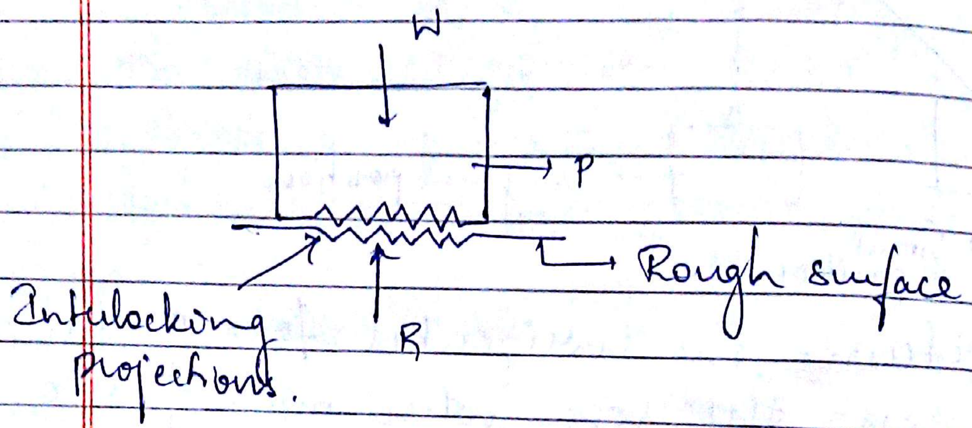
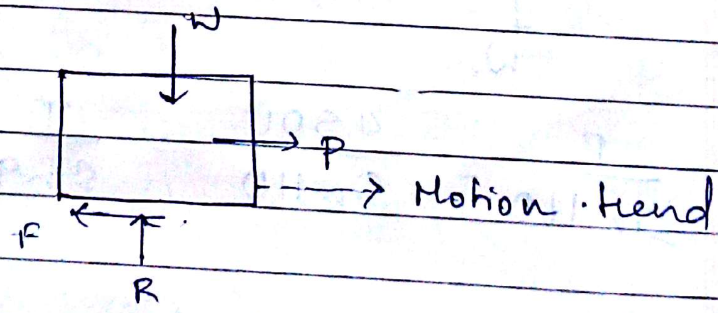
Unit - IV - Part - 1 - Start

Ch. 5. Friction



When a body tends to move in contact over the other body a resistance to it's motion is setup this resistance to movement of the body towards the direction of applied load is called as friction.



The figure shows the object of self weight 'W' 'R' is ~~resistance~~ reaction force normal to the surface, 'P' is the magnitude of force applied and 'F' is the frictional force which is always in the direction opposite to that of motion trend.

(*) Types of friction:

→ **Static friction:** Friction acting on a body which is at rest is called static friction.

→ **Dynamic friction:** Friction acting on a body which is in motion.

→ **Dry friction:** Friction acting on a body when contact surface are un-lubricated, then there is a tendency of relative motion ~~called~~ known as dry or columb friction.

It is divided into two types:

(i) ~~Solid~~ sliding friction - Friction acting on a body when two surfaces have tendency to slide relative to each other is called sliding friction.

(ii) Rolling friction - Friction acting on body when contact surfaces roll one over other is called rolling friction.

→ **Fluid friction** - Friction acting on body when contact surfaces are lubricated is called fluid friction.

It is divided into 2 type.

(i) Non viscous friction: Friction acting on body when contact surface are lubricated with extremely thin layer of lubricant which is non-viscous is called boundary / Non viscous

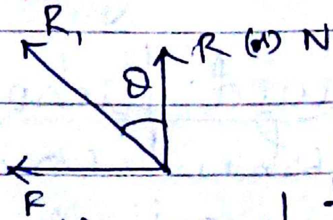
(ii) Viscous / film friction: Friction acting on body when surfaces are completely lubricated with a viscous type of fluid is called viscous or film friction

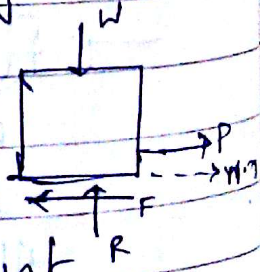
(*) → **Limiting friction: (f)**
Limiting friction is the friction acting on body which is just on point or regime of sliding is called as limiting friction.

(*) **Motion trend of a block on the surface**

Let a body of weight W is subjected to pulling force of magnitude P which tend to move towards right side as shown in the figure. The forces induced are:
Reaction (R) acting perpendicular to support and the force of friction (F) acting in the direction opposite to that of motion trend.

→ **Angle of friction:**





The angle made by the normal reaction and that of resultant is called angle of friction.

$$\boxed{\tan\theta = \frac{F}{R}}$$

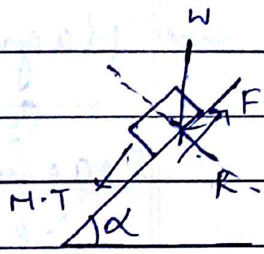
→ **Co-efficient of friction (μ);**

It is the ratio of limiting friction (f) to the normal reaction between the two surfaces.

$$f = \mu N$$

$$\Rightarrow \boxed{\mu = \tan\theta}$$

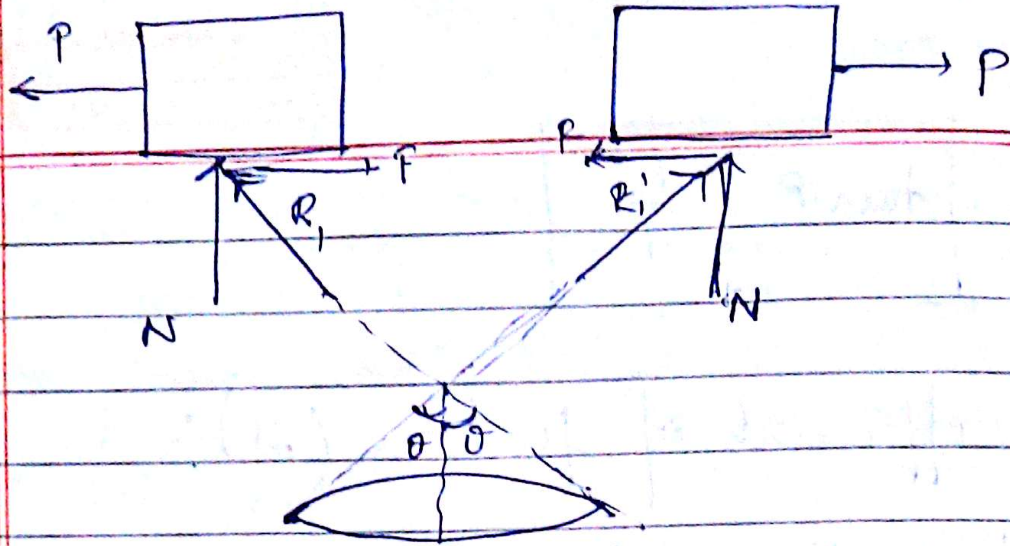
→ **Angle of repose : (α)**



If the body is placed on the inclined plane then the angle at which the body is just on the verge of sliding is called angle of repose.

→ **Cone of friction:**

When ever a body in contact with other and tend to move and also considering the two objects which is subjected to two forces of magnitude equal to P on each side then the cone of friction is obtained by projecting the resultant of the two objects.



Imp

(*) Laws of frictions:

→ Force of friction always acts in the direction opposite to that of motion tend.

→ Magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces.

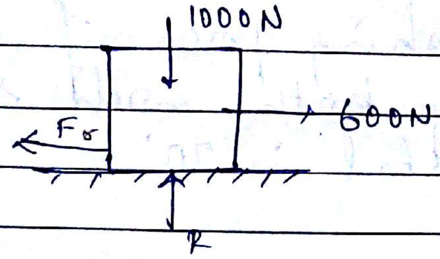
i.e $\mu = \frac{F}{R}$

→ Magnitude of force of friction is exactly equal to the force which tends the body to move as long as body is at rest (i.e $P = F$)

→ The force of friction ~~of~~ is independent of area of contact b/w two surfaces.

→ Force of friction depends on roughness of the surface.

Q.1 A Body of weight 1000N is placed on a rough surface, determine the co-efficient of friction due to the force 600N in the horizontal direction just causes the motion.



$\Sigma V = 0$

$\Rightarrow \quad N = 1000\,N$

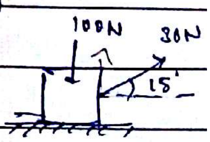
$\Sigma H = 0$

$600 - F = 0$

$F_r = 600\,N$

$$\mu = \frac{F}{N(R)} = \frac{\cancel{600}^{\,3}}{\cancel{1000}_{\,5}} = 0.6$$

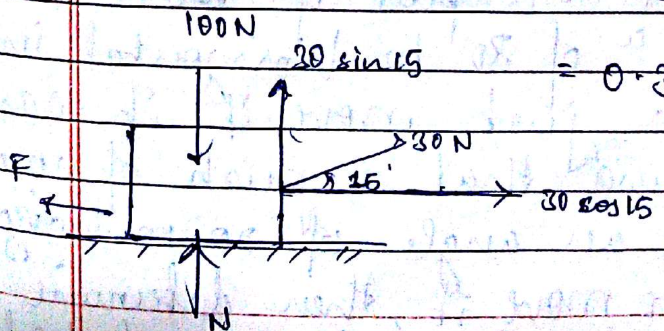
Q. A body weighing 100N is placed on a rough surface and pulled by the force of 30N inclined at an angle of 15° with horizontal, find co-efficient of friction.

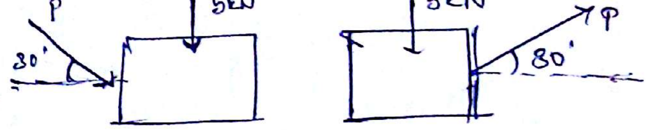


$$\mu = \frac{F\cos 15}{100} = \frac{30\cos 15}{100 - 30\sin 15}$$

$$= \frac{28.97}{92.23} =$$

$$= 0.31$$





Q. A block of 5kN rests on a horizontal surface and co-efficient of friction b/w them is 0.4 show that magnitude of force required to pull is less than magnitude of pushing force if angle of force made by both with respect to horizontal is 30°.

→ Pulling force

$$0.4 = \frac{P\cos 30°}{5 - P\sin 30}$$

$$2 - 0.2P = P(0.86)$$

$$P = 1.876\,kN$$

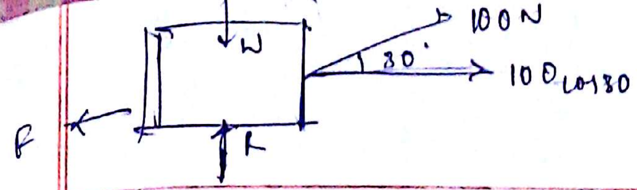
Pushing force:

$$0.4 = \frac{P\cos 30°}{5 + P\sin 30}$$

$$2 + 0.2P = P(0.86)$$

$$P = 3.03\,kN$$

Q. A body resting on an horizontal surface required a pull of 100 N inclined at an ∠ of 30 to horizontal in order to just move it it was also found that a push of 110 N inclined at an angle of 20° to horizontal in order to move it, then determine the weight of body & co-efficient of friction.



(i) Pulling force

$$\mu = \frac{100 \cos 30}{W - 100 \sin 30} \longrightarrow (1)$$

(ii) Pushing force

$$\mu = \frac{110 \cos 20}{W + 110 \sin 20} \longrightarrow (2)$$

From (1) & (2)

$$\frac{86.60}{W - 50} = \frac{103.36}{W + 37.62}$$

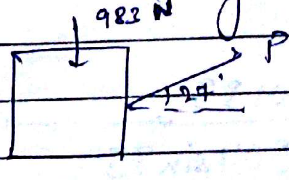
$$\cancel{86.6W + 3257.9 = 103.36W - 5168}$$

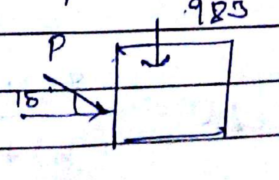
$$8426 = 16.76\,W$$

$$W = 502.7\ N$$

$$\Rightarrow \mu = 0.19$$

Q. A body of weight 983 N rests on a rough surface which is having coefficient of friction as 0.31. Determine the pulling & pushing force when the former make an angle of 27° & later makes an angle of 15° with x-axis.





Pulling

$$\mu = \frac{P\cos 27}{983 - P\sin 27} \longrightarrow (1)$$

Pushing

$$\mu = \frac{P'\cos 15}{983 + P'\sin 15} \longrightarrow (2)$$

from (1) & (2)

$P = 327.69\,N$ // (Pull)

$P' = 391.85\,N$ // (Push)

Q. The object lying on horizontal plane with some co-effrint of fricting lying in horizontal plane & having magnitude of pulling force is 283 N & pushing force is 304N. Determine the self weight of object. (Take the "L" of inc. for both the force with refunce to horizontal 33°).

→

$$\mu = \frac{283\cos 33}{W - 283\sin 33} \longrightarrow (1)$$

$$\mu = \frac{804\cos 33}{W + 804\sin 33} \longrightarrow (2)$$

$$237.34\,W + 39297.07 = 255W - 39297.07$$

$$78594.14 = 17.66\,W$$

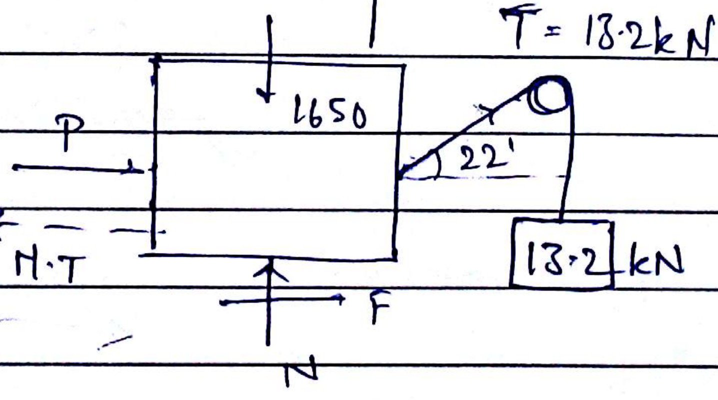
$$W = 4450.84\ N$$

$$\mu = 0.055$$

Q. Body weight 1650 N is attached to a weight passing through the frictionless pulley at $\angle$ of $22°$ if magnitude of pushing force is equal to P find the value of P. (Take $\mu = 0.23$)

(i) M.T in left direction

(ii) ⟵ right ⟶

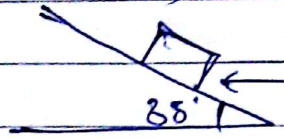


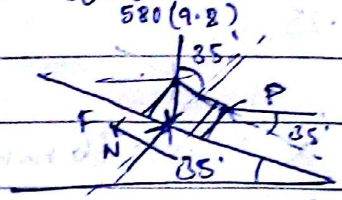
Unit-IV - Part-1- End.

Unit - IV - Part - 2. - Start

Friction

1. Q. A body as shown in figure has a mass of 580 kg, if P = 6000 N, find magnitude of frictional force which acts on the body. what value of P for the motion tend is in upward direction & also in downward direction ($\mu = 0.25$)





$\Sigma V = 0$

$-580(9.8)\cos 35° - 6000 \sin 35° + N = 0$

$N = 8097.52 \text{ N}$

$F = \mu N$

$= 2024.38 \text{ N}$

(i) Downward.

$-2024.38 - P\cos 35 + 580(9.8)\sin 35 = 0$

$P = 1508.6 \text{ N}$

(ii) Upward:

$-P\cos 35 + 2024.38 + 580(9.8)\sin 35$

$P = 6451.29 \text{ N}$

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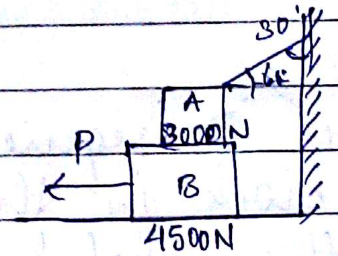
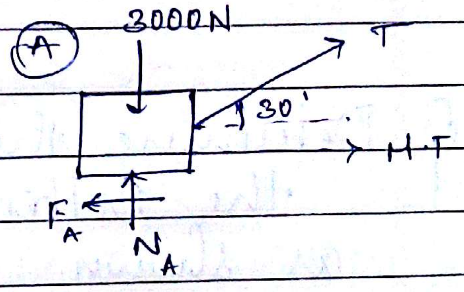
29. A block weighing 4500N resting on horizontal surface supports another block of 3000N as shown. Find horizontal force P required to just move the block in left direction. (Take $\mu = 0.3$)





$\Sigma V = 0$

$$-3000 + T(0.5) + N_A = 0$$

$$N_A = 3000 - T(0.5)$$

$\Sigma H = 0$

$$-F_A + T\cos 30 = 0$$

$$F_A = T\cos 30$$

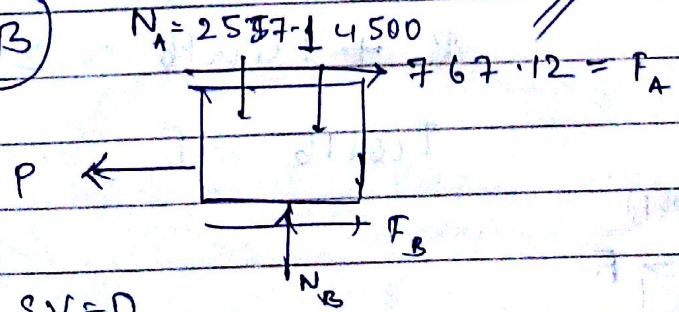
$$\frac{F_A}{N_A} = 0.3$$

$$T(0.866) = 900 - 0.15T$$

$$T(1.016) = 900$$

$$T = 885.8\ N$$

(B)



$\Sigma V = 0$

$$N_B = 2557.1 + 4500 = 7057.1\ N$$

$$-P + F_B + F_A = 0$$

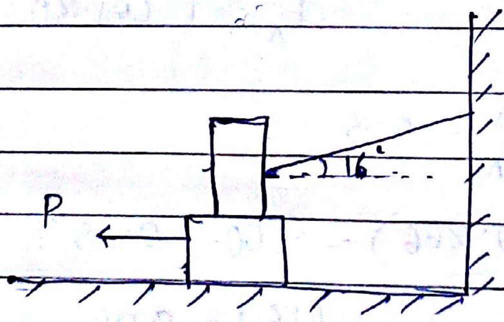
$$-F_A + P = F_B$$

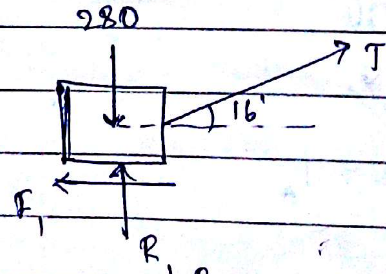
$$\frac{F_B}{N_B} = 0.3$$

$$\frac{-767.12 + P}{} = 0.3\,(7057.1)$$

$$P = 2884.25\,N$$

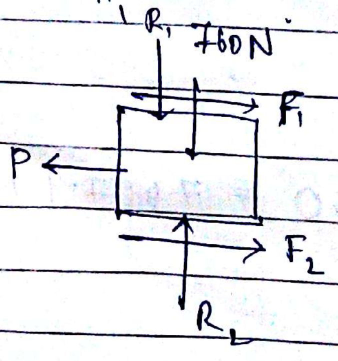
Q. Determine the force 'P' required to slide the bottom block in the arrangement as shown in the sketch, also find the tension in the string. Given self-weight of top block 280N, self weight of bottom block 760 N, μ b/w the contact of the block = 0.25, μ between floor & block is 0.3.





$$R_1 + T\sin 16 = 280$$

$$T\cos 16 = F_1$$



$$\mu_1 = \frac{F_1}{R_1}$$

$$0.25 = \frac{T\cos 16}{280 - T\sin 16}$$

$$70 - 0.069T = 0.96T$$

$$T = 67.94\ N$$

$$R_1 = 261.27\ N$$

$$F_1 = 65.30\ N$$

$$R_2 = 261.27 + 760$$

$$R_2 = 1021.27\ N$$

$$\mu_2 = \frac{F_2}{R_2} \Rightarrow F_2 = 306.38\ N$$

$$P = F_1 + F_2$$

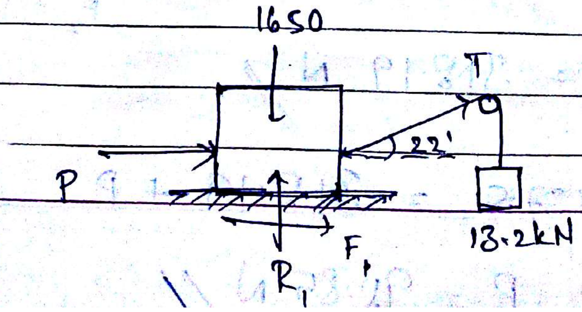
$$= 871.68\ N$$

Q. Body weighing 1650 N attached to a weight passing through frictionless pulley at an $\angle^{le}$ of $22°$. if the magnitude of pushing force on the body is 'P'. Find the magnitude of P.

(i) Body moving in left direction

(ii) —— " —— right —— " ——

Take $\mu = 0.8$



$\mu = \frac{F}{R} \Rightarrow F = 493.51\ N$

(i) $R_1 + (13.2)\sin 22 = 1650$ (Left side)

$R_1 = 1645.05\ N$

$P + F_1 + 13.2 \cos 22 = 0$

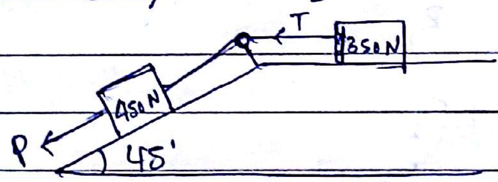
$P = -505.74\ N$

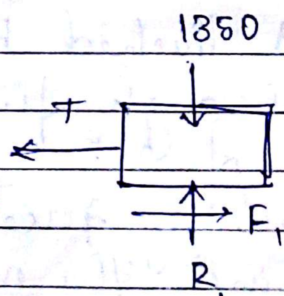
(Right side)

(ii) $P + 13.22 \cos 22 = F_1$

$\Rightarrow P = 481.25\ N$

Q. Determine force acting $\parallel^{lel}$ to plane, take $\mu = 0.25$.

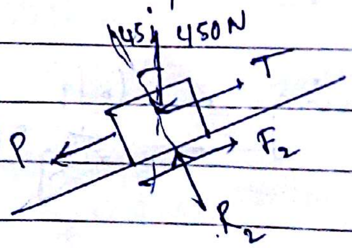




$R_1 = 1350\ N \qquad T = F_1$

$\mu = \frac{F_1}{R_1} \Rightarrow F_1 = 337.5\ N$

F_2



$\mu = \frac{F_2}{R_2}$

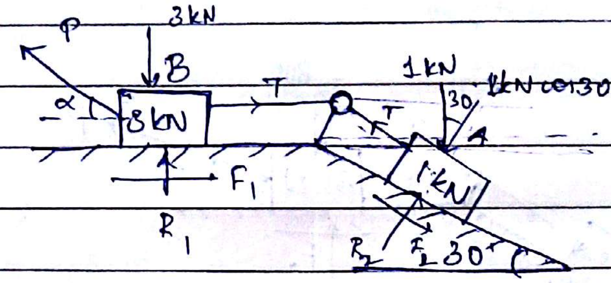
$F_2 = 79.54\ N$

$R_2 = 318.19\ N$

$79.54 + 337.5 = 318.19 + P$

$P = 98.85\ N$

Q. Find the least value of 'P' required to cause the motion of the system of block in left direction. (take $\mu = 0.2$)



$R_2 = 0.866\,kN$ $\quad$ $F_2 = \mu R_2 = 0.1732\,kN$

$T = F_2 + 1\sin 30$

$T = 0.6782\,kN$

$P\sin\alpha + R_1 = 8\,kN$

$R_1 = 8 - P\sin\alpha$

$P\cos\alpha = 0.6782 + F_1$

$\Rightarrow F_1 = P\cos\alpha - 0.6782$

$\mu = \frac{F_1}{R_1} \Rightarrow 0.2 = \frac{P\cos\alpha - 0.6732}{8 - P\sin\alpha}$

$0.6 - 0.2P\sin\alpha = P\cos\alpha - 0.6732$

$1.278 = P(0.2\sin\alpha + \cos\alpha) \longrightarrow (1)$

On diff (1) with respect to α

$\Rightarrow -\sin\alpha + 0.2\cos\alpha = 0$

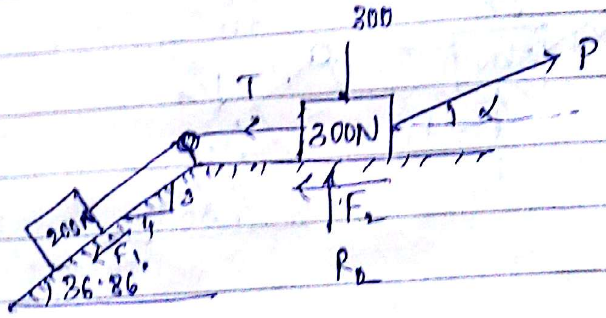
$0.2\cos\alpha = \sin\alpha$

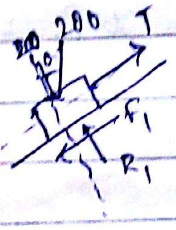
$0.2 = \tan\alpha$

$\alpha = 11.31°$

$\Rightarrow P = 1.248\,kN$

Q. Find the least value of P in which the ~~pulley~~ whole system block moves in right direction. (Take $\mu = 0.3$)





$R_1 = 160\text{ N}$ //

$F_1 = \mu R_1 = 48\text{ N}$ //

$48 + 120 = T \Rightarrow T = 168\text{ N}$ //

$R_2 = 200 - P\sin\alpha$

$F_2 = P\cos\alpha - 168$

$\Rightarrow \mu = \dfrac{F_2}{R_2}$

$\Rightarrow 0.3 = \dfrac{P\cos\alpha - 168}{300 - P\sin\alpha}$

$90 - 0.3P\sin\alpha = P\cos\alpha - 168$

$258 = P(0.3\sin\alpha + \cos\alpha)$

$\Rightarrow P = \dfrac{258}{(0.3\sin\alpha + \cos\alpha)} \longrightarrow ①$

On diff ① wrt α

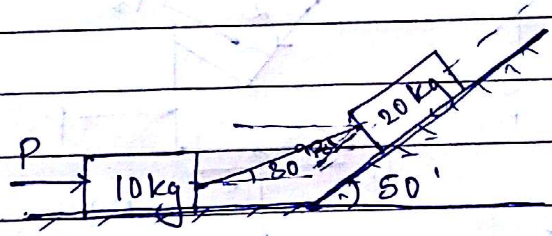
$0.3\cos\alpha - \sin\alpha = 0$

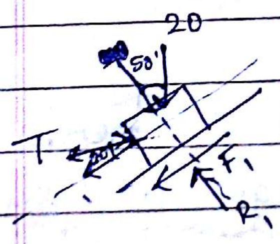
$\tan\alpha = 0.3$

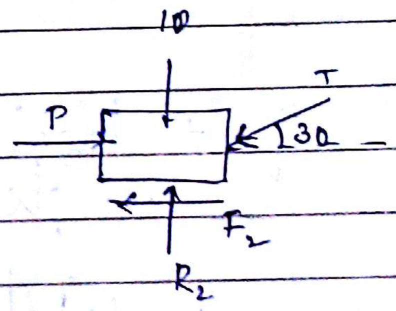
$\alpha = 16.7^\circ$

$\Rightarrow P = 247.12\ N$

Q. Calc. value of 'P' required to cause the motion of block as shown in the figure, (take μ) for all the contact surfaces as 0.2







$R_1 = 12.85 + T(0.342) = 20N$

$F_1 = T(0.94) - 15.32 = 4N$

$\mu = \frac{F_1}{R_1}$

$\Rightarrow 0.2\ (12.85 + T(0.342)) = T(0.94) - 15.32$

$2.57 + 0.0684T = 0.94T - 15.32$

$17.89 = 0.87T$

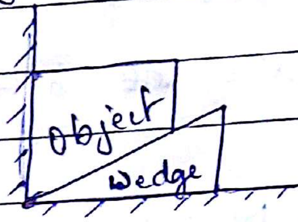
$T = 20.56 N$

$R_2 = 20.28\ N$

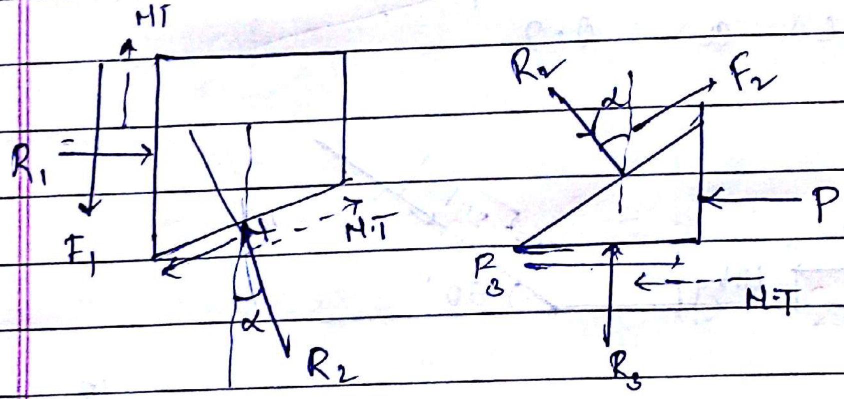
$F_2 = 4.056\ N$

$P = 21.86\ N$

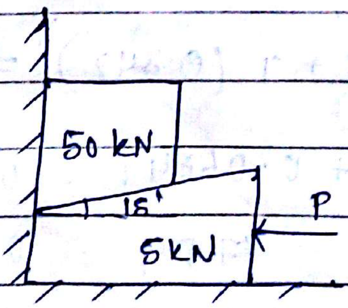
(*) Wedge friction

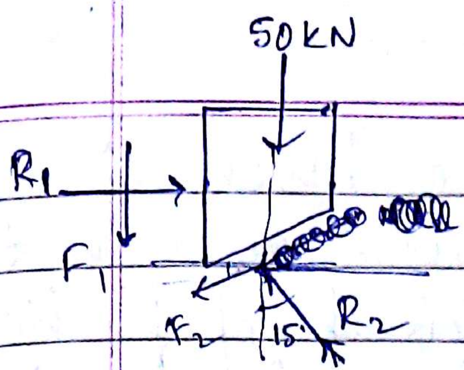


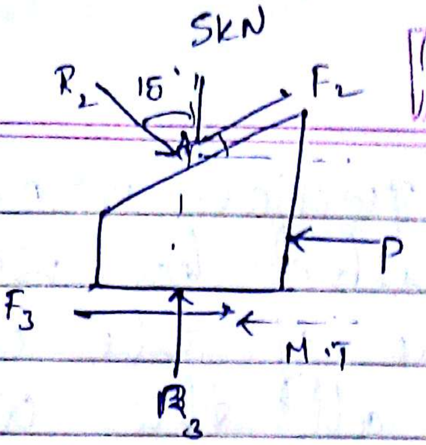
Wedge is an object which is triangular or trapizoidal in shape which is used to lift the heavy for different heights



Q. A block of weight 50kN is kept in equilibrium by a wedge as shown in figure. If the co-efficient of friction is 0.2 for all the contact surfaces determine the force 'P' necessary to cause a tendency to move the block in upward direction. Self weight of wedge is 5kN







Object

$\Sigma H = 0$

$$R_1 - F_2 \cos 15° - R_2 \sin 15° = 0$$

$\Sigma V = 0$

$$-F_1 - 50 - F_2 \sin 15 + R_2 \cos 15° = 0$$

Wedge

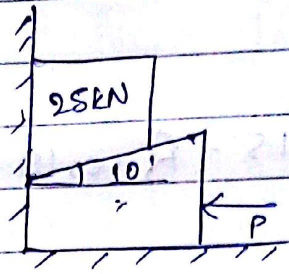
$\Sigma H = 0$

$$-P + F_3 + R_2 \sin 15° + F_2 \cos 15 = 0$$

$\Sigma V = 0$

$$-5 - R_2 \cos 15° + R_3 + F_2 \sin 15° = 0$$

The position of machine block is adjusted by wedge as shown in the figure. If the co-efficient of friction of all the contact surfaces is 0.25, determine the value of P, self weight of object is 25kN & self weight of wedge is neglected.



$$R_1 = F_2 \cos 10 + R_2 \sin 10$$

$$F_1 = R_2 \cos 10 - F_2 \sin 10 - 25$$

$$0.25 = \frac{R_2 \cos 10 - F_2 \sin 10 - 25}{F_2 \cos 10 + R_2 \sin 10}$$

$$0.246 F_2 + 0.043 R_2 = 0.984 R_2 - 0.173 F_2 - 25$$

$$0.419 F_2 = 0.941 R_2 - 25$$

(WKT $F_2 = 0.25 R_2$)

$$\Rightarrow \quad 0.104 R_2 = 0.941 R_2 - 25$$

$$R_2 = 29.86 \text{ kN}$$

$$\Rightarrow F_2 = 7.46 \text{ kN}$$

$$\Rightarrow R_1 = 12.58 \text{ kN}$$

$$\Rightarrow R_1 = 8.11 \text{ kN}$$

$R_3 = (29.86)\cos 10 - 7.46 \sin 10$

$R_3 = 28.11$ kN

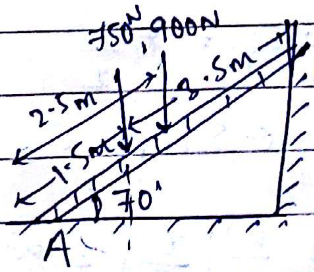
$R_3 = \cancel{\quad} 7.02$ kN

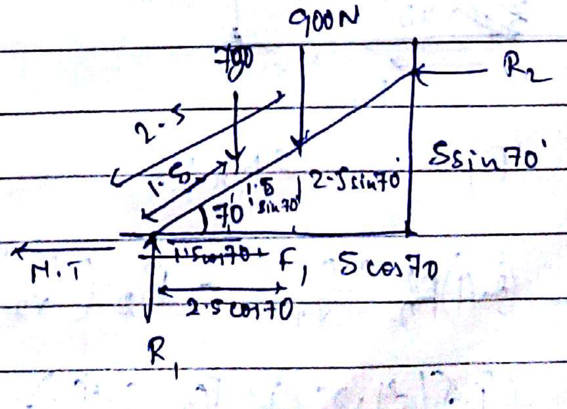
$P = \cancel{\quad} 7.027 + 29.86 \sin 10 + 7.46 \cos 10$

$P = 19.55$ kN

Ladder friction:

A ladder of 5m long rest on a horizontal ground & leans against smooth wall at an angle of 70' with reference to horizontal as shown in figure weight of ladder is 900N, man weighing 750N stands on ladder from 1.5m from bottom of ladder. Calc co-efficient of friction b/w ladder & ground, since it is smooth wall i.e no friction with wall.





$\Sigma H = 0$, $\Sigma V = 0$

$\Sigma H = 0 \longrightarrow F_1 - R_2 = 0$

$F_1 = R_2 \longrightarrow (1)$

$\Sigma V = 0$

$R_1 - 750 - 900 = 0 \Rightarrow R_1 = 1650$ N

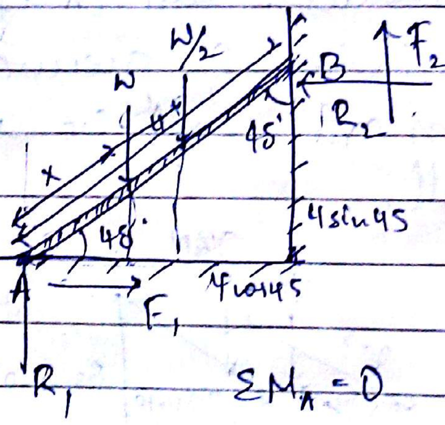
$\Sigma M_A = 0$

$-R_2 \times 4.69 + 900 \times 2.5\cos 70 + 750(1.5\cos 70) = 0$

$\Rightarrow R_2 = 246.12 = F_1$

$\frac{F_1}{R_1} = \mu = 0.15$

Q. A uniform ladder of 4m length rest against a vertical wall with which makes an angle of 45° as shown in fig. μ b/w ladder & wall = 0.4 and b/w ladder & floor = 0.5. If the man whose weight is ½ of the ladder & ascends it then how high he will be when ladder slips.



$\frac{F_1}{R_1} = 0.5$ $\frac{F_2}{R_2} = 0.4$

$F_1 = R_2$

$R_1 + F_2 = \frac{3W}{2}$

$\Sigma M_A = 0$ $0.5 R_1 = R_2$

$-(0.4)R_2\left(\frac{4}{\sqrt{2}}\right) - R_2\left(\frac{4}{\sqrt{2}}\right) + \frac{W}{2}\left(\frac{x}{\sqrt{2}}\right) + W\left(\frac{2}{\sqrt{2}}\right) = 0 \quad \text{—— (1)}$

$F_1 = 0.5 R_1$ $R_2 = \frac{F_2}{0.4}$

$\frac{F_2}{R_1} = 0.2$ $F_2 = 0.2 R_1$

$-(0.4)(0.625)\left(\frac{4}{\sqrt{2}}\right)w - (0.625)\left(\frac{4}{\sqrt{2}}\right)w + \frac{wx}{2\sqrt{2}} + \frac{2w}{\sqrt{2}} = 0$

From ① & ②

$\Rightarrow x = 3\,m$

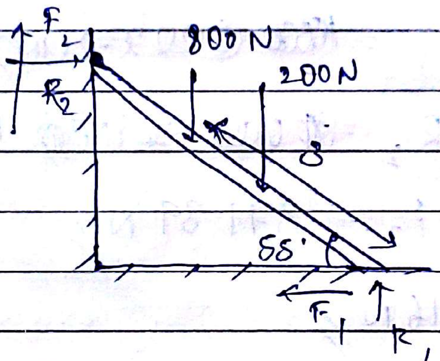
$1.2R_1 = \frac{3W}{2}$ correct.

$R_1 = 1.25W$

$R_2 = 0.5R_1 \Rightarrow R_2 = 0.625W \rightarrow$ ②

Q. A uniform ladder 4m long is supported by a wall & floor as shown in figure the co-efficient of friction b/w ladder & wall is 0.3 and floor & ladder is 0.5, weight of ladder is 200N, the ladder supports a vertical load of 800N at pt. C.

Determine the reactions at the floor and also wall



$\frac{F_1}{R_1} = 0.5$

$\frac{F_2}{R_2} = 0.3$

$F_1 = R_2$

$R_1 + F_2 = 1000N$

$\frac{F_1}{0.5} + 0.3R_2 = 1000$

$2.3R_2 = 1000N$

$R_2 = 434.78\,N = F_1$

$F_2 = 130.4N$

$R_1 = 869.56\,N$

~~$0.5R_2 + 0.4R_2 = \frac{3W}{2}$~~

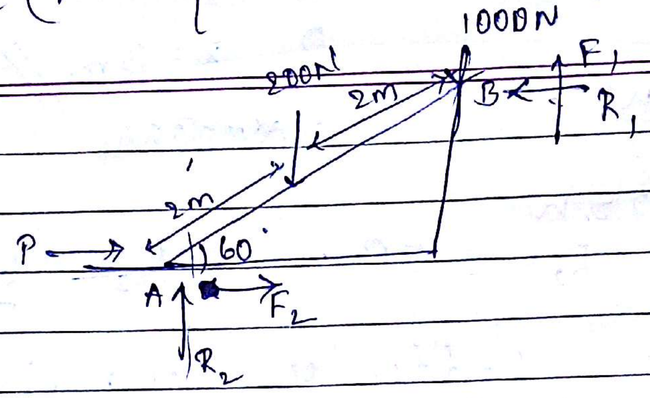
~~$0.9R_2 = \frac{3W}{2}$~~

~~$W = 0.6R$~~

$\frac{R_2}{F_2}$

μ (Wall & ladder) = 0.25 $\qquad$ μ (Ground & ladder) = 0.35

Q.



$\frac{F_1}{R_1} = 0.25$

$\frac{F_2}{R_2} = 0.35$

(i) $R_2 + F_1 = 1200$ $\qquad$ $P = R_1 - F_2$

$\Sigma M_A = 0$

$-F_1(2) - R_1(3.46) + 1000(2) + 200(1) = 0$

$-3.96 R_1 + 2200 = 0$

$R_1 = 555.55 N$ //

$F_1 = 138.88 N$ //

$R_2 = 1061.12 N$ //

$F_2 = 371.39 N$ //

$\Rightarrow P = 184.16 N$ //

(ii) if α

$F_2 = R_1$ $\qquad$ $R_2 + F_1 = 1200 N$

$\frac{F_2}{0.35} + 0.25 F_2 = 1200$

$3.10 F_2 = 1200$

$F_2 = 387.09 N$ // $= R_1$

$R_2 = 1105.97 N$ //

$F_1 = 96.77 N$ //

$$-F_1(4\cos\alpha) - R_1(4\sin\alpha) + 1000(4\cos\alpha) + 200(2\cos\alpha) = 0$$

$$-387.08\cos\alpha - 1548.36\sin\alpha + 4000\cos\alpha + 400\cos\alpha = 0$$

$$4012.92\cos\alpha = 1548.36\sin\alpha$$

$$\tan\alpha = \frac{4012.92}{1548.36}$$

$$\alpha = 68.90^{\circ}$$