$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}=e \rightarrow \boxed{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}=e}$$

$$\rightarrow \boxed{\lim_{x\to 0}\left(1+x\right)^{\frac{1}{x}}=e}$$

$$\boxed{\lim_{x\to 0}\frac{\log_e(1+x)}{x}} = \lim_{x\to 0}\log_e\Big[\underbrace{(1+x)^{\frac{1}{x}}}_{e}\Big]=\boxed{1}$$

$$\boxed{\lim_{x\to 0}\frac{e^{x}-1}{x}} = \lim_{y\to 0}\frac{y}{\log(1+y)}=\boxed{1}$$

$$\left[\,y=e^{x}-1 \rightarrow e^{x}=y+1 \rightarrow x=\log(y+1)\right]$$

$$\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$$

$$\iff \lim_{x\to 0} \underbrace{\left[\frac{\log(1+x)}{x} - 1\right]} = 0$$

$$\iff \frac{\log(1+x)}{x} - 1 = o(1) \quad \text{per } x \to 0$$

$$\iff \frac{\log(1+x)}{x} = 1 + o(1) \quad \text{per } x \to 0$$

$$\iff \boxed{\log(1+x) = x + o(x)} \quad \boxed{\text{per } x \to 0}$$

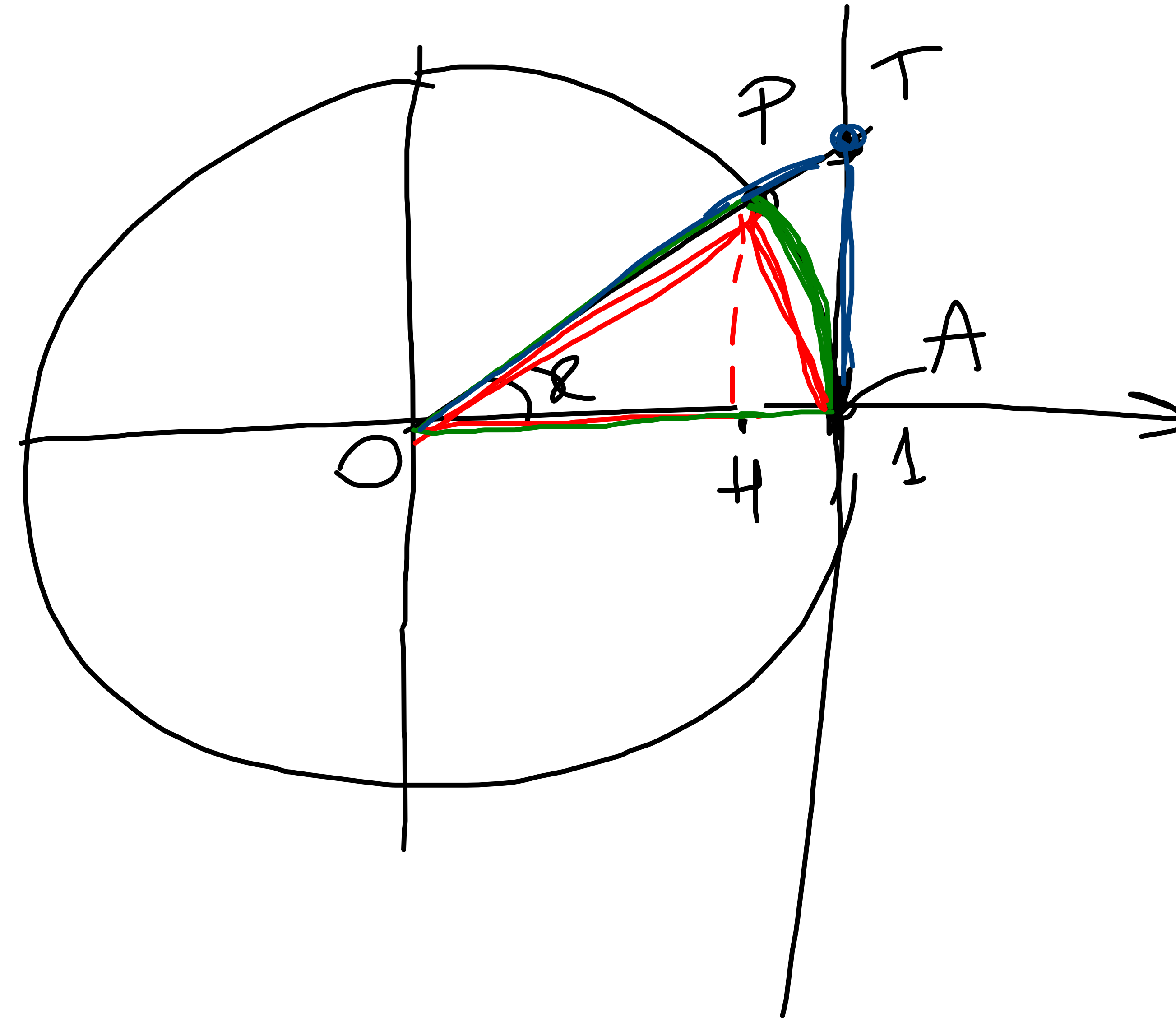
- $$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\iff \frac{e^x-1}{x} - 1 = o(1) \quad \text{per } x \to 0$$

$$\iff e^x - 1 = x + o(x) \quad \text{per } x \to 0$$

$$\iff \boxed{e^x = 1 + x + o(x) \quad \text{per } x \to 0}$$

• $\lim_{x \to 0} \frac{\operatorname{sen} x}{x}$



AREA $\overset{\triangle}{OPA}$: $\frac{\operatorname{sen} x}{2}$

AREA $\overset{\frown}{OPA}$ $\frac{x}{2}$

AREA $\overset{\triangle}{OTA}$: $\frac{\operatorname{tg} x}{2}$

Quindi $\frac{\operatorname{sen} x}{2} \le \frac{x}{2} \le \frac{\operatorname{tg} x}{2}$

Poiché sono tutte funzioni dispari

$\forall x \in]-\frac{\pi}{2}, \frac{\pi}{2}[\quad |\operatorname{sen} x| < |x| < |\tan x|$

$\forall x \in]-\frac{\pi}{2}, \frac{\pi}{2}[\quad 1 < \frac{|x|}{|\operatorname{sen} x|} < \frac{|\tan x|}{|\operatorname{sen} x|}$

$x \neq 0$

$$\Downarrow$$

$$\boxed{1 < \frac{x}{\operatorname{sen} x} < \frac{1}{\cos x}}$$

Passando al limite per $x \to 0$ si ha $1 \leq \lim_{x \to 0} \frac{x}{\operatorname{sen} x} \leq 1$

Per il Teor dei 2 carabinieri:

$$\lim_{x \to 0} \frac{x}{\operatorname{sen} x} = 1$$

$$\Rightarrow \boxed{\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1}$$

$$\Leftrightarrow \boxed{\operatorname{sen} x = x + o(x) \quad \text{per } x \to 0}$$

$$\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}\right)$$

$$=\lim_{x\to 0}\left(\frac{1-\cos^2 x}{x^2}\cdot\frac{1}{1+\cos x}\right)$$

$$=\lim_{x\to 0}\Bigg(\underbrace{\frac{\sin x}{x}}_{1}\Bigg)^2\cdot\underbrace{\frac{1}{1+\cos x}}_{\frac{1}{2}}=\frac{1}{2}$$

$$\Rightarrow\boxed{\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}}$$

$$\frac{1-\cos x}{x^2} - \frac{1}{2} = o(1) \quad \text{per } x \to 0$$

$$\frac{1-\cos x}{x^2} = \frac{1}{2} + o(1) \quad \text{per } x \to 0$$

$$1-\cos x = \frac{x^2}{2} + o(x^2) \quad \text{per } x \to 0$$

$$\boxed{\cos x = 1 - \frac{x^2}{2} + o(x^2) \quad \text{per } x \to 0}$$

$$\boxed{\lim_{x\to 0} \frac{\operatorname{tg} x}{x}} = \lim_{x\to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\downarrow\, 1} \cdot \underbrace{\frac{1}{\cos x}}_{\downarrow\, 1} = \boxed{1}$$

$$\lim_{x\to 0} \frac{e^x - 1}{\operatorname{sen} x} = \lim_{x\to 0} \frac{\cancel{1} + x + o(x) \cancel{-1}}{x + o(x)} = \boxed{1}$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(x^2)}{1-\cos x} = \lim_{x\to 0} \frac{x^2 + o(x^2)}{\cancel{1} - \cancel{1} + \frac{1}{2}x^2 + o(x^2)} = 2$$

$\operatorname{sen} x = x + o(x)$ per $x \to 0$

$\operatorname{sen}(x^2) = x^2 + o(x^2)$ per $x \to 0$

• $\lim_{x\to 2} \dfrac{e^{x-2}-1}{1-\cos(x-2)} =$

$= \lim_{y\to 0} \dfrac{e^{y}-1}{1-\cos y} =$

pongo $y = x-2$

$= \lim_{y\to 0} \dfrac{\cancel{1}+y+o(y)\cancel{-1}}{\cancel{1}-\cancel{1}+\frac{1}{2}y^2+o(y^2)}$

$= \lim_{y\to 0} \dfrac{2}{y}$ non ha limite

- $$\lim_{x\to+\infty} \frac{\operatorname{sen}\left(\frac{1}{x}\right)}{1-e^{\frac{1}{x}}} = \lim_{x\to+\infty} \frac{\frac{1}{x}+o\left(\frac{1}{x}\right)}{\cancel{1}-\cancel{1}-\frac{1}{x}+o\left(\frac{1}{x}\right)} = \boxed{-1}$$

- $$\lim_{x\to 2} \frac{\log(3-x)}{e^{x-2}-1} = \lim_{x\to 2} \frac{\log\left(1+(2-x)\right)}{e^{x-2}-1}$$

$$= \lim_{x\to 2} \frac{2-x+o(2-x)}{\cancel{1}+x-2+o(x-2)\cancel{-1}} = \boxed{-1}$$

$$\lim_{x \to \frac{1}{2}} \frac{e - e^{2x}}{x - \frac{1}{2}} = \lim_{x \to \frac{1}{2}} \frac{e\left[1 - e^{2x-1}\right]}{x - \frac{1}{2}}$$

$$= \lim_{x \to \frac{1}{2}} \frac{e\left[\cancel{1} - \cancel{1} - (2x-1) + o(2x-1)\right]}{x - \frac{1}{2}}$$

$$= \lim_{x \to \frac{1}{2}} \frac{-e2\cancel{\left(x - \frac{1}{2}\right)} + o(2x-1)}{\cancel{x - 1/2}} = \boxed{-2e}$$

- $$\lim_{x\to 1} \frac{e^2 - e^{2x}}{\operatorname{sen}(x-1)} = \lim_{x\to 1} \frac{e^2\left[1 - e^{2(x-1)}\right]}{\operatorname{sen}(x-1)}$$

$$= \lim_{x\to 1} \frac{e^2\left[1 - \left(1 - 2(x-1) + o(x-1)\right)\right]}{x-1+o(x-1)} = \boxed{-2e^2}$$

- $$\lim_{x\to +\infty} x^{\frac{1}{x}} = \lim_{x\to +\infty} e^{\log\left(x^{\frac{1}{x}}\right)}$$

$$= \lim_{x\to +\infty} e^{\frac{1}{x}\log x \to 0} = \boxed{1}$$

• $$\lim_{x \to 2} \frac{\text{sen}(x-2)}{x^2-5x+6} = \lim_{x \to 2} \frac{\text{sen}(x-2)}{(x-2)(x-3)} = \boxed{-1}$$

$$\frac{\text{sen}(x-2)}{(x-2)} \to 1$$

• $$\lim_{x \to 0} \frac{1-e^{x^2}}{x^2-3x} = \lim_{x \to 0} \frac{\cancel{1}-\cancel{1}-x^2+o(x^2)}{x(x-3)}$$

$$= \lim_{x \to 0} -\frac{x}{x-3} = \emptyset$$

a. $$\lim_{x\to 0}\frac{\operatorname{sen}(2x)}{x\log(1+x^2)}=\lim_{x\to 0}\frac{2x+o(x)}{x\left(x^2+o(x^2)\right)}$$

$$=\lim_{x\to 0}\frac{2\cancel{x}}{x^{\cancel{3}\,2}}=+\infty$$

b. $$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\operatorname{sen}x}\cdot\frac{\left(\sqrt{1+x}+1\right)}{\underbrace{\sqrt{1+x}+1}_{\to 2}}$$

$$=\lim_{x\to 0}\frac{\cancel{1}+x\cancel{-1}}{\left(x+o(x)\right)2}=\boxed{\frac{1}{2}}$$

$$\bullet \quad \lim_{x\to 1^+} \frac{\cos(x-1)-1}{\sqrt{x^2-1}} =$$

$$= \lim_{x\to 1^+} \frac{\cos(x-1)-1}{\underbrace{\sqrt{x+1}}\cdot\sqrt{x-1}}$$

$$= \lim_{x\to 1^+} \frac{\cancel{1}-\frac{1}{2}(x-1)^2+o\left((x-1)^2\right)\cancel{-1}}{\sqrt{x+1}\cdot\sqrt{x-1}} = \emptyset$$

$$\cdot \lim_{x \to +\infty} \frac{\overset{\frac{\pi}{2}}{\arctan x} \left(1 - e^{\frac{1}{x}}\right)}{\sqrt{1+\frac{1}{x}} - 1} \cdot \frac{\overset{\to 2}{\sqrt{1+\frac{1}{x}} + 1}}{\sqrt{1+\frac{1}{x}} + 1}$$

$$= \lim_{x \to +\infty} \frac{\frac{\pi}{2} \left(\cancel{1} - \cancel{1} - \frac{1}{x} + o\left(\frac{1}{x}\right)\right)}{\cancel{1} + \frac{1}{x} - \cancel{1}} \cdot 2$$

$$= \boxed{-\pi}$$

$$\bullet \lim_{x \to 0} \frac{1 - e^{x^3}}{\left(1 - \cos(2x)\right) \cdot \operatorname{sen} x}$$

$$= \lim_{x \to 0} \frac{\cancel{1} - \cancel{1} - x^3 + o(x^3)}{\left(\cancel{1} - \cancel{1} + \frac{(2x)^2}{2} + o(x^2)\right)\left(x + o(x)\right)}$$

$$= \lim_{x \to 0} \frac{-x^3 + o(x^3)}{2x^3 + o(x^3)} = \boxed{-\frac{1}{2}}$$

$$\bullet \lim_{x \to 0} \frac{1+\cos(x^2) - 2e^{x^4}}{\left(\operatorname{sen}(x^2)\right)^2}$$

$$= \lim_{x \to 0} \frac{1 + \left(1 - \frac{x^4}{2} + o(x^4)\right) - 2\left(1 + x^4 + o(x^4)\right)}{\left(x^2 + o(x^2)\right)^2}$$

$$= \lim_{x \to 0} \frac{-\frac{x^4}{2} - 2x^4 + o(x^4)}{x^4 + o(x^4)} = -\frac{5}{2}$$