Logic is Science of reasoning. Logic provides rules and techniques for determining whether a reasoning is valid or not.

Proposition or Statement:- A declarative sentence that is either true or false but not both is called a proposition or a st.

lower case letters - p, q ... used to rep st.

proposition Ex:-

Ex:- p: $4+7=9$ F

q: It is raining T

r: 5 is a prime no. T

Not propositions Exclamation or command. $x+7=10$

Getup & do Exercise.

What a beautiful evening!

primitive statements/atomic proposition/simple prop.

A St which cannot be further broken into simpler propositions is called pri. st.

Methods for obtaining new st.

1. Transform p into $\neg p$.

if p: Combinatorics is a req course for sophomores.

Then $\neg p$: " is not req " " ".

2. Combine two or more sts into a compound st. using logical connectives.

a) Conjunction - $p \wedge q$.

b) Disjun - $p \vee q$

c) Implication. $p \to q$

p implies q or impli of q by p.

$p \rightarrow q$ 1) if p then q

hypothesis Conclusion 2) p is sufficient for q.

3) p is " cond for q

4) p only if q.

$p \rightarrow q$ is F when $p=0$ & $q=1$.

5) q is necessary for p.

6) " " " cond for p.

d) Biconditional:- $p \leftrightarrow q$ or p iff q

p is the necessary & sufficient cond for q

$p \leftrightarrow q \Leftrightarrow$ XNOR.

$\Leftrightarrow \neg(p \veebar q)$

Ex:-

s: Phyllis goes out for a walk

t: The moon is out

u: it is snowing.

a) $(t \wedge \neg u) \rightarrow s$

b) $t \rightarrow (\neg u \rightarrow s)$

c) $\neg (s \leftrightarrow (u \vee t))$

it is not the case that phyllis goes out for a w iff it is snowing or the moon is out

d) "Phyllis will go out walking iff ~~if~~ the moon is out"

$s \leftrightarrow t$

e) "if it is snowing & moon is not out, then phy will not go out for walk"

$(u \wedge \neg t) \rightarrow \neg s$

f) "It is snowing but Phy will still go out for a walk"

$\boxed{\text{but} \Leftrightarrow \text{and}}$

$u \wedge s$.

g) if I weigh more than 120 pounds, then I shall ~~not~~ enroll in an exer. class.

p : I weigh more than 120 po

q : I shall enroll in an ex class.

$p \rightarrow q$.

T - Tautology - A compound st which is true for all truth value assignments for its component sts.

F - Contradiction - F for all

Tautology & implication are used to describe valid arg.

& it will help To prove mathematical theorems.

In general, an arg starts with a list of given sts called premises and a st called conclusion of the arg.

We examine premises $p_1, p_2 \ldots p_n$ & try to show that conclusion q follows logically from that given st ie we try to show if each of $p_i, 1 \le i \le n$ is true then q is true.

$(p_1 \wedge p_2 \ldots \wedge p_n) \rightarrow q$ — (1)

If any one of p_i is false, then irrespective of q, the implication (1) is true.

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p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

} if p is F, then irrespe[ctive] of q, $p \to q$ is T.

} if p is T, then $p \to q$ depen[ds] on q and is same as q

Consequently, if we start with premises $p_1, p_2 \ldots p$ each with 1 and find that under these circumstances q also has 1, then we have $(p_1 \wedge p_2 \ldots \wedge p_n) \to q$ is a tautology a valid arg.

2.2 Logical Equivalence: The laws of logic

$p \to q \Leftrightarrow \neg p \vee q$.

logical equivalence $\Leftrightarrow$:- Two sts s_1 and s_2 are equivalent if when s_1 is true iff s_2 is true & when s_1 is false iff s_2 is also false.

$(p \leftrightarrow q) \Leftrightarrow (p \to q) \wedge (q \to p)$

T.T. 2.7.

$(p \to q) \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p)$

$p \veebar q \Leftrightarrow (p \vee q) \wedge \neg (p \wedge q)$

$\Leftrightarrow (p \vee q) \wedge (\neg p \vee \neg q)$

Demorgan's law:- In algebra $-(a+b) = (-a) + (-b)$

$\neg (p \vee q) \Leftrightarrow \neg p \wedge \neg q$.

$\neg (p \wedge q) \Leftrightarrow \neg p \vee \neg q$.

Distributive law - In algebra $a \times (b+c) = (a \times b) + (a \times c)$

$a \wedge (b \vee c) \Leftrightarrow (a \wedge b) \vee (a \wedge c)$.

$a \vee (b \wedge c) \Leftrightarrow (a \vee b) \wedge (a \vee c)$.

But in alge' $a + (b \times c)$ is not equal to $(a+b) \times (a+c)$.

Ex:- $a=2$, $b=3$, $c=5$

$a + (b \times c) = 17$ but $(a+b) \times (a+c) = 35$.

Laws of Logic

1. $\neg\neg p \Leftrightarrow p$ - Law of double negation.
2. - Demorgan's law
3. - Commutative law
4. - Associative law
5. - Distributive law
6. $p \vee p \Leftrightarrow p$ - Idempotent law.
   $p \wedge p \Leftrightarrow p$.
7. $p \wedge T_0 \Leftrightarrow p$. - Identity law
   $p \vee F_0 \Leftrightarrow p$

8) Inverse law $p \vee \neg p \Leftrightarrow T_0$

$p \wedge \neg p \Leftrightarrow F_0$.

9) Domination law

$p \vee T_0 \Leftrightarrow T_0$

$p \wedge F_0 \Leftrightarrow F_0$.

10) Absorption law.

$p \vee (p \wedge q) \Leftrightarrow p$

$p \wedge (p \vee q) \Leftrightarrow p$.

Dual of a Statement -

If s is a st with $\wedge, \vee, T_0, F_0$. then dual of st is s^d by replacing $\wedge$ by $\vee$, $\vee$ by $\wedge$ resp. & T_0 by F_0 & F_0 by T_0.

Dual of a) $p \vee \neg p$ is $p \wedge \neg p$

b) $p \vee T_0$ is $p \vee F_0$.

c) $(p \wedge \neg q) \vee (r \wedge T_0)$ is $(p \vee \neg q) \wedge (r \vee F_0)$

Thm 1:- Principle of Duality -

Let s and t be statements that contain no logical connectives other than $\wedge$ and $\vee$. If $s \Leftrightarrow t$ then $s^d \Leftrightarrow t^d$.

If q, r, s are primitive sts,

To prove - $(r \wedge s) \rightarrow q \Leftrightarrow \neg (r \wedge s) \vee q$ is a tautology

T.T $\rightarrow$ 211

or W.K.T $p \rightarrow q \Leftrightarrow \neg p \vee q$

Substitution rules

1) Suppose that the compound statement P is a tautology & p is a primitive st that appears in P and we replace each occ of p by same q, then resulting compound st P_1 is also a tautology.

Ex:- P: $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology

replace p by $r \wedge s$, then

$P_1 : \neg [(r \wedge s) \vee q] \Leftrightarrow \neg (r \wedge s) \wedge \neg q$ is also a tautology

2) Let P be a compound st where p is an arbitrary st that appears in P & q be st such as $q \Leftrightarrow p$. Suppose that in P we replace one or more occurrences of p by q. Then this replacement yields the compound statement P_1 $P_1 \Leftrightarrow P$.

Ex:- 1) $p \wedge (p \rightarrow q] \rightarrow q$ is a tautology.

if r, s, t, u are any st, then by subst p by $r \rightarrow s$ & q by $\neg t \vee u$

$[(r \rightarrow s) \wedge [(r \rightarrow s) \rightarrow (\neg t \vee u)]] \rightarrow (\neg t \vee u)$

2) P_1: $p \rightarrow$ ~~...~~ $p \vee q$

Ex:- Negate & simplify $(p \vee q) \rightarrow r$.

ie to find $\neg[(p \vee q) \rightarrow r]$.

Soln:-

Step 1) $(p \vee q) \rightarrow r$.

$\Rightarrow \neg(p \vee q) \vee r$. $\because p \rightarrow q \Leftrightarrow \neg p \vee q$.

Then $(p \vee q) \rightarrow r \Leftrightarrow \neg(p \vee q) \vee r$.

Step 2) Negating ①.

$\neg[(p \vee q) \rightarrow r] \Leftrightarrow \neg[\neg(p \vee q) \vee r]$.

Step 3:- Applying demorgans & subst law.

ie RHS $\neg[\neg(p \vee q) \vee r] \Leftrightarrow (p \vee q) \wedge \neg r$.

$\Leftrightarrow \neg\neg(p \vee q) \wedge \neg r$.

Step 4:- Law of Double negation & second substit

$\neg\neg(p \vee q) \wedge \neg r \Leftrightarrow p \vee q \wedge \neg r$.

$$\boxed{\neg[(p \vee q) \rightarrow r] \Leftrightarrow (p \vee q) \wedge \neg r}$$

$$\boxed{\begin{array}{l} p \rightarrow q. \text{ is } \neg p \vee q \\ \neg(p \rightarrow q) \text{ is } p \wedge \neg q \end{array}}$$

$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$

Ex:- Negation of an implication

p: Joan goes to Lake George.

q: Mary pays for Joan's shopping spree.

Then

$p \rightarrow q$: If Joan goes to lake george, then Mary will pay for Joan's shopping spree.

$\neg(p \rightarrow q)$ is

$\Rightarrow \neg(\neg p \vee q)$

$\Rightarrow p \wedge \neg q$. Here Joan goes to lake george but Mary does not pay for Joan's shopping spree.

~~Dual~~ of implication.

$s : p \rightarrow q$. ie $\neg p \vee q$ — 2nd substitution.

$s^d : \neg p \wedge q$.

1) ~~Contrapositive~~ of implication — 2nd substitution

If $p \rightarrow q$ is a st, then $\neg q \rightarrow \neg p$ is its contrapositive

3) ~~Inverse~~ of implication (contrapositive of converse)

$\neg p \rightarrow \neg q$

2) ~~Converse~~ of implication

$q \rightarrow p$.

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1. Simplify $(p \vee q) \wedge \neg(\neg p \wedge q)$.

Applying Demorgan's law

$(p \vee q) \wedge (\neg\neg p \vee \neg q)$

By Law of double negation

$(p \vee q) \wedge (p \vee \neg q)$.

By distributive law of $\vee$ over $\wedge$:

$[(p \vee q) \wedge p] \vee [(p \vee q) \wedge \neg q]$

By absorption law

$p \vee [(p \vee q) \wedge \neg q]$

By distributive law

$p \vee [(p \wedge \neg q) \vee (q \wedge \neg q)]$

By Inverse law

$p \vee [(p \wedge \neg q) \vee F_0]$

By Domination law

$p \vee [(p \wedge \neg q)]$

Distributive law,

$(p \vee p) \wedge (p \vee \neg q)$

$p \wedge$

$p \vee (q \wedge \neg q)$.

$\Rightarrow$ By inverse law

$p \vee F_0$

By identity law

$\Rightarrow \underline{p}$.

2. Simplify

$\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$

By Demorgan's law

$\Leftrightarrow \neg\neg[(p \vee q) \wedge r] \wedge \neg\neg q$.

By Law of Double negation

$[(p \vee q) \wedge r] \wedge q$

By associative law of $\wedge$

$\Leftrightarrow (p \vee q) \wedge (r \wedge q)$

By commutative law of $\wedge$

$\Leftrightarrow (p \vee q) \wedge (q \wedge r)$

By associative law of $\wedge$

$\Leftrightarrow [(p \vee q) \wedge q] \wedge r$.

By Law of absorption

$\Rightarrow \underline{q \wedge r}$.

Ex:- 22

Q.20) a $\rightarrow$ $(p \wedge \neg r) \vee q$.

b $\rightarrow$ $p \wedge t$.

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Logical Implication: Rules of Inference

$p \to q$ is false only when $p=1$,

$$(P_1 \wedge P_2 \dots \wedge P_n) \to q$$

P_i - premises, q - conclusion

If all P_is are true, then q is likewise true.

If any of P_i is false then $P_1 \wedge P_2 \dots \wedge P_n$ is false

$\therefore$ $(P_1 \wedge P_2 \wedge \dots P_n) \to q$ is true irrespective of truth value of q.

To establish validity of a given statement is to show that st $(P_1 \wedge P_2 \dots \wedge P_n) \to q$ is a tautology

Ex:-
1)

p: Rogger studies
q: Roger plays tennis
r: Roger passes DMS.

$P_1 : p \to r$
$P_2 : \neg q \to p$.
$P_3 : \neg r$.

~~Exam~~ Determine whether arg $(P_1 \wedge P_2 \wedge P_3) \to q$ is valid.

Ans:- T.T. for $[(p \to r) \wedge (\neg q \to p) \wedge \neg r] \to q$

if it is a tautology, then arg is true.

2) S.T $[p \wedge ((p \wedge r) \to s)] \to (r \to s)$ is a tautology.

Rules of Inference

1. Modus Ponens or Rule of Detachment (method of

$$[p \wedge (p \to q)] \to q$$

ie

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

2. Law of Syllogism:- $[(p \to q) \wedge (q \to r)] \to (p \to r)$

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

3. Modus Tollens:- $[(p \to q) \wedge \neg q] \to \neg q$ (method of denying

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array}$$

Ex:- Prove validity of arg

Given, $p \to r$
$r \to s$
$t \vee \neg s$
$\neg t \vee u$
$\neg u$.

Step 1:-	$p \to r$, $r \to s$	Premises
2:	$p \to s$	1, Law of Syllogism
3:	$\neg s \vee t$	Commutative law
4.	$s \to t$	3, $3 \Leftrightarrow s \to t$
5.	$\neg t \vee u$	premises
6.	$t \to u$	
7.	$s \to u$	From 4 & 6 & law of syl
8.	$p \to t$	From 2 & 4
9.	$p \to u$	From 8 & 6 & Syllogism
10.	$\neg u$	
11.	$\therefore \neg p$	From 9, 10 & ~~law~~ of Modus To

4. Rule of Conjunction

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

5. Rule of disjunctive Syllogism: $[(p \vee q) \wedge \neg p] \rightarrow$

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

6. Rule of Contradiction

$$\begin{array}{c} \neg p \rightarrow F_0 \\ \hline \therefore p \end{array}$$

Using contradiction, if you want to establish va
$(P_1 \wedge P_2 \ldots \wedge P_n \wedge \neg q) \rightarrow F_0$.

7. Rule of conjunction Simplification

$$\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array} \qquad p \wedge q \rightarrow p$$

8. Rule of disjunctive amplification

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array} \qquad p \rightarrow (p \vee q)$$

9. Rule of conditional proof

$$\begin{array}{c} p \wedge q \\ p \rightarrow (q \rightarrow r) \\ \hline \therefore r \end{array} \qquad [(p \wedge q) \wedge (p \rightarrow (q \rightarrow r))] \rightarrow r.$$

10. Rule for proof by cases.

$$\begin{array}{c} p \rightarrow r \\ q \rightarrow r \\ \hline \therefore (p \vee q) \rightarrow r. \end{array} \qquad [(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$$

11. Rule of Constructive dilemma

$$\begin{array}{c} p \rightarrow q \\ r \rightarrow s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

12. Rule of Distructive dilemma

$$\begin{array}{c} p \rightarrow q \\ r \rightarrow s \\ \neg q \vee \neg s \\ \hline \therefore \neg p \vee \neg r. \end{array}$$

Ex:-

1) Prove the validity of arg:

$p \to r$

$\neg p \to q$

$q \to s$

$\therefore \neg r \to s$.

Steps

1. $p \to r$ Premises
2. $\neg r \to \neg p$ Contrapositive (1)
3. $\neg p \to q$ Premise
4. $\neg r \to q$ (2)(3) Syllogism.
5. $q \to s$. Premise
6. $\neg r \to s$. 4,5, Syllogism.

Or

1) $\neg p \to q$ Premise
2) $q \to s$ ''
3) $\neg p \to s$ 1, 2, Syllo
4) $p \to r$ Premise
5) $\neg r \to \neg p$ Contra
6) $\neg r \to s$. 5,3, S

2) PT

$p \to q$

$q \to (r \land s)$

$\neg r \lor (\neg t \lor u)$

$p \land t$

$\therefore u$.

Step

1) $p \to q$ Prem
2) $q \to (r \land s)$ ''
3) $p \to (r \land s)$ (1)(2) Syll.
4. $p \land t$ Premise
5. p. (4) Conjun Simp.
6. $r \land s$ (3)(5) Modus Pon
7. r Conj Simp
8. $\neg r \lor (\neg t \lor u)$ Premise
9. $\neg (r \land t) \lor u$ Asso, Demorg.
10. t (4) Conj Simp.
11. $r \land t$ (7) (8) Rule of Conj
12. u (9)(11), Disj. Syllogism.

2. S.T arg is valid.

$(\neg p \lor \neg q) \to (r \land s)$

$r \to t$

$\neg t$

$\therefore p$.

Step.

1) $r \to t$ Premise
2) $\neg t$ ''
3) $\therefore \neg r$ Modus Tollens of (1) (2).
4) $\neg r \lor \neg s$ Disjunctive amplification of (3).
5) $\neg (r \land s)$ Demorgans of (4).
6) $(\neg p \lor \neg q) \to (r \land s)$ Premise
7) $\neg (\neg p \lor \neg q)$ Modus Tollens, (5) & (6).
8) $\neg\neg p \land \neg\neg q$ Demorgans, (7).
9) $p \land q$ Law of double negation.
10) $\therefore p$ Conjunctive simplification of (9)

3. Use proof by contradiction to validate o:

$$\neg p \leftrightarrow q$$
$$q \rightarrow r$$
$$\neg r$$
$$\therefore p$$

$$[(\neg p \leftrightarrow q) \wedge (q \rightarrow r) \wedge \neg r \wedge \neg p] \Rightarrow F_0$$

Steps

1. $\neg p \leftrightarrow q$ Premise
2. $(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$ (1) $\because$ $p \leftrightarrow q \Leftrightarrow p \rightarrow q \wedge$
3. $\neg p \rightarrow q$ Conjun Simplifin, (2)
4. $q \rightarrow r$ Premise
5. $\neg p \rightarrow r$ Law of Syllogism 3,4
6. $\neg p$ Premise (Assumed)
7. r Modus Ponens (5),(6)
8. $\neg r$ Premise
9. $r \wedge \neg r \, (\Leftrightarrow F_0)$ (7),(8) order Conjunction
10. $\therefore p$ (6) (9) Proof by contradiction

$\because$ $\neg p \leftrightarrow q$, $q \rightarrow r$, $\neg r$ are given true, for $[\neg p \leftrightarrow q \wedge q \rightarrow r \wedge \neg r \wedge \neg p]$ to have 0, it st $\neg p = 0$ $\therefore p = 1$. $\therefore$ Conclusion p of arg

4. Establish validity of Arg.

$$u \rightarrow r$$
$$(r \wedge s) \rightarrow (p \vee t)$$
$$q \rightarrow (u \wedge s)$$
$$\neg t$$
$$\therefore q \rightarrow p$$

$\Rightarrow$

$$u \rightarrow r$$
$$(r \wedge s) \rightarrow (p \vee t)$$
$$q \rightarrow (u \wedge s)$$
$$\neg t$$
$$q$$
$$\therefore p$$

Steps

1. q Premise
2. $q \rightarrow (u \wedge s)$ "
3. $u \wedge s$ (1),(2) Modus Ponens
4. $\therefore u$ (3) Conjunctive Simp
5. $u \rightarrow r$ Premise
6. $\therefore r$ (4),(5) Modus Ponens
7. s (3) Conjun Simp
8. $r \wedge s$ (6)(7) & Rule of conjunction
9. $(r \wedge s) \rightarrow p \vee t$ Premise
10. $p \vee t$ (8)(9) Modus Ponens
11. $\neg t$ Premise
12. $\therefore p$ 10,11, Disjunctive Syllogism

S.T arg is invalid.

$p \wedge \neg q$
$p \to (q \to r)$
___
$\neg r$.

If arg is invalid $\neg r$ is 0 $\therefore r = 1$.

$p \to (q \to r)$ is true.

If $p = 0$, $q \to r$ can be 0 or 1

$r = 1$

$q \to r$ q can be 0 or 1 for $q \to r$ to

if $(q \to r) = 0$ then $q = 1, r = 0$ which cont

$\therefore q \to r = 1$

$\because (q \to r) = 1$ $p = 1$.

$\because p \wedge \neg q = 1$

$\therefore \neg q = 1$

$\therefore q = 0$.

$\therefore$ when $p = 1, q = 0, r = 1$, the premises are true & conclusion is False.

(or).

$\neg r = 0$ $\therefore r = 1$.

From $p \wedge \neg q$, $p = 1$, $\neg q = 1$ $\therefore q = 0$.

For (2) $p \to (q \to r)$ to be 1, $\because p = 1$, $q \to r$ to be 1

$\therefore q = 0$, r could be 0 or 1. If $r = 0$, then $\neg r$ which contradicts.

10.c)

$p \to q$
$\neg q$
$\neg r$
___
$\therefore \neg (p \vee r)$.

Steps

(1)	$p \to q$	premise
(2)	$\neg q$	Premise
3)	$\therefore \neg p$	(1), (2) Modus tollens.
4)	$\neg r$	Premise
5)	$\neg p \wedge \neg r$	3,4, Rule of Conjunction
6)	$\neg (p \vee r)$	(5), Demorgans.

10)(d).

$p \wedge q$
$p \to (r \wedge q)$
$r \to (s \vee t)$
$\neg s$
___
$\therefore t$.

Steps:

1.	$p \wedge q$	Premise
2.	$\therefore p$	(1), Conjunctive Simplification
3.	$p \to (r \wedge q)$	Premise
4)	$r \wedge q$	2, 3, Modus Ponens
5)	r	(4), Conj Simpl.
6)	$r \to (s \vee t)$	Premise

7). $s \vee t$ — 5, 6, Modus Ponens.

8) $\neg s$ — Premise

9) t. — 7, 8, Rule of Disjun Syllosis

(10)(e)

$$p \rightarrow (q \rightarrow r)$$
$$\neg q \rightarrow \neg p$$
$$p$$
$$\therefore r.$$

Steps

1.	p	Premise
2.	$p \rightarrow (q \rightarrow r)$	Premise
3.	$q \rightarrow r$	1, 2, Modus Ponens.
4.	$\neg q \rightarrow \neg p$	Premise
5.	$p \rightarrow q$	(4) Contrapositive $\Leftrightarrow$ implica
6.	q	1, 5, Modus Ponens
7.	r.	3, 6 Modus Ponens

Use of Quantifiers

S: "$x+2$ is an even integer"
if x is replaced by even integer, $S=1$ else $S=0$
Here S is called an open st.

Defn:- A declarative statement is open st if
a) it contains one or more variables &
b) it is not a statement but
c) it becomes a st when variable in it is replaced by certain allowable choices.

The allowable choices constitute Universe of Disclosure for open st. Universe consists of choices we wish to consider or allow for variables in open st.

Above open statement S is written as

$p(x)$: $x+2$ is even integer.
↳ one var.

$q(x,y)$: $y+2$, $x-y$, $x+2y$ are even integer
↳ two var.

$p(5) =$ False, $\neg p(x)$: $x+2$ is not even in
$\neg p(7) =$ True.
$q(4,2) =$ True.

For $p(x)$, $q(x,y)$ some substitute ... make $p(x)$, $q(x,y)$ true & some " "

∴ we make following true st.

1) for some x, $p(x)$

2) For some x, y, $q(x,y)$.

In this sit for some x, $\neg p(x)$ &
" " x, y $\neg q(x,y)$ also t...

For some x & for some x, y are said to quantify open sts $p(x)$, $q(x,y)$

Two types of quantifiers

1) Existential — For some x or for atleast ... or there exists an x. Symbol - $\exists$

$\exists x$, $p(x)$.

2) Universal - For all x, For any x, for eac... For every x. Symbol $\forall$.

$\forall x$, $p(x)$.

Let $r(x)$: $2x$ is an even integer.

$\forall x$ $r(x)$ is true.

$\neg r(x)$: $2x$ is not an even integer.

$\exists x$ $r(x)$ is true.

$\forall x$ $\neg r(x)$ is False.

$\exists x$ $\neg r(x)$ is also false.

Here in $p(x)$ & $q(x)$, x is called free variab... As x varies truth value of $p(x)$, $q(x)$ may vary.

$\exists x$ $p(x) \rightarrow x$ is bound variable

$\forall x$ $p(x) \rightarrow x$ is " " .

Quantifiers are used in conjunction with logical conne...

$p(x)$: $x \geq 0$ $r(x) = x^2 - 3x - 4 = 0$

$q(x)$: $x^2 \geq 0$ $s(x) = x^2 - 3 > 0$.

1) $\exists x$ $p(x) \wedge q(x)$ is true

$x = 4$. $p(4) = q(4) = \text{true}$.

2) $\forall x$ $[p(x) \rightarrow q(x)]$. but when $p(x)$ is ...

if $x < 0$, then $p(x)$ is false. $p(x) \rightarrow q(x)$ is true irrespective of $q(x)$ false

$p(x) \rightarrow q(x) = \text{true}$.

if $x \geq 0$, $p(x) = q(x) = \text{true}$. $\forall x$ $[p(x) \rightarrow q(x)]$ is true.

3) Also $\exists x\, [p(x) \to q(x)]$ is true.

4) ST $\forall x\, [q(x) \to s(x)]$ is false.

if $x = 1$, $q(x)$ is true

$s(x)$ is false.

$\therefore$ $q(1) \to s(1)$ is false.

$\therefore$ $\forall x\, [q(x) \to s(x)]$ is false.

$\forall x\ p(x) \Rightarrow \exists x\ p(x)$

but $\exists x\ p(x) \not\Rightarrow \forall x\ p(x)$.

$p(x), q(x)$ - open sts.

$\forall x\, [p(x) \to q(x)]$

1) Contrapositive is $\forall x\, [\neg q(x) \to \neg p(x)]$.

2) Converse is $\forall x\, [q(x) \to p(x)]$.

3) Inverse is $\forall x\, [\neg p(x) \to \neg q(x)]$.

Implication & Contra+ve are logically equivalent

Inverse & Converse are also " "

$\exists x\, [r(x) \wedge s(x)] \not\Leftrightarrow \exists x\ r(x) \wedge \exists\ s(x)$.

Also $[\exists x\, r(x) \wedge \exists x\, s(x)] \not\Rightarrow \exists x\, [r(x) \wedge s(x)]$.

But

1. $\exists x\, [p(x) \wedge q(x)] \Rightarrow [\exists x\ p(x) \wedge \exists x\ q(x)]$
2. $\exists x\, [p(x) \vee q(x)] \Leftrightarrow [\exists x\ p(x) \vee \exists x\ q(x)]$
3. $\forall x\, [p(x) \wedge q(x)] \Leftrightarrow [\forall x\ p(x) \wedge \forall x\ q(x)]$.
4. $[\forall x\ p(x) \vee \forall x\ q(x)] \Rightarrow \forall x\, [p(x) \vee q(x)]$.

Associativity over $\wedge$, and $\vee$ with $\forall$ is satisfied

Demorgans " $\wedge, \vee$ ~~ov~~ with $\forall$ is satisfied.

Double negation with $\forall$ is also satisfied.

Rules for negating sts with one quantifier

$\neg [\forall x \; p(x)] \Leftrightarrow \exists x \neg p(x)$.

$\neg [\exists x \; p(x)] \Leftrightarrow \forall x \; \neg p(x)$

$\neg [\forall x \; \neg p(x)] \Leftrightarrow \exists x \; p(x)$

$\neg [\exists x \neg p(x)] \Leftrightarrow \forall x \; p(x)$.

Find -ve of

1) $\forall x \; [p(x) \rightarrow q(x)]$

$\Leftrightarrow \forall x \; [\neg p(x) \vee q(x)]$.

Negation of above st is

$\neg \{\forall x \; [\neg p(x) \vee q(x)]$

$\Leftrightarrow \exists x \; [\neg(\neg p(x) \vee q(x)]$

$\Leftrightarrow \exists x \; [p(x) \wedge \neg q(x)]$

$\forall x, \forall y$, $\exists x, \exists y$ } Associative & commutative law holds.

But $\forall x \exists y \; p(x,y) \neq \exists y \forall x \; p(x,y)$.

2) Find -ve of

$\forall x \; \exists y \; [(p(x,y) \wedge q(x,y)) \rightarrow r(x,y)]$

Soln:-

$\neg [\forall x \; \exists y \; [(p(x,y) \wedge q(x,y)) \rightarrow r(x,y)]$

$\Leftrightarrow \exists x \; [\neg \exists y \; [(p(x,y) \wedge q(x,y)) \rightarrow r(x,y)]$

$\Leftrightarrow \exists x \; [\forall y \; \neg [(p(x,y) \wedge q(x,y)) \rightarrow r(x,y)]$

$\Leftrightarrow \exists x \; [\forall y \; \neg [\neg (p(x,y) \wedge q(x,y)) \vee r(x,y)]$

$\Leftrightarrow \exists x \; \forall y \; [p(x,y) \wedge q(x,y) \wedge \neg r(x,y)]$.

Exer:- pg 104

2.4

4) a) Every 3 sided or 4 sided polygon is either a quadrilateral or a triangle.

b) Every isosceles triangle is an equilateral Δle.

c) There exists Δle whose interior $\angle$le exceeds 180°

d) For every equilateral Δle, all interior $\angle$les are equal

e) There exists a 3 or 4 sided polygon such that it is quadrilateral but not a rectangle.

f) There exists a 3 or 4 sided polygon such that it is a rectangle but not a square.

Pg 104

Universe - real nos.

6. $p(x,y): x^2 \geq y$
$q(x,y): x+2 < y$.

$\pi = \frac{22}{7}$

a) $p(2,4)$: $4^2 \geq 4$ T.

b) $q(1,\pi)$: $3 \leq \pi$ F

c) $p(-3,8) \wedge q(1,3)$

$9 \geq 8$ T $\wedge$ $3<3$ F $=$ F.

d) $p(\frac{1}{2},\frac{1}{3}) \vee \neg q(-2,-3)$

$\frac{1}{4} \geq \frac{1}{9}$ T $=$ T.

e) $p(2,2) \rightarrow q(1,1)$

$4 \geq 2$ T $\rightarrow$ $3<1$ F $=$ F.

f) $p(1,2) \leftrightarrow \neg q(1,2)$

$1 \geq 2$ F, $3<2$ T $\neg$F $=$ F

7. a)

i) $\exists x \; q(x)$ $\rightarrow$ T

ii) $\exists x \; [p(x) \wedge q(x)]$ $\rightarrow$ T

iii) $\forall x \; [q(x) \rightarrow \neg t(x)]$ $\rightarrow$ F

iv) $\forall x \; [q(x) \rightarrow \neg t(x)]$ — F — x - 10

v) $\exists x \; [q(x) \wedge t(x)]$ — T

vi) $\forall x [q(x) \wedge r(x) \rightarrow s(x)]$ — T.

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8) $p(x): x^2 - 8x + 15 = 0$ Universe of integers.
$q(x)$: x is odd
$r(x)$: $x > 0$.

$x = 5$ $x = 3$.

a) $\forall x \; [p(x) \rightarrow q(x)]$ T.

b) $\forall x \; [q(x) \rightarrow p(x)]$ — F.

For $x = 7$ $q(x)$ is T
But $p(x)$: $49 - 56 + 15 \neq 0$.

c) $\exists x \; [p(x) \rightarrow q(x)]$ T.

d) $\exists x \; [q(x) \rightarrow p(x)]$ T.

e) $\exists x \; [r(x) \rightarrow p(x)]$ T.

f) $\forall x \; [\neg q(x) \rightarrow \neg p(x)]$ T.

g) $\exists x\,[P(x) \rightarrow q(x) \wedge r(x)]$ — T.

$P(x)$ is true for $x=3$, $x=5$.

$q(x)$ is " " " "

$r(x)$ is " " " "

$\therefore$

h) $\forall x\,[(p(x) \vee q(x)) \rightarrow r(x)]$ — T.

for all $x>0$, $r(x)$ is true.

$p(x)$ is true for $x=3$ & $x=5$

$q(x)$ " " " " "

$q(x)$ is true for all odd nos

if x is even, $q(x)$ is 0 & $p(x)$ for x: even is false.

If $\forall x\ p(x)$ is ~~then~~ true, then $\exists x\ p(x)$ is also

$\therefore$ $\forall x\ p(x) \Rightarrow \exists x\ p(x)$.

Convert to open sts:

1) If a number is rational, then it is a real no

Here presence of indefinite article "a" indicates that we are dealing with universally quantified sts

$\therefore$ Universal quantifier is implicit

Let $p(x)$, $q(x)$ be open sts

$p(x)$: x is a rational no.

$q(x)$: x is a real no.

$\forall x\,[p(x) \rightarrow q(x)]$

2) For universe of all $\triangle$les in plane

"An equilateral $\triangle$le has three angles of 60° & conversely"

Let $e(t)$: Triangle t is equilateral

$a(t)$: Triangle t has three $\angle$les of 60°.

$\forall t\,[e(t) \leftrightarrow a(t)]$

3) " Integer 41 is equal to sum of two perfect squares"

$\exists m\ \exists n\ [41 = m^2 + n^2]$

for n := 1 to 20 do

A[n] = n*n − n

i) $\forall n\,[A[n] \geq 0]$ (Every element of array is non-…

ii) $\exists n\,(A[n+1] = 2 \cdot A[n])$

iii) $\forall n\,[(1 \leq n \leq 19) \rightarrow (A[n] < A[n+1])]$. {Elements of array are in ascending order)

iv) $\forall m\,\forall n\,[(m \neq n) \rightarrow (A[m] \neq A[n])]$

or $\forall m\,\forall n\,[(m < n) \rightarrow (A[m] \neq A[n])]$

(Entries in array are distinct)

Statement	When it is true?	When it is fal…
$\exists x\, p(x)$	For some a in universe, p(a) is true	For every a in universe, p(a)…
$\forall x\, p(x)$	For every a in uni, p(a) is true.	For atleast one a for which p(a)…
$\exists x\, \neg p(x)$	For some a in uni, $\neg p(a)$ is true	For every a, p(a) is true
$\forall x\, \neg p(x)$	For every a, $\neg p(a)$ is true. (ie p(a) is false)	For some a, $\neg p(a)$ is false (…

p(x), q(x) open sts are logically equivalent when the biconditional $p(a) \leftrightarrow q(a)$ is true for each replacement of a from universe & is written as

$\forall x\,[p(x) \Longleftrightarrow q(x)]$

If $p(a) \rightarrow q(a)$ is true for each a in universe, then we write

$\forall x\,[p(x) \Rightarrow q(x)]$. logical implication.

For universe of real nos.

1) Every real no greater than 3 has magnitude (or absolute value) greater than 3.

$p(x):\ x > 3$

$q(x):\ |x| > 3$.

$\forall x\,[p(x) \rightarrow q(x)]$ → True

2) Converse of (1). is

$\forall x\,[q(x) \rightarrow p(x)]$. → False.

ie If magnitude of a real no is greater than 3 then the real no itself is greater than 3

Ex:- $x = -5$, $q(-5)$ = True but $p(-5)$ is False

3) Contrapositive of (1) is → True.

$\forall x\,[\neg q(x) \rightarrow \neg p(x)]$

If magnitude of real no is less than or equal to 3 then no itself is less than or equal to 3.

4) Inverse of (1) is

$$\forall x\,[\neg p(x) \rightarrow \neg q(x)] \quad \rightarrow \text{False}.$$

: If a real no is less than or equal to 3, then so is its magnitude.

→ Prove that $\exists x\,[p(x) \wedge q(x)] \Rightarrow [\exists x\; p(x) \wedge \exists x$

Proof:- Consider $\exists x\,[p(x) \wedge q(x)] \rightarrow [\exists x\, p(x) \wedge \exists x$

hypothesis

Conclusion.

When hypothesis $\exists x\,(p(x) \wedge q(x))$ is true, there is atleast one element c in the universe fo which $p(c) \wedge q(c)$ is true. By rule of conjun simplification $[p(c) \wedge q(c)] \Rightarrow p(c)$. From truth p(c) we have true statement $\exists x\; p(x)$. |||ly $\exists x\; q(x)$ is another true st. Therefore $\exists x$ $\exists x\; q(x)$ is a true statement. Since $\exists x\; p$ $\exists q(x)$ is true whenever $\exists x\,[p(x) \wedge q(x)]$ is it follows

$$\exists x\,[p(x) \wedge q(x)] \Rightarrow \exists x\; p(x) \wedge \exists$$

|||ly it can be shown as

$\exists x\,[p(x) \wedge q(x)]$

$\Rightarrow p(a) \wedge q(a)$ Universal Specification

$\therefore p(a)$ \ Conjunction Simplification

$\therefore q(a)$

$\exists x\; p(x)$ } Universal Specification

$\exists x\; q(x)$

$\therefore \exists x\; p(x) \wedge \exists x$

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III^{ly} it can be proved that

2) $\forall x\, [p(x) \wedge q(x)] \Leftrightarrow [\forall x\, p(x) \wedge \forall x\, q(x)]$

3) $\exists x\, [p(x) \vee q(x)] \Leftrightarrow [\exists x\, p(x) \vee \exists x\, q(x)]$.

4) $\forall x\, [p(x) \vee q(x)] \Rightarrow [\forall x\, p(x) \vee \forall x\, q(x)]$.

ie NOTE:

$\forall$ with $\vee$ is $\Rightarrow$	$\exists$ with $\vee$ is $\Leftrightarrow$
$\exists$ with $\wedge$ is $\Rightarrow$.	$\forall$ with $\wedge$ is $\Leftrightarrow$.

1) Find negation of if x is odd, then x^2-1 is even.

Soln:- $p(x)$: x is odd.

$q(x)$: x^2-1 is even.

$\forall x\, [p(x) \rightarrow q(x)]$.

$\neg\, [\forall x\, [p(x) \rightarrow q(x)]]$

$\Leftrightarrow\ \exists x\, [\neg\, [p(x) \rightarrow q(x)]]$

$\Leftrightarrow\ \exists x\, [\neg\, (\neg p(x) \vee q(x))]$ Demorgans law

$\Leftrightarrow\ \exists x\, [\neg\neg p(x) \wedge \neg q(x)]$ ~~Dem~~ law of double ne

$\Leftrightarrow\ \exists x\, [$

$\Leftrightarrow\ \exists x\, [p(x) \wedge \neg q(x)]$

The exists an integer such that x is odd and x^2-1 is odd.

2) Find negation of

$\exists x \, [r(x) \wedge s(x)]$.

Soln:-

$\neg \exists x \, [r(x) \wedge s(x)]$

$\Leftrightarrow \forall x \, (\neg [r(x) \wedge s(x)])$ Demorgans law.

$\Leftrightarrow \forall x \, (\neg r(x) \vee \neg s(x))$

$p(x,y)$: $x+y=17$.

$\forall x \, \exists y \cdot p(x,y)$ is true.

ie For every integer x, there exists an integer such that $x+y=17$.

$\exists y \, \forall x \, p(x,y)$ is false.

There exists an integer y so that for all integer $x+y=17$.

1) For universe of real no.,

$a+0=0+a=a$, for every real no a

Here 0 is called identity of addition

It is written as

$\exists z \, \forall a \, (a+z=z+a=a)$ Here z is additive identity.

Express

1) "Every real no has an additive inverse."

$\forall x \, \exists y \, [x+y=y+x=0]$. y is additive inverse

2) existence of multiplicative identity

$\exists z \, \forall a \, [a \cdot z = z \cdot a = a]$ z is multiplicative identity

3) Existance of multiplicative inverses for nonzero real

$\forall x \neq 0 \, \exists y \, [x \cdot y = y \cdot x = 1]$

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Ex 2.4

24) $\forall x \, \exists x \, [x+y=17]$..

a) integers $\rightarrow$ if x is fixed, $y=17-x$ is an integer

$\therefore$ True.

b) +ve integers $\rightarrow$ if $x>17$, y is $-$ve.

$\therefore$ False.

c) integers for x, +ve integers for y:- if $x>0$ & if $x>17$, y is $-$ve.

$\therefore$ False.

d) +ve integers for x, integers for y.

if $x > 0$, $y = 17 - x$ is also integer

$\therefore$ True.

Find negation of

$$\forall x \ \forall y \ [(x > y) \rightarrow (x - y > 0)]$$

$\neg(p \rightarrow q)$
$\Leftrightarrow \neg(\neg p \vee q)$
$\Leftrightarrow p \wedge \neg q$

Soln:-

$$\neg[\forall x \ \forall y \ [(x > y) \rightarrow (x - y > 0)]]$$

$$\Leftrightarrow \exists x \ [\neg(\forall y \ ((x > y) \rightarrow (x - y > 0)))]$$

$$\Leftrightarrow \exists x \ \exists y \ [\neg((x > y) \rightarrow (x - y > 0))]$$

$$\Leftrightarrow \exists x \ \exists y \ [(x > y) \wedge \neg(x - y > 0)]$$

$$\Leftrightarrow \exists x \ \exists y \ [(x > y) \wedge (x - y \leq 0)]$$

$$\forall x \ \forall y \ [(x < y) \rightarrow \exists z \ (x < z < y)]$$

Soln:- $\neg \forall x \ \forall y \ [(x < y) \rightarrow \exists z \ (x < z < y)]$

is $\exists x \ \exists y \ [(x < y) \wedge (\neg \exists z \ (x < z < y))]$

$$\Leftrightarrow \exists x \ \exists y \ [(x < y) \wedge \forall z \ \neg(x < z < y)]$$

$$\Leftrightarrow \exists x \ \exists y \ [(x < y) \wedge \forall z \ (x \geq z \vee z \geq y)]$$

Theorems are used only to describe major results that have many and varied consequences. Certain of these consequences that follow rather immediately from a theorem are called corollaries.

Rule of Universal Specification:-

:- If an open statement becomes true for all replac[ements] by the numbers in a given univese, then that open statement is true for each specific indiv[idual] number in that universe.

i.e. if $p(x)$ is an open st,
& $\forall x \; p(x)$ is true,
then $p(a)$ is true for each a.

$m(x)$: x is a mathematics professor
$c(x)$: x has studied calcus.

All mathematics professors have studied calcus.
Leona is a " "
∴ Leona has studied calcus.

$\forall x \; m(x) \to c(x)$ Premise
$m(l)$

∴ $c(l)$

1)	$\forall x \,[m(x) \to c(x)$	Premise
2)	$m(l)$	Premise
3)	$m(l) \to c(l)$	(1) Rule of Universal spn.
4)	∴ $c(l)$.	(2),(3), Modus Ponens

Given

$\neg r(c)$
$\forall t \; [p(t) \to q(t)]$
$\forall t \; [q(t) \to r(t)]$

∴ $\neg p(c)$.

Soln:-

Step		Reason
1)	$\forall t \,[p(t) \to q(t)]$	Premise
2)	$p(c) \to q(c)$	(1), Rule of Universal spn.
3)	$\forall t \,(q(t) \to r(t))$	Premise
4)	$q(c) \to r(c)$	(3), Rule of Universal Spn
5)	$p(c) \to r(c)$	(2), (4) Syllogism
6)	$\neg r(c)$	Premise
7)	∴ $\neg p(c)$	(5),(6), Modus Tollens

Rule of Universal generalization:-

St:- If an open st $p(x)$ is proved to be true when x is replaced by any arbitrarily chosen element from universe, $\forall x\ p(x)$ is true.
IIIly, $\forall x\ \forall y\ q(x,y)$ is true

1) Show that

$$\begin{array}{l} \forall x\,[p(x) \to q(x)] \\ \forall x\,[q(x) \to r(x)] \\ \hline \therefore \forall x\,[p(x) \to r(x)] \end{array}$$

is valid.

Soln

Steps	Reason
1) $\forall x\,[p(x) \to q(x)]$	Premise
2) $p(c) \to q(c)$	(1), Rule of Universal sp^n.
3) $\forall x\,[q(x) \to r(x)]$	Premise
4) $q(c) \to r(c)$	(3), Rule of Universal sp^n.
5) $p(c) \to r(c)$	(2),(4), Law of Syllogism
6) $\therefore \forall x\,[p(x) \to r(x)]$	(5), Rule of Universal gen

Paragraph style proof:-

Thm:- For all integers k, l, if k, l are both odd, $k+l$ is even.

Proof:- let $k = 2a+1$, $l = 2b+1$. for some integers

$k+l = 2a+1+2b+1 = 2(a+b+1)$.

$\because$ a, b are integers, $a+b+1 = c$ is also intege

$\therefore$ $k+l$ is even.

5)

P.T 1) $\forall x\ [p(x) \wedge q(x)] \Longleftrightarrow \forall x\ p(x) \wedge \forall x\ q(x)$

2) $\forall x\ [p(x)] \vee \forall x\ q(x) \not\Rightarrow \forall x\ [p(x) \vee q(x)]$

Proof:- pg: 98 (text) or

pg:- 18 (back side) of note.

(or)

Step	Reason
1. $\forall(x)\ [p(x) \wedge q(x)]$	Ag. Premise
2. $p(c) \wedge q(c)$	(1) Rule of Universal spn.
3. $\therefore p(c)$	(2), Rule of Conjunctive Simpln.
4. $\therefore q(c)$	(2). " "
5. $\forall x\ (p(x))$	(3), Rule of generalization.
6. $\forall x\ (q(x))$	(4) " "
7. $\forall x\ p(x) \wedge \forall x\ q(x)$	(5),(6) Law of Conjunction.

Simplify: $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$

Soln:-

Steps	Reason
1. $(p \to q) \wedge \neg q \wedge (r \vee \neg q)$	Premise
2. $\Longleftrightarrow (p \to q) \wedge (\neg q)$	(1), Absorption law
3. $(\neg p \vee q) \wedge (\neg q)$	(2) $\Longleftrightarrow (\neg p \vee q)$
4. $(\neg p \wedge \neg q) \vee (q \wedge \neg q)$	(4), Distribution law of
5. $(\neg p \wedge \neg q) \vee F_0$	$\because q \wedge \neg q = F_0$.
6. $\neg (p \vee q)$	

6

8) Simplify $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$

Soln:-

Steps	Reason
1. $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$	Premise
2. $(p \to q) \wedge [\neg q]$	(1), absorption law
3. $(\neg p \vee q) \wedge (\neg q)$	$\because p \to q \Leftrightarrow \neg p \vee q$.
4. $(\neg p \wedge \neg q) \vee (q \wedge \neg q)$	(3), Distributive law of $\wedge$ over
5. $(\neg p \wedge \neg q) \vee F_0$.	$\because q \wedge \neg q = F_0$.
6. $(\neg p \wedge \neg q)$	$\because p \vee F_0 = p$
7. $\neg (p \vee q)$.	

9) Consider

"If train arrives late & there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. Train did arrive late. Therefore there were taxis at the st

Translate into equivalent formal repⁿ & prove its validity using rules of inference.

Soln:-

p: train arrives late

q: taxis are at station.

r: John is late for meeting.

$(p \wedge \neg q) \to r$.

$\neg r$

p

$\therefore q$

7b) Disjunctive Syllogism.

$$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$$

To prove this we have to show that

$[(p \vee q) \wedge \neg p] \rightarrow q$ is a tautology

1. $\neg [(p \vee q) \wedge \neg p] \vee q$
2. $[\neg (p \vee q) \vee (\neg \neg p)] \vee q$ Law of double negation
3. $[\neg (p \vee q) \vee p] \vee q$
4. $\neg (p \vee q) \vee (p \vee q)$ Associative law of $\vee$.
5. T_0 $\because$ $p \vee \neg p = T_0$ Idempotent la

Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \underline{\neg q} \\ \therefore \neg p \end{array}$$

To prove this S.T

$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology.

1. $\neg [(p \rightarrow q) \wedge \neg q] \vee \neg q$. Demorgans
2. $\neg(\neg p \vee q) \vee (q \vee \neg q)$ Inverse Law.
3. $(p \wedge \neg q) \vee T_0 \Rightarrow T_0$: Domination

1. $\neg [(p \to q) \wedge \neg q] \vee \neg q$.

2.

Grimaldi → pg 75 in pdf, Q 10.

Quiz

13 (pg 75 pdf)