

Network QoS

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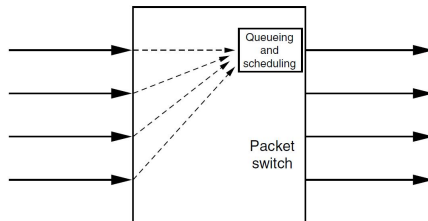
Lecture 5

Gabriel Scalosub

Outline

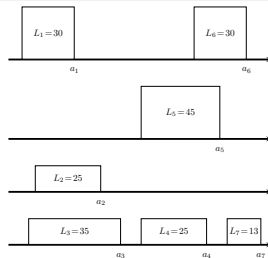
- 1 Network Calculus – Basics
 - Architecture and Notation
 - Reich's Equation
- 2 Network Calculus – Defining a New Calculus
 - Min-Sum Convolution
 - Service Curves
 - Envelopes and Regulators
- 3 Network Performance and Design
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 - Summary

Switch Architecture



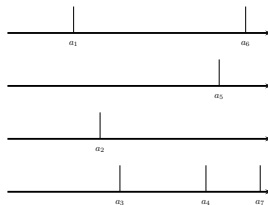
- input queues vs. *output queues*
- output queue + scheduling policy = *packet multiplexer*

Switch Architecture



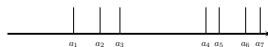
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 - output queue + scheduling policy = *packet multiplexer*
 - inter-arrival times
 - packet length L
 - arrival starts at time t
 - link of capacity C
- \Rightarrow arrival ends at time $t + \frac{L}{C}$

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- store-and-forward
 - packet *arrives* at time $t + \frac{L}{C}$

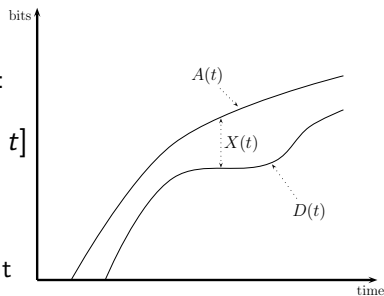
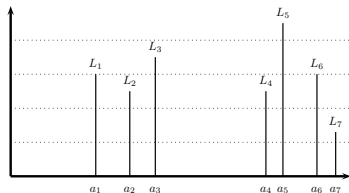
Switch Architecture



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- output queue + scheduling policy = *packet multiplexer*
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 - packet *arrives* at time $t + \frac{L}{C}$

Basic Processes

- traffic:
 - a sequence of packets p_1, p_2, \dots
 - arriving at an output queue
 - output link capacity C
- for each packet p_k
 - a_k : arrival time to queue
 - L_k : packet length
 - d_k : departure time from queue
- $A(t)$: cumulative arrivals in $[0, t]$
 - $A(t) = 0$ for $t < 0$
 - monotone, and right-continuous:
 - $\lim_{s \searrow t} A(s) = A(t)$
- $D(t)$: cumulative departures in $[0, t]$
 - $D(t) = 0$ for $t < 0$
 - $D(t) \leq A(t)$ for all t
- $X(t)$: queue length at t
 - includes partial packet being sent
 - $X(t) = A(t) - D(t)$ for all t

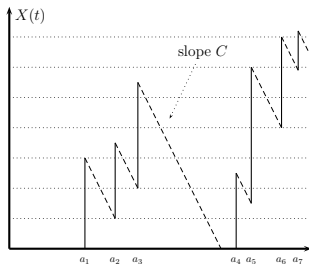
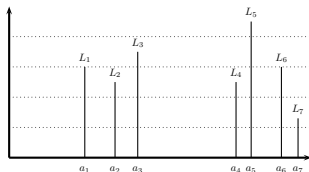


Work Conserving Schedulers

- *work conserving scheduler*
 - never idle when queue non-empty
 - buffer: empty or depletes at rate C
 - idle/busy periods
- $X(t)$ independent of scheduling policy
- busy period: same packets depart
 - *order* depends on scheduling policy
- $\bar{X}(t_1, t_2)$: time average queue size

$$\bar{X}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} X(u) du$$

- also independent of scheduler
 - $\bar{X}(t_1, t_2) \sim$ average delay
- $X(t) = \sum_{\text{sessions } s} X_s(t)$
- *conservation law*
 - $\bar{X}(t_1, t_2) = \sum_s \bar{X}_s(t_1, t_2)$
 - one goes down, another must go up
 - can be controlled by the scheduler



Reich's Equation

- Notation

- t_- : the moment just before t
- given a function f

$$f(t_-) = \lim_{s \nearrow t} f(s)$$

- Assume:

- work-conserving scheduler
- outgoing link capacity C

Reich's Equation

$$X(t) = \sup_{0 \leq s \leq t} (A(t) - A(s) - C(t - s))$$

- $A(t) - A(s)$: *actual* difference of arrivals
 - arrivals during $(s, t]$
- $C(t - s)$: *potential* service over interval of length $t - s$

Reich's Equation – Proof

Reich's Equation

$$X(t) = \sup_{0 \leq s \leq t} (A(t) - A(s) - C(t - s))$$

Proof of Reich's Equation.

- First note that for all $s \leq t$

$$X(t) \geq A(t) - A(s) - C(t - s)$$

- potentially, $X(s) \geq 0$
- service during $[s, t]$ upper bounded by $C(t - s)$

$$\Rightarrow X(t) \geq \sup_{s \leq t} (A(t) - A(s) - C(t - s))$$

- $A(s) = 0$ for $s < 0$

$$\Rightarrow X(t) \geq \sup_{0 \leq s \leq t} (A(t) - A(s) - C(t - s))$$

Reich's Equation – Proof

Reich's Equation

$$X(t) = \sup_{0_- \leq s \leq t} (A(t) - A(s) - C(t - s))$$

Proof of Reich's Equation.

- Define: $v = \sup \{0_- \leq s \leq t \mid X(s) = 0\}$

- beginning of “t's busy period”
- $X(v_-) = 0$
- queue non-empty over $[v, t]$
- depletes at rate C during $[v, t]$

$$\Rightarrow X(t) = A(t) - A(v_-) - C(t - v)$$

- $A(t) - A(v_-) - C(t - v) \leq \sup_{0_- \leq s \leq t} (A(t) - A(s) - C(t - s))$

$$\Rightarrow X(t) \leq \sup_{0_- \leq s \leq t} (A(t) - A(s) - C(t - s))$$

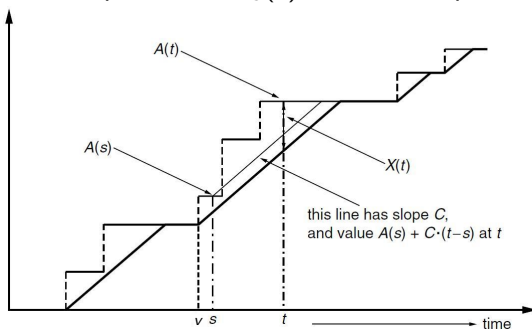


Reich's Equation - Geometric Interpretation

- rewriting Reich's equation

$$X(t) = \sup_{0 \leq s \leq t} (A(t) - \underbrace{(A(s) + C(t-s))}_{f_s(t)})$$

- our “goal”: to push down $f_s(t)$ as much as possible



Reich's Equation - Departure Process

Corollary

$$D(t) = \inf_{0 \leq s \leq t} (A(s) + C(t - s))$$

Proof.

$$\begin{aligned} D(t) &= A(t) - X(t) \\ &= A(t) - \sup_{0 \leq s \leq t} (A(t) - A(s) - C(t - s)) \\ &= \inf_{0 \leq s \leq t} (A(s) + C(t - s)) \end{aligned}$$



- FIFO scheduler
 - departures can be inferred from $D(t)$
 - $d_k = \inf \{u \mid D(u) \geq A(a_k)\}$

Massaging the Departure Process

- recall: $D(t) = \inf_{0 \leq s \leq t} (A(s) + C(t - s))$
- equivalently, $D(t) = \inf_{0 \leq s \leq t} (A(s) + C(t - s)^+)$
- since $A(s) = 0$ for $s < 0$ and $A(s)$ is non-decreasing

$$D(t) = \inf_{s \in \mathbb{R}} (A(s) + C(t - s)^+)$$

- Denote: $B(x) = Cx^+$
- We therefore have $D(t) = \inf_{s \in \mathbb{R}} (A(s) + B(t - s))$

Min-Sum Convolution

Given any two non-negative non-decreasing “real” functions $A(t)$, $B(t)$ (possibly also $+\infty$ valued), their *convolution* is defined by

$$(A * B)(t) = \inf_{\tau \in \mathbb{R}} (A(\tau) + B(t - \tau))$$

- A “real” function $A(t)$ is *causal* if $A(t) = 0$ for all $t < 0$.

Basic Observations

- for causal $A(t), B(t)$

$$(A * B)(t) = \inf_{0_- \leq \tau \leq t_+} (A(\tau) + B(t - \tau))$$

- $\tau < 0$: $A(\tau) = 0$ (causal), $B(t - \tau) \geq B(t)$ (non-decreasing)
 - $\tau > t$: $A(\tau) \geq A(t)$ (non-decreasing), $B(t - \tau) = 0$ (causal)
 - taking $0_-, t_+$ takes care of “jumps” of $A(t), B(t)$ at $t = 0$
- Why “convolution”?
 - recall the functional analysis convolution

$$(f * g)(t) = \int_{\tau \in \mathbb{R}} f(\tau) \cdot g(t - \tau) d\tau$$

- Sum (integral) \longrightarrow Min (infimum)
- Product \longrightarrow Sum
- distributive law maintained

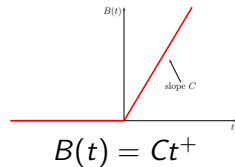
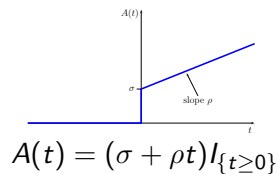
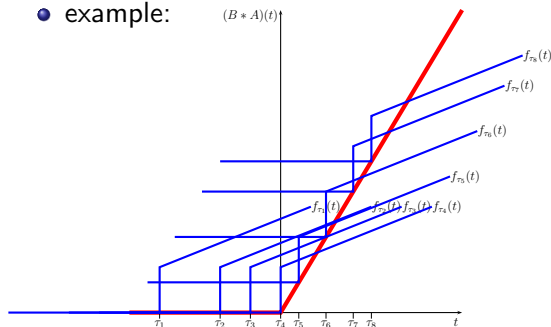
$$a \cdot (b + c) = a \cdot b + a \cdot c \quad \text{vs.} \quad a + \min \{b, c\} = \min \{a + b, a + c\}$$

Geometric Intuition

- alternative (more geometric) view of $B * A$:
 - define $f_{\tau}(t) = B(\tau) + A(t - \tau)$
 - hence,

$$(B * A)(t) = \inf_{\tau \in \mathbb{R}} f_{\tau}(t)$$

- example:

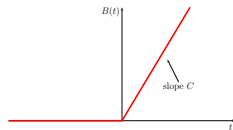
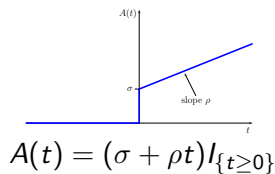
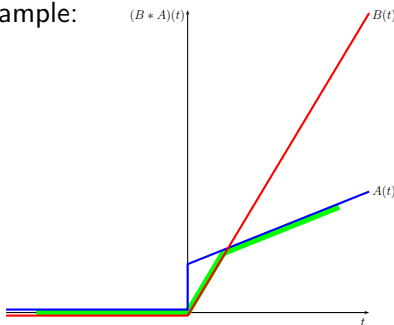


Geometric Intuition

- alternative (more geometric) view of $B * A$:
 - define $f_{\tau}(t) = B(\tau) + A(t - \tau)$
 - hence,

$$(B * A)(t) = \inf_{\tau \in \mathbb{R}} f_{\tau}(t)$$

- example:



- in this example, $(B * A)(t) = \min \{A(t), B(t)\}$
- is it always exactly the minimum?

Some Properties and Special Elements

- Additional properties of the $*$ operator
 - monotone: if $B \leq C$ then $A * B \leq A * C$
 - commutative: $A * B = B * A$
 - associative: $(A * B) * C = A * (B * C)$
 - distributive over min: $A * \min \{B, C\} = \min \{(A * B), (A * C)\}$
- Define

$$\delta(t) = \begin{cases} 0 & t < 0 \\ \infty & t \geq 0 \end{cases}$$

- unit element:

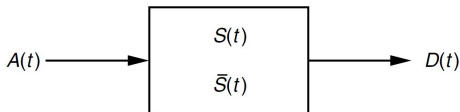
$$\begin{aligned} (A * \delta)(t) &= \inf_{\tau \in \mathbb{R}} (A(\tau) + \delta(t - \tau)) \\ &= \inf_{t < \tau} A(\tau) = A(t) \end{aligned}$$
- if B is causal then $B \leq \delta$: $A * B \leq A * \delta = A$
 - if A, B are causal then $A * B \leq \min \{A, B\}$
 - if A is causal then $A * A \leq A$
- Define

$$\delta_d(t) = \delta(t - d)$$

- delay element: $(A * \delta_d)(t) = A(t - d) \stackrel{?}{=} \min \{A, \delta_d\}$

Overview and Definition

- consider an arrival sequence serviced by a network element



- network element: dedicated link, link with cross traffic, path...
- single dedicated link: removes $\leq Ct$ bits in interval of length t
- main focus: analyze the departure process!!

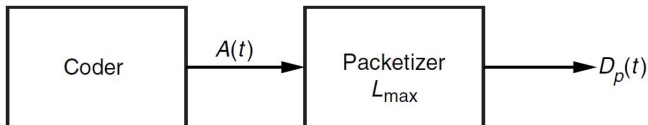
Definition. (Service Curves)

Let $S(t), \bar{S}(t)$ be non-negative, nondecreasing, causal functions

- if $A * S \leq D$, S is said to be a *lower service curve*
- if $A * \bar{S} \geq D$, \bar{S} is said to be an *upper service curve*
- if $A * S = D$, S is said to be the *service curve*

Examples

- Example 1: voice packetizer
 - $A(t)$: encoder generates bursts at byte rate r (during burst)
 - packetizer collects L_{\max} bytes, and sends packets
 - byte delay $\leq \frac{L_{\max}}{r}$
 - time to accumulate L_{\max} bytes arriving at rate r
 - packetized output $D_p(t) \geq A(t - \frac{L_{\max}}{r})$
- $\Rightarrow \delta_{\frac{L_{\max}}{r}}$ is a lower service curve for $D_p(t)$



Examples

- Example 2: constant-rate server (infinite buffer)
 - fluid model: we have seen $D(t) = \inf_{s \in \mathbb{R}} (A(s) + C(t - s)^+)$
 $\Rightarrow S(t) = Ct^+$ is *the* service curve
 - packetized model: a departure is considered at packet boundaries (size L_{\max})
 \Rightarrow a lower service curve:

$$\begin{aligned}(Ct^+ - L_{\max})^+ &= C(t - \frac{L_{\max}}{C})^+ \\ &= Ct^+ * \delta_{\frac{L_{\max}}{C}}\end{aligned}$$



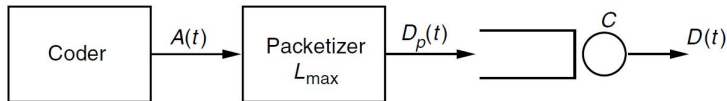
Examples

- Example 3: voice packetizer and constant-rate server
 - traffic entering the server is already packetized
 - no need to “pay” the delay twice
 - hence,

$$\begin{aligned}
 D &\geq D_p * Ct^+ \\
 &= (A * \delta_{\frac{L_{\max}}{r}}) * Ct^+ \\
 &= A * (\delta_{\frac{L_{\max}}{r}} * Ct^+)
 \end{aligned}$$

\Rightarrow a lower service curve:

$$\delta_{\frac{L_{\max}}{r}} * Ct^+$$



Examples

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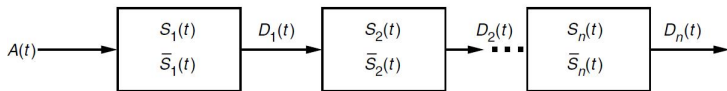
$$\delta_{\frac{L_{\max}}{r}} * Ct^+$$

Definition. (latency rate server)

A network element is a *latency rate server* with rate $r \in \mathbb{R}^+$ and delay $d \in \mathbb{R}^+$ if it has a lower service curve of the form

$$\delta_d * rt^+ = r(t - d)^+$$

Service Elements in Tandem



- concatenating service curves (e.g., lower service curves)

$$\begin{aligned}
 D_n &\geq D_{n-1} * S_n \\
 &\geq (D_{n-2} * S_{n-1}) * S_n \\
 &\geq \dots \\
 &\geq A * (S_1 * S_2 * \dots * S_n)
 \end{aligned}$$

- example – rate latency servers: $S_i(t) = r_i(t - d_i)^+$

$$\begin{aligned}
 (S_1 * \dots * S_n) &= (r_1(t - d_1)^+) * \dots * (r_n(t - d_n)^+) \\
 &= (r_1 t^+ * \delta_{d_1}) * \dots * (r_n t^+ * \delta_{d_n}) \\
 &= (r_1 t^+ * \dots * r_n t^+) * (\delta_{d_1} * \dots * \delta_{d_n}) \\
 &= r t^+ * \delta_d = r(t - d)^+
 \end{aligned}$$

where $r = \min_i r_i$ and $d = \sum_i d_i$

Obtaining a Better Departure Analysis

- recall: service curves aim at analyzing the departure process
- if we only know arrival's peak rate
 - results in over-provisioning
 - provides loose bounds
- goal: characterize *arrival* process better

Definition. (Envelope)

Given any function $E(t)$, arrival process $A(t)$ has *envelope* E if for all t , and all $\tau \leq t$,

$$A(t) - A(\tau) \leq E(t - \tau)$$

- envelope: invariant to “when”, depends only on “how long”
- example:
 - constant rate arrival: $A(t) = Rt^+$
 - A has envelope $E = A$:

$$A(t) - A(\tau) = Rt^+ - R\tau^+ \leq R(t - \tau)^+ = E(t - \tau)$$

From Envelopes to Regulators

- A has envelope E iff $A \leq A * E$
- if A has a causal envelope E then $A = A * E$
 - $A * E \leq A * \delta = A$
- A process E is **sub-additive** if for all $\tau \leq t$

$$E(t) \leq E(\tau) + E(t - \tau)$$

- if E is sub-additive, then $E \leq E * E$
- example – leaky bucket: $E(t) = \sigma + \rho t$

$$E(t) = \sigma + \rho t \leq 2\sigma + \rho t = E(\tau) + E(t - \tau)$$

Definition. (Regulator)

A network element is a **regulator** with envelope E if for *any* input process A , the departure process D has envelope E . I.e.,

$$\forall t, \forall \tau \leq t \quad D(t) - D(\tau) \leq E(t - \tau)$$

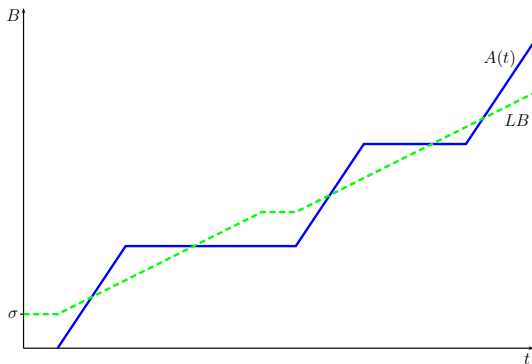
- equivalently, $D \leq D * E$

Leaky Bucket Example

Theorem.

For any arrival process $A(t)$ regulated by a leaky-bucket causal envelope $E(t) = \sigma + \rho t$, the departure process $D(t)$ satisfies

$$D = A * E$$

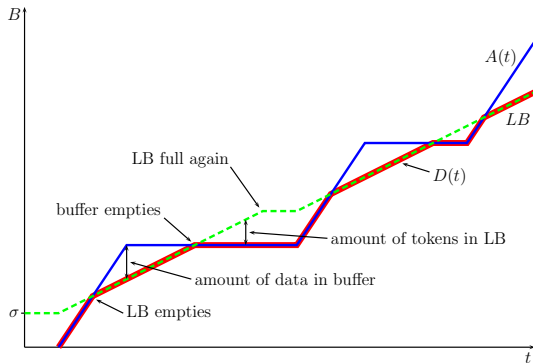


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Proof.

- for any t , and $\tau \leq t$

$$\begin{aligned} D(t) &= D(\tau) + D(t) - D(\tau) \\ &\leq D(\tau) + E(t - \tau) \\ &\leq A(\tau) + E(t - \tau) \end{aligned}$$

$$\Rightarrow D \leq A * E$$

- it remains to show the converse, i.e., that $D \geq A * E$
- consider any time t
 - if the data buffer is empty at t , then we have

$$D(t) = A(t) = (A * \delta)(t) \geq (A * E)(t)$$

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Proof.

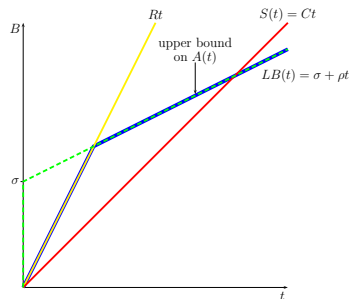
- assume the data buffer is not empty at t
 - the LB *bucket* must be empty at t
- let $v = \sup \{s \leq t \mid \text{the LB bucket is full at } s\}$
 (i.e., v is the “start” of a busy period)
 - \Rightarrow until v_- all arrivals have passed through
- during $[v, t]$, the LB constantly uses up tokens, hence

$$\begin{aligned} D(t) &= D(v_-) + \sigma + \rho(t - v) \\ &= A(v_-) + E(t - v) \\ &\geq \inf_{\tau} (A(\tau) + E(t - \tau)) = (A * E)(t) \end{aligned}$$



A New Dog Doing Old Tricks

- recall IntServ
 - single dedicated link with rate C
 - arrival process:
 - peak rate R
 - regulated by a (σ, ρ) LB envelope



A New Dog Doing Old Tricks

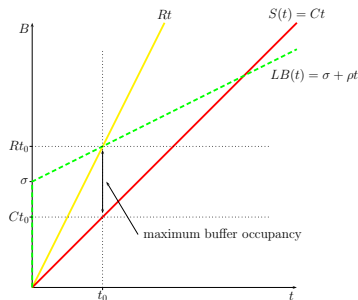
- recall IntServ
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- $R \cdot t_0 = \sigma + \rho \cdot t_0$

$$\Rightarrow t_0 = \frac{\sigma}{R-\rho}$$

- maximum buffer occupancy:

$$R \cdot t_0 - C \cdot t_0 = \frac{\sigma(R-C)}{R-\rho}$$



A New Dog Doing Old Tricks

- recall IntServ
 - single dedicated link with rate C
 - arrival process:
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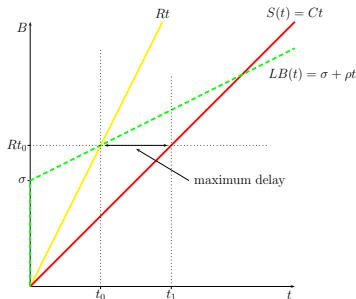
- maximum buffer occupancy:

$$R \cdot t_0 - C \cdot t_0 = \frac{\sigma(R-C)}{R-\rho}$$

- $Ct_1 = Rt_0 = \frac{R\sigma}{R-\rho}$

- maximum delay:

$$t_1 - t_0 = \frac{\sigma(R-C)}{C(R-\rho)}$$



A Network Calculus View

- Given:

- arrival process $A(t)$ with envelope $E(t)$
- lower service curve $S(t)$

define:

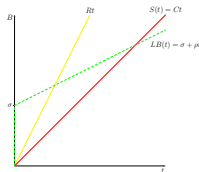
$$d_{\max} = \inf \{d \mid E * \delta_d \leq S\}$$

- d_{\max} : least right shift of E s.t. it's below S , or
- d_{\max} : least left shift of S s.t. it's above E
- For $S(t) = Ct^+$ it must hold for any t :

$$S(t + d_{\max}) = Ct + Cd_{\max} \geq E(t)$$

$$\Rightarrow d_{\max} = \sup_{t \geq 0} \left(\frac{E(t) - Ct}{C} \right)$$

- in the previous example:



$$\begin{aligned} E(t) &= \min(Rt^+, (\sigma + \rho t)I_{\{t \geq 0\}}) \\ &= (Rt^+ * (\sigma + \rho t)I_{\{t \geq 0\}})(t) \end{aligned}$$

Delay and Provisioning

- $D \geq A * \delta_{d_{\max}}$
 - S is a lower service curve: $D \geq A * S$
 - by the definition of d_{\max} : $A * S \geq A * (E * \delta_{d_{\max}})$
 - associativity + E is an envelope of A :

$$\begin{aligned} D &\geq (A * E) * \delta_{d_{\max}} \\ &\geq A * \delta_{d_{\max}} \end{aligned}$$

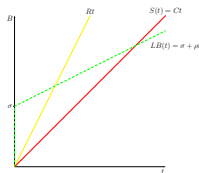
- Rate Provisioning for $S(t) = Ct^+$
 - assume we want the delay to be at most T :

$$d_{\max} = \sup_{t \geq 0} \left(\frac{E(t) - Ct}{C} \right) \leq T$$

- any C satisfying this: $E(t) - Ct \leq CT$

$$\Rightarrow C \geq \sup_{t \geq 0} \left(\frac{E(t)}{T + t} \right)$$

- C_{\min} : minimal rate satisfying this requirement
 - bound is tight



Generalizing Reich's Equation

Theorem. (Backlog bound)

If arrival A has envelope E , and S is a lower service curve on departures D , then the backlog at any time t satisfies

$$A(t) - D(t) \leq \sup_{\tau \geq 0} \{E(\tau) - S(\tau)\}$$

Proof.

By the definition of lower service curve S and envelope E

$$\begin{aligned} A(t) - D(t) &\leq A(t) - (A * S)(t) \\ &= A(t) - \inf_{0 \leq \tau \leq t} (A(t - \tau) + S(\tau)) \\ &= \sup_{0 \leq \tau \leq t} (A(t) - A(t - \tau) - S(\tau)) \\ &\leq \sup_{0 \leq \tau \leq t} (E(\tau) - S(\tau)) \end{aligned}$$



- i.e., the (max) vertical distance between E and S

A “Delay” Version of Reich’s Equation

Theorem. (Delay bound)

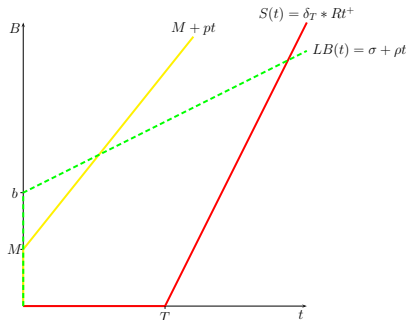
If arrival A has envelope E , and S is a lower service curve on departures D , then the delay $d(t)$ at any time t satisfies

$$d(t) \leq d_{\max} = \inf \{d \mid E * \delta_d \leq S\}$$

- i.e., the (max) horizontal distance between E and S

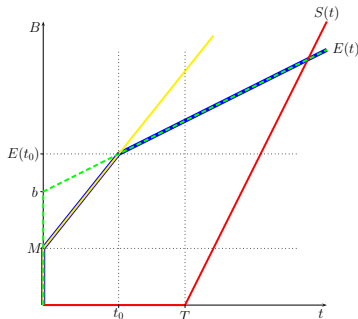
IntServ: Analyzing a (Path of) Latency-Rate Server(s)

- Recall IntServ TSpec: (M, p, r, b)
 - envelope $E(t) = (M + pt) * (b + rt) = \min \{M + pt, b + rt\}$
 - lower service curve $S(t) = \delta_T * Rt^+$



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 - Backlog $\leq \max \{E(T), E(t_0) - S(t_0)\}$
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- E.g., delay bound translates to

$$\frac{M + \frac{b-M}{p-r}(p-R)^+}{R} + T$$

- familiar???
- Recall:
 - concatenating latency-rate servers \equiv latency-rate server
 - effectively on a *path*!!
 - latency T = sum of latencies
 - rate R = minimum rate

Extensions and Current Research

- Stochastic Network Calculus
 - stochastic arrival curves, envelopes, algebra (min-sum operator), etc.
- Broad field (and thriving):
 - WFQ can be interpreted and analyzed within this framework
 - we'll see it later "stand-alone"
 - also related to Adversarial Queueing Theory
 - we'll see it later "stand-alone"
 - several Infocom papers every year
- Bibliography and recommended reading:
 - Kumar, Manjunath, and Kuri, "*Communication Networking: An Analytical Approach*", Elsevier, 2004.
 - Le Boudec and Thiran, "*Network Calculus: A Theory of Deterministic Queuing Systems for the Internet*", Springer, 2004.
 - many more...