Lösungen Testat STOC SW04

Daniel Winz

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1 Aufgabe 1

Allgemein:

$$P_\lambda(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

mit $\lambda = 200$

1.1 a

$$P_\lambda(200) = \frac{200^200}{200!} \cdot e^{-200} = 0.28198$$

R:

```
> dpois(200,200)
```

```
[1] 0.02819773
```

1.2 b

$$P_\lambda(\leq 3) = \sum_{j=0}^{210}(P_\lambda(j)) = 0.77271$$

R:

```
> sum(dpois(0:210,200))
```

```
[1] 0.772708
```

oder

```
> ppois(210,200)
```

```
[1] 0.772708
```

1.3 c

$$P_\lambda(\leq 3) = \sum_{j=190}^{210}(P_\lambda(j)) = 0.77271$$

R:

```
> sum(dpois(190:210,200))
```

```
[1] 0.5422097
```

oder

```
> ppois(210,200)-ppois(189,200)
```

```
[1] 0.5422097
```

2 Aufgabe 2

Allgemein:

$$P_\lambda(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

mit $\lambda = 2$

2.1 a

$$P_\lambda(0) = \frac{2^0}{0!} \cdot e^{-2} = 0.135$$

R:

```
> dpois(0,2)
```

```
[1] 0.1353353
```

2.2 b

$$P_\lambda(\leq 3) = \sum_{j=0}^{3}(P_\lambda(j)) = 0.857$$

R:

```
> sum(dpois(0:3,2))
```

```
[1] 0.8571235
```

oder

```
> ppois(3,2)
```

```
[1] 0.8571235
```

2.3 c

$$P_\lambda(> 3) = 1 - P_\lambda(\leq 3) = 0.143$$

R:

```
> 1-sum(dpois(0:3,2))
```

```
[1] 0.1428765
```

oder

```
> 1-ppois(3,2)
```

```
[1] 0.1428765
```

2.4 d

Poissonverteilung mit Erwartungswert $\lambda = 12 \quad Y \sim \text{Poisson}(\lambda)$

3 Aufgabe 3

3.1 a

$$P(X_1 = x_1 \cap X_2 = x_2) = P(X_1 = x_1) \cdot P(X_2 = x_2)$$

$$P(X_m = x_m) = \frac{n_m!}{x_m! \cdot (n_m - x_m)!} \cdot \pi^{x_m} \cdot (1-\pi)^{n_m - x_m}$$

$$P(X_1 = x_1) \cdot P(X_2 = x_2) = \frac{n_1!}{x_1! \cdot (n_1 - x_1)!} \cdot \pi^{x_1} \cdot (1-\pi)^{n_1 - x_1} \cdot \frac{n_2!}{x_2! \cdot (n_2 - x_2)!} \cdot \pi^{x_2} \cdot (1-\pi)^{n_2 - x_2}$$

3.2 b

$$\log(P(X_1 = x_1 \cap X_2 = x_2)) = \log(P(X_1 = x_1) \cdot P(X_2 = x_2))$$

$$= \log\left(\tfrac{n_1!}{x_1! \cdot (n_1 - x_1)!}\right) + \log(\pi^{x_1}) + \log((1-\pi)^{n_1 - x_1})$$
$$+ \log\left(\tfrac{n_2!}{x_2! \cdot (n_2 - x_2)!}\right) + \log(\pi^{x_2}) + \log((1-\pi)^{n_2 - x_2})$$

$$= \log(n_1!) - \log(x_1!) - \log((n_1 - x_1)!) + x_1 \cdot \log(\pi) + (n_1 - x_1) \cdot \log(1-\pi)$$
$$+ \log(n_2!) - \log(x_2!) - \log((n_2 - x_2)!) + x_2 \cdot \log(\pi) + (n_2 - x_2) \cdot \log(1-\pi)$$

3.3 c

$$\frac{d}{d\pi}\left(\log(P(X_1 = x_1 \cap X_2 = x_2))\right)$$

$$= \tfrac{d}{d\pi}\left(\log(n_1!) - \log(x_1!) - \log((n_1 - x_1)!) + x_1 \cdot \log(\pi) + (n_1 - x_1) \cdot \log(1-\pi)\right.$$
$$\left.+ \log(n_2!) - \log(x_2!) - \log((n_2 - x_2)!) + x_2 \cdot \log(\pi) + (n_2 - x_2) \cdot \log(1-\pi)\right)$$

$$= \frac{x_1}{\ln(10) \cdot \pi} + \frac{n_1 - x_1}{\ln(10) \cdot (1-\pi)} + \frac{x_2}{\ln(10) \cdot \pi} + \frac{n_2 - x_2}{\ln(10) \cdot (1-\pi)}$$

$$= log(e) \cdot \left(\frac{x_1}{\pi} + \frac{n_1 - x_1}{1-\pi} + \frac{x_2}{\pi} + \frac{n_2 - x_2}{1-\pi}\right)$$

$$= log(e) \cdot \left(\frac{x_1 + x_2}{\pi} + \frac{n_1 - x_1 + n_2 - x_2}{1-\pi}\right) \overset{!}{=} 0$$

$$\frac{x_1 + x_2}{\pi} + \frac{n_1 - x_1 + n_2 - x_2}{1-\pi} = 0$$

$$\frac{1-\pi}{\pi} = -\frac{n_1 - x_1 + n_2 - x_2}{x_1 + x_2}$$

$$\frac{1}{\pi} - 1 = -\frac{n_1 - x_1 + n_2 - x_2}{x_1 + x_2}$$

$$\frac{1}{\pi} = -\frac{n_1 - x_1 + n_2 - x_2}{x_1 + x_2} + 1$$

$$\pi = -\frac{1}{\frac{n_1 - x_1 + n_2 - x_2}{x_1 + x_2} + 1}$$

$$\pi = -\frac{1}{\frac{30 - 2 + 50 - 4}{2 + 4} + 1} = \frac{3}{40} = 0.075$$

4 Aufgabe 4

4.1 a

genau 2 Patienten:

```
> dbinom(2,size=10,prob=0.3)
```

```
[1] 0.2334744
```

mindestens 2 Patienten:

```
> pbinom(2,size=10,prob=0.3)
```

```
[1] 0.3827828
```

4.2 b

1. Modell:

$$X \sim \text{Bin}(n = 10, \pi)$$

2. Nullhypothese:

$$\begin{aligned} H_0 &: \pi_0 = 0.3 \\ H_A &: \pi > \pi_0 \end{aligned}$$

3. Teststatistik:

$$X : P(X = x|H_0) = \begin{pmatrix} 10 \\ x \end{pmatrix} \cdot 0.3^x \cdot 0.7^{10-x}$$

4. $\alpha = 0.05$

5. $P(X = x)$ in Abhängigkeit von x:

```
> dbinom(0:10,size=10,prob=0.3)
```

```
[1] 0.0282475249 0.1210608210 0.2334744405 0.2668279320 0.2001209490 0.1029193452
[7] 0.0367569090 0.0090016920 0.0014467005 0.0001377810 0.0000059049
```

$$K = \{6, 7, 8, 9, 10\}$$

6. $x = 4$

$$\Rightarrow H_0 \text{ behalten}$$

4.3 c

$\pi = 0.3$

```
> pbinom(6,size=10,prob=0.3)
```

```
[1] 0.9894079
```

$\pi = 0.6$

```
> pbinom(6,size=10,prob=0.6)
```

```
[1] 0.6177194
```

5 Aufgabe 5

1. Modell:

$$X \sim \text{Bin}(n = 50, \pi)$$

2. Nullhypothese:

$$\begin{aligned} H_0 &: \pi_0 = 0.1 \\ H_A &: \pi < \pi_0 \end{aligned}$$

3. Teststatistik:

$$X : P(X = x|H_0) = \begin{pmatrix} 50 \\ 3 \end{pmatrix} \cdot 0.1^3 \cdot 0.9^{50-3}$$

4. $\alpha = 0.05$

5. $P(X = x)$ in Abhängigkeit von x:

```
> dbinom(0:50,size=50,prob=0.1)

 [1] 5.153775e-03 2.863208e-02 7.794290e-02 1.385651e-01 1.809045e-01 1.849246e-01
 [7] 1.541038e-01 1.076281e-01 6.427788e-02 3.332927e-02 1.518333e-02 6.134680e-03
[13] 2.215301e-03 7.194996e-04 2.112816e-04 5.634176e-05 1.369418e-05 3.043151e-06
[19] 6.199011e-07 1.160049e-07 1.997862e-08 3.171209e-09 4.644701e-10 6.282687e-11
[25] 7.853358e-12 9.074992e-13 9.695504e-14 9.575807e-15 8.739824e-16 7.366901e-17
[31] 5.729812e-18 4.107392e-19 2.709738e-20 1.642265e-21 9.123696e-23 4.634258e-24
[37] 2.145490e-25 9.020078e-27 3.428685e-28 1.172200e-29 3.581722e-31 9.706563e-33
[43] 2.311087e-34 4.777440e-36 8.444969e-38 1.251107e-39 1.510998e-41 1.428840e-43
[49] 9.922500e-46 4.500000e-48 1.000000e-50
```

$$K = \{0, 1\}$$

6. $x = 3$

$$\Rightarrow H_0 \text{ behalten}$$

Um mit einem Signifikanzniveau 5 darf maximal ein Glas minderwertig sein.

6 Aufgabe 6

6.1 a

Macht:

```
> pbinom(1,size=50,prob=0.075)

[1] 0.1025006
```

6.2 b

Verwerfungsbereich:

```
> pbinom(0:50,size=150,prob=0.1)
```

```
 [1] 1.368915e-07 2.418416e-06 2.130437e-05 1.248274e-04 5.475463e-04 1.919034e-03
 [7] 5.601734e-03 1.401933e-02 3.073762e-02 6.004622e-02 1.059630e-01 1.708959e-01
[13] 2.544669e-01 3.530378e-01 4.602141e-01 5.681844e-01 6.694064e-01 7.580585e-01
[19] 8.308407e-01 8.870234e-01 9.279120e-01 9.560364e-01 9.743599e-01 9.856903e-01
[25] 9.923522e-01 9.960829e-01 9.980758e-01 9.990927e-01 9.995891e-01 9.998211e-01
[31] 9.999251e-01 9.999698e-01 9.999883e-01 9.999956e-01 9.999984e-01 9.999994e-01
[37] 9.999998e-01 9.999999e-01 1.000000e+00 1.000000e+00 1.000000e+00 1.000000e+00
[43] 1.000000e+00 1.000000e+00 1.000000e+00 1.000000e+00 1.000000e+00 1.000000e+00
[49] 1.000000e+00 1.000000e+00 1.000000e+00
```

$$K = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

Macht:

```
> pbinom(8,size=150,prob=0.075)
```

```
[1] 0.2000952
```