

$$\lim_{x \rightarrow 0} \frac{\sin(x^2 - x) - x^2 + x}{x^3}$$

$$\bullet \sin(x^2 - x) = x^2 - x + \underbrace{o(x^2 - x)}_{o(x)}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{x^2} - \cancel{x} + o(x) - \cancel{x^2} + \cancel{x}}{x^3}$$

Non posso
concludere

$$\text{Usa! } \sin(x^2 - x) = x^2 - x - \frac{1}{6}(x^2 - x)^3 + o(x^3)$$

$$\lim_{x \rightarrow 0} \frac{-\frac{1}{6}(x^2 - x)^3 + \cancel{o(x^3)}}{x^3} = \lim_{x \rightarrow 0} \frac{\frac{1}{6}x^3 + o(x^3)}{x^3} = \boxed{\frac{1}{6}}$$

$$\bullet \lim_{x \rightarrow 0} \frac{e^{1+x} (1 - \cos x + \log(1+x) - x)}{\sqrt{1-x^3} - 1}$$

$$\bullet (1+y)^2 = 1 + 2y + o(y) \quad y \rightarrow 0$$

$$\bullet \cos x = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 + o(x^5)$$

$$\bullet \log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + o(x^4)$$

$\lim_{x \rightarrow 0}$

$$e \left(\cancel{1} - \cancel{x} + \frac{1}{2}x^2 + \underbrace{o(x^3)} + \cancel{1} - \cancel{x} + \frac{1}{2}x^2 + \underbrace{\frac{1}{3}x^3 + o(x^3)} - \cancel{1} - \cancel{x} \right)$$

$$\cancel{1} - \frac{1}{2}x^3 + o(x^3) - \cancel{1}$$

$$= \boxed{1 - \frac{2}{3}e}$$

$$\lim_{x \rightarrow 0} \frac{1 - e^{x^2} + x}{\log(1 + x^2)}$$

$$e^{x^2} = e^{x + o(x^2)}$$

$$e^{x + o(x^2)} = 1 + \underbrace{x + o(x^2)}_{y} + \underbrace{o(x + o(x^2))}_{o(x)}$$

for $x \rightarrow 0$

and

$$\lim_{x \rightarrow 0} \frac{o(x)}{x^2 + o(x^2)}$$

you ~~poss~~
conclude

$$\begin{aligned}
 e^{x+o(x^2)} &= e^{\underline{x+o(x^2)}} \\
 &= 1 + \underline{x+o(x^2)} + \frac{1}{2} \left(\underline{x+o(x^2)} \right)^2 + o(x^2) \\
 &= 1 + x + \frac{1}{2} x^2 + o(x^2)
 \end{aligned}$$

Quindi:

$$\lim_{x \rightarrow \infty} \frac{-\frac{1}{2} x^2 + o(x^2)}{x^2 + o(x^2)} = \boxed{-\frac{1}{2}}$$

$$\lim_{x \rightarrow \underline{2}} \frac{e^{x-2} - 1 - \sin(x-2)}{1 - \sqrt{1 - (x-2)^2}}$$

$$= \lim_{x \rightarrow 2} \frac{1 + \cancel{x-2} + \frac{1}{2}(x-2)^2 + o(x-2)^2 - 1 - \left[\cancel{x-2} + o(x-2)^2 \right]}{1 - \left(1 - \frac{1}{2}(x-2)^2 + \underbrace{o(x-2)^2} \right)}$$

$$= \lim_{x \rightarrow 2} \frac{\frac{1}{2}(x-2)^2 + o(x-2)^2}{\frac{1}{2}(x-2)^2 + o(x-2)^2} = \boxed{1}$$

$$\lim_{x \rightarrow 0} \frac{\sec(x^2) - \log(1+x+x^2) + x - \frac{1}{2}x^2}{1 - e^{x^3}}$$

$$\bullet \sec(x^2) = \underline{x^2} + o(x^5)$$

$$\bullet -\log(1 + \underbrace{x+x^2}) = -x - \underline{x^2} + \frac{1}{2} \underbrace{(x+x^2)^2}_{\text{blue box}} - \frac{1}{3} \underbrace{(x+x^2)^3}_{\text{blue box}} + o(x^3)$$

$$Num: \cancel{x^2} - \cancel{x} - \cancel{x} + \frac{1}{2} \cancel{x^2} + \underline{x^3} - \frac{1}{3} \cancel{x^3} + \cancel{x} - \frac{1}{2} \cancel{x^2} + o(x^3)$$

$$N = \frac{2}{3} x^3 + o(x^3)$$

$$D: \cancel{x} - 1 - \cancel{x^3} + o(x^3)$$

$$\lim_{x \rightarrow 0} \frac{N}{D} = \boxed{-\frac{2}{3}}$$