* Damping rule (change of scale property)

If $Z_T[u_n] = U(z)$ then

i) $Z_T[k^n u_n] = U(z/k)$

ii) $Z_T[k^{-n} u_n] = U(kz)$

proof: i) $Z_T[u_n] = \sum_{n=0}^{\infty} u_n z^{-n}$ — (1)

$$= U(z)$$

Now $Z_T[k^n u_n] = \sum_{n=0}^{\infty} k^n u_n z^{-n}$

$$= \sum_{n=0}^{\infty} u_n (z/k)^{-n} \quad \text{— (2)}$$

$= U(z/k)$ [comparing (1) & (2)]

$$\boxed{Z_T[k^n u_n] = U(z/k)}$$

ii) $Z_T[k^{-n} u_n] = \sum_{n=0}^{\infty} k^{-n} u_n z^{-n}$

$$= \sum_{n=0}^{\infty} u_n (kz)^{-n}$$

$$\boxed{Z_T[k^{-n} u_n] = U(kz)}$$

Application of Damping rule

① $Z_T[k^n n] = \frac{zk}{(z-k)^2}$

$z[u_n] = \sum u_n z^{-n}$

u_n

LHS $= Z_T[k^n n]$

$= [Z_T(n)]_{z \to z/k}$

$Z_T[n] = \frac{z}{(z-1)^2}$

$Z_T[k^n u_n] = U(z/k)$

$z \to z/k$

$= \left[\frac{z}{(z-1)^2}\right]_{z \to z/k}$

$= \frac{z/k}{(z/k-1)^2} = \frac{z/k}{\frac{(z-k)^2}{k^2}} = \frac{z}{k} \cdot \frac{k^2}{(z-k)^2}$

$$Z_T[k^n n] = \frac{zk}{(z-k)^2}$$