CONIUGATO e MODULO $\mathbb{C}$

Sia $z = a + ib$, il suo complesso coniugato è

$$\bar{z} = a - ib$$

Si vede che:

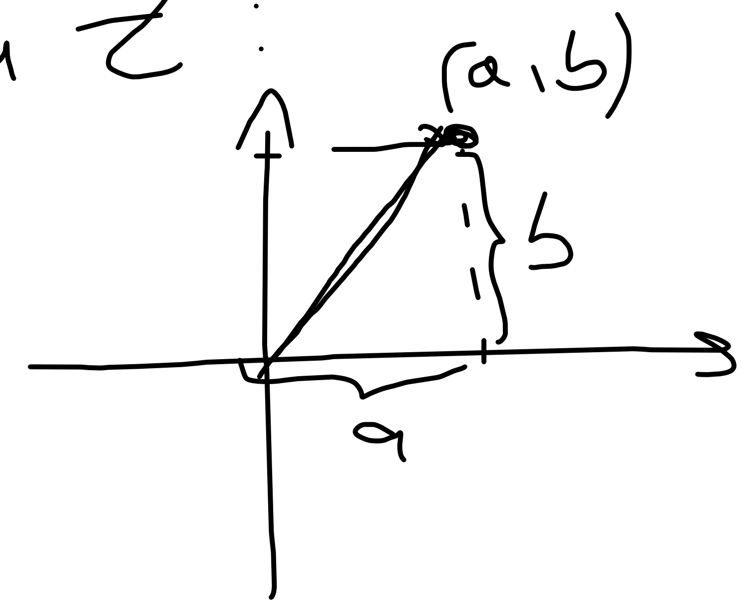
$$\operatorname{Re} z = \frac{z + \bar{z}}{2}$$

$$\operatorname{Im} z = \frac{z - \bar{z}}{2i}$$

- $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
- $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$
- $z \cdot \overline{z} = (a+ib)(a-ib) = a^2 + b^2$

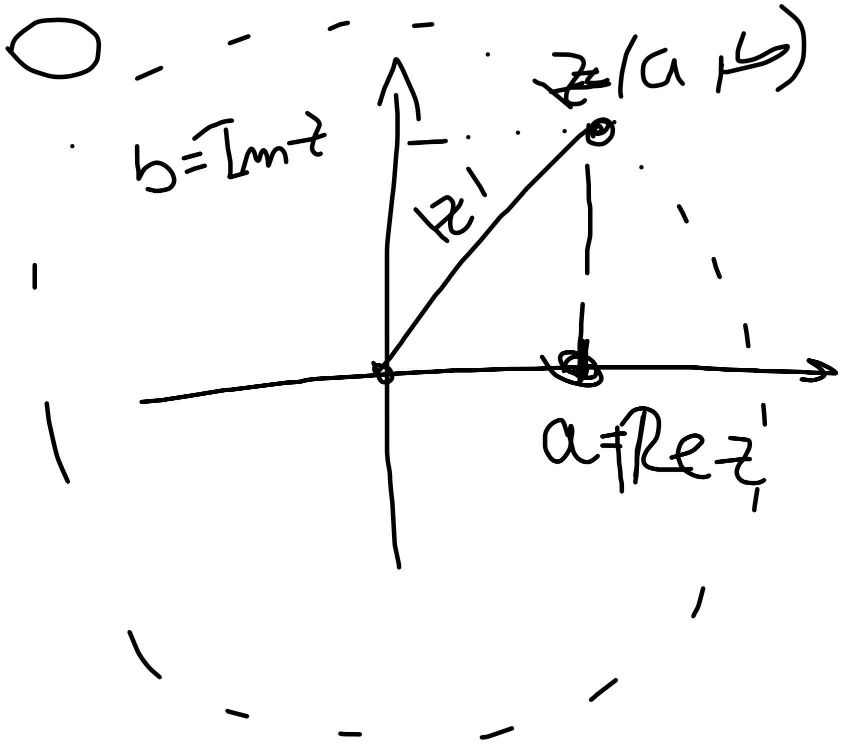
Chiamiamo <u>MODULO</u> di z:

$$\underline{|z| = \sqrt{z \cdot \overline{z}} = \sqrt{a^2+b^2}}$$



PROPRIETÀ

$\forall z \in \mathbb{C}$ $|z| \geq 0$, $|z| = 0 \iff z = 0$



- $|z| = |\bar{z}|$
- $|z| \geq \mathrm{Re}\, z$ $\quad |z| \geq \mathrm{Im}\, z$
- $|z| \leq \mathrm{Re}\, z + \mathrm{Im}\, z$

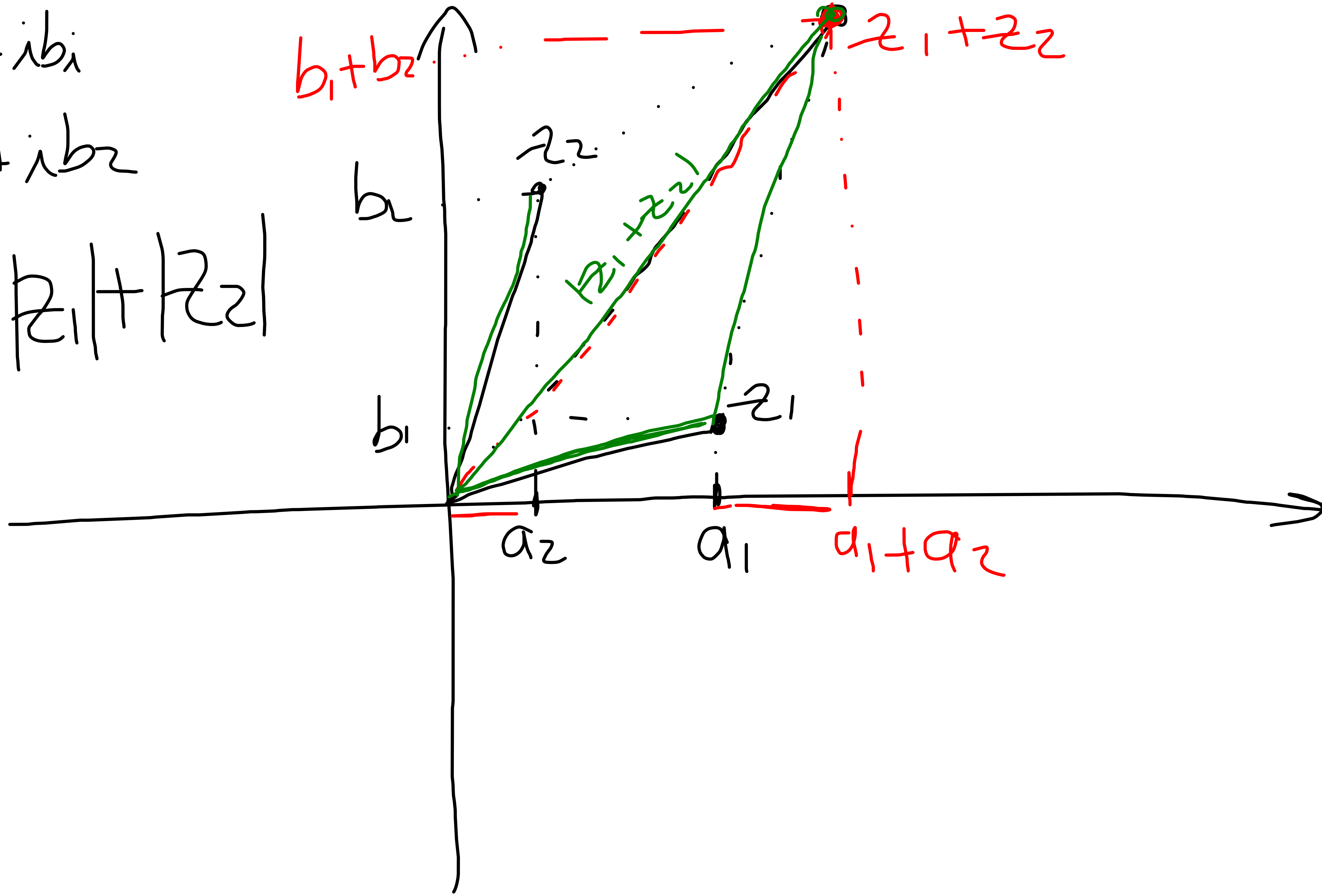
$\forall z_1, z_2 \in \mathbb{C}$

- $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 + z_2| \geq \big||z_1| - |z_2|\big|$

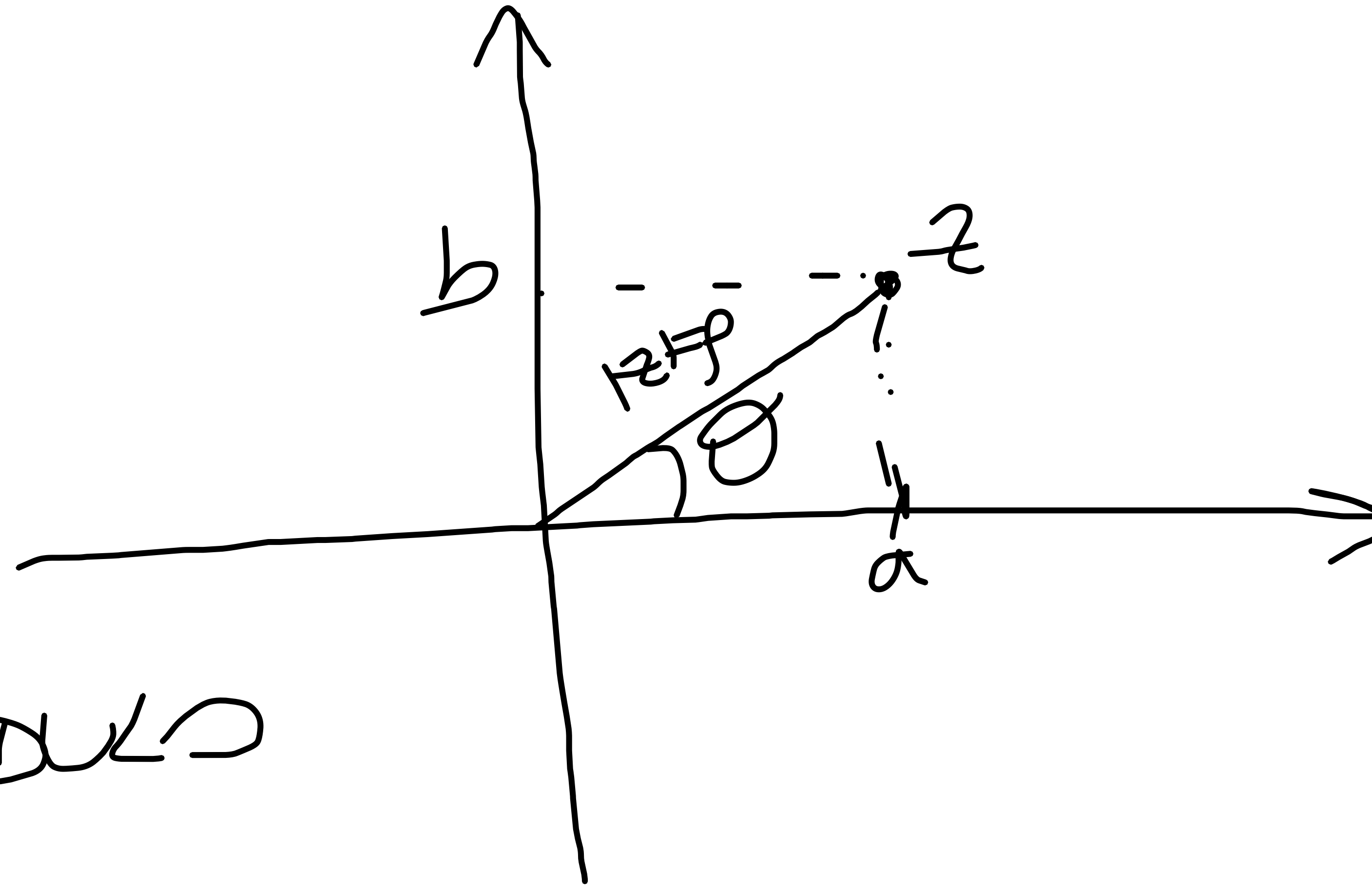
$z_1 = a_1 + ib_1$

$z_2 = a_2 + ib_2$

$|z_1 + z_2| \leq |z_1| + |z_2|$



FORMA TRIGONOMETRICA



COORDINATE POLARI

$z \in \mathbb{C}$ è individuato da i due numeri reali:

$$\rho = |z| = \text{MODULO}$$

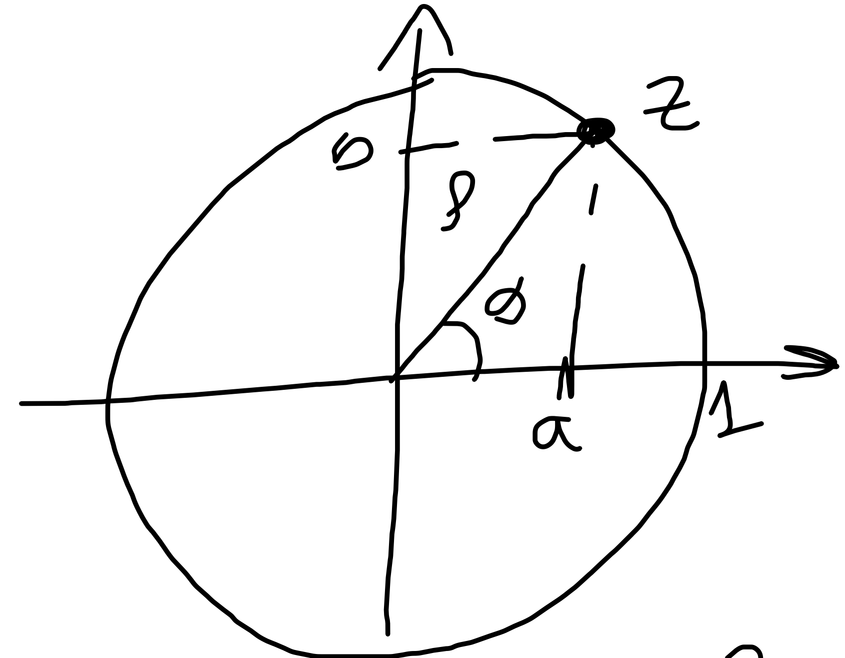
$$\theta = \text{ARGOMENTO di } z$$

Se scelgo $\theta \in [0, 2\pi[$ → ARGOMENTO PRINCIPALE di z

↳ $\arg z$

Supp. $\rho = |z| = 1$

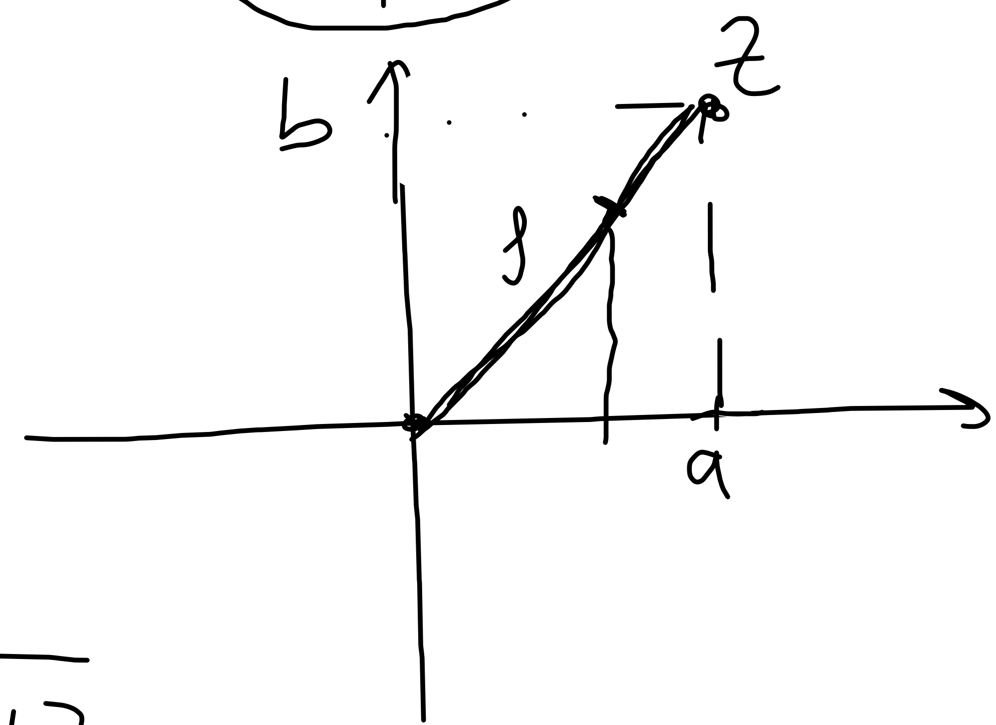
$$\begin{cases} a = \cos\vartheta \\ b = \operatorname{sen}\vartheta \end{cases}$$



Dato un qualsiasi z:

$$\boxed{\begin{cases} a = \rho\cos\vartheta \\ b = \rho\operatorname{sen}\vartheta \end{cases}}$$

$\hookrightarrow \rho^2 = a^2 + b^2 \Rightarrow \rho = \sqrt{a^2+b^2}$



$$\rho = \sqrt{a^2+b^2}$$

$$\cos\vartheta = \frac{a}{\sqrt{a^2+b^2}}, \quad \operatorname{sen}\vartheta = \frac{b}{\sqrt{a^2+b^2}}$$

FORMA TRIGONOMETRICA

$$\boxed{z = (a + ib) = \rho\left[\cos\vartheta + i\operatorname{sen}\vartheta\right]}$$

$\rho = |z|$

$\vartheta = \arg z$

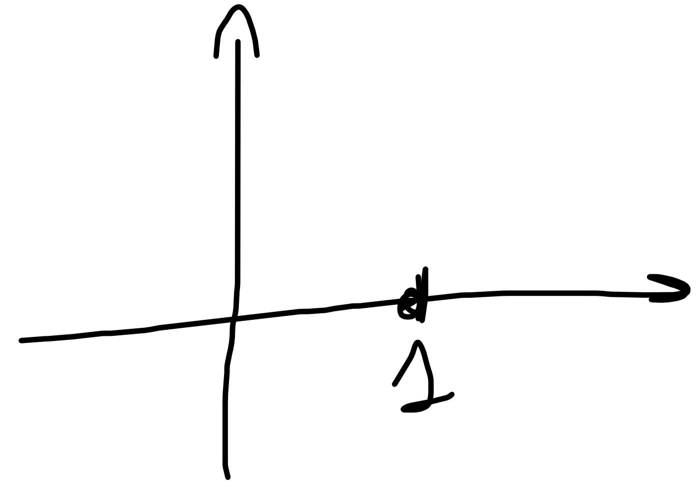
ESEMPI

$\rho = |z|$

$\theta = \arg z$

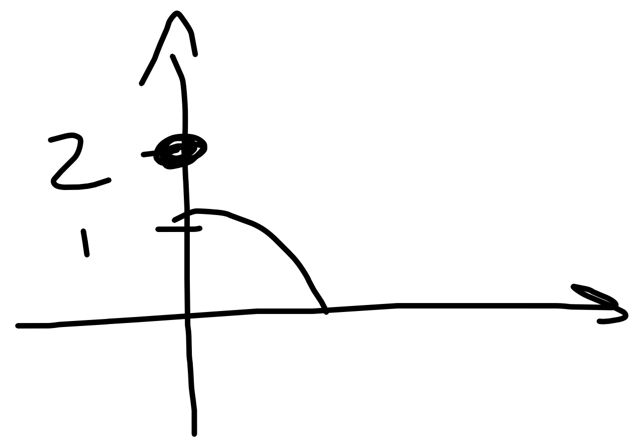
- $z = 1$

$|z| = 1 \qquad \theta = 0$



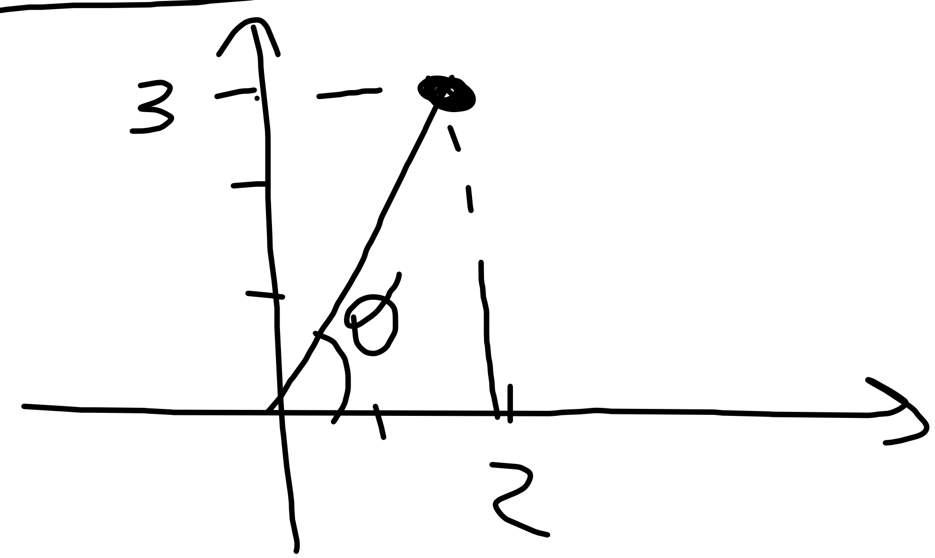
- $z = 2i$

$\rho = |z| = 2 \qquad \theta = \frac{\pi}{2}$



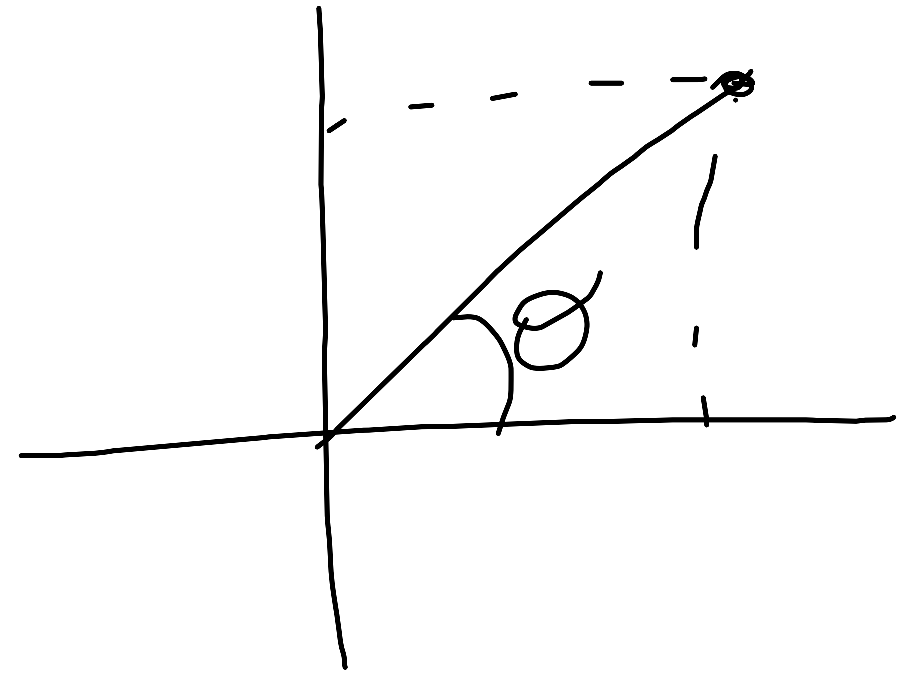
- $z = 2 + 3i$

$\rho = \sqrt{4+9} = \sqrt{13}$



Osservo che

$$tg\theta = \frac{sen\theta}{cos\theta} = \frac{b}{a} = \frac{3}{2}$$



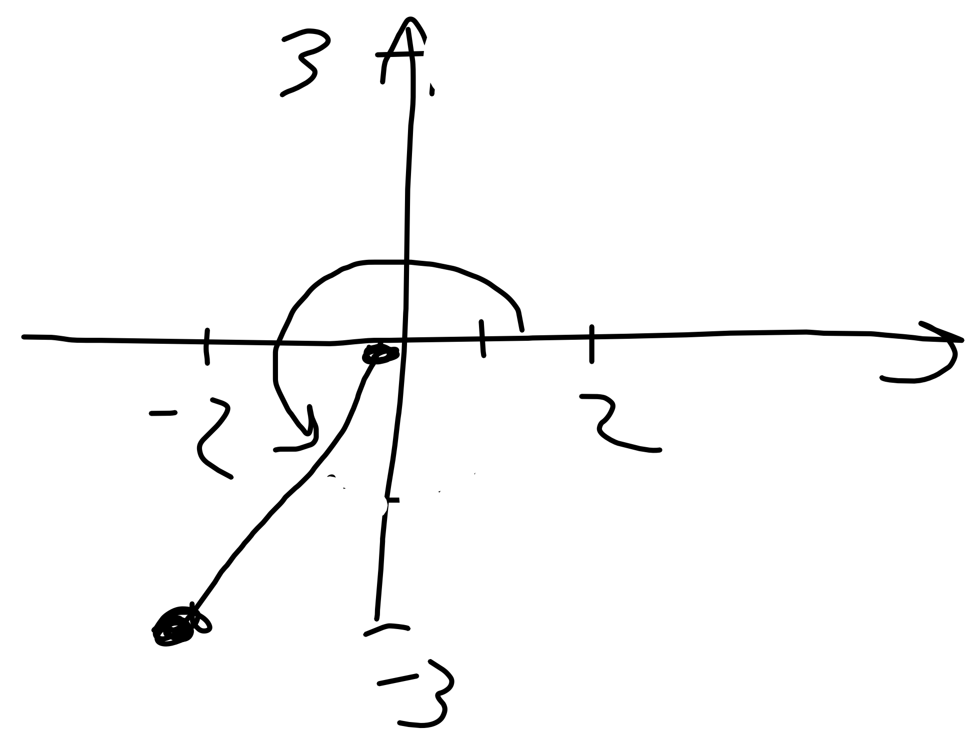
$$\Rightarrow \theta = arctg\frac{3}{2}$$

↳ perché $a>0, b>0$

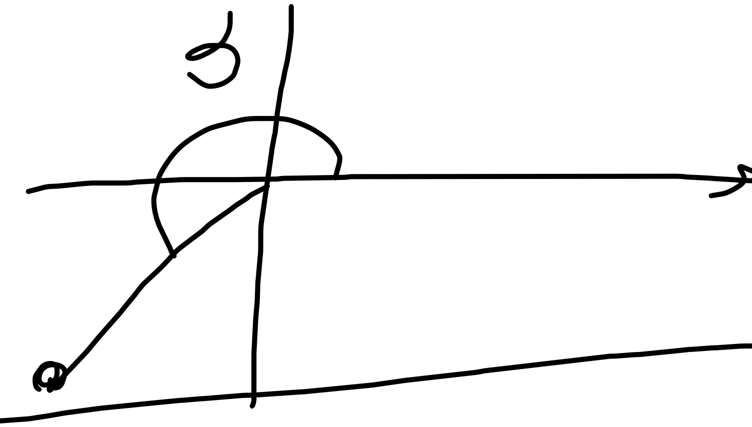
• $z = -2-3i$

$\rho = |z| = \sqrt{13}$

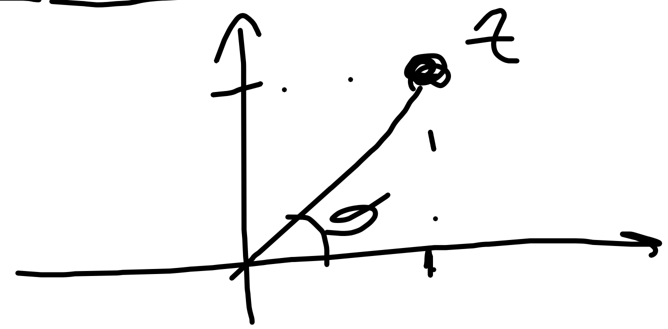
$tg\theta = \frac{3}{2}$



Ma ora $a<0$, $b<0$



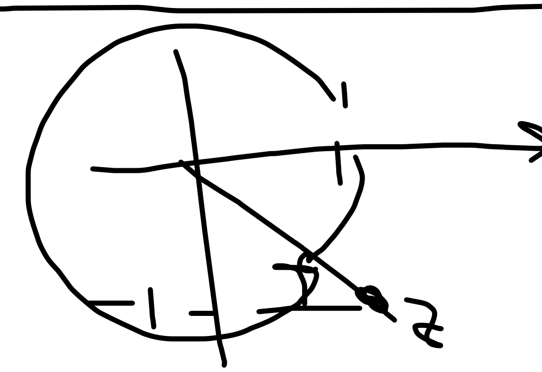
$\vartheta = \operatorname{arctg}\frac{3}{2} + \pi$



$z = 1 + i$

$\rho = \sqrt{2}$

$\vartheta = \frac{\pi}{4}$

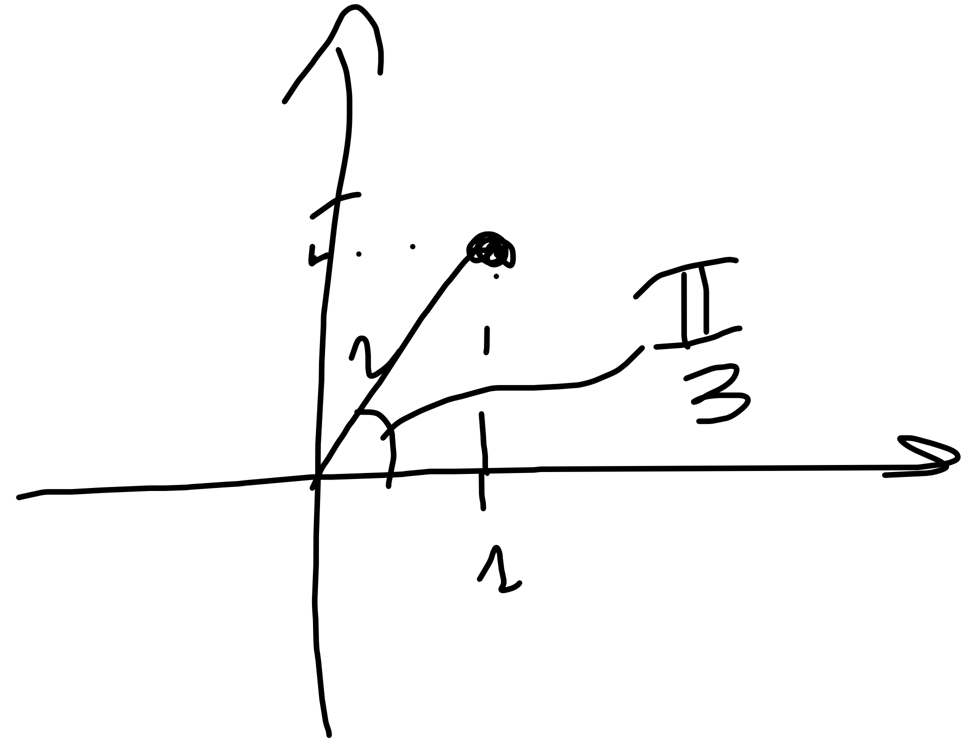


$z = 1 - i$

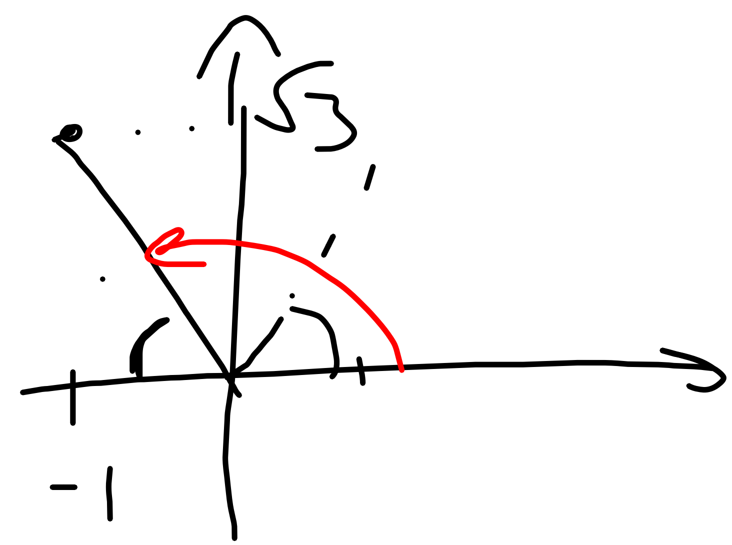
$\rho = \sqrt{2}$ $\vartheta = -\frac{\pi}{4}$

$$z = 1 + \sqrt{3}\, i$$

$$= 2\underbrace{\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)}$$



$$\rho = |z| = 2 \qquad \theta = \frac{\pi}{3}$$



$$z = -1 + \sqrt{3}\, i$$

$$\rho = |z| = 2$$

$$\theta = \pi - \frac{\pi}{3} = \boxed{\frac{2}{3}\pi}$$

FORMULE di DE MOIVRE

Sia $z_1=\rho_1(\cos\theta_1+i\,\mathrm{sen}\,\theta_1)$

$z_2=\rho_2(\cos\theta_2+i\,\mathrm{sen}\,\theta_2)$

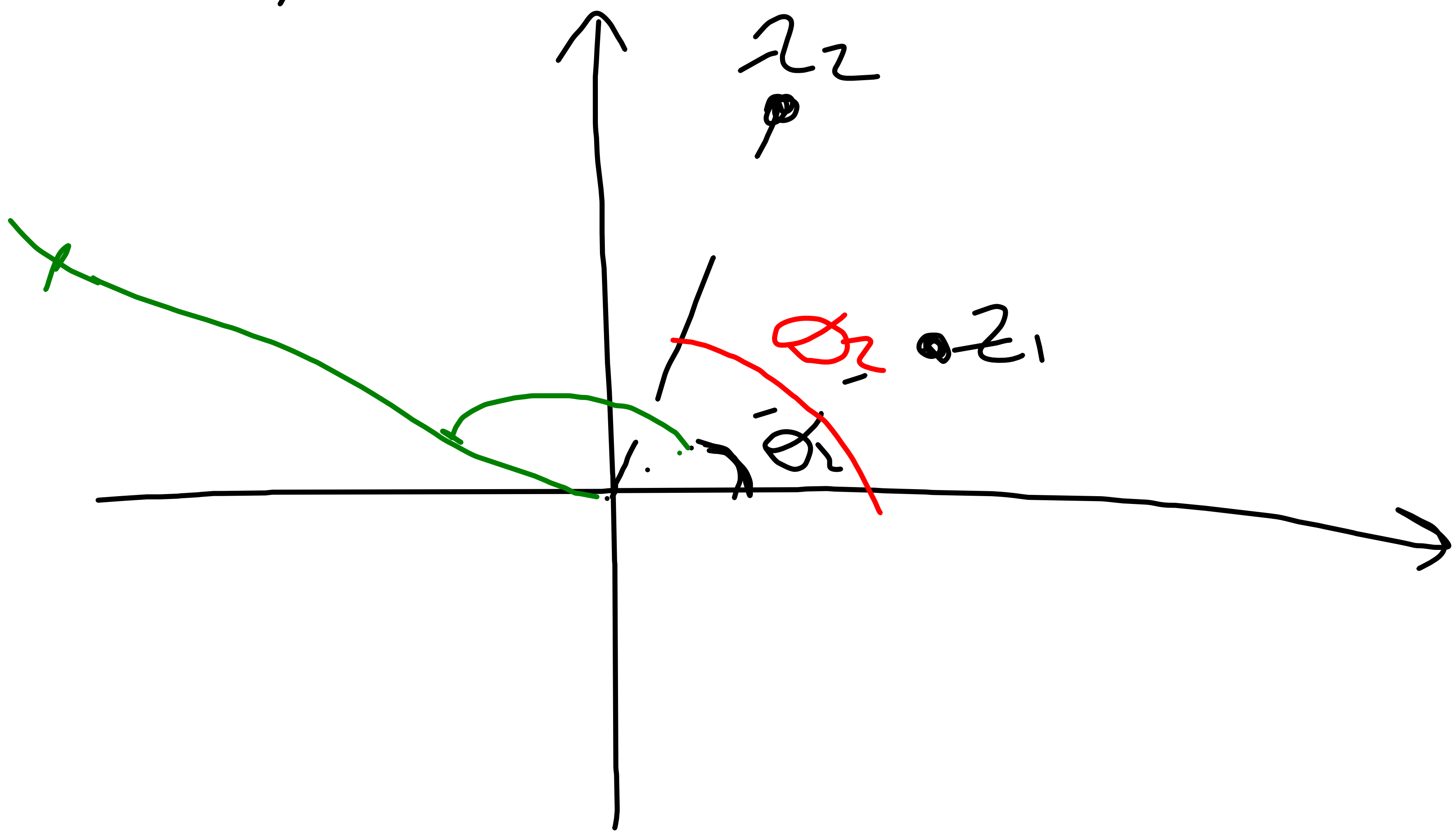
Considero il prodotto:

$$\boxed{z_1\cdot z_2}=\rho_1\rho_2\left(\cos\theta_1\cos\theta_2+i\cos\theta_1\,\mathrm{sen}\,\theta_2+i\cos\theta_2\,\mathrm{sen}\,\theta_1-\mathrm{sen}\,\theta_1\,\mathrm{sen}\,\theta_2\right)$$

$$=\rho_1\rho_2\left(\cos(\theta_1+\theta_2)+i\,\mathrm{sen}(\theta_1+\theta_2)\right)$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2| = \rho_1 \cdot \rho_2$$

$$\arg(z_1 \cdot z_2) = \theta_1 + \theta_2 = \arg z_1 + \arg z_2$$



Quindi, per $n \in \mathbb{N}^*$, $z = \rho(\cos\theta + i \operatorname{sen}\theta)$

$$z^n = \underbrace{\rho^n}_{|z|^n} \left(\cos(n\theta) + i \operatorname{sen}(n\theta)\right)$$

Notazione $\cos\theta + i\operatorname{sen}\theta =: e^{i\theta}$

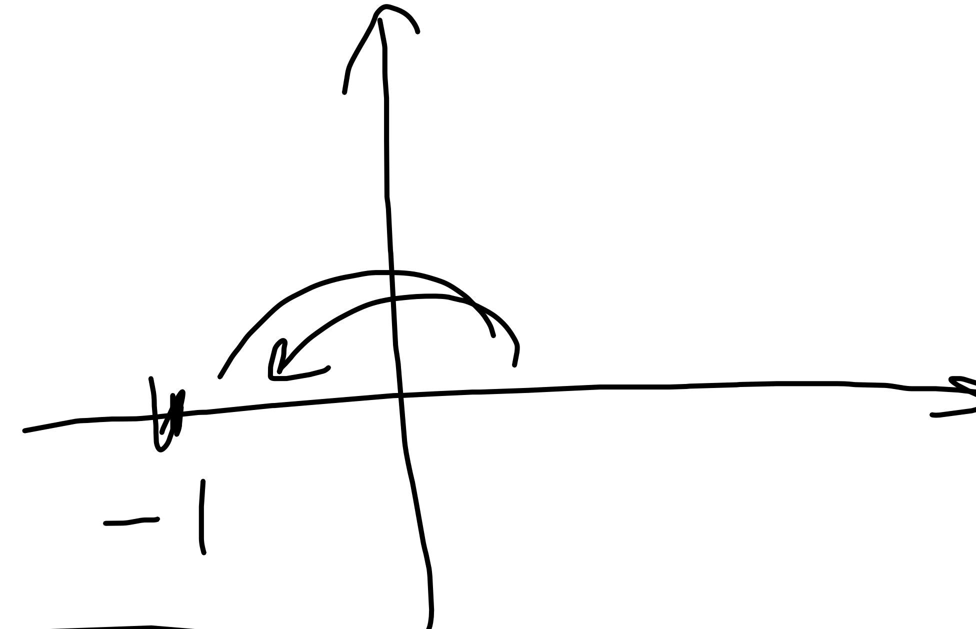
$$\left.\begin{array}{l} z_1 = e^{i\theta_1} \\ z_2 = e^{i\theta_2} \end{array}\right| \Rightarrow z_1 \cdot z_2 = e^{i\theta_1} \cdot e^{i\theta_2} \underset{?}{=} e^{i(\theta_1+\theta_2)}$$

Sia $z = \rho e^{i\theta}$

$$z^n = \rho^n e^{in\theta}$$

Es $z = -1$

$\rho = |z| = 1$ $\qquad \theta = \arg z = \pi$



$$-1 = e^{i\pi} \Rightarrow e^{i\pi} + 1 = 0$$