Unit - 2

INTRODUCTION TO ENGINEERING MECHANICS.

Mechanics

Statistics. (at rest) Dynamics. (at in motion).

Mechanics: is Branch of science that deals with study of bodies that do not undergo any deformation under the the applica^n of forces.

It is classified into

(i) statistics. (ii) Dynamics.

Statics: Branch of mechanics which deals with study of body in state of rest.

Dynamics: Branch of mechanics which deal with study of body in state of motion.

newton's laws of motion:

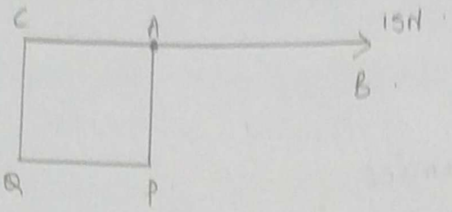
1st law: Every body continuous to be in state of rest or uniform motion along the straight line until and unless an external force is applied on it

2nd law: the rate of change of momentum is directly proportional to the impressed force and takes place in the direc^n of the force acting on it.

3rd law: For every action, there is an equal & opp. reac^n

Elements/ characteristics of forces

1) Magnitude
2) Direction
3) line of action
4) Point of application (From which point force starts)



1) magnitude: The length of the vector represents magnitude of force. considering the sketch magnitude of force is 15N.

2) Direction: The direcⁿ of the force is that which is represented by the direcn of the arrow Read. ~~Direcⁿ from right~~. In the fig. towards RHS or from A to B.

3) line of action: It is the line along which the force acts. In the fig. line of action is $\overrightarrow{CAB}$

4) Point of applicaⁿ: It is the initial point at which the force acts.

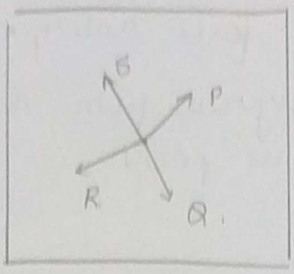
Force Systems:

1) coplanar
2) Non-coplanar.
3) collinear
4) non collinear.

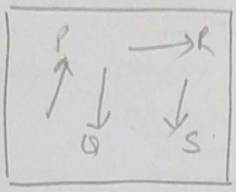
1) coplanar force system: the forces which are acting on the same plane is called as coplanar force system.

There are 3 types:-

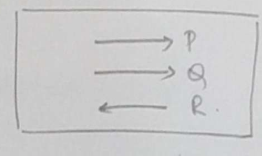
1) coplanar concurrent :- The forces which are acting on same plane and originating from same point is called as coplanar concurrent force sys



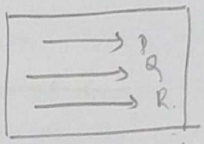
2) coplanar Non-Concurrent :- forces which same plane but not originating from the same point is called as coplanar non concurrent force sys.



3) coplanar parallel : Forces on the same plane and parallel to each other.



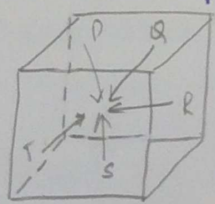
coplanar unlike parallel force sys



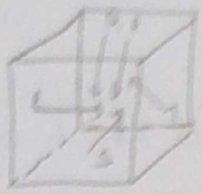
coplanar like parallel force sys.

4) non coplanar force system :- The forces which are acting on diff. planes is called as non-coplanar forces.

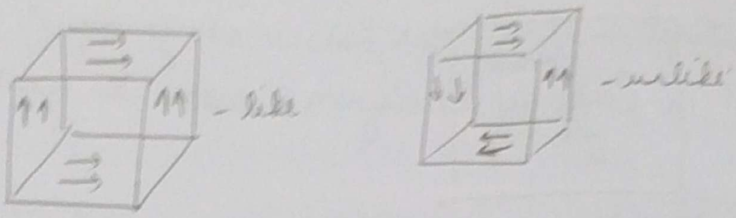
i) non-coplanar concurrent : the force which are acting ~~or coinciding~~ on diff. planes and originating from the same ~~plant~~ point or coinciding to the same point



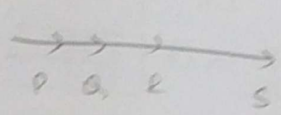
4) Non coplanar non concurrent: the force acting on diff. planes and which do not originate from the same point or coincide to the same point.



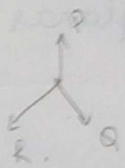
5) non-coplanar parallel: the forces acting on diff plane and which are parallel to each other.



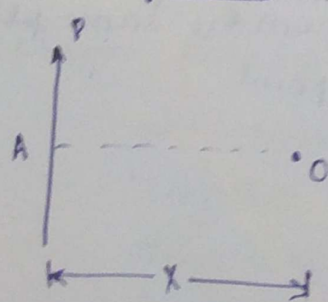
6) Collinear:- Two or more forces which is having same line of action.



7) Non-collinear force system: The force system which do not have common line of action.



Moment of Force:



$M = PX$

2 types of moments :-

1) clockwise (+ve)

2) Anticlockwise (-ve).

Moment of force : is defined as the rotational effect of the force about the point. the rotational effect may be clockwise or anticlockwise in direcn. In other words moment is defined the product of magnitude of force and ⊥r distance of the line of action from the point.

In figure, moment about point O is Px, which P is magnitude of force and x is ⊥r distance.

Sign conventions:

1) Clockwise direcn (+)
2) Anti " " (-ve)

S.I. unit of moment of force is Nm

Couple of force :

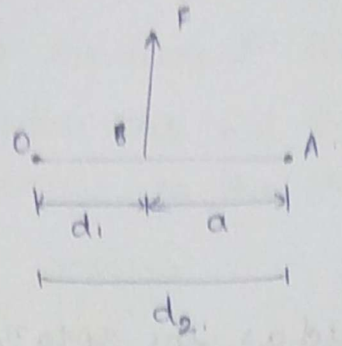
Two equal and opp. parallel forces act on a body separated by a finite distance apart constitutes a couple.

IMP
* * *

Characteristics of couple

1) The algebraic sum of forces constituting a couple = 0.

2) " " " " moments of force constituting the couple about any point is equal to moment of couple itself.



$+F(d_2) - F(d_1)$

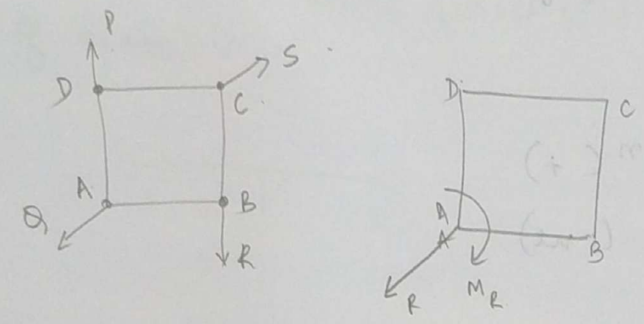
$= F(d_2 - d_1)$

$= F(a)$

1) A couple can be balanced only by an opp. couple in a same plane

2) Any no. of coplanar couple can be reduced to single couple whose mag. will be equal to algebraic sum of moments of all the couple.

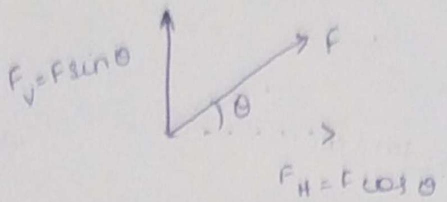
3) The moment of couple is constant for any point in the plane ~~of~~.



For a , coplanar force system an equivalent system is defined as the, the combinaⁿ of force, passing through ~~a~~ the given point & moment about the same point

In the above fig R is the resultant of forces PQRS and M_R is the resultant moment of the above menⁿed individual forces. A is the point of reference.

Resolution of forces: the method to resolve or to split a single force into the ⊥r direc^ns is called as resolution of forces



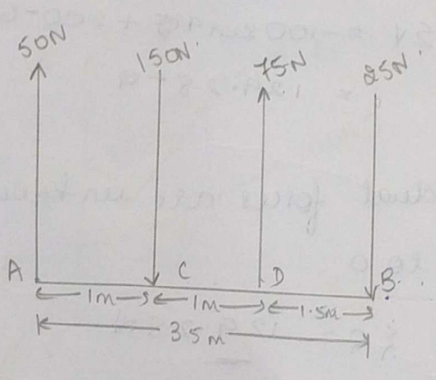
Composi^n of forces (Resultant force): It is a process of finding out the single resultant which produces same affect as that produced by no. of forces.

Problems on moment of forces and couple:

1) A system of forces are acting on a rigid bar as shown in the fig. find (i) single force

(ii) single force & couple at A.

(iii) " " & " " B.



↑ force → +ve

↓ force → −ve

↻ moment → +ve

↺ moment → −ve

(i) Single force

$R = +50 - 150 + 75 - 25$

$=$ ~~50~~ 50N in downward direc^n ($\because -50$)

(ii) Single force & couple at A:

$M_A = +25 \times 3.5 - 75 \times 2 + 150 \times 1 + 50 \times 0$

$= 87.5$ Nm (↻)

$M_A = Rx \Rightarrow x = M_A/R = 87.5/50 = 1.75$m

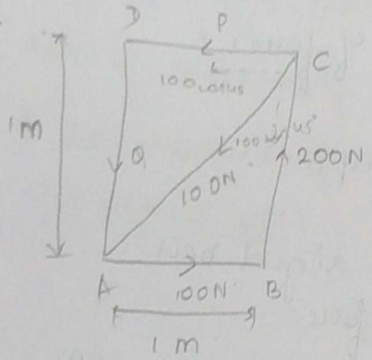
(iii) $M_B = 50 \times 3.5 - 150 \times 2.5 + 75(1.5)$

$= 87.5$ (A) N-m

$M_A = Rx$

$x = \frac{87.5}{50} = 1.75$ m

2) A square ABCD of side 1m × 1m acting along its sides as shown. Find the values of P



~~$R = 100\cos 45°$~~

$\Sigma H = -P - 100\cos 45 + 100$

$= -P - \frac{100}{\sqrt{2}}$

$\Sigma H = -P + 29.29$

$\Sigma V = -100\sin 45 + 200 - Q$

$= 129.28 - Q$

when individual forces are unknown equate ΣH & ΣV to 0.

$\therefore P = 29.29$N & $Q = 129.28$N

$\therefore R = 0$

$\Sigma M_A = Q(0) + 100(0) - 200(1) - P(1) - \cancel{100\cos 45(1)}$

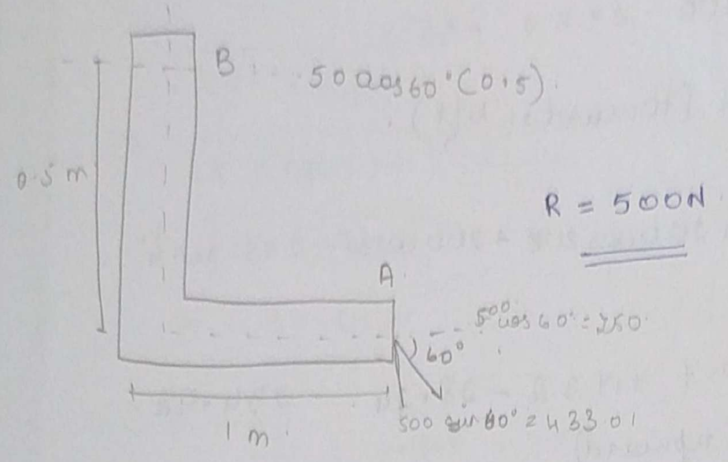
$+ \cancel{100\sin 45}$

$= -29.29 - 200$

$= 229.29$N-m in anticlockwise dirⁿ.

Scanned by CamScanner

3) If 500N force is applied to a point A which is of L shape find the equivalent system of force & moment abt the point B.

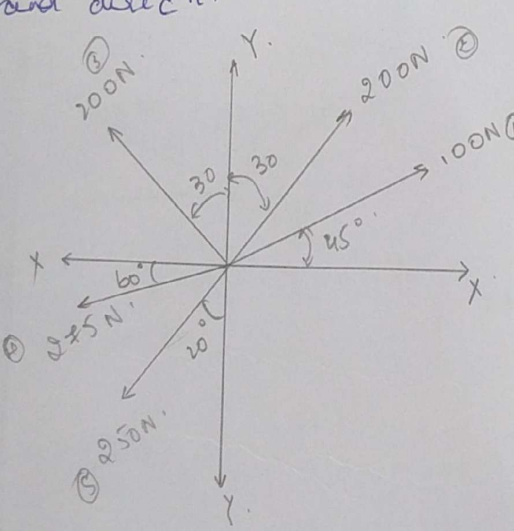


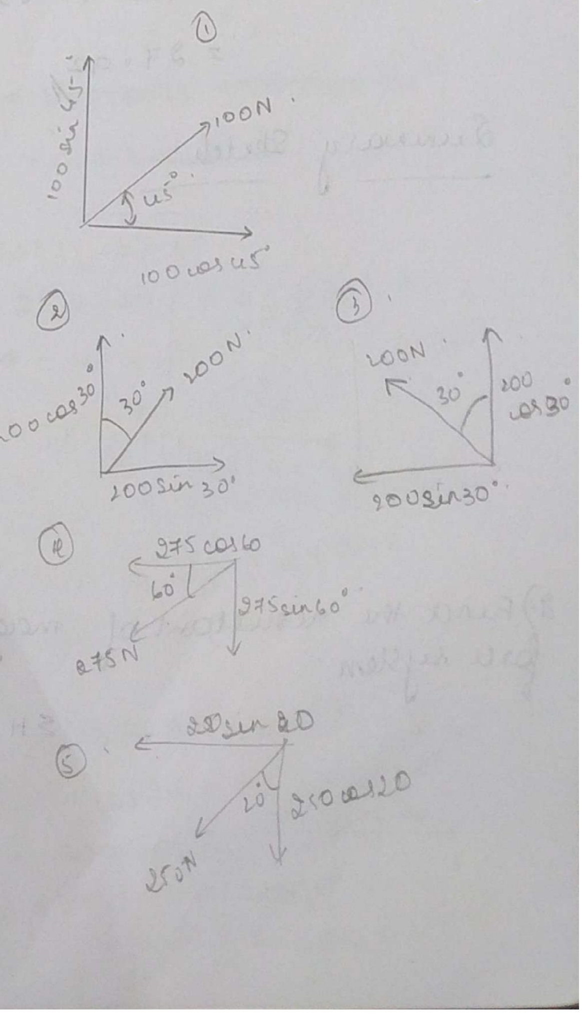
$M_B = -500\cos 60°(0.5) + 500\sin 60°(1)$

$= 308.01$ N-m

Coplanar concurrent force system :-

1) 5 coplanar forces are acting at a point as shown in the figure. determine the resultant of forces in both magnitude and direc^n.





$\Sigma H = +100\cos 45 + 200\sin 30 - 200\sin 30^\circ - 275\cos 6^\circ$
$- 250\sin 20^\circ$

$= 70.7 + 100 - 100 - 273.4 - 85.5$

$= -288.2N$ [towards. left).

$\Sigma V = +100\sin 45 + 200\cos 30^\circ + 200\cos 30^\circ - 275\sin 6^\circ$
$- 250\cos 20$
$= 173.2 + 70.7 + 173.2 - 28.74. - 234.92.$
$= 153.45N$ (upward).

$R = \sqrt{\Sigma H^2 + \Sigma V^2.}$

$= 326.5N.$

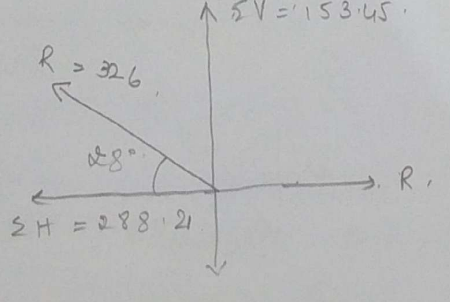
$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{153.45}{288.2.} =$

$= 0.53.$

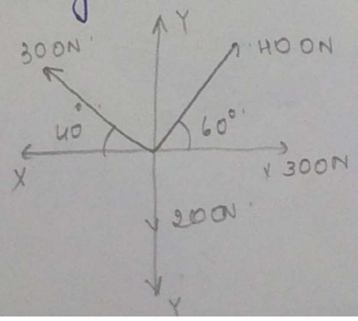
$\theta = \tan^{-1}(0.53).$

$= 27.92$

Summary Sketch:



2) Find the resultant of mag. of given coplanar concurrent force system.



$\Sigma H = 300 + 400\cos 60^\circ$
$- 300\cos 40^\circ.$
$= 300 + 200 - 229.8$
$= 270.186\ N\ (\rightarrow)$

$\Sigma V = 400 \sin 60° + 300 \sin 40 - 200$

$= 192.8 + 346.4 - 200$

$= 339.2N\ (\uparrow)$

$R = \sqrt{\Sigma V^2 + \Sigma H^2}$

$= \sqrt{73000 + 115056.64}$

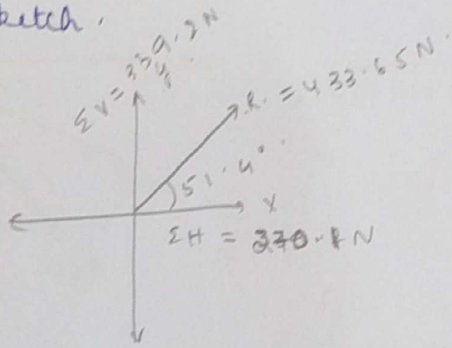
$= \sqrt{188056.64}$

$= 433.65N$

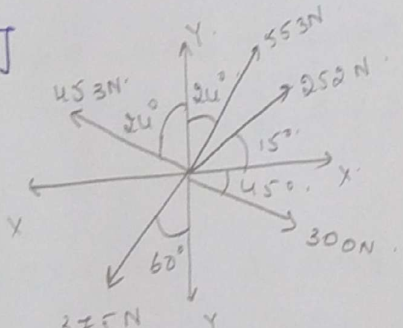
$\tan\theta = \dfrac{\Sigma V}{\Sigma H} = \cancel{1.25}\ 1.255$

$\theta = 51.4°$

Summary Sketch.



3.]



$\cancel{\Sigma H = 252 \cos 15 + 553 \cos 24°}$

$\cancel{- 453 \cos 2}$

$\Sigma H = 252 \cos 15 + 553 \sin 24°$

$- 453 \sin 24 - 375 \sin 60°$

$+ 300 \cos 45°$

$= 243.4 + 224.92 - 184.25$

$- 324.75 + 212.13$

$= 171.45N\ (\rightarrow)$

$\Sigma V = 252 \sin 15 + 553 \cos 24°$

$+ 453 \cos 24 - 375 \cos 60°$

$- 300 \sin 45°$

$= 65.2 + 505.1 - 212.1 - 187.5$

$+ 413.8 = 584.5N\ (\uparrow)$

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

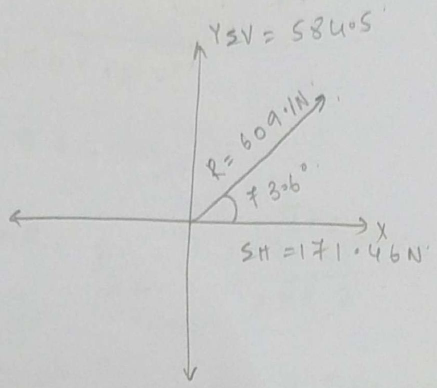
$= \sqrt{29\,3951 + 341640.25}$

$= \underline{609.1N}$

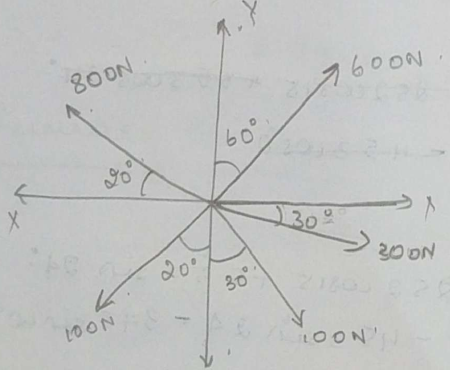
$\tan\theta = \dfrac{\Sigma V}{\Sigma H} = \dfrac{584.5}{171.45} = 3.40$

$\theta = \underline{73.6^\circ}$

Summary. Sketch



4.



$\Sigma H = \cancel{300 + 300}\ 600 \sin 60^\circ - 800 \cos 20^\circ$
$\quad - 100 \sin 20 + 100 \sin 30 + 300 \cos 30^\circ$

$= \underline{43.4N} \ (\longrightarrow)$

$\Sigma V = 600 \cos 60^\circ + 800 \sin 20^\circ - 100 \cos 20 - 100 \cos 30^\circ$
$\quad - 300 \sin 30^\circ$

$= 273.6 + \overset{300}{\cancel{519.6}} - 150 - 86.6 - 93.96$

$= \cancel{462.64}\ 243.04.$

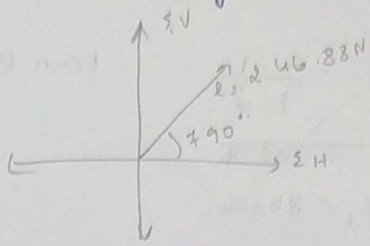
$\cancel{R} = \sqrt{18835.6 + 213998.76}$

$= \cancel{\sqrt{215882.37}}$

$R = \sqrt{1883.56 + 59068.4}$

$= 246.88N$

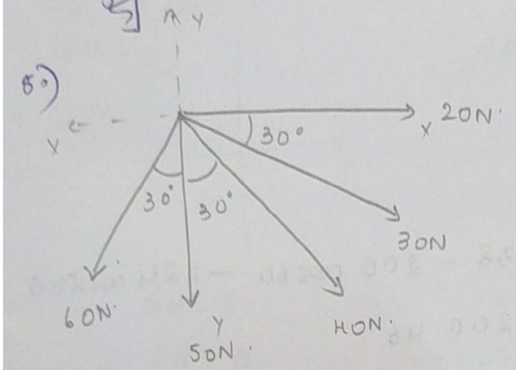
summary sketch



$\tan\theta = 5.6$

$\theta = 79^\circ$

5.)



$\Sigma H = 20\cancel{N} + 30\cos 30 + 40\sin 30 - 60\sin 30^\circ$

$= 20 + 25.9 + 20 - 30 = 35.9\ N(\rightarrow)$

$\Sigma V = -50 - 30\sin 30^\circ - 40\sin 30 - 60\cos 30.$

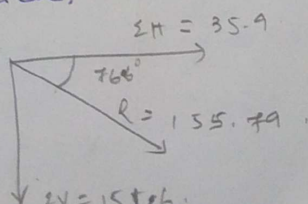
$= -50 - 15 - 34.64 - 51.96, = -151.6.$

$= \cancel{151}\ 151.6\ N(\downarrow).$

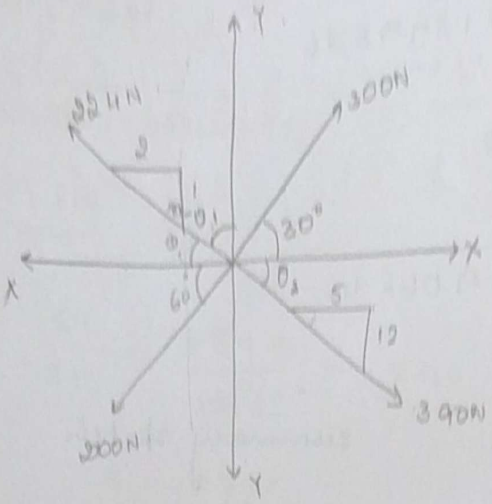
$R = \sqrt{\ } = 155.79\ N.$

$\theta = \tan^{-1}\left(\frac{151.6}{35.9}\right) = 76.67.$

Summary Sketch:



60



$\tan\theta_1 = \cancel{\frac{2}{1}}\ \frac{1}{2}$

$\cancel{\theta_1 = 63.4}$

$\theta_1 = 26.56$

$\tan\theta_2 = \frac{12}{5}$

$= 67.38$

$\tan\theta_1 = 1/2$

$\theta_1 = 26.5°$

$\Sigma H = 300\cos 30 + 390\cos 67.38 - 200\cos 60 - 224\cos$

$= 259.8 + 150 - 100 - 200.46$

$= 109.34\ N\ (\rightarrow)$

$\Sigma V = 224\sin 26.5 + 300\sin 30 - 200\sin 60 - 390\sin 6$

$= 99.94 + 150 - 173.2 - 360.7$

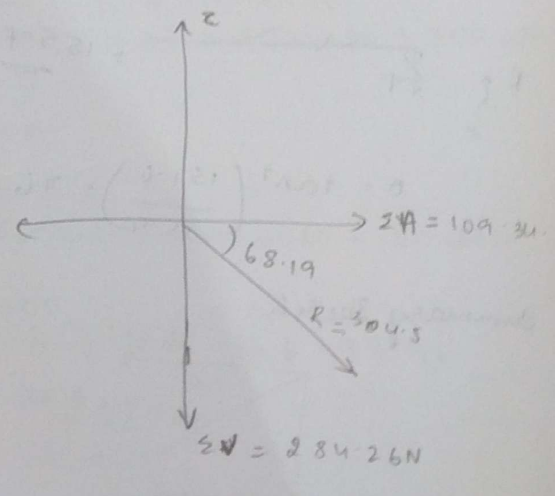
$= 284.24\ (\downarrow)$

$R = \sqrt{}$

$= 304.5\ N$

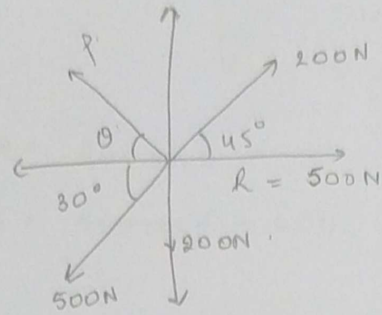
$\tan\theta = 2.5$

$\theta = 68.19°$



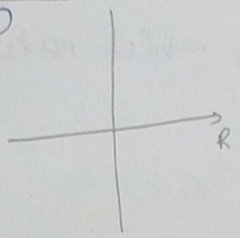
1) 4 coplanar forces acting,

One of the forces is unknown and its magnitude is shown by P. The resultant has the magnitude of 500N and is acting along x-axis determine the unknown force P and its angle of inclina^n θ with reference to x-axis,



Not for Quesⁿ:

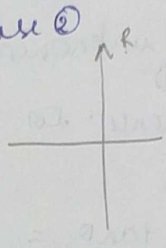
Case ①



$\Sigma H = R.$

$\Sigma V = 0$

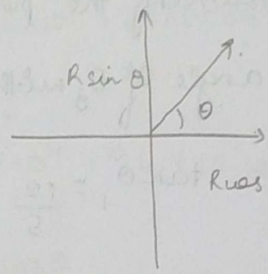
Case ②



$\Sigma H = 0$

$\Sigma V = R.$

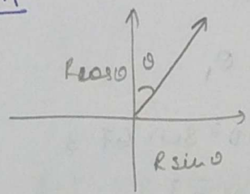
Case ③



$\Sigma H = R\cos\theta.$

$\Sigma V = R\sin\theta.$

Case 4



$\Sigma H = R\sin\theta.$

$\Sigma V = R\cos\theta.$

$\Sigma V = 200\sin 45 + P\sin\theta - 500\sin 30 = 0.$

$\Rightarrow 141.4 + P\sin\theta - 25\overset{+200}{0} = 0.$

$P\sin\theta = 308.6\,(\rightarrow).$

$\Sigma H = 200\cos 45 - P\cos\theta - 500\cos 30 = 500.$

$141.4 - 433.0 - 500 = P\cos\theta.$

$P\cos\theta = -791.6.$

$\dfrac{P\sin\theta}{P\cos\theta} = \dfrac{308.6}{-791.6}$

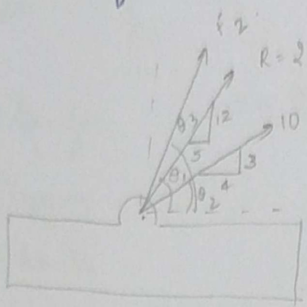
$\tan\theta = \cancel{-0.137}, \Rightarrow \theta = \tan^{-1}(-\overset{0.33}{\cancel{0.137}})$

$= -0.38 \qquad = \cancel{-7.8}°, \; -21.2°$

$$P = \frac{808.6}{\sin(-21.2)}$$

$= 853.3N$ (←)

8) The resultant of a force system is 26 kN, Determine the other unknown force



Assuming the posiⁿ of unknown force F_2 which makes an angle of θ_3 with reference to x-axis

$\tan\theta_1 = \frac{12}{5}$ $\tan\theta_2 = \frac{3}{4}$

$\theta_1 = \tan^{-1}\frac{12}{5}$ $\theta_2 = \tan^{-1} 3/4$

$= 67.3$ $= 36.86$

$\Sigma H = R\cos\theta_1$ $\Sigma V = R\sin\theta_1$

$= 26\times10^3 \cos 67.3$ $= 26\times10^3 \sin 67.3$

$= 26\times10^3 \times 0.38$ $= 23.9\times10^3$

$= 10^4 N$ $= ~~24~~\ 2.4\times10^4 N$

$10\cos 36.86 + F_2\cos\theta_3 = 10$ —①

$F_2\sin\theta_3 + 10\sin 36.86 = 23.9$ —②

$8 + F_2\cos\theta_3 = 10$

$6 + F_2\sin\theta_3 = 23.9$ ③/④ $= \tan\theta_3 = 18/2$

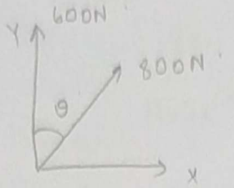
$F_2\cos\theta_3 = 2$ ④ ⇒ $= 9$

$F_2\sin\theta_3 = 18$ —③ $\theta_3 = 83.65$

Scanned by CamScanner

$F_2 = 18.12$ KN

9.] 2 forces 800N & 600N act at a point as shown in the fig. the resultant of 2 forces is 1200N. Determine θ between forces & direc^n of the resultant.



$\Sigma H = 800 \sin\theta$

$\Sigma V = 800 \cos\theta + 600$

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$1200^2 = 800^2 \sin^2\theta + (800\cos\theta + 600)^2$

$\Rightarrow 144 \times 10^4 = 64 \times 10^4 \sin^2\theta + 64 \times 10^4 \cos^2\theta + 36 \times 10^4 + 2 \times 800\cos\theta \times 600$

$144 \times 10^4 = 64 \times 10^4 (1) + 36 \times 10^4 + 96 \times 10^4 \cos\theta$

$144 \times 10^4 = 100 \times 10^4 + 96 \times 10^4 \cos\theta$

$144 = 100 + 96\cos\theta$

$44 = 96\cos\theta \Rightarrow \cos\theta = 0.458$

$\theta = \cos^{-1} 0.458$

$= 62.74°$

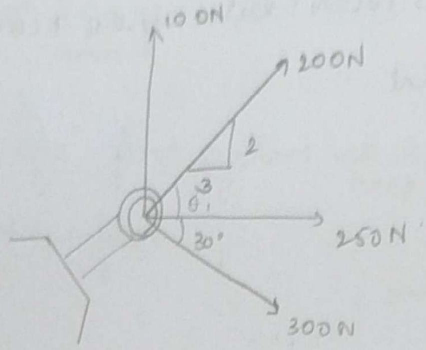
Let the angle of resultant be α

$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{966.67}{711} = 1.359$

~~ΣH = 800 sin 62.74~~ $\alpha = 53.66°$

=

10] 4 forces are acting on a bolt as shown in fig. Determine the magnitude & ~~resultant~~ posn of resultant force.



$\tan\theta_1 = \frac{2}{3}$

$\theta_1 = \tan^{-1}\frac{2}{3}$

$= 33.69°$ (→)

$\Sigma H = 250 + 200\cos 33.69 + \cancel{300}\ 300\cos 30°$

$= 676.2N$

$\Sigma V = 100 + 200\sin 33.69 - 300\sin 30°$

$= 60.93N$ (↑)

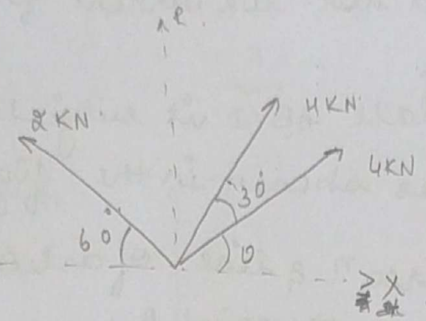
$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= 678.9N$

$\tan\theta = \frac{60.93}{676.2}$

$\theta = 5.085$

1) Resultant of a force system on a bracket as shown is acting vertically upwards. If the angle 30° b/w two 4KN force is fixed, find the angle θ & also determine the mag & dirⁿ of the resultant



$\Sigma H = 4\cos\theta + 4\cos(30+\theta) - 2\cos 60°$

$\Sigma H = 4\cos\theta + 4\cos(30+\theta) - 1$

Because resultant is ↑ $\Sigma H = 0$.

~~0 = 4 2 + 4cos (30+~~

$0 = 4\cos\theta + 4\cos(30+\theta) - 1$

$1 = 4\cos\theta + 4\cos(30+\theta)$ — ①

$\Sigma V = 4\sin\theta + 4\sin(30+\theta) + 2\sin 60$

$\Sigma V = 4\sin\theta + 4\sin(30+\theta) + \underline{1.73}$

$\Sigma V = R$. (given in q)

from ①

$\overset{A}{\cos\theta} + \overset{B}{\cos(30+\theta)} = 1/4.$

$2\cos\left(\frac{\theta+30+\theta}{2}\right) \cdot \cos\left(\frac{\theta-30-\theta}{2}\right)$

$2\cos(\theta+15) \cdot \cos(-15) = 1/4.$

$2\cos(\theta+15)\cos(15) = 1/4.$

$\cos(\theta+15) = 0.129$

$\theta+15 = \cos^{-1} 0.129.$

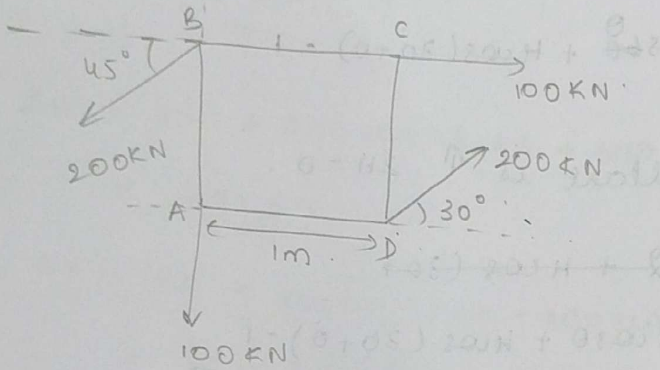
$\theta = 67.56$

$\therefore R = 4\sin 67.56 + 4\sin(30+67.56) + 1.73$

$= 9.39\ KN$

composi^n of coplanar non-con current force system

1) A plane rigid plate ABCD is subjected to the system of forces as shown in the figure. Determine the magnitude & direc^n & line of action of the resultant with refrence to point A.



$\Sigma H = 200\cos 30° + 100 - 200\cos 45$

$= \cancel{200}\ 173.205 + 100 - 141.42$

$= 131.785\ KN (\rightarrow)$

$\Sigma V = \cancel{-100 + 200\sin 30°} - 200\sin 45$

$= 141.42\ KN (\downarrow)$

$\tan\theta = \frac{141.42}{131.78} = 1.073$

$\theta = \tan^{-1} 1.073 = 47.01$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 193.30\ KN$

$= \sqrt{\quad}$

$M_A = (100 \times 0) + (\cancel{-200 \sin 30 \times 1}) + \cancel{100(1)}$

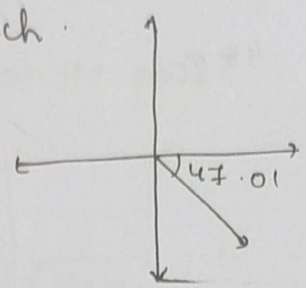
$- 200 \cos 45$

$= -141.42 \text{ KN-m}$ (↺)

$M_A = Rx$

$x = \dfrac{141.42}{193.3} = 0.73 \text{ m}$

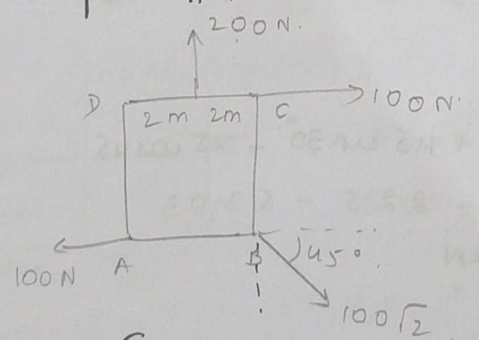
Summary Sketch.



X intercept $= \dfrac{\Sigma M_A}{\Sigma V}$.

Y intercept $= \dfrac{\Sigma M_A}{\Sigma H} = \dfrac{141.42}{131.78} = 1.073$

20) Determine the mag. direcn & resultant ~~of the~~ for w.r.t. point A.



$\Sigma H = 100\sqrt{2} \cos 45° + \cancel{100} - \cancel{100}$

$= 100 \text{ KN}.$

$\Sigma V = -100\sqrt{2} \sin 45 + 200$

$= 100 \text{ KN}$

$R = \sqrt{100^2 + 100^2} = 141.42$

$M_A = (100 \times 0) + (+100\sqrt{2}\sin 45^\circ \times 4) + \cancel{100}(4)$

$\cancel{-200(2)}$

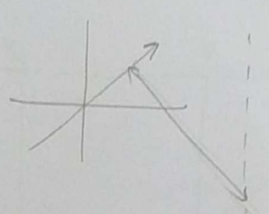
$= 400$ N-m (↻)

$X = \frac{M_A}{R} = \underline{\underline{2.82}}$ m.

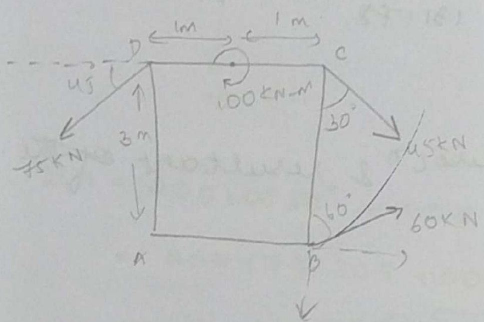
X intercept $= \frac{M_A}{\Sigma V} = \cancel{144m}$ 4m.

Y intercept $= \frac{M_A}{\Sigma H} = 4$ m

Summary Sketch:



3.]



$\Sigma H = 60\sin 60^\circ + 45\sin 30^\circ - 75\cos 45$

$= 51.96 + 22.5 - 53.03$

$= \underline{\underline{21.43}}$ kN

$\Sigma V = 60\cos 60 - 45\cos 30 - 75\,\cancel{\cos}\sin 45^\circ$

$= \underline{\underline{62}}$ kN

$R = \underline{\underline{65.59}}$ kN.

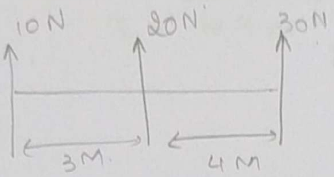
$M_A = -60 \cancel{\sin} 60 (2) + (+45 \cos 30)(2)$

$M_B = 45 \sin 30° (3) - 75 \cos 45 (3) - 75 \sin 45 (2)$,

$+100$

$=$

Q) Three like parallel forces 10; 20, 30 N acting on the line A, B, C resp. Find mag dir^n & distance of resultant from the point A.



$\Sigma H = 0$, $\Sigma V = 60N$ (↑↑) $R = 60N$.

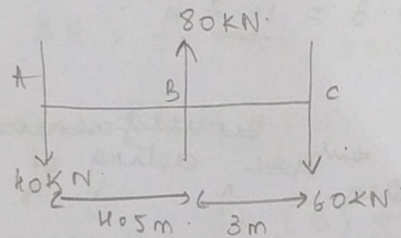
$M_A = -30(7) - 20(3)$

$= 270 \text{ N-m}$ (↺)

$M_A = Rx$

$x = 4.5 \underline{m}$

6)



$\Sigma H = 0$

$\Sigma V = -40 + 80 - 60$

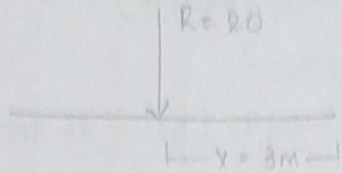
$= 20 \text{ KN} (\downarrow)$

$M_C = 0 + 80 \times 3 - 40 \times (7.5)$

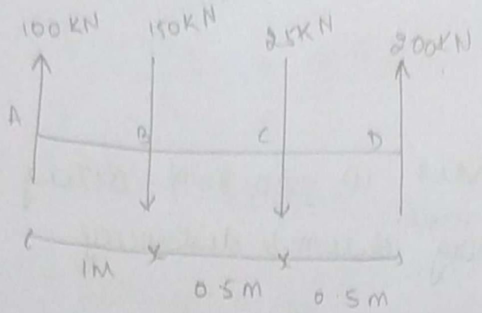
$= \cancel{6}\ 60 \text{ KN-m}$ (↺)

$x = \frac{M_C}{R} = \frac{60}{20} = 3 \text{ m}$

Resultant Sketch :-



6.)



$M_D = ?$

$x = ?$

$\Sigma H = 0$ $\quad \Sigma V = 100 - 150 - 25 + 200$

$= 300 - 175$

$= \underline{125 \text{ KN}}$

$R = \underline{\underline{125}}$

$M_D = -25(0.5) - 150(1) + 100(2)$

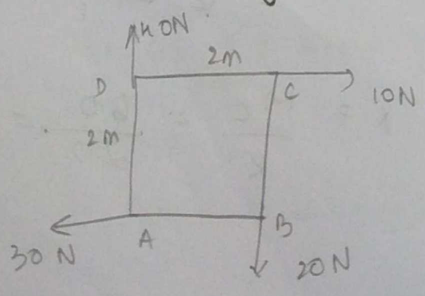
$= -12.5 - 150 + 200$

$= 37.5 \text{ KN-m}$

$x = \underline{\underline{0.3\text{m}}}$

$\therefore$ x from A is - $2 - 0.3 = \underline{1.7}$ m.

7) 4 forces are simullaer ~~simultaenously~~ acting of magnitude 10, 20, 30 & 40 along the four sides of square of dimension 2m x 2m. Determine the resultant, mag. and direcⁿ from the ref. point C.



$\Sigma H = 10 - 30 = -20N = 20N\ (\leftarrow)$

$\Sigma V = -20 + 40 = 20N\ (\uparrow)$

$\cancel{M_C = 40(2) + 30(2) - 20(0)}$

$M_C = 40(2) - 20(0) + 30(2)$

$= 140\ N\text{-}m\ (\circlearrowright)$

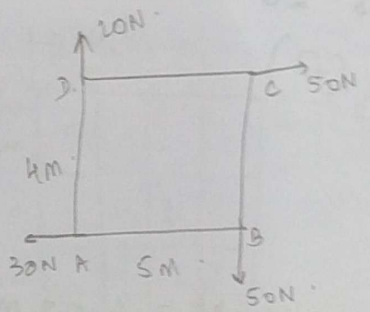
$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= \underline{28.28}$

$x = 49m$, x intercept $= \frac{\Sigma M_C}{\Sigma V} = 7$

y " $= \underline{7}$

3) Reduce the given system of forces to a single force and locate the resultant from the point A.



$\Sigma H = 50 - 30 = 20N\ (\rightarrow)$ $\Sigma V = 20 - 50 = -30N$

$= 30N\ (\downarrow)$

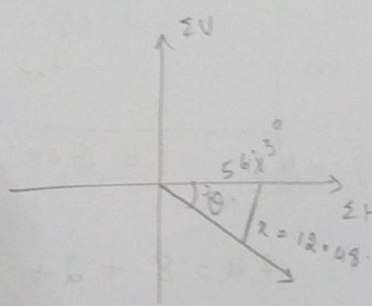
$R = \underline{36.05\ N}$

$M_A = 50(4) + 50(5)$

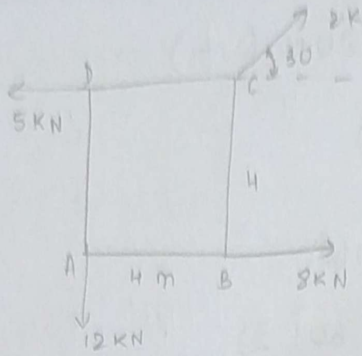
$= 200 + 250$

$= \underline{450\ N\text{-}m}\ (\circlearrowright)$

$x = \underline{12.48\ m}$



9)



$M_B = ?$

$x = ?$

X intercept $= ?$

Y " $= ?$

$\Sigma H = 8 - 5 + 2\cos 30$

$= 8 - 5 + 1.73$

$= \underline{4.73\ KN}\ (\rightarrow)$

$\Sigma V = -12 + 2\sin 30$

$= 11\ KN\ (\downarrow)$

$R = \underline{11.97}$

$M_B = 2\cos 30 (4) - 12(4) - 5(4)$

$= 1.73 - 48 - 20$

$= 66.27\ KN\text{-}m$ (↺)

X – V

Y = H

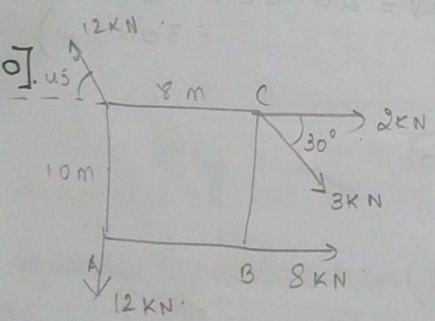
$\theta = \underline{66.73}$

$x = \underline{5.53}$

X intercept $= \underline{6.02}\ M$

Y " $= \underline{14.01}\ M$

10].



ref point A

$\Sigma H = 8 + 2 + 3\cos 30 - 12\cos 45$

$= \underline{4.11\ KN}\ (\rightarrow)$

$\Sigma V = -3\sin 30 + 12\sin 45 - 12.$

$= \cancel{6.98\ (\uparrow)}\ 5.02\ KN\ (\downarrow)$

$R = \cancel{8} \underline{\cancel{1}} 6.47 N$

$+ 3 \sin 30 (8$

$M_A = 2(10) + 3\cos 30(10) - 12\cos 45(10)$

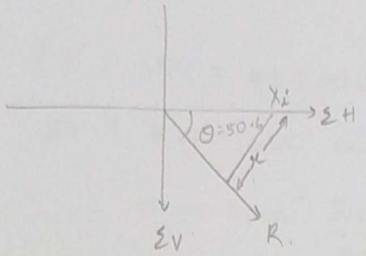
$= 20 + 25.98 - 84.85$

$= 26.87$ (C) KN-m

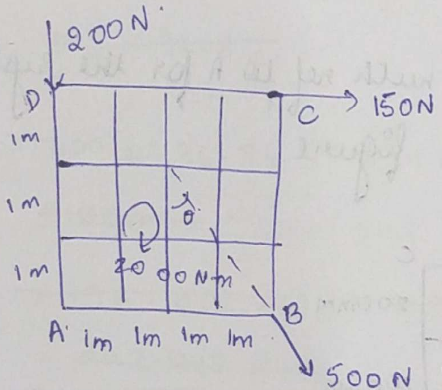
$X = \underline{4.15 m}$

X - inter cept $= \Sigma M_A / \Sigma V = 5.36 m$

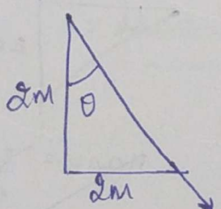
Y = intercept $\frac{\Sigma M_a}{\Sigma A} = \underline{6.537 m}$



11) to find the equilibrient with, refrence to A as origin for the system of forces as shown in the figure.



the force which is equal in magnitude & opp. in direc n to that of resultant.



$\tan\theta = 2/2$

$\theta = \underline{45°}$

$\Sigma H = 500 \sin 45 + 150 = 503.55 N (\rightarrow)$

$\Sigma V = -500\cos 45 - 200 = 553.55 N (\downarrow)$

$R = \underline{748.31 N}$

$\tan\phi = \frac{\Sigma V}{\Sigma H} \Rightarrow \phi = 47.70$

$M_A = +500\cos 45°(4) + 150(3) + 2000$

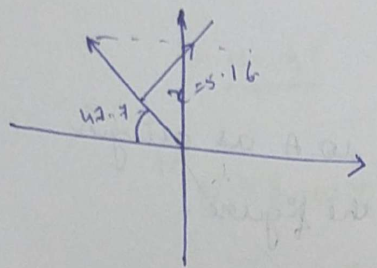
$= 1414.2 + 450 + 2000$

$= 3864.2$ N-m (↻)

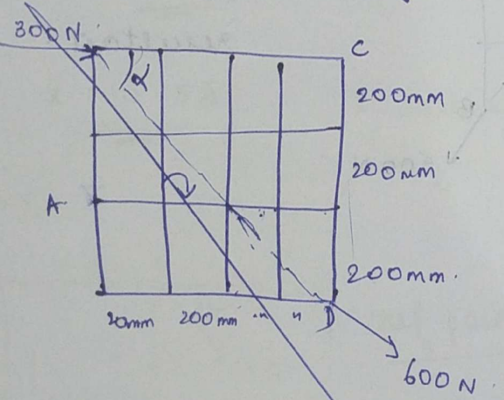
$d = \frac{M_A}{R} = \frac{3864.2}{748.31} = \cancel{2.49m}$ 5.16m.

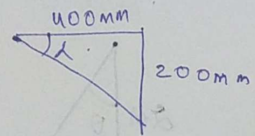
X int = 6.98m

Y int = 7.67m.



12) Find the equilibriant with ref. to A for the system of forces as shown in the figure.

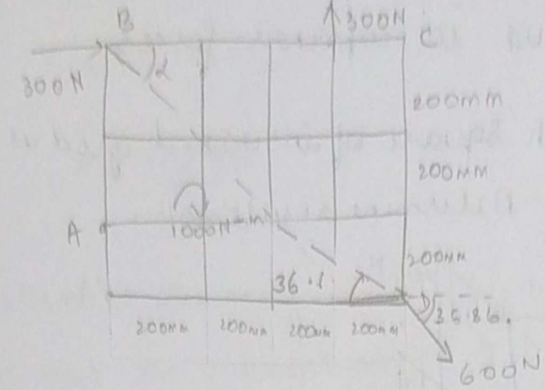




$\tan\alpha = \frac{200}{400} = \frac{1}{2}$

$\alpha =$

$\cancel{\Sigma H}$ $\tan\alpha = \frac{600}{800} = \frac{3}{4}$



$\tan\alpha = \frac{0.6}{0.8}$

$\Rightarrow \alpha = 36.86°$

$\Sigma H = 300 + 600\cos 36.86 = 780.06 \text{ N}$

$\Sigma V = 300 - 600 \sin 36.86 = 59.91 \text{ N}$

$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H} = 4.39$, $R = 782.35$ N

$M_A = \cancel{36.86}\ \cancel{600\cos 36.86} \cancel{+}\ 600 \sin \cancel{36.86} \cancel{+300} \cancel{-300}$

$= \cancel{839.97} \cancel{+} \cancel{100000}$ mm.

$\cancel{\frac{M}{R} = 36.86}$

$M_A = -600\cos 36.86 (0.2) + 600\sin 36.86 (0.8)$

$\cancel{+1000+} - 300(0.6) + 1000 + 300(0.4)$

$\cancel{= 96.012 + 287.93 - 180}$

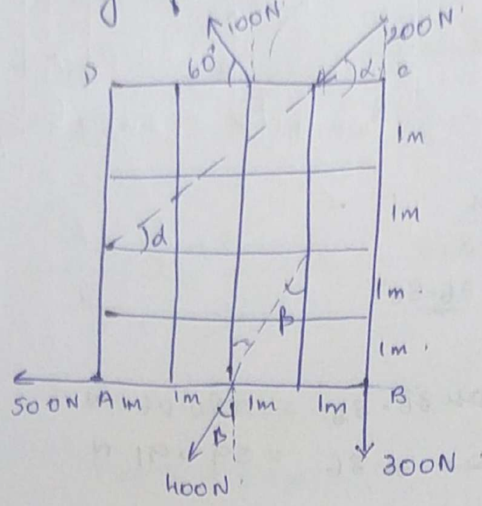
$= \cancel{203.942 \text{ N-m}}$

$= \cancel{1203.942}\ 1131.92$ N-m.

$X_{int} = 18.89$ m $Y_{int} = 1.45$ m.

$X = \cancel{14.4 m}\ \cancel{14.4}\ 1.44$

13) A flat plate subjected to coplanar forces as shown in fig. each square of inscribed grid is having the length of 1m. Determine resultant.



$\tan\alpha = \frac{2}{3} \Rightarrow 33.69 = \alpha$

$\tan\beta = \frac{1}{2}$

$\Rightarrow \beta = 26.86$

$\Sigma H = -400 \sin 26.86 - 500 - 100 \cos 60^\circ - 200 \cos 33.69$

$= 895.88 \text{ N} (\leftarrow)$

$\Sigma V = -300 - 400 \cos 26.86 + 100 \sin 60 - 200 \sin 33.69$

$= 681.18 (\downarrow)$

$R = 1125.43$ $\tan\theta = \frac{\Sigma V}{\Sigma H} \Rightarrow \theta = 37.3$

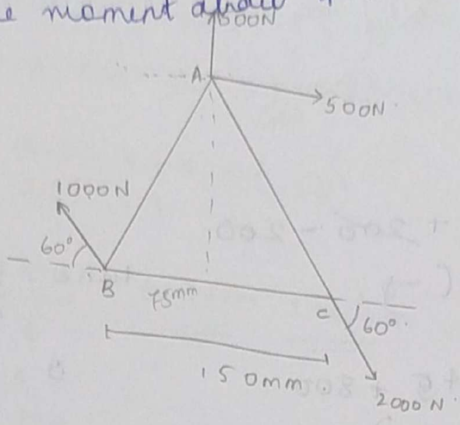
$M_A = 400 \cos 26.86 (2) + 300 (4) - 200 \cos 33.69 (4) + 200 \sin 33.69 (3) - 100 \sin 60 (2) - 100 \cos 60 (4)$

$= 1209.55 \text{ N-m}$ (↻)

$X_{int} = 1.07\,m$ $\quad Y_{int} = 1.3\,m$

$X = \underline{1.07\,m}$

14). Determine the mag. and direc^n of equilibrium & find out the moment about A.



$\Sigma H = 2000\cos 60 + 500 - 1000\cos 60$

$= 1000 + \cancel{500} - \cancel{500}$

$= 1000N\ (\rightarrow)$

$\Sigma V = -2000\sin 60 + 500 + 1000\sin 60°$

$= -1732.05 + 500 + 866.025$

$= 366.025N\ (\downarrow)$

$R = \cancel{134974}\ \underline{1064.882\ N}$

$\tan\theta = 0.366$

$\theta = \underline{20.1°}$

$M_A = -2000\cos 60°\,(129.9\times 10^{-3}) + 2000\sin 60°\,(75\times 10^{-3})$

$+ 1000\cos 60°\,(129.9\times 10^{-3}) + 1000\sin 60°\,(75\times 10^{-3})$.

$= -200\cos 60°\,(0.129) + 200\sin 60°\,(0.075)$

$+ 1000\cos 60°\,(0.129) + 1000\sin 60°\,(0.075)$.

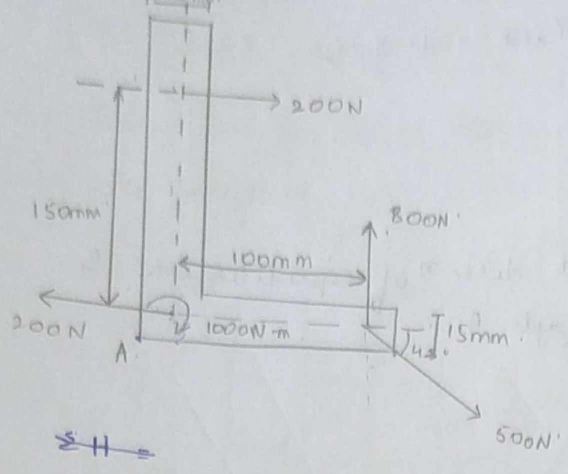
$= \cancel{-12.9} + \cancel{12.99} + 64.5 + 64.95$

$= \underline{129.45}\ N\text{-}m\ (\curvearrowright)$

$x = \underline{0.12\,m}$

$x_{int} = \underline{0.3\,m}$ $\quad Y_{int} = \underline{0.129}$

15)



~~ΣH =~~

$\Sigma H = 500\cos 45° + \cancel{200} - 200$

$= 353.55N$ (→)

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\Sigma V = -500 \sin 45 + 800$

$\theta = 51.62$

$= 446.4N$ (↑)

$R = 569.44N$

$M_A = 500 \sin 45 (0.1075) - 800(0.1075) + 200 (0.150)$

$= \cancel{38.006 - 86.7} + 500\cos 45 (0.075) - 200 (0.075) + 1000$

$= 38.006 - 86. + 30 + 26.5 - 15 + 1000$

$= \cancel{64.94}$ 993.506 (↻)

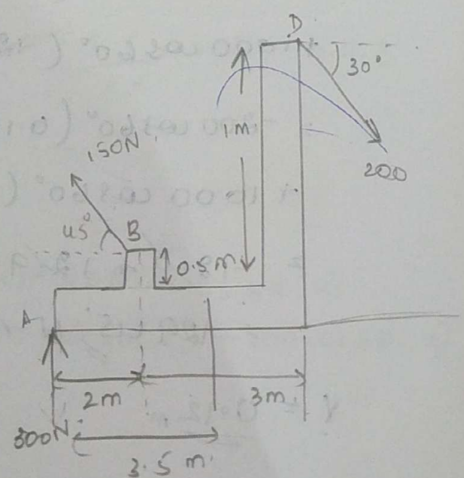
16) A bracket is subjected to system of forces as shown in figure. determine direcn, mag. & line of acn of the resultant from the point A.

$\Sigma H = 200\cos 30° - 150\cos 45$

$= 67.139N$ (→)

$\Sigma V = -200\sin 30 + 150 \sin 45 + 300$

$= 306.066$ (↑)



$R = 313.34\ N.$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 77.62.$$

$$\Sigma M_A = 300(0) - 150\sin 45(2) - 150\cos 45(0.5). + 200\sin 30°(5) + 200\cos 30°(1).$$

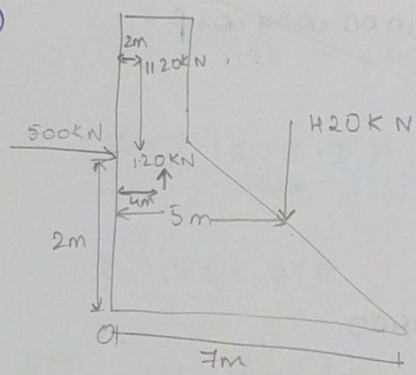
$$= \cancel{932.37} \, N-m\ (\curvearrowright). \quad 508.04.$$

$$X = \frac{\Sigma M_A}{R} = \cancel{2.97m} \ 1.62m$$

$$X_{int} = \frac{\Sigma M_A}{\Sigma V} = \frac{508.04}{304.066} = 1.67m$$

$Y_{int} = 7.56\ m$

17)



$\Sigma H = 500KN\ (\rightarrow)$

$\Sigma V = -420 + 120 - 1120$

$= 1420KN(\downarrow)$

$R = 1505.456KN.$

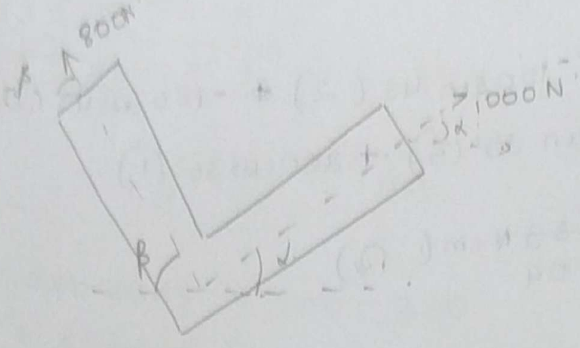
$$M_O = 420(5) - 120(4) + 1120(2) + 500(2)$$

$$= 2100 - 480 + 2240 + 1000$$

$$= 4860\ KN-m$$

$X = 3.22m.$

(8) Forces are transmitted by 2 member as shown. If the resultant of these force is 1400 N is directed vertically upward, determine the values of α & β.



$R = 1400N$, $\frac{\cos\alpha}{\cos\beta} = \frac{4}{5}$

$\Sigma H = 1000\cos\alpha - 800\cos\beta = 0$

$\Sigma V = 1000\sin\alpha + 800\sin\beta$

$R^2 = 1000^2\cos^2\alpha + 800^2\cos^2\beta + 1000^2\sin^2\alpha + 800^2\sin^2\beta$

$- 2\,1000\cos\alpha\,800\cos\beta + 2\,1000\cos\alpha\cos\beta$

$= 1000^2(1) + 800^2(1)$

$\Sigma V = R$

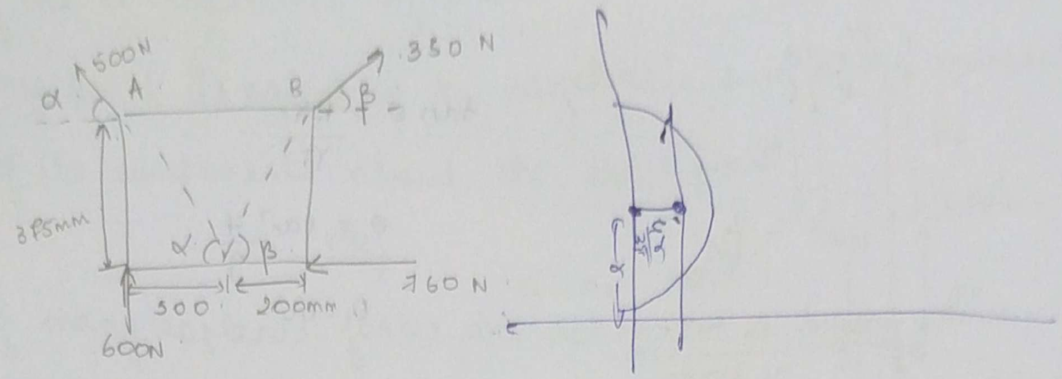
$800\sin\beta + 1000\sin\alpha = 1400$

$\sin\alpha = \frac{1400 - 800\sin\beta}{1000}$

$\sin\alpha = 1.4 - 0.8\sin\beta \rightarrow ①$

$\sin^2\alpha + \cos^2\alpha = 1$

19) 4 forces acting on 700 x 375mm plate. Find the resultant of these forces and locate the point where the resultant intersects the edge AB of the plate



$$\Sigma H = 350 \cos\beta - 500 \cos\alpha - 760$$

$$\Sigma V = 350 \sin\beta + 500 \sin\alpha + 600$$

$\tan\alpha = \frac{375}{500}$ $\qquad$ $\tan\beta = \frac{375}{200}$

$\alpha = \underline{36.86}$ $\qquad$ $= 61.92.$

$\Sigma H = 995.305N (\leftarrow)$

$\Sigma V = \underline{1208.7} N (\uparrow)$

$R = 1565.75N$

$\Sigma M_A = -350 \sin\beta (\cancel{0.700}) + 760 \cdot (0.375)$

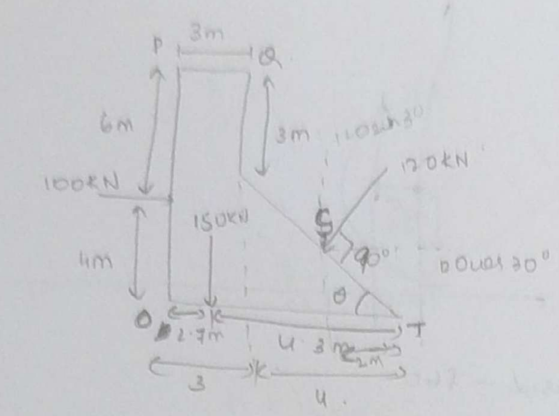
$= -216.161 + 285$

$= \underline{68.84 \quad KN-m} (\curvearrowright)$

$X_{int} = \underline{0.056} m$ $\qquad$ $Y_{int} = \underline{0.069} m$

$X = \underline{0.04}39 m$

1) Find the magnitude, direcⁿ & x-intercept of the resultant of system of forces shown in the figure. take S = mid point of R



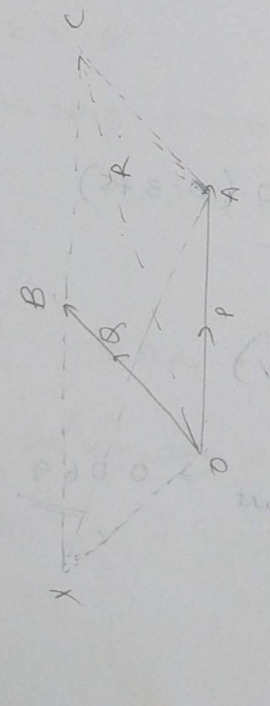
$\tan\theta = \frac{RY}{YT}$

$\theta = \tan^{-1}\frac{7}{4}$

$\theta = 60^\circ$

$\Sigma H = 120\cos 30^\circ + 100 = \underline{203.923}\ KN$

$\Sigma V =$



State and prove Varignon's theorem.

Varignon's theorem : the theorem states that the moment of the a resultant of two concurrent forces about any point is equal to the algebraic sum of the moments of its components about the same point.

OR.

If many coplanar forces are acting on a body, then algebraic sum of moments of all the forces about a point in the plane of the forces is equal to the moment of their resultant about the same point.

Proof :-

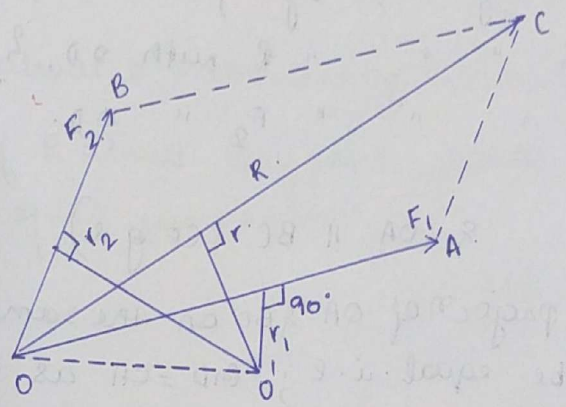


Fig 1

Fig. 1 shows 2 forces F_1 & F_2 acting at point O. these forces are represented in magnitude and direction by OA & OB. their resultant R is represented in mag. and direcn by OC which is the diagonal of parallelogram OACB. Let O' is the point in the plane about which moment of F_1, F_2 & R are to be determined. From point O', draw perpendiculars on OA, OC & OB.

Let r_1 = ⊥n distance b/w F_1 & O'.

r_2 = ⊥n distance " R & O'

r_3 = ⊥r distance b/w F_2 & O

then according to varignon's Principle

Moment of R about O' must be equal to algebraic sum of moments of F_1 & F_2 about O'.

$R \times r = F_1 \times r_1 + F_2 \times r_2$.

Now refer fig ② . Join OO' & produce it to D. From points C, A & B draw ⊥r on OD meeting at D, E & F resp. From A & B also draw ⊥r on CD meeting the line CD at G & H resp.

Let θ_1 = Angle made by F with OD,

θ = " " " R with OD, &

θ_2 = " " " F_2 " OD.

OA = BC & OA ∥ BC (cf fig 2)

hence the projecⁿ of OA & BC on the same vertical line CD will be equal. i.e; GD = CH as GD is the projecⁿ of OA on CD & CH is the projecⁿ of BC on CD.

Then from fig ②, $P_1 \sin\theta_1 = AE = GD = CH$

$F_1 \cos\theta_1 = OE$

$F_2 \sin\theta_1 = BF = HD$

$F_2 \cos\theta_2 = OF = ED$

OB = AC & OB ∥ AC hence projecⁿ of OB & AC on same horizontal line OD will be equal i.e OF = ED.

$R\sin\theta = CD$.

$R\cos\theta = OD$.

Let the length OO' = x.

Then $x \sin\theta_1 = r_1$, $x \sin\theta = r$ & $x \sin\theta_2 = r_2$.

Now moment of R about O' = R × (dist b/w O' & R) = R × r

$= R \times x \sin\theta$

$(r = x \sin\theta)$

$= R \sin\theta \times x$

$= CD \times x$ $\quad (R \sin\theta = CD)$

$= (CH + HD) \times x$

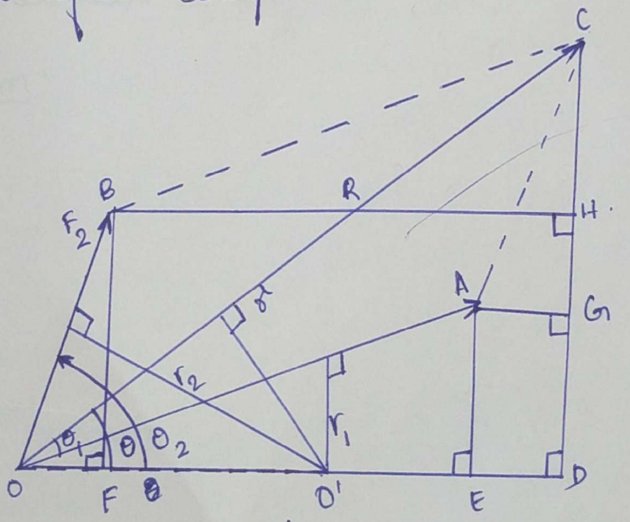
$= (F_1 \sin\theta_1 + F_2 \sin\theta_2) \times x$ $\quad (\because CH = F_1 \sin\theta_1$ & $HD = F_2 \sin\theta_2)$

$= F_1 \times x \sin\theta_1 + F_2 \times x \sin\theta_2$.

$= F_1 \times r_1 + F_2 \times r_2$. $\quad (x \sin\theta_1 = r_1$ & $x \sin\theta_2 = r_2)$

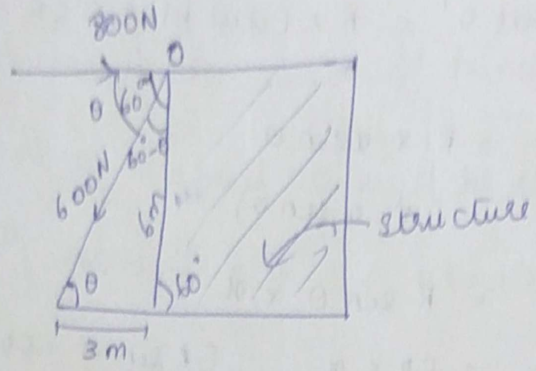
= Moment of F_1 about O' + Moment of F_2 about O'.

Hence moment of R about any point is the algebraic sum of moments of its components.



Fig(2)

1)



Determine the resultant force at O both in dir^n & magnitude

ΣH