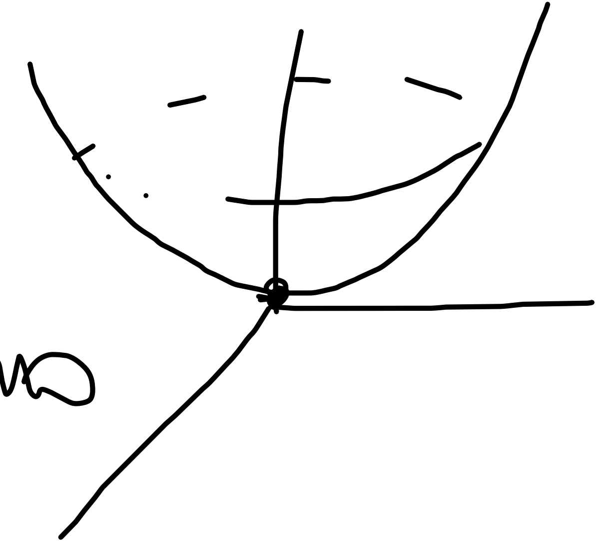
ES • $f(x,y) = x^2 + y^2$



Abbiamo visto che $(0,0)$ è pto di minimo

$$H_f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$\det H_f(0,0) = 4 > 0$

$a_{11} = 2 > 0$

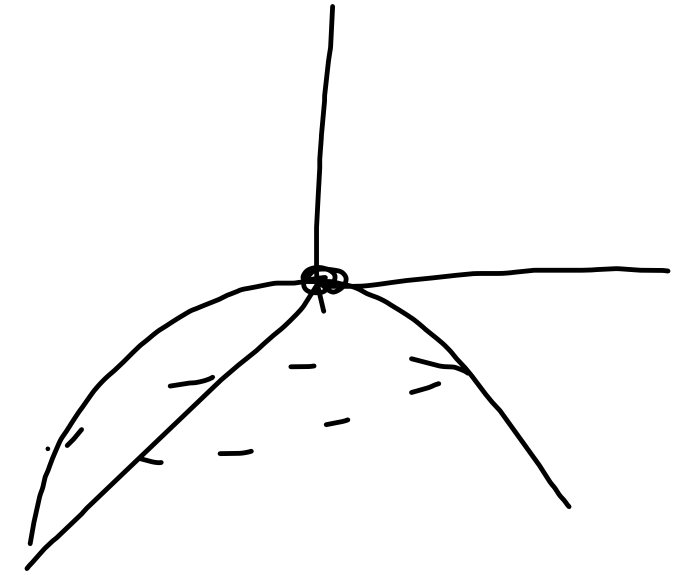
$\Downarrow$

$H_f(0,0)$ è def pos

$\Downarrow$

$(0,0)$ è pto di MIN

• $f(x,y) = -x^2 - y^2$



$$\begin{matrix} \partial_x f \\ \partial_y f \end{matrix} \begin{cases} -2x = 0 \\ -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$H_f(x,y) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\| \\ H_f(0,0)$$

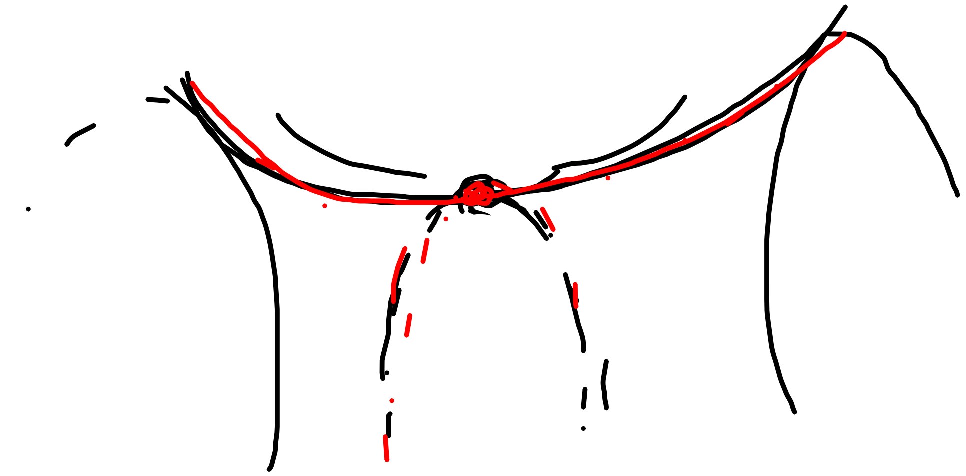
$\det H_f(0,0) = 4 > 0$

$a_{11} = -2 < 0$

$\Rightarrow H_f(0,0)$ def neg.

$\Rightarrow (0,0)$ pto di MAX

• $f(x,y) = \underline{x^2 - y^2}$



$$\begin{array}{l} \partial_x f \\ \partial_y f \end{array} \begin{cases} 2x = 0 \\ -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$H_f(x,y) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\| $$

$$H_f(0,0)$$

$\det H_f(0,0) = -4 < 0$

$\Rightarrow H_f(0,0)$ è indefinita

$\Rightarrow (0,0)$ p.to di sella

$$f(x,y) = x^3 - y^3 - xy$$

$\underline{\partial_x f}$ $\qquad \underline{\partial_y f}$

$$\begin{cases} \underline{3x^2 - y} = 0 \\ \underline{-3y^2 - x} = 0 \end{cases} \iff \begin{cases} y = 3x^2 \\ -27x^4 - x = 0 \end{cases}$$

$$\iff \begin{cases} y = 3x^2 \\ -\underline{x}\,\underbrace{(1 + 27x^3)}_{x^3 = -\frac{1}{27}} = 0 \end{cases} \iff \begin{cases} y = 0 \\ x = 0 \end{cases} \cup \begin{cases} y = \frac{1}{3} \\ x = -\frac{1}{3} \end{cases}$$

<u>PTI</u> <u>CRITICI</u> $\quad \underline{P_1 = (0,0)} \qquad \underline{P_2 = \left(-\frac{1}{3}, \frac{1}{3}\right)}$

$$H_f(x,y) = \begin{pmatrix} \underline{6x} & \underline{-1} \\ \underline{-1} & \underline{-6y} \end{pmatrix}$$

$$\boxed{H_f(x,y) = \begin{pmatrix} \partial_{xx}f(\cdot) & \partial_{xy}f(\cdot\cdot) \\ \partial_{xy}f(-) & \partial_{yy}f(-) \end{pmatrix}}$$

$\underline{P_1}$: $H_f(0,0) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ $\quad \det H_f(0,0) = -1 < 0$

$\Rightarrow P_1$ è PTO di SELLA

P_2: $H_f\left(-\frac{1}{3}, \frac{1}{3}\right) = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$ $\quad \det H_f\left(-\frac{1}{3}, \frac{1}{3}\right) = 4 - 1 = 3 > 0$

$a_{11} = -2 < 0 \Rightarrow H_f\left(\frac{1}{3}, \frac{1}{3}\right)$ def neg

$\Rightarrow P_2$ è PTO di MAX LOCALE

$$f(x,y) = 2(x^4+y^4+1) - (x+y)^2$$

$\underline{\partial_x f} \rightarrow \begin{cases} 8x^3 - 2x - 2y = 0 \\ 8y^3 - 2x - 2y = 0 \end{cases} \Leftrightarrow \begin{cases} \underline{8x^3} = 2x+2y \\ \underline{8y^3} = 2x+2y \end{cases}$

$\rightarrow \underline{\partial_y f}$

$\Leftrightarrow \begin{cases} x^3 = y^3 \\ 8y^3 = 2x+2y \end{cases} \Leftrightarrow \begin{cases} x = y \\ 8y^3 = 4y \end{cases} \Leftrightarrow \begin{cases} x = y \\ y(2y^2-1) = 0 \end{cases}$

$2y^3 = y$

$\underbrace{y(2y^2-1)=0}_{y=0,\ y=\frac{\sqrt{2}}{2},\ y=-\frac{\sqrt{2}}{2}}$

Pti critici: $P_1 = (0,0)$ $P_2 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $P_3 = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$$H_f(x,y) = \begin{pmatrix} 24x^2 - 2 & -2 \\ -2 & 24y^2 - 2 \end{pmatrix}$$

P_1) $H_f(0,0) = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$ $\det H_f(0,0) = 0$

(dopo lo studio)

$$H_f(P_2) = H_f(P_3) = \begin{pmatrix} 10 & -2 \\ -2 & 10 \end{pmatrix}$$

$$\det H_f(P_{2,3}) = 96 > 0$$

$$a_{11} = 10 > 0$$

$\Rightarrow H_f(P_2) = H_f(P_3)$ det. pos.

$\Downarrow$

P_2 e P_3 sono p^{ti} di MINIMO LOCALE.

Studiamo il punto $P_1 = (0,0)$

$$f(x,y) = 2(x^4+y^4+1) - (x+y)^2$$

- La restringo a $y = -x$:

$\to f(x,-x) = 2(2x^4+1)$ ha un minimo in $x = 0$

- La restringo a $y = x$

$$\to \underbrace{f(x,x)}_{g(x)} = 2(2x^4+1) - 4x^2 = 2(2x^4 - 2x^2 + 1)$$

$$\to g'(x) = 2(8x^3 - 4x) = 8(2x^3 - x)$$

$$g'(x) \geq 0 \iff \underbrace{x}_{F_1}\underbrace{(2x^2-1)}_{F_2} \geq 0$$

$$F_2 \geq 0 \iff x^2 \geq \frac{1}{2}$$

		$-\frac{\sqrt{2}}{2}$		0		$\frac{\sqrt{2}}{2}$	
F_1	$-$		$-$		$+$		$+$
F_2	$+$		$-$		$-$		$+$
			$+$		$-$		
			$\nearrow$		$\searrow$		

0 è pto di MAX LOCALE

per $g(x) = f(x, x)$

Quindi $P_1 = (0,0)$ NON può essere né pto di MAX né pto di MIN LOCALE

$$f(x,y) = xy\, e^{-\frac{(x^2+y^2)}{2}}$$

$$\begin{matrix} \to \partial_x f \\ \Rightarrow \underline{\partial_y f} \end{matrix} \begin{cases} y e^{-\frac{(x^2+y^2)}{2}} - x^2 y\, e^{-\frac{(x^2+y^2)}{2}} = 0 \\ x e^{-\frac{(x^2+y^2)}{2}} - x y^2 e^{-\frac{(x^2+y^2)}{2}} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} e^{-\frac{(x^2+y^2)}{2}}\, y\,[1-x^2] = 0 \\ e^{-\frac{(x^2+y^2)}{2}}\, x\,[1-y^2] = 0 \end{cases} \Leftrightarrow \begin{cases} y(1-x^2) = 0 \\ x(1-y^2) = 0 \end{cases} \leftarrow$$

$\underline{P_1 = (0,0)}$

$$\begin{cases} x = \pm 1 \\ y = \pm 1 \end{cases}$$

$\underline{P_2 = (1,1)}$

$\underline{P_3 = (1,-1)}$

$\underline{P_4 = (-1,1)}$

$\underline{P_5 (-1,-1)}$

$$\boxed{\partial_{xx} f(x,y)} = e^{-\frac{(x^2+y^2)}{2}} \left[\left(-xy(1-x^2) \right) - 2xy \right]$$

$$\boxed{= e^{-\frac{(x^2+y^2)}{2}} \underbrace{\left[\underline{-3xy + x^3y} \right]}}$$

- $\boxed{\partial_{xy} f(x,y)} = e^{-\frac{(x^2+y^2)}{2}}\left[-y^2(1-x^2)+1-x^2\right]$

$$= e^{-\frac{(x^2+y^2)}{2}}\left[1-x^2-y^2+x^2y^2\right]$$

- $\boxed{\partial_{yy} f(x,y)} = e^{-\frac{(x^2+y^2)}{2}}\left[-yx(1-y^2)-2xy\right]$

$$= e^{-\frac{(x^2+y^2)}{2}}\left[-3xy+y^3x\right]$$

Quindi $H_f(0,0) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\det H_f(0,0) = -1 < 0$

$\Rightarrow (0,0)$ è p.to di sella

$H_f(1,1) = \begin{pmatrix} -2/e & 0 \\ 0 & -2/e \end{pmatrix}$ $\det H_f(1,1) = \frac{4}{e^2} > 0$

$\parallel$

$H_f(-1,-1)$

$a_{11} = -2/e < 0$

$H_f(1,1)$ e $H_f(-1,-1)$ è def. neg.

$\Rightarrow (-1,-1)$ e $(1,1)$ p.to di MAX loc

$$H_f(1,-1) = H_f(-1,1) = \begin{pmatrix} 2/e & 0 \\ 0 & 2/e \end{pmatrix}$$

$$\left.\begin{array}{l} \det H_f(\ldots\ldots) > 0 \\ a_{11} = 2/e > 0 \end{array}\right\} \Rightarrow H_f(-1,1) = H_f(1,-1)$$

def pos.

$\Downarrow$

$(-1,1)$ e $(1,-1)$ sono

pti di MIN. LOC.

$$f(x,y) = \frac{x}{y} + \frac{8}{x} - y \qquad x \neq 0 \quad y \neq 0$$

$$\begin{aligned}
\partial_x f \quad & \begin{cases} \frac{1}{y} - \frac{8}{x^2} = 0 \\ -\frac{x}{y^2} - 1 = 0 \end{cases} \Longrightarrow \begin{cases} y = \frac{x^2}{8} \\ x = -y^2 \end{cases} \\
\partial_y f \quad &
\end{aligned}$$

$$\Longleftrightarrow \begin{cases} y = \frac{x^2}{8} \\ x = -\frac{x^4}{64} \end{cases} \Longleftrightarrow \begin{cases} y = \frac{x^2}{8} \\ x\left(1 + \frac{x^3}{64}\right) = 0 \end{cases} \qquad \begin{cases} y = 2 \\ x = -4 \end{cases}$$

$x = 0$, $y = 0 \rightarrow \notin$ DOMINIO

$$H_f(x,y) = \begin{pmatrix} \frac{16}{x^3} & -\frac{1}{y^2} \\ -\frac{1}{y^2} & \frac{2x}{y^3} \end{pmatrix}$$

$$\Rightarrow H_f(-4,2) = \begin{pmatrix} \frac{16}{-64} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{8}{8} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -1 \end{pmatrix}$$

$\det H_f(-4,2) > 0$ $\quad a_{11} = -\frac{1}{4} < 0 \Rightarrow (-4,2)$ è

PTO DI MAX LOC.