Maths Assignment- I

Q.1. Find the angle of intersection of the curves $r = 2 + 2\sin\theta$ and $r = 2 - 2\cos\theta$

$\rightarrow$

angle of intersection $= \phi = |\phi_2 - \phi_1|$

$\tan\phi_1 = r \cdot \frac{d\theta}{dr}$, $\frac{dr}{d\theta} = 2\cos\theta$

$\Rightarrow \tan\phi_1 = \frac{\cancel{2}(1+\sin\theta)}{\cancel{2}\cos\theta}$

$\tan\phi_2 = r\frac{d\theta}{dr}$, $\frac{dr}{d\theta} = 2\sin\theta$

$\Rightarrow \tan\phi_2 = \frac{\cancel{2}(1-\cos\theta)}{\cancel{2}\sin\theta}$

$$\tan\phi = \left| \frac{\tan\phi_2 - \tan\phi_1}{1 + \tan\phi_1 . \tan\phi_2} \right|$$

$$= \left| \frac{\frac{1-\cos\theta}{\sin\theta} - \frac{(1+\sin\theta)}{\cos\theta}}{1 + \frac{(1-\cos\theta)(1+\sin\theta)}{\sin\theta . \cos\theta}} \right|$$

$$= \frac{\cos\theta - \cos^2\theta - \sin\theta - \sin^2\theta}{1 - \cos\theta + \sin\theta}$$

$\tan\phi = |-\tan\theta| \Rightarrow \phi = \theta$ //

Q2. Find the pedal equation of the curve $r = a\sec t$, $\theta = \tan t - t$.

$\rightarrow$ given, $r = a\sec t$ ① & $\theta = \tan t - t$ - ②

$$\frac{dr}{dt} = a\sec t \cdot \tan t, \quad \frac{d\theta}{dt} = \sec^2 t - 1$$

$$\tan\phi = r\frac{d\theta}{dr} = \frac{a\sec t \cdot (\sec^2 t - 1)}{a\sec t \cdot \tan t} = r \cdot \frac{d\theta/dt}{dr/dt}$$

$$\tan\phi = \frac{\tan^2 t}{\tan t} = \tan t$$

$$\Rightarrow \phi = t$$

WKT, pedal eqn, $p = r\sin\phi$

$$p = r\sin t$$

squaring both side

$$p^2 = r^2\sin^2 t$$

using ①

$$\cos t = \frac{a}{r}, \Rightarrow \sin\theta = \sqrt{1 - \frac{a^2}{r^2}}$$

using above results

$$p^2 = r^2\left(1 - \frac{a^2}{r^2}\right)$$

$\Rightarrow \quad p^2 = r^2 - a^2$ //

Q. 3 if $u = \log(x^3 + y^3 - x^2y - xy^2)$ then prove

$$U_{xx} + 2U_{xy} + U_{yy} = \frac{-4}{(x+y)^2}$$

$\rightarrow$ completing cube

$$u = \log(x^3 + y^3 + 3x^2y + 3y^2x - 4(x^2y + xy^2))$$

$$= \log[(x+y)^3 - 4(x+y).xy]$$

$$= \log[(x+y)[(x+y)^2 - 4xy]]$$

$$u = \log[(x+y)(x-y)^2] \quad \because (x+y)^2 - 4xy = (x-y)^2$$

$$= \log(x+y) + 2\log(x-y)$$

$$U_x = \frac{1}{x+y} + \frac{2}{x-y}, \quad U_y = \frac{1}{x+y} - \frac{2}{x-y}$$

$$U_{xx} = \frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2}, \quad U_{yy} = \frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2}$$

$$2U_{xy} = \frac{-2}{(x+y)^2} + \frac{2x^2}{(x-y)^2}$$

now

$$v_{xx}+2v_{xy}+v_{yy} = -\frac{1}{(x+y)^2}-\frac{\cancel{2}}{(\cancel{x}-y)^2}-\frac{2}{(x+y)^2}+\frac{\cancel{4}}{(\cancel{x}-y)^2}$$

$$-\frac{1}{(x+y)^2}-\frac{\cancel{2}}{(\cancel{x}-y)^2}$$

$$= \frac{-4}{(x+y)^2}$$ $\therefore$ proved

Q4. If $u = \sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$ then find the value of $xu_x + yu_y + zu_z$.

→ by manipulating the given eqn

$$\sin u = \frac{x}{x^8}\left(\frac{1+2(y/x)+3(z/x)}{1+(y/x)^8+(z/x)^8}\right)$$

⇒ it's a homogeneous function of degree -7

∴ acc. to Euler's theorem

$$xv_x + yv_y + zv_z = (-7)u$$

$$x\frac{\partial \sin u}{\partial x} + y\cdot\frac{\partial \sin u}{\partial y} + z\cdot\frac{\partial \sin u}{\partial z} = -7\sin u$$

$$x.v_x + y.v_y + zv_z = -7\tan u$$

Q.5. If $f(x,y) = x^3y^2 + y\sin x$, where $x = \sin 2t$ & $y = \log t$, find $\frac{df}{dt}$

→ acc. to total derivatives

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} \quad - (1)$$

$$\frac{\partial f}{\partial x} = 3x^2y^2 + y\cos x \quad \& \quad \frac{\partial f}{\partial y} = 2x^3y + \sin x$$

$$\frac{dx}{dt} = 2\cos 2t \quad \& \quad \frac{dy}{dt} = \frac{1}{t}$$

using above results in (1)

$$\frac{df}{dt} = (3x^2y^2 + y\cos x)\, 2\cos 2t + (2x^3y + \sin x)\frac{1}{t}$$

on further substituting x & y

$$\frac{df}{dt} = 6\cos 2t \cdot \sin^2 2t (\log t)^2 + 2\cos 2t \cdot \log t \cdot \cos(\sin 2t) + \frac{2\sin^3 2t \cdot \log t + \sin(\sin 2t)}{t}$$

Q.6. Prove that the f^n $u = \frac{x-y}{x+y}$ & $v = \frac{xy}{(x+y)^2}$ are functionally dependent. Find the relation b/w them

→ To prove, $J = \frac{\partial(u,v)}{\partial(x,y)} = 0$

$$u_x = \frac{(x+y) - (x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$u_y = \frac{-(x+y) - (x-y)}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

$$V_x = \frac{y(x+y)^2 - 2(x+y).xy}{(x+y)^4} = \frac{y^2 - xy}{(x+y)^3}$$

$$V_y = \frac{x(x+y)^2 - 2(x+y).xy}{(x+y)^2} = \frac{x^2 - xy}{(x+y)^3}$$

acc to condn

$$\begin{vmatrix} \frac{2y}{(x+y)^2} & -\frac{2x}{(x+y)^2} \\ \frac{y^2-xy}{(x+y)^3} & \frac{x^2-xy}{(x+y)^3} \end{vmatrix} = 0$$

$$\Rightarrow \frac{2y(x^2-xy)}{(x+y)^5} + \frac{2x(y^2-xy)}{(x+y)^5}$$

$$\frac{\cancel{2x^2y} - \cancel{2xy^2} + \cancel{2xy^2} - \cancel{2x^2y}}{(x+y)^5} = 0$$

LHS = RHS $\therefore$ proved

- relatn b/w u & v

Squaring $U \Rightarrow \quad U^2 = \frac{(x-y)^2}{(x+y)^2}$ & $V = \frac{xy}{(x+y)^2}$

$\therefore (x+y)^2 = \frac{xy}{V} \quad - (1)$

Substituting (1) in $u2$

$$U^2 = \frac{(x-y)^2 . V}{xy.}$$

Q.7. Find the length of the curve $x = e^{\theta}(\sin\theta/_2 + 2\cos\theta/_2)$, $y = e^{\theta}(\cos\theta/_2 - 2\sin\theta/_2)$ measured from $\theta = 0$ to $\theta = \pi$

$\rightarrow$

~~dx~~ for parametric curves

$$s = \int_{0}^{\theta_2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \, . \, d\theta \quad - (1)$$

$$\frac{dx}{d\theta} = e^{\theta}(\sin\theta/_2 + 2\cos\theta/_2) + e^{\theta}\left(\tfrac{1}{2}\cos\theta/_2 - \sin\theta/_2\right)$$

$$\frac{dy}{d\theta} = e^{\theta}(\cos\theta/_2 - 2\sin\theta/_2) + e^{\theta}\left(-\tfrac{1}{2}\sin\theta/_2 - \cos\theta/_2\right)$$

$$\left(\frac{dx}{d\theta}\right)^2 = \frac{25}{4}(e^{\theta}.\cos\theta/_2)^2$$

$$\left(\frac{dy}{d\theta}\right)^2 = \frac{25}{4}(e^{\theta}.\sin\theta/_2)^2$$

using above results in ①

$$S = \int_0^{\pi} \sqrt{\frac{25}{4}(e^{\theta})^2 (\sin^2\theta/_2 + \cos^2\theta/_2)}\,.\,d\theta$$

$$\Rightarrow \quad S = \frac{5}{2}\int_0^{\pi} e^{\theta}.d\theta \qquad \because \sin^2\theta/_2 + \cos^2\theta/_2 = 1$$

$$\frac{5}{2}\left[e^{\theta}\right]_0^{\pi} = \frac{5}{2}\left[e^{\pi} - 1\right],$$

Q.8 Evaluate $\int_0^4 x^3\sqrt{4x - x^2}.\,dx$

$\rightarrow$

let $x = 4\sin\theta$

$dx = 4\cos\theta.d\theta$

new limits are

$\theta = 0$ to $\theta = \pi/2$

$$\Rightarrow \int_0^{\pi/2} 4^3\sin^3\theta.\sqrt{4^2(1-\sin^2\theta)}.\,4\cos\theta\,d\theta$$

$$= 4^5\int_0^{\pi/2} \sin^3\theta.\cos^2\theta.\,d\theta$$

acc. to derived result

$$\int_0^{\pi/2} \sin^m\theta.\cos^n\theta = \frac{(m-1)(m-3)..(n-1)(m-3)...\,k}{(m+n)(m+n-2)...}$$

& in this case $k \to 1$

$$\Rightarrow \frac{4^5(3-1)(2-1)}{5\times3\times1} = \frac{2.4^5}{15} = \frac{2048}{15}\,//$$

Q.9. find the length of the arc of the curve $y = e^x$ from point $(0,1)$ to $(1, e)$

→ $$S = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx$$

$$S = \int_0^1 \sqrt{1 + (e^x)^2} \cdot dx$$

let $1 + e^{2x} = t^2$

$$\cancel{2} e^{2x} dx = \cancel{2} t \cdot dt$$

$$\Rightarrow \quad dx = \frac{t \cdot dt}{t^2 - 1}$$

$$S = \int \frac{t^2}{t^2 - 1} \cdot dt = \int \left(\frac{t^2 - 1}{t^2 - 1}\right) + \left(\frac{1}{t^2 - 1}\right) dt$$

$$\Rightarrow \quad S = \int dt + \int \frac{1}{t^2 - 1} dt$$

$$= t + \frac{1}{2} \log \left| \frac{t - 1}{t + 1} \right|$$

Substituting t

$$\left. \sqrt{1 + e^{2x}} \right]_0^1 + \frac{1}{2} \log \left| \frac{\sqrt{1 + e^{2x}} - 1}{\sqrt{1 + e^{2x}} + 1} \right| \Bigg]_0^1$$

$$= \left(\sqrt{e^2 + 1} - \sqrt{2}\right) + \frac{1}{2} \log \left| \frac{\left(\sqrt{e^2 + 1} - 1\right) \cdot \left(\sqrt{2} + 1\right)}{\left(\sqrt{e^2 + 1}\right) \cdot \left(\sqrt{2} - 1\right)} \right|$$

Q10. Find the area common to circle $r = a\sqrt{2}$ & $r = 2a\cos\theta$

→ intersectⁿ points

$$\theta = \pi/4,\ 7\pi/4$$

required area = $2\,ar(OAC + ABC)$

$$= 2\left[\frac{1}{2}\int_0^{\pi/4}(a\sqrt{2})^2\,d\theta + \frac{1}{2}\int_0^{\pi/4}(2a\cos\theta)^2\,d\theta\right]$$

$$2a^2\left[\theta\right]_0^{\pi/4} + \underbrace{4a^2\int_0^{\pi/4}\cos^2\theta\,d\theta}_{I_1}$$

$$I_1 = 4a^2\int_0^{\pi/4}\left(\frac{\cos 2\theta + 1}{2}\right).d\theta$$

$$= a^2\left[\sin 2\theta + 2\theta\right]_0^{\pi/4}$$

$$= a^2\left[1 + \pi/2\right]$$

$$= \frac{\pi a^2}{2} + a^2\left[1 + \pi/2\right]$$

$$= a^2(\pi + 1)$$ //

Q.11. find the Volume of the Solid of Revolutⁿ generated by revolving the Curve $x = 2t + 3$, $y = 4t^2 - 9$ about the axis for $t = -3/2$ to $t = 3/2$

→ about x-axis

$$V = \pi \int_{t_1}^{t_2} y^2 \cdot \frac{dx}{dt} \cdot dt$$

$$= \pi \int_{-3/2}^{3/2} (2)(16t^4 + 81 - 72t^2) \cdot dt$$

$$= \pi \left[\frac{32t^5}{5} - \frac{144}{3}t^3 + 162t \right]_{-3/2}^{3/2}$$

$$= \pi \left[\frac{32}{5}\left[2 \times \frac{3^5}{2^5}\right] - \frac{144}{3}\left[2 \times \frac{3^3}{2^3}\right] + 162 \times 2 \times 3/2 \right]$$

$$= \frac{2916}{5} - 324 = \frac{1296\pi}{5}$$

about y-axis

$$V = \pi \int_{t_1}^{t_2} x^2 \cdot \frac{dy}{dt} \cdot dt$$

$$= \pi \int_{-3/2}^{3/2} [4t^2 + 9 + 12t](8t) \cdot dt$$

$$= \pi \int_{-3/2}^{3/2} [32t^3 + 96t^2 + 72t] \, dt$$

$$= \pi \left[\frac{32t^4}{4} + \frac{96t^3}{3} + \frac{72}{2}t^2 \right]_{-3/2}^{3/2}$$

$$= \pi (32)(2) \times \frac{3^3}{2^3}$$

$$= 8 \times 27 \times \pi = 216\pi$$

Q 12. The curve $r = e^{\theta/2}$ is revolved about initial line. prove that the ~~area~~ Surface area of revolutⁿ traced out by the part b/w the points $\theta = 0$ & $\theta = \pi$ is equal to $\frac{\pi}{2} \cdot \sqrt{5}\,(e^{\pi}+1)$

→

$$S.A = \int_0^{\pi} 2\pi (e^{\theta/2}) . \sin\theta . \frac{ds}{d\theta} . d\theta \quad , \quad \frac{ds}{d\theta} = \sqrt{(e^{\theta/2})^2 + \left(\frac{d e^{\theta/2}}{d\theta}\right)^2}$$

$$\frac{ds}{d\theta} = \sqrt{e^{\theta} + \frac{1}{4} e^{\theta}} = \frac{\sqrt{5e^{\theta}}}{2} = \frac{\sqrt{5}}{2} . e^{\theta/2}$$

$$SA = \cancel{2}\pi \int_0^{\pi} \frac{\sqrt{5}}{\cancel{2}} . e^{\theta/2} . e^{\theta/2} . \sin\theta . d\theta$$

let;

$$I = \sqrt{5}\pi \int_0^{\pi} e^{\theta} . \sin\theta . d\theta$$

$$= \sqrt{5}\pi \left[\left[\sin\theta . e^{\theta}\right]_0^{\pi} - \int_0^{\pi} \cos\theta . e^{\theta} d\theta \right]$$

$$= \sqrt{5}\pi \left[0 - \left(\left[\cos\theta . e^{\theta}\right]_0^{\pi} - \int_0^{\pi} (-\sin\theta) . e^{\theta} d\theta \right) \right]$$

$$2I = \sqrt{5}\pi \left[- \left[\cos\pi \, e^{\pi} - \cos 0 \, e^{0}\right] \right]$$

$$2I = \sqrt{5}\pi (e^{\pi}+1)$$

$$I = \pi/2 \, \sqrt{5} . (e^{\pi}+1)$$

Hence proved

Q 13. Show that the vector field given by $\vec{F} = xyz(yz\hat{i} + xz\hat{j} + xy\hat{k})$ irrotational. Also find it's scalar potential

$\rightarrow$ To prove $\nabla \times \vec{F} = 0$

L.H.S.
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2z^2 & yx^2z^2 & zy^2x^2 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(zy^2x^2) - \frac{\partial}{\partial z}(yx^2z^2)\right) - \hat{j}\left(\frac{\partial}{\partial x}(zy^2x^2) - \frac{\partial}{\partial z}(xy^2z^2)\right)$$
$$+ \hat{k}\left(\frac{\partial}{\partial x}(yx^2z^2) - \frac{\partial}{\partial y}(xy^2z^2)\right)$$

$$= \hat{i}\left(2x^2yz - 2x^2yz\right) - \hat{j}\left(2xy^2z - 2xy^2z\right)$$
$$+ \hat{k}\left(2xyz^2 - 2xyz^2\right)$$

$$= 0$$

LHS = RHS $\therefore$ proved

- Scalar potential (ϕ)

$$\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k} = xy^2z^2\hat{i} + yx^2z^2\hat{j} + zx^2y^2\hat{k}$$

$$\Rightarrow \quad \frac{\partial \phi}{\partial x} = xy^2z^2, \quad \frac{\partial \phi}{\partial y} = yx^2z^2, \quad \frac{\partial \phi}{\partial z} = zx^2y^2$$

integrating.

$$\phi = \int xy^2z^2\, \partial x = \frac{x^2y^2z^2}{2} + f(y,z) \quad -(1)$$

$\phi = \int y \cdot x^2 z^2 \cdot \partial y = \frac{x^2y^2z^2}{2} + f(x,z)$ — ②

$\phi = \int z \cdot x^2 y^2 \cdot \partial z = \frac{x^2y^2z^2}{2} + f(y,x)$ — ③

comparing ①, ② + ③

$f(y,z) = f(x,z) = f(x,y) = 0$

$\Rightarrow \quad \phi = \frac{x^2y^2z^2}{2}$ //

Q14. If $f = x^2+y^2+z^2$ & $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Show that div. $(f\vec{r}) = 5f$

$\longrightarrow$

$f \cdot \vec{r} = (x^3 + xy^2 + xz^2)\hat{i} + (y^3 + yx^2 + yz^2)\hat{j} + (z^3 + zx^2 + zy^2)\hat{k}$

$\nabla \cdot (f \cdot \vec{r}) = \frac{\partial}{\partial x}(x^3 + xy^2 + xz^2) + \frac{\partial}{\partial y}(y^3 + yx^2 + yz^2) + \frac{\partial}{\partial z}(z^3 + zx^2 + zy^2)$

$= 3x^2 + y^2 + z^2 + 3y^2 + x^2 + z^2 + 3z^2 + x^2 + y^2$

$= 5(x^2 + y^2 + z^2)$

$= 5f$ //

$\therefore$ proved //

Q. 15. A particle moves along curve $x = t^3 + 1$, $y = t^2$ & $z = 2t + 5$ where 't' is time. find the components of velocity & acceleratn at $t = 1$ in direction of $\hat{i} + \hat{j} + 3\hat{k}$

→ let 'ϕ' be a curve

$$\phi = (t^3 + 1)\hat{i} + t^2\hat{j} + (2t + 5)\hat{k}$$

• Velocity $= \frac{d\phi}{dt} = 3t^2\hat{i} + 2t\hat{j} + 2\hat{k}$

Component along $\hat{i} + \hat{j} + 3\hat{k}$ is

$$\vec{v}.\hat{n}, \quad \hat{n} = \frac{\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{1+1+9}} = \frac{\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{11}}$$

$$\therefore \vec{v}.\hat{n} = \frac{3t^2 + 2t + 6}{\sqrt{11}}$$

$$\vec{v}.\hat{n}\Big|_{t=1} = \frac{3(1) + 2(1) + 6}{\sqrt{11}} = \frac{11}{\sqrt{11}} = \sqrt{11} \quad //$$

• acceleratn $= \frac{d^2\phi}{dt^2} = 6t\hat{i} + 2\hat{j} + 0\hat{k}$

Component along $\hat{i} + \hat{j} + 3\hat{k}$ is

$$\vec{a}.\hat{n} = (6t\hat{i} + 2\hat{j})\frac{(\hat{i} + \hat{j} + 3\hat{k})}{\sqrt{11}} = \frac{6t + 2}{\sqrt{11}}$$

$$\vec{a}.\hat{n}\Big|_{t=1} = \frac{6(1) + 2}{\sqrt{11}} = \frac{8}{\sqrt{11}} \quad //$$

Q. 17. prove that $\nabla \cdot \left\{ r \cdot \nabla \left(\frac{1}{r^3} \right) \right\} = \frac{3}{r^4}$, where

$r = x^2 + y^2 + z^2$

$\rightarrow$ $r = \sqrt{x^2 + y^2 + z^2}$

$$\nabla \cdot \frac{1}{r^3} = -\frac{3}{2} \frac{(2x)}{(x^2+y^2+z^2)^{5/2}} - \frac{3}{2} \frac{(2y)}{(x^2+y^2+z^2)^{5/2}}$$

$$- \frac{3}{2} \frac{(2z)}{(x^2+y^2+z^2)^{5/2}}$$

$$r \cdot \nabla \cdot \frac{1}{r^3} = \frac{-3(x+y+z) \cdot r}{r^5} = \frac{-3(x+y+z)}{r^4}$$

$\nabla \cdot \left\{ r \cdot \nabla \left(\frac{1}{r^3} \right) \right\}$, let us consider first term $\frac{-3x}{r^4}$

$\hookrightarrow$ $-3 \left[\frac{r^4 - 4r^3 \cdot x \cdot dr/dx}{r^8} \right]$, $\frac{dr}{dx} = \frac{2x}{2r}$

$$= -3 \left[\frac{r^4 - 4r^2 x^2}{r^8} \right]$$

similarly,

$$\nabla \cdot \left\{ r \cdot \nabla \left(\frac{1}{r^3} \right) \right\} = -3 \left[\frac{r^4 - 4r^2x^2 + r^4 - 4r^2y^2 + r^4 - 4r^2z^2}{r^8} \right]$$

$$= -3 \left[\frac{3r^4 - 4r^2(x^2+y^2+z^2)}{r^8} \right]$$

$$= -3 \left[\frac{3r^4 - 4r^4}{r^8} \right]$$

$$= 3/r^4$$ $\therefore$ proved //

Q 18. Find the angle b/w the curves $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at point $(2,1,2)$

→ let $\phi_1 = x^2+y^2+z^2-9$ & $\phi_2 = x^2+y^2-z-3$ be the curves

$\nabla\phi_1 = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

$\nabla\phi_2 = 2x\hat{i} + 2y\hat{j} - \hat{k}$

$\nabla\phi_1]_{P(2,1,2)} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$\nabla\phi_2]_{P(2,1,2)} = 4\hat{i} + 2\hat{j} - \hat{k}$

angle b/w $\nabla\phi_1$ & $\nabla\phi_2$ is

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$$\cos\theta = \frac{16+4-4}{\sqrt{16+16+4}\sqrt{16+4+1}} = \frac{16}{6\sqrt{21}} = \frac{8}{3\sqrt{21}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$$

Q 19. Evaluate the double integral $\int_0^4 \int_{y^2/4}^{y} \frac{y}{x^2+y^2}\,dx\,dy$

→

$$I = \int_0^4 y \cdot \left[\frac{1}{y} \cdot \tan^{-1}\frac{x}{y}\right]_{y^2/4}^{y} \cdot dy$$

$$I = \int_0^4 \left[\tan^{-1} 1 - \tan^{-1}\frac{y}{4}\right] . dy$$

$= \int_0^4 [\pi/4 - \tan^{-1} y/4].dy$

let $y/4 = t \Rightarrow dy = 4dt$

& new limits are $t_1 = 0,\ t_2 = 1$

$\cdot\ 4\int_0^1 \pi/4 .dt - 4\int_0^1 \tan^{-1} t . dt$

$$I = 4 \times \frac{\pi}{4} [1-0] - 4\left[(t \tan^{-1} t]_0^1 - \underbrace{\int_0^1 \frac{t}{1+t^2} .dt}_{I_1} \right]$$

in I_1 let $1+t^2 = u$

$2t.dt = du \Rightarrow t.dt = \frac{du}{2}$

$$I_1 = \frac{1}{2}\int_0 \frac{1}{u}.du = \frac{1}{2}\log u$$

Substituting Value of u

$$I = \pi - 4\left[\pi/4 - [\tfrac{1}{2}\log|1+t^2|]_0^1\right]$$

$$= \pi - \pi + 2[\log|2| - \log|1|]$$

$$= 2\log 2 \ //$$

Q. 20. Evaluate $\iint (x^2+y^2)\,dx.dy$ over the area bounded by the curves $y = 4x$, $x+y = 3$, $y = 0$ & $y = 2$