Network QoS
371-2-0213

Lecture 10

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Outline

Classical Queueing Theory

- Single M/M/1 queue
  - arrival: Exponential with parameter λ
  - service: Exponential with parameter μ
- Concerns
  - utilization
    - fraction of time server is busy $\rho = \frac{\lambda}{\mu}$
  - stability
    - mean number of customers in system $\bar{N} = \frac{\rho}{1-\rho}$
    - $\lim_{\rho \to 1} \bar{N} = \infty$
    - tail probability $\Pr[N > k] = \rho^{k+1}$
  - delay/response time
    - Little's Law: mean delay $\bar{T} = \lambda^{-1}\bar{N}$
- Lessons learnt
  - overload $\rho \geq 1$ is not manageable
  - stability implies $\rho < 1$
  - stability related to delay

Queuing Networks

- Network of queues
  - open
    - customers arrive from outside
    - potentially to any queue in the network
    - potentially infinite customers population
    - concerns: stability, delay
  - closed
    - customers circulate in the network
    - finitely many customers
    - concerns: delay
  - multi-class
    - probabilistic transitions from queue to queue
    - different customers may follow different paths

System and Traffic Model

- Slotted time
- Directed graph $G = (V, E)$
  - each link $e \in E$ has rate 1
    - stochastic terms: "$\mu = 1$"
  - single queue Q_e at link tail
  - apriori, unlimited buffer space
  - employs scheduling policy $\mathcal{P}$
  - d: max path-length in G
  - m: number of edges
- Unit size packets
  - $p = (\pi_p, a_p)$
    - π_p: simple path in G (source routing)
    - a_p: arrival time of p at path source end
  - take a single time slot to traverse a link
- Adversary
  - Injects packets into the various nodes in the network
    - stochastic terms: similar to multi-class open queuing networks

Adversaries and Stability

- Notation:
  - $N(e, I)$: # packets injected during interval I with $e \in$ path
  - $Q_e(t)$: queue size at tail of e at time t
- Stability
  - $\sup_t \sum_{e \in E} Q_e(t) < \infty$
- For $M/M/1$, stability implies $\rho < 1$
- What about adversarial traffic?
  - average injection rate of packets traversing $e > 1$
    - system unstable
  - what about injection rate $= 1$? < 1?
- (r, b)-adversary
  - for any time interval I and edge $e \in E$

$$N(e, I) \leq r\,|I| + b$$

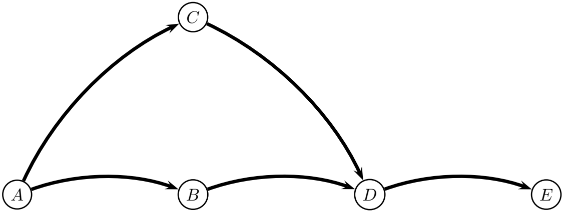
    - note: injections made throughout the network!

Elements of the Game

- Topology, traffic (adversary), protocol
- Recall concerns: delay and stability
- Questions we'd like to address
  - how each element affects these concerns
  - relation between stability and delay
    - "equivalent" to Little's Law?
  - identify stable triplets (topology,traffic,protocol)
  - are there protocols that are always stable
    - for any topology and traffic
    - universally stable
  - are there topologies that are always stable
    - for any work-conserving protocol and traffic
    - universally stable
  - are there protocols that are never stable
    - for any traffic (and appropriate topology)
    - for any topology (and appropriate traffic)

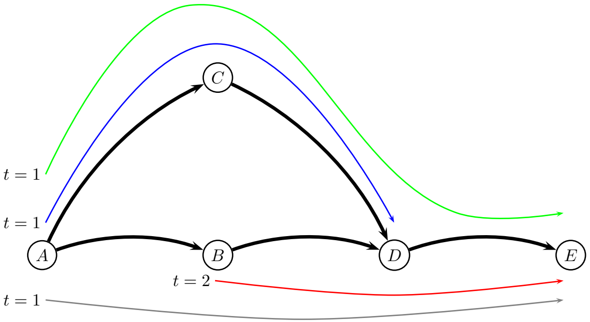
Protocols

- Work-conserving (WC)



Protocols

- Work-conserving (WC)



FIFO	Nearest-to-Go (NTG)	Farthest-to-Go (FTG)	Shortest-in-System (SIS)
LIFO	Farthest-from-Source (FFS)	Nearest-to-Source (NTS)	Longest-in-System (LIS)

Stability and Underload vs. Bounded Delay ($r < 1$)

Theorem

For any $(1-\varepsilon, b)$-adversary and any WC protocol, the system is stable iff it has bounded delay.

Proof.

- Stability: no queue ever holds more than k packets
    - in particular, $b \leq mk$
- Consider $Q_e(t) \neq \emptyset$ and show it empties during $I = [t, t+\frac{2mk}{\varepsilon}]$
    - by contradiction (and WC), $\frac{2mk}{\varepsilon} + 1$ packets traverse e during I
    - however,
        - at t there are $\leq mk$ packets in the system
        - arrivals during I at most $|I|(1-\varepsilon) + b = (\frac{2mk}{\varepsilon} + 1) - (2mk + \varepsilon) + b < (\frac{2mk}{\varepsilon} + 1) - mk + (b - mk) \leq (\frac{2mk}{\varepsilon} + 1) - mk$
        - strictly less than $\frac{2mk}{\varepsilon} + 1$ could have traversed e during I
- Every packet is absorbed in $\frac{2mk}{\varepsilon} \cdot d$ time

Stability and Underload vs. Bounded Delay ($r < 1$)

Theorem

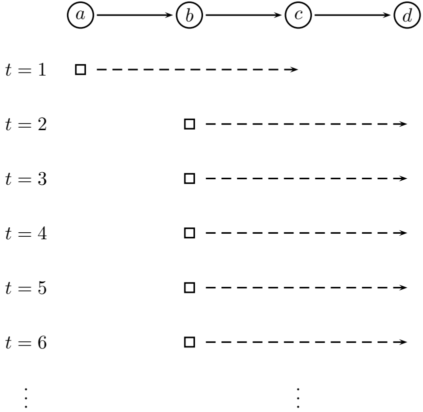
For any $(1-\varepsilon, b)$-adversary and any WC protocol, the system is stable iff it has bounded delay.

Proof.

- Assume every packet is absorbed in T time
- If $|Q_e(t)| > T$, the last packet would have delay $> T$
  - contradiction....
- Hence, network load $\leq mT$ □

What If $r = 1$?

- WC + Bounded delay implies stability?
  - proof unaffected
- WC + Stability implies bounded delay?



Traffic Characterization

Theorem

Given $G = (V, E)$, if traffic σ can be made stable by protocol $\mathcal{P}$ then σ is generated by an $(r \leq 1, b \geq 0)$-adversary

Proof.

- Stability: $\exists B$ s.t. network load under $\mathcal{P}$ is always $\leq B$
- Show that for every $e \in E$ and interval I

$$N(e, I) \leq |I| + B$$

- By contradiction, $N(e, I) > |I| + B$
  - during I at most $|I|$ packets traverse e
  - by end of I we have $> B$ packets pending for e. Contradiction.
- By definition, σ is generated by a $(1, B)$-adversary.

$\square$

$\mathcal{P}$ Is Stable iff All WC Protocols Are?

- Why should it matter *which* packets we forward?
  - any WC protocol forwards the same amount on any link
  - stability is determined by quantity, not "quality"

It matters!!

- Why?
  - local scheduling decisions affect load downstream
- Is there a good way to tell if a protocol is stable?
- Is it intuitive?

Protocol	Universally Stable?
FIFO	
LIFO	
Nearest-to-Go (NTG)	
Farthest-from-Source (FFS)	
Farthest-to-Go (FTG)	
Nearest-to-Source (NTS)	
Shortest-in-System (SIS)	
Longest-in-System (LIS)	

Farthest-To-Go (FTG)

Theorem (FTG stability)

For any $G = (V, E)$ and any $(r \leq 1, b)$-adversary, FTG is stable

- Notation
  - Recall: d = max path-length in G, $m = |E|$
  - $X_e^i(t)$: number of packets in $Q_e(t)$ with target-distance $\geq i$
  - sequence k_i:

$$k_i = \begin{cases} 0 & i > d \\ m \cdot k_{i+1} + b & 0 \leq i \leq d \end{cases}$$

    - non-increasing: $k_i \geq k_{i+1}$
    - $k_d = b$,
      $k_{d-1} = (m+1)b$,
      $k_{d-2} = (m(m+1)+1)b, \ldots$
- Plan
  - Show $X_e^i(t) \leq k_i$
  - network load $\leq \sum_e X_e^1(t) \leq m \cdot k_1 = O(m^d b)$

Farthest-To-Go is Universally Stable

Lemma

For any $e \in E$, time t and $0 \leq i \leq d$, $X_e^i(t) \leq k_i$

Proof (by reverse induction on i).

- Assume for simplicity system is empty at time 0
- $i > d$: trivial (no such packets in $Q_e(t)$)
- Assume true for $j > i$ and consider i (and some $e \in E$ and t)
- $t' = \arg\max \left\{ t'' \leq t \mid X_e^i(t'') = 0 \right\}$
  - X_e^i is "reset" at t'
  - exists since system is empty at time 0
- Consider packets "in" $X_e^i(t)$
  - some injected after t'
    - at most $(t - t') + b$ injected during $[t' + 1, t]$

Farthest-To-Go is Universally Stable

Lemma

For any $e \in E$, time t and $0 \leq i \leq d$, $X_e^i(t) \leq k_i$

Proof (by reverse induction on i).

- some in system at t'
  - none in $Q_e(t')$
  - all at target-distance $\geq i+1$
  - at most $m \cdot k_{i+1}$ (by I.H.)
- e delivers target-distance $\geq i$ packets throughout $[t'+1, t]$
  - $X_e^i > 0$ throughout by choice of t' + WC + FTG criteria
- Hence

$$
\begin{aligned}
X_e^i(t) &= X_e^i(t') + \text{type 1} + \text{type 2} - \text{departures} \\
&\leq 0 + \overbrace{(t-t') + b}^{} + \overbrace{m \cdot k_{i+1}}^{} - \overbrace{(t-t')}^{} \\
&= m \cdot k_{i+1} + b = k_i
\end{aligned}
$$

□

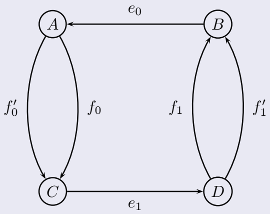
What about FIFO?

Theorem

FIFO is unstable against (r, b)-adversaries, for $r > 0.85$

Proof.

- Consider the "baseball" graph



- Assume s packets with path e_0 in B at t
  - all other queues empty
- Adversary forces $s' > s$ packets with path e_1 in C at $t' > t$
  - all other queues empty

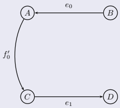
What about FIFO?

Theorem

FIFO is unstable against (r, b)-adversaries, for $r > 0.85$

Proof.

- Phase 1 (s time):
  - rs packets with path $e_0 f_0' e_1$ (set W)
  - end of phase 1: W in B
    - recall we start with s packets with path e_0 in B



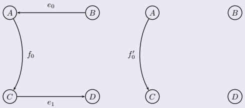
What about FIFO?

Theorem

FIFO is unstable against (r, b)-adversaries, for $r > 0.85$

Proof.

- Phase 2 (rs time):
  - $r^2 s$ packets with path $e_0 f_0 e_1$ (set X)
  - $r^2 s$ packets with path f_0' (set Y)
  - end of phase 2: X in B
    - since W are forwarded before
  - W arrive at rate 1 to A
    - serviced at rate $\frac{1}{r+1}$ by f_0'
    - $rs - rs\frac{1}{r+1} = \frac{r^2 s}{r+1}$ packets of W remain in A
  - Y arrive at rate r to A
    - serviced at rate $\frac{r}{r+1}$ by f_0'
    - $r^2 s - r^2 s\frac{r}{r+1} = \frac{r^3 s}{r+1}$ packets of Y remain in A



$\boxed{r^2 s \text{ waiting at } f_0' \text{ tail}}$

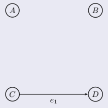
What about FIFO?

Theorem

FIFO is unstable against (r, b)-adversaries, for $r > 0.85$

Proof.

- Phase 3 ($r^2 s$ time):
  - $r^3 s$ packets with path e_1 (set Z)
  - remaining W arrive at rate $\frac{1}{r+1}$ to C
    - slowed down by remaining Y on f_0'
    - all $\frac{r^2 s}{r+1}$ remaining packets arrive
  - X arrive at rate 1 to C
    - flow freely through f_0
    - all $r^2 s$ packets arrive
  - Total arrivals to C (W, X, Z): $\frac{r^2 s}{r+1} + r^2 s + r^3 s$
  - packets served on e_1: $r^2 s$ (FIFO is WC)



What about FIFO?

Theorem

FIFO is unstable against (r, b)-adversaries, for $r > 0.85$

Proof.

- Pending packets for e_1 after phase 3

$$\phi(r,s) = \frac{r^2 s}{r+1} + r^3 s$$

- Requirement:

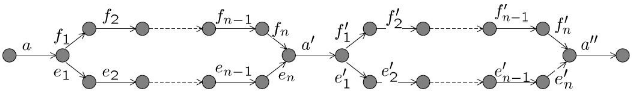
$$\phi(r,s) > s$$

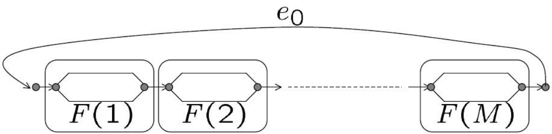
  - $\phi(1,s) = \frac{3}{2}s$
  - $\phi(0,s) = 0$
- Satisfied for $r > 0.85$
- Initialization: burst b

□

FIFO Instability (only for large enough r?)

Rate	Reference
> 0.85	Andrews et al. (2001)
> 0.8357	Díaz et al. (2001)
> 0.749	Koukopoulos et al. (2002)
> 0.5	Lotker et al. (2002)
any $\varepsilon > 0$	Bhattacharjee et al. (2003)





FIFO Instability (only for large enough r?)

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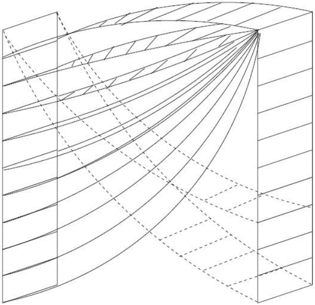

Universal Stability of WC Protocols

Protocol	Universally Stable?
FIFO	No
LIFO	
Nearest-to-Go (NTG)	
Farthest-from-Source (FFS)	
Farthest-to-Go (FTG)	Yes
Nearest-to-Source (NTS)	
Shortest-in-System (SIS)	
Longest-in-System (LIS)	

Towards Universally Stable Topologies

(or, Is FIFO Never Stable?)

- Next week
  - Alternative adversarial models
  - Easy traffic
    - are there rates for which any WC protocol is stable?
    - depends on topology?
  - Universally stable topologies
    - are there topologies for which any WC protocol is stable?
    - at any rate ≤ 1?

- References

  - Borodin, Kleinberg, Raghavan, Sudan and Williamson. Adversarial Queueing Theory, Journal of the ACM 48(1):13–38 (2001)
  - Andrews, Awerbuch, Fernández, Leighton, Liu, Kleinberg. Universal-Stability Results and Performance Bounds for Greedy Contention-Resolution Protocols, Journal of the ACM 48(1):39–69 (2001)
  - Cholvi and Echagüe. Stability of FIFO networks under adversarial models: State of the art, Computer Networks 51(15):4460–4474 (2007)