TEOR (de l'Hôpital)

Sia $I$ int. o int. forato di $\mathbb{R}$,
sia $c \in [\inf I, \sup I]$, $f, g : I \to \mathbb{R}$
derivabili in $I \setminus \{c\}$. Supp. $g$ e $g' \neq 0$ in $I \setminus \{c\}$

Se $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0$

o $\lim_{x \to c} f(x) = \pm\infty$ e $\lim_{x \to c} g(x) = \pm\infty$

e se $\exists \lim_{x \to c} \frac{f'(x)}{g'(x)} = \ell \Longrightarrow \lim_{x \to c} \frac{f(x)}{g(x)} = \ell$

$$\cdot \lim_{x \to 0} \frac{1 - e^{\cos x - 1}}{\operatorname{sen} x}$$

De l'Hôpital

$$\overset{DH}{=} \lim_{x \to 0} \frac{-\operatorname{sen} x \, e^{\cos x - 1}}{\cos x} = \emptyset$$

Con Taylor:

$$\cancel{1} - \frac{1}{2}x^2 + o(x^2) \cancel{-} 1$$

$$\lim_{x \to 0} \frac{1 - e}{x + o(x)} =$$

$$= \lim_{x \to 0} \frac{\cancel{1} - \cancel{1} + \frac{1}{2}x^2 + \underline{o(x^2)}}{x + o(x)} = \emptyset$$

$(1+y)^{\alpha} = 1+\alpha y + o(y)$

- $\displaystyle \lim_{x\to 0} \frac{e^{\operatorname{sen} x} - 1 - \operatorname{tg} x}{\sqrt{1+x^2} - 1}$

$\boxed{\operatorname{tg} x = x + \frac{1}{3}x^3 + o(x^3)}$

- $e^{\operatorname{sen} x} = e^{x+o(x^2)} = 1 + x + o(x^2) + \frac{1}{2}\left(x+o(x^2)\right)^2 + o(x^2)$

$= 1 + x + \frac{1}{2}x^2 + o(x^2)$

Num: $\cancel{1} + \cancel{x} + \frac{1}{2}x^2 + o(x^2) - \cancel{1} - \cancel{x}$

Den: $\cancel{1} + \frac{1}{2}x^2 + o(x^2) - \cancel{1}$

$\Rightarrow \displaystyle \lim_{x\to 0} \frac{\text{Num}}{\text{Den}} = \boxed{1}$

$$e^{\operatorname{sen} x} = e^{x - \frac{1}{6}x^3 + o(x^3)}$$

$$= 1 + x - \frac{1}{6}x^3 + o(x^3) + o(x$$

$$\frac{f(x)}{g(x)} : \frac{e^{\operatorname{sen} x} - 1 - \operatorname{tg} x}{\sqrt{1+x^2} - 1}$$

$$\boxed{\frac{0}{0}}$$

$$\frac{f'(x)}{g'(x)} = \frac{\cos x \, e^{\operatorname{sen} x} - 1 - \operatorname{tg}^2 x}{\frac{x}{\sqrt{1+x^2}}}$$

(da finire a casa)

$$\lim_{x \to 0} \frac{\operatorname{sen}(x^4)}{e^{\cos x - 1} + \frac{1}{2}\log(1+x^2) - 1}$$

$$\boxed{Num = x^4 + o(x^4)}$$

$\boxed{Den}$ :

$$e^{\cos x - 1} = e^{1 - \frac{1}{2}x^2 + o(x^3) + \frac{1}{24}x^4 + o(x^4) - 1}$$

$$= 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4) + \frac{1}{2}\left(-\frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)\right)^2 + o(x^4)$$

$$= 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{8}x^4 + o(x^4)$$

$$\frac{1}{2}\log(1+x^2) = \frac{1}{2}x^2 - \frac{1}{4}x^4 + o(x^4)$$

<u>Dem</u>: $$-\frac{1}{4}x^4 + \frac{x^4}{24} + \frac{x^4}{8} + o(x^4)$$

da finire

$$\left(\frac{N}{D} = \frac{x^4 + o(x^4)}{\underbrace{c}_{\text{da calcolare}}\, x^4 + o(x^4)} \xrightarrow[x \to 0]{} \frac{1}{c}\right)$$

$$\lim_{x\to 0}\left(\sqrt{9+2x}-2\right)^{\frac{1}{e^{2x}-1}}$$

$$=\lim_{x\to 0} e^{\log\left[\left(\sqrt{9+2x}-2\right)^{\frac{1}{e^{2x}-1}}\right]}$$

$$=\lim_{x\to 0} e^{\boxed{\frac{1}{e^{2x}-1}\log\left(\sqrt{9+2x}-2\right)}}$$

$$\lim_{x\to 0} \underbrace{\frac{1}{e^{2x}-1}}\cdot \underbrace{\log\left(\underbrace{\sqrt{9+2x}-2}\right)}$$

$$(1+y)^{\frac{1}{2}} = 1+\tfrac{1}{2}y+o(y)$$
per $y\to 0$

- $$e^{2x}-1 = \cancel{1}+2x+o(x)\cancel{-1} = 2x+o(x)$$
- $$\underbrace{\sqrt{9+2x}-2} = \underbrace{3\sqrt{1+\frac{2}{9}x}}-2$$

$$=\overbrace{3\left(1+\frac{1}{9}x+o(x)\right)}-2$$

$$= 1+\frac{1}{3}x+o(x)$$

$$\log\left(1+\frac{1}{3}x+o(x)\right)=\frac{1}{3}x+o(x)$$

da finire a casa