Determinare il DOMINIO e disegnarlo.

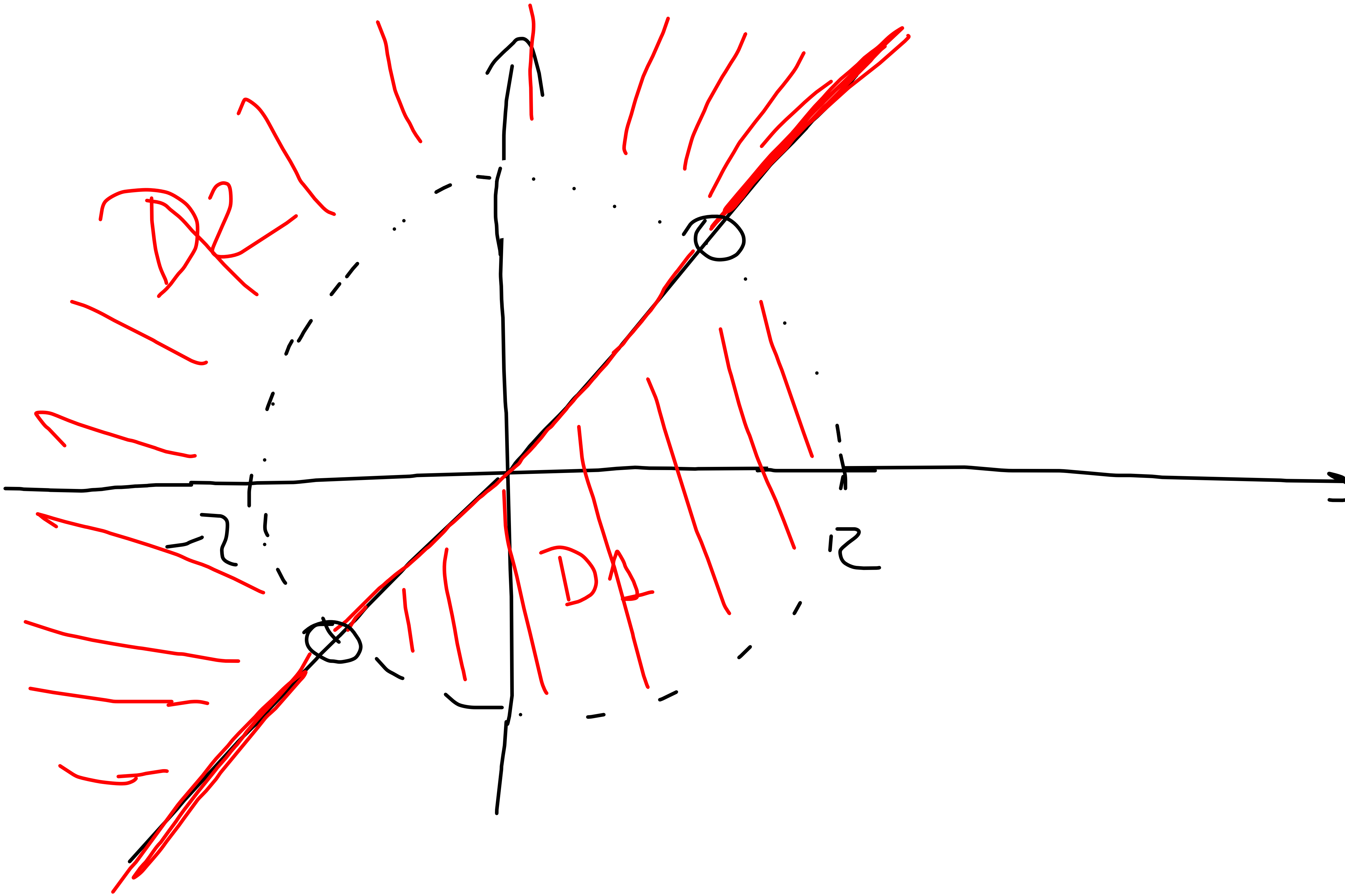
$$f(x,y) = \sqrt{\frac{x-y}{4-x^2-y^2}}$$

$$D: \begin{cases} \dfrac{x-y}{4-x^2-y^2} \geq 0 \\ x^2+y^2 \neq 4 \end{cases}$$

$$D = \{(x,y) \in \mathbb{R}^2 \mid \underbrace{y \leq x,\ x^2+y^2 < 4}_{D_1}\}$$

$$\cup$$

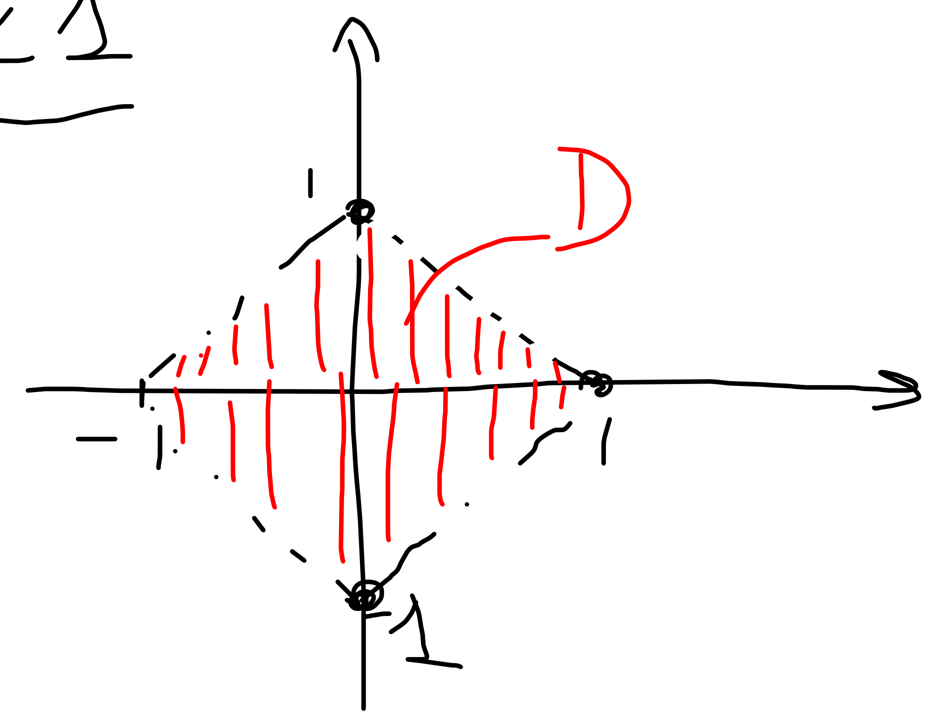
$$\{(x,y) \in \mathbb{R}^2 \mid \underbrace{y \geq x,\ x^2+y^2 > 4}_{D_2}\}$$



D2
D1
-2
2

• $f(x,y) = \log(1-|x|-|y|)$

D: $1-|x|-|y|>0 \iff \underline{|x|+|y|<1}$




Se $y \geq 0$ $\quad \underline{y < 1-|x|}$

• se $x \geq 0$ $\quad y < 1-x$

• se $x < 0$ $\quad y < 1+x$

Se $y<0$ $\quad y > |x|-1$

$D = \{(x,y)\in\mathbb{R}^2 \mid |x|+|y|<1\}$

ES $\circ$ $f(x,y) = \sqrt{y - x^2} - \log(1 + x - y)$

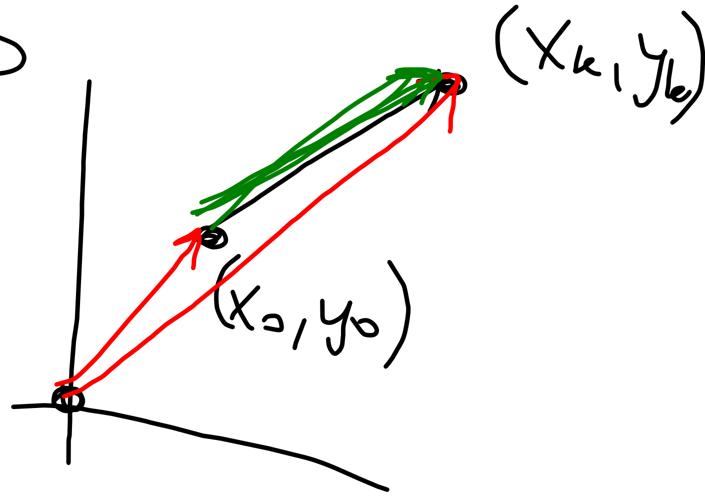
---

LIMITI di FUNZIONI di 2 VARIABILI

**Def** Sia $\{(x_k, y_k)\}_{k \in \mathbb{N}}$ successione in $\mathbb{R}^2$

Diciamo che $(x_k, y_k) \xrightarrow[k \to +\infty]{} (x_0, y_0)$

se $\underbrace{\left\| (x_k, y_k) - (x_0, y_0) \right\|} \xrightarrow[k \to +\infty]{} 0$



**Def** Sia $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$, $(x_0, y_0) \in \mathbb{R}^2$

Diciamo che esiste il limite

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = \ell \quad \text{se}$$

$\forall$ successione $\{(x_k, y_k)\}_{k\in\mathbb{N}}$ t.c. $\underline{(x_k, y_k) \xrightarrow[k\to+\infty]{} (x_0, y_0)}$

si ha $\lim_{k\to+\infty} \underline{f(x_k, y_k)} = \ell$

Equivalentemente

$\forall \varepsilon > 0, \exists \delta_\varepsilon > 0$ t.c. $\forall (x, y)$ : $|(x,y) - (x_0, y_0)| < \delta_\varepsilon$

si ha $|f(x,y) - \ell| < \varepsilon$

---

**Def** Diciamo che $f$ è continua in $(x_0, y_0)$

se $\lim_{(x,y) \to (x_0, y_0)} f(x,y) = f(x_0, y_0)$

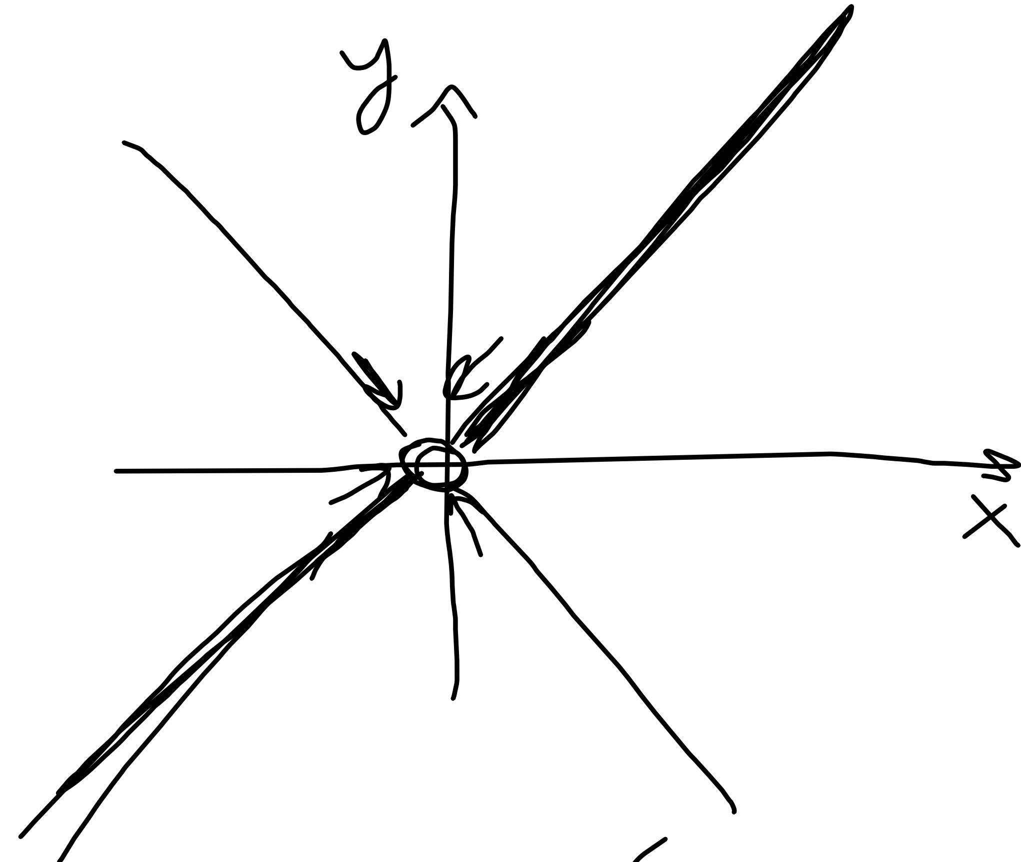
<u>OSS</u>

- Continuano a valere tutti i risultati sui limiti (unicità, operazioni, confronto)
- Somme, prodotti, composizione, rapporti di funz. continue sono continue

# ESEMPI (LIMITI)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

$D = \mathbb{R}^2 \setminus \{(0,0)\}$




- Se restringo $f$ alla retta $y = x$:

$$\lim_{x\to 0} \frac{x^2}{x^2+x^2} = \frac{1}{2}$$

$\neq$

- Se restringo $f$ alla retta $y = -x$

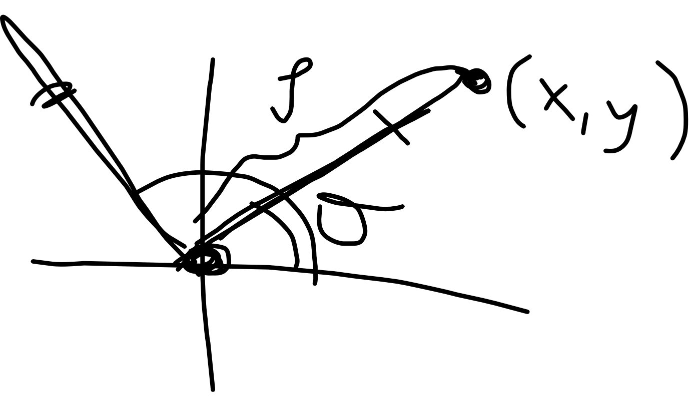
$$\lim_{x\to 0} -\frac{x^2}{2x^2} = -\frac{1}{2} \Rightarrow \nexists \lim$$

• $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+xy^2}{x^2+y^2}$

Passo a coordinate polari:

$$\begin{cases} x=\rho\cos\theta \\ y=\rho\,\mathrm{sen}\,\theta \end{cases} \Rightarrow \lim_{\rho\to 0} \frac{\rho^3\cos^3\theta+\rho^3\cos\theta\,\mathrm{sen}^2\theta}{\rho^2}$$

$$= \lim_{\rho\to 0} \frac{\cancel{\rho^3}\left(\cos^3\theta+\cos\theta\,\mathrm{sen}^2\theta\right)}{\cancel{\rho^2}}$$




$$= 0$$

$$\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}\left[(x^2+y^2)^2\right]}{2x^4+y^4}$$

coord. ~~p~~ polari:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases}$$

$$\lim_{\rho\to 0} \frac{\operatorname{sen}(\rho^4)}{2\rho^4(\cos\theta)^4+\rho^4(\operatorname{sen}\theta)^4}$$

$$= \lim_{\rho\to 0} \frac{\operatorname{sen}(\rho^4)}{\rho^4\left[2(\cos\theta)^4+(\operatorname{sen}\theta)^4\right]}$$

$$\to \nexists = \frac{1}{2(\cos\theta)^4+(\operatorname{sen}\theta)^4}$$ dipende da $\theta$

$\Rightarrow \nexists$ lim.

**Def** Sia $A \subseteq \mathbb{R}^2$ diciamo che

$A$ è limitato se

$$\exists \; B_R(x_0,y_0) = \{(x,y) \in \mathbb{R}^2 \mid |(x,y)-(x_0,y_0)| < R\}$$

t.c. $A \subset B_R(x_0,y_0)$