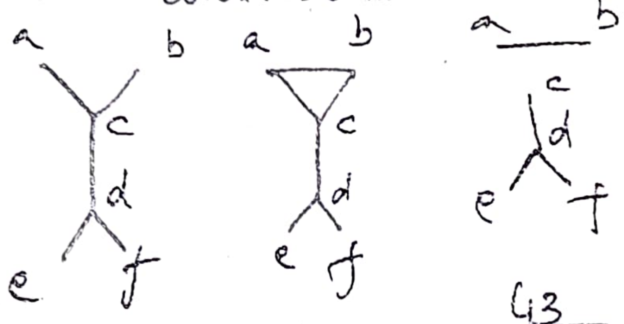
1

UNIT V - ~~GRAPH~~ TREES

13/12/17
Wednesday

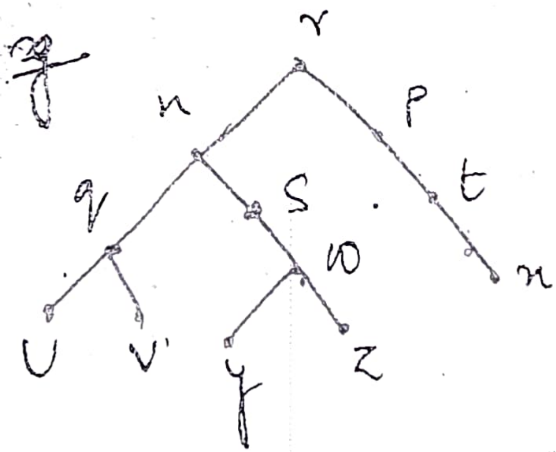
trees - no cycles
- connected



G1 ✓

G2 ✗

G3 forest (2 trees)

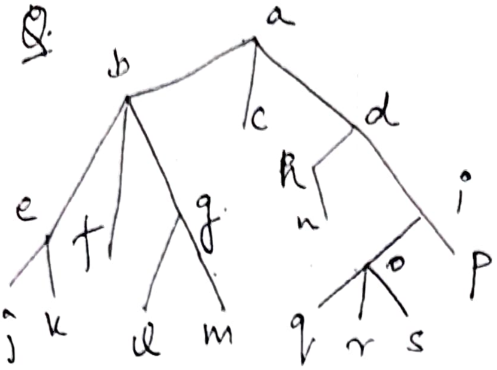


a graph G is called as a **rooted tree** if there is a unique vertex r called as root in G with indegree(r) = 0 and ∀ other vertices v, indegree(v) = 1

outdegree(v) = 0 ⇒ leaf nodes.

- level (s) = path from r (root) to s = 2.
- ancestors of w, y, z = s, n, r
  descendants s, n, r = w, y, z

Q.



(i) which vertex is root a
(ii) which vertices are internal = a, b, e, g, d, h, i, o
⇒ non leaf nodes
(iii) leaf = j, k, f, l, m, c, n, q, r, s, p
(iv) children (i) = o, p.
(v) parent (h) = d.
(vi) sibling (o) = p
(vii) ancestors (m)
= g, b, a
(viii) descendants (b) = e, j, k, f, g, l, m
(ix) level (f) = 2.
(x) at level 4 = q, r, s.

⇒ Inorder (LDR), preorder (DLR), postorder (LRD)

Optimal prefix code

prefix code - set P of binary sequences is called as a prefix code if no sequence in P is a prefix of any other sequence in P.

eg.

a : 0
b : 10
c : 11

P = {0, 10, 11}

lchild = 0
rchild = 1

0 is not prefix of 10 or 11
10 ______ 11

Q. a:01 e:0 n:1101 r:10 t:1

ata : $\underbrace{0}_{e}\underbrace{1}_{t}\underbrace{101}_{n}$ or $\underbrace{0}_{e}\underbrace{1}_{t}\underbrace{10}_{r}\underbrace{1}_{t}$

∴ not prefix code. (ata can be encoded as en or etrt)

Q. a:111 e:0 n:1100 r:1101 t:10

ata : $\underbrace{111}_{a}\underbrace{10}_{t}\underbrace{111}_{a}$ ✓

A = 01
B = 011
c = 1

∴ no other combination is possible.

Huffman's tree

Step 1 : Assign the given weights one each to set S of n isolated vertices. while |S| > 1 then perform the following:

① find 2 tree T and T' in S with smallest 2 root weights w and w' respectively.

② create a new complete binary tree T^* with root weight as $w^* = w + w'$ and having T and T' as its left & right subtrees respectively.

③ Place T^* in S and delete T and T'

8 mark
Q. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with the frequencies 20, 28, 4, 17, 12, 7

⇒ If frequency is not given, then a string will be given

Q. Construct an op. prefix code for symbols a to j that occur with the frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3.

A.

19

(i) 3 16 20 30 31 35 50 78 80 125

39

(ii) 19 20 30 31 35 50 78 80 125

3 16

61

(iii) 30 31 35 39 50 78 80 125

19 20

3 16

74

(iv) 35 39 50 61 78 80 125.

19 20

3 16

30 31

h: 010

i: 00

j: 110100

111

(v) 50 61 74 78 80 125

30 31 35 39

19 20

3 16

152

(vi) 74 78 80 111 125.

35 39

19 20

3 16

50 61

30 31

191

(vii) 80 111 125 152

50 61

30 31

74 78

35 39

19 20

3 16

277

(viii) 125 152

74 78

35 39

19 20

3 16

191

80 111

50 61

30 31

a: 111

b: 110101

c: 0110

d: 1100

e: 10

f: 0111

g: 11011.

468

(ix) 191 277

80 111 125 152

50 61 e 74 78

h 30 31 a

i f 35 39

d 19 20

g

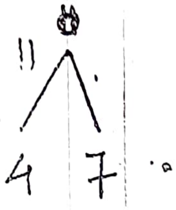
3 16

j b

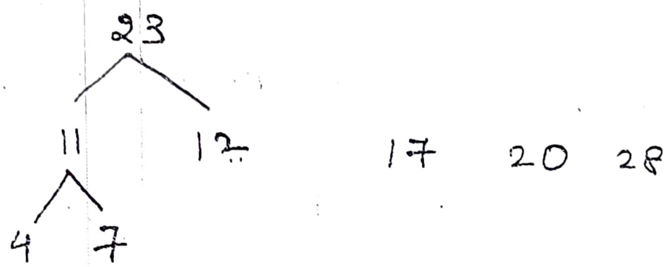
always write eg. with defn

A. (i) arrange all in ascending order.

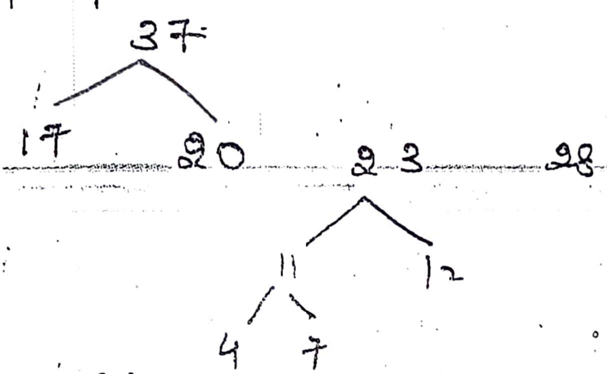
4 7 12 17 20 28.



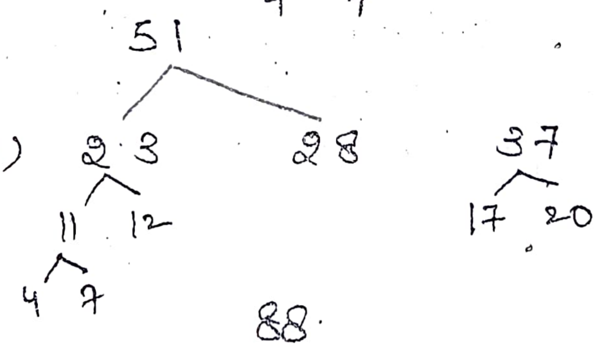
(ii)

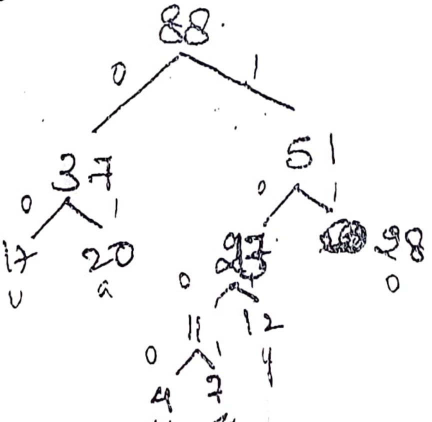


(iii)



(iv)





a: 01
u : 00
o : 11
y : 101
z : 1001
q : 1000

Q Find the value of x, y & z.

a: 00
b: 01
c: 101
d: x10
e: yz1

If x = 0, 01 is b.

A. (i) if x = 0, $\underbrace{01}_{b}0$ ×

if x = 1, 110 ✓

(ii) yz = 00, $\underbrace{00}_{a}1$ ×

yz = 01, $\underbrace{01}_{b}1$ ×

yz = 10, $\underbrace{101}_{c}$ ×

yz = 11, 111 ✓

∴ x = y = z = 1

Ford Fulkerson

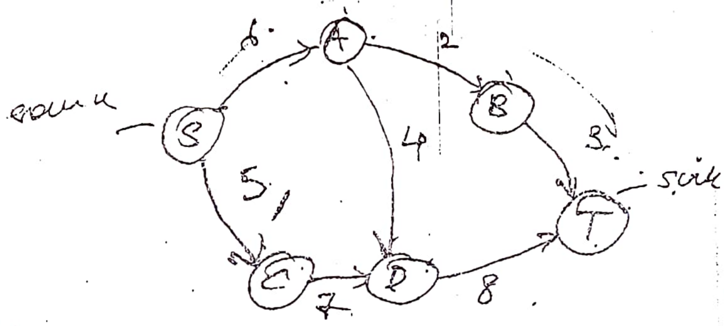
① If ∃ a path from source to sink, go to step ②. otherwise the optimal solution is determined by the backward arcs.

② Calculate the residual in flow network,

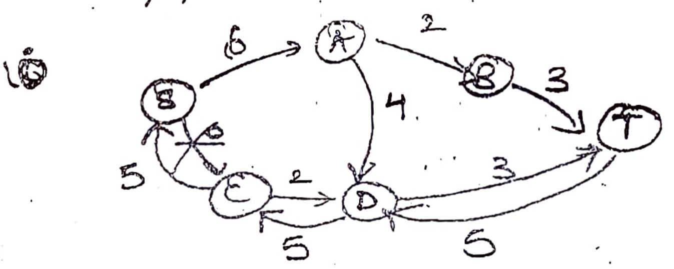
(i) find the smallest capacity of the arcs denoted by c

(ii) subtract c from the capacity of each arc on the path

(iii) add a backward edge for each arc on the path then add c to the capacity of each reverse arc & go back to ①

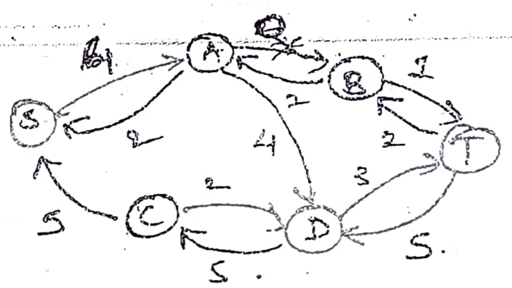


(i) choosing path $S \to C \to D \to T$

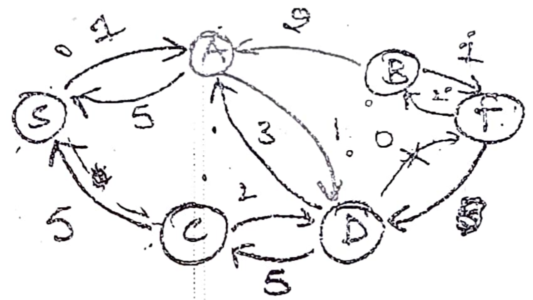


cycle

(ii) choosing $S \to A \to B \to T$



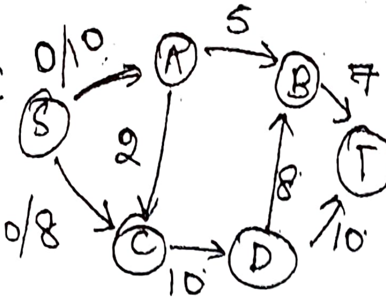
(iii) $S \to A \to D \to T$



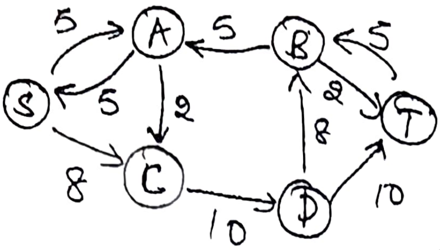
now no more paths possible.

maximum flow = $5 + 2 + 3 = \underline{10}$

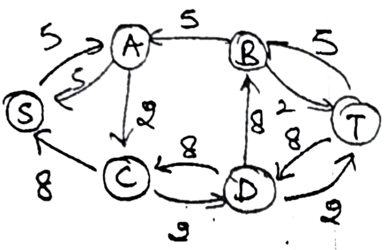
$\leq$ min capacity in all paths chosen.

Q. 

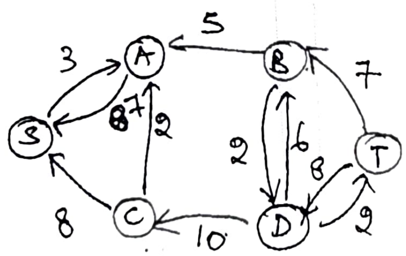
(i) $S \rightarrow A \rightarrow B \rightarrow T$ 5



(ii) $S \rightarrow C \rightarrow D \rightarrow T$ 2



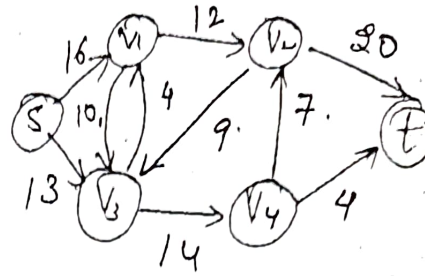
(iii) $S \rightarrow A \rightarrow B \rightarrow T$ 2



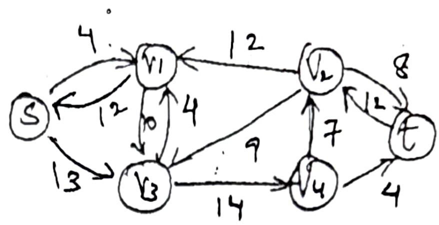
no more paths.

max. flow = 5+8+2 = 15

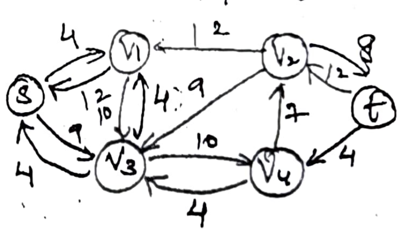
flow capacity

Q. 

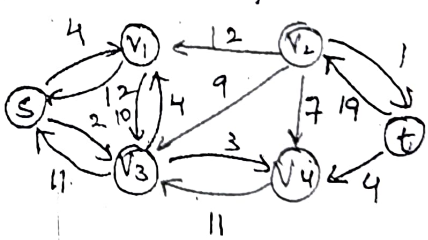
(i) $S \rightarrow V_1 \rightarrow V_2 \rightarrow t$ 12



(ii) $S \rightarrow V_3 \rightarrow V_4 \rightarrow t$ 4.



(iii) $S \rightarrow V_3 \rightarrow V_4 \rightarrow V_2 \rightarrow t$ 7



max flow = 12+4+7
= 23