28/1/14

MODULE :- 1

DIFFERENTIAL EQUATIONS - 1

LINEAR D.E With constant co-efficients :-

Homogenous D.E :-

consider the D.E $a_0 \frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \rightarrow (1)$

[Homogenous where Right hand side is zero]

$a_0 D^2 y + a_1 D y + a_2 y = 0$

$[a_0 D^2 + a_1 D + a_2] y = 0 \longrightarrow (1a)$

$L(D) y = 0 \longrightarrow (2)$

Where $L(D) = a_0 D^2 + a_1 D + a_2$ is known as Linear Differential operator.

To find the solution of (1), we write the auxiallary eq^n [AE]

[replacing $D \longrightarrow m$ in (1a)

ie $a_0 m^2 + a_1 m + a_2 = 0 \longrightarrow (3)$

The solution of (1) depends on the nature of the auxiallary eq^n given by (3)

Nature of roots of A.E (3)	sol^n of (1).
(1) roots are real & distinct say m_1, m_2.	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$.
(2) Roots are equal $m_1 = m_2 = m$	$y = (c_1 + c_2 x) e^{mx}$
(3) roots are complex say $\alpha \pm i\beta$	$y = e^{\alpha x} [c_1 \cos\beta x + c_2 \sin\beta x$

1. Solve $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = 0$.

$D^2y + 3Dy + 2y = 0$.

$[D^2 + 3D + 2]y = 0$.

$L(D)y = 0$.

$D \to m$,

$m^2 + 3m + 2 = 0$.

$m_1 = -1$, $m_2 = -2$

The roots are real & distinct.

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$= C_1 e^{-1x} + C_2 e^{-2x}$$ //

2. $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$.

$D^2y + 4Dy + 4y = 0$.

$[D^2 + 4D + 4]y = 0$.

$L(D)y = 0$

$D \rightarrow m$.

$m^2 + 4m + 4 = 0$

$m = -2, -2$.

The roots are equal

$\therefore$ the solution is

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}.$$

$$y = C_1 e^{-2x} + C_2 e^{-2x}$$ //

3. $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 6y = 0$.

$D^2y + 2Dy + 6y = 0$.

$\cancel{D^2 + 4D}$ $[D^2 + 2D + 6]y = 0$.

$L(D)y = 0$

$D \rightarrow m$.

$m^2 + 2m + 6 = 0$.

$m_1 = -1 \pm \sqrt{5}i$

The roots are complex

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$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

$$y = e^{-1x}\left[c_1 \cos\sqrt{5}x + c_2 \sin\sqrt{5}x\right]$$

4. $y'' + 5y' + 6y = 0$

$$D^2y + 5Dy + 6y = 0$$

$$[D^2 + 5D + 6]y = 0$$

$$L(D)y = 0$$

$$D \to m$$

$$m^2 + 5m + 6 = 0$$

$$m_1 = -2, \quad m_2 = -3$$

The roots are real & distinct.

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y = c_1 e^{-2x} + c_2 e^{-3x}$$

5. $y'' + 5y' - 6y = 0$

$$D^2y + 5Dy - 6y = 0$$

$$(D^2 + 5D - 6)y = 0$$

$$L(D)y = 0$$

$$D \to m$$

$$m^2 + 5m - 6 = 0$$

$$m_1 = 1, \quad m_2 = -6$$

The roots are real & distinct

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y = c_1 e^{1x} + c_2 e^{-6x}$$

6. $y'' - 5y' + 6y = 0$.

$D^2y - 5Dy + 6y = 0$

$[D^2 - 5D + 6]y = 0$.

$L(D)y = 0$

$D \to m$.

$m^2 - 5m + 6 = 0$.

$m_1 = 3$. $m_2 = 2$.

The roots are real & Distinct.

$y = c_1 e^{3x} + c_2 e^{2x}$ //.

7. $y'' + 6y' + 5y = 0$

$D^2y + 6Dy + 5y = 0$

$[D^2 + 6D + 5]y = 0$.

$L(D)y = 0$.

$D \to m$.

$m^2 + 6m + 5 = 0$.

$m_1 = -1$, $m_2 = -5$.

$y = c_1 e^{-1x} + c_2 e^{-5x}$ //.

8. $y'' - 6y' + 5y = 0$.

$D^2y - 6Dy + 5y = 0$.

$[D^2 - 6D + 5]y = 0$.

$L(D)y = 0$

$D \to m$.

$m^2 - 6D + 5 = 0$.

$m_1 = 5$, $m_2 = 1$

$y = c_1 e^{5x} + c_2 e^{x}$ //.

9. $\frac{d^2y}{dx^2} + y = 0$

$D^2y + 0 + y = 0.$

$(D^2 + 1)y = 0.$

A.E $m^2 + 1 = 0$

$m = \pm i$

complex.

$\alpha = 0, \beta = 1$

$y = e^{\alpha x}[C_1 \cos \beta x + C_2 \sin \beta x].$

$y = e^{0x}[C_1 \cos(1x) + C_2 \sin(1x)].$

$y = C_1 \cos x + C_2 \sin x$ //.

10. $y'' - y = 0$

$D^2y - 0 - y = 0.$

$(D^2 - 1)y = 0.$

A.E = $m^2 - 1 = 0$

$m = \pm 1$

$y = C_1 e^{x} + C_2 e^{-x}$ //.

Solution of non-Homogenous D.E

consider the D.E

$$a_0 \frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = X \rightarrow (1)$$

where a_0, a_1 & a_2 are constants
& X is fun^n of x alone.

The solution of one consist of two parts namely complementary fun^n [CF]
& particular integral [PI].

The sol^n of (1) is given by

$$\boxed{y = CF + PI}$$

Where CF is a solution of Homogenous part of (1) & PI depends on the nature of X

(1) $$\frac{d^2x}{dt^2} - \frac{3dx}{dt} + 2x = 0 \rightarrow (1)$$ given $\boxed{t=0}$ $x=0$ & $\frac{dx}{dt} = \phi$.

$D^2x - 3Dx + 2x = 0$

$[D^2 - 3D + 2]x = 0$

$f(D)x = 0$

$D \rightarrow m.$

A.E $m^2 - 3m + 2 = 0.$

$m_1 = 2 \quad m_2 = 1$

real & distinct

$$x = c_1 e^{2t} + c_2 e^{t} \rightarrow (2)$$

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now we use the given Initial conditions to find the constants c_1 & c_2

at $t=0$, $x=0$

Substitute in (2).

$$0 = c_1 e^0 + c_2 e^2$$

$$\boxed{0 = c_1 + c_2} \rightarrow (3)$$

Diff (2) w.r.t to 't'

$$\frac{dx}{dt} = c_1 e^t + 2c_2 e^{2t}$$

$\therefore$ at $t=0$, $dx/dt = 1$

$$\boxed{1 = c_1 + 2c_2} \rightarrow (4)$$

solving (3) & (4) we get.

$$\boxed{\begin{aligned} c_1 &= -1 \\ c_2 &= 1 \end{aligned}}$$

$$\boxed{x = -1e^t + e^{2t}}$$

(2) $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 3x = 0 \rightarrow (1)$

give $x(0) = 0$, $\frac{dx(0)}{dt} = 2$.

$$D^2x - 4Dx + 3x = 0$$

$$[D^2 - 4D + 3]x = 0.$$

$L(D)x = 0$.

$D \to m$.

$m^2 - 4m + 3 = 0$,

$m_1 = 1$, $m_2 = 3$,

$$\boxed{x = c_1 e^{t} + c_2 e^{3t}} \to (2)$$

$x = x(t)$. x is a function of t.

given $x(0) = 0$

$\therefore t = 0$,

at $t=0$, $x = 0$.

Substitute in (2).

$$\boxed{0 = c_1 + c_2} \to (3)$$

Diff (2) w.r.t t.

$$\frac{dx}{dx} = c_1 e^{t} + 3c_2 e^{3t}$$

at $t=0$, $dx/dt = 2$.

$$\boxed{2 = c_1 + 3c_2} \to (4)$$

Solve (3) & (4)

$$\therefore \boxed{\begin{array}{l} c_1 = -1 \\ c_2 = 1 \end{array}}$$

$$\therefore x = -e^{t} + e^{3t}$$

∴ Homogenous D.E of higher order :-

① Solve $y''' + 6y'' + 11y' + 6y = 0$.

$$D^3(y) + 6D^2y + 11Dy + 6y = 0$$

$$[D^3 + 6D^2 + 11D + 6]y = 0.$$

$$L(D)y = 0.$$

$D \rightarrow m$.

$A.E = m^3 + 6m^2 + 11m + 6 = 0$.

$m = -3, -2, -1$

real & Distinct :

$$\therefore y = C_1 e^{-3x} + C_2 e^{-2x} + C_3 e^{-1x};$$

② $y''' - 6y'' + 11y' - 6y = 0$

$$D^3y - 6D^2y + 11Dy - 6y = 0.$$

$$(D^3 - 6D^2 - 11D - 6]y = 0.$$

$A.E = m^3 - 6m^2 - 11m - 6 = 0$.

$m = 3, 2, 1$

$$y = C_1 e^{3x} + C_2 e^{2x} + C_3 e^{x}.$$

③ $\frac{d^4y}{dx^4} - 5\frac{d^2y}{dx^2} + 4y = 0$.

$$[D^4 - 5D^2 + 4]y = 0.$$

$AE = m^4 + 0 - 5m^2 + 0 + 4 = 0$

$m^4 - 5m^2 + 4 = 0$. [only even powers]

put $m^2 = t$.

$t^2 - 5t + 4 = 0$.

$t_1 = 4$, $t_2 = 1$.

$m_1^2 = 4$, $m_1 = \pm 2$.

$m_2^2 = 1$ $m_2 = \pm 1$

$\therefore m = -1, +1, -2, +2$.

$\therefore y = c_1 e^{-x} + c_2 e^{x} + c_3 e^{-2x} + c_4 e^{2x}$.

(3) $4\frac{d^4y}{dx^4} - 8\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$

$4D^4y - 8D^3y - 7D^2y + 11Dy + 6y = 0$.

$[4D^4 - 8D^3 - 7D^2 + 11D + 6]y = 0$.

A.E

$4m^4 - 8m^3 - 7m^2 + 11m + 6 = 0$

put $m = 1$ not a root.

put $m = 2$ is a root.

$$\begin{array}{c|ccccc} 2 & 4 & -8 & -7 & 11 & 6 \\ & 0 & 8 & 0 & -14 & -6 \\ \hline & 4 & 0 & -7 & -3 & 0 \end{array}$$

$4m^3 + 0m^2 - 7m - 3 = 0$

$m = 3/2, -1/2, -1$

$\therefore m = -1, -1/2, 3/2, 2$.

$\therefore y = c_1 e^{-x} + c_2 e^{-1/2 x} + c_3 e^{3/2 x} + c_4 e^{2x}$ //.

(4) $(D^4 - 2D^2 + 1)y = 0$.

$A.E = m^4 - 2m^2 + 1 = 0$.

put $m^2 = t$.

$t^2 - 2t + 1 = 0$

$t = 1, 1$

$m_1^2 = 1$, $m = \pm 1$

$m_2^2 = 1$, $m = \pm 1$

$m = +1, -1, +1, -1$

$y = (c_1 + c_2 x)e^{-x} + (c_3 + c_4 x)e^{x}$

(5) $[D^4 + 5D^3 + 6D^2 - 4D - 8]y = 0$

$A.E = m^4 + 5m^3 + 6m^2 - 4m - 8 = 0$.

at $m = 1$, it is a root

1	1	5	6	-4	-8
	0	1	6	12	8
	1	6	12	8	0

$= m^3 + 6m^2 + 12m + 8 = 0$

$m = -2, -2, -2$.

$\therefore m = -2, -2, -2, 1$

-2	1	6	12	8
	0	-2	-8	-8
	1	4	4	0

$m^2 + 4m + 4 = 0$

$y = c_1 e^{x} + \{c_2 + c_3 x + c_4 x^2\}e^{-2x}$

$y = c_1 e^{x} + [c_2 + c_3 x + c_4 x^2]e^{-2x}$

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6. $[D^4+18D^2+81]y=0$.

A.E $= m^4+18m^2+81=0$.

put $m^2=t$.

$t^2+18t+81=0$

$t=-9,-9$.

$m_1^2=-9 \quad m_1=\pm 3i$

$m_2^2=-9 \quad m_2=\pm 3i$

~~$y=-3+3, -3+3$..~~ $+3i \; +3i \; -3i \; -3i$

$m=\pm 3i \quad m=\pm 3i$ $(c_1+c_2x)\cos 3x+ (c_3+c_4x)\sin 3x$

Imaginary.

$y=e^{0x}[(c_1+c_2x)\cos 3x+(c_3+c_4x)\sin 3x]$

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Solution of non-homogenous D.E

consider a D.E $a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=X \rightarrow (1)$

where a_0, a_1, a_2 are constant & X is function of 'x' alone.

$[a_0D^2+a_1D+a_2]y=X$.

$L(D)y=X$

$L(D)=a_0D^2+a_1D+a_2$ is known as linear differential operator.

The sol^n of 1 consist of 2 parts namely complimentary fun^n [CF]

Particular integral [P.I].

The sol^n of (1) is given by

$y=CF+PI$

where CF is the solu^{-n} of homogenous part of eq^{-n} ①.

PI depends on the nature of Right hand side X & its given by $PI = \frac{1}{L(D)} X$

we have following types for P.I

i) $X = e^{ax}$

ii) $X = \sin ax$ or $\cos ax$.

iii) X = polynomial

iv) $X = e^{ax}.V$, V is fu^{-n} of 'x'

Type 1 :-

PI when $X = e^{ax}$:-

$$PI = \frac{1}{L(D)} X$$

$$= \frac{1}{L(D)} e^{ax}$$

$\frac{e^{ax}}{L(a)}$ if 'a' is not a root of A.E

$\frac{x\, e^{ax}}{L'(a)}$ if 'a' is a simple root of A.E

$\frac{x^2\, e^{ax}}{L''(a)}$ if 'a' is a double root.

1. Solve

$y'' + 5y' + 6y = e^{5x}$

$D^2y + 5Dy + 6y = e^{5x}$

$[D^2 + 5D + 6]\,y = e^{5x}$.

$m^2 + 5m + 6 = 0$

$m = -3, -2$

$CF = c_1 e^{-3x} + c_2 e^{-2x}$

$PI = \dfrac{e^{5x}}{D^2 + D5 + 6}$ $D \to a \Rightarrow 5$

$\dfrac{e^{5x}}{5^2 + 5(5) + 6}$

$= \dfrac{e^{5x}}{56}$

$y = c_1 e^{-3x} + c_2 e^{-2x} + \dfrac{e^{5x}}{56}$.

2.

2. $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 12y = e^{-4x}$

$D^2y + Dy - 12y = e^{-4x}$.

$[D^2 + D - 12]\,y = e^{-4x}$.

$AE = m^2 + m - 12 = 0$

$m = -4, 3$

$CF = c_1 e^{-4x} + c_2 e^{3x}$.

$PI = \dfrac{1}{f(D)} X$ $= \dfrac{e^{-4x}}{D^2 + D - 12}$

$$= \frac{x e^{-4x}}{2D+1}$$

$$= \frac{x e^{-4x}}{-7}$$

$$y = C_1 e^{-4x} + C_2 e^{3x} - \frac{x e^{-4x}}{7}$$

3. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$

$D^2y + 4Dy + 4y = e^{-2x}$

$[D^2+4D+4]y = e^{-2x}$

$= m^2+4m+4 = 0$

$m = -2, -2$

$CF = [C_1 + C_2 x]e^{-2x}$

$$PI = \frac{e^{-2x}}{D^2+4D+4} \qquad D \to -2$$

$$= \frac{e^{-2x}}{4-8+4} \quad \text{is a root}$$

$$= \frac{x e^{-2x}}{2D+4} \quad \text{is a root}$$

$$= \frac{x^2 e^{-2x}}{2}$$

$$y = [C_1 + C_2 x]e^{-2x} + \frac{x^2 e^{-2x}}{2}$$

$$= e^{-2x}\left[C_1 + C_2 x + x^2/2\right]$$

4. $y'' + 2y' + 2y = e^{2x}$

$[D^2y + 2Dy + 2y] = e^{2x}$

$[D^2 + 2D + 2]y = e^{2x}$.

$= m^2 + 2m + 2 = 0$

$m = -1 \pm i$

$CF = e^{-x}[c_1 \cos x + c_2 \sin x]$

$PI = \dfrac{e^{2x}}{D^2+2D+2}$ $\quad D \to 2$.

$PI = \dfrac{e^{2x}}{4+4+2}$

$= \dfrac{e^{2x}}{10}$

$y = CF + PI$

$y = e^{-x}[c_1 \cos x + c_2 \sin x] + \dfrac{e^{2x}}{10}$

5. $y'' + 8y' + 12y = e^{-6x}$.

$D^2y + 8Dy + 12y = e^{-6x}$

$[D^2 + 8D + 12]y = e^{-6x}$.

$= m^2 + 8m + 12 = 0$

$m = -6, -2$.

$CF = c_1 e^{-6x} + c_2 e^{-2x}$. $\quad 36 - 48 + 12$

$PI = \dfrac{e^{-6x}}{D^2+8D+12}$, $\quad D = -6$ is a root

$= \dfrac{x e^{-6x}}{2D+8} = \dfrac{x e^{-6x}}{-4}$.

$y = c_1 e^{-6x} + c_2 e^{-2x} - \dfrac{x e^{-6x}}{4}$

6. $y'' + 6y' + 9y = e^{-3x}$.

$D^2y + 6Dy + 9y = e^{-3x}$

$[D^2 + 6D + 9]y = e^{-3x}$.

$m^2 + 6m + 9 = 0$

$m = -3, -3$

$CF = (c_1 + c_2x)e^{-3x}$.

$$PI = \frac{e^{-3x}}{D^2 + 6D + 9} \qquad D \to -3$$

$$\frac{e^{-3x}}{9 - 18 + 9} \quad \text{is a root}$$

$$\frac{x e^{-3x}}{2D + 6}. \qquad D \to -3.$$

$$\frac{x e^{-3x}}{-6 + 6} \quad \text{is a root}.$$

$$PI = \frac{x^2 e^{-3x}}{2}.$$

$$y = (c_1 + c_2x)e^{-3x} + \frac{x^2 e^{-3x}}{2}.$$

7 $[D^3-5D^2+7D-3]y = e^{2x}\cosh x$.

G.S:- $y = CF + PI$

$m^3 - 5m^2 + 7m - 3 = 0$.

$m = 1, 1, 3$.

CF :- $[c_1 + c_2 x]e^{x} + c_3 e^{3x}$.

$$PI = y_p = \frac{e^{2x}\cosh x}{D^3-5D^2+7D-3}$$

$$= \frac{\frac{e^{2x}}{2}\left[e^{x}+e^{-x}\right]}{D^3-5D^2+7D-3}.$$

$$P.I = \frac{1}{2}\left[\frac{e^{3x}+e^{x}}{D^3-5D^2+7D-3}\right]$$

$$= \frac{1}{2}\left[\frac{e^{3x}}{D^3-5D^2+7D-3} + \frac{e^{x}}{D^3-5D^2+7D-3}\right].$$

$D \to 3$
$Dr = 0$

$D \to 1$
$Dr = 0$

$$\frac{1}{2}\left[\frac{x e^{3x}}{3D^2-10D+7} + \frac{x e^{x}}{3D^2-10D+7}\right]$$

$D \to 1$
$Dr = 0$,

$$\cancel{\frac{1}{2}\left[\frac{x e^{3x}}{3D^2-10D}\right]}$$

$$\frac{1}{2}\left[\frac{x e^{3x}}{4} + \frac{x^2 e^{x}}{6D-10}\right].$$

$$PI = \frac{1}{2}\left[\frac{x e^{3x}}{4} - \frac{x^2 e^{x}}{4}\right]$$

$$P.I. = \frac{1}{8}\left[xe^{3x} - x^2e^x\right].$$

$$\therefore G_s = CF + PI$$

$$G_s = [C_1 + C_2x]e^x + C_3e^{3x} + \frac{1}{8}\left[xe^{3x} - x^2e^x\right]$$

8) $y'' + 4y' + 5y = -3\sinh x.$

$\cancel{m^2+4m+5}$

$$D^2y + 4Dy + 5y = -3\sinh x.$$

$$[D^2 + 4D + 5]y = -3\sinh x.$$

$$m^2 + 4m + 5 = 0$$

$$m = -2 \pm i$$

$$CF = e^{-2x}\left[C_1\cos x + C_2\sin x\right]$$

$$PI = y_p = \frac{-3\sinh x}{D^2+4D+5}$$

$$y_p = \frac{\frac{-3}{2}\left[e^x - e^{-x}\right]}{D^2+4D+5}.$$

$$y_p = \frac{-3}{2}\left[\frac{e^x}{\underset{D\to 1}{D^2+4D+5}} - \frac{e^{-x}}{\underset{D\to -1}{D^2+4D+5}}\right]$$

$$= \frac{-3}{2}\left(\frac{e^x}{10} - \frac{e^{-x}}{2}\right).$$

$$P.I = \frac{-3}{2}\left[\frac{e^x}{10} - \frac{e^{-x}}{2}\right]$$

$G.S :- y = CF + PI$

$$e^{-2x}[C_1 \cos x + C_2 \sin x] - 3/2 \left[\frac{e^{x}}{10} - \frac{e^{-x}}{2}\right.$$

9. $y'' - 6y' + 9y = 6e^{3x} + 7e^{-2x} - 2^{x}$.

$D^2y - 6Dy + 9y = 6e^{3x} + 7e^{-2x} - 2^{x}$.

$(D^2 - 6D + 9)y = 6e^{3x} + 7e^{-2x} - 2^{x}$.

$m^2 - 6m + 9 = 0$.

$m = 3, 3$.

$CI = [c_1 + c_2 x]e^{3x}$.

$P.I = y = \dfrac{6e^{3x} + 7e^{-2x} - 2^{x}}{D^2 - 6D + 9}$. $\left\{ {}^{+}2^{x} = e^{\log 2 \times x} \right.$

$$= \frac{6e^{3x} + 7e^{-2x} - e^{\log 2 \times x}}{D^2 - 6D + 9}.$$

$$\left[\frac{6e^{3x}}{D^2 - 6D + 9}\right] + \left[\frac{7e^{-2x}}{D^2 - 6D + 9}\right] - \left[\frac{e^{\log 2\, x}}{D^2 - 6D + 9}\right].$$

$D \to 3$ $D \to -2$ $D \to \log 2$.

$Dr \to 0$. 25

$$\left[x\frac{6e^{3x}}{2D - 6}\right] + \left[\frac{7e^{-2x}}{25}\right] - \left[\frac{e^{\log 2 x}}{7.2844}\right]$$

$D \to 3$

$$\left[x^2\frac{6e^{3x}}{2}\right] + \left[\frac{7e^{-2x}}{25}\right] - \left[\frac{e^{\log 2x}}{7.2844}\right]$$

$y = CF + PI$

$$y = [c_1 + c_2 x]e^{3x} + x^2\frac{6e^{3x}}{2} + \frac{7e^{-2x}}{25} - \frac{e^{\log 2x}}{7.2844}$$

10 $y'' + 5y' + 6y = (e^{x}+1)^2$

$D^2y + 5Dy + 6y = e^{2x} + 1 + 2e^{x}$

$[D^2 + 5D + 6]y = e^{2x} + 1 + 2e^{x}$

$m^2 + 5m + 6 = 0$

$m = -2, -3$

$CI = C_1 e^{-2x} + C_2 e^{-3x}$

$$P.I :- \quad y = \frac{e^{2x} + 1e^{0x} + 2e^{x}}{D^2 + 5D + 6}$$

$$\therefore \left[\frac{e^{2x}}{D^2+5D+6}\right] + \left[\frac{1 \cdot e^{0x}}{D^2+5D+6}\right] + \left[\frac{2e^{x}}{D^2+5x+6}\right]$$

$D \to 2 \qquad D \to 0 \qquad D \to 2$

$20 \qquad 6 \qquad 12$

$$\therefore \left[\frac{e^{2x}}{20}\right] + \left[\frac{e^{0x}}{6}\right] + \left[\frac{2e^{x}}{12}\right]$$

$$\therefore y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{e^{2x}}{20} + \frac{e^{0x}}{6} + \frac{2e^{x}}{12} \; //$$

11). $y'' - 6y' + 9y = 3e^{-4x}$

12) $y''' + y'' + y' + y = e^{3x+4}$

11 A :- $y'' - 6y' + 9y = 3e^{-4x}$

$D^2y - 6Dy + 9y = 3e^{-4x}$

$(D^2 - 6D + 9)y = 3e^{-4x}$

$A.E = m^2 - 6m + 9 = 0$

$m = 3, 3$

$CF = (c_1 + c_2 x)e^{3x}$

$$P.I = 3\left[\frac{e^{-4x}}{D^2-6D+9}\right]$$

$$D \to -4.$$

$$P.I = 3\left[\frac{e^{-4x}}{49}\right]$$

$$y = CF + PI$$

$$y = (c_1 + c_2x)e^{3x} + \frac{3e^{-4x}}{49}.$$

12. $y''' + y'' + y' + y = e^{3x+4}$.

$$D^3y + D^2y + Dy + y = e^{3x+4}$$

$$(D^3 + D^2 + D + 1)y = e^{3x+4}.$$

$$m^3 + m^2 + m + 1 = 0$$

$$m = -1, \pm i$$

$$CF = C_1e^{-x} + C_2\cos x + C_3\sin x.$$

$$PI = \frac{e^{3x+4}}{D^3+D^2+D+1}$$

$$D \to 3$$

$$\frac{e^{3x+4}}{40}$$

$$PI = \frac{e^{3x+4}}{40}$$

$$y = C_1e^{-x} + C_2\cos x + C_3\sin x + \frac{e^{3x+4}}{40}$$

13 $\dfrac{d^2y}{dx^2} + \dfrac{6dy}{dx} + 7y = e^{-3x} + 2$.

$D^2y + 6Dy + 7y = e^{-3x} + 2$.

$(D^2 + 6D + 7)y = e^{-3x} + 2$.

A.E $m^2 + 6m + 7 = 0$

$m = -3 \pm \sqrt{2}$. → real & distinct

$CF = C_1 e^{(-3+\sqrt{2})x} + C_2 e^{(-3-\sqrt{2})x}$ //

P.I :- $y = \dfrac{e^{-3x} + 2}{D^2 + 6D + 7}$ $\left\{ PI = \dfrac{1}{L(D)} X \right.$

$= \dfrac{e^{-3x} + 2e^{0}}{D^2 + 6D + 7}$

$\left[\dfrac{e^{-3x}}{D^2 + 6D + 7}\right] + \left[\dfrac{2e^{0}}{D^2 + 6D + 7}\right]$.

$D \to -3$ $\quad$ $D \to 0$.

-2 $\quad$ 7.

$\left[\dfrac{e^{-3x}}{-2}\right] + \left[\dfrac{2e^{0}}{7}\right]$

$= \dfrac{e^{-3x}}{-2} + \dfrac{2}{7}$

$y = CF + PI$.

$y = C_1 e^{(-3+\sqrt{2})x} + C_2 e^{(-3-\sqrt{2})x} - \dfrac{e^{-3x}}{2} + \dfrac{2}{7}$ //

14. $y'' + 8y' + 6y = \sinh 2x$.

$D^2y + 8Dy + 6y = \sinh 2x$.

$(D^2 + 8D + 6)y = \sinh 2x$.

$A.E = m^2 + 8m + 6 = 0$.

$m = -4 \pm \sqrt{10}$.

$CF = c_1 e^{(-4+\sqrt{10})x} + c_2 e^{(-4-\sqrt{10})x}$.

$P.I = y = \frac{\sinh 2x}{D^2 + 8D + 6}$,

$y = \frac{1}{2}\left[\frac{e^{2x} - e^{-2x}}{D^2 + 8D + 6}\right]$.

$y = \frac{1}{2}\left[\frac{e^{2x}}{D^2 + 8D + 6}\right] - \left[\frac{e^{-2x}}{D^2 + 8D + 6}\right]$.

$D \to 2$. $\quad D \to -2$.

$26 \quad -6$

$y = \frac{1}{2}\left[\frac{e^{2x}}{26}\right] + \frac{1}{2}\left[\frac{e^{-2x}}{6}\right]$.

$y = CF + PI$.

$y = c_1 e^{(-4+\sqrt{10})x} + c_2 e^{(-4-\sqrt{10})x} + \frac{1}{2}\left[\frac{e^{2x}}{26} + \frac{e^{-2x}}{6}\right.$

15. $y'' - 6y' + 12y = \cosh^2 x + 2^x$.

$D^2y - 6Dy + 12y = \cosh^2 x + 2^x$.

$(D^2 - 6D + 12)y = \cosh^2 x + 2^x$.

$m^2 - 6m + 12 = 0$

$m = 3 \pm \sqrt{3}\, i$

$CF = e^{3x}\left[c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right]$.

$$PI = y = \frac{\cosh^2 x + 2^x}{D^2 - 6D + 12}$$

$$P.I = \frac{1}{4}\left[\frac{e^{2x} + e^{-2x} + 2e^{0} + e^{(\log 2)x}}{D^2 - 6x + 12}\right]$$

$\cosh^2 x = \frac{1}{4}\left[e^{2x} + e^{-2x} + 2e^{x}e^{-x}\right]$

$= \frac{1}{4}\left[e^{2x} + e^{-2x} + 2\right]$

$$P.I = \frac{1}{4}\left[\frac{e^{2x}}{D^2 - 6x + 12} + \frac{e^{-2x}}{D^2 - 6x + 12} + \frac{2e^{0}}{D^2 - 6x + 12} + \frac{e^{(\log 2)x}}{D^2 - 6x + 12}\right]$$

$$\frac{1}{4}\left[\frac{e^{2x}}{4}\right] + \frac{1}{4}\left[\frac{e^{-2x}}{28}\right] + \cancel{\frac{1}{4}}\left[\frac{2}{12}\right]$$

$$+ \cancel{\frac{1}{4}}\left[\frac{2^x}{(\log 2)^2 - 6(\log 2) + 12}\right].$$

$y = CF + PI$

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16. $y'' + 8y' + 6y = \sinh^2 2x$.

$D^2y + 8Dy + 6y = \sinh^2 2x$.

$(D^2 + 8D + 6)y = \sinh^2 2x$.

$m^2 + 8m + 6 = 0$

$m = 4 \pm \sqrt{10}$

$$\sinh^2 2x = \left[\frac{e^{2x} - e^{-2x}}{2}\right]^2.$$

$$= \frac{1}{4}\left[e^{4x} + e^{-4x} - 2e^{2x}e^{-2x}\right]$$

$$= \frac{1}{4}\left[e^{4x} + e^{-4x} - 2\right].$$

$CF = C_1 e^{(4+\sqrt{10})x} + C_2 e^{(4-\sqrt{10})x}$.

$$PI = y = \frac{\sinh^2 2x}{D^2 + 8D + 6}.$$

$$\frac{1}{4}\left[\frac{e^{4x} + e^{-4x} - 2}{D^2 + 8D + 6}\right].$$

$$\frac{1}{4}\left[\frac{e^{4x}}{D^2 + 8D + 6} + \frac{e^{-4x}}{D^2 + 8D + 6} - \frac{2e^{0}}{D^2 + 8D + 6}\right].$$

$$= \frac{1}{4}\left[\frac{e^{4x}}{54} + \frac{e^{-4x}}{-10} - \frac{2}{6}\right].$$

$$= \frac{1}{4}\left[\frac{e^{4x}}{54} - \frac{e^{-4x}}{10} - \frac{1}{3}\right].$$

$y = CF + PI$

$$C_1 e^{(4+\sqrt{10})x} + C_2 e^{(4-\sqrt{10})x} + \frac{1}{4}\left[\frac{e^{4x}}{54} - \frac{e^{-4x}}{10} - \frac{1}{3}\right]$$

17. $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = e^{2x} - e^{-2x} + \cosh 3x + 2^{-x} + m$.

$D^2y + 4Dy + 4y = e^{2x} - e^{-2x} + \cosh 3x + 2^{-x} + m$.

$(D^2 + 4D + 4)y =$ —— 11

A.E.

$m^2 + 4m + 4 = 0$

$m = -2, -2$

$CF = (C_1 + C_2 x)e^{-2x}$

$$P.I = y = \frac{e^{2x} - e^{-2x} + \cosh 3x + 2^{-x} + m}{D^2 + 4D + 4}$$

$$\cosh 3x = \frac{e^{3x} + e^{-3x}}{2}$$

$$2^{-x} = e^{(-\log 2)x}$$

$$P.I = \left[\frac{e^{2x}}{D^2+4D+4} - \frac{e^{-2x}}{D^2+4D+4} + \frac{1}{2}\left[\frac{e^{3x}}{D^2+4D+4} + \frac{e^{-3x}}{D^2+4D+4}\right] + \frac{e^{(-\log 2)x}}{D^2+4D+4} + \frac{me^0}{D^2+4D+4}\right]$$

$$\frac{e^{2x}}{16} - \frac{x^2 e^{-2x}}{2} + \frac{1}{2}\left[\frac{e^{3x}}{25} + e^{-3x}\right] + \frac{1 \cdot 2^{-x}}{(-\log 2)^2 + 4(-\log 2) + 4} + \frac{m}{4}$$

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Note :- some of the following cases under e^{ax}.

1. constant $k = k e^{0x}$.
2. $a^{x} = e^{(\log a)x}$.
3. $a^{-x} = e^{(-\log a)x}$.
4. $\cosh x = \frac{e^{x}+e^{-x}}{2}$, $\sinh x = \frac{e^{x}-e^{-x}}{2}$.
5. $\cosh^2 x = \frac{e^{2x}+e^{-2x}+2}{4}$. etc.

Type :- 2

To find the P.I for $x = \sin ax$ (or) $\cos ax$.

$$P.I = \frac{1}{f(D)} \sin ax \text{ or } \cos ax.$$

* $D^2 \rightarrow -a^2$

* multiply by conjugate
* $D^2 \rightarrow -a^2$
* simplify

1. solve $y'' + 2y' + y = \sin 3x$.

$$D^2y + 2Dy + y = \sin 3x.$$

$$[D^2 + 2D + 1]y = \sin 3x.$$

$$A.E = m^2 + 2m + 1 = 0.$$

$$m = -1, -1$$

$$CF = (C_1 + C_2 x)e^{-1x}.$$

$$P.I = \frac{\sin 3x}{D^2+2D+1} \qquad a = 3$$

$D^2 \to -a^2$

$D^2 \to -9.$

$$P.I = \frac{\sin 3x}{-9+2D+1}$$

$\times$ & $\div$ by $2D+8$

$$\cancel{P.I = \frac{\sin 3x}{2D+8} \times \frac{2D-8}{}}$$

$$P.I = \sin 3x \times \frac{1}{2D-8} \times \frac{2D+8}{2D+8}.$$

$$= \sin 3x \times \frac{2D+8}{4D^2-64}$$

$D^2 \to -a^2$.

$D^2 \to -9$.

$$= \sin 3x \times \frac{2D+8}{4(-9)-64}.$$

$$= \frac{-1}{100} \left[\sin 3x \,(2D+8)\right]$$

$$\frac{-1}{100}\left[2D\sin 3x + 8\sin 3x\right].$$

$$\frac{-1}{100}\left[2\times 3\cos 3x + 8\sin 3x\right]$$

$$= -1/100\left[6\cos 3x + 8\sin 3x\right].$$

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2. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2\sin^2 x$.

$D^2y + 3Dy + 2y = 2\sin^2 x$.

$[D^2 + 3D + 2]y = 2\sin^2 x$.

$m^2 + 3m + 2 = 0$

$m = -2, -1$

$$\boxed{CF = c_1 e^{-2x} + c_2 e^{-x}}$$

$P.I = \frac{2\sin^2 x}{D^2+3D+2}$.

$D^2 \to -a^2$.

$\frac{\cancel{2}\left[\frac{1-\cos 2x}{\cancel{2}}\right]}{D^2+3D+2}$.

$= \frac{1\, e^{0x}}{D^2+3D+2} - \frac{\cos 2x}{D^2+3D+2}$.

$\frac{1}{2} - \frac{\cos 2x}{-4+3D+2}$,

$D^2 \to -a^2$
$a = 2$
$-a^2 = -4^2$

$\frac{1}{2} - \frac{\cos 2x}{3D-2}$

$P.I = \frac{1}{2} - \frac{\cos 2x}{3D-2} \times \frac{3D+2}{3D+2}$

$\frac{1}{2} - \frac{\cos 2x \times 3D+2}{9D^2-4}$.

$D^2 \to -a^2$

$\frac{1}{2} - \frac{\cos 2x \times 3D+2}{-36-4}$

$\frac{1}{2} - \frac{\cos 3x \times 3D+2}{-40}$.

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$$= \frac{1}{2} + \frac{1}{40}(3D+2)\cos 2x$$

$$= \frac{1}{2} + \frac{1}{40}\left[-6\sin 2x + 2\cos 2x\right]$$

$$P.I = \frac{1}{2} + \frac{1}{20}\left[-3\sin 2x + \cos 2x\right]$$

$y = CI + PF$

$$C_1 e^{-2x} + C_2 e^{-x} + \frac{1}{2} + \frac{1}{20}\left[-3\sin 2x + \cos 2x\right]$$

3. $y'' + 4y' + 4y = 4\cos^3 x$

$D^2 y + 4Dy + 4y = 4\cos^3 x$

$(D^2 + 4D + 4)y = 4\cos^3 x$

$m^2 + 4m + 4 = 0$

$m = -2, -2$

$CI = [c_1 + c_2 x]e^{-2x}$

$$P.I = \frac{4\cos^3 x}{D^2 + 4D + 4}$$

$$\frac{\frac{4}{4}\left[3\cos x + \cos 3x\right]}{D^2 + 4D + 4}$$

$$\frac{3\cos x}{D^2 + 4D + 4} + \frac{\cos 3x}{D^2 + 4D + 4}$$

$= D^2 \to -a^2 \qquad D^2 \to -a^2$

$$= \frac{3\cos x}{-1 + 4D + 4} + \frac{\cos 3x}{-9 + 4D + 4}$$

$$= \frac{3\cos x}{4D + 3} + \frac{\cos 3x}{4D - 5}$$

$$= \left[\frac{3\cos x}{4D+3} \times \frac{4D-3}{4D-3}\right] + \left[\frac{\cos 3x}{4D-5} \times \frac{4D+5}{4D+5}\right].$$

$$\left[\frac{(4D-3)\,3\cos x}{16D^2-9}\right] + \left[\frac{(4D+5)\cos 3x}{16D^2-25}\right].$$

$D^2 \rightarrow -a^2$

$$\frac{[4D-3]\,3\cos x}{-16-9} + \frac{(4D+5)\cos 3x}{-144-25}.$$

$$\frac{-12\sin x - 9\cos x}{-25} + \frac{-12\sin 3x + 5\cos 3x}{-169}.$$

$$\left[\frac{12\sin x + 9\cos x}{25}\right] + \frac{12\sin 3x - 5\cos 3x}{169}.$$

$y = CF + PF$

$$[C_1 + C_2 x]e^{-2x} + \frac{12\sin x + 9\cos x}{25} + \frac{12\sin 3x - 5\cos 3x}{169}.$$

4. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 2\sin 4x\cos x.$

$D^2y + 6Dy + 9y = 2\sin 4x\cos x.$

$[D^2+6D+9]y = 2\sin 4x\cos x.$

$m^2 + 6m + 9 = 0.$

$m = -3, -3.$

$\therefore C.F = [C_1 + C_2 x]e^{-3x}.$

$$P.I = \frac{2\sin 4x\cos x}{D^2+6D+9}.$$

$\sin A \cdot \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$$= \frac{\sin 5x}{D^2+6D+9} + \frac{\sin 3x}{D^2+6D+9}.$$

$D^2 \rightarrow -a^2$ $\qquad$ $D^2 \rightarrow -a^2$

$$\frac{\sin 5x}{-25+6D+9} + \frac{\sin 3x}{6D}$$

$$\frac{\sin 5x}{6D-16} + \frac{\sin 3x}{6D} \qquad \frac{1}{D} = \int$$

$$= \frac{\sin 5x}{6D-16} \times \frac{6D+16}{6D+16} + \frac{1}{6}\int \sin 3x \cdot dx$$

$$= \frac{\cancel{\sin}\,(6D+16)\sin 5x}{36D^2-256} + \frac{1}{6}\left[-\frac{\cos 3x}{3}\right]$$

$D^2 \to -a^2$

$$\frac{30\cos 5x + 16\sin 5x}{-1156} - \frac{1}{18}\cos 3x$$

or

multiply & divide by D

$$\frac{D}{6D^2}(\sin 3x)$$

$$\frac{\cancel{3}\cos 3x}{2\cancel{6}(-9)}$$

$$= \frac{\cos 3x}{-18}$$

$\therefore y = CE + PI$

$$[c_1 + c_2 x]e^{-3x} - \frac{30\cos 5x + 16\sin 5x}{1156} - \frac{1}{18}\cos 3x$$

5. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 3\cos 3x \sin 2x \cdot \cos x$.

$D^2y + Dy + y = 3\cos 3x \sin 2x \cdot \cos x$.

$[D^2 + D + 1]y = 3\cos 3x \sin 2x \cdot \cos x$.

$m^2 + m + 1 = 0$.

$m = -\frac{1}{2} \pm \sqrt{3}/2\, i$

$C.F = e^{-\frac{1}{2}x}\left[C_1 \cos \sqrt{3}/2\, x + C_2 \sin \sqrt{3}/2\, x\right]$

$P.I = \frac{3\cos 3x(\sin 2x \cdot \cos x)}{D^2+D+1}$.

$\frac{3\cos 3x \ \frac{1}{2}[\sin 3x + \sin x]}{D^2+D+1}$.

$= \frac{\frac{3}{2}[\cos 3x \sin 3x + \cos 3x \sin x]}{D^2+D+1}$.

$\frac{3/2\left[\frac{\sin 6x}{2} + \frac{1}{2}[\sin 4x + \sin(-2x)]\right]}{D^2+D+1}$

$\frac{3}{4}\left[\frac{\sin 6x + \sin 4x - \sin 2x}{D^2+D+1}\right]$.

$= \frac{3}{4}\left[\frac{\sin 6x}{D^2+D+1} + \frac{\sin 4x}{D^2+D+1} - \frac{\sin 2x}{D^2+D+1}\right]$.

$D^2 \to -a^2$

$= \frac{3}{4}\left[\frac{\sin 6x}{-36+D+1} + \frac{\sin 4x}{-16+D+1} - \frac{\sin 2x}{-4+D+1}\right]$

$\frac{3}{4}\left[\frac{\sin 6x}{D-35} + \frac{\sin 4x}{D-15} - \frac{\sin 2x}{D-3}\right]$

$$\frac{3}{4}\left[\frac{\sin 6x}{D-35}\times\frac{D+35}{D+35}+\frac{\sin 4x}{D-15}\times\frac{D+15}{D+15}-\frac{\sin 2x}{D-3}\times\frac{D+3}{D+3}\right]$$

$$\frac{3}{4}\left[\frac{(D+35)\sin 6x}{D^2-1225}+\frac{(D+15)\sin 4x}{D^2-225}-\frac{(D+3)\sin 2x}{D^2-9}\right]$$

$$D^2\to -a^2$$

$$\frac{3}{4}\left[\frac{6\cos 6x+35\sin 6x}{-36-1225}+\frac{4\cos 4x+15\sin 4x}{-16-225}-\frac{2\cos 2x+3\sin 2x}{-4-9}\right]$$

$$=\frac{3}{4}\left[\frac{6\cos 6x+35\sin 6x}{-1261}+\frac{4\cos 4x+15\sin 4x}{-241}-\frac{2\cos 2x+3\sin 2x}{-13}\right]$$

$$= y = CF + PI$$

$$e^{-\frac{1}{2}x}\left[c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right]+\frac{3}{4}\left[\frac{6\cos 6x+35\sin 6x}{-1261}+\frac{4\cos 4x+15\sin 4x}{-241}-\frac{2\cos 2x+3\sin 2x}{-13}\right]$$

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6. $y'' + 3y' + 4y = \sin 3x$.

7. $y'' + 7y' + 6y = \cos^3 x \sin 2x$.

8. $y'' + 16y' + 8y = 2\sin^2 4x$.

9. $y'' - 7y' + 8y = e^{3x} + \sin 3x$.

10. $y'' + y = 3\sin x$

$D^2 y + 0 + y = 3\sin x$

$(D^2 + 1)\, y = 3\sin x$.

$m^2 + 1 = 0$

$m^2 = -1$

$m = \sqrt{-1}$

$m = \pm i$

$CF = e^{0x} \cdot (c_1 \cos x + c_2 \sin x)$.

$P.F = \dfrac{3\sin x}{D^2 + 1}$

$D^2 \to -a^2$

$\dfrac{3\sin x}{-1+1}$

Note :- $\dfrac{1}{L(D^2)} \sin ax$ or $\cos ax =$

If $L(-a^2) = 0$

the $\dfrac{x}{L'(-a^2)} (\sin ax$ or $\cos ax)$

$P.I = \dfrac{1}{D^2 + 1} 3\sin x$.

$$= \frac{x}{2'(-1)} 3\sin x .$$

$$= \frac{x}{2D}(3\sin x).$$

$$\frac{x}{2}\int 3\sin x .$$

$$\frac{3x}{2}\int \sin x .$$

$$P.I = \frac{3x}{2}[-\cos x]$$

$$P.I \quad -\frac{3x\cos x}{2}.$$

$$y = CF + PI$$

$$[c_1\cos x + c_2\sin x] + \frac{3x\cos x}{2}.$$

11 $y'' - y' = 3\cos 2x .$

$D^2y - Dy = 3\cos 2x . \quad 1, 0$

$[D^2 - D]y = 3\cos 2x . \quad c_1e^x + c_2$

$m^2 - m + 0 = 0 .$

$m = 1, 0 .$

$CF = \boxed{c_1e^x + c_2}.$

$$P.I = y = \frac{3\cos 2x}{D^2 - D}$$

$$D^2 \rightarrow -a^2 .$$

$$= \frac{3\cos 2x}{-4 - D} .$$

$$-\frac{3\cos 2x}{4 + D} \times \frac{4 - D}{4 - D} .$$

$$-\frac{[4-D]3\cos 2x}{16-D^2}$$

$D^2 \to -a^2$.

$$-\frac{12\cos 2x - 6\sin 2x}{16+4}.$$

$$-\frac{6\sin 2x - 12\cos 2x}{20}.$$

$$y = C_1 e^{x} + C_2 - \frac{6\sin 2x - 12\cos 2x}{20}.$$

12. $y'' - 6y' + 12y = 2\sinh x + \sin 2x$.

$D^2 y - 6Dy + 12y = 2\sinh x + \sin 2x$.

$[D^2 - 6D + 12]y = 2\sinh x + \sin 2x$,

$m^2 - 6m + 12 = 0$.

$m = 3 \pm \sqrt{3}\,i$

$CF = e^{3x}[C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x]$.

$$P.I = \frac{2\sinh x + \sin 2x}{D^2 - 6D + 12}.$$

$$= \frac{\cancel{2}\,\frac{e^{x} - e^{-x}}{\cancel{2}} + \sin 2x}{D^2 - 6D + 12}.$$

$$= \frac{e^{x} - e^{-x} + \sin 2x}{D^2 - 6D + 12}.$$

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$$\frac{e^{x}}{D^2-6D+12} - \frac{e^{-x}}{D^2-6D+12} + \frac{\sin 2x}{D^2-6D+12}$$

$D \to 1$ $\qquad$ $D^2 \to -a^2$

$$\frac{e^{x}}{6} - \frac{e^{-x}}{19} + \frac{\sin 2x}{-4-6D+12}$$

$$\frac{e^{x}}{6} - \frac{e^{-x}}{19} + \frac{\sin 2x}{-6D+8}$$

$$\frac{e^{x}}{6} - \frac{e^{-x}}{19} - \frac{\sin 2x}{6D-8}$$

$$-\frac{\sin 2x}{6D-8} \times \frac{6D+8}{6D+8}$$

$$-\frac{(6D+8)\sin 2x}{36D^2-64}$$

$$-\frac{12\cos 2x + 8\sin 2x}{36D^2-64}$$

$D^2 \to -a^2$

$$-\frac{12\cos 2x + 8\sin 2x}{-208}$$

$$P.I = \frac{e^{x}}{6} - \frac{e^{-x}}{19} + \frac{12\cos 2x + 8\sin 2x}{208}$$

$y = CF + PI$

$$e^{3x}[c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x] + \frac{e^{x}}{6} - \frac{e^{-x}}{19} + \frac{12\cos 2x + 8\sin 2x}{208}$$

Type - 3.

Polynomial :-

PI when x = polynomial.

eg :- x^2+1, $3x^2+2x+1$, $2x+3$.

$$PI = \frac{1}{L(D)} X$$

$$= [L(D)]^{-1} X$$

To find the PI when X is a polynomial we express $\frac{1}{L(D)}$ as any one of the following

~~1.) $(1+x^{-1})$~~

1. $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$
2. $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \cdots$
3. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 \cdots$
4. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$

$3-5/3$ $\quad 4-5/3$ $\quad 4/3$

$$\begin{array}{l|l} & \frac{1}{3}x + 4/18 \\ \hline 6+5D+D^2 & 2x+3 \\ & 2x+5/3 \\ & (-) \\ \hline & 4/3 + 0 \end{array}$$

1. Solve

$y'' + 5y' + 6y = 2x + 3.$

$D^2y + 5Dy + 6y = 2x+3$

$(D^2 + 5D + 6)y = 2x+5$

$\frac{1}{3}x + 4/18 \quad 4/5$

$m^2 + 5m + 6$

$m = -2, -3$

$CF = c_1 e^{-2x} + c_2 e^{-3x}.$

$\frac{1}{3}(3$

$$PI = \frac{1}{L(D)} X = \frac{1}{D^2+5D+6}(2x+3)$$

$$= \frac{1}{D^2+5D+6}(2x+3)$$

$$D^2+5D+6 = 6\left[1+\frac{(D^2+5D)}{6}\right]$$

$$PI = \frac{1}{6\left[1+\frac{(D^2+5D)}{6}\right]}(2x+3).$$

$$\frac{1}{6}\left[1+\left[\frac{D^2+5D}{6}\right]\right]^{-1}(2x+3)$$

$$(1+x)^{-1} = 1-x+x^2-x^3+x^4-x^5+\dots$$

$= \frac{1}{6}$ using the formula for $(1+x)^{-1}$ we expand the above as

$$= \frac{1}{6}\left[1-\left[\frac{D^2+5D}{6}\right]\right].(2x+3)$$

$$= \frac{1}{6}\left[(2x+3)-\frac{1}{6}(D^2+5D)(2x+3)\right]$$

$$= \frac{1}{6}\left[2x+3-\frac{1}{\cancel{6}_3}[0+5(\cancel{2})]\right]$$

$$= \frac{1}{6}(2x+3-5/3)$$

$D(2x+3)=2$
$D^2(2x+3)=0$

$$PI = 1/6\,(2x+4/3).$$

$$y = CF + PI$$

$$c_1e^{-2x}+c_2e^{-3x}+\frac{1}{6}(2x+4/3).$$

2. $y'' + 3y' + y = 4x + 3$.

$D^2y + 3Dy + y = 4x + 3$

$[D^2 + 3D + 1]y = 4x + 3$

$m^2 + 3m + 1 = 0$

$m = \dfrac{-3 \pm \sqrt{5}}{2}$ Real

$CF = c_1 e^{\left(\frac{-3+\sqrt{5}}{2}\right)x} + c_2 e^{\left(\frac{-3-\sqrt{5}}{2}\right)x}$.

$$\begin{array}{r|l} & 4x - 9 \\ \hline 1+3D+D^2 & 4x + 3 \\ & 4x + 12 \\ \hline & 0 - 9 \\ & -9 + 0 \\ \hline & 0 \end{array}$$

$PI = \dfrac{1}{L(D)} \cancel{X}$

$= \dfrac{1}{D^2 + 3D + 1} (4x + 3)$.

$D^2 + 3D + 1 = 1\left[\dfrac{1 + (D^2 + 3D)}{1}\right]$

$PI = \dfrac{1}{1\left[1 + \dfrac{(D^2 + 3D)}{1}\right]} (4x + 3)$

$PI = [1 + (D^2 + 3D)]^{-1} (4x + 3)$.

$PI = [1 - (D^2 + 3D)] (4x + 3)$.

$[(4x + 3) - (D^2 + 3D)(4x + 3)]$.

$[(4x + 3) - 0 + 12]$

$[(4x + 3) - 12]$.

$PI = [4x - 9]$

$y = CF + PI$ //

8. $y'' + 3y' + y = 4x^2 + 3$

$D^2y + 3Dy + y = 4x^2 + 3$

$[D^2 + 3D + 1]y = 4x^2 + 3$

$m^2 + 3m + 1 = 0$

$m = \frac{-3 \pm \sqrt{5}}{2}$

$C.F = c_1 e^{\left(\frac{-3+\sqrt{5}}{2}\right)x} + c_2 e^{\left(\frac{-3-\sqrt{5}}{2}\right)x}$

$PI = \frac{1}{L(D)} X$

$\frac{1}{D^2 + 3D + 1} \times (4x^2 + 3)$

$D \to 8x$
$D^2 \to 8$
$D^3 \to 0$
$D^4 \to 0$

$\frac{1}{1 + (D^2 + 3D)} \times (4x^2 + 3)$

$(1 + (D^2 + 3D))^{-1} \; (4x^2 + 3)$

$[1 - (D^2 + 3D) + (D^2 + 3D)^2] [4x^2 + 3]$

$[(4x^2 + 3) - (4x^2 + 3)(D^2 + 3D) + (D^2 + 3D)^2 [4x^2 + 3]]$

$\cancel{(4x^2+3) - 4[8 + 24] + [0 + 0]}$

$\cancel{4x^2 + 3 - 24}$

$\cancel{4x^2 - 21}$

$[(4x^2 + 3) - (8 + 24x) + (0 + 24)]$

$4x^2 + 3 - 8 - 24x + 24$

$4x^2 - 24x - 19$,

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4. $y'' + 2y' + 3y = e^{3x} + x + 2$

$D^2y + 2Dy + 3y = e^{3x} + x + 2$

$[D^2 + 2D + 3]y = e^{3x} + x + 2$

$m^2 + 2m + 3 = 0$

$m = -1 \pm \sqrt{2}\, i$

$CF = e^{-x}[c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x]$.

$PI = \frac{1}{f(D)} x$

$\frac{1}{D^2 + 2D + 3} x[e^{3x} + x + 2]$

$= \frac{e^{3x}}{D^2 + 2D + 3}$

$D \to 3$

$PI_1 = \frac{e^{3x}}{18}$

$PI_2 = \frac{(x+2)}{(D^2 + 2D + 3)}$

$\frac{1}{3\left[1 + \frac{(D^2 + 2D)}{3}\right]} (x+2)$

$\frac{1}{3}\left[1 + \frac{(D^2 + 2D)}{3}\right]^{-1} (x+2)$.

$\frac{1}{3}\left[(x+2) - \left(\frac{D^2 + 2D}{3}\right)(x+2)\right]$.

$= \frac{1}{3}\left[(x+2) - \frac{1}{3}[0 + 2(1)]\right]$.

$\frac{1}{3}\left[(x+2) - \frac{2}{3}\right]$.

$\frac{1}{3}\left[x + 2 - \frac{2}{3}\right] = \frac{1}{3}\left[x + \frac{4}{3}\right]$.

$y = CF + PI_1 + PI_2$

5. $y'' + 2y' - 2y = 2x^2 + x + 1$.

$D^2y + 2Dy - 2y = 2x^2 + x + 1$.

$[D^2 + 2D - 2]y = 2x^2 + x + 1$.

$m^2 + 2m - 2 = 0$.

$m = -1 \pm \sqrt{3}$

$$CF = C_1 e^{(-1+\sqrt{3})x} + C_2 e^{(-1-\sqrt{3})x}$$

$$PI = \frac{1}{D^2+2D-2}[2x^2+x+1]$$

$$\frac{1}{-2\left[1 + \frac{D^2+2D}{-2}\right]}[2x^2+x+1].$$

$$-\frac{1}{2}\left[1 - \frac{(D^2+2D)}{2}\right]^{-1} [2x^2+x+1].$$

$$-\frac{1}{2}\left[1 + \frac{(D^2+2D)}{2} + \frac{(D^2+2D)^2}{2}\right][2x^2+x+1]$$

$$-\frac{1}{2}\left[(2x^2+x+1) + \frac{1}{2}[4 + 2(4x+1)] + \frac{1}{4}[0 + 4(4) + 0]\right]$$

$D = 4x+1$

$D^2 = 4$

$D^3 \to 0$

$D^4 \to 0$

$$-\frac{1}{2}\left[2x^2+x+1 + 2 + 4x + 1 + 4\right].$$

$$-\frac{1}{2}\left[2x^2 + 5x + 8\right]$$

Type → 4.

PI. when $X = e^{ax} V$. wher V is fun^{n} of x.

$$PI = \frac{1}{L(D)} e^{ax} \cdot V.$$

$D \to D + a$.

$$= e^{ax} \frac{1}{L(D+a)} V.$$

Solve

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{x} \sin 3x.$$

$$D^2y + 4Dy + 4y = e^{x} \sin 3x.$$

$$[D^2 + 4D + 4]y = e^{x} \sin 3x.$$

$$m^2 + 4m + 4 = 0$$

$$m = -2, -2$$

$$CF = [c_1 + c_2 x] e^{-2x}.$$

$$P.I = \frac{1}{D^2 + 4D + 4} e^{x} \sin 3x. \quad a = 1$$

$D \to D + 1$

$$= e^{x} \frac{1}{(D+1)^2 + 4(D+1) + 4} \sin 3x.$$

$$= e^{x} \frac{1}{D^2 + 6D + 9} \sin 3x.$$

$D^2 \to -a^2 = -9$.

$$= e^{x} \frac{1}{-9 + 6D + 9} \sin 3x = e^{x} \frac{1}{6D} \sin 3x.$$

$$= \frac{e^x}{6}\int \sin 3x$$

$$= \frac{e^x}{6} \frac{-\cos 3x}{3}$$

$$= -\frac{e^x}{18}\cos 3x$$

$$y = CF + PI$$

$$[C_1 + C_2 x]e^{-2x} + \frac{e^x}{18}\cos 3x$$

2. $y'' + 7y' + 12y = e^{-x}\cos^2 x$

$$D^2y + 7Dy + 12y = e^{-x}\cos^2 x$$

$$[D^2 + 7D + 12]y = e^{-x}\cos^2 x$$

$$m^2 + 7m + 12 = 0$$

$$m = -4 - 3$$

$$CF = C_1 e^{-4x} + C_2 e^{-8x}$$

$$PI = \frac{1}{D^2 + 7D + 12}(e^{-x}\cos^2 x)$$

$$D \to D + a$$

$$D \Rightarrow D - 1$$

$$\frac{1}{(D-1)^2 + 7(D-1) + 12}$$

$$\frac{1}{D^2 + 1 - 2D + 7D - 7 + 12}$$

$$\frac{1}{D^2 + 5D + 6}$$

$$= \frac{1}{D^2 + 5D + 6} e^{-x}\cos^2 x$$

$$e^{-x}\frac{1}{D^2 + 5D + 6}\cos^2 x$$

$$e^{-x} \cdot \frac{1}{D^2+5D+6} \quad \frac{1+\cos 2x}{2}$$

$$= \frac{e^{-x}}{2}\left[\frac{1+\cos 2x}{D^2+5D+6}\right].$$

$$= \frac{e^{-x}}{2}\left[\frac{1\,e^{0x}}{D^2+5D+6} + \frac{\cos 2x}{D^2+5D+6}\right].$$

$D \to 0$ $\qquad$ $D^2 \to -a^2 = -4.$

$$\frac{e^{-x}}{2}\left[\frac{1}{6} + \frac{\cos 2x}{-4+5D+6}\right]$$

$$= \frac{e^{-x}}{2}\left[\frac{1}{6} + \frac{\cos 2x}{5D+2}\right]$$

$$= \frac{e^{-x}}{2}\left[\frac{1}{6} + \frac{\cos 2x}{5D+2} \cdot \frac{5D-2}{5D-2}\right]$$

$$= \frac{e^{-x}}{2}\left[\frac{1}{6} + \frac{(5D-2)\cos 2x}{(25D^2-4)}\right].$$

$D^2 \to -a^2 = -4$

$$\frac{e^{-x}}{2}\left[\frac{1}{6} + \frac{-10\sin 2x - 2\cos 2x}{-100-4}\right]$$

$$\frac{e^{-x}}{2}\left[\frac{1}{6} + \frac{10\sin 2x + 2\cos 2x}{104}\right].$$

$y = CF + PI.$

3. $y'' + 2y' + y = e^{3x}\sinh 2x.$

$D^2y + 2Dy + y = e^{3x}\sinh 2x$

$[D^2 + 2D + 1]y = e^{3x}\sinh 2x.$

$m^2 + 2m + 1 = 0$

$m = -1, -1$

$$CF = [c_1 + c_2 x] e^{-x}.$$

$$PI = \frac{1}{D^2+2D+1}[e^{3x} \sinh 2x].$$

$$e^{3x} \cdot \frac{1}{(D+3)^2 + 2(D+3)+1} \sinh 2x$$

$$= e^{3x} \frac{1}{D^2+9+6D+2D+6+1} \sinh 2x.$$

$$e^{3x} \frac{1}{D^2+8D+16} \sinh 2x.$$

$$\frac{e^{3x}}{2}\left[\frac{1}{D^2+8D+16} e^{2x} - e^{-2x}\right]$$

$$\frac{e^{3x}}{2}\left[\frac{e^{2x}}{D^2+8D+16} - \frac{e^{-2x}}{D^2+8D+16}\right]$$

$$D \to 2 \qquad D \to -2.$$

$$\frac{e^{3x}}{2}\left[\frac{e^{2x}}{4+8D+16} - \frac{e^{-2x}}{-2+8D+16}\right]$$

$$\frac{e^{3x}}{2}\left[\frac{e^{2x}}{8D+18} - \frac{e^{-2x}}{8D+14}\right].$$

$$\frac{e^{3x}}{2}\left[\frac{e^{2x}}{36} - \frac{e^{-2x}}{4}\right].$$

$$y = CF + PI$$

$$[c_1 + c_2 x] e^{-x} + \frac{e^{3x}}{2}\left[\frac{e^{2x}}{36} - \frac{e^{-2x}}{4}\right]$$

4. $(D^2-1)y = e^x(x^2+1)$.

$m^2-1=0$

$m^2=\sqrt{1}$

$m=\pm 1$ Imaginary.

$CF = c_1e^x + c_2e^{-x}$.

$PI = \dfrac{e^x(x^2+1)}{D^2-1}$

$D \to D+a = D+1$.

$e^x \dfrac{1}{(D+1)^2-1}(x^2+1)$.

$= e^x \dfrac{1}{D^2+2D}(x^2+1)$.

$e^x \dfrac{1}{2D\left[1+\left(\frac{D}{2}\right)\right]}(x^2+1)$

$e^x \times \dfrac{1}{2D}\left[1+(D/2)\right]^{-1}(x^2+1)$.

$\dfrac{e^x}{2}\times\dfrac{1}{D}\left[1-\dfrac{D}{2}+\dfrac{D^2}{4}\right](x^2+1)$.

$\dfrac{e^x}{2}\times\dfrac{1}{D}\left[x^2+1-\dfrac{1}{2}(2x)+\dfrac{1}{4}[2]\right]$

$\dfrac{e^x}{2}\times\dfrac{1}{D}\left[x^2+1-x+1/2\right]$

$\dfrac{e^x}{2}\times\dfrac{1}{D}\left[x^2-x+3/2\right]$ ~~dx~~

$\dfrac{e^x}{2}\times\int\left[x^2-x+3/2\right]dx$

$2D+D^2$

$x^3/6 + x^4/4$

x^2+1

x^2+x

$-x+1$

$-x-1/2$

$3/2$

$2\dfrac{2x}{4}$ $\quad x^2$ $\quad \dfrac{2x^2}{4}$

x^3

$3x^2$

$\dfrac{3x^2}{2}$ $\quad 3x^2 \quad 3/2$

$\dfrac{6x}{6}$ $\quad \dfrac{6x}{6}$ $\quad \dfrac{3x}{4}$

$\left\{\begin{array}{l} D \to 2x \\ D^2 \to 2 \\ D^3 \to 0 \\ D^4 \Rightarrow 0 \end{array}\right.$

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$$= \frac{e^x}{2}\left[\frac{x^3}{3} - \frac{x^2}{2} + \frac{3x}{2}\right].$$

5. $y'' + 5y' + 10y = e^{3x}(x+1) + \sin^2 x$.

$D^2y + 5Dy + 10y = 0$

$(D^2 + 5D + 10)y = 0$

$m^2 + 5m + 10 = 0$

$m = -\frac{5}{2} \pm \frac{\sqrt{5}}{2} i$

$$CF = e^{(-5/2)x}\left[C_1 \cos\left(\sqrt{5}/_2\right)x + C_2 \sin\left(\sqrt{5}/_2\right)x\right]$$

$$PI = \frac{e^{3x}(x+1) + \sin^2 x}{D^2 + 5D + 10}$$

$$\frac{e^{3x}(x+1)}{D^2+5D+10} + \frac{\sin^2 x}{D^2+5D+10}$$

$D \to D + a = D + 3$

$$= e^{3x}\frac{(x+1)}{(D+3)^2 + 5(D+3) + 10} + \frac{1 - \cos 2x}{2}$$

$$e^{3x}\frac{(x+1)}{D^2 + 9 + 6D + 5D + 15 + 10} + \frac{1/2\,[1 - \cos 2x]}{D^2+5D+10}$$

$$e^{3x}\frac{(x+1)}{D^2+11D+34} + \frac{1}{2}\left[\frac{1\,e^{0x}}{D^2+5D+10} - \frac{\cos 2x}{D^2+5D+10}\right]$$

$D^2 \to -4$

$$= e^{3x}\frac{(x+1)}{D^2+11D+34} + 1/2\left[1/10 - \frac{1}{5D+6}\cos 2x\right]$$

$$= e^{3x}\frac{1}{34\left[1 + \frac{(D^2+11D)}{34}\right]} x+1 + 1/2\left[1/10 - \frac{\cos 2x}{5D+6}\cdot\frac{5D-6}{5D-6}\right]$$

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$$= \cancel{\frac{e^{3x}}{34}} \cancel{1 + D^2 + 11}$$

$$= \frac{e^{3x}}{34}\left[\left[1 + \left(\frac{D^2 + 11D}{34}\right)\right]^{-1}(x+1)\right] + \frac{1}{2}\left[\frac{1}{10} + \frac{10\sin 2x}{25D^2 - 36}\right.$$

$$-6\cos 2x.$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3$$

$$= \frac{e^{3x}}{34}\left[1 - \frac{D^2 + 11D}{34}\right](x+1) + \frac{1}{2}\left[\frac{1}{10} + \frac{10\sin 2x + 6\cos 2x}{25D^2 - 36}\right.$$

$$D^2 \to -4,$$

$$\frac{e^{3x}}{34}\left[1 - \frac{D^2 + 11D}{34}\right](x+1) + \frac{1}{2}\left[\frac{1}{10} + \frac{(10\sin 2x + 6\cos 2x)}{-136}\right.$$

$$= \frac{e^{3x}}{34}\left(x + 1 - \frac{1}{34}(0 + 11(1))\right] + \ldots$$

$$= \frac{e^{3x}}{34}\left[x + 1 - \frac{1}{34}(11)\right] + \ldots$$

$$D(x+1) = 1$$

$$D^2 = 0$$

$$\cancel{\frac{1}{34}} e^{3x}\left(\cancel{x + 1 - 17}\right.$$

$$\frac{1}{34} e^{3x}\left(x + \frac{23}{34}\right] + \frac{1}{2}\left[\frac{1}{10} + \frac{(10\sin 2x + 6\cos 2x)}{-136}\right]$$

$$y = CF + PI$$

6. $y'' + 12y' + 10y = e^{3x+2} + 2\cos 2x.$

$$D^2y + 12Dy + 10y = \ldots$$

$$(D^2 + 12D + 10)y = 0$$

$$m^2 + 12m + 10 = 0$$

$$m = -6 \pm \sqrt{26}.$$

$$CF = C_1 e^{(-6+\sqrt{26})x} + C_2 e^{(-6-\sqrt{26})x}$$

$$PI_1 = \frac{e^{3x+2}}{D^2+12D+10}$$

$\cancel{D^2 \to 9}$

$D \to 3$

$$PI_1 \quad \frac{e^{3x+2}}{9+36+10}$$

$$PI_1 \quad \frac{e^{3x+2}}{55}$$

$$PI_2 = \frac{2\cos 2x}{D^2+12D+10}$$

$D^2 \to -a^2$

$$= \frac{2\cos 2x}{-4+12D+10}$$

$$= \frac{2\cos 2x}{12D+6} \times \frac{12D-6}{12D-6}$$

$$\frac{2\cos 2x\,(12D-6)}{144D^2-36}$$

$$= \frac{-24 \times 2\sin 2x - 12\cos 2x}{144D^2-36}$$

$D^2 \to -4$

$$\frac{-48\sin 2x - 12\cos 2x}{648}$$

$$PI_2 = \frac{-1}{648}(48\sin 2x + 12\cos 2x)$$

$y = CF + PI_1 + PI_2$.

7. $y'' + 8y' + 6y = 3^x + 4 + \sinh^2 2x$.

$D^2y + 8Dy + 6y =$

$(D^2 + 8D + 6)y = 0$

$m^2 + 8m + 6 = 0$.

$m = -4 \pm \sqrt{10}$.

$CF = e^{-4x}\left[c_1 \cos\sqrt{10}x + c_2 \sin\sqrt{10}x\right]$.

$$PI = \frac{3^x + 4 + \sinh^2 2x}{D^2 + 8D + 6}$$

$$PI_1 = \frac{3^x}{D^2 + 8D + 6}$$

$$= \frac{e^{(\log 2)x}}{D^2 + 8D + 6}$$

$D \to a$

$$\frac{e^{(\log 2)x}}{(\log 2)^2 + 8(\log 2) + 6} \to (1)$$

$$PI_2 \to \frac{4e^{0x}}{D^2 + 8D + 6} = \frac{4}{6}$$

$$PI_3 \to \frac{\sinh^2 2x}{D^2 + 8D + 6} = \frac{1}{4}\frac{\left[e^{2x} - e^{-2x}\right]^2}{D^2 + 8D + 6}$$

$$\frac{1}{4}\left[\frac{e^{4x} + e^{-4x} - 2\cancel{e^{2x}}\cancel{e^{-2x}}}{D^2 + 8D + 6}\right]$$

$$\frac{1}{4}\left[\frac{e^{4x}}{D^2+8D+6}\right]+\frac{1}{4}\left[\frac{e^{-4x}}{D^2+8D+6}\right] - 2e^{0x}$$

$$D \to 4$$

$$\frac{1}{4}\left[\frac{e^{4x}}{54}\right]+\frac{1}{4}\left[\frac{e^{-4x}}{-10}\right]+\frac{2}{6} \to (3)$$

$$y = CF + PI_1 + PI_2 + PI_3$$

$$= e^{-4x}\left[c_1\cos\sqrt{10}x + c_2\sin\sqrt{10}x\right] + \frac{3^x}{(\log 2)^2+8(\log 2)+6}$$

$$+\frac{4}{6}+\frac{1}{4}\left[\frac{e^{4x}}{54}-\frac{e^{-4x}}{10}\right]+\frac{2}{6}$$

8) $y'' - y = e^x + 8(x^2+1) + 8$

$D^2y - 1y = \cdot$

$(D^2-1)y = e^x + 8(x^2+1) + 8$

$m^2 - 1 = 0$

$m^2 = 1$

$m = \pm 1.$

$CF = c_1e^x + c_2e^{-x}$

$$PI = \frac{e^x + 8x^2 + 16}{D^2 - 1}$$

$$PI_1 = \frac{e^x}{D^2-1} = \frac{e^x}{0}$$

$$= \frac{xe^x}{2D} = \frac{xe^x}{2}\frac{1}{D} = \frac{xe^x}{2}.$$

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$$PI_2 = \frac{8x^2}{D^2-1} \quad (8x^2+1)$$

$$= \frac{8\ x^2}{-(1-D^2)}$$

$$= -8\ (1-D^2)^{-1} x^2$$

$$= -8(1+D^2+D^3)\ x^2$$

$$= -8\ (x^2+1+\ldots)$$

$$= -8x^2 - 8$$

$$PI_3 = \frac{16}{-1}$$

$$y = CF + PI_1 + PI_2 + PI_3$$

q. ~~y~~

Type - 5 :

PI when $X = xV$, V is func^{-n} of x.

$$PI = \left[x - \frac{L'(D)}{L(D)}\right] \frac{V}{L(D)}$$

1. Solve

$$y'' + 16y = x\sin 3x.$$

$$D^2y + 16y = x\sin 3x.$$

$$(D^2+16)y = x\sin 3x.$$

$$m^2 + 16 = 0.$$

$$m = \sqrt{\pm\ 16}$$

$$m = \sqrt{16}\ i$$

$$m = \pm 4i$$

$C.F = e^{0x}[C_1 \cos 4x + C_2 \sin 4x]$.

$PI = \left[\dfrac{x \sin 3x}{D^2+16}\right]$

$PI = \left[x - \dfrac{L'(D)}{L(D)}\right] \dfrac{V}{L(D)}$

$PI = \left[x - \dfrac{2D}{D^2+16}\right] \dfrac{\sin 3x}{D^2+16}$ $\quad D^2 \to -9$.

$PI = \left[x - \dfrac{2D}{D^2+16}\right] \dfrac{\sin 3x}{-9+16}$.

$PI = \left[x - \dfrac{2D}{D^2+16}\right] \dfrac{\sin 3x}{7}$

$\dfrac{x \sin 3x}{7} - \dfrac{2D}{D^2+16} \dfrac{\sin 3x}{7}$.

$\dfrac{x \sin 3x}{7} - \dfrac{2D}{7} \dfrac{\sin 3x}{7}$. $\quad D \to -9$

$\dfrac{x \sin 3x}{7} - \dfrac{2}{49} [3 \cos 3x]$

$PI = \dfrac{7x \sin 3x - 6 \cos 3x}{49}$.

$y = CF + PI$

2. $y'' - 2y' + y = x\cos x$.

$D^2y - 2Dy + y = x\cos x$

$m^2 + 2m + 1 = 0$

$m = 1, 1$

$CF = [c_1 + c_2 x]e^x$

$$PI = \frac{x\cos x}{(D^2 - 2D + 1)}$$

$$PI = \left[x - \frac{L'(D)}{L(D)}\right]\frac{V}{L(D)}.$$

$$PI = \left[x - \frac{2D - 2}{D^2 - 2D + 1}\right]\frac{\cos x}{D^2 + 2D + 1}$$

$D^2 \to -1$.

$$PI = \left[x - \frac{2D - 2}{D^2 + 2D + 1}\right]\frac{\cos x}{-2D}.$$

$$PI = \left[x - \frac{2D - 2}{D^2 - 2D + 1}\right]\frac{1}{-2}\sin x.$$

$$PI = \frac{x\sin x}{-2} + \frac{2D - 2}{D^2 - 2D + 1}\cdot\frac{\sin x}{2},$$

$$= \frac{x\sin x}{-2} + \frac{2D - 2}{-2D}\,\frac{\sin x}{2}$$

$$= -\frac{x\sin x}{2} - \frac{2D - 2}{2D}\cdot\frac{\sin x}{2}.$$

$$= -\tfrac{1}{2}\left[x\sin x + \frac{2(D-1)}{2}(-\cos x)\right].$$

$$-\tfrac{1}{2}\left[x\sin x - (D-1)\cos x\right].$$

$$-\tfrac{1}{2}\left[x\sin x + \sin x + \cos x\right]$$

$y = CF + PI$

3. $y'' - 2y' + y = xe^x \cos x$.

$D^2y - 2Dy + y = xe^x \cos x$

$(D^2 - 2D + 1)y = xe^x \cos x$.

$m^2 - 2m + 1 = 0$

$m = 1, 1$

$CF = (c_1 + c_2 x)e^x$.

$$PI = \frac{xe^x \cos x}{D^2 - 2D + 1} = \frac{e^x x \cos x}{D^2 - 2D + 1}$$

$D \to D + 1$

$$= \frac{e^x x \cos x}{(D+1)^2 - 2(D+1) + 1}$$

$$= e^x \left[\frac{x \cos x}{D^2} \right]$$

$$P.I = e^x \left[x - \frac{L'(D)}{L(D)} \right] \frac{V}{L(D)}$$

$\frac{1}{D^2} \cos x = \iint \cos x$

$\int \sin x$

$= -\cos x$

$$= e^x \left[x - \frac{2D}{D^2} \right] \frac{\cos x}{D^2}$$

$$PI = e^x \left[x - \frac{2D}{D^2} \right] - \cos x.$$

$$e^x \left[-x \cos x + \frac{2D}{\cancel{D^2}} \cos x \right]$$

$$e^x \left[-x \cos x + \frac{2}{D} \cos x \right]$$

$$e^x \left[-x \cos x + 2 \sin x \right].$$

4. $y'' + 4y' + 4y = e^{-2x} + \sin 3x + 4$

$D^2y + 4Dy + 4y = - -$

$(D^2 + 4D + 4)y = 0$

$m^2 + 4m + 4 = 0$

$m = -2, -2$

$CF = [c_1 + c_2 x]e^{-2x}$

$$PI = \frac{e^{-2x} + \sin 3x + 4}{D^2 + 4D + 4}$$

$$= \frac{e^{-2x}}{D^2+4D+4} + \frac{\sin 3x}{D^2+4D+4} + 1$$

$D \rightarrow -2 \quad D^2 \rightarrow -a^2$

$$\frac{x e^{-2x}}{(2D+4)} + \frac{\sin 3x}{4D-5}$$

$$\frac{x^2 e^{-2x}}{2} + \frac{\sin 3x}{4D-5} \times \frac{4D+5}{4D+5}$$

$$\frac{x^2 e^{-2x}}{2} + \frac{12\cos 3x + 5\sin 3x}{16D^2 - 25} + 1$$

$D^2 \rightarrow -a^2$

$$\frac{x^2 e^{-2x}}{2} + \frac{12\cos 3x + 5\sin 3x}{16(-9) - 25} + 1$$

$$\frac{x^2 e^{-2x}}{2} - \frac{1}{169}(12\cos 3x + 5\sin 3x) + 1$$

$y = CF + PI$

5. $y'' + 12y' + 12y = 2^{-x} + \sinh 3x$

$D^2y + 12Dy + 12y = - - -$

$(D^2 + 12D + 12)y = 2^{-x} + \sinh 3x$

$m^2 + 12m + 12 = 0$

$m = -6 \pm 2\sqrt{6}$

$CF = c_1 e^{(-6+2\sqrt{6})x} + c_2 e^{(-6-2\sqrt{6})x}$

$PI = \dfrac{2^{-x} + \sinh 3x}{D^2+12D+12}$

$\dfrac{2^{-x}}{D^2+12D+12} + \dfrac{\sinh 3x}{D^2+12D+12}$

$\dfrac{e^{-(\log 2)x}}{D^2+12D+12} + \dfrac{1}{2}\dfrac{e^{3x} - e^{-3x}}{D^2+12D+12}$

$D \to -\log 2$

$= \dfrac{e^{-\log 2\, x}}{(\log 2)^2 - 12(\log 2) + 12} + \dfrac{1}{2}\left[\dfrac{e^{3x}}{D^2+12D+12} - \dfrac{e^{-3x}}{D^2+12D+12}\right]$

$= \dfrac{e^{-\log 2\, x}}{\log 2} - - - + \dfrac{1}{2}\left[\dfrac{e^{3x}}{57} + \dfrac{e^{-3x}}{15}\right]$

$= \dfrac{e^{-\log 2\, x}}{(\log 2)^2 - 12(\log 2) + 12} + \dfrac{1}{2}\left[\dfrac{e^{3x}}{57} + \dfrac{e^{-3x}}{15}\right]$

6. $y'' + 5y' + 6y = 8(e^x + x^2 + 1)$.

$D^2y + 5Dy + 6y = 8 \ldots$

$(D^2 + 5D + 6)y = 8 \ldots$

$m^2 + 5m + 6 = 0$

$m = -3, -2$

$CF = c_1 e^{-3x} + c_2 e^{-2x}$

$D(x^2+1) = 2x$

$D^2 = 2$

$D^3 = 0$

$$PI = \frac{8e^x + 8(x^2+1)}{D^2 + 5D + 6}$$

$$= \frac{8e^x}{D^2+5D+6} + \frac{8(x^2+1)}{D^2+5D+6}$$

$D \to 1$

$$\frac{8e^x}{12} + 8\,\frac{1}{6}\left[\frac{(x^2+1)}{1 + \left(\frac{D^2+5D}{6}\right)}\right]$$

$$\frac{8e^x}{12} + \frac{8}{6}\left[\left(1 + \left(\frac{D^2+5D}{6}\right)\right)^{-1}(x^2+1)\right].$$

$$\frac{8e^x}{12} + \frac{8}{6}\left[1 - \frac{D^2+5D}{6} + \left(\frac{D^2+5D}{6}\right)^2\right](x^2+1).$$

$$\cancel{\frac{8e^x}{12} + \frac{8}{6}\left[(x^2+1) - \frac{2+10x}{6} + \frac{2+50x}{6}\right]}$$

$$\cancel{\frac{8e^x}{12} + \frac{8}{36}\left[(x^2+1)\right.}$$

$$\frac{8e^x}{12} + \frac{8}{6}\left[\left[1 - \frac{D^2+5D}{6} + \frac{1}{36}\left(D^4 + 10D^3 + 25D^2\right)\right](x^2+1)\right]$$

$$\frac{8e^x}{12} + \frac{8}{6}\left[(x^2+1) - \frac{2+10x}{6} + 0 + 0 + 50\right]$$

$$\frac{8e^x}{12} + \frac{8}{6}\left[(x^2+1) - \frac{2+10x}{6} + \frac{50}{36}\right].$$

$$\frac{8e^{x}}{12}+\frac{8}{6}\left[\frac{6(x^2+1)-2+10}{6}+\frac{80}{36}\right]$$

$$\frac{6x^2+6-2+10}{6}+\frac{50}{36}$$

$$=\frac{8e^{x}}{12}+\frac{8}{6}\left[\frac{6x^2+14}{6}+\frac{50}{36}\right]$$

7. $(D^4-3D^2-4)y = 5\sin 2x - e^{-2x}$.

A.E $m^4-3m^2-4=0$

$m^2=t$

$t^2-3t-4=0$

$t=4,-1$

$t=4$

$m^2=4$

$m=\pm 2$, $m^2=-1$

$m=\sqrt{-1}$

$m=\pm i$

$m, +2, -2, \pm i$

$CF = c_1e^{2x}+c_2e^{-2x}+e^{0x}[c_3\cos x+c_4\sin x]$

$= c_1e^{2x}+c_2e^{-2x}+c_1\cos x+c_2\sin x$

$$PI = \frac{5\sin 2x - e^{-2x}}{D^4-3D^2-4}.$$

$$= \frac{5\sin 2x}{D^4-3D^2-4} - \frac{e^{-2x}}{D^4-3D^2-4}.$$

$D^2 \to -a^2$

$D^2 \to -4$

$$= \frac{5\sin 2x}{D^4+12-4} - \frac{e^{-2x}}{16-32-4}$$

$$\frac{5\sin 2x}{(-4)^2+12-4} - \frac{e^{-2x}}{0}$$

$$\frac{5\sin 2x}{24} - \frac{x e^{-2x}}{4D^3-6D}$$

$$\frac{5\sin 2x}{24} - \frac{x e^{-2x}}{-20}$$

$$PI = \frac{5\sin 2x}{24} + \frac{x e^{-2x}}{20}$$

$$y = CF + PI$$

8. $(D^3+1)y = \cos^2(x/2) + e^{-x}$

$m^3 + 1 = 0$

$m = \sqrt[3]{-1}$

$m = -1, \quad 1/2 \pm \sqrt{3}/2\, i$

$$\therefore CF = c_1 e^{-x} + e^{1/2}\left(c_1\cos(\sqrt{3}/2)x + c_2\sin(\sqrt{3}/2)x\right)$$

$$PI = \frac{\cos^2(x/2) + e^{-x}}{D^3+1}$$

$$= \frac{\cos^2(x/2)}{D^3+1} + \frac{e^{-x}}{\cancel{\cos^2(x/2)^3}\; D^3+1}$$

$$\frac{\frac{1+\cos x}{2}}{D^3+1} + \frac{e^{-x}}{D^3+1}$$

$$1/2\left[\frac{1+\cos x}{D^3+1}\right] + \frac{e^{-x}}{D^3+1}$$

$$\frac{1}{2}\left[\frac{1+\cos x}{-D+1}\right] + \frac{x e^{-x}}{3D^2}$$

$$\frac{1}{2}\left[1-\left(\frac{\cos x}{D-1}\cdot\frac{D+1}{D+1}\right)\right] + \frac{x e^{-x}}{3}$$

$$\frac{1}{2}\left[1-\left[\frac{-\sin x+\cos x}{D^2-1}\right]\right] + \frac{x e^{-x}}{3}$$

$$\frac{1}{2}\left[1+\frac{(\sin x-\cos x)}{-2}\right]$$

$$\frac{1}{2}\left[1-\frac{(\sin x-\cos x)}{2}\right] + \frac{x e^{-x}}{3}$$

$$\frac{1}{2}\left[1+\frac{(\cos x-\sin x)}{2}\right] + \frac{x e^{-x}}{3}$$

$$\frac{1}{4}\left[2+(\cos x-\sin x)\right] + \frac{x e^{-x}}{3}$$

$y = CF + PI$

9. $(D^3-3D^2+4D-2)y = e^x + \cos x$

$m^3-3m^2+4m-2=0$

put $m=1$

1	1	-3	+4	-2
	0	1	-2	2
	1	-2	2	0

$m = 1,\ 1+i,\ 1-i$

m^2-2m+2

$CF = c_1 e^x + e^x(c_2\cos x + c_3\sin x)$

$$PI = \frac{e^x+\cos x}{D^3-3D^2+4D-2}$$

$$\frac{e^x}{D^3-3D^2+4D-2} + \frac{\cos x}{D^3-3D^2+4D-2}$$

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$$\frac{e^{x}}{1-3+4-2} + \frac{\cos x}{-D+3+4D-2} \qquad D^2 = -a^2$$

$$\frac{xe^{x}}{3D^2-6D+4} + \frac{\cos x}{3D+1}$$

$$xe^{x} + \frac{\cos x}{3D+1}\,\frac{3D-1}{3D-1}$$

$$xe^{x} + \left(\frac{-3\sin x - \cos x}{9D^2-1}\right)$$

$$xe^{x} + \left(\frac{3\sin x + \cos x}{9D^2-1}\right)$$

$$= xe^{x} + \frac{3\sin x + \cos x}{10}$$

$y = CF + PI$

10. $y''' + y'' + 4y' + 4y = 0$

$D^3y + D^2y + 4Dy + 4y = 0 \rightarrow (1)$

$(D^3 + D^2 + 4D + 4)y = 0$

$m^3 + m^2 + 4m + 4$

$m = -1, \pm 2i$

$CF = c_1e^{-x} + e^{0x}[C_2\cos 2x + C_3\sin 2x] \rightarrow (2)$

Case i

$y(0) = 0$

$x = 0$

$0 = c_1e^{-0} + C_2\cos 2(0) + C_3\sin 2(0)$

$0 = C_1 + C_2 \rightarrow (3)$

Diff (2) w.r.t x.

$\frac{dy}{dx} = -c_1e^{-x} + 2C_2(-\sin 2x) + 2C_3\cos 2x \rightarrow (4)$

$0 = -c_1 + 2c_3 \longrightarrow$ B

$y''(0) = -5$

Diff 4 w.r.t x.

$$\frac{d^2y}{dx^2} = c_1 e^{-x} + 4c_2(-\cos 2x) - 4c_3 \sin 2x$$

$-5 = c_1 - 4c_2 \longrightarrow$ C

Solve 3, B, C

$c_1 = -1, \; c_2 = 1, \; c_3 = -1/2$

substitute in 2

$$y = -1e^{-x} + \cos 2x - \frac{1}{2}\sin 2x$$

Note :- $\int \sin x \, dx = -\cos x + c$

$\int \cos x \, dx = \sin x + c$

$\int \tan x \, dx = \log|\sec x| + c$.

$\int \cot x = \log|\sin x| + c$.

$\int \operatorname{cosec} x = \log|\operatorname{cosec} x - \cot x| + c$.

$\int \sec x \, dx = \log(\sec x + \tan x)$.

$$\int \frac{f'(x)}{f(x)} dx = \log f(x).$$

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METHOD of variation of parameters

* consider D.E

$$a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_1 y = X \rightarrow (1)$$

* Find CF $= c_1 y_1 + c_2 y_2$
* Replace c_1 & c_2 by A & B & assume the Solution as $Ay_1 + By_2 \rightarrow (2)$.

where A & B are function of X.

* Find Wronskian $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$
* Find A & B using

$$A = -\int \frac{X}{W} y_2 \, dx$$

$$B = \int \frac{X}{W} y_1 \, dx.$$

* Substituting A & B in (2) we get the required solⁿ.

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1. Solve

$y'' + 4y = \tan 2x$ by method of variation of parameter

$(D^2+4)y = \tan 2x \rightarrow (1)$

A.E $= m^2 + 4 = 0$

$m = \pm 2i$

C.F $= c_1 \cos 2x + c_2 \sin 2x \rightarrow (2)$

$c_1 \rightarrow A \quad , \quad c_2 \rightarrow B$

let the sol^n be $A\cos 2x + B\sin 2x$.

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$2\cos^2 2x + 2\sin^2 2x$

$2[\cos^2 2x + \sin^2 2x]$

$W = 2$

<u>To find</u>

$$A = -\int \frac{X}{W} y_2 \, dx$$

$$= -\int \frac{\tan 2x}{2} (\sin 2x) \cdot dx$$

$$= -\frac{1}{2} \int \tan 2x \cdot \sin 2x \cdot dx$$

$$= -\frac{1}{2} \int \frac{\sin 2x}{\cos 2x} \sin 2x \cdot dx$$

$$= -\frac{1}{2}\int \frac{(\sin^2 2x)}{\cos 2x} \, dx$$

$$= -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x} \, dx$$

$$= -\frac{1}{2}\left[\int \frac{1}{\cos 2x} - \int \frac{\cos^2 2x}{\cos 2x}\right] dx$$

$$-\frac{1}{2}\left[\int \sec x - \int \cos 2x\right] dx$$

$$-\frac{1}{2}\left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right] + c_1$$

$$A = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right] + c_1 \rightarrow (2)$$

$$B = \int \frac{x}{w} y_1 \, dx$$

$$= \int \frac{\tan 2x}{2} \cos 2x + c$$

$$= \frac{1}{2}\int \frac{\sin 2x}{\cos 2x} \cdot \cos 2x + c$$

$$\frac{1}{2} - \frac{\cos 2x}{2} + c_2$$

$$B = -\frac{1}{4} \cos 2x + c_2 \; // \rightarrow (4)$$

$$CF = \left[-\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right] + c_1\right] \cdot \cos 2x$$

$$+ \left[-\frac{1}{4}\cos 2x + c_2\right] \sin 2x$$

$$= c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4}(\log(\sec 2x + \tan 2x) - \sin 2x) \times \cos 2x - \frac{1}{4} \cos 2x \sin 2x$$

$y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{4} \log[\sec 2x + \tan 2x]$
$x \cos 2x$.

2. $y'' + y = \operatorname{cosec} x$.

$(D^2+1)y = \operatorname{cosec} x$.

$m^2 + 1 = 0$.

$m = \pm i$

$CF = C_1 \cos x + C_2 \sin x$.

$C_1 \rightarrow A \quad C_2 \rightarrow B$

$\therefore = A\cos x + B \sin x$.

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}.$$

$W = (\cos^2 x + \sin^2 x)$

$W = 1$.

$A = -\int \frac{x}{W} y_2 \, dx$

$= -\int \frac{\operatorname{cosec} x}{1} \cdot \sin x \cdot dx$

$A = -\int \frac{1}{\sin x} \cdot \sin x \, dx$

$A = -x + C_1$

$B = \int \frac{x}{W} y_1 \, dx$.

$= \int \operatorname{cosec} x \cdot \cos x \cdot dx$.

$= \int \cot x \cdot dx$.

$= \log|\sin x| + C_2$.

$\therefore y = (-x + c_1)\cos x + (\log(\sin x) + c_2)\sin x$.

$y = c_1\cos x + c_2\sin x - x\cos x + \log(\sin x)\sin x$

3. $y'' + y = \sec x$.

$(D^2+1)y = \sec x$.

$m^2 + 1 = 0$

$m = \pm i$

$CF = c_1\cos x + c_2\sin x$.

$c_1 \to A \quad c_2 \to B$.

$A\cos x + B\sin x$.

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$W = 1$

$A = -\int \frac{X}{W} \cdot y_2\, dx$.

$= -\int \sec x \cdot \sin x \cdot dx$.

$= -\int \tan x \cdot dx$.

$A = -\log|\sec x| + c_1$.

$B = \int \frac{X}{W} y_1\, dx$.

$= \int \sec x \cdot \cos x \cdot dx + c_2$

$B = x + c_2$

$y = [-\log(\sec x) + c_1]\cos x + [x + c_2]\sin x$.

$y = c_1\cos x + c_2\sin x - \log(\sec x)\cos x + x\sin x$

4. $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ · by MOVOP

$(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$,

$m^2 - 6m + 9 = 0$

$m = 3, 3$.

$CF = [C_1 + C_2 x]e^{3x}$,

$[A + Bx]e^{3x}$

$Ae^{3x} + Bxe^{3x}$

$W = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & x\,3e^{3x} + e^{3x} \end{vmatrix}$

$= e^{3x} \cdot e^{3x}[3x+1] - e^{3x}[3x]$.

$e^{6x}[\cancel{3x} + 1 - \cancel{3x}]$

e^{6x}

$A = -\int \frac{X}{W} y_2 \, dx$

$= -\int \frac{\frac{e^{3x}}{x^2} \times e^{3x}}{e^{6x}}$.

$= -\int \frac{1}{x^2} \, dx$.

$\frac{x^{-2+1}}{-2+1}$

$+\frac{1}{x} + C_1$

$= \frac{x^{-1}}{-1}$

$= \left(\frac{-1}{x}\right)$

$$B = \int \frac{\frac{e^{3x}}{x^2} \cdot x e^{3x}}{e^{6x}}$$

$$\frac{x^{-1+1}}{-0}$$

$$= \int \frac{1}{x} \cdot dx$$

$$= \log x + c_2 .$$

$$y = \left[\frac{1}{x} + c_1 \right] e^{3x} + \left[\log x + c_2 \right] x e^{3x}$$

$$y = c_1 e^{3x} + c_2 x e^{3x} + \frac{e^{3x}}{x} + x e^{3x} (\log x) //.$$

5. $y'' + y = \sec x \tan x$.

$$[D^2 + 1] y = \sec x \tan x .$$

$$m^2 + 1 = 0 .$$

$$m = \pm i$$

$$CF = c_1 \cos x + c_2 \sin x .$$

$$c_1 \to A \quad c_2 \to B .$$

$$A \cos x + B \sin x .$$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$W = (\cos^2 x + \sin^2 x).$$

$$W = 1$$

$$A = -\int \frac{x}{W} y_2 \, dx .$$

$$= -\int \sec x \tan x \cdot \sin x \cdot dx$$

$$= -\int \cancel{\sec x} \frac{1}{\cos x} \frac{\sin x}{\cos x} \cdot \sin x \cdot dx .$$

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$$= -\int \tan^2 x \cdot dx.$$

$$= -\int (\sec^2 x - 1) \cdot dx.$$

$$= -\int \sec^2 x - \int dx + 1.$$

$$= -(\tan x - x + c_1$$

$$B = +\int \frac{X}{W} y_1 \, dx.$$

$$= \int \sec x \cdot \tan x \cdot \cos x.$$

$$= \int \frac{1}{\cancel{\cos x}} \cdot \frac{\sin x}{\cos x} \cdot \cancel{\cos x}.$$

$$= \int \tan x \cdot dx.$$

$$B = \log(\sec x) + c_2.$$

$y = \cancel{c_1 \cos x}$

$[-x - \tan x + c_1] \cos x + (\log(\sec x) + c_2) \sin x$

$y = c_1 \cos x + c_2 \sin x - (\tan x - x) \cdot \cos x + \log(\sec x) \sin x$

6. $y'' + 4y = 4\sec^2 2x.$

$[D^2 + 4]y = 4\sec^2 2x$

$m^2 + 4 = 0$

$m = \pm 2i$

$CF = c_1 \cos 2x + c_2 \sin 2x$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}.$$

$$2\cos^2 2x + 2\sin^2 2x.$$

$$2[\cos^2 2x + \sin^2 2x]$$

$$= 2 \,//.$$

$$A = -\int \frac{X}{W} y_2 \cdot dx.$$

$$= -\int \frac{\cancel{4}^{2} \sec^2 2x}{\cancel{2}} \cdot \sin 2x$$

$$= -2 \int \sec^2 2x \cdot \sin 2x.$$

$$= -2 \int \frac{1}{\cos^2 2x} \sin 2x.$$

$$\frac{\sin 2x}{\cos}$$

$$= -2 \int \tan 2x \cdot \sec 2x. \qquad 2\sec^2 2x \cdot dx = dt$$

$$= -\cancel{2}\int -\cancel{2}\frac{\sec 2x}{\cancel{2}} + c_1. \qquad dx = \frac{dt}{2\sec^2 2x}$$

$$-\sec 2x + c_1. \qquad \frac{dt}{\sec 2x}$$

$$B = \int \frac{X}{W} y_1 \, dx. \qquad 2\sec 2x \cdot \tan 2x \cdot dx = dt$$

$$\int \frac{\cancel{4}^{2} \sec^2 2x \cdot \cos 2x}{\cancel{2}} \qquad dx = \frac{\cancel{dt}}{2\sec 2x \cdot \cancel{\tan 2x}}$$

$$2 \int \frac{1}{\cancel{\cos^2 2x}} \cancel{\cos 2x}. \qquad \frac{dt}{t}$$

$$\log($$

$$2 \int \sec 2x$$

$$\log(\sec 2x + \tan 2x) + c_2.$$

$$y = c_1 \cos 2x + c_2 \sin 2x - (\sec 2x) \cos 2x + \log(\sec 2x + \tan 2x) \sin 2x.$$

7. $y'' - 2y' + y = e^{x} \log x$.

$(D^2 - 2D + 1)y = e^{x} \log x$.

$m^2 - 2m + 1 = 0$.

$m = 1, 1$.

$CF = (c_1 + c_2 x)e^{x}$.

W 7 $c_1 e^{x} + c_2 x e^{x}$.

$f.$ $Ae^{x} + Bxe^{x}$.

$$W = \begin{vmatrix} e^{x} & xe^{x} \\ e^{x} & x.e^{x} + e^{x} \end{vmatrix}.$$

$e^{x}.e^{x}(x+1) - e^{x}e^{x}.x$.

$e^{2x}[\cancel{x} + 1 - \cancel{x}]$

e^{2x}.

$$A = -\int \frac{X}{W}.y_2.dx.$$

$$= -\int \frac{e^{x}\log x}{e^{2x}}.xe^{x}.dx.$$

$$= -\int x \log x . dx.$$

Integrate by parts.

$$-\left[\log x . \frac{x^2}{2} - \int \frac{x^2}{2} \frac{1}{x}.dx\right].$$

$$-\left[\frac{x^2}{2} \log x - \frac{x^2}{4}\right] + c_1$$

$$B = \int \frac{X}{W} y_1 dx.$$

$$= \int \frac{e^{x} \log x}{e^{2x}}.e^{x}.dx$$

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$$B = \int \log x \cdot dx .$$

$$B = x\log x + x + c_2 .$$

$$\left[-\left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right] + c_1\right]e^x + (x\log x + x + c_2)x e^x$$

$$c_1 e^x + c_2 x e^x - \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]e^x + (x\log x + x)x e^x$$

8. $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$.

$$(D^2 - 3D + 2)y = \frac{1}{1+e^{-x}} .$$

$$m^2 - 3m + 2 = 0$$

$$m = 2, 1$$

$$CF = c_1 e^{2x} + c_2 e^{2x} .$$

$$c_1 \to A , \quad c_2 \to B .$$

$$A e^{2x} + B e^{2x} .$$

$$|W| = \begin{vmatrix} e^{2x} & e^{2x} \\ 2e^{2x} & 2e^{2x} \end{vmatrix}$$

$$2e^{3x} - 2e^{3x}$$

$$e^{3x}$$

$$A = -\int \frac{\frac{1}{1+e^{-x}}}{e^{3x}} \cdot e^{2x} dx .$$

$$= -\int \frac{1}{e^x(1+e^{-x})} \cdot dx$$

$$= -\int \frac{1}{e^x + 1} \cdot dx .$$

put $e^x = t$.

$e^x \cdot dx = dt$.

$dx = \frac{dt}{e^x} = \frac{dt}{t}$.

$A = -\int \frac{1}{(t+1)} \cdot \frac{dt}{t}$.

$= -\int \frac{1}{t(t+1)} \cdot dt$.

consider $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$

$1 = A(t+1) + Bt$.

put $t = -1$. $B = -1$

$t = 0$ $A = 1$.

$A = -\int \left(\frac{1}{t} - \frac{1}{t+1}\right) \cdot dt$.

$A = -\left[\log t - \log(t+1)\right]$,

$A = \log(t+1) - \log t$.

$= \log\left(\frac{t+1}{t}\right) + C_1$.

$A = \log\left[\frac{e^x+1}{e^x}\right] + C_1$

$B = \int \frac{x}{w} \cdot y \cdot dx$.

$= \int \frac{\frac{1}{1+e^{-x}}}{e^{3x}} \cdot e^x$.

$= \int \frac{1}{e^x(1+e^{-x})} \cdot dx$.

$$= \int \frac{1}{e^{2x} + e^{x}}.$$

$$= \int \frac{1}{e^{x}(e^{x}+1)} \cdot dx \qquad \text{put } e^{x} = t.$$

$$e^{x} \cdot dx = dt$$

$$dx = \frac{dt}{e^{x}}.$$

$$dx = \frac{dt}{t}.$$

$$= \int \frac{1}{t(t+1)} \cdot \frac{dt}{t}.$$

$$\int \frac{1}{t^2(t+1)}.$$

consider.

$$= \frac{1}{t^2(t+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+1}.$$

$=$

$$1 = At(t+1) + B(t+1) + Ct^2.$$

Put $t = 0$.

$\boxed{1 = B}$. ; $t = -1$ $\boxed{C = 1}$

Equate co-eff of $t^2 = A + C = 0$

$\boxed{A = -1}$

$$B = \int \left(\frac{-1}{t} + \frac{1}{t^2} + \frac{1}{t+1} \right) \cdot d\cancel{t}.$$

$$= -\log t - \frac{1}{t} + \log(t+1)$$

$$= -\log e^{x} - e^{-x} + \log(e^{x}+1) + C_2.$$

$$B = \log e^{-x} - e^{-x} + \log(e^{x}+1) + C_2.$$

$$y = \left[\log\left[\frac{e^{x}+1}{e^{x}}\right] + C_1 \right] e^{x} + \left[\log e^{-x} - e^{-x} + \log(e^{x}+1) + C_2 \right] e^{2x}.$$

$$y = C_1 e^{x} + C_2 e^{2x} + \log\left[\frac{e^{x}+1}{e^{x}}\right] e^{x} + \left[\log e^{-x} - e^{-x} + \log(e^{x}+1) \right] e^{2x}$$

9. $y'' - y = \frac{2}{1+e^x}$.

$[D^2 - 1]y = \frac{2}{1+e^x}$.

$m^2 - 1 = 0$

$m = \pm 1$

$CF = c_1 e^x + c_2 e^{-x}$.

$y = Ae^x + Be^{-x}$.

$$W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$$

$-1 - 1$

$W = -2$

$$A = -\int \frac{x}{W} y_2 \cdot dx$$

$$= -\int \frac{\frac{2}{1+e^x}}{-2} \cdot e^{-x} \cdot dx$$

$$= +\int \frac{2}{2(1+e^x)} e^{-x} dx$$

$$= \int \frac{e^{-x}}{(1+e^x)} dx$$

$$= \int \frac{1}{e^x(1+e^x)} \cdot dx \qquad e^x = t$$

$e^x \cdot dx = dt$

$dx = \frac{dt}{t}$

$$= \int \frac{1}{t(t+1)} \frac{dt}{t}$$

$$= \int \frac{1}{t^2(t+1)} dt$$

$$\frac{1}{t^2(t+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{(t+1)}$$

$1 = At(t+1) + B(t+1) + Ct^2$

put $t = 0$

$\boxed{B = 1}$

$t = -1$

$\boxed{C = 1}$

$\boxed{A = -1}$

$$= \int \left[-\frac{1}{t} + \frac{1}{t^2} + \frac{1}{t+1} \right] dt$$

$$= -\log t - \frac{1}{t} + \log(t+1)$$

$$-\log e^{x} - \frac{1}{e^{x}} + \log(e^{x}+1)$$

$$-\log e^{-x} - e^{-x} + \frac{1}{e^{x}+1}$$

$$B = \int \frac{x}{w} y \, dx$$

$$= \int \frac{\frac{\cancel{2}}{1+e^{x}} \cdot e^{x}}{-\cancel{2}}$$

$$- \int \frac{e^{x}}{1+e^{x}} \cdot dx \qquad 1+e^{x} = t \qquad e^{x} dx = dt$$

$$- \int \frac{dt}{t} \qquad e^{x} = t \qquad e^{x} \cdot dx = dt$$

$$- \log t + c_2$$

$$\frac{d}{dt} \log\left(\frac{t}{1+t}\right) = \frac{t}{1+t} \times \frac{1+t}{t}$$

$$-\log(1+e^{x}) + c_2$$

$$= \log(1+e^{x}) + c_2$$

10. $y'' + 2y' + 2y = e^{-x} \sec^3 x$

$D^2y + 2Dy + 2y = -$

$[D^2 + 2D + 2]y = e^{-x} \sec^3 x$.

$m^2 + 2m + 2 = 0$,

$m = -1, \pm i$

$CF = e^{-1x}[c_1 \cos x + c_2 \sin x]$

$c_1 \to A, \quad c_2 \to B$

$CF = Ae^{-x}\cos x + Be^{-x}\sin x$.

$$W = \begin{vmatrix} e^{-x}\cos x & e^{-x}\sin x \\ -e^{-x}\sin x + \cos x(-e^{-x}) & e^{-x}\cos x + \sin x(-e^{-x}) \end{vmatrix}$$

$e^{-x}\cos x[-e^{-x}\sin x + e^{-x}\cos x] - e^{-x}$

$e^{-x}\sin x[-e^{-x}\sin x - e^{-x}\cos x]$

$\cancel{e^{-x}}\{e^{-x}[-e^{-x}\cos x\sin x + e^{-x}\cos^2 x + e^{-x}\sin^2 x + e^{-x}\cos x\sin x]$

$e^{-2x}[-\cancel{\cos x\sin x} + \cos^2 x + \sin^2 x + \cancel{\cos x\sin x}]$

$e^{-2x}[1]$

$= e^{-2x}$.

$A = -\int \frac{X}{W} y_2 \, dx$.

$= -\int \frac{\cancel{e^{-x}}\sec^3 x}{\cancel{e^{-2x}}} \cancel{e^{-x}} \sin x \, dx$.

$$= -\int \frac{1}{\cos^3 x} \cdot \sin x \cdot dx$$

$$= -\int \tan x \cdot \sec^2 x \cdot dx$$

put $\tan x = t$

$\sec^2 x \cdot dx = dt$

$dx = \frac{dt}{\sec^2 x}$

$$= -\int t \cdot \cancel{\sec^2 x} \cdot \frac{dt}{\cancel{\sec^2 x}}$$

$$= -\int t \cdot dt$$

$$= -\frac{t^2}{2} + c_1$$

$$= -\frac{\tan^2 x}{2} + c_1$$

$$B = \int \frac{X}{W} y_1 \, dx$$

$$= \int \frac{\cancel{e^{-x}} \sec^3 x}{\cancel{e^{-2x}}} \cdot \cancel{e^{-x}} \cos x$$

$$= \int \frac{1}{\cos^{\cancel{3}} x} \cdot \cancel{\cos x}$$

$$= \int \sec^2 x \, dx$$

$B = \tan x + c_2$

$$y = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x - \left[\frac{\tan^2 x}{2}\right] e^{-x} \cos x + (\tan x) e^{-x} \frac{\sin x}{\cancel{\cos x}}$$

11. $y'' - 3y' + 2y = \cos(e^{-x})$.

$D^2y - 3Dy + 2y = - -$

$(m^2 - 3m + 2)\cancel{m} = 0$.

$m = 2, 1$.

$CF = c_1 e^{2x} + c_2 e^{x}$.

$c_1 \to A$, $c_2 \to B$

$CF \to Ae^{2x} + Be^{x}$.

$$W = \begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix}$$

$2e^{3x} - e^{3x}$

$W = e^{3x}$.

$A = -\int \frac{X}{W} y_2 \, dx$,

$= -\int \frac{\cos(e^{-x})}{e^{3x}} \cdot e^{2x}$.

$= -\int \frac{\cos(e^{-x})}{e^{x}} \cdot dx$

$= -\int \cos(e^{-x}) \cdot e^{-x} \cdot dx$.

put $e^{-x} = t$.

$-e^{-x} \cdot dx = dt$.

$dx = \frac{dt}{-e^{-x}}$.

$= \cancel{-}\int \cos t \cdot \cancel{t} \frac{dt}{\cancel{-t}}$.

$\int \cos t \cdot dt$

$= \sin t + c_1$

$= \sin(e^{-x}) + c_1$.

$$B = \int \frac{X}{W} y_1 \, dx$$

$$= \int \frac{\cos(e^{-x})}{e^{3x}} \cdot e^{x}$$

$$= \int \cos(e^{-x}) \cdot e^{-2x} \cdot dx = \int \cos(e^{-x}) \cdot e^{-x} \cdot e^{-x}$$

put $e^{-x} = t$

$e^{-2x} \cdot dx = dt$

$-e^{-x} dx = dt$

$dx = \frac{dt}{-t}$

$$= -\int t \cos t \cdot dt$$

$$= -\left[t \cdot \sin t - \int \sin t \, (1) \, dt\right]$$

$$= -t \sin t - \cos t + c_2$$

$$B = -\left[e^{-x} \sin(e^{-x}) + \cos(e^{-x})\right] + c_2$$

$$y = c_1 e^{2x} + c_2 e^{x} + e^{2x} \sin(e^{-x}) - \left[\sin(e^{-x}) + \cos(e^{-x})\right] c_2$$

12. $(D^2 + 3D + 2) y = e^{e^{x}}$

$m^2 + 3m + 2 = 0$

$m = -1, -2$

$CF = c_1 e^{-1x} + c_2 e^{-2x}$

$A e^{-x} + B e^{-2x}$

$$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$$

$-2e^{-3x} + e^{-3x}$

$= -e^{-3x}$

$$A = -\int \frac{e^{e^x}}{-e^{-3x}} \cdot e^{-2x} \cdot dx$$

$$\int e^x e^{e^x} \cdot dx$$

$e^x = t$

$e^x \cdot dx = dt$

$dx = \frac{dt}{e^x}$

$= \frac{dt}{t}$

$$\int \cancel{t}\, e^t \cdot \frac{dt}{\cancel{t}}$$

$$\int e^t \cdot dt$$

$$= e^t + c_1$$

$$= e^{e^x} + c_1$$

$$B = \int \frac{e^{e^x}}{-\cancel{e}^{-3x}} \cancel{e^x} \cdot dx$$

$$= -\int e^{2x} \cdot e^{e^x} \cdot e^x \cdot dx$$

Put $e^x = t$

$e^x \cdot dx = dt$

$dx = \frac{dt}{t}$

$$= -\int t \cdot e^t \cdot \cancel{t} \frac{dt}{\cancel{t}}$$

$$= \int t \, e^t \, dt$$

$$t \cdot e^t - \int e^t (1) \cdot dt$$

$$= t \cdot e^t - e^t$$

$$e^t(t-1)$$

$$e^{e^x}(e^x - 1) + c_2$$

$$y = c_1 e^{-x} + c_2 e^{-2x} + (e^{e^x}) e^{-x} + e^{e^x}(e^x - 1)e^{-2x}$$ //.

Method of Undetermined co-efficient :-

Consider the D.E

$$a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = X \rightarrow (1)$$

we find CF with the usual procedure & PI is assumed as per the table given below

X	assumption of PI	Restriction
1. Ke^{ax}	$C_1 e^{ax}$	a is not a root of A.E.
2. $\sin ax$ or $\cos ax$.	$C_1 \sin ax + C_2 \cos ax$	$\pm ia$ is not a root of AE
3. x (1st deg poly)	$ax+b$	0 is not a root of AE
x^2+x+1 (2nd deg poly)	ax^2+bx+c	0 is not a root of AE
4. $e^{mx}\cos nx$ (or) $e^{mx}\sin nx$.	$e^{mx}(a\cos nx + b\sin nx)$	$m \pm in$ is not a root of A.E
5. $x\cos nx$ (or) $x \sin nx$	$(ax+b)\cos nx + (cx+d)\sin nx$	$\pm in$ is not a root of AE

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Note:- In case of restriction the assumed PI is multiplied by x (or) x^2 or x^3 according as root is repeated once or twice or thrice respectively.

1. Solve

$$y'' + 2y' + y = 2e^{3x} \longrightarrow (1)$$

$$[D^2 + 2D + 1]y = 2e^{3x}.$$

$$m^2 + 2m + 1 = 0$$

$$m = -1, -1$$

$$CF = (C_1 + C_2 x)e^{-x}.$$

Assume PI $y = ae^{3x}. \longrightarrow (2)$

$$\left.\begin{aligned} y' &= 3ae^{3x} \\ y'' &= 9ae^{3x} \end{aligned}\right\} \longrightarrow (2)$$

Substituting (2) in (1).

$$9ae^{3x} + 6ae^{3x} + ae^{3x} = 2e^{3x}.$$

$$16a\cancel{e^{3x}} = 2\cancel{e^{3x}}$$

$$a = \frac{2}{16} = \frac{1}{8}$$

$$PI = y = \frac{1}{8}e^{3x}.$$

$$PI = [C_1 + C_2 x]e^{-x} + \frac{e^{3x}}{8}.$$

2. $y'' + 6y' + 5y = e^{-5x} \rightarrow (1)$

$(D^2 + 6D + 5)y = e^{-5x}$.

$m^2 + 6m + 5 = 0$.

$m = -1, -5$.

$CF = c_1 e^{-x} + c_2 e^{-5x}$.

Assume $y = a_1 e^{-x} + a_2 e^{-5x}$.

$y' = -a_1 e^{-x} - 5a_2 e^{-5x}$.

$y'' = a_1 e^{-x} + 25 a_2 e^{-5x}$

$\Bigg\} (2)$

Substitute (2) in (1).

$a_1 e^{-x}$

X

assume

$y = xae^{-5x}$.

$y' = -5ae^{-5x}$

$y'' = 25ae^{-5x}$.

$\therefore (1) \Rightarrow 25ae^{-5x} - 30ae^{-5x} + 5ae^{-5x} = e^{-5x}$.

assume PI ae^{-5x}

$y = ae^{-5x}$

$y' = -5ae^{-5x}$

$y'' = +25ae^{-5x}$.

$(1) \rightarrow 25ae^{-5x} - 30ae^{-5x} + 5ae^{-5x} = e^{-5x}$.

axe^{-5x}

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3. Solve by the method of undetermined co-efficient.

$y'' + 3y' + 2y = e^{3x} + e^{-x} \longrightarrow (1)$

$(D^2 + 3D + 2)y = (1)$

$m^2 + 3m + 2 = 0$

$m = -2, -1$

$CF = c_1 e^{-2x} + c_2 e^{-x}$.

let $y = ae^{3x} + bxe^{-x} \longrightarrow (2)$

$\because$ 3 is not root.

-1 is a simple root.

diff (2) w.r.t. x.

$y' = 3ae^{3x} + b(-xe^{-x} + e^{-x})$

~~$y'' = 9ae^{3x} + bxe^{-x} + e^{-x} - e^{-x}$.~~

~~$y'' = 9ae^{3x} + bxe^{-x}$.~~

$y'' = 9ae^{3x} + b(+x.e^{-x} - e^{-x} - e^{-x})$

$y'' = 9a^{3x} + b(+xe^{-x} - e^{-x} - e^{-x}) \longrightarrow (3)$.

Subs (2) & (3) in (1)

$9ae^{3x} + bxe^{-x} - 2be^{-x} + 3(3ae^{3x} - bxe^{-x} + be^{-x})$

$+ 2(ae^{3x} + bxe^{-x}) = e^{3x} + e^{-x}$.

$20ae^{3x} + be^{-x} = e^{3x} + e^{-x}$.

equate co-eff. $e^{3x} = 20a = 1 \Rightarrow a = 1/20$

" " $e^{-x} = b = 1$.

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$y_P = \frac{1}{20} e^{3x} + xe^{-x} = PI$.

$y = CF + PI$.

$y = c_1 e^{-2x} + c_2 e^{-x} + \frac{e^{3x}}{20} + xe^{-x}$ //.

4. $y'' + 4y' + 4y = e^{-2x} \longrightarrow$ ①

$D^2 y + 4Dy + 4y = e^{-2x}$.

$(D^2 + 4D + 4)y = e^{-2x}$.

$m = -2, -2$.

$CF = (c_1 + c_2 x)e^{-2x} \longrightarrow$ ②

$= c_1 e^{-2x} + c_2 x e^{-2x}$.

$y = x^2 a e^{-2x}$.

$y' = a[2x e^{-2x} + e^{-2x} \cdot 2x]$

$-2a[x^2 e^{-2x} - e^{-2x} x]$.

$y'' = -2a[x^2 e^{-2x}(-2) + e^{-2x} 2x - (e^{-2x}(1) + xe^{-2x}(-2))]$.

$y'' = -2a[-2x^2 e^{-2x} + 2xe^{-2x} - e^{-2x} + 2xe^{-2x}]$.

$4ax^2 e^{-2x} - 4axe^{-2x} + 2ae^{-2x} - 4axe^{-2x}$.

$4ax^2 e^{-2x} + 2ae^{-2x} - 8axe^{-2x}$.

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$$4ax^2e^{-2x} + 2ae^{-2x} - \cancel{8axe^{-2x}} +$$

$$4a[-2x^2e^{-2x} + \cancel{2xe^{-2x}}] + 4x^2ae^{-2x} = e^{-2x}.$$

$$2ae^{-2x} = e^{-2x}.$$

$$2a = 1$$

$$\boxed{a = 1/2}.$$

$$y_P = x^2(1/2)e^{-2x}.$$

$$Y = CF + PI.$$

$$y = (c_1 + c_2x)e^{-2x} + 1/2\, x^2 e^{-2x}$$

5. $y'' + 5y' + 6y = 4\sin 2x \rightarrow (1).$

$$CF = c_1e^{-3x} + c_2e^{-2x}$$

$$y = a\sin 2x + b\cos 2x.$$

$$\left.\begin{aligned} &\cancel{y' = 2a\sin 2a} \\ &y' = 2a\cos 2x - 2b\sin 2x. \\ &y'' = -4a\sin 2x - 4b\cos 2x \end{aligned}\right\} \rightarrow (2).$$

Substitute (2) in (1).

$$-4a\sin 2x - 4b\cos 2x + 5[2a\cos 2x - 2b\sin 2x] + 6[a\sin 2x + b\cos 2x].$$

$$-4a\sin 2x - 4b\cos 2x + 10a\cos 2x - 10b\sin 2x + 6a\sin 2x + 6b\cos 2x.$$

$$\cos 2x[-4b + 10a + 6b] + \sin 2x[-4a + 6a - 10b]$$

$$2b\cos 2x + 10a\cos 2x + 2a\sin 2x - 10b\sin 2x = 4\sin 2x.$$

$(2a-10b)\sin 2x + (10a+2b)\cos 2x = 4\sin 2x.$

Coeff of $\sin 2x$: $2a-10b=4$.

$\cos 2x$; $10a+2b=0$.

$a = 1/13$

$b = -5/13$.

$PI = 1/3 \sin 2x - 5/13 \cos 2x$.

$y = CF + PI$.

$y = c_1 e^{-3x} + c_2 e^{-2x} + 1/3 \sin 2x - 5/3 \cos 2x$

6. $y'' + 2y' + y = 2e^{x} + 3\cos x \longrightarrow (1)$

$CF = (c_1 + c_2 x)e^{-x}$. $m = -1, -1$ $ae^{x} + b\sin x$ $c\cos x$

$$\left.\begin{aligned} y_p &= ae^{x} + b\cos x + c\sin x \\ y' &= ae^{x} - b\sin x + c\cos x \\ y'' &= ae^{x} - b\cos x - c\sin x \end{aligned}\right\} (2)$$

Substitute (2) in (1).

$= (ae^{x} - b\cos x - c\sin x + 2(ae^{x} - b\sin x + c\cos x)$

$+ ae^{x} + b\cos x + c\sin x$.

$= ae^{x} - b\cos x - c\sin x + 2ae^{x} - 2b\sin x + 2c\cos x$

$+ ae^{x} + b\cos x + c\sin x$.

$4ae^{x} + (\cancel{-b} + 2c + \cancel{b})\cos x + (\cancel{-c} - 2b + \cancel{c})\sin x$

$4ae^{x} + 2c\cos x - 2b\sin x = 2e^{x} + 3\cos x$.

$\cancel{4}ae^{\cancel{x}} = \cancel{2}e^{x}$.

$2a = 1$ $a = 1/2$.

$$\left\{\begin{array}{l} 2c = 3 \\ c = 3/2 \\ -2b = 0 \\ b = 0 \end{array}\right\}$$

$ae^{x} + b\sin 3x + c\cos 3x$

$y_p = \frac{1}{2}e^{x} + \frac{3}{2}\sin x$

$y = CF + PI$

$(c_1 + c_2 x)e^{-x} + \frac{1}{2}e^{x} + \frac{3}{2}\sin x$ //.

7. $y'' + y' + y = 3e^{-x} + \sin 3x \rightarrow (1)$

$D^2y + Dy + y = \text{---}\ \text{---}$

$[D^2 + D + 1]y =$

$ae^{-x} + b\sin 3x + c\cos 3x$

$m^2 + m + 1 = 0$

$m = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$CF = e^{-\frac{1}{2}x}\left[c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right]$.

$y = ae^{-x} + b\sin 3x + c\cos 3x \rightarrow (2)$

$y' = -ae^{-x} + 3b\cos 3x - 3c\sin 3x$

$y'' = ae^{-x} - 9b\sin 3x - 9c\cos 3x$ $\Big\} \rightarrow (3)$

$ae^{-x} - 9b\sin 3x - 9c\cos 3x + [-ae^{-x} + 3b\cos 3x -$
$3c\sin 3x + ae^{-x} + b\sin 3x + c\cos x$
$= 3e^{-x} + \sin 3x$.

$e^{-x}(a - a + a) = 3e^{-x}$.

$\boxed{a = 3}$

$\sin 3x(-9b-3c+b) + (-9c+3b+c)\cos 3x$

$= 3e^{-x} + \sin 3x = \sin 3x$

$-8b-3c = 1$

$-8c+3b = 0$

$$\boxed{\begin{aligned} b &= \frac{-8}{73} \\ c &= \frac{-3}{73} \end{aligned}}$$

$PI = 3e^{-x} - \frac{8}{73}\sin 3x - \frac{3}{73}\cos 3x$

$y = CF + PI$

$y = e^{-1/2x}\left(c_1\cos\sqrt{3}/2 + c_2\sin\sqrt{3}/2\,x\right) + 3e^{-x} - \frac{8}{73}\sin 3x$
$- \frac{3}{73}\cos 3x$

8. $y'' + 3y' + 4y = x^2 + 2\cos 2x$. $(ax^2 + bx + c) + d\cos$

$D^2y + 3Dy + 4y =$

$(D^2 + 3D + 4)y =$

$m^2 + 3m + 4 = 0$

$m = -3/2 \pm \sqrt{7}/2\, i$

$CF = -3/2 \pm \sqrt{7}/2\, i$

$e^{-3/2}\left(c_1\cos\sqrt{7}/2\,x + c_2\sin\sqrt{7}/2\,x\right)$

$y = (ax^2 + bx + c) + d\cos 2x + e\sin 2x$

$y' = 2ax + b - 2d\sin 2x + 2e\cos 2x$

$y'' = 2a - 4d\cos 2x - 4e\sin 2x$

$2a - 4d\cos 2x - 4e\sin 2x + 3[2ax + b - 2d\sin 2x$
$+ 2e\cos 2x] + 4[(ax^2 + bx + c) + d\cos 2x + e\sin 2x]$
$= x^2 + 2\cos 2x$

x^2: $4a = 1 \Rightarrow a = 1/4$

x: $6a + 4b = 0 \Rightarrow b = -3/8$

const: $2a + 3b + 4c = 0 \Rightarrow c = 5/32$.

$\sin 2x$: $-\cancel{4e} - 6d + \cancel{4e} = 0. \Rightarrow d = 0$.

$\cos 2x$: $-\cancel{4d} + 6e + \cancel{4d} = 2. \Rightarrow e = 1/3$

$$PI = \frac{1}{4}x^2 - 3/8\, x + 5/32 + 1/3 \sin 2x.$$

$$y = CF + PI$$
$$= e^{-3/2 \pm \sqrt{7}/2\, x}\left(c_1 \cos \sqrt{7}/2\, x + c_2 \sin \sqrt{7}/2\right).$$
$$+ 1/4\, x^2 - 3/8\, x + 5/32 + 1/3 \sin 2x.$$

9. $y'' + y = x \sin 3x$.

$D^2y + 1 =$ —

$(m^2 + 1) = 0$

$m^2 = -1$

$m = \sqrt{-1}$

$m = \pm i$

$CF = (c_1 \cos x + c_2 \sin x)$

$PI = y = (ax + b)\cos 3x + (cx + d)\sin 3x$.

$y' = -3(ax + b)\sin 3x + 3(cx + d)\cancel{\sin 3x}\ \cos 3x$.

$y'' = -9(ax + b)\cos 3x - 9(cx + d)\sin 3x$.

$-9(ax + b)\cos 3x - 9(cx + d)\sin 3x$

$y' = 3(ax+b)\cos 3x + a\sin 3x - 3(cx+d)\sin 3x$.
$+ c\cos 3x$.

$= [3ax+3b+c]\cos 3x + [a-3cx-3d]\sin 3x$.

$y'' = -3[3ax+3b+c]\sin 3x + \cos 3x(3a)$
$+3[a-3cx-3d]\cos 3x + \sin 3x(-3)$..

$\therefore$ $-3[3ax+3b+c]\sin 3x + 3a\cos 3x + 3[a-3cx-3d]$
$\cos 3x + \sin 3x(-3) + (ax+b)\cos 3x$.
$+ (cx+d)\sin 3x = x\sin 3x$.

$\sin 3x$: $-9b - 3c - 3 + b = 0$.
$\cos 3x$: $3a - 9d + 3a + d = 0$ $\boxed{b = -3/8}$
$x\sin 3x$: $-9a + a = +1$ $\boxed{a = -1/8}$
$x\cos 3x$: $-9c + c = 0$. $\boxed{c = 0}$

Subs a & c.

$-8b - 3 = 0$

$b = -3/8$

$-3/8 - 8d + \frac{3}{8}$

$\frac{-6}{8} - 8d = 0$

$8d = \frac{-6}{8}$, $\frac{-6}{64}$, $= -3/32$.

$PI = (1/8 x + -3/8)\cos 3x + (-3/32)\sin 3x$.

$y = CF + PI$

$= c_1\cos x + c_2\sin x - (1/8 x - 3/8)\cos 3x - 3/32 \sin 3x$

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