CHAPTER 5

RATE-OF-RETURN CALCULATIONS

No law can reduce the common rate of interest below the lowest ordinary market rate at the time when that law is made.

Adam Smith, *The Wealth of Nations*, 1776

The rate of return is the last of the discounted cash flow comparison methods we need to consider. A *minimum acceptable rate of return (MARR)* is the lowest level at which an independent alternative (or, as we will see, an incremental cash flow between two mutually exclusive alternatives) is still attractive. It varies among and within organizations. Although there are a wide variety of recommendations for determining this lowest level of acceptability, it is generally agreed that the level should be no lower—and most likely considerably higher—than the cost of capital. How much higher depends on the circumstances, objectives, and policies of the organization. The purpose of establishing a minimum acceptable rate of return is to ration capital to the most deserving proposals.

Calculation of an *internal rate of return (IRR)* will allow us to determine, under possible reinvestment constraints, whether an alternative meets the MARR value. The IRR analysis may begin with an equivalent annual-worth (EAW) (covered in Chap. 4), present-worth (PW), or future-worth (FW) (both PW and FW are covered in Chap. 3) formulation. By definition, *the internal rate of return of an investment is the rate of interest earned on the unrecovered balance of an investment where the terminal balance is zero.* Given that the present and future worths of a cash flow are equivalent when either is \$0, the IRR computation frequently starts with the PW formulation of the cash flow, which is equal to FW = 0 through PW = $\$0[1/(1+i)^N]$.

There is no way to avoid trial-and-error computations in *manually* calculating the IRR for complex formulations, but as we will see, the structure of the cash flow can offer clues about where to begin. The computer program (CHEER) that is ancillary to this text allows the IRR to be calculated directly, as do many spreadsheet functions available to the analyst (Excel, Quatro Pro, Lotus, Supercalc, etc.). The spreadsheet programs provided with this text use Quatro Pro and Excel functions.

As mentioned in Chap. 4, consistent results are obtained from EAW, PW, and FW comparisons. This will also be true with the incremental IRR computation, assuming it is applied correctly, as we will see in Sec. 5.5.

We will see that a cash flow pattern that reverses signs (from negative to positive, or vice versa) more than once may have a present-worth equation that is a polynomial in terms of the rate of return. In such cases we can have multiple roots i^*, and we will not know right away whether one of these roots is the true IRR. Historically, one suggested way to eliminate all but one possibility for the IRR is to apply an external rate of return to a portion of the cash flow, in order to eliminate sign reversals. This external reinvestment rate is usually the MARR. Another historically suggested way, commonly called the *external rate-of-return (ERR) method,* to obtain just one root is to (1) apply an explicit interest rate i percent, probably equal to the MARR, in calculating the future worth of *receipts* and (2) equate this FW to the future worth of *expenditures* compounded at a yet unknown rate of return e'. The value of e' is then found, usually by a trial-and-error procedure, that results in FW = 0. Both methods recognize that the IRR should not be the rate of return used for *reinvestment,* although both methods have logical flaws. Examples will be given later to show how the approaches work. To avoid confusion with the true external rate of return (ERR), a term which will be used later, we will use the full name *historical external rate-of-return (HERR) method* instead of *external rate-of-return (ERR) method.*

A more logical approach is the *project balance method (PBM),* whereby an external rate of return is utilized when any *cash flow balance,* computed by using a potential IRR, is greater than zero. We will see in Sec. 5.3 that using the PBM or determining the rate of return simply by setting the PW equation to zero and solving for i is a function of the cash flow structure (type of investment) and whether the alternatives are independent (selection of one does not affect the selection of another) or mutually exclusive (selection of one alternative precludes the selection of another). Definitions and evaluation methodology will be delayed until Sec. 5.3.

RATES OF RETURN

The rate of return is a percentage that indicates the relative yield on different uses of capital. Since interest rates are well understood throughout the world of commerce, there should be little danger of misinterpreting rate-of-return figures.

IRR is a metric used in capital budgeting measuring profitability of an investment. It is the discount rate that makes net present value of all cash flows zero.

As indicated in the introductory remarks, three rates of return appear frequently in engineering economy studies:

- The *minimum acceptable rate of return* (MARR) is the rate set by an organization to designate the lowest level of return that makes an investment acceptable.
- The *internal rate of return* (IRR) is the rate on the unrecovered balance of the investment in a situation where the terminal balance is zero.
- The *external rate of return* (ERR) is the rate of return that is possible to obtain for an investment under current economic conditions. For example, suppose that analysis of an investment shows that it will realize an IRR of 50 percent. Rationally, it is not reasonable to expect that we can invest in the external market and get that high a rate. In engineering economy studies, the external interest rate most often will be set to the MARR.

All three rates of return are discussed in the following sections.

5.2 MINIMUM ACCEPTABLE RATE OF RETURN

The minimum acceptable rate of return, also known as the *minimum attractive rate of return,* is a lower limit for investment acceptability set by organizations or individuals. It is a device designed to make the best possible use of a limited resource, i.e., money. Rates vary widely according to the type of organization, and they vary even within the organization. Historically, government agencies and regulated public utilities have utilized lower required rates of return than have competitive industrial enterprises. Within a given enterprise, the required rate may be different for various divisions or activities. These variations usually reflect the risk involved. For instance, the rate of return required for cost reduction proposals may be lower than that required for research and development projects in which there is less certainty about prospective cash flows.

There is a wealth of literature on the subject but little agreement. The components included in the selection of a before-tax MARR include an inflation-free rate for the cost of using capital and the risk profile of a particular venture. A MARR to be used with *constant* dollars, the assumption we have adopted, is an *inflation-free interest rate* that represents the earning power of capital when inflation effects have been removed. A MARR value that includes the effect of inflation is referred to as the *market interest rate.* Use of this rate will require that all cash flows be in *actual* dollars. It is generally accepted that the lower bound for a minimum required rate of return should be the *cost of capital.* The constitution of this cost is also subject to controversy. As will be discussed in Sec. 5.6, the cost of capital for competitive industries must reflect the expense of acquiring funds from various sources; and as will be discussed in Chap. 8, the cost of safe government bonds is a basis for the lower bound of interest rates used to evaluate public investments.

How much above the cost of capital to set the minimum attractive rate of return depends on an organization's circumstances and aspirations. A small company strapped for cash and burdened by a low credit rating will have a higher cost of capital and so must have a very attractive proposal before it can consider investing. Larger, established companies tend to view the rate as a realistic expectation of how much their capital can earn when it is invested. This MARR is a typical figure promised (and later substantiated) for a large number of high-quality investment proposals available to the firm; it is assumed that the proceeds earned from current projects can be reinvested at comparable rates in future proposals. The rate so derived is sometimes called the *opportunity cost of capital* because any proposal funded to earn a lower rate precludes the opportunity to earn the minimum attractive rate of return.

The purpose of establishing a minimum attractive rate of return higher than the cost of capital is to ration capital. It is rationed to *divisions of an organization* and to the *whole organization* as a function of time. The purpose is to avoid unproductive investments in marginal activities, perhaps favored for political reasons, and to conserve capital during periods when fewer attractive proposals are submitted. Rationing capital by setting higher MARRs is simplistically seen in Sec. 6.5, where we discuss funding a *subset* of independent proposals from the *capital-budgeting viewpoint.*

INTERNAL RATE OF RETURN

The IRR is the best-known and most widely used rate-of-return method. It is also known as the *true rate-of-return method* and the *discounted cash flow method.* The latter term is indicative of the way interest rates were interpreted in previous chapters; the internal rate of return, represented by i in the traditional interpretation of interest rates, is the rate of interest earned by an alternative investment on the unrecovered balance of an investment. (For example, Table 4.1 illustrated the pattern of capital recovery at an IRR = i of 10 percent.)

The internal rate of return can be calculated by equating the annual, present, or future worth of cash flow to zero and solving for the interest rate (IRR) that allows the equality. It should be added that solving for the interest rate in this manner results in a polynomial equation that is a function of i, which may result in multiple roots of the equation (i^*). In such cases the IRR may or may not be one of the equation roots. This will be clarified and exemplified in Sec. 5.3.1.

Although both the EAW and FW approaches are legitimate, the rate of return is often defined in terms of present worth, under the constraints of possible i^* roots, where the IRR is

- The interest rate at which the present worth of the cash flow of a project is zero, *or,* to restate this in another way:
- The rate which when employed in computing the present worth of all costs and present worths of all returns will make them equal

Because rate-of-return computations usually begin with a problem expressed in terms of present worth or annual worth, it is necessary to pay attention to the guidelines for the EAW and PW methods. In particular, mutually exclusive alternatives (where selection of one precludes selection of others) must be compared on the basis of equivalent outcomes. In the case of independent alternatives (the choice of one does not affect the choice of another, except for limited capital availability), all costs and benefits must be explicitly stated. As in the previous discussions of discounted cash flow, we initially investigate the rate-of-return methods without considering the effects of income taxes.

5.3.1 Calculation of IRR

Determining the IRR is a function of the type of investment (simple, pure, and mixed) and the characteristics of the alternatives (mutually exclusive or independent). If we have independent projects, we may fund combinations of the projects since an independent project does not affect the funding of another project (except for capital availability limitations which are very real in most situations analyzed by the engineering economist). The cash flows of several independent alternatives that are being considered as a group may be summed to form the group's composite cash flow. Analysis can then be performed on this composite cash flow. Where capital limitations are apparent in a department and several independent alternatives are competing for funding, combinations of alternatives may be formed where each combination's first costs have to be equal to or less than the capital available. In this case, mutually exclusive *combinations* will usually be realized, where selection of one group of independent alternatives will preclude the selection of another. This can be due to alternatives being in more than one group and/or capital limitations.

We will see that ranking alternatives according to their IRR values is not consistent with the PW, FW, or AW rankings. Mutually exclusive alternatives may be analyzed by *incremental IRR analysis,* and the results will be found to be completely consistent with the PW, FW, and AW methods. Incremental analysis assumes that we start with a satisfactory low-investment alternative. Analysis of a higher-investment alternative is then based on the differences between the cash flows of the second alternative and the acceptable alternative. These differences in cash flows are *incremental* cash flows. The cash flow of the second alternative is equal to the cash flow of the first alternative plus the incremental cash flows. Thus, if the *incremental* cash flow is acceptable when compared to the MARR, then the larger investment has to be a better investment than the first alternative, which was also acceptable. Otherwise, remove the larger investment from consideration. This type of evaluation is continued until all alternatives have been evaluated; one of the mutually exclusive alternatives is then determined to be the best investment. This will be exemplified later.

As mentioned earlier, there is a possibility that the PW equation may be a polynomial in terms of i such that multiple roots i^* of polynomial PW(i) may result. Often, multiple i^*'s are assumed to be multiple IRR values. This is misleading since there is really only one true IRR for an investment, and so we will need to determine which i^*, if any, is the investment IRR. Classifying investments into *simple* and *nonsimple* investments will tell us if just one i^* exists which, in turn, tells us that we have found the IRR when we have found i^*. An investment is simple if there is only one cash flow sign change (minus to plus) from period to period. A simple investment is as follows:

Time period	Cash flow, $	Sign change
0	−1000	
1	−200	
2	500	Yes (− to +)
3	500	
4	500	

There will only be one i^* if the investment is simple.

A *nonsimple investment* will have more than one sign change in the cash flow sequence:

Time period	Cash flow, $	Sign change
0	−1000	
1	200	Yes (− to +)
2	−500	Yes (+ to −)
3	500	Yes (− to +)
4	500	

There may be multiple i^* values if the investment is nonsimple.

The next step in the analysis will be to determine value(s) for i^*. We will see that this is usually done by trial and error if manually accomplished, or for very simple manual cases we can find i^* directly. Or we can use a spreadsheet program (with some possible reservations for multiple i^* values). Better yet, we can use CHEER which will guarantee the correct IRR if there is a single root i^* and will give close approximate values if two values of i^* exist. All methods will be considered shortly.

Finally, if we have multiple i^* values (nonsimple investment), we will need to determine the true IRR. First, we determine whether the investment is *pure* or *mixed*. A *pure investment* occurs if the project cash flow balances, evaluated at i^*, are all less than or equal to zero. We should now realize that a simple investment has to be a pure investment as will be exemplified very shortly. If any of the project cash balances are positive (and some are negative), we have a need to use an external interest rate for reinvestment. This is called a *mixed investment* since we will "externally" reinvest at the external rate (MARR) when balances are positive and we will invest "internally" at the IRR rate when balances are negative (or zero). We do not consider mixed investments until we get to Sec. 5.5. Now we are ready to determine IRR values for a simple investment.

5.3.2 Single, Simple Investment

IRR for

The rate of return for a single, simple investment is determined by setting the present worth (or EAW) of receipts equal to the present worth (or EAW) of disbursements. Then an interest rate is sought that makes the discounted cash flows conform to the equality:

Find i so that

$$\boxed{\text{PW(receipts)} = \text{PW(disbursements)}}$$

The same relation obviously occurs when the discounted flows are subtracted from each other to equal zero:

Find i so that

$$\text{PW(receipts)} - \text{PW(disbursements)} = \text{net PW} = 0$$

For either PW formulation, the *manual* calculation of i is usually a trial-and-error procedure. We initially look at manually computing the IRR to foster understanding of the method, but it will be assumed that the analyst will use CHEER, a spreadsheet function, or a programmed calculator to determine the IRR values after the learning period.

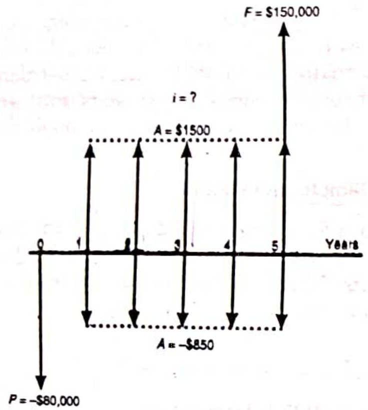
When a single proposal is for a cost reduction project, the receipts take the form of net savings from the method of operation used before the cost reduction investment. In effect, we get an *incremental investment* that is the difference between the do-nothing case and the single investment.

EXAMPLE 5.1 **Income-Producing Proposal**

A parcel of land adjacent to a proposed freeway exit is deemed likely to increase in value. It can be purchased now for $80,000 and is expected to be worth $150,000 within 5 years. During that period it can be rented for pasture at $1500 per year. Annual taxes are presently $850 and will likely remain constant. What rate of return will be earned on the investment if the estimates are accurate?

Solution

The conditions of the proposal are depicted in a cash flow diagram.



The income (positive cash flow) and disbursements (negative cash flow) can be equated according to their equivalent present worths:

$$PW_{inc} = PW_{disb}$$

or

$$\$150{,}000(P/F, i, 5) + \$1500(P/A, i, 5) = \$80{,}000 + \$850(P/A, i, 5)$$

or the positive and negative cash flows can be subtracted:

$$\$150{,}000(P/F, i, 5) - \$80{,}000 + \$1500(P/A, i, 5) - \$850(P/A, i, 5) = 0$$

which reduces to

$$\$150{,}000(P/F, i, 5) - \$80{,}000 + \$650(P/A, i, 5) = 0$$

The specific value of i, i^*, that conforms to the equation above is the rate of return on the remaining balance of the investment. As said earlier, manual computation usually requires the value to be determined by trial and error.

For a simple investment, a quick preliminary check to see whether the relation has a positive rate of return results from letting $i = 0$. At $i = 0$,

$$\$150{,}000 - \$80{,}000 + \$650(5) = \$70{,}000 + \$3250 = \$73{,}250$$

The positive value indicates that the investment will produce a positive rate of return because the total income is much greater than the outgo. The check also gives a very rough idea of how large the rate of return might be. For instance, the 72 rule suggests that a sum doubles in value every $72/i$ years. Since the $80,000 initial investment results in almost twice as much income at the end of 5 years, i should be near $72/5 = 14.4$ percent.

Letting $i = 15$ percent as the first trial, we have

$$\$150{,}000(P/F, 15, 5) - \$80{,}000 + \$650(P/A, 15, 5) \overset{?}{=} 0$$

$$\$150{,}000(0.49718) - \$80{,}000 + \$650(3.35216) = -\$3244.10 < 0$$

The negative value indicates that the trial i used was too large. Now it is known that i lies between 0 and 15 percent.

Letting $i = 14$ percent gives

$$\$150{,}000(P/F, 14, 5) - \$80{,}000 + \$650(P/A, 14, 5) \overset{?}{=} 0$$

$$\$150{,}000(0.51937) - \$80{,}000 + \$650(3.43308) = \$137.00 > 0$$

which shows that the IRR lies between 14 and 15 percent (since we now have a positive PW and the previous PW was negative), but much closer to 14 percent. The approximate value of i is determined by linear interpolation:

i	14%	?	15%
PW	$137.00	0	-$3244.10

$$i^* = 14\% + \frac{(15 - 14)\,\%\,(\$137.00 - \$0)}{\$137.00 - (-\$3244.10)} = 14\% + \frac{137 \times 1\%}{3381.10} = 14.04\%$$

A characteristic worth noting in the previous calculations is that whenever the present worth turns out to be positive, the next trial should employ a higher interest rate to approach the desired zero outcome. Conversely, lowering the interest rate in the present-worth formulation increases the resulting outcome.

If the interest tables at the back of this text are used, a small degree of error is introduced by linear interpolation between interest table values that are not linearly related. To keep the error as small as possible, interpolations should be conducted between adjacent interest tables. The error is naturally less between lower-interest-rate tables, separated by 0.5 percent, than at increments of 10 percent for the largest interest rates. For the purposes of this book, interpolated rates of return computed to the nearest 0.1 percent are adequate. The slight error that may be thus introduced will very seldom influence the choice among alternatives; this error is probably much less significant than are actual deviations from the cash flows estimated in the comparisons.

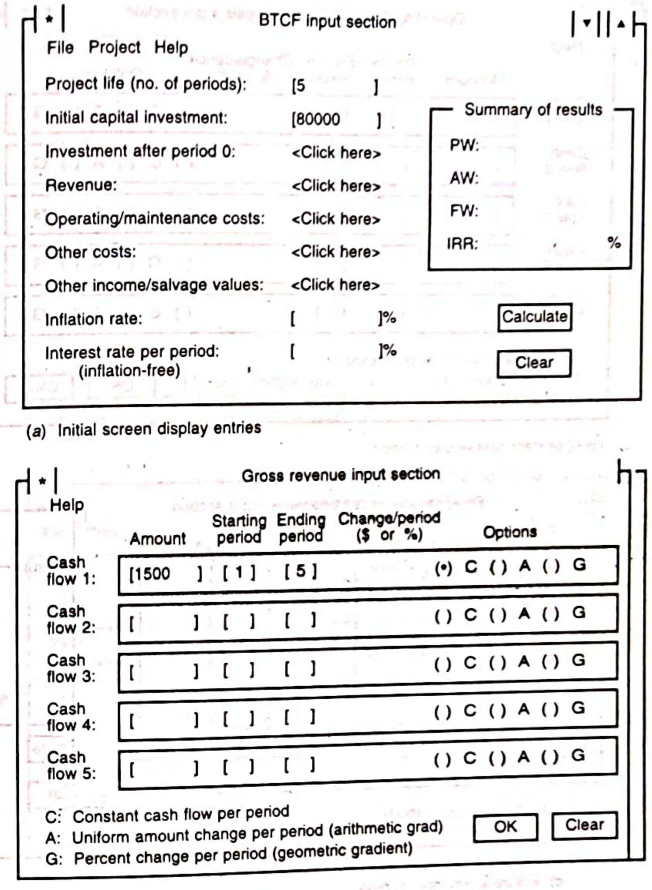
The reader should be familiar with manually computing the IRR values. CHEER will also calculate IRR values, but it can be dangerous to blindly use programs without fully understanding what they are computing and how. Obviously, it makes a lot of sense to use a program to compute IRR values, thus obviating the need for the manual trial-and-error process. To demonstrate the program's use in this regard, Example 5.2 will use the program to solve Example 5.1.

XAMPLE 5.2 **Using the Computer Program for Example 5.1**

Eventually, after mastering the basic concepts of rate of return, the user will want to use CHEER or some other automatic means to determine the IRR value. Details on using the computer program are given in App. B.

It will be assumed that the reader has been able to start the program according to App. B directions. The five stages in getting the IRR solution to Example 5.1 are given in Fig. 5.1.

1. Figure 5.1*a* shows the initial screen display. The user should access locations for data input by using a mouse or cursor manipulations (a mouse is much more convenient). The first two pieces of data to be input are the project life (5) and initial capital investment ($80,000). The initial screen display shows these two values having been input.
2. We have three more categories of data to input: annual taxes of $850 per year, annual rent (income) of $1500 per year, and a future worth of $150,000 after the 5-year period is over. We click for Revenue in the main screen, and the gross revenue input screen (Fig. 5.1*b*) comes up. Since we have a constant income per year, we can handle this through cash flow 1 only. The user inputs 1500 for the annual amount, 1 for the starting year, and 5 for the ending year. The option C is clicked to indicate that this is a constant income for each of the five periods. When the user is satisfied with the data input, OK is clicked. This returns the user to the initial screen.
3. Now we click to get the Operating/Maintenance Costs screen (Fig. 5.1*c*). The $850 annual tax value is a constant value per year, and so this is entered in the same manner as we entered the annual income. After we are satisfied with the data, we click OK to return to the main screen. We could have input the tax information by clicking for the



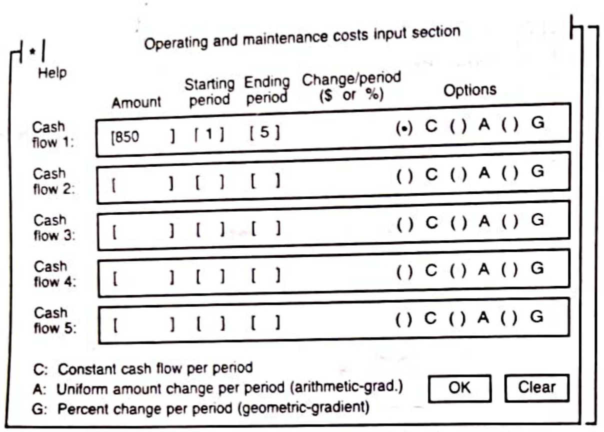
(*a*) Initial screen display entries

(*b*) Constant income entry screen

FIGURE 5.1 Initial CHEER example screen displays.

Other Costs menu in the main screen. This would have required five different values to be input even though the tax values would all be the same. The result would be the same either way, of course.

4. Finally, we input the future worth of $150,000 through the Other Income/Salvage Values screen request (Fig. 5.1*d*). We input 5 for the period and 150,000 for the worth value. We could have input the annual rent income of $1500 per year through this screen, but it was more convenient to accomplish this with the revenue screen.
5. After clicking OK on the last screen, we get the initial screen display once more. Now that we have all the data input, we can click the Calculate area and get the result shown in Fig. 5.1*e*. The IRR of 14.04 percent corresponds to the result we determined manually. Finally, once



(c) Constant cost entries screen

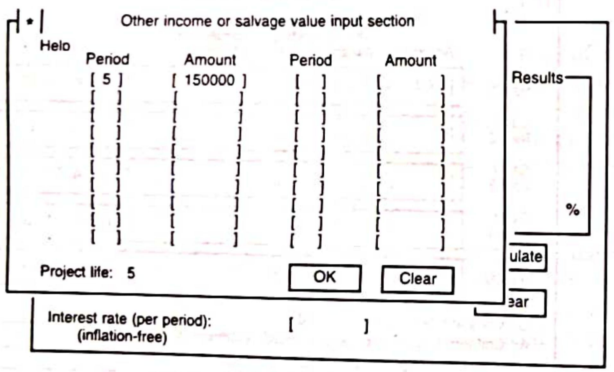


FIGURE 5.1 continued

(d) Future worth input screen

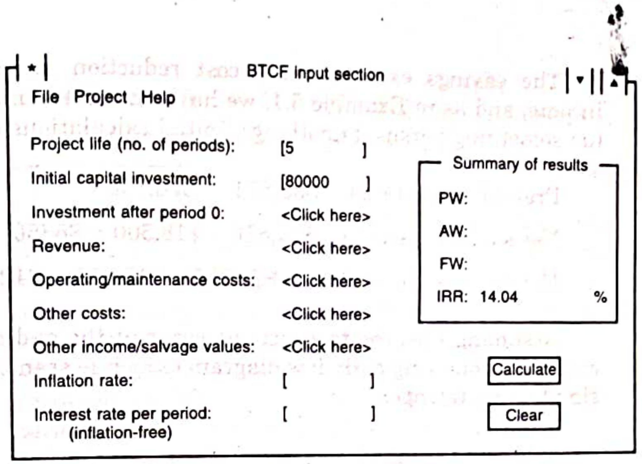
we are satisfied with the IRR result, we can click File on the initial screen and then click Print from the resulting menu to get the summary result in Fig. 5.1*f*.

A little practice using the program to compute IRR values will demonstrate the value of the computerized approach as contrasted to the trial-and-error manual approach.

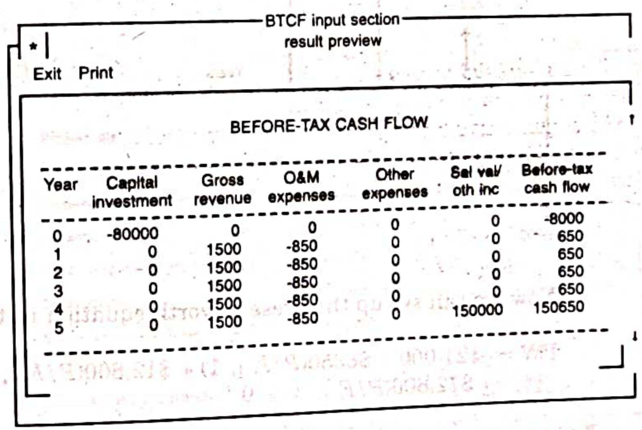
EXAMPLE 5.3

Cost Reduction Proposal

Subassemblies for a model IV scope are purchased for $71 apiece. The annual demand is 350 units and is expected to continue for 3 years, at



(e) Return to initial screen for IRR calculation



BEFORE-TAX CASH FLOW

Year	Capital investment	Gross revenue	O&M expenses	Other expenses	Sal val/ oth inc	Before-tax cash flow
0	-80000	0	0	0	0	-8000
1	0	1500	-850	0	0	650
2	0	1500	-850	0	0	650
3	0	1500	-850	0	0	650
4	0	1500	-850	0	0	650
5	0	1500	-850	0	150000	150650

FIGURE 5.1 continued

(f) Summary results screen

which time the model V scope now under development should be ready for manufacturing. With equipment purchased and installed for $21,000, the production costs to internally produce the subassemblies should be $18,500 for the first year and $12,250 in each of the last 2 years. The equipment will have no salvage value. Should the company make or buy the subassemblies?

Solution

This is an ideal problem for solution by CHEER. The objective of this text is to teach fundamentals, so this example will again stress manual solution to IRR problems even though in actual practice you will undoubtedly want to utilize the more efficient computer.

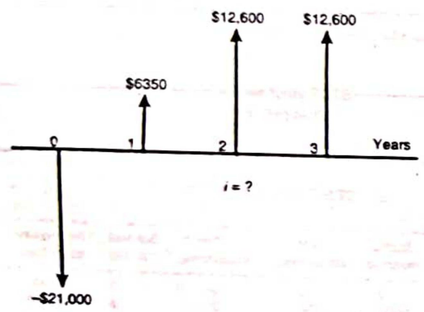
The savings expected in a cost reduction proposal are treated as income, and as in Example 5.1, we have an incremental analysis situation (do something versus do nothing). Initial calculations are as follows:

Present annual cost = 350($71) = $24,850

Net savings (year 1) = $24,850 – $18,500 = $6350

Net savings (years 2, 3) = $24,850 – $12,250 = $12,600

Assuming that the transactions occur at the end of each year, we can create the following cash flow diagram (which is seen to represent a single, simple investment):



Now we can set up the present-worth equation in terms of the IRR (i):

$$PW = -\$21{,}000 + \$6350(P/F, i, 1) + \$12{,}600(P/F, i, 2) + \$12{,}600(P/F, i, 3) \overset{?}{=} 0$$

Trying successively higher rates of return, we find at i = 10 percent,

$$PW = -\$21{,}000 + \$6350(0.90909) + \$12{,}600(0.82645) + \$12{,}600(0.75131) = \$4652.50 > 0$$

Similarly, using App. D tables, we find:

i = 15 percent	PW = $2333.89
i = 20 percent	PW = $ 333.21
i = 25 percent	PW = –$1404.80

The PW with i = 25 percent is negative while the PW with i = 20 percent is positive. Therefore, we can interpolate to find the rate of return on the $21,000 investment:

$$IRR = 20\% + (25\% - 20\%)\frac{\$333.21 - \$0}{\$333.21 - (-\$1404.80)} = 20\% + 0.96\% = 20.96\%$$

If CHEER had been utilized to directly determine the IRR value found in Example 5.3, we would find the IRR value much more easily than by the manual approach. Figure 5.2 shows the appropriate computer screens after solution. Figure 5.2b shows that the Other Income/Salvage Value screen was used rather than the Revenue screen used in Example 5.2. This is because we had three different values to input instead of the one constant-value input in Example 5.2. We could have used the Revenue screen to input $12,600 for periods 2 and 3 and the Other Income/Salvage Value screen for the $6350 value. This would give exactly the same results but would be a little cumbersome. The user can use the screens that seem best for a particular problem.

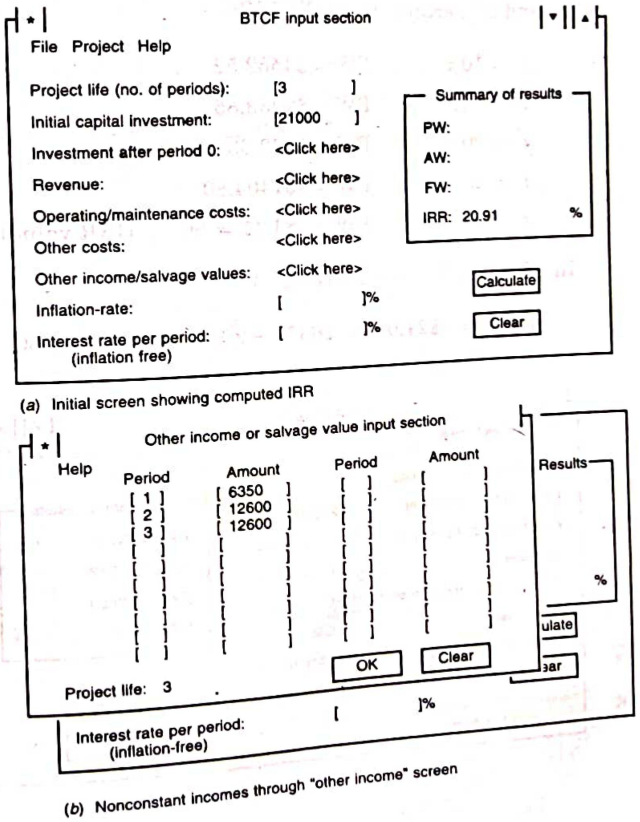


FIGURE 5.2 Computer solution for Example 5.3.

Figure 5.2*a* shows that the IRR value proves to be 20.91 percent as contrasted to the manually interpolated value of 20.96 percent. Logically, either one would give the same interpretation against a particular MARR value.

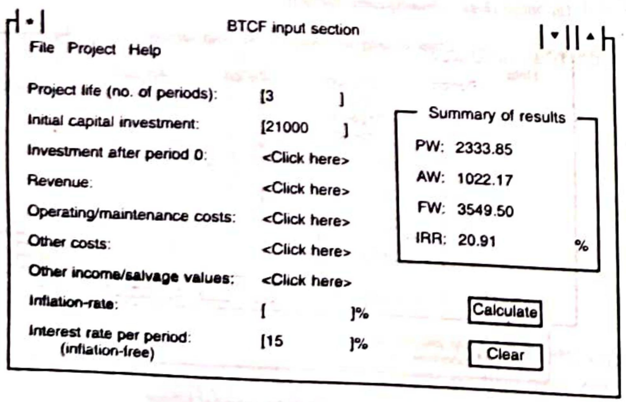
Figure 5.3 shows the computer program primary screen when Example 5.3 data were run with 15 percent, the first interest rate tried with the manual method. The IRR value is still computed, and, of course, this is still 20.91 percent (with roundoff error). Now, however, we have the PW, AW, and FW values computed since a specific interest rate was input. The PW was calculated to be \$2333.85—a very minor change from the manual result. The insignificant difference is probably due to rounding errors in the appendix tables.

The answer to the make-or-buy question depends on how large a return the firm expects on its invested capital. For Example 5.3, we can see the conditions for accepting the proposal to manufacture the subassembly internally by plotting the present worth of the proposal as a function of interest. Several interest rates were tried before we interpolated between 20 and 25 percent using CHEER:

i = 10%	PW = \$4562.52	
i = 15%	PW = \$2333.85	
i = 20%	PW = \$333.33	
i = 25%	PW = −\$1404.80	
i^* = 20.91%	PW = −\$1.11 ≃ \$0	(IRR value)

In addition, for i = 0 percent,

$$PW(0) = -\$21{,}000 + \$6350 + 2(\$12{,}600) = \$10{,}550$$



GURE 5.3
omputer solution
hen i = 15 percent
Example 5.3.

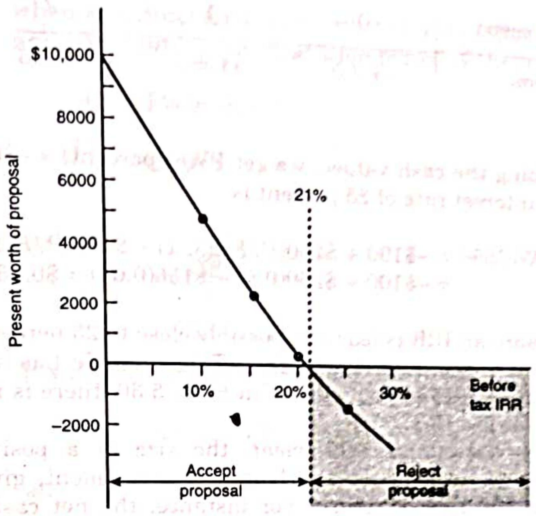


FIGURE 5.4
Present worth of cash flow for the proposal of Example 5.3 at different interest rates.

A plot of the PW(i) versus i is shown in Fig. 5.4. At any MARR value below 21 percent, the present worth of the proposal is positive and therefore acceptable. For a minimum attractive rate of return greater than 21 percent, the proposal exhibits a negative present worth and is therefore rejected.

5.3.3 Clues for IRR Calculations

Even though most analysts will use CHEER or spreadsheet functions to compute IRR values, we should comment on how we might possibly simplify the trial-and-error manual process, if only for completeness. There is really no way to avoid the trial-and-effort search procedure for manually determining the IRR in problems with complex cash flows, although an approach for simple problems will be mentioned shortly. A little preplanning may narrow the area of search.

The first maneuver to avoid unnecessary computations, if we have a simple investment, is to sum the cash flows. A negative total may indicate that the proposal being considered cannot meet even an i = 0 percent requirement, thereby eliminating it from further consideration whenever a positive IRR is required. This will happen only with the common simple investment case where we have investment at the beginning of the project followed by incomes. It is possible in unusual cases for PW(0 percent) to be negative while the PW for a higher interest rate is positive because of discounting the cash flow to the present. For example, consider the following cash flow:

End of period	0	1	2
Cash flow	−\$100	+\$250	−\$156

Summing the cash values, we get PW(0 percent) = −\$6. The present worth at an interest rate of 25 percent is

$$\begin{aligned} PW(25\%) &= -\$100 + \$250(P/F, 25, 1) - \$156(P/F, 25, 2) \\ &= -\$100 + \$250(0.8) - \$156(0.64) = \$0.16 \end{aligned}$$

Therefore an IRR is feasible, possibly close to 25 percent. Problem 5.30 will ask you to evaluate this further. This example has a cash flow that is a nonsimple, and as you will see in Prob. 5.30, there is more than one possibility for the IRR.

For the *simple investment*, the size of a positive net sum, with respect to the amount and length of investment, gives a rough suggestion of the rate of return. For instance, the net cash flow at $i = 0$ percent is \$10,550 in Example 5.3, and the initial investment is \$21,000 for 3 years. This 50 percent return on investment over 3 years suggests a substantial IRR.

A quick proximity fix on the IRR is possible for problems that have their major cash flows at the beginning and end of the study period, or those consisting largely of constant cash flow streams. When the salvage value is close to 100 percent of the first cost, the net annuity divided by the first cost gives a close approximation to i; that is, $A/P \simeq i$. Also, as demonstrated in Example 5.1, if most of the income is at year N and is about twice the initial outlay, then $i \simeq 72/N$.

The more variation in cash flows, the more difficult the guessing game gets. Sometimes irregular cash flows can be rounded to approximate an ordinary annuity, or individual transactions within short time intervals can be lumped together to allow gross preliminary calculations that suggest the vicinity of the IRR. For instance, in Example 5.3, the irregular receipts could be approximated by an average A of, say, \$10,000. Then, A/P = \$10,000/\$21,000 = 0.4761. With this figure as an entry to an interest table for a capital recovery factor at $N = 3$, $(A/P, 20, 3) = 0.47473$ gives a good place to begin IRR trial computations. Of course, with the computer program, we do not have to worry about the trial-and-error procedure. To repeat what was said earlier, the computer program should be used for real problems, but the user should have a firm understanding of what is entailed in IRR calculations. Only by working with the trial-and-error manual process can this understanding be attained.

It is possible to determine the IRR directly for very simple situations. For example, the most simple situation might have

$$PW = -CI + FW(P/F, i, N) = 0$$

when searching for IRR (i), where CI = capital investment at time period 0 and FW = future worth at time period N. Rearranging terms gives

$$CI = FW(P/F, i, N)$$

$$\frac{CI}{FW} = (P/F, i, N) = \frac{1}{(1+i)^N}$$

and

$$(1+i)^N = \frac{FW}{CI}$$

If we take natural logarithms of both sides of the equation, we get

$$N \ln(1+i) = \ln FW - \ln CI$$

and

$$\ln(1+i) = \frac{\ln FW - \ln CI}{N}$$

For example, let FW = \$150, CI = \$100, and $N = 3$. Then

$$\ln(1+i) = \frac{\ln 150 - \ln 100}{3} = \frac{5.01 - 4.61}{3} = 0.13$$

Finally, $1 + i = e^{0.13} = 1.14$, and

$$i^* = (1.14 - 1)(100\%) = 14\% = \text{IRR}$$

Unfortunately, the equation with i as an unknown is a polynomial with an order equal to the number of terms in the equation with i. This is an intractable situation for any problem of realistic complexity. Therefore, a trial-and-error procedure is often mandated.

A graphical approach can greatly simplify the problem, especially when it is used with a variety of IRR values in conjunction with the text's computer program to determine intermediate values of PW. Figure 5.6 will show such a graphical representation, and details for obtaining the graph are given later. Also, the computer program may be used directly to determine the solution for problems where only one IRR value exists.

5.4 CONSISTENCY OF IRR WITH OTHER ECONOMIC COMPARISON METHODS

The acceptability of alternative courses of action will be identical whether they are evaluated according to their annual worth, present worth, or

TABLE 5.1

Estimated cash flows (all costs) for alternative operating plans, $

Year	Plan A	Plan B	Plan C
0		30,000	25,000
1	20,000	15,000	12,000
2	20,000	15,000	12,000
3	20,000	15,000	12,000
4	20,000	15,000	12,000
5	20,000	15,000	37,000
6	20,000	15,000	12,000
7	20,000	15,000	12,000
8	20,000	15,000	12,000
9	20,000	15,000	12,000
10	20,000	15,000	12,000

incremental IRR. Ranking alternatives by individual IRR values will usually not be consistent with PW rankings as will be shown in Sec. 5.5.1. The important points to understand are the meaning of the measures of acceptability and the assumptions upon which they are based. Sample calculations applying all three comparison methods to the data given in Table 5.1 will reveal the consistency of results. They will also illustrate how the rate of return is calculated to compare investments when only the disbursements are known and how to interpret the outcomes.

Suppose that a certain function is currently being performed at an annual expense of $20,000. Three mutually exclusive alternatives are being considered. One alternative, plan *A*, is to leave the operation unchanged. In effect, this is the do-nothing alternative that is almost always present in a decision situation. A second alternative, plan *B*, is to invest in layout modifications that will allow the function to be performed at a reduced labor cost of $15,000. The expense of the renovations must be recovered in 10 years according to operating policy. Plan *C* is a proposal to install a labor-saving device that will cut the labor cost to $12,000. The device has a first cost of $25,000, will be worn out in 5 years, and has no salvage value. Table 5.1 is a year-by-year tabulation of the cash flow for the three plans. If the company's minimum acceptable rate of return is 8 percent, which plan offers the greatest economic benefit?

AW and PW comparisons

Using the already familiar procedures for computing the equivalent annual worth (cost) of a cash flow stream, we find that the annual cost for the current operating method, plan *A*, is read directly from the table:

$$\text{EAC(plan } A) = \text{labor expense} = \$20{,}000$$

In plan *B*, the initial investment is spread over the 10-year study period and is added to the annual labor expense to get

$$\begin{aligned}\text{EAC(plan } B) &= \$30{,}000(A/P,\ 8,\ 10) + \$15{,}000\\ &= \$30{,}000(0.14903) + \$15{,}000 = \$19{,}471\end{aligned}$$

Since the study period constitutes two cycles of the 5-year economic life of the labor-saving device in plan *C*, the annual cost will be the same over each 5-year period [assuming first costs have been corrected for inflation (constant dollars) and the technology is stable]:

$$\begin{aligned}\text{EAC(plan } C) &= \$25{,}000(A/P,\ 8,\ 5) + \$12{,}000\\ &= \$25{,}000(0.25046) + \$12{,}000 = \$18{,}262\end{aligned}$$

Thus, plan *C*, with the lowest annual cost, is preferred. Compared with the currently existing plan *A*, an investment of $25,000 in a labor-saving device will yield a return of 8 percent per year plus the equivalent receipt of $20,000 – $18,262 = $1738 each year from savings in labor expense for 5 years.

The equivalent present worth (cost) of the three plans is calculated by simply multiplying each EAC by the uniform series present-worth factor $(P/A,\ 8,\ 10) = 6.71008$:

$$\text{PW(plan } A) = \$20{,}000(6.71008) = \$134{,}202$$

$$\text{PW(plan } B) = \$19{,}471(6.71008) = \$130{,}652$$

$$\text{PW(plan } C) = \$18{,}262(6.71008) = \$122{,}539$$

Since a lower present cost is preferred, plan *C* again gets the nod, as expected. This means that over a 10-year period when money is worth 8 percent, plan *C* is expected to cost $134,202 – $122,539 = $11,663 less in *today's dollars* to accomplish the same operation now being done under plan *A*. This total saving is, of course, the present worth of the annual gain beyond the 8 percent return calculated in the EAC comparison:

$$\$1738(P/A,\ 8,\ 10) = \$1738(6.71008) = \$11{,}662$$

IRR comparison

The data given in Table 5.1 provide examples of IRR comparisons that do not have natural cash flows which are positive (all values are costs). This is handled with *incremental* analysis as defined earlier. The positive cash flow stream is developed from the savings generated by each additional increment of investment. A more detailed discussion of incremental analysis is presented in Chap. 6, but it is sufficient here to understand that *each increment of capital expended must be justified in itself.*

Logically, we start with the smallest investment over the do-nothing alternative (plan *A*), which is the $25,000 outlay for the labor-saving device in plan C. The "earnings" from this investment are the annual reductions in labor expense: \$20,000 − \$12,000 = \$8000. The incremental cash flow is seen to be a *simple* investment, so we do not have to be concerned with an external interest rate. By using the program CHEER, it is necessary to input only a first cost of \$25,000 and a revenue of \$8000 for each of years 1 through 5. The resultant IRR_{C-A} is 18.03 percent, indicating that plan C is preferred to plan A when the required rate of return is 8 percent. Why can we say this? From the way we compute incremental data it should be clear that

$$\text{Cash flow}(C) = \text{cash flow}(A) + \text{cash flow}(C - A)$$

Therefore, if the lowest investment plan is satisfactory (which we assume it is when we are dealing only with costs, as occurs with the do-nothing plan A), the next-lowest alternative has to be an even better alternative if the incremental cash flows satisfy the MARR since cash flow (C) will have the return from the increment (C − A) plus the return from cash flow *(A)*.

Although it is already known from the EAC and PW comparisons that plan C is preferred to plan B, it can be checked by determining the IRR for the \$30,000 investment in plan B over the currently accepted plan C. The incremental cash flow (B − C) has a \$30,000 − \$25,000 = \$5000 incremental *investment* in B over C followed by incremental cost *increases* of \$15,000 − \$12,000 = \$3000 per year with the exception of a net savings of \$25,000 in year 5. The costs so far outweigh the savings in year 5 that the PW will be less than zero for any interest rate. Therefore, IRR_{B-C} is less than the MARR of 8 percent, and plan B is rejected. Plan C should be recommended.

5.5 IRR MISCONCEPTIONS

list & explain

The consistency of AW and PW comparisons is above reproach, and both always agree with IRR evaluations when done correctly. Unfortunately, there are some misconceptions that should be clarified.

5.5.1 Ranking Alternatives by Individual IRR Values

choosing

ranking IRR values can conflict PW ranking

The reader might wonder why incremental analysis was used in the previous example when it might seem that ranking individual alternative IRR values would be the way to go. Ranking alternatives on their individual IRR values can conflict with PW rankings. For example, suppose we have two projects with the cash flows indicated in Table 5.2. Both proposals require the same \$1000 initial investment. The contrasting net annual returns are conspicuous; project X starts low and increases, whereas project Y has a high first-year flow followed by constant lower flows.

The two projects are first compared by their present worths when the minimum required rate of return is 10 percent:

TABLE 5.2

Cash flows for two mutually exclusive projects with 4-year lives and no salvage value

	End-of-Year Cash Flow, \$				
Project	**0**	**1**	**2**	**3**	**4**
X	−1000	100	350	600	850
Y	−1000	1000	200	200	200

$$\begin{aligned} PW(X) &= -\$1000 + [\$100 + \$250(A/G, 10, 4)]\,(P/A, 10, 4) \\ &= -\$1000 + [\$100 + \$250(1.38117)]\,(3.16987) \\ &= \$411.52 \end{aligned}$$

$$\begin{aligned} PW(Y) &= -\$1000 + [\$1000 + \$200(P/A, 10, 3)]\,(P/F, 10, 1) \\ &= -\$1000 + [\$1000 + \$200(2.48685)]\,(0.90909) \\ &= \$361.24 \end{aligned}$$

This *ranks project X higher than project Y.*

When a comparison is made that uses individual IRRs, the rankings are reversed, as shown by the following calculations:

For project X:

$$PW = -\$1000 + [\$100 + \$250(A/G, i, 4)]\,(P/A, i, 4) \stackrel{?}{=} 0$$

At i = 20 percent,

$$PW = -\$1000 + [\$100 + \$250(1.27422)]\,(2.58873) = \$83.53$$

and at i = 25 percent,

$$PW = -\$1000 + [\$100 + \$250(1.22493)]\,(2.36160) = -\$40.64$$

Interpolating for PW = 0, we get IRR(X) = 23.4 percent.

For project Y:

$$PW = -\$1000 + [\$1000 + \$200(P/A, i, 3)]\,[P/F, i, 1] \stackrel{?}{=} 0$$

This results in IRR = 34.26 percent that *ranks project Y ahead of project X.*

The net present-worth profiles for the two projects are shown in Fig. 5.5. This shows that the present worth of X exceeds that of Y at i = 10 percent, and the internal rate of return is higher for Y than X when PW = 0. An *incremental* analysis is the way to handle the problem.

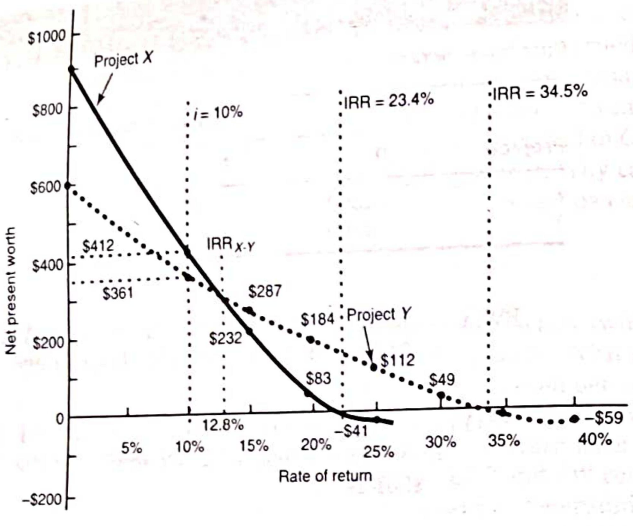


FIGURE 5.5 Relationship of net worth and different discount rates, showing how the rankings for two projects can change.

Using the original yearly data from Table 5.2, we can get year-by-year differences in X and Y cash flows. These are shown in Table 5.3. Now, using the data from Table 5.3, we can find the IRR that makes project X equivalent to project Y (use $X - Y$ column):

$$\text{PW} = -\$900 + \$150(P/F,\ i,\ 1) + \$400(P/F,\ i,\ 2) + \$650(P/F,\ i,\ 3) \overset{?}{=} 0$$

CHEER finds $\text{IRR}_{X\text{-}Y}$ to be 12.8 percent, which is greater than the MARR of 10 percent, and so alternative X is acceptable. The IRR of 12.8 percent is called the *incremental rate of return,* and it makes project X equivalent to project Y. We see in Fig. 5.5 that the intersection of the present-worth

TABLE 5.3

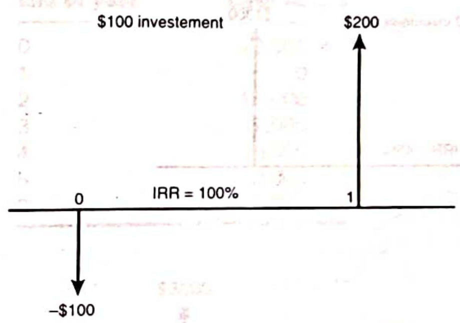
Yearly differences in X and Y cash flows, $

Year	Project X	Project Y	Project X − Y
0	−1000	−1000	0
1	100	1000	−900
2	350	200	150
3	600	200	400
4	850	200	650

curves occurs at the $\text{IRR}_{X\text{-}Y}$ of 12.8 percent ($\text{PW}_{X\text{-}Y} = 0$). This, of course, is the incremental rate of return we just calculated.

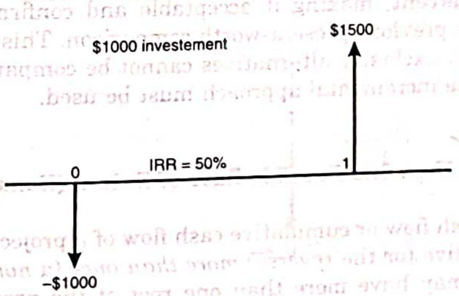
This intersection is the pivot point for selecting the superior alternative. Select X when the required rate of return is equal to or less than 12.8 percent, and select Y when the required rate is more than 12.8 percent but less than 35 percent. This selection rule is consistent with the PW comparison at $i = 10$ percent, which indicated a preference for X. A similar PW comparison at $i = 14$ percent indicates a preference for Y.

As another example to support the selection procedure, assume that an investment of $100 returns $200 at the end of 1 year; hence,



$$\text{PW(\$100 investment)} = -\$100 + \$200(P/F,\ i,\ 1)$$
$$= 0 \quad \text{at IRR} = 100\%$$

Let another investment of $1000 return $1500 in 1 year; hence,



$$\text{PW(\$1000 investment)} = -\$1000 + \$1500(P/F,\ i,\ 1)$$
$$= 0 \quad \text{at IRR} = 50\%$$

Thus the $100 investment seems to be more attractive if it is ranked by the internal rates of return. However, if the MARR is only 10 percent,

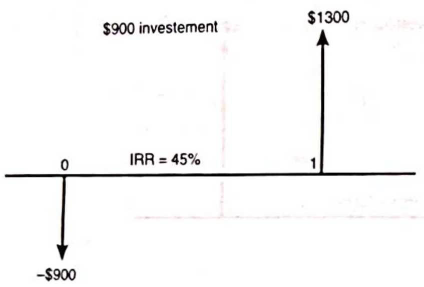
$$PW(\$100 \text{ investment}) = -\$100 + \$200(P/F, 10, 1) = \$81.82$$

and

$$PW(\$1000 \text{ investment}) = -\$1000 + \$1500(P/F, 10, 1) = \$363.64$$

which seems to indicate a preference for the $1000 investment.

In actual fact, the reason that the $1000 investment is superior may be better understood by considering the IRR on the additional $900 increment of the investment:



$$\begin{aligned} PW(\$900 \text{ increment}) &= -\$1000 - (-\$100) + (\$1500 - \$200)(P/F, i, 1) \\ &= 0 \quad \text{at IRR} \\ &= -\$900 + \$1300(P/F, 44.6, 1) = 2.78 \simeq 0 \end{aligned}$$

Thus, the additional $900 investment has an internal rate of return of 44.6 percent (found with CHEER), which well exceeds the minimum required rate of 10 percent, making it acceptable and confirming the preference shown by the previous present-worth comparison. This proves, by example, that mutually exclusive alternatives cannot be compared by their individual IRRs. The incremental approach must be used.

2

5.5.2 More Than One Possible Rate of Return (Nonsimple Investment)

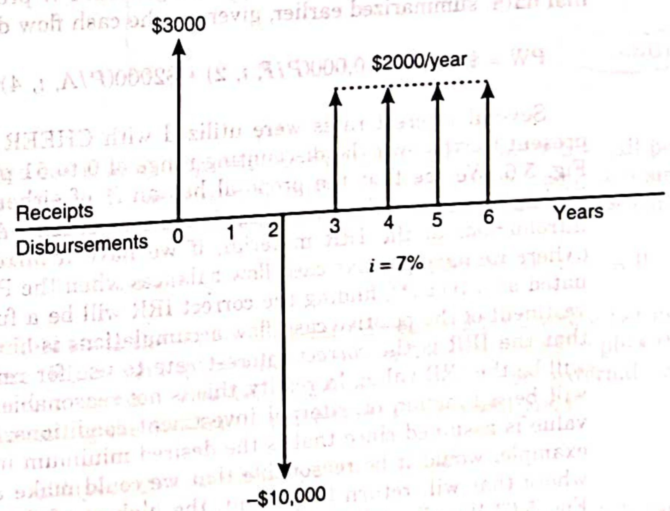
When the cash flow or cumulative cash flow of a project switches from negative to positive (or the reverse) *more than once* (a *nonsimple investment*), the project may have more than one root of the present-worth equation $PW(i) = 0$. In such cases we have to determine which root is, in fact, the true IRR value. In a single project such situations will occur relatively rarely in practice, although they do occur. When incremental analysis is used to compare mutually exclusive alternatives, especially for projects with unequal lives, this becomes much more common.

EXAMPLE 5.4 **Two Solutions for an IRR Evaluation**

To introduce the multiple-root problem, consider a rather contrived single-project situation. One of the alternatives for improving an operation is to do nothing to it for 2 years and then spend $10,000 on improvements. If this course of action is followed, the immediate gain is $3000 (income) followed by 2 years of break-even operations. Thereafter, annual income should be $2000 per year for 4 years. What rate of return can be expected from following this course of delayed action?

Solution

End of year	Cash flow, $
0	+3,000
1	0
2	-10,000
3	2,000
4	2,000
5	2,000
6	2,000



The casn flow suggests that there might be multiple roots for $PW(i)$, i^*, since the cash flow reverses from positive to negative at year 2 and again reverses signs at year 3. Since a double sign reversal is not always accompanied by dual rates of return, trials can be conducted at arbitrarily

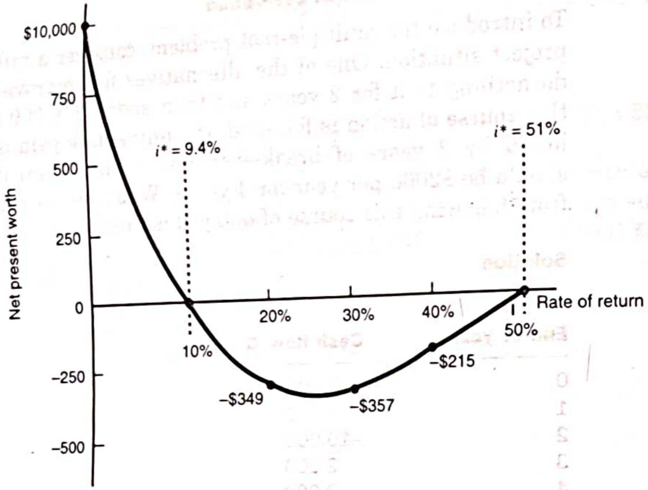


FIGURE 5.6
Net present worth of a proposal with multiple roots for PW (i) = 0.

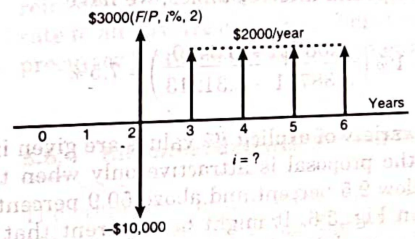
selected interest rates to determine the general PW profile. Using the original data, summarized earlier, given on the cash flow diagram, we get

$$PW = \$3000 - \$10{,}000(P/F, i, 2) + \$2000(P/A, i, 4)(P/F, i, 2) \overset{?}{=} 0$$

Several interest rates were utilized with CHEER to give a profile of present worths over the discounting range of 0 to 51 percent, as shown in Fig. 5.6. We see that the proposal has an i^* of either 9.4 or 51 percent. Which, if either, of the two values is the correct IRR? As mentioned in the introduction to the IRR material, if we have a *mixed* investment case (where we have positive cash flow balances when the PW equation is evaluated at a root i^*), finding the correct IRR will be a function of how reinvestment of the positive cash flow accumulations is handled. If we assume that the IRR is the correct interest rate to use for reinvestment, then i^* will be the IRR value. In reality, this is not reasonable since reinvestment will be a function of external investment conditions; usually the MARR value is assumed since that is the desired minimum investment rate. For example, would it be reasonable that we could make an investment elsewhere that will return 51 percent, the highest of the i^* values shown in Fig. 5.6? We all wish that could be true. The true rate might be in the vicinity of 10 percent or so. In Sec. 5.5.5 we introduce the recommended method, the project balance method (PBM), to use in finding the true IRR. First we will look at two historical approaches that preceded the PBM method.

5.5.3 Explicit Investment Rate

One approximate answer to the IRR question in Example 5.3 is developed by applying an *explicit interest rate* to a limited portion of the cash flow that will disturb the total cash flow pattern as little as possible while eliminating one of the sign reversals. An explicit reinvestment rate is a designated interest percentage appropriate for a specific application. The *explicit reinvestment* rate may be the minimum attractive rate of return employed by the organization or a rate suggested by the PW profile. As an example, let us assume that the $3000 receipt at the beginning of year 1 is assumed to be invested at an explicit interest rate for 2 years. Under this assumption, one sign reversal is avoided (at year 2), and the modified cash flow diagram is as follows:



We see that by applying the explicit interest rate to a small portion of the cash flow we have transformed a nonsimple investment to a simple investment. Now we can modify the original present-worth equation:

$$PW = \$3000(F/P, i\%, 2) - \$10{,}000 + \$2000(P/A, i, 4) \overset{?}{=} 0$$

where $i\%$ is the explicit interest rate applied for 2 years. Let us determine the IRR for an explicit interest rate of 5 percent. This implies that only a 5 percent return can be confidently obtained on external investments. Investing the $3000 at 5 percent for 2 years will yield

$$FW(\text{year } 2) = \$3000(F/P, 5, 2) = \$3308$$

But at the beginning of year 2 we also have a $10,000 disbursement, and so the cash flow at the beginning of year 2 is

$$-\$10{,}000 + \$3308 = -\$6692$$

The cash flow over years 1 through 6 would now be as follows:

Year	0	1	2	3	4	5	6
Cash flow, $	0	0	−6692	2000	2000	2000	2000
Cumulative cash flow, $	—	—	−6692	−4692	−2692	−692	+1308

The IRR for the modified cash flow pattern is simply the interest rate that makes an initial investment of \$6692 equivalent to a \$2000 annual annuity for 4 years:

$$PW = -\$6692 + \$2000(P/A, i, 4) = 0 \quad \text{at IRR}$$

$$(P/A, i, 4) = \frac{\$6692}{\$2000} = 3.3460$$

So, using the compound-interest tables, we have

$$IRR = 7\% + 1\%\left(\frac{3.38721 - 3.3460}{3.38721 - 3.31213}\right) = 7.5\%$$

The IRRs for a variety of explicit $i\%$ values are given in Table 5.4, and it is apparent that the proposal is attractive only when the explicit reinvestment rate is below 9.5 percent and above 50.9 percent, the same result as shown earlier in Fig. 5.6. It might be apparent that the text's computer program greatly simplifies the determination of the IRR values in Table 5.4. Using Table 5.4, we see that when funds can be invested externally at, say, 15 percent, there is no incentive to invest in a proposal that returns

TABLE 5.4

Different IRR percentages resulting from explicit reinvestment rates used on a limited portion of cash flow to convert dual rates of return to single IRR

Explicit reinvestment rate applied to \$3000 payment for 2 years, %	IRR on net investment when explicit reinvestment rate is utilized, %
0	5.6
5	7.5
9.4	9.5
15	12.3
20	15.6
30	22.6
40	33.1
51	51.2
60	77.6

only 12.4 percent. If the external funding rate were 5 percent, as we used earlier, the IRR (7.5 percent) would be attractive.

The use of an explicit interest rate may seem like an artificial device to alleviate a mathematical difficulty, but the concept is both realistic and reasonable. Funds received from an ongoing project are indeed reinvested in new projects that have passed the MARR criterion. These funds then earn at least the required rate, but it would be unrealistic to expect them to earn an enormously higher rate, such as 51 percent suggested by the sample problem. However, if 51 percent were the actual external reinvestment rate, then the given cash flow pattern would also meet that criterion.

One major problem with the use of the explicit interest rate just described is the fact that where the rate is applied is somewhat arbitrary. Our contrived example worked correctly because the only positive cash flow value in the original situation occurred at time 0. Now we will look at a second historical reinvestment method (the HERR method) that applies the explicit interest rate to all positive cash flows. This has a major flaw, but historically it is the precursor for the project balance method, covered in Sec. 5.5.5.

5.5.4 Historical External Rate-of-Return Method

The occurrence of multiple i^* roots with the nonsimple investment return can be avoided by using the *historical external rate-of-return (HERR) method* where the main appeal is its pragmatic assumption that receipts are actually reinvested at a generally available interest rate. This rate is typically taken to be the MARR. The flaw is that this method does not base the reinvestment on project cash flow balances; it is based solely on project receipts.

An unknown rate of return e' is found by equating the future worth of receipts (positive cash flows) compounded at an explicit interest rate ($i\%$) to the future worth of disbursements (negative cash flows) compounded at e':

$$\text{FW(receipts compounded at } i\%) = \text{FW(disbursements compounded at } e')$$

When $i\%$ is the MARR and e' exceeds $i\%$, the investment is assumed to be attractive because it promises a yield greater than the lower limit of acceptability.

EXAMPLE 5.5 **Example 5.4 Revisited with a HERR Evaluation**

Evaluate the cash flow described in Example 5.4 when receipts are reinvested at the MARR of 15 percent.

Solution

Given $i\%$ = 15 percent, the future worths of receipts and of disbursements are equated

$$\$3000(F/P, i\%, 6) + \$2000(F/A, i\%, 4) = \$10{,}000(F/P, e', 4)$$

from which

$$(F/P, e', 4) = \frac{\$3000(2.31306) + \$2000(4.99338)}{\$10{,}000}$$
$$= \frac{\$6939.18 + \$9986.76}{\$10{,}000}$$
$$= 1.69259$$

By interpolation, this produces an e' value of 14.1 percent. Since e' is less than the MARR, the investment is unacceptable under the HERR assumptions.

Now we will look at a technique where the external rate (MARR) is applied in a more systematic and logical manner.

5.5.5 Project Balance Method

...d to ...ulative cash balances

An approach more logical than the traditional HERR methodology is the *project balance method (PBM)* that has the HERR method as a historical precedent but that applies the IRR and ERR (external investment rate, usually the MARR) to *cumulative cash balances* that include interest to date. Note that the acronymn *ERR* is now used correctly, which is why we renamed the external rate-of-return method the *historical* rate-of-return method. The PBM has evolved over many years, but Park† has nicely summarized the steps in the reinvestment problem [only needed if we have a nonsimple investment case—the cash flow sequence has more than one sign change (plus to minus or minus to plus) from period to period]:

1. Determine i^* (or multiple i^* values for the cash flow sequence). If there are multiple i^* values, it is suggested that the one closest to the MARR value be used in the subsequent analysis.
2. Determine the *current balance (CB)* for each period, using any of the determined i^* values. These we will call $CB(i^*)_t$, where t is the end of the period for which CB is being calculated.

†C. S. Park, *Contemporary Engineering Economics Text with 3½" disk*, © 1993, Addison-Wesley Publishing Company, Inc., Reprinted by permission of the publisher.

If all $CB(i^*) \le 0$, then internal rate, true

3. If all $CB(i^*)_t$ are equal to or less than zero, receipts in all periods t are being used to pay off project investments and so are assumed to be used *internally;* thus i^* is the true IRR value.
4. If any $CB(i^*)_t$ is positive and some are negative (so this is not pure borrowing), then the IRR is still not known. The MARR or other acceptable external rate of return will be applied to the positive $CB(IRR/ERR)_t$, which is funds in excess of those being applied to the project's investment; and i^* will be applied to the negative $CB(IRR/ERR)_t$. We do know that we have a mixed investment—both internal and external investment.
5. Iterate through the periods with an assumed IRR and the explicit external interest rate. Apply the IRR to negative balances and the explicit rate to positive balances. If $CB(IRR/ERR)_N$ is equal to zero, we have the true IRR value which should be used in project evaluations.

Mixed +ve & -ve then IRR not known
external rate applied to +ve CB & i^* to -ve CB

If CB = 0, then true, used in project

As an example, let us apply this process to the original Example 5.4 data, which had the following cash flow sequence:

End of period	Cash flow, $
0	3,000
1	0
2	−10,000
3	2,000
4	2,000
5	2,000
6	2,000

As with the last two solutions, let us assume that the MARR is 15 percent. Also, suppose we just use an arbitrary first estimate for the IRR, close to the MARR, that is, 13 percent.

End of period	Cash flow, $	Current balance $CB(IRR/MARR)_t$†, $		Comments
0	3,000	3,000		$CB_0 > 0$ (use MARR for $t = 1$)
1	0	$3{,}000(1 + MARR)^1 = 3{,}450$		$CB_1 > 0$ (use MARR for $t = 2$)
2	−10,000	$3{,}450(1 + MARR)^1 - 10{,}000 =$	−6,032.5	$CB_2 < 0$ (use IRR for $t = 3$)
3	2,000	$-6{,}032.5(1 + IRR)^1 + 2{,}000$	= −4,816.73	All subsequent $CB_t \le 0$, so IRR is used
4	2,000	$-4{,}816.73(1 + IRR)^1 + 2{,}000 =$	−3,442.9	
5	2,000	$-3{,}442.9(1 + IRR)^1 + 2{,}000$	= −1,890.48	
6	2,000	$-1{,}890.48(1 + IRR)^1 + 2{,}000 =$	−1,36.25	

†Since we used MARR as the specific value for ERR.

Since CB(IRR/MARR)$_6$ is less than zero, the IRR is too high and needs to be reduced. By trial and error we find IRR = 12.34 percent. A spreadsheet approach that facilitates the PBM is given in App. C and is available on this text's diskette. We went through these iterations:

$$\begin{array}{lll} IRR_{est} = 12.8\% & CB(12.8)_6 = -\$95.12 & \\ IRR_{est} = 12.6\% & CB(12.6)_6 = -\$54.29 & \\ IRR_{est} = 12.4\% & CB(12.4)_6 = -\$13.77 & \\ IRR_{est} = 12.3\% & CB(12.3)_6 = \$6.39 & \\ IRR_{est} = 12.34\% & CB(12.34)_6 \simeq \$0 & \text{true IRR} = 12.34\% \end{array}$$

Earlier we found the external interest rate e' to be 14.1 percent, but MARR was applied to all revenues whereas the project balance method applies them correctly only to positive *balances*, which shows why the IRR is reduced. Since the IRR is less than the MARR, the project is not viable.

If we apply the project balance reinvestment procedure to the explicit return example (Example 5.4), using the original MARR of 5 percent and the original IRR of 7.5 percent we get the following:

Period	Cash flow, $	CB(7.5/5), $	CB(7.5/5)$_t$ calculations, $
0	3,000	3,000	Cash flow at period 0
1	0	3,150	3,000(1.05) + 0
2	−10,000	−6,692.5	3,150(1.05) − 10,000
3	2,000	−5,194.44	−6,692.50(1.075) + 2,000
4	2,000	−3,584.02	−5,194.44(1.075) + 2,000
5	2,000	−1,852.82	−3,584.02(1.075) + 2,000
6	2,000	8.22	−1,852.82(1.075) + 2,000

The current balance at period 6 is almost $0. In fact, if we had used IRR = 7.541 percent, we would have realized $0. For this case only, since the earlier explicit interest rate was applied only to the first two periods, as is the case with the reinvestment PBM, we get identical results.

To complete this section on IRR determination, we summarize the overall process with a flowchart (Fig. 5.7). When the incremental cash flow analysis is accomplished, as many passes through the flowchart as there are alternatives will be required. The same is true for independent alternative analysis. Only one mutually exclusive alternative can be accepted—the one with the highest investment whose incremental IRR is greater than MARR, assuming that analysis is accomplished from the lowest investment alternative monotonically to the highest investment alternative. All independent alternatives that satisfy the IRR > MARR criterion

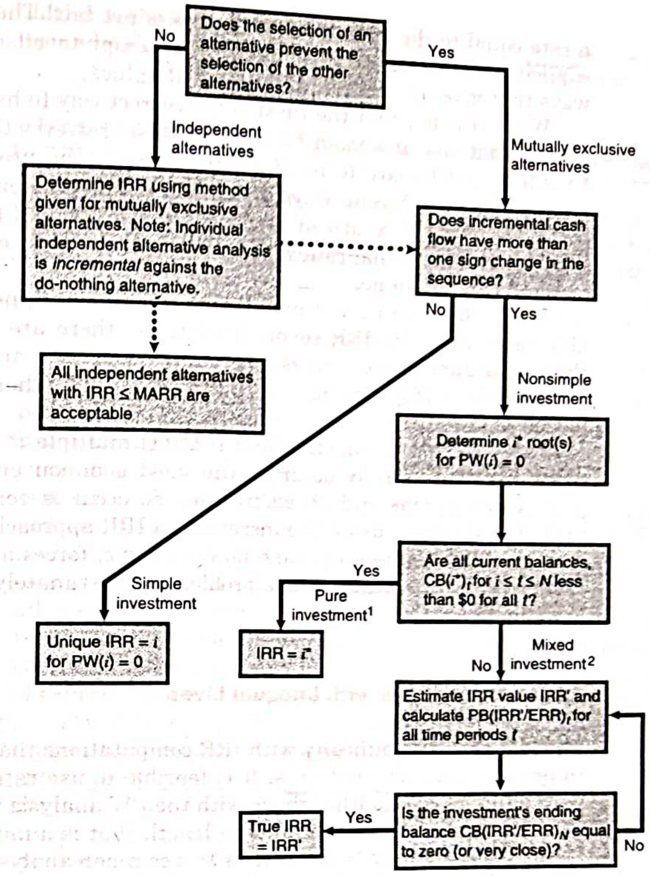


FIGURE 5.7 Flowchart of the IRR determination process. As many passes as there are alternatives will have to be made through the process.

[1]All CB(i) ≤ $0 means that receipts are being used to pay off project investments.
[2]MARR or other acceptable external rate of return (ERR) applied to positive CBs: IRRs to negative CBs.

are accepted. Management will have to apply different measures if there is insufficient capital to fund all the acceptable alternatives.

5.5.6 Reinvestment Question

We have now seen several approaches to the reinvestment question as it affects the calculation of the IRR. The basic question boils down to whether it is fair to assume that external balances from a project can be invested at a rate equal to the IRR. In general, this is not fair! The PBM, HERR, and explicit interest rate ($i\%$) approaches attempt to offset the problems in ways that give, in general, different IRR values.

While it is felt that the PBM is the correct way to handle mixed-investment situations, it should be of interest to review other points of view. Excellent articles are to be found that review the problem and in many cases suggest different ways to alleviate the problem.† All recommend using two rates of return in the calculations: the IRR that is being determined and an external rate which may or may not be equal to a more logical and current value.

Is IRR right? External rate?

While the logic for a reinvestment rate more aligned to current available rates than the IRR seems irrefutable, there are arguments against the use of dual interest rates, especially when both are used in the same period, which they are not with the PBM and other methods that we looked at.‡

Other logical arguments hold that (1) multiple i^* roots, the cause of many problems, rarely occur in the most common engineering economy project evaluations and (2) where they do occur is generally for multiple project evaluations using the incremental IRR approach. The evaluation of a single project, where the cash flow is *simple*, forces all $CB(i)_t$ to be negative or zero, thus negating the problem. Unfortunately, not all cash flows are simple.

dual i values rarely occur
whereas they do, is multiple project eval using inc approach

3

5.5.7 Alternatives with Unequal Lives

The last potential difficulty with IRR computations that we discuss relates to projects with unequal lives. It is feasible to use rate-of-return analyses with different lives, although, as with the PW analysis we did in Chap. 3, it is necessary to find an overall time length that is a multiple of the lives of the alternatives or to truncate lives to a common analysis period, the period of need. For example, if we had three alternatives with lives of 3, 4, and 6 years, we would use an overall study period of 12 years and the alternatives would be replicated 4, 3, and 2 times, respectively. It turns out that this gets very messy with *manual* IRR determinations, and it is recommended that if and when this type of situation occurs, one of the previously discussed evaluation techniques (PW, FW, EAW) be utilized instead. Using CHEER or a spreadsheet application will greatly simplify the problem.

LCM & truncation but

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†See, e.g., J. Lohmann, "The IRR, NPV and the Fallacy of the Reinvestment Assumptions," pp. 303–330, and R. G. Beaves, "Net Present Values and Rates of Return; Implicit and Explicit Reinvestment Assumptions," pp. 275–302, both in *The Engineering Economist*, vol. 33, no. 4, Summer 1988. See also R. H. Bernhard, "Income, Wealth Base and Rate of Return Implications of Alternative Evaluation Criteria," *The Engineering Economist*, vol. 38, no. 3, pp. 165–175, Spring 1993. Publisher is the Institute of Industrial Engineers.

‡E. L. Grant, W. G. Ireson, and R. S. Leavenworth, *Principles of Engineering Economy*, John Wiley & Sons, New York, 1990.

5.6 FINAL COMMENTS ON THEORY AND PRACTICE BEHIND INTEREST RATES

① Rate that an organisation sets for using its money depends on its economic cond & policies
② Supply & demand relationship
③ Diagram explanation

A minimum attractive rate of return sets a price on money. Prices are rationing devices. The rate (or price) that an organization sets for the use of its money reflects its internal economic conditions, such as the amount of capital accumulated from successful operations, and external factors, such as the total supply of loanable capital in the whole economy and current or anticipated government fiscal policies.

A supply-and-demand relationship is responsible for a "market" rate of interest under traditional capital theory. This single theoretical rate is the percentage return yielded by any riskless bond or other security. Since capital is assumed to be subject to the law of diminishing returns, its demand curve D takes the general shape shown in Fig. 5.8. At a certain time, the demand curve is intersected by an inelastic supply line S that represents the amount of capital then available. The intersection point is the temporary interest rate at which projects yielding that rate of productivity (measured as a percentage) are funded. This short-run equilibrium point implies that all projects with greater net productivity have already been funded and those that will yield a lower rate must await the accumulation of additional capital. The graph also suggests that interest rates will decline as the pool of capital increases. This, historically, has not always been the case.

④ The relationship of supply and demand in Fig. 5.8 is probably more representative of an individual firm than of the economy as a whole. Shifting expectations, technological innovations, inflation, and other factors impinge on traditional capital theory to thwart the stability of conceived ⑤ investment relations. Yet the theory does offer a simplified explanation of the purpose served by interest rates. Money is "hired" by businesses to buy capital goods. The productivity of these goods must at least pay the "wages" of the money used to acquire the goods, and attaining this wage level sets a lower limit on accepting investment proposals.

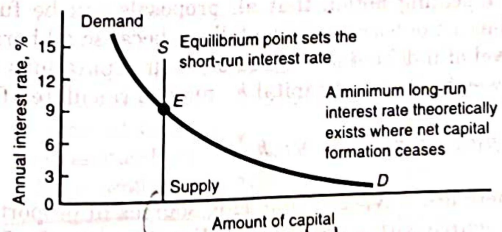


FIGURE 5.8 Traditional capital theory suggests that a current rate of interest is a function of the amount of capital available and the productivity of capital expected at that level of supply.

capital available at one pt of time

Although it is convenient to theorize a single interest rate for the whole economy, there exist, of course, many different interest rates. Reasons for the differences among loans include the following:

- Higher interest charges are applied when there is greater risk that a loan will not be repaid.
- Charges for long-term loans are usually greater than those for short-term loans because a lender forgoes the opportunity to take advantage of alternative uses of the money for a longer period and incurs greater risk from potential changes in the economic environment.
- Administrative costs of lending are a higher percentage of a smaller loan than a larger loan.
- Local money markets vary, owing to regional differences; interest rates are often higher in small towns than in large cities because borrowers find it less convenient to shop for better rates.

5.6.1 Cost-of-Capital Concepts

The cost of capital is seldom, if ever, determined by the engineering economist. Cost of capital will be determined by the chief financial officer (or someone reporting to the chief executive officer) in conjunction with the accounting department. The cost of capital is derived from the composition of the capital pool. The term *pool* is appropriately suggestive of capital from many sources, pooled for funding purposes. The proportion of capital from different sources obtained at different costs to a firm is represented by a *weighted cost of capital*. This weighted cost sets the lower bound for the minimum acceptable rate of return (MARR), as discussed earlier.

The actual rate of return expected from new investments is normally greater than the cost of capital. How much greater depends on the risk involved. Riskier proposals are subject to higher discount rates to compensate for the chance that they will not meet net-return expectations. But even a minimum-risk investment, such as the purchase of government bonds, must yield a return greater than the interest rate a firm is charged on its debts; otherwise, it would be sensible to direct currently available funds toward debt retirement in anticipation of more rewarding future investment opportunities.

The appealing notion that all proposals can be funded if their IRRs exceed the cost of borrowing is a fallacy, because all borrowers have a maximum level of indebtedness based on their reputations and resources.

The weighted cost of capital k_w may be calculated from the formula

$$k_w = p_1k_1 + p_2k_2 + \cdots + p_nk_n$$

where there are n types of financing sources in proportions p of total capital, each source with its own cost k. For instance, if a firm is financed from

bonds, preferred stock, and common stock with costs of 7, 9, and 12 percent and in proportions 20, 30, and 50 percent, respectively, then

$$\text{Weighted cost of capital } k_w = 7\%(0.2) + 9\%(0.3) + 12\%(0.5)$$
$$= 1.4\% + 2.7\% + 6\%$$
$$= 10.1\%$$

The cost of capital is troublesome to estimate, despite the apparent precision of its formula. There are differences of opinion about both costs and proportions, and naturally a number of variations to the general formula have been proposed.

The effect of taxes will be discussed in Chap. 9, but we should mention at this time that sources of capital are impacted differently by taxes and that all values stated here are assumed to be on an after-tax basis.

5.6.2 Sources of Funds

Even though engineers will generally not be responsible for obtaining funds for their enterprise, it will be an advantage for them to have an overview of fund sources. Financial management has traditionally been concerned with obtaining funds and learning how to use them. Although sources of funding are obviously important considerations in any private enterprise, since securing capital is a prerequisite to implementing an acceptable proposal, the primary focus of this text remains on the effective use of funds. Engineers and managers of operating units have responsibilities for proposing and evaluating investments that support the productivity of operations, be they highway construction, steel production, office services, mail delivery, or any other productive function. Proposals are usually generated and evaluated on the assumption that a given level of funding is available to carry out the best ones.

Diagram

The size of a pool of investment funds available at a particular time is a function of many financial decisions made previously. As shown in Fig. 5.9, the pool is fed by gross revenue resulting from sales, such other income as returns from investments and capital obtained by borrowing or selling equity in the organization; *debt* and *equity* sales are *external* sources of funds. A portion of the inflow is allocated to *depreciation*, a charge made for accounting purposes that spreads the cost of owning assets over several years. One effect of depreciation charges is to increase the net throughput of funds by decreasing the percentage of income paid in taxes. The flow attributable to depreciation (to be discussed in Chap. 9) plus the net income remaining after all outflows—retained earnings—is the sum available from *internal sources* to satisfy investment proposals. The best way to cultivate funding sources is to have profitable operations that yield large net earnings, or activities that strongly promise future profits, thereby easing borrowing costs and boosting stock prices.

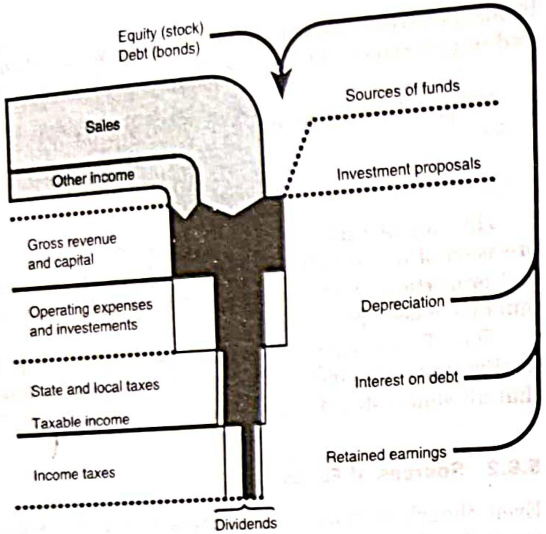


FIGURE 5.9 Flow of funds into, through, and out of an industrial organization. New funds are derived from borrowing or selling equity. Sources of internal funds are depreciation charges and retained earnings.

The management of an industrial enterprise works for the owners. In smaller firms the managers may be the owners. In the role of either an owner or someone representing owners, management has the responsibility of conducting operations in ways that financially benefit the equity holders. Owners receive gains from successful corporate operations in two forms: dividend payments and increases in the value of their stock. Each of the four main sources of funds shown in Fig. 5.9 has its own effect on returns to equity holders.

Depreciation charges are the least controversial source of funds for new investments. How a firm allocates the depreciation reserve shown on accounting records is not specified by legal requirements. However, there is an implied obligation that depreciation charges be applied to purchases that maintain the production capabilities of the firm. Investments in replacements for worn assets and new processes to update production functions are customary practices.

Retained earnings come from after-tax income that could be distributed to shareholders as dividends. Expenditures of retained earnings are judged by the effect they have on the market price of the stock. Stock prices typically reflect current earnings per share and the growth potential of a firm. If the stock market is confident that a firm is building up to a more profitable position, the stock price can rise even when the earnings are modest and no dividends are declared over a prolonged period.

Equity financing results from selling part ownership in a firm. When additional shares of stock are issued and sold and profits do not increase, there is a dilution of earnings that decreases the capitalization rate.

Debt financing does not dilute ownership, and it offers certain tax advantages, but the incurred indebtedness can be an onerous drain on finances during periods of low economic activity.

In conclusion to this section on the cost of capital, whatever theory one adopts to explain how interest rates depend on the supply and demand for capital, no one can deny the dominant influence of government on those rates. In effect, an interest rate is an administered price. Fiscal policies of government affect the supply of capital through the power to issue money and the demand through massive borrowing. The reason government purposely influences interest rates is to direct the economy toward national priorities. For instance, other things being equal, a lowering of interest rates will stimulate business expenditures and create more employment. However, other things are seldom equal, so fiscal policies do not always achieve the desired objectives by themselves. Interacting forces in the national economy (and indirect international influences) are awesomely complex; money management is only one contributor.

5.7 REVIEW EXERCISES AND DISCUSSIONS

EXERCISE 1

A $1000 utility bond with 14 years remaining before maturity can now be purchased for $760. It pays interest of $20 each 6-month period. What rate of return is earned by purchasing the bond at the current market price plus a brokerage charge of $20?

SOLUTION 1

This is a variation of the bond problem in Example 3.8. In the original version the question was, How much could be paid for the described bond in order to earn a rate of return of 8 percent compounded semiannually? The solution revealed that the PW of the bond at i = 4 percent per period was $666.74 − $780 = −$113.26. So we know that the actual rate of return must be smaller than 8 percent compounded semiannually.

Trying i = 5 percent compounded semiannually, or 2½ percent per period for 28 periods, gives

$$\begin{aligned}\text{PW} &= \$1000(P/F, 2\tfrac{1}{2}, 28) + \$20(P/A, 2\tfrac{1}{2}, 28) - (\$760 + \$20) \overset{?}{=} 0\\ &= \$1000(0.50089) + \$20(19.96489) - \$780 = \$120.19 > 0\end{aligned}$$

which indicates that i should be greater than $2\frac{1}{2}$ percent. When we try i = 6 percent compounded semiannually, we get

$$\begin{aligned}\text{PW} &= \$1000(P/F, 3, 28) + \$20(P/A, 3, 28) - \$780 \overset{?}{=} 0\\ &= \$1000(0.43708) + \$20(18.76411) - \$780 = \$32.36 > 0\end{aligned}$$

which is closer, but i is still too low. The present worth of the bond at i = 4 percent per period was −$113.28. Interpolating between 3 percent and 4 percent gives

$$i = 3\% + (4\% - 3\%)\frac{\$32.36 - \$0}{\$32.36 - (-\$113.28)} = 3.2\%$$

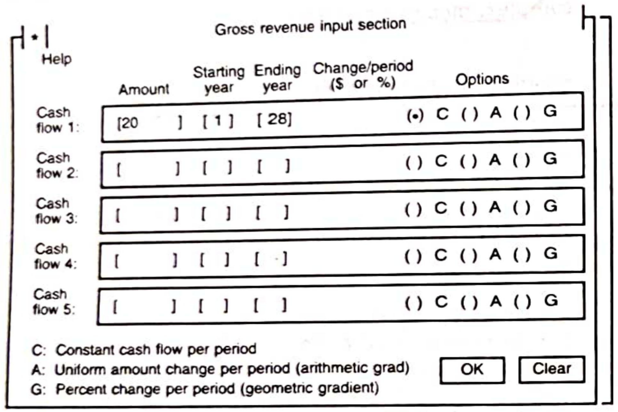
which means that the bond purchased for $780 will earn 6.4 percent compounded semiannually.

EXERCISE 2

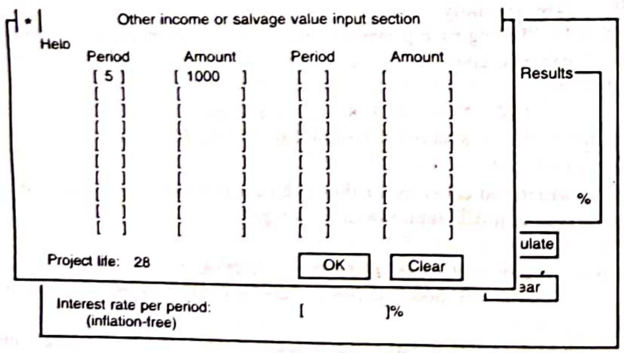
Use the computer program available with this text to solve directly for the bond return in Exercise 1.

SOLUTION 2

The solution to this exercise is given in the three screen displays of Fig. 5.10. The income data are given in Fig. 5.10*a* and *b*. First, we input the $20 per-period income through the Gross Revenue screen. The $1000 face value of the bond after

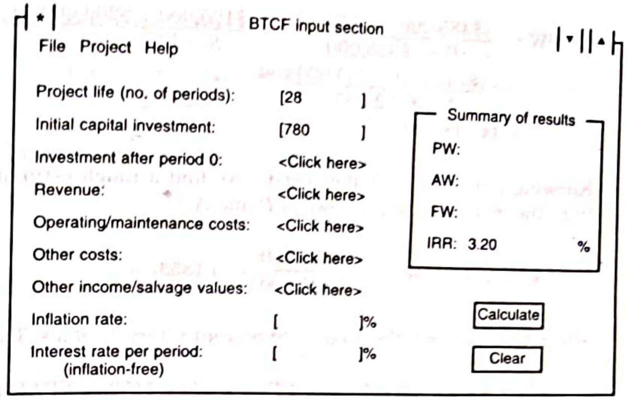


(*a*) $20 per period entered through gross revenue screen



(*b*) $1000 face value entered through "other income" screen

FIGURE 5.10 Computer solution for Exercise 2.



(*c*) Initial entry data and IRR solution for Exercise 2. Note: The IRR cannot be computed until all the cost and income data have been input (previous two screens). Because CHEER accomplishes some data entry error checking, the value *N* has to be input through this initial screen before any cost or income data screens can be used.

FIGURE 5.10 continued

28 periods is input through the Other Income/Salvage Value screen. Last, Fig. 5.10*c* gives the initial data—the project life is 28 periods, and the initial capital investment is $780 ($760 purchase price plus $20 commission). As can be seen on this screen, the calculated IRR is 3.20 percent, which is the value we tediously computed in Exercise 1.

EXERCISE 3

An old hotel was recently damaged by a fire. Since it has a desirable location in the old part of the city that is currently being rejuvenated by an urban-renewal project, it will be rebuilt and renovated as either a showroom and office building or a modern apartment building. Estimated receipts and disbursements for the 30-year life of the refurbished structure are shown below.

	Offices	Apartments
First cost of renovation	$340,000	$490,000
Increase in salvage value from renovation	120,000	190,000
Annual receipts	212,000	251,200
Annual disbursements	59,100	88,000
Present value of fire-damaged building	485,000	485,000
Expected salvage value of fire-damaged building after 30 years	266,000	266,000

If the required rate of return is 12 percent, which renovation plan is preferable?

SOLUTION 3

Investigating first the lowest-cost alternative, we check to see whether an office building will be profitable at $i = 0$.

$$PW = -\underbrace{(\$485{,}000 + \$340{,}000)}_{P = -\$825{,}000} + \underbrace{\$120{,}000 + \$266{,}000(1)}_{S = \$386{,}000}$$
$$+ \underbrace{(\$212{,}000 - \$59{,}000)(30)}_{A = \$152{,}900}$$
$$= \$4{,}148{,}000$$

Knowing a positive cash flow exists, we find a rough estimate of the IRR evident from the more significant flows of P and A:

$$(A/P,\ i,\ 30) \simeq A/P \simeq \frac{\$152{,}900}{\$825{,}000} \simeq 0.1853$$

which falls between the 15 and 20 percent interest tables. Then, by trial and error,

$$PW = -\$825{,}000 + \$386{,}000(P/F,\ i,\ 30) + \$152{,}900(P/A,\ i,\ 30) \overset{?}{=} 0$$

At IRR = 15 percent,

$$PW = -\$825{,}000 + \$386{,}000(0.01510) + \$152{,}900(6.56598)$$
$$= \$184{,}721 > 0$$

At IRR = 20 percent,

$$PW = -\$825{,}000 + \$386{,}000(0.00421) + \$152{,}900(4.97894)$$
$$= -\$62{,}095 > 0$$

Interpolation for PW = 0 gives IRR = 18.7 percent. If we had used the computer program, we would have obtained IRR = 18.5 percent. The slight discrepancy is due to interpolating over a 5 percent range in the tables (15 percent to 20 percent).

The conversion of the fire-damaged hotel to a showroom and office building is thus an acceptable alternative; the 18.7 percent IRR is greater than the required 12 percent.

Now we will check to see whether the additional expenditures and incomes for the apartment complex are justified. The alternative plan to convert to an apartment house has incremental additional values of

First cost:	$490,000 − $340,000 = $150,000
Salvage value:	$190,000 − $120,000 = $70,000
Net annual returns:	$251,200 − $88,000 − $152,900 = $10,300

The incremental rate of return is calculated as

$$PW = -\$150{,}000 + \$70{,}000(P/F,\ i,\ 30) + \$10{,}300(P/A,\ i,\ 30) \overset{?}{=} 0$$

At IRR = 6 percent,

$$PW = -\$150{,}000 + \$70{,}000(0.17411) + \$10{,}300(13.76483)$$
$$= \$3965$$

At IRR = 7 percent,

$$PW = -\$150{,}000 + \$70{,}000(0.13137) + \$10{,}300(12.40904)$$
$$= -\$12{,}991$$

Interpolating between 6 and 7 percent, we find

$$IRR = 6\% + (1\%)\left[\frac{\$3965 - 0}{\$3965 - (-\$12{,}991)}\right]$$
$$= 6.2\%$$

The computer value for IRR will also be found to be 6.2 percent

The IRR value's being lower than the required 12 percent rate disqualifies the additional investment needed to proceed from the office plan to the apartment plan. This assumes that the additional capital could earn at least the required 12 percent.

Note that the apartment plan still has a total IRR greater than the minimum required 12 percent:

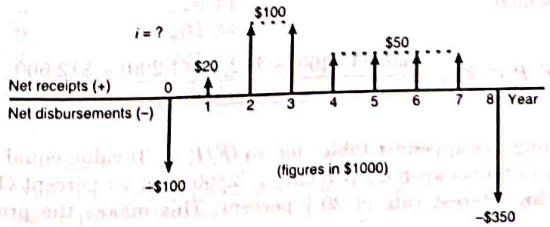
$$PW = -(\$490{,}000 + \$485{,}000) + (\$190{,}000 + \$266{,}000)(P/F,\ i,\ 30)$$
$$+ (\$251{,}200 - \$88{,}000)(P/A,\ i,\ 30) \overset{?}{=} 0$$

CHEER calculates the IRR value to be 16.66 percent. This tells us that if the apartment complex were the only alternative considered, then it would be viable. Since we had two alternatives, the 16.7 percent return is not viable since the costs and revenues additional to the office complex figures will return only 6.2 percent, which does not meet the required MARR of 12 percent. The results are summarized below:

	Office plan	→	increment	→	Apartment plan
First cost, $	825,000		150,000		975,000
Salvage value, $	386,000		70,000		456,000
Annual returns, $	152,900		10,300		163,200
Rate of return, %	18.5	→	6.2	→	16.7

EXERCISE 4

Expected cash flows for a strip-mining project are estimated as shown in the cash flow diagram.



A start-up cost is incurred immediately. Then income exceeds outlays for the next 7 years. During the eighth year the major cost is for landscape improvement. Does the strip-mining project appear to be a profitable investment?

SOLUTION 4
The cash flow pattern has a sign change (minus to plus) at period 1 and another change at period 8 (plus to minus):

Year	0	1	2	3	4	5	6	7	8
Cash flow, $1000	-100	20	100	100	50	50	50	50	-350

This pattern suggests dual rates of return. The suspicion of dual interest rates is confirmed by calculations to find i^* based on

$$\begin{aligned} PW = &-\$100{,}000 + \$20{,}000(P/F, i, 1) + \$100{,}000(P/F, i, 2) \\ &+ \$100{,}000(P/F, i, 3) + \$50{,}000(P/A, i, 4)(P/F, i, 3) \\ &- \$350{,}000(P/F, i, 8) \\ &\stackrel{?}{=} 0 \end{aligned}$$

CHEER was used in the trial-and-error procedure to find the IRR values. The income and disbursement data were set up in the various entry screens, and CHEER estimated values to be 50 and 4 percent (within 2 percent) automatically. Various interest rates were then tried with CHEER, first around 4 percent and then 50 percent, to find the i^* that resulted in getting PW $\neq$ 0. Values of 3.1 and 48.2 percent resulted. We will see in Exercise 6 how the project balance reinvestment method will find one true IRR value.

EXERCISE 5
Calculate the e' value, using the HERR method for the cash flow in Example 5.3 when i = MARR = 20 percent.

SOLUTION 5
Given receipts (savings) of $6350, $12,600, and $12,600 at the end of years 1, 2, and 3, respectively, and an initial disbursement (investment) of $21,000 at time 0, the future worths of the positive and negative cash flows are equated:

$$\$6350(F/P, 20, 2) + \$12{,}600(F/P, 20, 1) + \$12{,}600 = \$21{,}000(F/P, e', 3)$$

from which

$$(F/P, e', 3) = \frac{\$6350(1.4400) + \$12{,}600(1.200) + \$12{,}600}{\$21{,}000} = 1.7554$$

Scanning the appendix tables for an $(F/P, e', 3)$ value equal to 1.7544, we see that it has to lie between 20 percent (1.72800) and 25 percent (1.95313). Interpolation gives an interest rate of 20.1 percent. This makes the project barely acceptable when MARR = 20 percent. Remember, though, that the HERR method does not consider all positive cash *balances*. This is the fallacy of the method.

EXERCISE 6
Apply the project balanće method (PBM) to the data of Exercise 4. Use a MARR value of 20 percent as the external implicit rate. Start with the IRR values of 3.1 and 48.2 percent realized in Exercise 4.

SOLUTION 6
Checking the CB(i^*)$_t$ values, we get, using the spreadsheet program given in App. C, Example 6.†

Period	Cash flow, $	CB(3.1/3.1)$_t$, $	CB(48.2/48.2)$_t$, $
0	-100	-100.00	-100.00
1	20	-83.10	-128.20
2	100	14.32	-89.99
3	100	114.77	-33.37
4	50	168.33	0.55
5	50	223.54	50.81
6	50	280.47	125.30
7	50	339.17	235.70
8	-350	-0.32	-0.69

Both IRRs give some current balances greater than zero (and some less), so it follows that use of an external investment rate should be considered. Note that only one of the i^* values needs to be evaluated; both were shown to give an additional example of the results for the reader to check.

Now we set MARR = 20 percent and run the spreadsheet program with various potential IRR values until CB$_8$(IRR/20) = 0 (or we try to manually tackle the trial-and-error process). Finally we get, for IRR = 38.5 percent,

Period	CB(38.5/20), $
0	-100.00
1	-118.50
2	-64.12
3	11.19
4	63.43
5	126.11
6	201.34
7	291.60
8	-0.07

†Note that, for this situation only, we set the MARR in the program to i^* since we are checking for positive balances with just i^*.

We see that the i^* values of 3.1% and 48.2% found in Exercise 4 differ considerably from the true IRR value. Since IRR = 38.5 percent is far greater than the MARR of 20 percent, we have a potentially good investment.

PROBLEMS

5.1 Sometimes objects of art are respectable investments. In 1975 a marble bust of Benjamin Franklin, from which the engraving was made for $100 bills, was auctioned for $310,000. It was sculpted in France in 1778 by Jean-Antoine Houden. In 1939 the same bust sold for $30,000. What rate of return was earned by the collector who owned the statue from 1939 to 1975?

5.2 A $5000 bond matures in 10 years and pays 2.5 percent interest twice a year. If the bond sold for $5050, what is the actual investment rate?

5.3 A construction firm can lease a crane required on a project for 3 years for $180,000 payable now, with maintenance included. The alternative is to buy a crane for $240,000 and sell it at the end of 3 years for $100,000. Annual maintenance costs are expected to be $5000 the first 2 years and $10,000 the third year (payable at the end of each year). At what interest rate would the two alternatives be equivalent?

5.4 Additional parking space for a factory can be either rented for $15,000 per year on a 10-year lease or purchased for $160,000. The rental fees are payable in advance at the beginning of each year. Taxes and maintenance fees will be paid by the lessee. The land should be worth at least $95,000 after 10 years. What rate of return will be earned from purchase of the lot?

5.5 Proposal 1 has an initial cost of $1500 and a positive cash flow that returns $200 the first year and increases by $200 each of the following years until the end of the 5-year study period. Proposal 2 also has a 5-year life and an initial cost of $1500. Its positive cash flow is constant at $200 for the last 4 years. It also has another receipt in year 1. All receipts occur at the end of the year.

(*a*) What is the rate of return on proposal 1?

(*b*) If the two proposals are equally attractive at i = 15 percent annually, how large must proposal 2's unknown receipt be in period 1?

5.6 In 1994 a small apartment building was purchased for $200,000. Receipts from rent have averaged $30,200 a year; taxes, maintenance, and repair costs have totaled $8620 annually. The owner intends to hold the property until she retires in 2004. If at that time the property sells for $200,000, what rate of return will be obtained on the investment?

5.7 Stock in a corporation was purchased 10 years ago for $80 per share. For the first 6 years, the stock paid annual dividends of $11 per share and the market price climbed to $120. However, for the past 4 years, annual dividends have been only $5 per share and the price of the stock has dropped to $100.

(*a*) What rate of return would have been obtained by an investor who sold the stock at the end of the first 6 years?

(*b*) What rate of return would be obtained if an investor purchased the stock 4 years ago and sold it today?

(*c*) What rate of return would be obtained by an investor who purchased the stock 10 years ago and sold it today?

5.8 A company can purchase a new central computer for $17,500 or can lease it for 3 years with annual payments of $8400. Determine at what interest rate the leasing and purchasing costs would be equivalent

(*a*) If lease payments were due at the first of each year

(*b*) If lease payments were due at the end of each year

5.9 Determine the annual effective interest rate at which the lease and purchase costs in Prob. 5.8 would be equivalent if a payment of $700 were due at the first of each month over the 3-year period instead of $8400 annually.

5.10 An investor has an opportunity to purchase a commercial rental property for $300,000. The current occupants have signed a 10-year lease at a constant annual rent of $48,000. Maintenance costs and taxes on the structure are currently $12,000 and are expected to increase at a rate of $1500 per year over the 10-year period. Assuming that the property can be sold for at least the purchase price when the current lease expires, determine the investor's minimum expected rate of return.

5.11 A student renting an unfurnished apartment has decided to purchase some furniture from Fred's Fine Furniture. The total purchase price of the three-room set is $495. However, after a down payment of $95, Fred will finance the balance through a 2-year series of end-of-month payments of $19.98. Determine the nominal and effective annual rates of interest paid by the student.

5.12 A bookstore is considering expanding its facilities. The first costs are estimated to be $50,000 with additional maintenance and operating expenses of $15,000 per year. Additional income is anticipated to be $25,000 for the first year with a $2000-per-year gradient for subsequent years. The store's MARR value is 12 percent. Determine if the investment should be pursued, using an evaluation period of 5 years. Afterward, try an evaluation period of 10 years and comment on the results.

5.13 Two alternative investment proposals are under consideration for a vacant lot owned by Urban Development Corporation. Plan *A* would require an immediate investment of $120,000 and a first-year expenditure for property taxes, maintenance, and insurance of $4000, with this amount expected to increase at a rate of $1000 per year. Plan *B* would have a first cost of $170,000 and total first-year expenses of $9000, with an increase of $1000 per year. The economic life of each project is forecast to be 10 years; and at the end of this time, only the facilities from plan *B* with a value of $50,000 are expected to be salvaged. During the life of the project, the facility in plan *A* is expected to produce $34,000 annually, whereas plan *B* is expected to produce $42,000.

(*a*) Determine the rate of return of each plan.

(*b*) Determine the rate of return of the additional investment required in plan *B* compared with plan *A*.

(*c*) Which plan should Urban Development select if the company uses a MARR of 12 percent?

(*d*) If applicable, use the PBM with a MARR of 12 percent to determine which plan to recommend.

5.14 Two mutually exclusive programs are being considered for funding. Projected cash flows are as follows:

	Cash flow, $	
End of Year	Program A	Program B
0	−10,000	−15,000
1	3,000	5,000
2	5,000	5,000
3	2,000	5,000
4	4,000	5,000

Determine which program to recommend, assuming a MARR of 12 percent.

5.15 Two mutually exclusive projects are being considered: Project X requires \$500 now and results in a return amounting to a one-time-only profit of \$1000 in 5 years from now. Project Y also requires \$500 now, but will return \$170 per year for each of the next 5 years. Given a MARR of 14 percent, which project should be adopted?

5.16 In the next chapter we will see that through capital budgeting we might allow one department to fund two independent projects or several projects when the total expenditure does not exceed some budgeted amount. Suppose that the two projects in the previous problem are independent and your department can fund up to two projects if the capital outlay does not exceed \$1000. What would you suggest if the MARR is 17 percent?

5.17 The cash flow for a project is shown below:

End of year	0	1	2	3	4	5
Cash flow, $	3000	1000	−5000	−5000	2000	5000

The first two payments (in thousands of dollars) represent advance payments for distribution rights on a motion picture. The next 2 years show a negative cash flow from production costs, and the last 2 years are net receipts from the finished picture. If advance payments can be invested at the external interest rate of 7 percent until period 2, what rate of return can be expected from the project?

5.18 Determine the e' rate with the HERR method for the project given in Prob. 5.17, assuming an explicit interest rate of 7 percent is feasible.

5.19 Use the project balance method to find the IRR with the same situation given in Prob. 5.18; the external rate is still 7 percent.

5.20 This problem is a variant of Prob. 2.39: A new piece of materials handling equipment costs \$20,000 and is expected to save \$7500 in the first year of operation. Maintenance and operating cost increases are expected to reduce the net savings by \$500 per year for each additional year until the equipment is worn out at the end of 8 years. Evaluate the purchase against a MARR of 25 percent.

5.21 A bioengineering research laboratory has a patent that it is considering leasing for 10 years at \$40,000 for the first year with increments of \$4000 per year (arithmetic gradient) for the following 9 years. The accounting office says the company has \$250,000 in development investment costs that are considered to be a first cost. Is the lease reasonable if the laboratory's MARR is 20 percent?

5.22 Suppose the research laboratory in Prob. 5.21 now has a firm MARR of 25 percent. The company it is negotiating with will not consider a first-year lease cost that is more than \$30,000 due to uncertainty in the potential effectiveness of the patented process. The other company will, however, allow an increase in the arithmetic gradient that was originally proposed to be \$4000 per year. What is the minimum gradient that the laboratory should consider?

5.23 West Texas Oil has paid \$300,000 for a producing oil well. Field engineers estimate that net receipts will be \$120,000 for the first year of operation with a reduction of 15 percent per year in the following years (geometric gradient). It plans to sell the well after 5 years for \$80,000. How does this seem financially if their MARR is 20 percent?

5.24 Suppose that West Texas Oil, from Prob. 5.23, now has a MARR of 25 percent. What is the minimum salvage value that it should realize in 5 years to make its investment viable?

5.25 This problem is a variant of Prob. 3.18: A company is considering the purchase of a new piece of testing equipment that is expected to produce \$8000 additional profit during the first year of operation; this amount will probably decrease by \$500 per year for each additional year of ownership. The equipment costs \$20,000 and will have an estimated salvage value of \$3000 after 8 years of use. How does the proposal match up against a MARR of 18 percent?

5.26 Rotor Turbine Engine Company needs a new automated gear production machine. It has two bids with associated estimated data:

	Company A	Company B
Initial cost	\$85,000	\$110,000
Estimated net income for:		
Year 1	45,000	61,000
Year 2	40,000	53,000
Year 3	30,000	44,000
Estimated salvage value	15,000	21,000

The net income estimates include subcontracting necessary to offset different productivity levels. Determine which company to recommend, if any, given a MARR of 12 percent.

5.27 Suppose for the bids given in Prob. 5.26 that the salvage value for company B drops to \$0. Does this change your recommendation from that for Prob. 5.26?

5.28 Owing to perennial complaints from students and faculty about the lack of parking spaces on campus, a parking garage on university-owned property is being considered. Since there are no university funds available for the project, it will have to pay for itself from parking fees over a 15-year period. A 10 percent minimum rate of return is deemed reasonable for consideration of how large the structure should be. Based on the income and cost data shown below, determine how many levels should be built.

Number of levels	Cumulative construction costs, $	Annual operating cost, $	Income per year, $
1	600,000	35,000	100,000
2	2,200,000	60,000	350,000
3	3,600,000	80,000	570,000
4	4,800,000	95,000	810,000

5.29 A business property can be purchased today for $90,000; the expected resale value after 20 years is $60,000. If annual rental income is $11,800 and expenses are $4700, what before-tax rate of return will be earned by purchasing the property?

5.30 A cash flow pattern shows an income of $250 at the end of year 1 between expenditures of $100 now and $156 at the end of year 2.

(*a*) Calculate the roots i^* for PW(i) = 0 by trial and error and/or with an appropriate program.

(*b*) We looked at a way to directly solve IRR simple problems earlier in this chapter. It is possible to do the same thing for this problem. Write the PW equation for the cash flows in terms of the (P/F, i, N) factors $1/(1+i)^N$. Now set the PW equation to zero and multiply both sides by $(1+i)^2$. Gather terms to form a quadratic equation. You should be able to solve the quadratic equation directly for i. Logically, you should get the same results as you did in part (*a*).

(*c*) What is the true IRR value, given MARR = 18 percent?

5.31 Assume that the MARR of 18 percent can be earned on positive cash balances for Prob. 5.30. Use the project balance reinvestment method to determine the cumulative equation to set the FW to zero, as in part (*c*), but leave the internal rate as i. You should be able to solve directly for the IRR value instead of by trial and error.

5.32 The owner of a truck-weighing and lumber-scaling station has agreed to lease the facility for 15 years at $8000 per year under an agreement that the scales and other equipment will be overhauled and repaired by the owner at the end of the eighth year at a cost not to exceed $150,000. The lease payments occur at the end of each year.

(*a*) What potential rate(s) of return i^* will the owner receive for the station lease with the equipment repair agreement?

(*b*) After negotiations on the above lease, caused by concern that the equipment needed overhauling before 8 years, it was agreed that the owner would pay up to $90,000 for repairs at the end of year 4 instead of making the repairs at the end of year 8. What rate(s) of return i^* will the owner receive under the revised agreement?

(*c*) Check the revised agreement to see if it is acceptable at a MARR of 20 percent.

5.33 An interesting article about the reasons why Continental Oil Company switched in 1955 to the discounted-cash method for evaluating investments was written by John G. McLean, then vice president for international and financial operations.[†] One of the applications described was a water-flood project that exhibited dual rates of return.

The problem was to determine the profitability of acquiring a small oil-producing property in which the primary reserves were nearly exhausted. The owner of the property would receive a royalty of 12½ percent on all oil produced from the property, and the company would agree to water-flood the reservoir at an expected cost of $2.5 million. The injection of water into a reservoir is a method of *secondary recovery* that often increases the total amount of oil recovered after the free-flowing oil supply has diminished.

Estimated cash flows for the 10-year project are as follows:

Present worth of secondary oil recovery project at different discount rates (cash flows in $1000)

		Present worth of cash flow at:						
Year	**Cash flow**	**10%**	**20%**	**28%**	**30%**	**40%**	**49%**	**50%**
1	200	182	167	156	154	143	134	133
2	100	83	69	61	59	51	45	44
3	50	38	29	24	23	18	15	15
4	−1800	−1229	−868	−671	−630	−469	−365	−356
5	600	373	241	175	162	112	82	79
6	500	282	167	114	104	66	46	44
7	400	205	112	71	64	38	24	23
8	300	140	70	41	37	20	12	12
9	200	85	39	21	19	10	5	5
10	100	39	16	8	7	3	2	2
	650	198	42	0	−2	−8	0	1

The primary reserve of oil yields returns the first 3 years. The water flood is then expected to boost the company's income to $700,000 in the fourth year while it invests $2.5 million, for a net outlay of $1,800,000 that year. Thereafter, income decreases annually by $100,000. The present worths of the yearly cash flows are indicated for different discount rates. At i = 28 percent and i = 49 percent, the present worth of the cash flow is zero. Between these rates the present worth of the venture is negative.

Instead of settling for two rates of return, assume that the cash flows from the first 3 years can actually be reinvested at an annual interest rate of only 15 percent. What is the IRR on the resulting net investment, when this portion of the total is reinvested at the external 15 percent rate? How would you explain your solution to a group of investors not very familiar with discounted cash flow analysis?

5.34 Determine the PBM solution for the IRR for the cash flow given in Prob. 5.33, using the external rate of return equal to 15 percent.

[†]J. G. McLean, "How to Evaluate New Capital Investments," *Harvard Business Review*, vol. 36, no. 6, pp. 59–69, 1958.