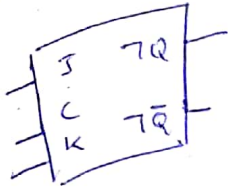
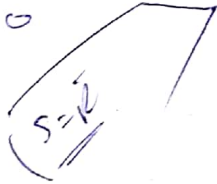


Master slave JK



$1 \rightarrow 0$

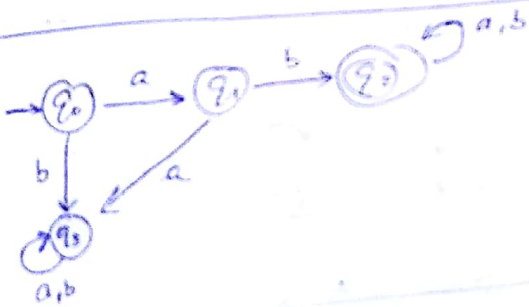


J	K	C	Q^+	\bar{Q}^+
0	0	1	Q	\bar{Q}
0	1	1	0	1
1	0	1	1	0
1	1	1	\bar{Q}	Q
X	X	0	Q	\bar{Q}

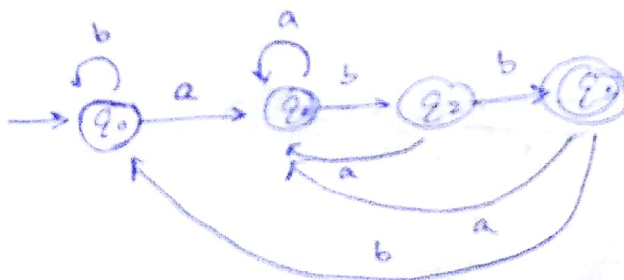
$$Q^+ = J\bar{Q} + K\bar{Q}$$



exactly one a

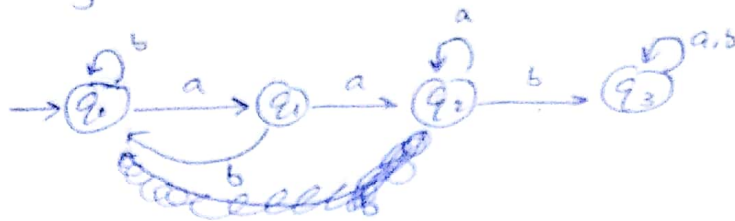


starts with ab

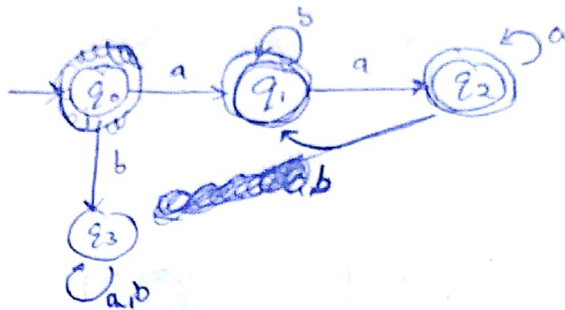


ending with abb

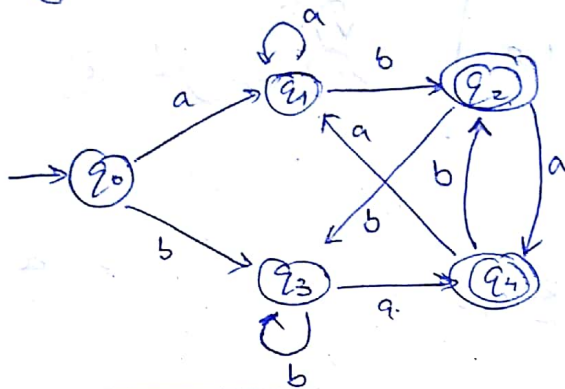
Substring aab



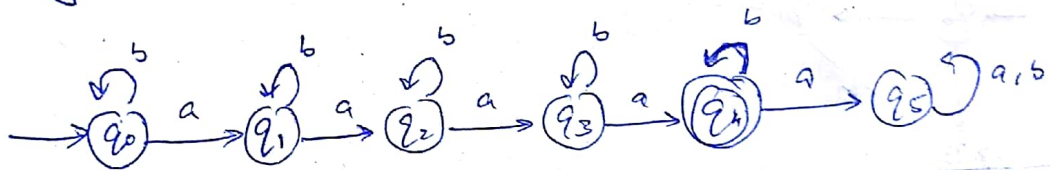
starting and ending with a



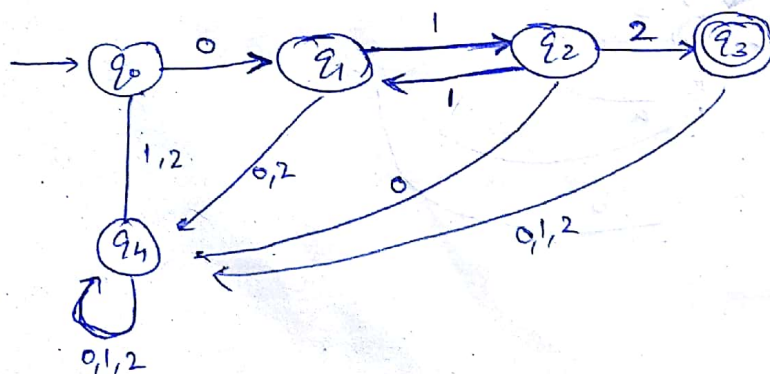
ending with ab or ba

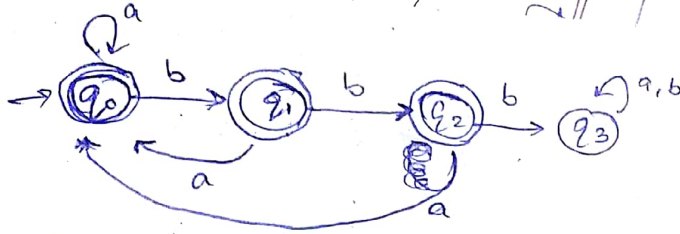


having 4 a's



0 followed by odd 1's ending with 2



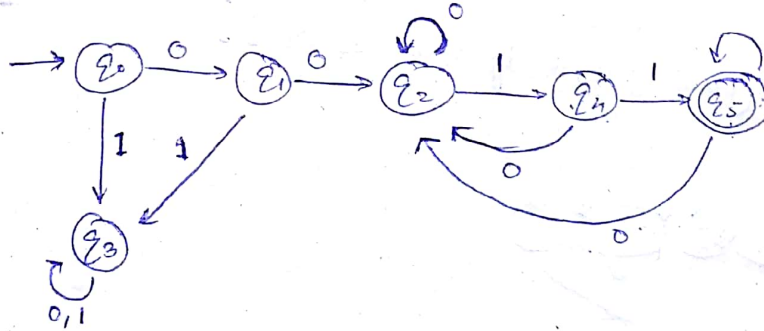


at most 2 consecutive b's

abbbaa
abbabb

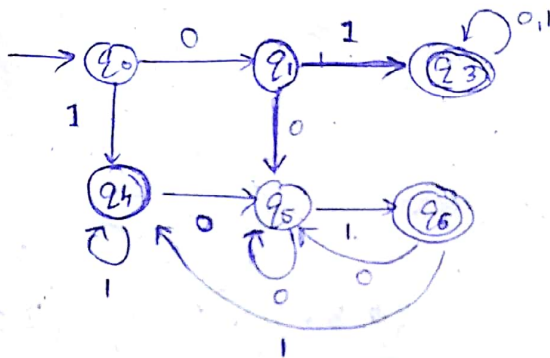
0101

Start with at least 2 0's ending with at least 2 1's



110

begin or end or both with 01



0110

1101

0101

110001

10101

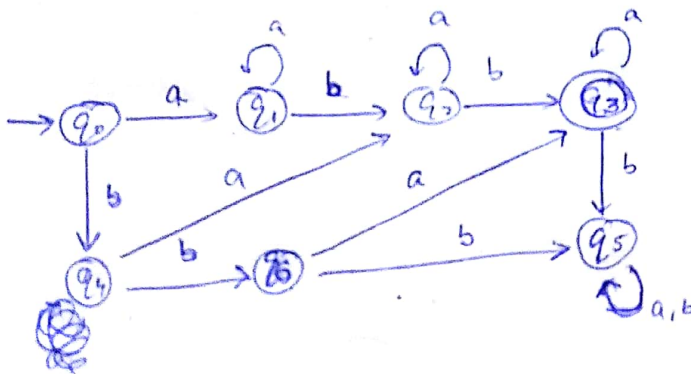
101101

0101

11101

$n_a \geq 1$

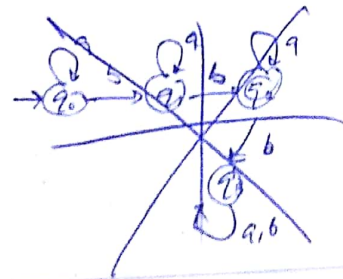
$n_b = 2$



babb

bba

bab



Bin i/p mod 5 is zero.

$$S(q_i, a) = q_j$$

$$j = (q \times i + d) \bmod k$$

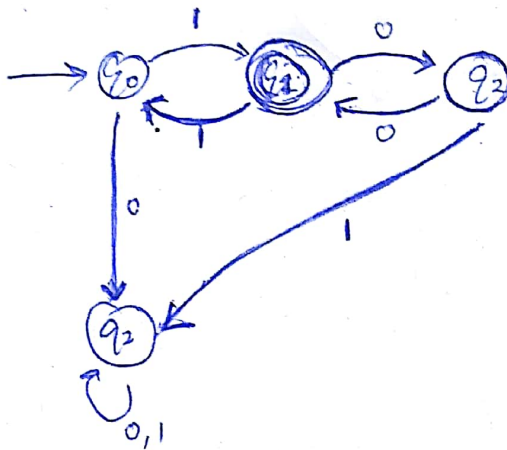
Remainder after
dividing by k

modin of $i/p = 2$ for binary
(base)

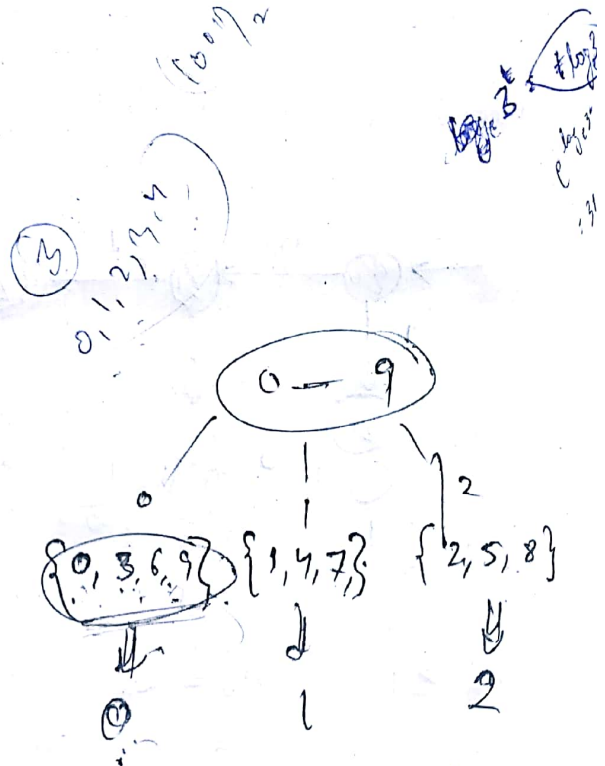
divisor

digits $\{0, 1\}$ for bin

odd no of 1's followed by even nos of 0's



11100
1001
1010
1100



Relations

• Antisymmetry :- $(aRb \wedge bRa) \rightarrow a=b$

If 2 objects are both related to each other, they are the same object.

* Partial order (\leq)

- Antisymmetric
- Reflexive
- Transitive

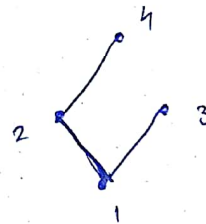
eg $n \leq y$ $2 \leq 4$ ref ✓
trans ✓
antisym ✓

eg alphabetical order

* Hasse Diagram (for Partial order)

eg $A = \{1, 2, 3, 4\}$ where xRy if $x \leq y$
 $R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

- don't show reflexive
- don't show direction
- remove transitivity

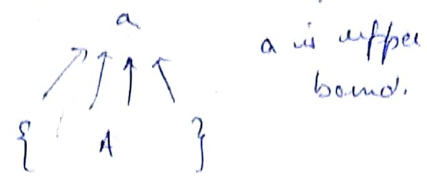


eg - $A = \{1, 2, 4, 8\}$ $xRy = x|y$



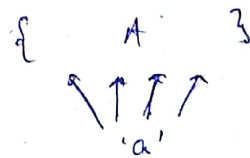
total order (if we can connect all the elements in a straight line)

→ Upper bound (A) means all the elements accessed by the all elements of set A below 'a'



least upper bound (A) : should be a unique element closest to ^{elements of} set A

→ lower bound (A) all the elements below set A which can access all elements of set A



upper lower bound (A) : should be a unique element closest to elements of A

→ maximal : - elements on the top level if only 1 element then aka "greatest"

→ minimal : - elements on the bottom level if only 1 element then aka "least"

Composition Relation:-

$$R_1 \circ R_2 = \{(x, z) \in R \mid \begin{array}{l} x \in A, z \in C \text{ there exists} \\ y \in B, (x, y) \in R_1, \\ (y, z) \in R_2 \end{array}\}$$

A, B & C are sets with $R_1 \subseteq A \times B$, $R_2 \subseteq B \times C$
then composite relation is a relation from $A \rightarrow C$ defined by $R_1 \circ R_2$

• Power of relation

$$R^1 = R$$

$$R^2 = R \circ R$$

$$R^3 = R^2 \circ R$$

$$R^{n+1} = R^n \circ R$$

Matrix Rep of Relation

$$M_{ij} = \begin{cases} 1 & (x_i, y_j) \in R \\ 0 & (x_i, y_j) \notin R \end{cases}$$

eg $A = \{1, 2, 3, 4\}$ x divides y
 $R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

$$M(R) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

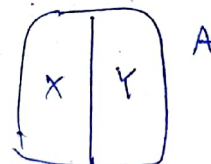
partition

For each subset X, Y

$$X \subseteq A, Y \subseteq A$$

i) $X \cap Y = \emptyset$

ii) $\cup X_i = A$



eg $A = \{a, b, c, d\}$

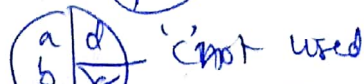
$P_1 = \{\{a, b\}, \{c\}, \{d\}\}$ ✓



$P_2 = \{\{a, b, c\}, \{c, d\}\}$ ✗



$P_3 = \{\{a, b\}, \{d\}\}$ ✗



DFA

It is 5-tuples of $M = (Q, \Sigma, \delta, q_0, F)$

Q : non-empty, finite set of states

Σ : set of input alphabets

δ : $Q \times \Sigma$ to Q is transition function mapping $Q \times \Sigma$ to Q

q_0 : $q_0 \in Q$ is the start state

F : $F \subseteq Q$ is set of final states

extended transition function. (δ^*): describes what happens to a state of machine when it is a string

$$\delta^*: Q \times \Sigma^* \rightarrow Q$$

$$\delta^*(q, \epsilon) = q$$

$$\delta^*(q, w) = \delta^*(q, xa) = \delta(\delta^*(q, x), a)$$

$$\delta^*(q, w) = \delta^*(q, au) = \delta^*(\delta(q, a), u)$$

Language accepted by DFA =

$$L(M) = \{ w \mid w \in \Sigma^* \text{ and } \delta^*(q_0, w) \text{ is in } F \}$$

insert not for language rejected by DFA

NFA

$M = (Q, \Sigma, \delta, q_0, F)$

Q : non-empty, finite set of states

Σ : set of ~~diff~~ alphabets

δ : $Q \times \Sigma \rightarrow 2^Q$

q_0 : $q_0 \in Q$ start state

$F \subseteq Q$: set of final states

$$\delta^* : Q \times \Sigma^* \rightarrow 2^Q$$

$$\delta^*(q, \epsilon) = \{q\}$$

$$\delta^*(q, w) = \delta^*(q, xa) = \{\delta(\{\delta^*(q, x)\}, a)\}$$

$$\delta^*(q, w) = \delta^*(q, an) = \{\delta^*(\{\delta^*(q, a)\}, n)\}$$

$$\delta^*(q, x) = \{p_1, p_2, \dots, p_m\}$$

$$\delta(\{p_1, p_2, \dots, p_m\}, a) = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k\}$$

$$\text{i.e. } \bigcup_{i=1}^m \delta(p_i, a) = \{\lambda_1, \lambda_2, \dots, \lambda_k\}$$

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for transfer

$$\underline{\epsilon\text{-NFA}} : M = (Q, \Sigma, \delta, q_0, F)$$

Q : non empty, finite set of states

Σ : input alphabet

$$\delta : Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$$

$$q_0 \in Q \text{ start state}$$

$$F \subseteq Q \text{ set of final states}$$

$$\delta^* : Q \times (\Sigma \cup \epsilon)^* \rightarrow 2^Q$$

$$\delta^*(q, \epsilon) = \epsilon\text{-closure}(q)$$

$$\delta^*(q, w) = \delta^*(q, na) = \epsilon\text{-close}(\delta(\delta^*(q, n), a))$$

$$\delta^*(q, w) = \delta^*(q, an) = \epsilon\text{-close}(\delta^*(\delta(q, a), n))$$



$$\neg [\neg [(p \vee q) \wedge r] \vee \neg q]$$

$$[(p \vee q) \wedge r \wedge q]$$

$$\underline{p \vee (p \wedge q)}$$

$$(p \vee q) \wedge r \wedge q$$

$$(p \vee q) \wedge (q \wedge r)$$

$$q \vee (q \wedge r) \wedge r$$

$$\underline{p \vee q}$$

1) Modus Ponens

$$[p \wedge (p \rightarrow q)] \rightarrow q$$

$$p$$

$$p \rightarrow q$$

$$\underline{\therefore q}$$

2) Syllogism

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\underline{p \rightarrow r}$$

3) Modus Tollens

$$p \rightarrow q$$

$$\neg q$$

$$\underline{\therefore \neg p}$$

$$(p \rightarrow q) \wedge \neg q$$

$$p \rightarrow (q \vee \neg q)$$

$$p \rightarrow F$$

$$\underline{\neg p}$$



6. Conjunction

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

7. Disjunctive Syllogism

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

$$\begin{array}{l} (p \vee q) \wedge \neg p \\ p \vee q \wedge \neg p \\ (p \vee \neg p) \wedge q \\ (p \wedge q) \\ \emptyset \end{array}$$

8. Contradiction

$$\begin{array}{c} \neg p \rightarrow F \\ \hline \therefore p \end{array}$$

9.

$$\begin{array}{c} p \wedge q \\ \hline p \end{array}$$

Conjunction
Simp.

10.

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$$

Disjunctive Amplification