Single-phase A.C. Circuits

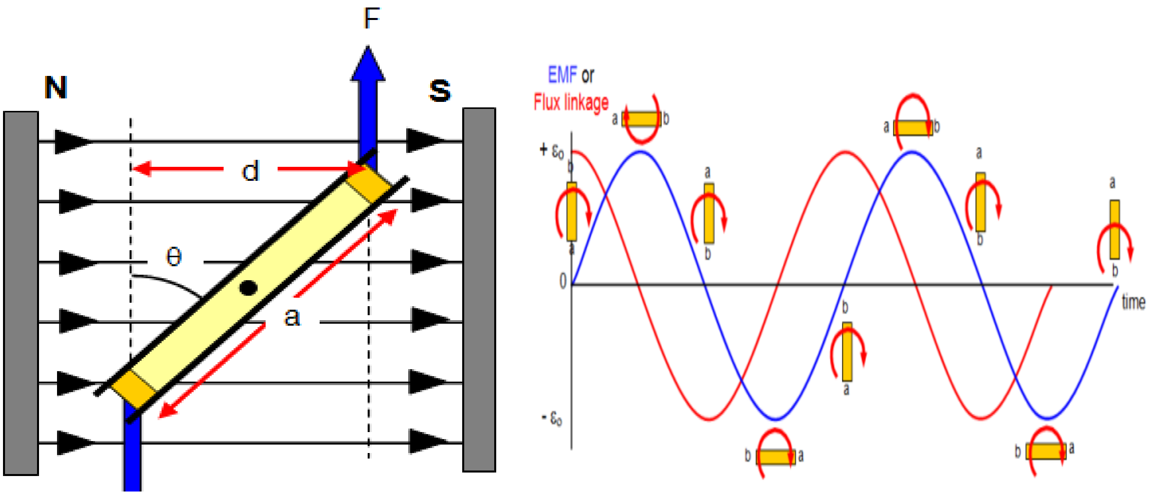
Syllabus: Generation of sinusoidal voltage, frequency of generated voltage, definition and numerical values of average value, root mean square value, form factor and peak factor of sinusoidally varying voltage and current, phasor representation of alternating quantities. Analysis, with phasor diagrams, of R, L, C, R-L, R-C and R-L-C circuits and, parallel and series- parallel circuits. Real power, reactive power, apparent power and power factor.

Introduction:

- An Alternating Current is one in which the magnitude and direction of an electrical quantity changes with respect to time.

Generation of sinusoidal voltage:

- Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure.



- The coil is rotating in the anticlockwise direction at a uniform angular velocity of ω rad/sec.
- When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero.
- When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil.
- When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum.
- When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces.
- Next when the coil comes to the vertical position, the emf induced becomes zero.
- After that the same cycle repeats and the emf is induced in the opposite direction.
- When the coil completes one complete revolution, one cycle of AC voltage is generated.
- An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time.

Important terms associated with an alternating quantity are defined below.

1. Amplitude
   - It is the maximum value attained by an alternating quantity. Also called as maximum or peak value.

2. Time Period (T)
   - It is the Time Taken in seconds to complete one cycle of an alternating quantity.

3. Instantaneous Value
   - It is the value of the quantity at any instant.

4. Frequency (f)
   - It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec.
   - The relationship between frequency and time period can be derived as follows.
   - Time taken to complete f cycles = 1 second
   - Time taken to complete 1 cycle = 1/f second

$$T = 1/f$$

5. Angular Frequency (ω)
   - Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil).
   - The unit of angular frequency is rad/sec.

   - The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil i.e $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$e = -N\, d\, \Phi_{max} \cos\omega t / dt$$

$$e = N\Phi_{max}\, \omega \sin \omega t$$

$$e = E_m \sin \omega t$$

Advantages of AC system over DC system

1. AC voltages can be efficiently stepped up/down using transformer.
2. AC motors are cheaper and simpler in construction than DC motors.
3. Switchgear for AC system is simpler than DC system.

Average Value

- The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

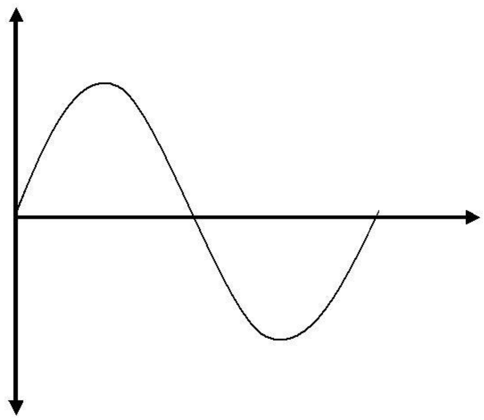
$$\text{Average value} = \frac{\text{Area under one cycle}}{\text{Base}}$$

- For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area.
- Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$V_{av} = \frac{1}{2\pi}\int_0^{2\pi} v\,d(\omega t)$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\,d(\omega t)$$

Average value of a sinusoidal current



$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Average value of a full wave rectifier output

$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Average value of a half wave rectifier output

$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i d(\omega t)$$

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t d(\omega t)$$

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

RMS or Effective Value

- The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 d(\omega t)}$$

RMS value of a sinusoidal current

$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Form Factor

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMS\ Value}{Average\ Value}$$

Peak Factor or Crest Factor

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

$$PF = \frac{Maximum Value}{RMS Value}$$

Phasor Representation

- An alternating quantity can be represented using

(i) Waveform

(ii) Equations

(iii) Phasor

- A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.**
- A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity.

Phase

- Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase Difference

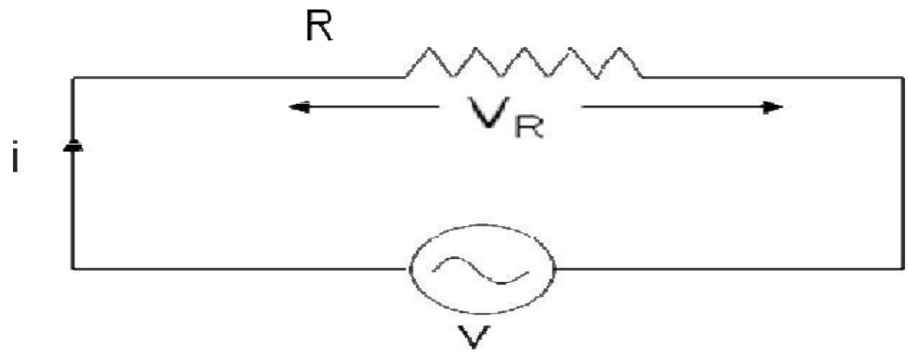
- When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

- Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same.

- The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \quad \text{----------} (1)$$

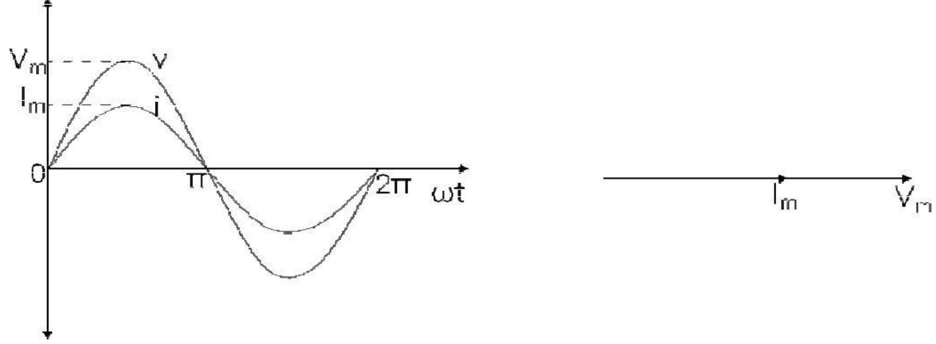
The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.
Using ohms law, we can write the following relations

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = I_m \sin \omega t \qquad (2)$$

Where $I_m = \frac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasor can be drawn as below.



Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$p = V_m I_m \sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

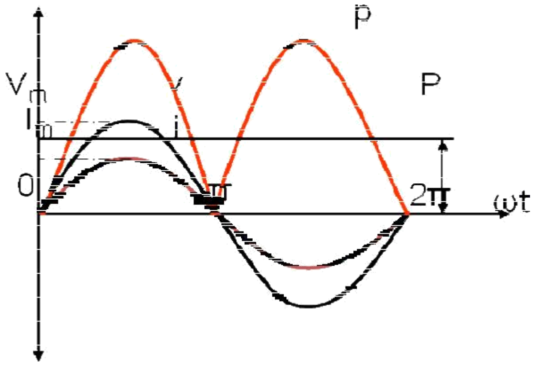
$$P = \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t \, d\omega t$$

$$P = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\cos 2\omega t \, d\omega t$$

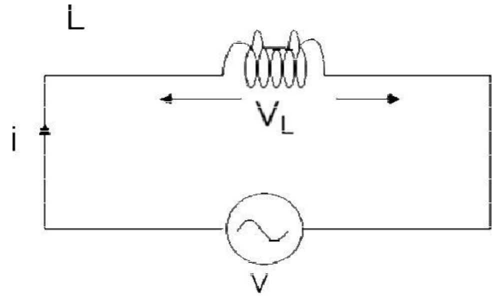
$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$P = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.



AC circuit with a pure inductance



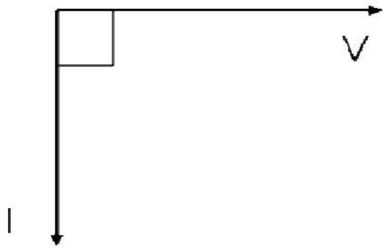
Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

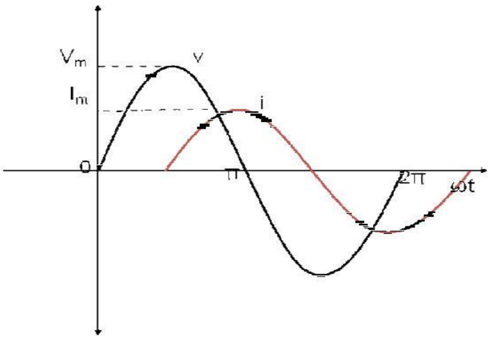
$$v = V_m \sin \omega t \quad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90^o. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = \omega L = 2\pi f L$$

$$I_m = \frac{V_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms ( )

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$P = vi$$
$$= (V_m \sin \omega t)(I_m \sin(\omega t - \pi/2))$$
$$= -V_m I_m \sin \omega t \cos \omega t$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

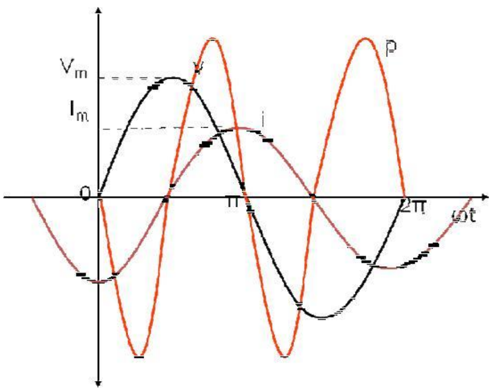
Average power

From the instantaneous power we can find the average power over one cycle as follows

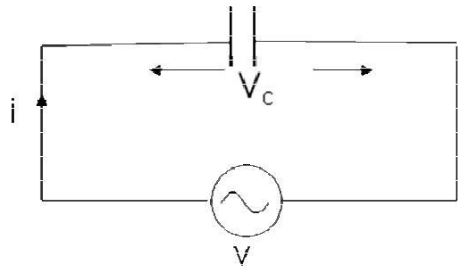
$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$
$$P = 0$$

The average power in a pure inductive circuit is zero. Or in other words, the power consumed by a pure inductance is zero.

The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.



AC circuit with a pure capacitance



$$q = Cv$$

$$q = CV_m \sin \omega t$$

$$i = \frac{dq}{dt}$$

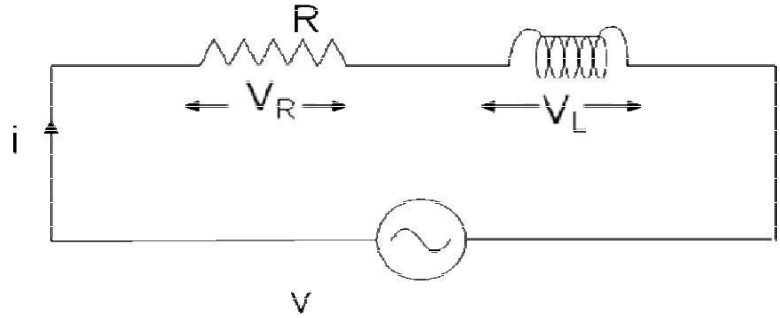
$$i = CV_m \omega \cos \omega t$$

$$i = \omega CV_m \sin(\omega t + \pi / 2)$$

$$i = I_m \sin(\omega t + \pi / 2) \quad \text{-------------------(2)}$$

Where $I_m = \omega CV_m$

R-L Series circuit



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L.

$V_R = IR$ is in phase with I

$V_L = IX_L$ leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.

The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_R = IR$$

$$V_L = IX_L$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_L^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms ( ).

Instantaneous power

The instantaneous power in an RL series circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \Phi))$$

$$p = \frac{V_m I_m}{2} \cos \Phi - \frac{V_m I_m}{2} \cos(2\omega t - \Phi)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

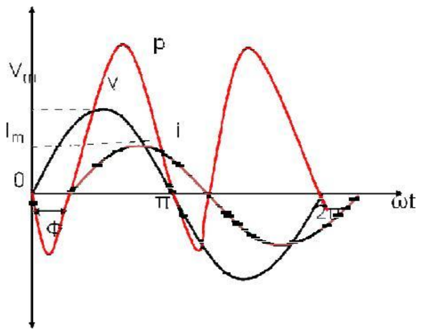
$$P = \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)\ d\omega t$$

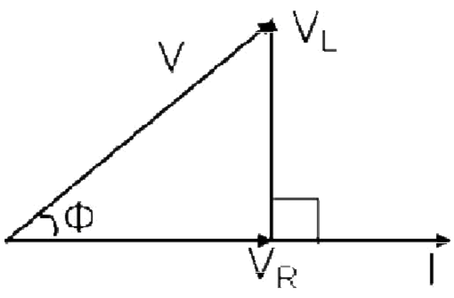
$$P = \frac{V_m I_m}{2}\cos\Phi$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$$

$$P = VI\cos\Phi$$

The voltage, current and power waveforms of a RL series circuit is as shown in the figure.





As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the load and when the power in negative, the power flows from the load to the source. The positive power is not equal to the negative power and hence the average power in the circuit is not equal to zero.

From the phasor diagram,

$$\mathrm{P} = VI\cos\Phi$$

$$\mathrm{P} = (IZ)\cdot I\cdot\frac{R}{Z}$$

$$P = I^2 R$$

Hence the power in an RL series circuit is consumed only in the resistance.
The inductance does not consume any power.

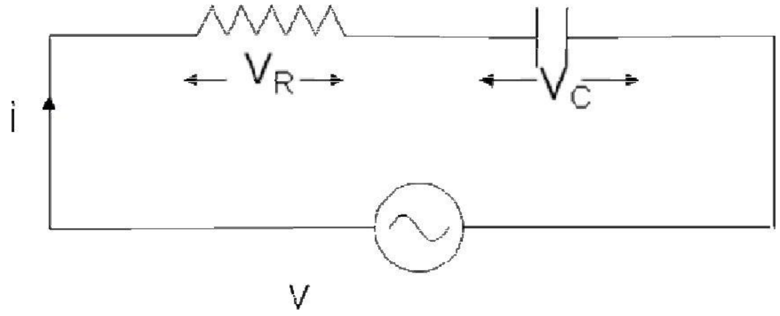
Power Factor

The power factor in an AC circuit is defined as the cosine of the angle between voltage and current ie cosΦ

$$P = VI\cos\Phi$$

The power in an AC circuit is equal to the product of voltage, current and power factor

R-C Series circuit

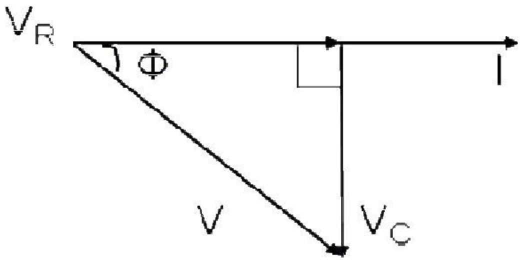


Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

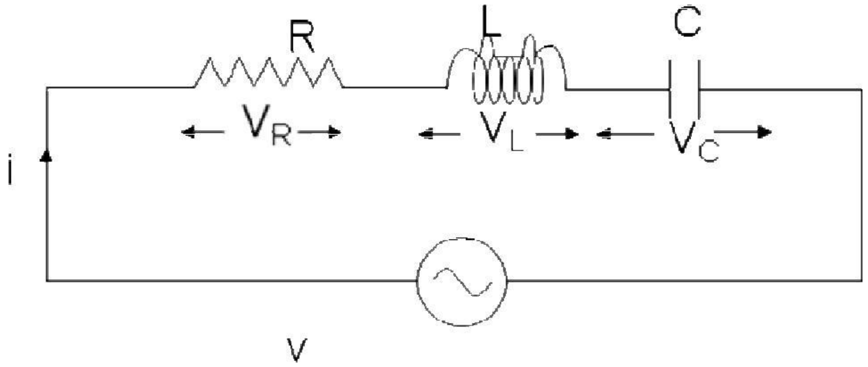
The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C.

V_R=IR is in phase with I
V_C=IX_C lags behind the current by 90 degrees
With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

R-L-C Series circuit



Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

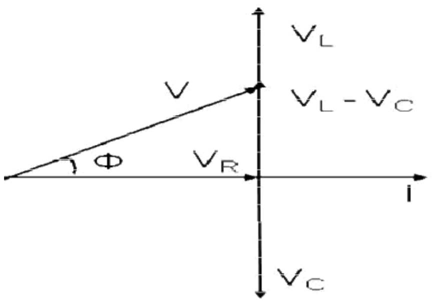
The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

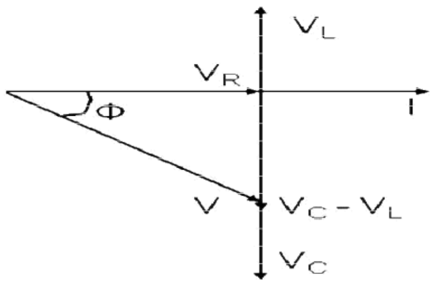
V_R=IR is in phase with I

V_L=IX_L leads the current by 90 degrees

V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L>V_C$ and $V_L<V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor V_L-V_C or V_C-V_L is drawn and then the resultant voltage V is drawn.





Numerical:

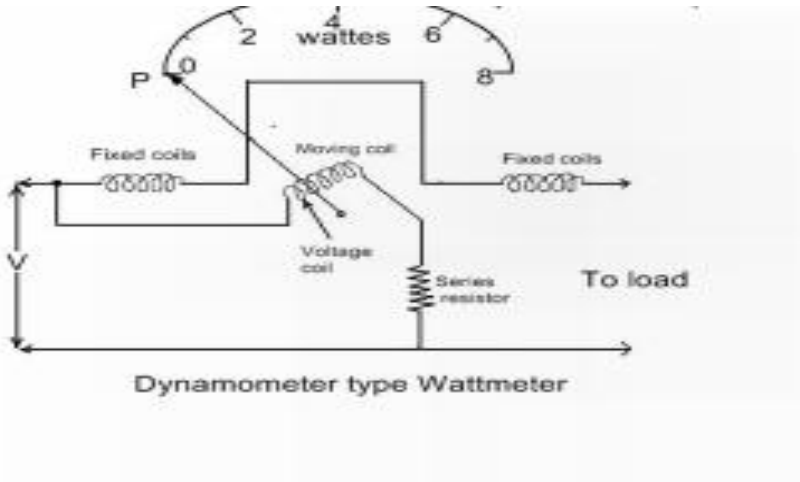
1. Two impedances Z_1=(10+j15) & Z_2=(5-j8) are connected in parallel across a voltage source. If the total current drawn is 10A, calculate currents in Z_1& Z_2 and power factor of the circuit.
2. A circuit consists of resistance 10 ohm, an inductance of 16mH & a capacitance of 150µF connected in series. A supply of 100V at 50Hz is given to the circuit. Find the current, power factor & power consumes by the circuit..
3. A parallel circuit comprises of a resistor of 20 ohm in series with an inductive reactance of 15 ohm in one branch & a resistor of 30 ohm in series with a capacitive reactance of 20 ohm in the other branch. Determine the current & power dissipated in each branch of the circuit if the total current drawn by the parallel circuit is $10\angle-30^0$ Amps
4. Two circuits A & B are connected in parallel across 200V, 50Hz supply. Circuit A consists of 10 ohm resistance & 0.12H inductance in series while circuit B consists of 20 ohm resistance in series with 40µF capacitance. Calculate (i) Current in each branch (ii) Total power factor (iii) Draw phasor diagram.

5. A 60 ohm resistor is connected in parallel with an inductive reactance of 80 ohm to a 240V, 50Hz supply. Calculate (i) the current through the resistor & inductor, (ii) the supply current, (iii) the circuit phase angle; (iv) draw the phasor diagram.
6. An inductor coil is connected to supply of 250V at 50Hz & takes a current of 5A. The coil dissipates 750W. Calculate power factor, resistance, & inductance of the coil.
7. Impedance in parallel with a 100μF capacitor is connected across a 200V, 50Hz supply. The coil takes a current of 4A & power loss in the coil is 600W. Calculate (i) resistance of the coil (ii) inductance of the coil (iii) the power factor of the circuit.

Measuring *Instruments*:

Syllabus: Construction and Principle of operation of dynamometer type wattmeter and single phase induction type energy meter.

Electrodynamometer Type Wattmeter:



Construction:

It consists of the following parts:

Moving coil - Moving coil moves the pointer with the help of spring control instrument In electrodynamometer type wattmeter, moving coil works as pressure coil. Hence moving coil is connected across the voltage and thus the current flowing through this coil is always proportional to the voltage.

Fixed coil - The fixed coil is divided into two equal parts and these are connected in series with the load, therefore the load current will flow through these coils. Now the reason is very obvious of using two fixed coils instead of one, so that it can be constructed to carry considerable amount of electric current. These coils are called the current coils of electrodynamometer type wattmeter. Earlier these fixed coils are designed to carry the current of about 100 amperes but now the modern wattmeter are designed to carry current of about 20 amperes in order to save power.

Control system - Out of two controlling systems i.e. gravity control and spring control, only spring controlled systems are used in these types of wattmeter. Gravity controlled system cannot be employed because they will contain appreciable amount of errors.

Damping system - Air friction damping is used, as eddy current damping will distort the weak operating magnetic field and thus it may lead to error.

Scale - There is uniform scale is used in these types of instrument as moving coil moves linearly over a range of 40 degrees to 50 degrees on either sides.

Working:

Let

v=supply voltage

i=load current and

R=resistance of the moving coil circuit

Current through fixed coils, $i_f = i$
Current through the moving coil, $i_m = v/R$
Deflecting torque,

$$T_d \propto (i_f * i_m) \propto \frac{iv}{R}$$

For a DC circuit the deflecting torque is thus proportional to the power.

For any circuit with fluctuating torque, the instantaneous torque is proportional to instantaneous power. In this case due to inertia of moving parts, the deflection will be proportional to the average power. For sinusoidal alternating quantities the average power is V.I.cosϕ, where

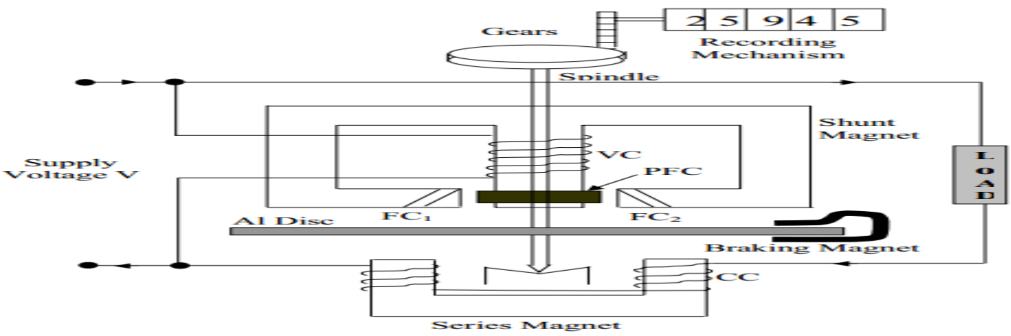
V= RMS value of voltage

I = RMS value of current, and

ϕ= phase angle between V and I

Hence an electrodynamometer instrument, when connected as shown in figure, indicates the power, irrespective of the fact it is connected in an AC or DC circuit.

Single Phase Induction Type Energy Meter:



Construction:

It consists of a pressure coil made of thin copper wire of many turns (also called shunt magnet); a current coil made of thick copper wire of one or two turns (also called series magnet), an aluminium disc mounted on spindle

A braking magnet is arranged on a disc to control its movement and to stop the movement under no load.

A phase difference of 90^0 is set between current coil and pressure coil with the help of copper shaded rings.

Working:

This instrument works on the principle of induction that when both the shunt and series coils are energized by ac, there will be tow alternative fluxes are in the shunt coil and one in the series coil these time varying fluxes are cut by a stationary disc.

These currents interacts with the fluxes and results in a torque.

The disc rotates in a particular direction and the number and speed of rotations depends on the energy consumed by the load.