MODULE -5

13 BASIC CONCEPTS AND APPLICATION IN KINEMA

SYLLABUS

Definitions - Displacement - Average velocity - Instantaneous velocity - Speed - Acceleratio - Average acceleration - Variable acceleration - Acceleration due to gravity - Newton's Law of Motion - Motion under gravity - Numerical problems.

Dynamics is the branch of science which deals with the study of behaviour of the bo particle in the state of motion under the action of force system. It is divided into two stre (i) Kinematics and (ii) Kinetics.

(1) Kinematics: Kinematics is the branch of dynamics which deals with the motio bodies without referring to the forces causing the motion of the body.

(2) Kinetics: It is the branch of dynamics which deals with the motion of bodies referr to the forces causing the motion of the body.

13.1 BASIC TECHNICAL TERMS

1. **Motion:** A body is said to be in motion, if it is changing it's position with respect to point of reference.

   Example: A person on a scooter is in motion with respect to the road, but is at rest wit respect to the scooter itself.

   For all engineering problems, any fixed point on the earth is taken as reference point.

2. **Distance:** It is a positive scalar quantity which represents the total length of the path covered by a body/object/particle.

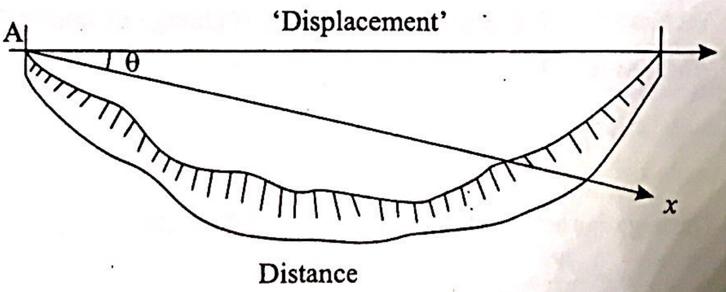


Fig. 13.1

Let a body moving along the path shown by hatched line in the fig 13.1. Let 't' be the time taken by the body to move from A to B, then the distance moved by the body in time 't' is the distance measured along the hatched line.

3. **Displacement:** It is defined as the change in position, it is a vector quantity.

   It is defined as the linear distance between the two position of the body in the beginning and at the end of the time interval.

   It is the linear distance 'AB' which makes an angle 'θ' with x-axis in time 't' as shown.

4. **Speed:** It is defined as the rate of change of distance with respect to time. It is a scalar quantity.

5. **Velocity:** It is defined as the rate of change of displacement with respect to time. It is a vector quantity.

6. **Average velocity:** If 'S' is the displacement in an interval 't'. The average velocity is given by

$$\boxed{v_{avg} = \frac{S}{t}}$$

7. **Instantaneous Velocity:** Velocity of a particle at a given Instant is called instantaneous velocity and given by the limiting value of the ratio $\frac{S}{t}$ at time 't' when both 'S' and 't' are very small.

   Let δS and δt be the small displacement and small interval then.

$$v = \lim_{\delta t \to 0} \frac{\delta S}{\delta t} = \frac{dS}{dt}$$

   In S.I Units, the unit of velocity is n/sec. or km/h or kmph

$$1\text{km/hr} = \frac{1000}{60 \times 60} = \frac{5}{18} = \text{m/sec}$$

8. **Acceleration:** It is defined as the **"Rate of change of velocity"** with respect to time.

   Mathematically,

$$\boxed{a = \frac{dV}{dt}}$$

   The negative acceleration is referred to as **"RETARDATION"** or **"DECELERATION"**

$$\therefore \quad a = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dS}{dt}\right) \qquad \therefore \quad \boxed{a = \frac{d^2S}{dt^2}}$$

   The unit of acceleration is m/sec^2.

9. Newton's Laws of motion

9.1 First Law: Every body continues in it's state of rest of or uniform motion, unless compelled by an external agency.

"A body acted by a balanced system of forces has no acceleration."

9.2 Second law: The rate of change of momentum it directly proportional to the impressed force and takes in the direction of the impressed force.

i.e., $F \propto ma$

$F = kma$, $k = \text{constant}, \; k = 1$

$$\boxed{F = ma}$$

Hence, it can be stated as.

"When an unbalanced system forces act on a particles, the particles moves with an acceleration proportional to the resultant force and it takes place in the direction of the resultant force".

Also, when resultant = 0, Acceleration = 0

9.3 Third law: It states that "for every action there is an equal and opposite reaction".

10. Newton's law of Gravitation

It states that "Any two particles or bodies of masses m_1 and m_2 separated by a distance 'r' attract each other with a force directly proportional to their masses and inversely proportional to square of their distance.

i.e., $F \propto \dfrac{m_1 m_2}{r^2} = G.\dfrac{m_1 m_2}{R}$

G = Universal constant of gravitation.

The force of attraction on a body by the earth is called "WEIGHT" of the body is given by

$$F = \frac{G_m M}{R^2}$$

where, G = Universal constant

m = mass of the body

M = mass of the earth

R = Radius of the earth

$$\boxed{F = mg = w}$$

where, $g = \dfrac{GM}{R^2} = 9.81 \text{ m/sec}^2$

If m = mass of the body

M = mass of the earth = 5.96506×10^{24} kg

R = Radius of the earth = 6371 km = 6371×10^3 m

G = 6.673×10^{-11} Nm²/kg² Universal constant

$F = mg$

and $$g = \frac{6.673 \times 10^{-11} \times 5.96506 \times 10^{24}}{(6371 \times 10^3)^2}$$

$$g = 9.806 \simeq 9.81 \text{m/sec}^2$$

$$\boxed{g = 9.81 \text{ m/sec}^2}$$

11. **Types of motion:** Motion in a single plane is called plane motion.

11.1 **Translation:** A motion is said to be translation. If a straight line drawn on a moving body remains parallel to it's original position at any time.

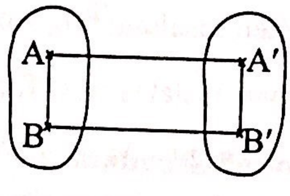


Fig. 13.2

During translation, If the path traced by a point is a straight line, It is called **'Rectilinear translation"** or **"Linear motion"** and of path is a curve one, then it is called as **"curvilinear translation"** or **"Curvilinear motion".**

11.2 **Rotation:** A motion is said to be rotation, if all particles of a rigid body move in a concentric line.

11.3 **General plane motion:** It is a combination of both translation and rotation.

Example: (1) Points on wheels of moving vehicles.

(2) A ladder sliding down from it's position against wall.

13.2 GENERAL CONCEPTS TO BE FOLLOWED TO SOLVE PROBLEMS

(1) Displacement and position are same when starting position is origin. Hence in situation where starting position is not known, it is convenient to take the starting position as origin.

(2) The magnitude of displacement is equal to distance traveled only when particle keeps traveling in the same direction.

(3) Whenever particles changes it's direction in rectilinear motion, its velocity at that instant becomes zero. At the point where particle changes it's direction reaches maximum or minimum value of position co-ordinate 'x'.

$$\frac{dx}{dt} = 0 \qquad \text{Where } x \text{ is position co-ordinate}$$

$$v = 0$$

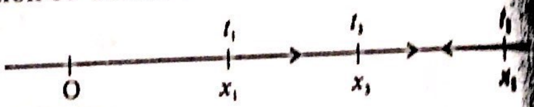


Fig. 13.3

(4) Condition for maximum or minimum velocity is

$$\frac{dV}{dt} = 0 \text{ i.e., } a = 0$$

(5) To calculate distance traveled, first put $v = 0$, to find whether particle changes direction or not in the given time interval.

(6) A linear relation between two variables, say when 'v' varies linearly with 't' is the form

$$v = mt + C$$

13.3 RELATED PROBLEMS

Problem 1

The acceleration of a particle moving along a straight line decreases uniformly from 14m/sec² to zero in line 12 seconds, at which time it's velocity is 12m/sec. Find the initial velocity and change in Position during 12 seconds interval.

Solution

As acceleration decreases uniformly with time, the relation between acceleration and time is linear.

$$a = mt + C \qquad .....(1)$$

at $t = 0$, $a = 14$ m/sec²., from Eqn, (1)

$$14 = m(0) + C$$

$$\boxed{C = 14}$$

at. $t = 12$ sec, $a = 0$

from Eqn. (1)

$$0 = m(12) + 14$$

$$m = \frac{-14}{12} = \frac{-7}{6} = -1.16$$

$$\therefore \qquad \boxed{a = -1.16t + 14}$$

To find initial velocity, 'v_0' we use

$$\because \quad a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = -1.16t + 14$$

$$\int_{v_0}^{12} dv = \int_0^{12} (-1.16t + 14).\,dt$$

$$[v]_{v_0}^{12} = \left[\frac{-1.16t^2}{2} + 14t\right]_0^{12}$$

$$12 - v_0 = -83.52 + 168$$

$$\boxed{v_0 = -72.48 \text{ m/sec}}$$

To find change in position, first we find a relation between 'v' and 't'

$$a = \frac{dv}{dt} = -\,1.16t + 14$$

$$\int_{-72.48}^{v} dv = \int_0^t (-16t + 14)dt$$

$$[v]_{-72.48}^{v} = \left[\frac{-1.16t^2}{2} + 14 \times t\right]_0^t$$

$$v + 72.48 = -\,0.58t^2 + 14t$$

$$v = -\,0.58t^2 + 14t - 72.48$$

$$v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = -0.58t^2 + 14t - 72.48$$

$$\int_{x_0}^{x_{12}} dx = \int_0^{12} (-0.58t^2 + 14t - 72.48).dt$$

$$x_{12} - x_0 = \left[\frac{-0.58t^3}{3} + \frac{14t^2}{2} - 72.48t\right]_0^{12}$$

$$x_{12} - x_0 = -\,0.19 \times 12^3 + 7(12)^2 - (72.48 \times 12)$$

$$\therefore \quad \boxed{x_{12} - x_0 = -190.1\text{m}}$$

Problem 2

An insect crawls along the edge of a rectangular swimming pool of length 30m and width 25m, if it crawls from the corner A to corner B in 40 minutes.

(1) What is it's averaged speed

(2) What is the magnitude of it's average velocity

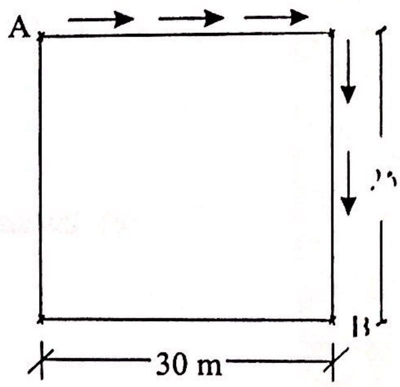
Solution

Total distance travelled by the insect = (30 + 25), $d = 55\text{m}$

Displacement $= x = \sqrt{(30)^2 + (25)^2} = 39.05$ m

(i) Average speed

$$\bar{v} = \frac{d}{t} = \left(\frac{55}{40 \text{ minutes}}\right) = \left(\frac{55}{40 \times 60 \text{ sec}}\right)$$



$$\boxed{\bar{v} = 0.022 \text{ m/sec}}$$

(ii) Average velocity, $\bar{v}_{avg} = \frac{x}{t} = \frac{39.05}{(40 \times 60)}$

$$\boxed{\bar{v}_{avg} = 0.0162 \text{ m/sec}}$$

Problem 3

The motion of the particle is defined by the relation $x = 2.4t^2 - 0.12t^3$ where x and 't' are in meters and second respectively. Determine

(i) Average velocity of particle at time interval $t = 0$ seconds and $t = 15$ seconds.

(ii) Instantaneous acceleration of particle at $t = 5$ seconds.

Solution

Given, $x = 2.4t^2 - 0.12t^3$

(a) at $t = 0$ seconds, $x_1 = 0$, $t = 15$ seconds

$$x_{15} = 2.4\,(15)^2 - 0.12\,(15)^3$$

$$\boxed{x_{15} = 135m}$$

Average velocity

$$\overline{V}_{avg} = \frac{(x_{15} - x_0)}{(t_{15} - t_0)} = \left(\frac{135-0}{15-0}\right)$$

$$\boxed{\overline{V}_{avg} = 9 \text{ m/sec}}$$

(b) Instantaneous velocity

$$v = \frac{dx}{dt} = \frac{d}{dt}(2.4t^2 - 0.12t^3)$$

$$v = \left(2.4 \times 2t - 0.12 \times \frac{3t^2}{2}\right)$$

$$v = 4.8t - 0.36t^2 \qquad .....(1)$$

at, $t = 5$ seconds,

$$v = (4.8 \times 5 - 0.36 \times 5^2) = 15\text{m/sec}$$

$$\boxed{v = 5\text{m/sec}}$$

(c) Instantaneous acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}(4.8t - 0.36t^2)$$

$$\boxed{a = 4.8 - 0.72t} \qquad .....(2)$$

at, $t = 5$ seconds

from Eqn. (2)

$$a = 4.8 - (0.72 \times 5)$$

$$\boxed{a = +1.2 \text{ m/sec}^2}$$

Problem 4

A train of weight 5000 KN starts from rest and reaches a velocity of 70 kmph after 80 seconds. If the frictional resistance of the track is 5.5 N per KN of the weight of the trained, find the tractive force developed by the engine.

Solution

Initial velocity = 0

Velocity after 80 sec = 70 kmph = $\frac{70 \times 100}{60 \times 60}$ = 19.45 m/sec

Acceleration, $a = \dfrac{\text{Change in velocity}}{\text{time}}$

$$a = \left(\frac{19.45-0}{80}\right) = 0.243\ m/\text{sec}^2$$

If the tractive force developed by the engine is 'P' newton Frictional resistance of train = 5.5 × 5000 = 27500 N

Mass of the train,

$$m = \frac{W}{g} = \frac{5000 \times 1000}{9.81} \text{ Newton}$$

$$\boxed{m = 509683.9 \text{ kg}}$$

From Newton's Second law,

$$\text{Force} = \text{Mass} \times \text{acceleration}$$

$$(P - 27500) = 509683.9 \times 0.243$$

$$P = 151353.2 \text{ N}$$

$$\boxed{P = 151.3 \text{KN}}$$

Problem 5

A Bus is moving with a uniform acceleration and covers 15m in 5 seconds and 25m in the 9th second calculate.

(1) The initial velocity of the train.

(2) Acceleration of the body

Solution

Distance traveled in 5th second = 15

Distance traveled in 9th second = 25*m*

Let, u = Initial velocity and

a = acceleration of the Bus,

Distance covered in the n^{th} second is given by equation is $u + \frac{a}{2}(2n-1)$

∴ Distance covered in 5th second = $u + \frac{a}{2}(2 \times 5 - 1)$

$$15 = u + 4.5a \qquad \text{....(1)}$$

Similarly, distance covered in the 9th second is

$$25 = u + \frac{a}{2}(2 \times 9 - 1)$$

$$25 = u + 8.5\,a \qquad .....(2)$$

Solving Eqn. (1) and (2)

$$\boxed{u = 3.75 \text{ m/sec}} \text{ and } \boxed{a = 2.5 \text{m/sec}}$$

PROBLEMS FOR PRACTICE

1. A train is moving with a uniform acceleration and covers 20m in 5 second and 30m in the 9th second. Calculate (i) The initial velocity of the train (ii) Acceleration of the train.

   [Ans. $a = 2.5$m/sec^2, $u = 8.75$ m/sec]

2. The car travels in a straight line along a road. It's distance is given as a function time 't' given by equation. $x = t^2 - (t-3)^2 m$, where 'x' and 't' are in meters and seconds respectively. Determine: (i) The time when velocity is maximum (ii) The position and maximum velocity (iii) The distance traveled at t = 12 second.

   [Ans. time = 3.33 seconds, position, $x = 11.05$m and $v_{max} = 6.33$ m/sec, distance traveled, $x = 636.54$m]

3. Rectilinear motion of a particle is described by equation, $a = -0.4$V, where a and v are in mm/sec^2 and mm/sec respectively. If $\theta = 30$mm/sec at $t = 0$, find the distance traveled by particle before coming to rest.

   [Ans. x = 75mm]

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14 RECTILINEAR MOTIO

SYLLABUS

Rectilinear Motion - Numerical problems

"Rectilinear motion is another name for straight-line motion. this type of motion describ the movement of a particle or a body. A body is said to be experience rectilinear motion, i any two particles of the body travel the same distance along two parallel straight lines".

14.1 DERIVATION OF EQUATION OF MOTION

14.1.1 Equations of Motion with uniform acceleration

Let, u = Initial velocity of body

v = Final velocity of body

t = time elapsed.

motion with uniform acceleration is given by,

$$a = \frac{dv}{dt}$$

$$dv = a\, dt$$

Integrating both sides,

$$\int_u^v dv = \int_0^t a.\, dt$$

$$[v]_u^v = a.[t]_0^t$$

$$v - u = a.\, t$$

(or) $$\boxed{v = u + at} \quad \checkmark \qquad .....(1)$$

14.1.2 Equations for distance traveled

We know that velocity

$$v = \frac{dx}{dt}$$

$$dx = v.\, dt \qquad .....(2)$$

from Eqn. (1),

$$\therefore \quad v = u + at$$

Eqn. (2) becomes

$$dx = (u + at) \, . \, dt$$

Integrating both sides

$$\int_0^s dx = \int_0^t (u + at) dt$$

Assuming that at, $t = 0$, $x = 0$

$$[x]_0^s = \left[ut + \frac{at^2}{2} \right]_0^t$$

$$s = ut + \tfrac{1}{2} at^2 \qquad \text{......(3)}$$

14.1.3 We know that acceleration

$$a = v . \frac{dv}{dx}$$

$$a . \, dx = v . \, dv$$

Integrating both sides

$$\int_0^s a . dx = \int_u^v v . dv$$

$$a [x]_0^s = \left[\frac{v^2}{2} \right]_u^v$$

$$a . \, [s - 0] = \frac{1}{2} (v^2 - u^2)$$

$$as = \frac{1}{2} (v^2 - u^2)$$

$$2as = v^2 - u^2$$

(or) $\boxed{v^2 - u^2 = 2as}$

.....(4)

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14.2 ACCELERATION DUE TO GRAVITY

A body which is free to move entirely under the influence of the earth's gravitation, will be subjected to an acceleration directed towards the centre of the earth of value is 9.81 m/sec².

Equation of motion for body freely falling vertically downwards.

Under gravity .....(5)

$$\left.\begin{array}{l} (i)\ v = u + gt \\ (ii)\ s = ut + \tfrac{1}{2} gt^2 \\ (iii)\ v^2 - u^2 = 2gs \end{array}\right]$$

For bodies projected vertically 'g' of is negative as it is, against the gravity. .....(6)

$$\left.\begin{array}{l} (i)\ v = u - gt \\ (ii)\ s = ut - \tfrac{1}{2} gt^2 \\ (iii)\ v^2 - u^2 = -2gs \end{array}\right]$$

↑ upward. -ve.

Equations for bodies just dropped ($\underline{u} = 0$) ......(7)

$$\left.\begin{array}{l} (i)\ v = gt \\ (ii)\ s = \tfrac{1}{2} gt^2 \\ (iii)\ v^2 = 2gs \end{array}\right]$$

↓ drop

14.2.1 Greatest height reached by a body and the time it takes

Let, 'u' be the initial velocity with which body is projected vertically up. When it reaches to the greatest height 'h' it's final velocity, $v = 0$.

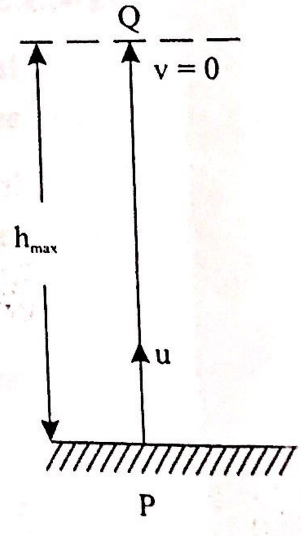

Fig. 14.1

From Eqn. (6)

$$v = u - gt$$

$$0 = u - gt$$

or $$\boxed{u = gt}$$

from Eqn. (6)

$$v^2 - u^2 = -2gs$$

$$-u^2 = -2gh \text{ max}$$

$$\boxed{h_{max} = \frac{u^2}{2g}}$$

14.3 MOTION CURVES

Motion curves are the graphical representation of the displacement, velocity and acceleration with time.

14.3.1 Displacement - time curve (s – t curve)

It is a curve with time as Abscissa (x – axis) and displacement as ordinate (y - axis).

At Instant of time 't' velocity is given by

$$v = \frac{ds}{dt} = \theta \text{ (slope)}$$

Velocity at any time may be found from the slope of the curve.

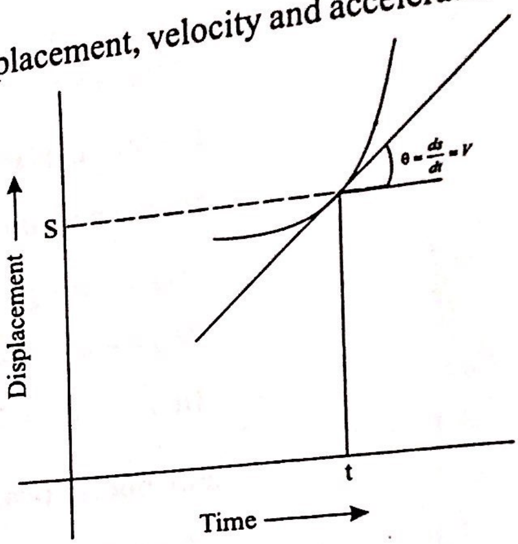

Fig. 14.2

14.3.2 Velocity - time curve (v - t curve)

It is a curve with time as abscissa and velocity as ordinate. Acceleration is given by the slope of the curve.

$$a = \frac{dv}{dt} = \theta$$

Area under the curve t_1 and t_2 represents the displacement.

(i) Acceleration by the slope of the curve.

(ii) Displacement by the area below the curve.

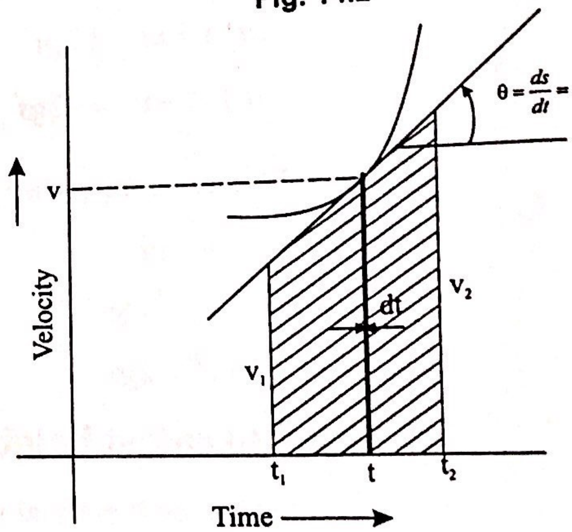

Fig. 14.3

14.3.3 Acceleration - time curve

It is a curve with time as abscissa and acceleration as ordinate.

It deals with the study of bodies moving with varying acceleration.

$$\therefore \quad a = \frac{dv}{dt}$$

$$dv = a.\,dt$$

$$\int dv = \int a.dt$$

$$\therefore \quad \boxed{v = \int a.dt}$$

∴ "Area Under the curve represents velocity".

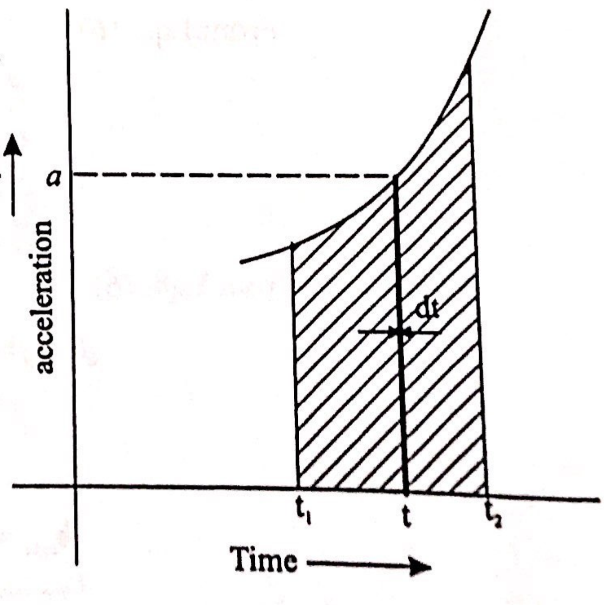

Fig. 14.4

14.4 PROBLEMS ON RECTILINEAR OR LINEAR MOTION

Problem 1

A body is moving with a velocity of 4m/sec. After 10 seconds the velocity of the bo reaches 8m/sec. Find the acceleration of the body.

Solution

Given, $u = 4$m/sec, $t = 10$ sec, $v = 8m$/sec.

Let 'a' be the acceleration then from Eqn.

$$v = u + at$$

$$8 = 4 + a \times 10$$

$$10a = 4$$

$$a = \frac{4}{10} = 0.4m/\text{sec}^2$$

$$\boxed{a = 0.4 \text{ m/sec}^2}$$

Problem 2

A car is moving with a velocity of 25m/sec. The car is brought to rest by applying brakes in 8 seconds. Determine,

(1) The retardation.

(2) The distance traveled by the car after applying the brakes.

Solution

Given: Initial velocity, $u = 25m$/sec.

Final velocity = 0

time, $t = 8$ seconds

$$v = u + at$$

$$0 = 25 + (a \times 8)$$

$$\boxed{a = -3.125 \text{ m/sec}^2} \text{ (retardation)}$$

Let 's' be the distance travelled by the car after applying the brakes.

$$s = ut + \tfrac{1}{2}at^2 = (25 \times 8) + \tfrac{1}{2}(-3.125)(8)^2$$

$$\boxed{s = 100m}$$

Problem 3

A car is accelerating at a constant rate of 4m/sec^2. If it travels a distance of 1000m in 20 sec.

(i) What must be it's initial velocity?

(ii) What will be it's final velocity?

Solution

Given: $a = 4m/\text{sec}^2$., $S = 1000m$, $t = 20$ secs.

(i) Initial velocity (u)

$$S = ut + \frac{1}{2}at^2$$

$$1000 = (u \times \overset{20}{25}) + \frac{1}{2} \times 4 \times (20)^2$$

$$1000 = 254 + 800$$

$$200 = 25u$$

$$\boxed{u = \cancel{8}\text{ m/sec}} \quad 10$$

(ii) Final velocity (v)

$$v = u + at$$

$$v = \overset{10}{\cancel{8}} + (4 \times 20)$$

$$\boxed{v = \cancel{88}\text{m/sec}} \quad 90$$

Problem 4

A police officer observes a car approaching at the unlawful speed of 60 kmph. He gets on his motor cycle and starts chasing the car, just as it passes in front of him. After accelerating for 10 secs, at a constant rate, the officer reaches his top speed of 75 kmph. How long does it take the officer to overtake the car from the time he started?

(V.T.U June/July - 2015)

Solution

Speed of Unlawful car, 60 kmph = $\dfrac{60 \times 1000}{(60 \times 60)} = 16.67m/\text{sec}$.

Let, 'S' be the distance travelled by car at constant speed.

$$S = ut + \frac{1}{2}at^2$$

But, $a = 0$

$\therefore$ $S = ut = 16.67t$

$\boxed{S = 16.67t}$ .....(1)

Let, 't' be the time taken by motor cycle to overtake the car. The motor cycle and car travel same distance S.

Motion of motor cycle

Initial velocity, $u = 0$

Maximum velocity after 10 secs = 75 kmph = $\frac{75 \times 1000}{60 \times 60}$ = 20.83m/sec

Let. 'a' be the acceleration to attain a speed of 20.83m/sec after 10 secs ($u = 0$)

$V = u + at$

$20.83 = 0 + a \times 10$

$[a = 2.083 \text{ m/sec}^2]$

Distance travelled by motor ~~car~~ cycle in 10 secs.

$S_1 = ut + \frac{1}{2}at^2$

$S_1 = 0 + \frac{1}{2} \times 2.083 \times (10)^2$

$\boxed{S_1 = 104.15\text{m}}$ .....(2)

Distance travelled by motor cycle in $(t - 10)$ secs, with constant speed of 20.83 m/sec ($a = 0$).

$S_2 = ut$

$\boxed{S_2 = 20.83 \text{ m } (t - 10)}$ .....(3)

Total distance travelled by motor cycle

$S = S_1 + S_2$

$S = 104.15 + 20.83\ (t - 10)$ .....(4)

Since, the motor cycle and car travel by same distance,

Equate Eqn. (1) = Equate Eqn. (2)

$16.67t = 104.15 + 20.83\ (t - 10)$

$4.16t = 104.15$

$\boxed{t = 25.04 \text{ secs}}$

Problem 5

Burglar's car starts at an acceleration of $2m/sec^2$. A police vigilant party came after 5 seconds. and continued to chase the burglar's car with a uniform velocity of $20m/sec$. Find the time take in which the police van will overtake the car.

(V.T.U Dec. 2014/Jan. 2015)

Solution

For Burglar's car, $u = 0$, $a = 2m/sec^2$

Let, t = time taken by police van to overtake burglar's car as the police vigilant came after 5 secs. Hence burglar's car will be in motion for $(t + 5)$ secs.

Uniform velocity of police van = 20m/sec.

When police van will overtake burglar's car, then distance travelled by both Van's will be same.

$\therefore$ Distance travelled by police van = Uniform velocity × time

$$S = 20t \qquad \text{.....(1)}$$

The distance travelled by burglar's car is $(t + 5)$ sec. is

$$S = ut + \tfrac{1}{2}at^2$$

$$S = 0 + \tfrac{1}{2} \times 2 \times (t+5)^2 \qquad \text{.....(2)}$$

Equate Eqn. (1) and Eqn. (2)

$$20t = (t + 5)^2$$

$$20t = t^2 + 25t + 10t$$

$$(t - 5)^2 = 0$$

$$\boxed{t = 5 \text{ secs}}$$

Problem 6

Two cars A and B accelerates from a standing starts. The acceleration of A is $1.2m/sec^2$ and that of B is 1.5 m/sec. If B was originally 5 m behind A, How long it takes to overtake A?

Solution

Let 't' be the time taken by car 'B' to overtake car 'A'.

Let, 'S' be the distance travelled by car A then distance travelled by car 'B' is (S + 5)m.

Car A: Initial velocity; u = 0

Acceleration, a = 1.2 m/sec²

Distance travelled,

$$S = ut + \frac{1}{2}at^2$$

$$S = 0 + \frac{1}{2} \times 1.2 \times t^2$$

$$\boxed{S = 0.6t^2} \quad \text{.....(1)}$$

Car B: Initial velocity; u = 0

Acceleration, a = 1.5m/sec²

Distance travelled

$$S = 0 + \frac{1}{2} \times 1.5t^2$$

$$\boxed{S = 0.75t^2} \quad \text{.....(2)}$$

Distance travelled,

$$d = S + 5 = 0.6t^2 + 5 \quad \text{.....(3)}$$

Equate Eqn. (2) = Eqn. (3)

$$0.6t^2 + 5 = 0.75t^2$$

$$0.15t^2 = 5$$

$$\boxed{t = 5.77 \text{ seconds}}$$

Problems : Motion Under Gravity

Problem 7

A stone is thrown upward with a velocity of 35m/sec. Determine the time of the stone when it is at a height of 8m and is moving downwards.

Solution

Upward motion is positive, Downward is negative.

$$\therefore \quad a = -g = -9.81 m/\text{sec}^2$$

Displacement. S = +8m as final point is above the initial point.

$$u = 35m/\text{sec}$$

$\therefore \quad S = ut + \frac{1}{2}at^2$

$\therefore \quad 8 = 35t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 - 35t + 8 = 0$

on solving [$t = 0.2364$ sec (or) 6.899sec.]

The smaller value of 't' is for upward motion and larger value for downward motion.

$\therefore \quad \boxed{t = 6.89 \text{ sec}}$

Problem 8

A stone is dropped into a well. After 4 sec the sound of splash is heard. If the velocity of sound is 280m/sec. Find the depth of the well upto the water surface.

Solution

Let, x = Depth of well.

Then displacement of stone = $S = -x$

Negative sign is for downward displacement.

$a = -g = -9.81 m/\text{sec}^2$

t = time and

$u = 0$ for the stone

$S = ut + \frac{1}{2}at^2$

$-x = 0 + \frac{1}{2}(-9.81)t^2$

$x = 4.905t^2 \quad .....(1)$

As the total time is u sec for the stone to reach the water surface and sound to come up, the time for sound will be $(4 - t)$.

Using distance = Velocity × time

$x = 280 \times (4 - t)$

$x = 1120 - 280t \quad .....(2)$

From Eqn. (1) and (2)

$4.905t^2 = 1120 - 280t$

$4.905t^2 + 280t - 1120 = 0$

On solving [$t = 3.75$sec. (or) 60.83 sec)

Time for stone cannot be greater than 4 sec.

$$\boxed{t = 3.75 \text{ sec}}$$

from Eqn. (1), $x = 4.905t^2$

$$x = 4.905 \times (3.75)^2$$

$$\boxed{x = 68.97 \text{ m}}$$

Problem 9

A stone is dropped from the top of the tower 50m high. At the same time another ston is thrown up from the tower with a velocity of 25m/sec. At what distance from the to and after how much time the two stones cross each other? **(V.T.U Dec. 2014/Jan. 2015**

Solution

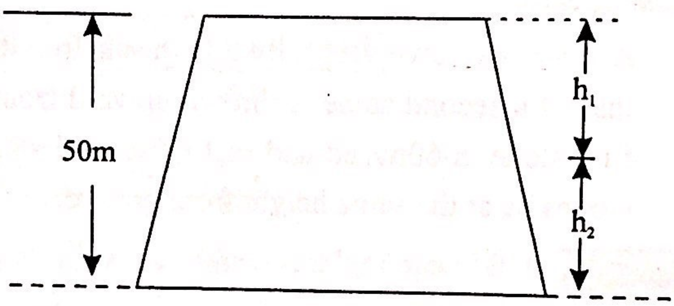

Fig. 14.4.1

$$h_1 + h_2 = 50 \quad .....(1)$$

Stone (1): Initial velocity, $u = 0$

height, $S = S_1$

gravity [$g = 9.81 m/sec^2$]

$$S = ut + \frac{1}{2}gt^2$$

$$S_1 = 0 + \frac{1}{2}(9.81)t^2$$

$$S_1 = 0 + 4.905t^2 \quad .....(2)$$

Stone (2): $u = 25m/sec$

$$S = S_2, g = -9.81 m/sec^2$$

$$S = ut + \frac{1}{2}gt^2$$

$$S_2 = 25t - \frac{1}{2}(9.81)t^2 = 25t - 4.905t^2 \quad .....(3)$$

Substitute Eqn. (2) and (3) in (1)

$$4.905t^2 + 25t - 4.905t^2 = 50$$

$$25t = 50$$

$$\boxed{t = 2 \text{ sec}}$$

Substitute 't' value in Eqn. (2) and in (3)

$$S_1 = 4.905(2)^2 = 19.6m$$

and $$S_2 = (25 \times 2) - 4.905 \times (2)^2$$

$$\boxed{\begin{aligned} S_1 &= 19.6\text{m} \\ S_2 &= 30.4\text{m} \end{aligned}}$$

Problem 10

A stone is thrown vertically into the air from the top of tower 150m high. At the same instant a second stone is thrown upward from the ground. The initial velocity of the first stone is 60m/sec and that of second stone is 80/sec. When and where will the stones be at the same height from ground.

Solution

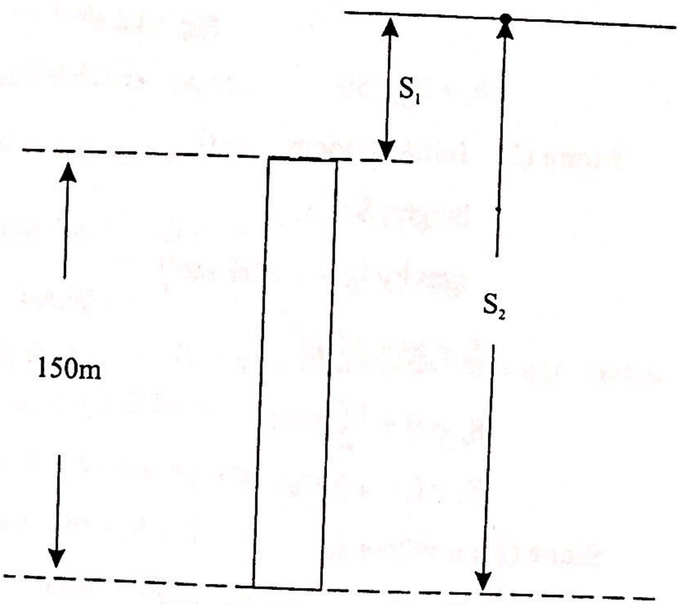

Fig. 14.4.2

Solution

Case (i): Object A refer Fig.

Let, 't_1' be the time taken by 'A' to reach the ground.

Initial velocity $u = 30$m/sec

$$S = -130m$$

$$S = ut_1 + \frac{1}{2}gt_1^2$$

But $g = -9.81\ m/sec^2$

$$-130 = 30t_1 - \frac{1}{2} \times 9.81t_1^2$$

$$4.90t_1^2 - 30t_1 - 130 = 0$$

$$t_1 = \frac{30 \pm \sqrt{30^2 + 4 \times 4.9 \times 130}}{2 \times 4.9}$$

$$\boxed{t_1 = 9.053 \text{ secs}}$$

Object B: Let 't_2' be the time taken by object 'B' to reach ground

$$S = 30t_2 + \frac{1}{2}(9.81)t_2^2$$

$$130 = 30t_2 + \frac{1}{2}(9.81)t_2^2$$

$$4.9t_2^2 + 30t_2 - 130 = 0$$

$$\boxed{t_2 = 2.929 \text{ secs}}$$

Case (ii): Let 'h' be the height at which the body 'A' is released from rest so that the two object reach the ground simultaneously.

$$h = \frac{1}{2}gt_2^2 = \frac{1}{2} \times 9.81 \times (2.929)^2$$

$$\boxed{h = 42.04 \text{ m}}$$

Consider the first stone,

$$u_1 = 60\ m/\text{sec},\ g = -9.81 m/\text{sec}^2$$

$$S_1 = u_1 t + \tfrac{1}{2} g t^2$$

$$S_1 = 60t - 4.905t^2 \qquad .....(1)$$

Consider the Second stone

$$u_2 = 80 m/\text{sec},\ g = -9.81 m/\text{sec}^2$$

$$S_2 = u_2 t + \tfrac{1}{2} g t^2$$

$$S_2 = 80t - 4.905t^2 \qquad ......(2)$$

$$S_2 - S_2 = 150$$

$$(80t - 4.905t^2) - (60t - 4.905t^2) = 150$$

$$80t - \cancel{4.905t^2} - 60t + \cancel{4.905t^2} = 150$$

$$20t = 150$$

$$\boxed{t = 7.5 \sec}$$

Substitute the value of 't' in Eqn. (1) and (2)

$$S_1 = (60 \times 7.5) - (4.905 \times (7.5)^2)$$

$$\boxed{S_1 = 174.09 \text{ mtr.}}$$

$$S_2 = (80 \times 7.5) - (4.905 \times (7.5)^2)$$

$$\boxed{S_2 = 324.09 \text{ mtr.}}$$

Problem 11

Two objects A and B are projected vertically at 130m above the ground level. A is projected up with a velocity of 30m/sec and 'B' is projected downwards with the same velocity. Find the time taken by each object to reach the ground.

Also find the height from which the object 'A' must be just released from rest in order the two objects hits the ground simultaneously.

(V.T.U Model Paper 2015/2016)

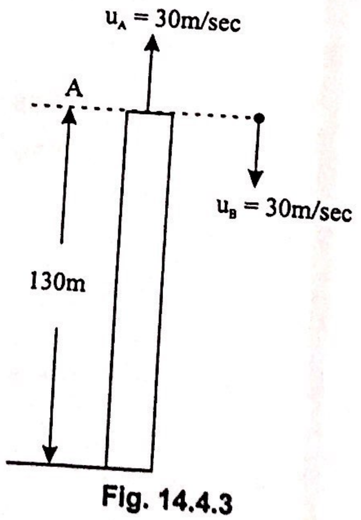

Fig. 14.4.3

PROBLEMS FOR PRACTICE

1. A stone is dropped into the well and a sound of splash is heard after 4 sec. Find the depth of well if the velocity of sound is 350 m/sec. **(Ans. h = 70.75m)**

2. A vehicle starts from rest is uniformly accelerated during the first 200m of it's run, after which it runs the next 800m at uniform speed acquired. It is than brought to rest in 60 sec under uniform retardation. If the time of entire journey is 300sec. Then find the acceleration with which the vehicle started and the retardation with which it stopped.

   (Ans. acceleration, $a_1 = 0.05 m/sec^2$, retardation $a_2 = 0.1 m/sec^2$, Distance travelled $S = 125 m$)

3. A Burglar's car starts with an acceleration of $2 m/sec^2$. A police van came after 10secs and continued to chase the burglar's car with an uniform velocity of 40m/sec. Find the time taken by the police van to overtake the burglar's car? **(Ans. t = 10 secs).**

4. A stone is thrown vertically upwards and return to the earth in 10sec. What was it' initial velocity and How high dit it go? **(Ans. V = 98.1 m/sec, S = 490.5m)**

5. A stone is dropped into a well and a sound of splash is heard after 4 sec. Find the depth or the well. **(Ans. d= 78.48m)**

❖ ❖ ❖ ❖

15 CURVILINEAR MOTION

SYLLABUS

Curvilinear Motion - Super Elevation - Projectile Motion - Relative motion - Numerical problems.

The motion of a particle along a curved path is said to be curvilinear motion. When a particle is projected upwards and at certain angle, the particle moves along a curved path before striking the ground and this path traced is known as trajectory and the projected particles as projectile. The path of the projectile will be parabolic.

Fig. shown below shows path traced by the projecticle.

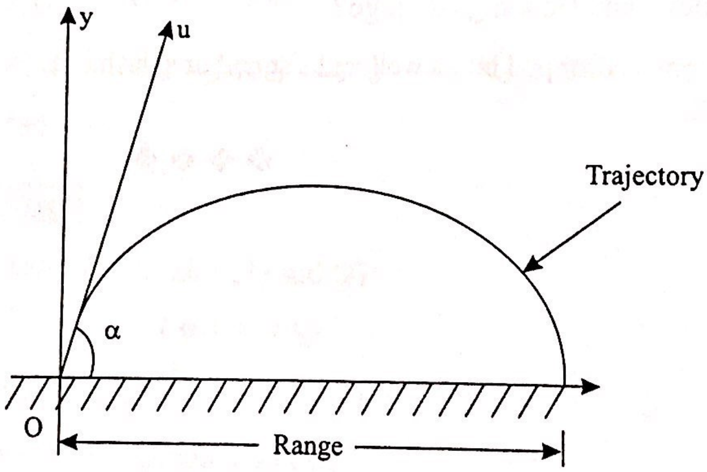

Fig. 15.1 Trajectory of a particle

15.1 DEFINITIONS

(i) **Angle of projection (α):** It is the angle between the direction of motion of the particle and the horizontal reference line.

(ii) **Trajectory:** It is the path traced by the projectile in space.

(iii) **Horizontal range:** It is the horizontal distance between the point of projection and the projectile meets Horizontal.

(iv) **Velocity of projection:** The velocity with which the particle projected is called as velocity of projection i.e., 'u' m/sec.

(v) **Time of flight:** The time interval during which the projectile is in motion is called the time of flight.

15.2 EQUATION FOR THE PATH OF PROJECTICLE

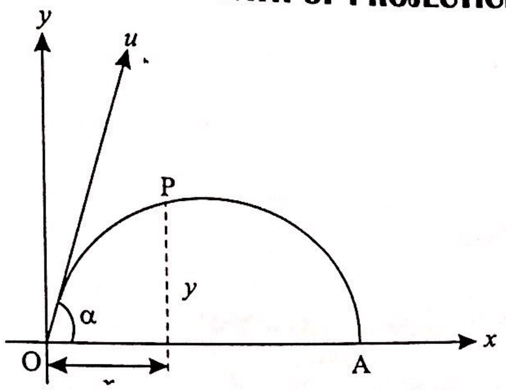

Fig. 15. 2

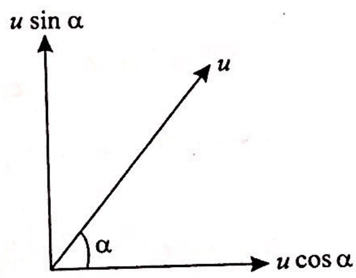

Fig. 15. 3

Fig. 15.2 shows the path of a projecticle and in the fig.

α = Angle of projection

u = Initial velocity of the projectile

Resolving u into Horizontal and vertical components shown in fig. 15.3

Horizontal components of velocity = $u \cos \alpha$

Vertical component of velocity = $u \sin \alpha$.

The Horizontal component of velocity will not be affected by the gravitational force while the vertical component gets affected.

The particle moves along path OPA and reaches the ground at A shown in fig. 15.2. Consider a point P(x, y) on the trajectory and let 't' be the time taken by the particle to travel from O to P.

∴ Horizontal distance travelled by particle in 't' seconds is given by

$x = u_x(t)$ where, $u_x = u \cos \alpha$

$$x = (u \cos \alpha)(t) \quad .....(1)$$

Similarly

Vertical distance travelled by particle in 't' seconds is given by,

$y = u_y(t) - \frac{1}{2}gt^2$ where, $u_y = u \sin \alpha$

$$y = (u \sin \alpha)(t) \quad ......(1)$$

We know that

$$\therefore S = u_y(t) - \frac{1}{2}gt^2$$

on simplifying, $t = \dfrac{x}{\cos\alpha}$ .....(3)

Substitute equation (3) in (1)

$$y = u\sin\alpha\left(\frac{x}{\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{\cos\alpha}\right)^2$$

$$\boxed{y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}}$$

15.3 EQUATION FOR MAXIMUM HEIGHT ATTAINED BY THE PROJECTILE

The particle is said to attain the maximum height, where it's vertical component of velocity $u.\sin\alpha$ is zero.

If h_{max} is maximum height attained, then from the equation.

$$V^2 = u^2 - 2as$$

where, V = Final velocity

u = Initial velocity

a = acceleration

S = height

In this case, V = 0, u = Initial velocity component = u sinα

$$S = h_{max} \text{ and } a = g$$

$$0 = \{(u\sin\alpha)^2 - 2gh_{max}\}$$

$$\boxed{\therefore h_{max} = \left(\frac{u^2\sin^2\alpha}{2g}\right)}$$

15.4 EQUATION FOR TIME OF FLIGHT (T)

Time of flight is the total time for which the projectile is in motion.

From fig. 15.1 it is clear that the y-co-ordinate of the projectile of any point having on the path of the projectile after a time 't' is given by.

$$y = \{(u\sin\alpha) \times t - \tfrac{1}{2}gt^2\}$$

However, when the projectile in at A, y = 0, t = T

$$0 = (u \sin \alpha)T - \tfrac{1}{2} g T^2$$

$$(u \sin \alpha)T = \tfrac{1}{2} g T^2$$

or, $$\boxed{T = \left\{ \frac{2u \sin \alpha}{g} \right\}}$$

15.5 EQUATION FOR HORIZONTAL RANGE (R) OF THE PROJECTILE

From fig. 15.1, 0A = R (Horizontal range)

But, R = (Horizontal component) × (Time taken by the particle from 0 To A)

$$R = (u \cos \alpha) \times \text{Time of flight}$$

$$R = u \cos \alpha \times \frac{2u \sin \alpha}{g}$$

$$R = \frac{2u^2 \sin \alpha \cos a}{g}$$

$$\boxed{R = \left(\frac{u^2 \sin 2\alpha}{g} \right)}$$

15.6 EQUATION FOR MAGNITUDE AND DIRECTION OF MAXIMUM RANGE

From the Horizontal range equation it is seen that for a given initial velocity u, R is maximum when sin 2α is maximum.

i.e, (sin 2α) = 1 = sin 90°

$$2\alpha = 90^\circ$$

(α = 45°) (Direction or Angle for maximum R)

and the value of R_{max} is given by

$$R_{max} = \left[\frac{u^2 \sin 2 \times 45^\circ}{g} \right]$$

$$\boxed{R_{max} = \left(\frac{u^2}{g} \right)}$$

15.7 EQUATION FOR TIME TO REACH THE MAXIMUM HEIGHT

When the particles reaches the highest or maximum point it's vertical component of velocity will be zero. If T_1 is the time taken by the projectile to reach the highest point then from the equation.

$$V = u + at$$

$$0 = u \sin \alpha - gT_1$$

or $$\boxed{T_1 = \left(\frac{u \sin \alpha}{g}\right)}$$

T_1 is half the time of flight for maximum range.

15.8 SOME DEFINITIONS

(1) Angular Displacement (θ) :The total angle traced by a particle with respect to its original position along circular path is called angular displacement (θ), measured in radians.

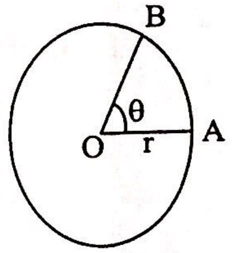

Fig. 15. 4

(2) Angle Velocity (ω) : It is defined as the rate of change of angular displacement (θ) with respect to time.

It is expressed in radians / second.

$$\omega = \frac{\theta}{t} \text{ or } \frac{d\theta}{dt}$$

The relation between linear velocity and angular velocity is.

$$v = r\omega$$

$$\omega = \frac{v}{r}$$

Linear velocity = Radius × Angular velocity.

Let 'T' be the time required for one revolution them, ($\theta = 2\pi$)

$$\therefore \boxed{\omega = \frac{2\pi}{R} = \frac{2\pi r}{v}}$$

(3) Angular acceleration (α): It is defined as the rate of change of angular velocity. It is measured in radians /sec²

$$\therefore \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d\omega}{dt}\cdot\frac{d\theta}{dt} = \omega.\frac{d\omega}{d\theta}$$

$$\therefore \boxed{\alpha = \frac{d\omega}{dt} = \omega.\frac{d\omega}{d\theta}}$$

(4) Linear acceleration: It is defined as product of radius and angular acceleration.

$$\boxed{a = r.\alpha}$$

(5) Equations of angular motion

Let, $S = \theta$, $u = \omega_0$, $v = \omega$, and $a = \alpha$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\omega t^2$$

$$\omega^2 - \omega_0^2 = 2\,\alpha\,\theta$$

(6) Relation between r.p.m (N) and ω

Angle covered by a body in one second = Angle covered in one revolution × Number of revolutions per second.

$$\omega = 2\pi \times \frac{N}{60} = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi N}{60}$$

Linear speed/velocity

$$\boxed{v = \omega r = \frac{2\pi N}{60}} \text{ (or) } \boxed{v = \frac{\pi d N}{60}}$$

15.9 PROBLEMS ON CURVILINEAR MOTION

Problem 1

A wheel is rotating about a fixed axis at 30r.p.m is uniformly accelerated for 80 seconds during which time at take 60 revolutions. Determine.

(i) Angular velocity at the end of this interval

(ii) Time required for the speed to reach 120 r.p.m.

Solution

Given, $N_1 = 30$ r.p.m, $t = 80$ seconds, $N_2 = 60$ r.p.m

(i) Angular velocity at the end of this interval

$$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi \times 30}{60} = 3.14 \text{ rad/second}$$

$$\theta = 2\pi \times N_2 = 2\pi \times 60 = 120\pi$$

$$\therefore \quad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$120\pi = (3.14 \times 80) + \tfrac{1}{2} \times \alpha \times (80)^2$$

$$\therefore \quad \boxed{\pi = 0.039 \text{ rad/sec}^2}$$

(ii) Time required for the speed to reach 120 r.p.m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(120)}{60} = 12.56 \text{ rad/sec.}$$

$$\omega = \omega_0 + \alpha t$$

$$12.56 = 3.14 + 0.039 \times t$$

$$\therefore \quad \boxed{t = 241.7 \text{ seconds}}$$

Problem 2

A pulley 300mm in diameter in wounded round by a rope with one of the ends of rope fixed to pulley and the other end is fixed to a weight freely hanging as shown in fig. 15.9.1. the weight moves down by 8m after starting from rest in 4 seconds. Find the angular velocity of the pulley. Find also the total distance moved by the weight to make the pulley to rotate 400 revolutions.

(V.T.U Model Paper 2015)(CBCS Scheme)

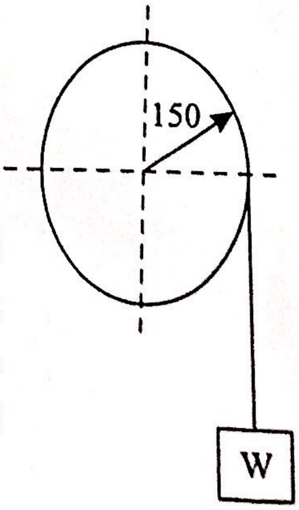

Fig. 15.9.1

Solution

Given: $\omega_0 = 0, t = 4$ seconds

$u = 0$, S = 8m = linear displacement

$S = ut + ½at^2$

$8 = 0 + ½\, a(4)^2$

∴ $\boxed{a = 1 \text{ m/sec}^2}$

Angular acceleration,

$$\alpha = \frac{a}{r} = \frac{1}{0.15} = 6.67 \text{ rad/sec}^2$$

∴ Angular velocity,

$$\omega = \omega_0 + \alpha t = 0 + (6.67 \times 4)$$

$$\boxed{\omega = 26.68 \text{ m/sec}}$$

When pulley rotates by 400 r.p.m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(400)}{60}$$

$$\boxed{\omega = 41.87 \text{ rad/sec}}$$

Time taken to attain w = 41.87 rad/sec is

$$\omega = \omega_0 + \alpha t$$

$$41.87 = 0 + (6.67 \times t)$$

∴ $\boxed{t = 6.27 \text{ secs}}$

∴ $\theta = \omega_0 t + ½at^2 = 0 + ½ \times 6.67 \times (6.27)^2$

$$\boxed{\theta = 131.10 \text{ radians}}$$

$$N = \frac{\theta}{2\pi} = \frac{131.1}{2\pi}$$

$$\boxed{N = 20.8 \simeq 21 \text{ revolutions}}$$

Problem 3

A flywheel rotates at 300 r.p.m. A uniform retardation is given for 10 seconds and the rotation reduces at 160 r.p.m.

(1) Determine the number of revolutions made by the flywheel before it comes to rest.

(2) Time taken by the flywheel before it comes to rest from 300 r.p.m.

Solution

$$\omega_0 = 300 \text{ rpm} = \frac{2\pi \times 300}{60} = 31.41 \text{ rad/sec}$$

$$\omega = 160 \text{ rpm} = \frac{2\pi \times 160}{60} = 16.75 \text{ rad/sec.}$$

$$\omega = \omega_0 + \alpha t$$

$$16.75 = 31.41 + \alpha(10)$$

$$\boxed{\alpha = -1.465 \text{ m/sec}^2}$$

Let the angular displacement be 'θ' before it comes to rest from 300 r.p.m.

$$\omega_0 = 31.41 \text{ rad/sec.}, \omega = 0,$$

$$\omega^2 - \omega^2_0 = 2\alpha\theta$$

$$0 - (31.41)^2 = 2(-1.465)\theta$$

$$\boxed{\theta = 336.71 \text{ radians}}$$

$$\therefore \qquad N = \frac{\theta}{2\pi} = \frac{336.71}{2\pi} = 53.59 \text{ r.p.m}$$

$$\boxed{N = 53.59 \text{ r.p.m}}$$

(ii) Time taken before it comes to rest 't'.

$$\omega = \omega_0 + \alpha t$$

$$0 = 31.41 - (1.465 \times t)$$

$$\boxed{t = 21.45 \text{ seconds}}$$

Problem 4

A body is projected at an angle of 30°to the Horizontal with a velocity of 100m/sec. Find the maximum height attained the Horizontal range and the time of flight, take $g = 9.8 \text{m/sec}^2$.

Solution

$$\alpha = 30°, u = 100 \text{m/sec.}$$

Maximum height attained

$$h_{max} = \left(\frac{u^2 \sin^2 \alpha}{2g}\right) = \left(\frac{100^2 \sin^2 30°}{2 \times 9.81}\right) = 127.55 \text{m}$$

Horizontal range,

$$R = \frac{2u^2 \sin\alpha \cos\alpha}{g} = \frac{u^2 \sin 2\alpha}{g}$$

$$R = \left(\frac{100^2 \times \sin(2 \times 30°)}{9.8}\right) = 883.7 \text{m}$$

Time of flight,

$$T = 2T_1 = 2\left(\frac{u \sin\alpha}{g}\right) = 2\left(\frac{100 \sin 30°}{9.8}\right)$$

$$\boxed{T = 10.20 \text{ seconds}}$$

Problem 5

A flywheel starts rotating from rest and is given an angular acceleration of 1 rad/sec². Determine the angular velocity and speed in r.p.m after 90 seconds.

If the flywheel is brought to rest with an uniform angular retardation of 0.5 rad/sec². Find the time required by the flywheel come to rest. **(V.T.U July 2015)**

Solution

$$\omega_0 = 0, \alpha = 1 \text{ rad/sec}^2$$

$$t = 90 \text{ sec}$$

$$\omega = \omega_0 + \alpha t = 0 + 1\ 5\ 90$$

$$\boxed{\omega = 90 \text{ rad/sec}}$$

Scanned with CamScanner

Also,

$$\omega = 90 \times \frac{60}{2\pi} \text{ r.p.m}$$

$$\boxed{\omega = 859.44 \text{ r.p.m}}$$

Now, for the next part.

$$\omega_0 = 90 \text{ rad/sec}, \omega = 0, \alpha = -0.5 \text{ rad/sec}^2$$

$$\omega = \omega_0 + \alpha t$$

$$0 = 90 - (0.5 \times t)$$

$$\boxed{t = 1.42 \text{ seconds}}$$

Problem 6

A Cricket ball thrown by a player from a height of 2.0m above the Horizontal ground at an angle of 30° to the Horizontal and with the velocity of 12m/sec. The ball hits the wicket at a height of 0.6m above the ground. How far is the player from the wicket?

(V.T.U Dec. 2015/Jan. 2016)

Solution

$$y_0 = -1.4\text{m}, \alpha = 30°, u = 12\text{m/sec}.$$

$$-y_0 = u \sin \alpha t - ½gt^2$$

$$-1.4 = 12 \sin 30° \times t - ½ \times 9.81 \times t^2$$

$$-1.4 = 6t - 4.905\, t^2$$

$$4.905t^2 - 6t - 1.4 = 0$$

On solving equation

$$\boxed{t = 1.42 \text{ seconds}}$$

∴ The distance of the fielder

$$R = u \cos \alpha \times t$$

$$R = 12 \cos (30°) \times 1.42$$

$$\boxed{R = 14.75m}$$

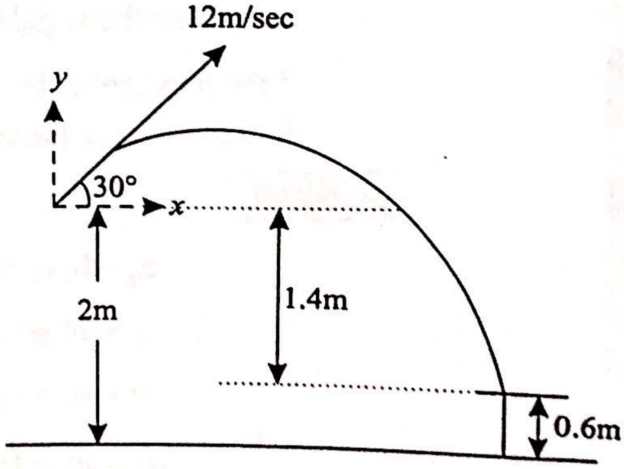

Fig. 15.9.2

Problem 7

Gravel is thrown into a bin from the top of a conveyor as shown in fig. 15.9.3 with a velocity of 5m/sec. Determine.

(1) Time it takes the gravel to hit the bottom of the bin.

(2) The Horizontal distance from the end of the conveyor to the bin where the ground strikes the bin.

(3) The velocity at which the gravel strikes the bin.

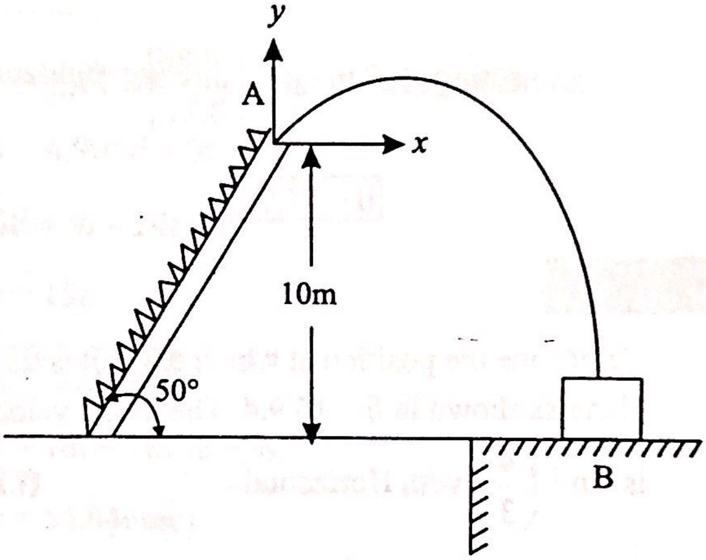

Fig. 15.9.3

Solution

$$u = 5\text{m/sec. } y_0 = -10\text{m}$$

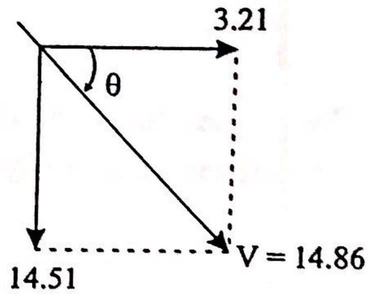

Fig. 15.9.3(a)

(i) $-y_0 = u \sin \alpha . t - \frac{1}{2}gt^2$

$$-10 = 5 \sin (50°)t - \frac{1}{2} \times 9.81 \times t^2$$

$$-10 = 3.83(t) - 4.905t^2$$

$$4.905t^2 - 3.83t - 10 = 0$$

$$\boxed{t = 1.87 \text{ seconds}}$$

(ii) $R = u \cos a.t = 5 \cos (50°) \times 1.87$

$$\boxed{R = 6.01 \text{ m}}$$

(iii) $v = \sqrt{v_H^2 + v_v^2}$

$\therefore\ v_H = u \cos\alpha = 5\cos(50°) = 3.21 \text{m/sec}$

$v_v = u \sin\alpha - gt = 5\sin(50°) - (9.81 \times 1.87)$

$\boxed{v_v = -14.51 \text{ m/sec}}$

$\therefore\ v = \sqrt{(3.21)^2 + (-14.51)^2}$

$\boxed{v = 14.86 \text{ m/sec}}$

acting at, $\theta = \tan^{-1}\left|\frac{14.86}{3.21}\right| = 77.5$

$\boxed{\theta = 77.5°}$

Problem 8

Determine the position at which the ball is thrown up the plane will strike the inclined plane as shown in fig. 15.9.4. The initial velocity is 30m/sec and angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$ with Horizontal. **(V.T.U Dec. 2014/Jan. 2015, Dec. 2015/Jan. 2016)**

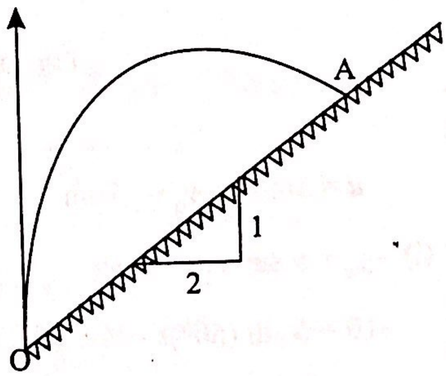

Fig. 15.9.4

Solution

By given slope, $x = 2y$

Horizontal component

$u_x = u \cos\theta$

$u_x = 30 \cos 53.13 = 18 \text{m/sec}$

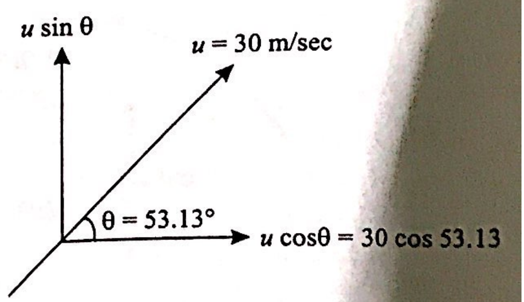

Fig. 15.9.4(a)

Vertical component,

$u_y = u \sin\theta = 30 \sin 53.13$

$u_y = 24$ m/sec.

$$y = u\sin\theta \times t - \frac{g}{2}t^2 = 24t - \frac{9.81}{2}t^2$$

$y = 24t - 4.905t^2$

$x = 2y = u \cos\theta .\ t = 18t$

$\boxed{y = 9t}$

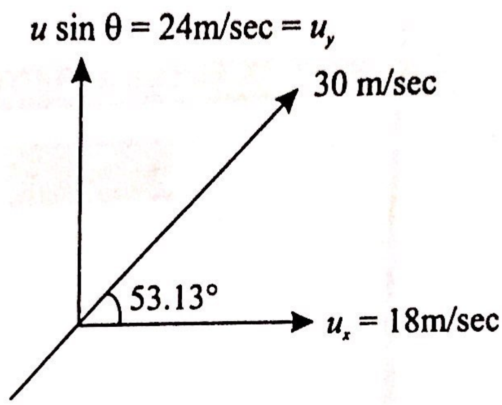

Fig. 15.9.4(b)

Substitute the value of y in above equation

$24t - 4.905t^2 = 9t$

$4.905t^2 + 9t - 24t = 0$

$4.905t^2 = 15t$

$\boxed{t = 3.058 \text{sec}}$

$\therefore \quad x = 18t = 18x\ (3.058)$

$[x = 55.046\text{sec}]$

and $\ y = 9t = 9 \times 3.058$

$\boxed{y = 27.523 \text{second}}$

Problem 9

A projectile is fired from the top of cliff 150m height with an initial velocity of 180m/sec at an angle of elevation of 30° to Horizontal. Neglecting air resistance; Determine.

(i) The greatest elevation above the cliff.

(ii) The great elevation above the ground reached by the particle.

(iii) The Horizontal distance from the gun to the point where the projectile strikes the ground. **(V.T.U June/July 2015, Model Paper 2015 CBCS scheme)**

Solution

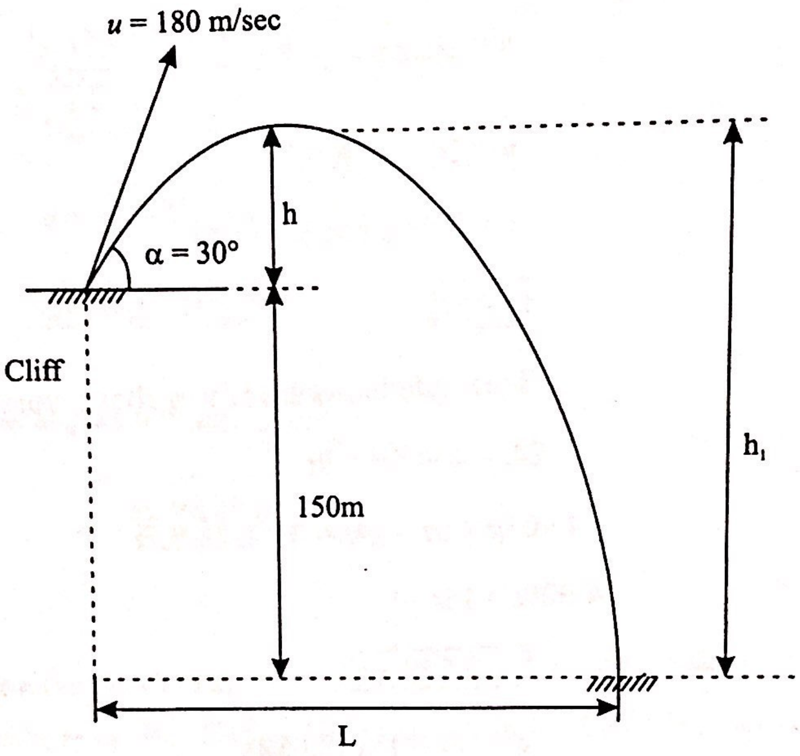

Fig. 15.9.5

(i) The greatest elevation above the cliff

$$\text{i.e., } h = \frac{u^2 \sin^2 \alpha}{2g} = \frac{(180)^2 \sin^2(30°)}{2 \times 9.81}$$

$$\boxed{h = 412.84\,\text{m}}$$

(ii) The greatest elevation above the ground reached by the particle

$$\text{i.e., } h_1 = 150 + h = 150 + 412.84$$

$$\boxed{h_1 = 562.84\,\text{m}}$$

(iii) Horizontal distance,

$$t_1 = \frac{u \sin \alpha}{g} = \frac{180 \sin 30°}{9.81} = 9.17 \text{ secs}$$

t_2 can be find as follows,

$$h_1 = 0 + \tfrac{1}{2} g t_2^2$$

$$562.84 = 0 + \tfrac{1}{2} \times 9.81 t_2^2$$

Solution

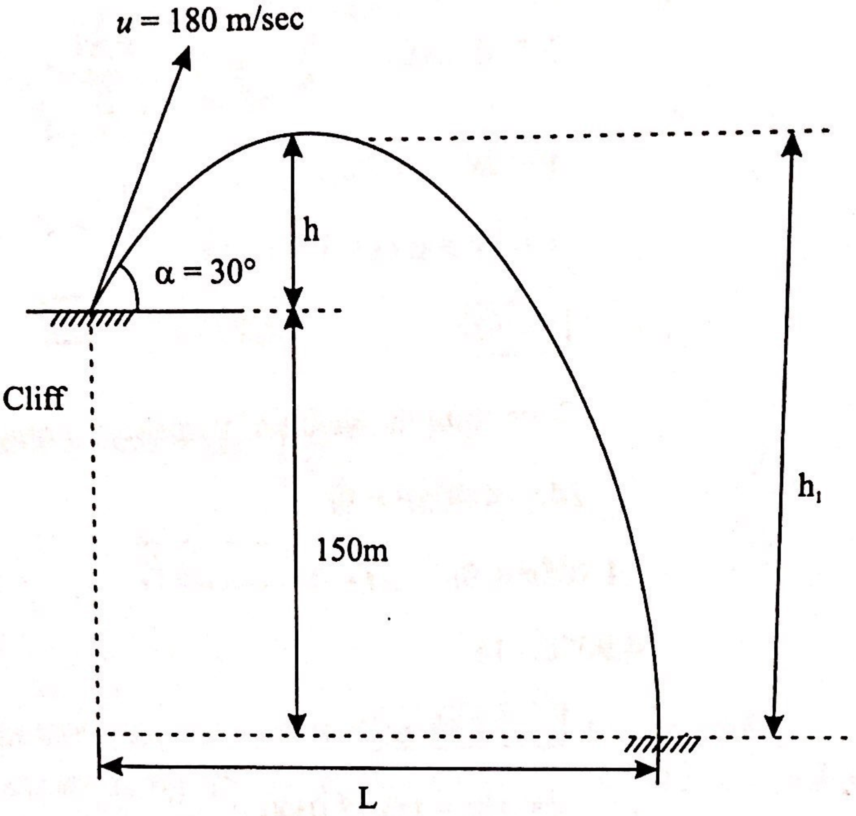

Fig. 15.9.5

(i) The greatest elevation above the cliff

$$\text{i.e., } h = \frac{u^2 \sin^2 \alpha}{2g} = \frac{(180)^2 \sin^2(30°)}{2 \times 9.81}$$

$$\boxed{h = 412.84\,\text{m}}$$

(ii) The greatest elevation above the ground reached by the particle

$$\text{i.e., } h_1 = 150 + h = 150 + 412.84$$

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(iii) Horizontal distance,

$$t_1 = \frac{u \sin \alpha}{g} = \frac{180 \sin 30°}{9.81} = 9.17 \text{ secs}$$

t_2 can be find as follows,

$$h_1 = 0 + \tfrac{1}{2} g t_2^2$$

$$562.84 = 0 + \tfrac{1}{2} \times 9.81 t_2^2$$

$$\boxed{t_2 = 10.71\,\text{sec}}$$

$$t = t_1 + t_2$$

$$t = 9.17 + 10.71$$

$$\boxed{t = 19.88\,\text{sec}}$$

$$R = u \cos \alpha \times t$$

$$R = 180 \cos 30° \times 19.88$$

$$R = 30.99\text{m} = 3.099 \text{ km}$$

$$\boxed{R = 3.099\text{km}}$$

Problem 10

A person can throw a ball at a maximum velocity of 30m/sec. If he wants to get maximum range on the plane inclined at 20° to horizontal, at what angle should the ball be projected and what would be the maximum range.

(i) Up the plane (ii) Down the plane.

Solution

For a maximum range

$\frac{\theta_1}{2}$ [The direction of projection must bisect the θ_1]

$$\theta = \frac{90 - 20}{2} = 35°$$

$$\theta_1 = 90 - 20° = 70°$$

$$\therefore \quad \theta = 35° \qquad \therefore\ \alpha = 20° + 35° = 55°$$

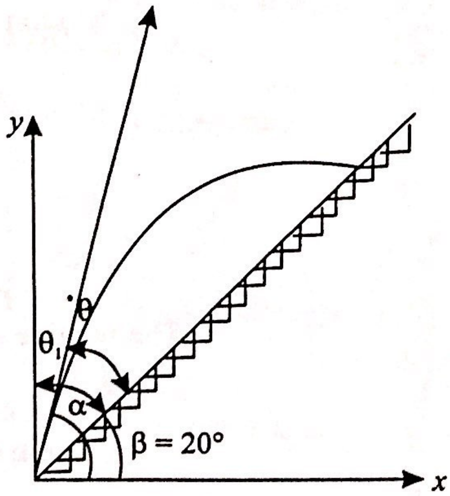
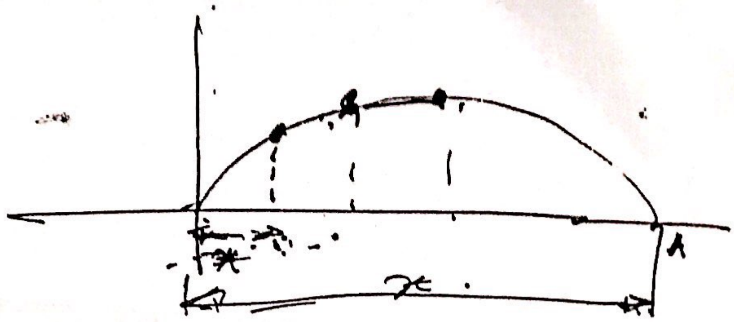


Fig. 15.9.6 (a)

$$R_{max} = \frac{2u^2 \cos\alpha}{g\cos^2\beta} \times \sin(\alpha - \beta)$$

$$R_{max} = \frac{2 \times (30)^2 \times \cos(55°)}{9.81 \times \cos^2(20°)} \times \sin(55° - 22°)$$

$$\boxed{R_{max} = 68.36\text{m}}$$



(ii) Down the plane

$$\therefore \quad \frac{\theta_1}{2} = \theta = \frac{90+20}{2} = 55°$$

$$\alpha = 55° - 20° = 35°$$

$$R = \frac{2u^2 \cos\alpha}{g\cos^2\beta} \times \sin(\alpha - \beta)$$

$$R = \frac{2(30)^2 \times \cos 35°}{9.81 \times \cos^2(20°)} \times \sin 55°$$

$$\boxed{R = 139.43 \text{ m}}$$

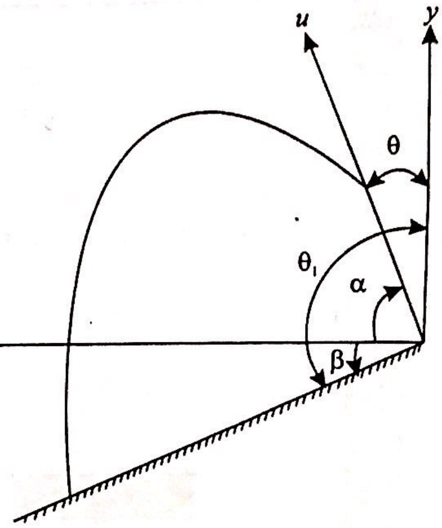


Fig. 15.9.6 (b)

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15.10 RELATIVE MOTION

The motion of an object as observed or felt by an observer in motion is known as relative motion and the velocity of the object with respect to the observer is known as relative velocity.

The concept of relative motion can be explained as follows.

1. Motion on parallel paths in like directions.

A ——→ - - - - - - - - - - - 60 kmph   70 kmph

- - - - - - - - - - - - Difference = (0) - - - - - - - - - - - - (10 kmph)

B ——→ - - - - - - - - - - - 60 kmph   60 kmph

The relative velocity of 'A' with respect to B is the vector difference between the velocities A and B.

2. Motion on parallel path in opposite direction.

A = 70 kmph

- - - - - - - → - - - - - - - - - - -

70 – (–60) = 130 kmph

- - - - - - - - ← - - - - - - - - - - -

B = 60 kmph

→ x