FORMULE

$\forall\, x_1, x_2 \in \mathbb{R},$

(1) $\cos(x_1 + x_2) = \cos x_1 \cos x_2 - \sin x_1 \sin x_2$

(2) $\sin(x_1 + x_2) = \sin x_1 \cos x_2 + \cos x_1 \sin x_2$

(1) $\Rightarrow \cos(2x) = \cos^2 x - \sin^2 x$

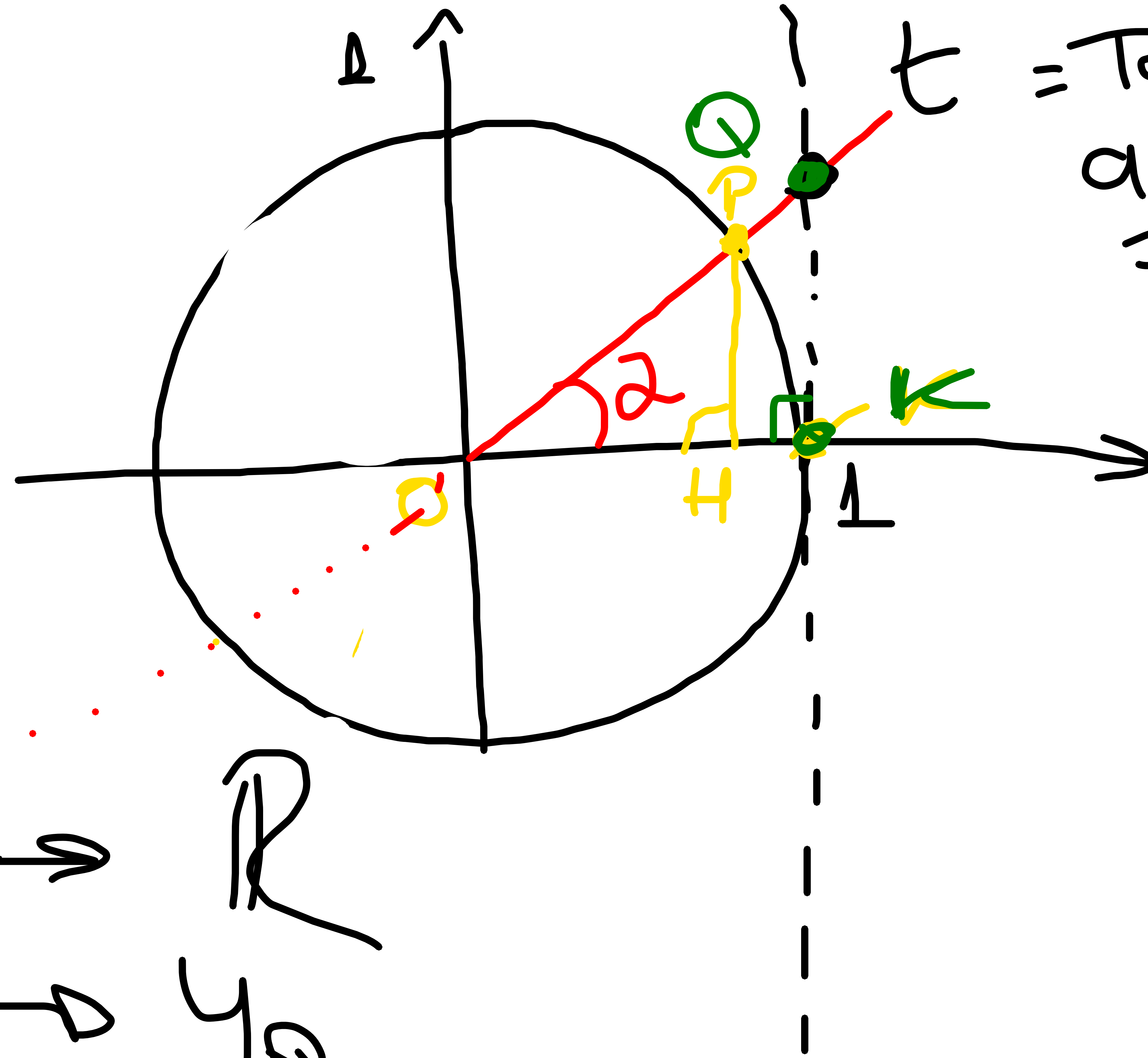
(2) $\Rightarrow \sin(2x) = 2 \sin x \cos x$

$$\left|\cos\left(\frac{x}{2}\right)\right| = \sqrt{\frac{1+\cos x}{2}}$$

$$\left|\sin\left(\frac{x}{2}\right)\right| = \sqrt{\frac{1-\cos x}{2}}$$

TANGENTE

U = CIRCONF. GONIOMETRICA



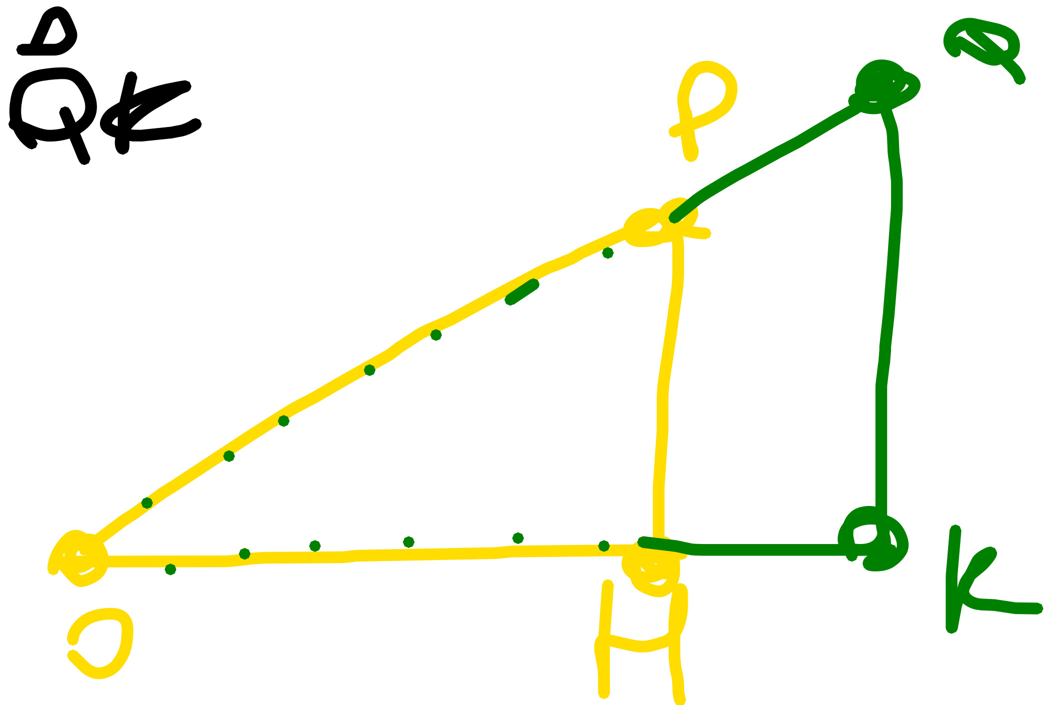
t = Tangente a U nel punto $(1,0)$

$Q = (1, y_Q)$

$$tg: D \longrightarrow \mathbb{R}$$
$$\alpha \longmapsto y_Q$$

$$D = \left\{ x \in \mathbb{R} \,\middle|\, x \neq \frac{\pi}{2} + k\pi ,\ k \in \mathbb{Z} \right\}$$

I due triangoli $O\hat{P}H$ e $O\hat{Q}K$ sono simili, dunque:



$$\frac{QK}{OK} = \frac{PH}{OH}$$

$$\frac{tg\alpha}{1} = \frac{sen\alpha}{\cos\alpha}$$

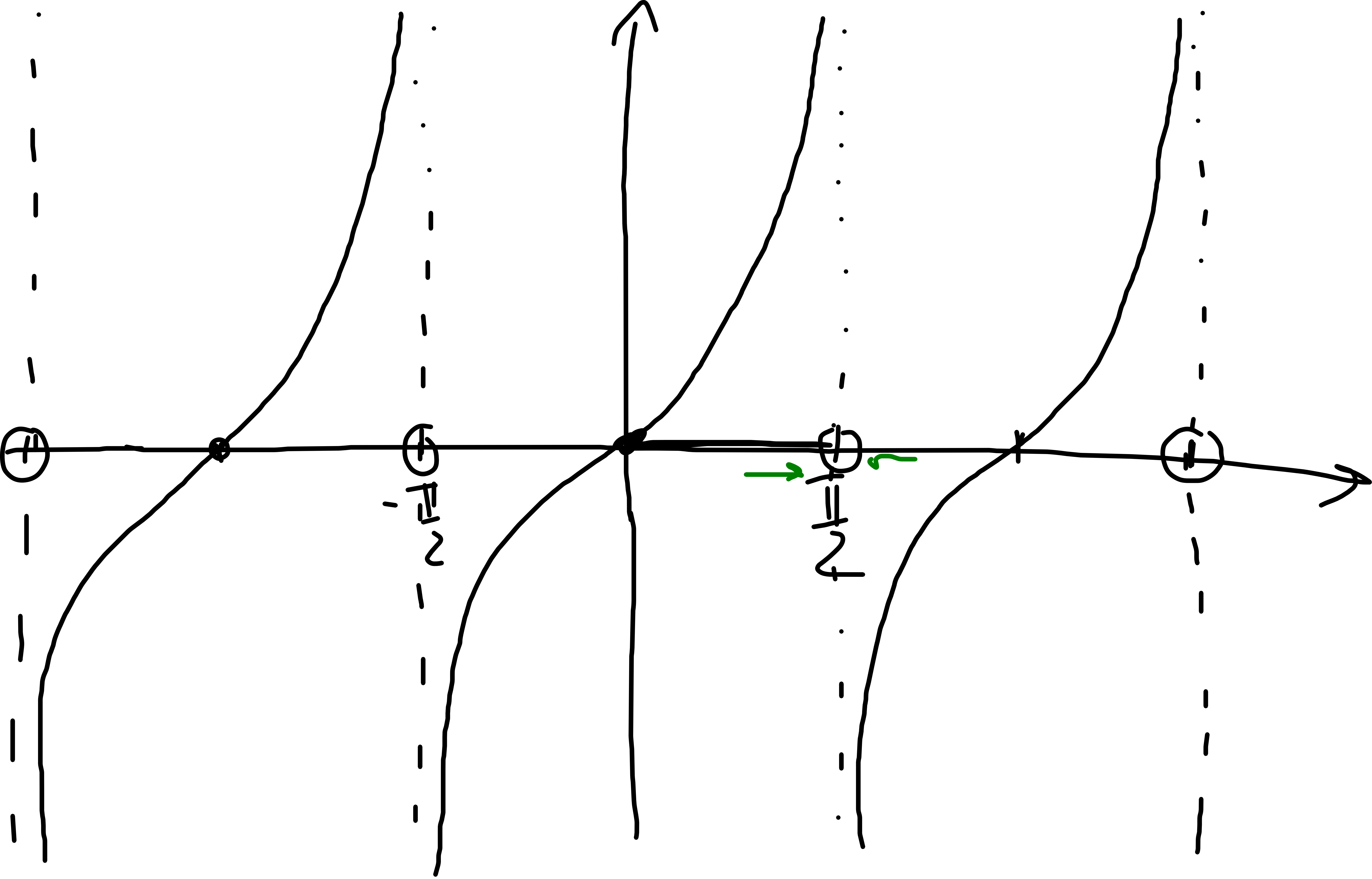
$$\Rightarrow Tg\alpha = \frac{sen\alpha}{\cos\alpha}$$

(es $Tg(-\alpha) = -Tg(\alpha)$)

Inoltre tg è PERIODICA di PERIODO π:

$$tg(\alpha+\pi)=tg\alpha$$

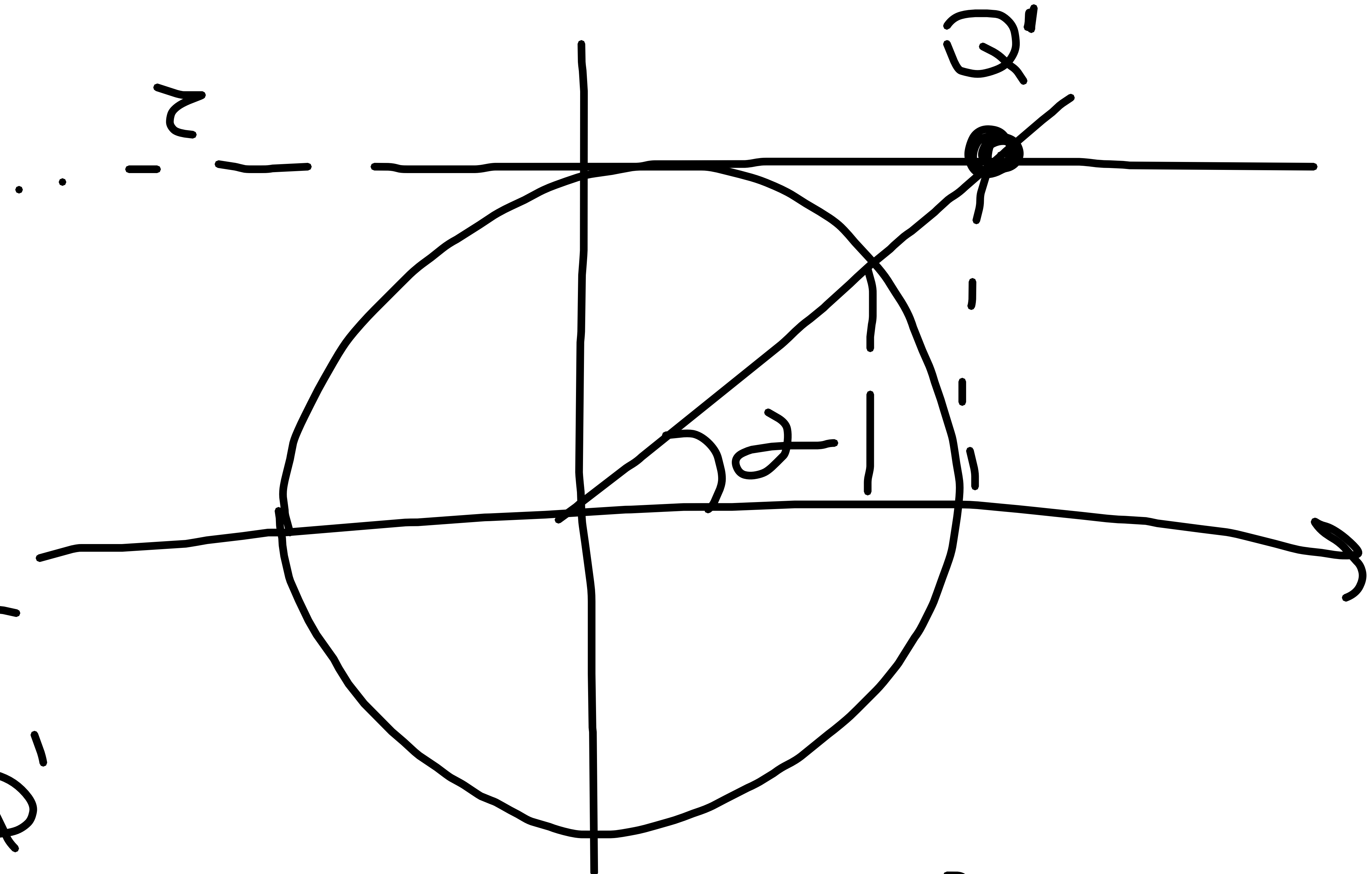
Dalla periodicità e dal fatto che Tg è dispari, basta studiarla nell'intervallo $[0,\frac{\pi}{2}[$



$-\frac{\pi}{2}$
$\frac{\pi}{2}$

COTANGENTE

$Q' = (x_{Q'}, 1)$



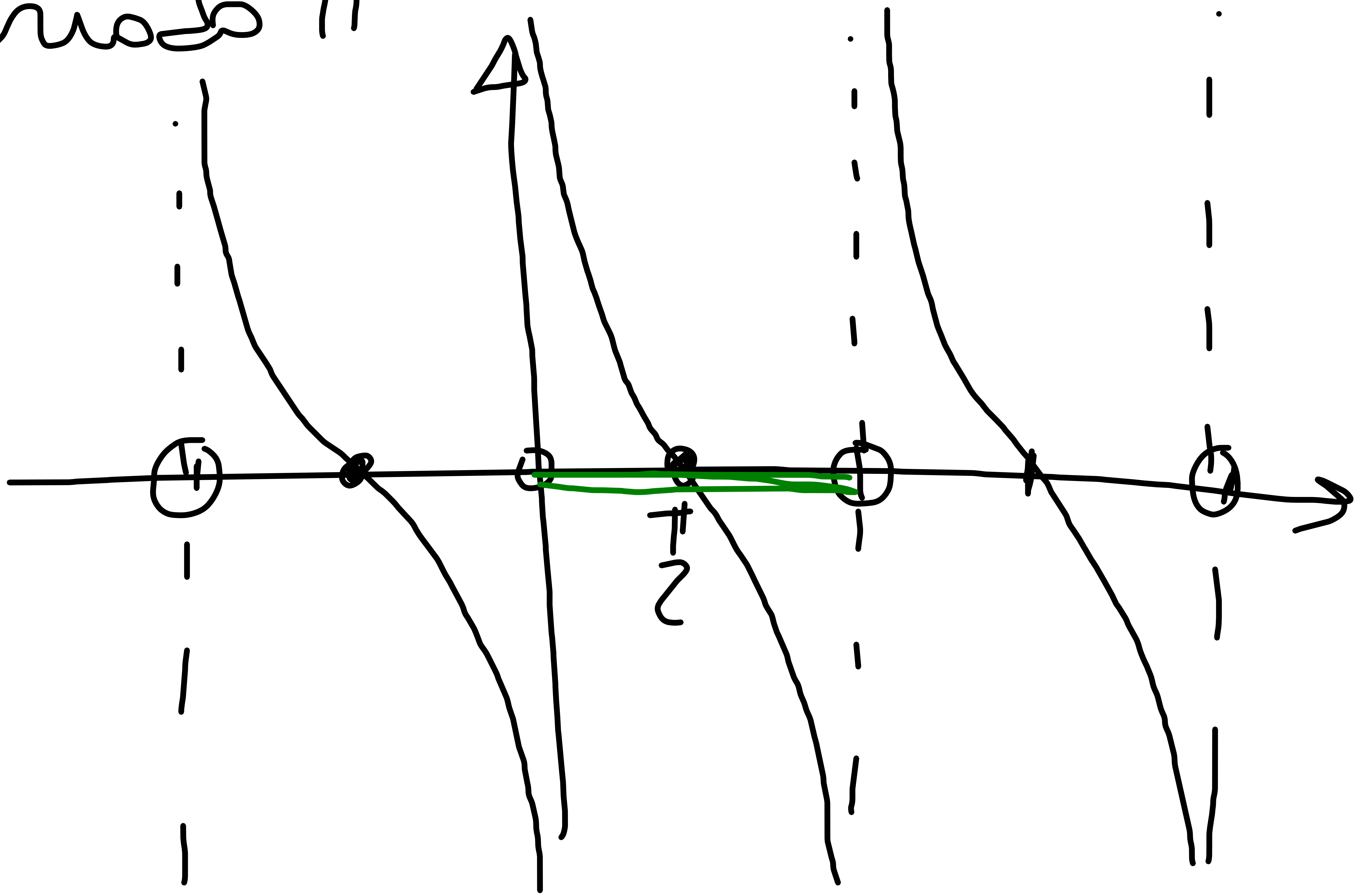
$\cot g : D \longrightarrow \mathbb{R}$

$\alpha \longmapsto x_{Q'}$

$D = \{x \in \mathbb{R} \mid x \neq k\pi,\ k \in \mathbb{Z}\}$

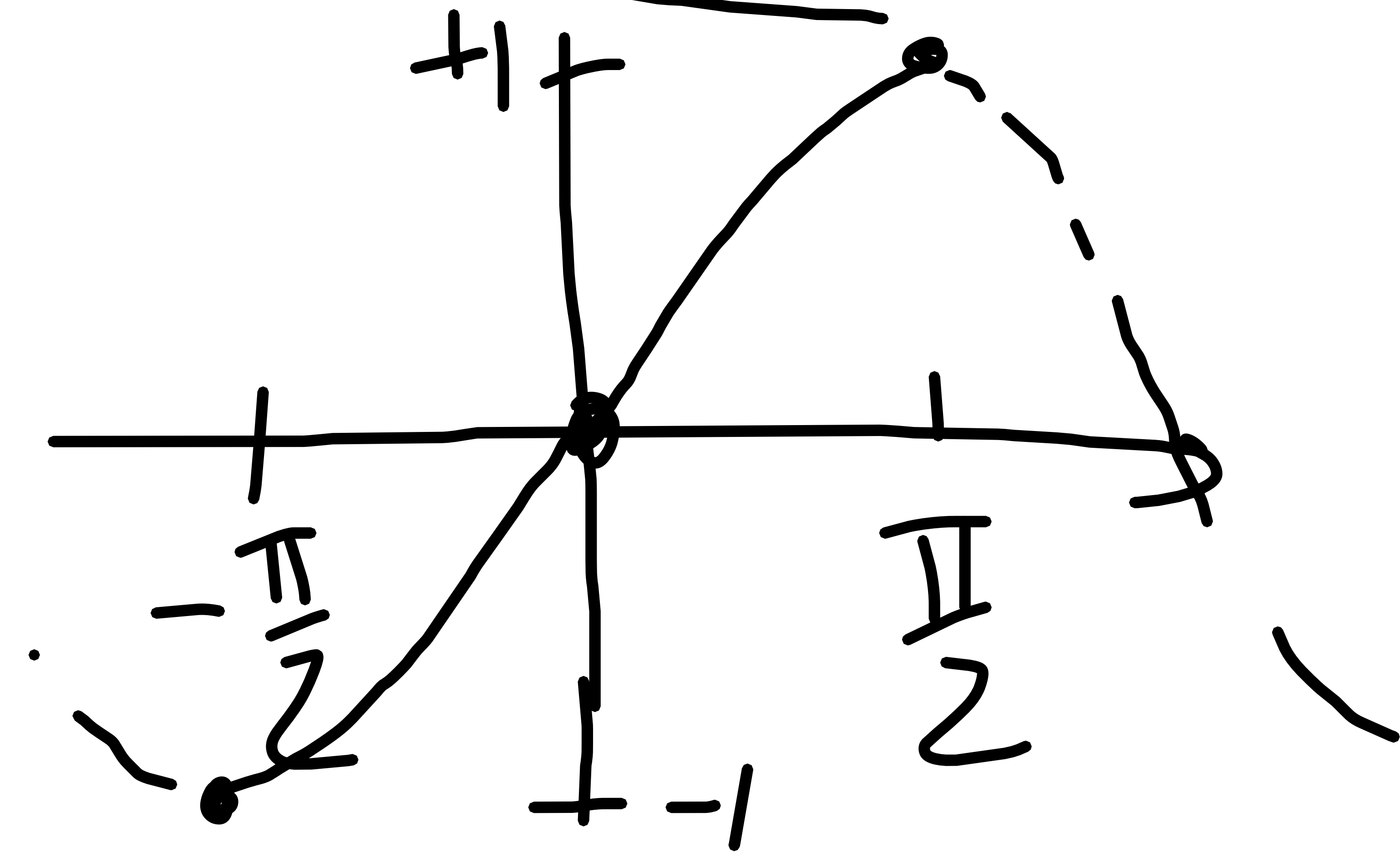
Analogamente a prima si vede che $\cot g\,\alpha = \dfrac{\cos\alpha}{\operatorname{sen}\alpha}$

cotg è dispari e periodica di periodo π



FUNZIONI TRIGONOMETRICHE INVERSE

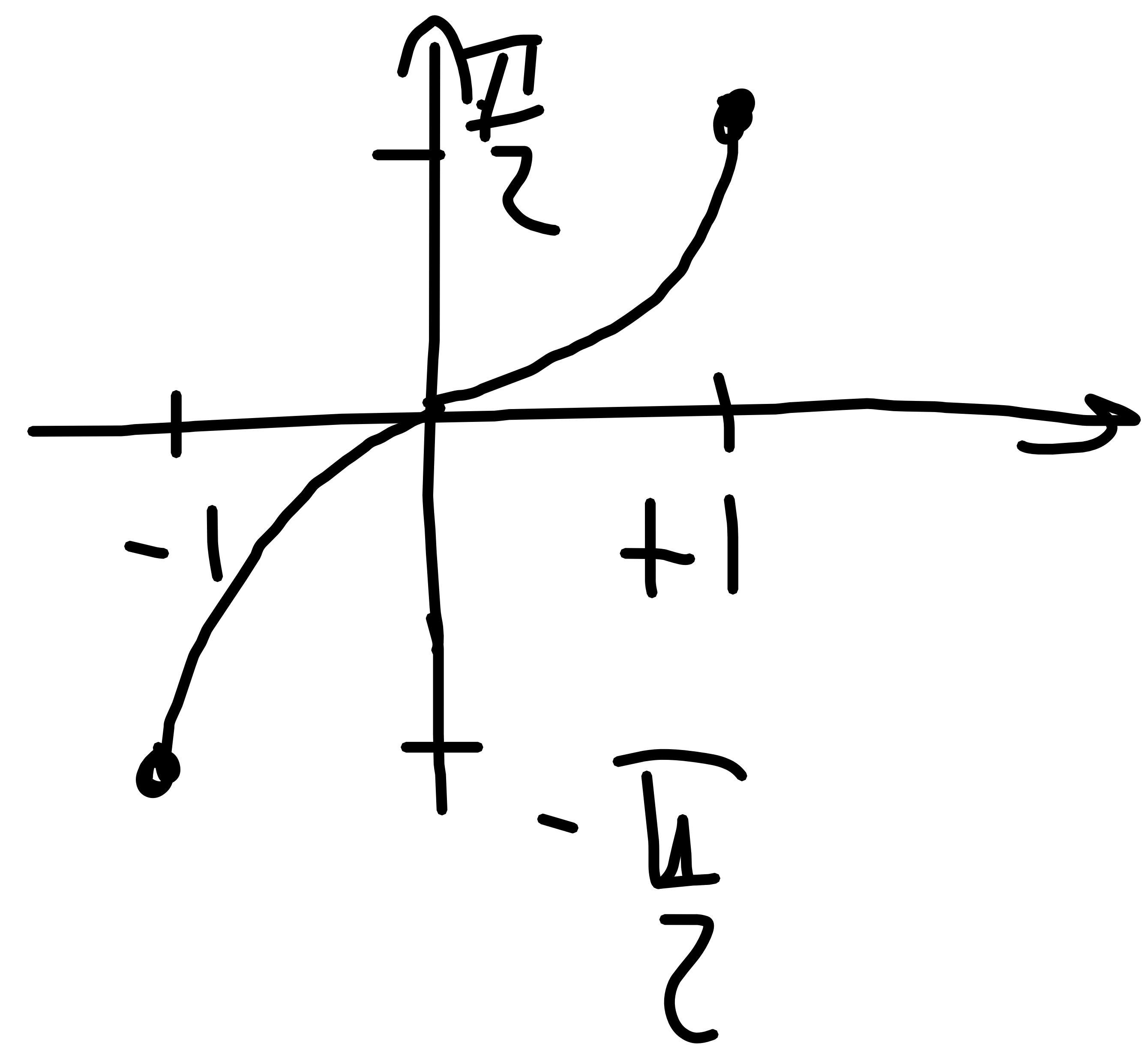
$\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \xrightarrow[\text{su}]{1-1} [-1, 1]$



(è invertibile)

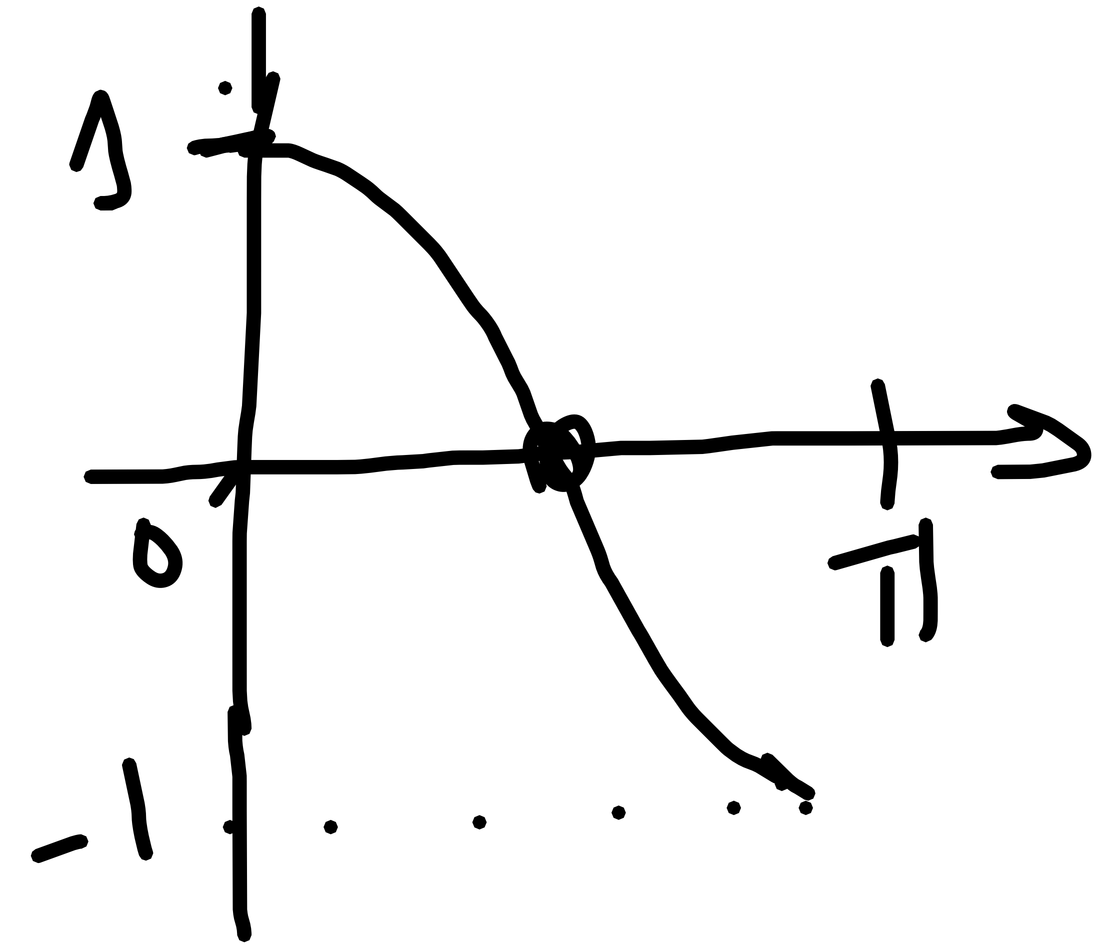
Chiamo (arcoseno)

$\arcsin : [-1, 1] \xrightarrow[\text{su}]{1-1} \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



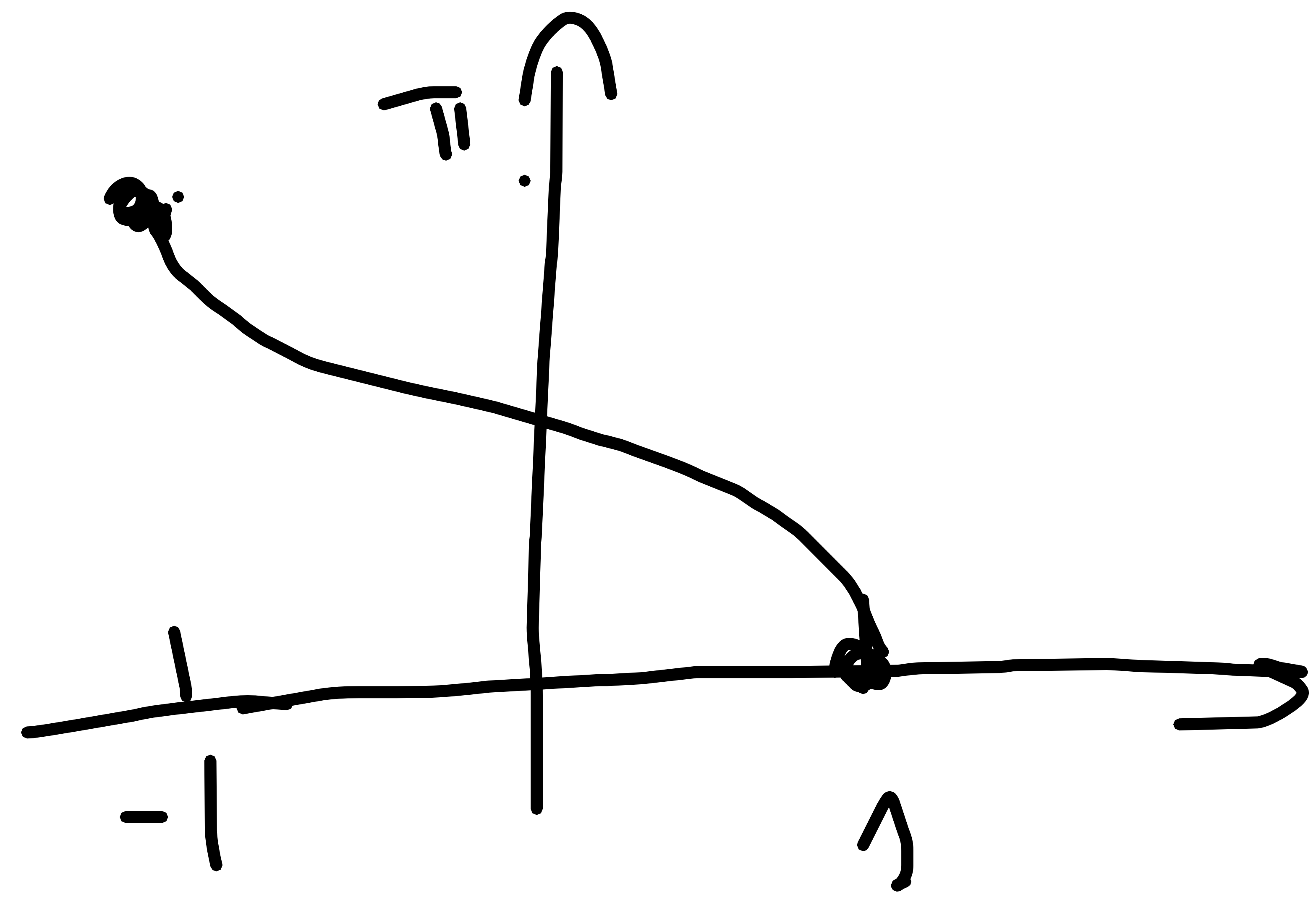
la sua INVERSA

$$\cos : [0, \pi] \xrightarrow[\text{su}]{1-1} [-1, 1]$$



La sua è inversa è

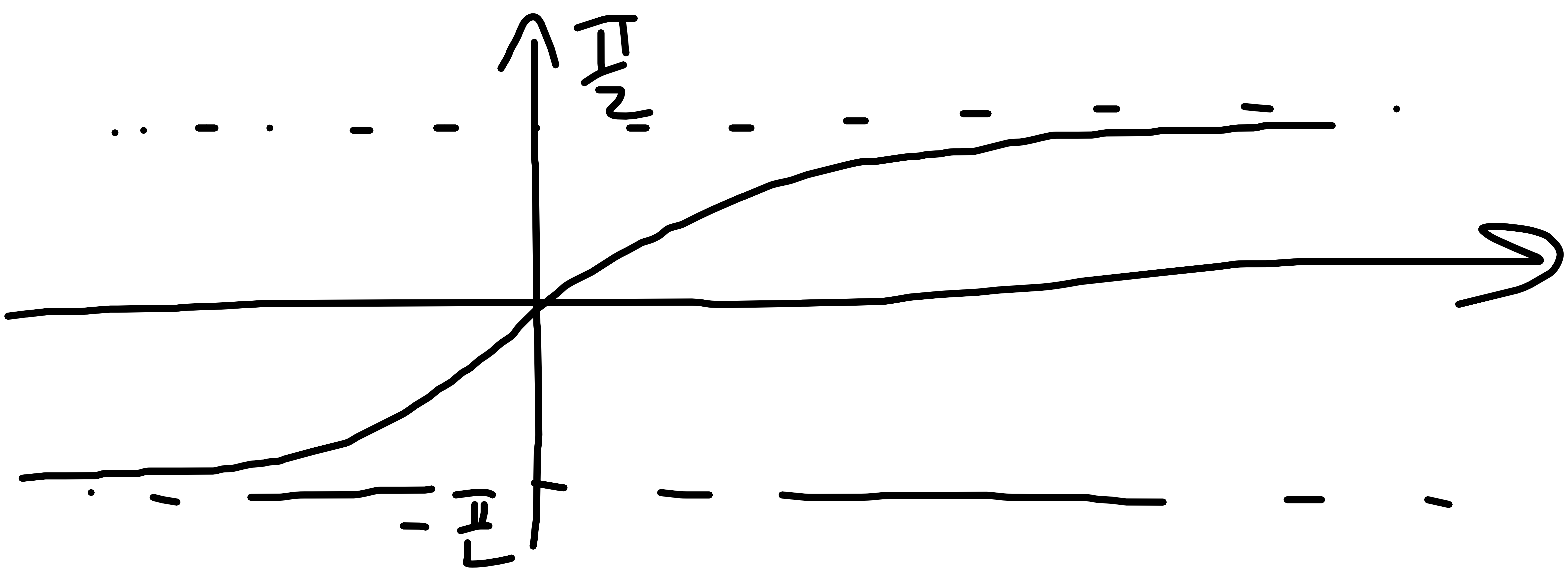
$$\underline{\arccos} : [-1, 1] \xrightarrow[\text{su}]{1-1} [0, \pi]$$



$tg : \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$

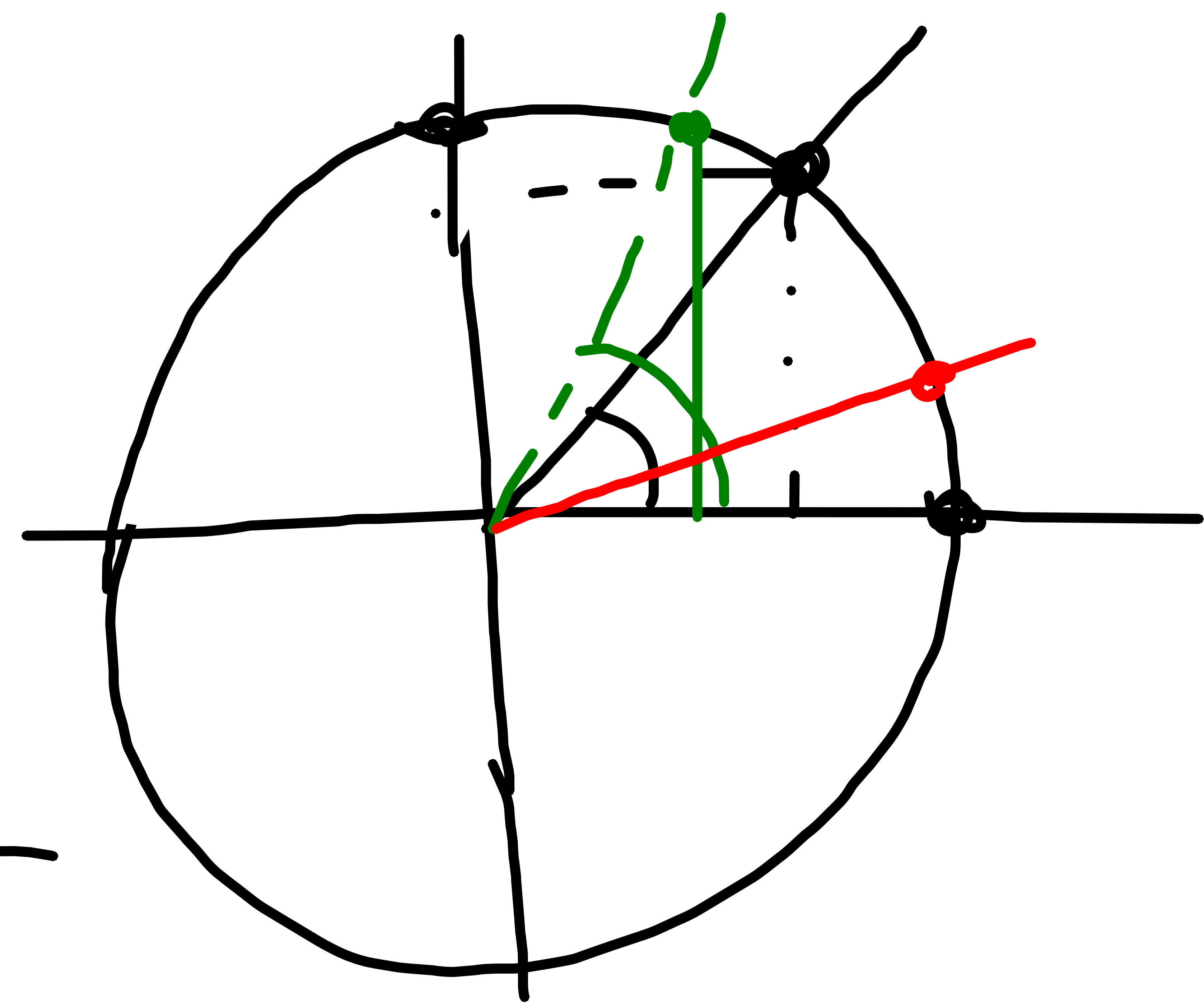
La sua inversa è

$\arctan : \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$



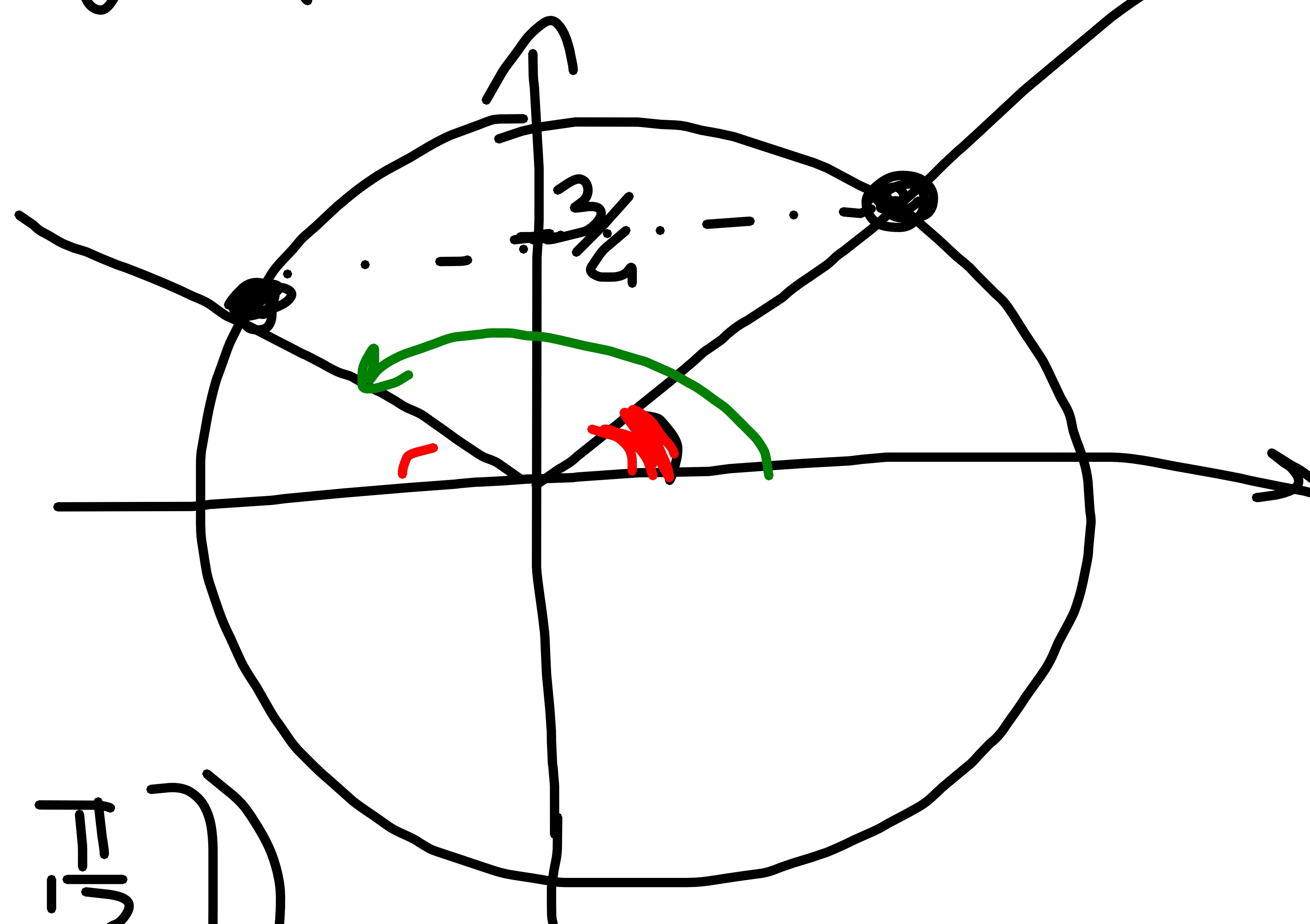
ESEMPI ed ESERCIZI

	0	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$
sin	0	1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
cos	1	0	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
tg					



• $\operatorname{sen} x = 2$ Non ha soluz.

$\operatorname{sin} x < 2 \quad \forall x \in \mathbb{R}$



• $\operatorname{sin} x = 3/4$

$x = \arcsin 3/4$

(è l'unica soluz in $[-\frac{\pi}{2}, \frac{\pi}{2}]$)

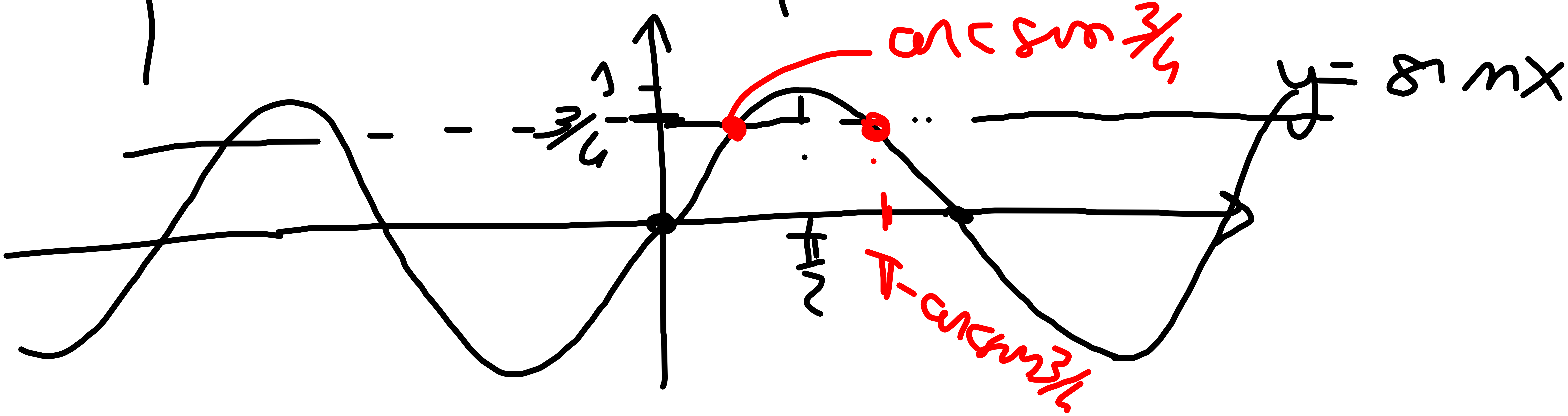
$x = \pi - \arcsin 3/4$

L'insieme di tutte le soluz. è

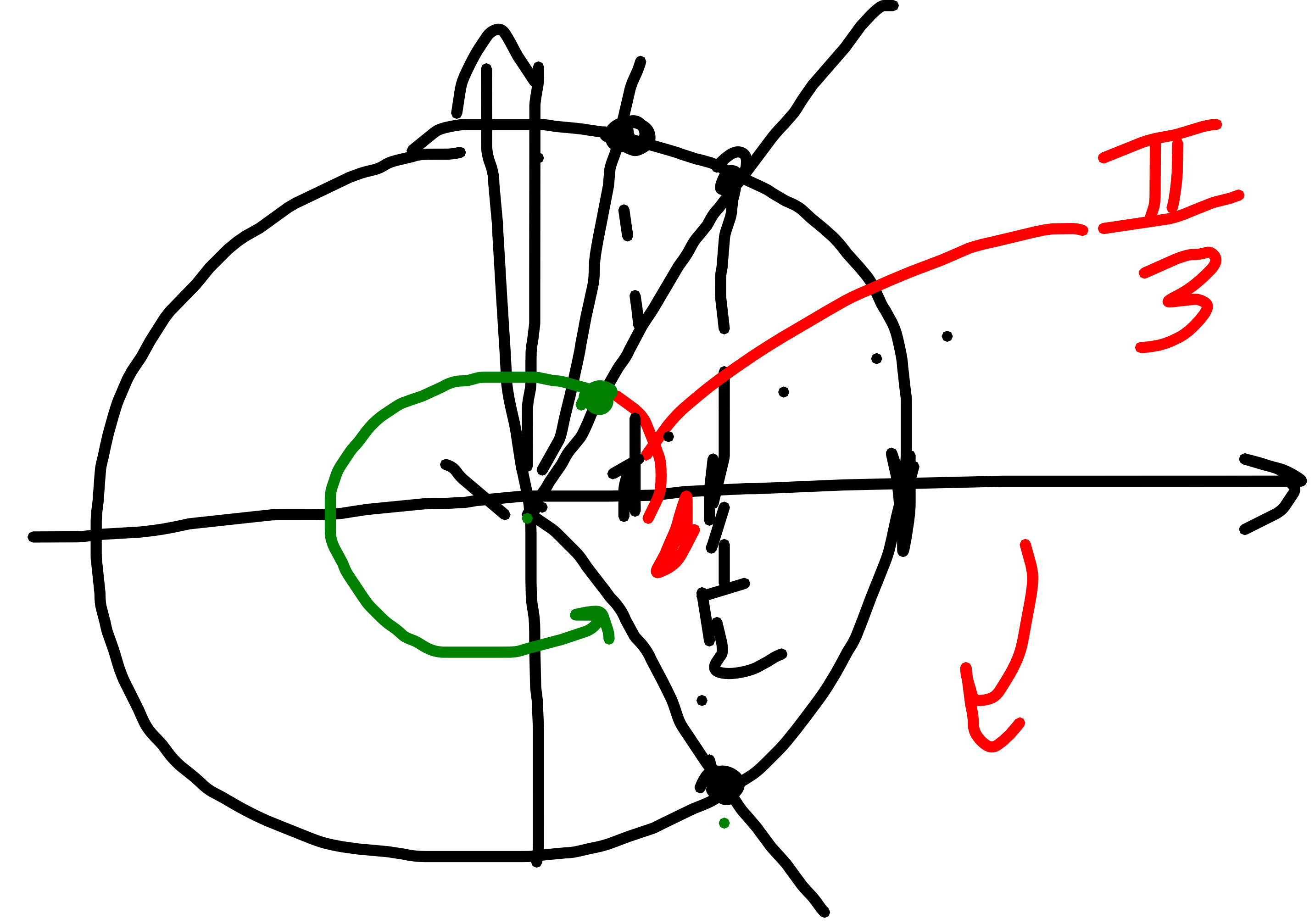
$$\left\{ \arcsin \frac{3}{4} + 2k\pi,\ k \in \mathbb{Z} \right\}$$

$$\cup$$

$$\left\{ \pi - \arcsin 3/4 + 2k\pi,\ k \in \mathbb{Z} \right\}$$



• $\cos x < \frac{1}{2}$



soluzioni in $[0, 2\pi]$ $\begin{cases} \frac{\pi}{3} < x < \frac{5}{6}\pi \\ x \in \left]\frac{\pi}{3}, \frac{5}{3}\pi\right[\end{cases}$

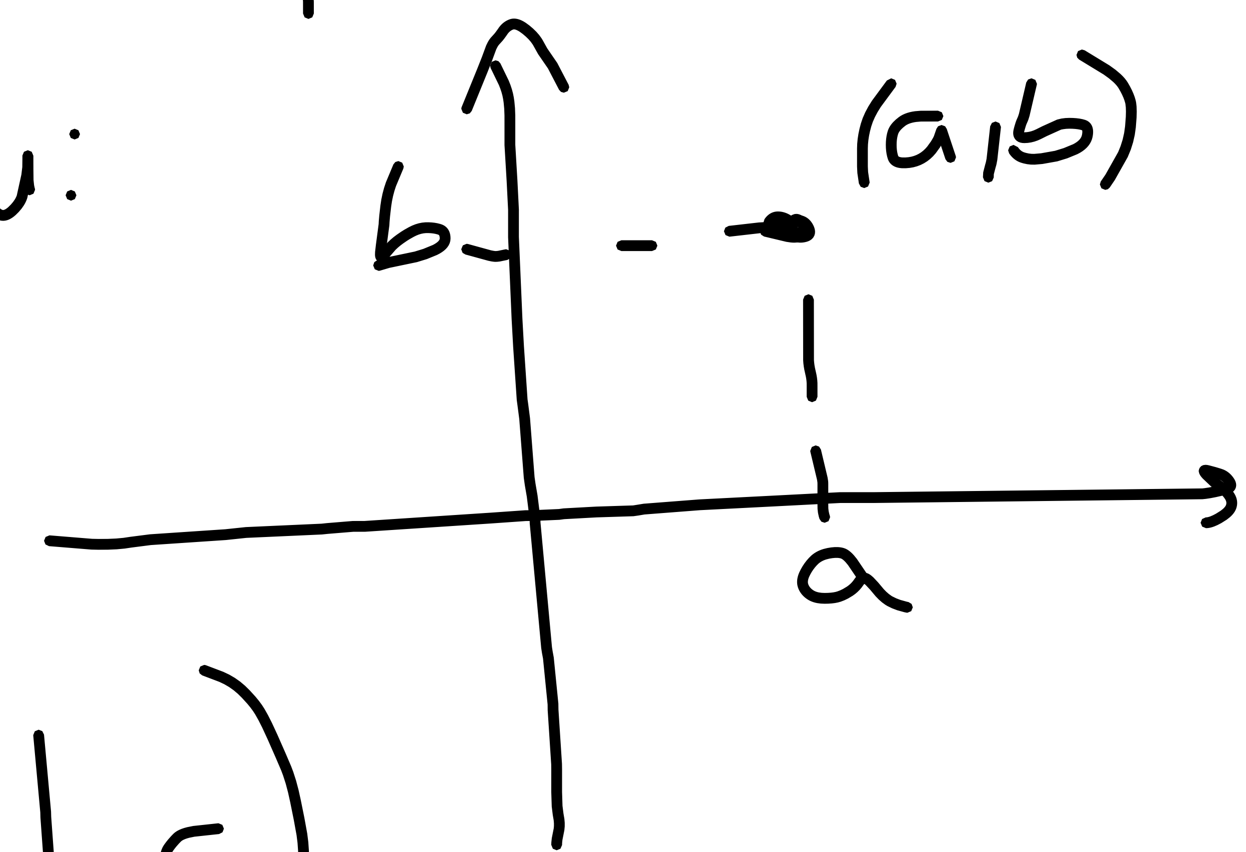
Tutte le soluzioni

$$\bigcup_{k \in \mathbb{Z}} \left]\frac{\pi}{3} + 2k\pi, \frac{5}{3}\pi + 2k\pi\right[$$

NUMERI COMPLESSI

Considero $\mathbb{R}^2 = \mathbb{R}\times\mathbb{R} = \{(a,b) \mid a\in\mathbb{R}, b\in\mathbb{R}\}$

con le seguenti operazioni:



SOMMA • $(a,b)+(c,d) = (a+c,\ b+d)$

PRODOTTO • $(a,b)\cdot(c,d) = (ac-bd,\ ad+bc)$

OSS. • $(a,b)+(0,0)=(a,b)$

$\rightarrow (0,0)$ ELEM. NEUTRO di $+$

• $(a,b)\cdot(1,0)=(a,b)$

$\rightarrow (1,0)$ ELEM. NEUTRO $\cdot$

• L'OPPOSTO di (a,b) [rispetto alla somma]

è $(-a,-b)$

- il RECIPROCO di (a,b) è

$$\left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right) \quad \left(\text{se } (a,b) \neq (0,0)\right)$$

Sono verificate le proprietà associativa, distributiva, commutativa.

$(\mathbb{R}^2, +, \cdot)$ è il CAMPO dei NUMERI COMPLESSI.

Lo indichiamo con $\mathbb{C}$

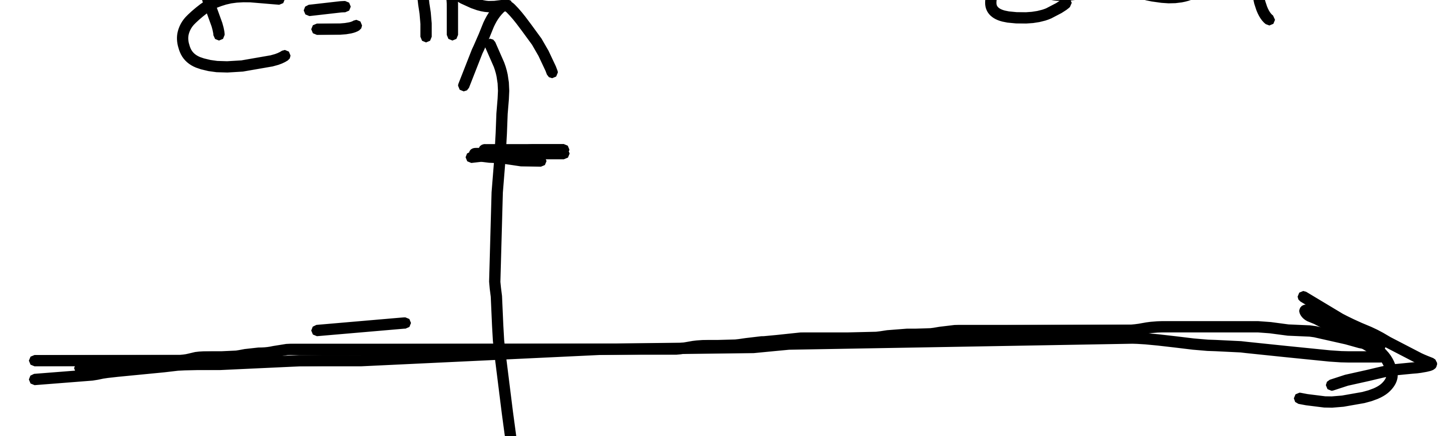
- Posso identificare un numero reale a col numero complesso $(a,0)$

- $(0,1)\cdot(0,1) = \underbrace{(-1,0)}_{\text{identifico con numero reale}} \equiv -1$

Quindi in $\mathbb{C}$

$$\boxed{(0,1)}\cdot(0,1) = -1$$

$\mathbb{C}=\mathbb{R}^2$



↳ Lo indichiamo con i = UNITÀ IMMAGINARIA

Quindi, possiamo scrivere

$$i^2 = -1$$

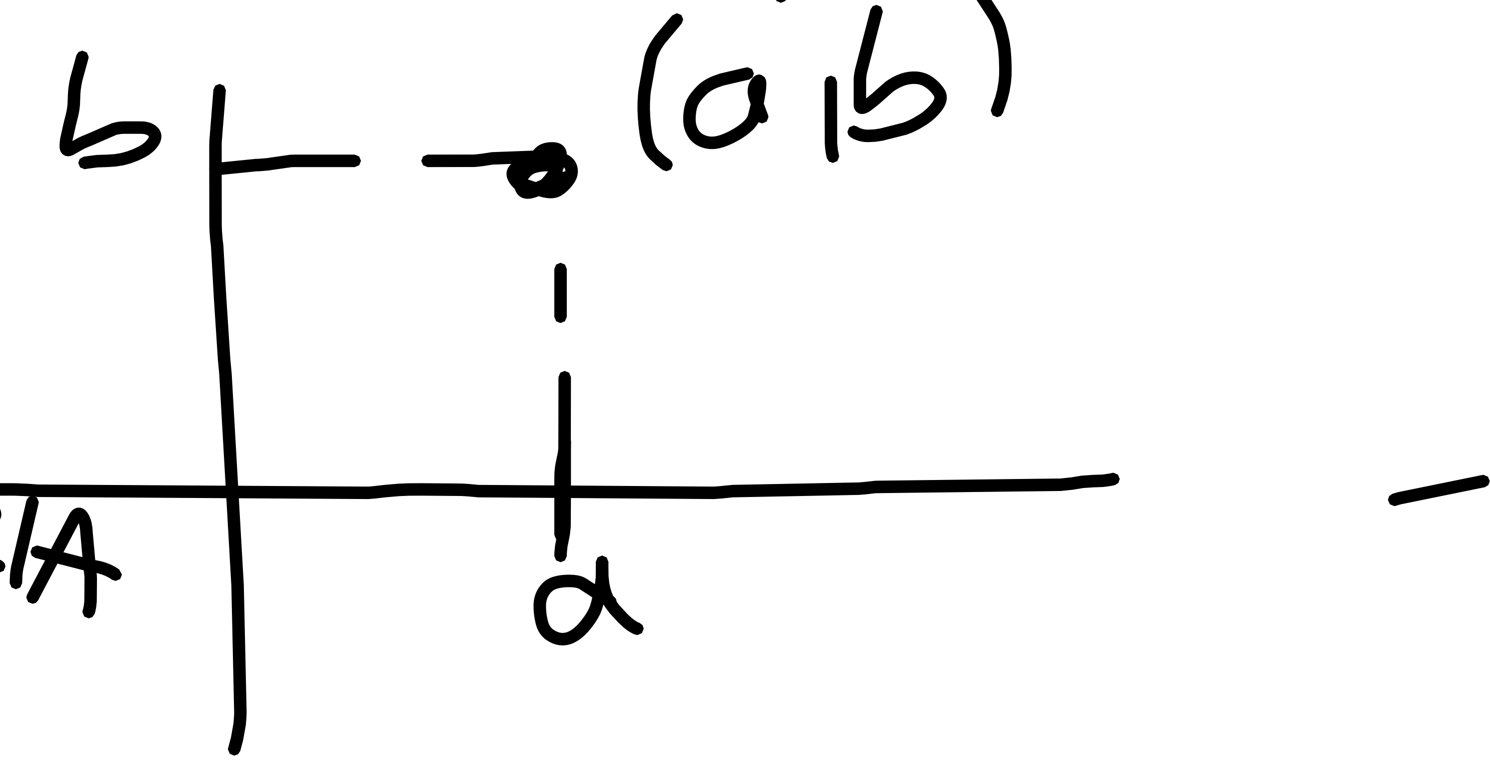
FORMA ALGEBRICA

$$(a,b) = (a,0) + (0,1) \cdot (b,0)$$

$$\sim \boxed{a + ib} \qquad \text{con } a,b \in \mathbb{R}$$

a: PARTE REALE

b: PARTE IMMAGINARIA



Vediamo che questa notazione è coerenti con l'operazione $\cdot$

$(a+ib)\cdot(c+id) =$ → uso $i^2=-1$

$= ac + \underline{i}\,ad + \underline{i}\,bc - bd$

$= ac - bd + i\,(bc+ad)$

$= (ac-bd,\ bc+ad)$

$z = a + ib$

$a = \mathbb{R}e\, z \rightarrow$ parte reale

$b = \mathrm{Im}\, z \rightarrow$ parte immaginaria.