Lösungen Testat STOC SW03

Daniel Winz

14. März 2013 13:22

Inhaltsverzeichnis

1 Aufgabe 1

1.1 a

$$S_{xy} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{n-1}$$
$$= \frac{\sum_{i=1}^{n}(x_i \cdot y_i - x_i \cdot \overline{y} - \overline{x} \cdot y_i + \overline{x} \cdot \overline{y})}{n-1}$$
$$= \frac{\sum_{i=1}^{n}(x_i \cdot y_i) - \sum_{i=1}^{n}(x_i \cdot \overline{y}) - \sum_{i=1}^{n}(\overline{x} \cdot y_i) + \sum_{i=1}^{n}(\overline{x} \cdot \overline{y})}{n-1}$$
$$= \frac{\sum_{i=1}^{n}(x_i \cdot y_i) - \overline{y} \cdot \sum_{i=1}^{n}(x_i) - \overline{x} \cdot \sum_{i=1}^{n}(y_i) + n \cdot \overline{x} \cdot \overline{y}}{n-1}$$
$$= \frac{\sum_{i=1}^{n}(x_i \cdot y_i) - \overline{y} \cdot n \cdot \overline{x} - \overline{x} \cdot n \cdot \overline{y} + n \cdot \overline{x} \cdot \overline{y}}{n-1}$$
$$= \frac{\sum_{i=1}^{n}(x_i \cdot y_i) - n \cdot \overline{x} \cdot \overline{y}}{n-1}$$

Neuer Versuch

$$\hat{a} = \bar{y} - \hat{b} \cdot \bar{x}$$
$$\hat{b} = \frac{s_{xy}}{s_x^2}$$
$$s_{xy} = \hat{b} \cdot {s_x}^2$$
$$s_{xy} = \frac{\bar{y} - \hat{a}}{\bar{x}} \cdot {s_x}^2$$

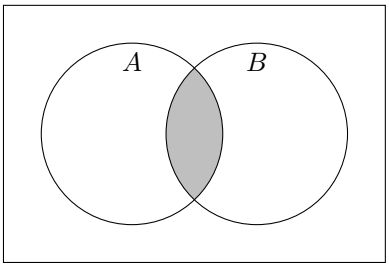
1.2 b

$$r = \frac{S_{xy}}{S_x \cdot S_y}$$

2 Aufgabe 2

2.1 a

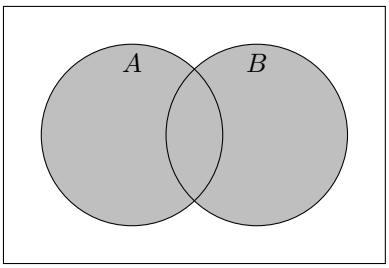
$$P(A \cap B) = P(A) \cdot P(B) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$



```
\begin{venndiagram2sets}
  \fillACapB
\end{venndiagram2sets}
```

2.2 b

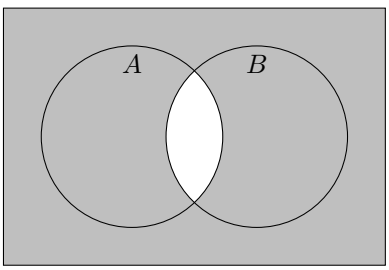
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{4} + \frac{2}{3} - \frac{2}{4} = \frac{11}{12}$$




```
\begin{venndiagram2sets}
  \fillA \fillB
\end{venndiagram2sets}
```

2.3 c

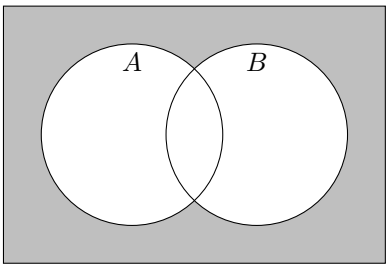
$$P(\overline{A \cap B}) = P(\Omega) - P(A \cap B) = 1 - P(A \cap B) = 1 - \frac{1}{2} = \frac{1}{2}$$




```
\begin{venndiagram2sets}
  \fillNotAorB \fillANotB \fillBNotA
\end{venndiagram2sets}
```

2.4 d

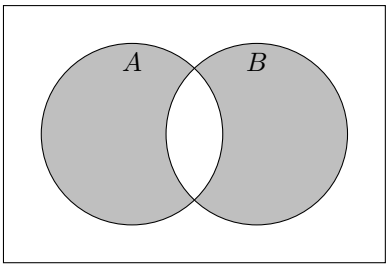
$$P(\overline{A \cup B}) = P(\Omega) - P(A \cup B) = 1 - P(A \cup B) = 1 - \frac{11}{12} = \frac{1}{12}$$




```
\begin{venndiagram2sets}
  \fillNotAorB
\end{venndiagram2sets}
```

2.5 e

$$P(A \cup B) = P(A) + P(B) - 2 \cdot P(A \cap B) = \frac{3}{4} + \frac{2}{3} - 2 \cdot \frac{2}{4} = \frac{5}{12}$$




```
\begin{venndiagram2sets}
  \fillANotB \fillBNotA
\end{venndiagram2sets}
```

3 Aufgabe 3

	F	KF
A	$\frac{0.95}{356}$	$\frac{0.01 \cdot 355}{356}$
KA	$\frac{0.05}{356}$	$\frac{351.45}{356}$

$\Rightarrow$

	F	KF
A	0.0026685	0.0099719
KA	0.00014045	0.98722

3.1 a

$$P(F|A) = \frac{P(A \cap F)}{P(A)} = 0.2111$$

3.2 b

$$P(KF|KA) = \frac{P(KF \cap KA)}{P(KA)} = 0.99$$

4 Aufgabe 4

4.1 a

$\{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36,$
$41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$

4.2 b

$$\frac{1}{36}$$

4.3 c

$$E_1 = \{16, 25, 34, 43, 52, 61\}$$

$$P(E_1) = \frac{6}{36} = \frac{1}{6}$$

4.4 d

$$E_2 = \{11, 12, 21\}$$

$$P(E_2) = \frac{3}{36} = \frac{1}{12}$$

4.5 e

$$E_3 = \{11, 13, 15, 31, 33, 35, 51, 53, 55\}$$

$$P(E_3) = \frac{9}{36} = \frac{1}{4}$$

4.6 f

$$E_2 \cup E_3 = \{11, 12, 13, 15, 21, 31, 33, 35, 51, 53, 55\}$$

$$P(E_3) = \frac{11}{36}$$

5 Aufgabe 5

5.1 a

$P(\text{Kopf}) + P(\text{Zahl})$ muss 1 ergeben.

5.2 b

Eine Wahrscheinlichkeit kann nicht negativ sein.

5.3 c

Da $\{S \cap M\}$ nicht existiert muss $P(S) + P(M) = P(S \cup M)$ sein.

6 Aufgabe 6

6.1 a

	E	N
w	$P(w \cap E) = 0.210226$	$P(w \cap N) = 0.303774$
m	$P(m \cap E) = 0.280908$	$P(m \cap N) = 0.205092$

6.2 b

$$P(w|E) = \frac{P(w \cup E)}{P(E)} = 0.428042$$

6.3 c

$$P(w|E) = \frac{P(w \cup E)}{P(E)} = 0.428042$$

$$P(m|E) = \frac{P(m \cup E)}{P(E)} = 0.571958$$

$$P(w|N) = \frac{P(w \cup N)}{P(N)} = 0.596963$$

$$P(m|N) = \frac{P(m \cup N)}{P(N)} = 0.403037$$