SYLLABUS

ENGINEERING MATHEMATICS – II

Unit I

Differential Calculus - II: Derivatives of arc length, curvature, radius of curvature. Taylor's series and Maclaurin's series (without proof), Taylor's and Maclaurin's series for functions of two variables (without proof), maxima and minima of functions of two variables, Lagrange's method of undetermined multipliers.

Unit II

Applications of first order and first degree differential equations: Applications of first order and first degree ODEs to solve LCR circuits, Newton's law of cooling and orthogonal trajectories.

Linear differential equations of higher Order-I: Linear differential equations of higher order with constant coefficients.

Unit III

Linear differential equations of higher order-II: Cauchy's and Legendre's linear differential equations, method of variation of parameters – Engineering applications.

Partial differential equations: Introduction to PDE, solutions of partial differential equations - direct integration method, Lagrange's method and method of separation of variables.

Unit IV

Beta and Gamma Function: Definition, Relation between Beta and Gamma Function, Problems.

Laplace transforms I: Definition, transforms of elementary functions, properties of Laplace transforms, existence conditions, transform of derivatives, integrals, multiplication by t^n, division by t, evaluation of integrals by Laplace transforms, unit–step function, unit–impulse function.

Unit V

Laplace transforms II: Laplace transforms of Periodic function, Inverse transforms, convolution theorem, solution of linear differential equations and simultaneous linear differential equations using Laplace transforms. Engineering applications.

Text Books:

1. G. B. Thomas and Finney – Calculus and Analytical Geometry, Pearson, 12th edition, 2017.
2. B.S. Grewal – Higher Engineering Mathematics, Khanna Publishers, 44th edition, 2017.

Reference Books:

1. Erwin Kreyszig –Advanced Engineering Mathematics, Wiley publication, 10th edition, 2015.
2. Peter V. O' Neil – Advanced Engineering Mathematics, Thomson Brooks/Cole, 7th edition, 2011.
3. Glyn James – Advanced Modern Engineering Mathematics, Pearson Education, 4th edition, 2010.
4. George B. Thomas, Maurice D. Weir, Joel Hass - Thomas' Calculus, Pearson, 13th edition, 2014.

LESSON PLAN

Lesson / Session No	Topics	No. of hours
Unit-I (09 Hours) **Differential Calculus-II**		
1	Derivatives of arc length – Cartesian, parametric and polar forms.	1 Hr
2	Problems on Lesson No.1.	1 Hr
3	Curvature, Radius of Curvature - Cartesian, parametric, polar and pedal forms.	1 Hr
4	Problems on Lesson No.3.	1 Hr
5	Taylor's theorem (Statement only) and Maclaurin's series, problems.	1 Hr
6	Taylor's and Maclaurin's theorem for functions of two variables (Statements only), problems.	1 Hr
7	Maxima and minima of functions of two variables and problems.	1 Hr
8	Problems continued on Lesson No. 7	1 Hr
9	Lagrange's method of undetermined multipliers and problems.	1 Hr
Unit-II (08 Hours) **Applications of first order and first degree differential equations & Linear differential equations of higher order-I**		
10	Application of ODEs to solve simple problems related to engineering applications.	1 Hr
11	Problems continued on Lesson No.10.	1 Hr
12	Orthogonal trajectories- Cartesian form, Problems.	1 Hr
13	Orthogonal trajectories- Polar form. Problems.	1 Hr
14	Introduction to LDE with constant coefficients. Solution of Homogeneous LDE with constant coefficients.	1 Hr
15	Particular integral for e^{ax}, $\sin ax$ or $\cos ax$, Problems.	1 Hr
16	Particular integral for x^m, $e^{ax}V(x)$ & $xV(x)$, Problems.	1 Hr
17	Problems continued on Lesson No.16.	1 Hr
Unit-III (08 Hours) **Linear differential equations of higher order-II &** **Partial differential equations**		
18	Cauchy's and Legendre's differential equation, Problems.	1 Hr
19	Problems continued on Lesson No.18.	1 Hr
20	Method of variation of parameters, Problems.	1 Hr
21	Initial and boundary value problems.	1 Hr
22	Introduction to Partial differential equations .	1 Hr
23	Solution of PDE by direct integration & problems.	1 Hr
24	Solution of PDE by Lagrange's method & problems.	1 Hr
25	Solution of PDE by the method of separation of variables & problems.	1 Hr

Unit-IV (09 Hours) Beta and Gamma Function & Laplace transforms I		
26	Beta and Gamma functions, Relation between Beta and Gamma functions, Problems.	1 Hr
27	Problems continued on Lesson No. 26.	1 Hr
28	Introduction to Transforms, Laplace transforms of elementary functions, Problems.	1 Hr
29	Properties of Laplace transform, Problems.	1 Hr
30	Problems continued on Lesson No.29.	1 Hr
31	Evaluation of integrals by Laplace transforms.	1 Hr
32	Problems continued on Lesson No.31.	1 Hr
33	Laplace transform of unit step and unit impulse functions, Problems.	1 Hr
34	Problems continued on Lesson No.33.	1 Hr
Unit-V (08 Hours) Laplace transforms II		
35	Laplace transform of periodic functions, Problems.	1 Hr
36	Problems continued on Lesson No.35.	1 Hr
37	Inverse transforms, Problems.	1 Hr
38	Problems continued on Lesson No.37.	1 Hr
39	Convolution theorem, Problems.	1 Hr
40	Problems continued on Lesson No.39.	1 Hr
41	Solution of initial value problems using Laplace transforms.	1 Hr
42	Solution of system of ODE's using Laplace transforms.	1 Hr

Internal Assessment Details

Test marks = **30 marks** (T_1 and T_2 – each carries 30 marks, average of T_1 & T_2)

Quiz = **10 marks**

Assignment = **10 marks**

CIE Total = **50 Marks**

Syllabus for Tests

Test	Unit	Lesson No.
Test - 1	Unit-I and Unit-II	Lesson 1 to Lesson 21
Test - 2	Unit-III and Unit-IV	Lesson 22 to Lesson 38

UNIT – I: DIFFERENTIAL CALCULUS-II

DERIVATIVE OF ARC LENGTH AND RADIUS OF CURVATURE	
Two & Four marks questions	
1.	Write the derivative of arc length of a curve in Cartesian and parametric forms.
2.	Write the derivative of arc length of a curve in polar form.
3.	Define (i) Curvature (ii) Radius of curvature.
4.	Write the expression for radius of curvature for curves in Cartesian and parametric forms.
5.	Write the expression for radius of curvature for curves in polar and pedal forms.
6.	Prove that the curvature of a circle is constant.
7.	With the usual notation, prove that $\sin\phi = r\frac{d\theta}{ds}$.
8.	With the usual notation, prove that $\cos\phi = \frac{dr}{ds}$.
9.	With usual notations show that $\frac{ds}{dx} = \sec\psi$ and $\frac{ds}{dy} = \cos ec\psi$.
10.	Find $\frac{ds}{dy}$ for the following curves: i) $a^2y^2 = a^3 - x^3$ at $(a,0)$ ii) $ax^2 = y^3$ iii) $y^2 = 4ax$ iv) $y = a\log\sec(x/a)$.
11.	Find $\frac{ds}{dx}$ for the following curves: $3ay^2 = x(x-a)^2$ ii) $x^{2/3} + y^{2/3} = a^{2/3}$ iii) $y = \ln\left(\frac{e^x - 1}{e^x + 1}\right)$ iv) $y = a\cosh(x/a)$.
12.	Find $\frac{ds}{dt}$ for the curves: i) $x = e^t \sin t$, $y = e^t \cos t$ ii) $x = a\cos t$, $y = b\sin t$.
13.	Find $\frac{ds}{dr}, \frac{ds}{d\theta}$ for the curve $r\theta = a$.
14.	Show that $\frac{ds}{d\theta} = \frac{a^2}{r}$ for the curve $r^2 = a^2\cos 2\theta$.
15.	Show that $\frac{ds}{d\theta} = r\sqrt{8r-3}$ for the curve $2r\cos^2\theta = 1$.
16.	Find the radius of curvature for the following curves: i) $xy^3 = a^4$ at (a,a) ii) $pa^2 = r^3$ iii) $x = a\cos\theta$, $y = a\sin\theta$ at $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$.
Seven marks questions	
17.	Find $\psi, \frac{ds}{dt}, \frac{ds}{dx}$ and $\frac{ds}{dy}$ for the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$.

18.	Obtain an expression for radius of curvature in Cartesian form.
19.	Obtain an expression for radius of curvature in parametric form.
20.	Obtain an expression for radius of curvature in polar form.
21.	Obtain an expression for radius of curvature in pedal form.
22.	Find the radius of curvature for the following curves: i) $x^3+y^3=3axy$ at $\left(\frac{3a}{2},\frac{3a}{2}\right)$ ii) $y=ax^2+bx+c$ at $x=\frac{1}{2a}\left[\sqrt{a^2-1}-b\right]$ iii) $r^n=a^n\sin n\theta$ iv) $x=a\left(\cos t+\log\tan\left(t/2\right)\right),\ y=a\sin t$ v) $x=a\cos^3 t,\ y=a\sin^3 t$.
23.	Prove that the radius of curvature of the curve $x^4+y^4=2$ at the point (1,1) is $\frac{\sqrt{2}}{3}$.
24.	Show that for the curve $r=ae^{\theta\cot\alpha}$, where a and α are constants, $\frac{\rho}{r}$ is a constant.
25.	Show that the radius of curvature of the curve $y=4\sin x-\sin 2x$ at $x=\frac{\pi}{2}$ is $\frac{5\sqrt{5}}{4}$.
26.	Show that the radius of curvature of the curve $x^2y=a(x^2+y^2)$ at $(-2a,2a)$ is 2a.
27.	Show that the radius of curvature of the curve $r^n=a^n\cos n\theta$ varies inversely as r^{n-1}.
28.	Show that for the curve $r(1-\cos\theta)=2a$, ρ^2 varies as r^3.
29.	If ρ_1 and ρ_2 are the radii of curvature at the extremities of any chord of the cardioid $r=a(1+\cos\theta)$ and which passes through the pole then show that $\rho_1^2+\rho_2^2=\frac{16a^2}{9}$.
TAYLOR'S SERIES AND MACLAURIN'S SERIES FOR FUNCTIONS OF ONE VARIABLES & TWO VARIABLES	
Two & Four marks questions	
30.	State Taylor's theorem for the function of one variable.
31.	State Taylor's theorem for the function of two variables.
32.	State Maclaurin's theorem for the function of two variables.
33.	State Maclaurin's theorem for the function of one variables.
34.	Using Maclaurin's series expand $\sqrt{1+\sin x}$ up to the term containing x^4.
35.	Obtain the first four terms of the Taylor's series of $\cos x$ about $x=\frac{\pi}{3}$.
36.	Expand $\sin^{-1}x$ in powers of x up to second degree term.
37.	Expand a^x in powers of x up to first three terms.
38.	Prove that $\log_e x=(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-......$ and hence evaluate $\log_e(1.1)$.

Seven marks questions	
39.	Expand $\tan^{-1}x$ in ascending powers of x upto the first three non-zero terms and hence show that $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - .....\right)$.
40.	Expand $\tan^{-1}x$ in powers of $(x-1)$ up to the term containing $(x-1)^4$.
41.	Using Maclaurin's series expand $\log(\sec x)$ up to the term containing x^6.
42.	Expand $e^{a\sin^{-1}x}$ in ascending powers of x upto the term containing x^4.
43.	Obtain the Maclaurin's expansion of $e^x \cos x$ up to x^4.
44.	Expand $\log(1+\sin 2x)$ in powers of x up to the term containing x^4.
45.	Expand the following functions in powers of x and y up to second degree terms: (i) $\sin x \sin y$ ii) $e^x \sin y$ iii) $e^{x^2-y^2}$ iv) $e^x \log(1+y)$.
46.	Expand the following functions at the given point up to second degree terms: (i) $xy^2+\cos(xy)$ about $\left(1, \pi/2\right)$ ii) $x^2y+3y-2$ about (1,-2) iii) x^y about (1,1).
MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES, LAGRANGE'S METHOD OF UNDETERMINED MULTIPLIERS	
Two & Four marks questions	
47.	Define maxima and minima for the function of two variables.
48.	Define stationary point and saddle point.
49.	Explain Lagrange's method of undetermined multipliers.
50.	Write the steps involved in finding the extreme values of $f(x,y)$.
51.	Examine x^3+y^3-3axy for extreme values.
52.	Show that $f(x,y) = xy(1-x-y)$ is maximum at the point $(1/3, 1/3)$.
Seven marks questions	
53.	Discuss the maxima and minima of $x^3y^2(1-x-y)$.
54.	Find the maximum value of $x^m y^n z^p$, when $x+y+z=a$.
55.	Find the extreme values of $f(x,y) = \sin x \sin y \sin(x+y);\ 0 < x < \frac{\pi}{2}, 0 < y < \frac{\pi}{2}$.
56.	Find the minimum and maximum values of $x^3+y^3-3y-12x+20=0$.
57.	Find the maximum and minimum distances of the point (1, 2, 3) from the sphere $x^2+y^2+z^2=56$.
58.	Find the extreme value of $x^2+y^2+z^2$, when $xy+yz+zx=p$.
59.	Find the minimum value of x^2yz^3 subject to $2x+y+3z=a$.
60.	A rectangular box open at the top is to have volume of 108 cubic ft. Find the dimension of the box if its total surface area is minimum.
61.	The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.

62.	Show that the volume of the greatest parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{8abc}{3\sqrt{3}}$.
63.	Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum.
64.	Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.
65.	In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$.

UNIT – II
APPLICATIONS OF FIRST ORDER AND FIRST DEGREE DIFFERENTIAL EQUATIONS & LINEAR DIFFERENTIAL EQUATIONS OF HIGHER ORDER-I

APPLICATIONS OF FIRST ORDER AND FIRST DEGREE ODE'S TO SOLVE LCR CIRCUITS &NEWTON'S LAW OF COOLING	
Two & Four marks questions	
1.	Write the DE of the closed circuit involving R and C along with a voltage source E.
2.	Write the DE of the closed circuit involving L and C both in series without applied e.m.f.
3.	Write the DE of the closed circuit involving L, C and R in series with applied e.m.f.
4.	The equation of an L-R circuit is given by $L\,i' + Ri = 10\sin t$ with $i = 0$ at $t = 0$. Express i as a function of t.
Seven marks questions	
5.	A resistance of 100 ohms, an inductance of 0.5 henry is connected in series with a battery of 20 volts. Find the current in a circuit as a function of t.
6.	According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 30^0 C and the substance cools from 100^0 C to 70^0 C in 15 minutes, at what time the temperature will be 40^0 C.
7.	A voltage Ee^{-at} is applied to a circuit of inductance L and resistance R. Show that the current at time t is $\dfrac{\mathrm{E}}{\mathrm{R - aL}}\left(e^{-at} - e^{-Rt/L}\right)$, if initial current is zero.
8.	The charge q on the plate of condenser of capacity C charged through a resistance R by a steady voltage E satisfies differential equation $\mathrm{R}\dfrac{\mathrm{dq}}{dt} + \dfrac{q}{C} = E$. If $\mathrm{q} = 0$ at $\mathrm{t} = 0$, then show that $\mathrm{q} = \mathrm{CE}\left(1 - \mathrm{e}^{-t/RC}\right)$. Find the current flowing into the plate.
9.	The temperature of a cup of coffee is 92^0 C, when freshly poured and the room temperature being 24^0 C. In one minute it was cooled to 80^0 C. How long a period must elapse, before the temperature of the cup becomes 65^0 C?
ORTHOGONAL TRAJECTORIES	
Two & Four marks questions	
10.	Define orthogonal trajectories.
11.	Write the steps involved in finding the orthogonal trajectories of the curve $f(x, y, c) = 0$.
12.	Write the steps involved in finding the orthogonal trajectories of the curve $f(r, \theta, c) = 0$.
13.	Define self-orthogonality of family of curves.
14.	Test for self-orthogonality of $r^n = a\sin(n\theta)$, where a is the parameter.
15.	If the stream lines of the flow in the channel are $\varphi(x, y) = xy - k$ then, find the orthogonal trajectories of the stream lines.
16.	Show that $r = b\sin\theta$ is the orthogonal trajectories of the family of curves $r = a\cos\theta$.
17.	Find the O.T of the family of astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

18.	Show that the family of curves $x^3 - 3xy^2 = a$ and $y^3 - 3x^2y = b$ are O.T's of each other.
Seven marks questions	
19.	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is the parameter.
20.	Find the O.T's of the family $r = a(1+\sin\theta)$, where a is the parameter.
21.	Find the O.T's of the family $r^n \cos n\theta = a^n$, where a is the parameter.
22.	If $\frac{dr}{d\theta} = r\cot(\theta/2)$ is the differential equation of the family of cardioids $r = a(1-\cos\theta)$, then find its orthogonal trajectory.
23.	If $\frac{xy}{a^2-x^2} + \frac{dy}{dx} = 0$ is the differential equation of the family of curves $f(x,y,c) = 0$, then find its orthogonal trajectory.
24.	If $x^2 + y^2 - 2a^2 \log x = c$ is the orthogonal trajectory of given family of curves then, find the differential equation of that family, if a is a fixed constant.
25.	If $r^n = k\cos(n\theta)$ is the orthogonal trajectory of given family of curves, then find the differential equation of that family.
26.	Show that the family of parabolas $y^2 = 4a(x+a)$ is self-orthogonal, where a is the parameter.
27.	Given $y = ke^{-2x} + 3x$, find member of the orthogonal trajectory passing through (0,3).
28.	Show that the orthogonal trajectories of the family of cardioids $r = a\cos^2(\theta/2)$ is the other family of cardioids $r = b\sin^2(\theta/2)$, where a and b are parameters.
29.	Find the OT's of the family of confocal and co-axial parabolas $r = 2a/(1+\cos\theta)$, where a is the parameter.
30.	Show that the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal, where λ is the parameter.
31.	Find the orthogonal trajectories of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter.
LINEAR DIFFERENTIAL EQUATIONS OF HIGHER ORDER WITH CONSTANT COEFFICIENTS	
Two & Four marks questions	
32.	Define the LDE and give an example of a second order LDE.
33.	Write the complementary function of the fourth order LDE $f(D)y = 0$, if the roots of the auxiliary equation are imaginary and repeated.
34.	If $D = \frac{d}{dx}$ and $X = X(x)$, then prove that $\frac{1}{D}X = \int X(x)dx$.
35.	If $D = \frac{d}{dx}$ and $X = X(x)$, then prove that $\frac{1}{D+a}X = e^{-ax}\int Xe^{ax}dx$.
36.	If $D = \frac{d}{dx}$ and $X = X(x)$, then prove that $\frac{1}{D-a}X = e^{ax}\int Xe^{-ax}dx$.

37.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when $X = e^{ax}$.
38.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when $X = \sin(ax+b)$ or $\cos(ax+b)$.
39.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when X is a polynomial function of degree n.
40.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when $X = e^{ax}V(x)$ and $V(x)$ is any function of x.
41.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when $X = xV(x)$ and $V(x)$ is any function of x.
42.	What is an initial value problem?
43.	What is a boundary value problem?
44.	Find the general solution of a homogeneous equation whose auxiliary equation is $\lambda^3(\lambda+4)^2(\lambda^2+2\lambda+5)^2 = 0$.
45.	If $k > 0$, then show that the general solution of $y^{iv} - k^4 y = 0$ can be expressed as $y = C_1 \cos kx + C_2 \sin kx + C_3 \cosh kx + C_4 \sinh kx$.
46.	Solve the following differential equations: (i) $(D^2 - 6D + 9)y = 0$, (ii) $y''' - 8y' + 8y = 0$, (iii) $4y''' + 4y'' + y' = 0$, (iv) $\frac{d^4y}{dx^4} + 4y = 0$.
47.	Find the P.I of the following differential equations: (i) $(D^2+4)y = x^2$, (ii) $(D^2+4)y = \sin 2x$ (iii) $(D^2 - 6D + 9)y = e^{2x}$.
48.	Solve the following initial value problems and boundary value problems i) $y'' - y = 0,\ y(0) = 3,\ y'(0) = -3;$ (ii) $y'' + y = 0,\ y(0) = 2,\ y\left(\frac{\pi}{2}\right) = -2;$ (iii) $y'' - 9y = 0,\ y(0) = 2,\ y\left(\frac{1}{3}\right) = \frac{2}{e}$.
Seven marks questions	
49.	Solve the following differential equations: a) $(D^3 + 3D^2 + 3D + 1)y = 5e^{2x} + 6e^{-x} + 7$ b) $y'' - 4y' + 13y = e^x \cosh 2x + 2^x$ c) $(D^3 - 1)y = (e^x + 1)^2$ d) $(D^3 + 2D^2 - D - 2)y = 2\cosh x$ e) $(D^3 - 3D^2 + 3D - 1)y = \sinh(x+2)$ f) $y'' + y' = x^2 + 2x + 4$ g) $y''' - 2y'' + y = x^4 + 2x + 5$ h) $y'' + 9y = \cos 2x . \cos x$ i) $(D^2 - 4D + 3)y = \sin 3x . \cos 2x$ j) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = \frac{\sin^2 x}{4}$ k) $(D^3 + D^2 - D)y = 2\cos^2 x$ l) $y''' + 2y'' + y' = e^{-x} + \sin 2x$ m) $(D^2 - 4)y = 8xe^x$ n) $(D^2 + 4D + 5)y = x^2 e^{2x}$ o) $(D^2 - 2D + 4)y = e^x \cos x$ p) $(D^3 - 7D - 6)y = e^{2x}(1+x)$ q) $(D^3 + 2D^2 + D)y = x^2 e^{2x} + \sin^2 x$ r) $(D^2 + 1)y = x\cos x$ s) $(D^2 - 2D + 1)y = xe^x \sin x$ t) $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$.

50.	Solve the following IVP/BVP: a) $y'' - 2y' + y = x$, $y(0) = 0,\ y(1) = 3$ b) $y'' + 4y' + 4y = 0$, $y(1) = 0,\ y'(0) = -1.$ c) $(D^2 + D)y = 2 + 2x + x^2$, $y(0) = 8,\ y'(0) = -1.$ d) $y''' - y'' + 100y' - 100y = 0,\ y(0) = 4,\ y'(0) = 11,\ y''(0) = -299.$

UNIT – III
LINEAR DIFFERENTIAL EQUATIONS OF HIGHER ORDER-II & PARTIAL DIFFERENTIAL EQUATIONS

CAUCHY'S AND LEGENDRE'S LINEAR DIFFERENTIAL EQUATIONS	
Two & Four marks questions	
1.	Write a second order general Legendre's linear differential equation.
2.	Write the steps involved in solving Cauchy's LDE.
3.	Write a second order general Cauchy's linear differential equation.
4.	Write the steps involved in solving Legendre's LDE.
5.	Reduce the following differential equations into a differential equation with constant coefficients: a) $x^2y''+9xy'+25y=0,$ b) $(x+1)^2y''+2(x+1)-y=0$ c) $(2x+3)^2y''+2(2x+3)-12y=0.$
6.	Solve the following linear differential equations: a) $(x^2D^2+7xD+9)y=0,$ b) $(1-2x)^2y''-2(1-2x)y'=0,$ c) $xy''+4y'=0$. d) $(x+1)^2y''+2(x+1)y'-12y=0,$ e) $4x^2y''-4xy'+3y=0.$
Seven marks questions	
7.	Solve the following differential equations: a) $x^2y''+xy'+y=\log x\sin(\log x)$ b) $x^2y''-xy'+2y=x\sin(\log x)$ c) $x^2y''-4xy'+6y=\cos[2\log x]$ d) $x^4y''''+2x^3y'''-x^2y'+xy=\sin(\log x)$ e) $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$ f) $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$ g) $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$ h) $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+y=(\log x)^2$ i) $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x\log x$ j) $(2x+1)^2y''-2(2x+1)y'-12y=6x+5$ k) $(3x+2)^2y''+3(3x+2)y'-36y=8x^2+4x+1$ l) $(2x-1)^2y''+(2x-1)y'-2y=8x^2-2x+3$.
METHOD OF VARIATION OF PARAMETERS AND ENGINEERING APPLICATIONS	
Two & Four marks questions	
8.	Write the steps involved in solving the LDE by the method of variation of parameters.
9.	The free oscillations of a galvanometer needle, as affected by the viscosity of the surrounding air which varies directly as the angular velocity of the needle, are determined by the equation $\theta''+k\theta'+\mu\theta=0,$ where θ is the angular deflection of the needle at time t . Obtain θ in terms of t .

Seven marks questions	
10.	Solve the following differential equations by the method of variation of parameters: a) $y''+y=\operatorname{cosec} x$ b) $y''+a^2y=\tan ax$ c) $y''+a^2y=\sec ax$ d) $y''-2y'+y=\dfrac{e^x}{x}$ e) $y''-3y'+2y=\dfrac{e^x}{1+e^x}$ f) $y''+3y'+2y=e^{e^x}$ g) $y''-y=\dfrac{2}{1+e^x}$ h) $y''-3y'+2y=\cos(e^{-x})$ i) $x^2y''+xy'-y=x^2e^x$ j) $y''-6y'+9y=\dfrac{e^{3x}}{x^2}$ k) $y''+y=\dfrac{1}{1+\sin x}$ l) $y''-2y'+2y=e^x\tan x$ m) $y''+2y'+2y=e^{-x}\sec^3 x$.
PARTIAL DIFFERENTIAL EQUATIONS	
Two & Four marks questions	
1.	Define partial differential equation and give an example.
2.	Write the general form of Lagrange's linear PDE.
3.	Write the steps involved in solving Lagrange's linear PDE.
4.	Write the steps involved in solving the PDE by the method of separation of variables.
5.	If $z=X(x).Y(y)$ is solution of $\dfrac{\partial z}{\partial x}=6\dfrac{\partial z}{\partial y}$ then find $Y(y)$.
6.	Solve the following PDE by direct integration method: a) $\dfrac{\partial^2 z}{\partial x^2}=2y^2$; b) $\dfrac{\partial^2 z}{\partial x^2}=6x$; c) $\dfrac{\partial^2 u}{\partial x\partial y}=x^2y$; d) $\dfrac{\partial^2 z}{\partial x^2}=\sin(xy)$; e) $\log_e\left(\dfrac{\partial^2 z}{\partial x\partial y}\right)=x+y$; f) $\dfrac{\partial^2 u}{\partial x\partial y}=e^y$.
Seven marks questions	
7.	Solve the following PDE's by direct integration: a) $\dfrac{\partial^2 z}{\partial x^2}=x+y$, given that $z=y^2$ when $x=0$ and $\dfrac{\partial z}{\partial x}=0$ when $x=2$. b) $\dfrac{\partial^2 u}{\partial x\partial t}=e^{-t}\cos x$, given that $u=0$ when $t=0$ and $\dfrac{\partial u}{\partial t}=0$ when $x=0$. c) $\dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{x}{y}+a$, given that $u=0$ when $x=0$ and $\dfrac{\partial u}{\partial x}=x$ when $y=1$.

8.	Solve the following PDE by Lagrange's method: a) $p\sqrt{x}+q\sqrt{y}=\sqrt{z}$ b) $\dfrac{y^2 z}{x}p+xzq=y^2$ c) $p\tan x+q\tan y=\tan z$ d) $pyz+qzx=xy$ e) $(z-y)p+(x-z)q=y-x$ f) $(x^2-y^2-z^2)p+2xyq=2xz$ g) $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$ h) $(y+z)p-(z+x)q=x-y$.
9.	Solve the following PDE by method of separation of variables: a) $\dfrac{\partial z}{\partial x}=2\dfrac{\partial z}{\partial y}+z$, $z(x,0)=6e^{-3x}$ b) $\dfrac{\partial^2 z}{\partial x^2}-2\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}=0$ c) $x^2\dfrac{\partial^2 u}{\partial x^2}+x\dfrac{\partial u}{\partial x}+\dfrac{\partial^2 u}{\partial y^2}=0$ d) $\dfrac{\partial z}{\partial x}=4\dfrac{\partial z}{\partial y}$; $z(0,y)=8e^{-3y}$ e) $\dfrac{\partial^2 z}{\partial t^2}=c^2\dfrac{\partial^2 z}{\partial x^2}$ f) $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=0$

UNIT – IV

BETA AND GAMMA FUNCTION & LAPLACE TRANSFORMS - I

BETA AND GAMMA FUNCTIONS	
Two marks and four marks questions	
1.	Define beta function and hence write the trigonometric form of beta function.
2.	Write the relation between beta and gamma function.
3.	Define Gamma function. Find the value of $\Gamma(-3.5)$.
4.	Prove that $\Gamma(1)=1$.
5.	Evaluate (i) $\Gamma(-1/2)$ (ii) $\beta(1/3, 2/3)$.
6.	Prove that $\Gamma(n+1)=n\Gamma(n)$.
7.	Prove that $\beta(m,n) = \beta(n,m)$.
8.	If n is a positive integer, then prove that $\Gamma(n)=(n-1)!$.
9.	Show that $\int_0^\infty e^{-a^2x^2}\, dx = \frac{\sqrt{\pi}}{2a}$.
10.	Evaluate $\int_0^\infty \frac{x^a}{a^x} dx$.
11.	Prove that $\int_0^1 x^m \left[\log\left(\frac{1}{x}\right)\right]^n dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}$.
12.	Prove that $\Gamma(1/2)=\sqrt{\pi}$ starting from the definition of gamma function.
13.	Evaluate the following: a) $\int_0^\infty x^3 e^{-x^2} dx$ b) $\int_0^\infty e^{-ax^{1/a}} dx$ c) $\int_0^1 x^a [\log(x)]^b dx$ d) $\int_0^1 xe^{-ax} \sin bx\, dx$.
Seven marks questions	
14.	If $p>-1, q>-1,$ then prove that $\int_0^{\pi/2} Sin^p\theta\, Cos^q\theta\, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$ and hence evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.
15.	Obtain the relationship between Beta and Gamma functions.
16.	Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$.

17.	Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx = \frac{\pi}{4\sqrt{2}}$.
18.	Show that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\,dx \times \int_0^\infty e^{-x^4}\, x^2\,dx = \frac{\pi}{4\sqrt{2}}$.
19.	Prove that $\int_a^b (x-a)^m (b-x)^n\,dx = (b-a)^{m+n+1}\beta(m+1,\, n+1)$ and hence deduce that $\int_5^9 (x-5)^{\frac{1}{4}}(9-x)^{\frac{1}{4}}\,dx = \frac{2\left(\Gamma\left(\frac{1}{4}\right)\right)^2}{3\sqrt{\pi}}$.
	LAPLACE TRANSFORMS-I ***TRANSFORMS OF ELEMENTARY FUNCTIONS AND PROPERTIES***
Two marks & four marks questions	
20.	Define Laplace Transform of a function.
21.	State the sufficient conditions for the existence of the Laplace transform of a function $f(t)$.
22.	Define first shifting property of Laplace transform.
23.	If $\mathrm{L}\{\mathrm{f(t)}\} = F(s)$, then show that $\mathrm{L}\{\mathrm{f'(t)}\} = sF(s) - f(0)$ if $\lim_{t\to\infty} e^{-st} f(t) = 0$.
24.	Prove that $\mathrm{L}\{1\} = \frac{1}{\mathrm{s}}$.
25.	Obtain the Laplace transform of (i) e^{at} (ii) $\sin at$ (iii) $\cos at$ (iv) $\sinh at$ (v) $\cosh at$.
26.	If $\mathrm{L}\{\mathrm{f(t)}\} = F(s)$, then prove that $\mathrm{L}(\mathrm{e}^{\mathrm{at}}\mathrm{f(t)}) = F(s-a)$.
27.	If $\mathrm{L}\{\mathrm{f(t)}\} = F(s)$, then prove that $\mathrm{L}\{\mathrm{sinhat\ f(t)}\} = \frac{1}{2}[F(s-a) - F(s+a)]$.
28.	If $\mathrm{L}\{\mathrm{f(t)}\} = F(s)$, then prove that $\mathrm{L}\{\mathrm{coshat\ f(t)}\} = \frac{1}{2}[F(s-a) + F(s+a)]$.
29.	Evaluate $\int_0^\infty t^2 \cos t\,dt$ using Laplace transform.
30.	Evaluate $\int_0^\infty e^{-2t}t^2 \sin t\,dt$ using Laplace transform.
31.	Find Laplace transform of the following: a) $L\{(t^2+4)^3\}$ b) $L\{e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t\}$ c) $L\{\sin \omega t - \omega t\cos \omega t\}$ d) $L\{1 + 2\sqrt{t} + 3/\sqrt{t} + 4t\}$ e) $L\{\sinh\, 2t \sin\, 4t\}$
32.	Write the formula of (i) $L\left\{\frac{f(t)}{t}\right\}$ (ii) $L\left\{\int_0^t f(t)\,dt\right\}$.
33.	Write the formula for: (i) $L\{f''(t)\}$ (ii) $L\{f'''(t)\}$.
34.	Write the formula of $L\{t^n\}$, where n is a fraction.

35.	If $L\{f(t)\}=F(s)$, then prove that $L\left\{\int_0^t f(t)dt\right\}=\frac{F(s)}{s}$.
36.	If $L\{f(t)\}=F(s)$, then prove that $L\{t\,f(t)\}=-\frac{d}{ds}(F(s))$.
37.	If $L\{f(t)\}=F(s)$, then prove that $L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(s)\,ds$.
Seven marks questions	
38.	If $L\{f(t)\}=F(s)$, then prove that $L\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}\{F(s)\}$, where n is a positive integer.
39.	If $L\{f'(t)\}=sF(s)-f(0)$, then prove that $L\{f''(t)\}=s^2F(s)-sf(0)-f'(0)$.
40.	Obtain the Laplace transform of $\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)^3$.
41.	Given $L\{Sin\sqrt{t}\}=\frac{\sqrt{\pi}}{4s^{3/2}}e^{-1/4s}$, then find $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.
42.	If $f(t)$ and its $(n-1)$ derivatives are continuous then prove that $L\{f^{(n)}(t)\}=s^n L\{f(t)\}-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots\ldots\ldots\ldots-f^{n-1}(0)$.
43.	Find i) $L\left\{\frac{\cos at-\cos bt}{t}\right\}$ and $L\left\{t^4e^{-\frac{3}{2}t}\right\}$ ii) $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$ and $L\{t\sin 3t\cos 2t\}$ iii) $L\left\{\frac{\cosh 2t\sin 2t}{t}\right\}$ and $L\{t^2\sin 7t\}$ iv) $L\left\{\frac{t-\sinh at}{t}\right\}$ and $L\{t\sin^2 3t\}$.
44.	Find the Laplace transform of $\cosh at$ and hence find $L\{e^{-t}t\cosh t\}$.
45.	Find (i) $L\left\{e^t\int_0^t\frac{\sin t}{t}dt\right\}$ (ii) $L\left\{\int_0^t\int_0^t(t\sin t)dt\,dt\right\}$.
46.	Find the Laplace transform of i) $e^{-3t}\cos 5t\sin 5t$ ii) $(1-\cos t)/t^2$.
47.	Find i) $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$ ii) $L\{te^{-4t}\sin 3t\}$ iii) $L\left\{\int_0^t te^{-t}\cos t\,dt\right\}$ iv) $L\left\{\frac{\sin t\sin 3t}{t}\right\}$ v) $L\{t^2e^{-2t}\sin 3t\}$
48.	Evaluate i) $\int_0^\infty\frac{e^{-t}\sin^2 t}{t}dt$ ii) $\int_0^\infty e^{-3t}t\sin t\,dt$ iv) $\int_0^\infty\frac{e^{-t}-e^{-3t}}{t}dt$ using Laplace transform.
49.	If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)\,dt=\frac{3}{8}$, find α.

UNIT STEP FUNCTION & UNIT IMPULSE FUNCTION	
Two marks & four marks questions	
50.	Define unit step function and represent graphically.
51.	Write Laplace Transform of Heaviside's function $H(t-a)$.
52.	Write Laplace Transform of Dirac-Delta function and find $\mathrm{L}\{\delta(\mathrm{t})\}$.
53.	Define the Dirac-Delta function and sketch its graph.
54.	Prove that $L\{H(t-a)\}=\dfrac{e^{-as}}{s}$.
55.	Prove that $L\{f(t)\delta(t-a)\}=e^{-as}f(a)$.
56.	Prove that $L\{f(t-a)H(t-a)\}=e^{-as}F(s)$.
57.	Find the Laplace transform of $(t^2+1)H(t-1)$.
58.	Find the Laplace transform of $\cos t\, U(t-\pi)$.
59.	Find $L\{(t^2-8t+16)\,e^{-(t-4)}\,U(t-4)\}$.
Seven marks questions	
60.	If $L\{f(t)\}=F(s)$, then prove that $L\{f(t-a)H(t-a)\}=e^{-as}F(s)$. Also, evaluate $e^{-t}\sin t\, H(t-\pi)$
61.	Express the following functions in terms of Heaviside function $f(t)=\begin{cases}A(t), & a<t<b\\ B(t), & c<t<d\\ C(t), & t>d\end{cases}$
62.	Express the following function in terms of the Heaviside function and hence find its Laplace transform: a) $f(t)=\begin{cases}5t, & 0<t<2,\\ t^2, & t>2.\end{cases}$ b) $f(t)=\begin{cases}\sin t, & 0<t<\pi/2,\\ \cos t, & t>\pi/2.\end{cases}$ c) $f(t)=\begin{cases}t-1, & 1<t<2,\\ 3-t, & 2<t<3\end{cases}$ d) $f(t)=\begin{cases}2, & 0<t\le 1,\\ \dfrac{t^2}{2}, & 1<t\le\dfrac{\pi}{2},\\ \cos t, & t>\dfrac{\pi}{2}\end{cases}$ e) $f(t)=\begin{cases}t^2, & 0<t<2,\\ t-1, & 2<t<4,\\ 7, & t>4.\end{cases}$ f) $f(t)=\begin{cases}\sin t, & 0<t<\pi,\\ \sin 2t, & \pi<t<2\pi,\\ \sin 3t, & t>2\pi.\end{cases}$

UNIT–V
LAPLACE TRANSFORMS - II

LAPLACE TRANSFORMS OF PERIODIC FUNCTION

Two marks & four marks questions

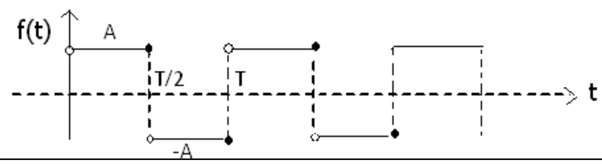
1. Define a periodic function with an example.

2. Draw the graph of the following periodic functions:

a) $f(t)=\begin{cases} t, & 0\le t\le a \\ 2a-t, & a\le t\le 2a \end{cases}$ b) $f(t)=\begin{cases} \sin \omega t, & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<2\pi/\omega \end{cases}$ c) $f(t)=\begin{cases} 1 & where\ 0\le t\le 1 \\ -1 & where\ 1\le t\le 2 \end{cases}$.

3. If $f(t)$ is a periodic function of period T then prove $L\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$.

4. Obtain the Laplace transform of the rectangular wave $f(t)$ given by following figure:



5. Draw the graph of the periodic function $f(t)=\begin{cases} t, & 0<t<\pi \\ \pi-t, & \pi<t<2\pi \end{cases}$ and find its Laplace transform.

Seven marks questions

6. A periodic function of period $2\pi/\omega$ is defined by $f(t)=\begin{cases} E\sin(\omega t) & for\ 0<t<\pi/\omega \\ 0 & for\ \pi/\omega<t<2\pi/\omega \end{cases}$

where E and ω are positive constants, then show that $L\{f(t)\}=\frac{E\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$.

7. A periodic function of period $2a$ is defined by $f(t)=\begin{cases} E & for\ 0\le t\le a \\ -E & for\ \ a\le t\le 2a \end{cases}$, find $L\{f(t)\}$.

8. A periodic square wave function $f(t)$, in terms of unit-step function is written as $f(t)=k\left[u_o(t)-2u_a(t)+2u_{2a}(t)-2u_{3a}(t)+\ldots\ldots\ldots\right]$where $u_i(t)=u(t-i)$,

show that $L\{f(t)\}=\frac{k}{s}\tanh\left(\frac{as}{2}\right)$.

9. Find the Laplace transform function of period 2π, given by $f(t)=\begin{cases} t & for\ 0<t<\pi \\ \pi-t & for\ \ \pi<t<2\pi \end{cases}$.

10. Find the Laplace transform of saw-tooth wave function of period T, given by $f(t)=\frac{t}{T}$ for $0<t<T$.

11. Find the Laplace transform of the triangular wave function of period $2a$, given by $f(t)=\begin{cases} t & for\ 0\le t\le a \\ 2a-t & for\ \ a\le t\le 2a \end{cases}$.

12.	Find the Laplace transform of square wave function of period a defined by $f(t)=\begin{cases}1 & for\ 0<t<a/2\\ -1 & for\ a/2<t<a\end{cases}$.
	INVERSE LAPLACE TRANSFORMS
Two marks & four marks questions	
13.	Define inverse Laplace transform.
14.	Write the formula to find $\mathrm{L}^{-1}\left\{\frac{\mathrm{F(s)}}{\mathrm{s}}\right\}$.
15.	Find the inverse Laplace transform of the following: a) $\frac{1}{s\sqrt{s}}+\frac{3}{s^2\sqrt{s}}-\frac{8}{\sqrt{s}}$ b) $\frac{s}{(s-2)^5}$ c) $\frac{3s-12}{s^2+8}$ d) $\frac{3s+1}{(s+1)^4}$ e) $\frac{e^{-\frac{s}{2}}}{s^4}$ f) $\frac{s^3+6s^2+12s+8}{s^6}$ g) $\frac{4s+12}{s^2+8s+16}$ h) $\frac{se^{-s/2}+\pi e^{-s}}{s^2+\pi^2}$ i) $\frac{1+e^{-3s}}{s^2+2s+1}$ j) $\frac{\cosh 2s}{e^{3s}s^2}$ k) $\frac{s}{(s+3)^2+4}$ l) $\frac{se^{-as}+1}{s^2+\omega^2}$ m) $\frac{s+2}{s^2-4s+13}$ n) $\frac{3(s^2-2)^2}{2s^5}$ o) $\frac{e^{-3s}}{(s-4)^2}$.
16.	Prove the following: a) $L^{-1}\left\{\frac{1}{s}\cos\frac{1}{s}\right\}=1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-......$ b) $L^{-1}\left\{\frac{1}{s}\sin\frac{1}{s}\right\}=\frac{t}{1!}-\frac{t^3}{(3!)^2}+\frac{t^5}{(5!)^2}-.......$
Seven marks questions	
17.	Evaluate the following: a) $L^{-1}\left\{\log\left(\frac{s^2+b^2}{s^2+a^2}\right)\right\}$ b) $L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\}$ c) $L^{-1}\left\{\log\frac{1+s}{s}\right\}$ d) $L^{-1}\left\{s\log\left(\frac{s+4}{s-4}\right)\right\}$ e) $L^{-1}\left\{\log\left(\frac{s^2+1}{s(s+1)}\right)\right\}$ f) $L^{-1}\left\{\log\left(\frac{s^2+4}{s(s+4)(s-4)}\right)\right\}$ g) $L^{-1}\left\{\frac{e^{-s}(s+2)}{(s+1)^2}\right\}$ h) $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$
18.	Find the inverse Laplace transform by the method of partial fraction: a) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ b) $\frac{s+2}{(s^2+4s+5)^2}$ c) $\frac{s+4}{s(s^2+4)(s-1)}$ d) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$
	CONVOLUTION THEOREM
Two marks & four marks questions	
19.	Define convolution of two functions.
20.	State convolution theorem.
21.	Show that convolution of two functions is commutative.

Seven marks questions	
22.	State and prove convolution theorem.
23.	Verify convolution theorem for the following pair of functions: a) $f_1(t)=t$ and $f_2(t)=te^{-t}$ b) $f_1(t)=\sin t$ and $f_2(t)=e^{-t}$ c) $\phi(t)=\cos at$ and $\psi(t)=\cos bt$ d) $f_1(t)=\dfrac{\sin 2t}{2}$ and $f_2(t)=e^{-t}$
24.	Using convolution theorem find the inverse Laplace transform of the following: a) $\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$ b) $\dfrac{1}{(s^2+4)(s+1)^2}$ c) $\dfrac{s^2}{(s^2+a^2)^2}$ d) $\dfrac{1}{(s+3)(s^2+2s+2)}$ e) $\dfrac{s}{(s+2)(s^2+9)}$
SOLUTION OF LINEAR DIFFERENTIAL EQUATION	
Two marks & four marks questions	
25.	Explain the procedure of solving initial value problem using Laplace transforms.
26.	Explain the procedure of solving simultaneous differential equations using Laplace transforms.
27.	Solve the following using Laplace transform: a) $y''-y=0;\ y(0)=3,\ y'(0)=-3$ b) $y'-5y=e^{5t};\ y(0)=0$ c) $y'-4y=1;\ y(0)=1$
Seven marks questions	
28.	Solve the following differential equation by the method of Laplace transform: a) $\dfrac{dy}{dt}+y=\cos 2t,\ y(0)=1$ b) $y''+4y'+8y=1$ given that $y(0)=0,\ y'(0)=1.$ c) $y'''+2y''-y'-2y=0$ given $y(0)=0,\ y'(0)=0, y''(0)=6$ d) $y''+y=H(t-1)$; given $y(0)=0,\ y'(0)=1.$ e) $x''+9x=\cos t$; given $x(0)=1,\ x\left(\dfrac{\pi}{2}\right)=-1.$ f) $tD^2y+(1-2t)Dy-2y=0$ given $y(0)=1;\ y'(0)=2$ g) $y''+4y=f(t)$; $y(0)=y'(0)=0,\ f(t)=\begin{cases}0 & \text{If } t<3\\ t & \text{if } t\ge 3\end{cases}$ h) $\dfrac{dy}{dt}+2y+\int_0^t y\,dt=\sin t$,given $y(0)=0;\ y'(0)=1$.
29.	Using Laplace transform, solve the following simultaneous equation: a) $\dfrac{dx}{dt}-2y=\cos 2t;\ \dfrac{dy}{dt}+2x=\sin 2t$, given $x(0)=1,\ y(0)=0.$ b) $\dfrac{dx}{dt}-y=e^t;\ \dfrac{dy}{dt}+x=\sin t$, given $x(0)=1,\ y(0)=0.$ c) $\dfrac{dx}{dt}+5x-2y=t,\ \dfrac{dy}{dt}+2x+y=0$ given $x(0)=0, y(0)=0$.

30.	The coordinates (x, y) of a particle moving along a plane curve at any point t, are given by $y' + 2x = \sin 2t$; $x' - 2y = \cos 2t$; $t > 0$. If at $t = 0$, $x = 1$ and $y = 0$ show by using Laplace Transforms that the particle moves along the curve $4x^2 + 4xy + 5y^2 = 4$.
31.	A voltage Ee^{-at} is applied at $t = 0$ to a circuit of inductance L and resistance R. Show that the current at time $t = 0$ is $\frac{E}{R - al}\left(e^{-at} - e^{-Rt/L}\right)$ using Laplace transforms.
32.	Determine the response of damped mass-string system under a square wave, given by the following equation $y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2)$; $y(0) = 0 = y'(0)$.
33.	The currents i_1 and i_2 in a mesh are given by the differential equations $i_1' - \omega i_2 = a\cos pt$; $i_2' - \omega i_1 = a\sin pt$. Find the currents i_1 and i_2 by Laplace transforms, if $i_1 =$ i_2 =0 at $t = 0$.

MOBILES ARE BANNED **USN:**

1	M	S							

DEPARTMENT OF MATHEMATICS
CIE MODEL QUESTION PAPER

Sub Code:	MA21	**Sub:**	Engineering Mathematics-II	**Test:**	01
Semester:	II	**Term:**	20-01-2020 to 09-05-2020	**Marks:**	30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q. No.		Questions	Blooms Level	CO's	Marks
1.	(a)	Define the term (i) Curvature (ii) Radius of curvature.	L1	CO1	2
	(b)	Show that $\frac{ds}{d\theta}=\frac{a^2}{r}$ for the curve $r^2=a^2\cos 2\theta$.	L2	CO1	3
	(c)	Using Maclaurin's series expand $\sqrt{1+\sin x}$ up to the term containing x^4.	L3	CO1	5
	(d)	Show that the orthogonal trajectories of the family of cardioids $\mathrm{r}=\mathrm{a}\cos^2(\theta/2)$ is the other family of cardioids $\mathrm{r}=\mathrm{b}\sin^2(\theta/2)$, where a and b are parameters.	L5	CO1	5
2.	(a)	Write the DE of the closed circuit involving L and C both in series without applied e.m.f.	L1	CO2	2
	(b)	Reduce the differential equation $x^2y''+9xy'+25y=0$ into a linear differential equation with constant coefficients.	L2	CO2	3
	(c)	Solve the differential equation $y''+a^2y=\tan ax$ by the method of variation of parameters.	L3	CO2	5
	(d)	Solve: $(D^2-2D+1)y=xe^x\sin x$.	L4	CO2	5
3.	(a)	Define self-orthogonality of family of curves.	L1	CO2	2
	(b)	If $D=\frac{d}{dx}$ and $X=X(x)$, then prove that $\frac{1}{D+a}X=e^{-ax}\int Xe^{ax}dx$	L2	CO1	3
	(c)	The temperature T at any point (x,y,z) in space is $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.	L3	CO2	5
	(d)	Solve: $(2x-1)^2y''+(2x-1)y'-2y=8x^2-2x+3$.	L4	CO3	5

MOBILES ARE BANNED USN: | 1 | M | S | | | | | | | |

DEPARTMENT OF MATHEMATICS
CIE MODEL QUESTION PAPER

Sub Code:	MA21	**Sub:**	Engineering Mathematics-II	**Test:**	02
Semester:	II	**Term:**	20-01-2020 to 09-05-2020	**Marks:**	30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q. No.		Questions	Blooms Level	CO's	Marks
1.	(a)	Write Laplace Transform of Heaviside's function $H(t-a)$.	L1	CO3	2
	(b)	Find the inverse Laplace transform of $\frac{s}{(s+3)^2+4}$.	L2	CO3	3
	(c)	Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx = \frac{\pi}{4\sqrt{2}}$.	L3	CO3	5
	(d)	Solve the PDE $x^2\frac{\partial^2 u}{\partial x^2}+x\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial y^2}=0$ by method of separation of variables.	L4	CO3	5
2.	(a)	Write the formula for $\mathrm{L}^{-1}\left\{\frac{\mathrm{F(s)}}{\mathrm{s}}\right\}$.	L1	CO4	2
	(b)	Solve the PDE $\frac{\partial^2 u}{\partial x \partial y}=e^y$ by Direct integration.	L2	CO4	3
	(c)	Express the function $f(t)=\begin{cases}\sin t, & 0<t<\pi/2,\\ \cos t, & t>\pi/2.\end{cases}$ in terms of the Heaviside function and hence find its Laplace transform.	L3	CO4	5
	(d)	Solve the PDE $(z-y)p+(x-z)q=y-x$ by Lagrange's method.	L4	CO5	5
3.	(a)	Write the relation between beta and gamma function.	L1	CO5	2
	(b)	Obtain the Laplace transform of $\int_0^\infty t^3 e^{-t}\,dt$.	L2	CO4	3
	(c)	Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$ by the method of partial fraction.	L3	CO3	5
	(d)	Find the Laplace transform function of period 2π, given by $f(t)=\begin{cases} t & for\ 0\le t\le a\\ 2a-t & for\ \ a\le t\le 2a\end{cases}$.	L4	CO4	5

RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE-560054

SEE MODEL QUESTION PAPER-I

Course & Branch	**: B.E. – Common to all Branches**	**Semester:**	**II**
Subject	**: Engineering Mathematics – II**	**Max. Marks:**	**100**
Subject Code	**: MA21**	**Duration:**	**3Hrs**

Instructions to the candidates:

Answer ONE full question from each unit.

		UNIT – I		
1.	a.	Write the derivative of arc length of a curve in polar form.	CO1	2
	b.	Show that $f(x,y)=xy(1-x-y)$ is maximum at the point $(1/3, 1/3)$.	CO1	4
	c.	Obtain the expression for radius of curvature in Cartesian form.	CO1	7
	d.	Expand the function $e^x \sin y$ in powers of x and y up to second degree term.	CO1	7
2.	a.	State Taylor's theorem for the function of one variable.	CO1	2
	b.	With usual notation, show that $\frac{ds}{dx}=\sec\psi$ and $\frac{ds}{dy}=\operatorname{cosec}\psi$.	CO1	4
	c.	If ρ_1 and ρ_2 are the radii of curvature at the extremities of any chord of the cardioid $r=a(1+\cos\theta)$ and which passes through the pole then show that $\rho_1^2+\rho_2^2=\frac{16a^2}{9}$.	CO1	7
	d.	A rectangular box open at the top is to have volume of 108 cubic ft. Find the dimension of the box if its total surface area is minimum.	CO1	7
		UNIT – II		
3.	a.	Define self-orthogonality of family of curves.	CO2	2
	b.	Explain the working rule to find the P.I. of the LDE $f(D)y=X$,when $X=\sin(ax+b)$ or $\cos(ax+b)$.	CO2	4
	c.	Solve: $(D^2-4D+4)y=8x^2e^{2x}\sin 2x$.	CO2	7
	d.	The temperature of a cup of coffee is 92^0 C, when freshly poured and the room temperature being 24^0 C. In one minute it was cooled to 80^0 C. How long a period must elapse, before the temperature of the cup becomes 65^0 C?	CO2	7
4.	a.	Write the differential equation of the closed circuit involving R and C along with a voltage source E.	CO2	2
	b.	If $D=\frac{d}{dx}$ and $X=X(x)$, then prove that $\frac{1}{D-a}X=e^{ax}\int Xe^{-ax}dx$.	CO2	4
	c.	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, where λ is the parameter.	CO2	7
	d.	Solve: $(D^2+D)y=2+2x+x^2$, $y(0)=8, y'(0)=-1$.	CO2	7

UNIT – III				
5.	a.	Write a second order general Cauchy's linear differential equation.	CO3	2
	b.	Solve the PDE $\frac{\partial^2 z}{\partial x^2} = \sin(xy)$ by direct integration.	CO3	4
	c.	Solve: $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 8x^2 + 4x + 1$.	CO3	7
	d.	Solve the PDE $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$, $z(x,0) = 6e^{-3x}$ by method of separation of variables.	CO3	7
6.	a.	Define partial differential equation and give an example.	CO3	2
	b.	Solve the differential equation: $(x^2D^2 + 7xD + 9)y = 0$.	CO3	4
	c.	Solve the differential equation $y'' + 2y' + 2y = e^{-x}\sec^3 x$ by the method of variation of parameters.	CO3	7
	d.	Solve: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.	CO3	7
UNIT – IV				
7.	a.	Prove that $\Gamma(n+1) = n\Gamma(n)$.	CO4	2
	b.	If $\mathrm{L}(\mathrm{f}(\mathrm{t})) = F(s)$ then prove that $\mathrm{L}(\mathrm{e}^{at}\mathrm{f}(\mathrm{t})) = F(s-a)$.	CO4	4
	c.	Find (i) $L\left\{\frac{\sin t \sin 3t}{t}\right\}$ (ii) $L\{t^2 e^{-2t} \sin 3t\}$.	CO4	7
	d.	Show that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}dx \times \int_0^\infty e^{-x^4} x^2\, dx = \frac{\pi}{4\sqrt{2}}$.	CO4	7
8.	a.	Define unit step function and represent graphically.	CO4	2
	b.	If $p > -1, q > -1$, then prove that $\int_0^{\pi/2} Sin^p\theta\, Cos^q\theta \; d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$.	CO4	4
	c.	Express the function $f(t) = \begin{cases} \sin t, & 0 < t < \pi, \\ \sin 2t, & \pi < t < 2\pi, \\ \sin 3t, & t > 2\pi. \end{cases}$ in terms of the Heaviside function and hence find its Laplace transform.	CO4	7
	d.	Find (i) $L\{te^{-4t}\sin 3t\}$ (ii) $L\left\{\int_0^t te^{-t}\cos t\, dt\right\}$.	CO4	7
UNIT – V				
9.	a.	Define a periodic function with an example.	CO5	2
	b.	Find the inverse Laplace transform of $\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$.	CO5	4
	c.	Find the inverse Laplace transform of $\frac{4s+5}{(s+1)^2(s+2)}$.	CO5	7
	d.	Solve the differential equation $y''' + 2y'' - y' - 2y = 0$ given $y(0) = 0,\ y'(0) = 0, y''(0) = 6$ by the method of Laplace transform.		7

10	a.	Define convolution of two functions.	CO5	2
	b.	Evaluate $L^{-1}\left(\cot^{-1}\left(\frac{s}{a}\right)\right)$.	CO5	4
	c.	Find the Laplace transform of the triangular wave function of period $2a$, given by $f(t)=\begin{cases} t & for\ 0\le t\le a \\ 2a-t & for\ \ a\le t\le 2a \end{cases}$.	CO5	7
	d.	Verify convolution theorem for the pair of functions $f_1(t)=\sin t$ and $f_2(t)=e^{-t}$.	CO5	7

Note: <u>students should not be under the impression that questions from model question paper will appear in SEE.</u>

RAMAIAH INSTITUTE OF TECHNOLOGY

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BANGALORE-560054

SEE MODEL QUESTION PAPER-II

Course & Branch	**: B.E. – Common to all Branches**	**Semester:**	**II**
Subject	**: Engineering Mathematics – II**	**Max. Marks:**	**100**
Subject Code	**: MA21**	**Duration:**	**3Hrs**

Instructions to the candidates:

Answer ONE full question from each unit.

		UNIT – I		
1.	a.	State Taylor's theorem for the function of one variable.	CO1	2
	b.	Find $\frac{ds}{dx}$ for the curve $y=\ln\left(\frac{e^x-1}{e^x+1}\right)$.	CO1	4
	c.	Obtain the expression for radius of curvature in polar form.	CO1	7
	d.	Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.	CO1	7
2.	a.	Write the derivative of arc length of a curve in parametric form.	CO1	2
	b.	Prove that the curvature of a circle is constant.	CO1	4
	c.	Expand the function $e^x \sin y$ in powers of x and y up to second degree terms.	CO1	7
	d.	Discuss the maxima and minima of $x^3y^2(1-x-y)$.	CO1	7
		UNIT – II		
3.	a.	Write the complementary function of the fourth order LDE $f(D)y=0$, if the roots of the auxiliary equation are imaginary and repeated.	CO2	2
	b.	Solve: $y''-y=0,\ y(0)=3,\ y'(0)=-3$.	CO2	4
	c.	Show that the family of parabolas $y^2=4a(x+a)$ is self-orthogonal, where a is the parameter.	CO2	7
	d.	Solve: $(D^3+2D^2+D)y=x^2e^{2x}+\sin^2 x$.	CO2	7
4.	a.	Define orthogonal trajectories.	CO2	2
	b.	If $k>0$, then show that the general solution of $y^{iv}-k^4y=0$ can be expressed as $y=C_1\cos kx+C_2\sin kx+C_3\cosh kx+C_4\sinh kx$.	CO2	4
	c.	The charge q on the plate of condenser of capacity C charged through a resistance R by a steady voltage E satisfies differential equation $R\frac{dq}{dt}+\frac{q}{C}=E$. If $q=0$ at $t=0$, then show that $q=CE\left(1-e^{-t/RC}\right)$. Find the current flowing into the plate.	CO2	7
	d.	Solve: $y''-4y'+13y=e^x\cosh(2x)+2^x$.	CO2	7

UNIT – III				
5.	a.	Write a second order general Legendre's linear differential equation.	CO3	2
	b.	If $z = X(x)Y(y)$ is solution of $\frac{\partial z}{\partial x} = 6\frac{\partial z}{\partial y}$ then find $Y(y)$.	CO3	4
	c.	Solve the differential equation $y'' - 3y' + 2y = \frac{e^x}{1+e^x}$ by the method of variation of parameters.	CO3	7
	d.	Solve: $(z-y)p + (x-z)q = y - x$.	CO3	7
6.	a.	Write the general form of Lagrange's linear PDE.	CO3	2
	b.	Solve the PDE $\frac{\partial^2 u}{\partial x \partial y} = x^2 y$ by direct integration method:	CO3	4
	c.	Solve the PDE $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by method of separation of variables:	CO3	7
	d.	Solve: $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$.	CO3	7
UNIT – IV				
7.	a.	Define the Dirac-Delta function and sketch its graph.	CO4	2
	b.	Show that $\int_0^\infty e^{-a^2x^2}\, dx = \frac{\sqrt{\pi}}{2a}$.	CO4	4
	c.	Find i) $L\{\cos\sqrt{t}\}$ ii) $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$.	CO4	7
	d.	Express the function $f(t) = \begin{cases} t, & 0 < t < 2, \\ t-1, & 2 < t < 3, \\ 7, & t > 3. \end{cases}$ in terms of the Heaviside function and hence find its Laplace transform.	CO4	7
8.	a.	Write the relation between beta and gamma function.	CO4	2
	b.	If $L(f'(t)) = sF(s) - f(0)$ then prove that $L(f''(t)) = s^2F(s) - sf(0) - f'(0)$.	CO4	4
	c.	Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi.$	CO4	7
	d.	If $L\{f(t)\} = F(s)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{F(s)\}$, where n is a positive integer.	CO4	7
UNIT – V				
9.	a.	Define inverse Laplace transform.	CO5	2
	b.	Draw the graph of the periodic function $f(t) = \begin{cases} t & where\, 0 \le t \le 1 \\ 2-t & where\ 1 \le t \le 2 \end{cases}$.	CO5	4
	c.	State and prove convolution theorem.	CO5	7

	d.	Using Laplace transform, solve the simultaneous differential equations $\frac{dx}{dt} = 2x - 3y$; $\frac{dy}{dt} = y - 2x$ given $x(0) = 8, y(0) = 3$.	CO5	7
10	a.	Define convolution of two functions.	CO5	2
	b.	Find the inverse Laplace transform of $\frac{3(s^2-2)^2}{2s^5}$.	CO5	4
	c.	Find the Laplace transform of the function of period $2\pi/\omega$ defined by $f(t) = \begin{cases} \sin \omega t & 0 < t < \pi/\omega \\ 0 & \pi/\omega < t < 2\pi/\omega \end{cases}$.	CO5	7
	d.	Solve the differential equation $y'' + 4y = f(t)$; $y(0) = y'(0) = 0$, $f(t) = \begin{cases} 0 & \text{If } t < 3 \\ t & \text{if } t \geq 3 \end{cases}$ by the method of Laplace transform.	CO5	7

Note: <u>students should not be under the impression that questions from model question paper will appear in SEE.</u>

ASSIGNMENT QUESTIONS	
1.	Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=4$ at the point where it cuts the line passing through the origin making an angle $\pi/4$ with the x-axis.
2.	Show that at the point where the curve $r=a\theta$ intersects the curve $r=\dfrac{a}{\theta}$ their curvatures are in the ratio 3:1.
3.	Expand the function $e^{ax}\cos(by)$ in powers of x and y up to second degree terms.
4.	Examine the function $\sin x+\sin y+\sin(x+y)$ for extreme values.
5.	If $x+y+z=a$, show that the maximum value of $x^m y^n z^p$ is $m^m n^n p^p\left(\dfrac{a}{m+n+p}\right)^{m+n+p}$.
6.	According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 30^0 C and the substance cools from 100^0 C to 70^0 C in 15 minutes, at what time the temperature will be 40^0 C.
7.	Find the Orthogonal trajectories of the family of curves $\left(r+\dfrac{k^2}{r}\right)\cos\theta=a$, where a being a parameter.
8.	A particle moves along the x-axis according to the law $\dfrac{d^2x}{dt^2}+\dfrac{dx}{dt}+25x=0$. If the particle is started at $x=0$ with an initial velocity of 12ft/Sec to the left, determine x in terms of t.
9.	Solve $y''+4y'+20y=23\sin t-15\cos t$, given $y(0)=0,\ y(0)=-1$.
10.	Solve $(D^2-4D+3)y=\sin 3x.\cos 2x.$
11.	Solve $(D^2-4D+4)y=6x^{-4}e^{2x}$ by the method of variation of parameter.
12.	Solve $(2x+1)^2y''-2(2x+1)y'-12y=x\log(2x+1)$.
13.	Solve $\dfrac{y-z}{yz}p+\dfrac{z-x}{zx}q=\dfrac{x-y}{xy}$.
14.	Solve $\dfrac{\partial z}{\partial x}=2\dfrac{\partial z}{\partial y}+z$, $z(x,0)=6e^{-3x}$ by method of separation of variables.
15.	Evaluate $\displaystyle\int_0^3\frac{x^{3/2}}{\sqrt{3-x}}dx\times\int_0^1\frac{dx}{\sqrt{1-x^{1/4}}}$.
16.	Find (i) $L\left\{\dfrac{\cosh 2t\sin 2t}{t}\right\}$ (ii) $L\{t^2\sin 7t\}$.
17.	Evaluate $\displaystyle\int_0^\infty\frac{\cos 6t-\cos 4t}{t}dx$ using Laplace transform.

18.	Express the function $f(t)=\begin{cases} t^2, & 0<t<2, \\ t-1, & 2<t<4, \\ 7, & t>4. \end{cases}$ in terms of the Heaviside function and hence find its Laplace transform.
19.	Find the Laplace transform of the function $f(t)=\dfrac{kt}{p}$ for $0<t<p$ and $f(t+p)=f(t)$.
20.	Find $L^{-1}\left\{\dfrac{3s^2+16s+26}{s(s^2+4s+13)}\right\}$.

IMPORTANT NOTE:
Department of Mathematics will conduct remedial classes for needy and interested students prior to Test-I and Test-II for a week. Test syllabus will be revised. Make the best use of the same.