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(Spanning Trees)

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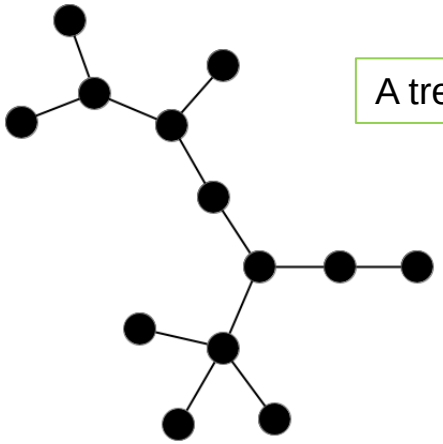
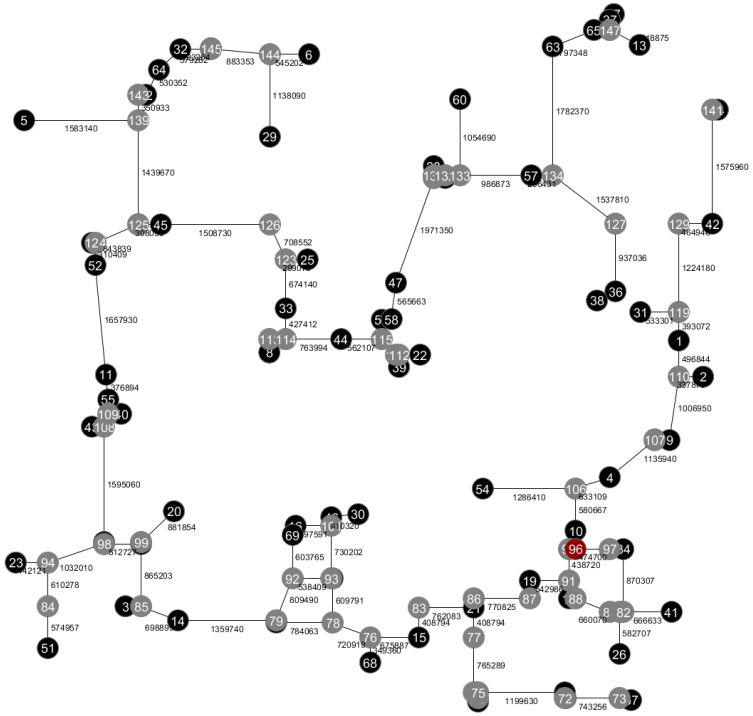
The Minimum Spanning Tree Problem

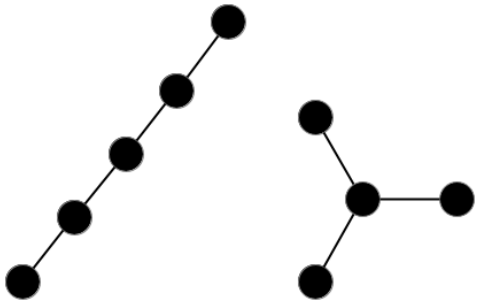
A **tree** is a graph such that each pair of nodes is connected by exactly one path.



A tree has no cycles!

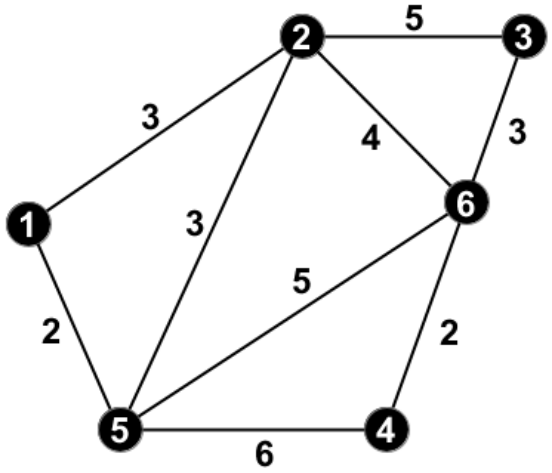


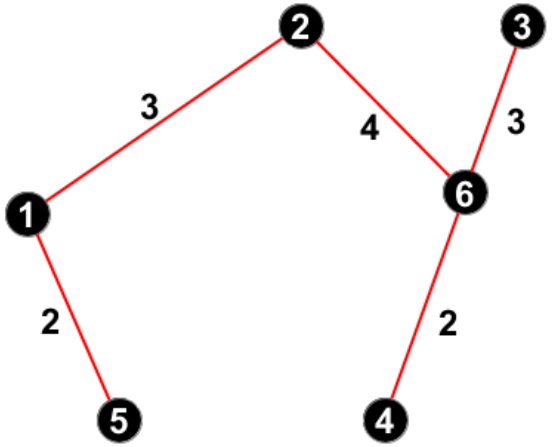
A tree-shaped telecommunication network.



The Minimum Spanning Tree Problem

The **minimum spanning tree problem** asks for a tree subgraph that spans all given nodes such that the total cost of tree edges is minimized.



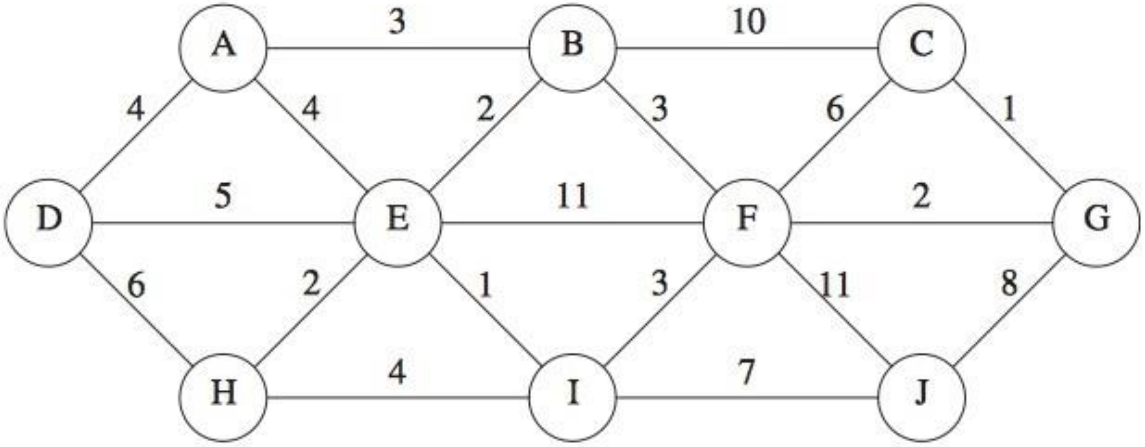


Prim's algorithm (exact; Vojtěch Jarník, 1930)**:**

1. Select network node.
2. Connect current partial tree to "cheapest tree neighbor".
3. Repeat 2. until all nodes are connected.

The Minimum Spanning Tree Problem

Find the minimum spanning tree for the following given network.



The Minimum Spanning Tree Problem

IP Model (non-compact)

Binary edge variable for each edge that could be part of the tree:

$$x_{i,j} = \begin{cases} 1 & \text{if edge } \{i,j\} \text{ will be used in the tree,} \\ 0 & \text{otherwise.} \end{cases}$$

Objective: Minimize $\sum_{\{i,j\} \in E} w_{i,j} x_{i,j}$

Number of overall edges:

$$\sum_{\{i,j\} \in E} x_{i,j} = |\mathrm{N}| - 1$$

No cycles allowed:

$$\sum_{\{i,j\} \in E(S)} x_{i,j} \leq |S| - 1 \qquad \text{for each subset } S \subset N.$$

The Minimum Spanning Tree Problem

Find a minimum spanning tree for the following given network.