ECE 513: Computer Assignment 4

Image Restoration using 2-D Wiener Filtering

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1. Introduction

Image restoration is simply the process of converting a corrupt or noisy image and reconstructing a clear image without any distortions. The image can be corrupted through a number of different ways like camera mis-focus, motion blur, external noise just to name a few. Similar to the enhancement technique, the function of the restoration technique is to improve the image. But image restoration is different from image enhancement in that the latter is designed to emphasize features of the image that make the image more pleasing to the observer, but not necessarily to produce realistic data from a scientific point of view. Image restoration is performed by reversing the process that blurred the image and such is performed by imaging a point source and use the point source image, which is called the Point Spread Function (PSF) to restore the image information lost to the blurring process. In this assignment, the 'Lena' image is used for learning the properties of 2-DWiener Filter for motion blur removal, Noise removal and Image Restoration.

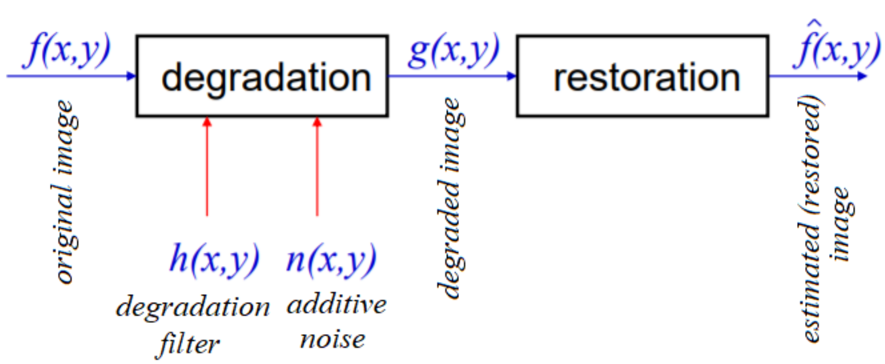

Fig 1: General Image Restoration Block Diagram

The above block diagram shows the working of a general image restoration process. Where f(x,y) is the original image, g(x,y) is the corrupted image, and f^(x,y) is the restored image.

2. Theory

Image Restoration

Images are often degraded during the data acquisition process. The degradation may involve blurring, information loss due to sampling, quantization effects, and various sources of noise. The purpose of image restoration is to estimate the original image from the degraded data. Applications range from medical imaging, astronomical imaging, to forensic science, etc. Often the benefits of improving image quality to the maximum possible extent far outweigh the cost and complexity of the restoration algorithms involved.

2-D Weiner Filter

The Wiener filter is a linear spatially invariant filter in which the point spread function h (n1, n2) is chosen such that it minimizes the mean-squared error (MSE) between the ideal and the restored image. This criterion attempts to make the restoration error between the ideal image and the restored image as small as possible on the average:

$$MSE = E[(f(n_1,n_2) - \hat{f}(n_1,n_2))^2]$$

In 2 where ˆf(n1, n2) is the restored image and f(n1, n2) is the actual image. The Wiener filter incorporates both the degradation function and statistical characteristics of noise into the restoration process. Wiener filter uses the 1st and 2nd. To get the idea of the 2-D wiener filter we start with a 1-D wiener filter. This restoration method assumes that noise which is present in the system is additive white Gaussian noise and it minimizes mean square error between original and restored images. Wiener filtering normally requires prior knowledge of the power spectra (spectral power densities) of the noise and the original image. Spectral power density is a function that describes power distribution over the different frequencies.

A simplified equation of the Wiener filter Rw is given below:

$$R_W(u,w) = \frac{H(u,w)^*}{|H(u,w)|^2 + \frac{S_n(u,w)}{S_f(u,w)}}$$

where $S_n(u,w)$ is the spectral power density of the noise and $S_f(u,w)$ is the spectral power density of the image. The inverse filter of a blurred image is a high pass filter. The parameter K of the Wiener filter is related to the low frequency aspect of the Wiener filter. The Wiener filter behaves as a band pass filter, where the high pass filter is an inverse filter and the low pass filter is specified by the parameter K. Note how Wiener filter becomes an inverse filter when K = 0.

$$\frac{S_n(u,w)}{S_f(u,w)} = const = K$$

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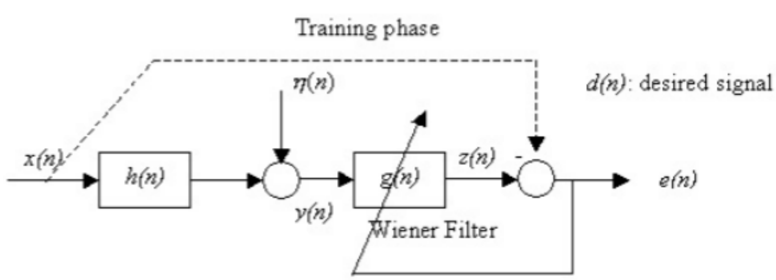

Fig 2: Wiener Filter

The general function for the wiener filter transfer function is:

$$G(e^{j\Omega}) = \frac{S_{dy}(e^{j\Omega})}{S_{yy}(e^{j\Omega})}$$

The $S_{dy}(e^{j\Omega})$ = DT FT $r_{dy}(K)$ is the Cross-power spectrum, $S_{dy}(e^{j\Omega})$ = DT FT $r_{dy}(K)$ is the power spectrum. The following is the Transfer function of a 1-D transfer function.

$$G(e^{j\Omega}) = \frac{H_*(e^{j\Omega})S_{xx}(e^{j\Omega})}{|H(e^{j\Omega})|^2 S_{xx}(e^{j\Omega}) + S_{\eta\eta}(e^{j\Omega})}$$

The following is the Transfer function of a 1-D transfer function.

$$G(k,l) = \frac{H_*(k,l)S_{xx}(k,l)}{|H(k,l)|^2 S_{xx}(k,l) + S_{\eta\eta}(k,l)}$$

where H(k, l) = 2-D DFT h(m, n), and $S_{xx}(k, l)$ and $S_{\eta\eta}(k, l)$ are power spectra of the original image x(m, n) and additive noise, η(m, n), respectively.

Using wiener filter is used in de-blurring, noise removal and reconstruction of the image in this assignment. The MATLAB function 'deconvwnr' is used for de-blurring and removing noise from the 'Lena' image. For this assignment we simulate a horizontal motion blur to the 'Lena' image by creating a filtering function. We add this motion blur using 'fspecial' function.

Noise Removal

Noise removal is done using the 2-d wiener filer with the MATLAB function 'wiener2' which is also used to estimate the variance of the output image. The function takes two inputs: The noisy Image and a Kernel block size of mxn. We then formulate an algorithm that using the power spectrum of the original image and the noisy image for finding the variance of the noisy image. The variance of the image is found by calculating the estimate of the local mean and variance around each pixel. This process is done before the filtering is done.

$$\mu = \frac{1}{NM} \sum_{n_1, n_2 \epsilon \eta} a(n_1, n_2)$$

$$\sigma^2 = \frac{1}{NM} \sum_{n_1, n_2 \epsilon \eta} a(n_1, n_2) - \mu^2$$

where η is the N-by-M local neighborhood of each pixel in the image A. Wiener2 then creates a pixelwise Wiener filer using these estimates.

$$b(n_1, n_2) = \nu + \frac{\sigma^2 - v^2}{\sigma^2}(a(n_1, n_2) - \mu)$$

where v^2 is the noise variance estimated by the filter if known.

De-Blurring

Deblurring is the process of removing blurring artifacts from images. Mathematically we represent

$$B = S*K$$

where B is blurred input image, we need to find out both sharp image S and K which is blur kernel and * is called convolution. We say that S is convolved with K to generate blurred image B, where K is the blur caused by defocus aberration, motion blur, gaussian blur or any kind of blur. So, our goal is now to recover S which is Sharp image and also K and the process is known as Deblurring.

The blurring, or degradation, of an image can be caused by many factors:

- Movement during the image capture process, by the camera or, when long exposure times are used, by the subject

- Out-of-focus optics, use of a wide-angle lens, atmospheric turbulence, or a short exposure time, which reduces the number of photons captured

- Scattered light distortion in confocal microscopy

A blurred or degraded image can be approximately described by this equation

$$g = Hf + n.$$

where g is the blurred image. H is the distortion operator, also called the point spread function (PSF). In the spatial domain, the PSF describes the degree to which an optical system blurs (spreads) a point of light. The PSF is the inverse Fourier transform of the optical transfer function (OTF). In the frequency domain, the OTF describes the response of a linear, position-invariant system to an impulse. The OTF is the Fourier transform of the point spread function (PSF). The distortion operator, when convolved with the image, creates the distortion. Distortion caused by a point spread function is just one type of distortion. F is the original true image and finally n is the additive noise, introduced during image acquisition, that corrupts the image.

De-blurring function used

Deconvwnr: Implements a least squares solution. You should provide some information about the noise to reduce possible noise amplification during deblurring. See Deblur Images Using a Wiener Filter for more information.

Signal to Noise Ratio

Since the reconstructed image has some Noise in it the SNR for the original image and Reconstructed image is calculated using the following formula.

$$SNR = 10log_{10}\frac{\sigma_o^2}{\sigma_e^2}$$

The σ^2_o is the variance of the original image and σ^2_e is the variance of the error image. Using this SNR value, we can calculate the loss in the compression of the image as well as the quality of the image.

3. Result

In this Assignment we use the 'Lena' image. Using the MATLAB function 'fspecial' we create a correlation kernel, which is the appropriate form to use with 'imfilter' using which, we create a motion blur on the image.

Fig 3: Original Lena Image

For the above image horizontal blur is applied.

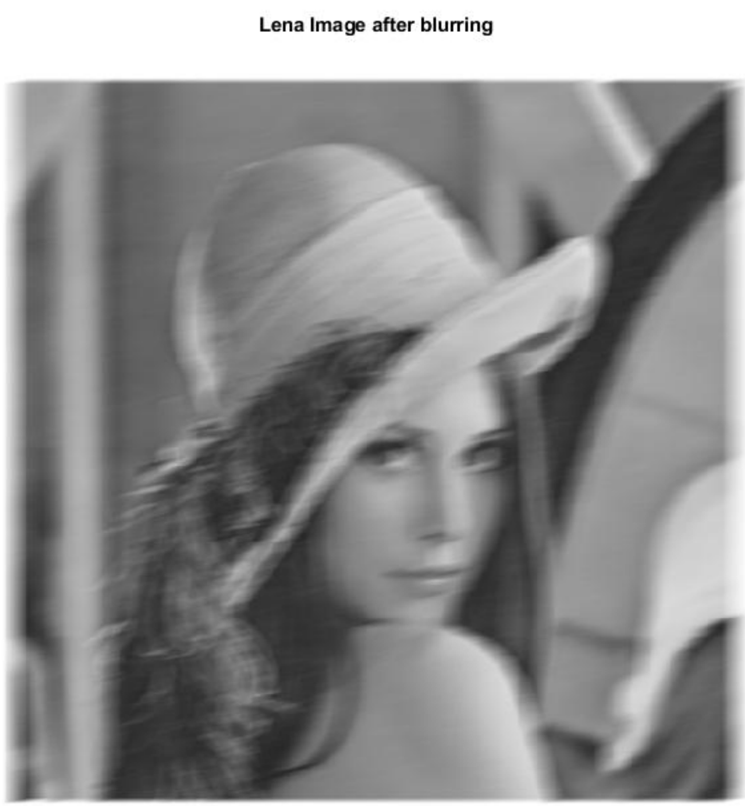

Fig 4: Lena image after blurring

The “fspecial” function in MATLAB uses motion blur with linear motion 21 and angular motion 11. The “imfilter” function in MATLAB uses a convolution and circular Padding which implicitly assume that the input array is periodic.

We will be performing de-blurring options on this blurred image. We will be implementing the “deconvwnr” function in MATLAB to view its results.

De-Blurring

The MATLAB function ’deconvwnr’ is used for de-blurring the blurred image using the appropriate PSF used for blurring.

The blurred Lena image is provided as input and the ’deconvwnr’ function is used to de-blur. The NSR (Noise to signal ratio) is used. Noise-to-signal ratio, specified as a nonnegative scalar or numeric array of the same size as the image, I. If nsr is an array, then it represents the spectral domain. Specifying 0 for the nsr is equivalent to creating an ideal inverse filter.

Fig 5: The input blurred image

The image is then set under de-blurring using the above-mentioned filters and NSR noise var value. This is an estimated value and this value help alter the blur of any image. The NSR is similar to SNR, this is used for image reconstruction quality measurement.

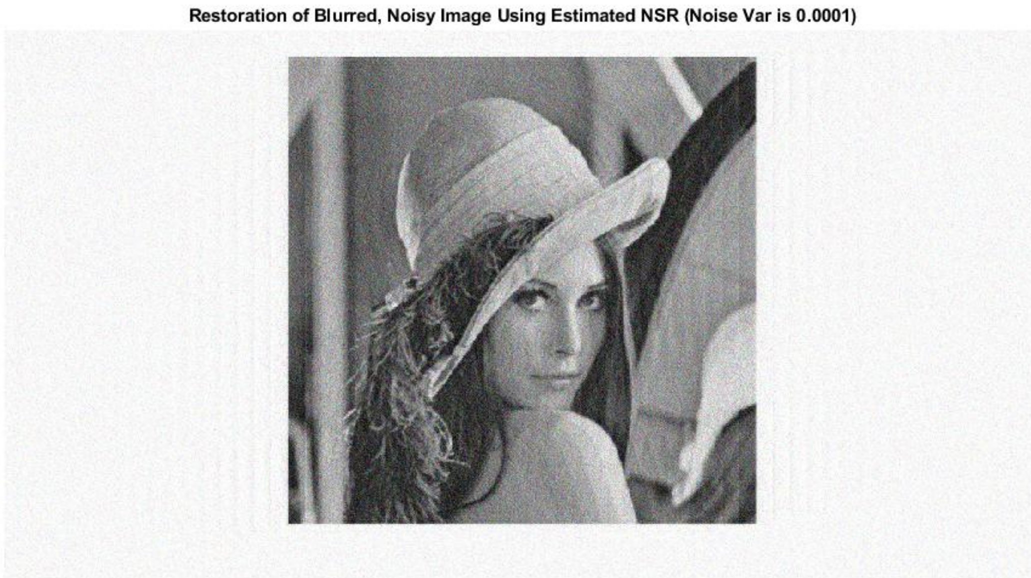

Fig 6: De-Blurred Image

It can be seen that the Wiener filter gives good reconstructed image for a motion blurred image. The filter size used is small, with linear camera motion of 21 and angular motion of 11. As we increase the filter size, it is observed that the reconstructed image still has some blurring. This is due to the assumption of Wide-sense stationary, that is, it has constant (and finite) mean and variance, and a correlation function which is a function of time shift only.

Noise Removal

The noise removal filters are tested by first adding a 5dB of gaussian white noise to the Lena image. This image is then used with the 2D wiener filter. The following is the image with 5db noise.

Fig 7: Lena Image with 5dB Gaussian Noise

The noisy image is the given as an input to the wiener 2 filter with a noise-to-signal ratio as zero. The following are the results of the Wiener filter.

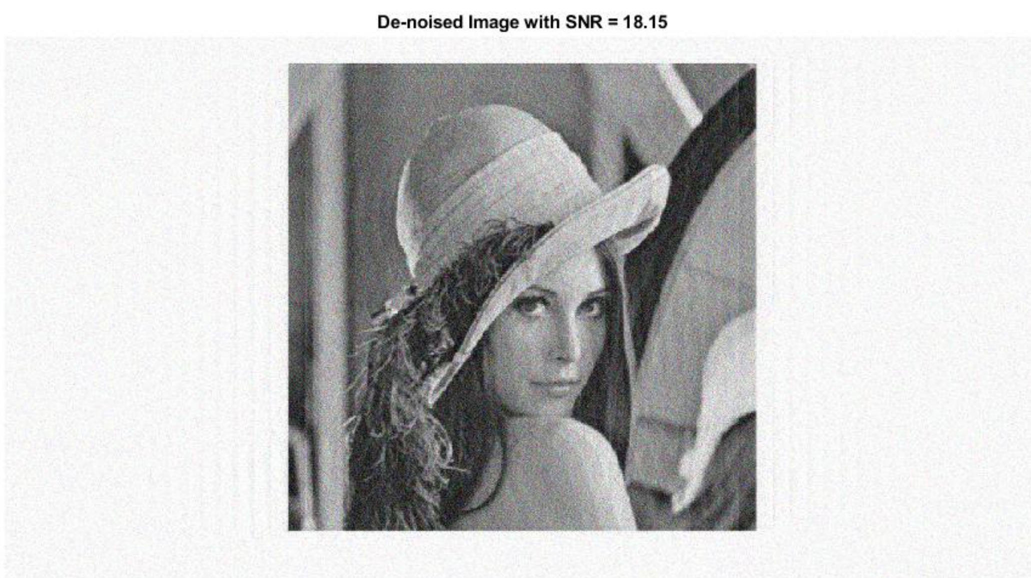

Fig 8: De-noised Image using Wiener Filter

To compare the Noise removal efficiency of the wiener2 filter we also use a spatial filter and the results are as follows:

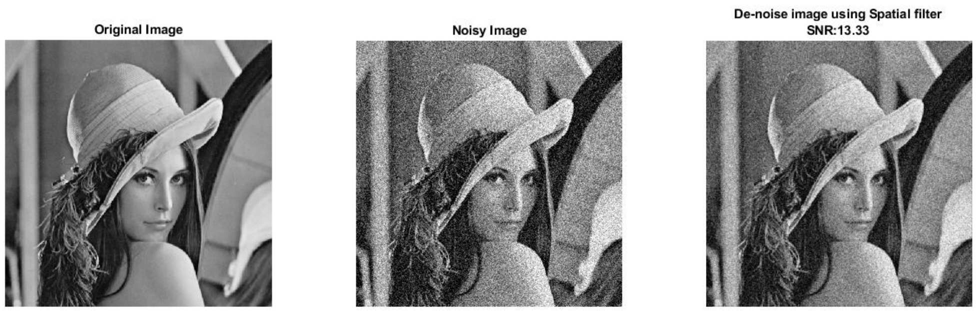

Fig 9: De-noised Image using Spatial Filter

We can see that the wiener filter does not show any visible noise elements due to the edge smearing, patchiness. It can also be seen that the snr of wiener filter 18.15 dB is higher than the snr of the spatial filter 13.33 dB so the wiener filter performs better than the spatial filter. We also use the algorithm that find the variance of the noisy image which can be used in the reconstruction of the image.

Image Restoration

For this part we use the horizontal motion blurred image and add a 5dB noise to it. The noise-to-signal ratio is calculated using the algorithm. The following is the output of the Reconstruction.

Fig 10: Noisy image for Reconstruction

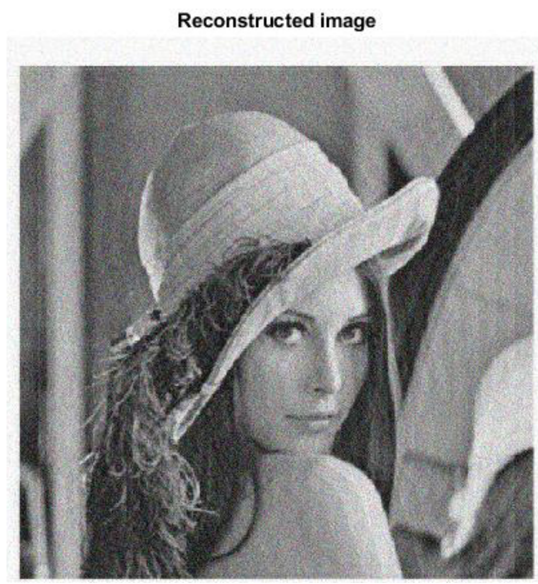

Fig 11: Reconstructed Image

We can see that the reconstructed image has some noise in it along with the edge smearing artifact and blur. This is due to the factor that Wiener filter assume a widespread stationarity of the image field. The quality of the reconstructed image degrades as the motion blur increases.

4. Data Tabulation

The reconstruction of the image using the Wiener filter gives better solution than the other enhancement techniques. The following table show the Snr value of the reconstructed image for different linear (LIN) and angular (THETA) motion added to the image keeping the White Gaussian noise constant.

LIN	THETA	SNR
21	11	**14.5547**
31	21	**13.9856**
41	31	**12.9789**

We can see that the snr decreases as the Linear and Angular motion increases, from this we can see that the Wiener filter better but the overall the smearing artifacts persists which is due to the wide-sense stationarity. The results of noise removal of Wiener filter were also better than the spatial filter.

5. Conclusion

In this assignment we studied the performance of Wiener filter for de-blurring, noise removal and total reconstruction. The Wiener filter was also compared with spatial filter from which we infer that Wiener filter is better than the spatial filter. The frequency domain 2-D Wiener filter was able to achieve a higher image quality (higher SNR) when compared to other filters. The Weiner filter was found effective when tested for image de-blurring, image noise removal, and Image restoration.

6. References

[1] Professor Azimi Lecture slides

[2] Rani, Shilpa Jindal, Sonika Kaur, Bhavneet. (2016). A Brief Review on Image Restoration Techniques. International Journal of Computer Applications. 150. 30-33. 10.5120/ijca2016911623.

[3] MATLAB Documentation/deconvwnr

[4] MATLAB Documentation/wiener2.

[5] https://en.wikipedia.org/wiki/Signal-to-noiseratio