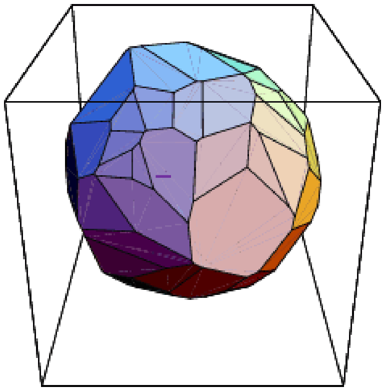
Spreadsheet and Online LP Solvers

Solving LPs With More Than 2 Variables



The 3-Variable Case (3D):

- A corner point in 3D is defined by 3 (in-)equalities.
- The good news: The corner property still holds!
- The downside: For and LP with m inequalities, there are $\binom{m}{3}$ potential corner points!

Important: Not every point induced by 3 equalities is a corner!

Additional condition: The point has to be feasible! (So check all remaining constraints!)

Exercise: Graph the feasible region of the following LP!

Max $Z = X + Y + Z$

Subject to: $2X + 4Y + Z \leq 4$

$X, Y, Z \geq 0$

Too much work for us, we use computers (and sophisticated algorithms) instead!

LP Solution with Computer Software

- Excel Solver is based on SIMPLEX method (algebraic version of the graphical version discussed earlier).
  Basically, it's evaluating a sequence of relevant corner points!

- In addition to the solution, Excel Solver also provides other useful information for sensitivity analysis.

- Unfortunately, Excel Solver is limited to 200 decision variables!

- Linear Programming (and other analytic) models are often solved using special purpose software in industry. (e.g., IBM ILOG CPLEX, GUROBI, FICO XPRESS)

- Some (good ones) can be used without purchase in research or education. (e.g., Soplex, COIN-OR, QSopt)

Solve high-dimensional LPs

Tasks: $J = J_{\text{in}} \cup J_{\text{out}}$

$$J_{\text{in}} = V_{\text{in}}$$
$$J_{\text{out}} = V_{\text{out}}$$

Modes: $M = \bigcup_{j \in J} M_j$

$$j \in J: \quad M_j = W$$

Resources: $R = R_Q \cup R_B \cup R_D$

$$\begin{aligned}
R_Q = R_{Q_{\text{in}}} \cup R_{Q_{\text{out}}}, \quad & R_{Q_{\text{in}}} = W \\
& R_{Q_{\text{out}}} = W \\
& R_B = W \\
R_D = R_{D_{\text{in}}} \cup R_{D_{\text{out}}}, \quad & R_{D_{\text{in}}} = W \\
& R_{D_{\text{out}}} = W
\end{aligned}$$

Capacities:

$$\begin{aligned}
r \in R_{Q_{\text{in}}}, t \in T_r: \quad & q_{r,t} = Q_{\text{in}}(r) \\
r \in R_{Q_{\text{out}}}, t \in T_r: \quad & q_{r,t} = Q_{\text{out}}(r) \\
r \in R_{Q_{\text{in}}}, t \in T \setminus T_r: \quad & q_{r,t} = 0 \\
r \in R_{Q_{\text{out}}}, t \in T \setminus T_r: \quad & q_{r,t} = 0 \\
r \in R_B, t \in T: \quad & q_{r,t} = B(r) \\
r \in R_{D_{\text{in}}}, t \in T: \quad & q_{r,t} = D(r,t) \\
r \in R_{D_{\text{out}}}, t \in T: \quad & q_{r,t} = D(r,t)
\end{aligned}$$

Resource utilization:

$$\begin{aligned}
r \in R_B, j \in J, m \in M_j: \quad & u_{m,r} = b(j) \\
r \in R_D, j \in J, m \in M_j: \quad & u_{m,r} = d(j) \\
r \in R_Q, j \in J, m \in M_j: \quad & u_{m,r} = 1
\end{aligned}$$

Durations:

$$\begin{aligned}
j \in J_{\text{out}}, m \in M_j: \quad & d_m = z_{\text{in}}(j,m) \\
j \in J_{\text{out}}, m \in M_j: \quad & d_m = z_{\text{out}}(j,m)
\end{aligned}$$

$$\min \sum_{j \in J} \sum_{m \in M_j} \sum_{t \in T} (t - EST_j) x_{j,m,t} \tag{1}$$

$$\text{subject to} \quad \sum_{m \in M_j} \sum_{t \in T} x_{j,m,t} = 1 \quad \forall j \in J, \tag{2}$$

$$\sum_{j \in J} \sum_{m \in M_j} \sum_{t < t' \le t + d_m} u_{m,r} x_{j,m,t'} \le q_{r,t} \quad \forall r \in R, \ t \in T, \tag{3}$$

$$EST_j \le \sum_{m \in M_j} \sum_{t \in T} (t - d_m) x_{j,m,t} \le LST_j \quad \forall j \in J, \tag{4}$$

$$x_{j,m,t} \in \{0,1\} \quad \forall j \in J, \ m \in M_j, \ t \in T. \tag{5}$$

Solve large LPs

Excel & Numerical Accuracy

Most computing tools (including Excel) have a limited numerical accuracy!

See Excel Sheet!

Typically, numerical issues arise when

- model is not formulated well,
- formulation is very large (many variables and/or constraints),
- variable coefficients are very large and small at the same time, or
- complicating input data is used (very small/large values).

Often the accuracy can be adjusted as a parameter.

Remember: Higher precision will be paid by solver run time!

Solver Ambiguity

Consider the following LP:

$$
\begin{aligned}
\textbf{Max } Z = \quad & X + Y \\
\textbf{Subject to:} \quad & X + Y \le 4 \\
& X \le 2 \\
& Y \le 3 \\
& X, Y \ge 0
\end{aligned}
$$

What optimal solution do you get both graphically, and from Excel Solver?

See Example Excel Sheet!

Solver performance AND the returned solution itself depend on

- the LP formulation (even when feasible regions are identical!),
- the order in which variables and its expressions are fed into the solver, and
- the solver parameters (e.g., solution method, numerical tolerances).

Excel Limitations

- How large can the LPs in Excel be?

Variable limit of 200
Constraint limit of 100

- How large can LPs be such that Excel Solver solves them efficiently?

Can you find an LP that takes a loooong time to solve?

- Are there harder and easier LPs, or is their practical complexity constant?

Open Solver*

- OpenSolver is an **Excel add-in** that extends Excel's built-in solver with a more powerful Linear Programming solver.
- OpenSolver uses the excellent, COIN-OR CBC optimization engine, to quickly solve large **Linear (including Integer) Programs**.
- **Compatible with existing Solver models**, so there is no need to change your spreadsheets.
- **No limits on the size** of problem you can solve.
- **OpenSolver is free**, open-source software licensed under the GPL.

*Developed by Andrew Mason in the Department of Engineering Science at the University of Auckland.

Open Solver

- A **built-in model visualizer** that highlights your model's decision variables, objective and constraints directly on your spreadsheet.

- A fast QuickSolve mode that makes it much **faster to re-solve your model** after making changes to a right-hand side.

- An **optional model building tool** that analyses your spreadsheet, and then fills in the Solver dialog automatically.

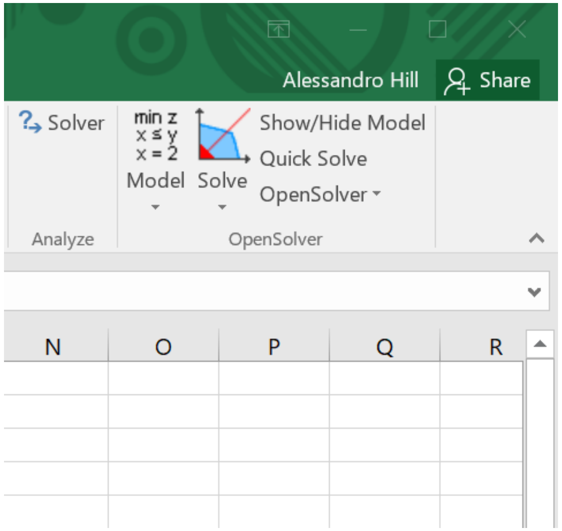
- OpenSolver has been developed for **Excel under Windows**. It should work with these or **later Excel versions**.

Open Solver

Installation

1. Download the `OpenSolver.zip file` from www.opensolver.org
2. Extract the files to a convenient location
3. Double click on `OpenSolver.xlam`
4. If asked, give Excel permissions to run OpenSolver

The OpenSolver commands will then appear under Excel's Data tab.



OpenSolver will be available until you quit Excel. If you wish to make OpenSolver always available in Excel, the files from the zip all need to be copied into the Excel add-in directory. This is typically either:

- `C:\Documents and Settings\"user name"\Application Data\Microsoft\Addins\`, or
- `C:\Users\"user name"\AppData\Roaming\Microsoft\Addins\`

Online Solvers

- https://www.zweigmedia.com/RealWorld/simplex.html
- http://www.phpsimplex.com/en/
- https://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/LPTools.htm
- http://online-optimizer.appspot.com
- http://www.mathstools.com/section/main/simplex_online_calculator
- https://www.wolframalpha.com/widgets/view.jsp?id=1e692c6f72587b2cbd3e7be018fd8960
- https://www.desmos.com/calculator/tdhxorkxgb (good for graphing feasible regions!)