1) $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ Ex 13.1 pg 474 grewal

Synthetic division

$4m^4 - 8m^3 - 7m^2 + 11m + 6 = 0$ - Auxilary equation

$m = -1$ $4 + 8 - 7 - 11 + 6 = 0$

root

$$\begin{array}{r|rrrrr} -1 & 4 & -8 & -7 & 11 & 6 \\ & 0 & -4 & 12 & -5 & -6 \\ \hline & 4 & -12 & 5 & 6 & \boxed{0} \end{array}$$

$4m^2 - 12m^2 + 5m + 6 = 0$

by inspection $m = 2$ is root

$32 - 48 + 10 + 6 = 0$

$$\begin{array}{r|rrrr} 2 & 4 & -12 & 5 & 6 \\ & 0 & 8 & -8 & -6 \\ \hline & 4 & -4 & -3 & \boxed{0} \end{array}$$

$4m^2 - 4m - 3 = 0$

$2m(2m-3) + 1(2m-3) = 0$

$m = -\frac{1}{2}, 3/2$

$\therefore$ roots are $-1, 2, -1/2, 3/2$

$\therefore$ g.s $\Rightarrow$ $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-x/2} + c_4 e^{3x/2}$

2) $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$ ex 13.1 general

$AE = (D^4 + 8D^2 + 16)\, y = 0$

$m^4 + 8m^2 + 16 = 0$

$(m^2+4)^2 = 0$

$(m^2+4)\,(m^2+4) = 0$

$m^2+4=0 \quad m = \pm 2i$

$\therefore$ roots are $m = \pm 2i, \pm 2i$ (repeated complex roots)

$\therefore$ g.s $y = (C_1 + C_2 x)\cos 2x + (C_3 + C_4 x)\sin 2x$

3) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$

AE $m^3 - 3m^2 + 3m - 1 = 0$

$(m-1)^3 = 0$

$m = 1, 1, 1$

g.s $y = (C_1 + C_2 x + C_3 x^2)\, e^x$

Ex 13.2 pg 486
grewal

1) $(D^2-1)y = x\sin x + (1+x^2)e^{x}$

AE $\quad m^2-1=0 \quad m=\pm 1$

$CF = c_1 e^{x} + c_2 e^{-x}$

$$PI = \frac{(1+x^2)e^{x} + x\sin x}{D^2-1} = \underset{(1)}{\frac{(1+x^2)e^{x}}{D^2-1}} + \underset{(2)}{\frac{x\sin x}{D^2-1}}$$

(1) $$\frac{(1+x^2)e^{x}}{D^2-1} = e^{x}\left\{\frac{1+x^2}{(D+1)^2-1}\right\}$$

$$= e^{x}\left\{\frac{1}{D^2+2D}(1+x^2)\right\}$$

$$\begin{array}{r|l}
 & \frac{x^3}{6} - \frac{x^2}{4} + \frac{3x}{4} \\
\hline
2D+D^2 & x^2+1 \\
 & \underset{(-)}{x^2} \underset{(-)}{+x} \\
\hline
 & -x+1 \\
 & -x-\frac{1}{2} \\
 & (+)\ (+) \\
\hline
 & \frac{3}{2} \\
 & \frac{3}{2} \\
\hline
 & 0
\end{array}$$

$$PI = e^{x}\left\{\frac{x^3}{6} - \frac{x^2}{4} + \frac{3x}{4}\right\}$$

(2) $$\frac{x\sin x}{D^2-1} \qquad \frac{xV}{f(D)} = \left[x - \frac{f'(D)}{f(D)}\right]\frac{V}{f(D)}$$

$$\left[x - \frac{2D}{D^2-1}\right]\frac{\sin x}{D^2-1}$$

$$\left[x - \frac{2D}{D^2-1}\right] \frac{\sin x}{-1-1} = -\frac{x\sin x}{2} + \frac{\cos x}{D^2-1} \qquad D^2 = -1$$

$$-\frac{x\sin x}{2} + \frac{\cos x}{-2} = -\frac{1}{2}\left(x\sin x + \cos x\right)$$

$GS \to y = y_c + y_p$

$$y = c_1 e^{x} + c_2 e^{-x} + \frac{x e^{3x}}{12}\left(2x^2 - 3x + 9\right)$$
$$-\frac{1}{2}\left(x\sin x + \cos x\right)$$

2) $$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$$

$$m^2 - 6m + 9 = 0$$
$$(m-3)^2 = 0 \qquad m = 3, 3$$
$$y_1 = (c_1 + c_2 x)e^{3x}$$

$$y_p = \frac{6e^{3x}}{D^2-6D+9} + 7\frac{e^{-2x}}{D^2-6D+9} - \frac{\log 2\, e^{0}}{D^2-6D+9}$$

$$= \frac{6e^{3x}}{3^2-18+9} + 7\frac{e^{-2x}}{(-2)^2-6(-2)+9} - \frac{\log 2\, e^{0}}{0-0+9}$$

$DV=0$

$$\frac{6xe^{3x}}{2D-6} + \frac{7e^{-2x}}{25} - \frac{\log 2}{9}$$

$DV=0$

$$\frac{6x^2e^{3x}}{2} = 3x^2e^{3x}$$

$$\therefore \; G.S = (C_1 + C_2x)e^{3x} + 3x^2e^{3x} + \frac{7e^{-2x}}{25} - \frac{\log 2}{9}$$

3) $y'' + 16y = x\sin 3x$

$$(D^2+16)y = x\sin 3x$$

$$m^2 + 16 = 0 \qquad m = \pm 4i$$

$$y_c = C_1\cos 4x + C_2\sin 4x$$

$$y_p = \frac{x\sin 3x}{D^2+16}$$

$$\left[x - \frac{f'(D)}{f(D)}\right]\frac{V}{f(D)} \qquad D^2 = -9$$

$$= \left(x - \frac{2D}{D^2+16}\right)\frac{\sin 3x}{D^2+16} = \frac{x\sin 3x}{7} - \frac{6\cos 3x}{7(D^2+16)}$$

$$= \frac{x\sin 3x}{7} - \frac{6\cos 3x}{7(-9+16)}$$

$$y_p = \frac{x\sin 3x}{7} - \frac{6\cos 3x}{49}$$

$$\therefore \; G.S = C_1\cos 4x + C_2\sin 4x + \frac{1}{49}(7x\sin 3x - 6\cos 3x)$$

4)

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$$

$AE \quad m^3 + 2m^2 + m = 0$

$m(m+1)^2 = 0 \qquad m = 0, -1, -1$

$CF = c_1 + (c_2 + c_3 x) e^{-x}$

$$PI = \underbrace{\frac{1}{D^3+2D^2+D} e^{-x}}_{1} + \underbrace{\frac{\sin 2x}{D^3+2D^2+D}}_{2}$$

1 $\quad \dfrac{e^{-x}}{D^3+2D^2+D} = \dfrac{e^{-x}}{-1+2-1} \quad Dr = 0$

$\dfrac{x e^{-x}}{3D^2+4D+1} = \dfrac{x e^{-x}}{3-4+1} \quad Dr = 0$

$\dfrac{x^2 e^{-x}}{6D+4} = \dfrac{x^2 e^{-x}}{-6+4} = -\dfrac{1}{2} e^{-x} x^2$

2 $\quad \dfrac{\sin 2x}{D^3+2D^2+D} \qquad D^2 = -4$

$\dfrac{\sin 2x}{D^2(D) + 2D^2 + D} = \dfrac{\sin 2x}{-4D - 8 + D} = \dfrac{\sin 2x}{-3D - 8}$

$= -\dfrac{\sin 2x}{3D+8} \times \dfrac{3D-8}{3D-8} = -\left\{\dfrac{3D-8\ (\sin 2x)}{9D^2-64}\right.$

$$= -\frac{1}{9(-4)-64} \cdot \frac{6\cos 2x - 8\sin 2x}{}$$

$$\frac{1}{100}(6\cos 2x - 8\sin 2x)$$

$$\therefore G.S = y = c_1 + (c_2 + c_3 x)e^{-x}$$

$$+ \frac{1}{2}x^2 e^{-x} + \frac{1}{50}(3\cos 2x - 4\sin 2x)$$

5) $y'' + 2y = x^2 e^{3x} + e^{x}\cos 2x$

$AE = D^2 + 2 = 0 \quad m^2 + 2 = 0 \quad m = \pm i\sqrt{2}$

$CF = A\cos\sqrt{2}x + B\sin\sqrt{2}x$

$$PI = \frac{1}{D^2+2} \cdot x^2 e^{3x} + \frac{1}{D^2+1} e^{x}\cos 2x$$

(1) (2)

$$(1) \Rightarrow e^{3x} \frac{1}{(D+3)^2+2} x^2$$

$$e^{3x} \frac{x^2}{D^2+6D+11} \qquad \therefore (1)\ \frac{e^{3x}}{11}\left(x^2 - \frac{12}{11}x + \frac{50}{121}\right)$$

$$11 + 6D + D^2 \;\Big)\; x^2 \;\Big(\; \frac{x^2}{11} - \frac{12}{11^2}x + \frac{50}{11^3}$$

$$\frac{x^2}{11} + \frac{12x}{11} + \frac{2}{11}$$

$$(-) \quad (-) \quad (-)$$

$$-\frac{12}{11}x - \frac{2}{11}$$

$$-\frac{12}{11}x - \frac{72}{11^2}$$

$$(+) \quad (+)$$

$$\frac{50}{121}$$

$$\frac{50}{121}$$

$$0$$

(2) $\frac{1}{D^2+2} e^x \cos 2x = e^x \left[\frac{1}{(D+1)^2+2} \cos 2x \right]$

$= e^x \frac{1}{D^2+2D+3} \cos 2x \qquad D^2 = -4$

$= e^x \frac{1}{2D-1} \cos 2x = e^x \frac{1 (2D+1)}{2D-1 \, (2D+1)} \cos 2x$

$= e^x \frac{(2D+1) \cos 2x}{4D^2-1} = \frac{e^x}{-17} \left(2(\sin 2x) \cdot 2 + \cos 2x \right)$

$= \frac{-e^x}{17} \left(-4 \sin 2x + \cos 2x \right)$

$\therefore y = A \cos\sqrt{2}x + B \sin\sqrt{2}x + \frac{e^{3x}}{11} \left(x^2 - \frac{12}{11}x + \frac{50}{121} \right)$

$+ \frac{e^x}{17} \left(4 \cos 2x - \cos 2x \right)$

6) $\frac{d^4 y}{dx^4} - y = \cos x \cosh x$

AE $m^4 - 1 = 0 \qquad (m^2-1)(m^2+1) = 0$

$m = \pm 1 \quad \pm i$

$CF = C_1 e^x + C_2 e^{-x} + C_3 \cos x + C_4 \sin x$

$PI = \frac{\cos x \left(\frac{e^x + e^{-x}}{2} \right)}{D^4 - 1} = \frac{1}{2} \left\{ \frac{e^x \cos x}{D^4-1} + \frac{e^{-x} \cos x}{D^4-1} \right\}$

$$= \frac{1}{2}\left\{ e^{x} \frac{\cos x}{((D+1)^2-1)((D+1)^2+1)} + e^{-x} \frac{1}{((D-1)^2+1)((D-1)^2+1)} \cos x \right]$$

$$= \frac{1}{2}\left\{ e^{x} \frac{\cos x}{(D^2+2D)(D^2+2D+2)} + e^{-x} \frac{\cos x}{(D^2-2D)(D^2-2D+2)} \right]$$

$D^2 = -1$

$$= \frac{1}{2}\left\{ e^{x} \frac{\cos x}{(2D-1)(2D+1)} + e^{-x} \frac{1}{(2D+1)(2D-1)} \cos x \right]$$

$$\frac{1}{2}\left\{ e^{x} \frac{\cos x}{4D^2-1} + e^{-x} \frac{1}{4D^2-1} \cos x \right]$$

$$\frac{1}{2}\left(e^{x}\left(-\frac{1}{5}\right)\cos x + e^{-x}\left(-\frac{1}{5}\right)\cos x \right] = -\frac{1}{5} \cos x \cosh x$$

G.S $y = c_1 e^{x} + c_2 e^{-x} + (c_3 \cos x + c_4 \sin x)$

$-\frac{1}{5} \cos x \cosh x$

2) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$

$(D^2+3D+2)y = 4\cos^2 x$

$m^2+3m+2=0 \qquad m = -1, -2$

$\cancel{y}$. CF $= c_1 e^{-x} + c_2 e^{-2x}$

$4\cos^2 x = 2(1+\cos 2x)$

$$PI = \underbrace{\frac{2}{D^2+3D+2}}_{1} + \underbrace{\frac{2\cos 2x}{D^2+3D+2}}_{2}$$

(1) $\dfrac{2e^{0x}}{D^2+3D+2} = \dfrac{2}{0+0+2} = 1$

(2) $\dfrac{2\cos 2x}{D^2+3D+2} = 2\left[\dfrac{\cos 2x}{-4+3D+2}\right] = 2\,\dfrac{\cos 2x}{3D-2}$

$2\dfrac{(3D+2)}{(3D-2)(3D+2)}\cos 2x = \dfrac{2(-6\sin 2x+2\cos 2x)}{9D^2-4}$

$= \dfrac{3\cos 2x - \cos 2x}{10}$

$\therefore y = c_1e^{-x} + c_2e^{-2x} + 1 + \dfrac{3\sin 2x - \cos 2x}{10}$

(8) $y'' + 16y = x\sin 3x$

$(D^2+16)y = 0 \qquad A\cdot E = m^2+16=0$

$m = \pm 4i \qquad y(CF) = c_1\cos 4x + c_2\sin 4x$

$\dfrac{x\sin x}{D^2+16} = \left[x - \dfrac{2D}{D^2+16}\right]\dfrac{\sin 3x}{D^2+16} \qquad D^2 = -9$

$= \left[x - \dfrac{2D}{D^2+16}\right]\dfrac{\sin 3x}{7}$

$= \dfrac{x\sin 3x}{7} - \dfrac{6\cos 3x}{49}$

$\therefore y = c_1\cos 4x + c_2\sin 4x + \dfrac{1}{49}(7x\sin 3x - 6\cos 3x)$

Solve by variation of parameters   ex 13.3 Pg 490 grewal

1) $\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}$

AE $m^2+1=0$ $m = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

G.S $y = A(x)\cos x + B(x)\sin x$

$A(x)\, y_1 + B(x)\, y_2$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$

$A = \int -\frac{y_2\,\phi(x)}{W}\,dx = -\int \frac{\sin x}{1}\,\frac{1}{1+\sin x}\,dx$

$A = -\int \frac{\sin x^{+1-1}}{1+\sin x}\,dx = \int -1 + \frac{1}{1+\sin x}\,\frac{1+\sin x}{1+\sin x}\,dx$

$A = -x + \int \frac{1+\sin x}{\cos^2 x}\,dx$

$A = -x + \int (\sec^2 x - \sec x \tan x)\,dx$

$A = -x + \tan x - \sec x + k_1$

$B = \int \frac{y_1\,\phi(x)}{W} = \int \frac{\cos x}{1+\sin x}\,\frac{(1-\sin x)}{1-\sin x}\,dx$

$= \int \frac{(1-\sin x)\cos x}{\cos^2 x} = \int \frac{1-\sin x}{\cos x}\,dx$

$$\int (\sec x - \tan x)\,dx -$$

$$\log(\sec x + \tan x) + \log(\cos x) + k_2$$

$\sec x = \frac{1}{\cos x}$

$\tan x = \frac{\sin x}{\cos x}$

$$B = \log\left(\frac{1+\sin x}{\cos x}\right) + \log(\cos x) + k_2$$

$$\therefore B = \log(1+\sin x) - \cancel{\log(\cos x)} + \cancel{\log(\cos x)} + k_2$$

$$B = \log(1+\sin x) + k_2$$

$$\therefore y = (-x + \tan x - \sec x + k_1)\cos x$$
$$+ (\log(1+\sin x) + k_2)\sin x$$

$$\therefore y = k_1 \cos x + k_2 \sin x - (x\cos x + 1)$$
$$+ \sin x \log(1+\sin x)$$

(2) $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$$ ex 13.2 pg 486 grewal

$$(D^2 + 3D + 2)y = e^{e^x}$$

AE $m^2 + 3m + 2 = 0$ $\quad m = -1, -2$

$$y_c = c_1 e^{-x} + c_2 e^{-2x}$$

g.s $$y = Ae^{-x} + Be^{-2x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}$$

$$A = \int -\frac{y_2\,\phi(x)}{W}\,dx = -\int \frac{e^{-2x} e^{e^x}}{-e^{-3x}}\,dx$$

$$= \int e^{x} e^{e^x}\,dx = \boxed{\begin{array}{l} e^x = t \\ e^x dx = dt \end{array}}$$

$$= \int e^t\,dt = e^t + k_1$$

$$A = e^{e^x} + k_1$$

$$B = \int \frac{y_1\,\phi(x)}{W}\,dx = \int \frac{e^{-x} e^{e^x}}{-e^{-3x}}$$

$$= -\int e^{2x} e^{e^x}\,dx$$

$$= -\int e^x e^x e^{e^x}\,dx = -\int t\,e^t\,dt$$

$$= B = -(te^t - e^t) + k_2$$

$$\therefore B = e^{e^x}(1 - e^x) + k_2$$

$$\therefore y = \left(e^{e^x} + k_1\right)e^{-x} + \left(e^{e^x}(1 - e^x) + k_2\right)e^{-2x}$$

$$\therefore y = k_1 e^{-x} + k_2 e^{-2x} + e^{-2x} e^{e^x}$$

(3)

$$y'' - 2y' + 2y = e^x \tan x$$

AE $\quad m^2 - 2m + 2 = 0 \qquad m = 1 \pm i$

$$Cf = e^x (c_1 \cos x + c_2 \sin x)$$

$$y = A e^x \cos x + B e^x \sin x$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x \cos x & e^x \sin x \\ e^x(-\sin x + \cos x) & e^x(\sin x + \cos x) \end{vmatrix}$$

$$= e^{2x}(\cos x \sin x + \cos^2 x) - e^{2x}(-\sin^2 x + \sin x \cos x)$$

$$= e^{2x}(\cos x \sin x + \cos^2 x + \sin^2 x - \sin x \cos x)$$

$$W = e^{2x}$$

$$A = \int \frac{-y_2 \phi(x)}{W} dx = -\int \frac{e^x \sin x \, e^x \tan x}{e^{2x}} dx$$

$$A = -\int \sin x \frac{\sin x}{\cos x} dx = -\int \frac{\sin^2 x}{\cos x} dx$$

$$A = -\int \frac{1 - \cos^2 x}{\cos x} dx = -\int \sec x - \cos x \, dx$$

$$A = -\{\log(\sec x + \tan x) - \sin x]$$

$$A = \sin x - \log(\sec x + \tan x) + k_1$$

$$B = \int \frac{y_1 \phi(x)}{W} dx = \int \frac{e^x \cos x \, e^x \tan x}{e^{2x}} dx$$

$$\int \sin x \, dx = -\cos x + k_2$$

$R_g = \left(\sin x - \log(\sec x + \tan x) + k_1\right) e^x \cos x$

$+ (-\cos x)\, e^x \sin x$

$y = e^x(k_1 \cos x + k_2 \sin x) - e^x \cos x \log(\sec x + \tan x)$

(4) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$ Ex 13.3 Pg 490 general

$AE = m^2 - 3m + 2 = 0$

$m = 1, 2$

$y_c = c_1 e^x + c_2 e^{2x}$

$y = A e^x + B e^{2x}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = e^{3x}$

$\phi(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\frac{1}{e^x}} = \frac{e^x}{e^x+1}$

$A = \int \frac{-y_2\,\phi(x)}{W}\,dx = -\int e^{2x} \frac{e^x}{1+e^x} \frac{1}{e^{3x}}\,dx$

$A = -\int \frac{1}{1+e^x}\,dx$ $e^x = t$ $e^x dx = dt$ $dx = \frac{dt}{t}$

$A = -\int \frac{1}{t+1}\frac{dt}{t}$ partial fraction or direct

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\therefore A = -\int \frac{1}{t} - \frac{1}{t+1}\,dt = \log(t+1) - \log t$$

$$= \log\left(\frac{t+1}{t}\right)$$

$$A = \log\left(\frac{e^x+1}{e^x}\right) + K_1$$

$$B = \int \frac{y_1\,\phi(x)}{w}\,dx = \int e^x \frac{e^x}{1+e^x} \frac{1}{e^{3x}}\,dx$$

$$B = \int \frac{dx}{e^x(e^x+1)} \qquad e^x = t$$

$$\therefore B = \int \frac{dt}{t^2(t+1)}$$

$$\frac{1}{t^2(t+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+1} \quad \text{partial fractions}$$

$$A = -1 \quad B = 1 \quad C = 1$$

$$\therefore \int \frac{1}{t^2(t+1)} = -\int \frac{dt}{t} + \int \frac{dt}{t^2} + \int \frac{dt}{t+1}$$

$$B = -\log t - \frac{1}{t} + \log(t+1) = \log\left(\frac{t+1}{t}\right) - \frac{1}{t}$$

$$B = \left(\log\left(\frac{e^x+1}{e^x}\right) - \frac{1}{e^x} + K_2\right)$$

y_2

$y = \underline{k_1 e^{x}} + k_2 e^{2x} + \log(1+e^{-x})(e^{x}+e^{2x}) \underline{- e^{x}}$

add

$y = k_1' e^{x} + k_2 e^{2x} + \log(1+e^{-x})(e^{x}+e^{2x})$

$k_1' = (k_1 - 1)$

Solve by method of undetermined co-efficients

1) $(D^2 - 2D)\, y = e^{x} \sin x$ Pg 490 ex 13.3 grewal

$m^2 - 2m = 0 \quad m = 0, 2$

$CF = c_1 e^{0x} + c_2 e^{2x} = c_1 + c_2 e^{2x}$

$PI = e^{x}(k_1 \sin x + k_2 \cos x) = y_p$ (say)

$y_p' = e^{x}(k_1 \cos x - k_2 \sin x) + e^{x}(k_1 \sin x + k_2 \cos x)$

$y_p' = e^{x}((k_1 - k_2)\sin x + (k_1 + k_2)\cos x)$

$y_p'' = e^{x}((k_1 - k_2)\cos x - (k_1 + k_2)\sin x) + e^{x}((k_1 - k_2)\sin x + (k_1 + k_2)\cos x)$

$y_p'' = e^{x}(k_1 - k_2 + k_1 + k_2)\cos x + (-k_1 - k_2 + k_1 - k_2)\sin x$

$y_p'' = e^{x}(2k_1 \cos x - 2k_2 \sin x)$

$y_p'' - 2y_p' = e^{x} \sin x$

$\Rightarrow e^{x}(2k_1\cos x - 2k_2\sin x) - 2e^{x}((k_1-k_2)\sin x$
$+ (k_1+k_2)\cos x) = e^{x}\sin x$

$e^{x}((2k_1 - 2k_1 - 2k_2)\cos x + (-2k_2 - 2k_1 + 2k_2)\sin x)$
$= e^{x}\sin x$

$-2k_2 e^{x}\cos x - 2k_1 e^{x}\sin x = e^{x}\sin x$

$-2k_2 = 0 \qquad -2k_1 = 1$

$k_2 = 0 \qquad k_1 = -\frac{1}{2}$

$PI = e^{x}\left(-\frac{1}{2}\sin x + 0\cdot\cos x\right) = -\frac{1}{2}e^{x}\sin x$

$\therefore y = c_1 + c_2 e^{2x} - \frac{1}{2}e^{x}\sin x.$

2) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x$

AE $m^2 + m - 2 = 0 \qquad m = 1, -2$

$CF = c_1 e^{x} + c_2 e^{-2x}$

$PI = k_1 x + k_0 + k_2\sin x + k_3\cos x = y_p$ (say)

$y_p' = k_1 + k_2\cos x - k_3\sin x$

$y_p'' = -k_2\sin x - k_3\cos x$

$y_p'' + y_p' - 2y_p = x + \sin x$

$-k_2 \sin x - k_3 \cos x + k_1 + k_2 \cos x - k_3 \sin x$

$- 2(k_1 x + k_0 + k_2 \sin x + k_3 \cos x) = x + \sin x$

$(-3k_2 - k_3)\sin x + (-3k_3 + k_2)\cos x - 2k_1 x$

$+ k_1 - 2k_0 = x + \sin x$

equating the co-efficients

① $-3k_2 - k_3 = 1$ $\qquad$ $-2k_1 = 1$

② $-3k_3 + k_2 = 0$ $\qquad$ $k_1 = \frac{-1}{2}$

$k_1 - 2k_0 = 0$

$-\frac{1}{2} - 2k_0 = 0$ $\qquad$ $k_0 = \frac{-1}{4}$

$-3k_2 - k_3 = 1$

$k_2 = 3k_3$

$-3(3k_3) - k_3 = 1$ $\qquad$ $-10k_3 = 1$

$k_3 = -\frac{1}{10}$ $\qquad$ $k_2 = \frac{-3}{10}$

$\therefore PI = -\frac{1}{2}x - \frac{1}{4} - \frac{3}{10}\sin x - \frac{1}{10}\cos x$

G.S $\qquad y = c_1 e^{x} + c_2 e^{-2x} - \frac{1}{2}x - \frac{1}{4} - \frac{3}{10}\sin x$

$- \frac{1}{10}\cos x$

3) $y'' + 4y = e^{-x} + x^2$

$m^2 + 4 = 0 \qquad m = \pm 2i$

$y_{c_f} = c_1 \cos 2x + c_2 \sin 2x$

PI let $y_p = k_3 e^{-x} + k_2 x^2 + k_1 x + k_0$

$y_p' = -k_3 e^{-x} + 2k_2 x + k_1$

$y_p'' = k_3 e^{-x} + 2k_2$

$\therefore \quad y_p'' + 4y_p = e^{-x} + x^2$

$k_3 e^{-x} + 2k_2 + 4(k_3 e^{-x} + k_2 x^2 + k_1 x + k_0)$
$= e^{-x} + x^2$

$5k_3 e^{-x} + 4k_2 x^2 + 4k_1 x + 2k_2 + 4k_0 = e^{-x} + x^2$

equating the co-efficients in the equation we get

$y_p'' + 4y_p = e^{-x} + x^2$

$k_3 e^{-x} + 2k_2 + 4(k_3 e^{-x} + k_2 x^2 + k_1 x + k_0)$
$= e^{-x} + x^2$

$5k_3 = 1 \qquad 4k_2 = 1 \qquad k_1 = 0 \qquad 2k_2 + 4k_0 = 0$

$k_3 = \frac{1}{5} \qquad k_2 = \frac{1}{4} \qquad 2\left(\frac{1}{4}\right) + 4k_0 = 0$

$k_0 = \frac{-1}{8}$

$PI = \frac{1}{5} e^{-x} + \frac{1}{4} x^2 - \frac{1}{8}$

G.S $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{5} e^{-x} + \frac{1}{4} x^2 - \frac{1}{8}$