Radius of Curvature in parametric form (2)

for $y = f(x)$ $\quad x = f(t)$, $y = g(t)$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \longrightarrow (1)$$

We express y' & y'' in terms of parameter t

$$y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'}{x'} \longrightarrow (2)$$ where $y' = \frac{dy}{dt}$, $x' = \frac{dx}{dt}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left[\frac{y'}{x'}\right]$$

$$= \left(\frac{x'y'' - y'x''}{(x')^2}\right) \cdot \frac{dt}{dx}$$

$$= \frac{x'y'' - y'x''}{(x')^2} \cdot \frac{1}{x'}$$

$$\frac{d^2y}{dx^2} = \frac{x'y'' - y'x''}{(x')^3} \longrightarrow (3),$$ where $y'' = \frac{d^2y}{dt^2}$, $x'' = \frac{d^2x}{dt^2}$