Chaotic Encryption Method Based on Life-Like Cellular Automata

Marina Jeaneth Machicao, Anderson G. Marco, Odemir M. Bruno
Instituto de Física de São Carlos - Universidade de São Paulo - Brasil
mj.machicao@gmail.com ; anderson.marco@gmail.com ; bruno@ifsc.usp.br

Abstract—We propose a chaotic encryption method based on Cellular Automata(CA), specifically on the family called the "Life-Like" type. Thus, the encryption process lying on the pseudo-random numbers generated (PRNG) by each CA's evolution, which transforms the password as the initial conditions to encrypt messages. Moreover, is explored the dynamical behavior of CA to reach a "good" quality as PRNG based on measures to quantify "how chaotic a dynamical system is", through the combination of the entropy, Lyapunov exponent, and Hamming distance. Finally, we present the detailed security analysis based on experimental tests: DIEHARD and ENT suites, as well as Fouriers Power Spectrum, used as a security criteria.

I. INTRODUCTION

Cryptography can be traced back over 2300 years as decisive in the course of human History. The sort of battles and kings were decided by the power or weakness of ciphers. Nowadays, its importance lies on: military communications, intelligence tool, banking system, and e-commerce foundation. However in the last 20 years many classic encryption algorithms had been broken, among them, DES algorithm in 1998[1], cryptography hash functions MD5 (2008)[2] and SHA1 (2009)[3]. Furthermore, Cryptography is constantly searching for new algorithms to encrypt and to make ciphers more secure, some new mathematical methodologies has to be considered.

Chaos Theory is a mathematical field that studies the behavior of complex systems with highly sensitiveness to initial conditions. In fact, chaos can be found in very simple systems that shows complex patterns, for instances, Chaos was first introduce in 1960 as a weather prediction by Edward Lorenz, furthermore it was involved with population growth, and also with cellular evolution introduced as Cellular Automata. In sum Chaos Theory deals with unpredictable systems, most of them with systems found in nature: weather, turbulence, stock market, population, and so on.

Cellular automata, first introduced by von Neumann in the early 1950s, was conceived as a model of biological evolution [4], and also used as a prototyping model for a large variety of natural systems, this models have attracted the attention of several groups of research as CA provide approximation to partial differential equations, and because CA are considered discrete dynamical systems and furthermore computational systems. Twenty years later, the mathematician Jhon Conway introduced his well-known Life game, which initially was intended to modeling natural cellular phenomena, consists of a regular grid of cells, with alive-dead states("On" and "Off"),

through out simple rules, which involves neighborhood's cells, they constantly evolved even to get unpredictable behavior[?], [5].

Over the past decade, there exists progress on research encryption techniques based on Chaos Theory[6], [7], [8]. This position was motivated by the similarity between Chaos and Cryptography, where were emphasized the sensitiveness of initial conditions trough the *key* and input(plaintext) dependency, also both Chaotic-System and Crypto-System has a "random-like" behavior[9], which is very desirable for cryptographic schemes. By extension all this relationship are inherited to Cellular Automata. Consequently cryptography can take advantages from those CA which shows chaotic behavior.

Cryptography based on CA gave first signals at [5], [10], [11] and recently dealing in [12], [13], it was also included CA as pseudo-random number generator in [14], [15], [16], [17], [18], [19]. Although the inclusion of CA in the field of cryptography is not new, this proposal follows a chaotic sensitivity throughout the process, and our results exceed expectations compared to others found in literature.

This motivated us to develop an chaos-based encryption scheme based on CA that takes advantage of chaos to encrypt in a "unpredictable" manner, this provides a very attractive alternative method as there exists CA rules with chaotic behavior that, can be employed as pseudo-random number generators, and because it shows efficiently hardware incorporation according to its dynamical properties and through its parallelism fundamentals. To understand what this "chaotic behavior" really means, where used the Lyapunov exponent as a measure that characterizes the rate of separation of infinitesimally close trajectories provided by the system, in advance it determines a notion of predictability for a dynamical system, where positives values is usually taken as a chaotic indicator. [20]

In Section 2, we mention the state of art of some drawbacks based on chaos-based cryptography. Section 3 introduces basic theory on the subject, as needed. In Section 4, the encryption method based on CA is proposed in detail.

Section 5, presents the security analysis based on experimental tests: DIEHARD[21] and ENT[22], as well as Fourier's Power Spectrum, used as a security criteria. Finally, in Section 6, the paper ends with a discussion about the method.

II. BACKGROUND

Modern cryptography is based on two approaches: Discrete Mathematics (Symmetric key) and Number Theory (Asym-

metric key)[23], [24]. Cryptography's algorithms stand on an elementary mathematics need to be extremely complex (Symmetric key Algorithm), performing many bit-wise operations and permutation between neighbors, on the other hand, having long keys (Asymmetric key Algorithm). However, for both strategies, computing complexity and the size of the keys, just make information to be secure for a certain time.

Since Baptista has introduced first chaos-based encryption[25], many researches have been interested in the relationship between chaos and cryptography, then many properties of chaotic systems have to be on mind e.g: ergodicity, sensitivity to initial conditions, deterministic dynamics, and structural complexity.

Cryptography of symmetric key systems based on CA, were first studied by Wolfram [5], Habutsu[10], Nandi et al.[11] and Gutowitz[26], and later by Tomassini et al.[12]. Recently one-dimensional CA was subject of study by Seredynski et al.[13]. This paper presents a new effort to involve Chaos Theory and Cellular Automata to implements an encryption method.

III. CELLULAR AUTOMATA

In this work a cellular automata (CA) is considered a discrete dynamical system defined on a discrete space which is governed by its local rules and by its immediate neighbors, which specifies how CA evolves in time.

Definition An homogeneous CA can be represented as a sextuple $\langle \mathbb{T}, S, s, s_0, N, \phi \rangle$ where:

1) $\mathbb{T}$ is a lattice of a n-dimensional Euclidean space $\mathbb{R}^n$, consisting of cells c_i, $i \in \mathbb{N}$
2) S is a finite set of k states, often $S \subset \mathbb{N}$
3) The output function $s : \mathbb{T} \times \mathbb{N} \to S$ maps the states of cell c_i at discrete time t, i.e., $s(c_i, t)$.
4) The function $s_0 : A \to S$ allocates the initial configuration of every cell c_i, i.e., $s(c_i) = s_0(c_i)$.
5) The neighborhood function $N : \mathbb{T} \to \bigcup_{p=1}^{\infty} \mathbb{T}^p$, yields every cell c_i to a finite sequence $N(c_i) = (c_{i_j})_{j=1}^{|N(c_i)|}$, with $|N(c_i)|$ distinct cells c_i.
6) The transition function $\phi : S^{|N(c_i)|} \to S$, describes the rules governing the dynamics of every cell c_i

$$s(c_i, t+1) = \phi((s(c_i, t))_{j=1}^{|N(c_i)|}) = \phi(\sigma_i)$$

with,

$$\sigma_i = \sum_{j=1}^{N(c_i)} s(c_i, t) \qquad (1)$$

A. Life-Like Cellular Automata

A well known two-dimensional CA was proposed by John Conway, called "Game of Life", which is based on a biological model, with cells either dead(0) or alive(1), in general the state of a cell at the next generation depends on its own state and the sum of cells, to Conway's CA a dead cell births when is surrounding for 3 alive neighbors and an alive cell can survive if there exists 2 or 3 active cells.

Using a standard convention of Golly simulator[27], a notation for naming CA, a rule is written in the form $By\backslash Sx$

where x and y is a sequence of distinct digits from 0 to 8, in numerical order. Thus, Conway's Game of Life is denoted $B3\backslash S23$, where "B" stands for "birth" and the "S" stands for "survival". It is common to refer to "Life-Like" or simply "Life Family" in the sense of similarity to Conway's, to all those CA in the format of Golly that have one of several rules, for instances $(B36\backslash S23)$, $(B23\backslash S36)$.

Formally, any "Life-Like" automata is described as $\langle \mathbb{Z}^2, S = \{0, 1\}, s, s_0, \text{Moore neighborhood}, \phi : S^9 \to S \rangle$, where Moore neighborhood considers 8 cardinal direction and the state of the own center. For instances, for binary cells $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8,$ and c_9 we say that the transition function, at any time t, for rule $B3\backslash S23$(Game-of-Life)[28] is of the form:

$$\phi \begin{pmatrix} c_1, & c_2, & c_3, \\ c_4, & c_5, & c_6, \\ c_7, & c_8, & c_9 \end{pmatrix} = \begin{cases} 1, & \text{if } \sum_{i=1}^{9} s(c_i, t) = 3 \\ 1, & \text{if } \sum_{i=1}^{9} s(c_i, t) = 2, i \neq 5 \\ 0, & \text{otherwise} \end{cases}$$

$$(2)$$

There are two main issues with this "Life-Family" CA:

- **Cellular growth and decay**: The population of cells of some rules of the Life-Family seems to decay when iterated considerable time. It is mean, they are "like-death" because most of their cells are *like* to arise death[29].
- **Boundary conditions**: By definition, a 2D-CA consists of an *infinite* plane, for instances, its space $\mathbb{T}$ can be a matrix of order $m \times n$. For computational reasons they are often simulated on a finite grid rather than an infinite one, although there is an evidently issue with the boundaries. There exists plenty methods to handled it, in this paper is used a hyper-toroidal method, where the grid is considered as their edges were touching on all boundaries. So far, this toroidal arrangement simulates an infinite periodic lattice.

B. Chaotic Cellular Automata

In some cases, CAs present chaotic behavior, then we are interested to use chaotic CAs in order to take advantage for cryptography. To measure the chaotic behavior in dynamical systems were announced several studies that involved both geometrical[30] and statistical[31] approaches. The statistical approach, seeks to characterize dynamical systems through the Lyapunov exponent, which has been proven to be the most useful to measure chaos[32], [33], [34], [35].

1) Lyapunov Exponent(LE): The Lyapunov exponent λ is a measure of the sensitive dependence on initial conditions, it is a measure of how chaotic a dynamical system is. There are several ways to estimate the LE, and several variants were suggested[36], [37], [31], [30]. Where, the information extracted from λ is:

$$\begin{cases} \lambda < 0 & \text{Stable Periodic} \\ \lambda > 0 & \text{Chaotic} \\ \lambda = 0 & \text{Neutrally Stable} \end{cases} \qquad (3)$$

For this purpose is considered a recent method to estimate the LE in CA, according with Baetens and De Baets [38], where is considered two initial configurations s_0 and s_0^* of CA with states $s = \{0, 1\}$. Is defined a "damage vector", $h(\cdot, t) = s(\cdot, t) \oplus s^*(\cdot, t)$, as the number of different cells or distance between both CA. Is also defined a **perturbed cell** c_i for which $s(c_i, t) \neq s^*(c_i, t)$ and the smallest perturbation of s_0 due to the state domain of CA, $\sum_{c_i} h(c_i, 0) = 1$.

Is is important to take into account all the perturbations originated from one initial perturbation, where can be defined as the maximum Lyapunov exponent $\lambda(t)$, with finite $t \in \mathbb{N}$.

$$\lambda(t) = \frac{1}{t} \log\left(\frac{\sum_{c_i} h(c_i, t)}{\sum_{c_i} h(c_i, 0)}\right) \tag{4}$$

2) Entropy: Entropy is a measure of disorder or randomness in a closed system. Entropy applied to CA can be estimated by:

$$H = -\sum_{i=0}^{k} P_i \log(P_i)$$
$$0 \leq H \leq 1 \tag{5}$$

Where P_i is the probability of k possible states of every cell in CA. By using entropy, we are interested on maximal values, thus makes 1 the maximum normalized entropy, which would mean a low level of redundancy or predictability of the encryption method.

3) Hamming Distance (HD): We employ the Hamming distance D_H in CA as a measure that estimates the minimum number of substitutions required to change one state c_i into another, this means the count of differences between the states of cells $s(\cdot, t)$ and $s(\cdot, t+1)$, then, as the higher disturbances the lower repetition of patterns.

$$D_H = \frac{\sum s(\cdot, t) \oplus s(\cdot, t+1)}{size(\mathbb{T})} \tag{6}$$

Finally, is considered the highest Hamming distance, when all cells are the opposed to next iteration, thus the maximal D_H is closed to 1.

4) Chaos Combined Measure (Max): It is notice that isolated high Lyapunov exponent, entropy or Hamming distance are necessary, but by no means enough to response the question: "which CA is more chaotic than the other?". Then, this new measure *Max* maximize the combination of the measures mentioned above, by simple multiplicity. Moreover, as the higher *Max* gets, the more chaotic a system is. this means:

$$\text{Max} = (\lambda * H * D_H) \tag{7}$$

IV. The proposed Algorithm

We propose a chaotic encryption method based on Cellular Automata(CA), specifically on two-dimensional "Life-like" CA, mentioned on section III, to design a symmetric key cryptography system.

In Figure 1 is showed the proposal, which is divided into 4 principal parts, where a chaotic CA is applied as a pseudo-random number generator (PRNG) which is employ during the encryption. With this process, is constantly taken pseudo-random numbers generated at every step during the CA's evolution, this numbers are composed in blocks of size of the plaintext, for later encryption. The initial conditions for this CA can be fulfilled in different ways, for this proposal was taken the Logistic Map (dynamical system) where the password to encrypt is taken as a seed for the system, thus, every cell takes binaries values as the Logistic Map iterates. At next, is showed each of the 4 the parts of our cryptosystem:

- Seed based on Logistic Map
- Chaotic Cellular Automata
- Pseudo-random number generator
- Encryption/Decryption

A. Seed based on Logistic Map

The Logistic Map is a well-known continuous dynamical systems, mathematically is written as

$$X_{h+1} = \mu X_h (1 - X_h) \tag{8}$$

Where $X_h \in [0, 1]$ for $h \in \mathbf{N}$, with parameter μ, which presents chaotic behavior for values among $3.9 \leq \mu \leq 4.0$.

This Logistic Map generates continuous values between $[0, 1]$, which are discretized(binarized) in order to fulfill the initial cellular automata to later encryption.

This discretization is allowed because of the equally distribution of the Logistic Map, this property is very important in order to be used in cryptography, thus, the probabilities of any password would also fit in an equally distributed region.

As described above, this map takes the password as a **seed** to generate discrete X_h, through it iterates until all cells in CA gets completed. Should be noted that have to be omitted first iterations in order to get rid of the transient α of the map, which presents non interested region of Logistic Map.

To transform X_h into discrete values, we have to consider the following definitions:

Definition Let $\vec{\pi}$ be a vector containing a string(password), in bytes, with $size(\vec{\pi}) := 16$, this means length of 128 bits. Is defined:

$$\Omega := \sum_{i=1}^{size(\vec{\pi})} 2^{8(i-1)} \pi_i \tag{9}$$

$$\Omega := \frac{\Omega}{2^{(8m+1)}}. \tag{10}$$

Where $X_0 = \Omega + \epsilon$, with $\Omega \in [0, 1]$ and for very small ϵ, as Logistic Map cannot take zero as an entrance value.

Definition Let the space of CA $\mathbb{T}$ be a matrix of order $m \times n$, with entries $1 \leq i \leq m; 1 \leq j \leq n$, which also means $size(\mathbb{T}) = m \times n$.

Let $\alpha \in \mathbf{N}$ be the transient of the Logistic map.

$$s_0(\cdot) = \begin{cases} 1 & \text{if } X_{((m(i-1)+j)+\alpha)} < 0.5 \\ 0 & \text{if } X_{((m(i-1)+j)+\alpha)} \geq 0.5 \end{cases} \tag{11}$$
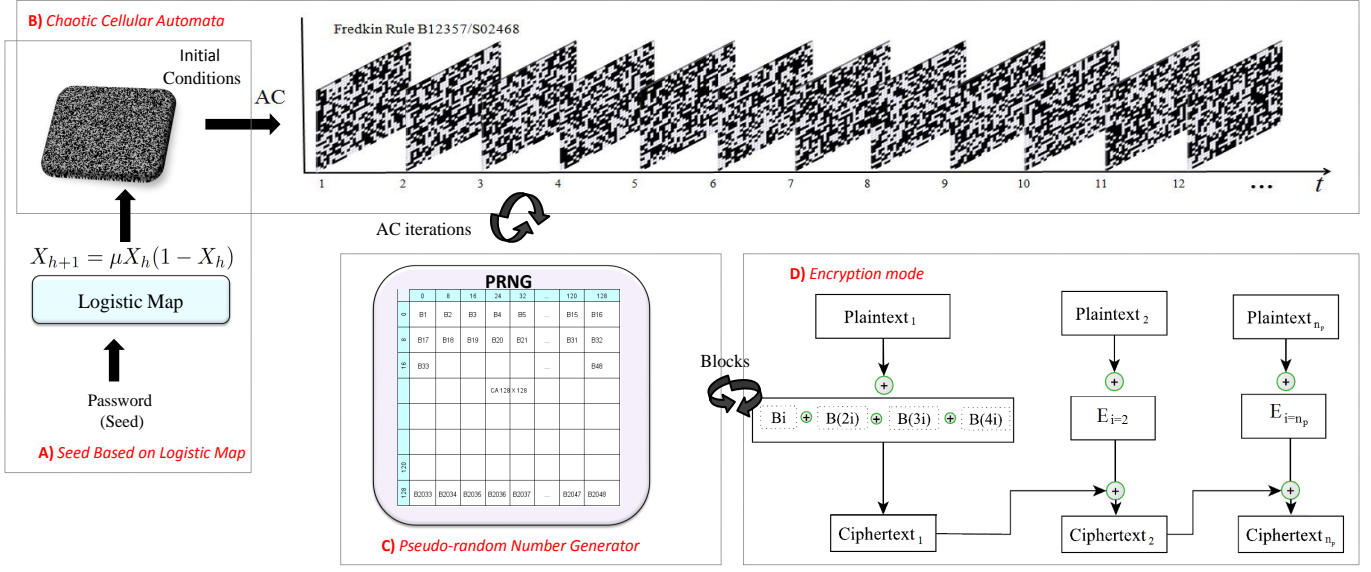
Fig. 1. Proposal of chaotic encryption method based on Cellular Automata(CA), for instances, it uses 4 Blocks as PRNG. Its design is divided into 4 principal parts. (A) The transformation of the password into a initial CA, (B) The CA is defined with a specific rule and a initial CA and is iterated as many as required (C) Transforms the CA into a PRNG to generate blocks, (D)The encryption mode takes blocks as many as needed. Note that the decryption uses the same design just than inverse the encrypt process.

B. Chaotic Cellular Automata

In this section is discussed the different types of rules of the "Life-Like" family, as several rules can be considered to cryptography. In Table I can be visually appreciated the random nature of 64x64 2D-CA "Fredkin" with rule $B1357\backslash S02468$, each of the 3 first figures present different percentage of initial seed of 10%, 50%, and 90% of alive cells. Curiously, as seen, this rule stabilizes to 50% of alive cells after few iterations and keep it constantly through time, even with seeds of 10% or 90%. Furthermore, the measure *Max* (combination of entropy, Lyapunov Exponent, and Hamming distance) mentioned in Section III, is then considered as a way to choose the most appropriate chaotic CA, which according to results showed in next section, the rule "Fredkin" is considered the most acceptable CA to cryptography.

C. Pseudo-random number generator(PRNG)

A pseudo-random number generator is an algorithm for generating sequences of numbers that approximates randomness, this property is required in different problems domains such as simulations, testing, games, security, and of course making it suitable for use in cryptography. This approach is not recent, and exists several implementations in literature, the most recents ones are mentioned here in [14], [15], [16], [18], [19], for this, a CA can be taken in several different ways. The states of cells c_i of an $m \times n$ CA are binary values, then it can be taken as an advantage, by **composing blocks** which is described at next:

- Let $\vec{\beta}$ be a vector of bits (0,1)
  - $size(\vec{\beta}) = floor(\frac{size(\vec{\beta})}{8})$

$$\beta_{(i-1)+((j-1)\times n)+((t-1)\times m \times n)+1} = s(\cdot, t) \quad (12)$$

TABLE I
2D-CA "FREDKIN" $B1357\backslash S02468$ EVOLUTION WITH DIFFERENT SEEDS

2D-CA Fredkin with random seed A)10%, B)50% and C)90% of alive cells
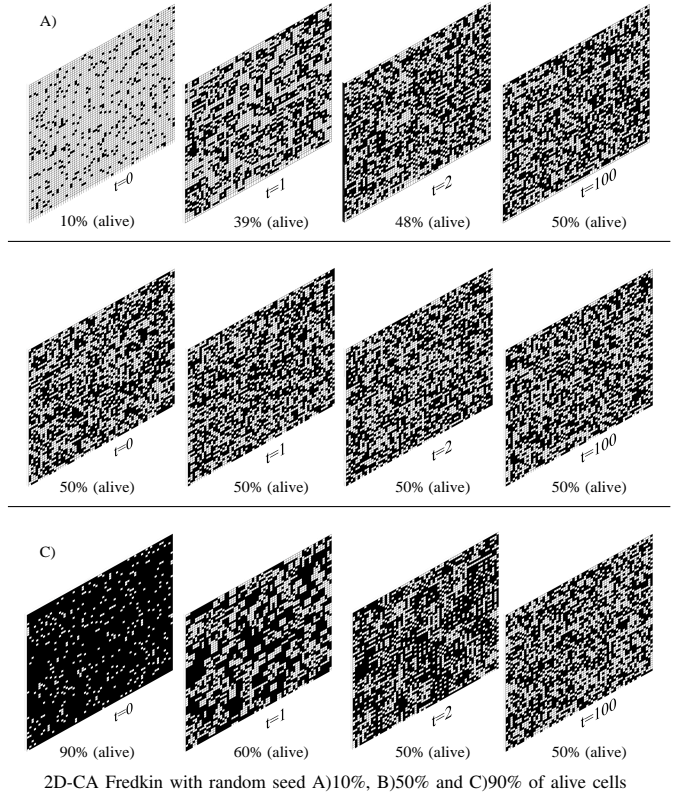
- Let $\vec{B}$ be a vector of blocks of bytes

$$B_i = \sum_{j=1}^{8} 2^{8(j-1)} \beta_{8(i-1)+j} \qquad (13)$$

- Let $\vec{Y}$ be a vector composed by ρ blocks.
  - $size(\vec{Y}) \leq floor(\frac{size(\vec{B})}{8})$

This "composing-block" process is defined as the consecutive XORed of the blocks $\vec{B}$ as follows:

$$Y_i = B_{(i \times \rho)+1} \oplus B_{(i \times \rho)+2} \oplus \ldots \oplus B_{(i \times \rho)+\rho} \qquad (14)$$

The symbol $\oplus$ represents the bitwise XOR logical operation, which is due to its reversible property.

D. Encryption and Decryption

Let $\vec{P}$ be the plaintext message of size n_p and E our ciphering algorithm, which represents any symmetrical cryptography algorithm using password $\vec{\pi}$. The fundamental transformation to obtain the ciphertext $\vec{C}$ of same size n_p, is by encrypting the combined blocks generated by the PRNG and then XORed with the previous plaintext sequence.

This procedural can ensure that similar blocks of the plaintext are codified as different ciphertext, making the cipher stronger.

For the Encryption mode:

$$\begin{aligned} C_i &= E_i(P_i \otimes C_{i-1} \otimes Y_i) \\ C_0 &= Y_1 \otimes P_1 \end{aligned} \qquad \text{for } i = 1, 2, \ldots, n_p \qquad (15)$$

For the Decryption mode:

$$\begin{aligned} P_i &= E_i^{-1}(C_i \otimes Y_i) \otimes C_{i-1} \\ C_0 &= Y_1 \otimes P_1 \end{aligned} \qquad \text{for } i = 1, 2, \ldots, n_p \qquad (16)$$

V. EXPERIMENTAL RESULTS

In this section, the efficiency of the chaotic encryption method based on Life-Like CA is analyzed, thus it was separated into parts to presents the results:

A. Chaotic Life-Like CA

The purpose of this experiment was to discover among an enlarged set of rules of CAs, which one presents the most chaotic behavior by having high entropy, Lyapunov Exponent, and Hamming distance, in order to produce very high quality of PRNG and furthermore a "good" cryptosystem.

In Table II are showed the results of the test with a 2D-CA of 128x128 cells of the Life-Like family, to estimate the average entropy were employed 20 million iterations, the LE was estimated under $T = 200$, and the average Hamming distance was calculated with $T = 1000$. Besides, the maximal values were taken as the highest combination Max, this means, by selecting the "Fredkin", "Amoeba", and "$B23 \backslash S36$" rules as some of the best chaotic CA to be employed for this encryption proposal.

For all subsequent experiment in this paper, is used the rule *Fredkin* $B1357 \backslash S02468$, as it posses constantly and highly entropy, that also presents chaotic behavior, and finally

TABLE II
COMPARISON OF LIFE-LIKE RULES, BASED ON ITS ENTROPY, LYAPUNOV EXPONENT, AND HAMMING DISTANCE

Life-Like rule		Entropy	LE	DH	Max
Fredkin	B1357/S02468	0,999940	0,943100	0,501850	0,473268
Amoeba	B357/S1358	0,983020	0,860380	0,439330	0,371569
B23/S36	B23/S36	0,968580	0,618600	0,546080	0,327190
Serviettes	B234/S	0,951260	0,351230	0,741730	0,247824
34 Life	B34/S34	0,984320	0,506940	0,460290	0,229680
Long life	B345/S5	0,904000	0,351750	0,679550	0,216081
Seeds	B2/S	0,743040	0,583270	0,419600	0,181852
WalledCities	B45678/S2345	0,997250	0,411290	0,421570	0,172911
Gnarl	B1/S1	0,775450	0,512600	0,381110	0,151489
Coagulations	B378/S235678	0,377740	0,672010	0,163430	0,041485
2x2	B36/S125	0,179990	0,705260	0,132300	0,016795
Pseudo life	B357/S238	0,095440	0,667450	0,093230	0,005939
Conway's Life	B3/S23	0,212000	0,552850	0,047720	0,005593
Coral	B3/S45678	0,694870	0,463520	0,015950	0,005138
HighLife	B36/S23	0,127020	0,618370	0,039710	0,003119
Day & Night	B3678/S34678	0,071400	0,672280	0,050880	0,002442
Mazectric	B3/S1234	0,999990	0,304500	0,003920	0,001193
Move	B368/S245	0,033030	0,779040	0,026460	0,000681
Assimilation	B345/S4567	0,793400	0,025290	0,002950	0,000059
Maze	B3/S12345	0,999990	0,019960	0,001080	0,000022
Flakes	B3/S012345678	0,949110	0,003010	0,000140	0,000000
Diamoeba	B35678/S5678	0,000000	-Infinity	0,019420	0,000000
Replicator	B1357/S1357	0,000000	0,894060	0,031960	0,000000
Stains	B3678/S235678	0,087480	-Infinity	0,002370	0,000000

guarantee a higher disturbance variation as it is iterated, then this model is applied in this proposal. the random nature of this *Fredkin* rule can be visually appreciated in Table III, where is presented the complete analyze of the chaotic measures presents above, the pictures of each CA, were take at $t = 1000$.
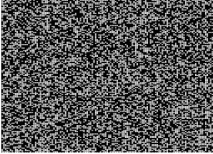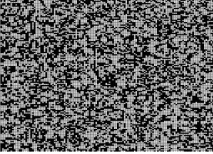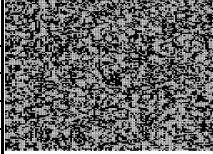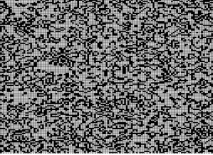
B. CA as PRNG
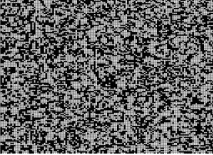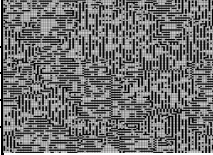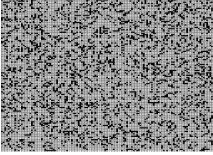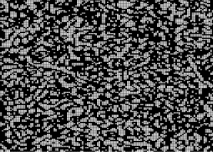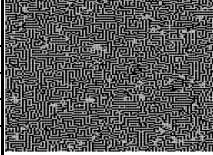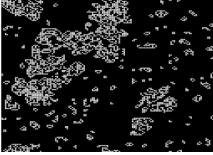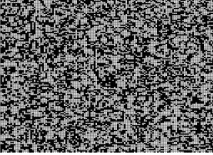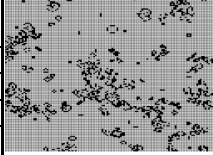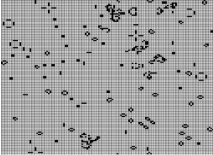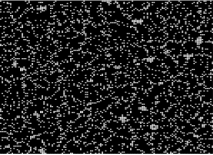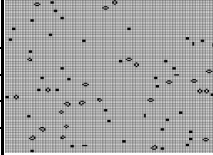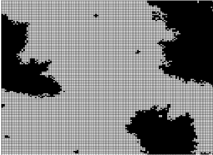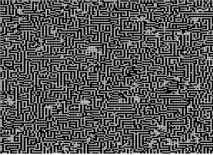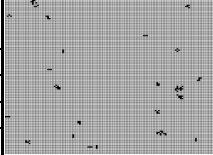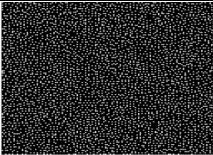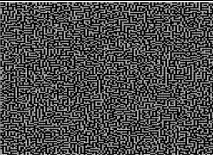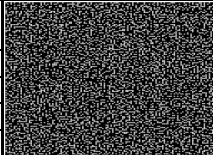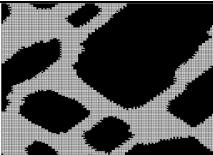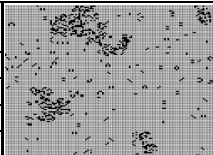
Most PRNGs are tested by suites or tools to analysis its behavior and truly randomness, thus according to Guan's experimental setup[16], all this experiments were tested with ENT[22] and DIEHARD[21] suite.

The ENT test consist of five tests which are the Entropy test, Chi-square test, Serial correlation coefficient (SCC) test, Arithmetic Mean test, and Monte Carlo value for Pi, perhaps the complete details of the ENT tests is beyond the scope of this paper. In the other hand, the DIEHARD contains 18 different and independent statistical tests. The result of each test is called $p - value$. Most of the tests return a uniform p-value on $[0, 1)$ and is considered a "Fail" test, if the p-value is 1 or 0, in other cases can be considered as a "Pass" test [21].

For both testes, was required a minimum of 10 Mb of random numbers, each PRNG received randomly chosen password and then executed until 10 Mb times. Were used 10 types of sets according to the number of blocks ρ, each one with 100 samples for testing.

In Table IV is showed the average results with its standard deviation respectively for different values of ρ, all results are considered "good" PRNG because of the close approximation to theoretical randomness. Thus, for this proposal, we could take the $\rho = 10$ blocks PRNG as the one with better results.

TABLE III
LIFE-LIKE RULES, BASED ON ITS ENTROPY, LYAPUNOV EXPONENT, AND HAMMING DISTANCE

Fredkin B1357/S02468			Amoeba B357/S1358			B23/S36		
Entropy	0,9999		Entropy	0,9830		Entropy	0,9686	
LE	0,9431		LE	0,8604		LE	0,6186	
DH	0,5019		DH	0,4393		DH	0,5461	
Max	0,4733		Max	0,3716		Max	0,3272	
Serviettes B234/S			34 Life B34/S34			Long life B345/S5		
Entropy	0,9513		Entropy	0,9843		Entropy	0,9040	
LE	0,3512		LE	0,5069		LE	0,3518	
DH	0,7417		DH	0,4603		DH	0,6796	
Max	0,2478		Max	0,2297		Max	0,2161	
Seeds B2/S			WalledCities B45678/S2345			Gnarl B1/S1		
Entropy	0,7430		Entropy	0,9973		Entropy	0,7755	
LE	0,5833		LE	0,4113		LE	0,5126	
DH	0,4196		DH	0,4216		DH	0,3811	
Max	0,1819		Max	0,1729		Max	0,1515	
Coagulations B378/S235678			2x2 B36/S125			Pseudo life B357/S238		
Entropy	0,3777		Entropy	0,1800		Entropy	0,0954	
LE	0,6720		LE	0,7053		LE	0,6675	
DH	0,1634		DH	0,1323		DH	0,0932	
Max	0,0415		Max	0,0168		Max	0,0059	
Conway's Life B3/S23			Coral B3/S45678			HighLife B36/S23		
Entropy	0,2120		Entropy	0,6949		Entropy	0,1270	
LE	0,5529		LE	0,4635		LE	0,6184	
DH	0,0477		DH	0,0160		DH	0,0397	
Max	0,0056		Max	0,0051		Max	0,0031	
Day & Night B3678/S34678			Mazectric B3/S1234			Move B368/S245		
Entropy	0,0714		Entropy	1,0000		Entropy	0,0330	
LE	0,6723		LE	0,3045		LE	0,7790	
DH	0,0509		DH	0,0039		DH	0,0265	
Max	0,0024		Max	0,0012		Max	0,0007	
Assimilation B345/S4567			Maze B3/S12345			Flakes B3/S012345678		
Entropy	0,7934		Entropy	1,0000		Entropy	0,9491	
LE	0,0253		LE	0,0200		LE	0,0030	
DH	0,0030		DH	0,0011		DH	0,0001	
Max	0,0001		Max	0,0000		Max	0,0000	
Diamoeba B35678/S5678			Replicator B1357/S1357			Stains B3678/S235678		
Entropy	0,0000		Entropy	0,0000		Entropy	0,0875	
LE	-Infinity		LE	0,8941		LE	-Infinity	
DH	0,0194		DH	0,0320		DH	0,0024	
Max	0,0000		Max	0,0000		Max	0,0000	

In Table V is showed a comparison of the average results using the ENT suite, applied to our proposal based on $\rho = 10$ blocks and others PRNG based on CA found in literature. As it can be seen the proposed scheme is superior to other schemes, in Entropy test and the Serial Correlation Coefficient, although for the Chi-square does not seems to reaches to Shin et. al, [18].

As with previous experiments, the experiments with different number of blocks on DIEHARD test, gave for each one a "Pass" result, thus it has been proven that the quality of randomness for $1, \ldots, 10$ blocks got good results. So, in Table VI is showed a comparison of the DIEHARD test for our proposal by employing $\rho = 10$ blocks and then compared with others PRNG based on CA found in literature.

Should be notice that the sequences of numbers generated by using just 4 blocks, presents a sustain number of blocks that improve cryptography thus by increasing the number of blocks will keeping this results, although will decrease the performance by slowing the encryption/decryption process.

TABLE IV
THE RESULTS OF THE ENT TEST SUIT, PERFORMED FOR DIFFERENT δ BLOCKS

		ENT									
		Proposal PRNG-CA according to the number of Blocks ρ									
Test No.	Test name	1	2	3	4	5	6	7	8	9	10
1	Entropy (Close to 8.0)	7,9999821	7,9999817	7,9999818	7,9999819	7,9999822	7,9999822	7,9999822	7,9999825	7,9999819	7,9999819
	Std Dvt	0,0000012	0,0000014	0,0000013	0,0000012	0,0000012	0,0000014	0,0000013	0,0000013	0,0000015	0,0000015
2	Chi-square (Close to 1.0)	0,997893	0,984747	0,988358	0,995630	0,988156	0,990519	0,990106	0,968390	0,998606	0,998784
	Std Dvt	0,068852	0,073852	0,070944	0,068508	0,065252	0,076053	0,066655	0,071157	0,080108	0,080108
3	SCC (Close to 0.0)	0,00401	0,00004	0,00008	-0,00002	0,00000	0,00005	-0,00004	0,00000	-0,00003	0,00002
	Std Dvt	0,00028	0,00023	0,00025	0,00027	0,00025	0,00023	0,00024	0,00027	0,00021	0,00021
4	Arithmetic Mean (random =0,5)	0,499998	0,499956	0,499991	0,499989	0,499978	0,499998	0,499991	0,499989	0,499988	0,499961
	Std Dvt	0,000069	0,000062	0,000069	0,000067	0,000068	0,000076	0,000081	0,000075	0,000070	0,000070
5	Monte Carlo Value for Pi	3,14173	3,14182	3,14137	3,14154	3,14172	3,14170	3,14163	3,14138	3,14156	3,14127
	Std Dvt	0,00092	0,00102	0,00105	0,00095	0,00097	0,00102	0,00105	0,00110	0,00103	0,00103

TABLE V
A COMPARISON OF THE ENT TEST SUITE RESULTS

		ENT				
Test No.	Test name	Tomassini et al.	Guan et al.	Shin et al.(3D)	Shin et al.	Proposal (10 Blocks)
1	Entropy (Close to 8.0)	7,999710	7,999830	7,999880	7,999910	7,999982
2	Chi-square (Close to 1.0)	0,989412	0,992002	0,999281	0,999283	0,998784
3	SCC (Close to 0.0)	0,000227	0,0001710	0,0000510	0,0000620	0,0000159

Tomassini et. al [14] according to [18];
Guan et. al. (3D) [16] according to [18]; Shin et. al. (3D) [17]; Shin et. al. [18]

TABLE VI
A COMPARISON OF THE RESULTS OF DIEHARD TEST IN P-VALUE PASS RATE 90%

		DIEHARD					
		The results of tests (Pass or Fail)					
Test No.	Test name	Chowdhurry et al.	Tomassini et al.	Guan et al.	Shin et al.(3D)	Shin et al.	Proposal (10 Blocks)
1	Birthday Spacing	Pass	Pass	Pass	Pass	Pass	Pass
2	Over. 5-Per.	Fail	Fail	Fail	Pass	Pass	Pass
3	Binary Rank 31×31	Pass	Pass	Pass	Pass	Pass	Pass
4	Binary Rank 32×32	Pass	Pass	Pass	Pass	Pass	Pass
5	Binary Rank 6×8	Fail	Pass	Pass	Pass	Pass	Pass
6	Bitstream	Pass	Pass	Pass	Pass	Pass	Pass
7	OPSO	Fail	Pass	Pass	Pass	Pass	Pass
8	OQSO	Fail	Fail	Fail	Pass	Pass	Pass
9	DNA	Fail	Pass	Pass	Pass	Pass	Pass
10	Count-The-1's 01	Fail	Fail	Pass	Pass	Pass	Pass
11	Count-The-1's 02	Fail	Pass	Fail	Pass	Pass	Pass
12	Parking Lot	Pass	Pass	Pass	Pass	Pass	Pass
13	Minimum Distance	Pass	Pass	Pass	Pass	Pass	Pass
14	3DS Spheres	Pass	Pass	Pass	Pass	Pass	Pass
15	Squeeze	Fail	Pass	Pass	Pass	Pass	Pass
16	Overlapping Sums	Pass	Pass	Pass	Pass	Pass	Pass
17	Runs	Pass	Fail	Pass	Pass	Pass	Pass
18	Craps	Pass	Pass	Pass	Pass	Pass	Pass

Chowdhury et al. [39] according to [18]; Tomassini et. al [14] according to [18];
Guan et. al. (3D) [16] according to [18]; Shin et. al. (3D) [17]; Shin et. al. [18]

C. Encryption Results

In this experiment the cipher was evaluated encrypting an image, on Figure 2 is presented the original image (left), and the resultant cipher image (right). Visibly the ciphertext is much more uniform and do not bears any resemblance of the plainimage.

Fig. 2. Left: The original plainimage. Right: The resultant cipherimage with the encryption method proposed

To analyze the statistical distribution of the PRNG, was generated an histogram of the plainimage and of the cipherimage which is shown on the left and right of Figure 3 respectively. As it is shown the histogram of the cipherimage is much more regular and almost plain, which demonstrated that all the 256 numbers(ASCCI code) generated with the CA are scattered all over, with almost same percentages regions, by means, hiding information.
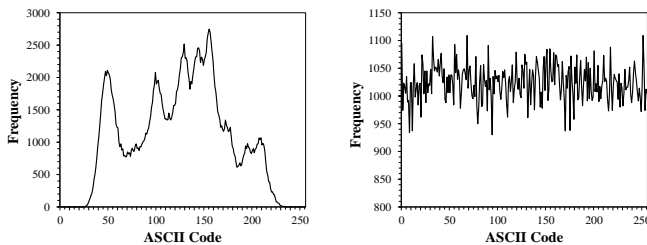

Fig. 3. Frequency Analysis of the plainimage(left) and cipherimage(right) encrypted with the proposal method

D. Fourier's Power Spectrum

On Figure 4, for both the plainimage and cipherimage, has been performed and compared to 2D Fourier power spectrum, as we can seen the spectrum of the cipherimage minimal presents a white noise, which it is said to presents any frequency information, demonstrating any compressible information can be achieved in the cipher image[7].

VI. Conclusions

In this paper, a chaotic encryption method based on the "Life-Like" Cellular Automata was proposed. The cryptosystem we have described is based on two-dimensional CA which is able to generate pseudo-random numbers of very good quality as measured by both: chaotic measures(Lyapunov Exponent, Entropy, Hamming distance, and combination of them) and statistical tests(DIEHARD and ENT test suites), as
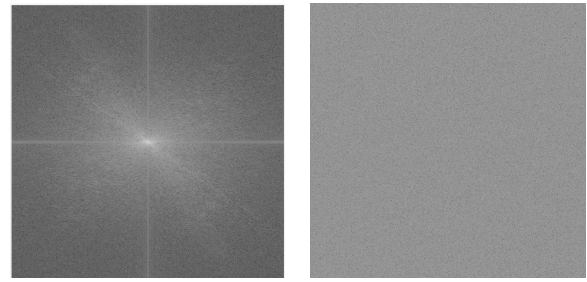

Fig. 4. 2D Fourier power spectrum for the plainimage(left) and cipherimage(right)

they achieve better randomness quality than others based on CA compared in literature.

Here were described chaotic properties of CA as good reasons to be employed in cryptography, furthermore, its simplicity and low cost of implementation as one of its main advantages.

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