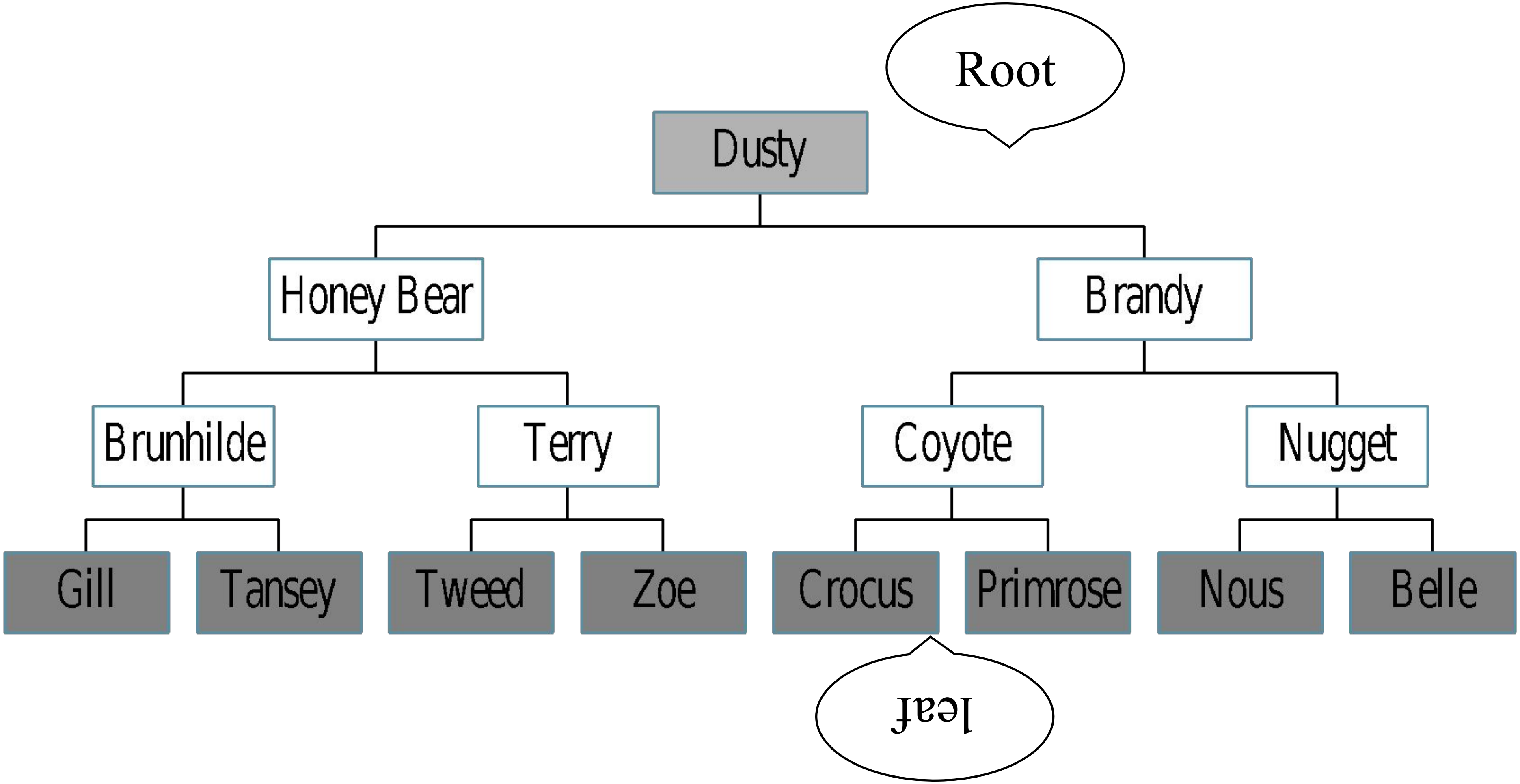
CHAPTER 5

Trees

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed
"Fundamentals of Data Structures in C",
Computer Science Press, 1992.

Trees

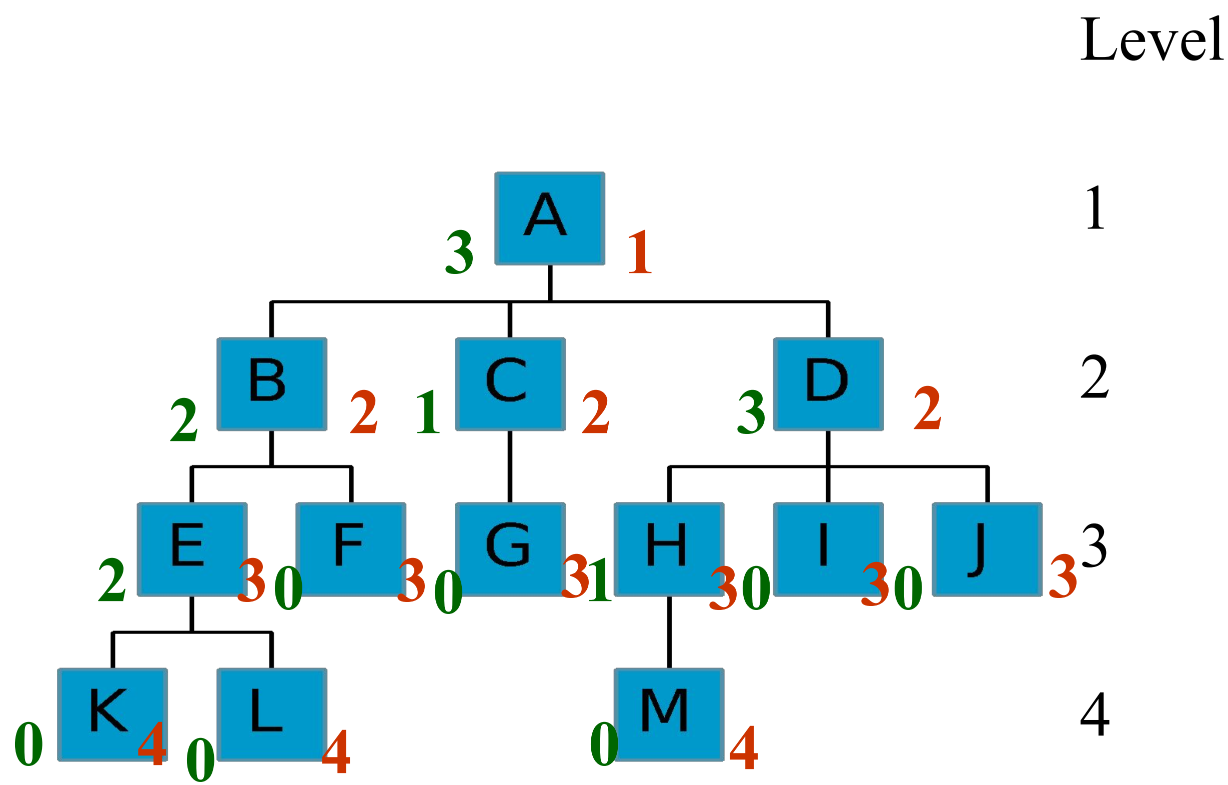

Root
Dusty
Honey Bear
Brandy
Brunhilde
Terry
Coyote
Nugget
Gill
Tansey
Tweed
Zoe
Crocus
Primrose
Nous
Belle
leaf

Definition of Tree

- A tree is a finite set of one or more nodes such that:
- There is a specially designated node called the root.
- The remaining nodes are partitioned into n>=0 disjoint sets $T_1, \ldots, T_n$, where each of these sets is a tree.
- We call $T_1, \ldots, T_n$ the subtrees of the root.

Level and Depth

node (13)
degree of a node
leaf (terminal)
nonterminal
parent
children
sibling
degree of a tree (3)
ancestor
level of a node
height of a tree (4)



Terminology

- The degree of a node is the number of subtrees of the node
  – The degree of A is 3; the degree of C is 1.
- The node with degree 0 is a leaf or terminal node.
- A node that has subtrees is the *parent* of the roots of the subtrees.
- The roots of these subtrees are the *children* of the node.
- Children of the same parent are *siblings*.
- The ancestors of a node are all the nodes along the path from the root to the node.

Representation of Trees

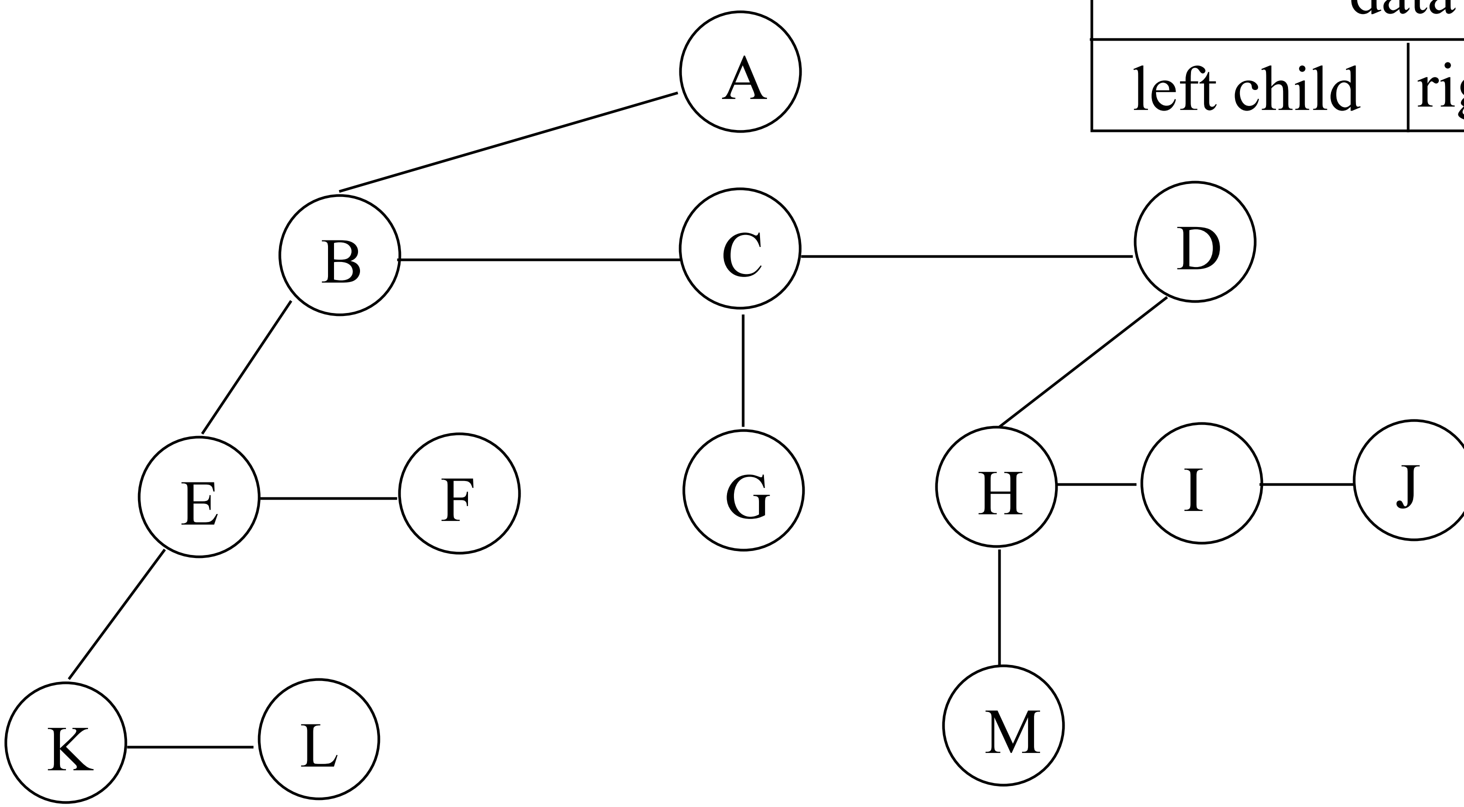
- List Representation
  - (A (B (E (K, L), F), C (G), D (H (M), I, J)))
  - The root comes first, followed by a list of sub-trees

data	link 1	link 2	...	link n

How many link fields are needed in such a representation?

Left Child - Right Sibling

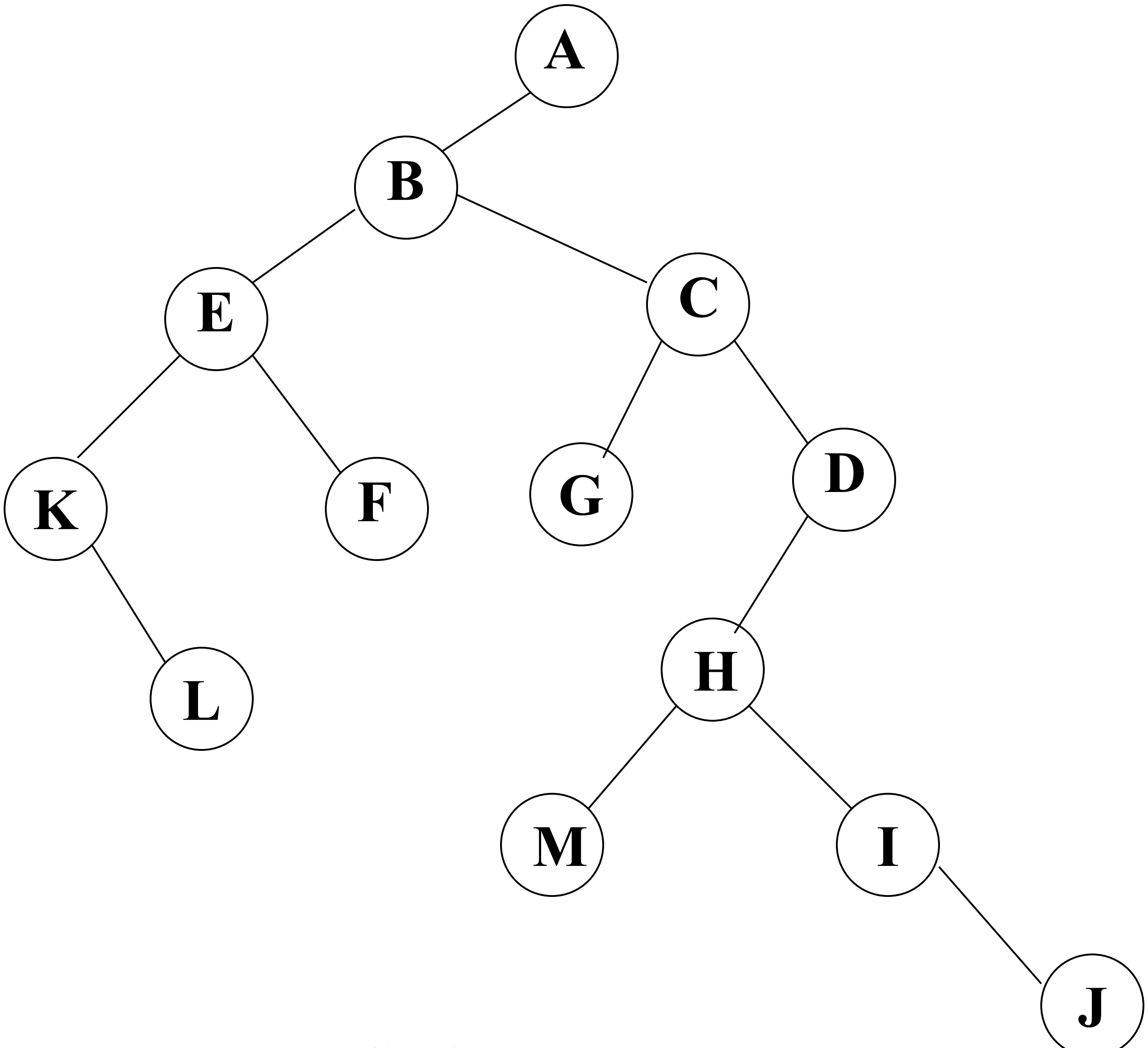
data	
left child	right sibling



Binary Trees

- A binary tree is a finite set of nodes that is either empty or consists of a root and two disjoint binary trees called *the left subtree* and *the right subtree*.
- Any tree can be transformed into binary tree.
  - by left child-right sibling representation
- The left subtree and the right subtree are distinguished.

***Figure 5.6:** Left child-right child tree representation of a tree (p.191)



Abstract Data Type Binary_Tree

structure *Binary_Tree*(abbreviated *BinTree*) is

objects: a finite set of nodes either empty or consisting of a root node, left *Binary_Tree*, and right *Binary_Tree*.

functions:

for all *bt*, *bt1*, *bt2* $\in$ *BinTree*, *item* $\in$ *element*

Bintree Create()::= creates an empty binary tree

Boolean IsEmpty(*bt*)::= if (*bt*==empty binary tree) return *TRUE* else return *FALSE*

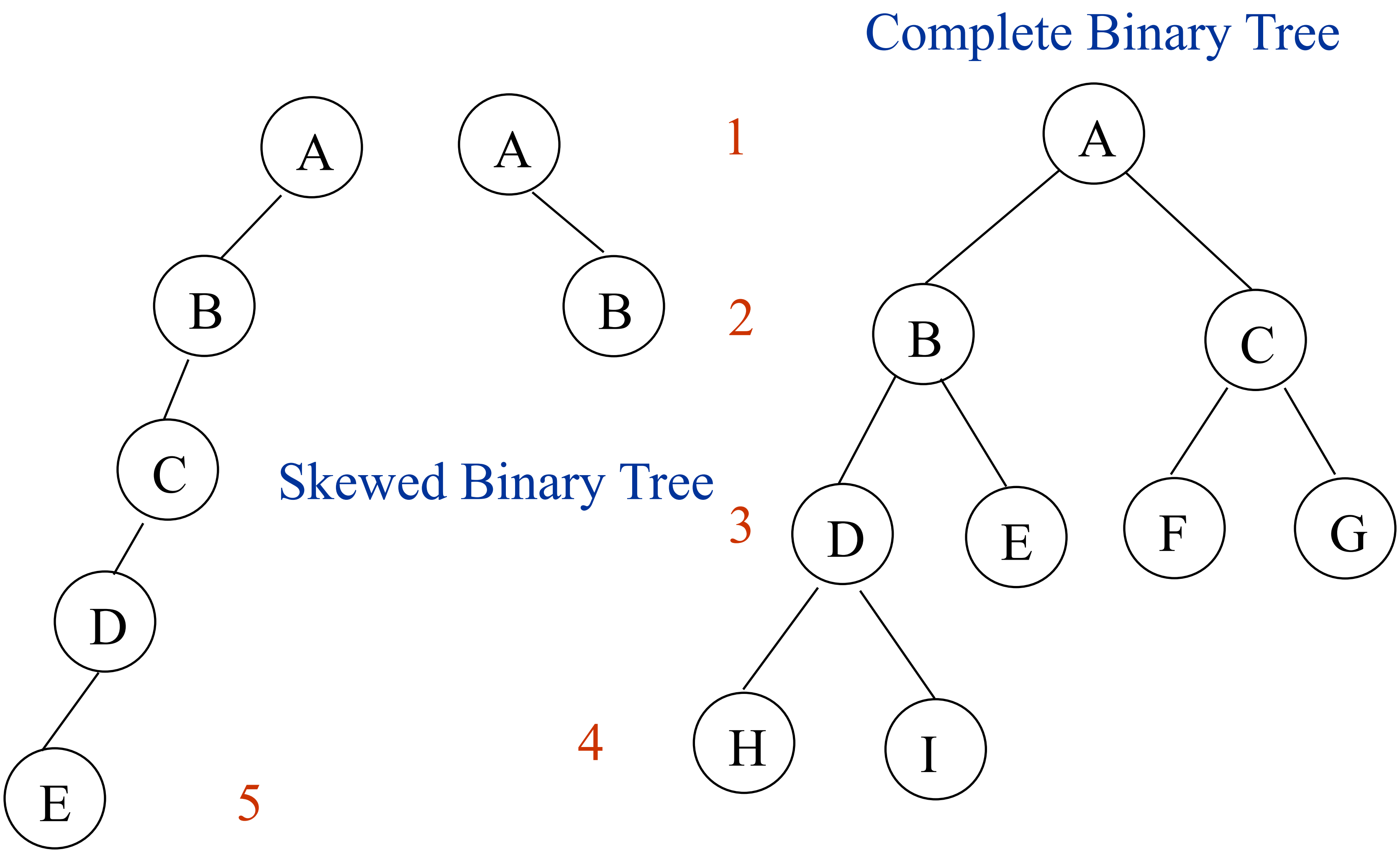
BinTree MakeBT(*bt1*, *item*, *bt2*)::= return a binary tree
whose left subtree is *bt1*, whose right subtree is *bt2*,
and whose root node contains the data *item*

Bintree Lchild(*bt*)::= if (IsEmpty(*bt*)) return error
else return the left subtree of *bt*

element Data(*bt*)::= if (IsEmpty(*bt*)) return error
else return the data in the root node of *bt*

Bintree Rchild(*bt*)::= if (IsEmpty(*bt*)) return error
else return the right subtree of *bt*

Samples of Trees



Maximum Number of Nodes in BT

- The maximum number of nodes on level i of a binary tree is 2^{i-1}, $i>=1$.
- The maximum nubmer of nodes in a binary tree of depth k is 2^k-1, $k>=1$.

i-1

Prove by induction.

$$\sum_{i=1}^{k} 2^{i-1} = 2^k - 1$$

Relations between Number of Leaf Nodes and Nodes of Degree 2

For any nonempty binary tree, T, if n_0 *is the number of leaf nodes and* n_2 *the number of nodes of degree 2, then* $n_0 = n_2 + 1$

proof:

Let n and B denote the total number of nodes & branches in T.

Let n_0, n_1, n_2 represent the nodes with no children, single child, and two children respectively.

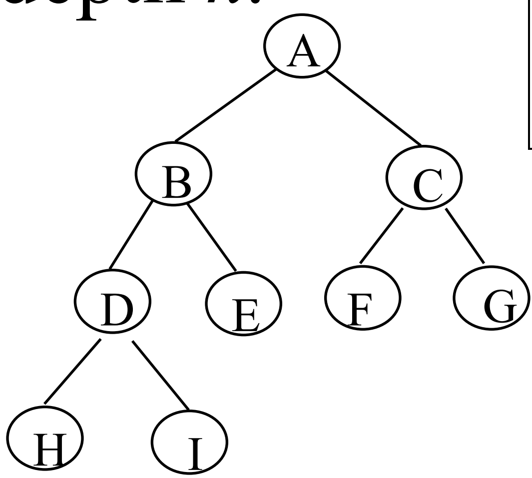
$n = n_0 + n_1 + n_2$, $B + 1 = n$, $B = n_1 + 2n_2 \Longrightarrow n_1 + 2n_2 + 1 = n$

$n_1 + 2n_2 + 1 = n_0 + n_1 + n_2 \Longrightarrow n_0 = n_2 + 1$

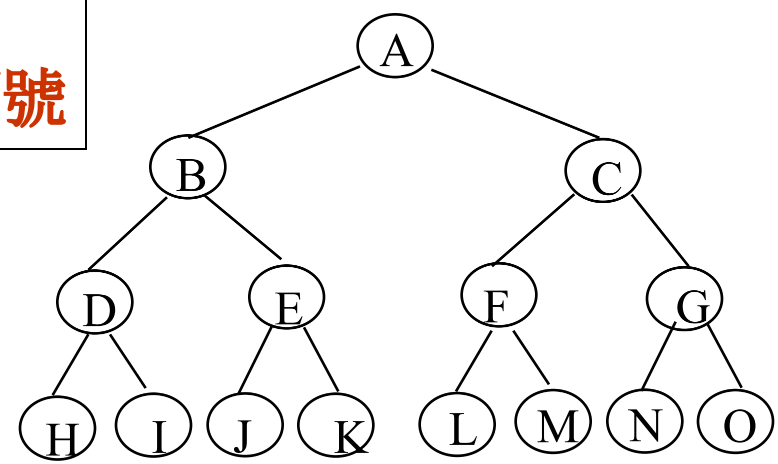
Full BT VS Complete BT

- A full binary tree of depth k is a binary tree of depth k having 2^k-1 nodes, k>=0.
- A binary tree with n nodes and depth k is complete *iff* its nodes correspond to the nodes numbered from 1 to n in the full binary tree of depth k.

由上至下，
由左至右編號



Complete binary tree

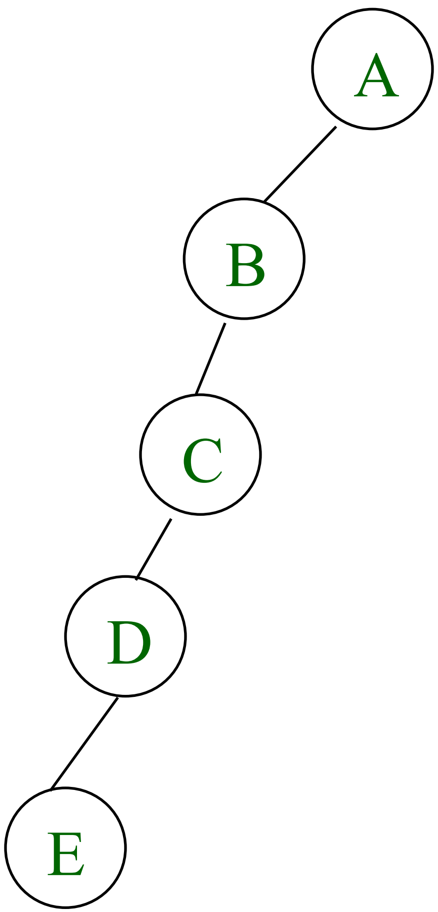


Full binary tree of depth 4

Binary Tree Representations

- If a complete binary tree with n nodes (depth = log n + 1) is represented sequentially, then for any node with index i, $1<=i<=n$, we have:
  - *parent*(i) is at $i/2$ if $i!=1$. If $i=1$, i is at the root and has no parent.
  - *left_child*(i) ia at $2i$ if $2i<=n$. If $2i>$n, then i has no left child.
  - *right_child*(i) ia at $2i+1$ if $2i+1<=n$. If $2i+1>$n, then i has no right child.

Sequential Representation

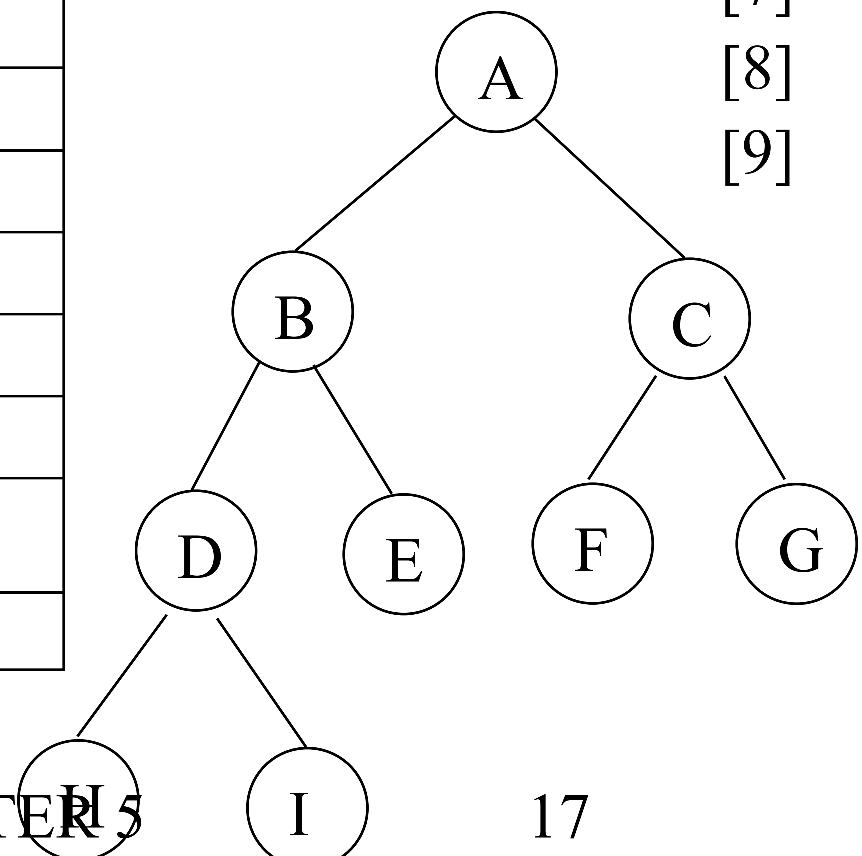


[1]	A
[2]	B
[3]	--
[4]	C
[5]	--
[6]	--
[7]	--
[8]	D
[9]	--
.	.
[16]	E

(1) waste space

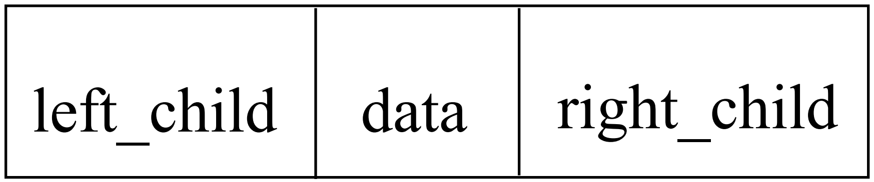
(2) insertion/deletion problem

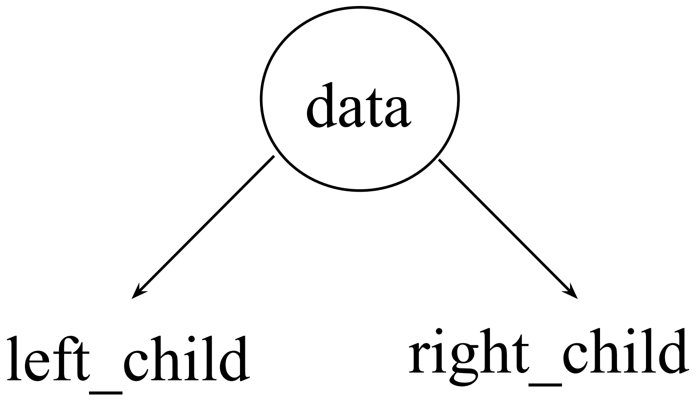
[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I



Linked Representation

```
typedef struct node *tree_pointer;
typedef struct node {
 int data;
 tree_pointer left_child, right_child;
};
```

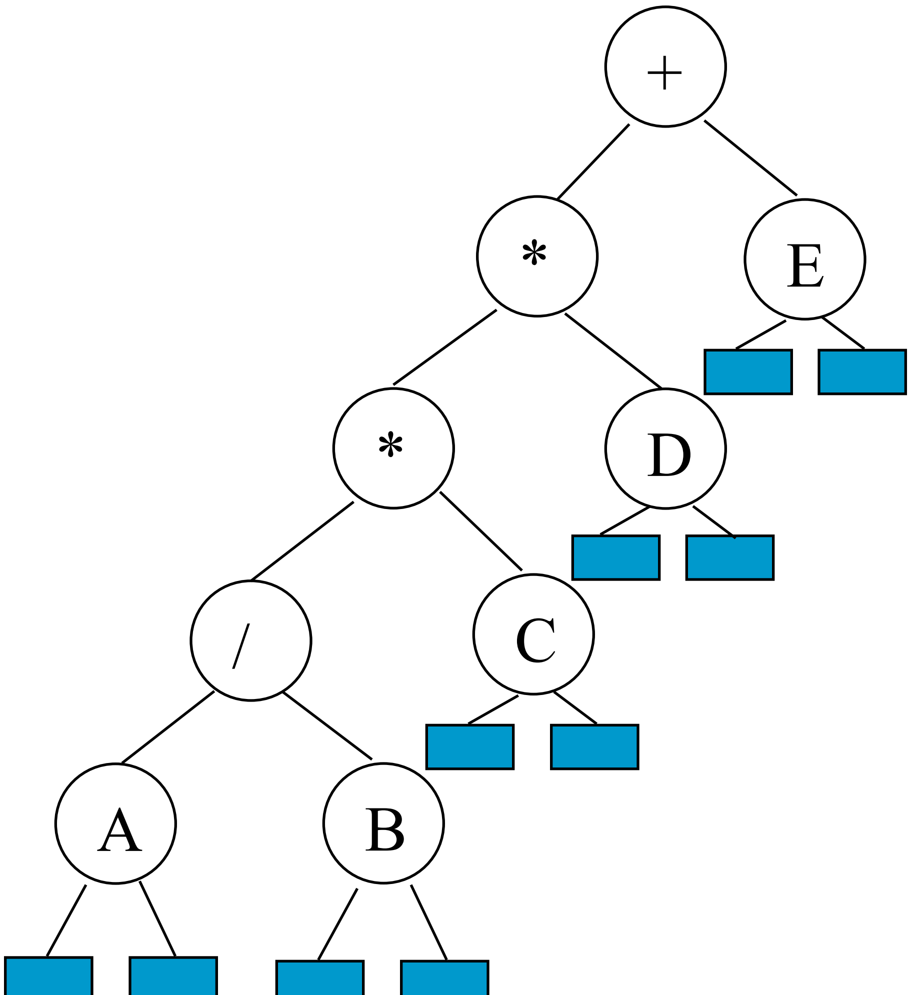




Binary Tree Traversals

- Let L, V, and R stand for moving left, visiting the node, and moving right.
- There are six possible combinations of traversal
  - LVR, LRV, VLR, VRL, RVL, RLV
- Adopt convention that we traverse left before right, only 3 traversals remain
  - LVR, LRV, VLR
  - inorder, postorder, preorder

Arithmetic Expression Using BT



inorder traversal

A / B * C * D + E

infix expression

preorder traversal

+ * * / A B C D E

prefix expression

postorder traversal

A B / C * D * E +

postfix expression

level order traversal

+ * E * D / C A B

Inorder Traversal (recursive version)

```
void inorder(tree_pointer ptr)
/* inorder tree traversal */
{
    if (ptr) {
        inorder(ptr->left_child);
        printf("%d", ptr->data);
        indorder(ptr->right_child);
    }
}
```

A / B * C * D + E

Preorder Traversal (recursive version)

```
void preorder(tree_pointer ptr)
/* preorder tree traversal */
{
    if (ptr) {
        printf("%d", ptr->data);
        preorder(ptr->left_child);
        predorder(ptr->right_child);
    }
}
```

+ * * / A B C D E

Postorder Traversal (recursive version)

```
void postorder(tree_pointer ptr)
/* postorder tree traversal */
{
    if (ptr) {
        postorder(ptr->left_child);
        postdorder(ptr->right_child);
        printf("%d", ptr->data);
    }
}
```

A B / C * D * E +

Iterative Inorder Traversal

(using stack)

```
void iter_inorder(tree_pointer node)
{
  int top= -1; /* initialize stack */
  tree_pointer stack[MAX_STACK_SIZE];
  for (;;) {
   for (; node; node=node->left_child)
     add(&top, node);/* add to stack */
   node= delete(&top);
                 /* delete from stack */
   if (!node) break; /* empty stack */
   printf("%D", node->data);
   node = node->right_child;
  }
}
```

O(n)

Trace Operations of Inorder Traversal

Call of inorder	Value in root	Action	Call of inorder	Value in root	Action
1	+		11	C	
2	*		12	NULL	
3	*		11	C	printf
4	/		13	NULL	
5	A		2	*	printf
6	NULL		14	D	
5	A	printf	15	NULL	
7	NULL		14	D	printf
4	/	printf	16	NULL	
8	B		1	+	printf
9	NULL		17	E	
8	B	printf	18	NULL	
10	NULL		17	E	printf
3	*	printf	19	NULL	

Level Order Traversal

(using queue)

```
void level_order(tree_pointer ptr)
/* level order tree traversal */
{
  int front = rear = 0;
  tree_pointer queue[MAX_QUEUE_SIZE];
  if (!ptr) return; /* empty queue */
  addq(front, &rear, ptr);
  for (;;) {
    ptr = deleteq(&front, rear);
```

```
    if (ptr) {
      printf("%d", ptr->data);
      if (ptr->left_child)
        addq(front, &rear,
                        ptr->left_child);
      if (ptr->right_child)
        addq(front, &rear,
                        ptr->right_child);
    }
    else break;
  }
}
```

+ * E * D / C A B

Copying Binary Trees

```
tree_poointer copy(tree_pointer original)
{
tree_pointer temp;
if (original) {
 temp=(tree_pointer) malloc(sizeof(node));
 if (IS_FULL(temp)) {
   fprintf(stderr, "the memory is full\n");
   exit(1);
 }
 temp->left_child=copy(original->left_child);
 temp->right_child=copy(original->right_child)
 temp->data=original->data;
 return temp;
}
return NULL;
}
```

postorder

Equality of Binary Trees

the same topology and data

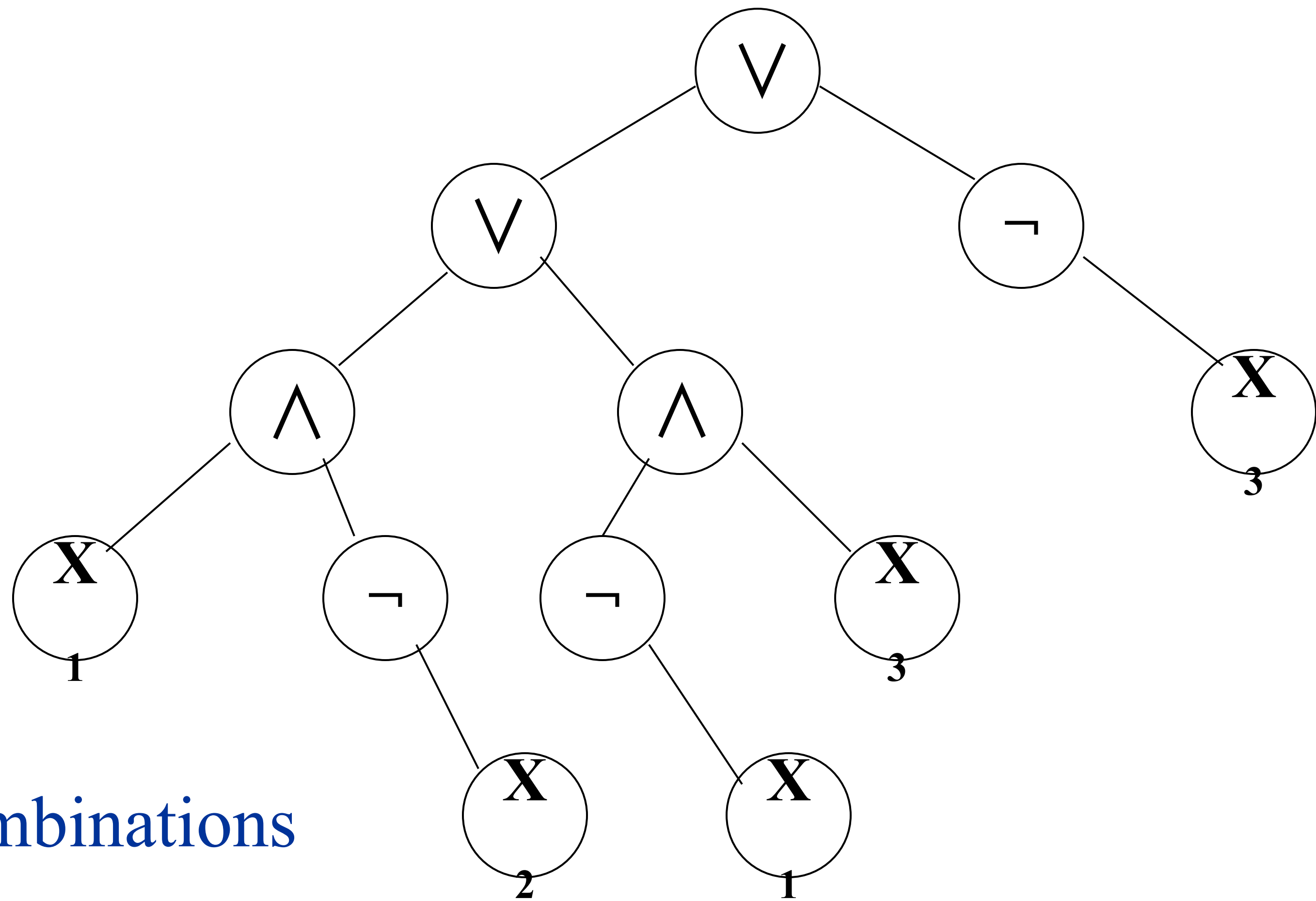
```
int equal(tree_pointer first, tree_pointer second)
{
/* function returns FALSE if the binary trees first and
   second are not equal, otherwise it returns TRUE */

  return ((!first && !second) || (first && second &&
       (first->data == second->data) &&
       equal(first->left_child, second->left_child) &&
       equal(first->right_child, second->right_child))
}
```

Propositional Calculus Expression

- A variable is an expression.
- If x and y are expressions, then $\neg x$, $x \wedge y$, $x \vee y$ are expressions.
- Parentheses can be used to alter the normal order of evaluation ($\neg > \wedge > \vee$).
- Example: $x_1 \vee (x_2 \wedge \neg x_3)$
- satisfiability problem: Is there an assignment to make an expression true?

$(x_1 \wedge \neg x_2) \vee (\neg x_1 \wedge x_3) \vee \neg x_3$

(t,t,t)
(t,t,f)
(t,f,t)
(t,f,f)
(f,t,t)
(f,t,f)
(f,f,t)
(f,f,f)



2^n possible combinations for n variables

postorder traversal (postfix evaluation)

node structure

left_child	***data***	***value***	***right_child***

```
typedef emun {not, and, or, true, false } logical;
typedef struct node *tree_pointer;
typedef struct node {
        tree_pointer  list_child;
        logical       data;
        short int     value;
        tree_pointer  right_child;
        } ;
```

First version of satisfiability algorithm

```
for (all 2^n possible combinations) {
      generate the next combination;
      replace the variables by their values;
      evaluate root by traversing it in postorder;
      if (root->value) {
            printf(<combination>);
            return;
   }
}
printf("No satisfiable combination \n");
```

Post-order-eval function

```
void post_order_eval(tree_pointer node)
{
/* modified post order traversal to evaluate a propositional
calculus tree */
    if (node) {
        post_order_eval(node->left_child);
        post_order_eval(node->right_child);
        switch(node->data) {
          case not:  node->value =
                  !node->right_child->value;
                  break;
```

```
case and:     node->value =
          node->right_child->value &&
          node->left_child->value;
          break;
    case or:        node->value =
          node->right_child->value | |
          node->left_child->value;
          break;
     case true:    node->value = TRUE;
          break;
     case false:  node->value = FALSE;
    }
  }
}
```

Threaded Binary Trees

- Two many null pointers in current representation of binary trees

  n: number of nodes
  number of non-null links: n-1
  total links: 2n
  null links: 2n-(n-1)=n+1

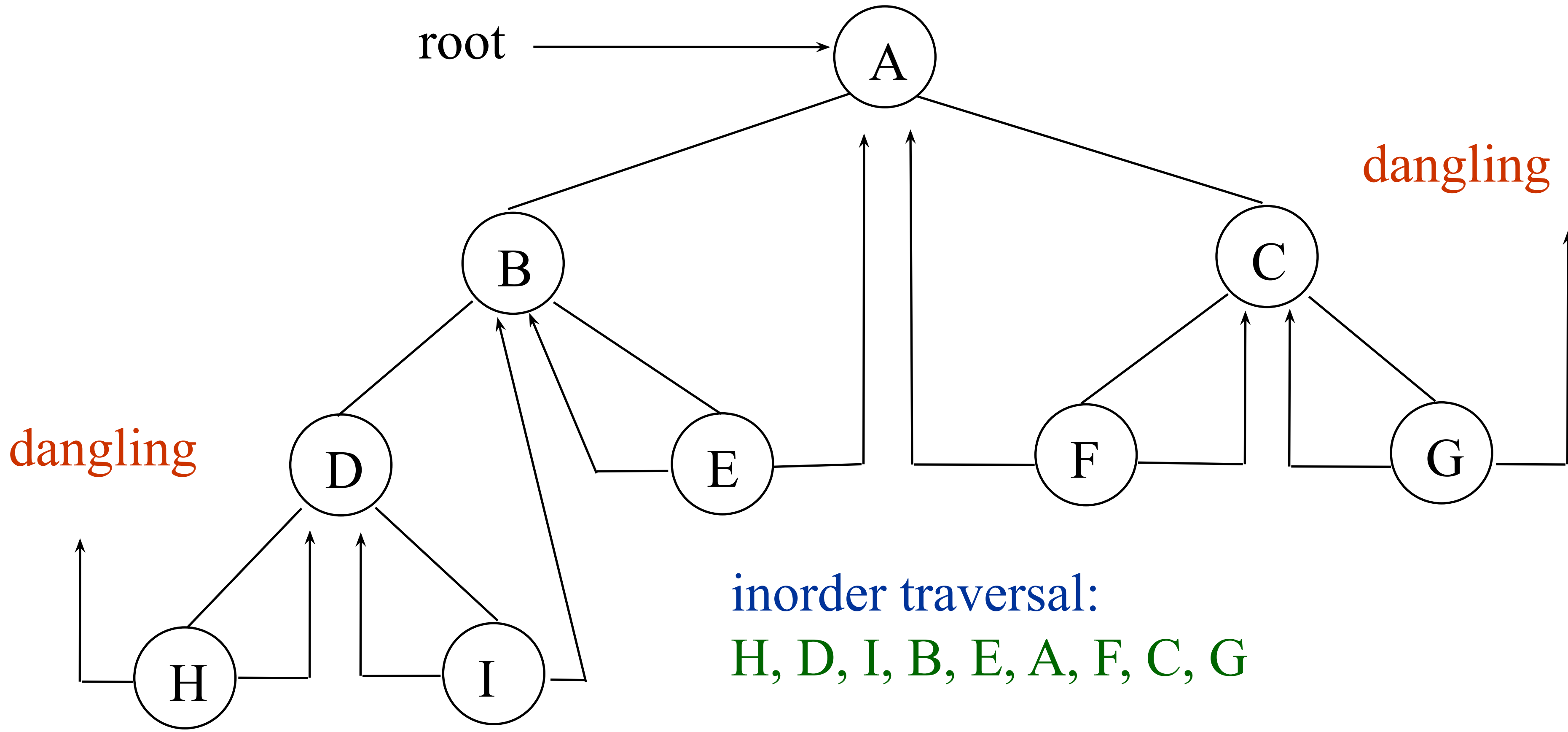
- Replace these null pointers with some useful "threads".

Threaded Binary Trees (*Continued*)

If `ptr->left_child` is null,
replace it with a pointer to the node that would be visited *before* `ptr` in an *inorder traversal*

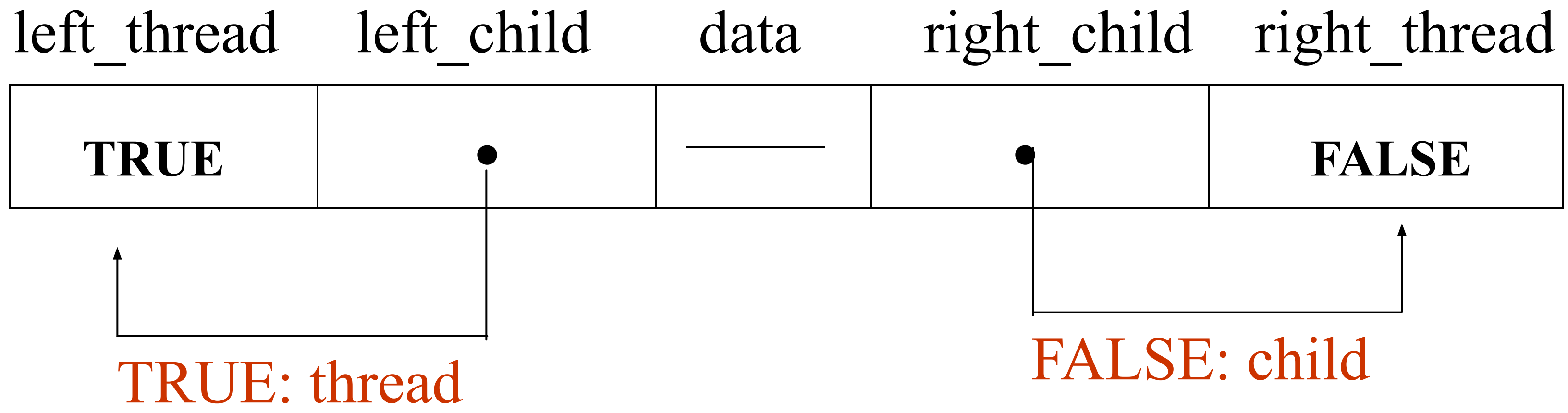
If `ptr->right_child` is null,
replace it with a pointer to the node that would be visited *after* `ptr` in an *inorder traversal*

A Threaded Binary Tree



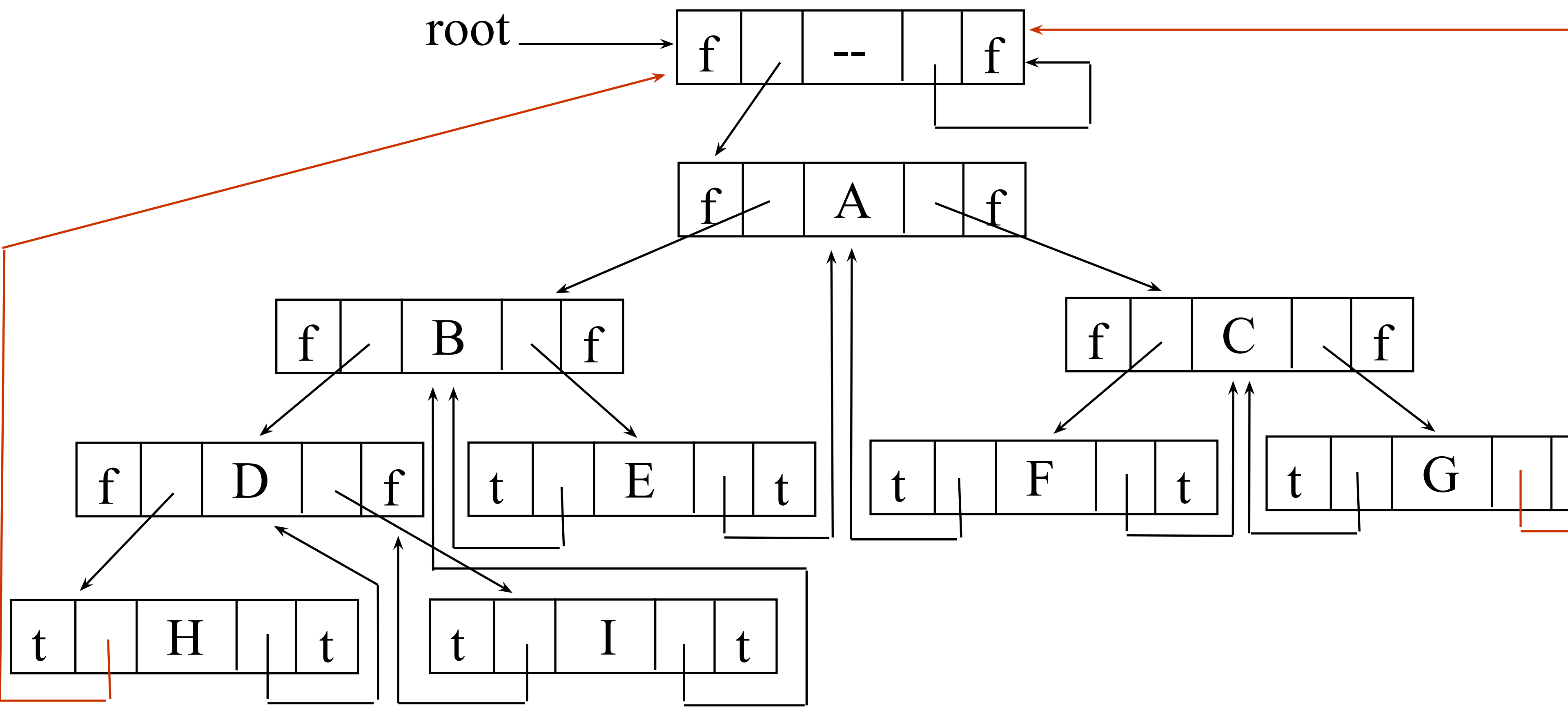
inorder traversal:
H, D, I, B, E, A, F, C, G

Data Structures for Threaded BT



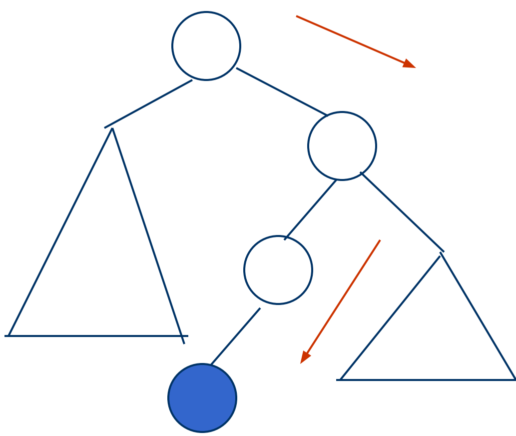

```
typedef struct threaded_tree
  *threaded_pointer;
typedef struct threaded_tree {
    short int left_thread;
    threaded_pointer left_child;
    char data;
    threaded_pointer right_child;
    short int right_thread; };
```

Memory Representation of A Threaded BT



Next Node in Threaded BT

```
threaded_pointer insucc(threaded_pointer
  tree)
{
  threaded_pointer temp;
  temp = tree->right_child;
  if (!tree->right_thread)
    while (!temp->left_thread)
      temp = temp->left_child;
  return temp;
}
```



Inorder Traversal of Threaded BT

```
void tinorder(threaded_pointer tree)
{
/* traverse the threaded binary tree
  inorder */
    threaded_pointer temp = tree;
    for (;;) {
        temp = insucc(temp);
O(n)    if (temp==tree) break;
        printf("%3c", temp->data);
    }
}
```

Inserting Nodes into Threaded BTs

- Insert `child` as the right child of node `parent`
  - change `parent->right_thread` to FALSE
  - set `child->left_thread` and `child->right_thread` to TRUE
  - set `child->left_child` to point to `parent`
  - set `child->right_child` to `parent->right_child`
  - change `parent->right_child` to point to `child`

Examples

Insert a node D as a right child of B.

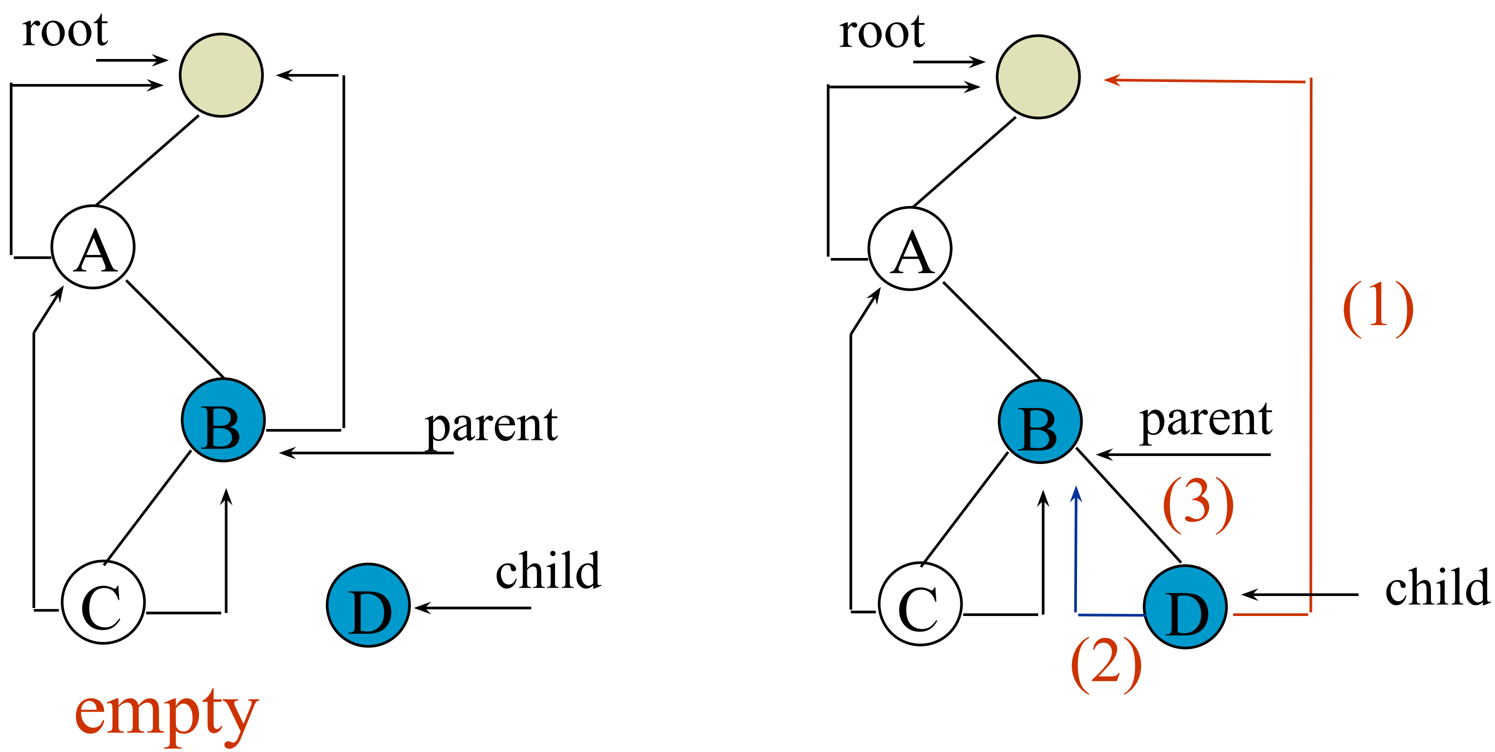
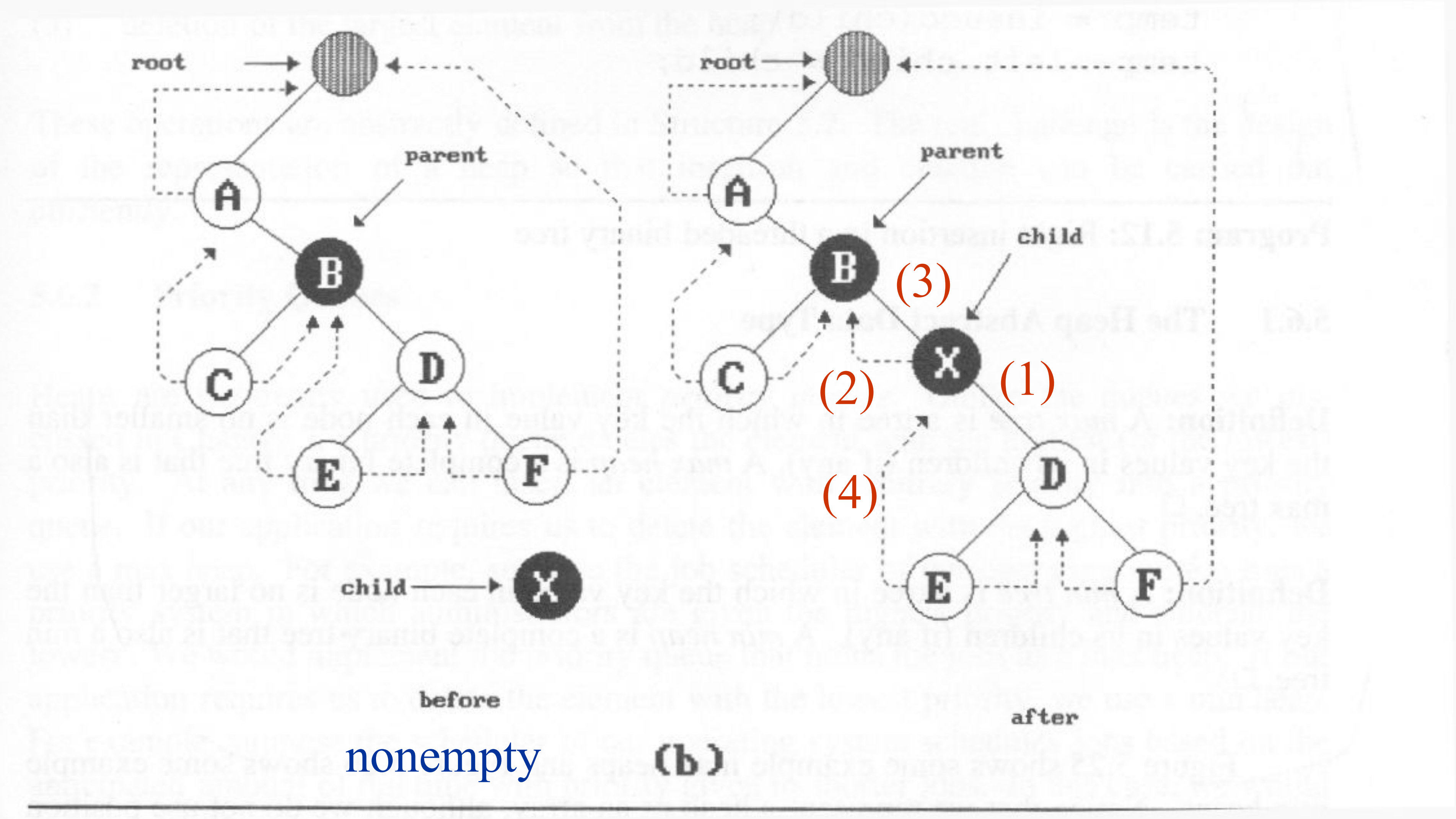


***Figure 5.24:** Insertion of child as a right child of parent in a threaded binary tree (p.217)*



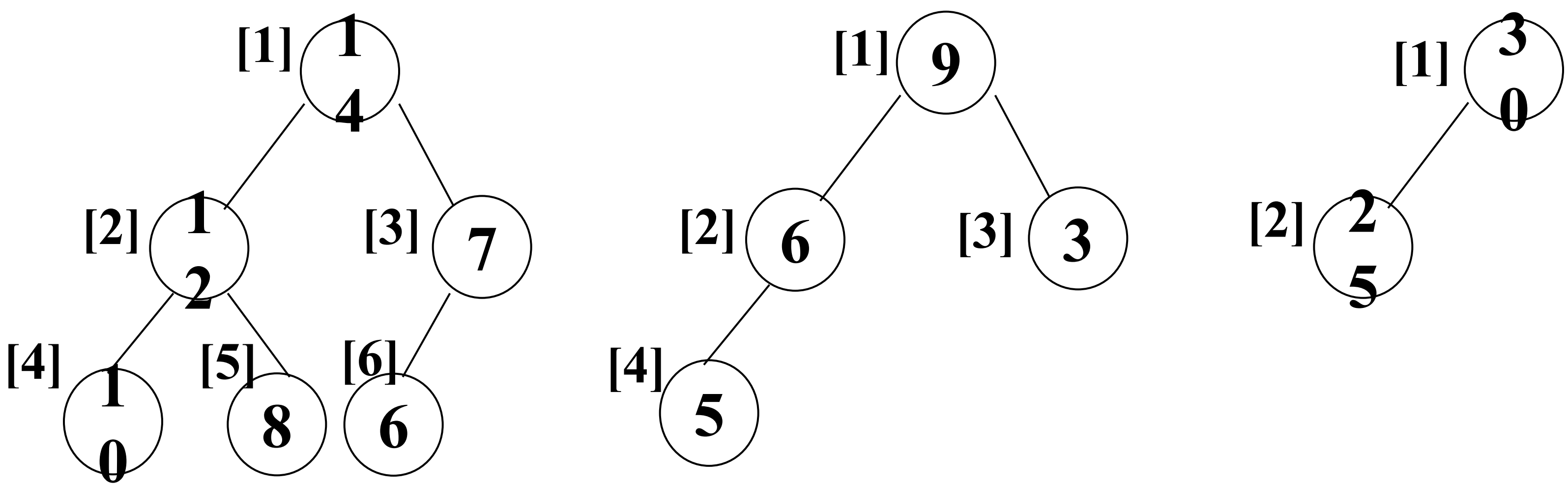
Right Insertion in Threaded BTs

```
void insert_right(threaded_pointer parent,
                  threaded_pointer child)
{
    threaded_pointer temp;
    child->right_child = parent->right_child;
(1) child->right_thread = parent->right_thread;
    child->left_child = parent;   case (a)
(2) child->left_thread = TRUE;
    parent->right_child = child;
(3) parent->right_thread = FALSE;
    if (!child->right_thread) { case (b)
      temp = insucc(child);
(4)   temp->left_child = child;
    }
}
```

Heap

- A *max tree* is a tree in which the key value in each node is no smaller than the key values in its children. A *max heap* is a complete binary tree that is also a max tree.
- A *min tree* is a tree in which the key value in each node is no larger than the key values in its children. A *min heap* is a complete binary tree that is also a min tree.
- Operations on heaps
  – creation of an empty heap
  – insertion of a new element into the heap;
  – deletion of the largest element from the heap

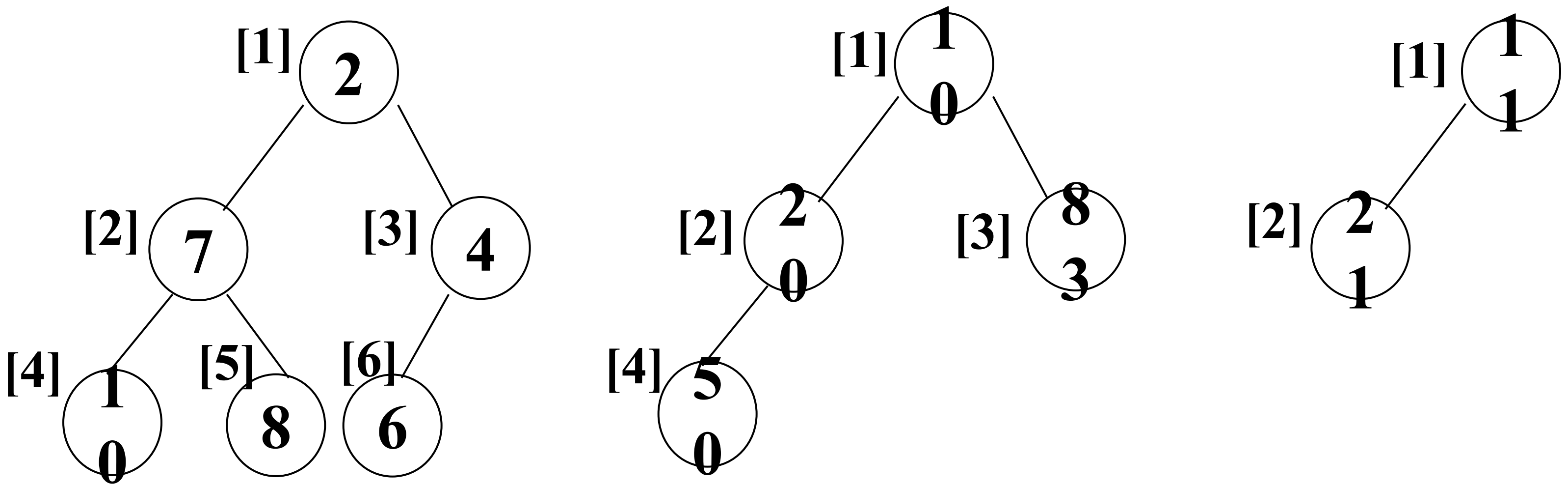
***Figure 5.25:** Sample max heaps (p.219)



Property:

The root of max heap (min heap) contains the largest (smallest).

***Figure 5.26:**Sample min heaps (p.220)



ADT for Max Heap

structure MaxHeap

objects: a complete binary tree of n > 0 elements organized so that the value in each node is at least as large as those in its children

functions:

for all *heap* belong to *MaxHeap*, *item* belong to *Element*, *n*, *max_size* belong to integer

MaxHeap Create(max_size)::= create an empty heap that can hold a maximum of max_size elements

Boolean HeapFull(heap, n)::= if (n==max_size) return TRUE
else return FALSE

MaxHeap Insert(heap, item, n)::= if (!HeapFull(heap,n)) insert item into heap and return the resulting heap
else return error

Boolean HeapEmpty(heap, n)::= if (n>0) return FALSE
else return TRUE

Element Delete(heap,n)::= if (!HeapEmpty(heap,n)) return one instance of the largest element in the heap and remove it from the heap
else return error

Application: priority queue

- machine service
  - amount of time (min heap)
  - amount of payment (max heap)
- factory
  - time tag

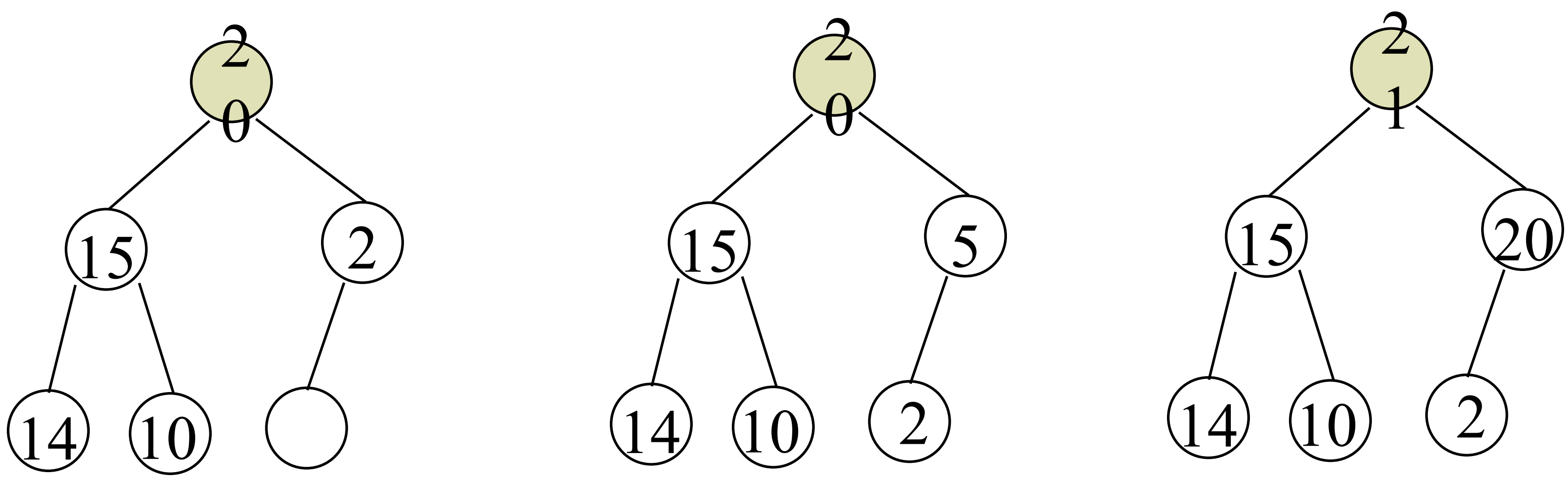
Data Structures

- unordered linked list
- unordered array
- sorted linked list
- sorted array
- heap

***Figure 5.27:** Priority queue representations (p.221)*

Representation	Insertion	Deletion
Unordered array	$\Theta(1)$	$\Theta(n)$
Unordered linked list	$\Theta(1)$	$\Theta(n)$
Sorted array	O(n)	$\Theta(1)$
Sorted linked list	O(n)	$\Theta(1)$
Max heap	$O(\log_2 n)$	$O(\log_2 n)$

Example of Insertion to Max Heap



initial location of new node   insert 5 into heap   insert 21 into heap

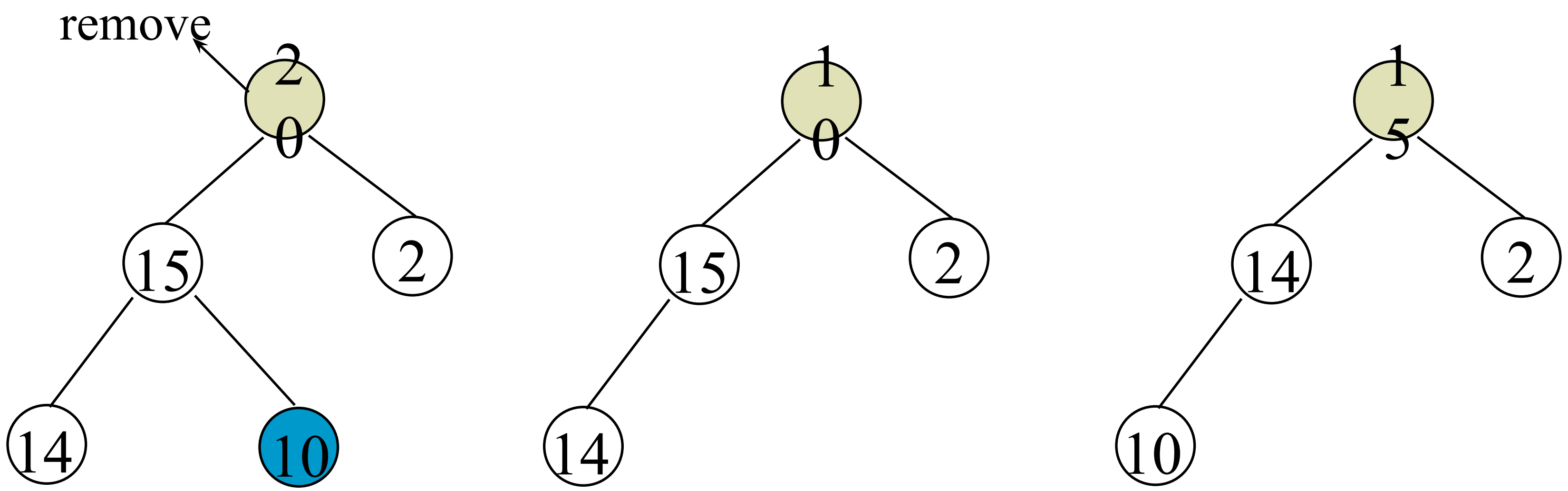
Insertion into a Max Heap

```
void insert_max_heap(element item, int *n)
{
  int i;
  if (HEAP_FULL(*n)) {
    fprintf(stderr, "the heap is full.\n");
    exit(1);
  }
  i = ++(*n);
  while ((i!=1)&&(item.key>heap[i/2].key)) {
    heap[i] = heap[i/2];
    i /= 2;
  }
  heap[i]= item;
}
```

$2^k-1=n \implies k=\lceil \log_2(n+1) \rceil$

$O(\log_2 n)$

Example of Deletion from Max Heap



Deletion from a Max Heap

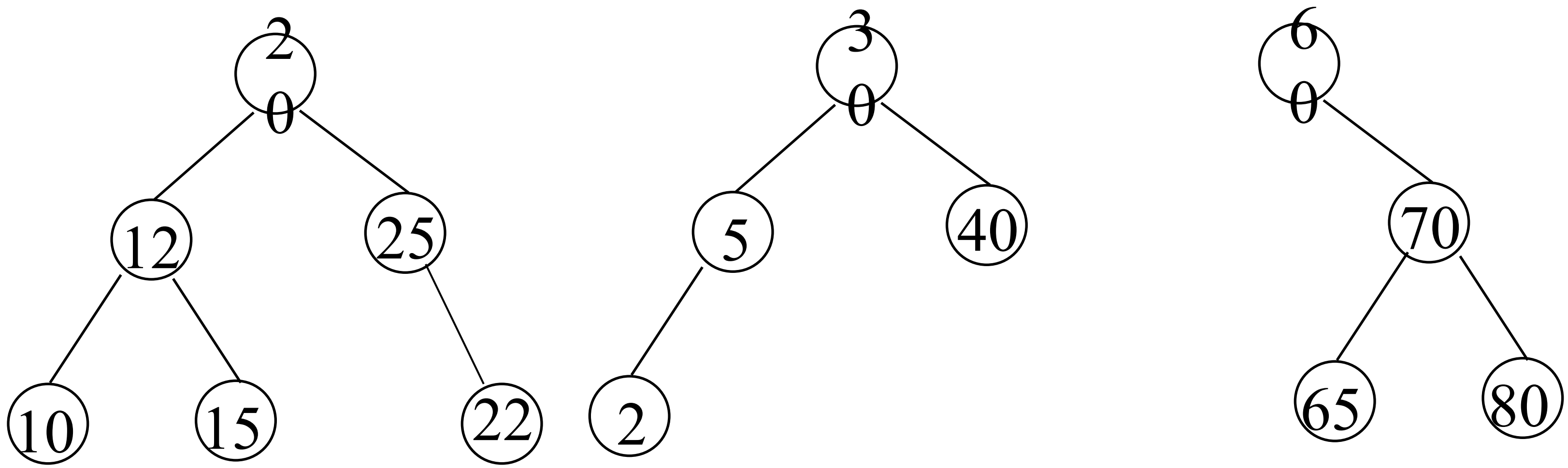
```
element delete_max_heap(int *n)
{
  int parent, child;
  element item, temp;
  if (HEAP_EMPTY(*n)) {
    fprintf(stderr, "The heap is empty\n");
    exit(1);
  }
  /* save value of the element with the
     highest key */
  item = heap[1];
  /* use last element in heap to adjust heap
  temp = heap[(*n)--];
  parent = 1;
  child = 2;
```

```
while (child <= *n) {
    /* find the larger child of the current
       parent */
    if ((child < *n)&&

(heap[child].key<heap[child+1].key))
      child++;
    if (temp.key >= heap[child].key) break;
    /* move to the next lower level */
    heap[parent] = heap[child];
    child *= 2;
  }
  heap[parent] = temp;
  return item;
}
```

Binary Search Tree

- Heap
  - a min (max) element is deleted. $O(\log_2 n)$
  - deletion of an arbitrary element $O(n)$
  - search for an arbitrary element $O(n)$
- Binary search tree
  - Every element has a unique key.
  - The keys in a nonempty left subtree (right subtree) are smaller (larger) than the key in the root of subtree.
  - The left and right subtrees are also binary search trees.

Examples of Binary Search Trees



Searching a Binary Search Tree

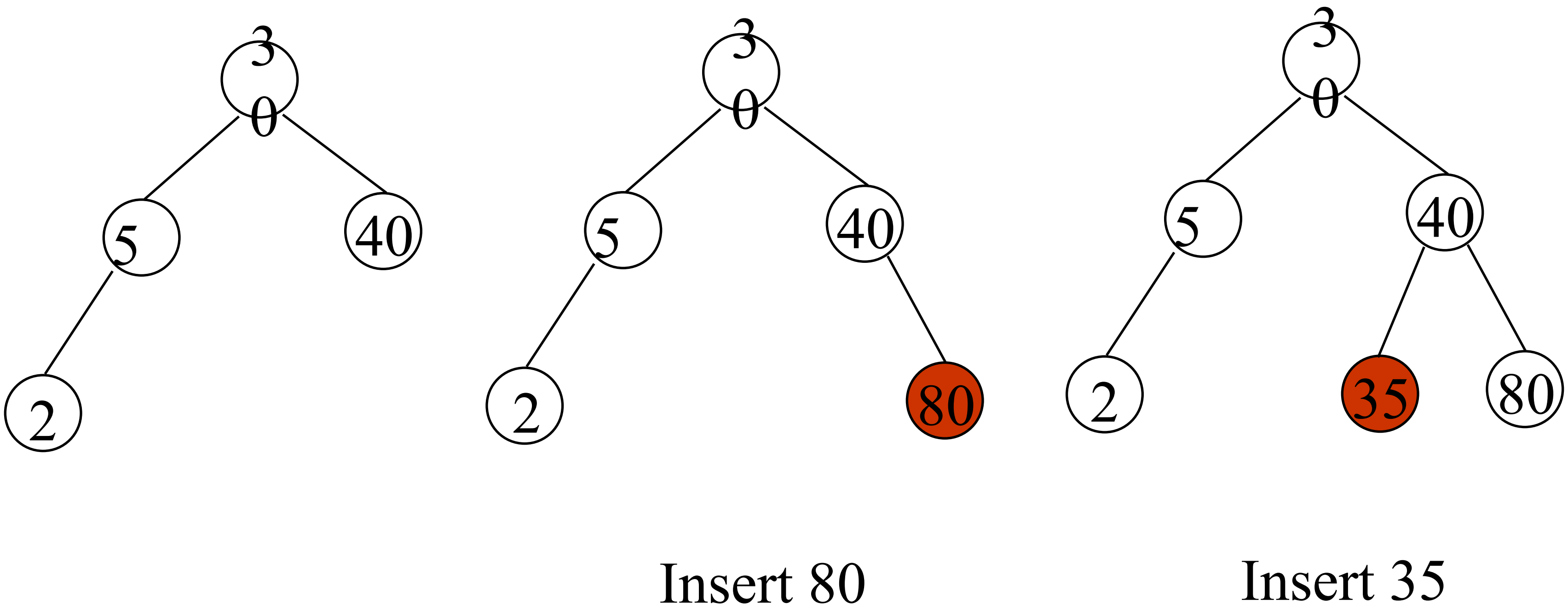
```
tree_pointer search(tree_pointer root,
               int key)
{
/* return a pointer to the node that
  contains key. If there is no such
  node, return NULL */

   if (!root) return NULL;
   if (key == root->data) return root;
   if (key < root->data)
       return search(root->left_child,
                    key);
   return search(root->right_child,key);
}
```

Another Searching Algorithm

```
tree_pointer search2(tree_pointer tree,
  int key)
{
  while (tree) {
    if (key == tree->data) return tree;
    if (key < tree->data)
        tree = tree->left_child;
    else tree = tree->right_child;
  }
  return NULL;
}
```

O(h)

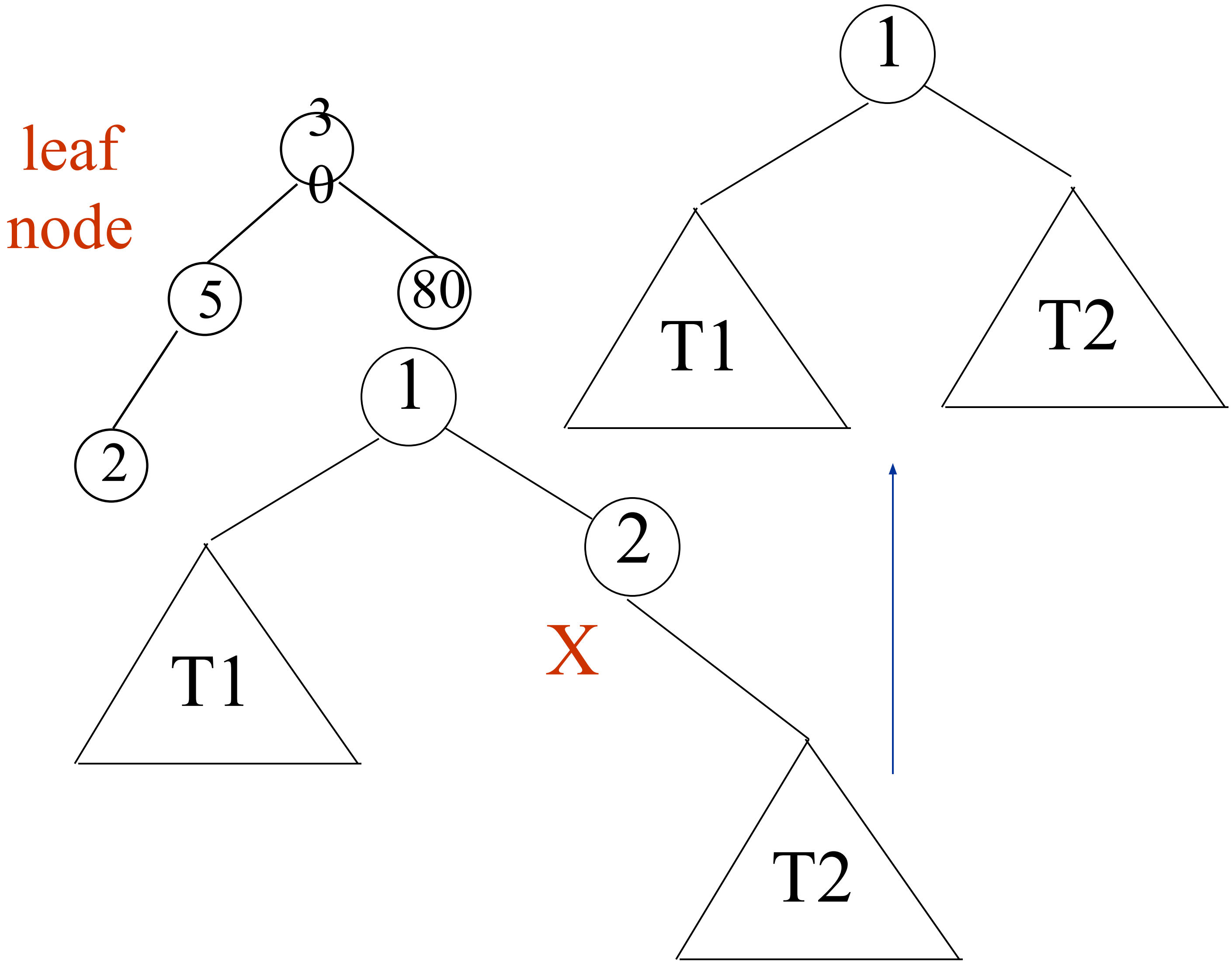
Insert Node in Binary Search Tree



Insertion into A Binary Search Tree

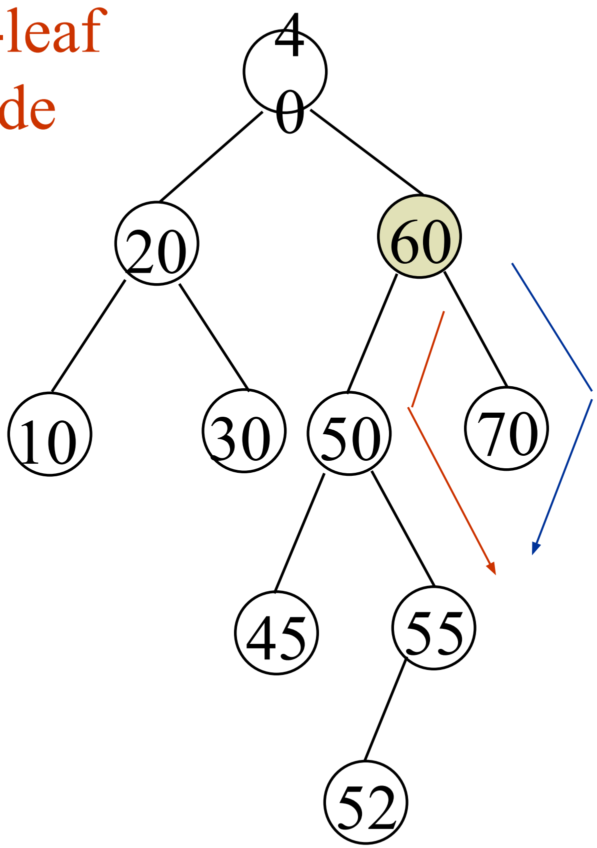
```
void insert_node(tree_pointer *node, int num)
{tree_pointer ptr,
      temp = modified_search(*node, num);
  if (temp || !(*node)) {
   ptr = (tree_pointer) malloc(sizeof(node));
   if (IS_FULL(ptr)) {
     fprintf(stderr, "The memory is full\n");
     exit(1);
   }
   ptr->data = num;
   ptr->left_child = ptr->right_child = NULL;
   if (*node)
     if (num<temp->data) temp->left_child=ptr;
        else temp->right_child = ptr;
   else *node = ptr;
  }
}
```

Deletion for A Binary Search Tree

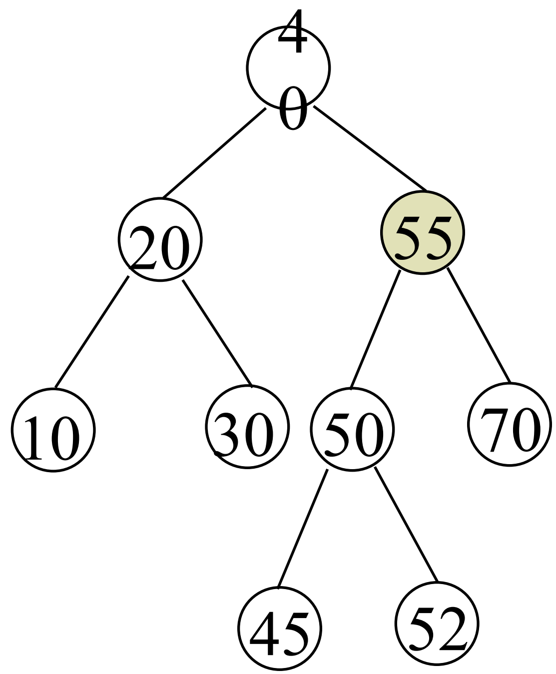


Deletion for A Binary Search Tree

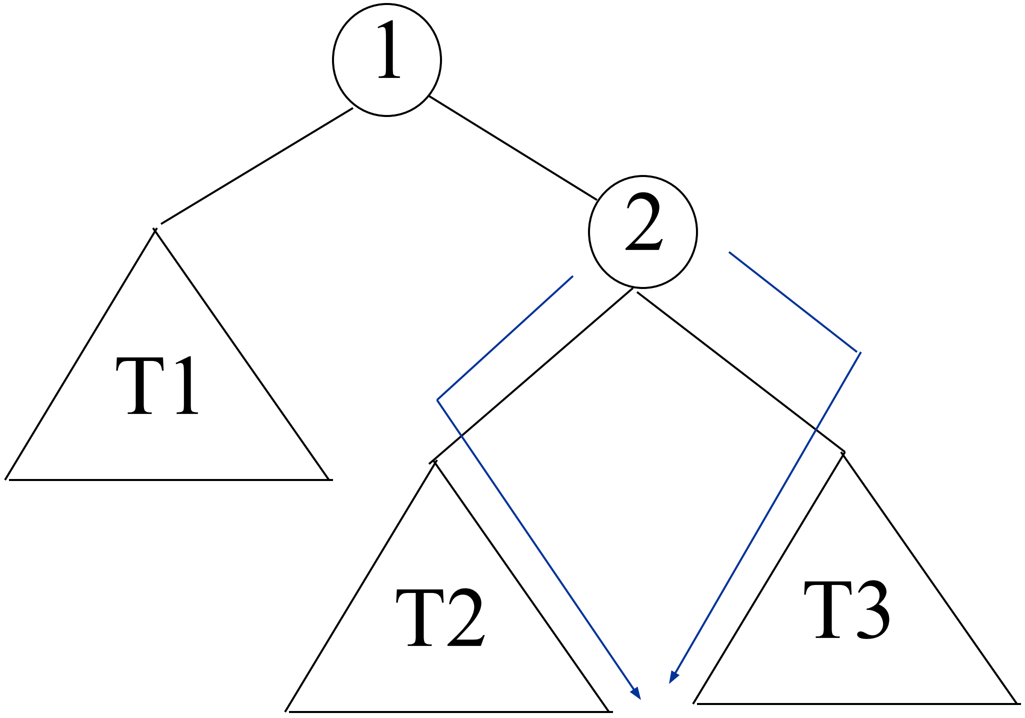
non-leaf node

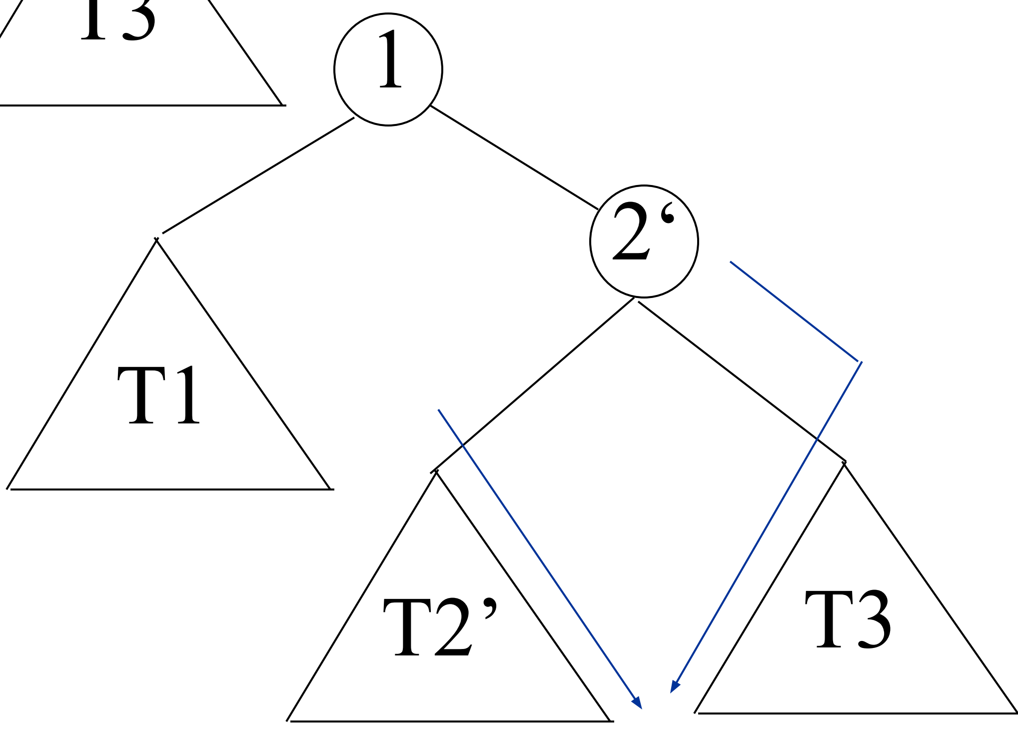


Before deleting 60



After deleting 60


1
2
T1
T2
T3


1
2‘
T1
T2’
T3

Selection Trees

(1) winner tree
(2) loser tree

winner tree

sequential allocation scheme (complete binary tree)

Each node represents the smaller of its two children.

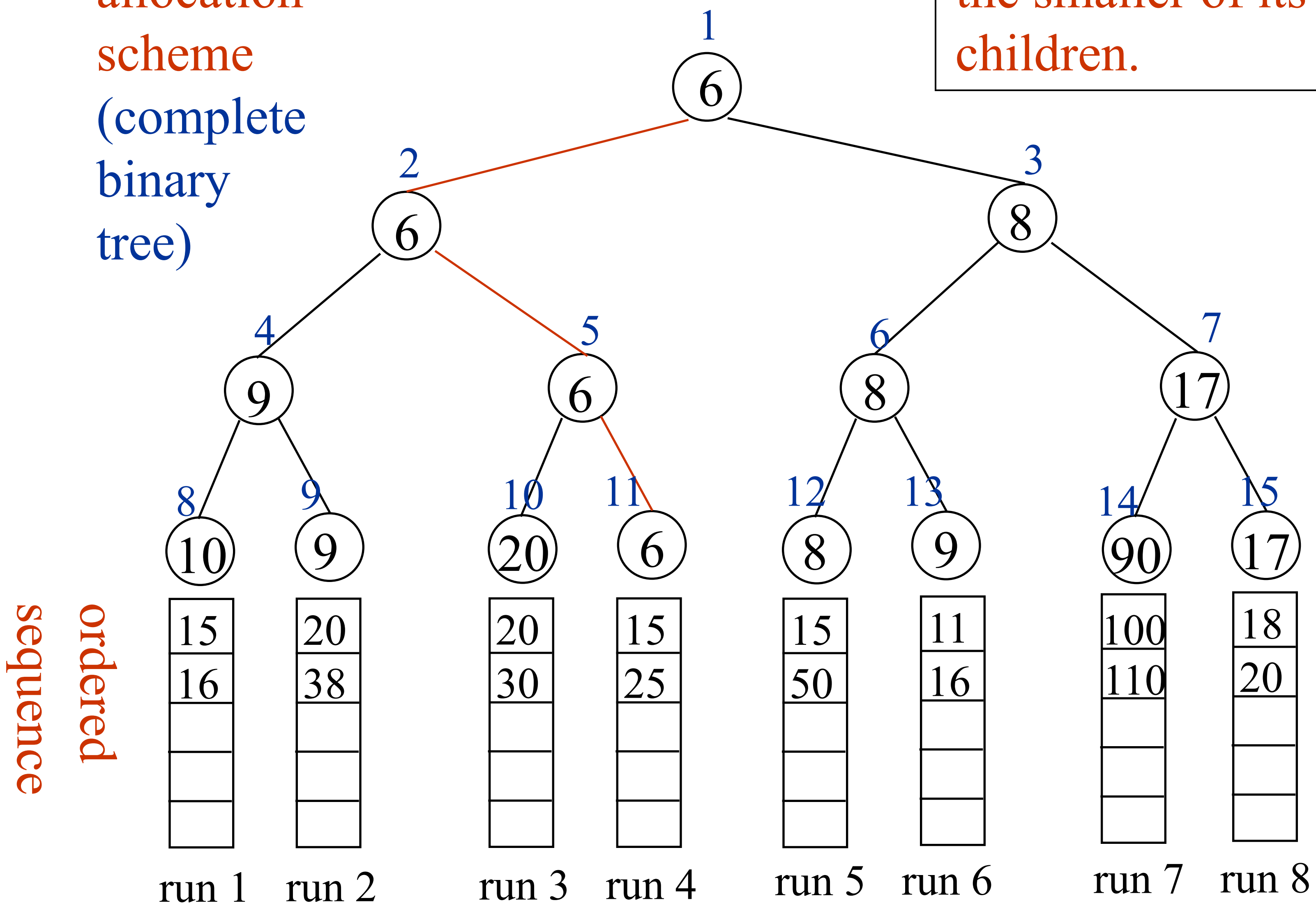
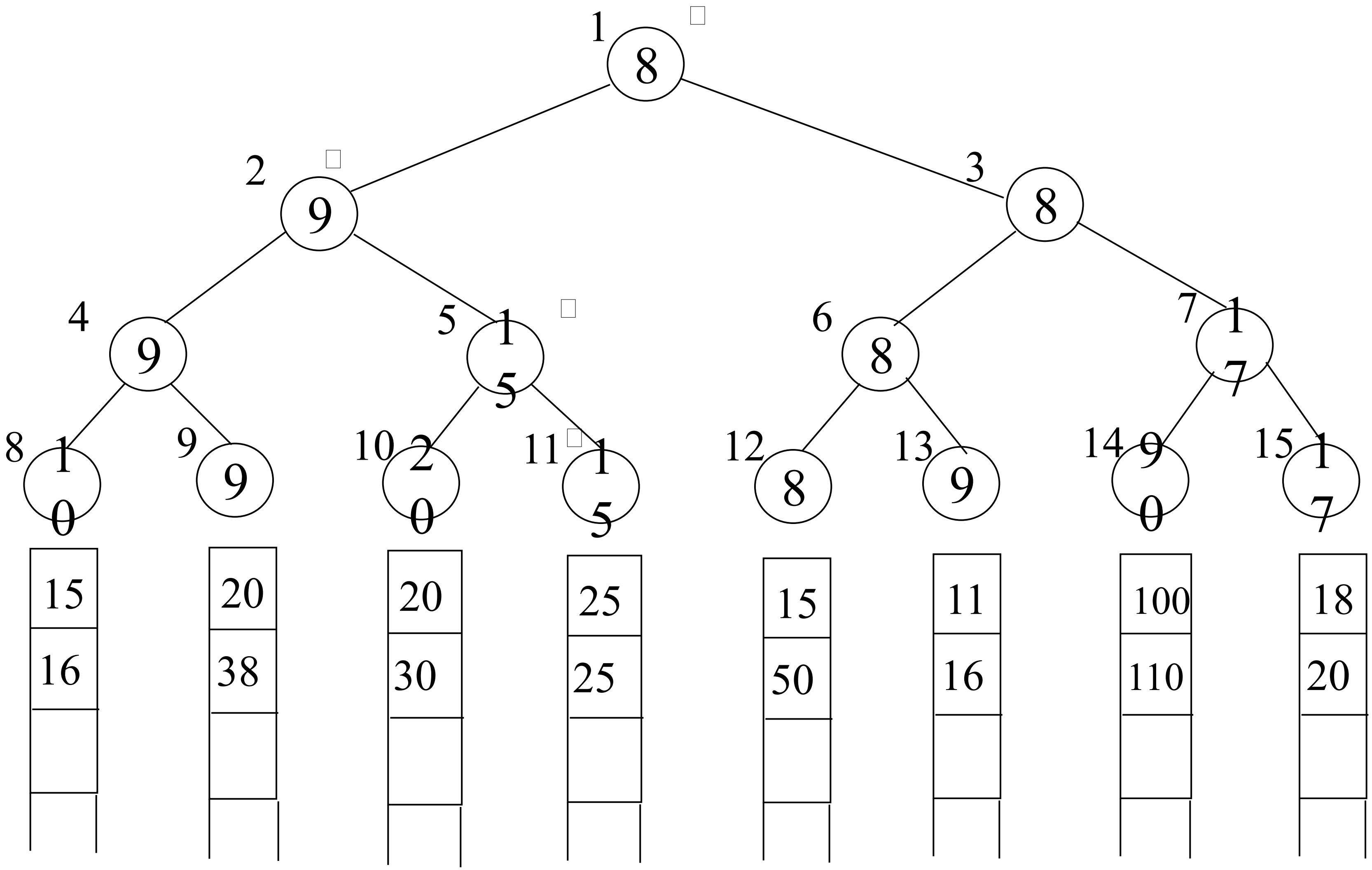


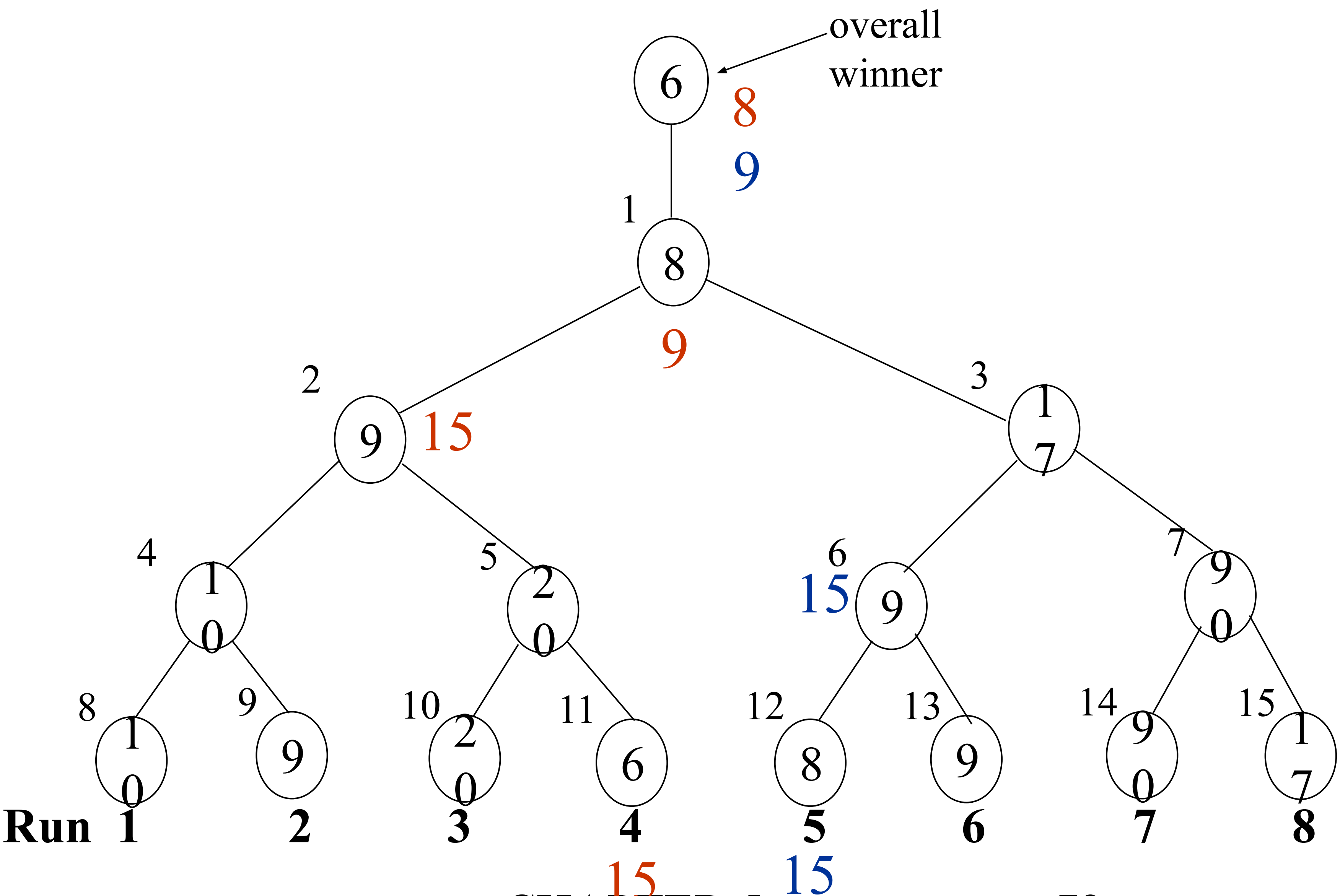
***Figure 5.35:** Selection tree of Figure 5.34 after one record has been output and the tree restructured(nodes that were changed are ticked)*



Analysis

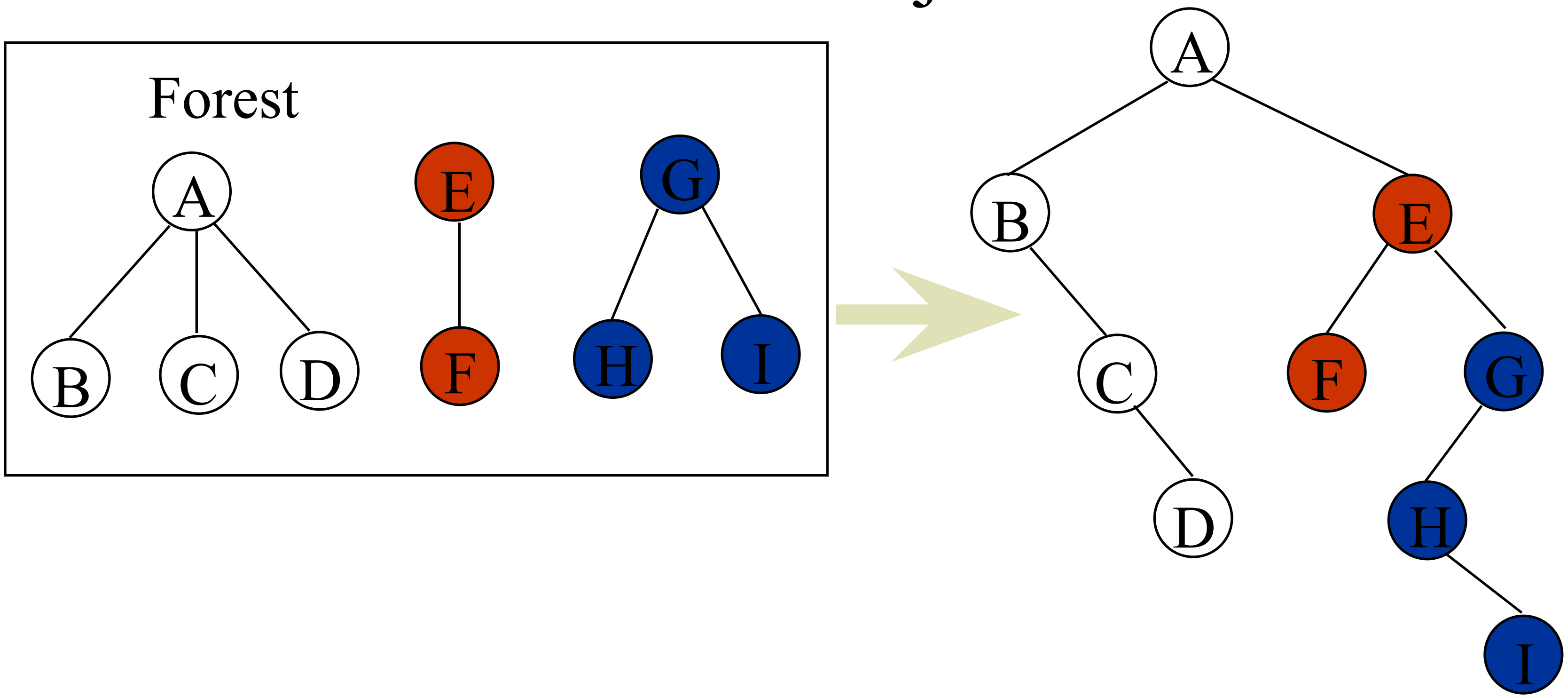
- K: # of runs
- n: # of records
- setup time: $O(K)$ $(K-1)$
- restructure time: $O(\log_2 K)$ $\lceil \log_2(K+1) \rceil$
- merge time: $O(n\log_2 K)$
- slight modification: tree of loser
  - consider the parent node only (vs. sibling nodes)

***Figure 5.36:** Tree of losers corresponding to Figure 5.34 (p.235)



Forest

- A forest is a set of n >= 0 disjoint trees



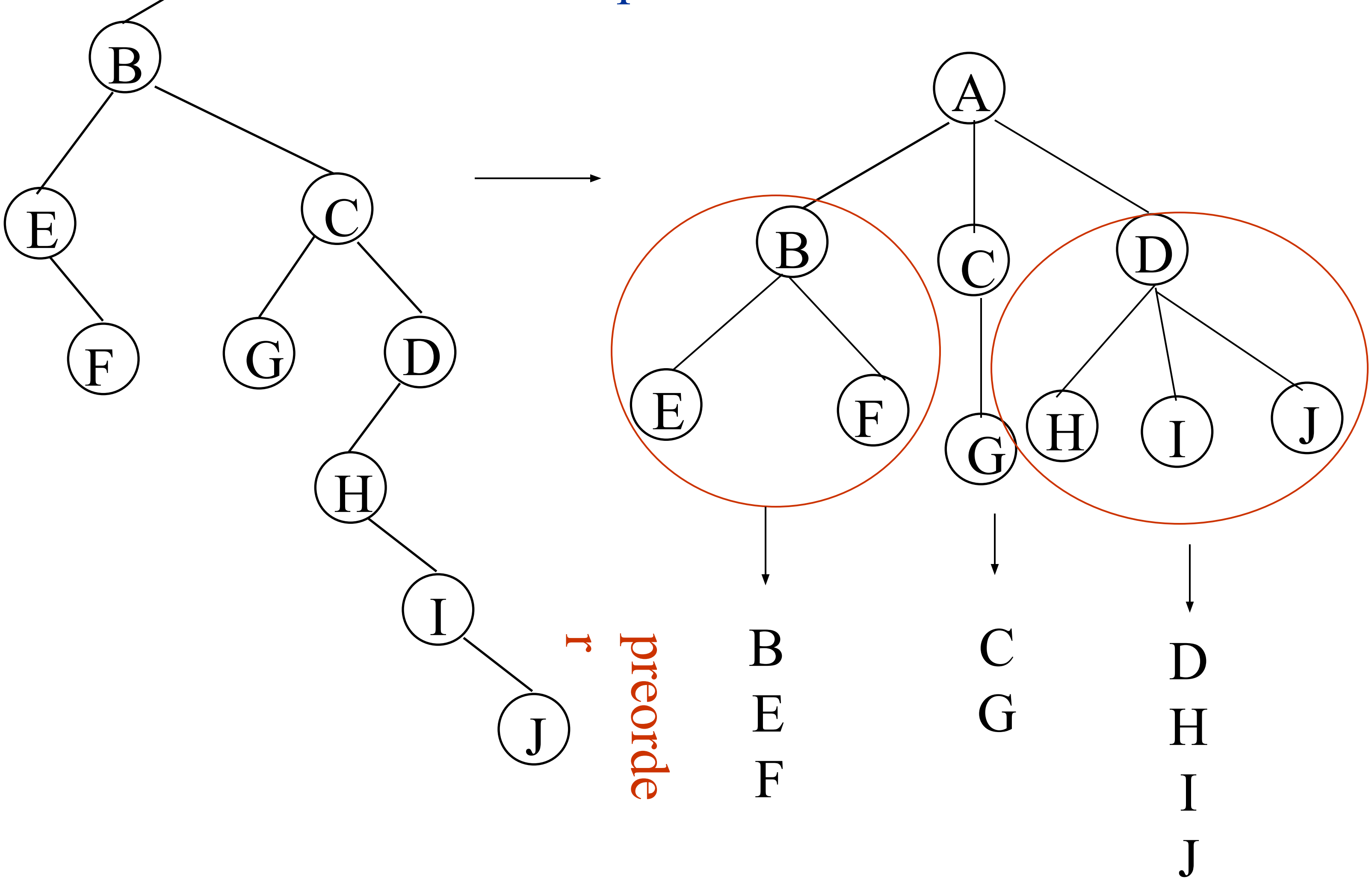
Transform a forest into a binary tree

- T1, T2, …, Tn: a forest of trees
  B(T1, T2, …, Tn): a binary tree corresponding to this forest
- algorithm
  (1) empty, if n = 0
  (2) has root equal to root(T1)
  has left subtree equal to B(T11,T12,…,T1m)
  has right subtree equal to B(T2,T3,…,Tn)

Forest Traversals

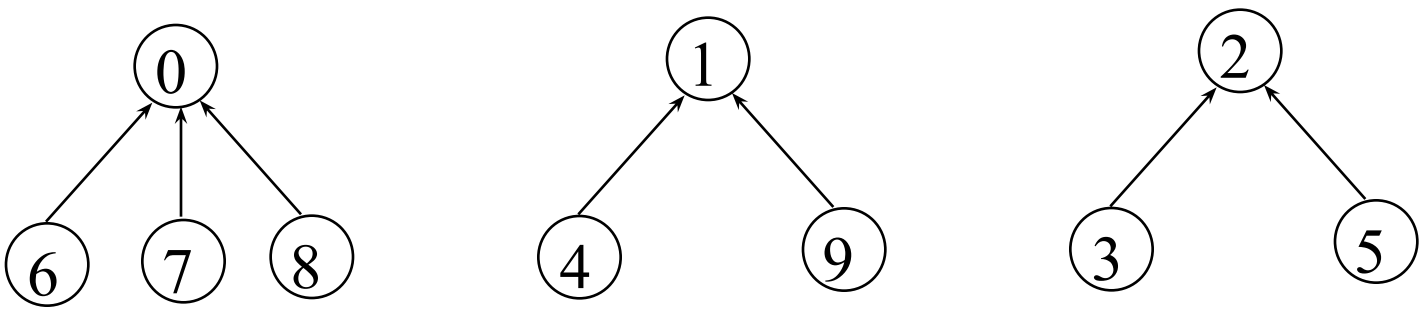
- Preorder
  - If F is empty, then return
  - Visit the root of the first tree of F
  - Taverse the subtrees of the first tree in tree preorder
  - Traverse the remaining trees of F in preorder
- Inorder
  - If F is empty, then return
  - Traverse the subtrees of the first tree in tree inorder
  - Visit the root of the first tree
  - Traverse the remaining trees of F is indorer

inorder: EFBGCHIJDA
preorder: ABEFCGDHIJ



Set Representation

- $S_1=\{0, 6, 7, 8\}$, $S_2=\{1, 4, 9\}$, $S_3=\{2, 3, 5\}$



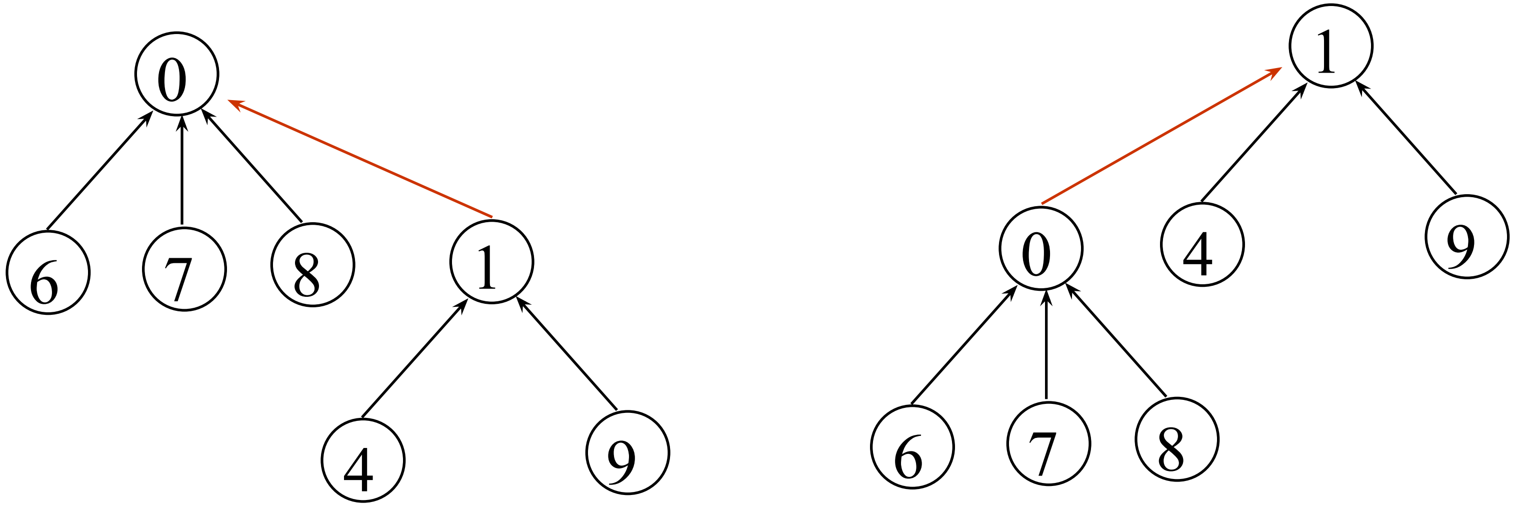
$S_i \cap S_j = \varphi$

- Two operations considered here
  - *Disjoint set union* $S_1 \cup S_2=\{0,6,7,8,1,4,9\}$
  - *Find*(i): Find the set containing the element i.

    $3 \in S_3$, $8 \in S_1$

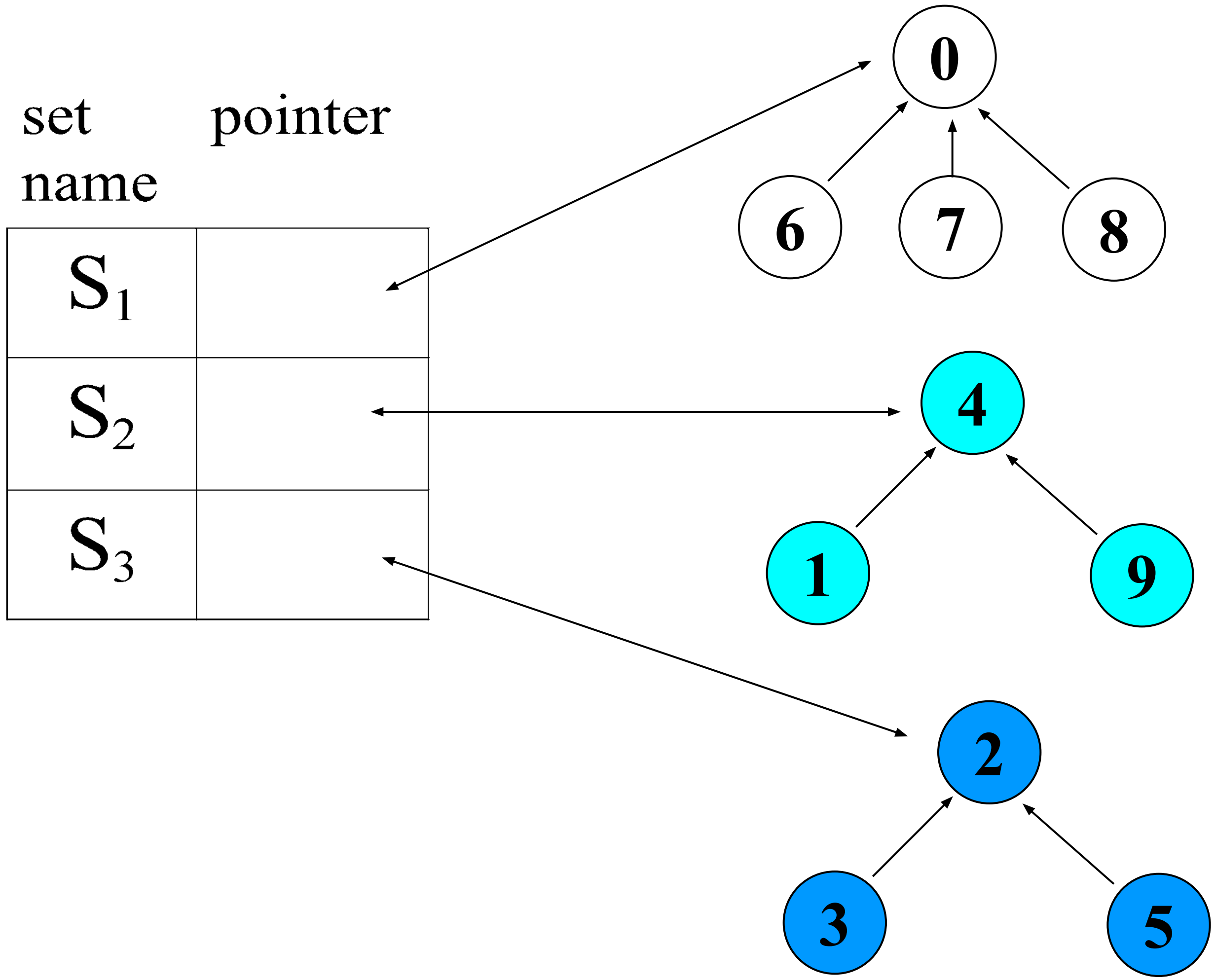
Disjoint Set Union

Make one of trees a subtree of the other



Possible representation for S_1 *union* S_2

***Figure 5.41:**Data Representation of $S_1 S_2$ and S_3 (p.240)



Array Representation for Set

i	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
parent	-1	4	-1	2	-1	2	0	0	0	4

```
int find1(int i)
{
    for (; parent[i]>=0; i=parent[i]);
    return i;
}

void union1(int i, int j)
{
    parent[i]= j;
}
```

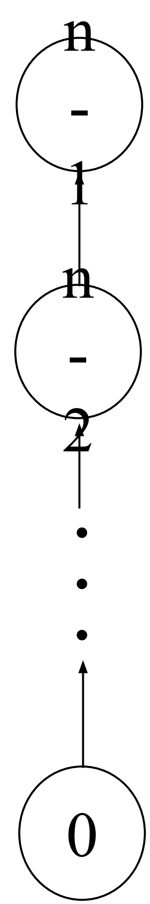
***Figure 5.43:** Degenerate tree (p.242)

union operation

O(n) n-1

find operation

$O(n^2)$ $\sum_{i=2}^{n} i$



degenerate tree

union(0,1), find(0)

union(1,2), find(0)

.

.

.

union(n-2,n-1),find(0)

__Figure 5.44:__ Trees obtained using the weighting rule(p.243)

weighting rule for union(i,j): if # of nodes in i < # in j then j the parent of i

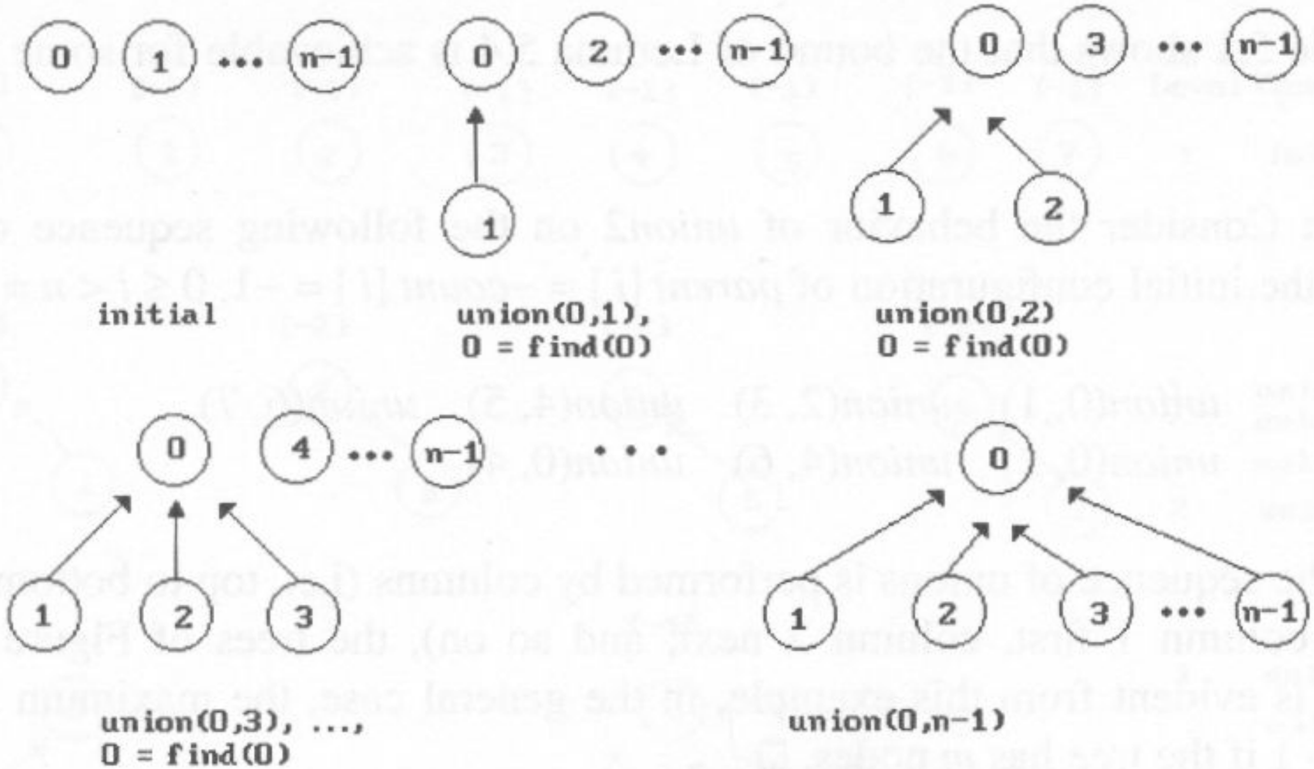


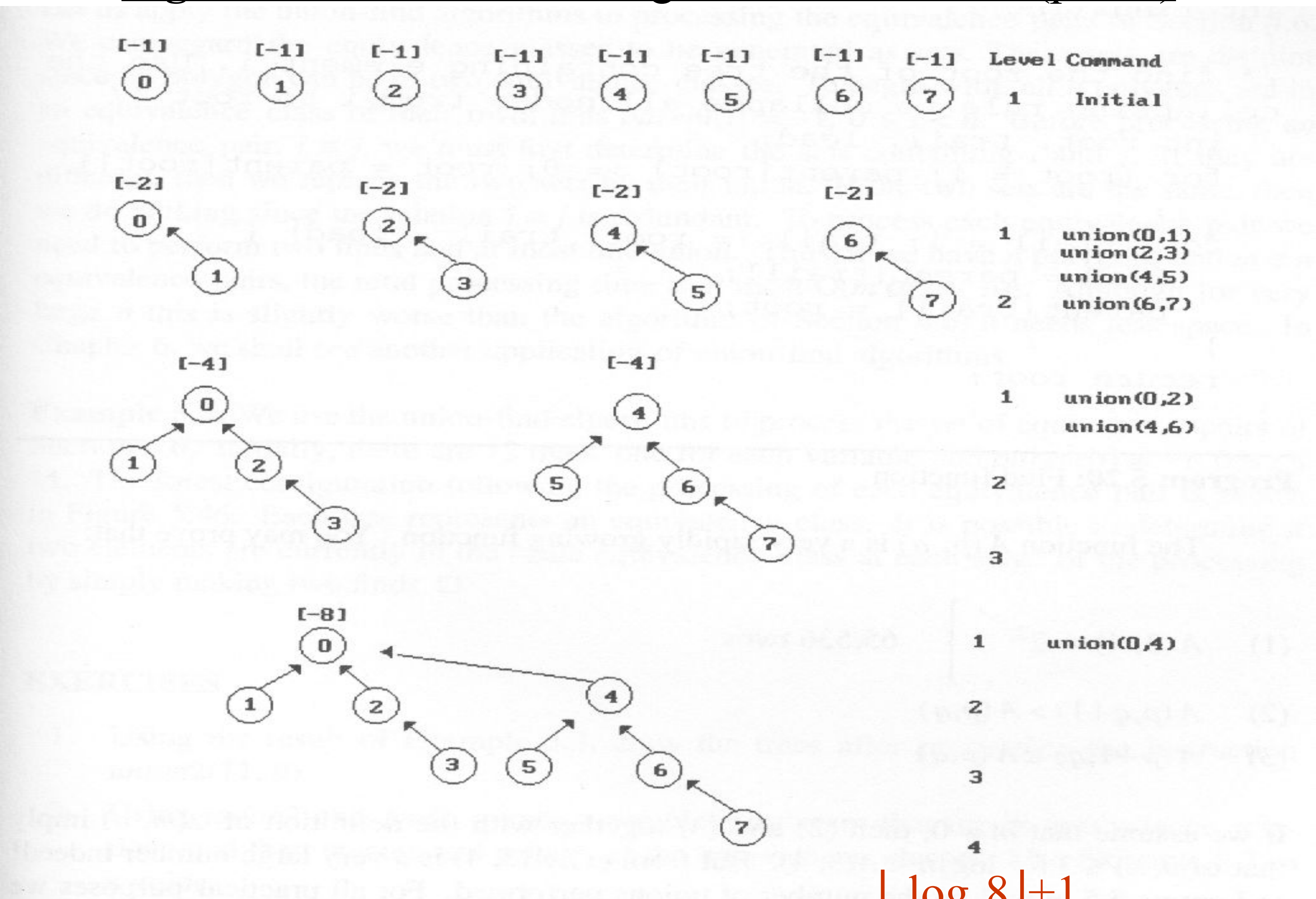
Figure 5.44: Trees obtained using the weighting rule

Modified Union Operation

```
void union2(int i, int j)
{     Keep a count in the root of tree
      int temp = parent[i]+parent[j];
      if (parent[i]>parent[j]) {
            parent[i]=j;      i has fewer nodes.
            parent[j]=temp;
      }
      else { j has fewer nodes
            parent[j]=i;
            parent[i]=temp;
      }
}
```

If the number of nodes in tree i is less than the number in tree j, then make j the parent of i; otherwise make i the parent of j.

Figure 5.45: Trees achieving worst case bound (p.245)

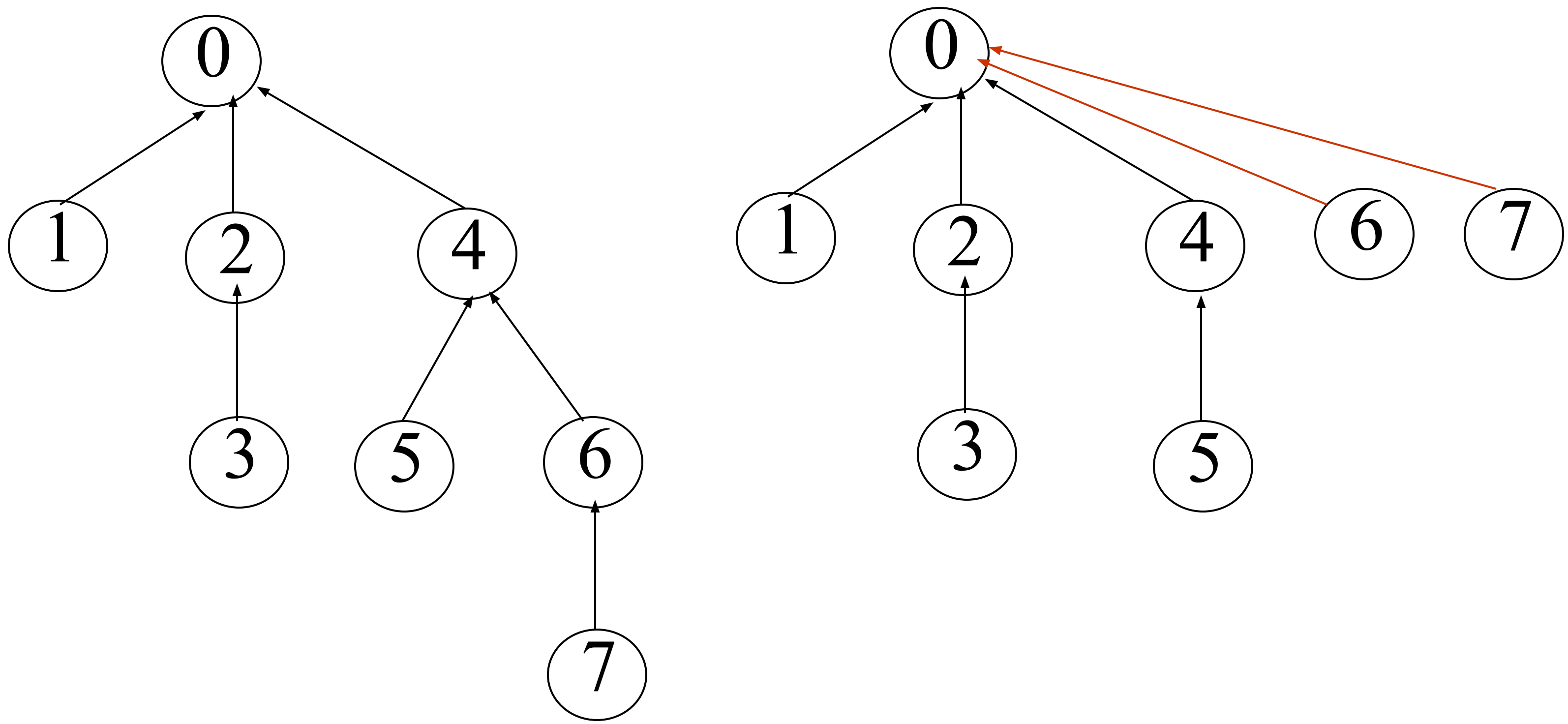


$\lfloor \log_2 8 \rfloor + 1$

Modified *Find*(*i*) Operation

```
int find2(int i)
{
    int root, trail, lead;
    for (root=i; parent[root]>=0;
                        root=parent[root]);
    for (trail=i; trail!=root;
                        trail=lead) {
        lead = parent[trail];
        parent[trail]= root;
    }
    return root:
}
```

If j is a node on the path from i to its root then make j a child of the root



	find(7)	find(7)	find(7)	find(7)	find(7)	find(7)	find(7)	find(7)
go up	3	1	1	1	1	1	1	1
reset	2							

12 moves (vs. 24 moves)

Applications

- Find equivalence class $i \equiv j$
- Find S_i and S_j such that $i \in S_i$ and $j \in S_j$ (two finds)
  - $S_i = S_j$ do nothing
  - $S_i \neq S_j$ union(S_i , S_j)
- example
  $0 \equiv 4$, $3 \equiv 1$, $6 \equiv 10$, $8 \equiv 9$, $7 \equiv 4$, $6 \equiv 8$,
  $3 \equiv 5$, $2 \equiv 11$, $11 \equiv 0$
  {0, 2, 4, 7, 11}, {1, 3, 5}, {6, 8, 9, 10}

preorder: A B C D E F G H I

inorder: B C A E D G H F I