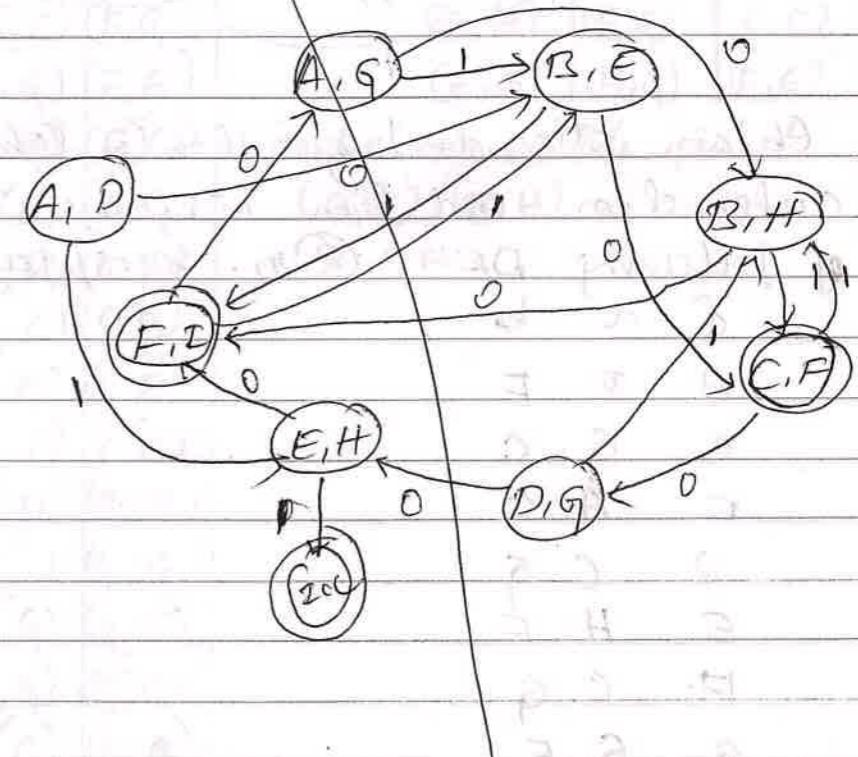
~~(A, D), (A, G), (B, E), (B, H), (C, F), (D, G), (E, H), (F, I), (C, I)~~

distinguishable:

Compute the transition table:

$\delta'$	(A, D)	(A, G)	(B, E)	(B, H)	(C, F)

Make non-final 1 final as groups



Distinguishable pairs:

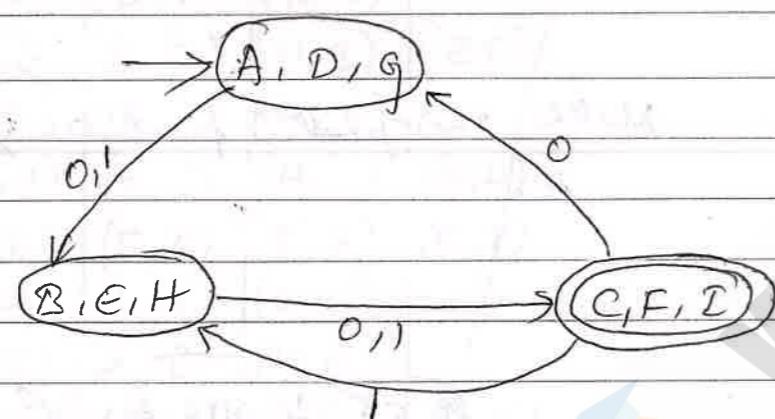
$$(A,D), (A,G), (D,G) = (A,D,G)$$

$$(B,E), (B,H), (E,H) = (B,E,H)$$

$$(C,F), (C,I), (F,I) = (C,F,I)$$

Distinguishable state: does not exist.

	0	1
0	(A,D,G)	(B,E,H)
1	(B,E,H)	(C,F,I)
*	(C,F,I)	(A,D,H)



Ques 2 Obtain the distinguishable table for the automaton and then minimize the states of following DFA. (M. Decos/Janoz [July 07])

f a b

A B F

B G C

C A C

D C G

E H F

F C G

G G E

H G C

Obtain the distinguishable table for the automaton and then minimize the states of following DFA.

f	a	b
→ A	B	F
B	G	C
*	A	C
C	C	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

	B	C	D	E	F	G
B	0	x	x	x	x	x
C	x	x	x	x	x	x
D	x	x	x	x	x	x
E	x	x	x	0	x	x
F	x	x	x	x	x	x
G	x	x	x	x	0	x
H	x	x	x	x	x	0

	a	b
(A, B)	(B, G)	(F, C)
(A, D)	(D, C)	(E, G)
(A, E)	(E, H)	(F, P)
(A, F)	(B, C)	(F, G)
(A, G)	(B, G)	(F, E)
(A, H)	(B, G)	(F, C)
(B, D)	(G, C)	(C, G)
(B, E)	(G, H)	(C, F)
(B, F)	(G, C)	(C, G)
(B, G)	(G, G)	(C, E)
(B, H)	(G, G)	(C, C)
(D, E)	(C, H)	(G, F)
(D, P)	(C, C)	(G, G)
(D, G)	(C, G)	(G, E)
(D, H)	(C, G)	(G, C)
(E, F)	(H, C)	(F, G)
(E, G)	(H, G)	(F, E)
(E, H)	(H, G)	(F, C)
(F, G)	(C, G)	(G, E)
(F, H)	(C, G)	(G, C)

	a	b
(A, E)	(B, H)	(F, F)
(A, G)	(B, G)	(F, E)
(B, E)	(G, H)	(C, F)
(B, H)	(G, G)	(C, C)
(E, G)	(H, G)	(F, E)

Distinguishable pairs  
(A, E), (B, H)

	a	b
(A, B)	(B, G)	(F, C)
(A, D)	(D, C)	(E, G)
(A, E)	(E, H)	(F, P)
(A, F)	(B, C)	(F, G)
(A, G)	(B, G)	(F, E)
(A, H)	(B, G)	(F, C)
(B, D)	(G, C)	(C, G)
(B, E)	(G, H)	(C, F)
(B, F)	(G, C)	(C, G)
(B, G)	(G, G)	(C, E)
(B, H)	(G, G)	(C, C)
(D, E)	(C, H)	(G, F)
(D, P)	(C, C)	(G, G)
(D, G)	(C, G)	(G, E)
(D, H)	(C, G)	(G, C)
(E, F)	(H, C)	(F, G)
(E, G)	(H, G)	(F, E)
(E, H)	(H, G)	(F, C)
(F, G)	(C, G)	(G, E)
(F, H)	(C, G)	(G, C)

Minimize the following DFA using Table filtering algo.

	0	1	
$\rightarrow A$	B	A	B
B	A	C	C
C	D	B	D
D	D	A	x
E	D	F	x
F	G	E	x
G	F	G	A
			B
			C
			D
			E
			F

(A,B)

(A,C)

(A,E)

(A,F)

(E,G)

CFG'8

Simplification of CFG'8

Elimination of useless symbols :-

Algorithm

$$OV = \emptyset$$

$$nv = OV \cup \{ A/A \rightarrow y \text{ and } y \in (OV \cup T)^* \}$$

while (OV != nv)

$$\{ \quad OV = nv;$$

$$\{ \quad nv = OV \cup \{ A/A \rightarrow y \text{ and } y \in (OV \cup T)^* \}$$

$$Y_1 = OV$$

Done

i) Eliminate the useless symbols from the grammar

$$S \rightarrow aA/bB$$

$$A \rightarrow aA/a$$

$$B \rightarrow bB$$

$$D \rightarrow ab/Ea$$

$$E \rightarrow ac/d.$$

OV	nv	P	grammar
$\emptyset$	A,D,E	$A \rightarrow a$ $D \rightarrow ab$ $E \rightarrow d$	$S \rightarrow aa$ $A \rightarrow a/aA$ $D \rightarrow Ea/ab$ $E \rightarrow d$ .
A,D,E	A,D,E,S	$S \rightarrow aa$ $A \rightarrow aa$ $D \rightarrow Ea$	
A,D,E,S	A,D,E,S	-	

The symbols D S E are not reachable from start state.  $\therefore$  The resulting grammar is

$$\begin{cases} S \rightarrow aa \\ A \rightarrow aa/a \end{cases}$$

Q2) Eliminate useless prodns from the following grammar

$$A \rightarrow bA \mid Bba \mid aa$$

$$B \rightarrow aBa \mid b \mid D$$

$$C \rightarrow CA \mid AC \mid B$$

$$D \rightarrow a \mid C$$

Soln

OV	NV	P
$\emptyset$	$A, B, D$	$A \rightarrow aa$
		$B \rightarrow b$
		$D \rightarrow a$
$A, B, D$	$A, B, D, C$	$A \rightarrow bA$
		$A \rightarrow Bba$
		$B \rightarrow aBa$
		$D \rightarrow D$
		$C \rightarrow B$
$A, B, D, C$	$A, B, D, C$	$C \rightarrow CA$
		$C \rightarrow AC$

The gr obtained is

$$A \rightarrow aa \mid bA \mid Bba$$

$$B \rightarrow b \mid aBa \mid D$$

$$C \rightarrow CA \mid AC \mid B$$

$$D \rightarrow a$$

Step 2 The symbols  $c \cancel{\rightarrow}$  are not reachable from the start state.

i. The resultant grammar is

$$A \rightarrow Bba \mid bA \mid aa$$

$$B \rightarrow b \mid aBa \mid D$$

$$D \rightarrow a$$

### Elimination of Unit productions

Any prodn in Q of the form  $A \rightarrow B$  where  $A, B \in V$  is a unit prodn.

It is undesirable as one var simply replaces other var.

A unit prodn can be replaced by

① Removing all prodns of the form  $A \rightarrow A$

② Add all non-unit prodns to P,

③ For each var A find all var B such that

$$A \not\Rightarrow B$$

4) Obtain the dependency graph

$$\text{ex: } A \rightarrow B$$

$$B \rightarrow C$$

$$C \rightarrow B$$

the graph will be



5) Note from graph

a.  $A \not\Rightarrow B$ , B can be obtained from A.

So all non-unit prodns generated from B can also be generated from A.

Similarly for other variables.

6) Finally, the unit prodns. can be deleted from the grammar Q.

7) The resulting gr generates the same language as accepted by Q.

Soln Eliminate Unit prodns from the gr.  $\Rightarrow$

i)  $S \rightarrow AB$   $\Rightarrow$  Le unit prodns

$$A \rightarrow a$$

$$S \rightarrow AB$$

$$B \rightarrow C/b$$

$$A \rightarrow a$$

$$C \rightarrow D$$

$$B \rightarrow b$$

$$D \rightarrow E/bC$$

$$D \rightarrow bC$$

$$E \rightarrow d/AB$$

$$E \rightarrow d/AB$$

$\Rightarrow$  Le unit prodns are

$$B \rightarrow C$$

$$C \rightarrow D$$

$$D \rightarrow E$$



Item dependency graph

$$D \not\Rightarrow E$$

$\therefore$  Le non unit prodns generated from E can also be generated from D.

$E \rightarrow d/AB$  can also be obtained from D  
 $\therefore D \rightarrow d/AB$

$\Rightarrow$  Le resulting D prodns are

$$D \rightarrow bc$$

$$D \rightarrow d/AB$$

Item graph  $C \not\Rightarrow D$

$$\therefore C \rightarrow d/AB \cdot C \rightarrow bC$$

$$C \rightarrow d/AB$$

Item graph  $B \not\Rightarrow C$

$$B \rightarrow b$$

$$B \rightarrow d/AB$$

$$B \rightarrow bC$$

$$V' = \{B, A, B, C, D, E\}$$

$$T' = \{a, b, d\}$$

$$P' = \{S \rightarrow AB\}$$

$\Rightarrow$  Le final  $\mu$  is

$$\begin{aligned}
 & A \rightarrow a \\
 & B \rightarrow b/d/AB/bC \\
 & C \rightarrow bC/d/AB \\
 & D \rightarrow bC/d/AB \\
 & E \rightarrow d/AB
 \end{aligned}$$

ii)  $S \rightarrow Aa/B/ca$

$$B \rightarrow aB/b$$

$$C \rightarrow Db/D$$

$$D \rightarrow E/d$$

$$E \rightarrow ab$$

$\Rightarrow$  Le unit prodns are

$$S \rightarrow B$$

$$C \rightarrow D$$

$$D \rightarrow E$$

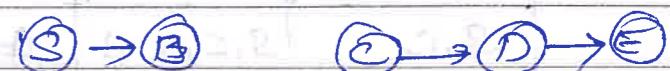
Non unit prodns

$$S \rightarrow Aa/ca$$

$$B \rightarrow aB/b$$

$$C \rightarrow Db$$

$$D \rightarrow d, E \rightarrow ab$$



$$S \not\Rightarrow B \quad B \rightarrow aB/b$$

$$\therefore S \rightarrow aB/b$$

$$D \not\Rightarrow E \quad E \rightarrow ab$$

$$\therefore D \rightarrow ab \quad C \rightarrow ab/d$$

$\Rightarrow$  Le final gr is

$$S \rightarrow aA/a|ca|ab/b$$

$$B \rightarrow aB/b$$

$$C \rightarrow Db/ab/d$$

$$D \rightarrow d/ab$$

$$E \rightarrow ab$$

Q)  $S \rightarrow aAa/bBb/c$

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$$A \rightarrow c/a$$

$$B \rightarrow c/b$$

$$C \rightarrow cDE/e$$

$$D \rightarrow A/B/ab$$

i) Eliminate  $e$ -prodns

ii)  $\rightarrow$  any unit prodns in resulting gr

iii)  $\rightarrow$  any useless symbols in  $\rightarrow$

PTO

To eliminate E-prods

$$S \rightarrow E \quad \{ \text{C and S are nullable variables}$$

$$C \rightarrow E.$$

Step 1

OV	nr	P
$\emptyset$	S, C	$S \rightarrow E$ $C \rightarrow E$
S, C	S, C, A, B	$A \rightarrow C$ $B \rightarrow C$
S, C, A, B	S, C, A, B, D	$S \rightarrow aAg$ $S \rightarrow bBb$ $D \rightarrow A$ $D \rightarrow B$
S, C, A, B, D	S, C, A, B, D	-

The nullable vars are S, C, A, B, D.

Step 2: Make each nullable var as null & find the grammar.

$$S \rightarrow aAa|aa|bBb|bb$$

$$A \rightarrow c|a$$

$$B \rightarrow c|b$$

$$C \rightarrow CDE|DE|CE|E$$

$$D \rightarrow A|B|ab$$

In this gr, the unit prods are

$$A \rightarrow C$$

$$\textcircled{A} \rightarrow \textcircled{C} \rightarrow \textcircled{E}$$

$$B \rightarrow C$$

$$\textcircled{B} \rightarrow \textcircled{C}$$

$$C \rightarrow E$$

$$\textcircled{D} \rightarrow \textcircled{A}$$

$$D \rightarrow A$$

$$D \rightarrow B$$

$$\textcircled{D} \rightarrow \textcircled{B}$$

The non-unit prods are

$$S \rightarrow aAa|aa|bBb|bb$$

$$A \rightarrow a, B \rightarrow b$$

$$C \rightarrow CDE|DE|CE$$

$$A \not\Rightarrow C$$

$$\therefore A \rightarrow CDE|DE|CE$$

$$B \not\Rightarrow C$$

$$\therefore B \rightarrow CDE|DE|CE$$

$$D \not\Rightarrow A$$

$$\therefore D \rightarrow CDE|DE|CE$$

$$D \not\Rightarrow B$$

$$D \rightarrow CDE|DE|CE$$

The gr is

$$S \rightarrow aAa|bBb|aa|bb$$

$$A \rightarrow CDE|DE|CE|a$$

$$B \rightarrow CDE|DE|CE$$

$$C \rightarrow CDE|DE|CE$$

$$D \rightarrow CDE|DE|CE|ab$$

Step 3: By eliminating useless prods,

$$S \rightarrow aAa|bBb|aa|bb$$

OV	nr	P
$\emptyset$	S, A, D	$S \rightarrow aa bb$ $A \rightarrow a$ $D \rightarrow ab$
S, A, D	S, A, D	$S \rightarrow aAg$

The gr is

$$S \rightarrow aa|bb|aa|a$$

$$A \rightarrow CDE|DE|CE|a$$

$$D \rightarrow ab$$

The final gr is  $S \rightarrow aAa|bBb$