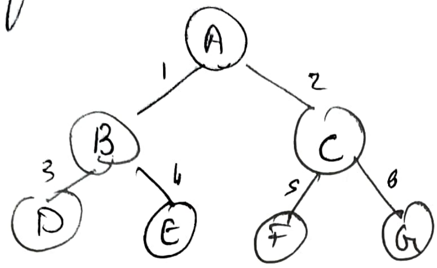
UNIT - V

1) Trees and its concepts
2) Weighted trees and prefix codes
3) Network flow graphs
   - Ford-fulkerson algorithm for network flow.

Trees and its concepts

A tree is a graph with $n-1$ edges (max), where 'n' is the no/: of nodes in the tree.

Ex:



$|V| = 7$

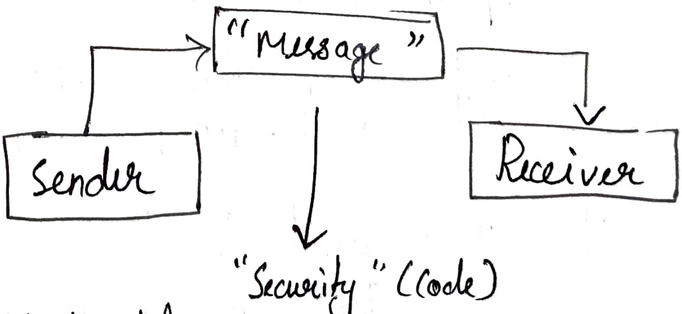
$|E| = 6$

Concepts — leaf, child, sibling, descendants, left node, right node etc.

Weighted trees and Prefix Code :

Coding theory:-

It deals with assigning some security related concept to the message between the sender and the receiver.



Ex:

Let there be a set of alphabets $S = \{a, e, r, w, t\}$

and code a:01 , e:0 , r:10 , w:101 , t:1

then we can send "awt" as 011011

But it can be read as attatt also. This is a problem ☹

This is where Prefix Codes come in.

A set P of binary sequences is called a prefix code if no sequence in P is the prefix of any other sequences in P.

Eg: $S = \{a, e, n, r, t\}$

Codes :- a : 111 , e : 0 , n : 1100 , r : 1101 , t : 10

Message = "ata" $\longrightarrow$ 111 10 111
Sender end

a t a

Receiver end

Problems on prefix code

1) A code for $\{a, b, c, d, e\}$ is given as
a : 00 , b : 01 , c : 101 , d : x10 , e : yz1
$x, y, z \in \{0, 1\}$.
What are the values of x, y, z so that it forms a prefix code?

Soln wkt
x can be either 0 or 1,

if $x = 0$, then b is a prefix to 010
if $x = 1$, then 110 has no prefix in P
$\therefore d = 110$, $x = 1$

if $y = 0$ and $z = 1$
then b is a prefix to 011
$y = 0$ and $z = 0$
then a is a prefix to 001
$y = 1$ and $z = 0$
then c = 101 = e
$y = 1$ and $z = 1$
then there's no prefix

$\therefore$ $\boxed{x = 1,\ y = 1,\ z = 1}$

Weighted Prefix ~~Codes~~ Tree

we will be given a set of alphabets and their corresponding frequencies. Using these we should find the prefix tree.

Method of solving

Given A – set of alphabets

F – their frequencies

Step-1: Sort F in increasing order.

Step-2: Pick the first 2 lower frequencies, add it and place it accordingly.

Step-3: Repeat prev step until there is a single element remaining

Step-4: Label 0 on all the left ~~nodes~~ branches and 1 on all right ~~nodes~~ branches, ~~table~~ ~~(except leaf nodes)~~ and mark the leaf nodes with respective alphabets

Step-5: U can read the prefix code of an alphabet from top to bottom.

Example:

Given $A = \{a, b, c, d, e\}$

$F = \{20, 30, 10, 5, 35\}$

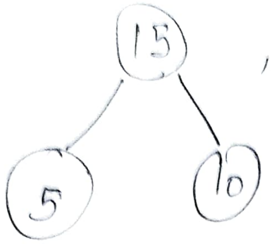
Step-1: Sort

$F = \{5, 10, 20, 30, 35\}$

(d) (c) (a) (b) (e)

Step - 2:

Pick 5 and 10

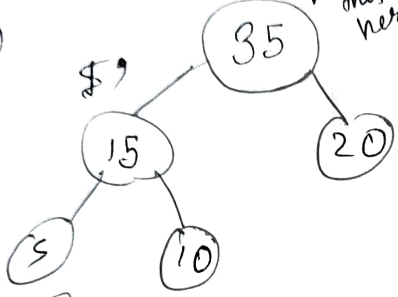


, 20, 30, 35

Step - 3: Repeat

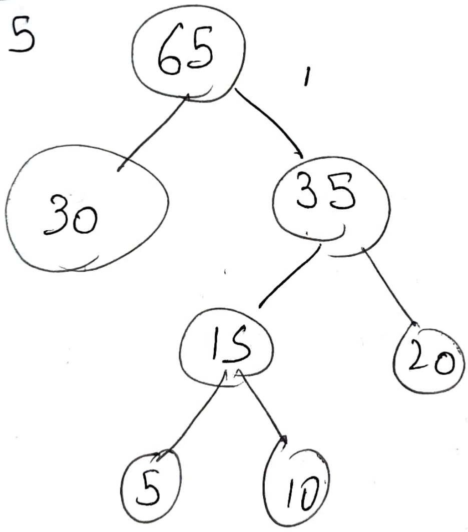
(you can place this here)

30 , 35 (or here)



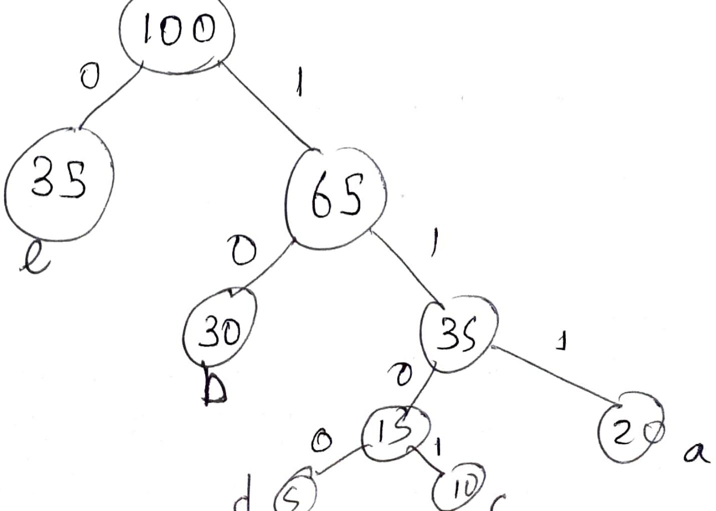
Repeat

35 ,



Repeat

Step - 4:



Prefix codes

a:	111
b:	10
c:	1101
d:	1100
e:	0

Problem

Determine the prefix code for the symbols in the word "HELLO".

Soln

$A = \{H, E, L, O\}$

$F = \{ ? \}$

Total length $= 5$ (Σf)

$f_H = 1/5 = 0.2$

$f_E = 1/5 = 0.2$

$f_L = 2/5 = 0.4$

$f_O = 1/5 = 0.2$

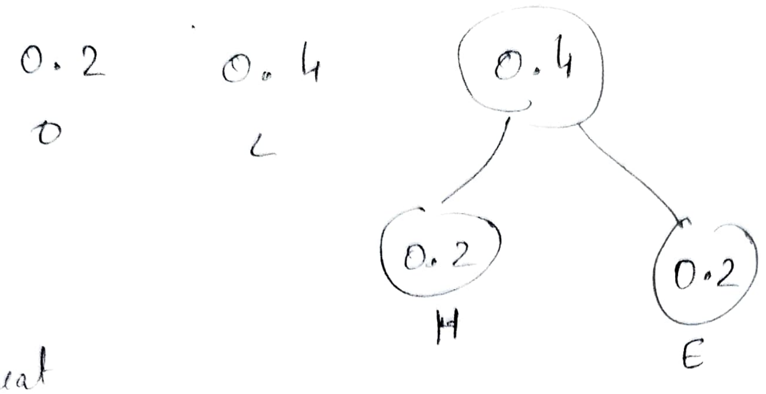
$\Sigma f = 1$

$\therefore F = \{0.2, 0.2, 0.4, 0.2\}$

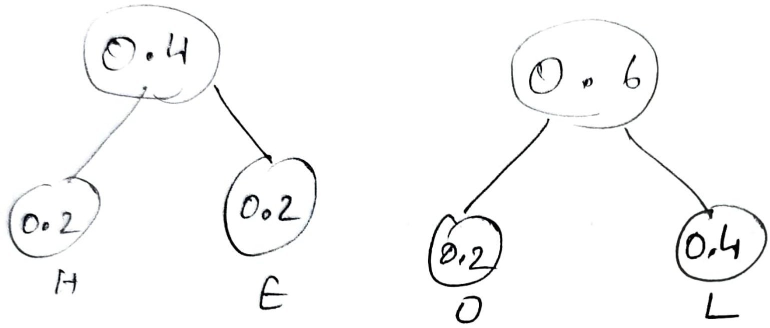
Step - 1: Arrange in increasing order

$$F = \{\underset{H}{0.2}, \underset{E}{0.2}, \underset{O}{0.2}, \underset{L}{0.4}\}$$

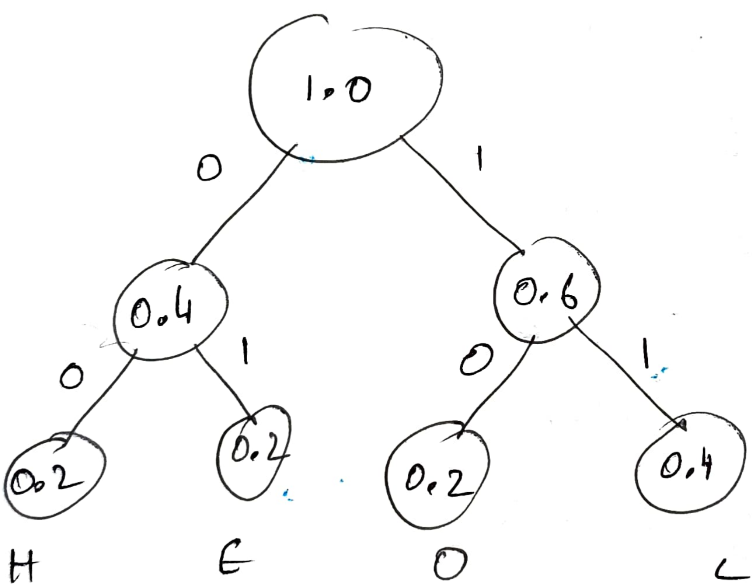
Step-2:



Repeat



Step-4



Step-6:

H: 0 0

E: 0 1

O: 1 0

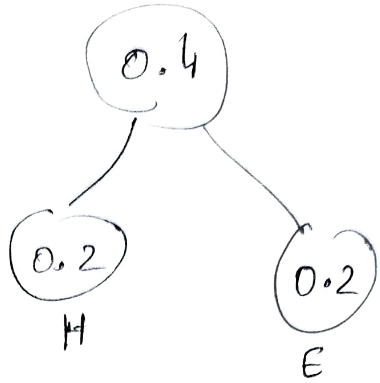
L: 1 1

∴ "HELLO" → 0001111110

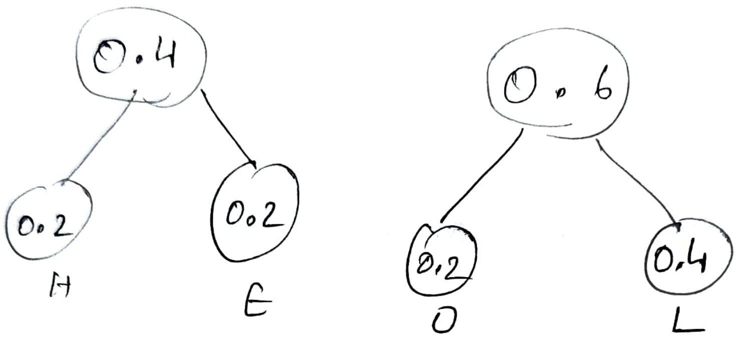
Step - 2 :

0.2 0.4

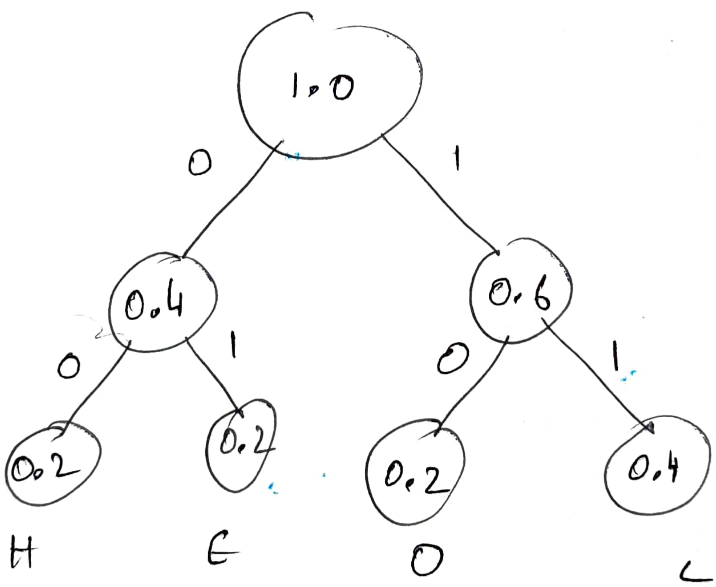
O L



Repeat



Step - 4 :



Step - 6 :

H : 0 0

E : 0 1

O : 1 0

L : 1 1

∴ "HELLO" → 0001111110

Maximum flow problem and Ford-Fulkerson Algorithm

The Ford-Fulkerson algorithm is an algorithm which computes the maximum flow in a flow network.

The idea behind the algorithm is simple. As long as there is a path from the source (start node) to the sink (end node), with available capacity on all edges in the path, we send flow along one of these paths. Then we find another path and so on.

Note: A path with available capacity is called an 'augmenting path.'.

Network Flow problems

- A highway system in which the edges are highways and nodes are interchanges.
- A computer network in which edges are links ~~or~~ that can carry packets and nods are switches.
- A fluid network in which edges are pipes that carry liquid and the nodes are the junctions.

These models ~~co~~ contain the following specs –

- Capacities
- Source
- Sink

Traffic as flow – An abstract entity that is generated at source nodes, transmitted across edges, and absorbed at the sink nodes.

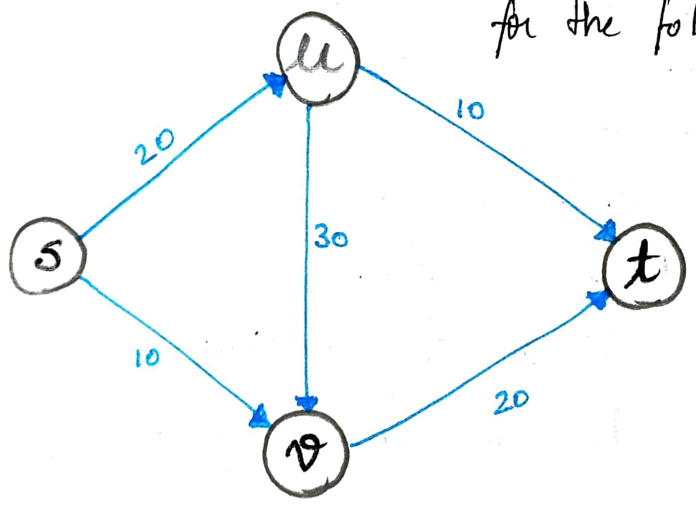
A flow network is a directed graph $G=(V,E)$ with the following features —

- Associated with edge e is a capacity, which is a non-negative number c_e
- There is a single source node $s \in V$
- There is a single sink node $t \in V$.
- Nodes other than s and t are internal nodes.

Two assumptions about the flow network are —

- ~~Ato~~ There is atleast one edge incident to each node.
- All capacities are integers.

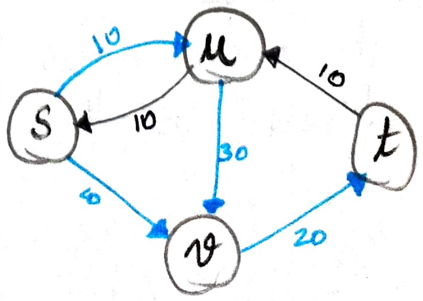
Ex: Determine the maximum flow for the following graph :-

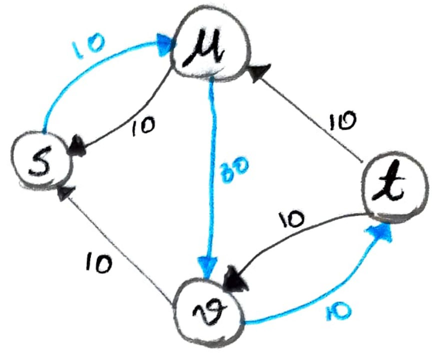
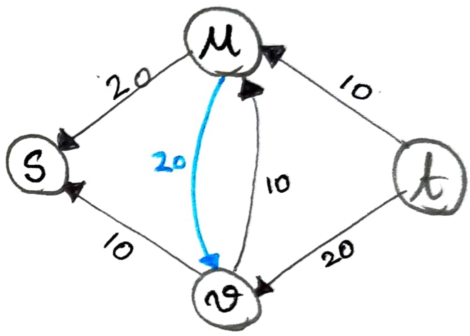


Soln

$s \rightarrow$ source (start)

$t \rightarrow$ sink (end)

s - t Path	Minimum edge cut	Residual ~~Bdeph~~ Graph
s - u - t	10	

s - v - t	10	
s - u - v - t	10	

We can see there are no remaining paths from s to t.

So, Maximum flow = 10 + 10 + 10 = 30

according to the Max-flow min-cut theorem.

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