CHAPTER - 2

ELECTROMAGNETISM

2.1 Introduction:

Any material, which has both *attractive* and *directive* properties is called a magnet. When a magnet is freely suspended in air, it always points in the North–South direction. A magnet always attracts iron filings. In nature, natural magnet is found in the form of an iron ore, which is an oxide of iron Fe_3O_4 (ferric oxide) known as *magnetite*. Natural magnets are very weak magnets and hence does not have many practical applications.

Artificial magnets, also known as electromagnets of any strength may be built which have many practical applications. Iron, Cobalt, Nickel and their alloys such as Silicon Steel are magnetic materials. When bars of such magnetic materials are wound with a coil and current is passed though them, they become electromagnets.

The strength of such an electromagnet depends on the number of turns in the coil and the magnitude of the current passing through it. Hence, by suitably increasing the number of turns of the coil and the current passing through it, an electromagnet of any strength can be built.

2.2 Important Definitions:

i) **Magnetic Field**: The region or space around a magnet in which the magnetic effects are felt is known as the *magnetic field*.

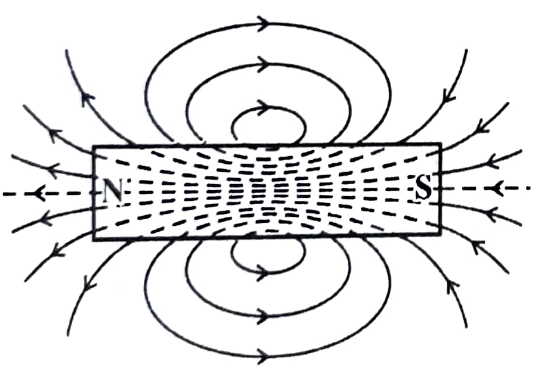

Fig. 2.1 Magnetic field

Fig. 2.1 represents a bar magnet and the magnetic field around it. The magnetic field is represented by magnetic lines of force, which start from the north pole and go into the south pole, completing their path in the surrounding medium and the material of the magnet. Hence, magnetic lines of force are always closed lines.

The magnetic lines of force are purely imaginary lines. They do not intersect each other. They are like elastic bands which always try to shorten themselves.

ii) **Magnetic Flux (ϕ)**: The entire magnetic lines of force representing a magnetic field is known as the *magnetic flux*. Its unit is *weber*, abbreviated as Wb, named after Wilhelm Eduard Weber (1804-91), a German Physicist.

iii) **Magnetic Flux Density (B)**: The magnetic flux per unit area, the area being normal to the lines of flux is known as the *flux density*. The unit is *weber per square metre* (Wb/m^2) or Tesla (T).

$$B = \frac{\phi}{a} \text{ Wb/m}^2 \text{ or T} \tag{2.1}$$

iv) Magnetomotive Force or M.M.F.: M.M.F. is defined as the magnetic force, which creates magnetic flux is a magnetic material. The unit is *ampere turns* (AT)

$$\text{M.M.F.} = N\,I, \tag{2.2}$$

Where, N = Number of turns in the coil

I = Current through the coil

Another equation for M.M.F. is,

$$\text{M.M.F.} = \text{Flux} \times \text{Reluctance} = \phi \times \mathcal{R} \tag{2.3}$$

v) Reluctance ($\mathcal{R}$): *Reluctance* is the property of a magnetic material by virtue of which, it opposes the creation of magnetic flux in it. The unit is *ampere turns per weber* (AT/Wb). The reluctance of a magnetic material is directly proportional to the length of the magnetic material and inversely proportional to its area of cross section.

$$\mathcal{R} \propto \frac{\ell}{a} = \frac{1}{\mu}\frac{\ell}{a} = \frac{\ell}{\mu_0 \mu_r a} \tag{2.4}$$

Where, ℓ = length of the magnetic material

a = area of cross section

μ = a constant known as the absolute permeability of the magnetic material = $\mu_0\,\mu_r$

Where, μ_0 = permeability of free space or air = $4\pi \times 10^{-7}$ henry per metre (H/m)

μ_r = relative permeability of the magnetic material.

vi) Permeability: *Permeability* is basically the property of the magnetic material by virtue of which the magnetic flux can be easily created in it. For any magnetic material, there are two permeabilities (i) absolute permeability and (ii) relative permeability.

(i) **Absolute Permeability** (μ): The *absolute permeability* of a magnetic material is defined as the flux induced in the magnetic material per unit magnetising force.

$$\mu = \frac{B}{H}, \quad \text{Where, } H = \text{magnetising force} \tag{2.5}$$

(ii) **Relative Permeability** (μ_r): For defining the relative permeability of a magnetic material, the permeability of free space or air is taken as reference. Hence, the relative permeability of free space or air is taken as unity.

$\mu_r = 1$, for free space or air

I law: *Whenever a magnetic flux linking an electric circuit changes, an e.m.f. is induced in the electric circuit.*

II law: *The magnitude of the induced e.m.f. is equal to the rate of change of f' x linkages.*

The direction of the induced e.m.f. was given by Heinrich Friedrich Emil Lenz (1804–1865) a Russian geologist and physicist.

Lenz's law: *The direction of the induced e.m.f. is such as to oppose the very cause of it.*

All the above three laws can be represented by the following equation.

$$e = -N \frac{d\phi}{dt} \tag{1.9}$$

Where, e = induced e.m.f. in volts.

N = Number of turns in the coil.

$\frac{d\phi}{dt}$ = rate of change of flux

The –ve sign indicates that the induced e.m.f. opposes the very cause of it i.e., the applied voltage.

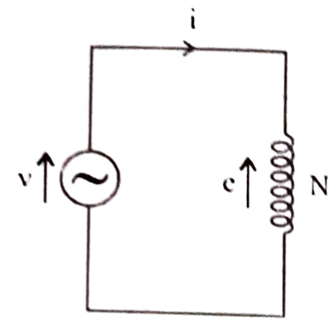

Fig. 2.7

Explanation: Consider a coil of N turns as shown in Fig. 2.7, to which an alternating voltage v is applied, due to which an alternating current i flows through the coil. This alternating current, produces an alternating flux ϕ, which links the coil. Hence, an e.m.f. is induced in the coil, which is given by the equation.

$$e = -N \frac{d\phi}{dt}$$

This induced e.m.f. opposes its own cause. The cause of the induced e.m.f. is the changing flux, which is due to the changing current, which in turn is due to the alternating voltage applied. Hence, we say that the induced e.m.f. opposes the applied voltage, which is the very cause of it.

2.5 Changing of Flux:

The change of flux linking an electric circuit can take place in two ways.

i) When a conductor cuts across a magnetic field of constant flux density, the flux changes and an e.m.f. is induced in the conductor. This type of e.m.f. induced is known as *dynamically induced e.m.f.*

ii) When the electric circuit is in the form of a coil, which is stationary and when an alternating current is passed through it, an alternating flux is produced, which links the coil. Hence, an e.m.f. is induced in the stationary coil. This type of e.m.f. induced is known as *statically induced e.m.f*

2.6 Dynamically Induced E.M.F.:

Consider a magnetic field of constant flux density B Wb/m^2, which is represented by the magnetic lines of flux as shown in fig. 2.8. Let a conductor of length ℓ and area of cross section **a** be placed perpendicular to the lines of flux.

When the conductor moves with a velocity v in the direction I, it moves parallel to the lines of flux and hence does not cut any flux. Hence, the e.m.f. induced in the conductor is zero.

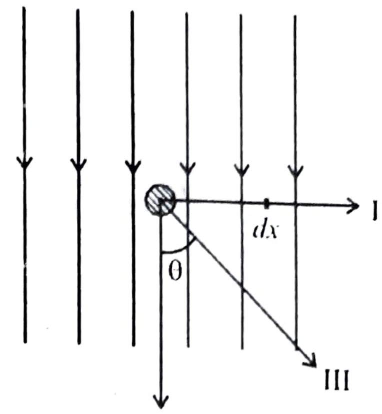

Fig. 2.8

When the conductor moves with a velocity v in direction II, it moves perpendicular to the lines of flux and cuts maximum flux. Hence, the e.m.f. induced in the conductor is maximum. To find an expression for this maximum induced e.m.f., let the conductor move through a small distance dx in dt seconds. Then the flux cut by the conductor is given by

$$d\phi = B \times \ell\, dx$$

The rate at which the flux is cut, is given by,

$$\frac{d\phi}{dt} = B \times \ell \frac{dx}{dt} = B\, \ell\, v$$

According to Faraday's laws of electromagnetic induction, the above equation is nothing but the e.m.f. induced in the conductor, which is maximum

$$\therefore \quad e = B\, \ell\, v = E_m \text{ volts} \qquad (2.10)$$

When the conductor moves in direction III, making an angle θ, with the direction of the lines of flux with a velocity v, the component of velocity perpendicular to the direction of flux is $v \sin\theta$, as shown in Fig. 2.9. Hence, The e.m.f. induced in the conductor is $B\, \ell\, v \sin\theta$

$$\therefore \quad e = B\, \ell\, v \sin\theta = E_m \sin\theta \qquad (2.11)$$

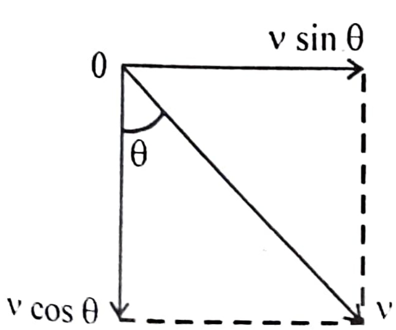

Fig. 2.9

The component of velocity $v \cos\theta$, which is in the direction of the lines of flux, does not contribute anything for the e.m.f. induced. The direction of the dynamically induced e.m.f. is given by Fleming's Right Hand Rule.

2.7 Fleming's Right Hand Rule:

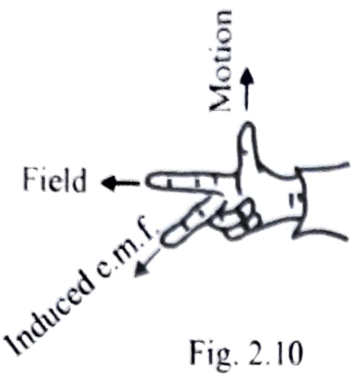

Fig. 2.10

This rule gives the direction of the induced e.m.f.. When the thumb, the fore finger and the middle finger of the right hand are held mutually perpendicular to each other, the thumb in the direction of the motion of the conductor, the fore finger in the direction of the magnetic field, then the direction of the middle finger gives the direction of the induced e.m.f. This is best illustrated from the Fig. 2.10.

2.8 Statically Induced E.M.F.:

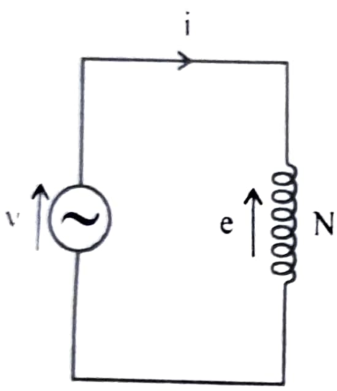

Fig. 2.11

Consider a coil of N turns which is connected to an alternating voltage v due to which, an alternating current i flows through the coil. This alternating current, produces an alternating flux ϕ, which links the coil. Hence, an e.m.f. e is induced in the coil. This e.m.f. induced is known as *statically induced e.m.f.* and is given by,

$$e = -N\frac{d\phi}{dt} = -N\frac{d\phi}{dt}\times\frac{di}{di} = -L\frac{di}{dt} \qquad (2.12)$$

$$\text{Where,}\quad L = N\frac{d\phi}{di} = N\frac{\phi}{I} \qquad (2.13)$$

L is a constant known as the *self inductance* of the coil.

$\frac{d\phi}{di} = \frac{\phi}{I}$ is a constant because, $\phi \propto i$

2.9 Self Inductance (L):

The self inductance of a coil is its property by virtue of which, it always opposes any change in the value of the current flowing through it.

The self inductance of a coil may also be defined as its property by virtue of which, an e.m.f. is induced in it, whenever an alternating current flows through it.

The self inductance of a coil is also defined as the number of weber turns produced per ampere in the coil as per the equation 2.13.

$$L = \frac{N\phi}{I} = \frac{N\,N\,I}{I\,\mathcal{R}} = \frac{N^2}{\ell/\mu_0\mu_r a} = \frac{\mu_0\,\mu_r\,a\,N^2}{\ell} \qquad (2.14)$$

This work done is stored in the coil in the form of an electromagnetic field. Hence, the energy stored in a coil of inductance L henrys in the form of an electromagnetic field is given by $\frac{1}{2}LI^2$.

2.11 Mutual Inductance (M):

Two coils, which are placed close to each other are said to be mutually coupled, when a part of the alternating flux produced in one coil links the other coil. As the flux is of alternating type, e.m.f. is induced in both the coils. The e.m.f. induced in the first coil, where the flux is produced, is called as *self induced e.m.f* and the e.m.f. induced in the second coil, which links a part of the flux produced in the first coil, is known as *mutually induced e.m.f.*

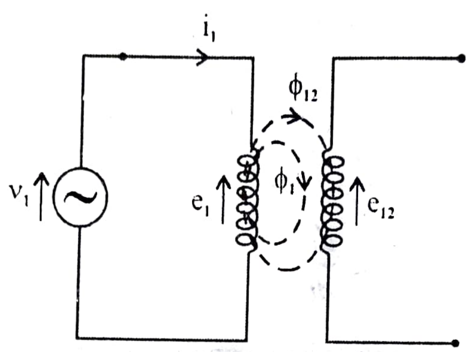

Fig. 2.13

Consider two coils of turns N_1 and N_2, which are placed very close to each other, as shown in Fig. 2.13. When an alternating voltage v_1 is applied to the first coil, an alternating current i_1 flows through it, producing an alternating flux ϕ_1. This flux ϕ_1, links coil 1 and hence an e.m.f. e_1 is induced in it, which is given by

$$e_1 = -N_1 \frac{d\phi_1}{dt} \tag{2.17}$$

This is known as the self induced e.m.f. in coil 1.

A part of the flux ϕ_1, produced in coil 1, links the coil 2 also. This flux ϕ_{12} which links both coil 1 and coil 2, is called as the *mutual flux* between the two coils. The flux ϕ_{11} links only coil 1. Hence the flux ϕ_1 is the sum of the two fluxes ϕ_{11} and ϕ_{12}.

$$\phi_1 = \phi_{11} + \phi_{12} \tag{2.18}$$

The mutual flux ϕ_{12} linking coil 2, induces an e.m.f. e_{12} in that coil. This e.m.f. is known as the mutually induced e.m.f. and is given by

$$e_{12} = -N_2 \frac{d\phi_{12}}{dt} \tag{2.19}$$

The equation for e_{12} may also be written as

$$e_{12} = -M_{12}\frac{di_1}{dt} \quad (2.20)$$

= e.m.f. induced in coil 2, due to the current flowing in coil 1.

M_{12} is known as the *mutual inductance* between coil 1 and coil 2. The equation for the mutual inductance M_{12} may be written as

$$M_{12} = N_2\frac{d\phi_{12}}{di_1} \quad (2.21)$$

Similar equations can be written, when coil 2 is energised by an alternating current i_2, producing a total flux ϕ_2 in it, as shown in Fig. 2.14.

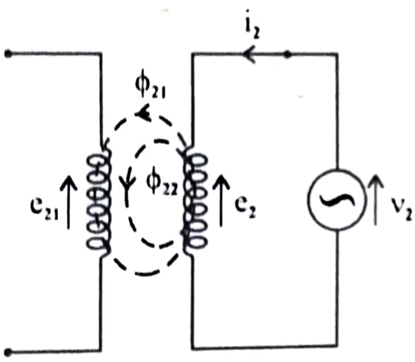

Fig. 2.14

$$\phi_2 = \phi_{22} + \phi_{21} \quad (2.22)$$

Where, ϕ_2 = total flux produced in coil 2.

ϕ_{22} = flux that links only coil.2

ϕ_{21} = flux that links both coil 2 and coil 1.

The self induced e.m.f. in coil 2 is given, by

$$e_2 = -N_2\frac{d\phi_2}{dt} \quad (2.23)$$

The mutually induced e.m.f. in coil 1 is given by

$$e_{21} = -N_1\frac{d\phi_{21}}{dt} = -M_{21}\frac{di_2}{dt} \quad (2.24)$$

Where, $$M_{21} = N_1\frac{d\phi_{21}}{di_2} \quad (2.25)$$

M_{21} is the mutual inductance between coil 2 and coil 1.

As the coupling between the two coils is bilateral, which means that, the coupled circuit has the same characteristics in both directions,

$$M_{12} = M_{21} = M \quad (2.26)$$

Hence, the mutual inductance between any two coils, which are placed close to each other, may be defined as the ability of one coil to induce an e.m.f in the other coil, when an alternating current flows through one of the coils.

$$M = N_2\frac{d\phi_{12}}{di_1} = N_1\frac{d\phi_{21}}{di_2} \quad (2.27)$$

2.12 Co-efficient of Coupling (K) :

The Co-efficient of coupling is the ratio of the mutual flux to the total flux.

$$\therefore \quad K_{12} = \frac{\phi_{12}}{\phi_1} \quad \text{and} \quad K_{21} = \frac{\phi_{21}}{\phi_2} \tag{2.28}$$

As the coupling is bilateral $K_{12} = K_{21} = K$ (2.29)

$$\therefore \quad \phi_{12} = K\,\phi_1 \quad \text{and} \quad \phi_{21} = K\,\phi_2 \tag{2.30}$$

From equation (2.26) $M^2 = M_{12} \, . \, M_{21} = N_2 \dfrac{d\phi_{12}}{di_1} \times N_1 \dfrac{d\phi_{21}}{di_2}$

$$= N_1 N_2 \frac{d(K\phi_1)}{di_1} \times \frac{d(K\phi_2)}{di_2} = K^2 N_1 \frac{d\phi_1}{di_1} \times N_2 \frac{d\phi_2}{di_2}$$

$$= K^2 L_1 L_2$$

$$\therefore \quad K = \frac{M}{\sqrt{L_1 L_2}} \tag{2.31}$$