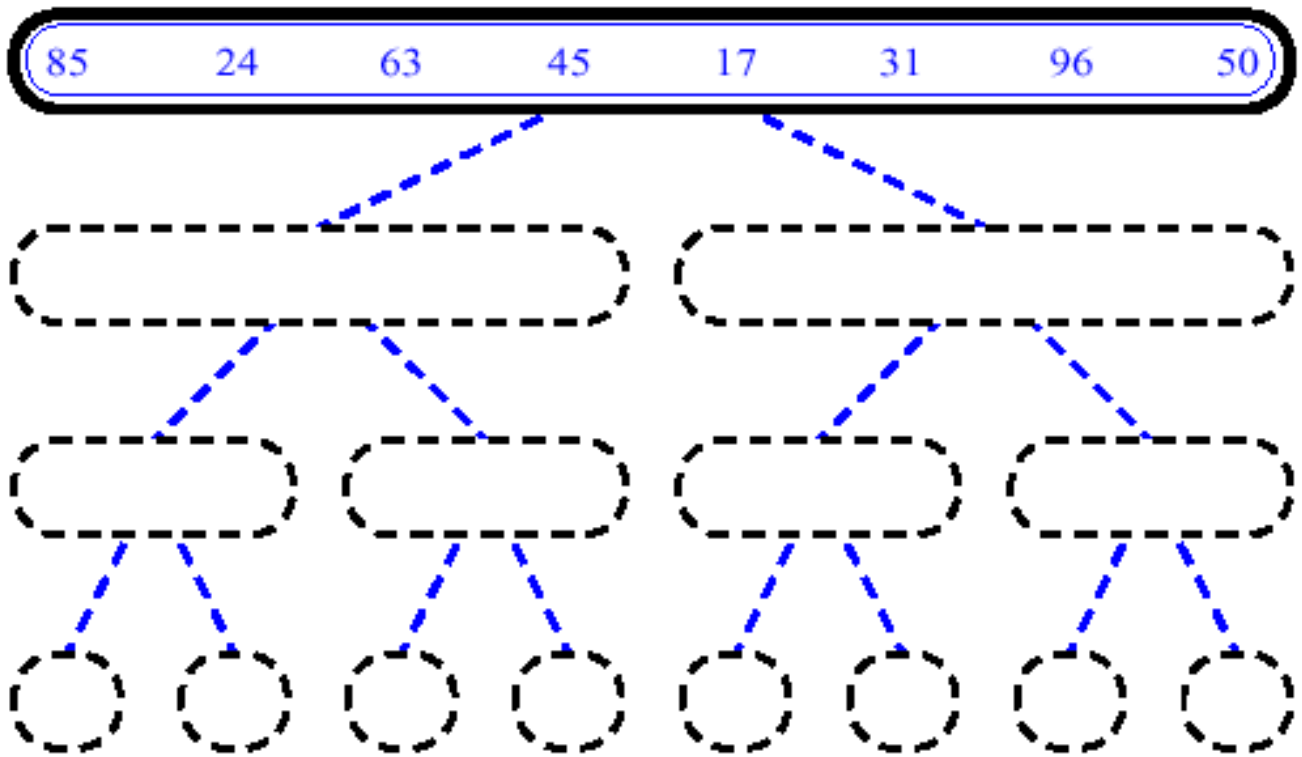
Prepared by,

Chetan Shetty
Assistant Professor
Department of CSE
MSRIT
Bangalore

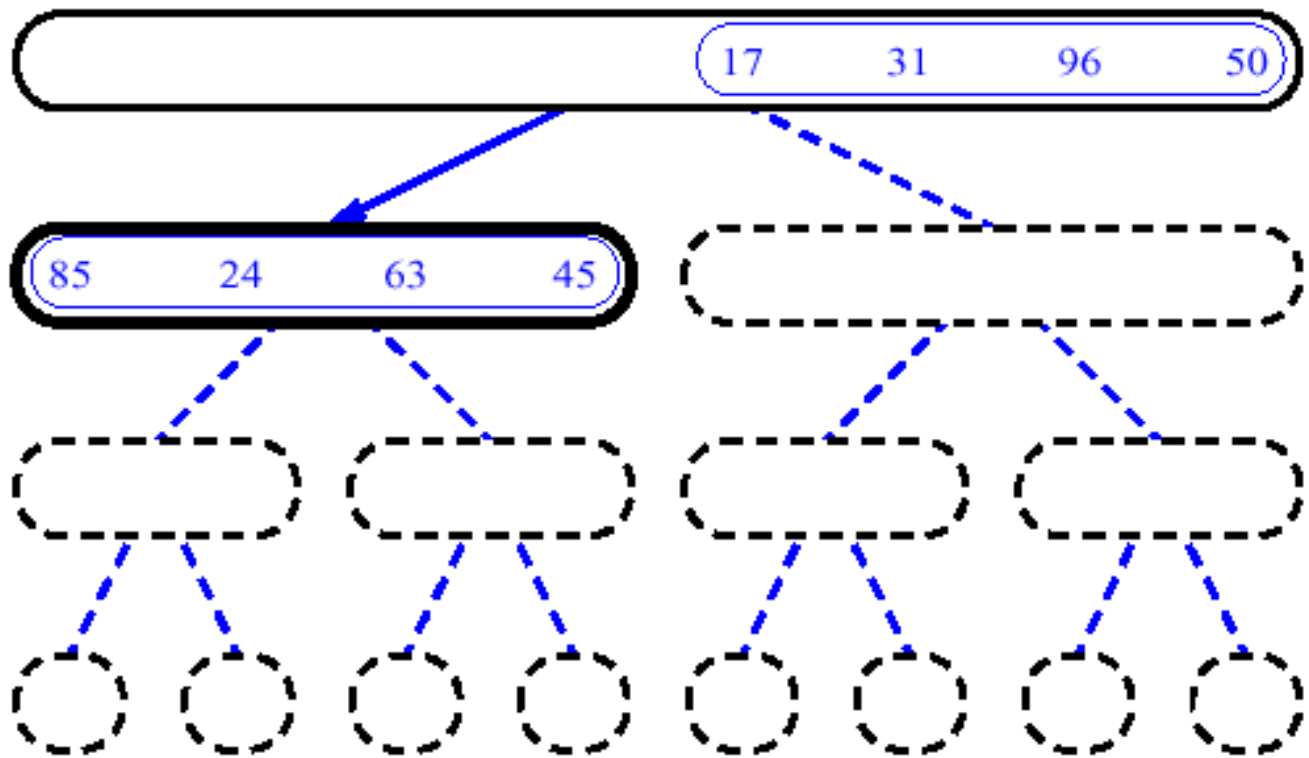
DIVIDE AND CONQUER

- **Divide** the problem into sub-problems that are similar to the original but smaller in size.
- **Conquer** the sub-problems by solving them recursively. If they are small enough, just solve them in a straightforward manner.
- **Combine** the solutions to create a solution to the original problem

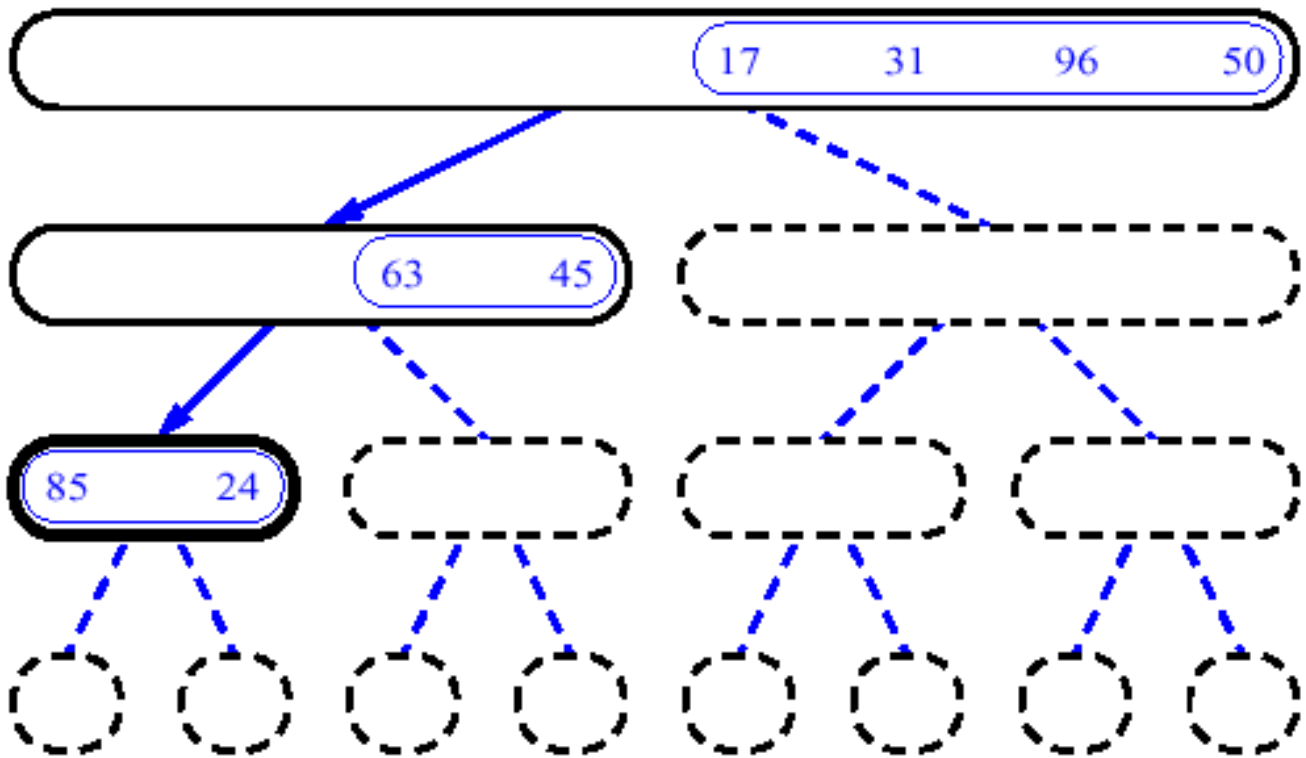
MERGESORT



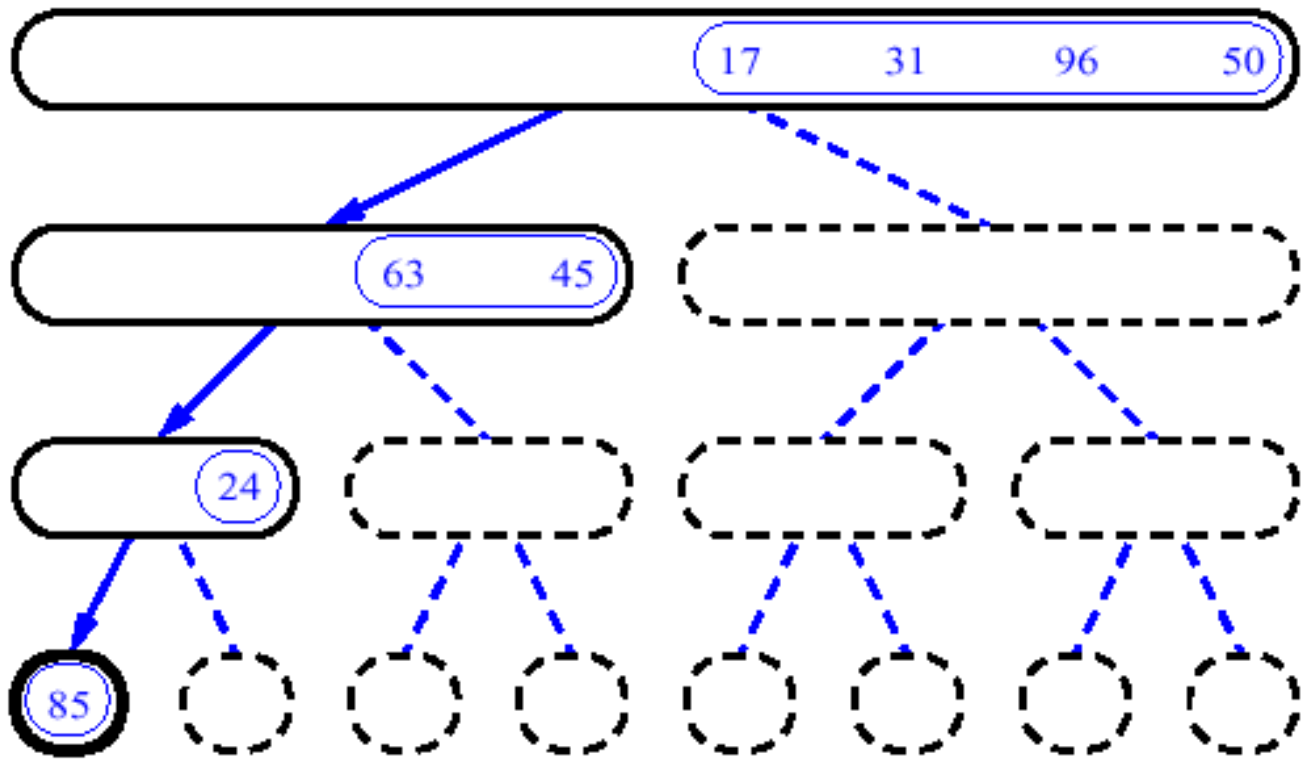
CONT…



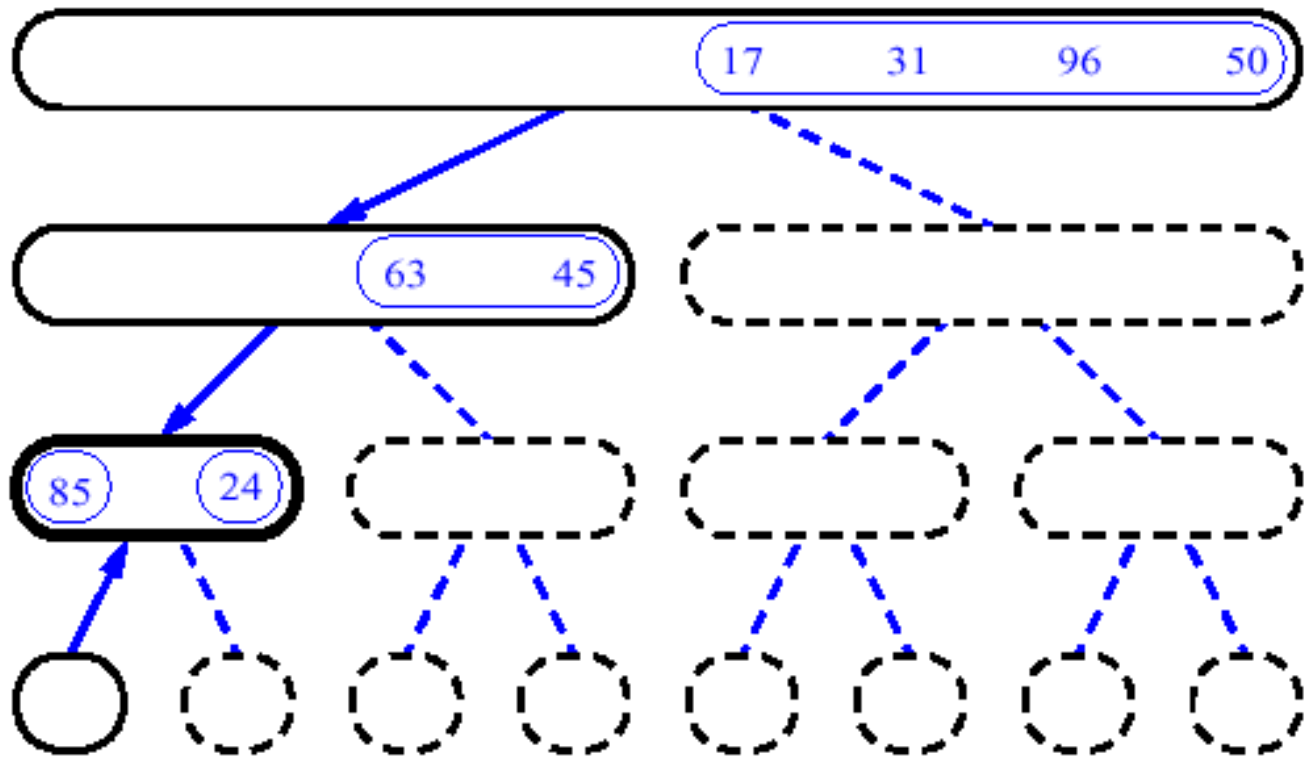
CONT…



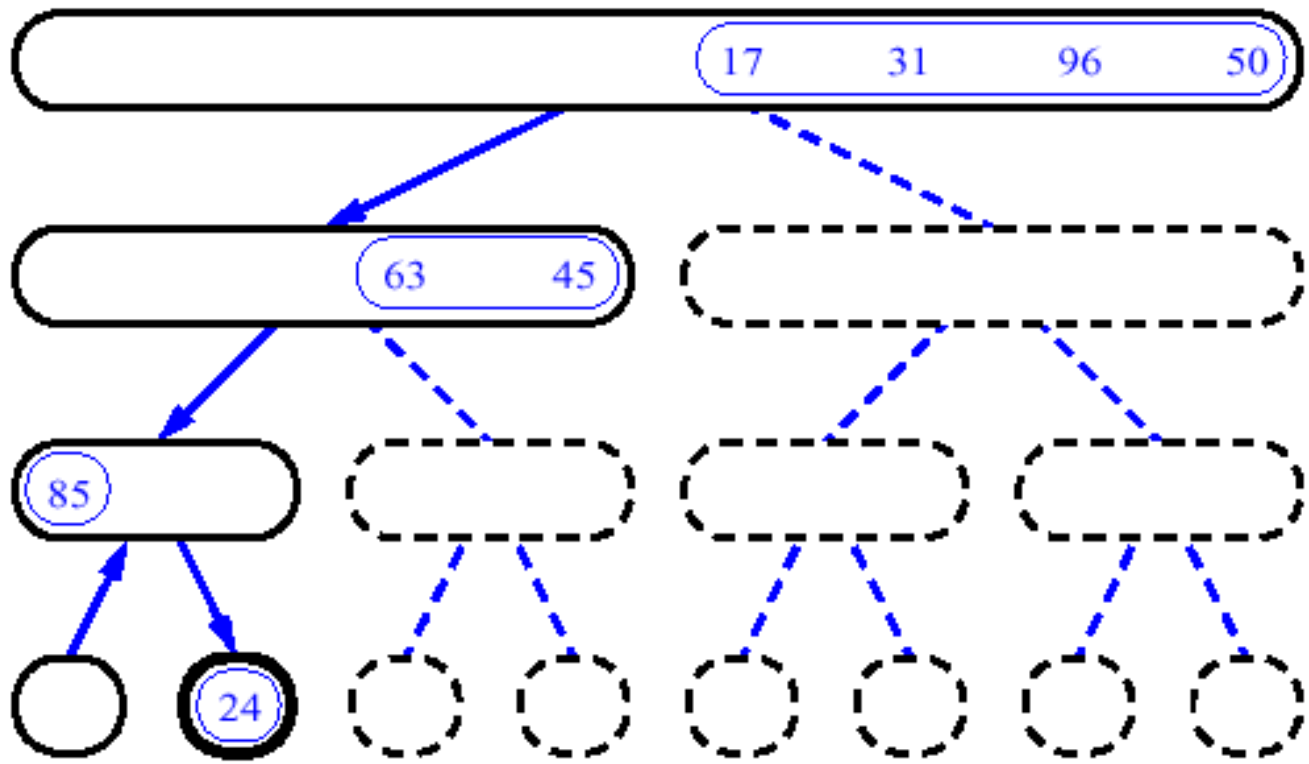
CONT…



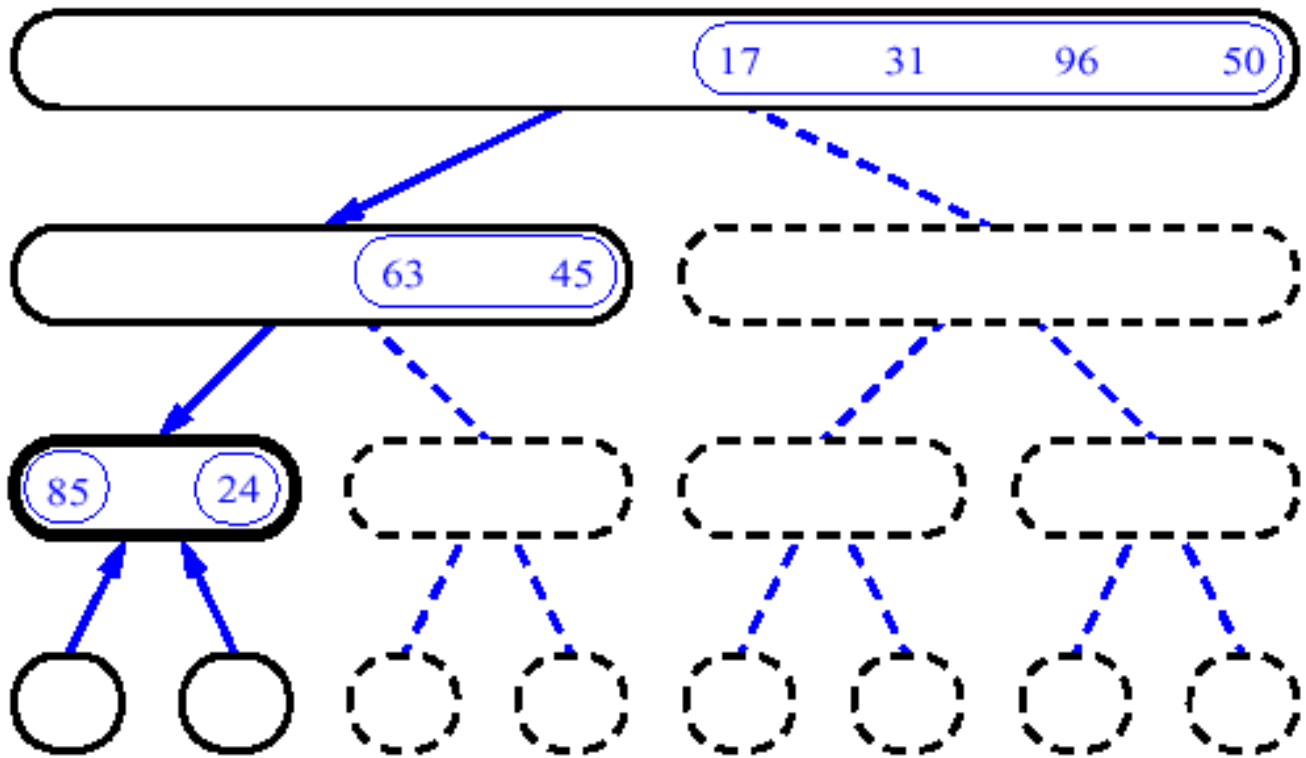
CONT…



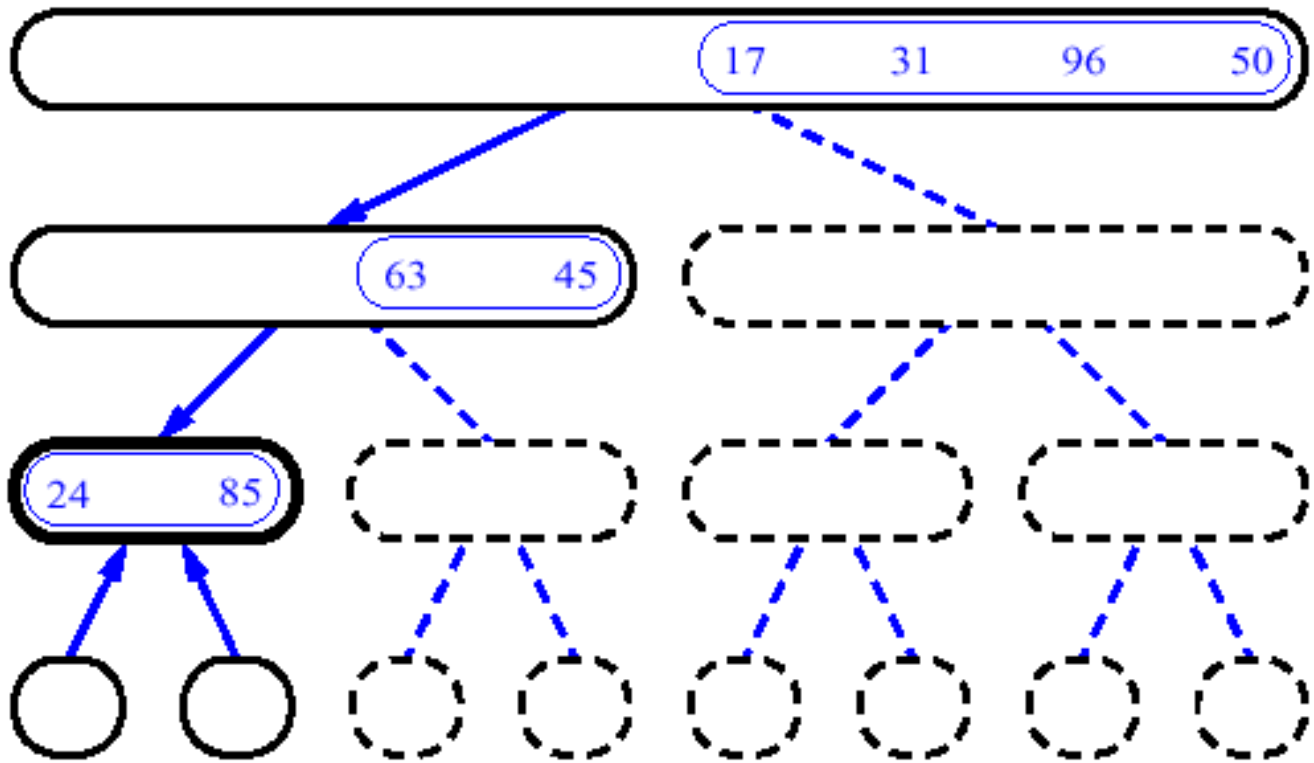
CONT…



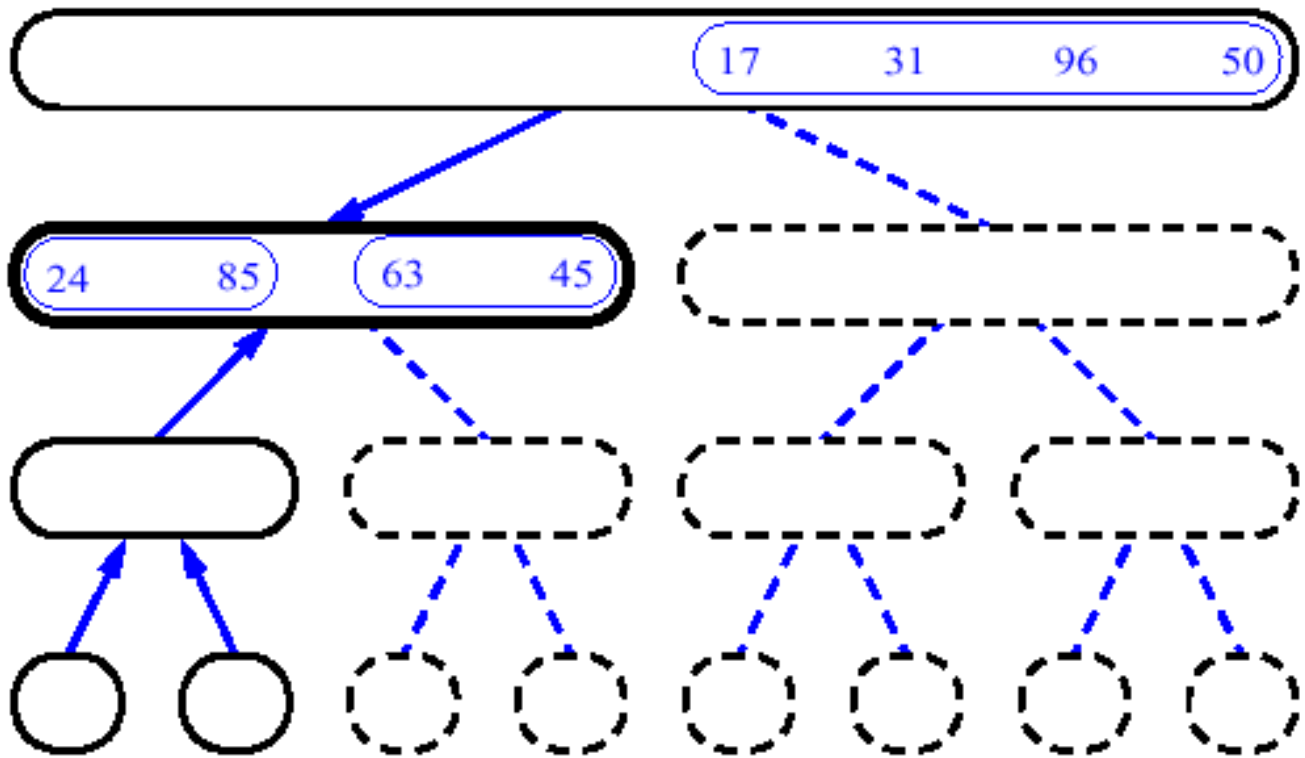
CONT…



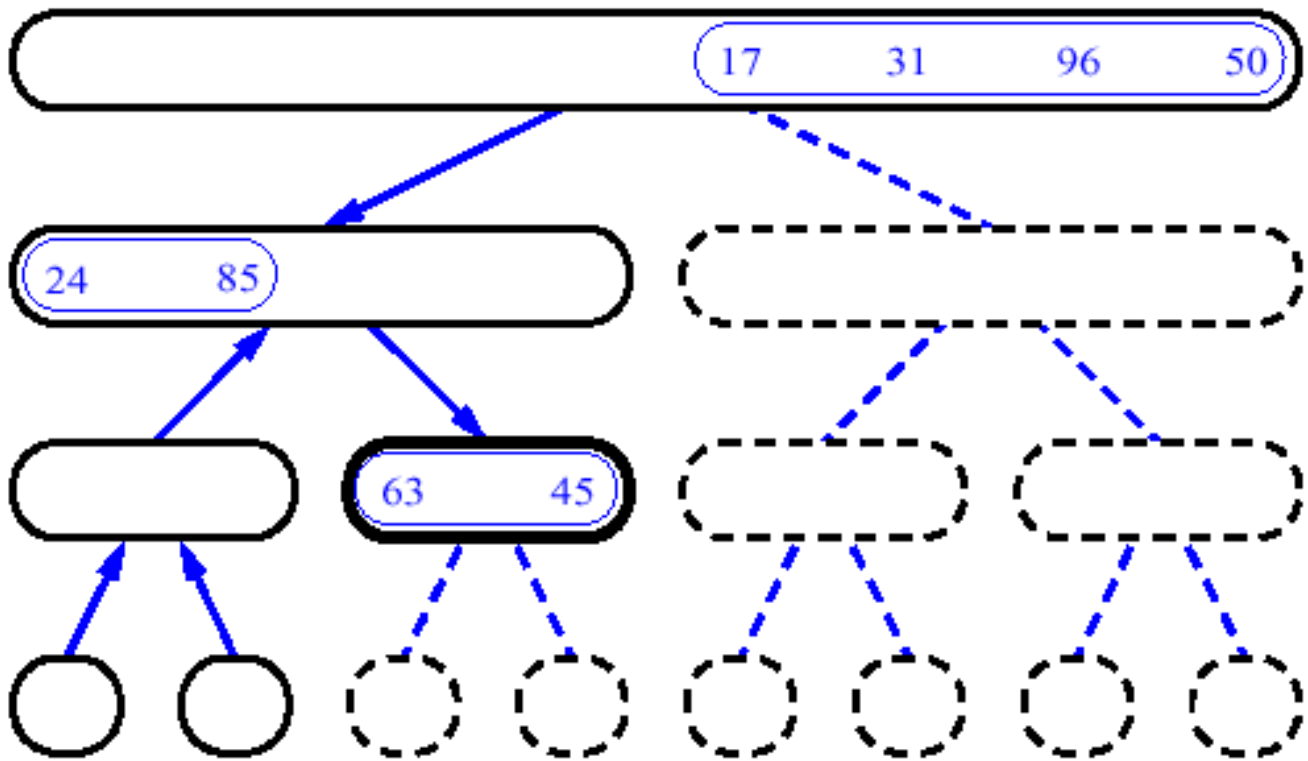
CONT…



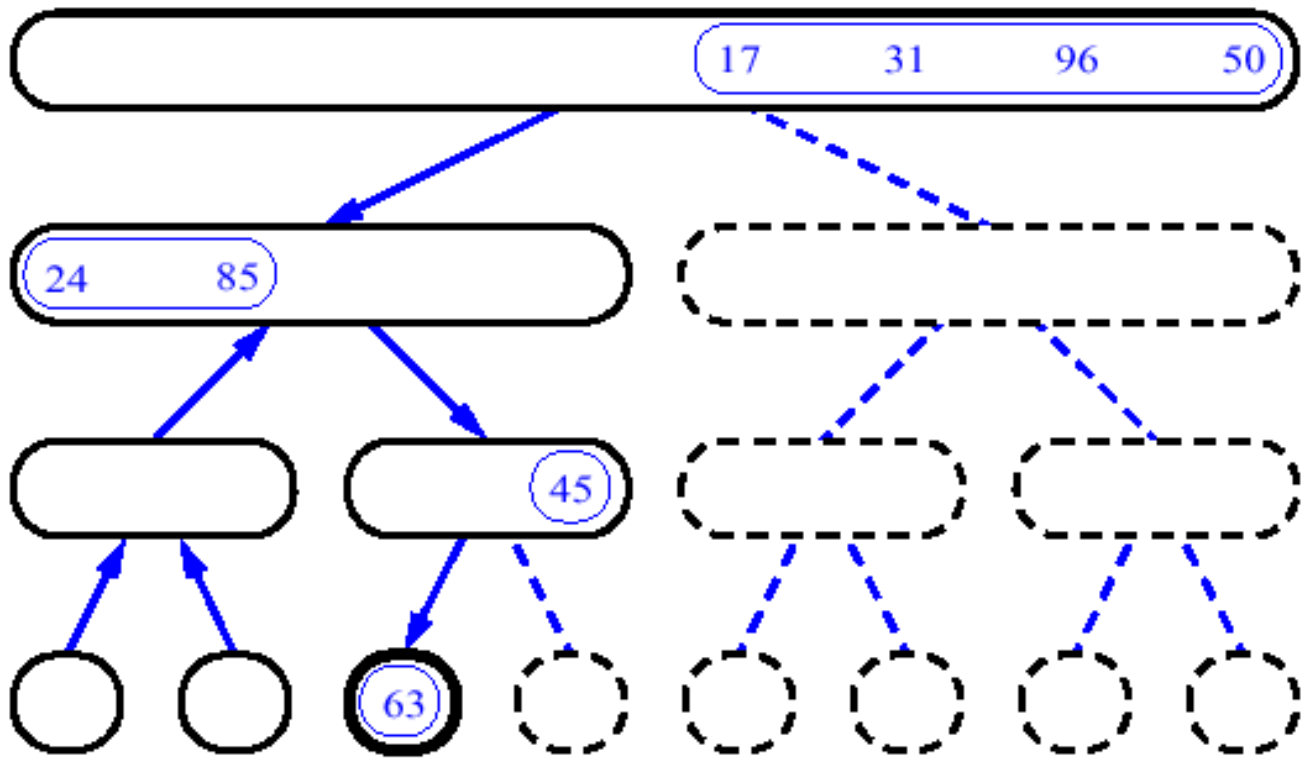
CONT…



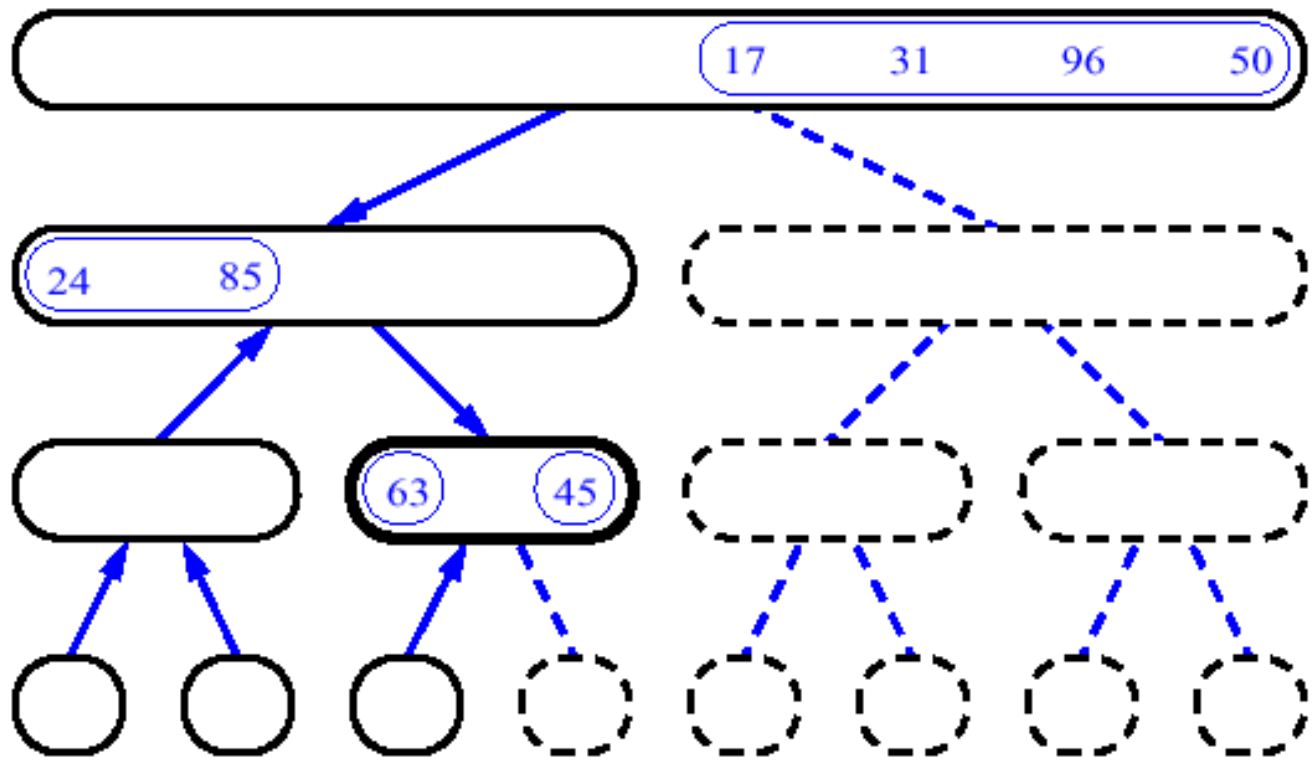
CONT…



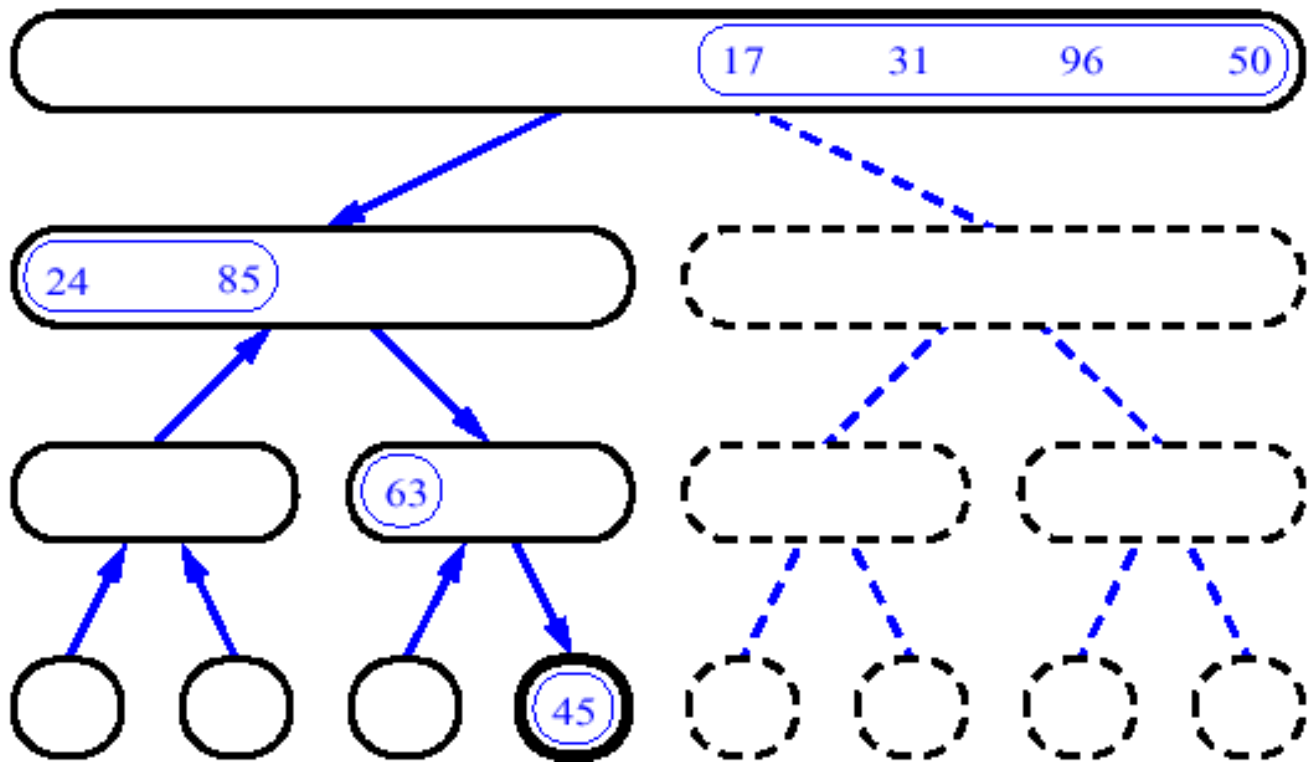
CONT…



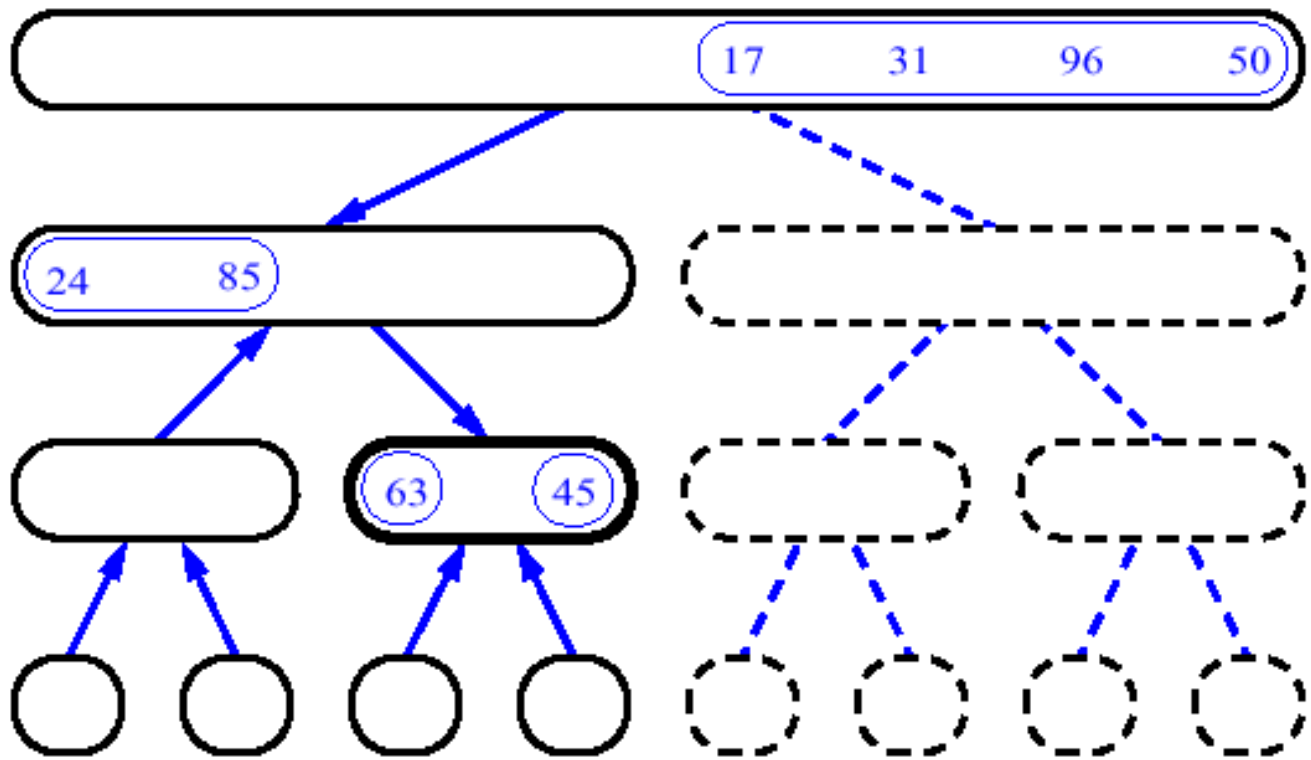
CONT…



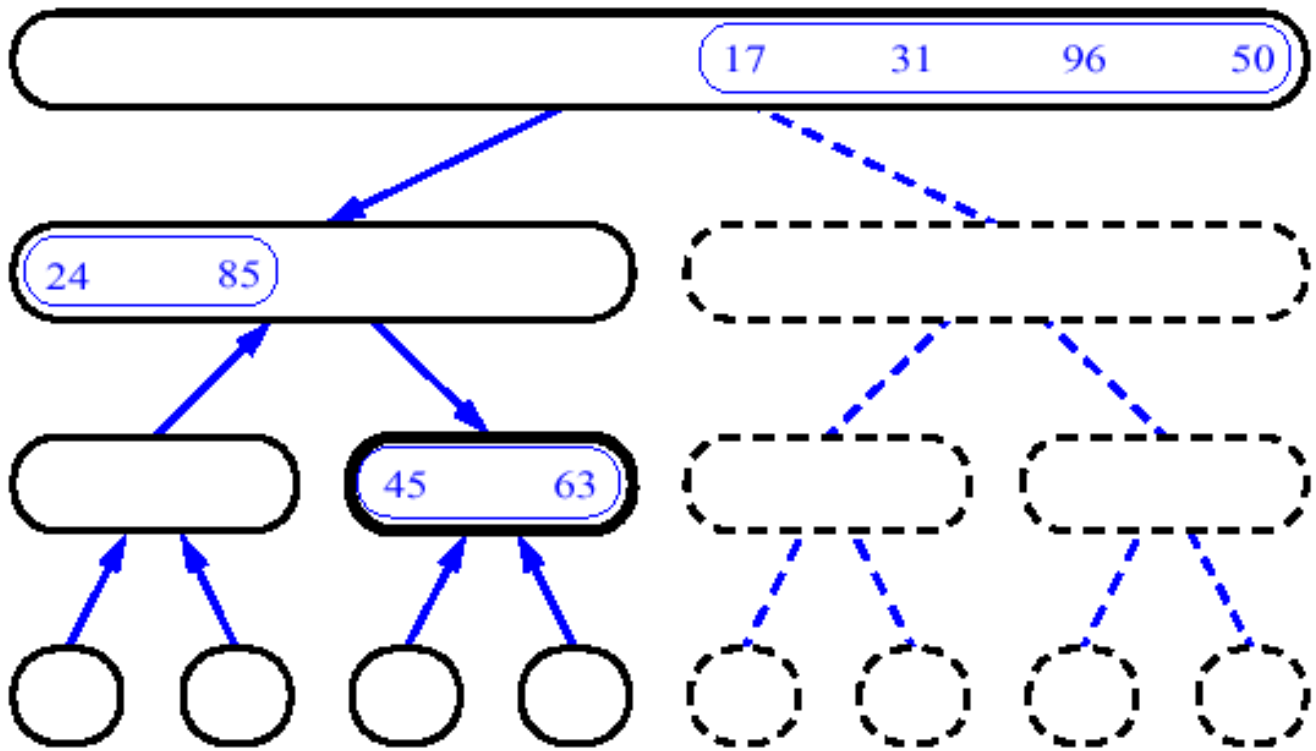
CONT…



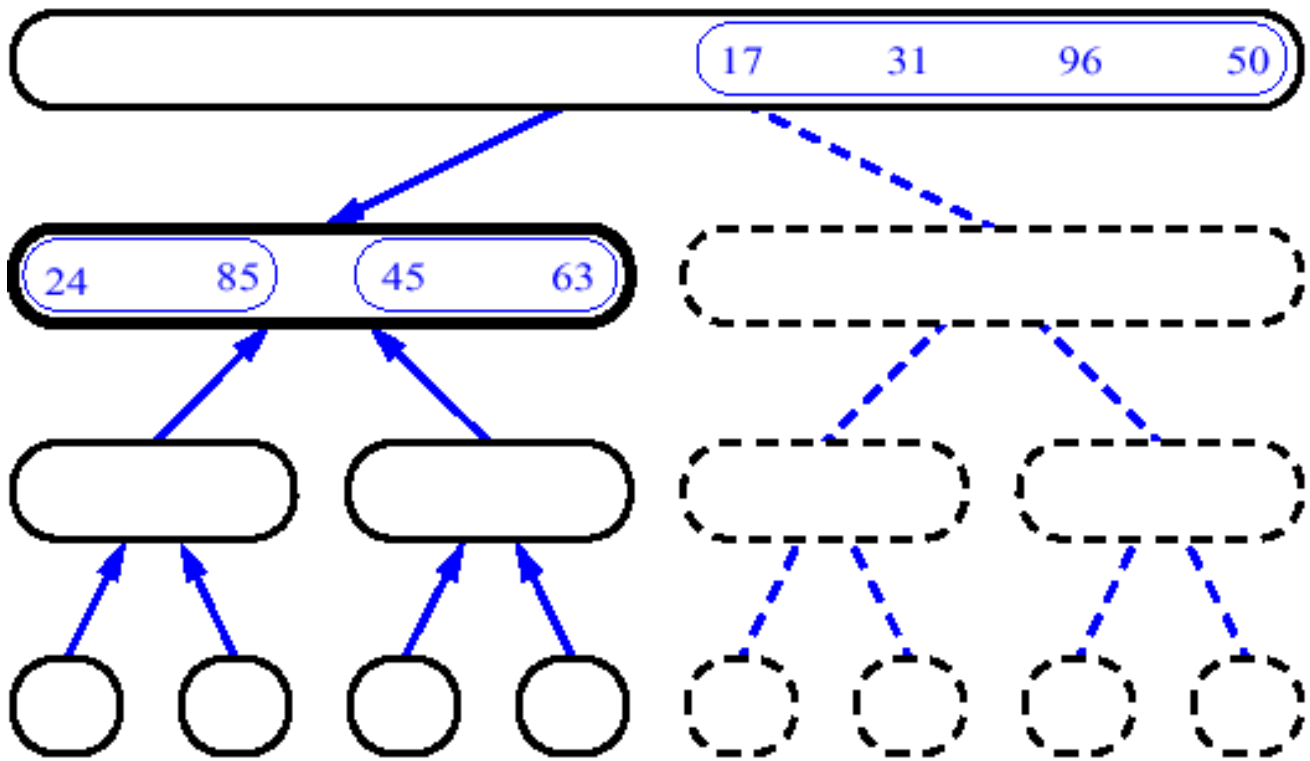
CONT…



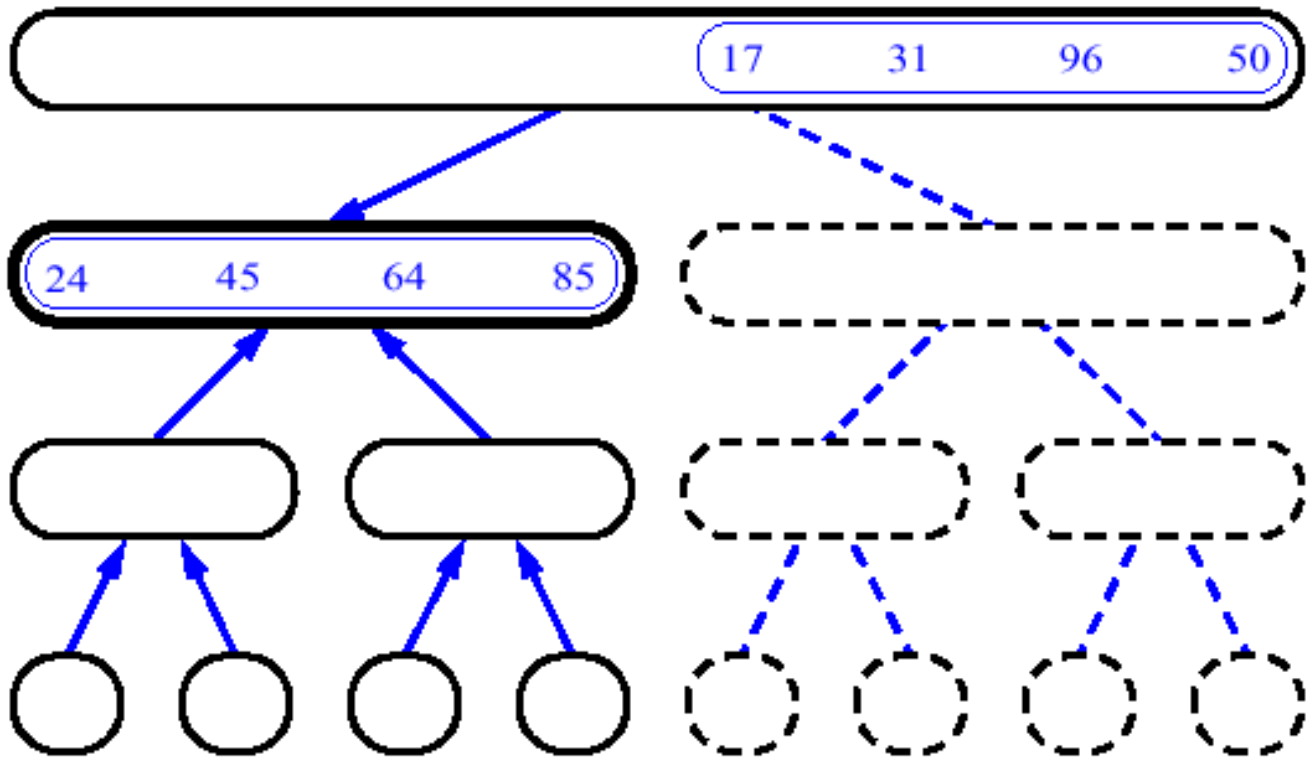
CONT…



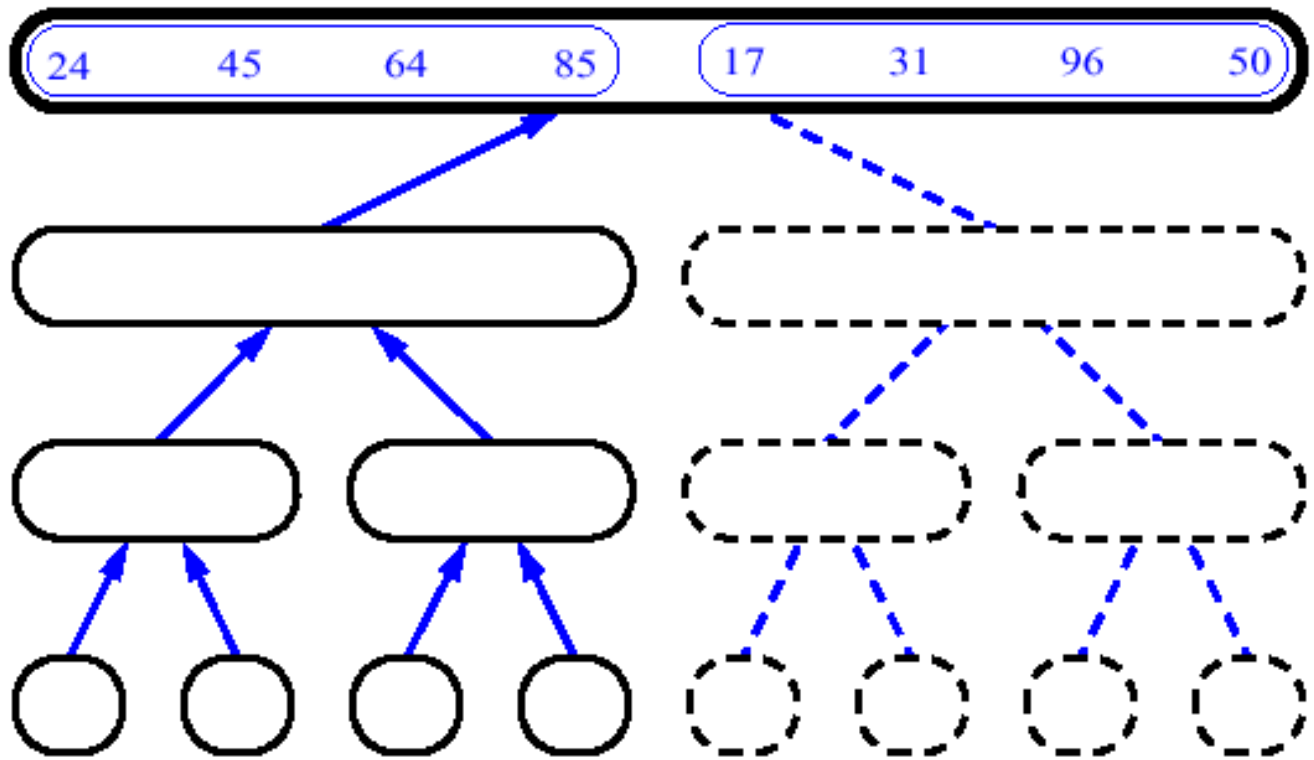
CONT…



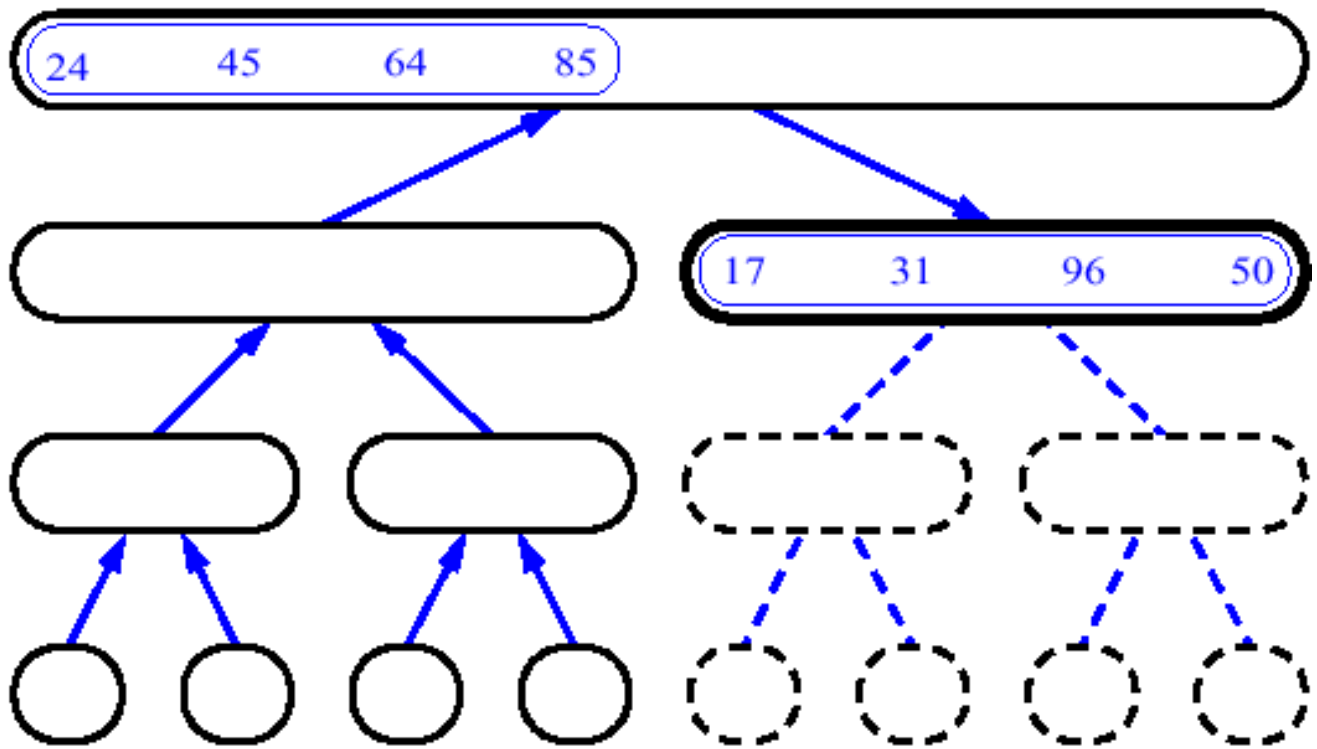
CONT…



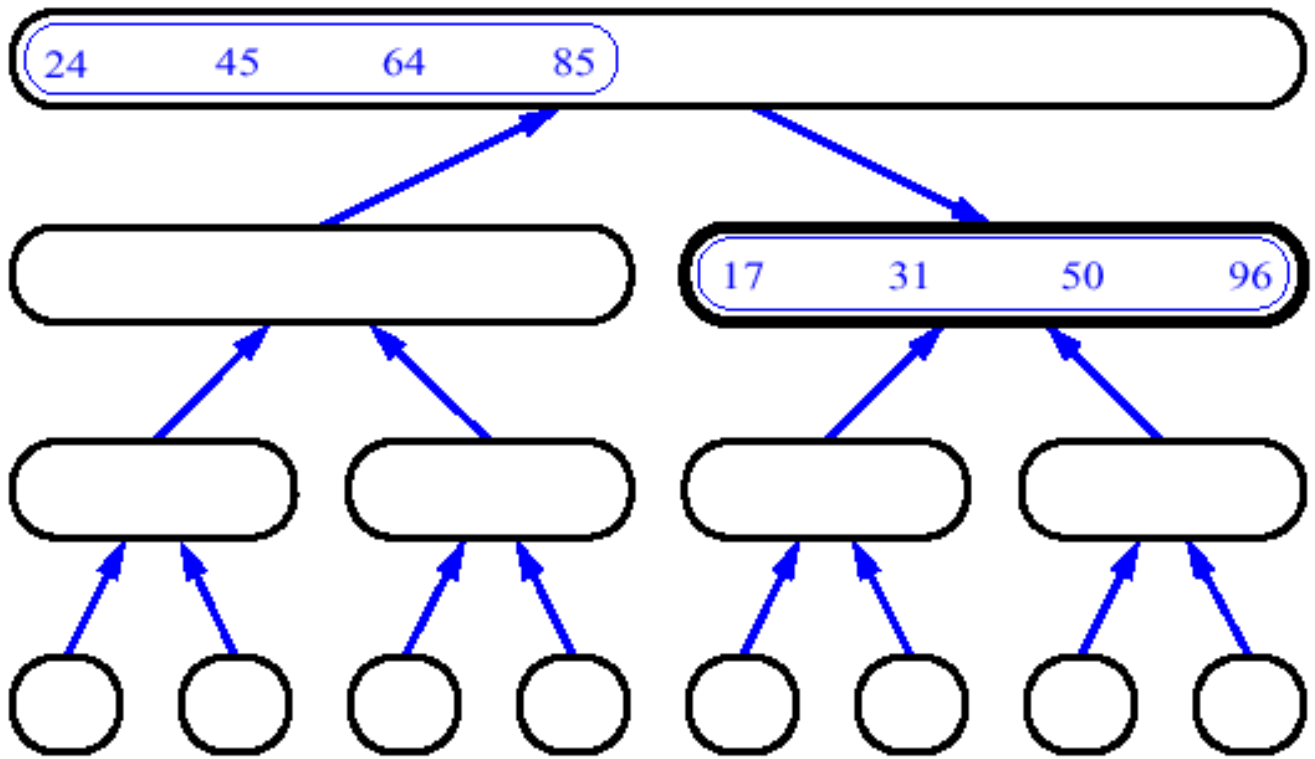
CONT…



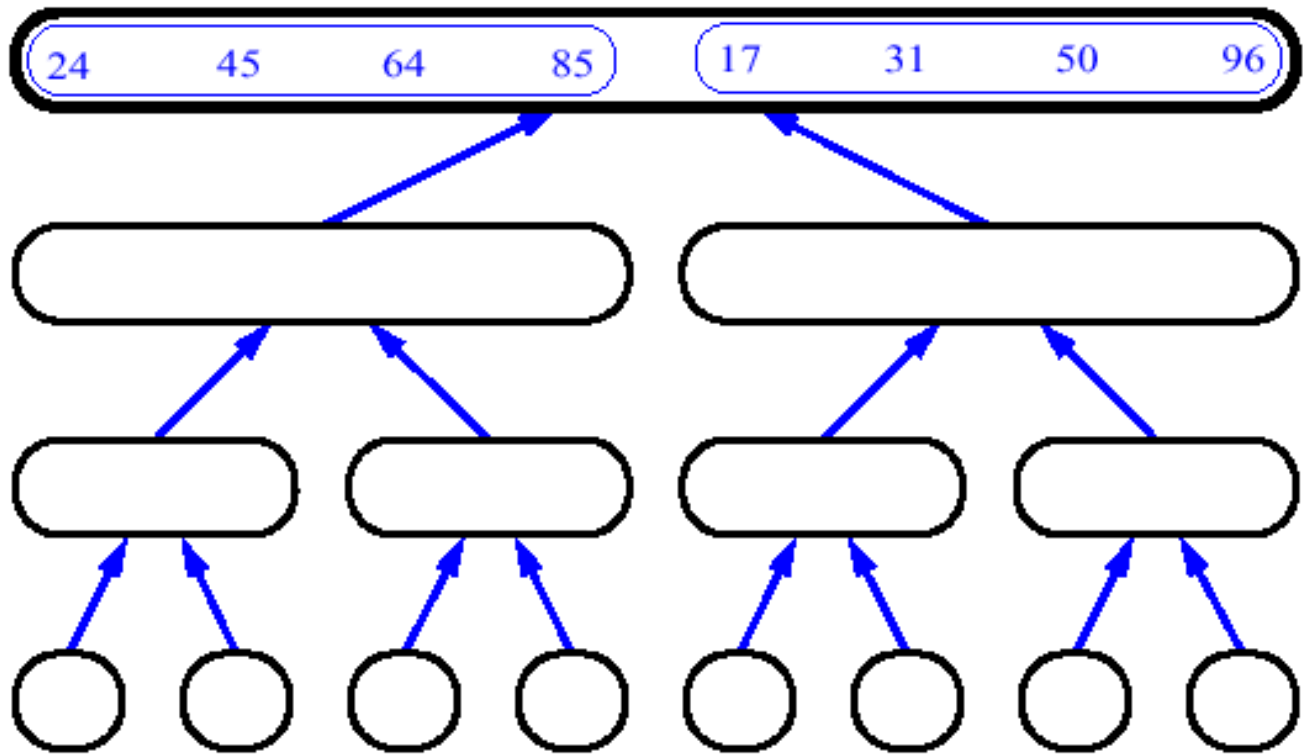
CONT…



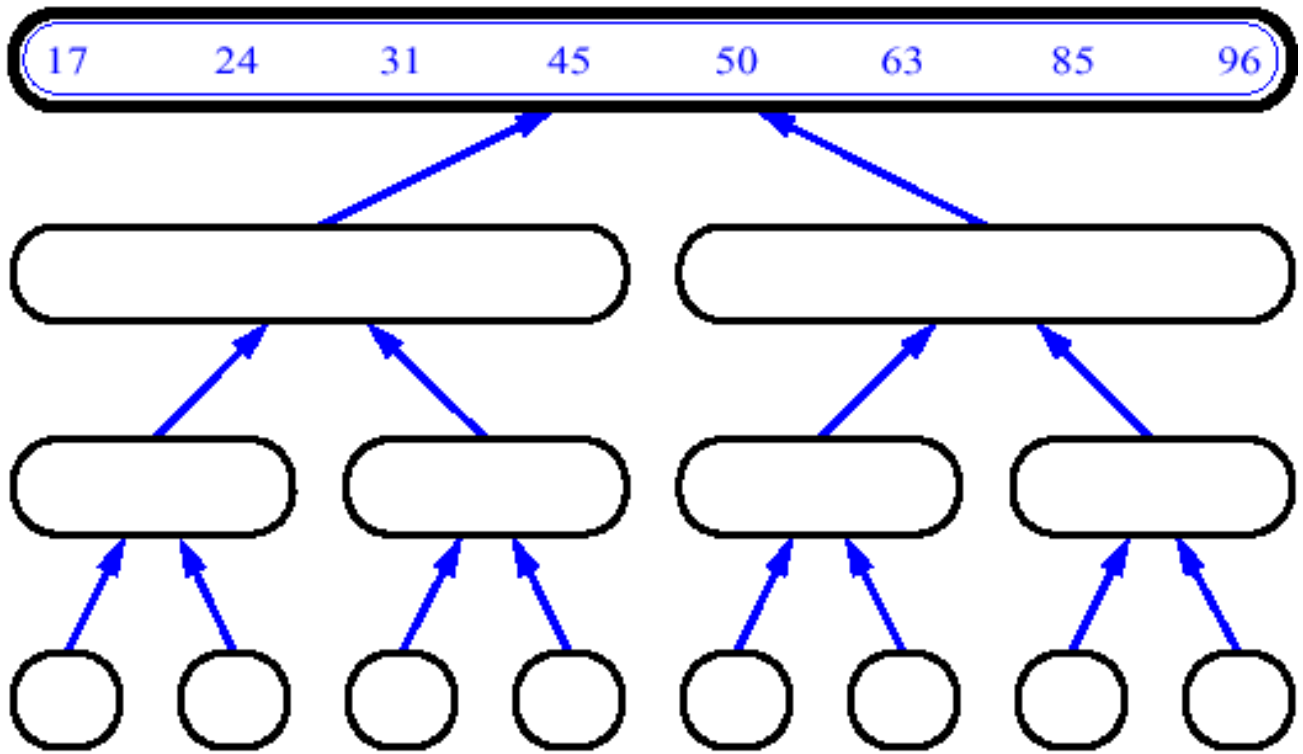
CONT…



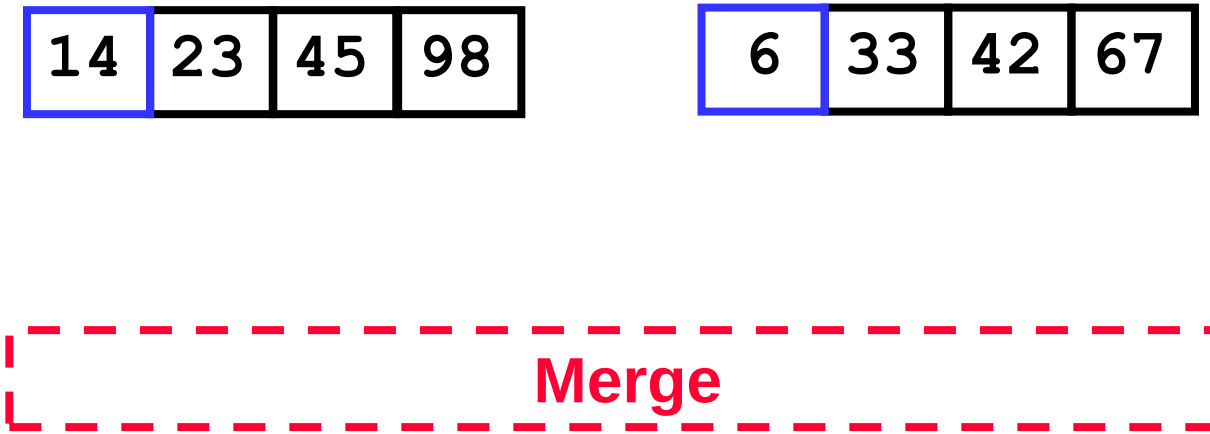
CONT…

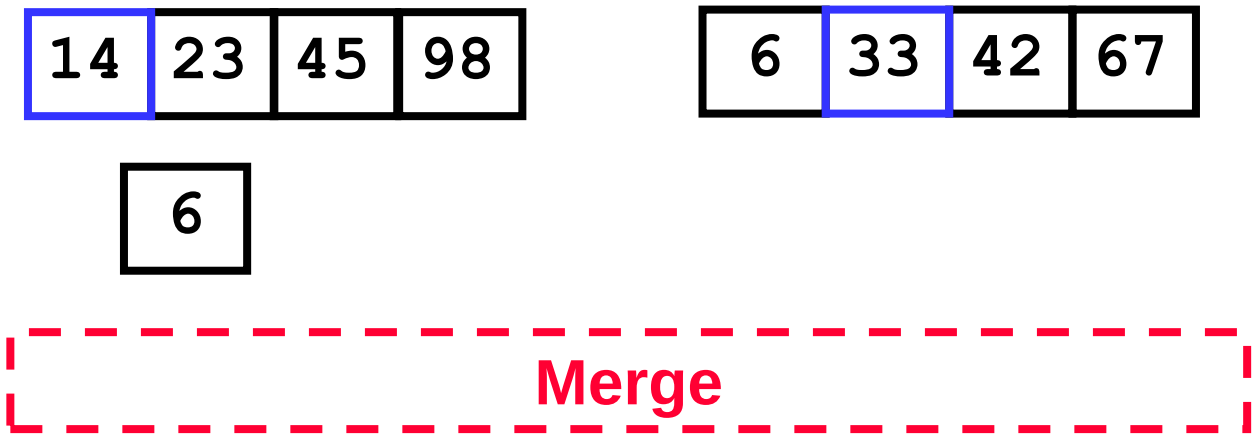


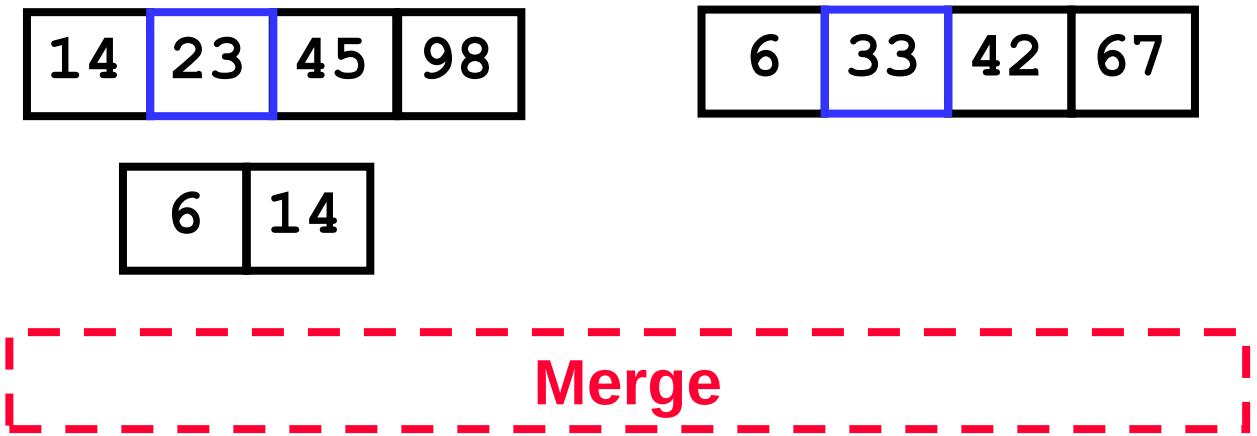
CONT…

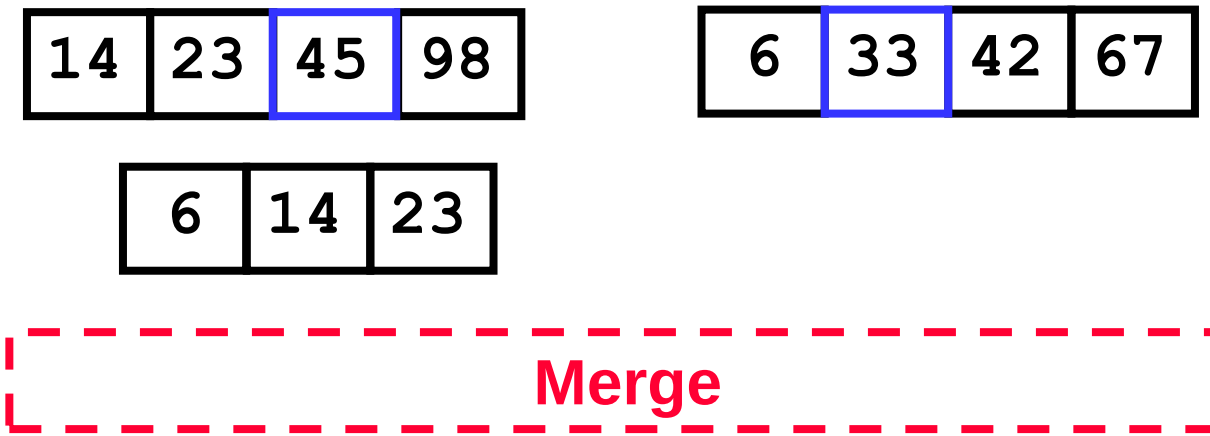


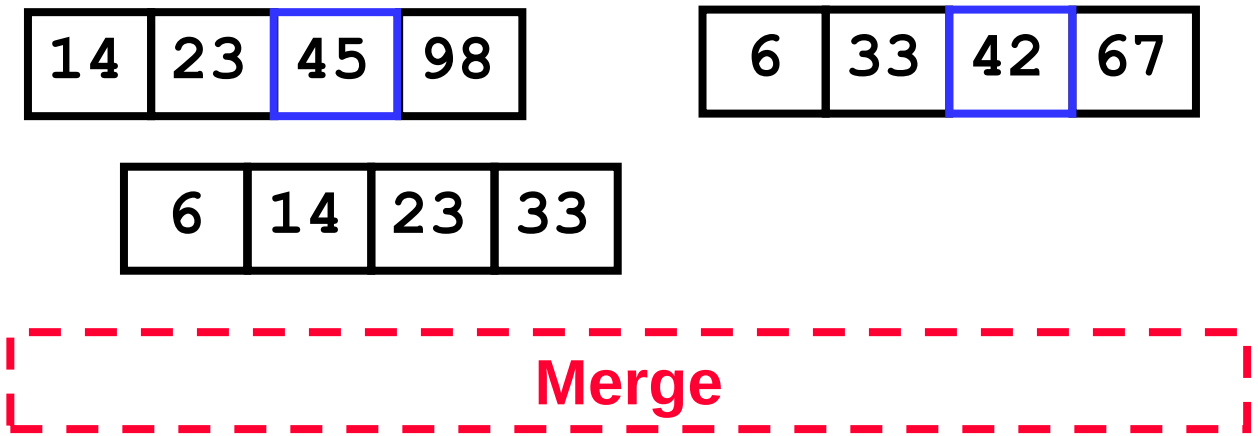

14
23
45
98
6
33
42
67

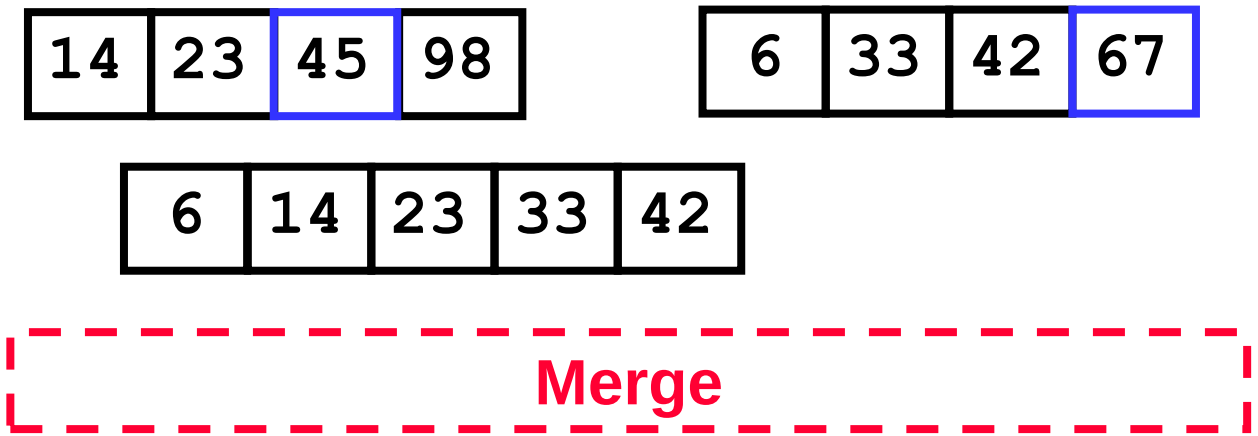

14
23
45
98
6
33
42
67
Merge

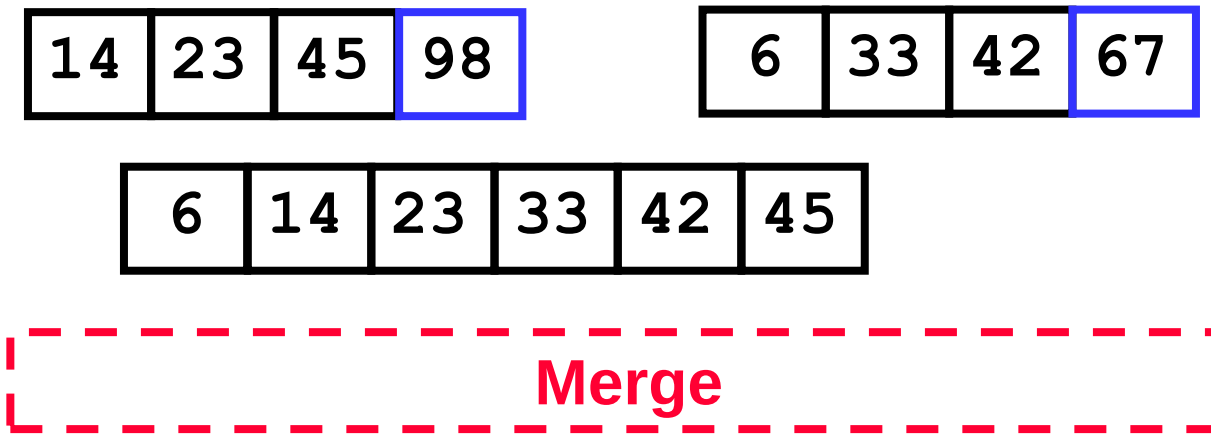

14
23
45
98
6
33
42
67
6
Merge

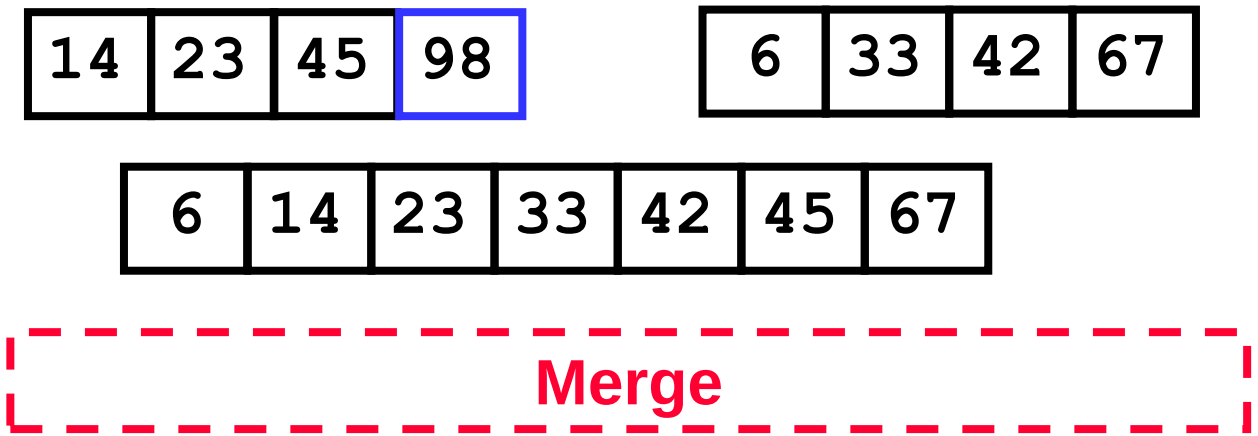

14 23 45 98
6 33 42 67
6 14
Merge

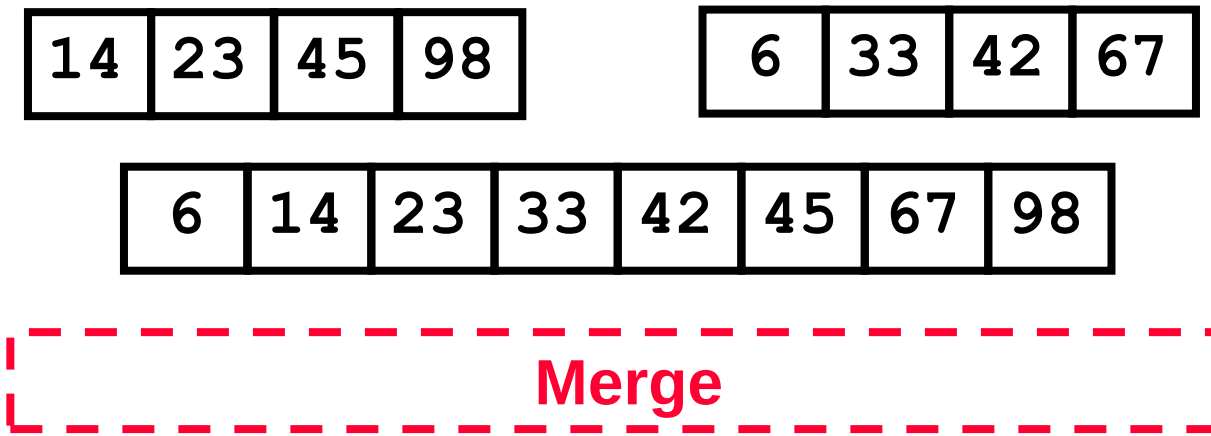

14 23 45 98
6 33 42 67
6 14 23
Merge


14 23 45 98
6 33 42 67
6 14 23 33
Merge


14 23 45 98
6 33 42 67
6 14 23 33 42
Merge


14 23 45 98
6 33 42 67
6 14 23 33 42 45
Merge


14 23 45 98
6 33 42 67
6 14 23 33 42 45 67
Merge


14 23 45 98
6 33 42 67
6 14 23 33 42 45 67 98
Merge

ALGORITHM

```
MergeSort(A[0....n-1])
  if n>1
      Copy A[0.... (n/2) -1] to B[ 0.... (n/2) -1]
      Copy A[(n/2).....n-1] to C[ 0.... (n/2) -1]
MergeSort(B[0....n-1])
MergeSort(C[0....n-1])
Merge(B,C,A)
```

```
Merge(B[0....p-1], C[0....q-1], A[0....p+q-1])
 i←0, j←0, k←0
while i<p & j<q do
 if B[i] <= C[j]
   A[K] ← B[i] ; i++;
 else
   A[K] ← C[j] ; j++;
K←K+1;

//Copy left over elements
if i==p
 copy C[j.....q-1] to A[k..... p+q-1]
else
 copy B[j.....p-1] to A[k..... p+q+1]
```


8
3
2
9
7
1
5
4

MergeSort(A[0....n-1]) → T(n)

if n>1

Copy A[0.... (n/2) -1] to B[0.... (n/2) -1]

Copy A[0.... (n/2) -1] to C[0.... (n/2) -1]

MergeSort(B[0....n-1]) → T(n/2)

MergeSort(C[0....n-1]) → T(n/2)

Merge(B,C,A) → T(n)

$$T(n) \in \begin{cases} 0 & \text{if } n=1 \\ \underbrace{T(\lceil n/2 \rceil)}_{\text{solve left half}} + \underbrace{T(\lfloor n/2 \rfloor)}_{\text{solve right half}} + \underbrace{n}_{\text{merging}} & \text{otherwise} \end{cases}$$

T(n)= 2T(n/2) +cn

APPROACHES TO SOLVE RECURSION

Approach 1:

1. Intuitive solution to recurrence is to “unroll” the recursion, accounting for the running time of first few levels.

2. Identify a pattern that can be continued as the recursion expands.

3. Sum the running times over all levels of the recursion and thereby arrives at a total running time.

Step1: Analyze the first few levels.

- 1st level of recursion → Single problem of size n → O(n)
- 2nd level of recursion → 2 problems each of size n/2 → O(n/2)
- 3rd level of recursion → 4 problems each of size n/4 → O(n/4)

Step 2: Identifying the pattern.

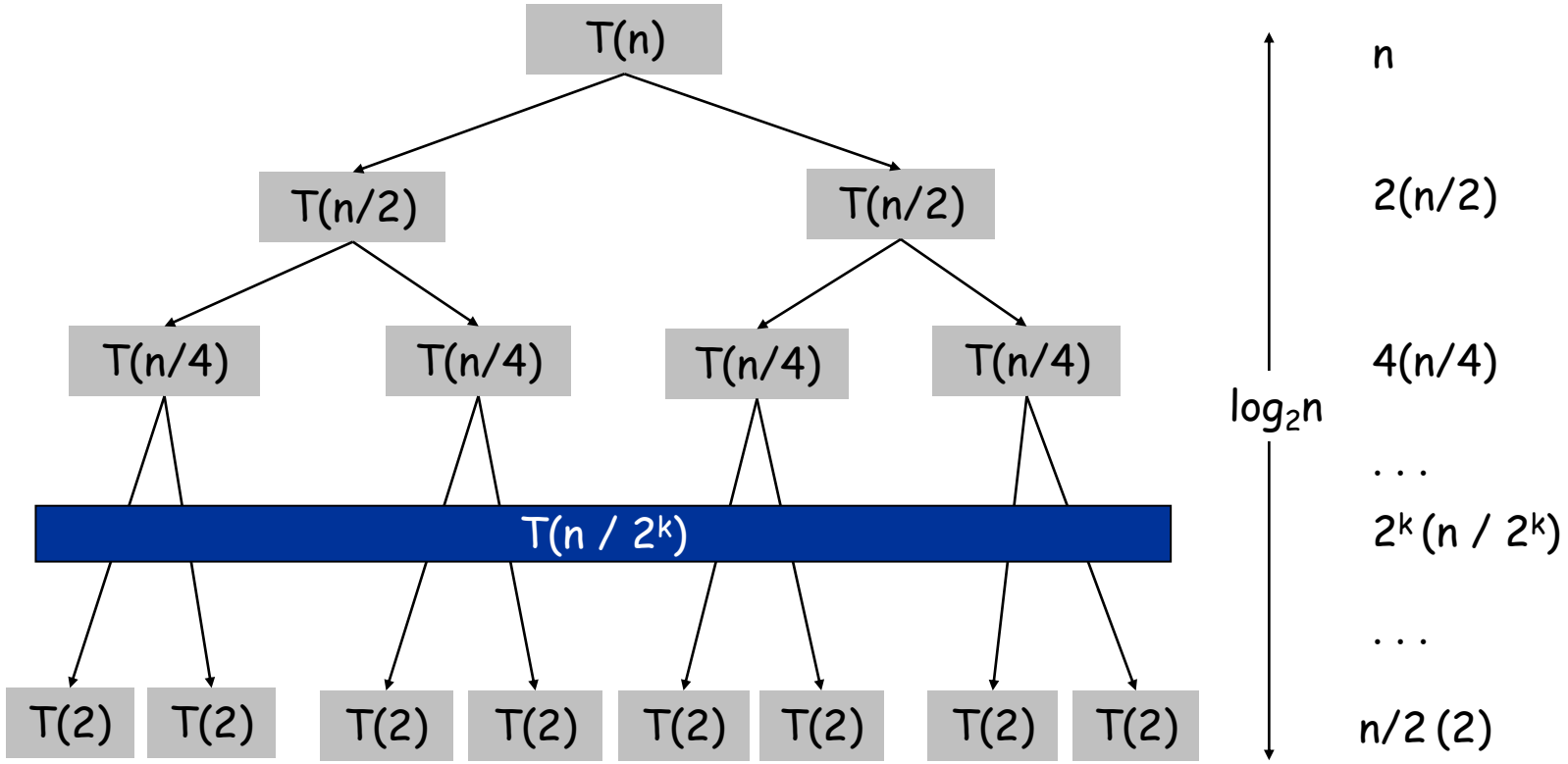
- At level *j* of the recursion, the number of subproblems are now a total of $\mathbf{2^j}$
- Each problem has shrunk in size by factor of 2 "j" time → $n/\mathbf{2^j}$

Step 3: Summing overall levels of recursion.

- The number of times the input must be halved to reduce the size of n to 2 is **log n**
- There are totally "n" levels of recursion → **O(n logn)**

ANALYSIS

$$T(n) = \begin{cases} 0 & \text{if } n = 1 \\ \underbrace{2T(n/2)}_{\text{sorting both halves}} + \underbrace{n}_{\text{merging}} & \text{otherwise} \end{cases}$$



SUBSTITUTING A SOLUTION INTO THE MERGESORT RECURSION

$T(n) = O(n \log n)$

$T(n) = c.\ n \log n$

$T(n/2) = c.\ n/2 \log n/2$

$T(n) = 2T(n/2) + O(n)$

$T(n) = 2T(n/2) + cn$

$T(n) = 2.\ \mathbf{c.\ n/2 \log n/2} + cn$

$T(n) = cn.\ [\log n - 1] + cn$

$T(n) = cn.\log n - cn + cn$

$T(n) = cn.\log n$

$\mathbf{T(n) = O(n \log n)}$