SUCC. ESPONENZIALE

$\boxed{a>1}$ $\quad a_n = a^n$

posso scrivere $a = 1+h$ con $h>0$

$$\underline{a^n} = (1+h)^n \underset{\text{dis. Bernoulli}}{\geq} \underbrace{1+nh}_{b_n}$$

Poiché $b_n = 1+nh \xrightarrow[n\to+\infty]{} +\infty$

allora (per il Teor. del confronto), $a^n \longrightarrow +\infty$

Per es: verificare che $a^n \xrightarrow[n\to+\infty]{} 0$ se $0<a<1$.

Richiamo il caso: $\frac{\ell}{0}$

Sia $a_n \to \ell \in \overline{\mathbb{R}} \setminus \{0\}$

$b_n \to 0$

i) Se $\exists\, m \in \mathbb{N}$ t.c. $\forall n \geq m$ $\frac{a_n}{b_n} > 0 \Rightarrow \frac{a_n}{b_n} \to +\infty$

ii) Se $\exists\, m \in \mathbb{N}$ t.c. $\forall n \geq m$ $\frac{a_n}{b_n} < 0 \Rightarrow \frac{a_n}{b_n} \to -\infty$

ES
- $a_n = 1$ $\quad b_n = \left(\frac{1}{2}\right)^n$

$$\lim_{n\to+\infty} \frac{a_n}{b_n} = \left(\lim_{n\to+\infty} 2^n\right) = +\infty$$

- $a_n = 1$ $\quad b_n = \left(-\frac{1}{2}\right)^n$

$$\lim_{n\to+\infty} \frac{a_n}{b_n} = \lim_{n\to+\infty} (-2)^n \to \text{NON HA LIMITE}$$

$\boxed{ES}$ ① $a_n = 2n^4 - 3n^2 + n - 7$

FORMA INDET: $+\infty - \infty$

$$\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} 2n^4 \left(1 - \underbrace{\frac{3}{2n^2}}_{0} + \underbrace{\frac{1}{n^3}}_{0} - \underbrace{\frac{7}{2n^4}}_{0}\right)$$

$$= \lim_{n\to+\infty} 2n^4 = \underline{+\infty}$$

② $\lim_{n\to+\infty} n^2 - 7n^4 + 2n - 2 = -\infty$

② $a_n = \dfrac{n^4 - 7n^3}{2n^2 - 3n + 1}$

$$\lim_{n \to +\infty} a_n = \lim_{n \to +\infty} \frac{n^4\left(1 - \underbrace{\frac{7}{n}}_{\to 0}\right)}{2n^2\left(1 - \underbrace{\frac{3}{2n}}_{\to 0} + \underbrace{\frac{1}{2n^2}}_{\to 0}\right)}$$

$$= \lim_{n \to +\infty} \frac{n^2}{2} = +\infty$$

3 $a_n = \dfrac{3n^4 - 7n^2 + 1}{2n^4 - 3n + 7}$

$$\lim_{n \to +\infty} a_n = \lim_{n \to +\infty} \frac{3\cancel{n^4}}{2\cancel{n^4}} = \frac{3}{2}$$

4 $a_n = \dfrac{3n^2 + 2n + 1}{3n^3 - 2n}$

$$\lim_{n \to +\infty} a_n = \lim_{n \to +\infty} \frac{3\cancel{n^2}}{3n^{\cancel{3}}} = \lim_{n \to +\infty} \frac{1}{n} = \varnothing$$

Regola generale

Sia $a_n = \sum_{j=0}^{p} a_j n^j = a_0 + a_1 n + a_2 n^2 + \dots + \underline{a_p n^p}$ $(a_p \neq 0)$

$b_n = \sum_{j=0}^{q} b_j n^j = b_0 + b_1 n + \dots + b_q n^q$ $(b_q \neq 0)$

Allora:

$$\lim_{n\to+\infty} \frac{a_n}{b_n} = \begin{cases} \pm\infty \ \left(\text{con la regola dei segni}\right) & p > q \\ \dfrac{a_p}{b_q} & p = q \\ 0 & p < q \end{cases}$$

Se $a_n \xrightarrow[n\to+\infty]{} 0$, b_n limitata

Allora: $a_n \cdot b_n \longrightarrow 0$

es $\lim_{n\to+\infty} \dfrac{(-1)^n}{n^3+2n-7} = 0$ b_n

$b_n = (-1)^n$ è limitata

$a_n = \dfrac{1}{n^3+2n-7} \longrightarrow 0$

$$a_n = \frac{\sqrt[3]{n^7 + 2n^3 - 2}}{n^{3/2} + n + 1}$$

$$\lim_{n \to +\infty} a_n = \lim_{n \to +\infty} \frac{\sqrt[3]{n^7\left(1 + \underbrace{\frac{2}{n^4}}_{\to 0} - \underbrace{\frac{2}{n^7}}_{\to 0}\right)}}{n^{3/2}\left(1 + \underbrace{\frac{1}{n^{1/2}}}_{\to 0} + \underbrace{\frac{1}{n^{3/2}}}_{\to 0}\right)}$$

$$= \lim_{n \to +\infty} \frac{n^{7/3}}{n^{3/2}} = +\infty$$

- $$\lim_{n\to+\infty} \frac{\sqrt{n+1}}{(-1)^n \cdot n} = \lim_{n\to+\infty} \frac{\sqrt{n\left(1+\frac{1}{n}\right)}}{(-1)^n\, n}$$

$$= \lim_{n\to+\infty} \frac{\sqrt{n}}{(-1)^n \cdot n} = \boxed{0}$$

poichè $\frac{1}{(-1)^n}$ è limitata

e $\frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} \longrightarrow 0$

$$\circ \lim_{n \to +\infty} \left(\sqrt{n+1} - \sqrt{n-1}\right) \cdot \frac{\left(\sqrt{n+1} + \sqrt{n-1}\right)}{\sqrt{n+1} + \sqrt{n-1}}$$

$$= \lim_{n \to +\infty} \frac{\cancel{n}+1 - \cancel{n}+1}{\sqrt{n+1} + \sqrt{n-1}}$$

$$= \lim_{n \to +2} \frac{2}{\sqrt{n+1} + \sqrt{n-1}} = \emptyset$$

$$\lim_{n\to+\infty} \frac{1}{n\left(\sqrt{n-2}-\sqrt{n}\right)}$$

$$= \lim_{n\to+\infty} \frac{1}{n\left(\sqrt{n-2}-\sqrt{n}\right)} \cdot \frac{\sqrt{n-2}+\sqrt{n}}{\sqrt{n-2}+\sqrt{n}}$$

$$= \lim_{n\to+\infty} \frac{\sqrt{n-2}+\sqrt{n}}{n\left(\cancel{n}-2-\cancel{n}\right)} =$$

$$= \lim_{n\to+\infty} \frac{2\sqrt{n}}{-2n} = 0$$

- Se avessi avuto

$$\lim_{n\to+\infty} \frac{1}{\sqrt{n}\left(\sqrt{n-2}-\sqrt{n}\right)}$$

ripercorrendo i calcoli, avrei ottenuto

$$= \lim_{n\to+\infty} \frac{2\sqrt{n}}{-2\sqrt{n}} = -1$$

- Analogamente

$$\lim_{n\to+\infty} \frac{1}{\sqrt[3]{n}\left(\sqrt{n-2}-\sqrt{n}\right)} = \lim_{n\to+\infty} \frac{2\sqrt{n}}{-2\sqrt[3]{n}} = -\infty$$

- $$\lim_{n \to +\infty} \frac{\sqrt{n^2+2n} - 2n}{n}$$

$$= \lim_{n \to +\infty} \frac{\sqrt{n^2\left(1+\frac{2}{n}\right)} - 2n}{n}$$

$\to 0$

$$= \lim_{n \to +\infty} \frac{n - 2n}{n} = \lim_{n \to +\infty} -\frac{n}{n} = -1$$

SUCCESSIONE TRASCURABILE

Def (o-piccolo)

Siano (a_n), (b_n) succ. reali, $b_n \neq 0 \ \forall n \in \mathbb{N}$

Diciamo che (a_n) è TRASCURABILE rispetto a (b_n) se

$$\lim_{n\to+\infty} \frac{a_n}{b_n} = 0$$

e scriviamo $\boxed{a_n = o(b_n)}$ per $n \to +\infty$

ES • $a_n = n^2$ $\quad b_n = n^3$

$$\lim_{n \to +\infty} \frac{n^2}{n^3} = 0 \Rightarrow n^2 = o(n^3) \text{ per } n \to +\infty$$

• $a_n = \frac{1}{n^3}$ $\quad b_n = \frac{1}{n^2}$

$$\lim_{n \to +\infty} \frac{n^2}{n^3} = 0 \Rightarrow \frac{1}{n^3} = o\left(\frac{1}{n^2}\right) \text{ per } n \to +\infty$$

• Se sono $a_n = o(1)$ per $n \to +\infty$

significa $a_n \to 0$ per $n \to +\infty$

ES

$$\lim_{n \to +\infty} \frac{n^{3/2} - 7n + 1}{\sqrt{n+1}}$$

$$= \lim_{n \to +\infty} \frac{n^{3/2}\left(1 + o(1)\right)}{\sqrt{n\left(1 + o(1)\right)}} = +\infty$$