

Unit - 4

Electrical conduction in metals and semiconductors

Metals are good electrical and thermal conductors. The outermost electrons in an atom are loosely bound and are readily shared with other atoms to form metallic bond. Some of these valence electrons are free to move within the physical dimensions of the metal. These free electrons are called conduction electrons because they are responsible for electrical and thermal conduction in metals.

Classical free electron theory (Drude-Lorentz theory):

- **Assumption of classical free electron theory:**

1. Metals contain a large number of free electrons which move freely through the entire volume of the metal.
2. The free electrons are treated as equivalent to molecules of an ideal gas and thus obey the laws of kinetic theory of gases. The kinetic energy associated with the each electron at a temperature T is given by

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$$

where k is the Boltzmann's constant and v_{th} is the thermal velocity of free electrons.

3. The electric potential due to positive ionic cores is considered to be constant inside the metal.
4. The Coulomb repulsion between the free electrons is considered to be negligible.
5. Electric current flows in a metal in the presence of an external electric field due to drift velocity of the free electrons in a direction opposite to the applied field.

Definitions of some terminology:

- **Mean collision time (τ):** The average time taken between two consecutive collisions of an electron with the lattice points (ionic cores) is called mean collision time. It is given by $\tau = \lambda/v_{th}$; where λ is the mean free path.
- **Mean free path (λ) :** The average distance travelled by the conduction electrons between two successive collisions is known as mean free path. $\lambda = \tau v_{th}$.

- **Relaxation time (τ_r):** The time required for the average velocity of the conduction electrons to exponentially decay to $1/e$ times its value just when the electric field is turned off, is called relaxation time.

Note: If the probability of scattering is same in all directions, then the scattering is called **isotropic**. For isotropic scattering, relaxation time is same as mean collision time (i.e. $\tau_r = \tau$).

- **Current density (J) :** is the current per unit area of cross section of an imaginary plane held normal to the direction of current in a current carrying conductor. If I is the current, A is the area of cross section then current density $J = I/A$; unit A/m^2
- **Electric field (E):** It is the potential drop/unit length of a conductor. If L is the length of a conductor of uniform cross section and V is the potential difference between its two ends, then electric field, $E = V/L$.
- **Drift velocity (v_d):** The constant average velocity of free electrons in the steady state in an applied electric field is called the drift velocity v_d .

If no electric field is applied, the free electrons in a conductor move in random directions. Since the motion is completely random, average velocity in any direction is zero. When an electric field is established, the electrons experience a force $F = -eE$ due to which they move in opposite direction to the applied field. These electrons undergo frequent collisions with positive ions. In each such collision, direction of motion of electrons undergoes random changes. As a result, the electrons are subjected to a very slow directional motion called drift motion and the constant average velocity of this motion is called drift velocity (v_d).

➤ Expression for drift velocity:

In the presence of an electric field, the conduction electron acquires a constant velocity called the drift velocity in the steady state.

If m is the mass of the electron, v_d the drift velocity, τ the mean collision time, then the resistive force ' F_r ' offered for the motion of electrons is

$$F_r = m \frac{v_d}{\tau}$$

If E the strength of the applied field, then the driving force acting on the electron is,

$$F = eE$$

In steady state, the force due to applied field is balanced by the resistive force.

$$\text{i.e. } F_r = F$$

$$(\text{or}) \quad m \frac{v_d}{\tau} = eE$$

$$\therefore v_d = \left(\frac{e\tau}{m} \right) E$$

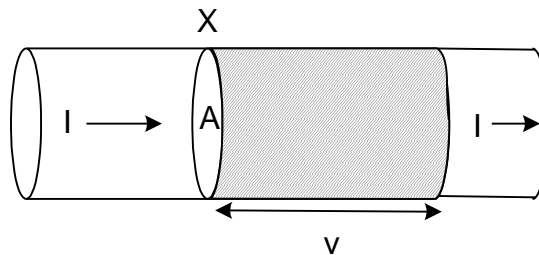
- **Mobility of electrons (μ):** The magnitude of the drift velocity acquired by the electrons in unit electric field is called the mobility of electrons, $\mu = \frac{v_d}{E}$; unit: $\text{m}^2\text{V}^{-1}\text{s}^{-1}$.

$$\therefore \mu = \left(\frac{e\tau}{m} \right)$$

Mobility is a measure of the ease with which the free electrons can drift in the conductor.

➤ Expression for electrical conductivity based on classical free electron theory:

Consider a conductor of uniform area of cross section ' A ' subjected to an electric field E and carrying current I . Let ' v_d ' be the drift velocity of electrons. Now consider an imaginary plane at ' X ' normal to the current direction. The electrons which are at X at a certain instant will travel a distance ' v_d ' in unit time. Thus the electrons are sweeping a volume ' $v_d \cdot A$ ' in the conductor in unit time.



If 'n' is the number of electrons per unit volume, then the number of electrons in volume 'v_d . A' = n.(v_d . A)

(or) the number of electrons crossing any cross-section in unit time = n. (v_d . A)

If 'e' is the charge on an electron, then the quantity of charge crossing any section per second = e.(n.v_d . A)

The quantity of charge crossing any section per second is the same as the current.

$$\therefore I = n e A v_d$$

$$\text{But } v_d = \left(\frac{e\tau}{m}\right) E$$

$$\therefore I = ne \left(\frac{e\tau}{m}\right) EA = \left(\frac{ne^2\tau}{m}\right) EA$$

$$\text{(or)} \quad \left(\frac{I}{A}\right) = J = \left(\frac{ne^2\tau}{m}\right) E \dots\dots\dots(1)$$

$$\text{By Ohm's law, } J = \sigma E \dots\dots\dots(2)$$

where 'σ' is electrical conductivity of the conductor.

From equations (1) and (2), we get

$$\sigma = \frac{ne^2\tau}{m}$$

$$\text{Note: } \sigma = \frac{ne^2\tau}{m} = ne \left(\frac{e\tau}{m}\right) = ne\mu$$

• Drawbacks of classical free electron theory:

1. Specific heat: In classical free electron theory, the electrons are treated equivalent to the molecules of ideal gas.

$$\text{Molar specific heat of gas at constant volume is } C_v = \frac{3}{2} R.$$

Hence the same must apply for free electrons in the metal also.

But experimentally it is found that the specific heat of a metal is $C_v = 10^{-4} RT$.

Thus, the experimental value of specific heat is much lower than the theoretical value. Also, C_v is proportional to temperature where as the theory predicts that C_v is independent of temperature.

- 2. Temperature dependence of electrical conductivity:** According to classical free electron theory, $\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT \Rightarrow v_{th}^2 \propto T$ (or) $v_{th} \propto \sqrt{T}$

$$\text{Also, } \sigma = \frac{ne^2\tau}{m} \quad \text{and} \quad \tau = \frac{\lambda}{v_{th}}$$

$$\therefore \sigma = \frac{ne^2\lambda}{mv_{th}}$$

$$\Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

But, It was experimentally observed that for metals, the electrical conductivity is inversely proportional to temperature.

$$\sigma \propto \frac{1}{T}$$

From this, it is clear that the prediction of classical free electron theory is not agreeing with the experimental observation.

- 3. Dependence of electrical conductivity on electron concentration (n) :** From classical free electron theory, the conductivity is given by

$$\sigma = \frac{ne^2\tau}{m} \quad \text{where 'n' is the free electron concentration.}$$

From the above equation, we see that $\sigma \propto n$

The electron concentration for trivalent aluminium is $18.06 \times 10^{28} / \text{m}^3$ which is much higher than that of monovalent copper ($8.45 \times 10^{28} / \text{m}^3$). But copper has higher conductivity than that of aluminium. Hence, $\sigma \propto n$ does not always hold good.

- 4.** The large variation in mean free path also could not be explained by classical theory. It has been found that the predicted mean free path is about one order (ten times) less compared to that of experimental value.

Quantum free electron theory (Sommerfeld theory):

Assumptions of quantum free electron theory:

1. The energy values of the free electrons are quantised.
2. The filling up of free electrons in the discrete energy levels happens in accordance with Pauli's exclusion principle.
3. The distribution of energy among the free electrons is according to **Fermi-Dirac statistics**.
4. The electrons travel in a constant potential inside a metal but stay confined within the boundaries.
5. The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are neglected.

❖ Comparison of assumptions:

	Classical Free electron theory	Quantum Free electron theory
1	Energy of the conduction electron is continuous	Energy is quantised, $E = \frac{n^2 h^2}{8ma^2}$
2	There is no restriction on the number of electrons possessing a given energy.	Pauli's exclusion principle is applicable. Hence only two electrons with opposite spin states can have the same energy.
3	The energy distribution among the free electrons obeys Maxwell-Boltzmann statistics .	The distribution of energy among the free electrons is according to Fermi-Dirac statistics .
4	The electric potential due to ionic cores inside the metal is considered to be a constant.	The electric potential due to ionic cores inside the metal is considered to be a constant.
5	The mutual repulsion between the electrons and the force of attraction between the electrons and ions are neglected.	The mutual repulsion between the electrons and the force of attraction between the electrons and ions are neglected.

Density of Energy States:

“It is defined as the number of available energy states per unit volume per unit energy range centered at a given energy E ”.

It is denoted by $g(E)$. It is a continuous function and the product $g(E)dE$ gives the number of states in the energy interval dE at E .

➤ Expression for Density of States:

We know the allowed energy for a particle confined in one dimensional box are given by,

$$E = \frac{n^2 h^2}{8ma^2} \quad \dots\dots\dots(1) \quad \text{where } n=1,2,3,\dots\dots$$

‘ a ’ is length over which the particle is free to move in one dimension.

Considering the free electrons in a metal as a particles in 3-dimensional box, the energy of the free electrons can be written as,

$$E = \frac{h^2}{8ma^2} (n_x^2 + n_y^2 + n_z^2) \quad \dots\dots\dots(2)$$

where n_x, n_y, n_z are +ve integers greater than zero. m --- mass of the electron.

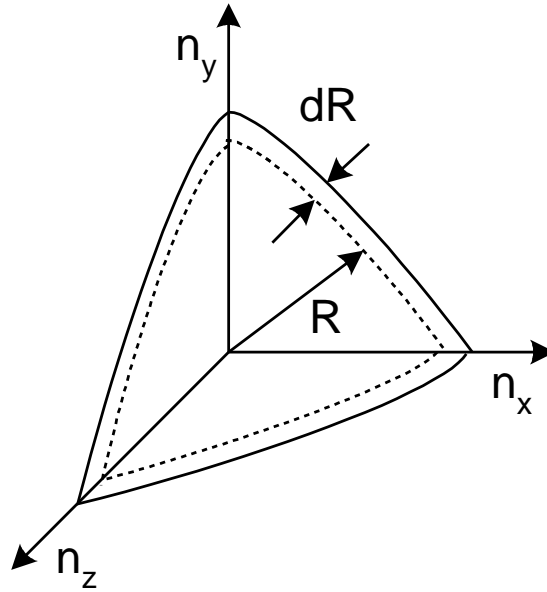
$$\text{Let } E_0 = \frac{h^2}{8ma^2} \quad \text{and} \quad R^2 = n_x^2 + n_y^2 + n_z^2$$

$$\text{Then eqn (2) becomes, } E = E_0 R^2 \quad \dots\dots\dots(3)$$

If we take a coordinate system with n_x, n_y and n_z along three mutually perpendicular axes, then each set of values consisting of three integers can be represented as a point in the system. For a very large number of such sets of values we get a spherical distribution of points in the plot.

Since the values of n_x, n_y and n_z are restricted to be positive, there is only one octant of the sphere of radius ‘ R ’ where in each point corresponds to only positive values of n_x, n_y and n_z .

Hence, we have the number of allowed energy values upto an energy E = number of points in the octant of sphere radius ‘ R .’



If we consider a small energy range between E and $E+dE$, then the number of allowed energy values in the energy range E and $E+dE$

= number of points in the space between the two octants of radii R and $R+dR$.

= (volume of space between the octants of radii R and $(R+dR)$) \times (Number of points per unit volume)

$$= \left(\frac{1}{8} 4\pi R^2 dR \right) \cdot (1) = \frac{1}{2} \pi R^2 dR$$

Now each energy value is applicable to two energy states, one for an electron, with spin up and other for spin down.

Thus, the number of allowed energy states in the range E and $E+dE = 2 \cdot \frac{1}{2} \pi R^2 dR$

$$\text{i.e.} \quad g(E)dE = \pi R^2 dR \quad \dots\dots\dots(4)$$

Now, differentiating eqn.(3) we have $dE = 2E_0 R dR$ or $R dR = \frac{dE}{2E_0}$

also eqn(3) can be rewritten as $R = \left(\frac{E}{E_0}\right)^{1/2}$

Substituting these in (4), we get

$$g(E)dE = \pi \left(\frac{E}{E_0}\right)^{1/2} \frac{dE}{2E_0} = \frac{\pi}{2E_0^{3/2}} E^{1/2} dE$$

$$\text{Here, } E_0 = \frac{h^2}{8ma^2}$$

$$\therefore g(E)dE = \frac{\pi (8ma^2)^{3/2}}{2 (h^2)^{3/2}} E^{1/2} dE$$

On simplifying this, we get

$$g(E)dE = \frac{8\sqrt{2}\pi a^3 m^{3/2}}{h^3} E^{1/2} dE$$

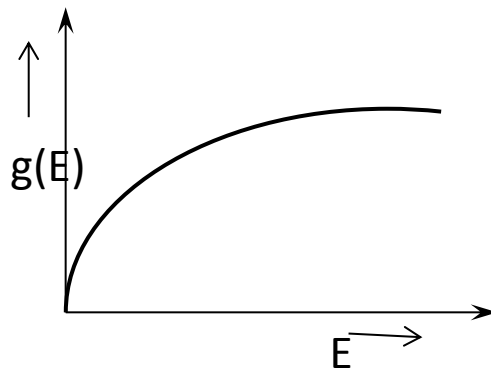
a^3 represents the volume of solid. If we consider the solid to be of unit volume, i.e, $a^3=1$, then

$$g(E)dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} dE$$

This equation represents the density of states.

Thus, $g(E) \propto E^{1/2}$

A plot of $g(E)$ Vs E is as shown. The shape of the curve is a parabola



- **Fermi energy:** The Energy corresponding to the highest occupied level at absolute zero temperature (i.e. 0 K) is called Fermi Energy and the energy level is referred to as Fermi level. It is denoted by E_F .
- **Fermi Factor (or) Fermi-Dirac distribution Function {f(E)}** : It gives the probability of occupancy of a given energy level by the electrons at thermal equilibrium.

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1} \quad \text{where } k \text{ is the Boltzmann constant.}$$

Variation of Fermi factor with temperature:

1. Probability of occupancy for $E < E_F$ at $T=0K$: In this case, $(E-E_F)$ is -ve.

$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1+0} = 1$$

i.e. $f(E) = 1$ for $E < E_F$, This means all the energy levels below the Fermi level are occupied.

2. Probability of occupancy for $E > E_F$ at $T=0K$:

In this case, $(E-E_F)$ is +ve.

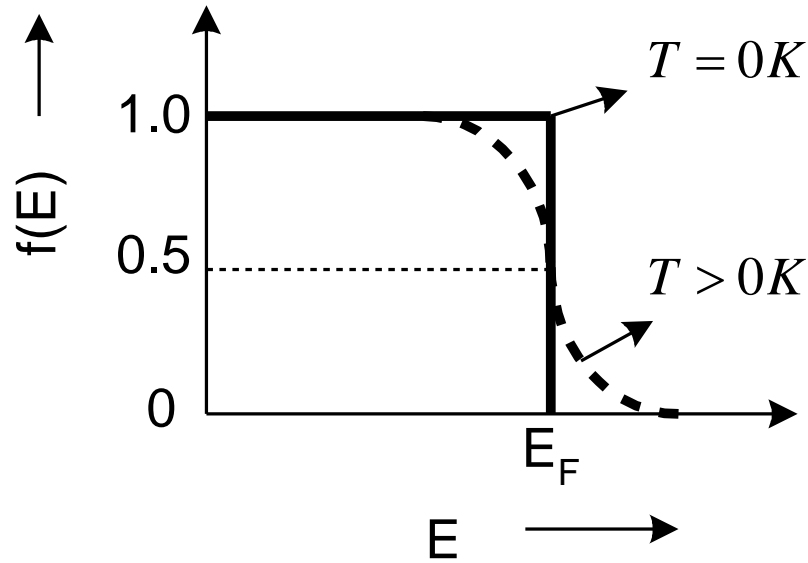
$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{1 + e^{\infty}} = \frac{1}{1 + \infty} = \frac{1}{\infty} = 0$$

i.e, $f(E) = 0$ for $E > E_F$. This means, all the energy levels above Fermi level are empty.

3. Probability of occupancy at ordinary temperature; $E=E_F$ at $T>0K$:

$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{1 + e^0} = \frac{1}{1+1} = \frac{1}{2} = 0.5 \quad \text{i.e probability of occupation is 50\%}.$$

At ordinary temperatures, though the value of probability remains one for $E < E_F$, it starts decreasing from one as the value of the E becomes closer to E_F and becomes $\frac{1}{2}$ at $E=E_F$. Further for $E > E_F$, the probability value falls off to zero rapidly.



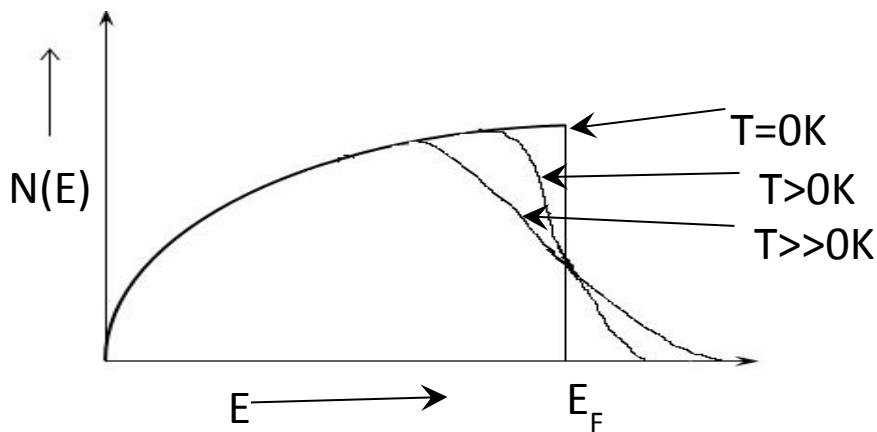
- Fermi Dirac distribution:**

If $N(E)dE$ is the number of those electrons which possess energy only in the range E and $E+dE$, then the number of electrons occupy the energy levels in an energy range dE is,

$$N(E)dE = g(E)dE \times f(E).$$

where $f(E)$ is the fermi factor.

A plot of $N(E)dE$ Vs E , represents the actual distribution of electrons among the available states. The distribution is known as Fermi-Dirac distribution.



In the plot, at $T = 0\text{ K}$, $N(E)$ increases with E up to $E = E_F$. For all values of $E > E_F$, $N(E) = 0$. At ordinary temperatures ($T > 0\text{ K}$), there are slight variations in $N(E)$ for energy values near the Fermi energy one either side of it. Thus, the electrons which possess energy quite lower than E_F , and quite higher than E_F are unaffected at ordinary temperatures.

- **Fermi energy (E_F) at $T=0K$**

Let E_{F0} be the maximum energy the electron can possess at $T=0K$, and n be the number of free electrons/unit volume.

‘ n ’ can be obtained by integrating $N(E)dE$ in the limit $E=0$ to E_{F0} .

$$\text{i.e. } n = \int_{E=0}^{E_{F0}} N(E)dE = \int_0^{E_{F0}} g(E)f(E)dE,$$

where $g(E)dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} dE$ and at $T=0K$,

For $E < E_F$, $f(E)=1$

$$\text{Hence } n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_0^{E_{F0}} E^{1/2} dE$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \left[\frac{2}{3} E^{3/2} \right]_0^{E_{F0}}$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \frac{2}{3} E_{F0}^{3/2}$$

$$n = \frac{\pi}{3} \frac{82^{3/2} m^{3/2}}{h^3} E_{F0}^{3/2}$$

$$\therefore E_{F0}^{3/2} = \frac{h^3}{(8m)^{3/2}} \frac{3n}{\pi}$$

$$(\text{or}) \quad E_{F0} = \frac{h^2}{8m} \left(\frac{3n}{\pi} \right)^{2/3}$$

This can be written as $E_{F0} = Bn^{2/3}$ where $B = \frac{h^2}{8m} \left(\frac{3}{\pi} \right)^{2/3} = 5.85 \times 10^{-38} \text{ Jm}^2$.

- **Fermi velocity (v_F):** It is the velocity of the electrons which occupy fermi level.

$$\text{Hence, } E_F = \frac{1}{2} m v_F^2 \quad \text{or} \quad v_F = \sqrt{\frac{2E_F}{m}}$$

According to quantum free electron theory, conductivity, $\sigma = \frac{ne^2\tau}{m} = \frac{ne^2\lambda}{m^*v_F}$; where m^* is the effective mass of the electron.

- **Success of quantum free electron theory:**

- 1. Specific heat at constant volume:**

According to quantum free-electron theory, only those electrons occupying energy levels close the Fermi energy can absorb the heat energy. Such electrons constitute a very small portion of the total free electrons, thus the specific heat value becomes very small. Further, it was found that

$$C_v = \frac{2k}{E_F} RT$$

Taking typical values of $E_F=5$ eV (for metal E_F lies in the range 1 to 10 eV), we get,

$$\frac{2k}{E_F} \approx 10^{-4}$$

So $C_v = 10^{-4} RT$

This is in agreement with the experimentally observed values.

- 2. Temperature dependence of electrical conductivity:**

As per quantum free electron theory, the electrical conductivity for a metal is

$$\sigma = \frac{ne^2 \lambda}{m^* v_F}$$

Also, as per the theory, E_F and v_F are essentially independent of temperature and λ is

inversely proportional to temperature i.e. $\lambda \propto \frac{1}{T}$.

$$\text{Therefore, } \sigma \propto \frac{1}{T}.$$

The nature of dependence of λ on T can be analyzed as follows.

We know that the electrons are subjected to scattering by the vibrating ions of the lattice. If r is the amplitude of vibration, the ions can be considered to present effectively

in a cross section of area πr^2 . When scattering increases, it results in reduction of mean free path of the electrons. $\therefore \lambda \propto \frac{1}{\pi r^2}$

Considering the fact that (i) The energy of vibrating body is proportional to square of amplitude (ii) The energy of ions is due to thermal energy (iii) The thermal energy is proportional to temperature, we can write, $r^2 \propto T$ and $\lambda \propto \frac{1}{T}$

According to quantum free-electron theory, $\sigma \propto \lambda$

$$\therefore \sigma \propto \frac{1}{T}$$

Thus, dependence of conductivity on temperature is correctly explained.

3. Conductivity and electron concentration:

According to quantum free electron theory, only those electrons which occupy energy states near to E_F can participate in conduction all moving with Fermi velocity.

The electrical conductivity is given by, $\sigma = \frac{ne^2}{m^*} \left(\frac{\lambda}{v_F} \right)$

Thus, ' σ ' depends both on ' n ' and $\left(\frac{\lambda}{v_F} \right)$.

So $v_d = v_F$. Therefore, mean collision time $\tau_c = \frac{\lambda}{v_F}$

If we compare the case of copper and aluminum, the value of ' n ' for Al is 2.13 times higher than that of Cu. But the value of (λ/v_F) for Cu is 3.73 times higher than that for Al. Thus the conductivity of Al is less than that of Cu.

Semiconductors:

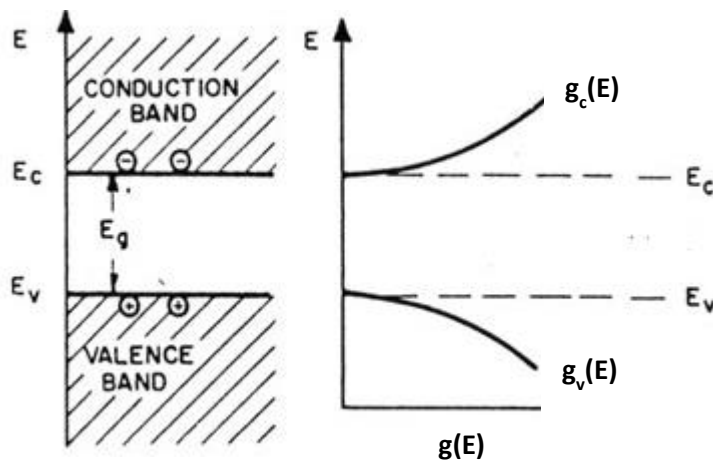
In semiconductors, both electrons in the conduction band and holes in the valance band are mobile charge carriers which contribute for electrical conduction.

The density of allowed energy states for electrons in conduction band is given by

$$g_c(E) dE = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} (E - E_c)^{1/2} dE$$

where, m_e^* is the effective mass of electron and E_c is bottom most energy level of conduction band.

The figure below shows density of energy states in the conduction band and the density of energy states in the valence band as a function of energy.



➤ Expression for Electron Concentration in conduction band at thermal equilibrium.

The density of allowed energy states for electrons in conduction band is given by,

$$g_c(E) dE = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} (E - E_c)^{1/2} dE$$

The distribution of electrons in the conduction band (CB) is given by the density of allowed quantum states times the probability that a state is occupied by an electron.

$$\text{i.e.} \quad n(E) = g_c(E) dE \times f(E) \quad \text{-----(1)}$$

where $f(E)$ is the Fermi-Dirac probability function.

The total electron concentration at thermal equilibrium in the conduction band is then found by integrating eqn (1) over the entire conduction band energy.

$$\text{i.e.} \quad n = \int_{E_c}^{\infty} g_c(E) dE \times f(E) \quad \text{-----(2)}$$

For electrons in the CB, at room temperature, $(E-E_F) \gg kT$. so Fermi probability function reduces to Boltzmann approximation

$$\text{i.e.} \quad f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \approx \exp\left(-\frac{(E-E_F)}{kT}\right) \quad \text{-----(3)}$$

Substituting (3) in (2) we get,

$$n = \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} (E - E_c)^{1/2} \exp\left[-\frac{(E-E_F)}{kT}\right] dE$$

By adding and subtracting E_c in the exponential term, we can write

$$n = \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} (E - E_c)^{1/2} \exp\left[-\frac{(E - E_c + E_c - E_F)}{kT}\right] dE$$

$$n = \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} (E - E_c)^{1/2} \exp\left[-\frac{(E_c - E_F)}{kT}\right] \exp\left[-\frac{(E - E_c)}{kT}\right] dE$$

$$n = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} \exp\left[-\frac{(E_c - E_F)}{kT}\right] \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left[-\frac{(E - E_c)}{kT}\right] dE \quad \text{.....(4)}$$

$$\text{Let us take,} \quad \left(\frac{E - E_c}{kT}\right) = x$$

Then $dE = (kT) \cdot dx$

Also, as $E \rightarrow E_c$; $x \rightarrow 0$ and as $E \rightarrow \infty$, $x \rightarrow \infty$

Then eqn (4) becomes

$$n = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} \exp\left[-\frac{(E_c - E_F)}{kT}\right] (kT)^{3/2} \int_0^\infty \sqrt{x} \exp[-x] dx$$

But $\int_0^\infty \sqrt{x} \exp[-x] dx = \frac{\sqrt{\pi}}{2}$ (Gamma function)

$$\therefore n = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} \exp\left[-\frac{(E_c - E_F)}{kT}\right] (kT)^{3/2} \frac{\sqrt{\pi}}{2}$$

$$= \frac{4\sqrt{2}(\pi kT m_e^*)^{3/2}}{h^3} \exp\left[-\frac{(E_c - E_F)}{kT}\right]$$

$$n = \frac{2(2\pi m_e^* kT)^{3/2}}{h^3} \exp\left[-\frac{(E_c - E_F)}{kT}\right]$$

(or) $n = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$

This is electron concentration at thermal equilibrium in the conduction band.

where $N_c = \frac{2(2\pi m_e^* kT)^{3/2}}{h^3}$ and it is called the effective density of states function for electrons in the conduction band.

Note: Similarly, thermal equilibrium hole concentration in the valance band is given by

$$p = \frac{2(2\pi m_h^* kT)^{3/2}}{h^3} \exp\left[-\frac{(E_F - E_v)}{kT}\right]$$

(or) $p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$

where $N_v = \frac{2(2\pi m_h^* kT)^{3/2}}{h^3}$ and it is called the effective density of states function for holes in the valance band.

Law of mass action in semiconductors:

Statement: “In a semiconductor under thermal equilibrium, the product of electron and hole concentration is a constant and is equal to the square of intrinsic carrier concentration”.

$$\text{i.e.} \quad np = n_i^2$$

The law of mass action holds for both intrinsic and extrinsic semiconductors.

In an intrinsic semiconductor, the electron concentration in conduction band is identical to hole concentration in valence band at any given temperature.

$$n = p = n_i \quad \text{where } n_i \text{ is intrinsic carrier concentration.}$$

According to the law of mass action in semiconductors, if ‘n’ is increased by doping, then ‘p’ decreases such that the product ‘np’ remain a constant irrespective of doping concentration.

The electron and hole concentration in an intrinsic semiconductor is given by

$$n = N_c \exp \left[-\frac{(E_c - E_F)}{kT} \right] \text{-----(1) and}$$

$$p = N_v \exp \left[-\frac{(E_F - E_v)}{kT} \right] \text{-----(2).}$$

For an intrinsic semiconductor concentration of electrons in the conduction band is equal to concentration of holes in the valence band. i.e, $n = p = n_i$, intrinsic carrier concentration.

$$\therefore np = n_i^2 = N_c N_v \exp \left[-\frac{(E_c - E_F)}{kT} \right] \exp \left[-\frac{(E_F - E_v)}{kT} \right]$$

$$= N_c N_v \exp \left[-\frac{(E_c - E_v)}{kT} \right]$$

$$n_i^2 = N_c N_v \exp \left[-\frac{(E_g)}{kT} \right]$$

$$\therefore n_i = \sqrt{N_c N_v} \exp \left[-\frac{(E_g)}{2kT} \right]$$

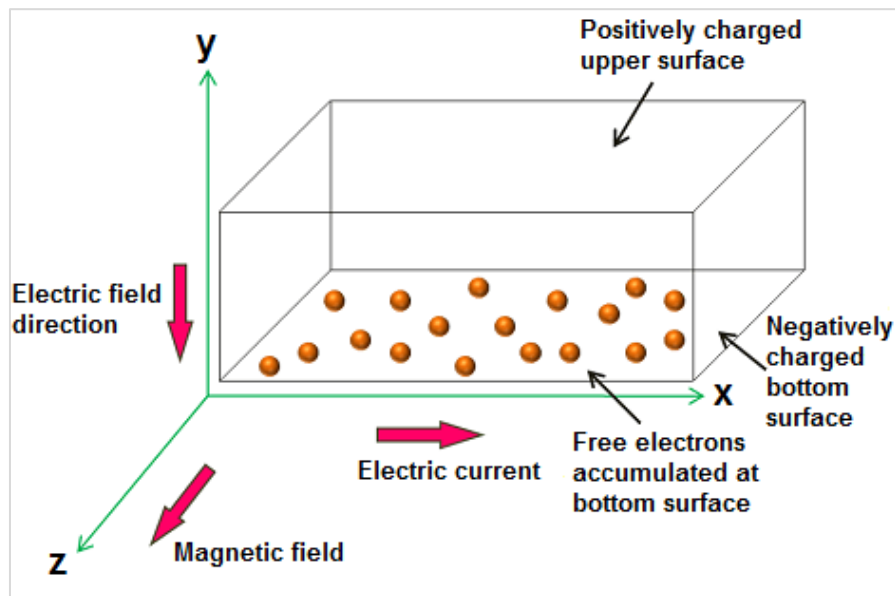
where E_g is the bandgap energy of semiconductor.

Thus for a given semiconductor material at a constant temperature the value of $\sqrt{np} = n_i$ is a constant and independent of Fermi energy. It is called the law of mass action in semiconductors.

HALL EFFECT

When a magnetic field is applied normal to a conductor carrying current, a potential difference V_H is developed in a direction normal to both the magnetic field and current. This is known as **Hall Effect** and generated voltage is called as **Hall voltage**.

Consider a rectangular slab (a metal or semiconductor) carrying current I_x in X-direction. If a uniform magnetic field B_z is applied along Z-direction, then a potential difference is developed along Y-direction (i.e. in a direction perpendicular to both I_x and B_z). This voltage is called Hall voltage (V_H).



Suppose the material is an n-type semiconductor, then current flow is entirely due to majority electrons. Since current is in +ve x-direction, the conduction electrons are drifting with velocity ' v ' in the -ve X-direction.

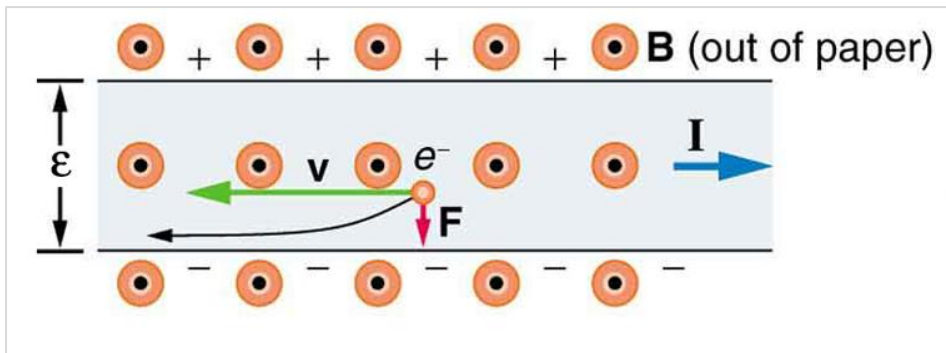
The Lorentz force $\{F_L = q (v \times B)\}$ causes the electron to bend downward as shown in the figure. As a result electrons accumulate on the bottom surface producing a net negative charge there. Simultaneously a net positive charge appears on the upper surface. This creates a downward electric field E_H called **Hall field** and thus Hall voltage V_H is developed between bottom and upper surface.

Expression for Hall voltage (V_H) and Hall coefficient (R_H):

The Lorentz force on moving electrons in the magnetic field acts downwards (as shown in the figure) and is given by,

$$F_L = e (v \times B_z) = e (v B_z \sin\theta) \quad \text{where } v \text{ is the velocity of electrons.}$$

$$F_L = e v B_z \dots\dots\dots(1) \quad (\text{since } v \text{ and } B_z \text{ are perpendicular, } \theta=90^\circ)$$



Now the Hall field (E_H) created by surface charges opposes this Lorentz force. The force due to Hall field acts upwards and is given by,

$$F_H = e E_H \dots\dots\dots(2)$$

In steady state, the net force on electrons is zero.

$$\text{i.e.} \quad F_H = F_L \quad (\text{They are in opposite directions}).$$

$$eE_H = e v B_z$$

$$E_H = vB_z \dots\dots\dots(3)$$

If J_x is the current density in X-direction, then

$$J_x = nev \quad \text{where 'n' is the concentration of charge carriers.}$$

$$(\text{or}) \quad v = \frac{J_x}{ne}$$

Substituting this in eqn. (3), we get

$$E_H = \frac{J_x B_z}{ne}$$

Thus Hall field is proportional to current density and magnetic field.

$$E_H = R_H J_x B_z \dots\dots\dots(4)$$

where $R_H = \frac{1}{ne}$ and it is called **Hall coefficient**.

From eqn. (4), we can write $R_H = \frac{E_H}{J_x B_z}$

For n-type semiconductors, electric field is developed in –ve Y- direction. Hence R_H is –ve.

$$R_H = -\left(\frac{1}{ne}\right) = -\left(\frac{E_H}{J_x B_z}\right)$$

In case of p-type semiconductors, R_H is +ve.

$$R_H = \left(\frac{1}{pe}\right)$$

where ‘p’ is hole concentration.

If ‘d’ is thickness and ‘w’ is width of semiconductor , then $A = (w.d)$ is the area of cross section of the slab. If V_H is the Hall voltage across bottom and upper surfaces of slab, then Hall field is

$$E_H = \frac{V_H}{d}$$

From eqn. (4), we get

$$\frac{V_H}{d} = \frac{R_H I_x B_z}{A} = \frac{R_H I_x B_z}{wd}$$

$$V_H = \frac{R_H I_x B_z}{w}$$

(or) $R_H = \frac{V_H w}{I_x B_z}$

The quantities V_H , B_z and I_x can be measured experimentally, so the Hall coefficient (R_H) and carrier concentration can be determined.

Applications of Hall Effect:

- **Determination of semiconductor type:** For an n-type semiconductor, the Hall coefficient is -ve whereas for p-type semiconductor it is +ve. Thus the sign of Hall coefficient can be used to determine whether a given semiconductor is n or p-type.
- **Calculation of Carrier concentration:** Hall coefficient ' R_H ' is measured experimentally.

$$R_H = \left(\frac{1}{ne} \right) \quad \therefore n = \left(\frac{1}{e R_H} \right)$$

Thus the concentration of charge carriers can be calculated.

- **Determination of mobility:** If the conduction is due to one type of carriers, eg., electrons, we have $\sigma = n e \mu_n = \frac{\mu_n}{R_H}$

$$\therefore \mu_n = \sigma R_H$$

Thus by knowing σ and measuring R_H , the mobility can be determined.