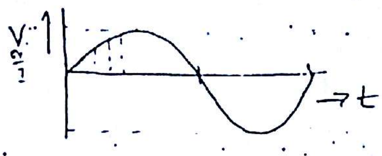
Unit - 2 [Single Phase AC].

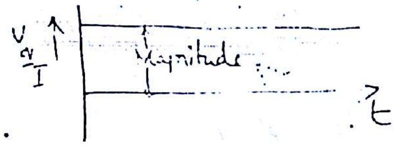
AC FUNDAMENTALS :

Difference between AC & DC :

In Alternating Current, the direction and magnitude both change with respect to time. It is represented using a sine wave.

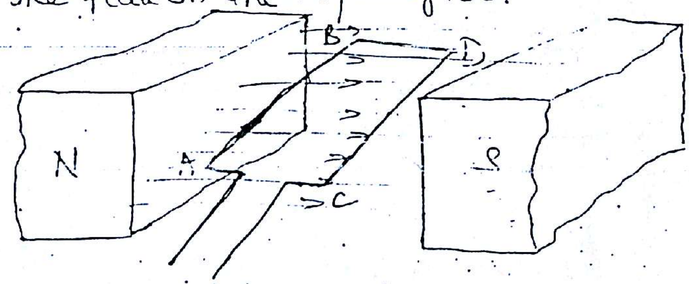


In Direct current, the direction and magnitude both donot change with respect to time and is represented as below.



Generation of Sine Wave :

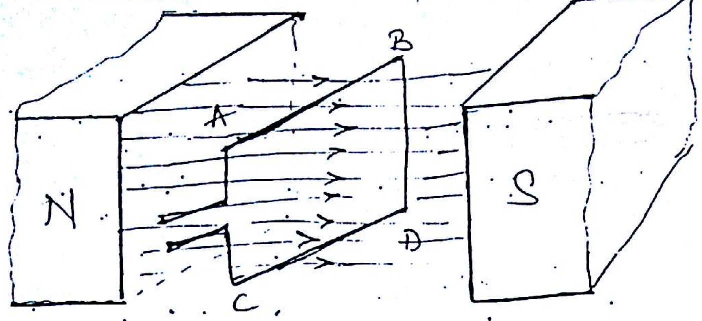
Consider a magnetic field with N and S pole and magnetic field lines starting from N and terminate at S. Let ABCD is a single turn rectangular coil to be rotated in the magnetic field. where AB and CD are the coilsides or active length of the coil which are responsible for cutting the flux in the magnetic field.



To understand the generation sine wave three important parameters are to analysed at every instance or position of coil in the magnetic field. The three important parameters are

1) Flux linking with coil
2) Change in flux linking with coil
3) EMF induced in the coil

Case 1> $\theta = 0$ (Initial position of Coil in the magnetic field)



The position of coil in the magnetic field shown above is taken as initial position hence $\theta = 0$. where θ is the angle through which the coil rotates in the magnetic field.

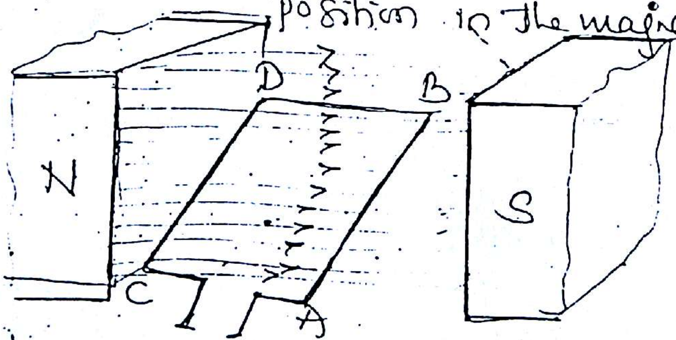
Now at this position the three parameters are

1> Flux linkage: Since coil is in vertical position in mag. field the flux linking with coil is maximum.

2> Change in flux linkage: Coil is about to start rotating hence no change in flux linking with coil, therefore change in flux linkage is minimum.

3> Emf induced: when change in flux linkage is minimum obviously the emf induced will be minimum

Case 2> $\theta = 90°$ (Coil is rotated from verticle position to horizontal position in the magnetic field)



The position of coil in the magnetic field is horizontal as it has travelled from verticle to horizontal through 90°. Hence $\theta = 90°$

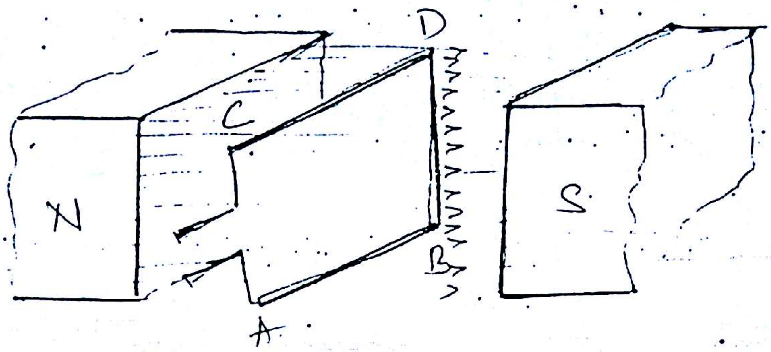
Now at this position,

1> Flux linkage: As coil is in horizontal position in mag. field the flux linkage is minimum.

2> Change in flux linkage: As coil has travelled through 90° during this maximum flux is cut and hence

3) EMF induced : As change in flux linkage is maximum the emf induced will be maximum.

Case : 3) $\theta = 180^\circ$ (coil rotates from horizontal to verticle position in mag. field)

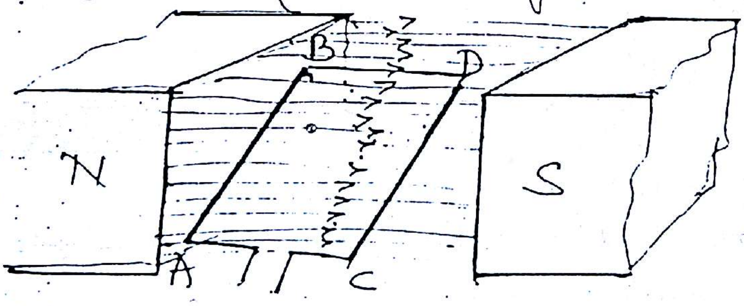


The position of coil in the magnetic field is verticle as it has travelled from $\theta = 90$ to $\theta = 180^\circ$.. Hence θ is 180°

At this position the three parameters are

1) Flux linkage : Maximum
2) Change in flux linkage : Minimum
3) EMF induced : Minimum

Case : 4) $\theta = 270^\circ$ (coil rotates from verticle to horizontal in mag. field)



The position of coil in the magnetic field is horizontal as it has travelled from $\theta = 180$ to $\theta = 270$. Hence θ is 270°.

At this position the three parameters are

1) Flux linkage : Minimum
2) Change in flux linkage : Maximum
3) EMF induced : Maximum.

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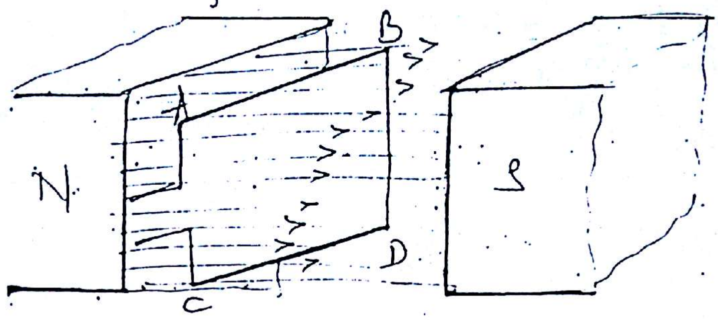
Case :- 5> θ = 360° (As coil travelled from horizontal to verticle position in magnetic field)



The position of the coil in the magnetic field is verticle as it has travelled through θ = 270° to θ = 360°. Hence θ = 360°

The three parameters at this position of coil are :-

1> Flux linkage : Maximum

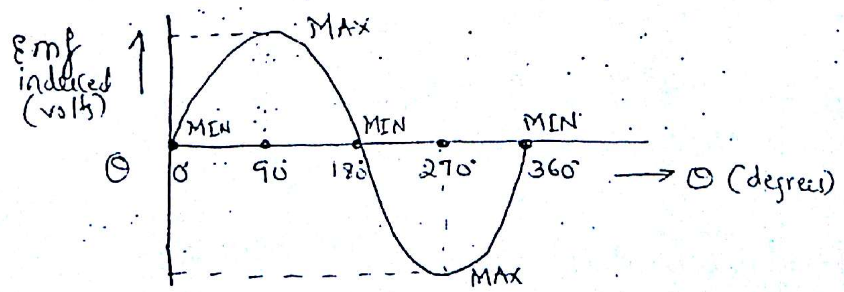
2> Change in flux linkage : Minimum

3> EMF induced : Minimum.

Now at different positions of coil in the magnetic field the emf induced was different which is listed below.

θ (Degrees)	EMF Induced (volts)
0°	Min
90°	Max
180°	Min
270°	Max
360°	Min

When a graph is plotted with emf induced along Y-axis and θ along x-axis then following wave form is obtained.



Thus generation of sinewave.

Derivation:

The above said concept of generation of sinewave is a dynamically induced emf process.

We have, dynamically induced emf as

$$E = Blv \sin\theta \longrightarrow (1)$$

For $E \rightarrow E_{max}$, $\sin\theta = 1$ or $\theta = 90°$

then $E_{max} = Blv \longrightarrow (2)$

∴ Putting equation (2) in equation (1).

$$\boxed{E = E_{max} \sin\theta} \text{ volts}$$

If θ is mentioned in radians then

$$\boxed{E = E_{max} \sin\omega t} \text{ volts}$$

Where ω is the angular frequency & $\omega = 2\pi f$

$$\boxed{E = E_{max} \sin 2\pi f t} \text{ volts}$$

As $f = \frac{1}{T}$

$$\boxed{E = E_{max} \sin \frac{2\pi}{T} t} \text{ Volts}$$

Note:- The above written equations for voltage also holds good for Current hence

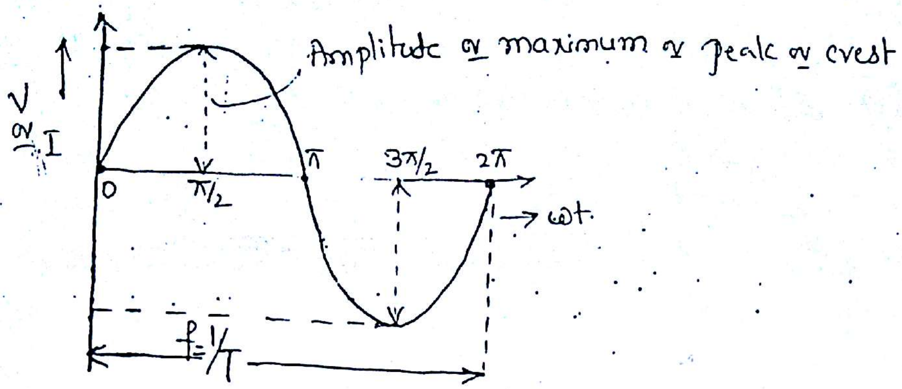
$$\boxed{I = I_m \sin\theta} \text{ Amp}$$

$$\boxed{I = I_m \sin\omega t} \text{ Amp}$$

$$\boxed{I = I_m \sin 2\pi f t} \text{ Amp}$$

$$\boxed{I = I_m \sin \frac{2\pi}{T} t} \text{ Amp.}$$

Basic Terms of an Alternating Current :



Frequency : It is defined as no. of cycles per second and its unit is Hz.

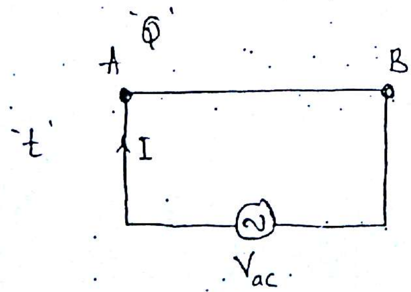
Waveform : The graph of instantenous values of an alternating quantity plotted against time is called its waveform.

Time Period (T) : It is defined as time taken by alternating quantity to complete its one cycle ~~is known~~ as time period.

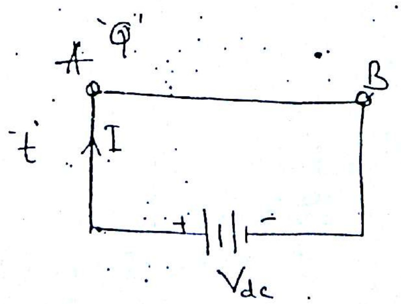
Amplitude : The maximum value attained by an alternating quantity during positive or negative half cycle is called its amplitude.

Instantaneous value : The value of an alternating at a perticular instant is known as its instantaneous value.

AVERAGE VALUE OF AN ALTERNATING QUANTITY :→



(1)



(2)

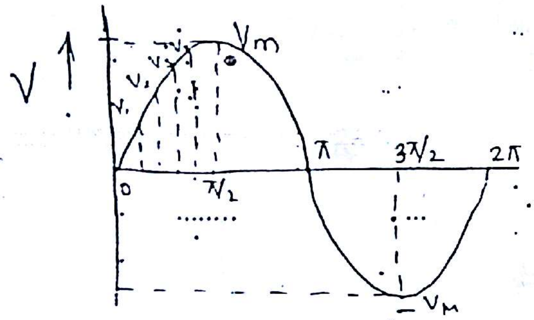
1. In fig(1) an AC voltage V_{ac} is applied to a given conductor AB for a given period of time 't' seconds. Then there are 'Q' no. of charges are transfered from point A to B of the conductor.

In fig (2), the same conductor AB is taken a DC voltage V_{dc} is applied such that it is applied for same 't' seconds and there are same 'Q' charges are transfered across conductor AB. Then the applied DC voltage V_{dc} is known as average value of an alternating quantity V_{ac}.

Definition:

The average value of an alternating quantity is that value of DC voltage which when applied to same conductor for same period of time, transfers same amount of charges as would have been transfered by the AC voltage. Then this DC voltage is called as average value of an alternating quantity.

Derivation:



The average value of alternating quantity over one full cycle is zero as in one cycle equal positive and negative instantaneous voltages will be there, which causes sum of instantaneous values of one full cycle will be zero. Hence to find average value of an alternating quantity only half cycle is considered.

$$\text{Average Value} = \frac{\text{Sum of all the values}}{\text{Total no. of values}}$$

From the above wave form,

$$\text{Average value} = \frac{V_1 + V_2 + V_3 + \cdots V_\pi}{\pi}$$

Therefore average value of an alternating quantity over half cycle is given by

$$V_{avg} = \frac{1}{\pi}\int_0^\pi V \, d\theta$$

$$= \frac{1}{\pi}\int_0^\pi V_m \sin\theta \cdot d\theta$$

$$= \frac{V_m}{\pi}\left[-\cos\theta\right]_0^\pi \Rightarrow \frac{V_m}{\pi}\left[-[(\cos\pi) - (\cos 0)]\right]$$

$$V_{avg} = \frac{V_m}{\pi}\left[-[(-1) - (1)]\right] \Rightarrow \frac{V_m}{\pi}[1+1]$$

$$\therefore V_{avg} = \frac{V_m}{\pi} \cdot 2$$

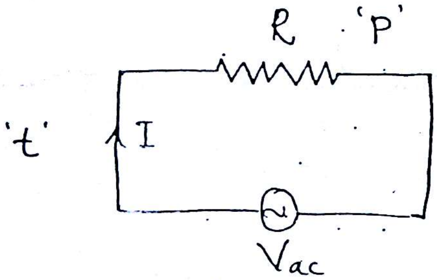
$$\therefore \boxed{V_{avg} = \frac{V_m}{\pi/2}}$$

$$\therefore \boxed{V_{avg} = 0.637\, V_m}$$

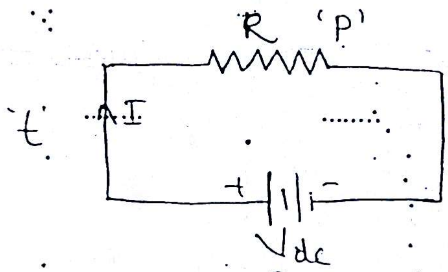
or

$$\boxed{V_{avg} = 63.7\% \text{ of } V_m}$$

ROOT MEAN SQUARE VALUE OF AN ALTERNATING QUANTITY :→
(Effective Value of an alternating Quantity)



(1)



(2)

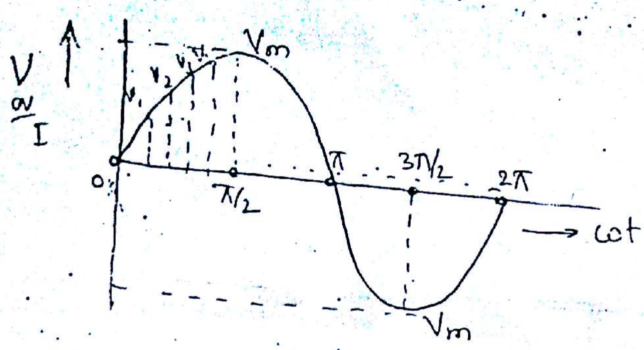
In fig ① an alternating quantity voltage is applied ~~to a resistor~~ 'R' for a period of time 't' sec, then 'P' is the amount of heat dissipated.

In fig ② a DC voltage Vdc is applied to the same resistor 'R' for same period of time 't' seconds such that same 'P' amount of heat is dissipated. Then Vdc is called as RMS value of Vac.

Definition:

RMS value of an alternating quantity is that value of DC voltage which when applied to the same resistor for same period of time such that the resistor dissipates same 'P' amount of heat. Then that value of DC voltage is called RMS value of an alternating quantity.

Derivation :→



Root mean Square value =

$$\text{RMS value} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \cdots V_{2\pi}^2}{2\pi}}$$

RMS value of an alternating quantity over one full cycle is given as

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} V^2 \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta \cdot d\theta$$

$$= \frac{V_m^2}{2\pi}\int_0^{2\pi} \frac{1-\cos 2\theta}{2} \, d\theta$$

$$= \frac{V_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{V_m^2}{4\pi}\left[\left[2\pi - \frac{\sin 4\pi}{2}\right] - 0\right]$$

$$= \frac{V_m^2}{\cancel{4\pi}_2} \cdot \cancel{2\pi}$$

$$V_{rms}^2 = \frac{V_m^2}{2}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}} \quad \text{or} \quad \boxed{V_{rms} = 0.707\, V_m}$$

or

$$\boxed{V_{rms} = 70.7\% \text{ of } V_m}$$

Form factor: It is the ratio of RMS value to the average value

$$\therefore FF = \frac{V_{rms}}{V_{avg}}$$

$$= \frac{V_m/\sqrt{2}}{V_{avg}/2\pi/2} \Rightarrow \frac{V_m}{\sqrt{2}} \times \frac{2\pi/2}{V_m}$$

$$= \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\boxed{FF = 1.11}$$

Peak factor or Crest factor: It is the ratio of peak value to the rms value.

$$\therefore \text{Peak factor} = \frac{V_m}{V_{rms}}$$

$$= \frac{V_m}{V_m/\sqrt{2}}$$

$$= \sqrt{2}$$

$$= 1.412$$

$$\boxed{\text{Peak factor} = 1.412}$$

Phasor representation of an alternating quantity :-

Phasor: It is a line of finite length, which represents a complex electrical quantity which is a vector.

If a phasor is used to represent an alternating quantity following assumptions are made.

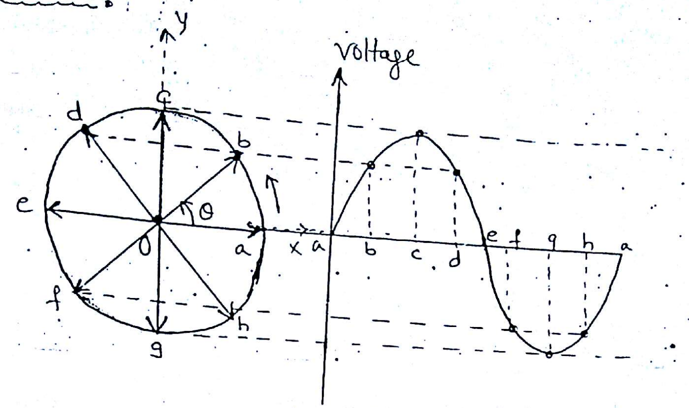
1) Always the phasor must rotate in the anticlockwise direction and is taken as positive.

2) A phasor must have a length to some scale, which will represent the Amplitude of the alternating quantity it is representing.

3) A phasor must take same time for completing one rotation as alternating quantity takes to complete one cycle.

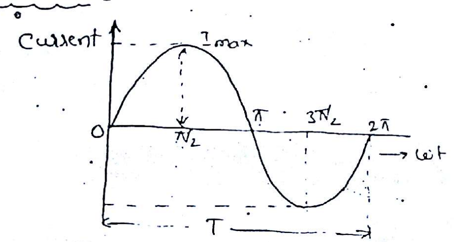
4) A phasor must lie horizontally when alternating quantity is crossing the zero on the reference access.

Explanation :



Consider a phasor $\overline{Oa}$, rotating in anticlockwise direction with uniform angular velocity, with its starting position 'a' as shown above.
If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ' we get a sine wave.

Phase :→



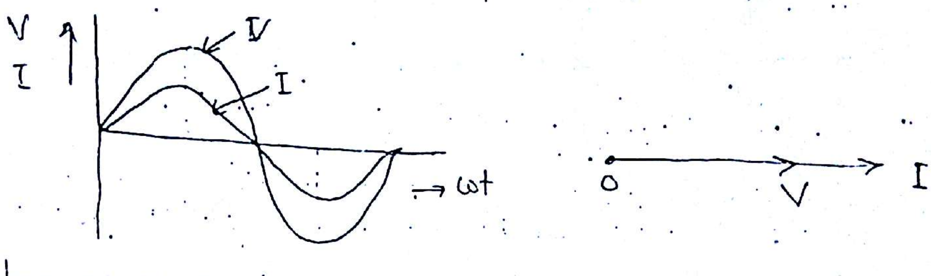
An alternating voltage or current changes its magnitude and direction at every instant. So it is necessary to know the condition of the alternating quantity at a particular instant.

The location of the condition of the alternating quantity at any particular instant is called its phase.

Definition:→
The phase of an alternating quantity at any particular instant is the fractional part of a period or cycle through which the quantity has advanced from the selected origin.

Phase Difference :→

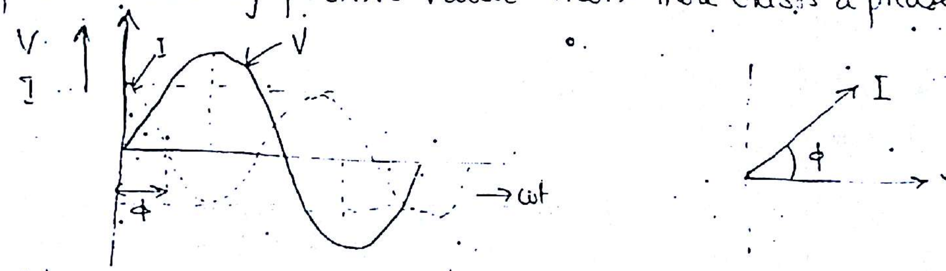
Two alternating quantities having same frequency, reaching maximum positive and negative values and zero values at the same time are said to in phase. The phase angle difference between them will be zero.



where $V = V_m \sin\theta$ & $I = I_m \sin\theta$.

The phase angle between V & I phases is zero hence are said to be in phase.

In AC Systems or analysis, it is not necessary that all alternating quantities must be always in phase. Suppose if one phase is achieving its zero, at the same time or instant other phase is having positive value then there exists a phase difference.

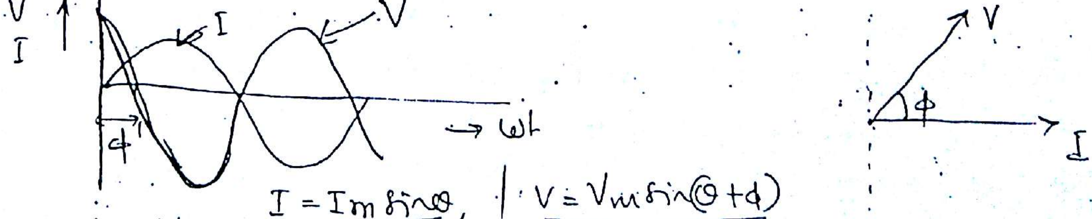


where $V = V_m \sin\theta$ & $I = I_m \sin(\theta + \phi)$

The current phase is leading the voltage phase by some angle ϕ

or The voltage phase lagging the current phase by same angle ϕ

Similarly if one phase is achieving its zero, at the same instant, other phase is having negative value then also there exist phase difference.



$I = I_m \sin\theta$, $V = V_m \sin(\theta + \phi)$

Voltage phase leads the current phase by ϕ or current phase lags voltage phase by ϕ.

Definition:
The difference between the phases of the two alternating quantities is called the phase difference.

A phasor can be represented in four different forms and they are

1) Complex form = $A+jB$

2) Trignometric form = $M(\cos\theta + j\sin\theta)$ ($\because M=\sqrt{A^2+B^2}$ & $\theta = \tan^{-1}\frac{B}{A}$)

3) Exponential form = $Me^{j\theta}$

4) Polar form = $M\angle\theta^\circ$

* To carry out addition and substraction operations the phasors must be in complex form only.

* To carry out multiplication and division operations the phasors must be in polar form only.

Addition:

$(A_1+jB_1)+(A_2+jB_2)$ Then addition of phasors can be given as

$= A_1+jB_1+A_2+jB_2$

$= (A_1+A_2)+j(B_1+B_2)$

Substraction:

$(A_1+jB_1)-(A_2+jB_2)$

$= A_1+jB_1-A_2-jB_2$

$= A_1 + A_2 + j(B_1-B_2)$

Multiplication:

$M_1\angle\theta_1 \times M_2\angle\theta_2$

$= M_1\times M_2 \angle\theta_1+\theta_2$

Division:

$M_1\angle\theta_1 \div M_2\angle\theta_2$

$= M_1/M_2 \angle\theta_2-\theta_1$

Active and Passive Components:→

Active Components:

Active Components are those which generate and deliver the energy. Ex:- Generator, Battery

Passive Components:

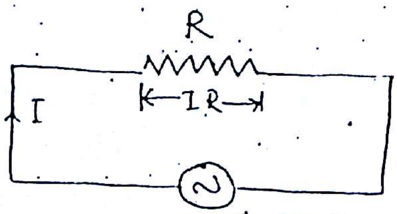
Passive Components are those which receive and store the energy. Ex:- Inductor, Capacitor.

-: SINGLE PHASE AC CIRCUITS :-

Resistance, Inductance and Capacitance are the basic elements in any electrical network. Therefore it is essential to analyze these circuits independently and a combination of these. The following cases are analysed.

1) Circuit containing pure resistance only.
2) Circuit containing only pure inductance.
3) Circuit containing only pure capacitance
4) Circuit containing Resistance and inductance connected in series
5) Circuit containing Resistance and Capacitance connected in series
6) Circuit containing Resistance, Inductance and capacitance connected in series.

1) Circuit containing pure resistance:→



$V = V_m \sin\theta$ ⟶ (1)

Consider a pure ohmic resistance R only in the circuit with $V = V_m \sin\theta$ is the voltage applied to it and I be the current flowing in the circuit.

Applying Ohm's law,

$$I = \frac{V}{R}$$

$$\therefore V = IR$$

Since only one element is present in the circuit then the voltage drop across it must be equal to applied voltage.

$$\therefore V = V_R$$

$$V_m \sin\theta = IR$$

$$\therefore I = \frac{V_m}{R} \cdot \sin\theta \rightarrow (2)$$

For current to become maximum $\sin\theta = 1$

$$\therefore I_{max} = \frac{V_m}{R} \rightarrow (3)$$

Substituting I_{max} in equation (2) ....

$$\therefore \boxed{I = I_m \sin\theta} \rightarrow (4)$$

Considering Eq:n (3). and dividing $\sqrt{2}$ on both sides

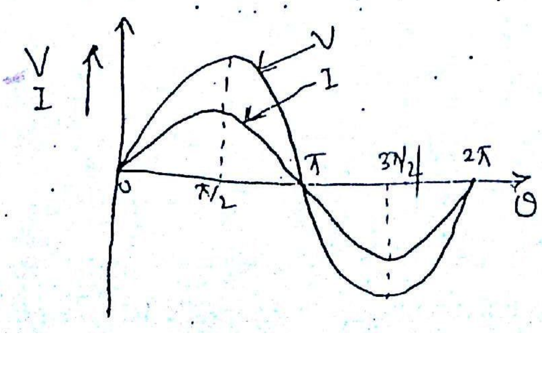
$$\frac{I_{max}}{\sqrt{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{1}{R} \qquad \left(\because I_{rms} = \frac{I_{max}}{\sqrt{2}} \text{ or } V_{rms} = \frac{V_{max}}{\sqrt{2}}\right)$$

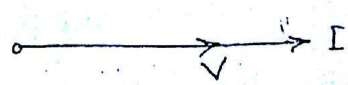
$$\therefore \boxed{I = \frac{V}{R}}$$

Rewriting Eq:n (1) & (4)

$$\boxed{\begin{aligned} V &= V_m \sin\theta \\ I &= I_m \sin\theta \end{aligned}}$$

From the two equations it is clear that both voltage and current in a pure resistance circuit are in phase i.e the phase angle difference is zero, and can be shown as below.





Power Consumed in pure resistance Circuit:

$P = V\,I$

$= V_m \sin\theta \cdot I_m \sin\theta$

$= V_m I_m \sin^2\theta$

$= V_m I_m \left(\frac{1-\cos 2\theta}{2}\right)$

$$= \underbrace{\frac{V_m I_m}{2}}_{\mathrm{I}} - \underbrace{\frac{V_m I_m}{2}\cos 2\theta}_{\mathrm{II}}$$

I - This term does not have variable parameter 'θ' hence is a steady power.

II - This term has 'θ' in the equation. hence is variable or fluctuating power.

∴ Fluctuating Power over one full cycle is given by

$$\int_0^{2\pi} \frac{V_m I_m}{2} \cdot \cos 2\theta \cdot d\theta$$

$$= \frac{V_m I_m}{2}\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{2}\left[\frac{\sin 4\pi}{2} - \frac{\sin 0}{2}\right]$$

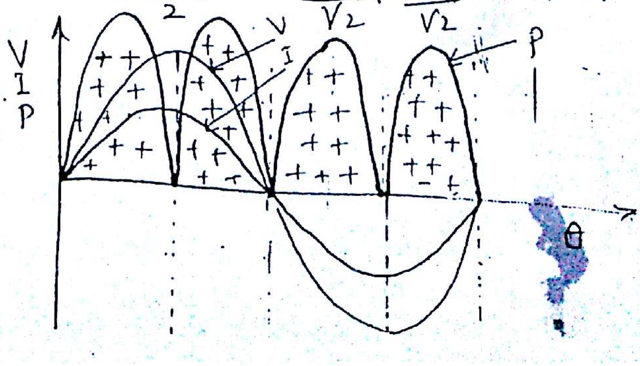
$$= \frac{V_m I_m}{2}\left[0-0\right]$$

$$= \underline{\underline{0}}$$

∴ Fluctuating power over one full cycle is Zero.

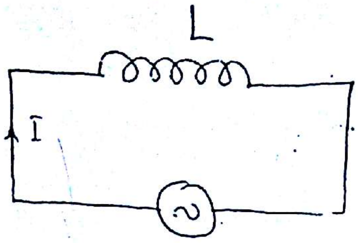
∴ Power: $P =$ steady power $= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$

∴ $\boxed{P = VI}$



(E)

2) Circuit Containing Pure Inductance :→



$V = V_m \sin\omega t \longrightarrow (1)$

Consider a circuit having pure inductance only in the circuit. with V be the voltage applied and current I be flowing in the circuit.. A pure inductance will have zero resistance. when voltage is applied to the inductor, there will be an emf induced in it called as self induced emf, and is given by

$$e = L\frac{dI}{dt}$$

Since only one element in the circuit the emf induced is equal in magnitude and opposite in direction with applied voltage.

$$\therefore V = e$$

$$V_m \sin\omega t = L\cdot\frac{dI}{dt}$$

$$\therefore \frac{dI}{dt} = \frac{V_m}{L}\cdot\sin\omega t$$

To get current I above Eq.n must be integrated

$$\int \frac{dI}{dt} = \int \frac{V_m}{L}\cdot\sin\omega t\cdot d\omega t$$

$$\therefore I = \frac{V_m}{L}\cdot\left[-\frac{\cos\omega t}{\omega}\right]$$

$$= \frac{V_m}{L\omega}\left[-\sin(\pi/2-\omega t)\right] \quad (\because \cos\omega t = \sin(\pi/2-\omega t))$$

$$= \frac{V_m}{L\omega}\left[-\sin(-(\omega t-\pi/2))\right]$$

$$= \frac{V_m}{L\omega}\left[\sin(\omega t-\pi/2)\right]$$

$$\therefore I = \frac{V_m}{L\omega}\sin(\omega t-\pi/2)$$

$$I = \frac{V_m}{L\omega}\sin(\omega t - \pi/2) \longrightarrow (2)$$

Now for I to become Imax $\sin(\omega t - \pi/2) = 1$

$$\therefore I_{max} = \frac{V_m}{L\omega} \longrightarrow (3)$$

dividing $\sqrt{2}$ on both sides

$$\therefore \frac{I_{max}}{\sqrt{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{1}{L\omega}$$

$$\therefore I = \frac{V}{L\omega} \Rightarrow \frac{V}{X_L}$$

$$\therefore \boxed{I = \frac{V}{X_L}}$$

where $X_L \rightarrow$ Inductive Reactance Ω

$X_L = 2\pi f L = L\omega$

$\therefore$ Inductive reactance is the opposition to flow of current due to inductance.

Now Substituting (3) in (2).

$$\therefore \boxed{I = I_m \sin(\omega t - \pi/2)} \longrightarrow (4)$$

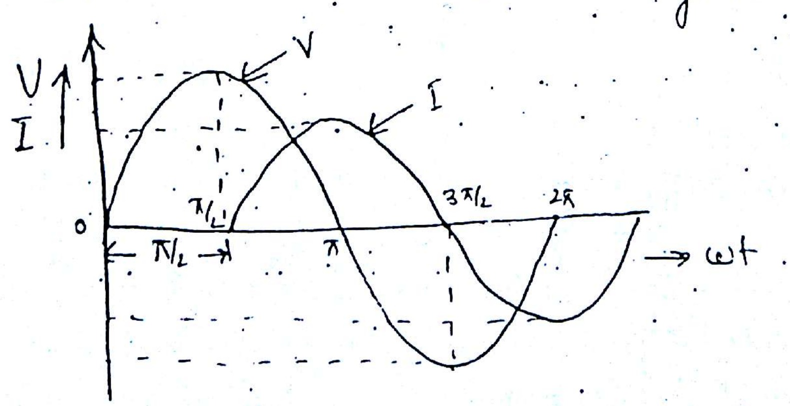
$\therefore$ Rewriting Eqn (1) & (4)

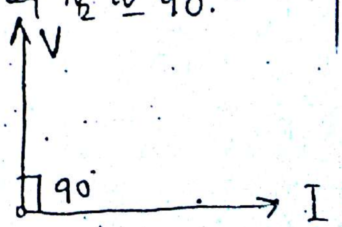
$$\boxed{\begin{array}{l} V = V_m \sin(\omega t) \\ I = I_m \sin(\omega t - \pi/2) \end{array}}$$

From the above equations it is clear that

In a pure inductive Circuit, Voltage leads the Current by $\pi/2$ or $90°$

or Can also be said as current lags the voltage by $\pi/2$ or $90°$.





Power Consumed in a pure inductive Circuit is given by

$$P = V\, I$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t - \pi/2)$$

$$= V_m I_m \sin\omega t (-\cos\omega t) \quad (\because \sin(\omega t - \pi/2) = -\cos\omega t)$$

$$= -V_m I_m \frac{\sin 2\omega t}{2}$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

Power over one full cycle is given by

$$P = \int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t$$

$$= -\frac{V_m I_m}{2}\left[-\frac{\cos 2\omega t}{2\omega}\right]_0^{2\pi}$$

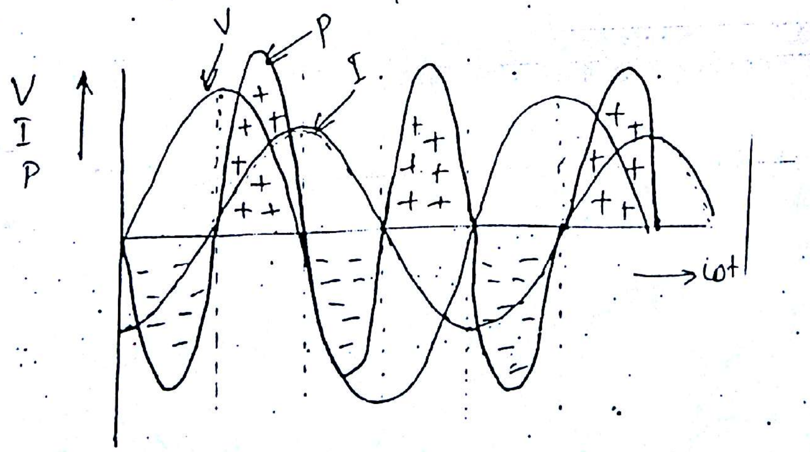
$$= \frac{V_m I_m}{2}\left[\frac{\cos 2\omega t}{2\omega}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{2}\left[\frac{\cos(4\pi)}{2\omega} - \frac{\cos(0)}{2\omega}\right]$$

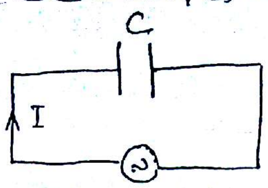
$$= \frac{V_m I_m}{2}\left[\frac{1}{2\omega} - \frac{1}{2\omega}\right]$$

$$\boxed{P = 0}$$

Power consumed in a pure inductive Circuit is Zero.



3) A Circuit Containing Pure Capacitance :→



$V = V_m \sin\omega t \longrightarrow (1)$

Consider a circuit having a pure capacitor with 'c' as capacitance and 'V' be the voltage applied and I be the current flowing in the circuit.

A pure capacitor is the one which will have zero dielectric loss.

When voltage is applied to the circuit, the current 'I' charges the capacitor 'c'. The instantaneous charge 'q' on the plates of the capacitor is given by

$$q = CV$$

$$\therefore q = C V_m \sin\omega t$$

Now to get current above eq: is differentiated w.r.t dt.

$$\therefore \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin\omega t)$$

But $\frac{dq}{dt} = I$, $\quad \therefore I = C V_m \frac{d}{dt}\sin\omega t$

$$\therefore I = \omega C V_m \cos\omega t$$

$$\therefore I = \omega C V_m \sin(\omega t + \pi/2) \quad (\because \sin(A+\theta) = \cos A)$$

Now for simplification.

$$I = \frac{V_m}{1/C\omega} \sin(\omega t + \pi/2) \longrightarrow (2)$$

For I to become I_{max} $\quad \sin(\omega t + \pi/2) = 1$

$$\therefore I_m = \frac{V_m}{1/C\omega} \longrightarrow (3)$$

dividing $\sqrt{2}$ on both sides

$$\frac{I_m}{\sqrt{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{1}{C\omega}$$

$$I = \frac{V}{C\omega}$$

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$$\therefore \boxed{I = \frac{V}{X_C}}$$

Where $X_C = 1/2\pi f C$ where $X_C \rightarrow$ Capacitive Reactance Ω

Capacitive reactance is the opposition to flow of current by the capacitor.

Substituting (3) in (2)

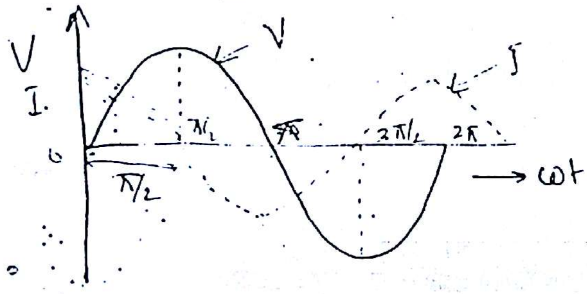
$$\boxed{I = I_m \sin(\omega t + \pi/2)} \rightarrow (4)$$

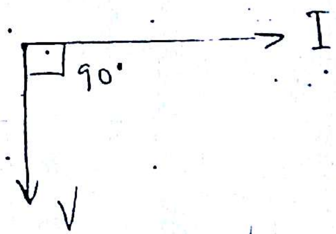
Rewriting Eq:1s (1) & (4)

$$\left.\begin{aligned} V &= V_m \sin\omega t \\ I &= I_m \sin(\omega t + \pi/2) \end{aligned}\right\}$$

From the above equations it is clear that.

In a pure Capacitive Circuit, The current leads the voltage by $\pi/2$ or $90°$ and is shown below.





Power Consumed in pure Capacitance Circuit is given by

$$P = V \cdot I$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2)$$

$$= +V_m I_m \sin\omega t \cos\omega t$$

$$= +V_m I_m \frac{\sin 2\omega t}{2}$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

power over one full cycle is

$$P = \int_0^{2\pi} \frac{V_m I_m}{2} \cdot \sin 2\omega t \cdot d\omega t$$

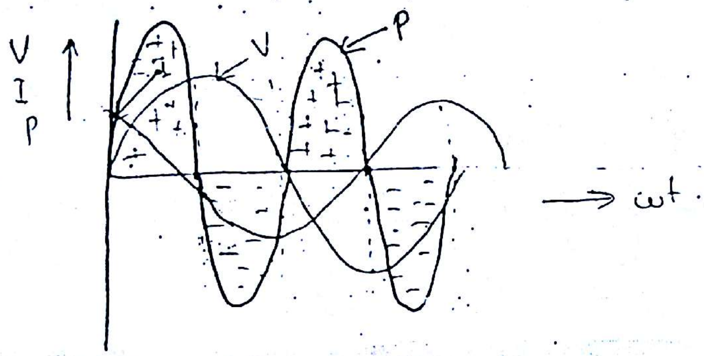
$$\therefore P = \frac{V_m I_m}{2 \times 2\omega}\left[-\cos 2\omega t\right]_0^{2\pi}$$

$$= \frac{-V_m I_m}{2} \cdot \left[\cos(4\pi) - \cos 0\right]$$

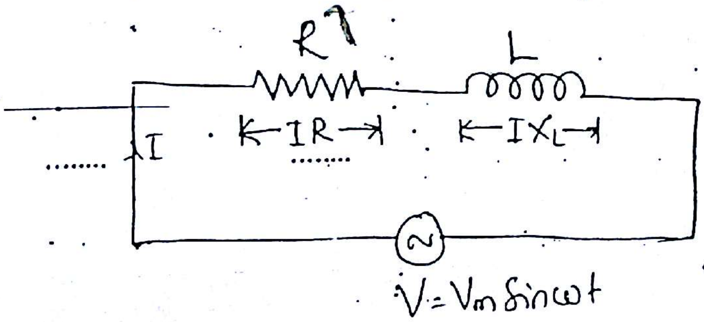
$$= -\frac{V_m I_m}{2}\left[1-1\right]$$

$$\boxed{P = 0}$$

The power consumed in a pure capacitive circuit is zero.



4> R-L Series Circuit :→



Consider a circuit having resistor and inductor connected in series and V be the voltage applied and I be the current flowing in the circuit.

As both R & L are connected in series, same current 'I' flows through both R and L. Then drop across these elements are given as

Drop across pure resistance = $V_R = IR$

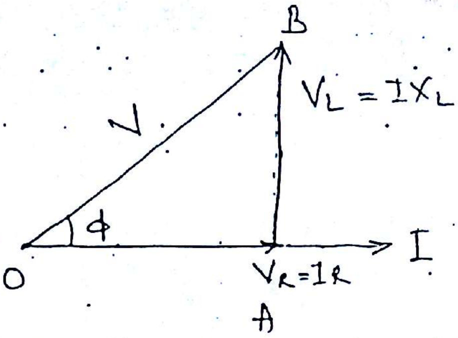
Drop across pure inductance = $V_L = IX_L$

The vector sum of these drops is equal to applied voltage V. i.e

$$\overline{V} = \overline{V_R} + \overline{V_L}$$

$$\overline{V} = \overline{IR} + \overline{IX_L}$$

By considering the elements in circuit the phasor diagram is drawn as shown below. by taking current as reference phasor.



From Δ^{le} OAB,

$$OB^2 = OA^2 + AB^2$$

$$V^2 = V_R^2 + V_L^2$$

$$V^2 = (IR)^2 + (IX_L)^2$$

$$\therefore V^2 = I^2(R^2 + X_L^2)$$

$$I^2 = \frac{V^2}{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$\boxed{I = \frac{V}{Z}}$$ where 'Z' Impedance Ω

$$\boxed{Z = \sqrt{R^2 + X_L^2}}$$ or $$\boxed{Z = R + jX_L}$$

Impedance (z) is the total circuit opposition to flow of current.

From Δ^{le} OAB

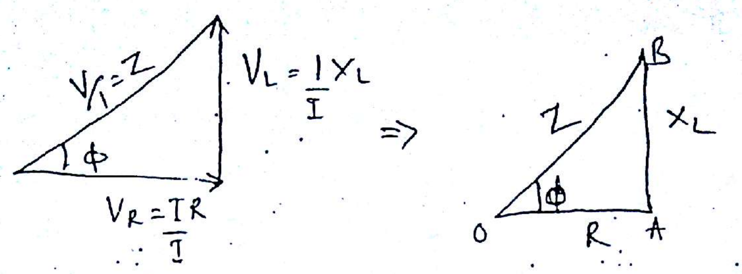
$$\boxed{\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}}; \quad \boxed{\cos\phi = \frac{V_R}{V} = \frac{R}{Z}};$$

$$\boxed{\sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}}$$

From the above triangle OAB, it is clear the voltage leads the current by an angle 'ϕ'. Hence equations are

$$\boxed{\begin{aligned} V &= V_m \sin\omega t \\ I &= I_m \sin(\omega t - \phi) \end{aligned}}$$

To the Δ^{le} OAB divide all sides by I then



(1) (2)

Δ^{le} (2) is called impedance triangle where

$\boxed{\cos\phi = \frac{R}{Z}}$ $\boxed{\sin\phi = \frac{X_L}{Z}}$

Power Consumed in R-L Series Circuit is given by

$$P = V\,I$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$$

$$= V_m I_m \left[\underset{A}{\sin\omega t} \cdot \underset{B}{\sin(\omega t - \phi)}\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$= \underbrace{\frac{V_m I_m}{2}\cos\phi}_{\text{Steady Power}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t - \phi)}_{\text{Fluctuating power}}$$

Fluctuating Power over one full cycle is given by

$$\int_0^{2\pi} \frac{V_m I_m}{2}\cos(2\omega t - \phi)\, d\omega t$$

$$= \frac{V_m I_m}{2}\left[\frac{\sin(2\omega t - \phi)}{2\omega}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{4\omega}\left[\sin 4\pi - \phi - \sin(0 - \phi)\right]$$

$$= \underline{\underline{0}}$$

Fluctuating power over one full cycle is given by Zero.

Steady power is

$$P = \frac{V_m I_m}{2} \cdot \cos\phi$$

$$\therefore P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi$$

$$\therefore \boxed{P = VI\cos\phi}$$

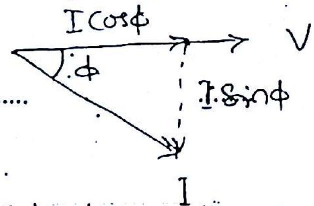
Power Consumed in Series R-L Series Circuit is given by $P = VI\cos\phi$.

Where $\cos\phi$ is Called as power factor

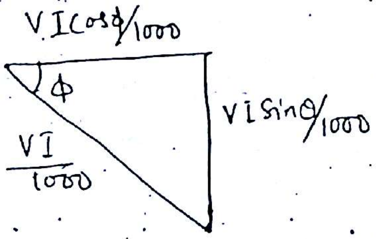
Power factor is defined as Cosine of the angle between voltage and Current. OR

It can be also defined as ratio of resistance to Impedance.

Power Triangle:



Current I split into Horizontal & Vertical Component: & Multiply all sides by $\frac{V}{1000}$



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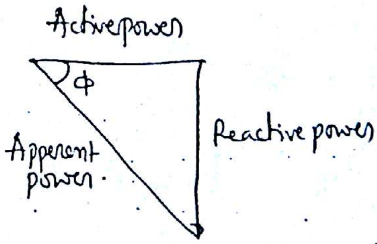

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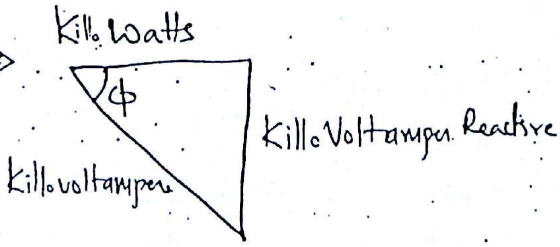
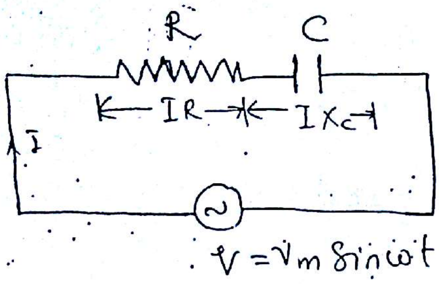


Active power = $P = VI\cos\phi$	K Watts
Reactive power = $Q = VI\sin\phi$	KVAR
Apparent power = $S = VI$	KVA

5) R–C Circuit :→



$v = V_m \sin\omega t$

Consider a circuit having resistor and capacitor connected in series and V be the voltage applied and I be the current flowing in the circuit.

As both R & C are connected in series, same current 'I' flows through both R & C. Then drop across these elements are given by

Drop across pure resistance = $V_R = IR$

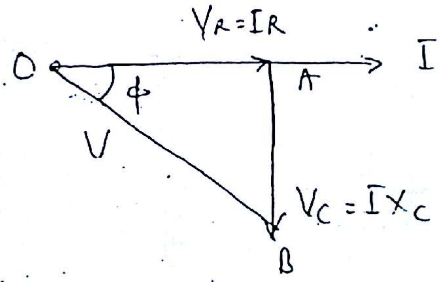
Drop across pure capacitance = $V_c = IX_c$

The vector sum of these drops is equal to the applied voltage 'V' i.e

$$\overline{V} = \overline{V_R} + \overline{V_c}$$

$$\overline{V} = \overline{IR} + \overline{IX_c}$$

By considering the elements in the circuit, the phasor diagram is drawn as shown below by taking current as reference phasor.



From the triangle OAB,

$$OB^2 = AB^2 + OA^2$$

$$V^2 = V_c^2 + V_R^2$$

$$V^2 = (IX_c)^2 + (IR)^2$$

$$V^2 = I^2(R^2 + X_c^2)$$

$$V^2 = I^2(R^2 + X_c^2)$$

$$I = V/\sqrt{R^2 + X_c^2}$$

$$\boxed{I = \frac{V}{Z}}$$

where Z - Impedance

$Z = \sqrt{R^2 + X_c^2}$

$\boxed{Z = R - jX_c}$ Ω

From the phasor diagram, It is clear that current leads the voltage by an angle 'ϕ'. Hence the equations are.

$$\boxed{\begin{aligned} V &= V_m \sin \omega t \\ I &= I_m \sin(\omega t + \phi) \end{aligned}}$$

Power Consumed in R-C Series Circuit is given by

$$P = VI.$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t + \phi)$$

$$= V_m I_m \sin\omega t \sin(\omega t + \phi)$$

$$= V_m I_m \left[\frac{\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)}{2} \right]$$

$$= \frac{V_m I_m}{2} \left[\cos\phi - \cos(2\omega t - \phi) \right]$$

$$= \underbrace{\frac{V_m I_m}{2} \cos\phi}_{\text{Steady power}} - \underbrace{\frac{V_m I_m}{2} \cos(2\omega t - \phi)}_{\text{Fluctuating power}}$$

Fluctuating power over one full cycle is zero i.e

$$\int_0^{2\pi} \frac{V_m I_m}{2} \cos(2\omega t - \phi) \cdot d\omega t$$

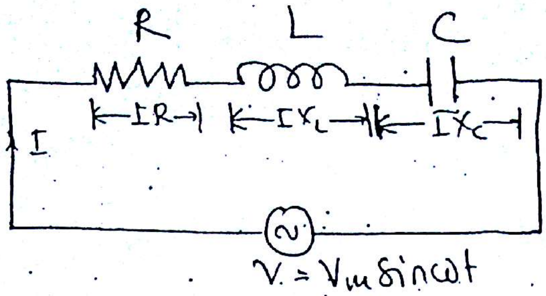
$$= 0$$

Steady power : $P = \frac{V_m I_m}{2} \cos\phi$

$$\therefore P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi.$$

$$\boxed{P = VI\cos\phi}$$

6) R-L-C Series Circuit :→



Consider a series R-L-C circuit with 'V' be the voltage applied and I be the current flowing through the circuit.

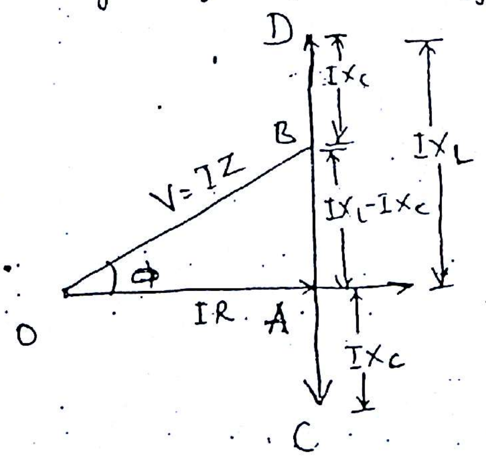
- The drop across R is IR
- The drop across L is IX_L
- The drop across C is IX_C

The vector sum of drops across all the three elements is equal to the applied voltage 'V'.

The analysis ~~of the circuit~~ can be done with following cases.
1) when $X_L > X_C$ 2) when $X_C > X_L$ 3) when $X_L = X_C$.

Case:-1) When $X_L > X_C$:→

- IR drop is in phase with I. IX_L drop is quadrature leading I and IX_C drop is quadrature lagging behind I.



As X_L is greater than X_C, $IX_L > IX_C$. A point B is choosen on line AD such that BD = AC

$AB = IX_L - IX_C$ the phasor difference of the reactances.

$OB = OA + AB$, the phasor sum = $IR + (IX_L - IX_C) = E$ the applied voltage.

From Δ^{le} OAB,

$$OB^2 = OA^2 + AB^2$$

$$V^2 = (IR)^2 + (IX_L - IX_C)^2$$

$$V^2 = I^2[R^2 + (X_L - X_C)^2]$$

$\therefore V = I\sqrt{R^2 + X^2}$ where X net reactance Ω

$X = X_L - X_C$

$\therefore I = \frac{V}{\sqrt{R^2+X^2}}$ ∴ Here net reactance is inductive.

$$\boxed{I = \frac{V}{Z}}$$

From the phasor diagram, it is clear that voltage leads the current I by angle φ. Hence current & voltage equations are as follows.

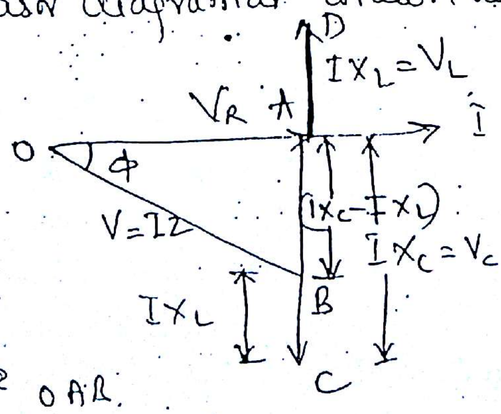
$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t - \phi)$$

Power consumed ~~in this case~~ is given by

$\boxed{P = VI\cos\phi}$ (After simplification of $P = VI$)

Case: 2> when $X_C > X_L$:

The phasor diagram is drawn as shown.



From Δ^{le} OAB.

$$OB^2 = OA^2 + AB^2$$

$$V^2 = (IR)^2 + (IX_C - IX_L)^2 \Rightarrow V^2 = I^2(R^2 + (X_C - X_L)^2)$$