$$\lim_{n\to+\infty} \frac{n^2 \log\left(1+\frac{1}{n^2}\right)}{n^3}$$

$$= \lim_{n\to+\infty} \frac{n^2\left(\frac{1}{n^2} + o\left(\frac{1}{n^2}\right)\right)}{n^3}$$

$$= \lim_{n\to+\infty} \frac{n^{2-2}}{n^3}$$

$$\boxed{\log(1+x) = x + o(x) \text{ per } x\to 0}$$

$$\boxed{\begin{array}{lll} \text{se } 2>5 & & l=+\infty \\ \text{se } 2=5 & & l=1 \\ \text{se } 2<5 & & l=0 \end{array}}$$

$$\int_1^2 \frac{1-2\log x}{x\cdot x}\,dx \qquad \text{Pongo } t=\log x$$

$$dt = \frac{1}{x}\,dx$$

$$x = e^t$$

$$\left(\int_0^{\log 2} \frac{1-2t}{e^t}\,dt\right.$$

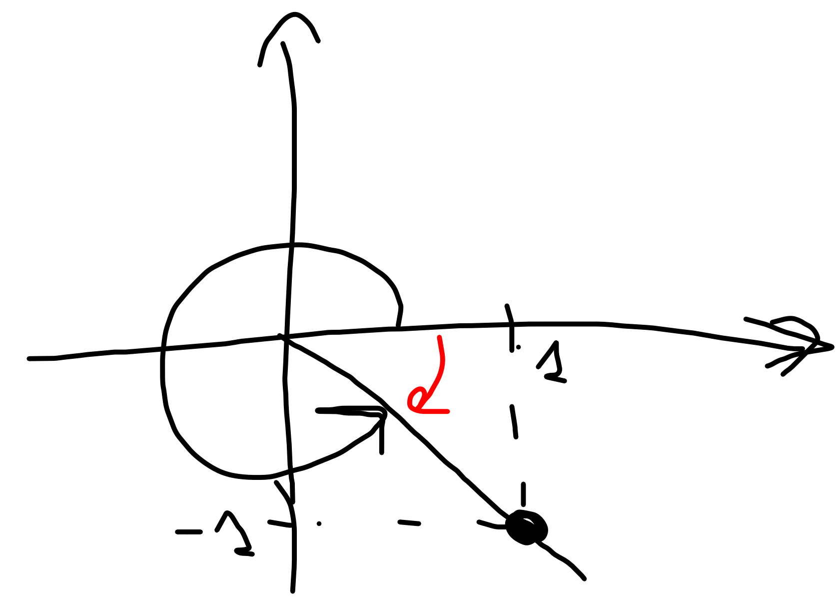
$$= \int_0^{\log 2} (1-2t)e^{-t}\,dt = \left[-e^{-t}\right]_0^{\log 2}$$

$$+2\left[te^{-t}\right]_0^{\log 2} - \int_0^{\log 2} 2e^{-t}\,dt \rightarrow \left[+2e^{-t}\right]_0^{\log 2}$$

b) $\rightarrow i z^3 = i + 1$

$$\boxed{z^3 = \frac{i+1}{i} \cdot \frac{i}{i} = \frac{-1+i}{-1} = \underbrace{1 - i}_{W}}$$

$W = 1 - i$



$\rho = |W| = \sqrt{2}$

$\theta = -\frac{\pi}{4}$

$z = r e^{i\varphi}$

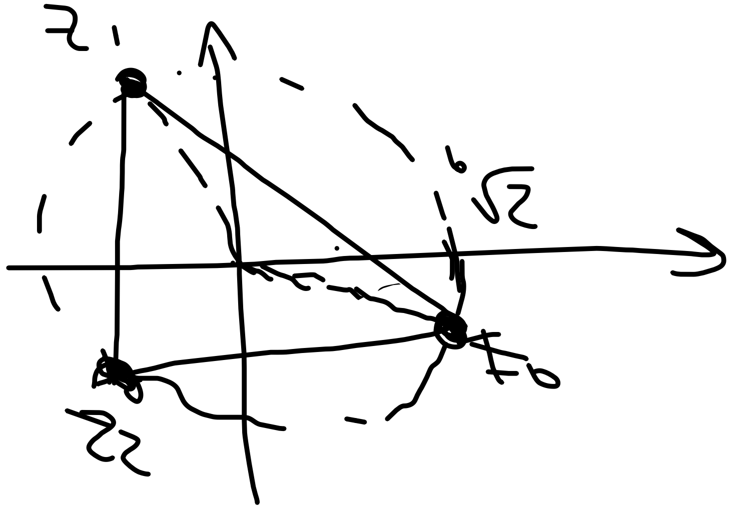
$z^3 = \sqrt{2}$

$3\varphi = -\frac{\pi}{4} + 2k\pi \quad k = 0, 1, 2$

$$\begin{cases} r = \sqrt[6]{2} \\ \varphi_k = \dfrac{-\frac{\pi}{4} + 2k\pi}{3} \end{cases} \qquad k = 0, 1, 2$$

$$\varphi_0 = -\frac{\pi}{12} \qquad \varphi_1 = \frac{7}{12}\pi \qquad \varphi_2 = \frac{5}{4}\pi$$

$$\Rightarrow z_0 = \sqrt[6]{2}\, e^{-i\frac{\pi}{12}} \qquad z_1 = \sqrt[6]{2}\, e^{i\frac{7}{12}\pi} \qquad z_2 = \underbrace{\sqrt[6]{2}}\, e^{i\frac{5}{4}\pi}$$



$$\lim_{x\to 0} \frac{\log(1+x)-1-\operatorname{sen}^2 x + e^{x^2-x}}{\operatorname{tg}x - x}$$

$\boxed{D}$: $\operatorname{tg}x = x + \frac{1}{3}x^3 + o(x^3)$

$\boxed{\text{Den}}$: $\operatorname{tg}x - x = \boxed{\frac{1}{3}x^3 + o(x^3)}$

$\boxed{N}$

- $\log(1+x) = \cancel{x} - \frac{1}{2}x^2 + \frac{1}{3}x^3 + o(x^3)$
- $-\operatorname{sen}^2 x = -\left(x - \frac{x^3}{6} + o(x^3)\right)^2 = \cancel{-x^2} + o(x^3)$
- $\underbrace{e^{x^2-x}-1}$ $= \cancel{x^2} - \cancel{x} + \frac{1}{2}(x^2-x)^2 + \frac{1}{6}(x^2-x)^3 + o(x^3)$

- $$\int_0^1 \frac{e^x+e^{2x}}{e^{2x}+1}\,dx = \int_0^1 \frac{(1+e^x)\,\underbrace{e^x\,dx}}{e^{2x}+1} \qquad e^x = t$$

$$= \int_1^e \frac{1+t}{t^2+1}\,dt = \int_1^e \left(\frac{1}{t^2+1} + \underbrace{\frac{t}{t^2+1}}\right)dt$$

$$= \Big[\arctan t\Big]_1^e + \frac{1}{2}\Big[\log(t^2+1)\Big]_1^e$$

$$\lim_{n\to+\infty} \frac{|a|^n}{(n+2)^n} \cdot \frac{n^n}{2^n}$$

$$= \lim_{n\to+\infty} \underbrace{\left(\frac{|a|}{2}\right)^n}\cdot \underbrace{\left(\frac{n}{n+2}\right)^n}_{\underbrace{\left(\cancel{\frac{n+2}{n+2}} - \frac{2}{n+2}\right)^n}_{\frac{1}{e^2}}}$$

Se $|a| > 2$

$\leftrightarrow$ $a < -2 \lor a > 2$ $\Rightarrow$ $\ell = +\infty$

Se $a = \pm 2$ $\Rightarrow$ $\ell = \frac{1}{e^2}$

Se $-2 < a < 2$ $\Rightarrow$ $\ell = 0$

$$e^{x^2-x}-1 = \frac{1}{2}\left(x^2-x\right)^2 + \underline{\frac{1}{6}\left(x^2-x\right)^3} + o(x^3)$$

si semplifica

$$= \underbrace{-\cancel{2}\,\frac{1}{\cancel{2}}\,x^3 - \frac{1}{6}x^3}\; + o(x^3)$$

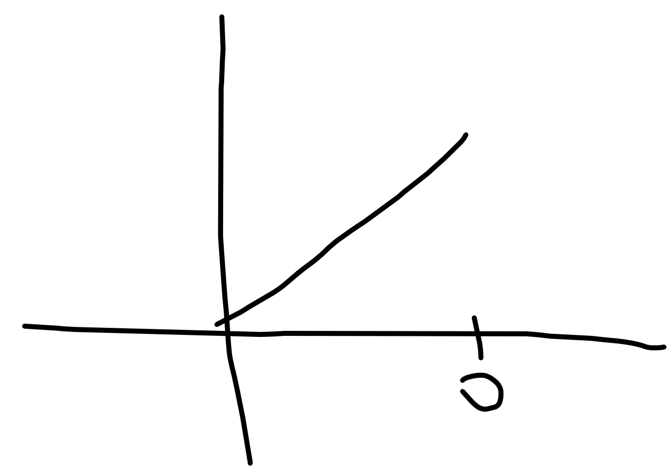
$$\boxed{N} : \; \underline{-\frac{7}{6}x^3} + \frac{1}{3}x^3 + o(x^3)$$

$$\lim_{x\to 0} \frac{N}{D} = \frac{-\frac{7}{6}+\frac{1}{3}}{\frac{1}{3}} = \boxed{-\frac{5}{2}}$$

$$\int_0^1 x\,tg^2 x\,dx = \underbrace{\int_0^1 x(tg^2 x+1)\,dx}_{(*)} - \underbrace{\int_0^1 x\,dx}$$

$$(*) \underset{\text{per parti}}{=} \Big[x\,tg\,x\Big]_0^1 - \underbrace{\int_0^1 tg\,x\,dx}_{(**)}$$

$$(**) = \underbrace{\int_0^1 \frac{-sen\,x}{cos\,x}\,dx} = \Big[\log(cos\,x)\Big]_0^1$$



$$\lim_{n\to+\infty} \frac{\left(1-e^{\frac{1}{n^{2}}}\right)(n+n^{a})}{e^{\frac{1}{n}}}$$

$1-e^{x} = 1-(1-x+o(x))$

$$= \lim_{n\to+\infty} -\frac{1}{n^{2}} \cdot \left(n^{1}+n^{a}\right)$$

Se $\boxed{a>1}$

$$\lim_{n\to+\infty} -\frac{n^{a}}{n^{2}}$$

$a>2 \quad \ell = -\infty$

$a=2 \quad \ell = -1$

$1<a<2 \quad \ell = 0$

Se $\boxed{a \leq 1} \rightarrow \ell = 0$