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MODULE :- 2

DIFFERENTIAL EQUATION - 2

Legendre's Linear D.E

A D.E of the form

$$a_0(ax+b)^2 d^2y/dx^2 + a_1(ax+b)\,dy/dx + a_2 y = X \rightarrow (1)$$

* to Solve 1 we put $\log(ax+b) = z$.

$$\left.\begin{aligned}(ax+b)\,dy/dx &= a\,dy/dz = aDy, \quad D = \frac{d}{dz}\\ (ax+b)^2 d^2y/dx^2 &= a^2 D(D-1)y\end{aligned}\right\} \rightarrow (2)$$

Substituting (2) in (1), eqn 1 reduces to a linear eqn with const co-efficient & Hence it can be solved.

1. Solve :-

$$(2x+3)^2 d^2y/dx^2 - 2(2x+3)\,dy/dx + y = x \rightarrow (1)$$

Soln put $\log_e(2x+3) = z$.

$$2x+3 = e^z$$

$$2x = e^z - 3$$

$$x = \frac{e^z-3}{2}.$$

$$2^2 D[D-1]y - 2[2Dy] + y = \frac{e^z-3}{2}.$$

$$[4D^2-4D-4D+1]y = \frac{e^z-3}{2}.$$

$$[4D^2-8D+1]y = \frac{e^z-3}{2}. \rightarrow (2)$$

$$4m^2 - 8m + 1 = 0$$

$$m = 1 \pm \sqrt{3}/2$$

$$CF = c_1 e^{(1+\sqrt{3}/2)x} + c_2 e^{(1-\sqrt{3}/2)x}.$$

$$PI = \frac{e^{z} - 3}{2}\left[\frac{1}{4D^2 - 8D + 1}\right]$$

$$\frac{1}{2}\left[\frac{-e^{z}}{3} - 3\right]$$

$$y = c_1 e^{(1+\sqrt{3}/2)z} + c_2 e^{(1-\sqrt{3}/2)z} + 1/2\left[\frac{-e^{z}}{3} - 3\right]$$

where $z = \log(2x+3)$.

2. $(x+2)^2 y'' + 3(x+2)y' + 3y = 0$

put $\log(x+2) = z$.

$$x + 2 = e^{z}$$

$$x = e^{z} - 2$$

$$D[D-1]y + 3Dy + 3y = 0.$$

$$[D^2 - D + 3D + 3]y = 0$$

$$[D^2 + 2D + 3]y = 0$$

$$m^2 + 2m + 3 = 0$$

$$m = -1 \pm \sqrt{2}i$$

$$CF = e^{-z}(c_1 \cos(\sqrt{2})z + c_2 \sin(\sqrt{2})z).$$

where $z = \log_e(x+2)$

3. $(2-3x)^2 y'' - (2-3x)y' - 6y = 0$

$\log_e(2-3x) = z$.

$(-3)^2 D(D-1)y - (-3Dy) - 6y = 0$

$[9D^2 - 9D + 3D - 6]y = 0$

$[9D^2 - 6D - 6]y = 0$

$AE = 9m^2 - 6m - 6 = 0$

$m = \dfrac{1 \pm \sqrt{7}}{3}$.

$y = c_1 e^{\left(\frac{1+\sqrt{7}}{3}\right)z} + c_2 e^{\left(\frac{1-\sqrt{7}}{3}\right)z}$.

where $z = \log_e(2-3x)$.

4. $(x-1)^2 y'' + 2(x-1)y' - 3y = \sin(\log(x-1))$

put $\log(x-1) = z$.

$D(D-1)y + 2Dy - 3y = \sin z$.

$[D^2 - D + 2D - 3]y = \sin z$

$[D^2 + D - 3]y = \sin z$

$m^2 + m - 3 = 0$

$m = \dfrac{-1 \pm \sqrt{13}}{2}$.

$CF = c_1 e^{\left(\frac{-1}{2} + \frac{\sqrt{13}}{2}\right)z} + c_2 e^{\left(-\frac{1}{2} - \frac{\sqrt{13}}{2}\right)z}$

$PI = \dfrac{\sin z}{D^2 + D - 3} \quad D^2 \to -1^2$

$$\frac{\sin z}{-1+D-3}$$

$$\frac{\sin z}{D-4}$$

$$\frac{(D+4)\sin z}{D^2-16} \qquad D^2 \to -1$$

$$\frac{\cos z + 4\sin z}{-17}$$

$$y = c_1 e^{\left(\frac{-1+\sqrt{13}}{2}\right)z} + c_2 e^{\left(-1-\sqrt{13}/2\right)z}$$

$$- \frac{\cos z + 4\sin z}{17}$$

where $z = \log_e(x-1)$.

5. $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$ → ①

Put $\log_e(1+x) = z$ → ② $a=1$

$(1+x)\frac{dy}{dx} = aDy = Dy$, $D = d/dz$.

$(1+x)^2\frac{d^2y}{dx^2} = a^2D(D-1)y = D(D-1)y$.

Subs ② in ①.

$D(D-1)y + Dy + y = 2\sin z$

$(D^2-D)y + D + y = 2\sin z$.

$y[D^2 - \cancel{D} + \cancel{D} + 1] = 2\sin z$

$[D^2+1]y = 2\sin z$.

$m^2+1 = 0$

$m = \pm i$

$CF = c_1\cos z + c_2\sin z$.

$PI = \frac{2\sin z}{D^2+1}$.

$D^2 \to -a^2$

$= \frac{2\sin z}{-1+1}$.

$= \frac{z\,2\sin z}{2D}$.

$= z\int \sin z$.

$PI = -z\cos z$.

$\frac{1}{9}\left[e^{2z} - 4\,\cancel{4}e^{z}\right] + \frac{1}{3}\left[e^{z} - 2\right]$

$= \frac{4e^z}{9} + \frac{e^z}{3}$

$\frac{3e^z - 4e^z}{9}$ $= \frac{-1e^z}{9}$

$y = CF + PI$.

$= c_1 \cos z + c_2 \sin z - z \cdot \cos z$.

where $z = \log(1+x)$.

6. $(3x+2)^2 \frac{d^2y}{dx^2} - 2(3x+2)\frac{dy}{dx} + 4y = x^2 + x + 1$

put $\cancel{\log(1+x)} = z$.

$\log(3x+2) = z$. $a = 3$.

$(3x+2)\frac{dy}{dx} = 3.Dy = D = \frac{d}{dz}$.

$e^z = 3x+2$.

$3x = e^z - 2$.

$x = \frac{e^z - 2}{3}$.

$(3x+2)^2 \frac{d^2x}{dy^2} = 3^2 D(D-1)y =$
$9D(D-1)$.

$9D(D-1)y - 2(3Dy) + 4y = \left(\frac{e^z-2}{3}\right)^2 + \left(\frac{e^z-2}{3}\right) + 1$

$[9D^2 - 9D - 6D + 4]y = \frac{1}{9}[e^{2z} + 4 - 4e^z] + \frac{1}{3}[e^z - 2] + 1$

$[9D^2 - 15D + 4]y = - - - -$

$9m^2 - 15m + 4 = 0$

$m = 4/3, 1/3$.

$CF = c_1 e^{4/3 z} + c_2 e^{1/3 z}$

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$$(9D^2-15D+4)y = \frac{e^{2z}}{9} - \frac{e^z}{9} + \frac{7}{9}$$

$$y = \frac{\frac{e^{2z}}{9}}{9D^2-15D+4} - \frac{\frac{e^z}{9}}{9D^2-15D+4} + \frac{7}{9\times 4}$$

$\frac{-4}{9} + 2/3$

$\frac{-4+3}{9} = -1/9$

$4/9 - 2/3 + 1 = 7/3$

$$\frac{1}{9}\left[\frac{e^{2z}}{9D^2-15D+4}\right] - \frac{1}{9}\left[\frac{e^z}{9D^2-15D+4}\right] + \frac{7}{36}$$

$$\frac{1}{9}\left[\frac{e^{2z}}{36-30+4}\right] - 1/9\left[\frac{e^z}{9-15+4}\right] + \frac{7}{36}$$

$$\frac{1}{9}\left[\frac{e^{2z}}{10}\right] - 1/9\left[\frac{e^z}{-2}\right] + \frac{7}{36}$$

$$PI = \frac{1}{9}\left[\frac{e^{2z}}{10} + \frac{e^z}{2} + \frac{7}{4}\right].$$

$$y = CF + PI$$

$$= c_1 e^{4/3 z} + c_2 e^{1/3 z} + \frac{1}{9}\left[\frac{e^{2z}}{10} + \frac{e^z}{2} + \frac{7}{4}\right]$$

where $z = \log(3x+2)$

7. $(2x-3)^2 y'' + 4y = 2\cos[3\log(2x-3)]$

put

$\log(2x-3) = z.$

$$4D(D-1)y + 4y = 2\cos[3\log(2x-3)].$$

$$[4D^2-4D+4]y = 2\cos[3z].$$

$4m^2 - 4m + 4 = 0$

$m = 1/2 \pm \sqrt{3}/2\, i$

$$CF = e^{1/2 z}\left(c_1 \cos \sqrt{3}/2\, z + c_2 \sin \sqrt{3}/2\, z\right).$$

$$PI = \frac{2\cos 3z}{4D^2 - 4D + 4}.$$

$$D^2 \to -a^2 = -9.$$

$$\frac{2\cos 3z}{-36 - 4D + 4}.$$

$$\frac{2\cos 3z}{-4D - 32}.$$

$$-\frac{2\cos 3z}{4D + 32}.$$

$$-\frac{2\cos 3z}{4D+32} \times \frac{4D - 32}{4D - 32}.$$

$$\frac{-2\cos 3z\,(4D - 32)}{16D^2 - 1024}.$$

$$\frac{+24\sin 3z + 64\cos 3z}{16D^2 - 1024}.$$

$$D^2 \to -a^2$$

$$\frac{+24\sin 3z + 64\cos 3z}{-1168}.$$

$$\frac{24\sin 3z + 64\cos 3z}{-1168}.$$

$$-\frac{3}{146}\sin 3z - \frac{4}{73}\cos 3z.$$

$$y = e^{1/2 z}\left(c_1 \cos \sqrt{3}/2\, z + c_2 \sin \sqrt{3}/2\, z\right) - \frac{3}{146}\sin 3z - \frac{4}{73}\cos 3z$$

$$z = \log(2x - 3)$$

$\frac{2}{3} - \frac{4}{9}$

$\frac{6 - 4}{9}$

$\frac{2}{9} + 1$

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Cauchy's Linear D.E.

A. D.E of the form

$$a_0 x^2 \frac{d^2y}{dx^2} + a_1 x \frac{dy}{dx} + a_2 y = X \rightarrow (1)$$

Put $\log x = z$.

$$\left.\begin{aligned} x \cdot \frac{dy}{dx} &= Dy. \\ x^2 \frac{d^2y}{dx^2} &= D(D-1)y \end{aligned}\right\} \rightarrow (2)$$

Substituting (2) in (1) the eq^{-n} reduces to. constant co-efficient hence it ~~reduces~~ ~~to~~ can be solved.

1. $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 4y = x^2$.

Put $\log x = z$.

$x = e^z$.

$$D(D-1)y - 2 \cdot Dy + 4y = x^2.$$

$$(D^2 - D - 2D + 4)y = e^{2z}.$$

$$m^2 - 3m + 4 = 0.$$

$$m = 3/2 \pm \sqrt{7}/2 \, i$$

$$CF = e^{3/2z}\left(C_1 \cos \sqrt{7}/2 z + C_2 \sin \sqrt{7}/2 z\right).$$

$$PI = \frac{e^{2z}}{D^2 - 3D + 4}.$$

$$\frac{e^{2z}}{4-6+4}$$

$$PI \quad \frac{e^{2z}}{2}.$$

$$y = e^{3/2z}\left(c_1 \cos \sqrt{7}/2\, z + c_2 \sin \sqrt{7}/2\, z\right) + \frac{e^{2z}}{2}$$ //

where $z = \log x$.

2. $xy'' + y' = x^2 + 1/x$.

~~put $\log x = z$.~~

~~$x = e^z$.~~

multiply B.s by x.

$$x^2 y'' + xy = x^3 + 1$$

put $\log x = z$.

$$x = e^z.$$

$$D(D-1)y + Dy = x^3 + 1$$

$$(D^2 - D + D)y = x^3 + 1$$

$$D^2 = x^3 + 1 = e^{3z} + 1$$

$$m = 0, 0.$$

$$CF = (c_1 + c_2 z)e^0$$

$$= (c_1 + c_2 z).$$

$$\left\{ \begin{array}{l} L(D) = D^2 \\ L' = 2D. \\ L'' = 2. \end{array} \right.$$

$$PF = \frac{e^{3z} + 1}{D^2}.$$

$$= \frac{e^{3z}}{D^2} + \frac{1e^{0z}}{D^2}$$

$$PI = \frac{e^{3z}}{4} + \frac{z^2}{2}.$$

$y = CF + PI$

$= (c_1 + c_2 z) + \frac{e^{3z}}{9} + \frac{1}{2}$ //

3. $xy'' + y' + 4y/x = 10(x + 1/x)$.

multiply by x.

$x^2y'' + xy' + 4y = 10x^2 + 10$

$= 10(x^2 + 1)$.

put $\log x = z$.

$x = e^z$.

$D(D-1)y + Dy + 4y = 10(e^{2z}+1)$ $\cancel{10(x^2+1)}$.

$(D^2 - \cancel{D} + \cancel{D} + 4)y = 10(e^{2z} + 1)$

$(D^2 + 4)y$.

$m^2 + 4 = 0$

$m^2 = -4$.

$m = \pm 2i$

$CF = c_1\cos 2z + c_2\sin 2z$.

$PI = \frac{10(e^{2z}+1)}{D^2+4}$.

$\frac{10e^{2z}}{D^2+4} + \frac{10}{D^2+4}$.

$\frac{\cancel{10}^5 e^{2z}}{\cancel{8}4} + \frac{\cancel{10}^5}{\cancel{4}2}$

$PI = \frac{5}{\cancel{16}4}e^{2z} + 5/2$. where $z = \log x$.

$y = c_1\cos 2z + c_2\sin 2z + \frac{5}{4}e^{2z} + 5/2$ //

11. $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 2y = 2\sin(\log x)$

Put $\log x = z$.

$D(D-1)y + 3Dy - 2y = 2\sin z$

$(D^2 - D + 3D - 2)y = 2\sin z$.

$m^2 + 2m - 2 = 0$

$m = -1 \pm \sqrt{3}$.

$CF = C_1 e^{(-1+\sqrt{3})z} + C_2 e^{(-1-\sqrt{3})z}$

$PI = \frac{2\sin z}{D^2 + 2D - 2}$.

$D^2 \to -a$.

$\frac{2\sin z}{-1 + 2D - 2}$.

$\frac{2\sin z}{2D - 3} \times \frac{2D+3}{2D+3}$.

$\frac{2\sin z\,(2D+3)}{4D^2 - 9}$.

$\frac{4\cos z + 6\sin z}{4D^2 - 9}$. $D^2 \to -a^2 = -1$

$\frac{4\cos z + 6\sin z}{-13}$.

$y = C_1 e^{(-1+\sqrt{3})z} + C_2 e^{(-1-\sqrt{3})z} - \frac{4\cos z}{13} - \frac{6\sin z}{13}$.

where $z = \log x$.

5. $(3-2x)^2 y'' + 2(3-2x)y' - 4y = 3\cos(\log(3-2x))$

Put $\log(3-2x) = z$.

$3 - 2x = e^z$.

$-2x = e^z - 3$.

$2x = 3 - e^z$.

$x = \frac{3-e^z}{2}$.

$4D(D-1)y + 2(2 \cdot Dy) - 4y = 3\cos(z)$.

$(4D^2 - 4D \cancel{+4D} - 4)y = 3\cos z$.

$4D^2 - 4 = 3\cos z$

$4m^2 - 4 = 0$

$(4D^2 - 8D - 4)y = 3\cos z$

$4m^2 - 8m - 4 = 0$

$m = 1 \pm \sqrt{2}$

$CF = c_1 e^{(1+\sqrt{2})z} + c_2 e^{(1-\sqrt{2})z}$.

$PI = \frac{3\cos z}{4D^2 - 8D - 4}$.

$\frac{3}{4}\left[\frac{\cos z}{D^2 - 2D - 1}\right]$ $D^2 \to -a^2$

$-\frac{3}{4}\left[\frac{\cos z}{2D + 2}\right] \times \frac{2D+2}{2D+2}$

$-\frac{3}{4}\frac{(-2\sin z + 2\cos z)}{4D^2 - 4}$, $D^2 \to -a^2$

$\boxed{-\frac{3}{16}\left(\frac{-2\sin z + \cos z}{}\right) \quad -\frac{3}{4}\left(\frac{2\sin z + \cos z}{5}\right)}$

$$\frac{-3}{16}\left(\frac{-2\sin z+2\cos z}{D^2-1}\right) \qquad D^2 \to -a^2$$

$$\frac{-3}{16}\left(\frac{-2\sin z+2\cos z}{-2}\right)$$

$$PI = \frac{-3}{16}(\cos z+\sin z)$$ //

$y = CF + PI$

6. $xy'' + y' + \frac{3y}{x} = 10\left(x^3 + \frac{1}{x^3}\right)$

multiply by x.

$$x^2y'' + xy' + 3y = 10\left(x^4 + \frac{1}{x^2}\right).$$

put $\log x = z$.

$x = e^z$.

$$D(D-1)y + Dy + 3y = - -$$

$$(D^2 - \cancel{D} + \cancel{D} + 3)y = 10(e^{4z} + e^{-2z}).$$

$$(D^2+3)y$$

$m^2 + 3 = 0$

$m^2 = -3$

$m = \pm 3i$

$CF = c_1 \cos 3z + c_2 \sin 3z$.

$$PI = 10\left[\frac{e^{4z}}{D^2+3} + \frac{e^{-2z}}{D^2+3}\right].$$

$$PI = 10\left[\frac{e^{4z}}{19} + \frac{e^{-2z}}{7}\right].$$

$$y = c_1\cos\sqrt{3}z + c_2\sin\sqrt{3}z + 10\left[\frac{e^{4z}}{19} + \frac{e^{-2z}}{7}\right]$$ //

where $z = \log x$.

7. $x^2y'' - 6y = x^3 + \cos(3\log x)$

Put $\log x = z$

$x = e^z$

$D(D-1)y - 6y = e^{3z} + \cos 3z$

$(D^2-D)y - 6y = - -$

$(D^2-D-6)y - -$

$m^2 - m - 6 = 0$

$m = \cancel{\frac{1}{2} \pm \frac{\sqrt{23}}{2} i}\ 3, -2$

$CF = C_1 e^{3z} + C_2 e^{-2z}$

$PI = \frac{e^{3z}}{D^2-D-6} + \frac{\cos 3z}{D^2-D-6}$

$D \to a$ $\qquad D^2 \to -a^2$

$\frac{e^{3z}}{9-3-6\ (0)} + \frac{\cos 3z}{-9-D-6}$

$\frac{ze^{3z}}{2D-1} + \frac{\cos 3z}{-D-15}$

$\frac{ze^{3z}}{5} + -\frac{\cos 3z}{D+15} \cdot \frac{D-15}{D-15}$

$\frac{ze^{3z}}{5} - \frac{(3\sin 3z - 15\cos 3z)}{D^2 - 225}$

$D^2 \to -a^2$

$\frac{ze^{3z}}{5} + \frac{(3\sin 3z + 15\cos 3z)}{-234}$

$\frac{ze^{3z}}{5} - \frac{(3\sin 3z + 15\cos 3z)}{234}$

$y = C_1 e^{3z} + C_2 e^{-2z} + \frac{ze^{3z}}{5} - \frac{(3\sin 3z + 15\cos 3z)}{234}$

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Differential eq^n of 1st order & Higher degree

1. Equation Solving for 'p'

conside a D.E in p given by $\longrightarrow$ (1)

$$A_0p^n + A_1p^{n-1} + A_2p^{n-2} + \cdots + A_n = 0$$

where A_i's are all are eigther constants or functions of x & i

$$p = \frac{dy}{dx}$$

* Factorise the L.H.S of eq^n (1) or we the formula $p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

* Let the factors be $[p - f_1(x,y)] = 0$

$$[p - f_2(x,y)] = 0. \text{ etc.}$$

* Let the Solution of the above D.E be

$$F_1(x,y,c_1) = 0, \quad F_2(x,y,x_2) = 0 \text{ etc}$$

* Then the solution of (1) is given by the product of all the above solutions.

$$F_1(x,y,c_1) \cdot F_2(x,y,c_2) = 0$$

[p^2 & p terms are present in the given equation]

1. Solve.

$p^2 - 5p + 6 = 0$

$p(p-3) - 2(p-3) = 0$

$(p-2) = 0, \quad (p-3) = 0$

$p - 2 = 0, \quad p = 2$

$p - 3 = 0 \quad p = 3.$

$$\begin{array}{c} +6 \\ \wedge \\ -3 \quad -2 \end{array}$$

$\because p = \frac{dy}{dx}$.

$\therefore \frac{dy}{dx} - 2 = 0 \Rightarrow \frac{dy}{dx} = 2. \to (2)$

$\frac{dy}{dx} - 3 = 0 \Rightarrow \frac{dy}{dx} = 3 \to (3)$

$(2) \Rightarrow \frac{dy}{dx} = 2.$

$dy = 2dx.$

$\int dy = 2\int dx.$

$y = 2x + c_1$

$y - 2x - c_1 = 0. \to (4)$

$(3) \Rightarrow \frac{dy}{dx} = 3.$

$dy = 3dx.$

$\int dy = 3\int dx.$

$y = 3x + c_2$

$y - 3x - c_2 = 0. \to (5)$

(4) & (5).

$$(y-2x-c_1)(y-3x-c_2)=0.$$

2. $p^2+p(x+y)+xy=0$.

$a=1,\quad b=x+y,\quad c=xy$

$$p=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\frac{-(x+y)\pm\sqrt{(x+y)^2-4xy}}{2}.$$

$$\frac{-(x+y)\pm\sqrt{x^2+y^2+2xy-4xy}}{2}.$$

$$\frac{-(x+y)\pm\sqrt{x^2+y^2-2xy}}{2}$$

$$\frac{-(x+y)\pm\sqrt{(x-y)^2}}{2}$$

$$\frac{-(x+y)\pm(x-y)}{2}.$$

$\therefore p=\dfrac{-\cancel{x}-y+\cancel{x}-y}{2}$ (or) $\dfrac{-x-\cancel{y}-x+\cancel{y}}{2}.$

$=\dfrac{-2y}{2}$ (or) $\dfrac{-2x}{2}$

$=-y$ $\qquad = -x.$

* $\dfrac{dy}{dx}=-y.$

$\displaystyle\int\frac{dy}{y}=\int dx$

$=\log y=-x+c_1$ $\quad \log y+x-c_2$

* $\displaystyle\int\frac{dy}{dx}=-x$

$\int dy=-\int x\,dx$

$=y=-\dfrac{x^2}{2}+c_2 \Rightarrow y+\dfrac{x^2}{2}-c_2$

$$(\log y + x - c_1)(y + \frac{x^2}{2} - c_2) = 0$$

3. $x^2p^2 + 3xyp + 2y^2 = 0$

$a = x^2,\ b = 3xy,\ c = 2y^2$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3xy \pm \sqrt{9x^2y^2 - 8x^2y^2}}{2x^2}$$

$$= \frac{-3xy \pm \sqrt{x^2y^2}}{2x^2}$$

$$\frac{-3xy \pm xy}{2x^2}$$

$$\frac{-3xy + xy}{2x^2} \text{ or } \frac{-3xy - xy}{2x^2}$$

$$p = \frac{-\cancel{4x}y}{\cancel{2}x^2} = -4/x \qquad \frac{-4xy}{\cancel{2}x^2} = \frac{2y}{x}$$

$$\frac{dy}{dx} = -4/x.$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}.$$

$$= \log y = -\log x + C_1$$

$$\log y + \log x - C_1$$

$$\frac{dy}{dx} = \frac{-2y}{x}.$$

$$\int \frac{dy}{y} = \int \frac{-2\,dx}{x}.$$

$$= \log y = -2\log x + C_2$$

$$\log y + 2\log x - C_2.$$

$$\left(\log y + \log x - c_1\right)\left(\log y + 2\log x - c_2\right) = 0.$$

$$\left(\log\left(\frac{y}{x}\right) + c_1\right)\cdot\left(\log y + 2\log x - c_2\right) = 0$$

4. $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$

$a = xy \quad b = 3x^2 - 2y^2 \quad c = -6xy.$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\frac{-(3x^2-2y^2) \pm \sqrt{(3x^2-2y^2)^2 - 4xy(6xy)}}{2xy}.$$

$$\frac{-(3x^2-2y^2) \pm \sqrt{9x^4 + 4y^4 - 12x^2y^2 + 24x^2y^2}}{2xy}.$$

$$-(3x^2-2y^2) \pm \sqrt{9x^4 + 4y^4 + 12x^2y^2}$$

$$\frac{-(3x^2-2y^2) \pm (3x^2+2y^2)}{2xy}.$$

$$\frac{-3x^2 + \cancel{2y^2} - 3x^2 - \cancel{2y^2}}{2xy} \text{ or } \frac{-3x^2 + 2y^2 + 3x^2 + 2y^2}{2xy}$$

$$\frac{-6x^2}{2xy} \text{ or } \frac{4y^2}{2xy}$$

$$\frac{-2(3x^2 - y^2)}{2xy}$$

$$\left(\frac{3x}{y} + \frac{1}{x}\right) \text{ or } \frac{3y}{x}.$$

$\therefore$

$$\frac{-\overset{3}{\cancel{6}}x^2}{\cancel{2}xy} \quad \text{or} \quad \frac{\overset{2}{\cancel{4}}y^2}{\cancel{2}xy}$$

$$-\frac{3x}{y} \quad \text{or} \quad \frac{2y}{x}.$$

$$p = -\frac{3x}{y} \quad \text{or} \quad p = \frac{2y}{x}.$$

$$\frac{dy}{dx} = -\frac{3x}{y}, \quad \frac{dy}{dx} = \frac{2y}{x}.$$

$$\int dy\,(y) = -\int 3x\,.dx, \quad \int \frac{dy}{2y} = 2\int \frac{dx}{x}.$$

$$\frac{y^2}{2} = -\frac{3x^2}{2} + c_1 \qquad \log y = 2\log x + c_2.$$

$$y^2 + 3x^2 + 2c_1 = 0. \qquad \log y - 2\log x - \log c_2$$

$$(y^2 + 3x^2 + 2c_1)(\log y - 2\log x - \log c_2) = 0$$

5. $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0.$

$$yp^2 + (x-y)p - x = 0$$

$$a = y, \quad b = (x-y) \quad c = -x.$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\frac{-(x-y) \pm \sqrt{(x-y)^2 + 4xy}}{2y}.$$

$$\frac{-(x-y) \pm \sqrt{x^2 + y^2 - 2xy + 4xy}}{2y}$$

$$\frac{-(x-y) \pm \sqrt{(x+y)^2}}{2y}$$

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$$\frac{-(x-y) \pm (x+y)}{2y}$$

$$\frac{-(x-y)+x+y}{2y} \quad \text{or} \quad \frac{-x+y-x-y}{2y}$$

$$\frac{2y}{2y} = 1 \quad \text{or} \quad \frac{-2x}{2y} = -x/y$$

$$\frac{dy}{dx} = 1 \qquad\qquad \frac{dy}{dx} = -x/y$$

$$\int dy = \int dx \qquad\qquad \int dy\,(y) = -\int x\,dx$$

$$y = x \qquad\qquad \frac{y^2}{2} = -\frac{x^2}{2}$$

$$y - x = 0 \qquad\qquad \frac{y^2}{2} + \frac{x^2}{2} = 0$$

$$(y-x)\left(\frac{y^2}{2} + \frac{x^2}{2}\right) = 0 \,//$$

6. $x^2\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{dy}{dx}\right) - 6y^2 = 0$.

$x^2 p^2 + xyp - 6y^2 = 0$

$a = x^2, \quad b = xy, \quad c = -6y^2$.

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-xy \pm \sqrt{x^2y^2 + 24x^2y^2}}{2x^2}$$

$$= \frac{-xy \pm \sqrt{25x^2y^2}}{2x^2}$$

$$= \frac{-xy \pm 5xy}{2x^2}$$

$$= \frac{-xy + 5xy}{2x^2} \quad \text{or} \quad \frac{-xy - 5xy}{2x^2}$$

$$\frac{{}^{2}\cancel{4}xy}{\cancel{2}x} \quad \text{or} \quad \frac{-{}^{3}\cancel{6}xy}{\cancel{2}x^2}$$

$$2\frac{y}{\cancel{x}} \quad \text{or} \quad \frac{-{}^{3}\cancel{2}y}{x}$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\int\frac{dy}{y} = 2\int\frac{dx}{x}$$

$$\therefore \log y = 2\log x + \log c_1$$

$$\log y - \log x^2 = \log c_1$$

$$\log\left[y/x^2\right] = \log c_1$$

$$\frac{dy}{dx} = \frac{-2y}{x}$$

$$\int\frac{dy}{y} = -2\int\frac{dx}{x}$$

$$\log y = -\cancel{3}\log x + c_2$$

$$\log y + \log x^3 = \log c_2$$

$$\log(y \cdot x^2) = \log c_2$$

$$\left(y/x^2 - c_1\right)\left(yx^2 + c_2\right) = 0$$

7. $p(p+y) = x\cdot(x+y)$

$$p^2 + py = x^2 + xy$$

$$p^2 + py - (x^2 + xy) = 0$$

$$a = 1, \quad b = y, \quad c = -(x^2+xy)$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-y \pm \sqrt{y^2 + 4(x^2+xy)}}{2}$$

$$\frac{-y \pm \sqrt{y^2 + 4x^2 + 4xy}}{2}$$

$$\frac{-y \pm \sqrt{(y+2x)^2}}{2}$$

$$= \frac{-y \pm y + 2x}{2}$$

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$$= \frac{-y + y + 2x}{2} = x.$$

$$-y - y - 2x.$$

$$\frac{-2y - 2x}{2} = \frac{-2(x+y)}{2}$$

$$= -(x+y).$$

$$\frac{dy}{dx} = x,$$

$$\int dy = \int x \cdot dx$$

$$y = \frac{x^2}{2} + C_1 \longrightarrow ①$$

$$\frac{dy}{dx} = -(x+y).$$

$$dy = -(x+y)\,dx.$$

$$\int dy = \int -x\,dx - \int y \cdot dx.$$

$$y = \frac{-x^2}{2} - yx + C_2$$

$$\frac{dy}{dx} = -x - y.$$

$$\boxed{\frac{dy}{dx} + y = -x} \longrightarrow ②$$

$$IF = e^{\int P dx} = e^{\int 1 dx} = e^x.$$

$$Sol^n = y[IF] = \int Q\,[IF]\,dx + C_2.$$

$$ye^x = -\int x \cdot e^x\,dx + C_2.$$

$$ye^x = -\left[x e^x - \int e^x \cdot 1\,dx\right] + C_2.$$

$$ye^x = -xe^x + e^x + C_2.$$

$$ye^x + xe^x - e^x - C_2 \longrightarrow ③$$

∴ ① × ③

$$\left(y - x^2/2 - c_1\right)\left(ye^x + xe^x - e^x - c_2\right) = 0$$

8. $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$.

$$p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$$

$$\frac{p^2 - 1}{p} = \frac{x^2 - y^2}{xy}.$$

$$(p^2 - 1)xy = p(x^2 - y^2)$$

$$xyp^2 - xy - p(x^2 - y^2)$$

$$xyp^2 - (x^2 - y^2)p - xy = 0$$

$$a = xy, \quad b = -(x^2 - y^2), \quad c = -xy.$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\frac{(x^2 - y^2) \pm \sqrt{x^4 + y^4 - 2x^2y^2 + 4x^2y^2}}{2xy}.$$

$$\frac{(x^2 - y^2) \pm \sqrt{x^4 + y^4 + 2x^2y^2}}{2xy}.$$

$$\frac{(x^2 - y^2) \pm \sqrt{(x^2 + y^2)^{\cancel{2}}}}{2xy}.$$

$$\frac{x^2 - \cancel{y^2} + x^2 + \cancel{y^2}}{2xy} \quad \text{or} \quad \frac{\cancel{x^2} - y^2 - \cancel{x^2} - y^2}{2xy}$$

$$\frac{\cancel{2}x^{\cancel{2}}}{\cancel{2}\cancel{x}y} \quad \text{or} \quad \frac{-\cancel{2}y^{\cancel{2}}}{\cancel{2}x\cancel{y}}$$

$$\frac{x}{y} \quad \text{or} \quad -\frac{y}{x}.$$

$$\frac{dy}{dx} = \frac{x}{y} \quad \Big| \quad \frac{dy}{dx} = -y/x.$$

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$$\int y \cdot dy = \int x \cdot dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c_1$$

$$\frac{y^2}{2} - \frac{x^2}{2} - c_1$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}.$$

$$\log y = -\log x + \log c_2$$

$$\log y + \log x = \log c_2$$

$$xy = c_2.$$

$$\left(\frac{y^2}{2} - \frac{x^2}{2} - c_1\right)(xy - c_2) = 0 //$$

9. $p^2 + 2py\cot x = y^2.$

$$p^2 + 2py\cot x - y^2 = 0.$$

$$a = 1, \quad b = 2y\cot x, \quad c = -y^2.$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$= \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2a}.$$

$$-2y\cot x \pm \sqrt{4y^2(\cot^2 x + 1)}.$$

$$-2y\cot x \pm 2y\sqrt{\operatorname{cosec}^2 x}.$$

$$\frac{-2y\cot x \pm 2y\operatorname{cosec} x}{2a}$$

$$p \cdot \left[\frac{-y\cot x \pm y\operatorname{cosec} x}{\cancel{2}}\right]$$

$$p = y(-\cot x - \operatorname{cosec} x) \quad \text{or}$$

$$p = y(-\cot x + \operatorname{cosec} x).$$

$$\frac{dy}{dx} = y(-\cot x + \operatorname{cosec} x).$$

$$\int \frac{dy}{y} = \int (-\cot x + \operatorname{cosec} x)\, dx$$

$$\log y = -\log|\sin x| + \log|\operatorname{cosec} x - \cot x| + \log c_1$$

$$y + \log|\sin x| - \log|\operatorname{cosec} x - \cot x| \pm \log c_2$$

$$y = \log \frac{|\operatorname{cosec} x - \cot x|}{\sin x} + c_1$$

$$\frac{y \sin x}{\log|\operatorname{cosec} x - \cot x|} = \log c_1$$

$$\frac{\sin x}{\operatorname{cosec} x - \cot x} = c_1$$

$$\frac{dy}{dx} = y(-\cot x - \operatorname{cosec} x)$$

$$\int \frac{dy}{y} = (-\cot x - \operatorname{cosec} x)\,dx$$

$$\log y = -\log|\sin x| - \log|\operatorname{cosec} x - \cot x| + \log c_2.$$

$$\log y + \log|\sin x| + \log|\operatorname{cosec} x - \cot x| = \log c_2$$

$$\log y \; \sin x(\operatorname{cosec} x - \cot x) = \log c_2.$$

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[y appears in first degree only]

II) Equation solvable for 'y' :-

Consider the D.E which can be in the form

* $y = f(x, p) \rightarrow (1)$, $p = \frac{dy}{dx}$.

* put $\frac{dy}{dx} = p$

* Let the sol^n of above D.E

$$F(x, p, c) = 0 \rightarrow (2)$$

* eliminate 'p' b/w (1) & (2) to get the required sol^n of (1).

1. $y = -px + p^2x^4 \rightarrow (1)$

The given eq^n is solved for 'y'

Diff w.r.t x.

$$\frac{dy}{dx} = -\left[p + x\frac{dp}{dx}\right] + \left[p^2 4x^3 + x^4 2p\frac{dp}{dx}\right]$$

$$p = \left[-p + 4p^2x^3\right] + \left[-x + 2px^4\right]\frac{dp}{dx}.$$

$$-2p + 4p^2x^3 + x\left[2px^3 - 1\right]\frac{dp}{dx} = 0$$

$$2p\left[2px^3 - 1\right] + x\left[2px^3 - 1\right]\frac{dp}{dx} = 0.$$

~~$2p[2px^3-1] + x[2px^3$~~

$$\left[2px^3 - 1\right]\left[2p + x\frac{dp}{dx}\right] = 0.$$

Ignoring the term does not contain $\frac{dp}{dx}$ we get

$$2p + x\frac{dp}{dx} = 0.$$

$$2p = -x\frac{dp}{dx}.$$

$$-2\int\frac{dx}{x} = \int\frac{dP}{P}$$

$$-2\log x = \log P + \log c$$

$$\log x^{-2} + \log c = \log P.$$

$$cx^{-2} = P.$$

$$\boxed{P = \frac{c}{x^2}} \rightarrow (2)$$

To eleiminate P b/w (1) & (2)

$$\therefore\ y = \frac{-c}{x^2}\,x + \frac{c^2}{x^4}\,x^4$$

$$\boxed{y = \frac{-c}{x} + c^2}$$

2. $x^2\left(\frac{dy}{dx}\right)^4 + 2x\frac{dy}{dx} - y = 0$

$$x^2P^4 + 2xP - y = 0$$

$$y = x^2P^4 + 2xP$$

$$\frac{dy}{dx} = \left[x^2 4P\frac{dP}{dx} + P^4 2x\right] + 2\left[x\frac{dP}{dx} + P\right]$$

$$P = \left[P^4 2x + 2P\right] + \left[4x^2P + 2x\right]\frac{dP}{dx}.$$

$2P[$

$$= P\left[1 + 2P^3x\right] + 2x\left[1 + \right.$$

$$p = 2xp^4 + 4x^2p^2\frac{dp}{dx} + 2p + 2x\frac{dp}{dx}$$

$$p + 2xp^4 + 4x^2\frac{dp}{dx} + 2x\frac{dp}{dx} = 0$$

$$p(1+2p^3x) + 2x[2p^3x + 1]\frac{dp}{dx} = 0$$

$$2p^3x\frac{dp}{dx}$$

$$(1+2p^3x)\left(p + 2x\frac{dp}{dx}\right) = 0$$

$$p + 2x\frac{dp}{dx} = 0$$

$$2x\frac{dp}{dx} = -p$$

$$2\frac{dp}{p} = -\frac{dx}{x}$$

$$2\log p = -\log x + \log c$$

$$\log p^2 = \log x^{-1} + \log c$$

$$p^2 = x^{-1}c$$

$$\therefore\ \cancel{x^2}(\cancel{x^{-2}}c^2) + 2x\left(\frac{\sqrt{c}}{\sqrt{x}}\right) - y = 0$$

$$c^2 + 2x\sqrt{\frac{c}{x}} - y = 0$$

3. $y = 2px + \tan^{-1}(xp^2)$

$$\frac{dy}{dx} = 2\left[p + x\frac{dp}{dx}\right] + \frac{1}{1+(xp^2)^2} \times \left[2xp\frac{dp}{dx} + p^2\right]$$

$$p = 2p + 2x\frac{dp}{dx} + \frac{2xp\frac{dp}{dx} + p^2}{1 + x^2p^4}$$

$$0 = \left[p + 2x\frac{dp}{dx}\right]\left[1 + x^2p^4\right] + \left[2x\,p\frac{dp}{dx} + p^2\right]$$

$$p + x^2p^5 + 2x\frac{dp}{dx} + 2x^3p^4\frac{dp}{dx} + 2xp\frac{dp}{dx} + p^2$$

$$\left(p + p^2 + x^2p^5\right) + \left(2x + 2x^3p^4 + 2xp\right)\frac{dp}{dx}$$

$$p\left(1 + p + x^2p^4\right) + 2x\left(1 + x^2p^4 + p\right)\frac{dp}{dx}$$

$$\left(1 + p + x^2p^4\right) + \left(\cancel{2x + p}\right)\frac{dp}{dx} = 0$$

$p + 2x$

$$\left(\cancel{2x + p\,\frac{dp}{dx}}\right) = 0$$

$$\left(p + 2x\frac{dp}{dx}\right) = 0$$

$$p = -2x\frac{dp}{dx}$$

$$-\int\frac{dx}{x} = 2\int\frac{dp}{p}$$

$$-\log x + \log c = 2\log p$$

$$x^{-1}c = p^2$$

$$p = \frac{\sqrt{c}}{\sqrt{x}}$$

$$p = \sqrt{\frac{c}{x}}$$

$$y = 2\sqrt{\frac{c}{x}}\;x + \tan^{-1}\left(x\cdot\frac{c^2}{x^2}\right)$$

$$y = c\sqrt{x} + \tan^{-1}(c^2)$$

4. $xp^2 - 2yp + ax = 0$.

$$x^{2}p^{2} + ax = 2yp$$

$$\frac{x\cancel{p}^{2}}{2p} + \frac{ax}{2p} = y$$

$$\frac{xp}{2} + \frac{ax}{2p} = y$$

$$\frac{dy}{dx} = \frac{1}{2}\times\left[x\frac{dy}{dx} + p\right] + \frac{a}{2}\left[\frac{p - x\frac{dp}{dx}}{p^2}\right]$$

$$p\,\cancel{\frac{dy}{dx}} = \frac{1}{2}\left[x\frac{dy}{dx} + p\right] + \frac{a}{2}\left[\frac{\cancel{p}}{\cancel{p^2}p} - \frac{x}{p^2}\frac{dp}{dx}\right]$$

$$0 = \left[\frac{-p}{2\cancel{p}} + \frac{a}{2p}\right] + \left[\frac{1}{2} - \frac{a}{2p^2}\right]x\frac{dp}{dx}$$

$$\frac{1}{2}\left[\frac{a}{p} - p\right] - \frac{1}{2}\left[\frac{a}{p^2} - 1\right]x\frac{dp}{dx} = 0$$

$$\frac{1}{2}p\left[\frac{a}{p^2} - 1\right] - \frac{1}{2}\left[\frac{a}{p^2} - 1\right]x\frac{dp}{dx} = 0$$

$$\frac{1}{2}\left[\frac{a}{p^2} - 1\right]\left[p - x\frac{dp}{dx}\right] = 0$$

$$\cancel{\frac{1}{2}}\quad p - x\frac{dp}{dx} = 0$$

$$p = x\frac{dp}{dx}$$

$$\int\frac{dx}{x} = \int\frac{dp}{p}$$

$$\log c + \log x = \log p$$

$$xc = p$$

$\underline{\text{sol}^{n}}$

$$c^2x^3 - 2cxy + ax = 0$$

5 Equations solvable for 'x'

['x' appers only in Ist degree]

conside the D.E which can be put in the form

$$x = f(y, p) \rightarrow (1)$$

Diff (1) w.r.t $\underline{y}$ we gett the D.E of the form

* $\phi\left(y, p, \frac{dp}{dy}\right) = 0 \rightarrow (2)$ $\quad \left\{ \frac{dx}{dy} = 1/p \right.$

* obtain the solution of (2) in the form

$$F(y, p, c) = 0 \longrightarrow (3)$$

· Eliminate p b/w the above solution & the given eq^n (1) to get the required solution.

1. Solve

$y = 2px + y^2p^3$ $\quad$ { it is not solvable for p becoz of p^3 & not solvable for y becoz of y^2

$2px = y - y^2p^3$

$x = \frac{y}{2p} - \frac{y^2 p^3}{2p}$

$x = \frac{y}{2p} - \frac{y^2p^2}{2}$.

diff w.r.t y.

$$\frac{dx}{dy} = \frac{1}{2}\left[\frac{p - y\, dp/dy}{p^2}\right] - \frac{1}{2}\left[2y^2 \cdot 2p\frac{dp}{dy} + p^2 \cdot 2y\right]$$

$$\frac{1}{p} = \frac{1}{2}\left[\frac{p}{p^2} - \frac{y}{p^2}\frac{dp}{dy}\right] - \frac{1}{2}\left[y^2 2\frac{dp}{dy} + p^2 2y\right]$$

$$0 = -1/p + 1/2p - p^2 y + \left[-\frac{1}{p^2} - 2py\right]\frac{y}{2}\frac{dp}{dy} = 0$$

$$= \left(\frac{-1}{2p} - p^2 y\right) + \left[-\frac{1}{2p^2} - py\right] y \frac{dp}{dy}$$

$$-p\left(\frac{1}{2p^2} + py\right) - \left(\frac{1}{2p^2} + py\right) y \frac{dp}{dy} = 0$$

$$\left(1/2p^2 + py\right)\left(-p - y\frac{dp}{dy}\right] = 0$$

$$-p - y\frac{dp}{dy} = 0$$

$$-p = y\frac{dp}{dy}$$

$$\int \frac{dy}{y} = -\int \frac{dp}{p}$$

$$\log y + \log c = \log p$$

$$yc = p$$

$$\therefore y = 2ycx + y^2 (y \cdot c)^3$$

$$y = 2ycx + y^5 c^3 //$$

2). $y = p^2 y + 2px$

$$y - p^2 y = 2px$$

$$\frac{y}{2p} - \frac{\cancel{p^2} y}{2\cancel{p}} = x$$

$$\frac{y}{2p} - \frac{py}{2} = x$$

$$\frac{1}{2}\frac{p - y\frac{dp}{dy}}{p^2} - 1/2\left[p + y\frac{dp}{dy}\right] = \frac{1}{p}$$

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$$\frac{1}{2}\left[\frac{1}{P}-\frac{y}{P^2}\frac{dP}{dy}\right]-\frac{1}{2}\left[P+y\frac{dP}{dy}\right]=\frac{1}{P}$$

$$\frac{1}{2}\left[\frac{1}{P}-\frac{y}{P^2}\frac{dP}{dy}\right]-\frac{1}{2}\left[P+y\frac{dP}{dy}\right]=\frac{\phi}{P}$$

$$\frac{1}{2}-\frac{P}{\cancel{2}}$$

$$=\frac{-1}{P}+\frac{1}{2P}-\frac{y}{2P^2}\frac{dP}{dy}-\frac{1}{2}P-\frac{y}{2}\frac{dP}{dy}=0$$

$$0=\frac{-P}{2}\left[\frac{1}{P^2}+1\right]-\left[\frac{1}{P^2}+1\right]\frac{y}{2}\frac{dP}{dx}$$

$$\left[\frac{-1}{P^2}+1\right]\left[\frac{P}{2}-\frac{y}{2}\frac{dP}{dy}\right]=0$$

$$=-\frac{1}{2}\left[\frac{1}{P^2}+1\right]\cdot\left[P-y\frac{dP}{dy}\right]=0$$

$$P-y\frac{dP}{dy}=0$$

$$P=y\frac{dP}{dy}$$

$$\int\frac{dy}{y}=\int\frac{dP}{P}$$

$$\log y+\log c=\log P$$

$$yc=P$$

$$y=y^3c_1+2xyc_1$$

3. $y = 3px + 6p^2y^2$

$y - 6p^2y^2 = 3px$

$\frac{y}{3p} - 2py^2 = x$

$\frac{dx}{dy} = \frac{1}{3}\left[\frac{p - y\frac{dp}{dy}}{p^2}\right] - 2\left[p2y - y^2\frac{dp}{dy}\right]$

$\frac{1}{p} = \frac{1}{3p} - \frac{y}{p^2}\frac{dp}{dy} - 4py \mp 2y^2\frac{dp}{dy}$

$\left[\frac{-2}{3p} - 4py\right] + \left[\frac{-1}{3p^2} \mp 2y\right] y\frac{dp}{dy}$

$\left[\frac{-2 - 12p^2y}{3p}\right] + \left[\frac{-1 \mp 6p^2y}{p^2}\right] y\frac{dp}{dy}$

$-2)$

Solvable for p	Solvable for y	Solvable for 'x'
p^2 & p term in given eq^n	* 'y' appears in first degree only * 'y' appears only in one term	* x appears in 1^{st} degree only. * x appears in only one term.
$p = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	* $y = \frac{x}{x} + \frac{x}{x}$ $\frac{dy}{dx} = ----$ $p = ..$ $(\quad)(\quad)\frac{dP}{dx} = 0$	$x = \frac{y}{x} + \frac{x}{x}$ $\frac{dx}{dy} = .$ $1/p =$ $(\quad)(\quad)\frac{dP}{dy} =$

$y = 3px + 6p^2y^2$

Solve for x

$3px = y - 6p^2y^2$

$$x = \frac{y}{3p} - \frac{6p^2y^2}{3p}$$

$$= \frac{y}{3p} - \frac{6py^2}{3}$$

$$x = \frac{y}{3p} - 2py^2$$

$$\frac{dx}{dy} = \frac{1}{3}\left(y\left(-\frac{1}{p^2}\right)\frac{dp}{dy} + \frac{1}{p}\right) - 2\left[2py + y^2\frac{dp}{dy}\right]$$

$$\frac{1}{p} = \frac{1}{3}\left(-\frac{y}{p^2}\frac{dp}{dy} + \frac{1}{p}\right) - 2\left[2py + y^2\frac{dp}{dy}\right]$$

$$0 = \frac{-1}{p} + \frac{1}{3p} - 4py + \left[\frac{-y}{3p^2} - 2y^2\right]\frac{dp}{dy}$$

$$\frac{-2}{3p} - 4py - \left[\frac{y + 6p^2y^2}{3p^2}\right]\frac{dp}{dy}$$

$$-\left[\frac{2 + 12p^2y}{3p}\right] - y\left[\frac{1 + 6p^2y}{3p^2}\right]\frac{dp}{dy}$$

$$-2\left[\frac{1 + 6p^2y}{3p}\right] - \frac{y}{p}\left[\frac{1 + 6p^2y}{3p^2}\right]\frac{dp}{dy}$$

$$\left[1 + \frac{6p^2y}{3p}\right]\left[-2 - \frac{y}{p}\frac{dp}{dy}\right] = 0$$

$$-2 - \frac{y}{p}\frac{dp}{dy} = 0$$

$$-2 = \frac{y}{p}\frac{dp}{dy}$$

$$-2\int\frac{dy}{y} = \int\frac{dp}{p}$$

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$-2\log y + \log c = \log p$

$cy^{-2} = p$.

① $\Rightarrow$ $y = 3x\, c/y^2 + 6y^2(c^2/y^4)$.

$p^3 - 4xyp + 8y^2 = 0$

$p^3 + 8y^2 = 4xyp$.

$\frac{p^3}{4yp} + \frac{\overset{2}{\cancel{8}}y^2}{\cancel{4}yp}$.

$x = \frac{p^2}{4y} + \frac{2y}{p}$.

$\frac{dx}{dy} = \frac{1}{4}\left[p^2 \frac{-1}{y^2} + \frac{1}{y} 2p\frac{dp}{dy}\right] + 2\left[y \frac{-1}{p^2}\frac{dp}{dy} + \frac{1}{p}\right]$

$0 = \frac{-1}{p} - \frac{p^2}{4y^2} + \frac{2}{p} + \left[\frac{2p}{4y} - \frac{2y}{p^2}\right]\frac{dp}{dy}$

$\frac{1}{p}\left[-1 + 2\right] - \frac{p^2}{4y^2} + \left[\frac{p}{2y} - \frac{2y}{p^2}\right]\frac{dp}{dy}$

$\frac{1}{p} - \frac{p^2}{4y^2} + \Big[$

$$\frac{1}{4}\left[\frac{y\,2p\,dp/dy - p^2}{y^2}\right] + 2\left[\frac{p - y\,dp/dy}{p^2}\right] = \frac{dx}{dy}$$

$$\frac{1}{4}\left[\frac{2p}{2y}\frac{dp}{dy} - \frac{p^2}{y^2}\right] + 2\left[\frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy}\right] = \frac{1}{p}$$

$$\frac{p}{2y}\frac{dp}{dy} - \frac{p^2}{4y^2} + \frac{1}{p} - \frac{2y}{p^2}\frac{dp}{dy} = 0$$

$$\left[\frac{-p^2}{4y^2} + \frac{1}{p}\right]\left[\frac{p}{2y} - \frac{2y}{p^2}\right]\frac{dp}{dy}$$

$$-\frac{1}{2y}\left[\frac{p^3 + 4y^2}{4py^2}\right]\left[\frac{p^3 - 4y^2}{2yp^2}\right]$$

$$\frac{-1}{2y}\left[\frac{p^3 - 4y^2}{2py}\right]\frac{1}{p}\left[\frac{p^3 - 4y^2}{2yp}\right]\frac{dp}{dy}$$

$$\frac{-1}{2y} + \frac{1}{p}\frac{dp}{d}$$

$$\frac{1}{p}\frac{dp}{dy} = \frac{1}{2y}$$

$$\frac{dp}{p} = \frac{1}{2}\frac{dy}{y}$$

$$\log p = \log y^{1/2} + \log c_2$$

$$p = \sqrt{y}^{1/2} c_2$$

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Clairaut's eqⁿ

A. D.E of the form $y = xp + f(P) \rightarrow (1)$
where $f(P)$ is funⁿ of P alone is known as clairaut's eqⁿ

The General solⁿ of 1 is obtained by replacing $p \rightarrow c$.

Solⁿ $\boxed{y = cx + f(c)} \rightarrow (2)$

Singular solⁿ :-

Diff (2) partially w.r.t 'c' & find the value of 'c'.

Substitute the value of c in 2 to get the required Singular solⁿ.

1. Solve :-
obtain the general & singular solⁿ

$$y = xp + p^2 \rightarrow (1)$$

$$y = xp + f(P)$$

$$p \rightarrow c.$$

$$\boxed{y = cx + c^2} \rightarrow (2)$$

Diff (2) partially w.r.t c.

$$0 = x(1) + 2c$$

$$\boxed{c = -x/2}.$$

$$y = -x/2\, x + (-x/2)^2$$

$$y = \frac{-x^2}{2} + \frac{x^2}{4}.$$

$$\boxed{y = \frac{-x^2}{4}} \rightarrow \text{Singular sol}^{n}$$

2. $P = \log(px - y) \rightarrow (1)$

$$e^{P} = Px - y$$

$$y = xp - e^{P}$$

$$G_s = \quad p \rightarrow c.$$

$$\boxed{y = cx - e^{c}} \rightarrow G.S \rightarrow (2)$$

Diff (2) partially w.r.t c.

$y =$

$\cancel{0 = P -}$

$$0 = x - e^{c}.$$

$$x = e^{c}.$$

$$\log x = c.$$

$e^{\int P dx}$ $e^{2\log x^{2}}$

$$y = x\log x - e^{\log x}$$

$$y = x\log x - x.$$

$$y = x[\log x - 1]$$

3. Reduce the following to clairauts eq^{n} & hence Find its general & singular Sol^{n}.

$$(x^2 - 1)p^2 - 2xyp + y^2 - 1 = 0$$

$$p^2x^2 - p^2 - 2xyp + y^2 - 1 = 0.$$

$$p^2x^2 + y^2 - 2xyp - p^2 - 1 = 0$$

$$(Px - y)^2 = 1 + P^2$$

$$px - y = \sqrt{1 + P^2}$$

$$y = px - \sqrt{1+p^2}$$

$$y = px + f(p)$$

$$p \to c$$

$$y = cx - \sqrt{1+c^2}$$

diff. partially w.r.t c.

$$0 = x - \frac{1}{2\sqrt{1+c^2}} 2c$$

$$0 = x - \frac{c}{\sqrt{1+c^2}}$$

$$x = \frac{c}{\sqrt{1+c^2}} \quad \text{Sq B.s}$$

$$x^2 = \frac{c^2}{1+c^2}$$

$$\frac{1}{x^2} = \frac{1+c^2}{c^2}$$

$$\frac{1}{x^2} = \frac{1}{c^2} + 1$$

$$\frac{1}{c^2} = \frac{1}{x^2} - 1$$

$$\frac{1}{c^2} = \frac{1-x^2}{x^2}$$

$$c^2 = \frac{x^2}{1-x^2}$$

$$c = \frac{x}{\sqrt{1-x^2}}$$

$$y = \frac{x^2}{\sqrt{1-x^2}} - \sqrt{1 + \left(\frac{x}{\sqrt{1-x^2}}\right)^2}$$

$$y = \frac{x^2}{\sqrt{1-x^2}} - \sqrt{1 + \frac{x^2}{1-x^2}}$$

$$= \frac{x^2}{\sqrt{1-x^2}} - \sqrt{\frac{1-x^2+x^2}{1-x^2}}$$

$$= \frac{x^2}{\sqrt{1-x^2}} - \sqrt{\frac{1}{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}}\left[x^2-1\right]$$

$$\frac{(x^2-1)}{\sqrt{1-x^2}}$$

$$y = -\frac{(1-x^2)}{\sqrt{1-x^2}}$$

4. $xp^3 - yp^2 + 1 = 0$

$$xp^3 + 1 = yp^2$$

$$\frac{xp^3}{p^2} + \frac{1}{p^2} = y$$

$$xp + \frac{1}{p^2} = y.$$

$$\boxed{y = px + \frac{1}{p^2}}$$

$p \to c$.

$$y = cx + \frac{1}{c^2}.$$

Diff partially w.r.t c

c^{-2}

c^{-2-1}

-2

$$0 = x + \frac{-2}{c^3}$$

$$x = \frac{2}{c^3}$$

$$c = \sqrt[3]{\frac{2}{x}}$$

$$\therefore y = x\sqrt[3]{\frac{2}{x}} + \frac{1}{\sqrt[2/3]{\frac{2}{x}}}$$

$1 - \frac{1}{3}$

$\frac{2}{3}$

$$y = x\left(\frac{2}{x}\right)^{1/3} + \frac{1}{\left(\frac{2}{x}\right)^{2/3}}$$

5) by reducing it in to clairauts form solve the eq^n by taking $X = x^2$, $Y = y^2$.

$$(px - y)(py + x) = 2p \rightarrow (1)$$

A:- $X = x^2$. $Y = y^2 \rightarrow (2)$

$dX = 2x\,dx \rightarrow (3)$ $\quad dY = 2y\,dy \rightarrow (4)$

$$\frac{dY}{dX} = \frac{\cancel{2}y\,dy}{\cancel{2}x\,dx}$$

$$P = \frac{y}{x}\,p$$

$$\boxed{p = \frac{x}{y}P} \rightarrow (5)$$

$(1) \Rightarrow$

$$\left[\frac{x}{y}P \cdot x - y\right]\cdot\left[\frac{x}{y}P \cdot y + x\right] = 2\frac{x}{y}P$$

$$\left[\frac{x^2P - y^2}{\cancel{y}}\right]\cdot \cancel{x}(P+1) = 2\frac{\cancel{x}}{\cancel{y}}P$$

$$[XP - Y][P+1] = 2P \quad \{\text{by } (2)$$

$$[XP - Y] = \frac{2P}{P+1}$$

$XP - \frac{2P}{P+1} + XP$

$$\boxed{Y = XP - \frac{2P}{P+1}} \text{ clairauts } eq^n$$

$$gs = \boxed{y = cx - \frac{2c}{c+1}}$$

$y = cx + f(x)$ $\quad y^2 = cx^2 - \dfrac{2c}{c+1}$

$y = px + f(p)$

Diff partially w.r.t c.

$$0 = x^2 - 2\left[\frac{c+1 - 2c}{(c+1)^2}\right.$$

6. Find the general & singular sol^n of $(p-1)e^{3x} + p^3 e^{2y} = 0$ by taking

$u = e^x, \quad v = e^y \rightarrow (1)$

$u = e^x \qquad v = e^y$

$du = e^x . dx \qquad dv = e^y dy$

$$\frac{dy}{dx} = \frac{dv}{du} = \frac{e^y}{e^x}\frac{dy}{dx}$$

$$\frac{dv}{du} = \frac{e^y}{e^x} p$$

$$\boxed{P = \frac{e^y}{e^x} p}$$

$(1) \Rightarrow \quad p = \dfrac{e^x}{e^y} P$

$$\left[\frac{e^x}{e^y} P - 1\right] u^3 + \left[\frac{e^{3x}}{e^{3y}} P^3\right] v^2 = 0$$

$$\left[\frac{e^x}{e^y} P - 1\right] u^3 = -\left[\frac{e^{3x}}{e^{3y}} P^3\right] v^2$$

$$\left[\frac{e^x}{e^y} P - 1\right] e^{3x} + \left[\frac{e^{3x}}{e^{3y}} P^3\right] v^2$$

$\div$ by e^{3x}.

$$\left(\frac{u}{v}p-1\right)+\left(\frac{p^3}{v}\right)=0$$

$$\left(\frac{up-v}{v}\right)+\frac{p^3}{v}=0.$$

$$up-v+p^3=0.$$

$$\boxed{v=pu+p^3} \rightarrow \quad c-eq^n$$

G.S. $\boxed{v=cu+c^3}$

Diff partially w.r.t. c.

$$0=u+3c^2$$

$$3c^2=-u$$

$$c^2=\frac{-u}{3}$$

$$c=\sqrt{-u/3}.$$

$$\boxed{e^y=ce^x+c^3}$$