years because of the warranty but are expected to be $1000 in year 3, $1500 in year 4, and $2000 in year 5. What is the minimum desired annual economic benefit of the device, assuming that these benefits will just offset the annual costs? The company uses an interest rate of 10 percent for economic evaluations.

2.38 Maintenance records of a certain type of machine indicate that the first-year maintenance cost of $350 increases by $70 per year over the 5-year life of the machine. Answer the following, if the maintenance cost is considered to occur at the end of the year and the firm's interest rate is 8 percent.

(*a*) What equal annual payments could the firm justify making to a service organization to carry out the maintenance for 20 machines?

(*b*) How much additional could be paid for a new type of machine with the same service life that required no maintenance in the first 2 years and $125 per year for each of the last 3 years?

2.39 A new piece of materials handling equipment costs $20,000 and is expected to save $7500 the first year of operation. Maintenance and operating cost increases are expected to reduce the net savings by $500 per year for each additional year of operation until the equipment is worn out at the end of 8 years. Determine the net present worth of the equipment at an interest rate of 12 percent.

2.40 Office equipment can be purchased for $6000 cash or a down payment of $1000 followed by 24 end-of-month payments of $220 each. At what effective interest rate are these terms equivalent?

2.41 Assume that you sold property today for $2421 and that you had purchased the property 4 years ago with $2000 withdrawn from your savings account. During the 4-year period your savings would have earned 6 percent compounded quarterly. For a comparison of the investments, calculate the nominal interest rate received from your property purchase.

2.42 Discuss briefly how the material we have covered so far might assist you in determining how to pay for your next automobile purchase. Could the material affect how you choose the type of automobile to buy?

CHAPTER 3

PRESENT-WORTH COMPARISONS

One today is worth two tomorrows.
Benjamin Franklin, *Poor Richard's Almanack*, 1758

We can infer quite a lot about engineering economic evaluations from the introductory quotation by Benjamin Franklin. First, any cash flows that we have at present are known and certain and from an analysis point of view can be more valuable than those uncertain values at some other time. Also, as we have learned from the previous chapter, an investment of $1000 now is currently worth more than an investment of the same amount sometime in the future. Part of the future amount will be consumed by the effect of intervening interest rates. Therefore, it is not surprising that we often make engineering economic decisions based on the expected present worth of all incomes and outlays associated with those decisions, regardless of when those activities occurred. The purpose of this chapter is to apply what we have learned in Chap. 2 in making present-worth comparisons.

Present-worth (PW) comparisons are made only between coterminated proposals, to ensure equivalent outcomes. *Cotermination* means that the lives of the involved assets end at the same time. When assets have unequal lives, the time horizon for an analysis can be set by a common multiple of asset lives or by a study period that ends with the disposal of all assets.

Net present worth, the difference between the present worths of benefits and of costs, is the most widely used present-worth model. A capitalized cost model is used when an asset is assumed to have infinite life. We discuss this concept in Sec. 3.4. Other PW models include deferred investments and valuation of property, stocks, and bonds.

Many economists prefer a present-worth analysis because it reveals the sum in today's dollars that is equivalent to a future cash flow stream, and PW models are less subject to misinterpretation. It is therefore important to become familiar with the assumptions upon which PW calculations are based and the types of comparisons that can be made.

3.1 CONDITIONS FOR PRESENT-WORTH COMPARISONS

In Chap. 2 we introduced present-worth comparisons. A present worth resulted from each calculation of P. In this chapter, the present worth of a cash flow stream is denoted by PW—the present worth of receipts and disbursements associated with a particular course of action. Each alternative is thus represented by a PW, a measure of economic merit.

The ingredients of a present-worth comparison are the amount (in dollars) and timing (N) of cash flow and the interest rate i at which the flow is discounted. Listed below are assumptions used in this and the next two chapters in introducing the basic comparison methods; also indicated are topics from subsequent chapters that make comparisons less restrictive.

1. *Cash flows are known.* The accuracy of cash flow estimates is always suspect because future developments cannot be anticipated completely. Transactions that occur now, at time 0, should be accurate, but future flows become less distinct as the time horizon is extended. Ways to evaluate riskier cash flows are discussed in Chaps. 13 and 14.
2. *Cash flows are in constant-value dollars.* The buying power of money is assumed to remain unchanged during the study period. Ways to include the effect of inflation are presented in Chap. 10. Because this is such an important assumption and the concept of *constant dollars* is paramount to the evaluation of alternatives in the next few chapters, we will summarize the salient points necessary for understanding inflation in Sec. 3.1.1.
3. *The interest rate is known.* Different interest rates have a significant effect on the magnitude of the calculated present worth, as illustrated in Fig. 3.1. The rate of return i required by an organization is a function of its cost of capital, attitude toward risk, and investment policy. Alternative courses of action for the same proposal are normally compared by using the same interest rate, but different proposals may be evaluated at different required rates of return. The sensitivity of interest rates is explored in Chap. 11.
4. *Comparisons are made with before-tax cash flows.* Inclusion of income taxes greatly expands the calculation effort for a comparison and correspondingly increases reality. Since the workings of comparison methods are featured here and in the next few chapters, inclusion of income taxes is delayed until Chap. 9.
5. *Comparisons do not include intangible considerations. Intangibles* are difficult-to-quantify factors that pertain to a certain situation. For instance, the "impression" created by a design is an intangible factor in evaluating that design and an important one for marketing, but it would not be included in a present-worth comparison unless its economic consequences could be reasonably estimated. (If a dollar value can be assigned, a factor is no longer intangible.) Ways to include intangible factors in the decision-making process are considered in Chap. 15.
6. *Comparisons do not include consideration of the availability of funds to implement alternatives.* It is explicitly assumed that funds will be found to finance a course of action if the benefits are large enough. Although financing is not a direct input for computations, the output computed can be appraised with respect to available funding. For instance, an old, inefficient machine could be kept in operation because there appears to be insufficient capital available in the organization to afford a replacement, but an engineering economic analysis might point out that the savings from replacing the outdated machine would be so great that the organization could not afford to not find funds for a replacement.

EXAMPLE 3.1 Present Worth by the "72 Rule"

The present worth of a future amount drops off rapidly as the time between *now* and *then* increases, particularly at higher interest rates. The pattern of this decrease in present value is shown in Fig. 3.1. Examination of the curves supports an interesting rule of thumb: The *72 rule* indicates the approximate number of years N^* at which the PW is one-half the future worth (FW) at the annually compounded interest rate i:

$$72 \text{ rule: } N^* = \frac{72}{(i)(100)}$$

Another way to view the 72 rule is that $(1 + i)^N$ doubles about every N^* years. Thus, as is clear in Fig. 3.1, the present worth is one-half of the future worth in about 14 years at $i = 5$ percent, 7 years at $i = 10$ percent, and 4 years at $i = 20$ percent.

3.1.1 Inflation and the Constant-Dollars Assumption

It was mentioned in the introductory material to this chapter that cash flows will be assumed to be in constant-value dollars where the buying

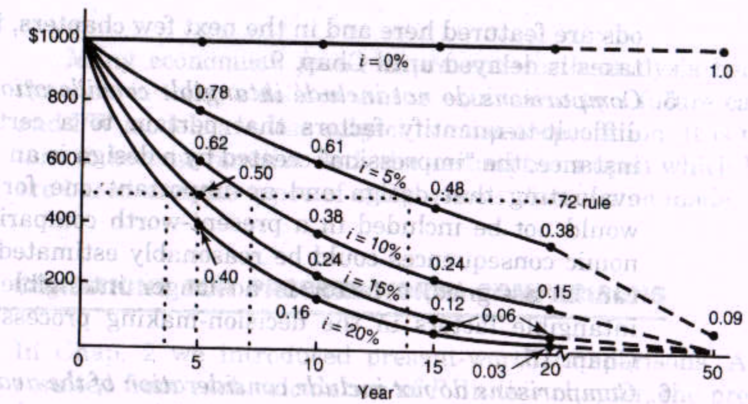


FIGURE 3.1 Present worth of $1000 as a function of time and interest rates when interest is compounded annually.

power of money is assumed to remain unchanged during the study period. Another way to state this is to say that cash flows are in *constant dollars*. We need to clarify what this means and its effect on the analyses we will be considering in the next few chapters, primarily this chapter and Chaps. 4 through 8. Therefore it is reasonable to summarize material covered in Chap. 10 (inflation) now.

We have all felt the effects of inflation. A house bought around 1960 might have cost $18,000. Today, that same house is selling for at least $80,000. Inflation causes prices to rise and decreases the purchasing power of money with the passage of time. In engineering economics studies, we have a related topic that we might need to consider: *technological stability*. Research and development cost money, and that money has to be returned in terms of increased product selling costs and/or increased volume of sales. Further, we have seen consumer products where technological improvements have been made commensurate with *reductions in prices*. Computer and many other electronic products can fall into this category. Unless stated in the associated material, we assume that we have a condition of *technological stability:* The same or similar product in a few years will have the same cost structure as it does now in constant dollars (when inflation effects are removed).

A common way to eliminate inflation effects is to convert all cash flows to money units that have constant purchasing power, called *constant*, or *real, dollars*. This approach is most suitable for before-tax analysis, when all cash flow components inflate at uniform rates. We assume that this is the way inflation has been removed in our pretax analyses through Chap. 8. Another way to handle the situation is to actually perform the analyses with the cash flows in the estimated amount of money exchanged at the time of transaction. These money units are called *future, then-current*, or *actual* dollars. The use of this approach in economic analysis is introduced in Chap. 10.

We mentioned in Chap. 2, when the geometric gradient was discussed, that inflation is an example of such a growth pattern (and deflation would follow a negative geometric gradient pattern). This means that we have a *percentage compounding* effect over a period of years. If f is the *inflation rate* over the next few years, actual dollars in year N can be converted to constant dollars by

$$\text{Constant dollars} = \frac{\text{actual dollars}}{(1+f)^N}$$

Since $1/(1+i)^N$ is the single-payment PW factor, we can rewrite the conversion equation as

$$\text{Constant dollars} = (\text{actual dollars})(P/F, f, N)$$

The reverse occurs if we want to convert constant dollars to actual dollars at time period N:

$$\text{Actual dollars} = (\text{constant dollars})(F/P, f, N)$$

Now we can show a cash flow series in actual dollars and the cash flow series after we have removed the inflationary effects:

End of year	Actual dollars		4% Annual inflation		Constant dollars
0	−5000	×	$1/1.04^0$	=	5,000
1	1000	×	$1/1.04^1$	=	962
2	800	×	$1/1.04^2$	=	740
3	600	×	$1/1.04^3$	=	533
4	400	×	$1/1.04^4$	=	342
5	200	×	$1/1.04^5$	=	164

If we were to perform a present-worth analysis under our constant-dollars assumption, we would use the rightmost column of cash flow data, not the second.

We will show in Chap. 10 that the *minimum attractive rate of return (MARR)* used by firms to evaluate their economic investments includes the effect of inflation. Use of this rate, referred to as the *market interest rate*, requires that all cash flows be in actual dollars. The MARR used with constant dollars is the inflation-free interest rate that represents the earning power of capital when inflation effects have been removed. Also in Chap. 10 we will show that CHEER can automatically handle the effects of an inflation rate. Until we get to Chap. 10, we assume that inflation has be removed from the cash flows and that the MARR is the inflation-free inter est rate.

3.2 BASIC PRESENT-WORTH COMPARISON PATTERNS

The present worth of a cash flow over time is its value today, usually rep resented as time 0 in a cash flow diagram. Two general patterns are appar ent in present-worth calculations: present-worth equivalence and net pre sent worth. The former has investments in the future while the latter has an investment at time 0 followed by possible receipts and expenditures in future time periods. Both types of analysis will give the same results and interpretation; they will be presented as separate concepts due to common acceptance in the field of engineering economics.

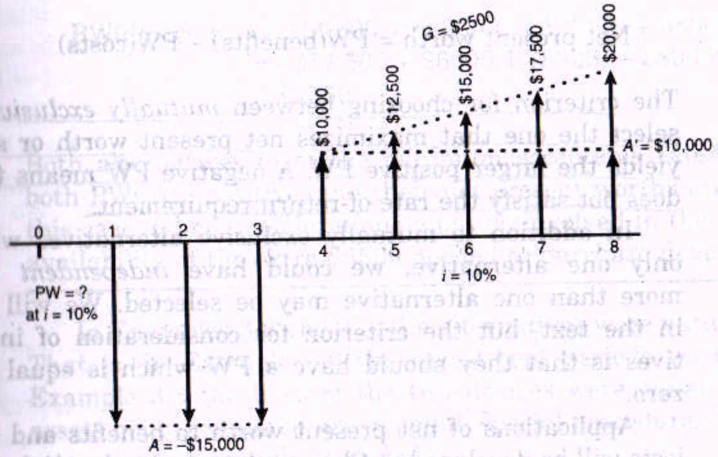
3.2.1 Present-Worth Equivalence

One pattern determines the present-worth equivalence of a series of future transactions. The purpose is to secure one figure that represents all the transactions. This figure can then be compared with a corresponding figure that represents transactions from a competing option, or it can be com pared with the option of doing nothing. A *do-nothing option* is always an alternative, even if it results only in procrastination. Often there is a go/no go situation where each alternative is selectively weighed to decide whether it is worth exercising. For instance, a series of expenses that will occur in the future can be discounted to obtain its PW, and then a decision can be made about whether an investment of the PW amount should be made *now* to avoid the expenses. Similar reasoning guides the equivalence comparison in the following example.

EXAMPLE 3.2 **Equivalent PW of an Option**

An investor can make three end-of-year payments of $15,000, which are expected to generate receipts of $10,000 at the end of year 4 that will increase annually by $2500 for the following 4 years. If the investor can earn a rate of return of 10 percent on other 8-year investments, is this alternative attractive?

Solution



From the cash flow diagram, it is apparent that the receipts and disbursements each constitute an annuity, one positive and the other negative. Both are discounted to time 0 at 10 percent:

$$\begin{aligned}
PW &= -\$15{,}000(P/A,\ 10,\ 3) \\
&\quad + [\$10{,}000 + \$2500(A/G,\ 10,\ 5)]\ (P/A,\ 10,\ 5)\ (P/F,\ 10,\ 3) \\
&= -\$15{,}000(2.48685) \\
&\quad + [\$10{,}000 + \$2500(1.81013)]\ (3.79079)\ (0.75131) \\
&= -\$37{,}303 + \$41{,}369 = \$4066
\end{aligned}$$

The interpretation of the $4066 present worth is that the transactions provide a return of 10 percent on the investment *plus* a sum of $4066. This investment is therefore preferable to one that would return exactly 10 percent over 8 years.

It is common practice to drop the minus sign when the cash flow is composed of only costs. The calculated PW is then called the *present worth of costs.* This is purely a convention of convenience and is unlikely to cause confusion as long as the alternatives in a comparison are treated in the same way. We will handle each situation in the most logical manner for the case at hand.

3.2.2 Net Present Worth

The second general pattern for PW calculations has an initial outlay at time 0 followed by a series of receipts and disbursements. This is the most frequently encountered pattern, which leads to the fundamental relation

$$\text{Net present worth} = \text{PW(benefits)} - \text{PW(costs)}$$

The criterion for choosing between *mutually exclusive* alternatives is to select the one that maximizes net present worth or simply the one that yields the larger positive PW. A negative PW means that the alternative does not satisfy the rate-of-return requirement.

In addition to mutually exclusive alternatives where we can select only one alternative, we could have *independent* alternatives where more than one alternative may be selected. We will consider this later in the text, but the criterion for consideration of independent alternatives is that they should have a PW which is equal to or greater than zero.

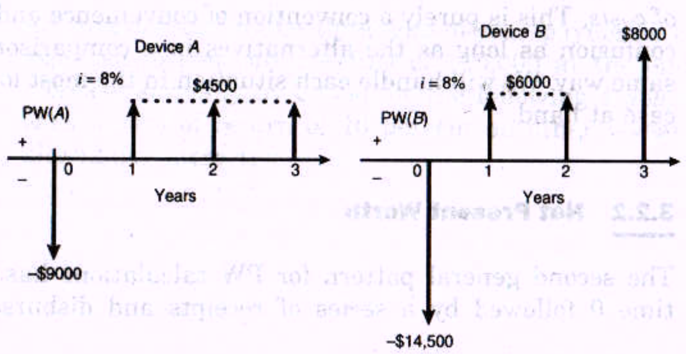
Applications of net present worth to benefits and costs of public projects will be developed in Chap. 8. A typical industrial application might be as follows.

EXAMPLE 3.3 **Net Present-Worth Comparison**

Two devices are available to perform a necessary function for 3 years. The initial cost (negative) for each device at time 0 and subsequent annual savings (positive), both in dollars, are shown in the following table. The required interest rate is 8 percent.

	Year 0	1	2	3
Device *A*	9,000	4,500	4,500	4,500
Device *B*	14,500	6,000	6,000	8,000

Solution



$$\begin{aligned}\text{PW (device } A) &= -\$9000 + \$4500(P/A, 8, 3)\\ &= -\$9000 + \$4500(2.5771) = \$2597\end{aligned}$$

$$\begin{aligned}\text{PW(device } B) &= -\$14{,}500 + \$6000\,(P/A, 8, 2) + \$8000(P/F, 8, 3)\\ &= -\$14{,}500 + \$6000(1.78326) + \$8000(0.79383)\\ &= \$2550\end{aligned}$$

Both alternatives meet the minimum acceptable rate of return, because both PWs are positive, and their net present worths are close in value. In this case, other considerations must be involved in the choice, such as the availability of the extra \$5500 needed to purchase device *B*.

In Examples 3.2 and 3.3, the alternatives were naturally coterminated. That is, in Example 3.2 the investment periods were identical, and in Example 3.3 the lives of the two devices were equal. When the lives of assets being compared are unequal, special procedures must be followed to coterminate the analysis periods.

COMPARISON OF ASSETS THAT HAVE UNEQUAL LIVES

The utilization of present-worth comparisons for *coterminated projects* implies that the lives involved have a common endpoint. The necessity for cotermination is readily apparent when a familiar decision is considered, such as the choice between paying \$30 for a 3-year subscription to a magazine and paying \$40 for a 5-year subscription to the same publication. A simple comparison of \$30 to \$40 for a subscription is inaccurate, because the extra \$10 buys 2 more years of issues. *Alternatives must be compared on the basis of equivalent outcomes.*

Several variations have been proposed to accommodate present-worth comparisons of unequal-life assets.† Two prominent methods are described below.

1. *Common-multiple method.* Alternatives are coterminated by selecting an analysis period that spans a common multiple of the lives of the involved assets. For instance, if assets had lives of 2, 3, 4, and 6 years, the least common multiple is 12 years, which means that the asset with a life of 2 years would be replaced 6 times during the analysis period. The assets with 3-, 4-, and 6-year lives would be replaced 4, 3, and 2 times, respectively.

Legitimate use of a least common multiple of lives depends on the validity of the assumption that assets will be repeatedly replaced by

†D. J. Kulonda, "Replacement Analysis with Unequal Lives," *The Engineering Economist,* vol. 23, no. 3, pp. 171–179, Spring 1978. Published by Institute of Industrial Engineers.

successors having identical cost characteristics. In other words, constant dollars are assumed where the purchasing power does not change with passage of time. This assumption is more often reasonable when the least common multiple of alternative lives is small. Then there is less likelihood that a technologically better asset will become available during the analysis period.

2. *Study-period method.* A more justifiable analysis is based on a specified duration that corresponds to the length of a project or the period of time the assets are expected to be in service. An appropriate study period reflects the replacement circumstances. Some of the options are to set the study period as the length of:
   (a) *The shortest life of all competing alternatives*—a protection against technological obsolescence.
   (b) *The known duration of required services*—a project philosophy in which each new undertaking is considered to start with new assets, continue to use similar replacements, and dispose of used assets when the project is completed. Again, as mentioned earlier, constant dollars are assumed.
   (c) *The time before a better replacement becomes available*—an attempt to minimize cost by purposely upgrading assets as improvements are developed.

A study period comparison presumes that all assets will be disposed of at the end of the period. Therefore it is usually necessary to estimate the income that can be realized from the sale of an asset which can still provide useful service. This *salvage value* can be established somewhat arbitrarily by prorating the investment in the asset over its normal service life and then calculating the size of the unrecovered amount at the time of disposal. A better estimate is obtained when it is possible to accurately appraise an asset's market value at the end of the study period. The most precise study would result from also having explicit knowledge of operating costs as a function of the asset's age and future capital costs for each successor. These conditions are explored more completely in Chap. 7.

EXAMPLE 3.4 **PW Comparisons of Alternatives with Unequal Service Lives**

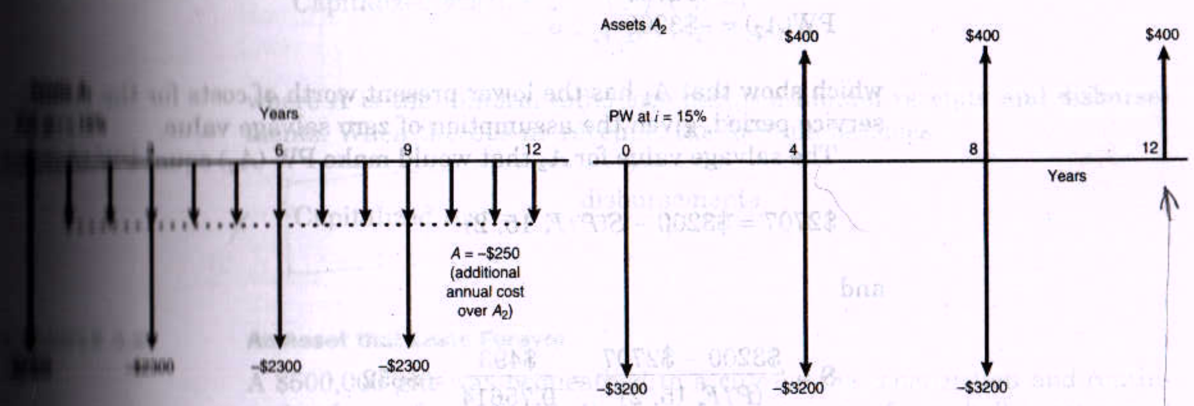
salvage value = +ve

Assets A_1 and A_2 have the capability of satisfactorily performing a required function. Asset A_2 has an initial cost of \$3200 and an expected salvage value of \$400 at the end of its 4-year service life. Asset A_1 costs \$900 less initially, with an economic life 1 year shorter than that of A_2; but A_1 has no salvage value, and its annual operating costs exceed those of A_2 by \$250. When the required rate of return is 15 percent, state which alternative is preferred when comparison is by:

(a) The repeated-projects method
(b) A 2-year study period (assuming the assets are needed for only 2 years)

Solution

(a) The repeated-projects method is based on the assumption that assets will be replaced by identical models possessing the same costs. Equivalent service results from comparing costs over a period divisible evenly by the service lives of the alternatives; in this case the least common multiple is 12 years.



$$
\begin{aligned}
\text{PW }(A_1) &= -\$2300 - \$2300(P/F, 15, 3) - \$2300(P/F, 15, 6) \\
&\quad - \$2300(P/F, 15, 9) - \$250(P/A, 15, 12) \\
&= -\$2300 - \$2300(0.65752) - \$2300(0.43233) \\
&\quad - \$2300(0.28426) - \$250(5.42062) \\
&= -\$6816
\end{aligned}
$$

$$
\begin{aligned}
\text{PW }(A_2) &= -\$3200 - \$2800(P/F, 15, 4) - \$2800(P/F, 15, 8) \\
&\quad + \$400(P/F, 15, 12) \\
&= -\$3200 - \$2800(0.57175) - \$2800(0.32691) \\
&\quad + \$400(0.18691) \\
&= -\$5642
\end{aligned}
$$

The net present-worth advantage of A_2 over A_1 for 12 years of service is –\$5642 – (–\$6816) = \$1174.

(b) A service period comparison is utilized when a limited period of ownership of assets is set by specific operational requirements. A 2-year study period for A_1 and A_2 indicates that the service required from either asset will be needed for only 2 years and will be disposed of at that time. If possible, estimates of the worth of assets at the end of the study period should be secured. These salvage values, which we will symbolize by S, may be quite large when the service period is only a small fraction of the service life. When it is difficult to secure reliable estimates of worth after use during the ownership period, minimum resale levels can be calculated to make the alternatives equivalent.

Then only a judgment is needed about whether the market value will be above or below the minimum level.

For instance, assuming $S = 0$ (certainly the minimum value) for alternatives A_1 and A_2 after 2 years of service, we have

$$\begin{aligned} PW(A_1) &= -\$2300 - \$250(P/A, 15, 2) \\ &= -\$2300 - \$250(1.62571) \\ &= -\$2707 \\ PW(A_2) &= -\$3200 \end{aligned}$$

which show that A_1 has the lower present worth of costs for the 2-year service period, given the assumption of zero salvage value.

The salvage value for A_2 that would make PW (A_2) equal PW (A_1) is

$$\$2707 = \$3200 - S(P/F, 15, 2)$$

and

$$S = \frac{\$3200 - \$2707}{(P/F, 15, 2)} = \frac{\$493}{0.75614} = \$652$$

which means that A_2 is preferred to A_1 when the resale value of A_2 at the end of 2 years is more than \$652 greater than the resale value of A_1 at the same time. The advantage of calculating this *aspiration* level is to avoid making an estimate of S; only a judgment is required about whether S will exceed a certain amount, the aspiration value, or \$652 in this case.

3.4 COMPARISON OF ASSETS ASSUMED TO HAVE INFINITE LIVES

The sum of the first cost and the present worth of disbursements assumed to last forever is called a *capitalized cost.* This type of evaluation is essentially limited to long-lived assets. It is not used as extensively as it once was, but it is still favored by some for studies of dams, railway rights-of-way, tunnels, and similar structures that provide extended service.

Capitalized cost is calculated in the same way as in a present-worth comparison, where N equals infinity. This makes the analysis very sensitive to the selected rate of return. As with all present-worth calculations, the final figure is usually an impressive amount. As such, it could appear discouragingly high if it were not properly interpreted.

We found in Chap. 2 that

$$(P/A, i, N) = \frac{1}{i}\,\frac{(1+i)^N - 1}{(1+i)^N}$$

The limit of $(P/A, i, N)$ as N approaches infinity is

$$\boxed{(P/A, i, \infty) = \frac{1}{i}}$$

So if P represents the first cost, we find

$$\begin{aligned} \text{Capitalized worth} &= P + A(P/A, i, \infty) \\ &= P + A(1/i) = P + A/i \end{aligned}$$

where A is the uniform difference between annual receipts and disbursements. When there is no revenue, the formula becomes

$$\boxed{\text{Capitalized cost} = P + \frac{\text{disbursements}}{i}}$$

EXAMPLE 3.5 **An Asset that Lasts Forever**

A \$500,000 gift was bequeathed to a city for the construction and continued upkeep of a music shell. Annual maintenance for a shell is estimated at \$15,000. In addition, \$25,000 will be needed every 10 years for painting and major repairs.

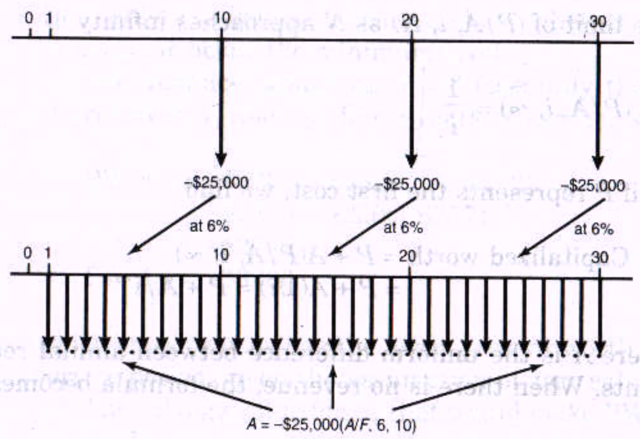
How much will be left for the initial construction costs, after funds are allocated for perpetual upkeep? Deposited funds can earn 6 percent annual interest, and these returns are not subject to taxes.

Solution

The total capitalized cost is known to be \$500,000. From the capitalized-cost formula,

$$\text{First cost} = \text{capitalized cost} - \frac{\text{annual disbursements}}{i}$$

The equivalent annual disbursements are \$15,000 for maintenance plus the annual payments necessary to accumulate \$25,000 every 10 years. The funds will earn interest at 6 percent. The \$25,000 every 10 years is annualized as follows:



$$\text{First cost} = \$500{,}000 - \frac{\$15{,}000 + \$25{,}000(A/F,\ 6,\ 10)}{0.06}$$

$$= \$500{,}000 - \frac{\$15{,}000 + \$25{,}000(0.07587)}{0.06}$$

$$= \$500{,}000 - \$281{,}613 = \$218{,}387$$

This means that the interest earned on the amount left after allowing $218,387 for construction will cover all the anticipated upkeep indefinitely, provided that the interest rate remains at 6 percent or more or else averages about 6 percent. Also, this assumes that painting and other costs will not increase over time, which is possibly an unrealistic assumption.

3.5 COMPARISON OF DEFERRED INVESTMENTS

An occupational hazard for engineers is the habitual appeal to "get it done today, or preferably yesterday." The accustomed response to such an appeal is a workable solution that admittedly may not be the most economical long-run course of action. But keeping an operation going with a less-than-optimum solution is often less costly than the wait caused by a search for something better. For instance, suppose that flooding at a construction site was the motivation for an engineer to remedy the situation quickly. The emergency purchase and installation of a pump plus sandbags for revetments cost $4000. The pumping facility was used off and on during the 2-year construction project for a total cost discounted to the time of purchase (assuming no salvage value and i = 12 percent) of

First cost of pump and revetment	$4000
Pumping cost and maintenance ($460/year)	
$460(P/A, 12, 2) = $460(1.69005)	777
PW of emergency pumping operation	$4777

A postproject review shows that a smaller pump could have been purchased to perform the same work adequately:

First cost of smaller pump and revetment	$3100
Pumping cost and maintenance ($640/year)	
$640(P/A, 12, 2) = $640(1.69005)	1082
PW of lower-cost solution to flooding	$4182

However, if it had taken only a week longer to select and get delivery of the lower-cost pump, it is likely that the cost of 7 days of flooding would have exceeded the $4777 − $4182 = $595 potential saving.

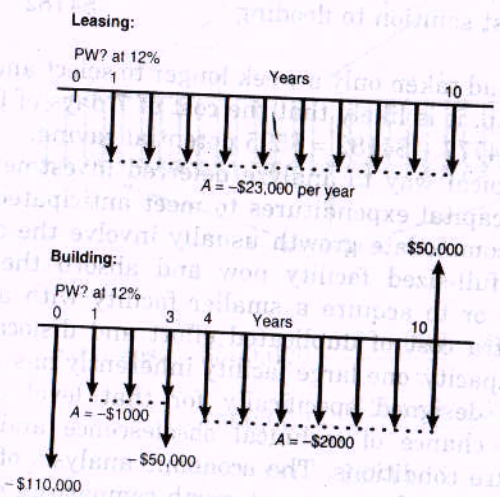
A more typical way to analyze deferred investments is to determine the timing of capital expenditures to meet anticipated activity increases. Designs to accommodate growth usually involve the question of whether to acquire a full-sized facility now and absorb the temporary cost of unused assets or to acquire a smaller facility with a later addition and accept the extra cost of duplicated effort and dislocation inconvenience. For a given capacity, one large facility inherently has a lower per-unit cost because it is designed specifically for that level of operation, but it increases the chance of technical obsolescence and idleness, owing to changing future conditions. The economic analysis of a deferred addition is usually conducted by a present-worth comparison of the options. Piecemeal additions to existing capacity are seldom an efficient expansion program. A planning horizon too short to accommodate growth is one example of shortsighted suboptimization. Careful planning and timing are always required for such activities.

EXAMPLE 3.6 **Immediate and Deferred Investments for Identical Capacity**

A small novelty manufacturing company must acquire storage space in order to reduce production costs by stabilizing employment. Ninety percent of the products produced are sold during the Christmas holiday season. A resource utilization study has shown that producing at a constant rate during the year and storing output will reduce the overall manufacturing costs.

The products produced by the novelty company have been well received, and sales have increased each year. Increased capacity will be needed in the future; two alternatives have been identified. A large warehouse with sufficient space to meet all needs for 10 years can be leased for that period at $23,000 per year. Since there is some doubt about how much business will increase in the future and since the company is reluctant to go into debt deeply enough to build a warehouse as large as the one available for leasing, the other feasible alternative is to build a small warehouse now for $110,000 and make an addition to it in 3 years for $50,000. Annual costs for taxes, insurance, maintenance, and repairs are expected to be $1000 for the first 3 years and $2000 for the next 7 years. The added-to warehouse should have a resale value of $50,000 in 10 years. Based on a study period equal to the lease contract and a 12 percent cost for capital, which alternative is preferable?

Solution



$$\text{PW(lease)} = -\$23{,}000(P/A, 12, 10) = -\$23{,}000(5.65022) = -\$129{,}955$$

$$\begin{aligned}\text{PW(build and add)} &= -\$110{,}000 - \$50{,}000(P/F, 12, 3) \\ &\quad + \$50{,}000\,(P/F, 12, 10) - \$1000(P/A, 12, 3) \\ &\quad - \$2000(P/A, 12, 7)\,(P/F, 12, 3) \\ &= -\$110{,}000 - \$50{,}000(0.71178) + \$50{,}000(0.32197) \\ &\quad - \$1000(2.40183) - \$2000(4.56376)(0.71178) \\ &= -\$138{,}389\end{aligned}$$

The analysis indicates that the present worth of storage costs for the next 10 years will be –$129,955 – (–$138,389) = $8434 less from leasing than building the required warehousing facility.

It should be mentioned that since we were considering only costs in this exercise, we could have treated them as positive values for positive costs and then picked the alternative with the smaller present worth. The analyst should handle the problem in the way that is most comfortable to her or him.

The major potential problem with situations involving estimates of future costs and factors relates to the accuracy of those estimates. Home mortgage rates in the early 1960s were on the order of 6 to 7 percent. Several years later there was a large increase in mortgage rates, and they were on the order of 15 percent. The early 1990s saw mortgage rates dropping close to those of the early 1960s. How then, can uncertain costs and factors be taken into consideration? The same problem that was addressed in Example 3.6 will be examined from a different perspective in Chap. 14, where it will no longer be assumed that future demands are known with certainty. If reliable probabilities for future capacity needs can be generated, they should be included in the analysis. The difficulty lies with finding a reliable way to peek into the future.

FUTURE-WORTH COMPARISONS

It is evident from the equivalence concept that a present worth can be translated to a future worth at any given time at a given interest rate. The mechanics of the translation are elementary, but the change in perspective of the analyst can be significant. For certain situations, the future worth of cash flow may be more meaningful to a decision maker than the present worth. The amount of debt or surplus accumulated at a future date could be a milestone in a financial plan or a goal that directs current activities. The potential benefits for a person about to retire certainly could be more meaningful at a future date.

Attention to future cash flows may yield more accurate estimates of receipts and disbursements because thinking tends to be in *then-current* dollars. Because the purchasing power of a unit of currency is eroded by inflation, it may be advantageous to focus on how many currency units must be available to meet needs at a later date—the quantity of then current dollars. Consequently, future-worth calculations are frequently utilized in escalation analyses, particularly for evaluating the effects of inflation.

Given the cash flow for an alternative, the future worth at any time can be calculated by

$$\text{FW} = \text{PW}(F/P, i, N) = \text{PW}(1 + i)^N$$

The decision rule based on FW has the same results as for PW since, for the same i and N, both of which are positive, the FW ranking of alternatives has to be the same as the ranking based on a PW analysis.

The future-worth perspective can be illustrated by reference to the cash flow in Example 3.2. The 8-year cash flow in Example 3.2 is composed of two annuities: annual $15,000 costs for the first 3 years followed by five equal step receipts ranging from $10,000 to $20,000.

The future value of disbursements, treated as debts for which interest at 10 percent must be paid, builds in 8 years to a total indebtedness of

$$\begin{aligned}\text{FW(disbursements)} &= -\$15{,}000(F/A, 10, 3)(F/P, 10, 5) \\ &= -\$15{,}000(3.31000)(1.61051) \\ &= -\$79{,}962\end{aligned}$$

A complementary future worth at the end of year 4 will show the accu-lated indebtedness before any income is generated as

$$FW(4) = -\$15{,}000(F/A, 10, 3)(F/P, 10, 1) = -\$54{,}615$$

This amount is reduced by the receipt of $10,000 at the end of year 4, it reveals the maximum liability level.

The positive arithmetic gradient cash inflow reaches a maximum at the end of year 8, amounting to

$$\begin{aligned} FW(\text{receipts}) &= [\$10{,}000 + \$2500(A/G, 10, 5)](F/A, 10, 5) \\ &= [\$10{,}000 + \$2500(1.81013)](6.10510) \\ &= \$88{,}677 \end{aligned}$$

The balance on hand at the end of the study period is then

$$\text{Net FW} = \$88{,}677 - \$79{,}962 = \$8715$$

This same result (differing only by rounding error from the interest table) could be obtained from the previously calculated (in Example 3.2) PW $4066 as

$$\text{Net FW} = \$4066(F/P, 10, 8) = \$4066(2.14359) = \$8716$$

The relative usefulness of PW and FW comparisons depends on which figure is more valuable for decision making. If knowledge of the dollars available at some impending date is critical, then a future-worth calcula-tion is called for. Knowing when to answer such a call is the mark of a com-petent engineering economist.

In the majority of engineering economy cases, the present-worth analy-sis would be more meaningful to the decision maker for reasons given ear-lier in this chapter's introductory material. In many cases, annualized ben-efits will make more sense, for management might want to track annualized values. For example, what are the anticipated increases in annual operating expenses due to a new process being implemented? We look at annualized evaluations in the next chapter.

3.7 VALUATION

Value is a measure of the worth of something in terms of money or goods. A barter system uses a personally directed trade of goods to establish equivalent values. In an auction, the measure of worth is established by competitive monetary bids. To some extent, the selling price in an openly competitive market sets a value on goods as a result of the amount cus-tomers are willing to pay. Value is more difficult to determine before a

transaction occurs. Expert appraisers are familiar with prices from previ-ous transactions and interpolate to set values on goods that have not been exposed to the market. Works of art and land properties are subject to sometimes controversial evaluations.

Appraisals regularly encountered in financial practice are known by specific names. The *going value* is how much the assets of an organization are worth as an operating unit. It is opposite to a *liquidating value,* which is the amount that could be realized if the assets were sold separately from the organization that is using them. The going value is normally greater than the liquidating value, in recognition of the "organizational" value of a unit still in operating condition; accountants term this difference *goodwill.* The worth of an asset for accounting purposes is its *book value,* which may be quite different from its *market value*—the price at which it can be sold. The book value reflects historical cost, whereas market value is dependent upon earnings.

When the value of property depends upon its earning capacity, valua-tion results from discounting future probable earnings to their present worth. When risk is ignored, the "value" of such property is simply its pre-sent worth at the interest rate deemed appropriate by the appraiser. Examples of properties whose values tend to be a function of future cash flows are bonds, stocks, and rental assets.

3.7.1 Bond Valuation[†]

A bond is sold by an organization to raise money. Bonds represent a debt to the bondholders rather than a share of ownership in the organization. Most bonds bear interest semiannually and are redeemable for a specified maturity value at a given date. There are many variations designed to make bonds more attractive to purchasers, such as an option to convert them to common stock under specified conditions, or to make the bond debt more manageable for the issuing organization, as in callable bonds that may be paid off prior to maturity according to a printed repurchase sched-ule. Some public organizations are allowed to issue bonds for which the interest payments are not taxable income to the bondholders.

The value of a bond of a given denomination depends on the size and timing of the periodic dividends and the duration before maturity. The bond valuation is thus the present worth[‡] of the cash flow stream of divi-

[†]The engineer interested in applying economic analysis only to equipment replacement deci-sions and similar engineering evaluations may feel that a discussion of stocks and bonds is not worthwhile. Granted, this material may be bypassed for someone in that category. At least a cursory review of the material is recommended to (1) provide a better understanding of PW calculations, (2) equip oneself for possible personal financial decisions, and (3) understand some of the financial considerations at the corporate level of the enterprise.

[‡]The procedure for calculating a value is sometimes called *capitalization of income.*

dends plus the discounted value of the redemption payment. The key valuation is the rate of return expected by the bond purchaser. Lower are reasonable when there is very little risk of default. For instance, a Treasury security would have less risk of nonrepayment than one issu Fly-by-Nite Corporation; consequently, a lower discount rate would appropriate.

Each bond has a stated redemption amount, called the *face valu* which it can be redeemed at maturity. Bonds sell for less than their value when buyers are not satisfied with the rate of interest promi called the *bond rate*, and more than the face value when market inte rates drop below the bond's face rate. Interest is paid in the form of lar premiums; the flow of premiums constitutes an annuity where

$$\boxed{A = \text{premium} = (\text{face value})(\text{bond rate})}$$

and the number of payments per year is a function of the bond rate ture (e.g., four payments per year for a quarterly bond rate). A prese worth purchase price for a bond that satisfies a buyer seeking a rate greater or less than the bond rate is calculated from the bond's matur value F and the annuity composed of premium payment A, both discoun at the buyer's desired rate of return.

EXAMPLE 3.7

To Buy a Bond

A 10-year corporate bond has a face value of \$5000 and a bond rate of percent payable quarterly. A prospective buyer desires to earn a nomin rate of 12 percent on investments. What purchase price would the buyer be willing to pay?

Solution

The bond can be redeemed at \$5000 in 10 years from now. During the period, four premium payments will be paid each year in the amount A = \$5000(0.08/4) = \$5000(0.02) = \$100. The buyer desires a return of 12/4 = percent per period. To meet the buyer's requirement, the purchase price must be the present worth of income from the bond discounted at 3 percent for 40 periods:

$$\begin{aligned}\text{PW} &= \$5000(P/F,\,3,\,40) + \$100(P/A,\,3,\,40)\\ &= \$5000(0.30656) + \$100(23.11477)\\ &= \$3844\end{aligned}$$

Bonds are traded regularly through financial markets. Depending on the prevailing interest rate, a bond may sell for a price that is less than, more than, or equal to its face value. When the owner of a bond purchased previously seeks to sell it before maturity, the original purchase price and

premiums already received have no bearing on its market value; only future cash flow has consequence.

EXAMPLE 3.8

Evaluation of a Bond Purchase

A utility company sold an issue of 4 percent bonds 6 years ago. Each bond has a face value (value at maturity) of \$1000, is due in 14 years, and pays interest twice a year (2 percent per period). Because interest rates on savings have climbed in recent years, the bond can now be sold on the bond market for only \$760. If a buyer wants his or her investment to earn 8 percent compounded semiannually and must pay a brokerage charge of \$20 to purchase each bond, is the current market price low enough to provide the desired return?

Solution

Semiannual interest payments amount to 2 percent of the face value of the bond, or 0.02 × \$1000 = \$20, and \$1000 will be redeemed in 14 years, or 28 half-year periods. Therefore, the present value of the cash flow when the desired interest rate is 4 percent per 6-month period is

$$\begin{aligned}\text{PW} &= \$20(P/A,\,4,\,28) + \$1000(P/F,\,4,\,28)\\ &= \$20(16.66306) + \$1000(0.33348)\\ &= \$666.74\end{aligned}$$

Since the price of the bond is \$760 + \$20 = \$780, prospective bond purchasers should look elsewhere to obtain their desired rate of return on investments.

Assuming that an original purchaser of the utility bond in Example 3.8 paid the face value (\$1000) 6 years ago and accepted \$760 for it now, we find that only the original investment was recovered and no interest was earned during the 6 years it was held:

$$\begin{aligned}\$1000 &= \$20(P/A,\,i,\,12) + \$760(P/F,\,i,\,12)\\ &= \$20(P/A,\,0,\,12) + \$760(P/F,\,0,\,12)\\ &= \$20(12) + \$760(1) = \$1000\end{aligned}$$

The current market rate of interest strongly affects bond prices. Higher market rates tend to lower bond prices by decreasing the present worth of the future stream of payments promised by the bond. Figure 3.2 displays the valuation at different market rates of interest for a \$1000, 20-year bond which pays \$40 semiannually (4 percent) and a 3-year bond with the same face value and dividend rate.

As is apparent in Fig. 3.2, the longer the maturity of a security, the greater its price change in response to a change in the market rate of inter-

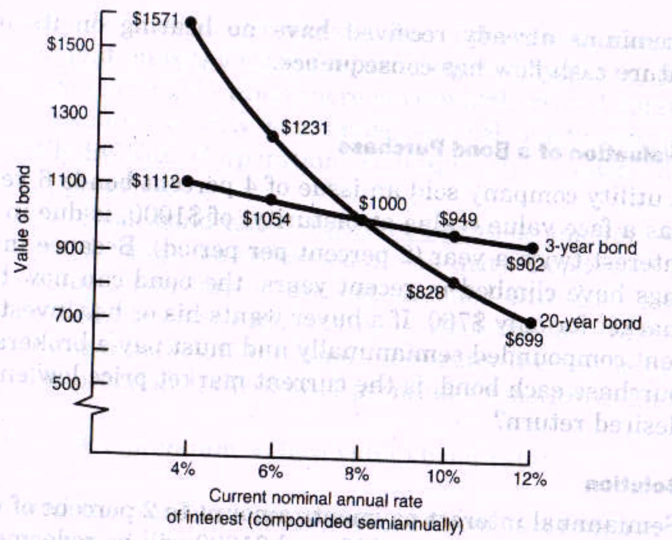


FIGURE 3.2 Value of $1000 long-term (20-year) and short-term (3-year) bonds at different market interest rates. Both bonds promise semiannual dividends of $40.

est. The fact that future interest rates are uncertain is an element of ris[k]. Investments in short-term securities expose the investor to less chance [of] severe fluctuations in the market rate of interest than do comparable lon[g]-term investments.

3.7.2 Stock Valuation

Stock in a company represents a share of ownership, as opposed to a bond, which is essentially a promissory note. There are many types of stocks and bonds, varying with respect to their degree of security and associated spe[-]cial privileges. In general, all bonds have claims on a company's assets before stock in case of a business failure. Stock is still a popular invest[-]ment because it has the potential of increasing in value and may pay higher dividends than bonds when the company is very successful. Some stockholders have voting privileges that allow major investors to have some say in company policy.

Preferred stocks usually entitle owners to regular, fixed dividend pay[-]ments similar to bond interest. Characteristically, preferred stock has no voting rights and initially has a higher yield than bonds. The *par value* of a preferred stock is the amount due to the stockholder in the event of liq[-]uidation, and the *annual yield* is often expressed as a percentage of the par value. Since preferred stock has no maturity date, it may be treated as a perpetuity whose value is

$$PW = \frac{\text{annual dividend on preferred stock}}{\text{annual rate of return expected by investor}}$$

Common stocks—the most common form of equity shares in a company—are more difficult to value than preferred stocks or bonds because dividends and prices of common stocks are not constant; investors hope that they will increase over time. It is therefore necessary to forecast future earnings, dividends, and stock prices. If reliable forecasts could be made (and that is a highly questionable assumption), stock valuation would result from discounting the forecast cash flow.

To illustrate, suppose that a share of Sumplex, Inc., has a current market price of $40. It is expected to pay a $3 dividend by the end of 1 year. Since the company went public in 1985, the value of its stock has been rising at an average rate of 4 percent. Using a 1-year study period, we find that

?

$$\text{Present worth} = \frac{\text{dividend}}{(F/P,\, i,\, 1)} + \frac{\text{market price at end of year}}{(F/P,\, i,\, 1)}$$

$$\text{Present worth} = \frac{\text{dividend}}{1+i} + \frac{\text{present price} \times 1.04}{1+i}$$

Rearranging and substituting numerical data give

$$1 + i = \frac{\$3 + \$40(1.04)}{\$40} = \frac{\$44.60}{\$40} = 1.115$$

to reveal the discount rate

$$i = 1.115 - 1 = 0.115 \text{ or } 11.5\%$$

If an investor is satisfied with an 11.5 percent rate of return after considering the risk involved, shares in Sumplex, Inc., could be purchased. The same expected rate of return could be calculated as

$$i = \text{dividend rate} + \text{growth rate} = \frac{\$3}{\$40} + 0.04$$

$$= 0.075 + 0.04 = 0.115$$

PAYBACK COMPARISON METHOD

The *payback method* is an extremely simple method used to obtain a rough estimate of the time that an investment will take to pay for itself. The reader should be warned that because of its simplicity, it does not give results that are equivalent to the present-worth evaluation or the other accepted techniques that we will be looking at in Chaps. 4 and 5. The payback method (sometimes called the *payout method*) avoids the need to calculate the cost of capital and still recognizes financial concern for limited resources. It guards against unexpected price (cost) increases and uti… a cutoff criterion by requiring proposals to return their original investm… from the savings they generate in a specified period, usually of short du… tion. It is typically applied to relatively small investment proposals th… originate from operating departments. A department manager often h… the authority to accept proposals up to a given ceiling without subject… them to outside review.

The formula for obtaining a rough measure of the time an investme… takes to pay for itself is simple to use and understand:

$$\text{Payback period} = \frac{\text{required investment}}{\text{annual receipts} - \text{annual disbursements}} = \frac{\text{first cost}}{\text{net annual savings}}$$

Data utilized in applying the formula are usually direct, *not discounted* cash flow amounts, and no salvage values are included. The result tells u… how long it will be before the amount invested is recovered in actual do… lars.

Claims such as "This investment will pay for itself in 18 months" ar… commonly heard in industry; they usually indicate anxiety about th… elapsed time before a proposed investment begins to show a profit. Th… payback period is an extremely popular investment criterion in the Unit… States and throughout the world. Polls consistently reveal that the pay… back method is used more than any other comparison method by U.… industry to rate investments, particularly proposals from operating unit… for relatively small capital expenditures to improve operations.

In actual practice, the simple payback formula is sometimes modifie… to recognize capital recovery through depreciation charges and to includ… some discounted values. The simple payback formula is still widely use… without elaborations, although it yields ratings that may lead to incorre… conclusions. Its deficiencies arise from failing to give recognition to cas… flows occurring *after* the payback period has passed and from ignoring th… time value of money.

For instance, as an extreme illustration of payback period deception, an investment of \$1000 in an asset with a life of 1 year and an associated net return of \$1000 will yield

$$\text{Payback period} = \frac{\$1000}{\$1000 \text{ per year}} = 1 \text{ year}$$

Another investment of \$1000 promises to return \$250 per year during its economic life of 5 years and yields

$$\text{Payback period} = \frac{\$1000}{\$250 \text{ per year}} = 4 \text{ years}$$

Favoring the alternative with the shortest payback period would rate the first alternative as better, but this alternative actually earns nothing: \$1000 − \$1000 = 0. Meanwhile, the spurned second alternative would have provided an annual return of 8 percent:

$$P/A = \frac{\$1000}{\$250} = 4 = (P/A, 8, 5)$$

Only when independent proposals have uniform returns and equal lives do the payback and internal rate-of-return (IRR) comparison methods indicate the same preferences.

If the results of payback calculations are questionable, why are they used? There are at least two apparent reasons. First, they are simple calculations. Since both depreciation and interest effects are usually ignored, the calculations are quick and easy and the results are intuitively logical. Second, the other reason stems from a preoccupation with the flexibility of capital. If the money spent on an improvement is recovered rapidly, the funds can be allocated again to other desired projects. This concept tends to engender a false sense of security, with reasoning such as "If the project can quickly pay for itself, it must be good" or "Only the best projects can meet our short-payback-period requirement."

Even though the payback period criterion is not always appropriate, it does address the problem of working-capital management by attempting to protect a firm's liquidity position. During times of constricted income, when a firm may have trouble meeting operating expenses and may have very limited capacity for funding new investments, an extremely short payback period (as low as 6 months) ensures that only quick-profit projects will be endorsed. In such exceptional situations, cash availability considerations may be equal to or more important than total earnings. As an auxiliary criterion, requiring a short payback period guards against the chance of losses due to new technological developments. Since the payback period criterion has serious weaknesses, it should never be applied alone; it should only be applied as an aid in decision making.

EXAMPLE 5.9 **Payback Period Comparisons for Alternatives with Different Lives**

The supervisor of a small machine shop has received three suggestions for reducing production costs. Suggestion *A* is to buy new jigs and fixtures; *B* is to rebuild an existing machine to improve its performance; and *C* is to purchase a new machine to replace some manual labor. Estimates have been made for the three alternative investments:

	Alternative	
	A	**B**
First cost, $	1800	2350
Economic life, years	4	4
Net annual saving, $	645	840
Payback period, years	2.8	2.8

The supervisor selects alternative *B*, explaining that because of limited capital for investments, shorter payback periods are preferable. With alternatives *A* and *B* having the same payback period, *B* is favored because the annual savings are greater than for *A*. What are the fallacies in this reasoning?

Solution

The flaws in the reasoning stem from a strict reliance on the payback criterion. It surely does not take into consideration the fact that alternatives *A* and *B* would have to reinvest in new equipment after 4 years had elapsed while alternative *C* would not. Also, an interest rate of 0 percent was assumed. We can see the flaws by computing the present worth of each alternative over an 8-year period, assuming that alternatives *A* and *B* have equipment costs at the end of year 4 that are identical, in constant dollars, to the costs at the end of time period 0. We will assume a desired rate of interest equal to 8 percent. For alternative *A*:

$$\begin{aligned}\text{PW}_A &= -\$1800 - \$1800(P/F, 8, 4) + \$645(P/A, 8, 8)\\ &= -\$1800 - \$1800(0.73503) + \$645(5.74664) = \$583.53\end{aligned}$$

Alternative *B* has the same structure:

$$\text{PW}_B = -\$2350 - \$2350(0.73503) + \$840(5.74664) = \$749.86$$

Finally alternative *C* has

$$\begin{aligned}\text{PW}_C &= -\$4200 + \$1100(P/A, 8, 8)\\ &= -\$4200 + \$1100(5.74664) = \$2121.30\end{aligned}$$

As expected, alternative *C* prevails due primarily to just one investment cost in 8 years.

This explains why the simple payback period method should not be applied to different life alternatives. It also leads to the idea of a *discounted* payback period analysis. As we will see, this also only works with situations where all lives are equal. We compute discounted cumulative cash flows instead of cumulative cash flows determined with 0 percent interest. Let's use alternative *C* as an example:

End of year	Cash flow, $	Discounted cumulative cash flow, $
0	−4200	−4200
1	1100	−4200(1.08) + 1100 = −3436
2	1100	−3436(1.08) + 1100 = −2611
3	1100	−2611(1.08) + 1100 = −1720
4	1100	−1720(1.08) + 1100 = −757
5	1100	−757(1.08) + 1100 = +282
6	1100	+282(1.08) + 1100 = +1405
7	1100	+1405(1.08) + 1100 = +2617
8	1100	+2617(1.08) + 1100 = +3926

We see that the discounted payback period occurs in the fifth year. Interpolating, we get

$$\begin{aligned}\text{Discounted payback period} &= 4 + 1\left(\frac{-757 - 0}{-757 - 282}\right)\\ &= 4.73 \text{ years}\end{aligned}$$

With 0 percent interest, by the conventional payback method this period is 3.8 years. Therefore, the conventional method tends to be optimistic in addition to other problems.

If we had performed a similar calculation for alternative *A* or alternative *B* we would have arrived at the same discounted payback period for either: 3.28 years. This tells us that we would have 0.72 year of profit, but then we would be investing in new equipment at the end of the fourth year! Having a net investment after period 0 is causing trouble with the method. This backs up the earlier statement that we should consider only alternatives with equal lives. CHEER will handle discounted payback period problems, and by inputting an interest rate of 0 percent, we can handle the original simple payback period method also.

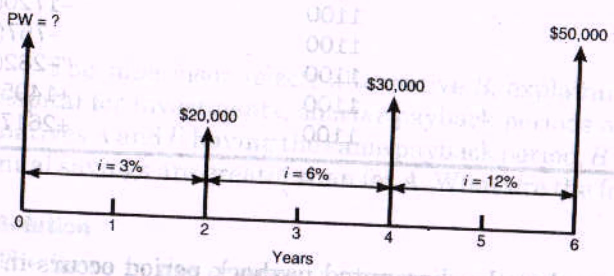
REVIEW EXERCISES AND DISCUSSION

EXERCISE 1

An entrepreneur intending to start a new business knows that the first few years are the most difficult. To lessen the chance of failure, a loan plan for start-up capital is proposed in which interest paid during the first 2 years will be at 3 percent, at 6 percent for the next 2 years, and at 12 percent for the final 2 years of the 6-year loan. How large a loan can be justified for proposed repayments at the end of years 2, 4, and 6 of, respectively, $20,000, $30,000, and $50,000?

SOLUTION 1

It is not unusual for interest rates to change during a study. Time-value equivalence is calculated by translating individual cash flows through each time period at the interest rate applicable during that period. The procedure is illustrated by considering each loan repayment independently and calculating its present worth:



$$\text{PW}(\$20{,}000\text{ payment}) = \$20{,}000(P/F,\ 3,\ 2) = \$20{,}000(0.94260) = \$18{,}852$$

$$\text{PW}(\$30{,}000\text{ payment}) = \$30{,}000(P/F,\ 6,\ 2)(P/F,\ 3,\ 2) = \$30{,}000(0.89000)(0.94260) = \$25{,}167$$

$$\text{PW}(\$50{,}000\text{ payment}) = \$50{,}000(P/F,\ 12,\ 2)(P/F,\ 6,\ 2)(P/F,\ 3,\ 2) = \$50{,}000(0.79719)(0.89000)(0.94260) = \$33{,}439$$

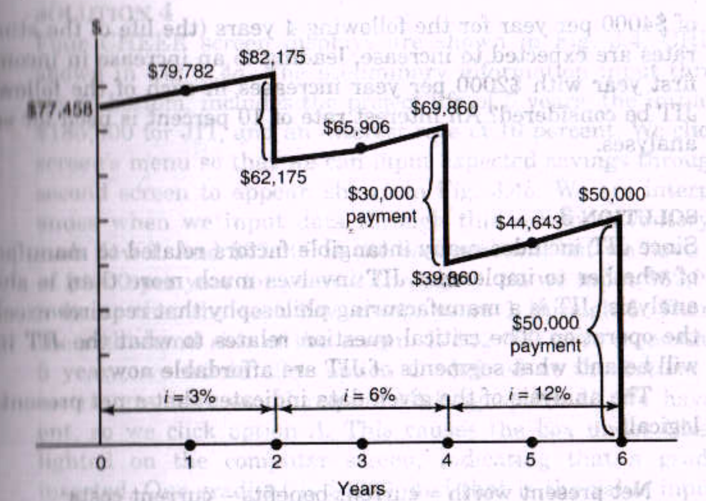
The loan that can be repaid by the given repayment amounts at the given interest rates is then

$$\text{PW(total loan)} = \$18{,}852 + \$25{,}167 + \$33{,}439 = \$77{,}458$$

A graph of the loan's balance over the 6-year period is shown in Fig. 3.3. From an initial value of $77,458 at time 0, the amount owed increases to $82,175 in 2 years with interest at 3 percent. Then the $20,000 payment is recorded, reducing the amount owed to $62,175. The steeper rates of increase during the later stages of the loan result from greater earnings generated by higher interest rates.

EXERCISE 2

A new rock pit will be operated for a construction project that will last 5 years. It can be loaded from an elevated box loader served by a conveyor from the pit or by mobile shovel loaders. The box loader and conveyor have an initial cost of $264,000 and will have no salvage value at the end of the project. Two shovel loaders, each priced at $42,000, can provide the same capacity, but their operating costs together will be $36,000 per year more than the box loader. Normal service life for a shovel loader is 3 years with zero salvage value, but a 2-year-old machine can likely be sold for $10,000. Which alternative is preferred when the interest rate is 13 percent?



SOLUTION 2

The study period is the duration of the project, 5 years. The present worth of the box-loader alternative is its initial cost: PW(box loader) = −$264,000.

Since the first two shovel loaders are replaced at the end of 3 years and the replacements will not be in service for their entire useful life, the estimated salvage value must be included as a cash inflow at the end of year 5. The additional operating costs incurred for the shovel loader alternative form a 5-year annuity of annual $36,000 payments:

$$\begin{aligned}\text{PW(shovel loaders)} &= 2(-\$42{,}000) + 2(-\$42{,}000)(P/F,\ 13,\ 3) - \$36{,}000 \\ &\quad \times (P/A,\ 13,\ 5) + 2(\$10{,}000)(P/F,\ 13,\ 5) \\ &= -\$84{,}000 - \$84{,}000(0.69305) - \$36{,}000(3.51723) \\ &\quad + \$20{,}000(0.54276) \\ &= -\$84{,}000 - \$58{,}216 - \$126{,}619 + \$10{,}856 \\ &= -\$257{,}979\end{aligned}$$

The shovel loaders appear to be more economical if the machines can be sold for the indicated salvage value after 2 years of use. Also, leverage might now be available for negotiation on the price of the box loader and conveyor.

EXERCISE 3

Kiddytoyz is a small novelty toy manufacturing company that is considering moving to a *just-in-time (JIT)* approach. JIT is a manufacturing philosophy where everything needed arrives at exactly the right time, equipment is supposed to operate without failure, and quality is perfect (zero defects). It is estimated for Kiddytoyz that the JIT first costs for equipment and tooling modifications will be $185,000. Annual maintenance and operating costs will increase due to the need for reliability improvements. These increases over the current operation are estimated to be a constant $8000 per year. Potential inventory reductions are estimated to be $32,000 for the first year with further arithmetic gradient reductions of \$4000 per year for the following 4 years (the life of the study). Also, production rates are expected to increase, leading to an increase in income of \$21,000 for the first year with \$2000 per year increases in each of the following 4 years. Should JIT be considered? An interest rate of 10 percent is used for economic justification analyses.

SOLUTION 3

Since JIT includes many intangible factors related to manufacturing, the decision of whether to implement JIT involves much more than is shown in an economic analysis. JIT is a manufacturing philosophy that requires excellence at all levels of the operation. The critical question relates to what the JIT implementation cost will be and what segments of JIT are affordable now.

The analysis of the given data indicates that a net present-worth evaluation is logical:

Net present worth = current benefits − current costs

PW (benefits) = [\$32,000 + \$4000 $(A/G, 10, 5)$]$(P/A, 10, 5)$
inventory savings with annual linear increase brought to present
+ [\$21,000 + \$2000$(A/G, 10, 5)$]$(P/A, 10, 5)$
potential increase in income with annual linear increase brought to present
= [\$32,000 + \$4000(1.81013)](3.79079)
+ [\$21,000 + 2000(1.81013)](3.79079)
= \$148,747 + \$93,327 = \$242,083

PW(costs) = \$185,000 + \$8000$(P/A, 10, 5)$
first costs plus linear increase in maintenance and operating costs
= \$185,000 + \$8000(3.79079)
= \$185,000 + \$30,326 = \$215,326

So,

Net present worth = \$242,083 − \$215,326 = \$26,757

Assuming that the estimated future data are reasonable, JIT should be more than economically sound. This brings up an interesting thought: Shouldn't we be able to evaluate such alternatives even if we know that the estimates of future values are subject to variation? The answer, logically, is yes! Through sensitivity analysis we can see the effect of changing estimate values on the final solution. The computer program available with this text has that capability. We will discuss sensitivity analysis later in Chap. 11.

EXERCISE 4

The computer program CHEER can be used to solve Exercise 3. This exercise will demonstrate the process.

SOLUTION 4

Four CHEER screen displays are shown in Fig. 3.4. CHEER's initial screen is shown in Fig. 3.4*a*. The preliminary information input through this screen, from top to bottom, includes the project life of 5 years, the initial capital investment of \$185,000 for JIT, and an interest rate of 10 percent. We click Revenue on the first screen's menu so that we can input expected savings through JIT. This causes the second screen to appear, shown in Fig. 3.4*b*. We are interpreting savings as revenues when we input data through this screen. Inventory savings are input as cash flow 1. And \$32,000 is the base amount, but we have an arithmetic gradient of \$4000 per year for years 2 through 5. So we input \$32,000 for the amount and indicate that the cash flow covers years 1 through 5. There are three options at the right-hand side of the screen. If \$32,000 were a constant flow for each of the 5 years, we would click option *C* (defined at the bottom of the screen). A geometric gradient would require clicking option *G*. We have an arithmetic gradient, so we click option *A*. This causes the box under Change/period to be highlighted on the computer screen, indicating that a gradient value has to be inserted. Our gradient is \$4000, and that is the value input. If we had a geometric gradient, we would need to input the percentage gradient rather than the constant-dollar gradient. The second cash flow is the expected increase in income to result from JIT.

After completing the screen 2 inputs, we click OK, and the first computer screen will appear (not shown again in the figure). Since we have additional maintenance costs to input, we click Operating/Maintenance costs and get the third screen (Fig. 3.4*c*). Now we have to input the expected \$8000 per year additional income expected, and this is input with option *C* since we do not expect increases per year. We click on OK and return to the original screen.

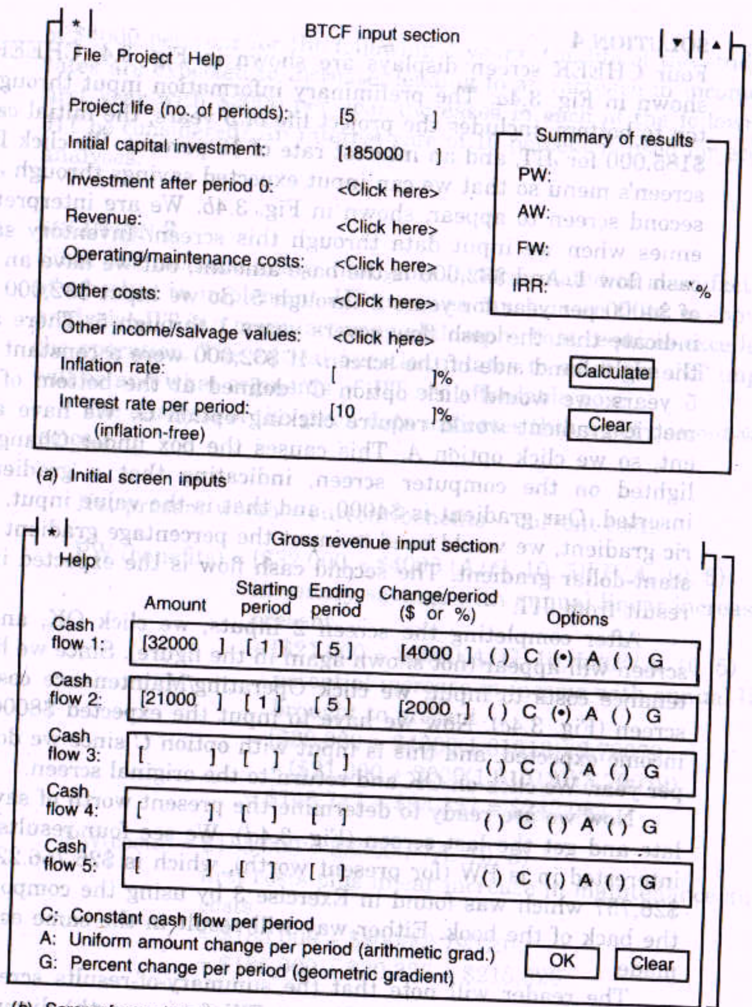
Now we are ready to determine the present worth of savings. We click Calculate and get the last screen (Fig. 3.4*d*). We see four results, and the one we are interested in is PW (for present worth), which is \$26,756.22. This compares with \$26,757 which was found in Exercise 3 by using the compound-interest tables at the back of the book. Either way will result in the same economic decision being made.

The reader will note that the summary-of-results screen also gives the AW (annual worth), discussed in Chap. 4, FW (future worth), discussed earlier in Sec. 3.6, and IRR (internal rate of return), deferred to Chap. 5.

EXERCISE 5

Metalix Company is planning for the manufacture of an assembly that will be used in a new product for which the company has a 5-year military contract. This assembly could be produced in the company's own facility with the purchase of specialized numerically controlled equipment; or the assembly could be subcontracted to a low bidder. The cost estimates for producing the assembly in-house are as follows:

First costs	\$825,000
Annual maintenance and operating costs	\$125,000 for first year with 4 percent per year additional costs for subsequent years
Tooling and material costs	\$525 per assembly
Equipment salvage value after 5 years	\$45,000



(a) Initial screen inputs

(b) Screen for savings input

FIGURE 3.4 CHEER screens for Exercise 4.

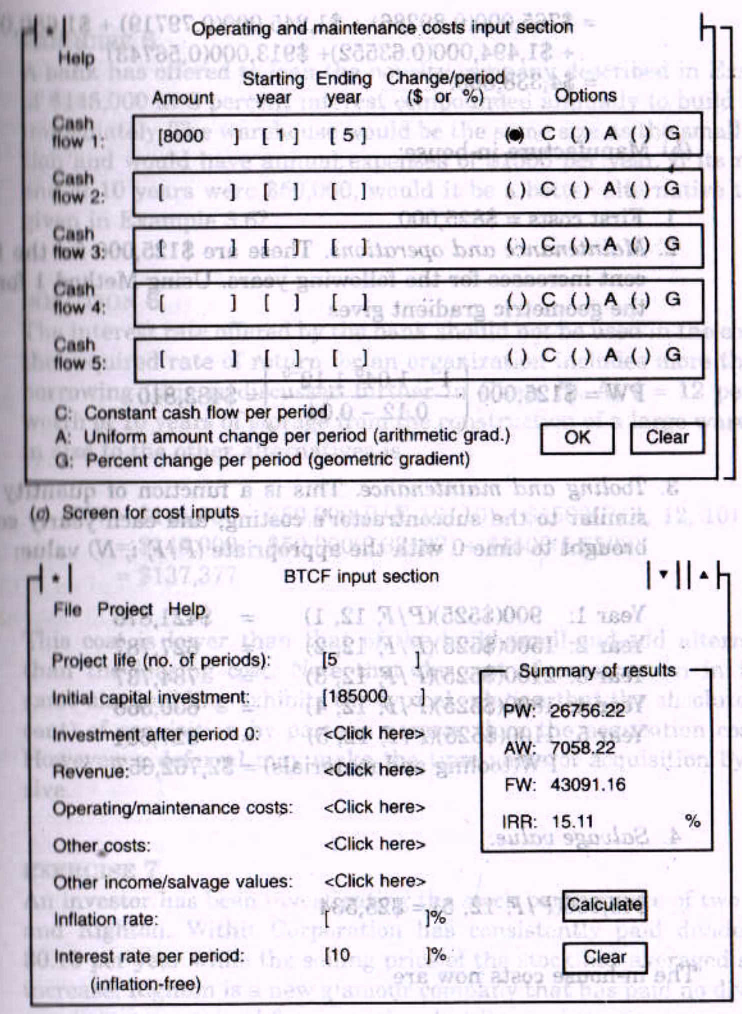
The low-bid subcontractor bids as follows:

500– 999 assemblies in 1 year	$850 per assembly
1000–1999 assemblies in 1 year	$830 per assembly
2000–2999 assemblies in 1 year	$800 per assembly

The bidder already has the specialized equipment needed to produce the assemblies.

The Metalix contract requires the following delivery schedule:

Year	1	2	3	4	5
Number of assemblies	900	1500	2100	1800	1100



(c) Screen for cost inputs

(d) Final results screen

If an interest rate of 12 percent is used by Metalix for its economic evaluations, would you recommend that Metalix (*a*) subcontract or (*b*) manufacture in-house?

SOLUTION 5

(*a*) Subcontract:

$$\begin{aligned} PW = {} & \underbrace{900(\$850)(P/F, 12, 1)}_{\text{first-year cost brought to time 0}} + \underbrace{1500(\$830)(P/F, 12, 2)}_{\text{second-year cost brought to time 0}} \\ & + \underbrace{2100(\$800)(P/F, 12, 3)}_{\text{third-year cost brought to time 0}} + \underbrace{1800(\$830)(P/F, 12, 4)}_{\text{fourth-year cost brought to time 0}} \\ & + \underbrace{1100(\$830)(P/F, 12, 5)}_{\text{fifth-year cost brought to time 0}} \end{aligned}$$

$$= \$765{,}000(0.89286) + \$1{,}245{,}000(0.79719) + \$1{,}680{,}000(0.71178)$$
$$+ \$1{,}494{,}000(0.63552) + \$913{,}000(0.56743)$$
$$= \$4{,}338{,}860.$$

(b) Manufacture in-house:

1. First costs = $825,000.
2. *Maintenance and operations.* These are $125,000 for the first year with 4 percent increases for the following years. Using Method 1 for Chap. 2 part (a) the geometric gradient gives

$$PW = \$125{,}000\left(\frac{1 - 1.04^5\ 1.12^{-5}}{0.12 - 0.04}\right) = \$483{,}810$$

3. *Tooling and maintenance.* This is a function of quantity produced in a year similar to the subcontractor's costing, and each yearly cost would have to be brought to time 0 with the appropriate $(P/F, i, N)$ value:

Year 1: 900($525)(*P*/*F*, 12, 1)	=	$421,876
Year 2: 1500($525)(*P*/*F*, 12, 2)	=	627,787
Year 3: 2100($525)(*P*/*F*, 12, 3)	=	784,737
Year 4: 1800($525)(*P*/*F*, 12, 4)	=	600,566
Year 5: 1100($525)(*P*/*F*, 12, 5)	=	327,691
PW(tooling and materials)	=	$2,762,657

4. *Salvage value:*

$$\$45{,}000(P/F,\ 12,\ 5) = \$25{,}534$$

The in-house costs now are

$$\$825{,}000 + \$483{,}810 + \$2{,}762{,}534 - \$25{,}534 = \$4{,}045{,}933$$

The net present worth difference for the two alternatives is

$$\$4{,}338{,}860 - \$4{,}045{,}657 = \$293{,}927$$

It looks as though manufacturing in-house will be more economical than subcontracting. Possible uncertain characteristics could make management think twice about this. The state of the military complex with cost reductions might force a cancellation of the project before the 5-year period is completed. If this probability is small, management would opt to manufacture. If it is large, then management might decide to subcontract. Uncertainty characteristics are discussed later in this text.

EXERCISE 6
A bank has offered to loan the novelty company described in Example 3.6 the sum of $145,000 at 8 percent interest compounded annually to build a large warehouse immediately. The warehouse would be the same size as the small one plus the addition and would have annual expenses of $1500 per year. If its resale value at the end of 10 years were $50,000, would it be a better alternative than the other two given in Example 3.6?

SOLUTION 6
The interest rate offered by the bank should not be used in the comparison, because the required rate of return for an organization includes more than just the cost of borrowing (this is discussed further in Chap. 5). At i = 12 percent, the present worth of 10 years of storage from the construction of a large warehouse comparable in size to the other alternatives is

$$PW = \$145{,}000 - \$50{,}000(P/F,\ 12,\ 10) + \$1500(P/A,\ 12,\ 10)$$
$$= \$145{,}000 - \$50{,}000(0.32197) + \$1500(5.6502)$$
$$= \$137{,}377$$

This cost is lower than that of the build-small-and-add alternative but is more than the leasing cost. Note that the cost of construction in two phases of the same-size building exhibits the typical relation that the absolute cost (at i = 0 percent) of acquisition by parts is greater than the acquisition cost all at one time. However, a deferral may make the time value of acquisition by parts less expensive.

EXERCISE 7
An investor has been investigating the stock performance of two companies: Withit and Righton. Withit Corporation has consistently paid dividends that increase $0.10 per year while the selling price of the stock has averaged a 2 percent annual increase. Righton is a new glamour company that has paid no dividends because all earnings are retained for expansion, but its market price is expected to increase by $10 per year. Moreover, in about 5 years Righton is expected to start paying dividends equal to 2 percent of its price per share (a price-to-earnings ratio of 50:1). Current data about the two companies are summarized below.

	Withit corporation	Righton corporation
Dividend	$2.25 (10¢/year increase)	0 (2% of market price after 5 years)
Market price	$28 (2% annual increase)	$65 ($10/year increase)
Capitalization rate	9%	12% (risk-adjusted)

Since it is generally believed that Righton's stock is less stable than Withit's, the extra risk of investing in Righton is recognized by requiring a higher rate of return for the valuation—12 percent versus 9 percent. Note that this might be a poor way to handle risk. Later chapters will discuss probability concepts that can be with future cash flows to reflect risk possibilities. Also, sensitivity analysis introduced in a later chapter to show how slightly changing values, such as return, can reveal whether there is a need for extremely accurate values.

Disregarding tax effects and brokerage commissions to buy or sell, dete which stock has the greater valuation for an anticipated 10-year ownership.

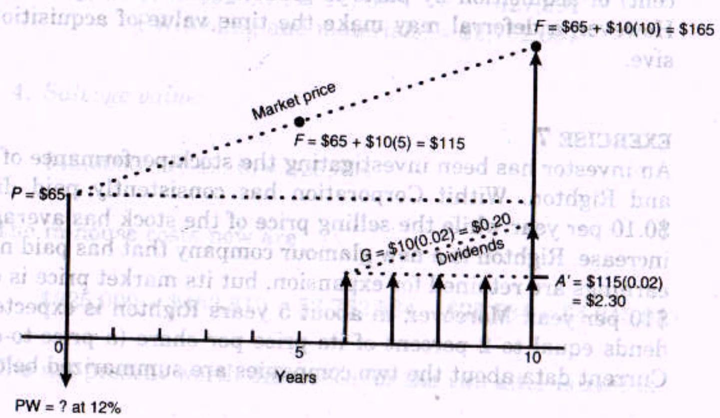
SOLUTION 7

A 10-year study period is used to calculate the present worth of each stock alte tive, assuming that dividends are paid at the end of the year. The valua Withit stock at $i = 9$ percent is

$$\begin{aligned} PW(\text{Withit}) &= [\$2.25 + \$0.10(A/G, 9, 10)] (P/A, 9, 10) + \$28 (F/P, 2, 10) \times (P/F, 9, 10) \\ &= [\$2.25 + \$0.10(3.79777)] (6.41766) + \$28(1.21899)(0.42241) \\ &= \$16.88 + \$14.42 = \$31.30 \end{aligned}$$

which makes the current market price of $28 appear attractive.

The valuation of Righton Company's stock based on a 12 percent desired of return and the assumption that dividends are paid annually after a 5-year is diagrammed as



$$\begin{aligned} PW(\text{Righton}) &= [\$2.30 + \$0.20(A/G, 12, 5)] (P/A, 12, 5)(P/F, 12, 5) + \$165(P/F, 12, 10) \\ &= [\$2.30 + \$0.20(1.77459)] (3.60478)(0.56743) + \$165(0.32197) \\ &= \$5.43 + \$53.13 = \$58.56 \end{aligned}$$

Since the $65 market price for Righton stock exceeds the calculated valuation it appears that Withit stock is a better investment opportunity, assuming that risk return ratings and forecast cash flows are reasonably accurate.

PROBLEMS

3.1 The lease on a warehouse amounts to $5000 per month for 5 years. If payments are made on the first of each month, what is the present worth of the agreement at a nominal annual interest rate of 12 percent, compounded monthly?

3.2 Determine the present worth of the lease in Prob. 3.1 if the payments are to increase at the beginning of each year equal to $250 per month for each month of occupancy.

3.3 What is the future worth at the end of 5 years of the payments to be made in Prob. 3.1? Do *not* use your PW results from Prob. 3.1!

3.4 A company borrowed $100,000 to finance a new product. The loan was for 20 years at a nominal interest rate of 8 percent compounded semiannually. It was to be repaid in 40 equal payments. After one-half the payments were made, the company decided to pay the remaining balance in one final payment at the end of the 10th year. How much was owed?

3.5 A proposed improvement in an assembly line will have an initial purchase and installation cost of $175,000. The annual maintenance cost will be $6000; periodic overhauls once every 3 years, excluding the last year of use, will cost $11,500 each. The improvement will have a useful life of 9 years, at which time it will have no salvage value. What is the present worth of the 9-year costs of the improvement at $i = 8$ percent?

3.6 The assembly line in Prob. 3.5 will have potential income increases, due to higher production volumes, of $29,000 in the first year with $3000 per year additional increases in years 2 through 5. These increases can be expected to decrease from the year 5 value at $5000 per year until the increase in year 9 is $21,000. Are the future expected increases sufficient to justify the expenditures given in Prob. 3.5?

3.7 Amjay Company is currently renting a parking lot for employee and visitor use at an annual cost of $9000, payable on the first of each year. The company has an opportunity to buy the lot for $50,000. Maintenance and taxes on the property are expected to cost $2500 annually. Given that the property will be needed for 10 more years, determine what sales price must be obtained at the end of that period in order for Amjay to break even, when the interest rate is 12 percent.

3.8 A bakery is thinking of purchasing a small delivery truck that has a first cost of $18,000 and is to be kept in service for 6 years, at which time the salvage value is expected to be $2500. Maintenance and operating costs are estimated at $2500 the first year and will increase at a rate of $200 per year. Determine the present worth of this vehicle, using an interest rate of 12 percent.

3.9 A small dam and an irrigation system are expected to cost $300,000. Annual maintenance and operating costs are expected to be $40,000 the first year and will increase at a rate of 10 percent per year. Determine the equivalent present worth of building and operating the system with interest of 10 percent over a 30-year life.

3.10 It is estimated that additional reinforcement of the dam in Prob. 3.9 in the first year at a cost of $65,000 will reduce annual maintenance costs to $25,000 in the first year with an increase of 5 percent per year over the rest of the 30-year Should the civil engineer in charge of the project seriously consider the proposal do the reinforcing?

3.11 A newly developed electric car will cost $21,000 to purchase. Operating maintenance costs, including home charging of the batteries, are estimated $350 for the first year with annual increases thereafter of $50 per year. Salvage value after 5 years is estimated to be $6500. A new gasoline runabout will $16,000 and will average 30 miles per gallon. Gasoline costs $1.26 per gallon is expected to increase at a rate of $0.05 per year for each of the next 4 years. Maintenance costs are estimated to be $300 per year including warranty coverage vage value is estimated to be $1500 after 5 years of service. If the vehicles expected to be driven for 20,000 miles per year, determine which option will the lower cost over 5 years. Use present-worth analysis with a 10 percent rate interest.

3.12 Assume that the estimates for the gasoline car in Prob. 3.11 are reasonable due to past experience but that the salvage value is somewhat unclear for the electric car. What is the minimum salvage value that should be obtained after 5 years to justify consideration of the electric car over the gasoline vehicle?

3.13 Machine *A* has a first cost of $9000, no salvage value at the end of its 6-year useful life, and annual operating costs of $5000. Machine *B* costs $16,000 new has an expected resale value of $4000 at the end of its 9-year economic life. Operating costs for machine *B* are $4000 per year. Compare the two alternatives on the basis of their present worths, using the repeated-projects assumption at 10 percent annual interest.

3.14 A commercial rental property is for sale at $100,000. A prospective buyer estimates that the property would be held for 12 years, at the end of which it could be sold for $90,000. During the ownership period, annual receipts from rents would be $15,000, and average disbursements for all purposes in connection with ownership would be $6000. If a rate of return of 9 percent is expected, what is the maximum bid that the prospective purchaser should make to buy the property?

3.15 A manufacturer requires an additional 10,000 square feet (929 square meters) of warehouse space. A reinforced-concrete building added to the existing main structure will cost $850,000, whereas the same amount of space can be constructed with a galvanized building for $595,000. The life of the concrete building is estimated at 25 years with a yearly maintenance cost of $23,800. The life of the galvanized building is estimated to be 15 years, and the annualized maintenance cost is estimated to be $53,000. Average annual property taxes are 1.2 percent of first costs for the concrete building and 0.5 percent for the metal building. Assume that the salvage value of the concrete building will be zero after 25 years. Compare the present worths of the two warehouse additions, using 12 percent interest for a 25-year study period, in determining the minimum salvage value of the galvanized building after the 25-year period to make it economically comparable to the concrete building, assuming that its salvage value after 15 years is zero.

3.16 Perpetual care for a small shrine in a cemetery is estimated to be available for $500 per year. The long-term interest rate is expected to average about 5 percent. If the capitalized cost is estimated at $15,000, what amount is anticipated for the first cost of the shrine?

3.17 A proposed mill in an isolated area can be furnished with power and water by a gravity-feed system. A stream high above the mill will be tapped to provide flow for water needs and power requirements by connecting it to the mill with a ditch-and-tunnel system or with a wood-and-concrete flume that winds its way down from the plateau. Either alternative will meet current and future needs, and both will utilize the same power-generating equipment.

The ditch-and-tunnel system will cost $500,000 with an annual maintenance cost of $2000. The flume has an initial cost of $200,000 and a yearly maintenance cost of $12,000. In addition, the wood portion of the flume will have to be replaced every 10 years at a cost of $100,000.

Compare the alternatives on the basis of capitalized costs with an interest rate of 6 percent.

3.18 A company is considering the purchase of a new piece of testing equipment that is expected to produce $8000 additional income during the first year of operation; this amount will probably decrease by $500 per year for each subsequent year of ownership. The equipment costs $20,000 and will have an estimated salvage value of $3000 after 8 years of use. For an interest rate of 15 percent, determine the net present worth of this investment. Neglect taxes in your computations.

3.19 To attract industry, a city has made an offer to a corporation. The city will install all roads and services for the plant site at no immediate cost to the corporation, but the corporation will be expected to bear the cost of operation and maintenance on a cost-sharing schedule whereby $65,000 is paid the first year of the project (2 years from now), each subsequent yearly payment will be $5000 less, and the corporation's obligation will end with the last $5000 payment. What is the present worth today of this agreement, if the annual interest rate is 9 percent?

3.20 A marina has two alternative plans for constructing a small-boat landing on a lake behind the sales building; one is a wooden dock, and the other is a metal-and-concrete wharf. Data for the two plans are as shown.

	Wood	Metal and concrete
First cost	$35,000	$55,000
Period before replacement	10 years	15 years
Salvage value	$5000	0
Annual maintenance	$6000	$3200

Using a minimum attractive rate of return of 10 percent, compare the present worths of the two plans. Assume that both will provide adequate service and that replacement costs will be the same as the original cost.

3.21 A refining company entered into a contract for raw materials with an agreement to pay $600,000 now and $150,000 per year beginning at the end of the fifth year. The contract was made for 10 years. At the end of the third year, because of unexpected profits, the company requested that it be allowed to make a lump-sum payment in advance for the rest of the contract. Both parties agreed that 7 percent compounded annually was a fair interest rate. What was the amount of the lump sum?

3.22 A machine can be repaired today for $2000. If repairs are not made, the operating expenses will increase by $200 each year for the next 5 years. Assume that the expenses will occur at the end of each year and that the machine will h value under either alternative at the end of the 5-year period. The mi acceptable rate of return is 12 percent. Compare the present worths of th alternatives.

3.23 The following alternatives are available to accomplish an objective o years' duration:

	Plan *A*	**Plan *B***	**Pla**
Life cycle	6 years	3 years	4 ye
First cost	\$2000	\$8000	\$10
Annual cost	\$3200	\$700	\$90

Compare the present worth of the alternatives, using an interest rate of 7 pe

3.24 The lining of a chemical tank must be replaced every 3 years at a c \$3500. A new type of lining is available that is more resistant to corrosion. The lining costs \$5100. If the minimum rate of return required is 12 percent and t and insurance are 4 percent of the first cost annually, how long must the impr lining last to be more economical than the present lining?

3.25 A single underground transmission circuit is needed immediately, and l studies indicate the need for a second circuit in 6 years. If provision is made f second conduit when the conduit for the first circuit is installed, there will b future need for reopening, trenching, backfilling, and repaving. The cos installing a single circuit with minimum preparation for the eventual second cir is \$850,000. The installation of the second circuit will be considered to \$800,000 at the end of year 6 in order to be in operation by the beginning of y 7. If the second circuit is installed immediately, the total cost will be \$1.4 milli

Constant annual operating and maintenance costs of the circuits are 8 perc of the first cost. The average life of a circuit is 20 years. The required rate of ret on such investments is 10 percent before taxes.

(*a*) Compare the deferred investment with the immediate investment, using 20-year study period.

(*b*) Compare the two conduit plans on an infinite study period. Then comp this solution with that in part *(a)*. Which is more reasonable? Why?

3.26 Autocon Company is evaluating three robots for possible use in its assem operations. (Only one robot will be purchased.) Data associated with these rob are as follows:

	Robot *A*	**Robot *B***	**Robot *C***
First cost, \$	55,000	58,000	53,000
Operating and maintenance costs, \$	3000/year	4500/year	4000/yea
Expected income, \$	40,000/year	44,000/year	38,000/ye
Estimated salvage value, \$	4000	6000	4000

Assuming a technological life of 3 years and a desired interest rate of 12 percent, which robot seems to be preferable, assuming all other factors are equal? Use a net present-worth evaluation.

3.27 What would the "net future" economic evaluations be, at the end of period 3, for the robots in Prob. 3.26? Would this future evaluation make much sense when presented to the manufacturing manager?

Note: First solve this problem *without* considering your PW results from Prob. 3.26. Next, directly use your PW results from Prob. 3.26 in verifying your FW solutions.

3.28 Suppose the technological lives for robots *A* and *B* in Prob. 3.26 are both 2 years while robot *C* still has a life of 3 years. Perform the economic evaluations, assuming that the salvage values of all three robots will be \$6000 after their useful lives. Also, robot first costs are expected to increase at a rate of 5 percent per year due to technological enhancements.

3.29 A wealthy industrial economist dies, and her will specifies that \$5 million of her estate will go to Arizona State University (ASU) to fund a small engineering economy building as well as 20 graduate scholarships per year over the next 20 years. The scholarships are to have a value of \$12,000 per year for the first year and should increase at a rate of \$1500 per year over the following 19 years. ASU requires that \$15,000, starting with the third year of the bequest, be reserved for building maintenance and operating costs. These costs are to have a linear increase of \$2000 per year, starting with year 4. Assuming that a 10 percent interest rate is used for such analyses, determine how much will be available for building first costs.

3.30 For the building specified in Prob. 3.29, determine the effect on potential first costs if the maintenance and operating costs increase by 12 percent per year instead of having a linear increase.

3.31 What is the maximum amount that you could afford to bid for a bond with a face value of \$5000 and a coupon rate of 8 percent payable semiannually, if your minimum attractive rate of return is 10 percent? The bond matures in 6 years.

3.32 Swampwater Flood Control District is selling 20-year tax-free bonds with a face value of \$2 million and a stated yield of 6 percent with semiannual interest payments. Printing and legal costs associated with issuing the bonds have totaled \$50,000, and costs associated with paying the dividends are expected to be \$1000 per period. What is the minimum amount the bonds must sell for such that the utility district's cost of capital does not exceed a nominal 8 percent per annum?

3.33 A bond with a face value of \$5000 pays quarterly interest of $1\frac{1}{2}$ percent each period. Twenty-six interest payments remain before the bond matures. How much would you be willing to pay for this bond today, if the next interest payment is due now and you want to earn 8 percent compounded quarterly on your money?

3.34 Bonds of Overightors Corporation are perpetuities bearing 7 percent annual interest. Their par value is \$1000.

(*a*) If bonds of this type currently are expected to yield 6 percent, what is the market price?

(*b*) If interest rates rise to the level at which comparable bonds return a yield of 8 percent, what would the market price be for Overightor bonds?

(*c*) How would the prices change in parts *(a)* and *(b)* if the bonds had a definite maturity date in 20 years?