

Booths Algorithm

Booths recoding scheme



bit i	bit i-1	
0	0	0 x M
0	1	+1 x M
1	0	-1 x M
1	1	0 x M

Eg 1)
$$\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 0 & 0 & [0 \\ -1 & +1 & 0 & -1 & 0 & 0 & \end{array}$$

a)
$$\begin{array}{ccccccc} 0 & 1 & 1 & 0 & 0 & 1 & [0 \\ +1 & 0 & -1 & 0 & +1 & -1 & \end{array}$$

1) Multiplicand +13 \rightarrow 1101 \Rightarrow 01101
 Multiplier -6 \rightarrow 0110

Add sign bit

Take 2's complement

$$\begin{array}{r} 2 \overline{) 13} \\ 2 \overline{) 61} \\ 2 \overline{) 30} \\ 11 \end{array}$$

Add sign bit

$$\begin{array}{r} \rightarrow \text{1010} \\ 11010 \end{array}$$

$$\begin{array}{r} 0110 \\ 1001 \\ \hline 1010 \end{array}$$

$$\begin{array}{r} 11010 \\ \hline 0-1+1-10 \end{array}$$

$$\begin{array}{r} 01101 \\ \times 0010 \\ \hline 10011 \end{array}$$

$$\begin{array}{l} n=5 \\ 2n=10 \end{array}$$

						0	1	1	0	1
						0	-1	1	-1	0
1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	0	0	1		
1	1	1	0	0	1	1	0	1		
0	0	0	0	0	0	0				

Carry
1 1 1 1 0 1 1 0 0 1 0 -78

\downarrow
2's complement

$$\begin{array}{r} 0001001101 \\ \hline 0001001110 \\ 2^4 + 2^3 + 2^2 + 2^1 = 78 \end{array}$$

2) Multiplicand = -13
 Multiplier = -20

$-13 \Rightarrow 1101 \Rightarrow 01101 \Rightarrow 10011 \Rightarrow 110011$ (2's comp, Add sign bit)
 $-20 \Rightarrow 10100 \Rightarrow 10100 \Rightarrow 01100 \Rightarrow 101100$

$010010101100 \begin{bmatrix} 0 \\ -110100 \end{bmatrix}$
 Recoding

$\begin{array}{r} 110011 \\ 001100 \\ \hline 001101 \end{array}$ 2's complement

$\begin{array}{r} 101100 \\ -110100 \\ \hline 011000 \end{array}$

$\begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & & & & \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & & & & & \end{array}$

$\left\{ \begin{array}{cccccccccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array} \right\} + 260$

any ignore.

\Downarrow
 $2^8 + 2^2 = 260$

3) Perform Multiplication for -13 and +09 using Booth's algorithm

$$\begin{array}{r} -13 \quad 1101 \\ +9 \quad 1001 \end{array} = \begin{array}{r} 0011 \\ 0100 \end{array} \longrightarrow \begin{array}{r} 10011 \\ 01001 \end{array}$$

$$\begin{array}{r} 10011 \\ +11011 \\ \hline 10011 \\ 01100 \\ \hline 01101 \end{array}$$

11110001011 = -117

⇓

2^{13} complement

$$\begin{array}{r} 0001110100 \\ \hline 0001110101 \end{array}$$

$$2^6 + 2^5 + 2^4 + 2^2 + 2^0 = 64 + 32 + 16 + 4 + 1 = 117$$

Q) Multiply the following signed 2's complement numbers using Booth's algorithm

multiplicand = $(010111)_2$
 multiplier = $(110110)_2$

$1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad \boxed{0}$
 $0 \quad -1 \quad +1 \quad 0 \quad -1 \quad 0$

$2^5 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1$
 $1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0$
 $\hline 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1$

$0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1$
 $0 \quad -1 \quad +1 \quad 0 \quad -1 \quad 0$

0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	1	0	0	1	
0	0	0	0	0	0	0	0	0	0		
0	0	0	0	1	0	1	1	1			
1	1	1	0	1	0	0	1				
0	0	0	0	0	0	0					

$\boxed{1} \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0$

— 230



2's complement

0	0	0	0	1	1	1	0	0	1	0	1
0	0	0	0	1	1	1	0	0	1	1	0

$$2^7 + 2^6 + 2^5 + 2^2 + 2^1 = 230$$

5.) Multiply 14×-8 using Booths Algorithm

Multiplicand = 14

Multiplier = -8

$$14 = 1110 \Rightarrow 2^{\text{'s complement}}$$

$$-8 = 1000 \Rightarrow 1000 \Rightarrow$$

Add sign bit

$$01110$$

$$11000$$

$$\begin{array}{r} 11000 \\ 0-1000 \end{array} \left[\begin{array}{l} 0 \\ 0 \end{array} \right]$$

$$\begin{array}{r} 01110 \\ 0-1000 \end{array}$$

$$\begin{array}{r} 2^{\text{'s}} \quad 01110 \\ \quad 10001 \\ \hline \quad 10010 \end{array}$$

$$\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc} 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc} 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array} \quad -112$$

↓

$$\begin{array}{cccccc} 2^{\text{'s}} \text{ Com} & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{array}$$

$$\begin{array}{cccccc} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}$$

$$2^6 \quad 2^5 \quad 2^4 = 112$$

Fast Multiplication

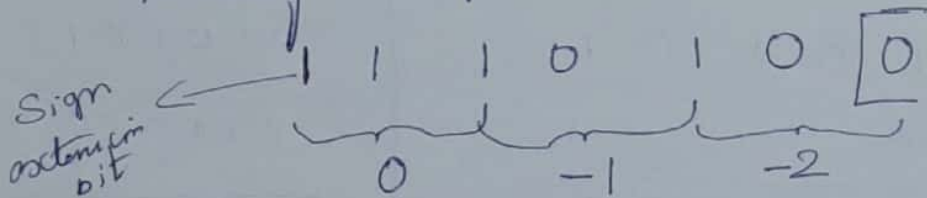
- ↳ 2 Technique
- 1) Bit Pair recoding of Multiplier
 - 2) Carry Save addition of Summands

Bit Pair recoding of Multiplier

$$n = 6 \text{ bits}$$

$$\text{recoding} = n/2 \text{ bits} = 6/2 = 3 \text{ bits}$$

Recoding Example



① Multiply -11 & $+27$

Multiplicand - 11
Multiplier 27

$-11 \Rightarrow 01011 \Rightarrow 10101 \Rightarrow 110101$
 $27 \Rightarrow 11011 \xrightarrow{\hspace{1cm}} 011011$

Recoding of Multiplier

$$\begin{array}{ccccccc}
 & +2 & & -1 & & -1 & \\
 \hline
 0 & 1 & 1 & 0 & 1 & 1 & [0]
 \end{array}$$

$$\begin{array}{cccccccccccc}
 & & & & & & & & 1 & 1 & 0 & 1 & 0 & 1 \\
 & & & & & & & & +2 & & -1 & & -1 & \\
 \hline
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
 \hline
 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\
 & & & & & & & & & & -297
 \end{array}$$

$36 - 1$ 2^5 of Multiplicand
 110101
 001010
 1

 001011

df + 2 \downarrow

~~110101~~ Binary value = 10

Now Multiply multiplicand bits with 10

110101	X 10
<hr/>	
000000	
110101	
<hr/>	
110101	0
<hr/>	

↓

0 0 0 1 0 0 1 0 1 0 0 0 1

0 0 0 1 0 0 1 0 1 0 0 1

$2^8 + 2^5 + 2^3 + 2^0$
 $256 + 32 + 8 + 1$

② Multiply +13 & -6

$$\begin{array}{lcl}
 +13 \Rightarrow 1101 & \xrightarrow{2^s \text{ complement}} & \text{Add Sign bit} \\
 -6 \Rightarrow 0110 \Rightarrow 1010 & \Rightarrow & 11010
 \end{array}$$

Recoding

$$\begin{array}{cccccc}
 1 & 1 & 1 & 0 & 1 & 0 \\
 \hline
 & 0 & -1 & -2 & &
 \end{array}$$

← Add Sign extended bit

If (-1)
Take 2's of Multiplicand

$$\begin{array}{r}
 01101 \\
 10010 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 01101 \\
 0-1-2
 \end{array}$$

$$\begin{array}{r}
 11111 \\
 00110
 \end{array}$$

$$\begin{array}{r}
 11110 \\
 0011
 \end{array}$$

$$\begin{array}{r}
 00000 \\
 0000
 \end{array}$$

$$\begin{array}{r}
 1110110010 \\
 \hline
 -78
 \end{array}$$

Carry ignore

↓ 2's complement

$$\begin{array}{r}
 0001001101 \\
 1
 \end{array}$$

$$\begin{array}{r}
 0001001110 \\
 \hline
 2^6 + 2^3 + 2^2 + 2^1 = 78
 \end{array}$$

If (-2)
Take 2's Complement of Multiplicand

$$\begin{array}{r}
 01101 \\
 10010 \\
 \hline
 1
 \end{array}$$

Then Multiply with 10

$$\begin{array}{r}
 10011 \times 10 \\
 \hline
 00000 \\
 10011 \\
 \hline
 100110
 \end{array}$$

③ Multiply the following pair of signed 2's complement number using bit pair recording of the Multiplier $A = 01011$, $B = 101100$

$$A = 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ (+2^3)$$

$$B = 101100 (-20)$$

Reading $\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{array} \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right]$

$$\begin{array}{cccccccccccc} & & & & & & 0 & 1 & 0 & 1 & 1 & 1 \\ & & & & & & & -1 & & -1 & & 0 \\ \hline 1 & \leftarrow & 1 & \leftarrow & 1 & \leftarrow & 1 & \leftarrow & 1 & \leftarrow & 1 & \leftarrow & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & & 1 & & 1 & & 1 & 0 & 1 & 0 & 0 & 1 & \\ 1 & & 1 & & 1 & 0 & 1 & 0 & 0 & 1 & & & \end{array}$$

1 1 1 0 0 0 1 1 0 1 0 0 $\Rightarrow -460$

Ignore
Cany

$$\begin{array}{cccccccccccc}
 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
 \hline
 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
 & & & 2^8 & 2^7 & 2^6 & & & 2^3 & 2^2 & & \\
 & & & 256 & 128 & 64 & & & 8 & 4 & & \\
 & & & 256 & + & 128 & + & 64 & + & 8 & + & 4 & = & 460
 \end{array}$$

$16 - 1$
 2^4 complement
 of Multiplicand

0	1	0	1	1	1
1	0	1	0	0	0
					1
<hr/>					
1	0	1	0	0	1

Integer Division

↳ 2 Method — 1) Restoring Division
a) Non restoring Division.

Restoring Division - Steps

- 1) $A \leftarrow 0$, $M \leftarrow \text{Divisor}$, $Q \leftarrow \text{Dividend}$
 $\text{count} \leftarrow n$
- 2) Shift A & Q Left one binary position
- 3) $A \leftarrow A - M$
- 4) If A is ≥ 0
 - ↳ Yes — Assign $Q_0 \leftarrow 0$
 $A \leftarrow A + M$ (Restore)
 - ↳ No — $Q_0 \leftarrow 1$
- 5) $\text{Count} \leftarrow \text{count} - 1$
- 6) Repeat 2, 3, 4, 5 n times

Example ①

Dividend = 1010 (Q) (10)

Count = 4/3/2

Divisor = 0011 (M) (3)

$$A \doteq n+1(\text{bits}) = 5$$

Initially

0 0 0 0 0

Shift left

0 0 0 0 1

$A \leftarrow A - M$

1 1 1 0 1

Add [2's Complement of M] add to A

1 1 1 1 0

Check sign of (1, then $q_0 = 0$)

restore $A \leftarrow A + M$

0 0 0 1 1

Restored A value

0 0 0 0 1

both same

Shift left

0 0 0 1 0

$A \leftarrow A - M$

1 1 1 0 1

1 1 1 1 1

Add $A \leftarrow A + M$

0 0 0 1 1

0 0 0 1 0

2 cycle

1 0 0 0

Q

1 0 1 0

0 1 0

Add one 0

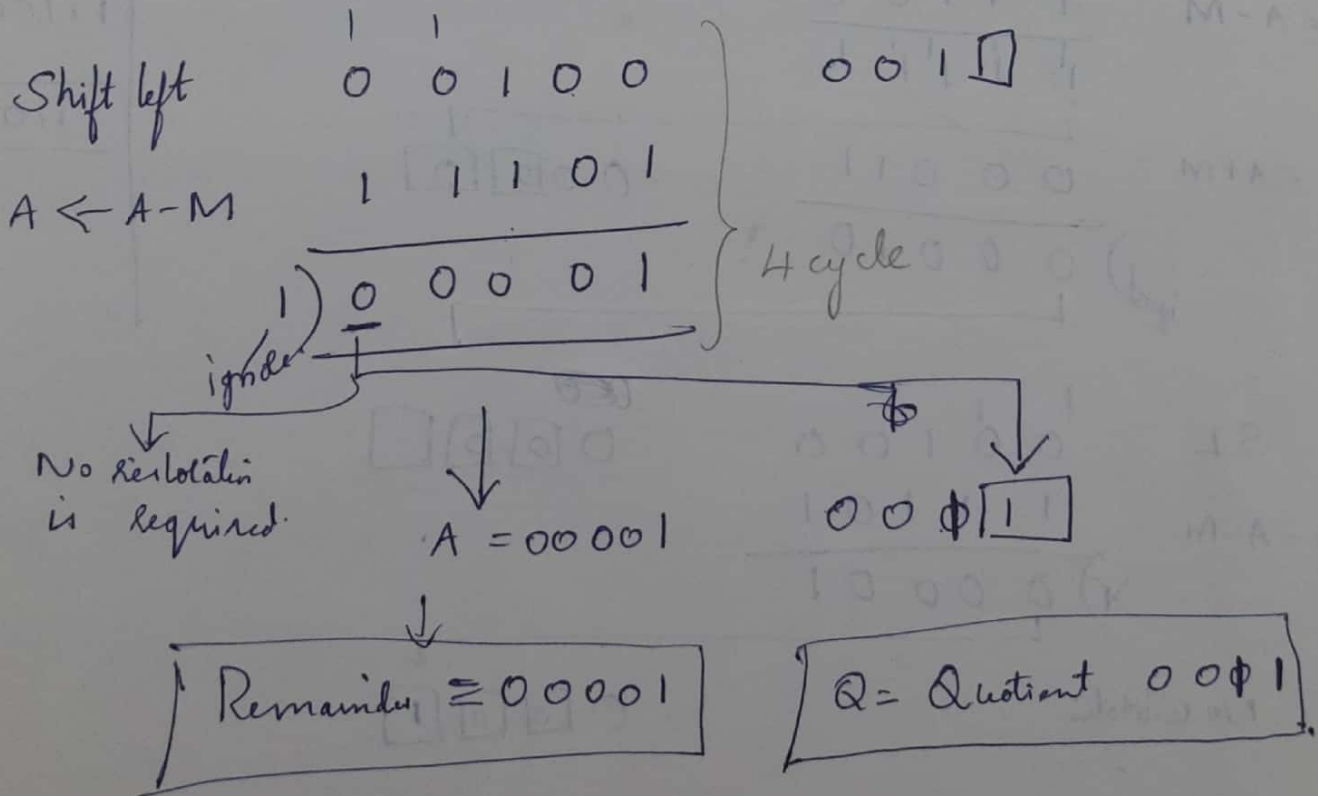
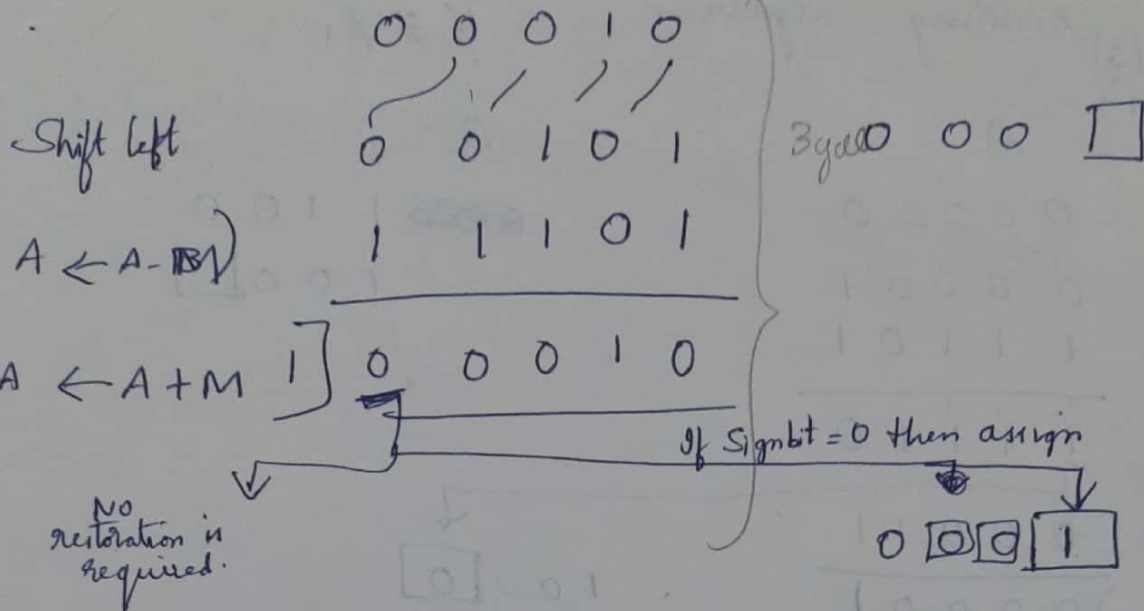
2's Complement

0 0 0 1 1

1 1 1 0 0

1

1 1 1 0 1



Example ② Perform $1100 \div 11$ using

restoring algorithm $4 \neq 1$

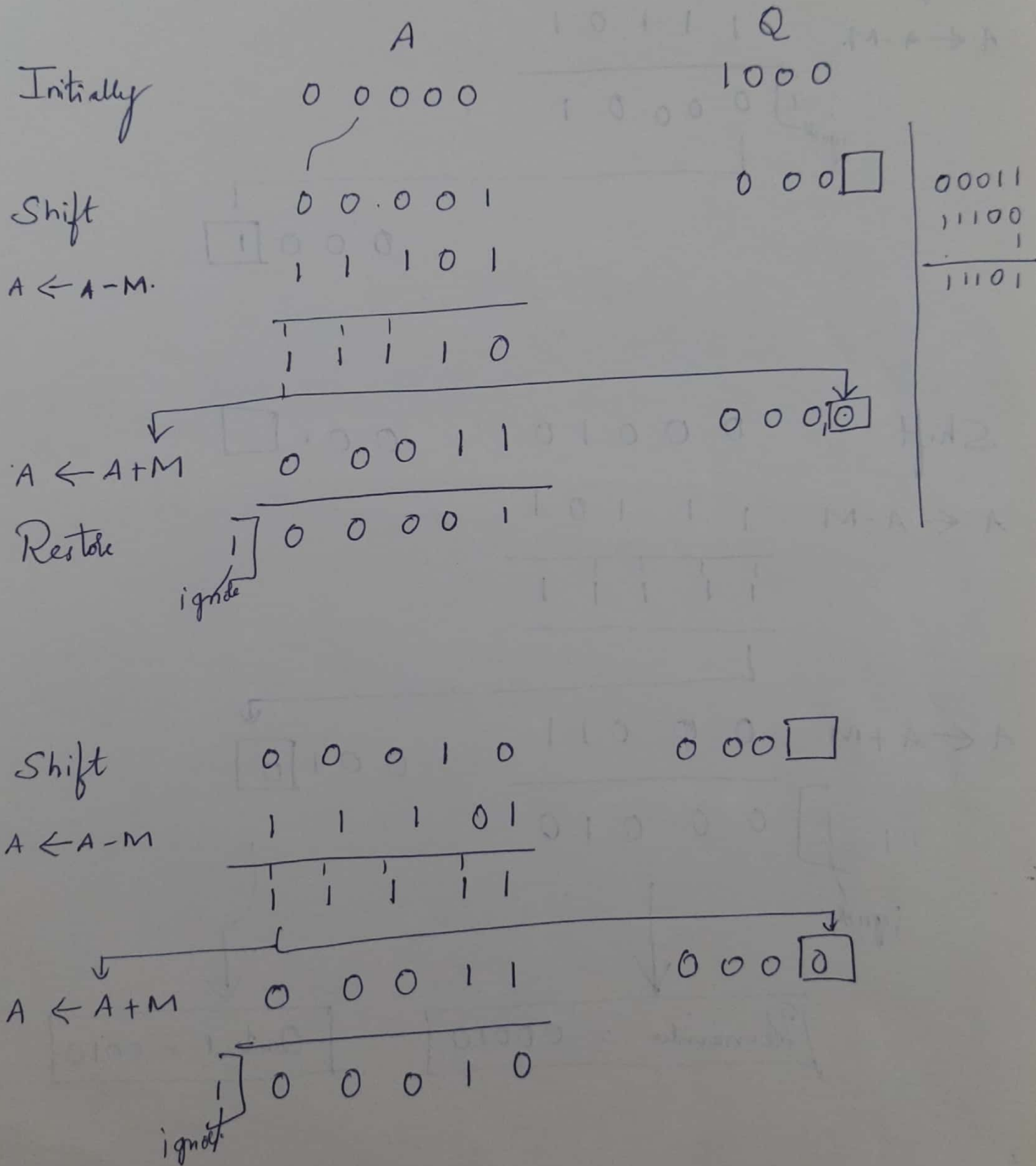
Dividend = 1100 (Q)

Divisor = 11 (M)

	A	Q	
	0 0 0 0 0	0 0 0 0 1 1 0 0	
SL	0 0 0 0 1	1 0 0 	
$A = A - M$	1 1 1 0 1		
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">1 1 1 0</div>		
$A = A + M$	0 0 0 1 1	1 0 0 	
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">0 0 0 0 1</div>	0 0 	
SL	0 0 0 1 1		
$A = A - M$	1 1 1 0 0		
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">1 1 1 1 1</div>	0 0 	
$A = A + M$	0 0 0 1 1	0 0 	
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">0 0 0 1 0</div>		
SL	0 0 1 0 0	0 	
$A = A - M$	1 1 1 0 1		
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">X) 0 0 0 0 1</div>	0 	
No restoration		0 	
SL	0 0 0 1 0	0 	
$A = A - M$	1 1 1 0 1		
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">1 1 1 1 1</div>	0 	
$A = A + M$	0 0 0 1 1	0 	
	<div style="border-top: 1px solid black; border-bottom: 1px solid black; padding: 2px 0;">X) 0 0 0 1 0</div>		
	Remainder = 00010	Quotient = 0010	

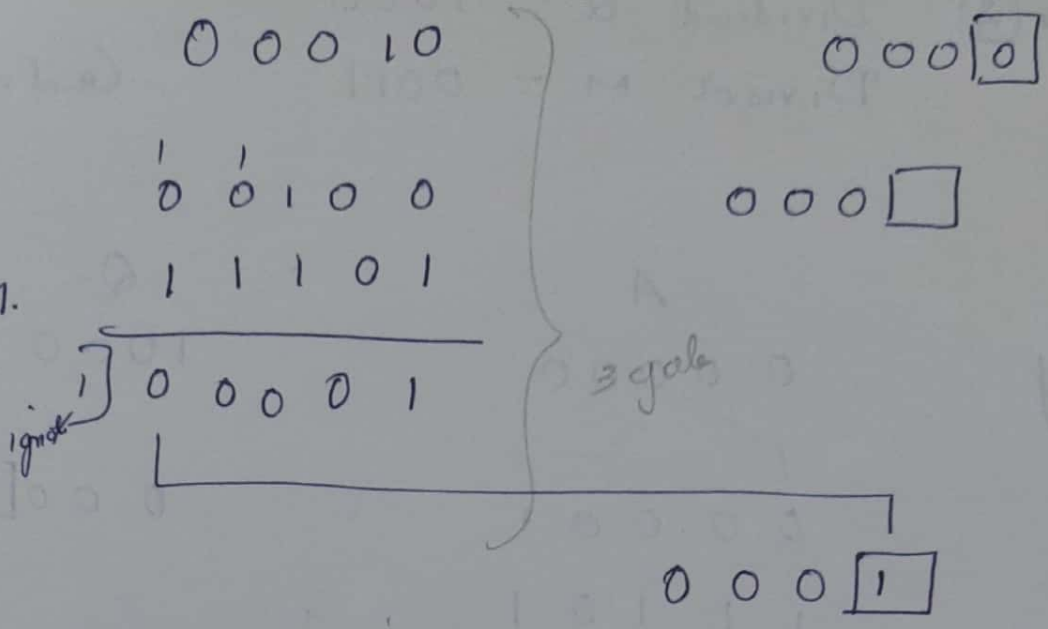
Example ③ Dividend $Q = 1000$
 Divisor $M = 0011$

Count = $4/3/2/1/0$



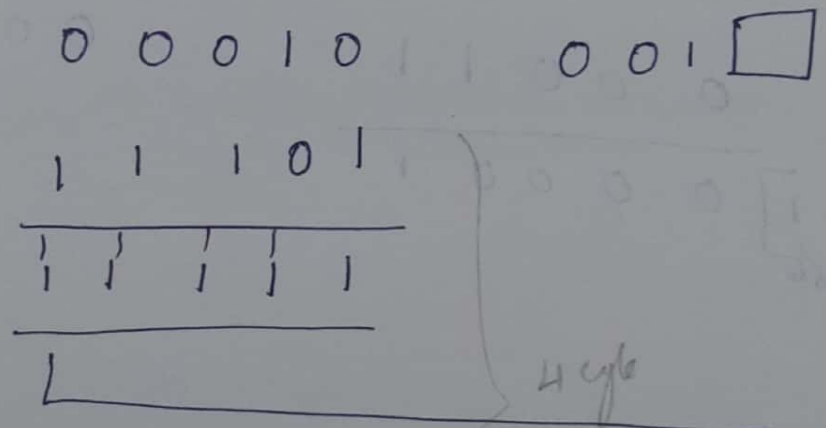
Shift

$A \leftarrow A - M.$

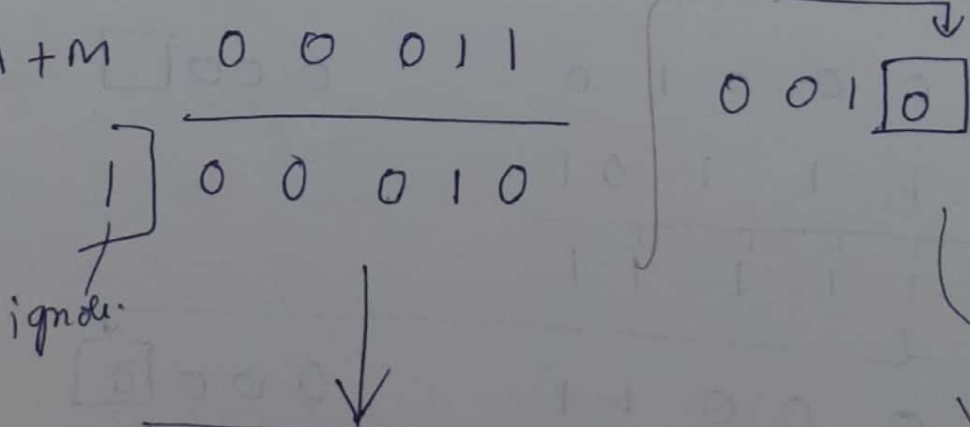


Shift

$A \leftarrow A - M$



$A \leftarrow A + M$



Remainder = 00010

Quotient = 0010

Example ①

Dividend = 1010 (Q)

Divisor = 0011 (M)

Count = 4

M = $\begin{array}{cccc} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 \end{array}$

Shift left
A ← A - M

A	Q
0 0 0 0 0	1 0 1 0
0 0 0 0 1	0 1 0
1 1 1 0 1	
1 1 1 1 0	0 1 0 0

1st cycle

SL
A ← A + M

A	Q
1 1 1 0 0	1 0
0 0 0 1 1	1 0 0 0
1 1 1 1 1	

2nd cycle

SL
A ← A + M

A	Q
1 1 1 1 1	0
0 0 0 1 1	0 1
0 0 0 1 0	

3rd cycle

SL
A ← A - M

A	Q
0 0 1 0 0	 1
1 1 1 0 1	 1 1
0 0 0 0 1	

4th cycle

Remainder = 0 0 0 0 1

Quotient = 0 0 1 1

Example ② = 1011(Q)
 Divisor = 0101 (M)

Count = 4

M = 00101
 11010
 11011

Step 1:

SL
 $A \leftarrow A - M$

A	0	0	0	0	0	Q	1	0	1	1
	0	0	0	0	1		0	1	1	□
	1	1	0	1	1		0	1	1	0
	<hr/>						0	1	1	0
	1	1	1	0	0					↑

Step 2:

SL
 $A \leftarrow A + M$

A	1	1	0	0	0	Q	1	1	0	□
	0	0	1	0	1		1	1	0	0
	1	1	1	0	1		1	1	0	0
	<hr/>									↑
	1	1	1	0	1					↑

Step 3:

SL
 $A \leftarrow A + M$

A	1	1	0	1	1	Q	1	0	0	□
	0	0	1	0	1		1	0	0	1
	0	0	0	0	0		1	0	0	1
	<hr/>									↑
	0	0	0	0	0					↑

Step 4:

SL
 $A \leftarrow A - M$

A	0	0	0	0	1	Q	0	0	1	□
	1	1	0	1	1		0	0	1	0
	1	1	1	0	0		0	0	1	0
	<hr/>									↑
	1	1	1	0	0					↑

1, then add divisor with A

Remainder = 00001

Quotient = 0010

00100(A)
 0101(divisor)
 00001

Example ③ - Using non restoring division algorithm, compute $23/5$

$M' 2^{15} \text{ comp}$
 $\begin{array}{r} 000101 \\ 111010 \\ \hline 111011 \end{array}$

Dividend $\rightarrow 23 \Rightarrow 10111(Q)$

Divisor $\rightarrow 5 \Rightarrow 00101(M)$

Count = 74/11

A
 000000
 SL 000001
 $A \leftarrow A - M$
 $\begin{array}{r} 110011 \\ \hline 111100 \end{array}$

Q
 10111
 01110

SL 111000
 $A = A + M$
 $\begin{array}{r} 000101 \\ \hline 111101 \end{array}$

11100

SL 111011
 $A = A + M$
 $\begin{array}{r} 000101 \\ \hline 000000 \end{array}$

11001

SL 000001
 $A \leftarrow A - M$
 $\begin{array}{r} 111011 \\ \hline 111100 \end{array}$

10010

SL 111001
 $A = A + M$
 $\begin{array}{r} 000101 \\ \hline 111100 \end{array}$

00100

Divisor 000101

$\times) 000011$

Quotient = 00100

Remainder = 000011