Esercizi del 18 Maggio

(1)

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{-2} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{-\frac{1}{3}} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & \frac{7}{3} \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-\frac{2}{3}} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 1 & 0 \\ 0 & 0 & \frac{7}{3} & -\frac{2}{3} & -\frac{2}{3} & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{2}{7} & -\frac{2}{7} & \frac{3}{7} \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & \frac{5}{7} & \frac{9}{7} & -\frac{3}{7} \\ 0 & 0 & 1 & \frac{2}{7} & -\frac{2}{7} & \frac{3}{7} \end{array}\right) \xrightarrow{-\frac{2}{3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{11}{21} & -\frac{18}{21} & \frac{6}{21} \\ 0 & 3 & 0 & \frac{5}{7} & \frac{9}{7} & -\frac{3}{7} \\ 0 & 0 & 1 & \frac{2}{7} & -\frac{2}{7} & \frac{3}{7} \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{11}{21} & -\frac{18}{21} & \frac{6}{21} \\ 0 & 1 & 0 & \frac{5}{21} & \frac{9}{21} & -\frac{3}{21} \\ 0 & 0 & 1 & \frac{2}{7} & -\frac{2}{7} & \frac{3}{7} \end{array}\right)$$

$$\det\begin{pmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix} = 1 \cdot \det\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} - 2 \det\begin{pmatrix} -1 & 1 \\ 0 & 3 \end{pmatrix}$$

$$= 1 - 2 \cdot (-3) = 7$$

Esercizio 2

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & k+1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\det A = \det\begin{pmatrix} -1 & k+1 \\ 1 & 2 \end{pmatrix} + \det\begin{pmatrix} 1 & k+1 \\ 0 & 2 \end{pmatrix}$$

$$= -\cancel{2} - k - 1 + \cancel{2}$$

A è invertibile per $k \neq -1$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & k+1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & k+1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & k+1 & 2(k+1) & 0 & 0 & k+1 \\ 0 & 0 & 2(k+1) & -2 & 2 & 0 \end{array}\right) \to \left(\begin{array}{ccc|ccc} k+1 & -(k+1) & 0 & k+1 & 0 & 0 \\ 0 & k+1 & 0 & 2 & -2 & k+1 \\ 0 & 0 & 2(k+1) & -2 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} k+1 & 0 & 0 & k+3 & -2 & k+1 \\ 0 & k+1 & 0 & 2 & -2 & k+1 \\ 0 & 0 & 2(k+1) & -2 & 2 & 0 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} \frac{k+3}{k+1} & -\frac{2}{k+1} & 1 \\ \frac{2}{k+1} & -\frac{2}{k+1} & 1 \\ -\frac{1}{k+1} & \frac{1}{k+1} & 0 \end{pmatrix}$$

Esercizio 3

$$\det\begin{pmatrix} 1 & 0 & k \\ k+1 & 2k+2 & -k-1 \\ k & 1 & 1 \end{pmatrix} = (k+1)\det\begin{pmatrix} 1 & 0 & k \\ 1 & 2 & -1 \\ k & 1 & 1 \end{pmatrix}$$

$$= (k+1)\left(\det\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} + k\det\begin{pmatrix} 1 & 2 \\ k & 1 \end{pmatrix}\right)$$

$$= (k+1)\left(3 + k(1-2k)\right. = (k+1)(-2k^2+k+3)$$

$$\left(-2k^2+k+3\right)$$

$$\|$$

$$(k+1)\left(-2k+3\right)$$

$$\begin{array}{r|rr|r} & -2 & 1 & 3 \\ -1 & & 2 & -3 \\ \hline & -2 & 3 & 0 \end{array}$$

$$k\det\begin{pmatrix} 1 & 2 & 3 & 4 \\ 12 & 25 & -34 & 41 \\ -1\cancel{k} & -2\cancel{k} & -3\cancel{k} & -4\cancel{k} \\ \sqrt{2} & \pi & e & -12 \end{pmatrix} = 0$$

perché la 1^e e la 3^e righe sono proporzionali.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \overset{\frac{1}{2}}{} - 4$$

$$\det B = 2 \cdot \det A$$

$$\det \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} = 2 \cdot \det \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Esercizio 5

$$\det\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} = \det\begin{pmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{pmatrix}$$

$$= \det\begin{pmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{pmatrix}$$

$$= (b-a)(c-a)\det\begin{pmatrix} 1 & 1 \\ b+a & c+a \end{pmatrix} = (b-a)(c-a)\det\begin{pmatrix} 1 & 1 \\ b & c \end{pmatrix}$$

$$= (b-a)(c-a)(c-b)$$

$$\det\begin{pmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{pmatrix} = \det\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & b-a & c-a & d-a \\ 0 & b^2-a^2 & c^2-a^2 & d^2-a^2 \\ 0 & b^3-a^3 & c^3-a^3 & d^3-a^3 \end{pmatrix}$$

$$= (b-a)(c-a)(d-a)\det\begin{pmatrix} 1 & 1 & 1 \\ b+\cancel{a} & c+\cancel{a} & d+\cancel{a} \\ b^2+\cancel{ab}+\cancel{a^2} & c^2+\cancel{ac}+\cancel{a^2} & d^2+\cancel{ad}+\cancel{a^2} \end{pmatrix}$$

$$= (b-a)(c-a)(d-a)(c-b)(d-c)(d-b)$$

Esercizio 6

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

(a) A è invertibile e calcolare $\det A^{-1}$.

$$\det A = \det \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix} = \det \begin{pmatrix} 1 & 2 & -1 \\ 0 & -2 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \det \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= -1$$

$\Rightarrow A$ è invertibile e $\det(A^{-1}) = -1$

(b) $H = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 0 \end{pmatrix}$

$$\det(H^{-1} A H) = \cancel{\det H^{-1}} \cdot \det A \cdot \cancel{\det H}$$

$$= -1$$

Matrici simili hanno lo stesso determinante,

Esempio

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\det A = \det B = 1$$

Sono simili? $\exists H$ t.c.

$$H^{-1} A H = B \;?$$

$$\begin{array}{c} H^{-1} A H \\ \| \\ H^{-1} \cdot H = I \end{array}$$

L'unica matrice simile a I è I stessa!