$U = \langle (1,1,1), (1,2,1) \rangle$ $\quad V = \{(x,y,z) \in \mathbb{R}^3 \mid x - y - z = 0\}$

$$(x,y,z) = \alpha(1,1,1) + \beta(1,2,1)$$

$$\begin{cases} x = \alpha + \beta \\ y = \alpha + 2\beta \\ z = \alpha + \beta \end{cases} \qquad \begin{cases} \alpha = x - \beta \\ y = x - \beta + 2\beta \\ z = x \cancel{-\beta} \cancel{+\beta} \end{cases} \qquad \begin{cases} \alpha = z - \beta \\ y = z + \beta \\ \boxed{z = x} \end{cases}$$

$U = \langle (1,1,1),(1,2,1) \rangle \quad V = \{(x,y,z) \in \mathbb{R}^3 \mid x-y-z=0\}$

$U \cap V$

$$\alpha(1,1,1)+\beta(1,2,1)$$

$$(\alpha+\beta,\ \alpha+2\beta,\ \alpha+\beta) \longrightarrow (-\beta,\ 0,\ -\beta)$$

$$(\cancel{\alpha}+\cancel{\beta})-(\cancel{\alpha}+2\beta)-(\alpha+\cancel{\beta})=0$$

$$\underline{-2\beta-\alpha=0}$$

$$\alpha=-2\beta$$

$$(-1,\ 0,\ -1)$$

$B = (-1, 0, -1) =$

$\dim U = 2$

$C = ((-1, 0, -1), (0, 1, 0))$

$W \quad x - y - z = 0$

$\dim W \quad D = ((-1, 0, -1), (1, 1, 0))$

$$U+V=\langle(-1,0,-1),(0,1,0),(1,1,0)\rangle$$

$$\begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Esercizio 2 $\quad \mathbb{R}^5$

$$U = \langle (1,2,1,4,5), (2,-1,3,-1,3), (0,5,-1,9,7) \rangle$$

(a) Eq. parametriche di U

$$U = \{ \alpha(1,2,1,4,5) + \beta(2,-1,3,-1,3) + \gamma(0,5,-1,9,7) : \alpha,\beta,\gamma \in \mathbb{R} \}$$

$$= \{ (\alpha+2\beta, 2\alpha-\beta-\gamma, \dots \quad) : \alpha,\beta,\gamma \in \mathbb{R} \}.$$

$$\begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 2 & -1 & 3 & -1 & 3 \\ 0 & 5 & -1 & 9 & 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 0 & -5 & 1 & -9 & -7 \\ 0 & 5 & -1 & 9 & 7 \end{pmatrix}$$

ha rango 2 $\Rightarrow$

$\dim U = 2$ e una sua base è

$$B = ((1,2,1,4,5), (0,5,-1,9,7))$$

(b) equazioni cartesiane

Metodo del rango

$$\begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 0 & 5 & -1 & 9 & 7 \\ x_1 & x_2 & x_3 & x_4 & x_5 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 0 & 5 & -1 & 9 & 7 \\ 0 & x_2-2x_1 & x_3-x_1 & x_4-4x_1 & x_5-5x_1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 0 & 5 & -1 & 9 & 7 \\ X_1 & X_2 & X_3 & X_4 & X_5 \end{pmatrix} \xrightarrow{\text{riga}} \begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 0 & 5 & -1 & 9 & 7 \\ 0 & X_2-2X_1 & X_3-X_1 & X_4-4X_1 & X_5-5X_1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 2 & 1 & 4 & 5 \\ 0 & 5 & -1 & 9 & 7 \\ 0 & 5(X_2-2X_1) & 5(X_3-X_1) & 5(X_4-4X_1) & 5(X_5-5X_1) \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & - & - & - & - \\ 0 & 5 & - & - & - \\ 0 & 0 & 5(X_3-X_1)+X_2-2X_1 & 5(X_4-4X_1)-9(X_2-2X_1) & 5(X_5-5X_1)-7(X_2-2X_1) \end{pmatrix}$$

dobbiamo sottrarre alla 3^a riga la 2^a moltiplicata per X_2-2X_1

$$\begin{cases} 7x_1 + x_2 + 5x_3 = 0 \\ 2x_1 - 9x_2 + 5x_4 = 0 \\ 11x_1 - 7x_2 + 5x_5 = 0 \end{cases}$$

1 2 1 4 5

$-7 + 2 + 5 = 0$

$-2 - 18 + 20 = 0$

$-11 - 14 + 25 = 0$ ✓

$5(x_3 - x_1) + x_2 - 2x_1$

$5(x_4 - 4x_1) - 9(x_2 - 2x_1)$

$5(x_5 - 5x_1) - 7(x_2 - 2x_1)$

Esercizio 3 In $\mathbb{R}^5$

$$U_k = \langle (1,2,k,1,0), (0,1,1,-1,1) \rangle$$

$$W \quad \begin{cases} x_1 - x_2 + x_3 - 2x_5 = 0 \\ x_3 - x_4 = 0 \end{cases}$$

(a) $\dim U_k = 2$ per ogni k.

(b) base di W

Siccome i due pivot dell' equazioni sono nelle colonne 1 e 3 possiamo scegliere come variabili libere x_2, x_4, x_5

$$x_3 = x_4$$

$$x_1 = x_2 - x_3 + 2x_5$$

$$= x_2 - x_4 + 2x_5$$

$$\{(x_2 - x_4 + 2x_5, x_2, x_4, x_4, x_5) : x_2, x_4, x_5 \in \mathbb{R}\}$$

$$= \langle (1,1,0,0,0), (-1,0,1,1,0), (2,0,0,0,1) \rangle$$

(c) Per quali k l'intersezione $U_k \cap W$ è banale?

$$\langle (1,2,k,1,0),(0,1,1,-1,1)\rangle$$

$(\alpha, 2\alpha+\beta, \alpha k+\beta, \alpha-\beta, \beta)$ ← vettore generico di U_k

$$\begin{cases} \alpha-(2\alpha+\beta)+\alpha k+\beta-2\beta=0 \\ \alpha k+\beta-(\alpha-\beta)=0 \end{cases}$$

$$\begin{cases} (k-1)\alpha-2\beta=0 \\ (k-1)\alpha+2\beta=0 \end{cases}$$

sottraendo le equazioni otteniamo $\beta=0$

$$\Rightarrow (k-1)\alpha=0 \quad \begin{cases} \text{se } k\neq 1 & \alpha=0 \\ \text{se } k=1 & \alpha \text{ è libero} \end{cases}$$

L'intersezione è banale per ogni $K \neq 1$

Esercizio 5

$u_1 = (1,1,1,1)$
$u_2 = (2,0,-4,-4)$
$w_1 = (1,2,0,0)$
$w_2 = (2,3,3,3)$
$w_3 = (3,3,2,5)$

$U = \langle u_1, u_2 \rangle$

$W = \langle w_1, w_2, w_3 \rangle$

a) $\dim U = 2$ (perché generato da 2 vettori non proporzionali)

$\dim W$?

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 3 & 3 \\ 3 & 3 & 2 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & -1 & 3 & 3 \\ 0 & -3 & 2 & 5 \end{pmatrix}$$

ha rango 3 $\Rightarrow$ $\dim W = 3$

$u_1 = (1,1,1,1)$

$u_2 = (2,0,-4,-4)$

Abbiamo bisogno di 2 equazioni:

Ad occhio:

$$\begin{cases} z - t = 0 \\ z + 2x - 3y = 0 \end{cases}$$

Eliminazione parametri

$$\begin{cases} x = \alpha + 2\beta \\ y = \alpha \\ z = \alpha - 4\beta \\ t = \alpha - 4\beta \end{cases}$$

$$\begin{cases} x = y + 2\beta \\ z = y - 4\beta \\ t = y - 4\beta \end{cases}$$

$$\begin{cases} z = y - 2(x-y) \\ t = y - 2(x-y) \end{cases}$$

Rango

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & -4 & -4 \\ x & y & z & t \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -\cancel{2}^{1} & -\cancel{6}^{3} & -\cancel{6}^{3} \\ 0 & y-x & z-x & t-x \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -3 & -3 \\ 0 & 0 & \underbrace{z-x-3(y-x)}_{=0} & \underbrace{t-x-3(y-x)}_{=0} \end{pmatrix}$$