Basics of computing

18/11/17
Saturday

Sum Rule - If an event can occur in m ways and another event can occur in n ways and if these events cannot occur simultaneously then one of the 2 events can occur in m+n ways.

Product Rule - If an event H is decomposed into 2 diff. tasks, then if an event A can occur in m ways and event B can occur in n diff. ways then the task H can be performed in m×n ways.

Q. A book shelf has 6 different English books, 8 diff French books & 10 diff German books. Find how many ways of selecting:-
(i) 3 books from each language
(ii) 1 book in any one of the lang.

A. (i) $6 \times 8 \times 10 = 480$
(ii) $6 + 8 + 10 = 24$

Q. Suppose a person has 3 shirts and 5 ties, how many diff. ways of choosing a shirt and a tie?

A. $3 \times 5 = 15$

Q. Suppose we wish to construct a sequence of 4 symbols in which first 2 are English letters, next two are single digit nos. If no letter or digit can be repeated. Find the no. of diff sequence we can construct?

A. | | | | |
26 25 10 9

$26 \times 25 \times 10 \times 9 = 58500$

Q. Suppose a restaurant sells 6 south Indian dishes, 4 North Indian, 3 hot drink & 2 cold drinks for a breakfast. A student wishes to buy 1 SI dish and 1 hot drink or 1 NI dish & 1 cold drink. In how many ways can he buy his breakfast?

A. $6 \times 3 + 4 \times 2$

$= 18 + 8 = 26$

Q. There are 20 married couples in a party. Find the no. of ways of choosing 1 woman & 1 man from the party s.t. the 2 are not married to each other.

A. $20 \times 19 = 380$

Q. There are 4 bus routes b/w A and B and 3 bus routes b/w B & C. Find the no. of ways a person can make a round trip from A to A through B & C with repeating the routes.

A. $4 \times 3 \times 4 \times 3 = 144$ (repetition allowed)

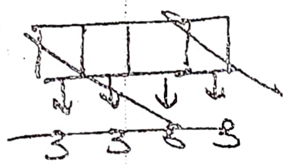
Q. A licence plate consists of 2 English letters which has only a, e, i, o, u followed by 4 digits. If repetitions are allowed then how many licence plates will have a e i o u and even digits.

5 5 5 5 5 5

5^6

~~12500~~

Find the no. of linear arrangement of 4 letters in BALL.



~~81~~

if there are repeated letters, then

$$\frac{n!}{n_1!\, n_2! \ldots}$$

n → no. of spaces

n_1 → no. of repeated letters

$$\frac{4!}{2!} = 12$$

Q. No. of ~~...~~ linear arrangement of 6 letters in PEPPER.

$$\frac{6!}{3!\,2!} = \frac{6\times\cancel{4}^{\,2\times5}}{\cancel{2}} = \underline{\underline{12}}\times5 = 60$$

Q. ~~Find the no.~~ DATABASES

$$\frac{9!}{3!\,2!} = 9\times8\times7\times6\times5\times\cancel{4}\,2 = 30240$$

Q. Find the no. of permutations of the letters of the word MASSASAUGA. In how many of these all 4 A's are together? How many of them begin with S.

(i) $\frac{10!}{3!\,4!} = 25200$

(ii) $\frac{7!}{3!} = 840$ — consider the 4 A's as 1 block

(iii) $\frac{9!}{2!\,4!} = 7560$

Q It is required to seat 5 men & 4 women in a row so that women occupy even places. How many such arrangements are possible?

A.

$\cancel{5\times4\times4\times}\ 5!\times4! = 2880$

$5\times4\times4\times3\times3\times2\times2\times1\times1$

Q. In how many ways can 6 men and 6 women be seated in a row

(i) if any person may sit next to any other

(ii) if men and women must occupy alternate seats.

A. (i) $12! = 479,001,600$

(ii) $6!\,6!\times2 = 1,036,800$

Q. 4 diff maths books, 5 diff cs books, 2 diff control theory books are to be arranged in a shelf. How many arrangements are possible

(i) if the books in each particular subject must be together? (ii) only math books must be together

A. (i) $4!\times5!\times2!\times3!$ → to arrange the blocks

(ii) $4!\times8!$

Q Suppose we are selecting r objects from a set of n, the set of r objects being selected is called combination of r —— which is represented by ${}^nC_r = \frac{n!}{(n-r)!\,r!}$.

Q An air hostess is having a dinner party for some members of her committee. Because of the size of her home, she can invite only 11 out of 20.

How many possibilities are there?

$$^{20}C_{11} = \frac{20!}{11!\,9!} = 167,960.$$

Q. A gym teacher must select 9 girls from seniors and juniors if there are 25 seniors and 28 juniors.
(i) How many selections can be done?
(ii) if 2 juniors & 1 senior must be in a team
(iii) team must have 4 juniors and 5 seniors.

A. (i) $^{53}C_9$

(ii) $\cancel{^{25}C_1 \times {}^{28}C_2}$ $^{50}C_6$ =

(iii) $^{25}C_5 \times {}^{28}C_4$

Q. A gym teacher must make 4 volley balls team of 9 girls each from 36 girls in how many ways can she select these 4 teams.

A. $^{36}C_9 \times {}^{27}C_9 \times {}^{18}C_9 \times {}^{9}C_9$

=

Q. Find out all ~~combinations~~ arrangements of the word.
TALLAHASSEE and how many should not have alternate adjacent A's.

A. (i) $\frac{11!}{3!\,2!\,2!\,2!}$

T L L H S S E E

(ii) $\cancel{\frac{9!}{2!\,2!\,2!}}$

$\frac{8!}{2!\,2!\,2!} \times {}^{9}C_3$

Q A woman 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following ways:-
(i) there are no restrictions on choices
(ii) 2 particular persons will not attend separately.
(iii) 2 ——————————————— together.

A. (i) $^{11}C_5$

(ii) $^{9}C_{3} + {^9}C_5$

(iii) $^{9}C_5 + {^{10}}C_5$ $\quad$ $2 \times {^9}C_4 + {^9}C_5$

378 $\quad$ 378

Combination with repetition

21/11/17
Tuesday

Q. 3 scoops of ice cream from 5 flavours

b	c	d	s	v

$^{r+n-1}C_r$

c c c → 000 → → →
b d v 0 → → 0 → → 0
b v v 0 → → → 0 → 00

Q A donut shop offers 20 kinds of donuts assuming that there are atleast a dozen of each kind. In how many ways we can select a dozen of donuts?

$C^{20+12-1}_{12} - C^{31}_{12} = 141,120,525$

Q In how many ways we distribute 7 bananas & 6 oranges among 4 children so that each child recieves atleast 1 banana.

selecting r items for a seln

$$C^{3+4-1}_{3} \times C^{6+4-1}_{6}$$

$20 \times 84 = 1680$

Q A message is made up of 12 different symbols and is to be transmitted through a comm. channel. In add. to 12 symbols, the transmitter will also send 45 blank spaces b/w the symbols with atleast 3 spaces b/w each pair of consecutive symbols. In how many ways can transmitter send such a mssg?

A. $12! \times C^{(11+12-1)}_{12} = 12! \times C^{22}_{12} = \underline{3.0974 \times 10^{14}}$

There are 12! ways to arrange 12 diff. symbols and for each of these arrangements, there are 11 positions b/w the 12 symbols because there must be atleast 3 spaces b/w successive symbols, we use and must now locate the upto 33 of 45 spaces and remaining 12 spaces. This is now selecti with repetiti of size 12 from a collecti of size 11.

Q Consider the following prog. segment where i, j, k are the integer variables.

```
for i = 1 to 20 do
for j = 1 to i do
for k = 1 to j do
print (i * j * k);
```

How many times the print statement is executed?

i 1 20
j 1 i
k 1 j

$1 \times (2+1) \times 1$ (

selecti of 3 loops 20 tim

$1 \le k \le j \le i \le 20$

$${}^{20+3-1}C_3 = {}^{22}C_3 = 1540$$

Q. Determine all integer solution of the equation $x_1 + x_2 + x_3 + x_4 = 7$, $x_i \geq 0 \ \forall\ 1 \le i \le 4$

A. Selecti of 7 items from a set of 4.

$${}^{7+4-1}C_7$$

Q. In how many ways one can distribute 10 marbles amongst 6 different containers.

A. $${}^{10+6-1}C_{10} = {}^{15}C_{10} = 3003$$

16 mark $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$

Q. Determine the no. of integer solution of $x_1 + x_2 + x_3 + x_4 = 32$

a) $x_i \geq 0$ $1 \le i \le 4$

b) $x_i > 0$ $1 \le i \le 4$

c) $x_1, x_2 \geq 5$ $x_3, x_4 \geq 7$

d) $x_i \geq 8$ $1 \le i \le 4$

A. a) C^{32+4-1}_{32}

b) $- (4+5+3)$

c) $C^{28+4-1}_{28} = 4495$

$n = 4 \quad r = 8$

x_1	x_2	x_3	x_4
5	5	7	7

$\Rightarrow$ these values already assigned. the rest is to be distributed

d) 1

Q. In how many ways 10 marbles be distributed among 5 children.
(i) no restriction
(ii) if each child gets atleast 1 marble

C^{5+5-1}

A. (i) $C^{10+5-1}_{10} = 1001$

(ii) $C^{5+5-1}_{5} = 126.$

$(x+y)^n = \sum_{r=1}^{n} {}^nC_{n-r}\, x^n \quad {}^nC_r\, y^{n-r}$

Binomial theorem

Q. Find the binomial coeff of x^5 & y^2 in $(x+y)^7$

A. $(x+y)^7 = \sum_{r=0}^{7} {}^7C_{7-r}\, x^{7-r} y^r$

for x^5 $\quad 7-r=5$
$r=2$

${}^7C_5 = 14$

for y^2 $\quad r=2 \quad {}^7C_5 = 14$

Q coeff of a^5 & b^2 in the exp. of $(2a-3b)^7$

A. $(2a-3b)^7 = \sum_{r=0}^{n} {}^nC_{n-r}\,(2a)^{n-r}(-3b)^r$

$7-r=5$

$r=2$

${}^7C_5 \times 2^5(-3)^2 = 14\times$

23/11/17
Thursday

Q $(x+y)^4$

$\sum_{j=0}^{4}\binom{4}{j}x^{4-j}y^j$

Q $x^{12}y^{13}$ in $(x+y)^{25}$

$\binom{25}{12} = 5200300$

Q. Explain multinomial theorem.

For the integers, n, t the coeff. of $x_1^{n1}, x_2^{n2}, x_3^{n3} \dots x_t^{nt}$

in the expansion $(x_1+x_2+\dots+x_n)^n$ is

$\dfrac{n!}{n_1!\,n_2!\dots n_t!}$ - where each n_i is an integer with $0\le n_i\le n \;\forall\; 1\le i\le t$ and.

$n_1+n_2+\dots n_t = n$

Q Find the coef. of $x^2y^2z^3$, xyz^5 and x^2z^4 in the expansion of $(x+y+z)^7$

A $x^2y^2z^3$ $\binom{n}{n_1\,n_2\,n_3} = \binom{7}{2\;2\;3} = \dfrac{7!}{2!\,2!\,3!} = 210$

$xy^5 \quad \binom{7}{1\ 1\ 5} \quad \frac{7!}{5!1!1!} = 42.$

$x^3z^4 \quad \binom{7}{3\ 0\ 4} = \frac{7!}{3!4!} = 35.$

Q. find the term which contains $x^{11}y^4$ in the exp. of $(2x^3 - 3xy^2 + z^2)^6$

A. general term $\binom{6}{n_1\ n_2\ n_3}(2x^3)^{n_1}(-3xy^2)^{n_2}(z^2)^{n_3}$

$\binom{6}{n_1\ n_2\ n_3} 2^{n_1} x^{3n_1+n_2} (-3)^{n_2} y^{2n_2} z^{2n_3}$

$3n_1 + n_2 = 11 \qquad n_1 = 3$

$2n_2 = 4 \qquad n_2 = 2$

$2n_3 = 0 \qquad n_3 = 0$] no need to eliminate z, can be given a power

$\binom{6}{3\ 2\ 0} 2^3 x^{11}\ 9\ y^4 z^0 = \frac{6!\times 2^3\times 9}{3!\,2!} x^{11}y^4$

$= \underline{4320}\, x^{11}y^4$

Q. xyz^2 in $(2x - y - z)^4$

$\binom{4}{n_1\ n_2\ n_3}(2x)^{n_1}(-y)^{n_2}(-z)^{n_3}$

$n_1 = 1 \quad n_2 = 1 \quad n_3 = 2.$

$\frac{-4!}{1!1!2!}\, 2\, xyz^2 = \underline{-24xyz^2}$

Q. $a^2 b^3 c^2 d^5$ $(a+2b-3c+2d+5)^{16}$.

$$\binom{16}{n_1\, n_2\, n_3\, n_4\, n_5} \cdot a^{n_1} (+2b)^{n_2} (-3c)^{n_3} (2d)^{n_4} 5^{n_5}$$

$n_1 = 2$ $n_2 = 3$. $n_3 = 2$ $n_4 = 5$.

$n_5 = 16 - 12 = 4$

$$\frac{16!}{2!\,3!\,2!\,5!\,4!} \quad 2^3 (-3)^2\, 2^5\, 5^4\, a^2 b^3 c^2 d^5$$

$= 4.3589 \times 10^{14} a^2 b^3 c^2 d^5$

Principle of Inclusion and exclusion

$|A \cup B| = |A| + |B| - |A \cap B|$ for inclusion.

Consider a sets with cardinality of $S = n$ and the conditions c_i st. $1 \le i \le t$ satisfied by some elements of S, the no. of elements of sets that satisfy none of the conditions c_i is denoted by $\bar{N} = N(\bar{C_1}\bar{C_2}\bar{C_3} \ldots \bar{C_t})$, given by the formula

$$\bar{N} = N - [N(C_1) + N(C_2) + \ldots + N(C_t)] +$$
$$[N(C_1C_2) + N(C_1C_3) + \ldots + N(C_1C_t) \ldots$$
$$\ldots + N(C_{t-1}C_t)] - [N(C_1C_2C_3) + N(C_1C_2C_4) + \ldots$$
$$\ldots + N(C_{t-2}C_{t-1}C_4)] + \ldots (-1)^t N(C_1C_2 \ldots C_t)].$$

OR

$\bar{N} = N - \sum_{1 \le i \le t} N(C_i) + \sum_{1 \le i \le j \le t} N(C_i C_j) + \ldots$

$\ldots + \sum_{1 \le i \le j \le k \le t} N(C_i C_j C_k) + \ldots\ldots (-1)^t N(C_1 C_2 \cdots C_t)$

$N(C_1) = |A|$

$N(C_2) = |B|$

$N(C_1 C_2) = |A \cap B|$

$|\bar{A} \cap \bar{B}| = N - |A \cup B|$

Q: Among the student in a hostel 12 students study maths, 20 study chem & 8 study biology, 20 physics.

$|A| = 12$ $\quad |A \cap B| = 5$ $\quad |B \cap D| = 4$

$|B| = 20$ $\quad |A \cap C| = 7$ $\quad |C \cap D| = 3$

$|C| = 20$ $\quad |A \cap D| = 4$ $\quad |A \cap B \cap C| = 3$

$|D| = 8$ $\quad |B \cap C| = 16$ $\quad |A \cap B \cap D| = 2$

$|\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| = 71$

$|B \cap C \cap D| = 2$ $\quad |A \cap B \cap D \cap C| = 2$

$|A \cap C \cap D| = 3$ $\quad$ Find total no. of student.

A: $N = |\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| + |A \cup B \cup C \cup D|$

$= 71 + 12 + 20 + 20 + 8 - 5 - 7 - 4 - 16 - 4 - 3 + 3 + 2 + 2 + 3 - 2$

$= 100$

Q. How many integers between b/w 1 & 300
(i) divisible by atleast one of
5, 6, 8 (ii) none of 5, 6, 8.

A $|A \cup B \cup C| = S_1 - S_2 + S_3 \cdots$
$S_1 = \Sigma N(c_i)$, $S_2 = \Sigma N(c_i c_j)$, $S_3 = \Sigma N(c_i c_j c_k)$

$S_1 = N(C_1) + N(C_2) + N(C_3)$
$= 60 + 50 + 37$

$S_2 = N(C_1C_2) + N(C_1C_3) + N(C_2C_3)$
$= 10 + 12 + 7$

$S_3 = N(C_1C_2C_3) = 2$

$N = 120$

(ii) $300 - 120 = 180$

27/11/17
Monday

Q. Determine the no. of +ve integers b/w $1 \le i \le 100$ which are not divisible by 2, 3 or 5.

A. 26

$N(C_1) = 50$
$N(C_2) = 33$
$N(C_3) = 20$

$N(C_1C_2) = 16$
$N(C_2C_3) = 6$
$N(C_1C_3) = 10$
$N(C_1C_2C_3) = 3$

$\overline{N} = 100 - (50 + 33 + 20 - 16 - 6 - 10 + 3)$
$= 26$

Q. In how many ways can 26 letters of an alphabet be permuted so that none of the pattern car, dog, pun or byte occurs

A. $N = 26!$

$N(C_1) = (23+1)!$ considering car as 1 letter & removing it from the 26 letters

$N(C_2) = 24!$

$N(C_3) = 24!$

$N(C_4) = 23!$

$N(C_1C_2) = (26-6+2)! = 22!$

$N(C_2C_3) = 22!$

$N(C_3C_4) = 21!$

$N(C_1C_3) = 22!$

$N(C_1C_4) = 21!$

$N(C_2C_4) = 21!$

$N(C_1C_2C_3) = 20!$ 26-9+3

$N(C_2C_3C_4) = 19!$

$N(C_1C_2C_4) = 19!$

$N(C_1C_2C_3C_4) = 17!$

$\overline{N} = N - |A \cup B \cup C \cup D|$

$= N - [24! \times 3 + 23! - (3 \times 22! + 3 \times 21!)$

$+ 19! \times 2 + 20! - 17!]$

$= 4.014 \times 10^{26}$

Q. Find the no. of permutations from a to z from which none of the patterns spin, game, path or net occurs.

A. $N = 26!$

$N(C_1) = 23!$ $\qquad$ $N(C_1 C_2) = 20!$

$N(C_2) = 23!$ $\qquad$ $N(C_1 C_3) = 0 \Rightarrow$ when letters are repeated

$N(C_3) = 23!$ $\qquad$ $N(C_1 C_4) = 0$

$N(C_4) = 24!$ $\qquad$ $N(C_2 C_3) = 0$

$N(C_2 C_4) = 0$

$N(C_3 C_4) = 0$

$N(C_1 C_2 C_3) = 0$

$N(C_2 C_3 C_4) = 0$ $\qquad$ $N(C_1 C_2 C_3 C_4) = 0$

$N(C_1 C_3 C_4) = 0$

$\bar{N} = 26! - (23! \times 3 + 24! - 20!)$

$= 4.0259 \times 10^{26}$

Q. Determine no. of permutations of the letters JNUISGREAT such that none of the words JNU, IS, GREAT occur as consecutive letters such as TSJNUGREAT, UNJGREATIS, JNUTUISGREA etc are not allowed

$N = 10!$

$N(C_1) = 8!$ $\quad N(C_1C_2) = 7!$ $\quad N(C_1C_2C_3) = 3!$

$N(C_2) = 9!$ $\quad N(C_2C_3) = 5!$

$N(C_3) = 6!$ $\quad N(C_1C_3) = 4!$

$$\overline{N} = 10! - (8! + 9! + 6! - 7! - 5! - 4! + 3!)$$

$= 3230058$

Q Find the no. of non-negative integer soln. of the eq. $x_1 + x_2 + x_3 + x_4 = 18$ s.t. $x_i \leq 7$ for $1 \leq i \leq 4$.

A. $N = {}^{18+4-1}C_{18}$.

$x_i \geq 8$

$N(C_1) = x_i \geq 8$

$= (18-8)$ $\quad {}^{10+4-1}C_{10}$

$N(C_2) =$

$N(C_3) =$

$N(C_4) =$ $\quad {}^{13}C_{10}$

$N(C_1C_2) = N(C_2C_3) = N(C_3C_4) = N(C_1C_3) = N(C_2C_4)$
$N(C_1C_4)$
$= {}^5C_2$

$N(C_1C_2C_3) = N(C_2C_3C_4) = N(C_1C_3C_4) = 0$
$N(C_1C_2C_3C_4) = 0$.

$$\overline{N} = {}^{21}C_{18} - (4 \times {}^{13}C_{10} - 6 \times {}^{5}C_2)$$

$= 246$

$\therefore$ 246 solutions are there for the given condition

Q. Determine how many integer solutions are there
$x_1 + x_2 + x_3 + x_4 = 19$

a) $0 \le x_i$ for all $1 \le i \le 4$

b) $0 \le x_i \le 8$ for all $i \le i \le 4$

A. a) $^{19+4-1}C_{19} = 1540$

b) $N = 1540$

$N(C_1) = {}^{13}C_{10} = N(C_2) = N(C_3) = N(C_4)$

$N(C_1C_2) = N(C_2C_3) = N(C_3C_4) = N(C_1C_3) = N(C_1C_4)$
$= N(C_2C_4) = {}^4C_1$

$N(C_1C_2C_3) = N(C_2C_3C_4) = N(C_1C_3C_4) = 0$

$N(C_1C_2C_3C_4) = 0$

$\overline{N} = 1540 - [4 \times {}^{13}C_{10} - 6 \times {}^4C_1]$
$= 372$

Q.① 6 married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband.

Q.② In certain areas of the countryside there are 5 villages. An engg. is to devise a system of 2 way roads so that after the system is completed no village will be isolated. In how many ways can he ~~do~~ this?

① 12 people

$N(C_i)$ = ith couple is sealed together

$= 10!$

$N(C_1) = 10!$ $N(C_3) = N(C_4) = N(C_5) = N(C_6)$

$S_1 = 6 \times 10!$

$11!$

$N(C_2) = 9!$ $S_2 = 15 \times 9!$

$N(C_1C_2C_3) = 8!$ $S_3 = 20 \times 8!$

$N(C_1C_2C_3C_4) = 7!$ $S_4 = 15 \times 7!$

$N(C_1C_2C_3C_4C_5) = 6!$ $S_5 = 6 \times 6!$

$N(C_1C_2C_3C_4C_5C_6) = 5!$ $S_6 = 5!$

$\bar{N} = [\ldots 2\,6 \times 10! - 2\,15 \times 9! + 2\,20 \times 8! - 2\,15 \times 7! + 2\,6 \times 6! - 5!\,]$

12

$10! \times 2$

$09! \times 2^5$

②

$N(C_1) = {}^4C_2$

$S_1 = 5 \times {}^4C_2$

$N(C_1C_2) = {}^3C_2$ $S_2 = 10 \times {}^3C_2$

$S_3 = 10$

$S_4 = 0$

$N(C_1C_2C_3C_4C_5) = 0$ $S_5 = 0$

$\bar{N} = \ldots - [\ldots + 10]$

$= 26$

$\bar{N} = 26$

28/11/17
Tuesday

Generalization of principal

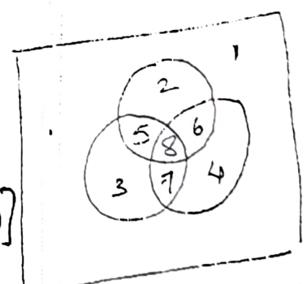
keywords: exactly, atleast.

Consider a set S with cardinality of $|S| = N$ with conditions $C_1, C_2 \ldots C_t$ satisfied by some of the elements of S. The inclusion & exclusion provides a way to determine $N(\overline{C_1}\,\overline{C_2} \ldots \overline{C_t})$ i.e. no. of elements in S that satisfies none of the t conditions

Now we want to determine E_m which denote no. of elements in S that satisfies exactly m of t conditions

$$E_1 = N(C_1 \overline{C_2}\,\overline{C_3} \ldots \overline{C_t}) + N(\overline{C_1} C_2 \overline{C_3} \ldots \overline{C_t}) \ldots + N(\overline{C_1}\,\overline{C_2} \ldots \overline{C_{t-1}} C_t)$$

$$E_2 = N(C_1 C_2 \overline{C_3} \ldots \overline{C_t}) + N(\overline{C_1} C_2 C_3 \overline{C_4} \ldots \overline{C_t}) + \ldots + N(\overline{C_1}\,\overline{C_2} \ldots C_{t-1} C_t)$$



$$E_1 = N(C_1) + N(C_2) + N(C_3) - 2[N(C_1C_2) + N(C_2C_3) + N(C_1C_3)] + 3N(C_1C_2C_3)$$

$\Rightarrow$ $\boxed{E_1 = S_1 - 2S_2 + 3S_3}$

$\boxed{E_2 = S_2 - 3S_3}$

for $t = 3$

count elements in region 5, 6, 7 twice & and those in region 8 thrice.

six times

$$\boxed{E_m = S_m - \binom{m+1}{1} S_{m+1} + \binom{m+2}{2} S_{m+2} - \cdots + (-1)^{t-m}\binom{t}{t-m} S_t}$$

for $t=4$,

$$E_1 = E_1 - \binom{2}{1} S_2 + \binom{3}{2} S_3 - \binom{4}{2} S_4$$

$$\boxed{L_m = S_m - \binom{m}{m-1} S_{m+1} + \binom{m+1}{m-1} S_{m+2} - \cdots (-1)^{t-m}\binom{t-1}{m-1} S_t}$$

(least) is atleast 2 conditions

Q. In how many ways can the letters in the word ARRANGEMENT be arrayed so that

(i) there are exactly 2 pairs of consecutive identical letters

(ii) atleast ... letters

A. (i) $N = \dfrac{11!}{2!\,2!\,2!\,2!}$

$N(C_1) = 2$ consecutive A's $= \dfrac{10!}{(2!)^3}$

$N(C_2) =$ R's $= \dfrac{10!}{(2!)^3}$

$N(C_3) =$ E's $= \dfrac{10!}{(2!)^3}$

$N(C_4) =$ N's $= \dfrac{10!}{(2!)^3}$

$S_1 = \dfrac{10!}{(2!)^3} \times 4$

$N(C_1C_2) = \frac{9!}{(2!)^2}$

$N(C_1C_3) =$

$N(C_1C_4) =$

$N(C_2C_3) =$

$N(C_2C_4) =$

$N(C_3C_4) =$

$S_2 = 6 \times \frac{9!}{(2!)^2}$

$N(C_1C_2C_3) = 8!/2!$

$N(C_1C_2C_4) =$

$N(C_2C_3C_4) =$

$S_3 = 3 \times \frac{8!}{2!}$

$N(C_1C_2C_3C_4) = 7! = S_4$

(i) $E_2 = S_2 - \binom{3}{1} S_3 + \binom{4}{2} S_4$

$= \cancel{282120}\ 544320 - 3 \times 60480 + \cancel{4}\ 6 \times 5040$

$= \cancel{126960}\ 332640$

(ii) $L_m = S_2 - \binom{2}{1} S_3 + \binom{3}{2} S_4$

$= \cancel{272160}\ 544320 - 2 \times 60480 + 3 \times 5040$

$= \cancel{166320}\ 398160$

Q. In how many ways can one arrange the letters in the word C~~O~~R~~R~~E~~S~~PO~~N~~DE~~N~~TS so that
(i) there is no pair of consecutive identical letters
(ii) there are exactly 2 pair ————
(iii) atleast 3 pair of consecutive ————

A. $N = \dfrac{14!}{2!2!2!2!2!}$

$$\left.\begin{array}{l} N(C_1) \\ N(C_2) \\ N(C_3) \\ N(C_4) \\ N(C_5) \end{array}\right\} = \frac{13! \times 5}{(2!)^4}$$

$$\left.\begin{array}{l} N(C_1C_2) \\ N(C_1C_3) \\ N(C_1C_4) \\ N(C_1C_5) \\ N(C_2C_3) \\ N(C_2C_4) \\ N(C_2C_5) \\ N(C_3C_4) \\ N(C_3C_5) \\ N(C_4C_5) \end{array}\right\} = \frac{12! \times 10}{(2!)^3}$$

${}^5C_3$ $\quad \frac{5!}{3!2!}$

$$\left.\begin{array}{l} N(C_2C_4C_5) \\ N(C_1C_4C_5) \\ N(C_1C_2C_3) \\ N(C_1C_2C_4) \\ N(C_1C_2C_5) \\ N(C_2C_3C_4) \\ N(C_2C_3C_5) \\ N(C_3C_4C_5) \end{array}\right\} S_3 = \frac{11! \times 10}{(2!)^2}$$

$$\left.\begin{array}{l} N(C_1C_2C_3C_4) \\ N(C_1C_2C_3C_5) \\ N(C_2C_3C_4C_5) \\ N(C_1C_3C_4C_5) \\ N(C_1C_2C_4C_5) \end{array}\right\} = \frac{10! \times 5}{(2!)}$$

$N(C_1C_2C_3C_4C_5) = 9!$

(i) $\overline{N} = N - [S_1 - S_2 + S_3 - S_4 + S_5]$

$$= \frac{14!}{(2!)^5} - \left[\frac{5 \times 13!}{(2!)^4} - \frac{10 \times 12!}{(2!)^3} + \frac{10 \times 11!}{(2!)^2} - \frac{5 \times 10!}{(2!)} + 9!\right]$$

$= 1286046720.$

(ii) $E_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 - \binom{5}{3}S_5$

$= 350,179,200.$

(iii) $L_3 = S_3 - \binom{3}{2}S_4 + \binom{4}{2}S_5$

$= 74753280.$

~~Pigeonhole Principle~~

Q. In how many ways can one arrange all of the letters in the word information so that no pair of consecutive letters occurs more than once.
(Here we want to count the arrangement such as (IINNOOFRMTA, FORTMAIINON) but not
(INFORINMOTA (where IN occurs twice)
NORTFNOIAM (.. NO ....)

N N
I O O

A.
N
O
I

Pigeon hole Principle

If m pigeons occupy n pigeonholes and $m > n$, then atleast 1 pigeonhole must contain 2 or more pigeons in it.

General

If m pigeons occupy n ph then atleast 1 pigeonhole must contain $p+1$ or more pigeons where $p = \frac{m-1}{n}$

Q. A bag contains 12 pairs of socks (each pair in diff. colour). If a person draws the socks 1 by 1 at random, determine atmost how many draws are required to get atleast 1 pair of matching socks.

A. 13

Q. Prove that in any set of 29 persons atleast 5 persons must have been born on the same day of the week

A. no. of days = 7

~~every~~

$m = 29$ (5)

$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = 5$

$\left\lfloor \frac{28}{n} \right\rfloor = 4$

$n = 7$

Rearrangement Dearrangement

30/11/17
Thursday

A permutation of n distinct objects in which none of the objects is in its natural place is called dearrangement.

Q. For eg. permutation of integers of 1 to n which 1 is not in 1st place, 2 is not in 2nd place & so on. Find the no. of dearrangement of 1, 2, 3, 4.

$d_4 = 4!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\}$

$= 24(1 - 1 + 1/2 - 1/6 + 1/24)$

$= 12 - 4 + 1 = \underline{\underline{9}}$

$$\boxed{d_n = n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots \frac{(-1)^n}{n!}\right\} = n! \times \sum_{k=0}^{n} \frac{(-1)^k}{k!}}$$

(write all)

2413

3421

Q. Find d_5, d_6, d_7 & d_8.

A. $d_5 = 5!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right\} = 44$

$d_6 = 265$

$d_7 = 1854$

$d_8 = 14833$.

Rook Polynomial

formula:

$$\boxed{r(C, x) = 1 + r_1 x + r_2 x^2 + r_3 x^3 + \dots r_n x^n}$$

Q.

1	2	3
4		
	5	6

$r(C, x) = 1 + 6x + 8x^2 + 2x^3$

r_2

$\{1, 5\}$ $\{2, 4\}$ $\{3, 4\}$ $\{4, 2\}$ $\{4, 5\}$.

$\{1, 6\}$ $\{2, 6\}$ $\{3, 5\}$ $\{4, 5\}$ $\{4, 6\}$

r_3

~~{1, 5, 6}~~ $\{3, 4, 5\}$ $\{2, 4, 6\}$

Q.

1	2
3	4

$r(c, x) = 1 + 4x + 2x^2$

non taking rooks = not present in same row or column.

$r_0 = 1$ (always)

$r_1 = 1$ non taking rooks

$r_2 = 2$ ______

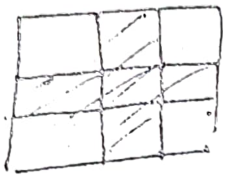
$r_3 = 3$ ______

Q. Find the rook polynomial of 3×3 chessboard.

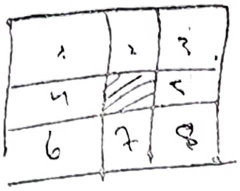
Expansion formula. $r(C,x) = x\,r(D,x) + r(E,x)$.

mark a star as you wish
D board is made by removing that row & column.

→ removing the cell with star



D



E

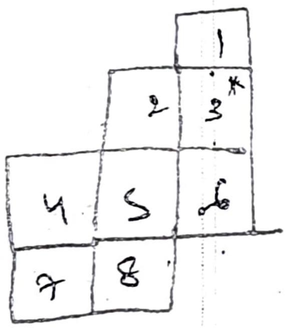
$r(E,x) = 1+8x+14x^2 + 4x^3$

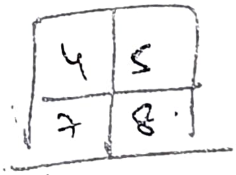
$r(D,x) = 1+4x+2x^2$

$r(C,x) = 1+8x+14x^2+4x^3 + x+4x^2+2x^3$
$= 1+9x+18x^2+6x^3$

(1,2) (1,4) (1,5) (1,7) (1,8) (1,5,7), (2,4,8)
(2,4) (2,6) (2,7) (4,8)
(5,2) (6,7) (6,4) (3,4,7) (6,2,5)

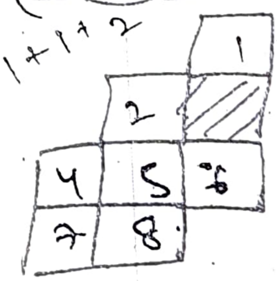
Q.





D

5+3+1+1+2



$r(C,x) = 1+4x+2x^2$

$r(C,x)$
$= 1+7x+12x^2 + 5x^3$

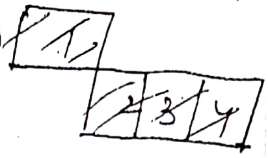
(1,2,4) (1,4,8)
(1,2,7) (2,6,7) (1,5,7)

$r(C,x) = 1+8x+16x^2+7x^3$

Q. Find the rook polynomial for the shaded chess boards C_1 & C_2.

(i) 

C_1

(ii) 

C_2

$r(C,x) = 1 + 4x + 3x^2$ $\qquad$ $r(C,x) = 1 + 4x + \cancel{6x} \; 3x^2$

Q. In making seating arrangement for their son's wedding reception Grace & Nick are down to 4 relatives R_i, $1 \le i \le 4$ who do not get along with one another. There is a single open seat at each of the 5 tables T_j $1 \le j \le 5$. Because of the family differences

- R_1 will not sit at T_1 or T_2. $\quad {}^4P_3$
- R_2 ______ T_2
- R_3 ______ T_3 or T_4
- R_4 ______ T_4 or T_5

How many no. of arrangement can be done?

using principle of inclusion & exclusion

A.

	T_1	T_2	T_3	T_4	T_5
R_1	X	X	1	2	3
R_2	4	X	5	6	7
R_3	8	9	X	X	10
R_4	11	12	13	X	X

~~[illegible]~~

$N = {}^5P_4 = 5!$

$N(C_1) = 2 \times 4!$

$N(C_2) = 4!$

$N(C_3) = 2 \times 4!$

$N(C_4) = 2 \times 4!$

$N(C_1 C_2) = \cancel{[illegible]} \; 3!$

$N(C_1 C_3) = 4 \times 3!$

i) R_1 at T_1 R_3 at T_3

(ii) R_1 at T_2 R_3 at T_3

(iii)

(iv)

$N(C_3 C_4) = 3 \times 3!$

$N(C_2 C_4) = 3 \times 3!$

$N(C_2 C_3) = 3 \times 3!$

$N(C_1 C_2 C_3) = 2 \times 2!$

A. $= 25$.

$1 + 7x + 16x^2 + 13x^3$
$+ 3x^4$