

LOGIC GATES

→ Logic Blocks:

1. Combinational logic block
O/P depends only on I/P values. (All ALU functions)

2. Sequential logic block

→ O/P depends on I/P and state (stored info)



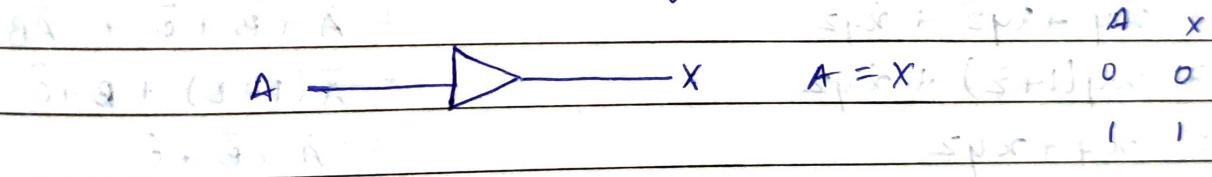
→ Functions of gates can be described by:

◦ Truth table

◦ Boolean functions

◦ Karnaugh map.

* NOTE: Buffer combinational gate: I/P = (A + B) X



II. BOOLEAN ALGEBRA

1. → I/P and O/P signals can be represented by Boolean variables
- Function of digital logic circuits can be represented by logic ops i.e., boolean functions

TRUTH TABLE

- Table that describes the O/P values for all combinations of the input values called MINTERMS

- n I/P variables $\rightarrow 2^n$ minterms

BASIC IDENTITIES OF BOOLEAN ALGEBRA

$$x + 0 = x$$

$$x \cdot 0 = 0$$

$$x + 1 = 1$$

$$x \cdot 1 = x$$

$$x + x = x$$

$$x \cdot x = x$$

$$x + x' = 1$$

$$x \cdot x' = 0$$

$$x + y = y + x$$

$$xy = yx$$

$$x(y+z) = xy + xz$$

$$x(yz) = (xy)z$$

$$x(y+z) = xy + xz$$

$$x + yz = (x+y)(x+z)$$

$$(x+y)' = x'y'$$

$$(xy)' = x'y'$$

$$x'' = x$$

De Morgan's Law

$$1) A \cdot \bar{A}C$$

$$= 0 \cdot C$$

$$= 0$$

$$2) ABCD + ABD$$

$$= ABDC + ABD$$

$$= ABD(C+1)$$

$$= ABD$$

$$3) AB + ABC + ABD + C$$

$$= AB(1+C) + C(ABD+1)$$

$$= AB + C$$

$$4) xy + xyz + xy\bar{z} + \bar{x}yz$$

$$5) \overline{ABC} + \overline{A}B\bar{C} + \overline{A}\bar{B}C$$

$$= xy(1+z) + y(x\bar{z} + \bar{x}z)$$

$$= \bar{A} + \bar{B} + \bar{C} + \bar{A}\bar{B}\bar{C}$$

$$= xy + x\bar{y}\bar{z} + \bar{x}yz$$

$$= \bar{A} + \bar{B} + \bar{C}$$

$$= xy(1+\bar{z}) + x\bar{y}z$$

$$= \bar{A}(1+B) + \bar{B} + \bar{C}$$

$$= xy + \bar{x}yz$$

$$= \bar{A} + \bar{B} + \bar{C}$$

$$= y(x + \bar{x}z)$$

$$= \overline{ABC}$$

$$= y((x+\bar{x})(x+z))$$

$$= \overline{ABC}$$

$$= y(x+z)$$

$$= \overline{ABC}$$

$$6) AB + \overline{AC} + A\overline{B}C(AB + C)$$

$$7) \overline{AB} + \overline{A} + AB$$

$$= \overline{A}(B+1) + AB$$

$$= \overline{\overline{A} + AB}$$

$$= \overline{\overline{A} + B}$$

$$= \overline{AB}$$

$$= 1$$

$$= 1$$

$$\begin{aligned}
 8) & \overline{ABC\bar{D}} + BC\bar{D} + B\bar{C}\bar{D} \\
 & = BC\bar{D}(\bar{A} + 1) + B\bar{C}\bar{D} + B\bar{C}\bar{D} \\
 & = BC\bar{D} + B\bar{C}\bar{D} + B\bar{C}\bar{D} \\
 & = B(C\bar{D} + \bar{C}\bar{D} + \bar{D}) + B\bar{C}\bar{D} \\
 & = B(\bar{C} + \bar{D} + \bar{D}) + B\bar{C}\bar{D} \\
 & = B(\bar{C} + \bar{D}) + B\bar{C}\bar{D} \\
 & = B\bar{C}\bar{D} \quad B(\bar{C} + \bar{D} + \bar{C}\bar{D}) \\
 & = \overline{B(\bar{C} + \bar{D})} \\
 & = \underline{\underline{B\bar{C}\bar{D}}}
 \end{aligned}$$

$$\begin{aligned}
 9) & ABC + A\bar{B}C + AB\bar{C} \\
 & = AC(B + \bar{B}) + AB\bar{C} \\
 & = AC + AB\bar{C} \\
 & = A(C + B\bar{C}) \\
 & = A(C + B)
 \end{aligned}$$



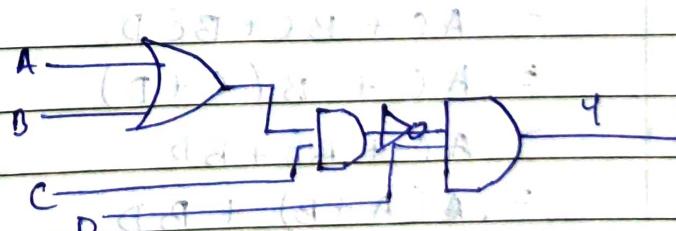
$$\begin{aligned}
 10) & \overline{ABC} + \overline{A}B\bar{C} + ABC \\
 & = \bar{A} + \bar{B} + \bar{C} + \overline{A}B\bar{C} + ABC \\
 & = \bar{A} + \bar{B} + \bar{C} + ABC \\
 & = \bar{A} + BC + \bar{B} + \bar{C} \\
 & = \bar{A} + \bar{B} + C + \bar{C} \\
 & = \underline{\underline{1}}
 \end{aligned}$$

A	B	C	Q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

NOTE: The logic functions should be simplified to minimum no. of gates as it is more favourable in terms of speed and design but is more expensive.

$$11) (\overline{A+B})C P$$

$$\begin{aligned}
 & = (\overline{A+B} + \bar{C})D \\
 & = \bar{A} \cdot \bar{B} D + \bar{C} D \\
 & = D(\bar{A} \cdot \bar{B} + \bar{C}) \\
 & = \underline{\underline{D(\overline{A+B} + \bar{C})}}
 \end{aligned}$$



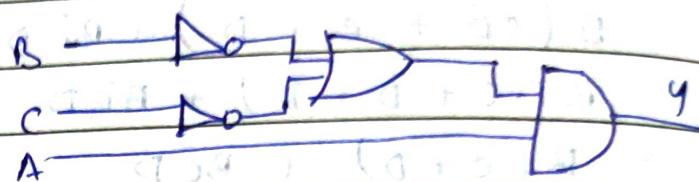
Review questions

Q1) Why NAND & NOR Gates are called Universal Gates?

Q2) (a) $AB\bar{C} + A\bar{B}$

$$= A(B\bar{C} + \bar{B})$$

$$= (\bar{B} + \bar{C})A$$



(b) $\bar{A}\bar{B}CD + BD + A\bar{C}D$

$$= (\bar{A} + \bar{B})CD + BD + A\bar{C}D$$

$$= \bar{A}CD + \bar{B}CD + BD + A\bar{C}D$$

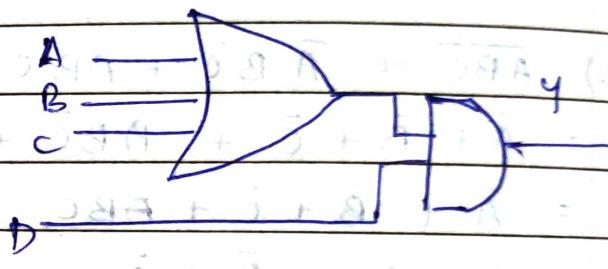
$$= \bar{A}CD + D(B + C) + A\bar{C}D$$

$$= D(\bar{A}C + A\bar{C}) + D(B + C)$$

$$= D(\bar{A}C + A\bar{C} + B + C)$$

$$= D(C + A\bar{C} + B)$$

$$= D(C + A + B)$$



Q3) Execute using NAND/NOR Gate

$$= \bar{A}BC + AC + B\bar{C}D$$

$$= C(A + \bar{A}B) + B\bar{C}D$$

$$= C(A + B) + B\bar{C}D$$

$$= AC + BC + B\bar{C}D$$

$$= AC + B(C + D)$$

$$= AC + BC + BD$$

$$= C(A + B) + BD$$

4) EX NOR Gate using NOR.

$$F = AB + A'B'$$

$$\text{NOR: } \overline{A+B}$$

VARIABLES

A	B	C	M _n	M _n
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	($\bar{A}B\bar{C}$) _{AA}	$A(\bar{B}C) \bar{A} = m_5 + (\bar{A}+\bar{B})B+\bar{C} = M_5$	
1	1	1	$\bar{A}BC = m_6$	$\bar{A}+\bar{B}+C = M_6$
			$\bar{A}B\bar{C} = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

$$1) Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC \\ = m_0 + m_1 + m_3 + m_6 \\ = \sum m(0, 1, 3, 6)$$

$$2) Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C) \\ = (M_1)(M_3)(M_6) \\ = \prod M(1, 3, 6) = \sum m(1, 3, 6)$$

$$1) AB + ABC + AB(D+E) \\ = AB(1+C) + AB(D+E) \\ = AB(1+C+D+E) \\ = \underline{AB}$$

$$2) \bar{ABC} + \bar{A}\bar{B}\bar{C} + \bar{A}BC \\ = \bar{A}+\bar{B}+\bar{C} + \bar{A}B(C+\bar{C}) \\ = \bar{A}(1+B) + \bar{B}+\bar{C}$$

$$\therefore = \bar{ABC}$$

$$\begin{aligned} 3) \quad & (A+B)(A+C) \\ &= A + BA + AC + BC \\ &= \underline{\underline{A+BC}} \end{aligned}$$

$$\begin{aligned} 4) \quad & ABC + A\bar{B}C + A\bar{B}\bar{C} \\ &= AC(B+\bar{B}) + A\bar{B}\bar{C} \\ &= AC + A\bar{B}\bar{C} \\ &= A(C + \bar{B}\bar{C}) \\ &= \underline{\underline{A(B+C)}} \end{aligned}$$

(Q) Convert the given expression in the standard S.O.P. & POS.

$$\begin{aligned} (a) \quad Y &= AC + BC + BA \\ &\equiv AAB + B\\ &\equiv (A+B)C \\ &= \underline{\underline{ABC+BC}} \end{aligned}$$

$$\begin{aligned} Y &= AC(B+\bar{B}) + BC(A+\bar{A}) + AB(C+\bar{C}) \\ &= ABC + A\bar{B}C + ABC + \bar{A}BC + ABC + A\bar{B}\bar{C} \\ &= ABC + A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} \\ \text{SOP} &= \underline{\underline{\Sigma m(1, 3, 5, 6)}} \end{aligned}$$

$$\begin{aligned} (b) \quad Y &= (A+B)(B+C)(A+C) \\ &= (A+B+C\bar{C})(A\bar{A} + B + C)(A + B\bar{B} + C) \\ &= (A+B+C)(A+B+\bar{C})(A+B+C)(\bar{A}+B+C) \\ &= (A+B+C)(A+\bar{B}+\bar{C}) \end{aligned}$$

$$= \underline{\underline{\Sigma m(0, 1, 4, 8)}}$$

Convert minterms

$$= \underline{\underline{\Sigma m(3, 5, 6, 7)}}$$

$$= ABC + A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$$

$$= BC + A\bar{B}C + AB\bar{C}$$

$$= BA + BC + A\bar{B}C = AB + BC$$

KARNAUGH MAP

→ Uses S.O.P.

→ Max 5 variables allowed

2 bit (variables)

	0	1
0		
1		

LSB ↑
BC | 3 variables:
↓ MSB

00	01	11	10	
0	000	001	011	010
Ā 0	4	5	7	6
A 1	100	101	110	111
↓				

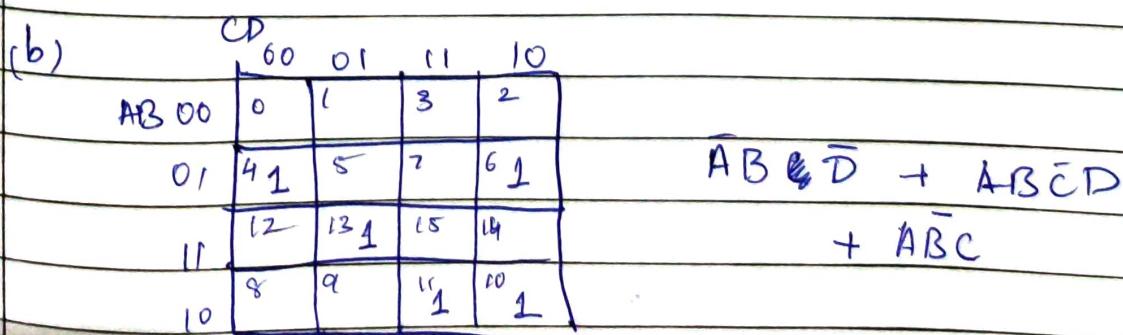
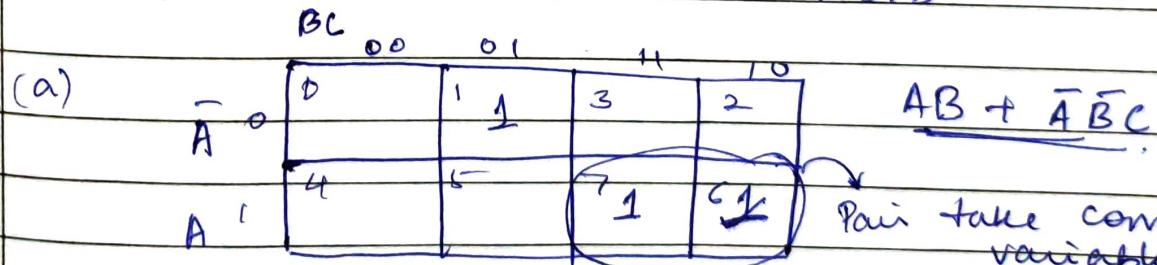
4 variables

AB	CD	00	01	11	10
00	0	1	3	2	
01	4	5	7	6	
11	12	13	15	14	
10	8	9	11	10	

(a) Plot the boolean expression on Kmap

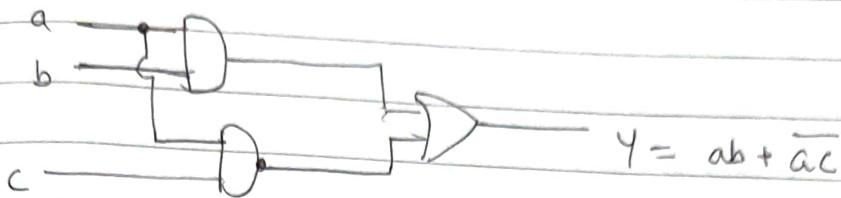
$$(a) Y = AB\bar{C} + ABC + \bar{A}\bar{B}C$$

$$(b) Y = \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}CD + ABC\bar{D}$$



$$\bar{A}\bar{B}\bar{D} \rightarrow \bar{A}\bar{B}C + A\bar{B}CD$$

Logic Circuits



	I/P			O/P		
	a	b	c	ab	$\bar{a}c$	$ab + \bar{a}c$
m_0	0	0	0	0	1	1
m_1	0	0	1	0	1	1
m_2	0	1	0	0	1	1
m_3	0	1	1	0	1	1
m_4	1	0	0	0	1	1
m_5	1	0	1	0	0	0
m_6	1	1	0	1	1	1
m_7	1	1	1	1	0	1

$$Y = \sum m (0, 1, 2, 4, 5, 6, 7) \quad \text{or} \quad \prod M(3)$$

min terms are nothing but product terms where
input variable appear in true form / false form
and for all min

max terms are sum terms

$$m_0 \rightarrow \bar{a} \bar{b} \bar{c}$$

$$M_0 \rightarrow a + b + c$$

$$m_1 \rightarrow \bar{a} \bar{b} c$$

$$M_1 \rightarrow a + b + \bar{c}$$

$$m_2 \rightarrow \bar{a} b \bar{c}$$

$$M_2 \rightarrow a + \bar{b} + c$$

$$m_3 \rightarrow \bar{a} b c$$

$$M_3 \rightarrow a + \bar{b} + \bar{c}$$

$$m_4 \rightarrow a \bar{b} \bar{c}$$

$$M_4 \rightarrow \bar{a} + b + c$$

$$m_5 \rightarrow a \bar{b} c$$

$$M_5 \rightarrow \bar{a} + b + \bar{c}$$

$$m_6 \rightarrow a b \bar{c}$$

$$M_6 \rightarrow \bar{a} + \bar{b} + c$$

$$m_7 \rightarrow a b c$$

$$M_7 \rightarrow \bar{a} + \bar{b} + \bar{c}$$

abc

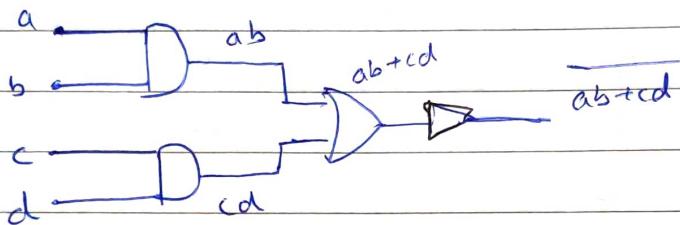
$$Y = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}\bar{c} + abc$$

$$+ abc$$

Pos:-

$$y = (\bar{a} + b + \bar{c})$$

Q A cht has 4 input variable . Draw cht with 4_{gab}
write exp for Y,
input of Y, write all min terms , maxterms



K Map 1:

It is a tabular representation of the function. any boolean function will have 1 or multiple output. For some of ip combination the op is 1 and for others op is 0.

These 1 and 0 are mapped to Kmap. using fixed method the number of 1's & 0's are grouped to get a minimal cost expression for the function.

$$Y = ab + a\bar{c}$$

$$Y = \sum m(0, 1, 2, 3, 4, 6, 7)$$

$$Y' = \sum \pi m(5) = \bar{a} + b + \bar{c}$$

a \ b\ c	00	01	11	10
0	1	1	1	1
1	1	0	1	1

a \ b\ c	00	01
00	0	1
01	2	3
11	6	7
10	4	5

a \ b\ d	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

The K map technique is suitable if number of variable is 3, 4, 5. If more than 5, different technique are used to get optimum solution.

$$Y = \sum m(0, 1, 2, 3, 4, 6, 7)$$

	$\bar{a}b\bar{c}$	$b\bar{c}$	$\bar{b}c$	$b\bar{c}$
\bar{a}	0	1	1	1
a	1	0	1	1

$$Y = \bar{a} + b + \bar{c}$$

2) $Y = \sum m(0, 2, 3, 5, 6)$

	$\bar{b}\bar{c}$	$\bar{b}c$	$b\bar{c}$	$b\bar{c}$
\bar{a}	1	0	1	1
a	0	1	0	1

SOP $\rightarrow Y = b\bar{c} + \bar{a}(b) + \bar{a}\bar{c} + ab\bar{c}$

POS $\rightarrow Y = ab\bar{c} + \bar{a}\bar{b}c + abc$
 $= (\bar{a} + b + c)(a + b + \bar{c})(\bar{a} + \bar{b} + \bar{c})$

(3) $F(w, x, y, z) = \sum m(0, 2, 4, 6, 7, 8, 9, 11, 15)$

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1	0	0	1
$\bar{a}b$	1	0	1	1
$a\bar{b}$	0	0	1	0
ab	1	1	1	0

$$Y = \bar{a}\bar{c} + ab + bc + abc$$

$$Y = \bar{a}\bar{d} + bcd + acd + a\bar{b}\bar{c}$$

$$Y = (\bar{a} + \bar{b} + c)(a + c + \bar{d})(a + b + \bar{d})(\bar{a} + \bar{c} + d)$$

3)

$$\Sigma_m(\cdot) \rightarrow \text{mark } 1\ddagger$$

) $y = \Pi m(2, 4, 6)$ $\Pi m(\cdot) \rightarrow \text{mark } 0\ddagger$

$a\bar{a}$	$b+c$	$b+\bar{c}$	$\bar{b}+c$	$\bar{b}+\bar{c}$
1	1	1	0	
0	1	0	0	

$$Y = a(\bar{b}+c)(\bar{a}+c)$$

4)

$$= \Sigma D(\cdot)$$

$$(w, x, y, z) = \Sigma m(6, 7, 8, 9) + D(12, 13, 14, 15)$$

$w\bar{x}$	$y\bar{z}\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
0	0	0	0	0
0	0	1	1	1
x	x	x	x	x
1	1	0	0	0

 \rightarrow Sol

$$Y = (w\bar{y}) + (xy)$$

$$\begin{matrix} Y \\ \text{pos} \end{matrix} = (\bar{w}\bar{y}) \\ (w+y)(x+\bar{y})$$

) $f(w, x, y, z) = \Pi m(6, 7, 8, 9) + D(12, 13, 14, 15)$

	$y+z$	$y+\bar{z}$	$\bar{y}+\bar{z}$	$\bar{y}+z$
$w+x$	1	1	1	1
$w+\bar{x}$	1	1	0	0
$\bar{w}+\bar{x}$	x	x	x	x
$\bar{w}+x$	0	0	1	1

$$Y = (\bar{w}+y)(\bar{x}+\bar{y})$$

HW

1) $f(a,b,c,d) = \sum m(1, 2, 6, 7, 8, 13, 14, 15) + D(3, 5, 12)$

2) $f(w,x,y,z) = \sum m(2, 3, 8, 10, 11, 12, 14, 15) + D(0, 5, 9)$

3) $f(a,b,c,d) = \Sigma \bar{b}\bar{c}\bar{d} + a\bar{b}\bar{d} + ab\bar{d} + abd$

1)

$$\sum m(1, 2, 6, 7, 8, 13, 14, 15) + D(3, 5, 12)$$

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	0	1	1	x
$\bar{a}b$	0	x	1	1
$a\bar{b}$	x	1	1	1
ab	0	0	0	0

$$y = bc + \bar{a}d + bd$$

$$y = (\bar{a}+b)(\bar{c}+d)$$

(2)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	x	0	1	1
$\bar{a}b$	0	x	0	0
$a\bar{b}$	1	0	1	1
ab	1	x	1	1

$$y = ac + \bar{b}c + a\bar{c}\bar{d}$$

$$y = (\bar{a}+c)(a+b)(c+\bar{d})$$

(3)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1	0	0	0
$\bar{a}b$	0	0	0	0
$a\bar{b}$	1	1	1	1
ab	1	0	0	1

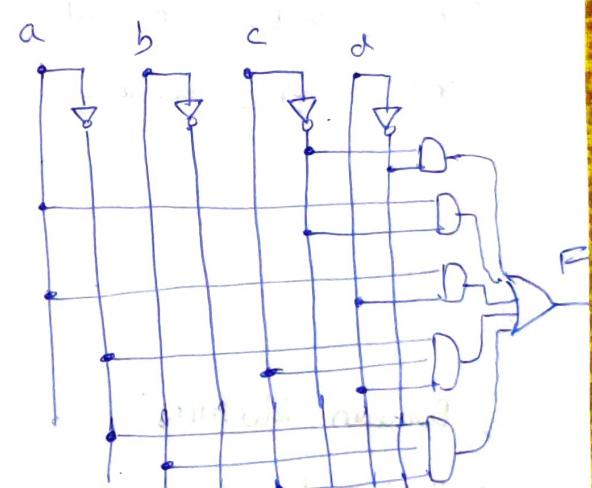
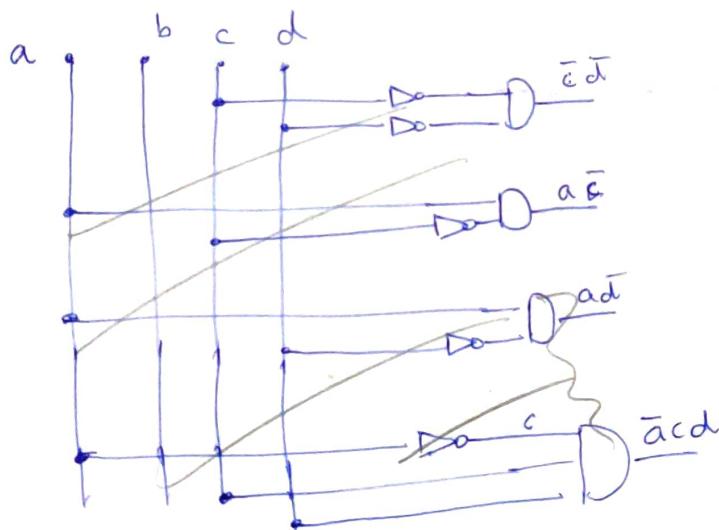
$$y = ab + \bar{b}\bar{c}\bar{d} + ac\bar{d}$$

Q) Simplify minterms using Kmap and draw o/p using simple logic gates.

$$1) F = \Sigma m(0, 3, 6, 7, 9, 10, 12, 14) + D(4, 8, 13)$$

$\bar{a}\bar{b}$	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}b$	1	0	1	0
$a\bar{b}$	X	0	1	1
ab	1	X	0	1
$a\bar{b}$	X	1	0	1

$$2) Y = \bar{c}\bar{d} + a\bar{c} + a\bar{d} + \bar{a}cd + \bar{a}bc$$



An SOP expression is implemented using 2 level ckt
in level 1 is AND array, each AND gate a for 1 fan
& level 2 OR gates are used to sum the product term.

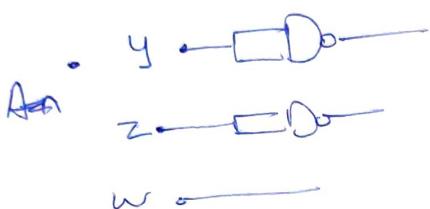
In order to implement POS expression, the 2 level circuit will have 1st OR array,

For NAND gate

- 1) double compliment, \rightarrow demorgan

$$Y = \overline{y}\overline{z} + w\overline{y}$$

$$= \overline{\overline{y}\overline{z} + w\overline{y}} = \overline{\overline{y}\overline{z} \cdot \overline{w}\overline{y}}$$



Any SOP can be converted using NAND gate

Step 1: double compliment

Step 2: apply demorgan to inside bar

Q

- 1) Boolean identities
- 2) Boolean Laws
- 3) Solving exp using identities & laws
- 4) solving exp using demorgan thm
- 5) given complex exp solve using Kmap
- 6) Kmap, dont care term,
- 7) convert SOP to POS & POS to SOP & canonical form

Unit-2

Half adder

It is a combination circuit which adds only 2 bits and produces sum and carry.



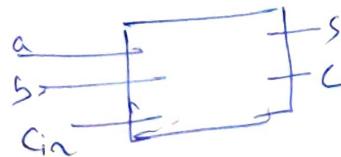
I/P		O/P	
a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

Draw ~~stair~~ using Basic gate & NAND gate also

Full adder

It is a combinational ckt which adds 3 bits giving x, y , and C_{in} from another circuit.

Full adder produces 2 outputs sum & carry

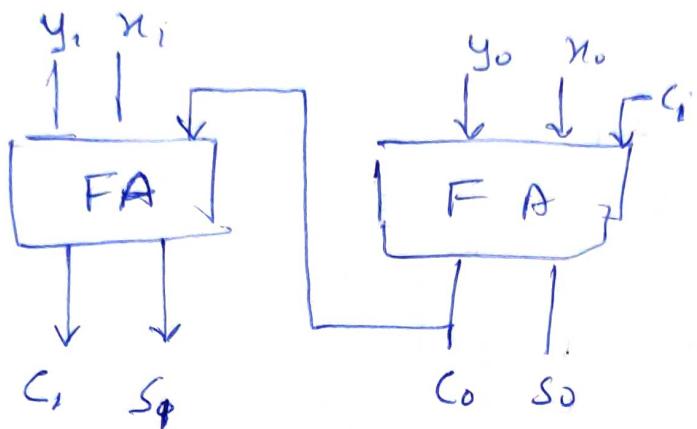


I/P			O/P		
a	b	c	s	Co	
0	0	0	0	0	
0	0	1	1	0	
0	1	0	1	0	
0	1	1	0	1	
1	0	0	1	0	
1	0	1	0	1	
1	1	0	0	1	
1	1	1	1	1	

Add 2 numbers of 2 digit (Parallel adder)

$$\text{Num1} = y_1 \cdot x_1, y_0 \cdot x_0$$

$$\text{Num2} = y_1 \cdot y_0$$



Parallel adder adds any 2 number, binary digit if there are 2 numbers with 4 digits each. It requires 4 full adder connected in cascade. Such adder circuit are called as parallel adder (or Ripple carry adder).

- a) difference b/w sequential & combination logic gates.
- b) Advantages & disadvantages of sequential & combination gates