

## TEOR (INTEGRAZIONE ~~per~~ PARTI)

Siano  $f \in C([a, b], \mathbb{R})$ ,  $g \in C^1([a, b], \mathbb{R})$ .

Sia  $F$  una primitiva di  $f$ .

Allora:

$$\int_a^b f(x)g(x)dx = \left[ F(x)g(x) \right]_a^b - \int_a^b F(x) \cdot g'(x)dx$$

DM

$\forall x \in [a, b]$ :

$$\underline{(F(x)g(x))' = F'(x)g(x) + F(x)g'(x)}$$

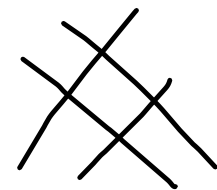
perché  $F' = f \Rightarrow$   $f(x)g(x) + F(x)g'(x)$

Integrazione per parti:

$$\int_a^b (F(x)g(x))' dx = \int_a^b f(x)g(x) dx + \int_a^b F(x)g'(x) dx$$

$[F(x)g(x)]_a^b$

per il  
Teorema



ES 1)  $\int_0^1 \underbrace{x}_g \cdot \underbrace{e^x}_f dx = \left[ e^x \cdot x \right]_0^1 - \underbrace{\int_0^1 e^x \cdot 1 dx}_{[e^x]_0^1}$

2)  $\int_{\pi/4}^{\pi/2} \underbrace{x}_g \underbrace{\cos x}_f dx = \left[ x \sin x \right]_{\pi/4}^{\pi/2} - \underbrace{\int_{\pi/4}^{\pi/2} x \sin x dx}_{[-x \cos x]_{\pi/4}^{\pi/2}}$

$$3) \int_2^4 \log x \, dx = \left[ x \cdot \log x \right]_2^4 - \int_2^4 \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

4) Calcolare una primitiva di  $f(x) = \cos^2 x$

$$\int_0^x \cos^2 t \, dt = \left[ \sin t \cos t \right]_0^x + \int_0^x \sin^2 t \, dt$$

$$\rightarrow = \sin x \cos x + \int_0^x (1 - \cos^2 t) \, dt$$

$$= \sin x \cos x + x - \int_0^x \cos^2 t \, dt$$

$$\int_0^{\pi/2} \cos^2 t \, dt = \frac{\sin 2t \cos t + t}{2}$$

$$\int_{1/3}^{2/3} \sqrt{1-x^2} \, dx = \left[ x \sqrt{1-x^2} \right]_{1/3}^{2/3} - \int_{1/3}^{2/3} \frac{-x \cdot 2x}{2\sqrt{1-x^2}} \, dx$$

$$= \dots = \int_{1/3}^{2/3} \left( \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right) \, dx$$

$\underbrace{\hspace{10em}}_{\sqrt{1-x^2}}$ 
 $\underbrace{\hspace{10em}}$

$$\int_{-\frac{1}{3}}^{\frac{1}{3}} \sqrt{1-x^2} dx = \dots + \left[ \arcsin x \right]_{-\frac{1}{3}}^{\frac{1}{3}}$$

$$\int_0^1 \arctan x dx = \left[ x \cdot \arctan x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} dx$$

$$\int_2^4 \log^2 x dx = \left[ x \cdot \log^2 x \right]_2^4 - \int_2^4 \frac{2 \log x}{x} dx$$

$-\frac{1}{2} \left[ \log(1+x^2) \right]_0^1$

$$\int_0^1 t^2 \arctan t \, dt = \left[ \frac{t^3}{3} \arctan t \right]_0^1 - \int_0^1 \frac{t^3}{3} \cdot \frac{1}{1+t^2} dt$$

$$= \frac{1}{3} \int_0^1 \frac{t^3 + t - t}{1+t^2} dt$$

$$= \frac{1}{3} \left[ \int_0^1 \frac{t(1+t^2)}{1+t^2} dt - \frac{1}{2} \int_0^1 \frac{2t}{1+t^2} dt \right]$$

$$= \frac{1}{3} \left[ \left[ \frac{t^2}{2} \right]_0^1 - \frac{1}{2} \left[ \log(1+t^2) \right]_0^1 \right]$$

# TEOR (INTEGRAZIONE per SOSTITUZIONE)

Siano  $I, J$  int. di  $\mathbb{R}$ ,  $f \in C(I, \mathbb{R})$ ,

$\phi \in C^1(J, I)$ . Allora:

→ 1) Se  $\alpha, \beta \in J$ :

$$\int_{\alpha}^{\beta} f(\phi(t)) \cdot \underline{\phi'(t)} dt = \int_{\phi(\alpha)}^{\phi(\beta)} f(x) dx \quad (x = \phi(t))$$

2) Se  $\phi$  è invertibile allora:

$$\int_a^b f(x) dx = \int_{\phi^{-1}(a)}^{\phi^{-1}(b)} f(\phi(t)) \phi'(t) dt$$



DM Dato che  $f$  è continuo, per il II Teor. fond.  
ammette una primitiva  $F$ .

Consideriamo  $F \circ \phi$ :

$$\underline{(F \circ \phi)'(t) = F'(\phi(t)) \cdot \phi'(t)}$$

$$= \underline{f(\phi(t)) \cdot \phi'(t)}$$

Integro tra  $a$  e  $b$ ,

I Teor. fond.

$$\int_a^b f(\phi(t)) \cdot \phi'(t) dt = \int_a^b (F \circ \phi)'(t) dt = F(\phi(b)) - F(\phi(a)) = \int_{\phi(a)}^{\phi(b)} f(x) dx$$

IS

$$\int_0^1 \sqrt{1-x^2} dx$$

$$= \int_0^{\frac{\pi}{2}} \underbrace{\cos t}_{\cos t} \cdot \cos t dt$$

$$x = \cos t$$

$$(x = \phi(t))$$

$$\underline{\phi'(t) = \cos t}$$

$$\frac{1}{3} \int_0^1 \frac{3e^{3t}}{1+e^{3t}} dt = \frac{1}{3} \left[ \log(1+e^{3t}) \right]_0^1$$

$x = e^{3t}$   
 $\phi'(t) = 3e^{3t}$

$$= \frac{1}{3} \int_1^{e^3} \frac{dx}{1+x} = \frac{1}{3} \left[ \log(1+x) \right]_1^{e^3}$$

$$\frac{1}{3} \int_0^1 \frac{3e^{3t}}{1+e^{6t}} dt = \frac{1}{3} \int_1^{e^3} \frac{dx}{1+x^2} = \frac{1}{3} \left[ \arctan x \right]_1^{e^3}$$

cambio  
 $x = e^{3t}$   
 $dx = 3e^{3t} dt$

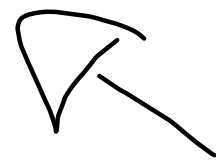
$$\textcircled{\bullet} \int_0^1 \frac{\log(1+\sqrt{x})}{\sqrt{x}+x} dx = 2 \int_0^1 \frac{\log(1+\sqrt{x})}{2\sqrt{x}(1+\sqrt{x})} dx$$

Pongp  
 $t = \sqrt{x}$   
 $dt = \frac{1}{2\sqrt{x}} dx$

$$= 2 \int_0^1 \frac{\log(1+t)}{1+t} dt = 2 \cdot \frac{1}{2} \left[ (\log(1+t))^2 \right]_0^1$$

2<sup>o</sup> mod!  $x = t^2 \Rightarrow t = \sqrt{x} \rightarrow x = t^2 = \phi(t)$   
 $dx = 2t dt$

$$\int_0^1 \frac{\log(1+\sqrt{x})}{\sqrt{x}(1+\sqrt{x})} dx = \int_0^1 \frac{\log(1+t)}{t(1+t)} \cdot 2t dt$$



$$\int_0^1 \underbrace{2x \sqrt{x+7}}_{f(x)} dx$$

$$= \int_7^{10} 2(t^2-7) \cdot t \cdot \underbrace{2t}_{\phi'(t)} dt$$

fimire

pong

$$t = \sqrt{x+7}$$

$$\sqrt{x} = \underbrace{t^2 - 7}_{\phi(t)}$$

$$dx = \phi'(t) dt = 2t dt$$

$$= \int_{\log 4}^{\log 8} \frac{\log^2(2x) + 3}{x \log(2x)} dx$$

ponyo  $t = \log(2x)$

$$dt = \frac{1}{x} dx$$

$$= \int_{\log 4}^{\log 8} \left( t + \frac{3}{t} \right) dt$$

$$= \left[ \frac{t^2}{2} + 3 \log t \right]_{\log 4}^{\log 8}$$

$$\int \frac{e^{3x}}{2+e^x} dx$$

peng  $e^x = t$   
 $dt = e^x dx$

$$\int \frac{e^{2x} \cdot e^x dx}{2+e^x}$$

$$\int \frac{t^2}{2+t} dt = \int \left( \frac{t^2-4}{t+2} + \frac{4}{t+2} \right) dt$$

$$\boxed{t-2}$$

$$\rightarrow \left[ \frac{t^2}{2} - 2t + 4 \log(t+2) \right]_1^e$$

