IMS20CS037
— Danish Mahajan
IV A

Tutorial - 1

Q1 Find the missing term

x	0	1	2	3	4	5	6
y	5	11	22	40	–	140	–

Q2 Using newton backward interpolation formula find the interpolating polynomial for $y = f(x)$
$f(2) = 4$, $f(4) = 56$, $f(6) = 204$, $f(8) = 496$

Find $f(7)$

Q3 A survey is conducting in a factory reveals that the following information. Estimate the probable number of persons income group 20 and 25

Income per hours :	10 - 20	20 - 30	30 - 40	40 - 50
No of person :	45	115	210	115

Q4 The table gives the distance in nautical miles of the visible horizon for the given height in feet above the earth's surface. Find y when $x = 12$ feet, $x = 42$ feet

height (x)	10	15	20	25	30	35	40
Distance (y)	10.63	~~0.0~~ 13.03	15.04	16.81	18.42	19.90	21.27

Q5 Apply Lagrange's formula to find a root of equation $f(x)$ given

$f(30) = -30, \quad f(34) = -13, \quad f(42) = 18$

Sol!

x	0	1	2	3	4	5	6
y	5	11	22	40	–	140	–
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	5	6	5	2	y_4-67	$370-5y_4$
1	11	11	7	y_4-65	$303-4y_4$	y_6+10y_4-1001
2	22	18	y_4-58	$238-3y_4$	y_6+6y_4-698	
3	40	y_4-40	$180-2y_4$	y_6+3y_4-460		
4	y_4	$140-y_4$	y_6+y_4-280			
5	140	y_6-140				
6	y_6					

$\Delta^5 y = 0$

$\Rightarrow \quad 370 - 5y_4 = 0$

$5y_4 = 370$

$y_4 = 74$

Also

$y_6 + 10y_4 - 1001 = 0$

$y_6 = 1001 - 740$

$y_6 = \underline{261}$

x	$f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$
2	4			
		52		
			96	
4	56			48
		148		$\nabla^3 y_n$
			144	
			$\nabla^2 y_n$	
6	204	292		
		∇y_n		
8	496			
	y_n			

$$p = \frac{x - x_n}{h} = \frac{7-8}{2} = -1/2$$

newton backword's difference formula

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$y_p = 496 + (0.5)292 + \frac{(-0.5)(-0.5+1)}{2} \times 144 + \frac{(-0.5)(-0.5+1)(-0.5+2)\times 48}{6}$$

$$y_p = 496 - 146 - 18 - 3$$

$$y_p = 329$$

polynomial $p = \frac{x-8}{2}$

$$y_p = 496 + \left(\frac{x-8}{2}\right)292 + \frac{(x-8)}{2}\frac{\left(\frac{x-8}{2}+1\right)}{2}\times 144 + \frac{\left(\frac{x-8}{2}\right)\left(\frac{x-8}{2}+1\right)\left(\frac{x-8}{2}+2\right)\times 48}{6}$$

$$= 496 + 146x - 1168 + (18x - 126) + (x-8)(x-8+2)(x-8+4)$$

$$= 496 + 146x - 1168 + 18x - 126 + (x-8)(x-6)(x-4)$$

$$= x^3 - 2x$$

$y(7) = 7^3 - 2 \times 7$
$= 329$

Q3

income per hr :	10 - 20	20 - 30	30 - 40	40 - 50
No of person :	45	115	210	115

⇒

income less than, x per hr :	20	30	40	50
No of person :	45	160	370	485

$p = \frac{25 - 20}{10} = 0.5$

Newton forward table :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	45			
		115	95	
30	160	210		-190
	370		-95	
40		115		
	485			
50				

using newton forward difference

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0$$

$$y_p = 45 + 0.5 \times 115 + \frac{0.5(-0.5)}{2} 95 + \frac{(0.5)(-0.5)(-1.5)(-190)}{6}$$

$$y_p = 45 + 57.5 - 11.875 - 11.875$$

$$y_p = 78.75 \approx 79$$

No of person whose salary is less than 25 = ~~28~~ 79

No of person whose salary is less than 20 = 45

∴ No of person whose salary per hour 20 – 25 = 79 – 45
= 34

Q sol 9 ~~forward table~~

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
10	10.63	2.4	-0.39	0.15	-0.07	0.02	0.02
15	13.03	2.01	-0.24	~~0.40~~ 0.08	-0.05	0.04	
20	15.04	1.77	-0.16	0.03	-0.01		
25	16.81	1.61	-0.13	0.02			
30	18.42	1.48	-0.11				
35	19.90	1.37					
40	21.27						

For $x = 12$ $\qquad p = \frac{12-10}{5} = 0.4$

using newton forward difference formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)}{6!}\Delta^6 y_0$$

$$y_p = 10.63 + 0.96 + \overset{0.0468}{\cancel{0.0624}} + 0.0096 + 0.002912$$

$$+ 0.000599 \; \cancel{0.00148} \; 0.00045926$$

$$y(12) = 11.6495$$

for $x = 42$

$$p = \frac{42-40}{5} = 0.4$$

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!}\nabla^6 y_n$$

$$y_p = 21.27 + (0.4)(1.37) + \frac{0.4(1.4)(-0.11)}{2} + \frac{(0.4)(1.4)(2.4)0.02}{6}$$

$$+ \frac{0.4(1.4)(2.4)(3.4)(-0.01)}{24} + \frac{(0.4)(1.4)(2.4)(3.4)(4.4)0.04}{120}$$

$$+ \frac{0.4(1.4)(2.4)(3.4)(4.4)(5.4)(0.02)}{720}$$

$$y(42) = 21.27 + 0.548 - 0.0308 + 0.00448$$

$$- 0.001904 + 0.0067028 + 0.003015936$$

$$y(42) = \underline{\underline{21.79949}}$$

Q5

x	30	34	42
y	-30	-13	18
	y_0	y_1	y_2

$$x = \frac{(y-y_1)(y-y_2)x_0}{(y_0-y_1)(y_0-y_2)} + \frac{(y-y_0)(y-y_2)x_1}{(y_1-y_0)(y_0-y_2)} + \frac{(y-y_0)(y-y_1)x_2}{(y_2-y_0)(y_2-y_1)}$$

$$= \frac{(30)(-18)(30)}{(-17)(-48)} + \frac{(30)(-18)(34)}{(17)(-31)} + \frac{(30)(13)(42)}{(48)(31)}$$

$$= -8.6029 + 34.8387 + 11.0081$$

$$= 37.2439$$

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Tutorial-2

Q1 Find $f'(1.1)$, $f''(1.1)$ for the following table

x	1	1.2	1.4	1.6	1.8	2.0
y	0.00	0.128	0.544	1.296	2.432	4

Q2 Find $f(8)$ and $f(15)$ using newton's devided difference formula

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

Q3 Find a cubic polynomial volume passing through (2,4), (4,56), (9,711) and (10,980) using devided difference formula and hence find $y(1.5)$

Q4 Find $f'(0)$, $f''(0)$ from the following data.

x	0	10	20	30	40
y	45	60	65	54	42

Q5 The following table gives corresponding values of pressure p and specific volume V of a superheated stream. Find the rate of change of p w.r.t V at $V = 2$

V	2	4	6	8	10
p	105	42.7	25.3	16.7	13

Sol 1

x	y	order Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	0.00				
		0.128			
1.2	0.128		0.288		
		0.416		0.048	
1.4	0.544		0.336		0
		0.752		0.048	
1.6	1.296		0.384		0
		1.136		0.048	
1.8	2.432		0.432		
		1.568			
2.0	4				

$$p = \frac{1.1 - 1}{0.2} = 0.5$$

$$f'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y\right]$$

$$= \frac{1}{0.2}\left[0.128 + (1-1)\Delta^2 y_0 + \frac{(-0.625)}{6} \times 0.048\right]$$

$$= \frac{1}{0.2}(20.128 -$$

$$= 0.603$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\Delta^3 y_0\right]$$

$$= \frac{1}{(0.2)^2}\left[0.288 + (0.5-1)\,0.048\right]$$

$$= 6.6$$

x	y	1st order	2nd order	3rd order
4	48			
		52		
			15	
5	100			1
		97		
7	294		21	
				1
		202		
10	900		27	
		310		1
11	1210			
			33	
13	2028	409		

$$y = f(x) = f(x_0) + (x - x_0) f(x_0 x_1) + (x - x_0)(x - x_1) f(x_0 x_1 x_2)$$
$$+ (x - x_0)(x - x_1)(x - x_2) f(x_0 x_1 x_2 x_3)$$

For $f(8)$

$$f(8) = 48 + (8-4)(52) + (8-4)(8-5)(15) + (8-4)(8-5)(8-7)(1)$$

$$= 48 + 208 + 180 + 12$$

$$= 448$$

For $f(15)$

$$f(15) = 48 + (15-4)(52) + (15-4)(15-5)(15) + (15-4)(15-5)(15-7)(1)$$

$$= 48 + 572 + 1650 + 880$$

$$= 3150$$

sol #3

x	2	4	9	10
y	4	56	711	980

x	y	1st order	2nd	3rd
2	4			
		26		
4	56		15	
				1
		131		
9	711		23	
		269		
10	980			

$$y = f(x_0) + (x-x_0) f(x_0 x_1) + (x-x_0)(x-x_1) f(x_0 x_1 x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0 x_1 x_2 x_3)$$

$$= 4 + (x-2)26 + (x-2)(x-4)15 + (x-2)(x-4)(x-9)\cdot 1$$

$$= 4 + 26x - 52 + (x^2-6x+8)15 + (x^2-6x+8)(x-9)$$

$$= 4 + 26x - 52 + 15x^2 - 90x + 120 + x^3 - 9x^2 - 6x^2 + 54x + 8x - 72$$

$$= x^3 + \cancel{9x^2} - 2x$$

$$= x^3 - 2x$$

$$f(1.5) = (1.5)^3 - 2(1.5)$$

$$= 0.375$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	45	15	-10	-6	21
10	60	5	-16	-15	
20	65	-11	-1		
30	54	-12			
40	42				

$$y'(x_0) = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)\Delta^2 y_0}{2!} + \frac{(3p^2-6p+2)\Delta^3 y_0}{3!} + \frac{(4p^3-18p^2+22p-6)\Delta^4 y_0}{4!}\right]$$

$$p = \frac{x_0 - x_0}{h} = \frac{0-0}{10} = 0$$

$$y'(0) = \frac{1}{10}\left(15 - \frac{(-1)0}{2} + \frac{2}{6}(-6) + \frac{(-6)(21)}{24}\right)$$

$$= \frac{1}{10}\left(15 + 5 - 2 - \frac{21}{4}\right)$$

$$= 1.275$$

$$y''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\Delta^3 y_0 + (12p^2 - 36p + 22)\Delta^4 y_0\right]$$

$$y''(0) = \frac{1}{100}\left[-10 + (-1)(-6) + \frac{22}{24} \times 21\right]$$

$$= 0.1525$$

Q5

V	P	Δp	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$
2	105				
		-62.3			
4	42.7		44.9		
		-17.4		-36.1	
6	25.3		8.8		32.2
		-8.6		-3.9	
8	16.7		4.9		
		-3.7			
10	13				

$$p = \frac{x - x_0}{h} = \frac{2-2}{2} = 0$$

$$\frac{dp}{dv} = f'(v) = \frac{1}{h}\left[\Delta p_0 + \frac{(2p-1)}{2!}\Delta^2 p_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 p_0 + \frac{(4p^3+8p^2+22p-6)}{4!}\Delta^4 p_0 + \cdots\right]$$

$$f'(2) = \frac{1}{2}\left[-62.3 - \frac{1}{2}(44.9) + \frac{1}{3}(-36.1) - \frac{6}{24}\times(32.2)\right]$$

$$= \frac{1}{2}\left[-104.833\right] = -52.4165$$

$\therefore \frac{dp}{dv}$ at $v=2$ is -52.4165

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Tutorial - 3

Q1 Evalute $\int_0^{\pi} t.\sin t \, dt.$ using trapezoidal rule (6 sub)

Q2 Evalute $\int_0^1 \frac{dx}{1+x^2}$ using simpson's ~~rule~~ $1/3^{rd}$ rule taking four equal steps

Q3 Evalute $\int_0^{\pi} \frac{dx}{2+\cos x}$ using simpson's $3/8^{th}$ rule taking 6 sub intervals

Q4 A curve is drawn to pass through the points given by the following table

x	4.47	7.48	7.49	7.50	7.51	7.52
y	1.93	1.95	1.98	2.01	2.03	2.06

Find the area bounded by the curve the X-axis & the line $x = 7.47$, $x = 7.52$

Q5 The following tables shows the velocities of the car at various interval of times ~~found~~ Find the distance covered by the car using simpson's $1/3^{rd}$ rule

Time	0	2	4	6	8	10	12
velocity	0	22	30	22	18	7	0

<u>Sol 1</u>

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+ \cdots y_{n-1})\right]$$

$$h = \frac{\pi - 0}{6} = \pi/6$$

$$f(t) = t \cdot \sin t.$$

t	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
~~f(t)~~	~~0~~	~~−0.517~~	~~−0.319~~	~~1.4033~~	~~1.2154~~	~~−1.8705~~	~~−2.5156~~
f(t)	0	0.2616	0.9064	1.57	1.8128	1.308	0

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{\pi}{12}\left[(0+0) + 2\left[0.2616 + 0.9064 + 1.57 + 1.8128 + 1.308\right]\right]$$

$$= 3.06$$

<u>Sol 2</u> $f(x) = \frac{1}{1+x^2}$

$$h = \frac{1-0}{4} = 0.25$$

x	0	0.25	0.5	0.75	1
f(x)	1	~~0.9444~~ 0.9417	0.8	0.64	0.5

$$\int f(x)dx = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right)$$

$$= \frac{0.25}{3}\left[(1 + 0.5) + 4(0.9417 + 0.64) + 2(0.8)\right)$$

$$= \frac{0.25}{3}\left[1.5 + 6.326 + 1.6\right]$$

$$= 0.7855$$

Sol 3

$$f(x) = \frac{1}{2+\cos x}$$

$$h = \frac{\pi - 0}{6} = \pi/6$$

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$\frac{5\pi}{6}$	π
$f(x)$	0.33	0.3489	~~0.667~~ 0.4	0.5	0.667	0.8819	1

$$\int_a^b f(x)dx = \frac{3 \times \pi}{8 \times 6}\left((y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right)$$

$$= \frac{3\pi}{16}\left((0.33 + 1) + 3(0.3489 + 0.4 + 0.667 + 0.8819) + 2 \times 0.5\right)$$

$$= \frac{\pi}{16}\left(1.33 + 1 + 6.8934\right)$$

$$= \cancel{1.8800}\ 1.813$$

Q4

$h = 0.01$

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right]$$

$$= \frac{0.01}{2}\left[(1.93 + 2.06) + 2(1.95 + 1.98 + 2.01 + 2.03)\right]$$

$$= \frac{0.01}{2}(3.99 + 15.94)$$

$$= 0.09965$$

Q5

$h = 2$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{2}{3}\left[(0 + -) + 4(22 + 27 + 7) + 2(30 + 18)\right]$$

$$= \frac{2}{3}(224 + 96)$$

$$= \cancel{200} \quad 213.33$$

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Tutorial- 4

Q1 The probability distribution of a finite random variable X is given by the following table

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	2k	0.3	k

Find (i) k (ii) $P(x<1)$ (iii) $P(x>-1)$
(iv) $P(-1<x\leq 1)$
(v) Calculate the mean and variance

sol (i) we know

$$\sum p(x_i) = 1$$

$$\Rightarrow 0.1 + 0.2 + k + 2k + 0.3 + k = 1$$

$$0.6 + 4k = 1$$

$$4k = 0.4$$

$$k = 0.1$$

(ii)
$$P(x<1) = 1 - P(x\geq 1)$$
$$= 1 - [P(x=2) + P(x=3)] + P(x=1)]$$
$$= 1 - [0.3 + 0.1] + 2\times 0.1]$$
$$= 1 - 0.6 = \cancel{0.16}\ 0.4$$

(iii)
$$P(x>-1) = 1 - P(x\leq -1)$$
$$= 1 - [P(x=-2) + P(x=-1)]$$
$$= 1 - [0.1 + 0.1]$$
$$= 0.8$$

(iv) $P(-1 \le x \le 1) = P(x=-1) + P(x=0) + P(x=1)$

$= 0.1 + 0.2 + 2 \times 0.1$

$= 0.5$

(v)

mean $= E(x) = \sum x_i \, P(x_i)$

$= (-2 \times 0.1) + (-1 \times 0.1) + 0 \times 0.2 + 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.1$

$= 0.8$

$E(x^2) = \sum x_i^2 \, P(x_i)$

$= 4 \times 0.1 + 1 \times 0.1 + 0 \times 0.2 + 1 \times 0.2 + 4 \times 0.3 \times 9 \times 0.1$

$= 2.8$

Variance $= \sigma^2 = E(x^2) - (E(x))^2$

$= 2.8 - (0.8)^2$

$= 2.16$

Q2 A random variable x has be the density function

$$f(x) = \begin{cases} kx^2, & -3 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Find
(i) k (ii) $P(1 \le x \le 2)$ (iii) $P(x \le 2)$ (iv) $P(x > 1)$

Sol (i) we know

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-3}^{3} kx^2 = 1$$

$$\frac{k}{3}\left(x^3\right)_{-3}^{3} = 1$$

$$\frac{k}{3}\left[27 - (-27)\right] = 1$$

$$\frac{k}{3}(54) = 1$$

$$k = 1/18$$

(ii) $P(1 \leq X \leq 2) = \int_1^2 f(x)\,dx$

$$= \int_1^2 \frac{1}{18} x^2\,dx$$

$$= \frac{1}{18} \int_1^2 x^2\,dx$$

$$= \frac{1}{18} \times \frac{1}{3} \left[x^3\right]_1^2$$

$$= \frac{1}{18} \times \frac{1}{3} [8 - 1]$$

$$= \frac{7}{54}$$

(iii) $P(X \leq 2) = \int_{-3}^{2} f(x)\,dx = \int_{-3}^{2} \frac{1}{18} \cdot x^2\,dx$

$$= \frac{1}{18} \times \frac{1}{3} \left[x^3\right]_{-3}^{2}$$

$$= \frac{1}{54} [8 + 27)$$

$$= \frac{35}{54}$$

(iv) $P(X > 1) = \int_1^3 f(x)\,dx = \int_1^3 \frac{1}{18} \cdot x^2\,dx$

$$= \frac{1}{18} \cdot \frac{1}{3} \left[x^3\right]_1^3$$

$$= \frac{1}{18} \times \frac{1}{3} [9 - 1] = \frac{8}{54}$$

Q3 Find $E(X)$, $E(X^2)$ & σ^2 for the probability function $P(x)$ defined by the following table, where k is an appropriate constant.

x_i	1	2	3	- - - - - - -	n
$P(x_i)$	k	$2k$	$3k$	- - -	nk

Sol

we know

$$\sum p(x_i) = 1$$

$$\Rightarrow k + 2k + 3k + \cdots nk = 1$$

$$k(1+2+3+ \cdots n) = 1$$

$$k \times \frac{n(n+1)}{2} = 1$$

$$k = \frac{2}{n(n+1)}$$

$$E(X) = \sum x_i P(x_i)$$

$$= 1.k + 2.2k + 3.3k + 4.4k + \cdots n.nk$$

$$= k(1 + 2^2 + 3^2 + 4^2 + \cdots n^2)$$

$$= k \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{2}{n(n+1)} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{2n+1}{3}$$

$$E(x^2) = \Sigma x_i^2 P(x_i)$$

$$= 1.k + 2^2.2k + 3^2.3k + \cdots\cdots n^2.nk$$

$$= k(1 + 2^3 + 3^3 + \cdots n^3)$$

$$= k\left(\frac{n(n+1)}{2}\right)^2$$

$$= \frac{2}{n(n+1)} \cdot \frac{n^2(n+1)^2}{4}$$

$$= \frac{n(n+1)}{2}$$

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$= \frac{n(n+1)}{2} - \left(\frac{2n+1}{3}\right)^2$$

$$= \frac{n^2+n}{2} - \frac{4n^2+4n+1}{9}$$

$$= \frac{9n^2+9n - 8n^2 - 8n + 2}{18}$$

$$= \frac{n^2 + n + 2}{18}$$

Q4 A continous random variable X has pdf given by

$$f(x) = \begin{cases} 2e^{-2x}, & 0 \le x \le \infty \\ 0, & x \le 0 \end{cases}$$

Evaluate

(i) $E(x)$

(ii) $E(x^2)$

(iii) Standard deviation.

(i) $E(x) = \int_0^\infty x_i f(x_i)\,dx$

$$= \int_0^\infty x \cdot 2e^{-2x}\,dx$$

$$= 2\int_0^\infty x \cdot e^{-2x}\,dx$$

$$= 2\left[\frac{x \cdot e^{-2x}}{-2} - \frac{1 \cdot e^{-2x}}{4}\right]_0^\infty$$

$$= 2\left((0-0) - \left(0 - \frac{e^0}{4}\right)\right)$$

$$= 2\left(0 + 1/4\right)$$

$$= 1/2$$

$$E(x^2) = \int_0^\infty x^2 f(x)\,dx = 2\int_0^\infty x^2 e^{-2x}\,dx$$

$$= 2\left[\frac{x^2 e^{-2x}}{-2} + \frac{x e^{-2x}}{-2} - \frac{1 \cdot e^{-2x}}{4}\right]_0^\infty$$

$$= 2\left[0 - 0 - \left(0 - \frac{e^0}{4}\right)\right]$$

$$= 2 \times 1/4$$

$$= 1/2$$

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$= 1/2 - 1/4 = 1/4$$

Standard Deviation $= \sigma = \sqrt{1/4}$

$$= 1/2$$

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4 A

Tutorial - 5

Q1 The probability that bomb dropped from a plane will strike the target is 1/5. If six bombs are dropped, find the probability that
(i) exactly two will strike the target
(ii) at least two will strike the target.

Ans

The probability of success $= p = 1/5$

The probability of failure $= q = 1 - 1/5$
$= 4/5$

No of bomb dropped $= 6$

(i) $P(X=2) = {}^nC_x \, p^x \, q^{n-x}$

$= {}^6C_2 \, (1/5)^2 \, (4/5)^4$

$= 15 \times \frac{1}{25} \times \frac{256}{625}$

$= \frac{768}{3125}$

(ii) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [{}^6C_0 (1/5)^0 (4/5)^6 + {}^6C_1 (1/5)^1 (4/5)^5]$

$= 1 - \frac{4^6}{5^6} - \frac{6 \times 4^5}{5^6}$

$$= 1 - \frac{4^6}{5^6} - \frac{6 \times 4^5}{5^6}$$

$$= \frac{5^6 - 4^6 - 6 \times 4^5}{5^6}$$

$$= \frac{1077}{3125}$$

Q2 The number of telephone line busy at on instant of time is a binomial variate with probability 0.2. If at an instant 10 lines are chosen at random, what is the probability that

(i) 5 line are busy

(ii) At most 2 lines are busy

Sol

Let p be the probability that a line is busy $= 0.2$

q be the probability that a line is not busy $= 1 - 0.2 = 0.8$

(i) $P(X=5) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$$= {}^{10}C_5\, 0.2^5\, 0.8^5$$

$$= \cancel{252 \times}\ 0.0267$$

(ii) $P(X \geq 2) = 1 - P(X<2)$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - {}^{10}C_0 (0.2)^0 (0.8)^{10} - {}^{10}C_1 (0.2)^1 (0.8)^9$$

$= 1 - 0.1073 - 0.268$

$= 0.6277$

Q3 In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packed of 10, use poisson distribution to calculate the approximate numbers of packed containing (i) no defective, (ii) Two defective blade (iii) one defective blades in a consignment of 10,000 pockets.

Sol Let p be the probability of defected blades
$p = 0.002$

no of blades in a packed $= n = 10$

$m = np = 0.02$

$$p(x) = \frac{e^{-m} . m^{x}}{x!}$$

(i) $$P(x=0) = \frac{e^{-0.02} . (0.02)^{0}}{0!} = 0.98019$$

In 1 packed $= 0.98019$

For 10000 packets $= 0.98019 \times 10000$

$= 9801$ packets

(ii) ~~P(x)~~
$$P(x=1) = \frac{e^{-0.02} (0.02)^{1}}{1!}$$

$= 0.01960$

For 10000 packets = 0.01960 × 10000
= 196 packets

(iii) $P(x=2) = \dfrac{e^{-0.02}(0.02)^2}{2!}$

$= 1.9603 \times 10^{-4}$

For 10000 packets = $1.9603 \times 10^{-4} \times 10000$
= 1.9603
≅ 2 packets

Q4 A communication channel recieves independent pulse at the rate of 12 pulse per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probability of
(i) No error (ii) One error
(iii) at lest One error (iv) two error
(iv) At most two errors

Sol

no of pulse (n) = 12

probability of transmission error = 0.001
p = 0.001

m = np
~~m = 12 × 0.001~~ m = 0.012

$$P(x) = \frac{e^{-m} \cdot m^x}{x!}$$

(i) $P(x=0) = \frac{e^{-0.012} \cdot (0.012)^0}{0!}$

$= 0.988$

(ii) $P(x=1) = \frac{e^{-0.012} \, (0.012)^1}{1!}$

$= 0.0118$

(iii) $P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(x=0)$

$= 1 - 0.988$

$= 0.012$

(iv) $P(x=2) = \frac{e^{-0.012} \, (0.012)^2}{2!}$

$= 0.0059$

(v) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= 0.988 + 0.0118 + 0.0059$

$= 0.9977$

Q5 The probability of a poisson variate taking the values 3 and 4 are equal. Calculate the probability of the variate taking values 0 and 1 sol

$$P(x) = \frac{e^{-m} \cdot m^{x}}{x!}$$

$$P(x=3) = P(x=4)$$

$$\frac{e^{-m} \cdot m^{3}}{3!} = \frac{e^{-m} \cdot m^{4}}{4!}$$

$$\Rightarrow \quad m = 4$$

$$P(x=0) = \frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-4} \cdot 4^{0}}{0!}$$

$$= 0.0183$$

$$P(x=1) = \frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-4} \cdot (4)^{1}}{1!}$$

$$= 0.0732$$

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Unit-5

Q In an investigation on the machine performance, the following results are obtained.

	No of units inspected	No of defectives	Total
Machine 1	375	17	392
Machine 2	450	22	472
Total	825	39	864

Sol H_0 : There is a significant performance of two machines

H_1 : There is no significant performance

Expected frequencies

(1) $\frac{825 \times 392}{864} = 374.3$ (2) $\frac{39 \times 392}{864} = 17.69$

(3) $\frac{825 \times 472}{864} = 450.69$ (4) $\frac{39 \times 472}{864} = 21.3$

$(O_i - E_i)^2 = 0.49$

0.4761

0.4761

0.49

$\frac{(O_i - E_i)^2}{E_i} = 0.00131$

$= 0.0269$

$= 0.0011$

$= 0.023$

0.05231

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 0.05231$$

$\chi^2_{0.05}$ $(2-1)(2-1) = 1$ d.f at 5% is 3.84

$\chi^2 < \chi^2_{0.05}$

H_0 is accepted

In this dos. $(r-1)(s-1)$

r = No. of rows

s = No. of columns.

Q Two random sample gave the following data

sample	size	Mean	variance
1	8	9.6	1.2
2	11	16.5	2.5

Can we conclude that the two samples have been draw from the same normal population having same variances?

sol:

$n_1 = 8$ $\quad$ $n_2 = 11$

$S_1^2 = 1.2$ $\quad$ $S_2^2 = 2.5$

$$\sigma_1^2 = \frac{n_1 S_1^2}{n_1 - 1} = \frac{8 \times 1.2}{7} = 1.37$$

$$\sigma_2^2 = \frac{n_2 S_1^2}{n_2 - 1} = \frac{11 \times 2.5}{10} = 2.75$$

$H_0 : \sigma_1^2 = \sigma_2^2$, $H_1 : \sigma_1^2 \neq \sigma_2^2$

$$F = \frac{\sigma_1^2}{\sigma_2^2} = 2.007$$

$F_\alpha = (10, 7) = 3.64$

$|F| < F_\alpha$, H_0 is accepted

we can conclude that the two samples have been drawn from the same normal population having same variances.