TUTORATO ATTIVO

VENERDÌ h: 14-16

AULE: $\underbrace{3.10}_{\text{CANALE A}}$ - $\underbrace{2.5}_{\text{CA. B}}$

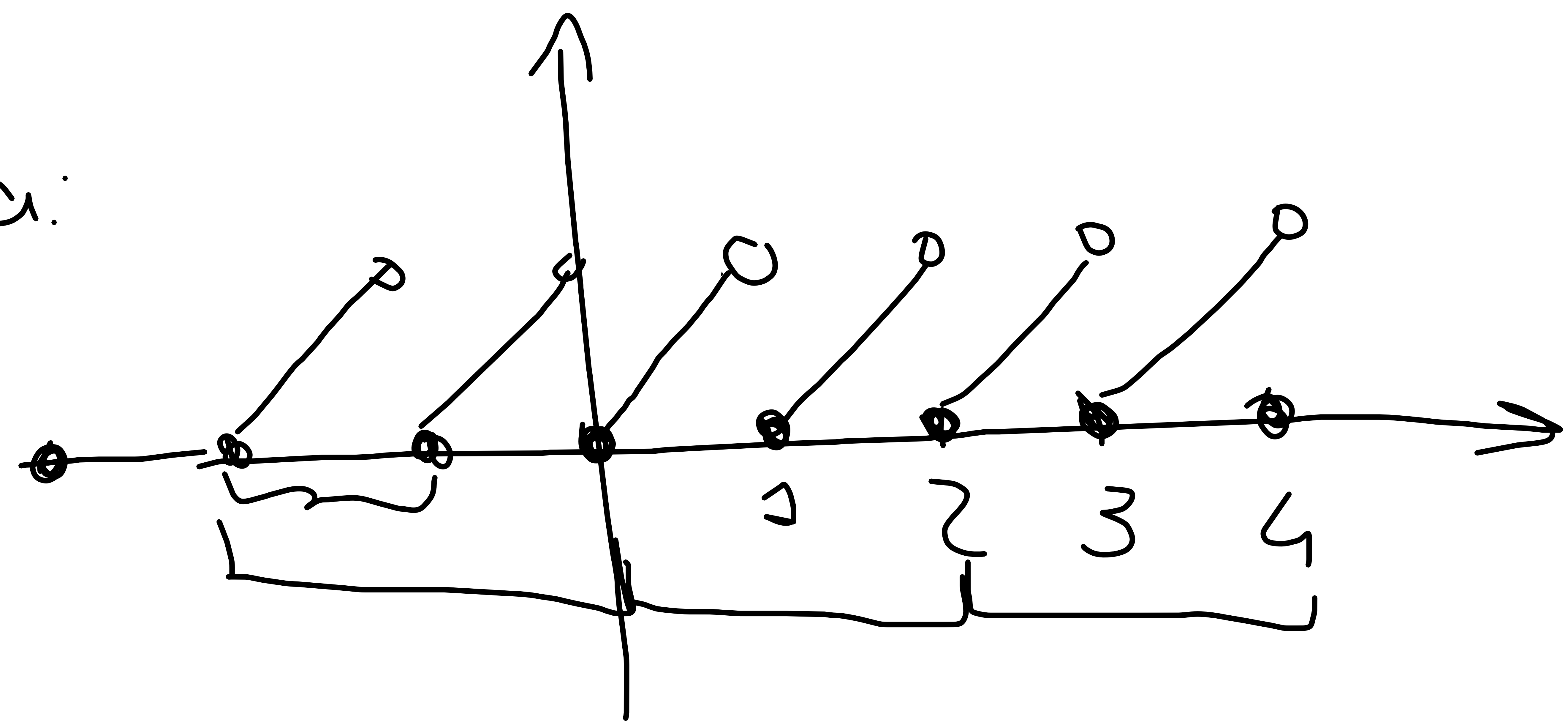
Def Sia $T \in \mathbb{R}_+^*$. Sia $A \subseteq \mathbb{R}$.
Diciamo che A è T-periodico
se $\forall x \in A$, $\forall k \in \mathbb{Z}$
$$\Rightarrow x + kT \in A.$$

Def Sia $A \subseteq \mathbb{R}$ T-periodico,
Sia $f: A \to \mathbb{R}$. Diciamo che
f è T-periodica se
$\forall x \in A$ $f(x) = f(x+kT)$ $\forall k \in \mathbb{Z}$.

[Il PERIODO è il più piccolo T
per cui vale la proprietà
$$f(x) = f(x+T)$$]

ES $f(x) = x - [x]$ → PARTE FRAZIONARIA

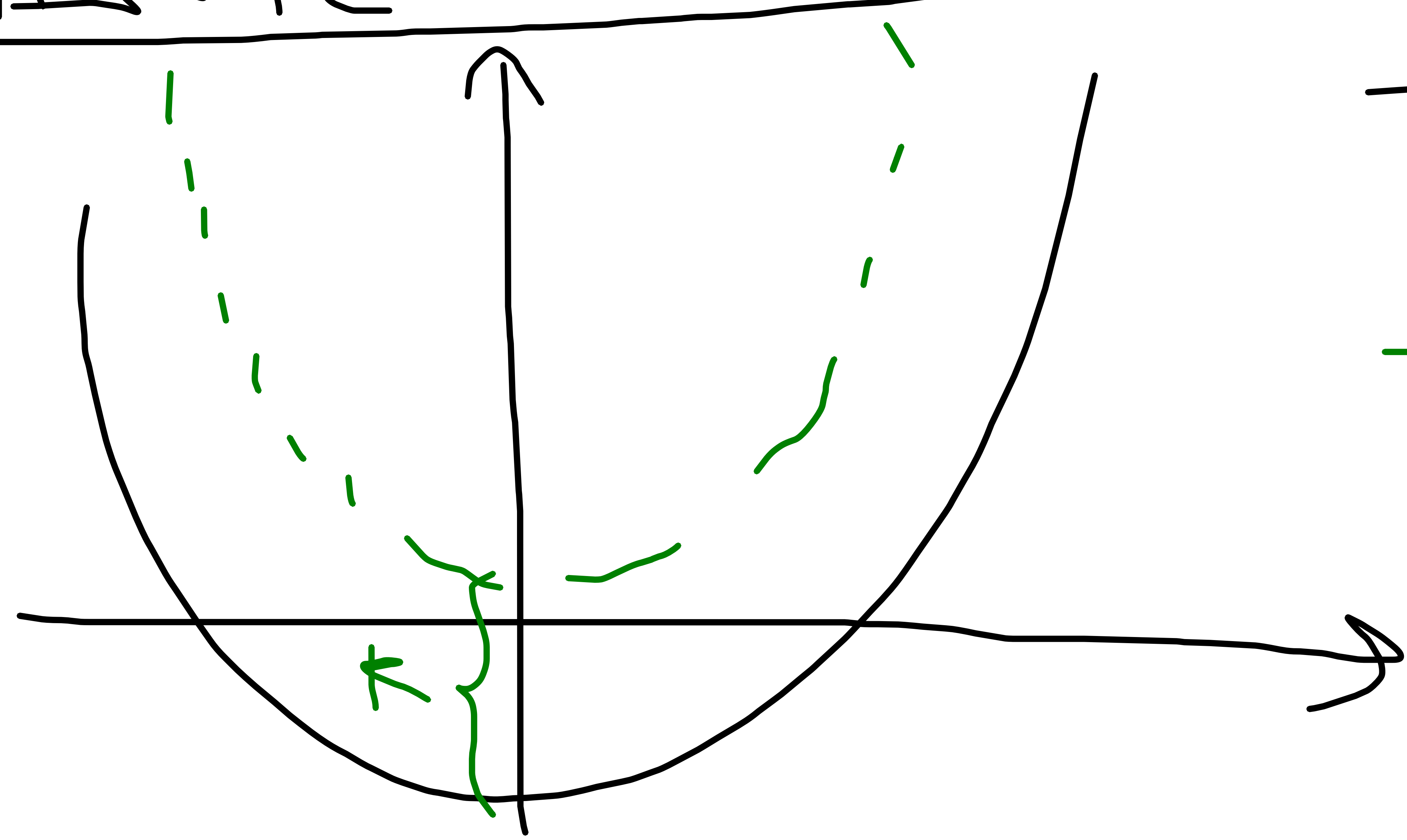
è 1-periodica:



Verifico $f(x) = f(x+1)$

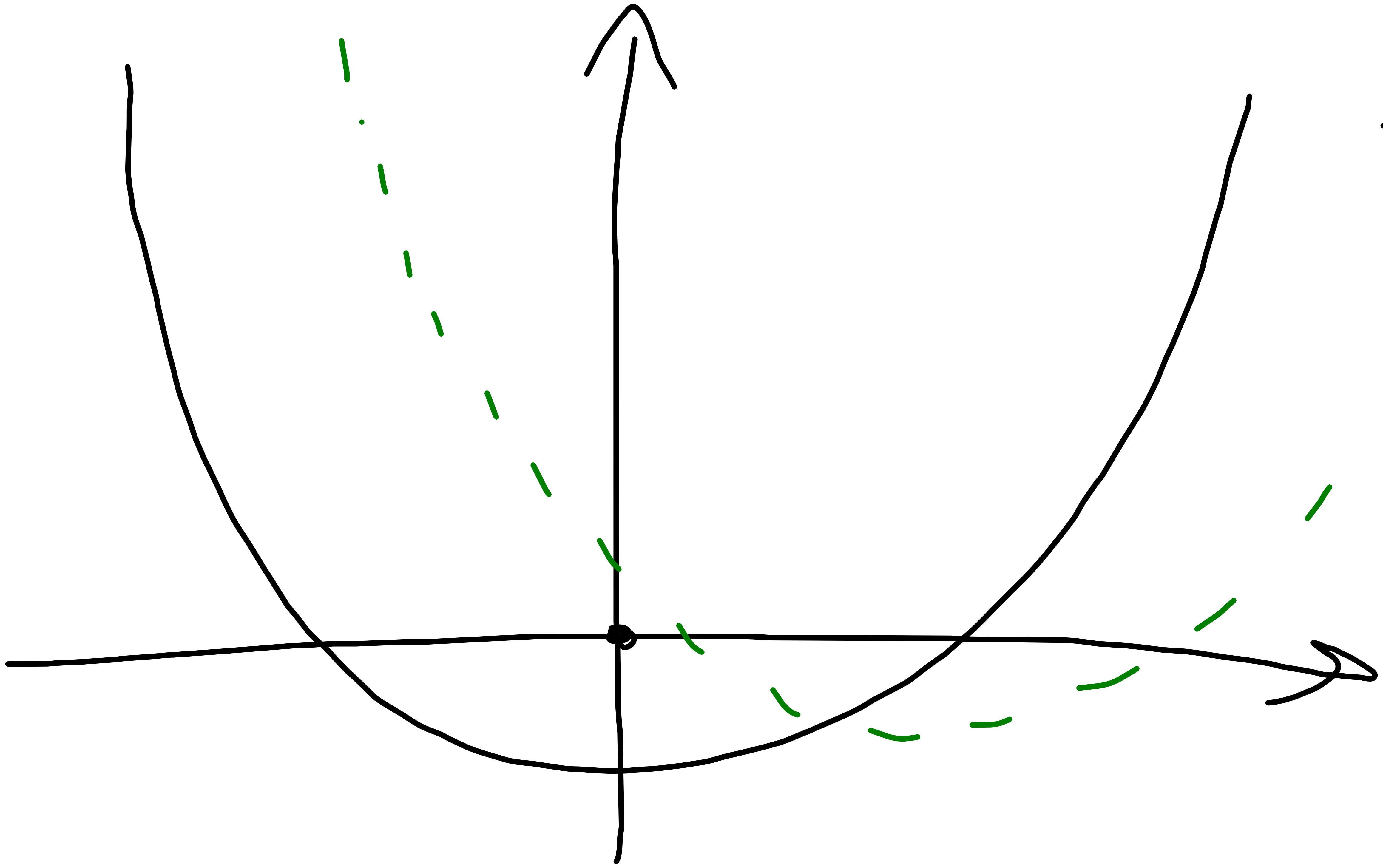
$$f(x+1) = x+1-\underbrace{[x+1]}_{[x]+1} = x-[x] = f(x)$$

OPERAZIONI sul GRAFICO



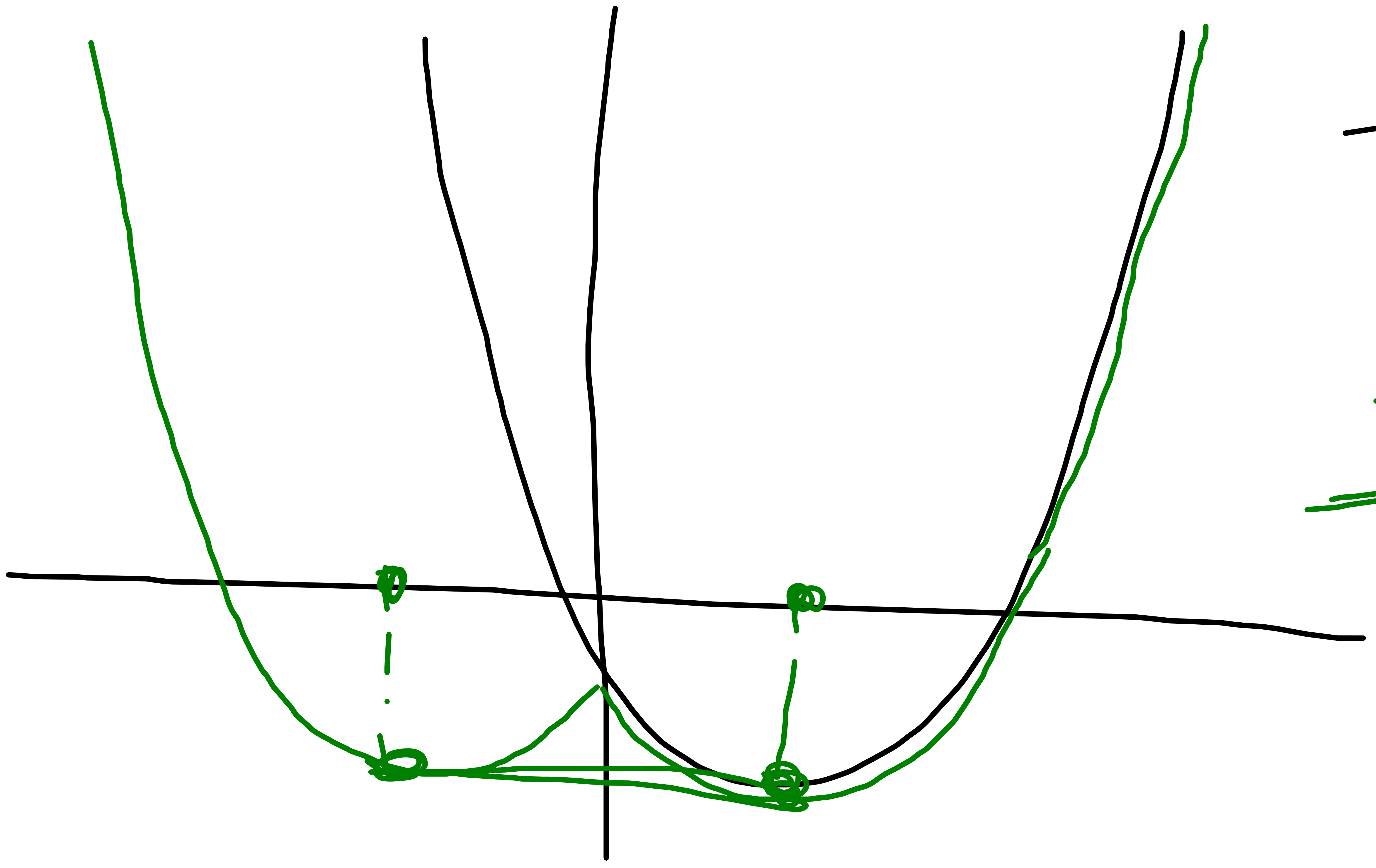
$f(x)$

$f(x)+K \quad K>0$



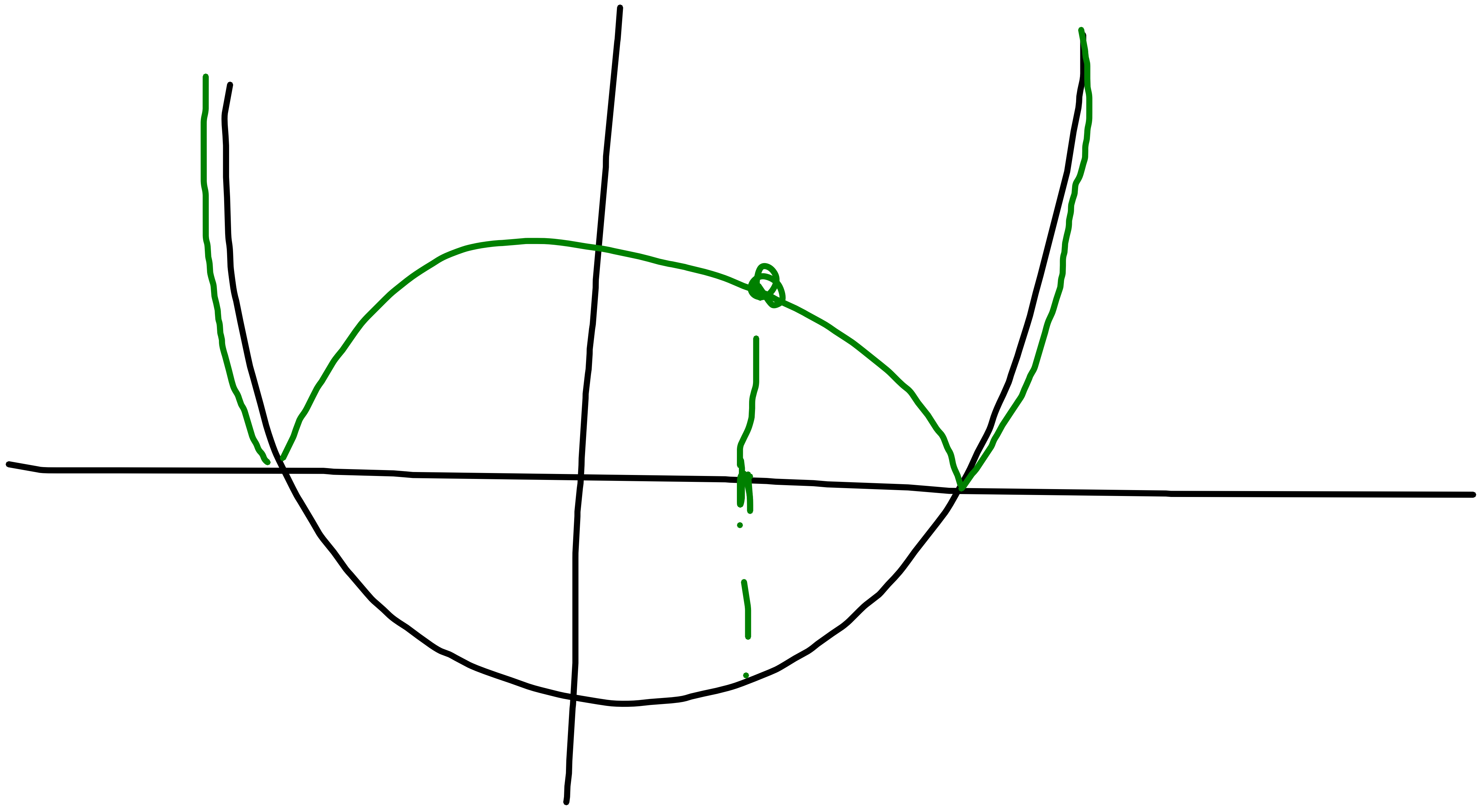
$f(x)$

$f(x-k) \quad k>0$



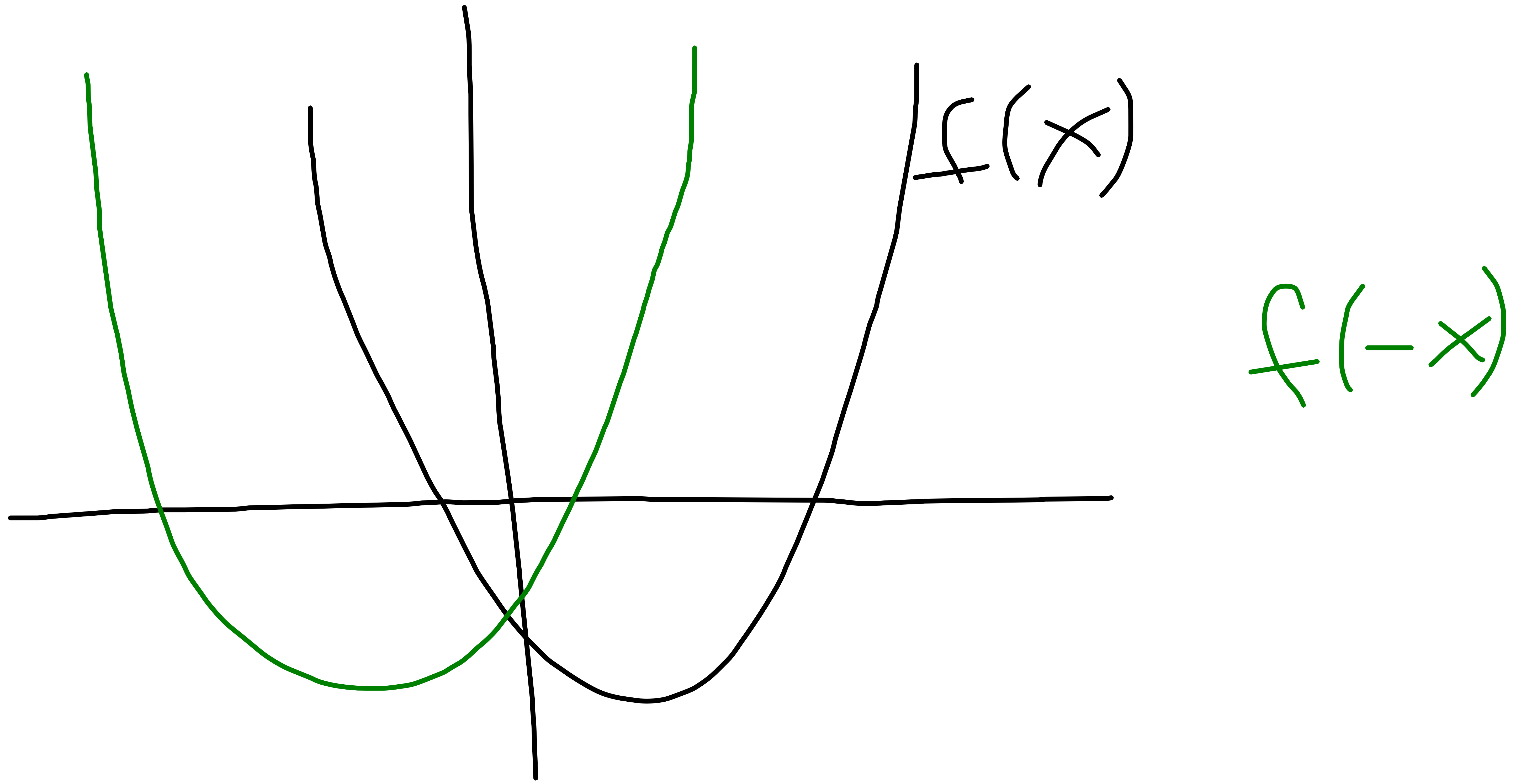
$f(x)$

$\underline{f(|x|)}$



$f(x)$

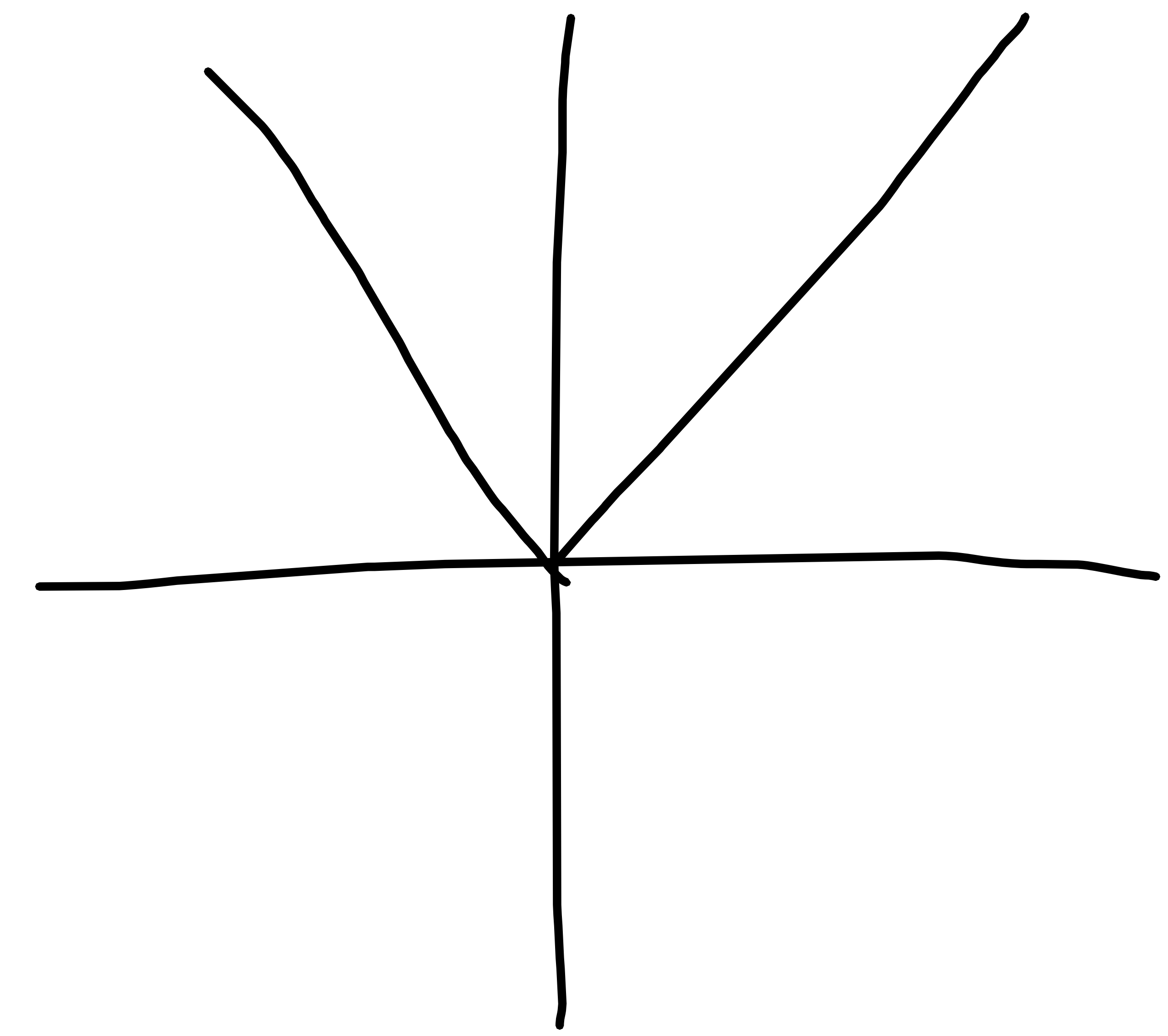
$|f(x)|$



$f(x)$
$f(-x)$

FUNZIONI ELEMENTRI

$$f(x) = |x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

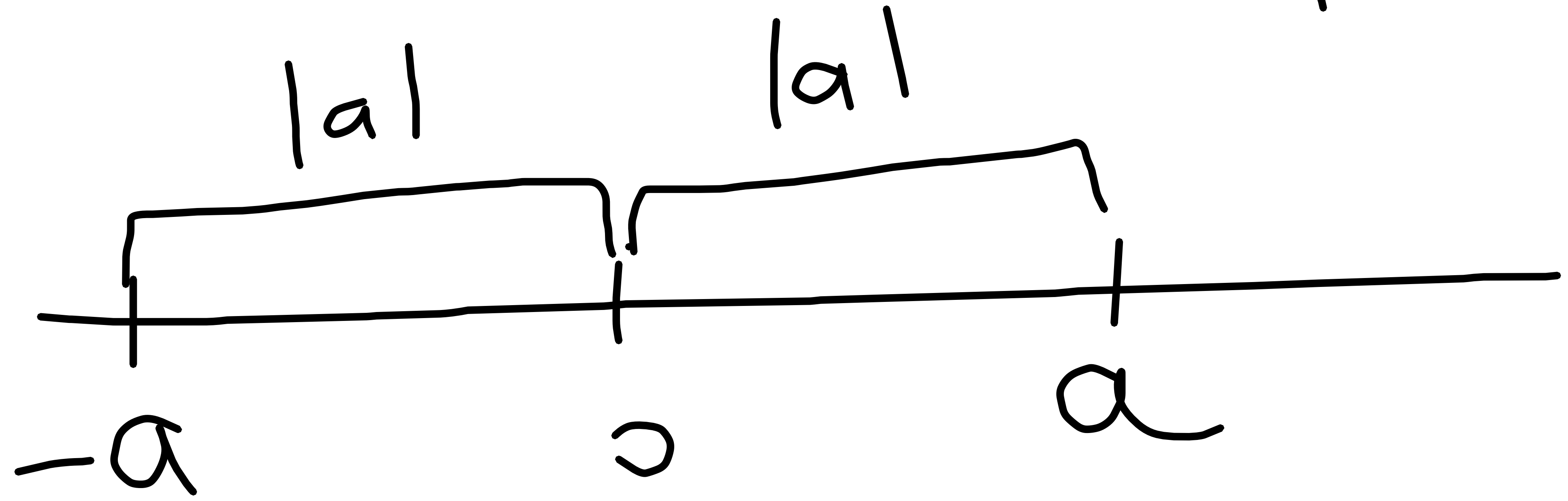


PROPRIETÀ

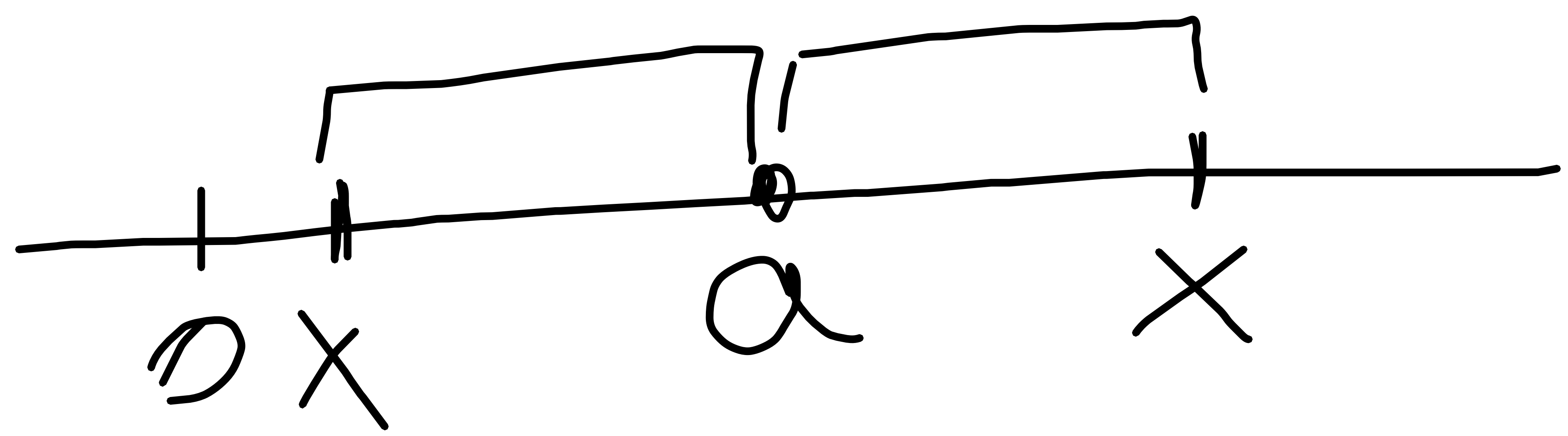
1) $|x| \geq 0$, $|x| = 0 \Leftrightarrow x = 0$
2) $|x \cdot y| = |x| \cdot |y|$
3) $|x + y| \leq |x| + |y|$
4) $|x - y| \geq \big| |x| - |y| \big|$

$\Big\}$ DISUGUAGLIANZA TRIANGOLARE

Dato $a \in \mathbb{R}$ $\qquad |a| = \begin{cases} a & \text{se } a \geq 0 \\ -a & \text{se } a < 0 \end{cases}$

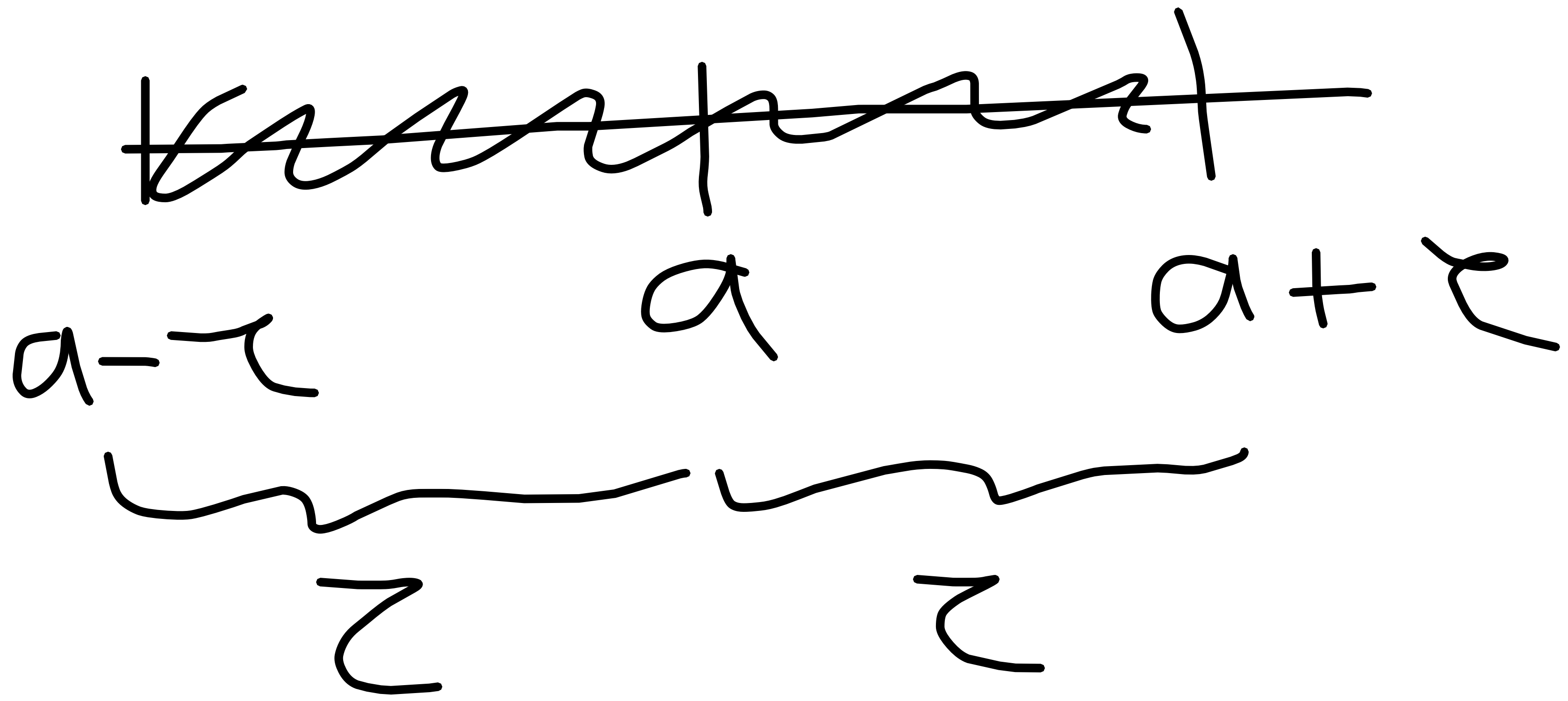


$$\underbrace{|x-a|}_{\text{distanza tra } x \text{ e } a}$$



$$\underbrace{|x-a|}<\varepsilon$$

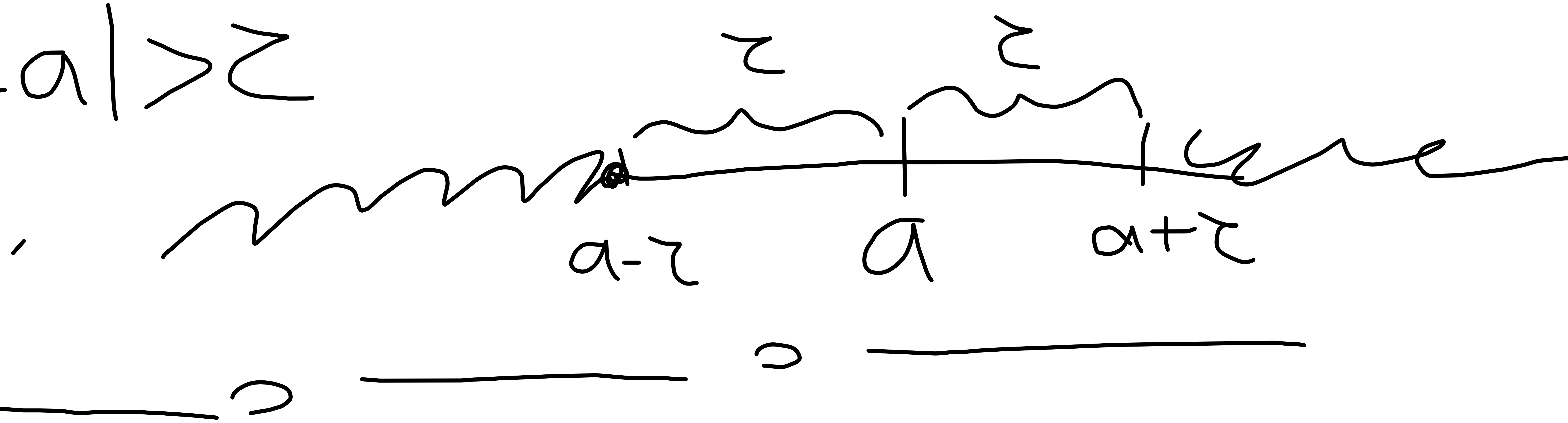
a fissato



$$-\varepsilon < x-a < \varepsilon$$

$$\boxed{a-\varepsilon < x < a+\varepsilon}$$

$|x-a|>\varepsilon$



RADICE n-esima

Sia $a \in \mathbb{R}_+$, $n \in \mathbb{N}^* \setminus \{1\}$

Diciamo che $x \in \mathbb{R}_+$ è RADICE n-esima di a se

$$x^n = a$$

e la indichiamo con $\sqrt[n]{a}$

PROPRIETÀ

i) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

ii) $\sqrt[n]{a+b} \leq \sqrt[n]{a} + \sqrt[n]{b}$

iii) Se n è pari: $\sqrt[n]{x^n} = |x|$

POTENZA / ESPONENZIALE

- **ESPONENTE $\in \mathbb{N}$**

$$a^0 = 1 \quad \forall a \in \mathbb{R}_+$$

$$a^n = a \cdot a^{n-1}$$

PROPRIETÀ

- $a^{n+m} = a^n \cdot a^m$
- $(a^n)^m = a^{n \cdot m}$
- $(ab)^n = a^n \cdot b^n$

OSS

- $1 = a^{n-n} = \underbrace{a^{n} \cdot a^{-n}}$

$$\boxed{a^{-n} = \frac{1}{a^{n}}} \Rightarrow \text{estendere a } n \in \mathbb{Z}$$

- $\left(a^{\frac{1}{n}}\right)^{n} = a^{\frac{1}{n} \cdot n} = a$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$\hookrightarrow \boxed{a^{\frac{p}{q}}} = \left(a^{p}\right)^{\frac{1}{q}} = \boxed{\sqrt[q]{a^{p}}} \quad p, q \in \mathbb{Z}$

Per densità posso definire a^y $\forall y \in \mathbb{R}$

<u>Def</u> Sia $a \in \mathbb{R}^*_+ \setminus \{1\}$

chiamiamo <u>FUNZIONE ESPONENZIALE</u> in base a la funzione

$$\exp_a : \mathbb{R} \to \mathbb{R} \qquad \exp_a(x) = a^x$$

$$\underline{TEGR}\begin{cases} (1) & \forall x\in\mathbb{R},\ a^x>0 \\ (2) & \forall x,y\in\mathbb{R}\ \ a^{x+y}=a^x\cdot a^y \\ (3) & \forall x,y\in\mathbb{R}\ \ (a^x)^y=a^{xy} \\ (4) & (a\cdot b)^x=a^x\cdot b^x \end{cases}$$

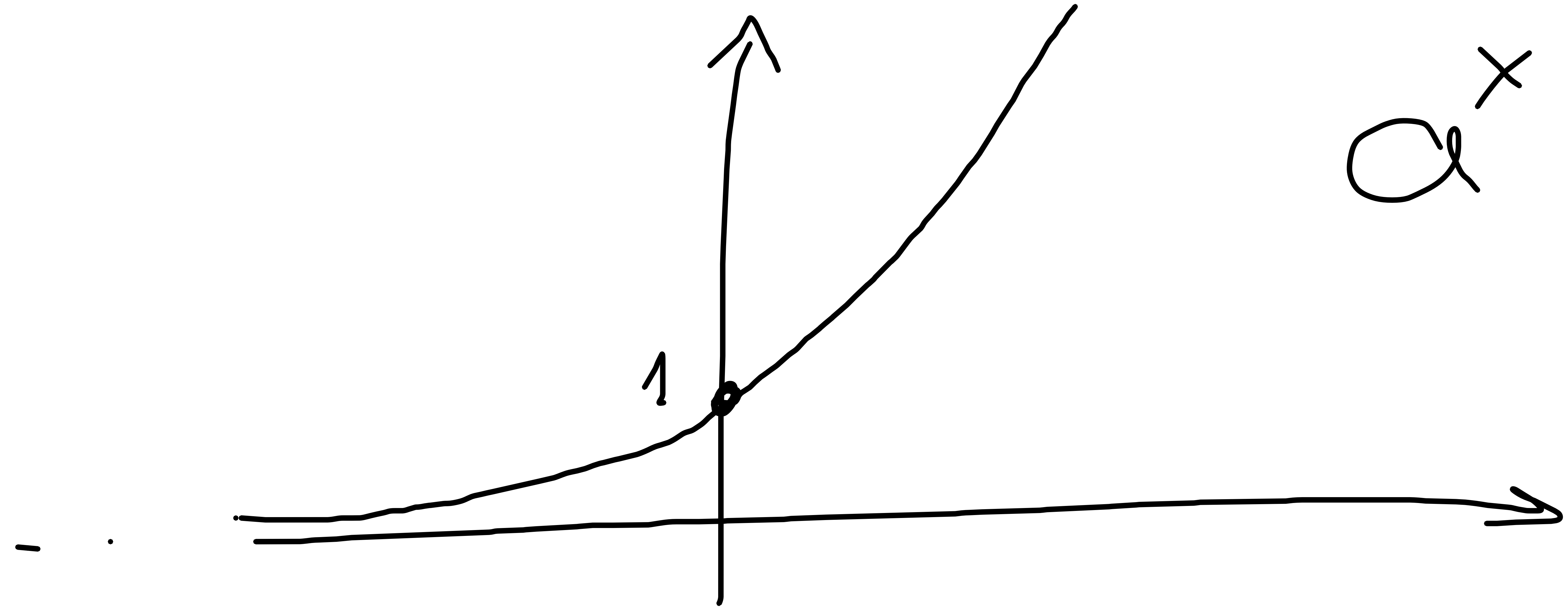
TEOR Sia $a \in \mathbb{R}_+^* \setminus \{1\}$

1) Se $a > 1 \Rightarrow exp_a$ è STRETT. CRESCENTE

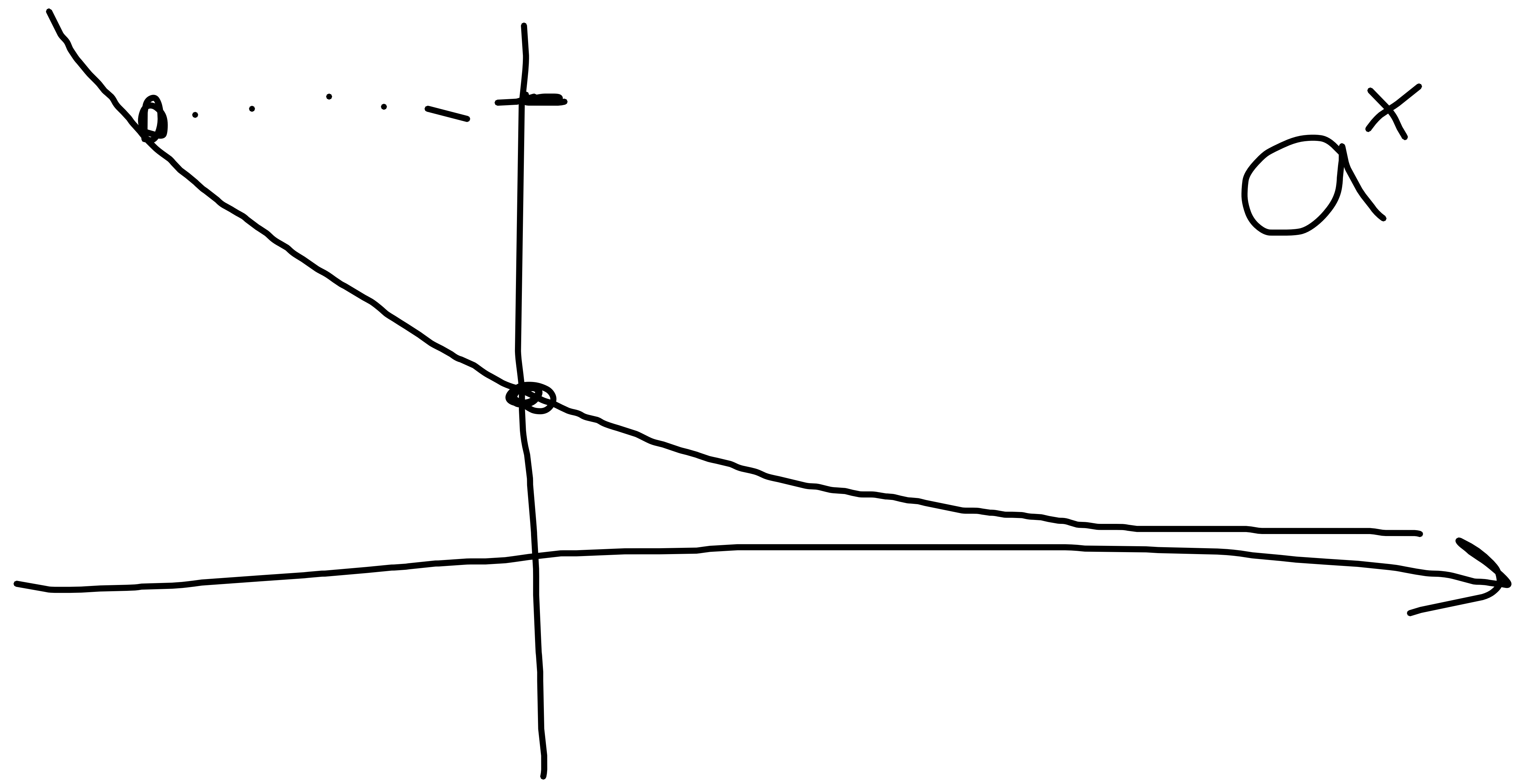
2) Se $0 < a < 1 \Rightarrow exp_a$ è STRETT. DECRESC.

3) $exp_a : \mathbb{R} \xrightarrow[su]{1-1} \mathbb{R}_+^*$ (invertibile)

$\underline{a > 1}$



$\underline{0 < a < 1}$



<u>Def</u> Sia $a \in \mathbb{R}^*_+ \setminus \{1\}$.

Chiamiamo LOGARITMO in base a l'inversa di $\exp_a$:

$$\log_a = (\exp_a)^{-1}$$

$$\log_a : \mathbb{R}^*_+ \longrightarrow \mathbb{R}$$

TEOR Sia $a \in \mathbb{R}^*_+ \setminus \{1\}$

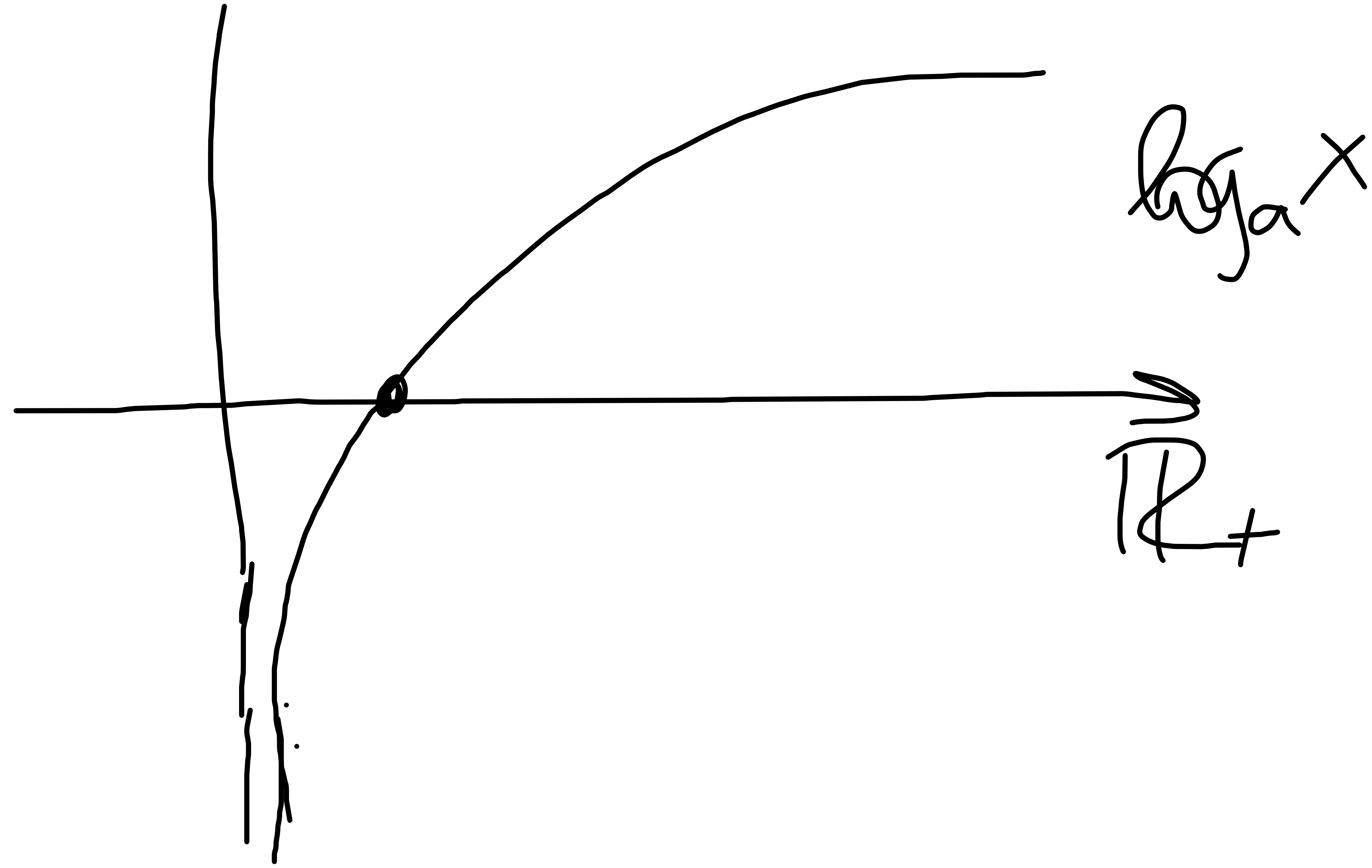
(1) Se $a > 1 \Rightarrow \log_a$ è STRETT $\nearrow$

(2) Se $0 < a < 1 \Rightarrow \log_a$ è STRETT $\searrow$

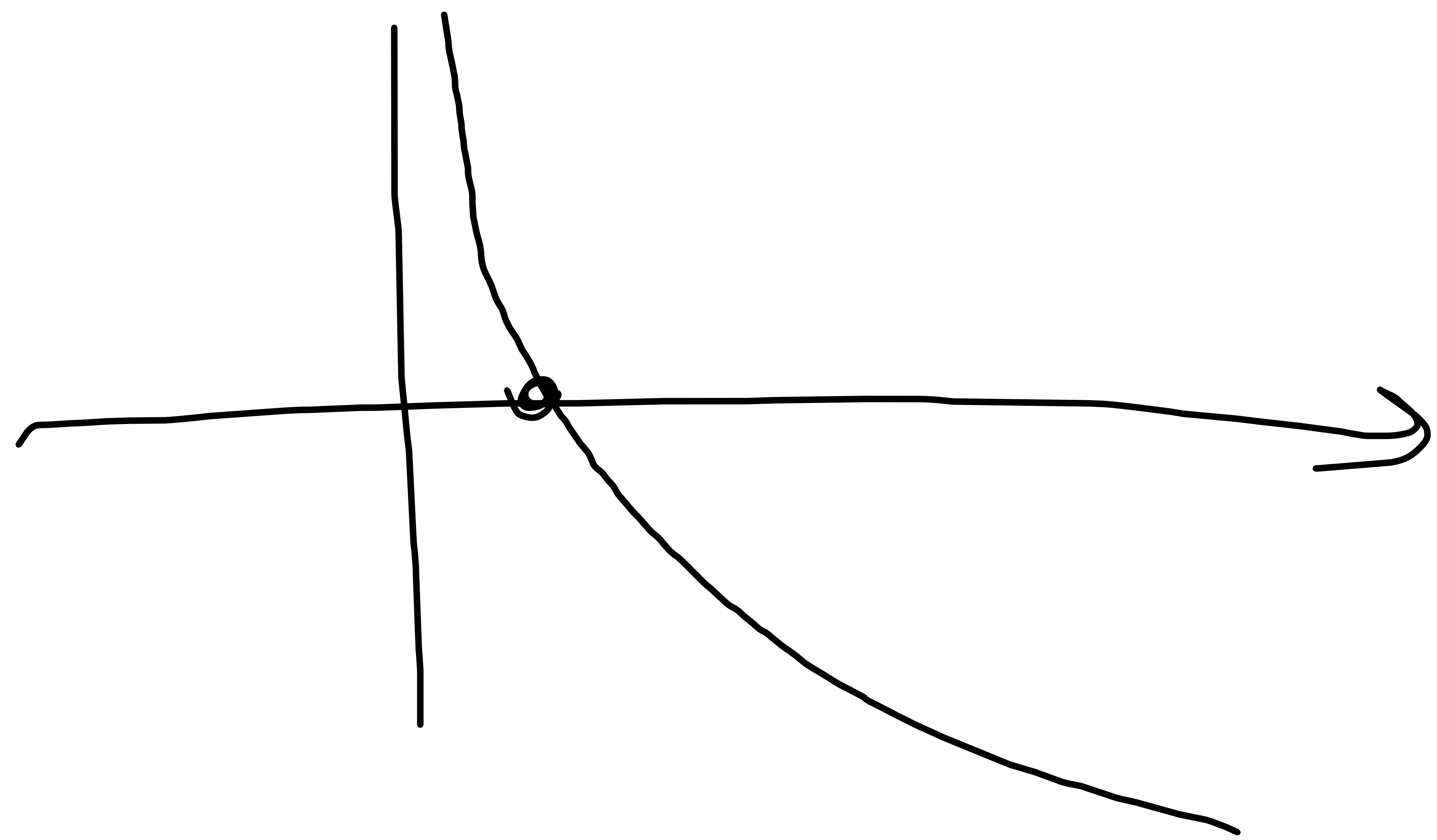
(3) $\log_a : \mathbb{R}^*_+ \xrightarrow[\text{su}]{1-1} \mathbb{R}$

$\log_a y$ è l'esponente da dare ad a per ottenere y, cioè l'unico x t.c. $a^x = y$

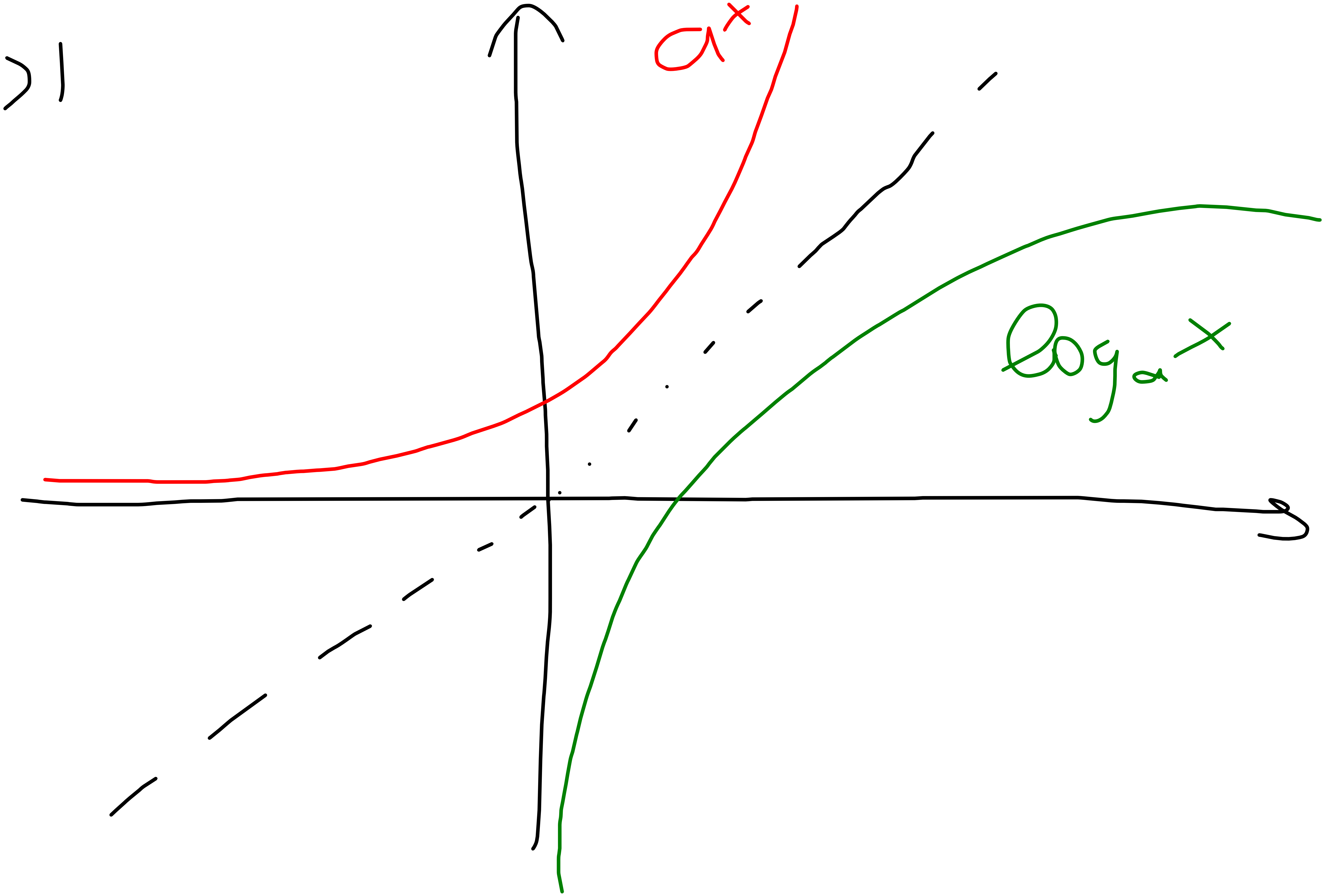
$a > 1$



$0 < a < 1$



$a > 1$



PROPRIETÀ

Sia $a \in \mathbb{R}_+^* \setminus \{1\}$

$\rightarrow$ 1) $\forall x, y \in \mathbb{R}_+^*$ $\quad \log(xy) = \log x + \log y$

2) $\forall x, y \in \mathbb{R}_+^*$ $\quad \log\left(\frac{x}{y}\right) = \log x - \log y$

3) $\forall x \in \mathbb{R}_+^*, \alpha \in \mathbb{R}$ $\quad \log(x^\alpha) = \alpha \log x$

DIM (1) $a^{\log_a(xy)} = xy = a^{\log_a x} \cdot a^{\log_a y}$

$= a^{\log_a x + \log_a y}$

(exp$_a$ è 1-1)

$\Longrightarrow$ $\boxed{\log_a(xy) = \log_a x + \log_a y}$

FORMULA del CAMBIAMENTO di BASE

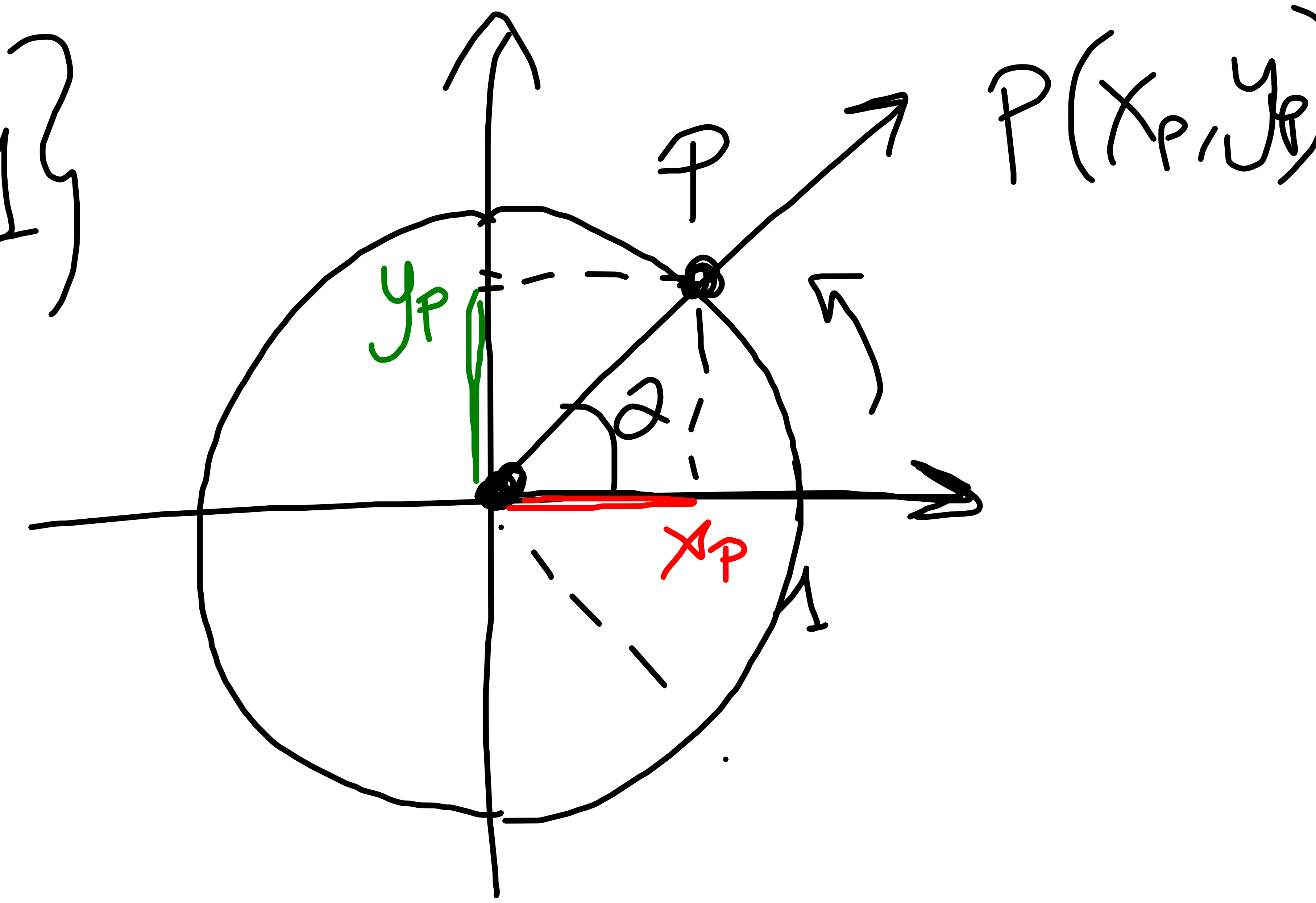
$a, b \in \mathbb{R}^+_* \setminus \{1\}$

$$\boxed{\log_b y = \frac{\log_a y}{\log_a b}}$$

FUNZIONI CIRCOLARI

Sia $U = \{(x,y) \in \mathbb{R} \mid x^2+y^2=1\}$

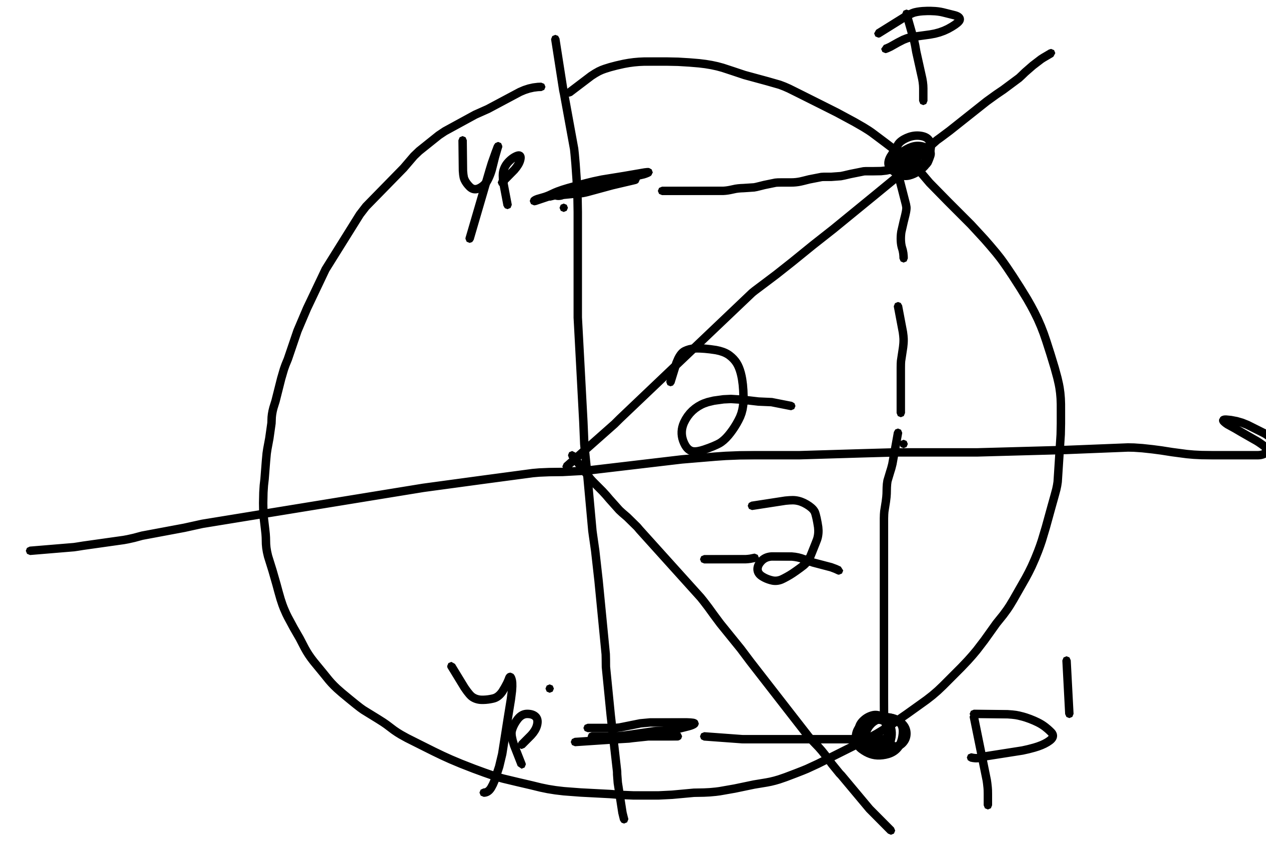
CIRCONFERENZA GONIOMETRICA



$$\cos : \mathbb{R} \longrightarrow \mathbb{R}$$
$$\alpha \longmapsto x_P$$

$$\sin : \mathbb{R} \longrightarrow \mathbb{R}$$
$$\alpha \longmapsto y_P$$

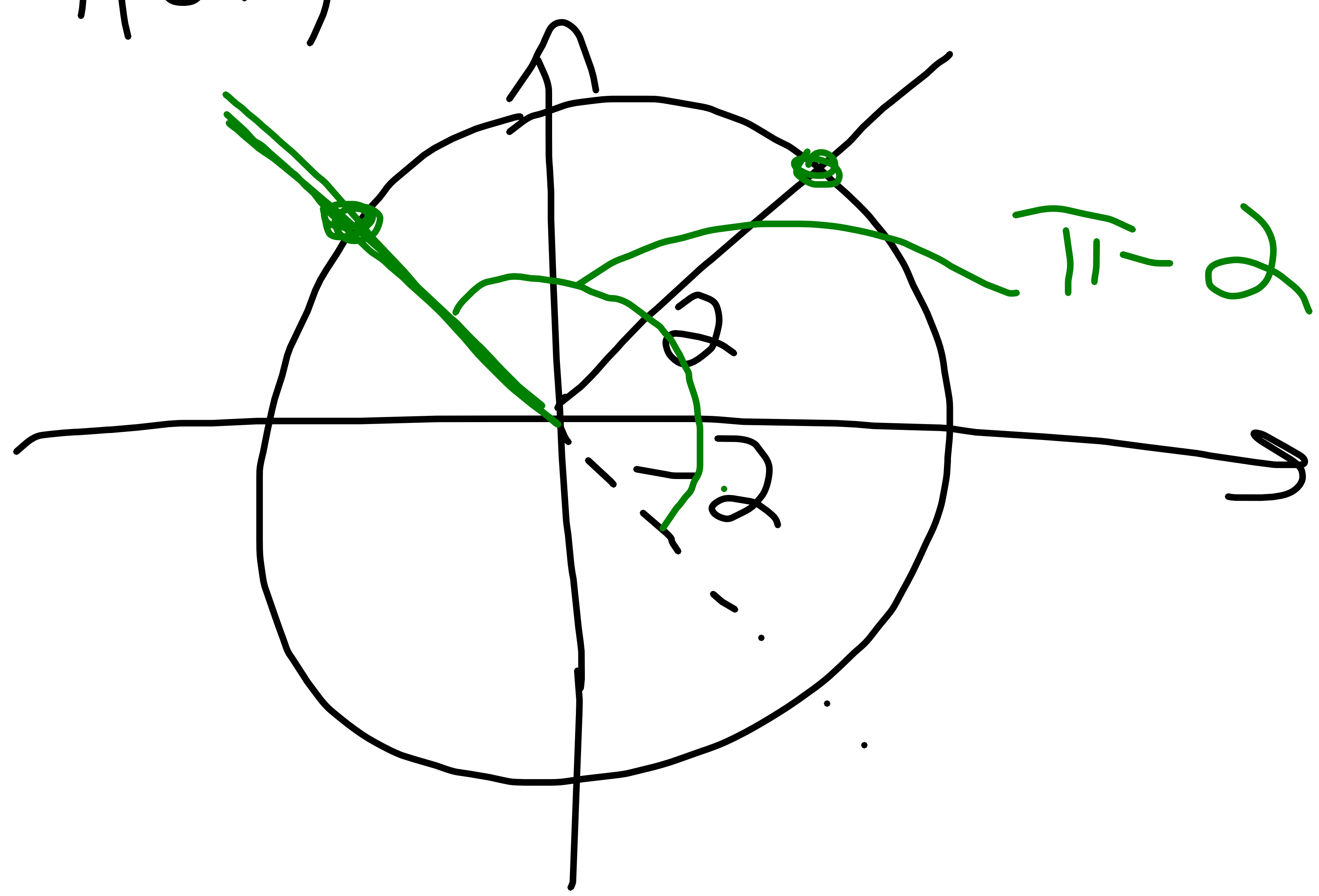
<u>OSS</u> • sen, cos sono PERIODICHE di periodo 2π



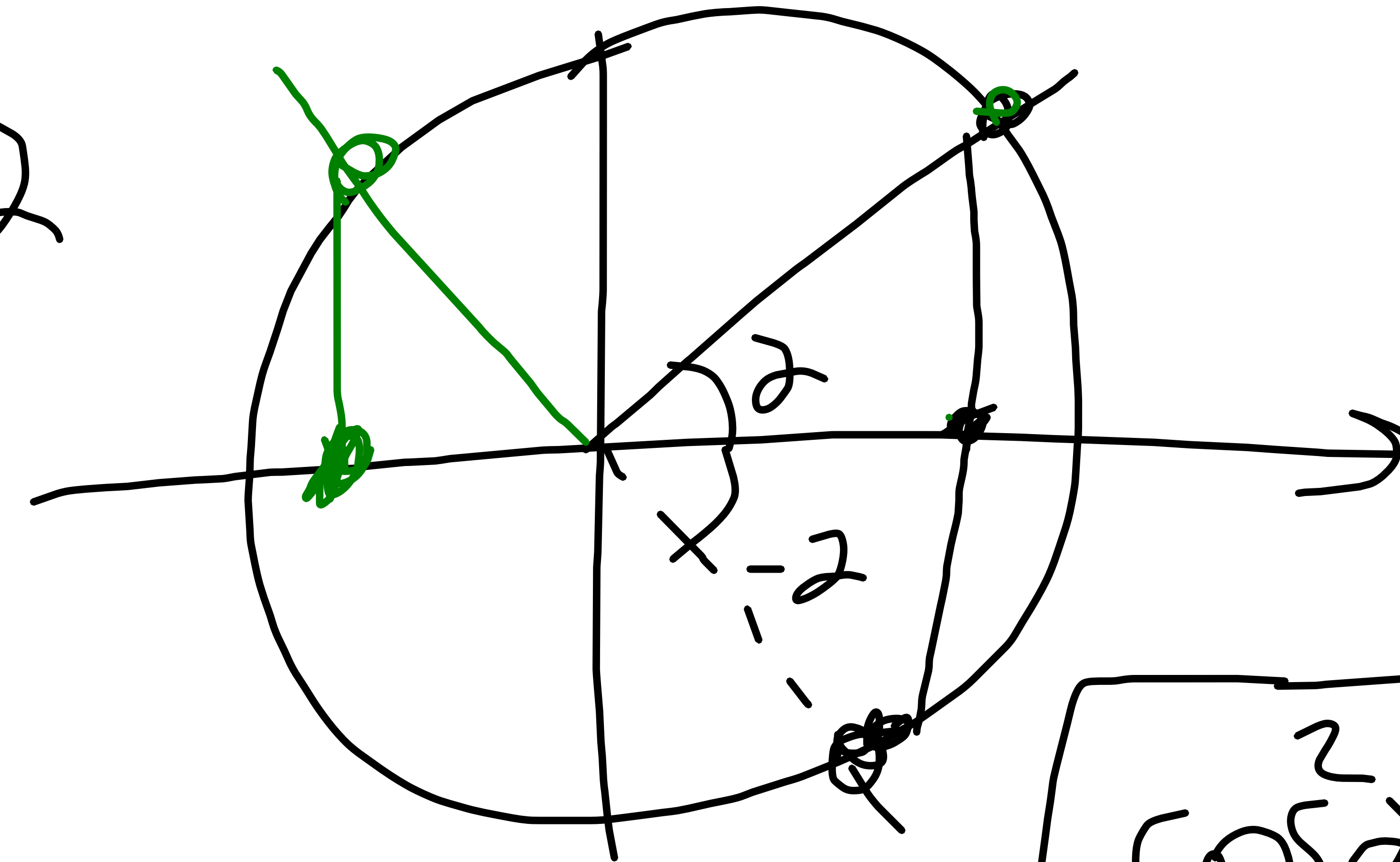
$$y_P = -y_{P'}$$

• $\operatorname{sen}(-2) = -\operatorname{sen}(2)$

• $\operatorname{sen}(\pi - 2) = \operatorname{sen}(2)$



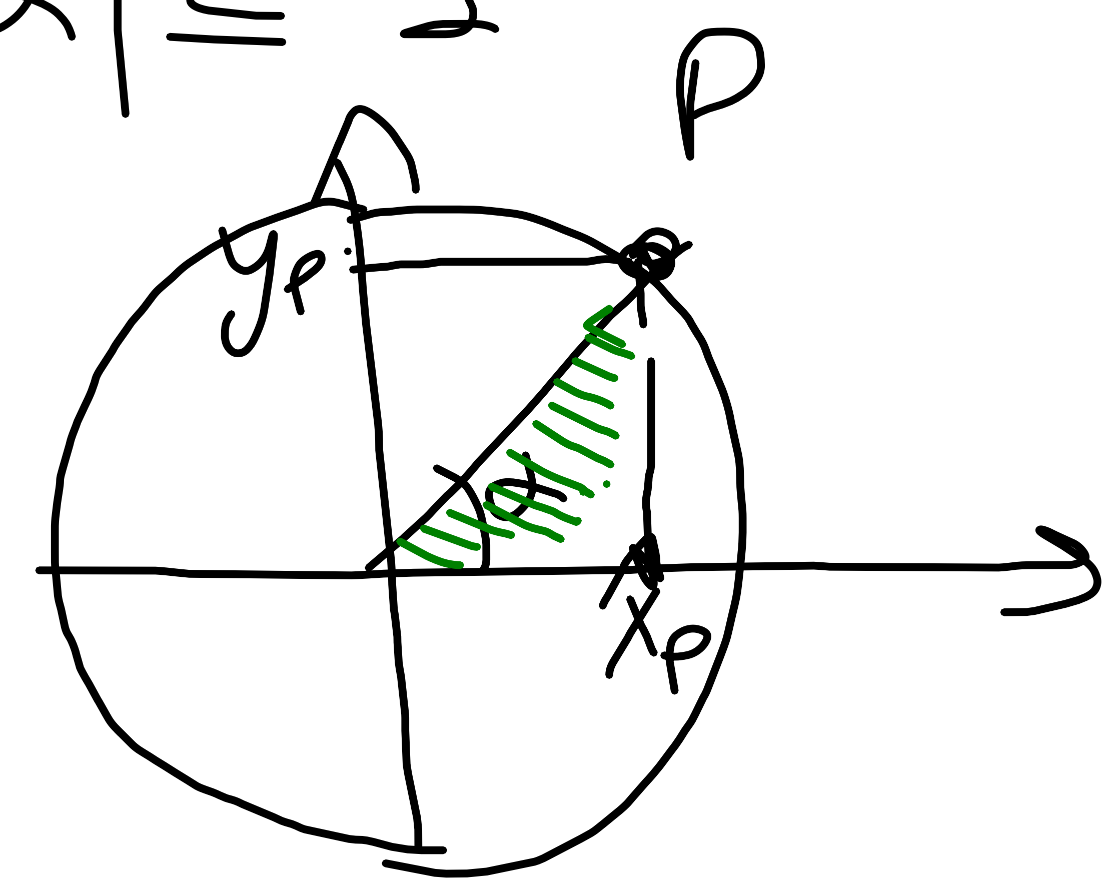
- $\cos(-\alpha) = \cos(\alpha)$
- $\cos(\pi - \alpha) = -\cos\alpha$



$\cos^2\alpha = (\cos\alpha)^2$

cos è PARI
sen è DISPARI

- $|\cos\alpha| \le 1$, $|\mathrm{sen}\,\alpha| \le 1$

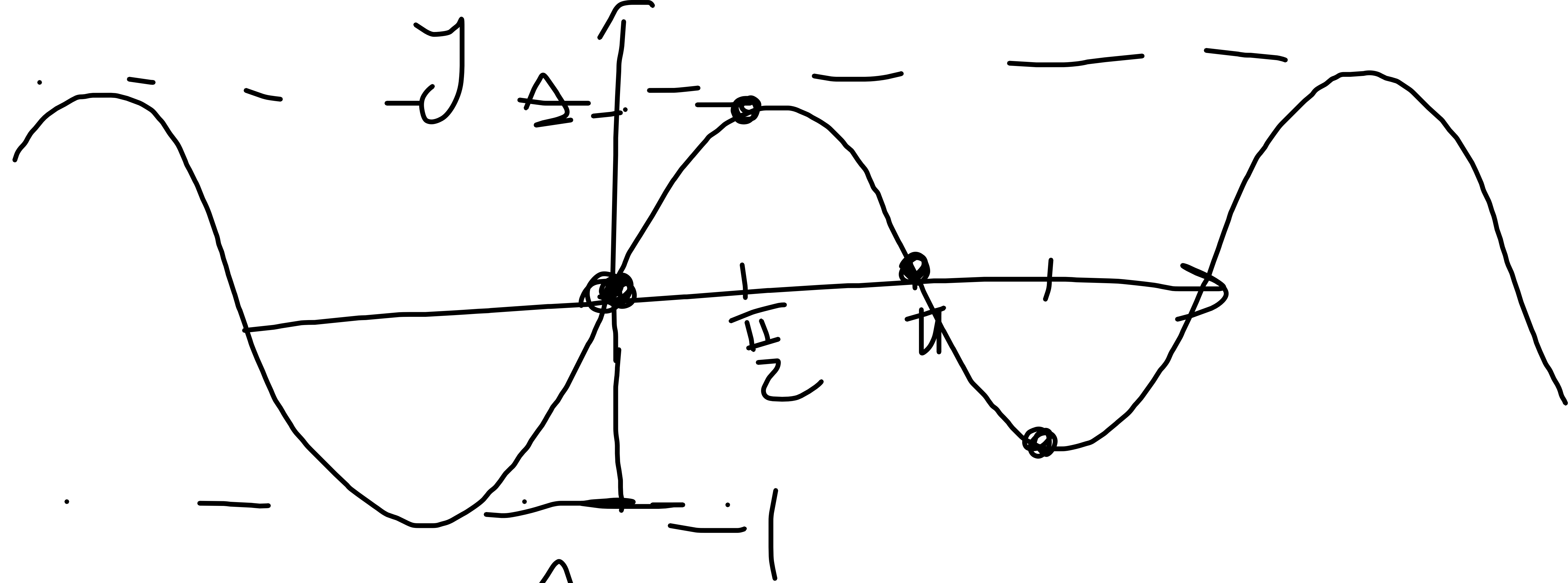


- (dal Teor di Pitagora):

$$\boxed{\cos^2\alpha + \mathrm{sen}^2\alpha = 1}$$

GRAFICI

- $f(x) = \sin x$



- $f(x) = \cos x$