$$f(x) = x\,e^{\frac{|x-1|}{x-2}} \qquad D = \mathbb{R} \setminus \{2\}$$

LIMITI

a) $\lim_{x \to +\infty} f(x) = +\infty$

b) $\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to 2^+} f(x) = +\infty$ A-

$\lim_{x \to 2^-} f(x) = 0$

EVENTUALI ASINTOTI OBLIQUI

a) $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\cancel{x}\,e^{\frac{|x-1|}{x-2}}}{\cancel{x}} = \underbrace{e}_{m}$

$$\lim_{x\to+\infty} x\left(e^{\frac{|x-1|}{x-2}} - e\right) =$$

$$= \lim_{x\to+\infty} x e\left(e^{\frac{x-1}{x-2}-1} - 1\right)$$

$$= \lim_{x\to+\infty} x e\left(e^{\frac{\cancel{x}-1-\cancel{x}+2}{x-2}} - 1\right)$$

$$= \lim_{x\to+\infty} x e\left(e^{\frac{1}{x-2}} - 1\right) =$$

$$= \lim_{x\to+\infty} x e\left(\frac{1}{x-2} + o\left(\frac{1}{x-2}\right)\right) = \boxed{e}$$

ASINT. OBLIQUO per $x \to +\infty$:

$$\Longrightarrow \boxed{y = e(x+1)}$$

b) $\lim_{x\to-\infty} \frac{f(x)}{x} = \underline{\lim_{x\to-\infty}} e^{\frac{|x-1|}{x-2}} = \boxed{\frac{1}{e}} \to m$

$\left(|x-1| = 1-x \quad \text{per } x\to-\infty \right)$

$\lim_{x\to-\infty} f(x) - \frac{x}{e}$

$= \lim_{x\to-\infty} x\left(e^{\frac{1-x}{x-2}} - \frac{1}{e}\right)$

$= \lim_{x\to-\infty} \frac{x}{e}\left(e^{\frac{1-x}{x-2}+1} - 1\right) = \lim_{x\to-\infty} \frac{x}{e}\left(e^{-\frac{1}{x-2}} - 1\right)$

$-\frac{1}{x-2} + o\left(\frac{1}{x-2}\right)$

$= \lim_{x\to-\infty} \frac{x}{e}\left(-\frac{1}{x-2}\right) = \boxed{-\frac{1}{e}}$

ASINT. OBLIQUO per $x\to-\infty$

$\boxed{y = \frac{1}{e}(x-1)}$

STUDIO di f':

$$f(x) = \begin{cases} x e^{\frac{x-1}{x-2}} & x \geq 1 \\ x e^{\frac{1-x}{x-2}} & x < 1 \end{cases}$$

$\boxed{x > 1}$

$(x \neq 2)$

$$f'(x) = e^{\frac{x-1}{x-2}} \left[1 + x \cdot \frac{\cancel{x}-2 - \cancel{x}+1}{(x-2)^2} \right]$$

$$= e^{\frac{x-1}{x-2}} \left[\frac{x^2+4-4x-x}{(x-2)^2} \right] \geq 0 \iff \boxed{x^2-5x+4 \geq 0}$$

$$(x-4)(x-1) \geq 0$$

Se $\boxed{x>1}$ $\quad \cancel{x\leq 1} \ \vee \ x\geq 4$

$f'(x)\leq 0$ per $1<x\leq 4$

$f'(x)\geq 0$ per $x\geq 4$

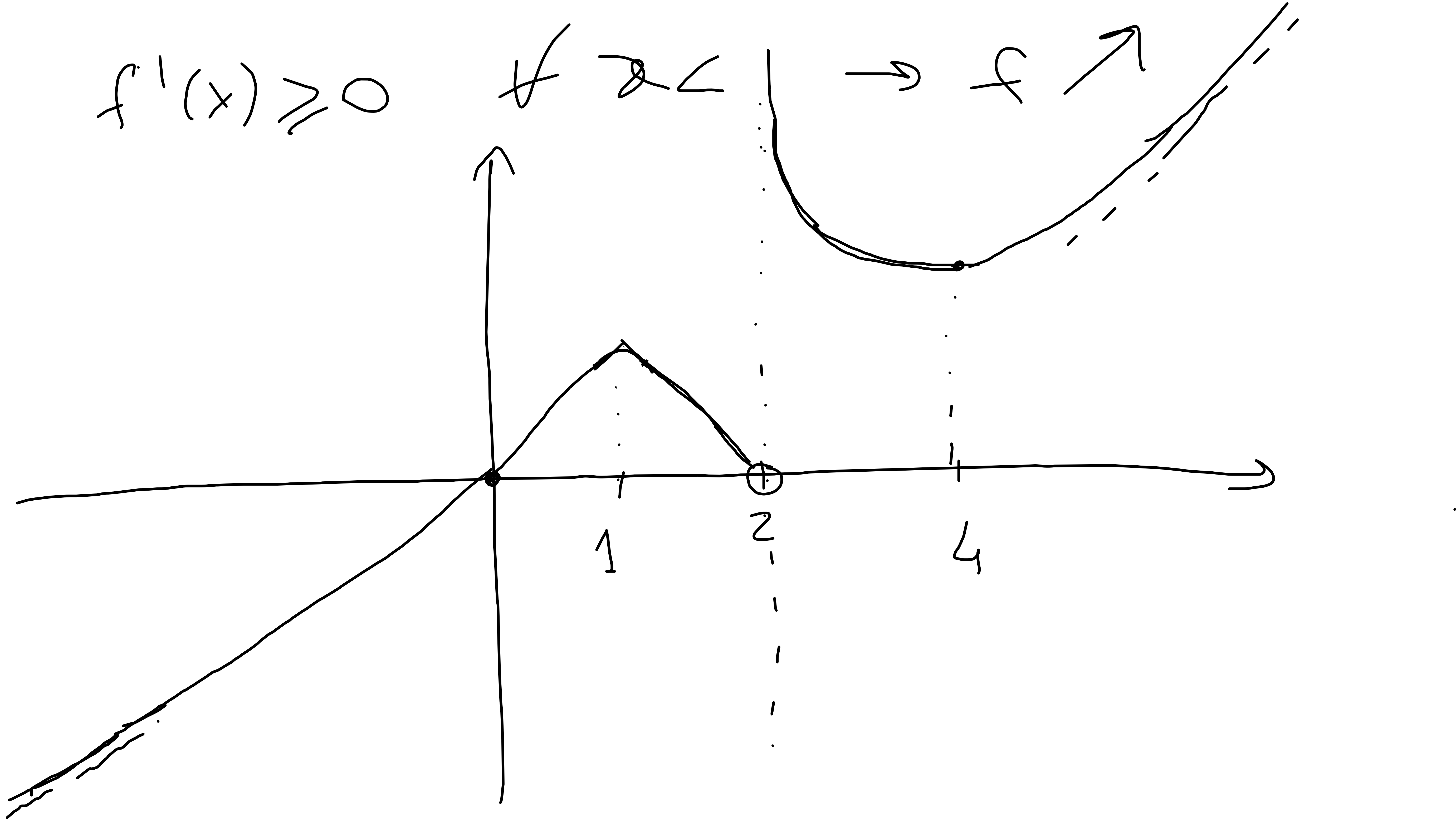
$x=4$ PTO di MIN LOCALE

Se $\boxed{x<1}$ $\quad f'(x)=e^{\frac{1-x}{x-2}}\left[1+\frac{x(-\cancel{x}+2-1+\cancel{x})}{(x-2)^2}\right]>0$

$\Leftrightarrow x^2-4x+4+x=x^2-3x+4\geq 0$

$\Delta=b^2-4ac=9-16<0$

$f'(x) \geq 0 \quad \forall x < \quad \rightarrow f \nearrow$



$$f(x) = e^{-\frac{x}{2\log|x|}} \qquad D = \mathbb{R} \setminus \{-1, 0, 1\}$$

LIMITI

- $$\lim_{x \to +\infty} f(x) = 0$$

- $$\lim_{x \to -\infty} f(x) = +\infty$$

- $$\lim_{x \to 0} f(x) = 1$$

$$\left(\frac{x}{\log|x|} \to 0 \right)$$

$-$ $\lim_{x\to 1^+} e^{-\frac{\cancel{x}}{2\log|x|}} = 0$

$-$ $\lim_{x\to 1^-} f(x) = +\infty$

$\lim_{x\to -1^+} f(x) = 0$

$\lim_{x\to -1^-} f(x) = +\infty$

$\left(x \to -1^+ \Rightarrow |x| \to 1^- \right.$

$\left. \Rightarrow \log|x| \to 0^- \right)$

$-1{,}1 \quad -1 \quad \underline{-0{,}9}$

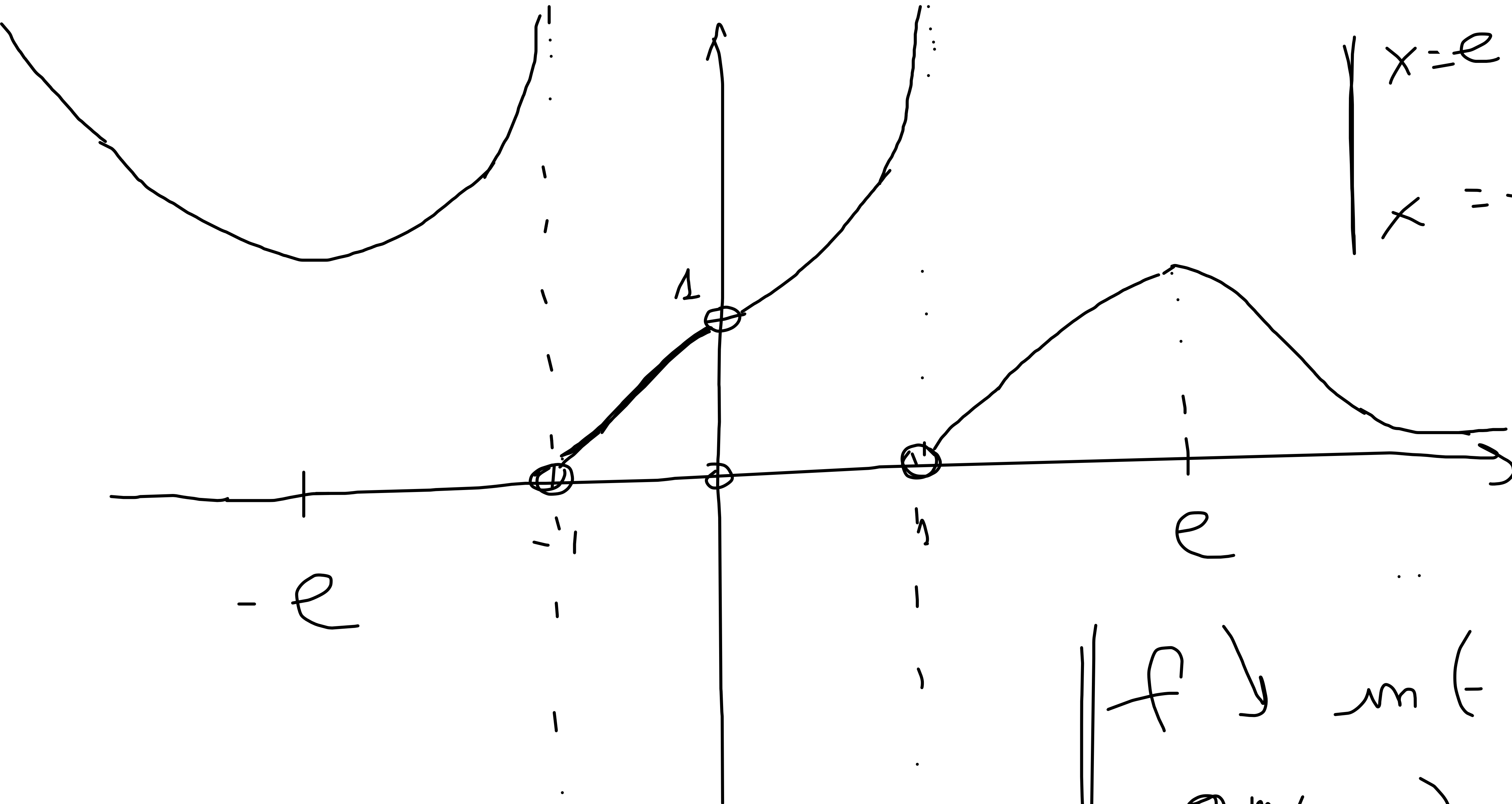
STUDIO di f'

$$f'(x) = e^{-\frac{x}{2\log|x|}} \left[\frac{-2\log|x| + \cancel{x}\,\frac{2}{\cancel{x}}}{(2\log|x|)^2} \right] \geq 0$$

$$\iff 1 - \log|x| \geq 0 \iff \log|x| \leq 1$$

$$\iff |x| \leq e \iff -e \leq x \leq e$$

$f \nearrow$



$x = e$ pto di max locale

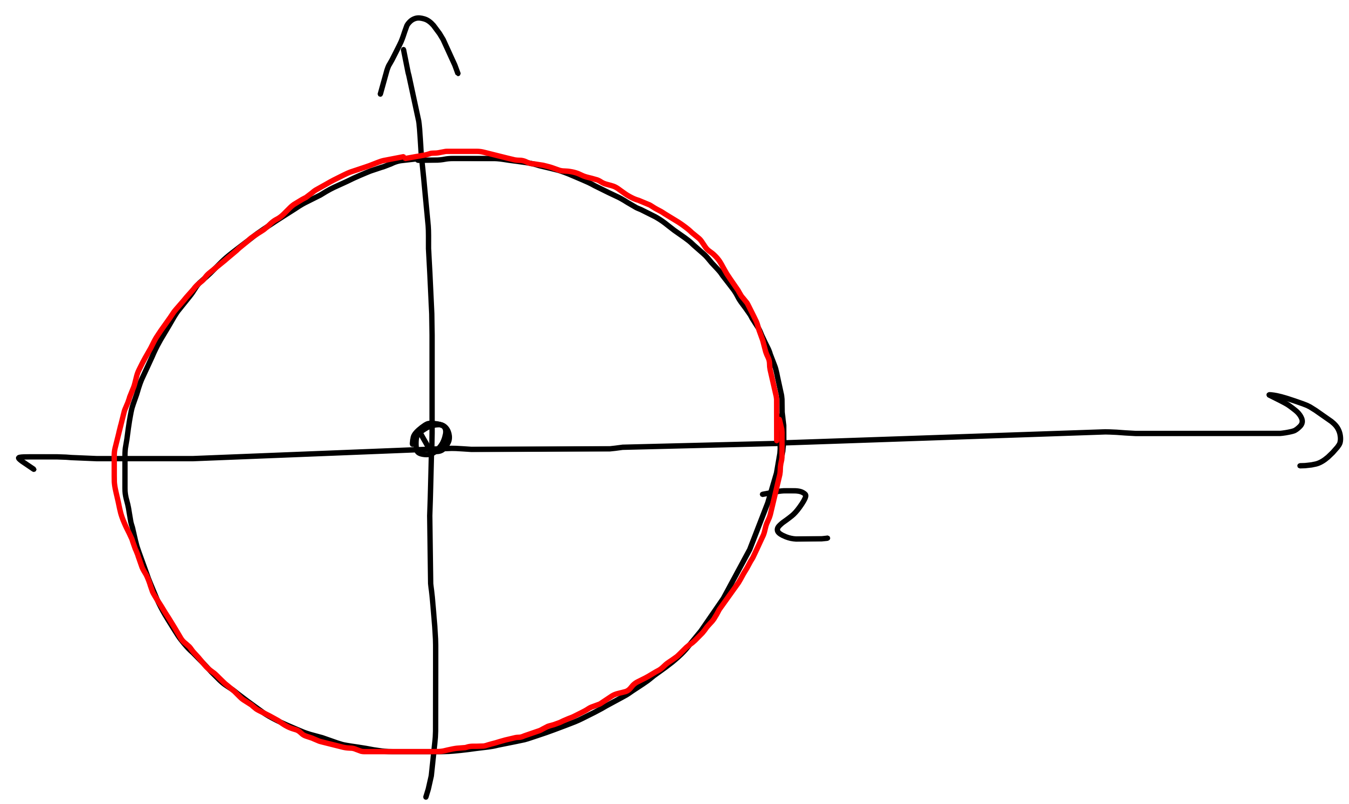
$x = -e$ pto di min locale

$f \searrow$ in $(-\infty, -e) \cup (e, +\infty)$

$f \nearrow$ in $(-e, e) \setminus \{0, 1, -1\}$

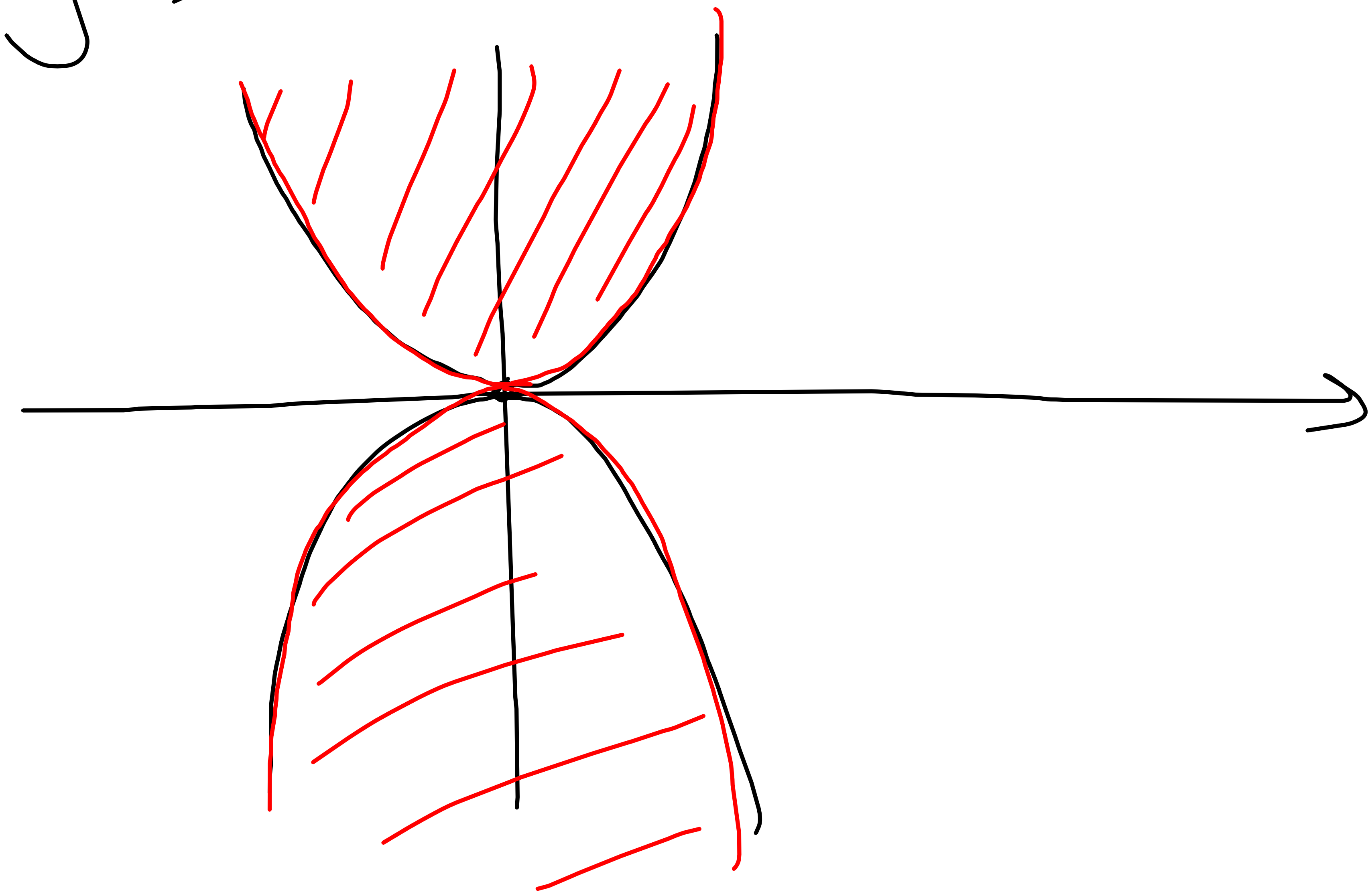
Determinare
<u>DOMINIO</u> : $f(x,y) = \sqrt{-|x^2+y^2-4|}$

D : $x^2+y^2-4 = 0 \rightarrow x^2+y^2 = 4$



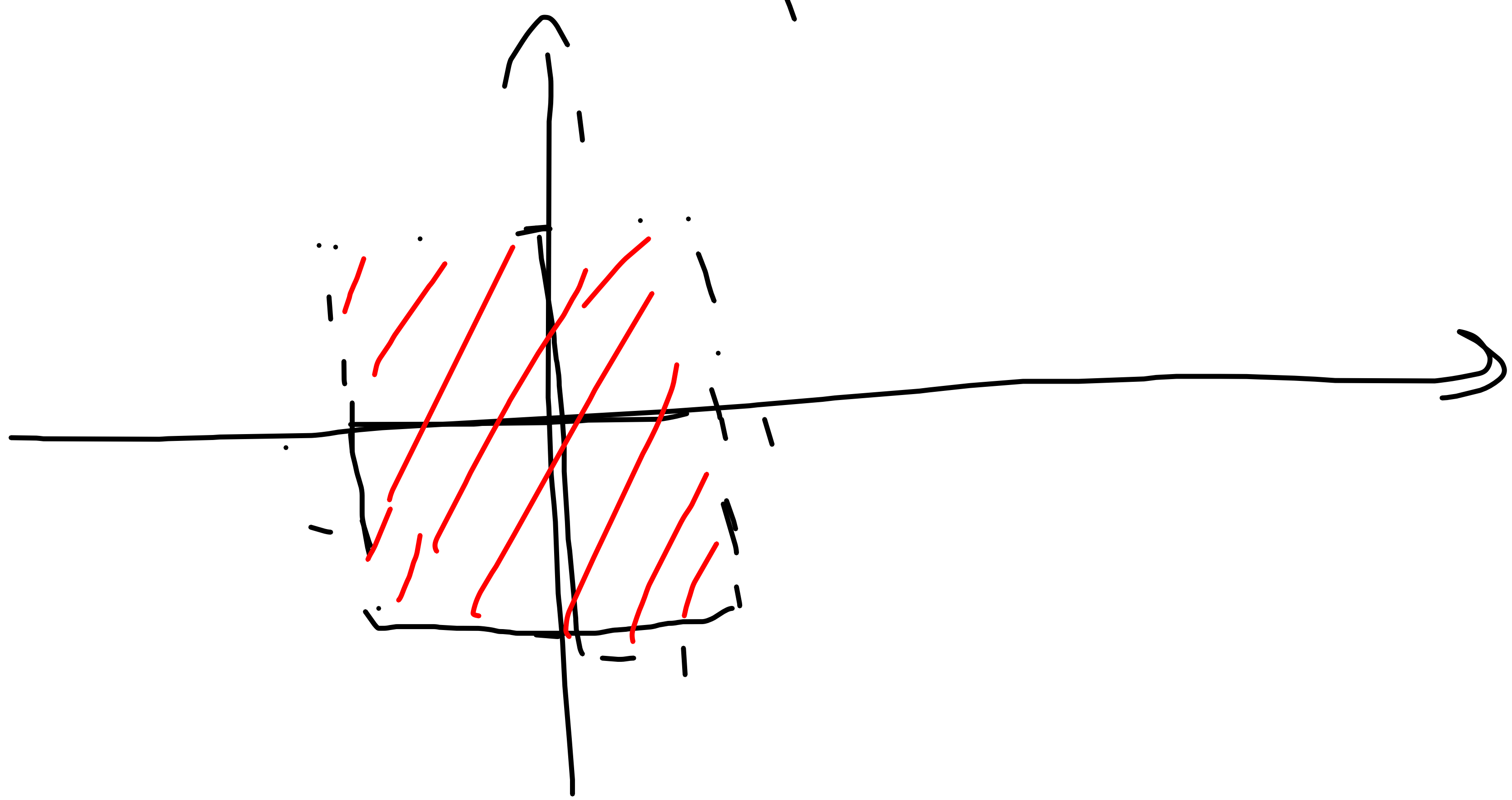
$f(x,y) = \sqrt{y^2 - x^4}$

$D: \; y^2 \geq x^4 \Rightarrow |y| \geq x^2 \Rightarrow \underline{y \geq x^2 \vee y \leq -x^2}$



$$f(x,y) = \log(1-y^2) + \log(1-x^2)$$

$$D: \begin{cases} 1-y^2 > 0 \\ 1-x^2 > 0 \end{cases} \quad \begin{cases} y^2 < 1 \\ x^2 < 1 \end{cases} \quad \begin{cases} -1 < y < +1 \\ -1 < x < 1 \end{cases}$$



TROVARE L'EQ. del PIANO TANGENTE

$$f(x,y) = \frac{x^2 y}{x+y} \qquad P(1,2,f(1,2))$$

$$z = f(1,2) + \partial_x f(1,2)(x-1) + \partial_y f(1,2)(y-2)$$

- $f(1,2) = \frac{2}{3}$
- $\partial_x f(x,y) = \frac{2xy(x+y) - x^2 y}{(x+y)^2} = \frac{x^2 y + 2xy^2}{(x+y)^2}$

$$\rightarrow \boxed{\partial_x f(1,2)} = \frac{2+8}{9} = \boxed{\frac{10}{9}}$$

$$\partial_y f(x,y) = \frac{x^2(x+y) - x^2 y}{(x+y)^2} = \frac{x^3}{(x+y)^2}$$

$$\boxed{\partial_y f(1,2) = \frac{1}{9}}$$

EQ. PIANO TANGENTE

$$z = \frac{2}{3} + \frac{10}{9}(x-1) + \frac{1}{9}(y-2)$$

. . . . .

DERIVATA DIREZIONALE

$f(x,y) = x + 4x^2 e^y \qquad P=(0,1) \qquad V=\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$g(t) = f\left(\underline{0 + t\frac{\sqrt{3}}{2}}, \underline{1 + \frac{t}{2}}\right)$$

$$= t\frac{\sqrt{3}}{2} + \cancel{4}\cdot\frac{3}{\cancel{4}}t^2 \; e^{1+\frac{t}{2}}$$

$$g'(t) = \frac{\sqrt{3}}{2} + 6t e^{1+\frac{t}{2}} + \frac{1}{2}e^{1+\frac{t}{2}} \cdot 3t^2$$

$$g'(0) = \boxed{\frac{\sqrt{3}}{2}}$$

Verificare che vale

$\nabla f(1,2) \cdot V = \frac{\sqrt{3}}{2}$

A CASA

$$f(x,y) = (x+y)^2$$

$P_1(1,2)$ $\quad V = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

RISULTATO $\rightarrow$ $\boxed{6\sqrt{2}}$

PT I
CRITICI

$$f(x,y) = \left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\left(\frac{1}{x}+\frac{1}{y}\right) \qquad \begin{array}{l} x \neq 0 \ \vee \\ y \neq 0 \end{array}$$

$$\begin{cases} \partial_x f & -\frac{1}{x^2}\left[\left(1+\frac{1}{y}\right)\left(\frac{1}{x}+\frac{1}{y}\right) + \left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\right] = 0 \\ \partial_y f & -\frac{1}{y^2}\left[\left(1+\frac{1}{x}\right)\left(\frac{1}{x}+\frac{1}{y}\right) + \left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\right] = 0 \end{cases}$$

$$\Longrightarrow \begin{cases} -\frac{1}{x^2}\left(1+\frac{1}{y}\right)\left[1+\frac{1}{y}+\frac{2}{x}\right] = 0 \\ -\frac{1}{y^2}\left(1+\frac{1}{x}\right)\left[1+\frac{1}{x}+\frac{2}{y}\right] = 0 \end{cases}$$

$$\partial_x f \begin{cases} \ldots \\ \dfrac{1}{x} = -1 \end{cases} \vee \begin{cases} \\ 1 + \dfrac{1}{x} + \dfrac{2}{y} = 0 \end{cases}$$

$$\begin{cases} \partial_x f & -1 \underbrace{\left(1 + \dfrac{1}{y}\right)}_{y=-1} \underbrace{\left(\dfrac{1}{y} - 1\right)}_{y=1} = 0 \\ \partial_y f & x = -1 \end{cases}$$

$P_1 = (-1, 1) \quad P_2 (-1, -1)$

Risolvete:

$P_3 = (1, -1) \quad P_4 = (-3, -3)$

SOLUZIONI P_1, P_2, P_3 pti SELLA

P_4 pto MIN LOC.

A CASA

$$f(x,y) = 2xy - x^3 - y^2$$

2 pti CRITICI $P_1 = (0,0)$ $P_2 = \left(\frac{2}{3}, \frac{2}{3}\right)$

P_1 → SELLA

P_2 → PTO di MAX