geometric Series	$\Sigma x^n = 1 + x + x^2 + \ldots$ $U_n = x^n$ & ΣU_n	converges if $\lvert x \rvert < 1$ or $-1 < x < 1$ diverges if $x \geq 1$ oscillates if $x < -1$
P-series	$\Sigma \frac{1}{n^p} = \frac{1}{p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$ $U_n = \frac{1}{n^p}$ & series ΣU_n	converges if $p > 1$ diverges if $P < 1$
① Comparision test	given U_n choose V_n If $\lim_{n \to \infty} \frac{U_n}{V_n} = l \neq 0$	If V_n converges, the U_n conv. V_n div.. then U_n div..

Rules to choose V_n

1) $V_n = \dfrac{\text{highest degree term in Nr of } U_n}{\text{highest degree term in Dr of } U_n}$

2) If U_n is of form $\sqrt{f(x)} - g(x)$ or $\sqrt{f(x)} - \sqrt{g(x)}$ then rationalize the term by multiplying $\sqrt{f(n)} + g(n)$ or $\sqrt{f(n)} + \sqrt{g(n)}$ resp

② D Alembert's ratio test	$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = l$ Then series ΣU_n	converges, $l < 1$ diverges, $l > 1$ test fail $l = 1$

Test

Test		
(3) Raabe's Test	$\lim_{n\to\infty} \left(\frac{U_n}{U_{n+1}} - 1\right) n = l$	Then serie $\sum U_n$ converge : $l > 1$ diverge : $l < 1$
(4) Cauchy's Root test	$\lim_{n\to\infty} (U_n)^{1/n} = l$	Then series $\sum U_n$ converges if $l < 1$ diverges if $l > 1$ Test fails if $l = 1$

Note :-

1) $\boxed{\lim_{n\to\infty} n^{1/n} = 1}$

3) $\boxed{\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x}$

2) $\boxed{\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e}$

4) $$\lim_{n\to\infty} x^n = \begin{cases} 0 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ \infty & \text{if } x > 1 \end{cases}$$

5) $1 + 2 + 3 + \dots n = \frac{n(n+1)}{2}$

(6) $1^2 + 2^2 + 3^2 + \dots n^2 = \frac{n(n+1)(2n+1)}{6}$

(7) $1^3 + 2^3 + 3^3 + \dots n^3 = \frac{n^2(n+1)^2}{4}$

Arithmetic Mean	geometric Mean
$T_n = a + (n-1)d$	$T_n = a r^{n-1}$
$S_n = \frac{n}{2}[2a + (n-1)d]$	$S_n = \frac{a(1-r^n)}{1-r} \quad r<1$
$S_n = \frac{n}{2}[2a + l]$	$S_n = \frac{a(r^n-1)}{r-1} \quad r>1$
	$S_\infty = \frac{a}{1-r}$

Note:-

① The necessary & sufficient condition for the convergence of positive sum series, ΣU_n given that the $\{S_n\}$ should converge

② (a) Comparision test [different form]

Let ΣU_n be ΣV_n be 2 series of +ve terms such that ΣV_n is convergent. $U_n \leq K V_n$, where $K > 0$

Then ΣU_n is also convergent.

(b) Let ΣU_n & ΣV_n be 2 series of +ve terms such the ΣV_n is divergent and $U_n > K V_n$.

The ΣU_n is also divergent.

(4) use comparision test to decide the ~~Nature~~ Nature of following series.

(1) $\frac{1}{1.3.5} + \frac{2}{3.5.7} + \frac{3}{5.7.9} + \cdots\cdots$

$1+2+3+\ldots$

$a=1,\ d=1$

$T_n = a+(n-1)d = \cancel{+}1+(n-1)$
$= n$

$1.3.5 \Rightarrow a=1,\ d=2$

$T_n = 1+(n-1)2 = 1+2n-2 = 2n-1$

$3.5.7 \Rightarrow$

$T_n = 3+(n-1)2 = 3+2n-2 = 2n+1$

$5.7.9$

$T_n = 5+(n-1)2 = 2n+3$

$$U_n = \frac{n}{(2n-1)(2n+1)(2n+3)}$$

$$V_n = \frac{n}{n^3} = \frac{1}{n^2}$$

$\cancel{\Sigma\frac{1}{n}}$

$$\Sigma V_n = \Sigma \frac{1}{n^2}$$

$\Sigma \frac{1}{n^p}$ converge if $p>1$
div $p<1$

$\Rightarrow p=2$

$\therefore$ From p-series Test, ΣV_n converges

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \lim_{n\to\infty} \frac{\frac{n}{(2n-1)(2n+1)(2n+3)}}{1/n^2} = \lim_{n\to\infty} \frac{n}{(2n-1)(2n+1)(2n+3)} \cdot n^2$$

$$= \lim_{n\to\infty} \frac{n^3}{n(2-\frac{1}{n})n(2+\frac{1}{n})n(2+\frac{3}{n})}$$

$\frac{1}{\infty} = 0$

$$= \frac{1}{(2-0)(2+0)(2+0)} = \frac{1}{8} \neq 0$$

$\Rightarrow \Sigma U_n$ & ΣV_n behave alike

since ΣV_n is conv $\rightarrow \Sigma U_n$ is conv