Fitness-Landscape

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$$\Psi(v) = \frac{1}{k} \sum \phi_i(v)$$

$$\phi_i(v) = \frac{M}{1 + p\underbrace{(v_i - \mu_i(v))^2}}$$

$$\mu_i(v) = c_i + \sum b_{ij} v_j \longrightarrow$$ this is relevant ①

gradient walk stops if $\frac{\partial \Psi(v)}{\partial v^j} = 0 \quad \forall j \in \{1, n\}$

① can be rewritten in Matrix form

$$\underbrace{v - (c + Bv)}$$

with v the vector of trait values, c the vector of trait optima and B the $n \times n$ trait "interaction" matrix.

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} - \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} - \begin{pmatrix} 0 & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & 0 & & & \\ b_{n1} & b_{n2} & & \cdots & 0 \end{pmatrix} \begin{bmatrix} v_1 \\ \\ v_n \end{bmatrix}$$

* (By definition, $B_{ii} = 0$)

$$= \begin{bmatrix} v_1 & -b_{12}v_2 - b_{13}v_3 & \cdots & -b_{1n}v_n - c_1 \\ -b_{21}v_1 & v_2 - b_{23}v_3 & \cdots & -b_{2n}v_n - c_2 \\ \vdots & & \vdots & \vdots \\ -b_{n1}v_1 & -b_{n2}v_2 & \cdots & v_n - c_n \end{bmatrix}$$

Incorporating the ^{2} in ① and considering the derivative (Each row is squared and we are interested in the derivative of row i with respect to v_i)

$$\downarrow {}^2_{\text{row}}$$

$$\begin{bmatrix} v_1^2 - 2b_{12}v_1v_2 - 2b_{13}v_1v_3 \cdots - 2b_{1n}v_1v_n - v_1c_1 \\ -2b_{21}v_1v_2 \quad v_2^2 - 2b_{23}v_2v_3 \cdots - 2b_{2n}v_2v_n - v_2c_1 \\ \vdots \qquad \vdots \\ -2b_{n1}v_1v_n \quad -2b_{n2}v_2v_n \quad \cdots \quad v_n^2 - v_nc_n \end{bmatrix} \xrightarrow[\text{to } v_i]{\text{Derivative with respect}} \begin{bmatrix} 2v_1 - 2b_{12}v_2 - 2b_{13}v_3 \cdots - 2b_{1n}v_n - c_1 \\ -2b_{21}v_1 \quad 2v_2 - 2b_{23}v_3 - 2b_{2n}v_n - c_2 \\ \\ -2b_{n1}v_1 \quad -2b_{n2}v_2 \quad \cdots \quad 2v_n - c_n \end{bmatrix} = 0$$

$$\boxed{(-B + I)v = c}$$

Solve to get trait fix points.

<u>Problem:</u> I guess this linear system of equations can never have more than one optimum (hence Landscape is not rugged)