ESERCIZI

$$f(x) = |x|\sqrt{x+2}$$

- DOMINIO : $D = [-2, +\infty)$

- LIMITI : - $\lim_{x \to -2^+} f(x) = 0$

  - $\lim_{x \to +\infty} f(x) = +\infty$

STUDIO di f'

$$\forall x \in D \setminus \{0, -2\}$$

$$f'(x) = \operatorname{sgn} x \sqrt{x+2} + \frac{|x|}{2\sqrt{x+2}}$$

$$= \operatorname{sgn} x \left[\sqrt{x+2} + \frac{x}{2\sqrt{x+2}} \right]$$

$$|x| = \operatorname{sgn} x \cdot x$$

$$= \operatorname{sgn} x \left[\frac{2x+4+x}{2\sqrt{x+2}} \right] \geq 0$$

$$\Leftrightarrow \boxed{\operatorname{sgn} x \cdot (3x+4) \geq 0}$$

$$f'(x) \geq 0 \Leftrightarrow \overbrace{\text{Sgm}\,x}^{F1}\,\overbrace{(3x+4)}^{F2} \geq 0$$

	-2		$-4/3$		0	
F1		$-$		$-$		$+$
F2		$-$		$+$		$+$
		$+$		$-$		$+$
		$\nearrow$		$\searrow$		$\nearrow$

STUDIO di EVENTUALI PUNTI di NON DERIVABILITÀ

$x=0$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{|x|\sqrt{x+2}}{x}$$

$$=\lim_{x\to 0}\sqrt{2}\,\frac{|x|}{x}\quad \text{NON ESISTE}$$

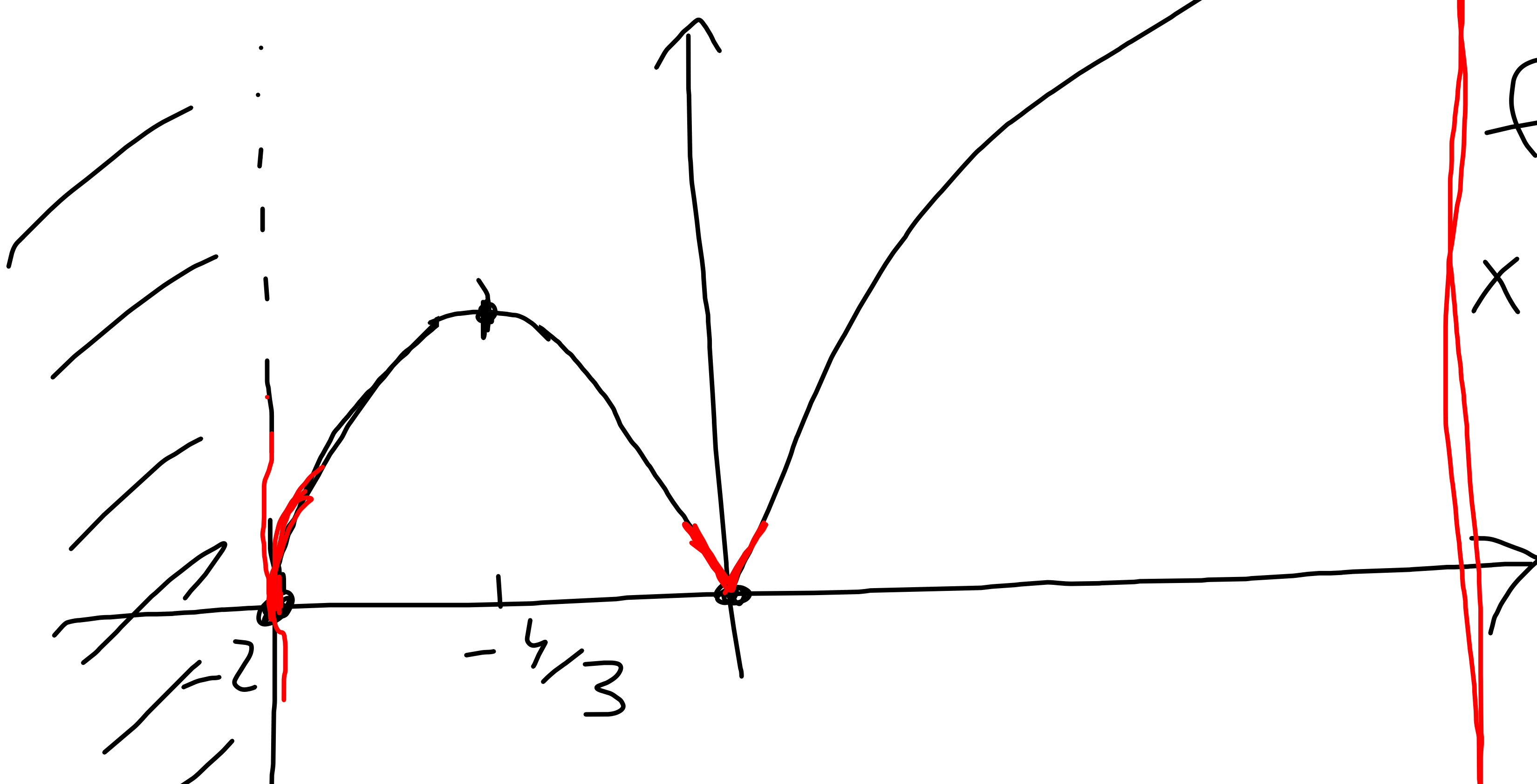
poiché

- $\lim_{x\to 0^+}\frac{\sqrt{2}|x|}{x}=\sqrt{2}$
- $\lim_{x\to 0^-}\frac{\sqrt{2}|x|}{x}=-\sqrt{2}$

$\neq \Rightarrow$ f NON è DERIVABILE in 0

$\underline{x=-2}$ $\quad \lim_{x \to -2^+} \frac{f(x)-f(-2)}{x+2} =$

$= \lim_{x \to -2^+} \frac{|x|\sqrt{x+2}}{x+2} = +\infty$



$f \nearrow$ in $\left[-2, -\frac{4}{3}\right] \cup [0, +\infty)$

$f \searrow$ in $\left(-\frac{4}{3}, 0\right)$

$x = -\frac{4}{3}$ PTO di MAX LOCALE

$x=-2$, $x=0$ PTI di MINIMO ASSOLUTO

- $\sup_D f = +\infty$
- $x=0$, $x=-2$ pti di NON DERIVABILITÀ

$$f(x) = |x+2|\, e^{2/x}$$

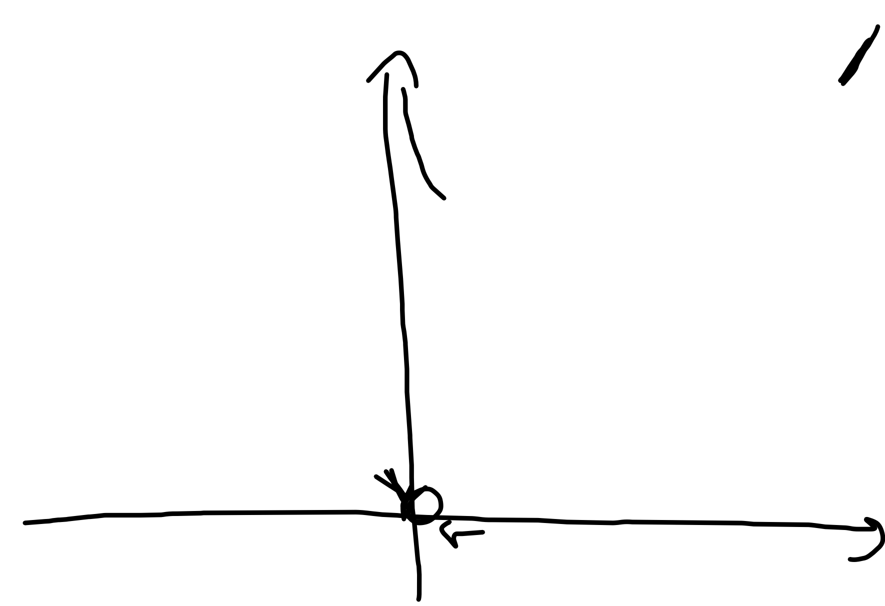
DOMINIO $\quad D = \mathbb{R}^* = \mathbb{R} \setminus \{0\}$

LIMITI

- $\lim_{x \to \pm\infty} f(x) = +\infty$

- $\lim_{x \to 0^+} f(x) = +\infty$

- $\lim_{x \to 0^-} f(x) = 0$



STUDIO di f'

$$f'(x) = \mathrm{sgn}(x+2)\, e^{2/x} - \frac{2}{x^2} e^{2/x} |x+2| \quad \forall x \in \mathbb{R}^* \setminus \{-2\}$$

$$= \mathrm{sgn}(x+2)\, e^{2/x} \left[1 - \frac{2}{x^2}(x+2) \right]$$

$$= \frac{\mathrm{sgn}(x+2)\, e^{2/x}}{x^2} \left[x^2 - 2x - 4 \right] \geq 0$$

$$\Leftrightarrow \underbrace{\mathrm{sgn}(x+2)}_{F1} \underbrace{(x^2 - 2x - 4)}_{F2} \geq 0$$

$$F1 \geq 0 \iff x > -2$$

$$F2 \geq 0 \iff x^2 - 2x - 4 \geq 0$$

$$x_{1,2} = 1 \pm \sqrt{1+4} \quad \begin{cases} 1+\sqrt{5} \\ 1-\sqrt{5} \end{cases}$$

$$F2 \geq 0 \iff x \leq 1-\sqrt{5} \ \vee \ x \geq 1+\sqrt{5}$$

		-2		$1-\sqrt{5}$		$1+\sqrt{5}$	
$F1$	$-$		$+$		$+$		$+$
$F2$	$+$		$+$		$-$		$+$
f'	$-$		$+$		$-$		$+$

$f \searrow$ in $(-\infty, -2) \cup (1-\sqrt{5}, 1+\sqrt{5}) \setminus \{0\}$

$f \nearrow$ in $(-2, 1-\sqrt{5}) \cup (1+\sqrt{5}, +\infty)$

$x = -2$ e $x = 1+\sqrt{5}$ sono PTI di MIN. LOCALE

$x = -2$ PTO di MIN. ASSOLUTO

$x = 1-\sqrt{5}$ PTO di MAX LOCALE

$\sup_D f = +\infty$

EVENTUALI pti di NON DERIVABILITÀ

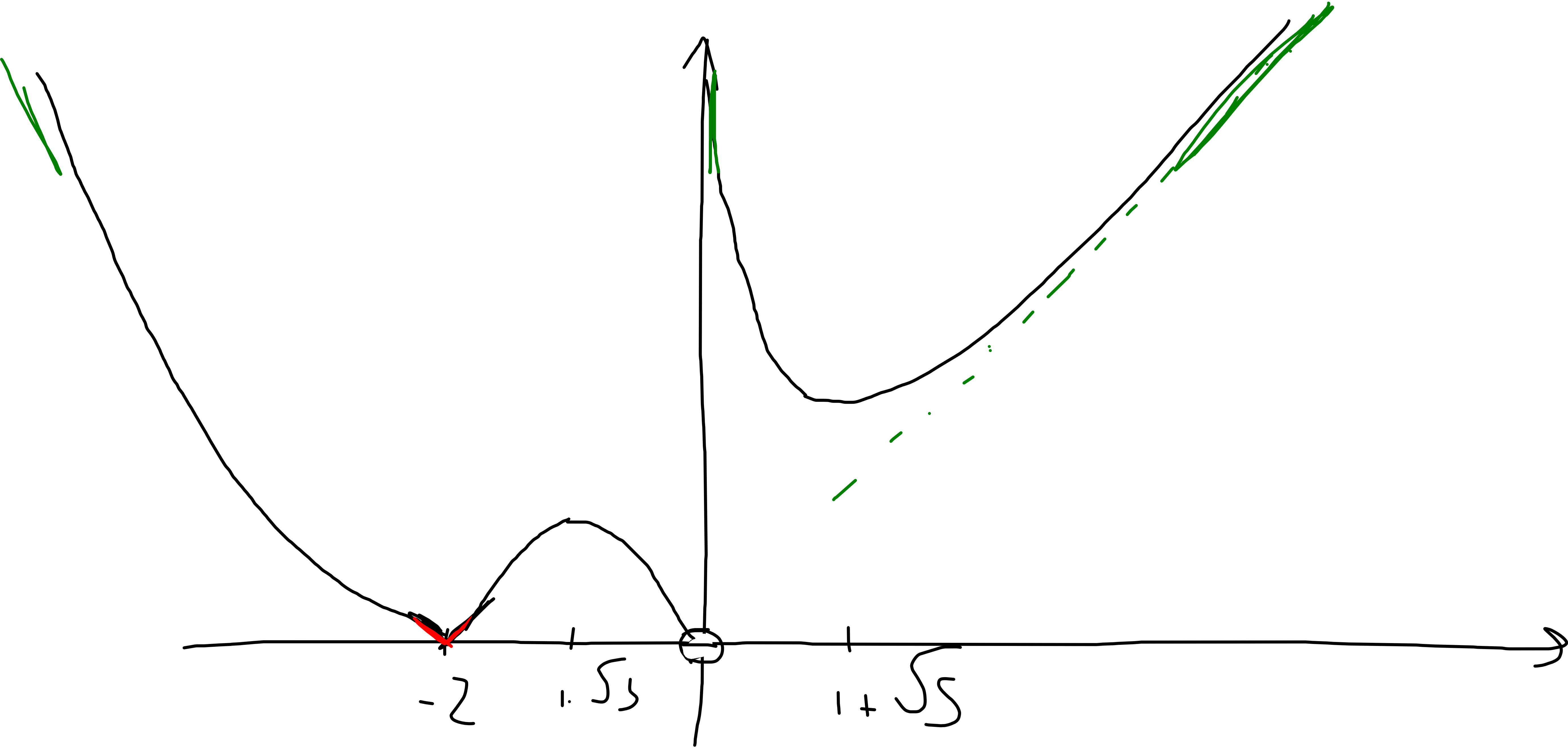
$x = -2$ $\qquad \lim_{x \to -2} \frac{f(x) - f(-2)}{x+2} =$

$$= \lim_{x \to -2} \frac{|x+2| e^{2/x}}{x+2}$$ NON ESISTE

$$\left.\begin{array}{l} \lim_{x \to -2^+} \frac{|x+2| e^{2/x}}{x+2} = e^{-1} \\ \lim_{x \to -2^-} \frac{|x+2| e^{2/x}}{x+2} = -e^{-1} \end{array}\right) \neq$$

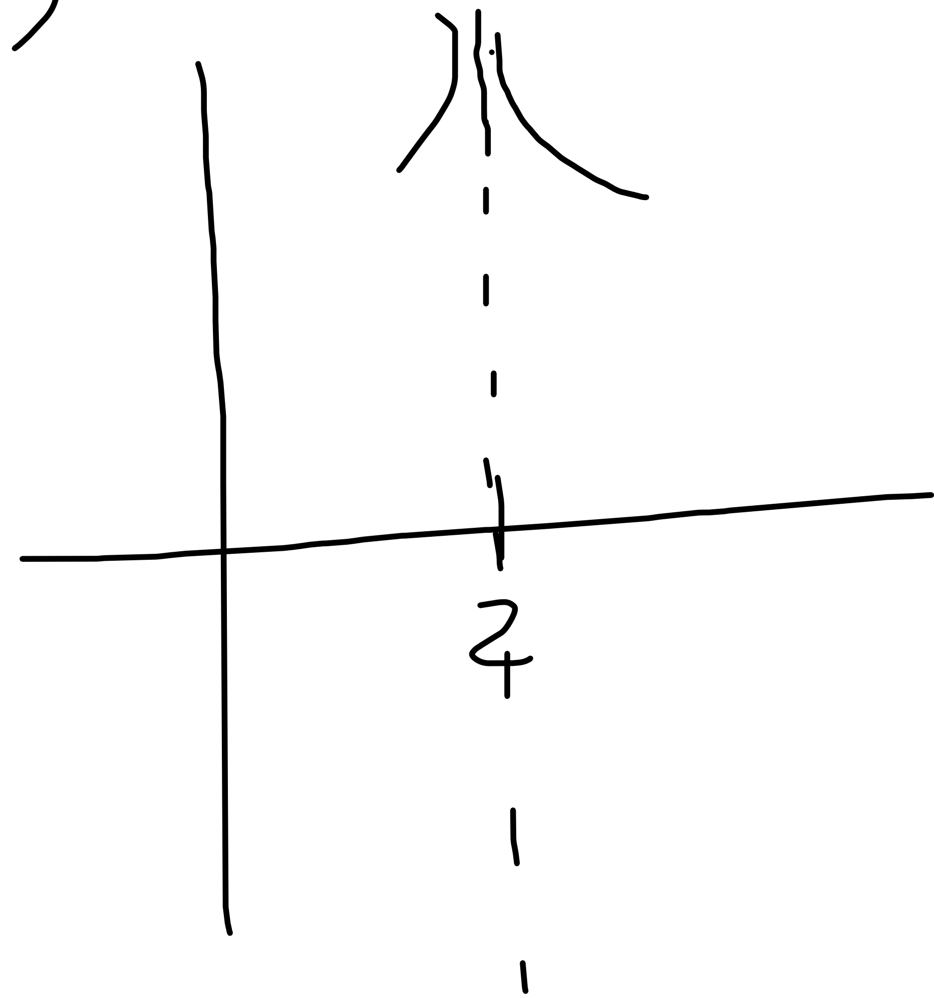
poiché

$x = -2$ è pto di NON DERIVABILITÀ


-2
1.53
1+√5

ASINTOTI VERTICALI

se $\lim_{x \to c^{\pm}} f(x) = +\infty$ $(-\infty)$ $\quad c \in \mathbb{R}$



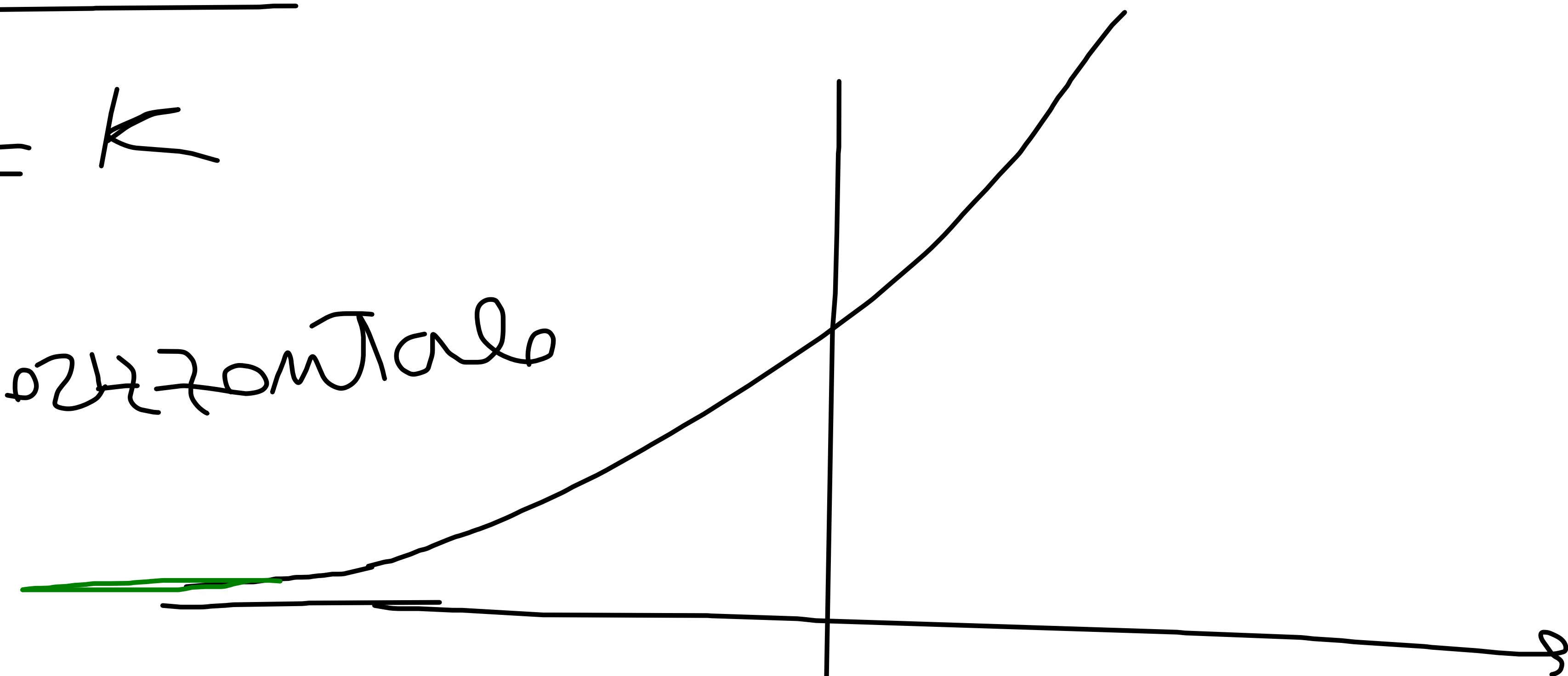
ES $\lim_{x \to 2} \frac{1}{(x-2)^2} = +\infty$

$x = 2$ è asintoto verticale

10 ASINTOTO ORIZZONTALE

$$\lim_{x \to \pm\infty} f(x) = k$$

$y = k$ asintoto orizzontale



Es $f(x) = e^x$

$$\lim_{x \to -\infty} e^x = 0 \longrightarrow y = 0 \text{ è asintoto orizzontale}$$

ASINTOTO OBLIQUO

Supponiamo:

$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

Per capire se c'è un asintoto obliquo, procedo come segue:

$(m \in \mathbb{R} \setminus \{0\})$

1) $$\lim_{x \to \pm\infty} \frac{f(x)}{x} = m$$

2) $$\lim_{x \to \pm\infty} f(x) - mx = q \in \mathbb{R}$$

$\Rightarrow$ $y = mx + q$ è asint. obliquo

ES $f(x) = |x+2| e^{2/x}$

Avevamo visto che

$$\lim_{x \to \pm\infty} f(x) = +\infty$$

Calcolo:

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{(x+2) e^{2/x}}{x} = 1 \quad (= m)$$

$$\lim_{x \to +\infty} (f(x) - x) = \lim_{x \to +\infty} (x+2) e^{2/x} - x$$

$$= \lim_{x\to+\infty} x\left(e^{2/x}-1\right) + \underbrace{2e^{2/x}}_{\downarrow\, 2} =$$

$$= \lim_{x\to+\infty} x\left[1+\frac{2}{x}+o\left(\frac{2}{x}\right)-1\right]+2 = 4 \quad (= q)$$

Quindi la retta: $y = x + 4$ è asint. obliquo (per $x \to +\infty$)

Per esercizio, studiare il caso $x \to -\infty$

$$f(x) = \frac{2x^2}{1-4x-x|x|}$$

DOMINIO: $1-4x-x|x| \neq 0$

- $\boxed{\text{Se } x \geq 0}$ $1-4x-x^2=0 \longleftrightarrow$

$$x_{1,2} = \frac{2 \pm \sqrt{4+1}}{-1} = \frac{2 \pm \sqrt{5}}{-1} = -2 \mp \sqrt{5}$$

solo $x=-2+\sqrt{5}$ è ≥ 0

- $\boxed{\text{Se } x < 0}$: $1-4x+x^2=0$, $x_{1,2} = 2 \pm \sqrt{3} = \underbrace{2 \pm \sqrt{3}}_{\text{nessuna è} < 0}$.

$$D = \mathbb{R} \setminus \{-2+\sqrt{5}\}$$

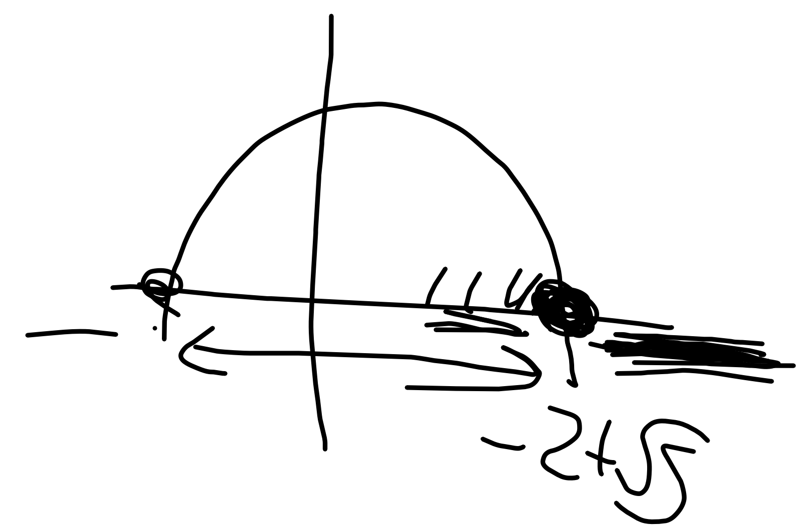
LIMITI:

- $\displaystyle \lim_{x \to +\infty} \frac{2x^2}{1-4x-x|x|} = \lim_{x \to +\infty} \frac{2x^2}{1-4x-x^2} = -2$

- $\displaystyle \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x^2}{1-4x+x^2} = 2$

- $\displaystyle \lim_{x \to (-2+\sqrt{5})^+} \frac{2x^2}{1-4x-x^2} = -\infty$

- $\displaystyle \lim_{x \to (-2+\sqrt{5})^-} \frac{2x^2}{1-4x-x^2} = +\infty$



STUDIO f'

$\boxed{x>0}$

$$f'(x) = \frac{4x(1-4x-x^2)-(-4-2x)2x^2}{(1-4x-x^2)^2}$$

$$= \frac{4x-16x^2-\cancel{4x^3}+8x^2+\cancel{4x^3}}{(\ .\ \ .\ -\ .\ )^2}$$

$$= \frac{4x(1-2x)}{(\ -\ -\ -\ .\ )^2} \geq 0 \iff 4x(1-2x) \geq 0$$

	0		$\frac{1}{2}$	
~~−~~		+		+
~~+~~		+		−
~~−~~		+		−
		↗		↘

$\boxed{x<0}$

$$f'(x)=\frac{4x(1-4x+x^2)-(-4+2x)\cdot 2x^2}{(\ldots\ldots)^2}$$

$$=\frac{4x-16x^2+\cancel{4x^3}+8x^2-\cancel{4x^3}}{(\quad)^2}$$

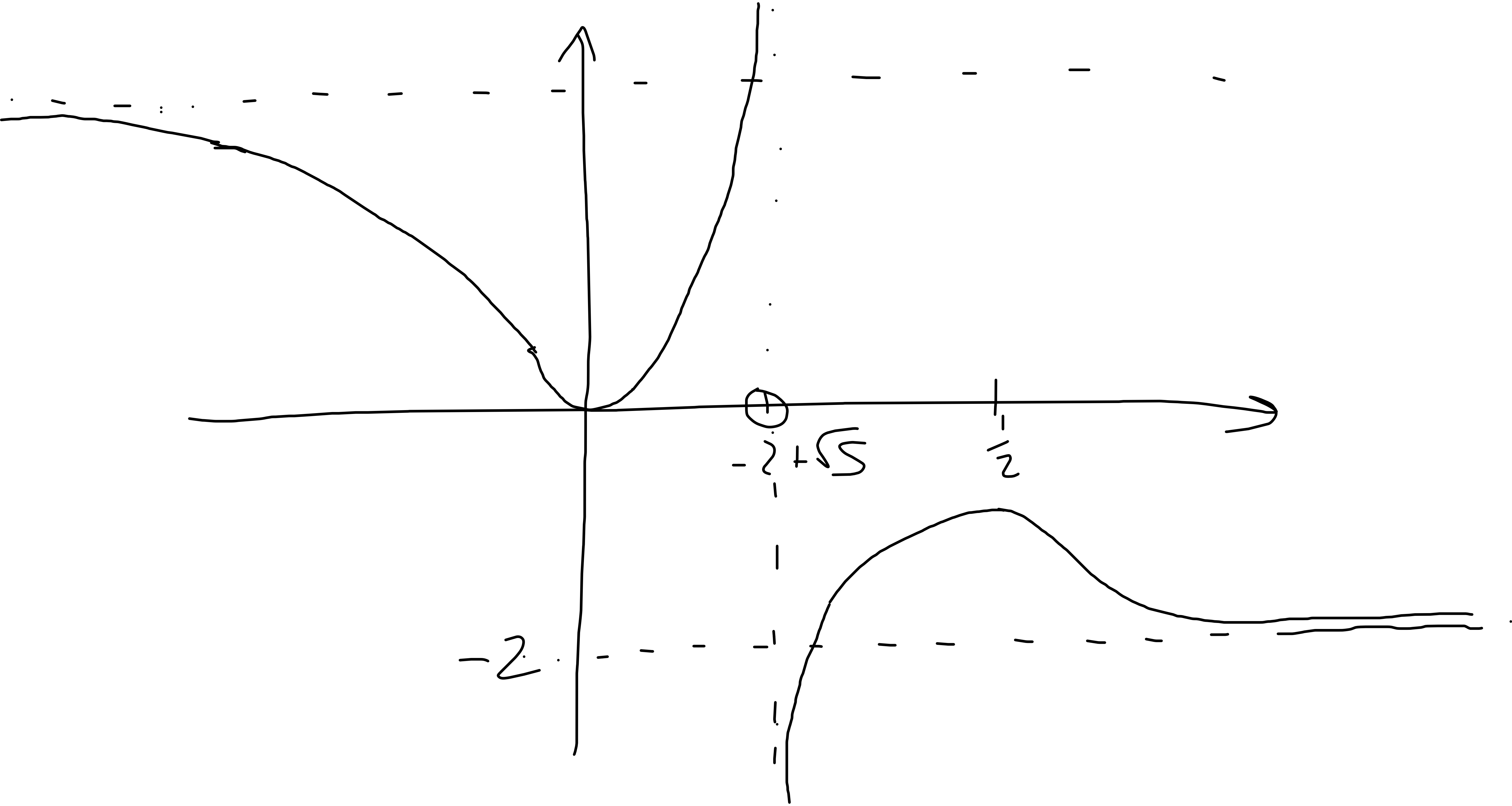
$$=\frac{4x(1-2x)}{(\ldots)^2}\geq 0 \iff 4x(1-2x)\geq 0$$

		0
	$-$	
	$+$	
f'	$-$	
	$\searrow$	

EVENTUALE PTO di NON DERIVABILITÀ

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x} =$$

$$= \lim_{x \to 0} \frac{2x^2}{\underbrace{|-4x - x|x||}_{\substack{\downarrow \\ 0 \\ \downarrow}}} \cdot \frac{1}{x} = 0$$



$-2+\sqrt{5}$
$\frac{1}{2}$
-2