1.	b)	Determine the effective interest rate for a nominal annual rate of 6% that is compounded 1. Semi annually 2. Quarterly 3. Monthly 4. daily	(10)	CO1

Sol: **Given:** $r = 6\%$ annual

Effective interest rate $\boxed{i_{eff} = \left(1+\frac{r}{m}\right)^m - 1}$ (For continuous compounding)

1. **Semi Annual:**

$m = 2$

$$i_{eff} = \left(1 + \frac{0.06}{2}\right)^2 - 1$$
$$= (1.03)^2 - 1$$
$$= 1.0609 - 1 = 0.0609$$
$$= 6.09\%$$

2. **Quarterly:**

$m = 4$

$$i_{eff} = \left(1 + \frac{0.06}{4}\right)^4 - 1$$
$$= 1.06136 - 1 = 0.06136$$
$$= 6.136\%$$

3. **Monthly:**

$m = 12$

$$i_{eff} = \left(1 + \frac{0.06}{12}\right)^{12} - 1$$
$$= 0.061678$$
$$= 6.1678\%$$

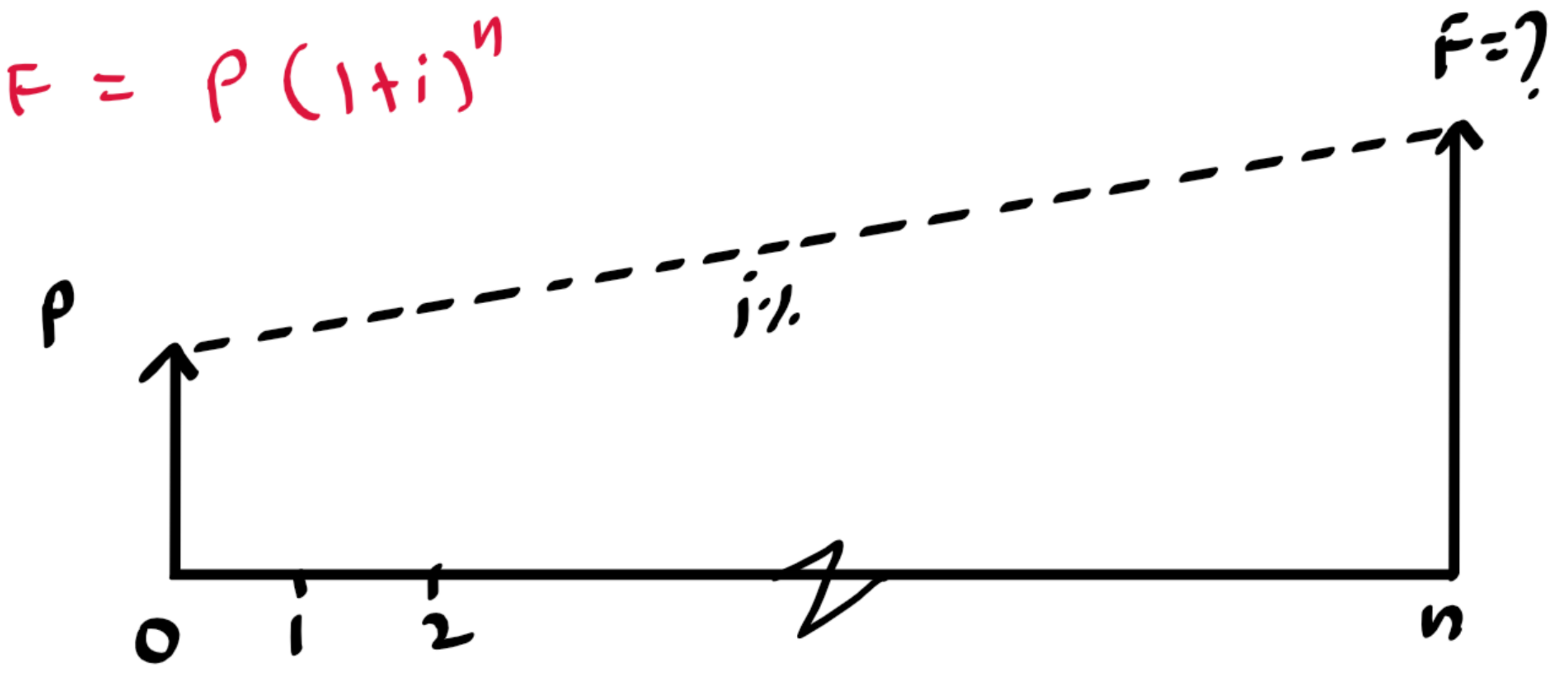
4. **Daily:**

$m = 365$

$$i_{eff} = \left(1 + \frac{0.06}{365}\right)^{365} - 1$$
$$= 0.06183$$
$$= 6.183\%$$

2	c	Derive the formula for capital recovery factor (uniform series). Firstly derive $F = P(1+i)^n$	8	CO1

Sol: To derive: Compound-amount factor: (single payment) $F = P(1+i)^n$



Find F, given P $(F/P, i\%, n)$

F → Future amount after n time periods

P → Present amount

i → Interest rate per period

n → Time periods

Future amount after 1 time period, $F_1 = P + Pi = P(1+i)$

Future amount after 2 time periods, $F_2 = F_1 + F_1 i = F_1(1+i)$

$F_2 = P(1+i)(1+i) = P(1+i)^2$

Future amount after 3 time periods, $F_3 = F_2 + F_2 i = F_2(1+i)$

$F_3 = P(1+i)^2(1+i) = P(1+i)^3$

⋮

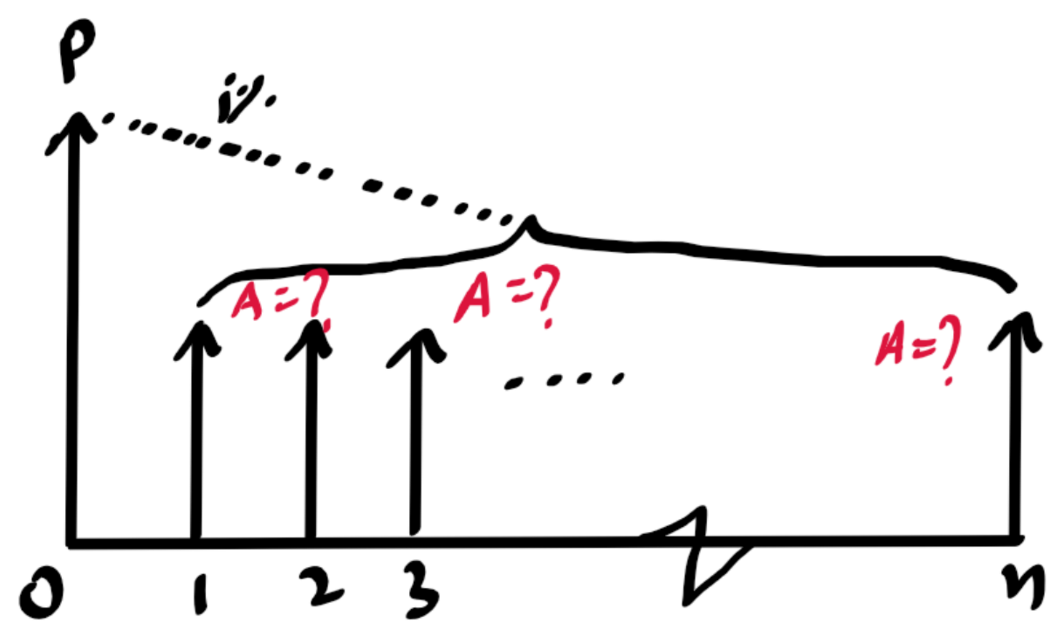
Future amount after n time periods, $F_n = \boxed{F = P(1+i)^n}$

To derive: Capital recovery factor: (uniform series) $A = P\left[\dfrac{i(1+i)^n}{(1+i)^n - 1}\right]$

Given P, find A. $(A/P, i\%, n)$

$P = \dfrac{F}{(1+i)^{-n}}$ as derived earlier

Year	A	P at beginning
1	1000	$1000(1.08)^{-1} = 926$
2	1000	$1000(1.08)^{-2} = 857$
3	1000	$1000(1.08)^{-3} = 793$
4	1000	$1000(1.08)^{-4} = 735$
		$P = 3311$



P → Present amount

A → Annuity

i → Interest rate per period

n → Time periods

Assuming each A as F, and calculating P

∴ From table,

$$P = A(1+i)^{-1} + A(1+i)^{-2} + \dots + A(1+i)^{-n}$$

$$\Rightarrow P = A\left[(1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + \dots + (1+i)^{-n}\right]$$

Multiply B.S by $(1+i)^{-1}$

$$\Rightarrow P(1+i)^{-1} = A\left[(1+i)^{-2} + (1+i)^{-3} + \dots + (1+i)^{-(n+1)}\right]$$

$$P = A\left[(1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + \cdots + (1+i)^{-n}\right] \quad -①$$

$$P(1+i)^{-1} = A\left[(1+i)^{-2} + (1+i)^{-3} + \cdots + (1+i)^{-(n+1)}\right] \quad -②$$

② − ①

$$\Rightarrow \quad P(1+i)^{-1} - P = A\left[(1+i)^{-n-1} - (1+i)^{-1}\right]$$

$$\Rightarrow \quad P\left[(1+i)^{-1} - 1\right] = A\left[(1+i)^{-n-1} - (1+i)^{-1}\right]$$

$$(1+i)^{-1} - 1 = \frac{1}{1+i} - 1 = \frac{1 - 1 - i}{1+i} = \frac{-i}{1+i}$$

$$\Rightarrow \quad P\left[\frac{-i}{(1+i)}\right] = A\left[\frac{1}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right]$$

Multiply B.S by $-(1+i)$

$$\Rightarrow \quad Pi = A\left[1 - \frac{1}{(1+i)^n}\right]$$

$$\Rightarrow \quad Pi = A\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$$

$$\Rightarrow \quad \boxed{A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]}$$

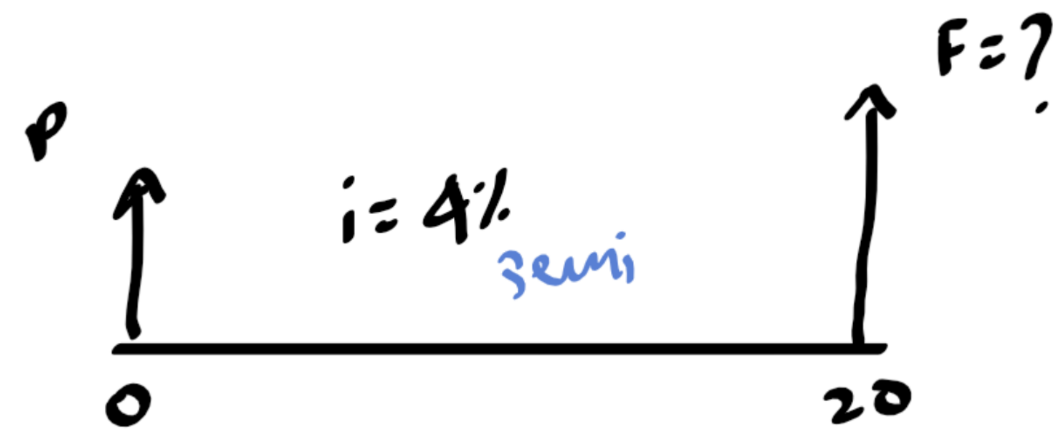
2	d	What will be the amount accumulated by each of these present investments? i)Rs.6750 in 20 years at 4% compounded semi-annually. ii)Rs. 11,000 in 10 years at 12% compounded quarterly	6	CO1

Sol:

i) $P = 6750$

$N = 20$

$i = 4\%$ compounded semi-annually



$F = P(1+i)^n$

$\Rightarrow F = 6750 \left[1 + \frac{0.04}{2}\right]^{20\times 2}$

$\Rightarrow F = 14904.2677$

method 2

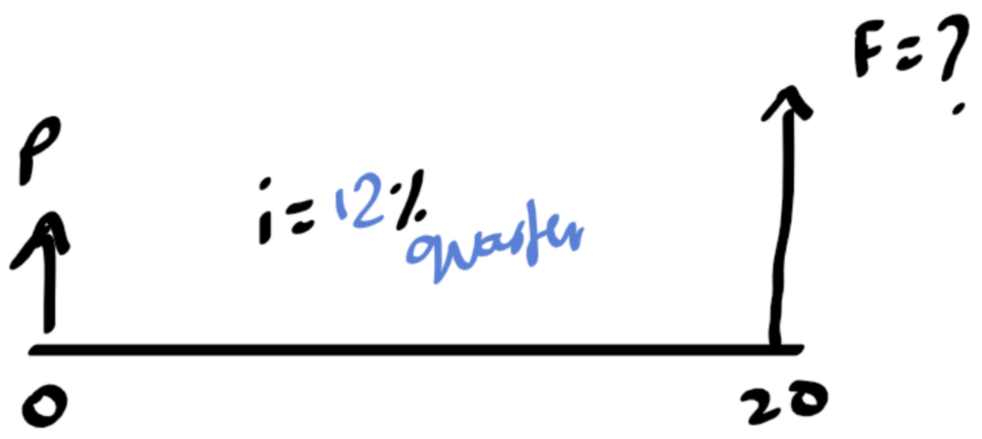
$i_{eff} = \left(1 + \frac{0.04}{2}\right)^2 - 1$

Then use $F = P(1+i_{eff})^{20}$

ii) $P = 11000$

$N = 10$

$i = 12\%$ compounded quarterly



$F = P(1+i)^n$

$\Rightarrow F = 11000 \left[1 + \frac{0.12}{4}\right]^{10\times 4}$

$\Rightarrow F = 35\,882.4157$

3	a)	The interest rate for a loan of Rs.2000 is 10% per year. If interest had not been paid each year, but has been allowed to compound how much interest would be due to the lender as lump sum at the end of 6 years	(10)	CO1

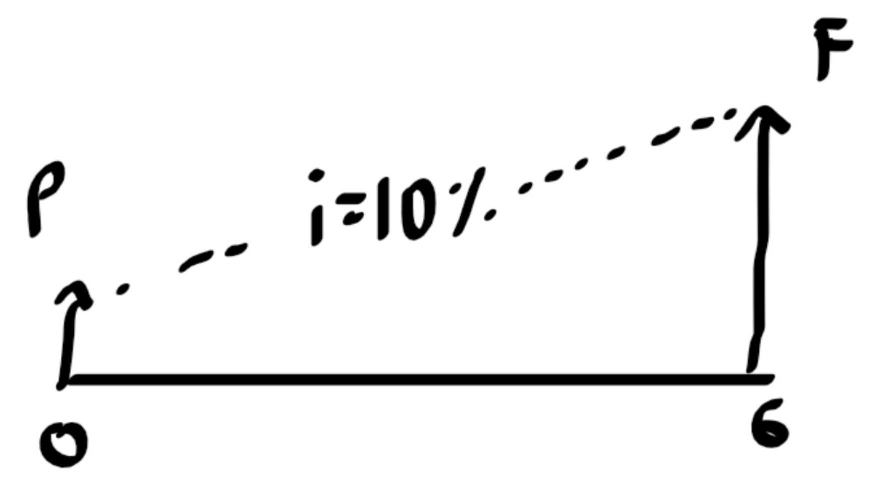
Sol: Given:

$P = 2000$

$i = 10\%$ per year

$N = 6$

Interest amount $= ?$



Interest amount = Future amount − Present amount

$= F - P$

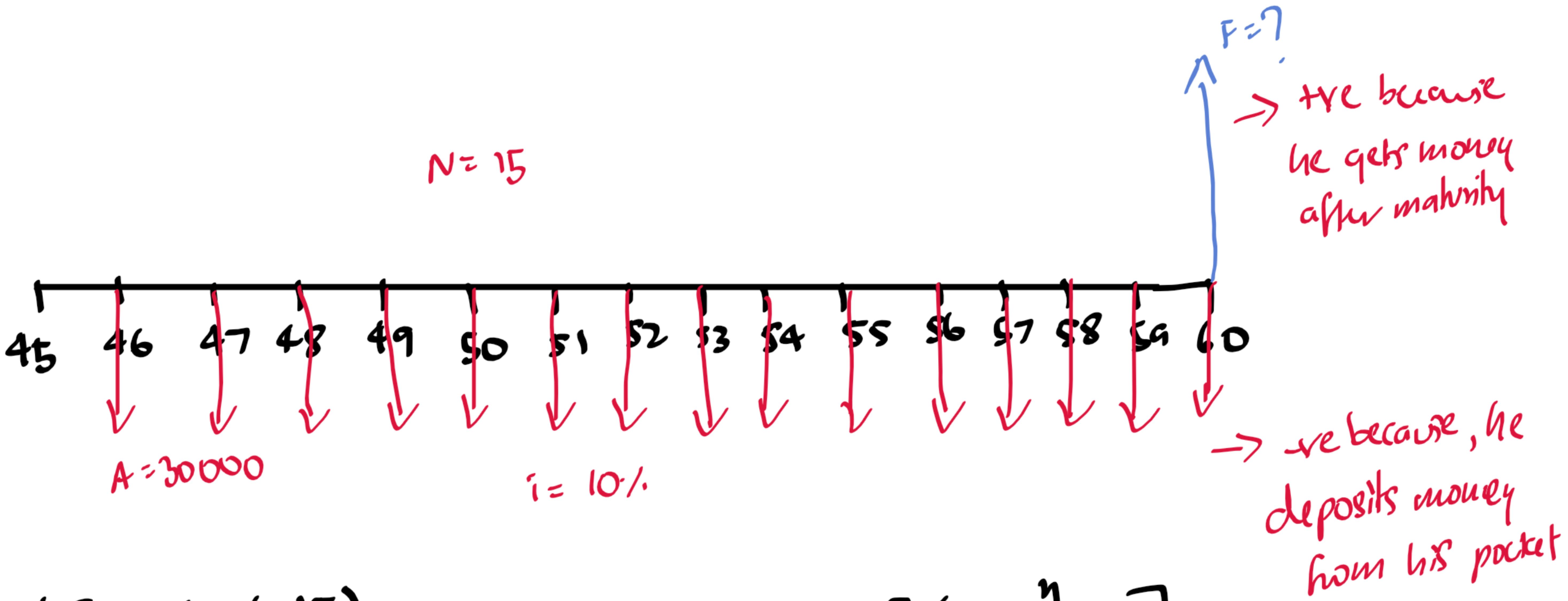
$= P(1+i)^n - P$

$= 2000(1+0.1)^6 - 2000$

$= 1543.122$

3	b)	A 45 year old person is planning for his retired life. He plans to divert Rs. 30000/- from his bonous as investment every year for the next 15 years. The bank gives 10% interest rate compounded annually. Find the maturity value of his account when he is 60 year old. Draw cash flow diagram	(10)	CO1

Sol:



$$F = 30000\,(F/A, 10\%, 15)$$

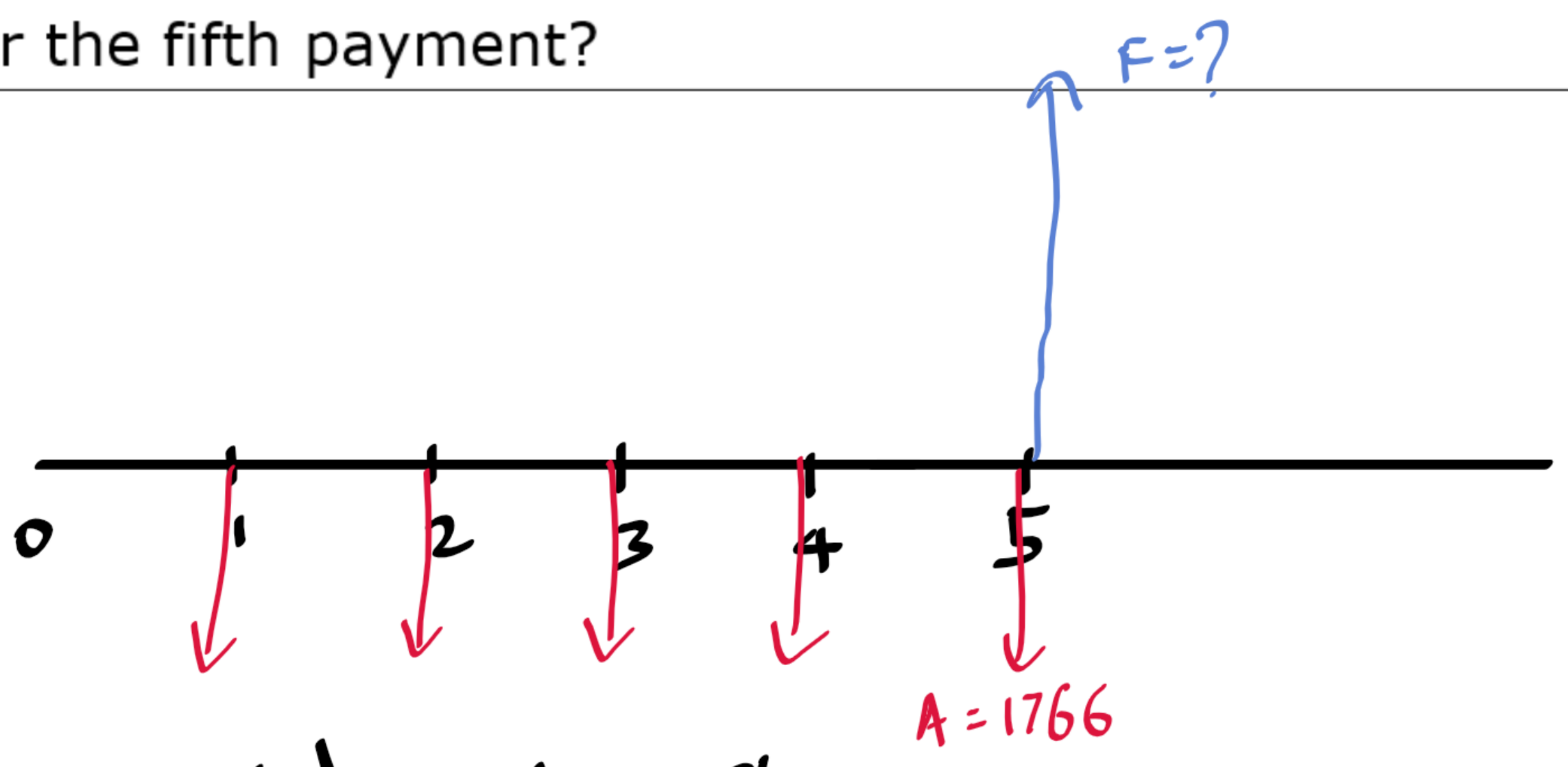
$$F = A\left[\frac{(i+1)^n - 1}{i}\right]$$

$$\Rightarrow F = 30000 \times \left[\frac{(1.1)^{15} - 1}{0.1}\right]$$

$$\Rightarrow F = 953174.451$$

4	a	A single payment of Rs.1766 is deposited in an account each year that earns 6 percent compounded continuously. What is the amount in the account immediately after the fifth payment?	6	CO1

Sol:



i=6% compounded continuously

N=5

$$A = F\left[\frac{e^{i}-1}{e^{in}-1}\right]$$ for continuous compounding

$$F = \frac{A\,(e^{in}-1)}{e^{i}-1} = \frac{1766\times(e^{0.06\times5}-1)}{e^{0.06}-1}$$

$\Rightarrow$ F = 1766 × 5.6618

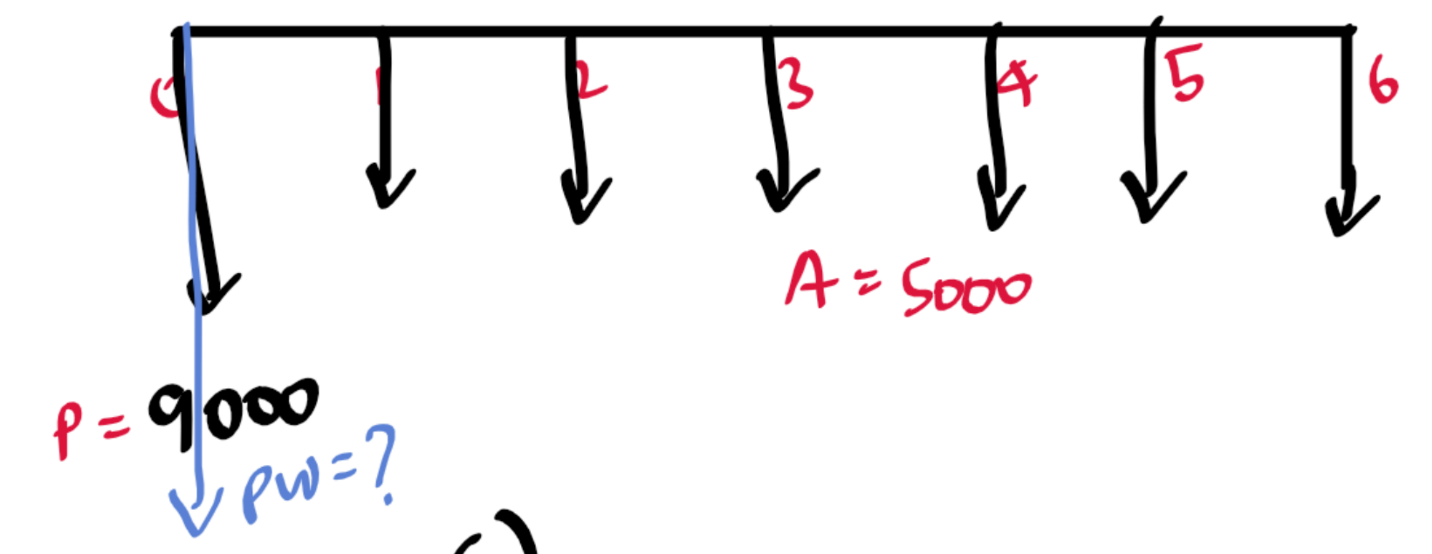
$\Rightarrow$ F = 9999

Always take 4 digits after decimal

4	c	Machine `X' has a first cost of Rs.9000 and no salvage value at the end of 6 years of useful life and has an annual operating cost of Rs.5000. Machine `Y' costs Rs.16,000 now and has an expected resale value of Rs.4,000 at the end of its life of 9 years and operating cost of machine Y is Rs. 4,000. Compare the two alternatives on the basis of their PW using repeated project assumption at 10% annual interest. Use CFD for year analysis.	8	CO1

Sol:

Machine X

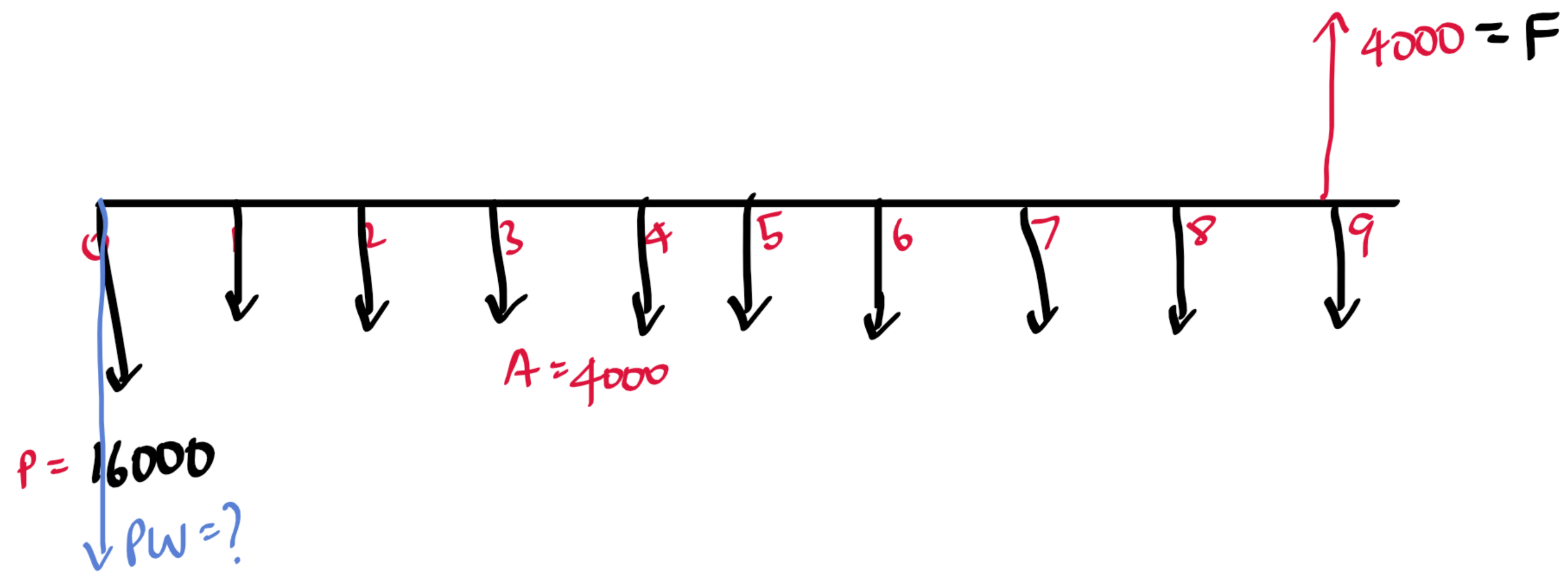


$$PW = P + A(P/A, 10\%, 6)$$

$$= -9000 - 5000 \times \left[\frac{(1.1)^6 - 1}{(0.1)(1.1)^6}\right]$$

$$PW = -30776.3035$$

Machine Y



$$PW = P + A(P/A, 10\%, 9) + F(P/F, 10\%, 9)$$

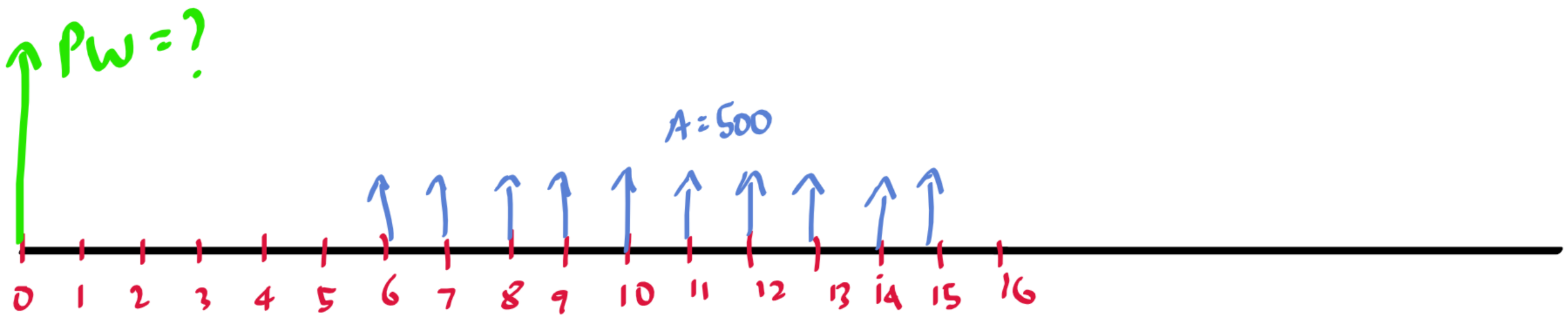
$$= -16000 - 4000 \times \left[\frac{(1.1)^9 - 1}{0.1(1.1)^9}\right] + \frac{4000}{(1.1)^9}$$

$$= -16000 - 23036.0952 + 1696.39$$

$$= -37339.7047$$

5	c	A deffered annuity is to pay $ 500 per year for 10 years with the first payment coming after 6 years from today. Determine the present worth	4	CO1

Sol:



Assume $i = 6\%$

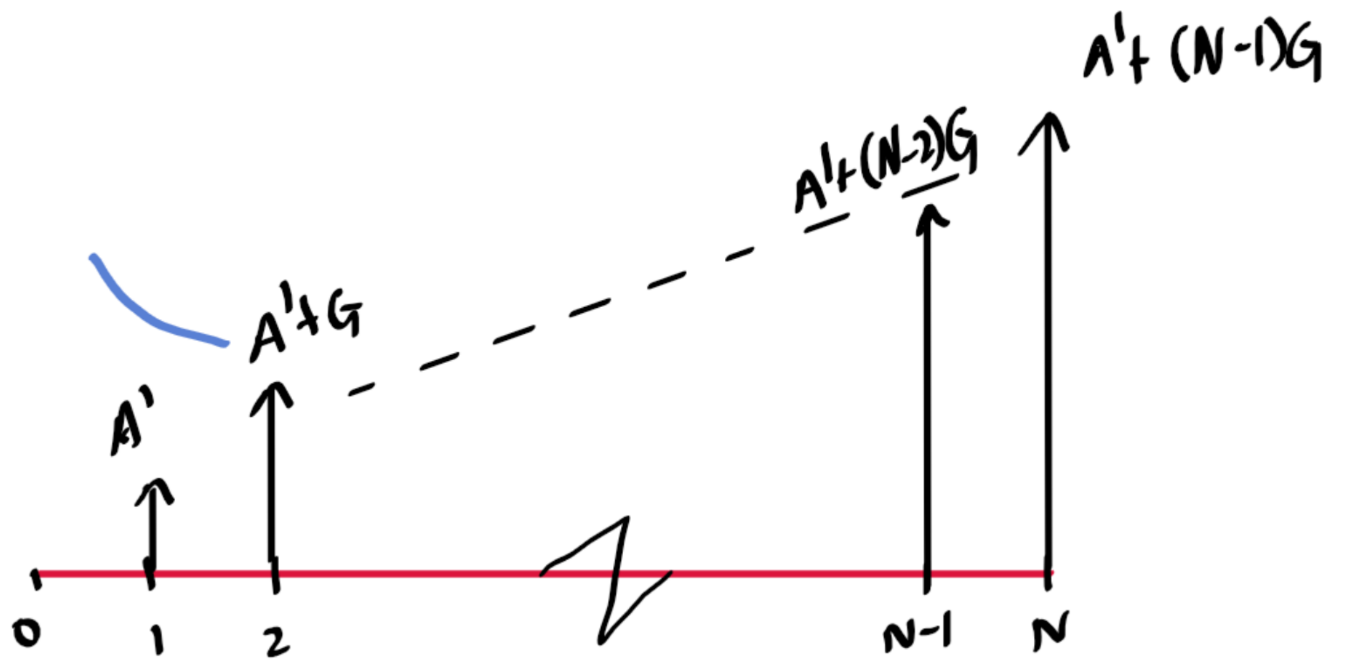
$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = 500\left[\frac{(1+0.06)^{10} - 1}{0.06}\right] = 6590.3975$$

$$P = \frac{F}{(1+i)^n} = \frac{6590.3975}{(1.06)^{15}} = 2749.9426$$

6	a	Apply the concepts of interest rate to derive a formula for arithmetic gradient conversion factor. Use the formula to compute the following problem. Receipts from an investment will decline by \$150 for each quarter for 2 years from a level of \$ 10000 at the end of the first quarter. For a nominal interest rate of 12% calculate a constant annual series amount that is equivalent to the gradient over 2 years period	8	CO1

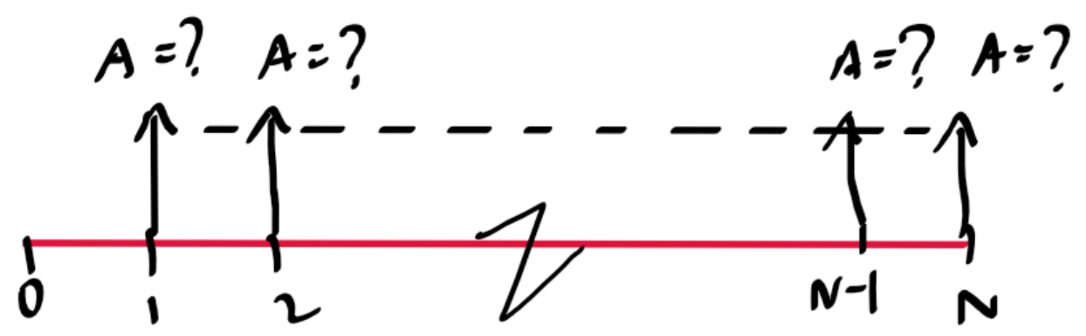
Sol:

Given:



At i% $\Rightarrow$

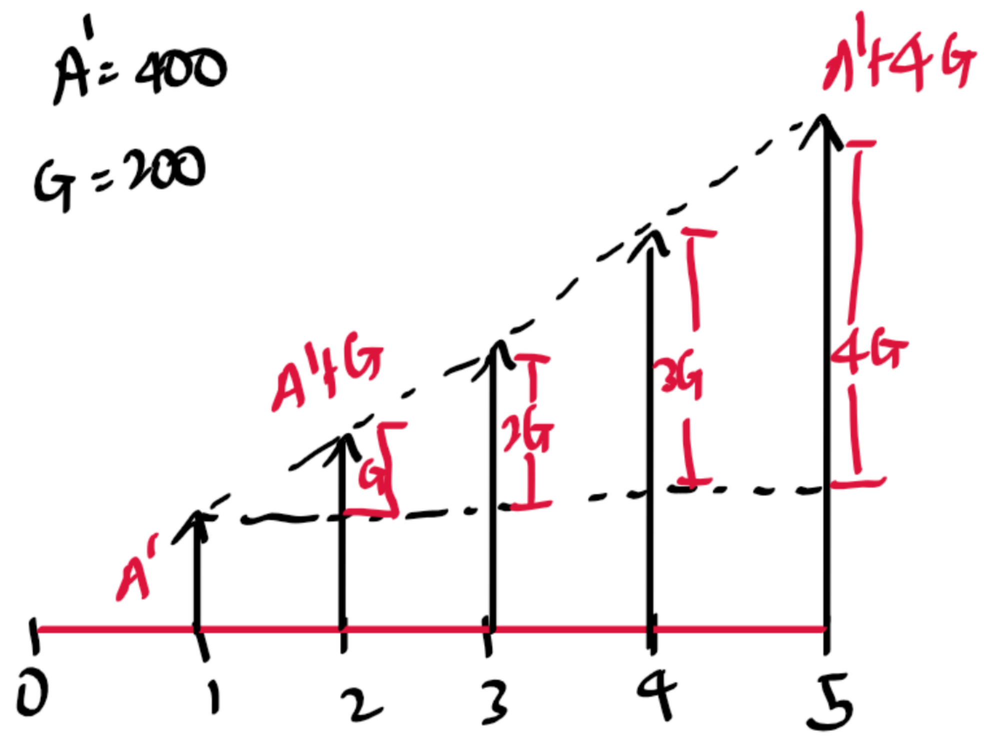
To convert to:



Gradient series: $A', \; A'+G, \; A'+2G, \; \ldots\ldots, \; A'+(N-1)G$

Eg:

$A' = 400$
$G = 200$



From the diagram,
Future worth of G values is

$$F = 200(F/P, i\%, 3) + 400(F/P, i\%, 2) + 600(F/P, i\%, 1) + 800$$

$$F = G(1+i)^3 + 2G(1+i)^2 + 3G(1+i) + 4G \quad -(1)$$

Multiply BS by $(1+i)$

$$\Rightarrow F(1+i) = G(1+i)^4 + 2G(1+i)^3 + 3G(1+i)^2 + 4G(1+i) \quad -(2)$$

(1) - (2)

$$\Rightarrow -Fi = -G(1+i)^4 - G(1+i)^3 - G(1+i)^2 - G(1+i) + 4G$$

$$\Rightarrow Fi = G(1+i)^4 + G(1+i)^3 + G(1+i)^2 + G(1+i) - 4G$$

For N periods

$$\Rightarrow Fi = G(1+i)^{N-1} + G(1+i)^{N-2} + \ldots\ldots + G(1+i) - (N-1)G$$

$$\Rightarrow Fi = G\Big[\underbrace{(1+i)^{N-1} + (1+i)^{N-2} + \ldots\ldots + (i+1) + 1}_{\text{series compound amount factor}}\Big] - NG$$

$$\Rightarrow Fi = G(F/A, i, N) - NG$$

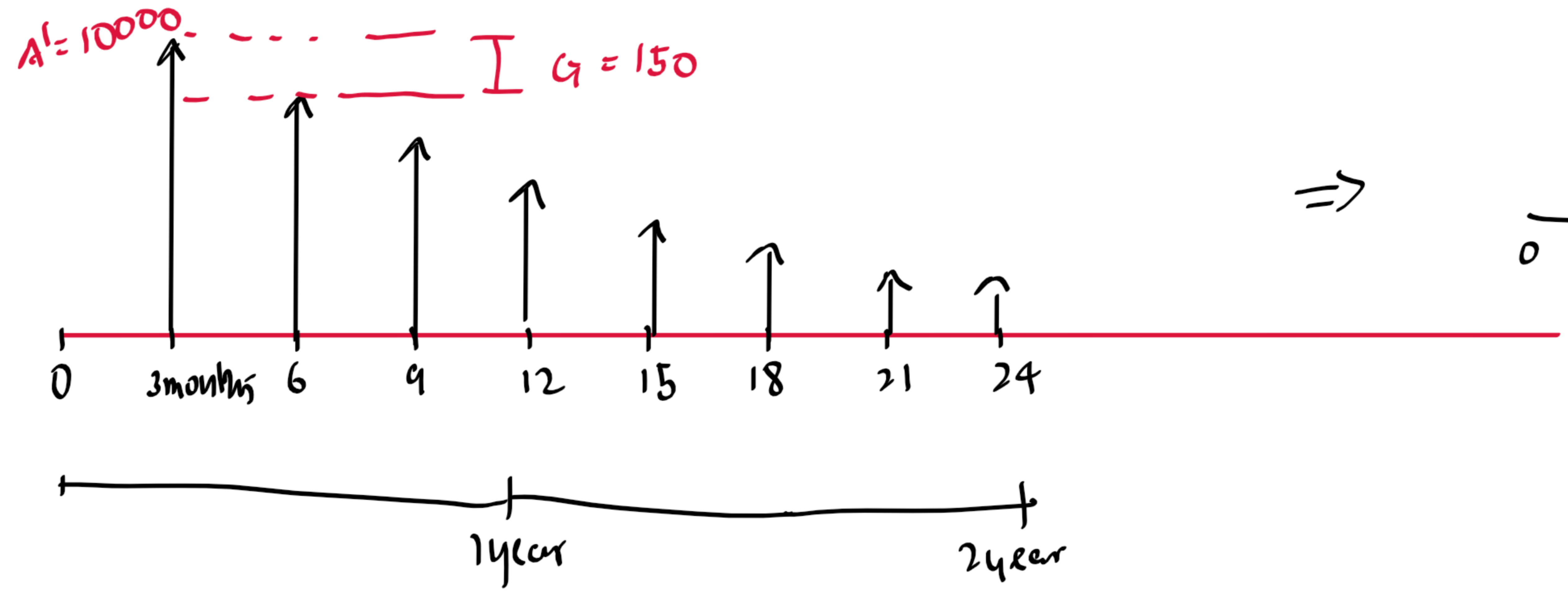
$$\Rightarrow Fi(A/F, i, N) = G - G(A/F, i, N)N$$

$F(A/F, i, N) = A$

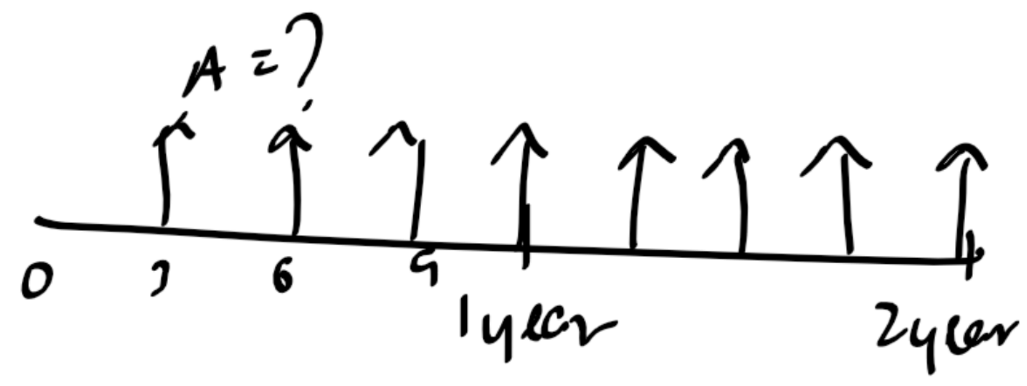
$$\Rightarrow Ai = G - GN(A/F, i, N)$$

$$\Rightarrow A = G\left[\frac{1}{i} - \frac{N\cancel{i}}{\cancel{i}((1+i)^N - 1)}\right] \Rightarrow \boxed{A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]} \Rightarrow$$

Payment per year
Add A'



$\Rightarrow$



A' = 10000

G = 150

i = 12% annual (Must convert to Quarter)

N = 2 years

Formula:

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right] + A'$$

$$= 150\left[\frac{1}{\left(\frac{12\%}{4}\right)} - \frac{2\times4}{\left(1+\frac{12\%}{4}\right)^{2\times4} - 1}\right] + 10000$$

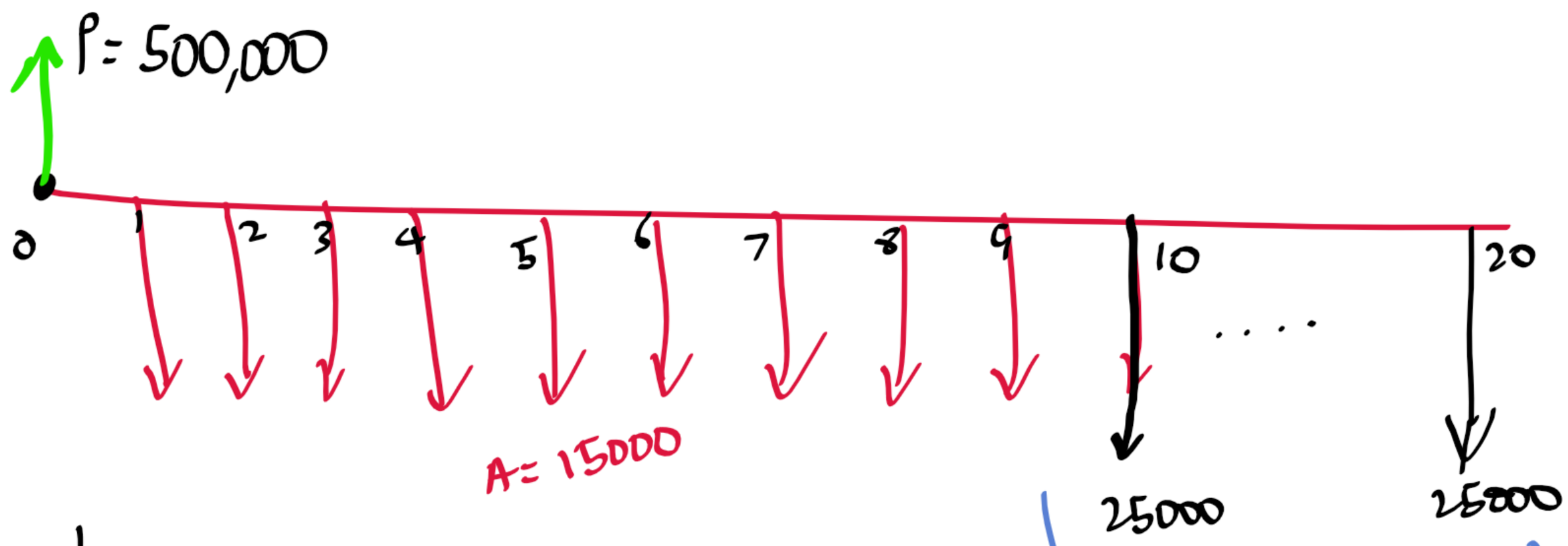
$$= 150\left[33.3333 - \frac{8}{0.26677}\right] + 10000$$

$$= 501.738 + 10000$$

$$= \boxed{10501.738}$$

7	c	Explain the different conditions for present worth comparison. A $500,000 gift was bequeathed to a city for the construction and continued upkeep of music shell. Annual maintenance for a shell is estimated at $15,000. In addition, $25,000 will be needed every 10 years for painting and major repairs. How much will be left for the initial construction costs, after funds are allocated for perpetual upkeep? Deposited funds can earn 6% annual interest, and these returns are not subject to taxes	5	CO1

Sol:



(Every 10 years) → convert to A

$N = \infty$

$(P/A, i\%, \infty) = \frac{1}{i}$

$$\text{Amount left} = P - (A + A')(P/A, 6\%, \infty)$$

$$= 500000 - (15000 + 1896.6989) \times \frac{1}{6\%}$$

$$= 500000 - 281611.64833$$

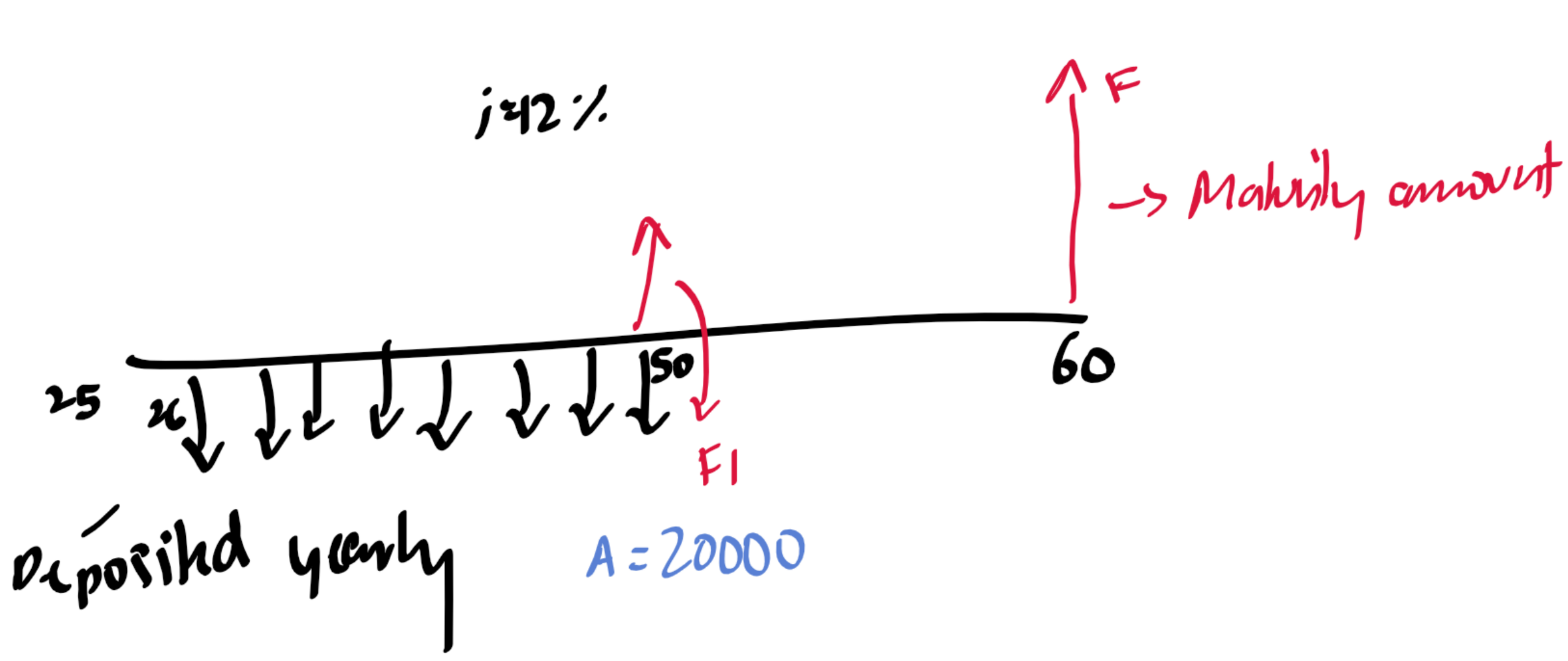
$$= 218388.35166$$

$$A' = 25000\,(A/F, 6\%, 10)$$

$$= 25000 \times \frac{0.06}{(1.06)^{10} - 1}$$

$$= 1896.6989$$

8	b	A 25 year old person is planning for his retired life. He plans to divert Rs. 20000/- from his bonous as investment every year for the next 25 years. The bank gives 12% interest rate compounded annually. Find the maturity value of his account when he is 60 year old. Draw cash flow diagram	10	CO1



$$F1 = A(F/A, 12\%, 25)$$

$$= 20000\left[\frac{(1+i)^n - 1}{i}\right] = 20000\left[\frac{(1.12)^{25} - 1}{0.12}\right]$$

$$F1 = 2666677.4$$

$$F = F1\,(1+i)^n$$

$$= 2666677.4\,(1.12)^{10}$$

$$\boxed{F = 8282295.23}$$