Resource-Constrained Project Scheduling

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The Resource-Constrained Project Scheduling Problem

We are given:

- Resources **R** = {1,2,…}
  Per-period availability for resource **r**: $q_r > 0$

- Jobs **J** = {1,2,…}
  Duration of job **j**: $d_j > 0$
  Per-period consumption of resource **r**: $u_{j,r} > 0$

- Time horizon **T** = {0,1,2,…}

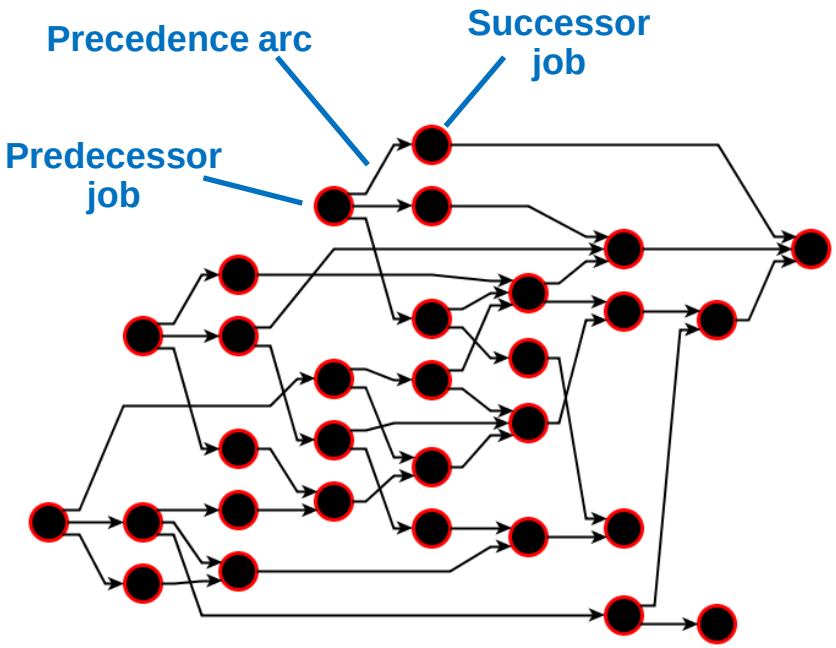
- Job precedences **A** = {(1,2),(2.3),(3,7),…}

The Resource-Constrained Project Scheduling Problem (RCPSP)* asks for a schedule of minimum duration (makespan) such that
- all jobs are scheduled within the time horizon,
- all job precedences are satisfied, and
- the resource availability is respected for every resource in every time period.

*NP-hard (extremely difficult to optimize)

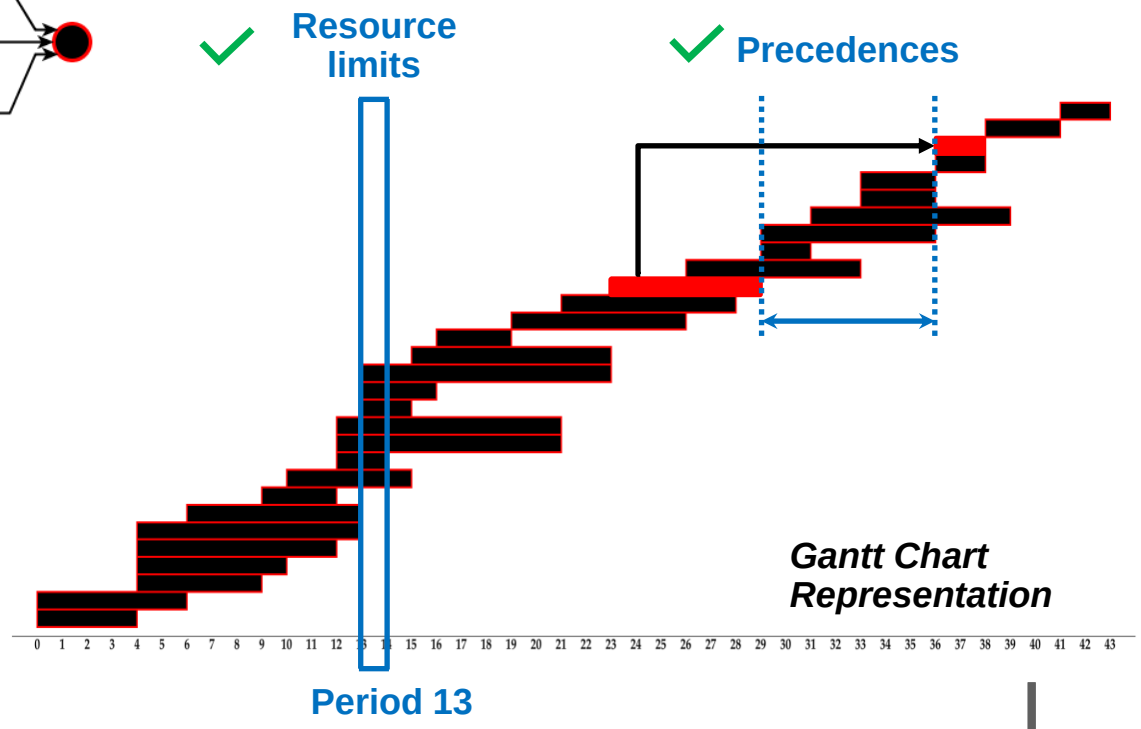
The Resource-Constrained Project Scheduling Problem

Precedence Relation Digraph



Assumption: *Directed graph (J, A) is acyclic!*

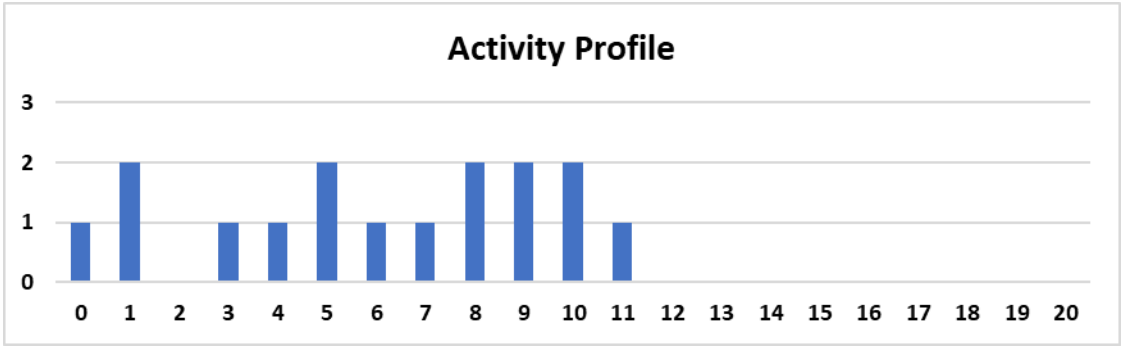
Solution Schedule



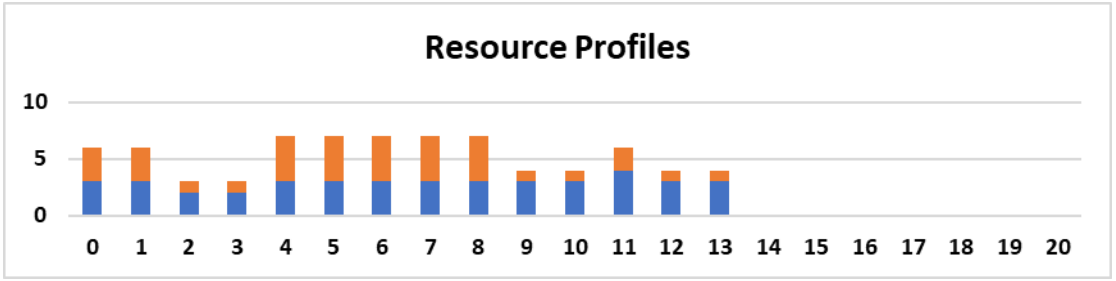
Gantt Chart Representation

The Resource-Constrained Project Scheduling Problem

An activity profile for a schedule **S** shows the number of active jobs for every time period.



A resource profile for a schedule **S** and a resource **r** shows the consumption of **r** in every time period.



The Resource-Constrained Project Scheduling Problem

IP Model

Binary job end-at-time variables:

$$x_{j,t} = \begin{cases} 1 & \text{if job } j \text{ will end at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Minimize $\sum_{t \in T} x_{n,t}$ (n is the terminal job)

$$\sum_{t \in T} x_{j,t} = 1 \qquad \forall\, j \in J \qquad \text{(Schedule Jobs)}$$

$$\sum_{t \in T} t \cdot x_{i,t} \leq \sum_{t \in T} t \cdot x_{j,t} - d_j \qquad \forall\, (i,j) \in A \qquad \text{(Precedences)}$$

$$\sum_{j \in J} \sum_{t'=t}^{t+d_j-1} u_{j,r} \cdot x_{j,t'} \leq q_r \qquad \forall\, t \in T, r \in R \qquad \text{(Resource Availability)}$$

Es. Capitol Construction (2a. Revisited)

La Capitol Construction Company deve completare la ristrutturazione del suo attuale ufficio il più rapidamente possibile…

Compito	Simbolo	Precedenca	Durata	Persone	Costi (in 1000)
Preparare opzioni di finanziamento	A	-	2	**3**	**3**
Preparare schizzi preliminari	B	-	3	**2**	**1**
Delineare le specifiche	C	-	1	**1**	**3**
Preparare disegni	D	A	4	**3**	**4**
Scrivere le specifiche	E	C, D	5	**3**	**1**
Eseguire le stampe	F	B	1	**1**	**1**

a) Costruire il modello in Excel che minimizza il «makespan» usando Excel Solver.
b) Visualizzare il «activity profile».
c) Visualizzare il «resource profile».

Exercise 1)

Find an optimal schedule for the model in Exercise 1 using CP in MiniZinc.
(Model on Virtuale)

Exercise 2)

In Table 1.1, a RCPSP instance is given with $n = 10$ real activities and $|\mathcal{R}| = 2$ resources with availabilities $B_1 = 7$ and $B_2 = 4$.

A_i	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
p_i	0	6	1	1	2	3	5	6	3	2	4	0
b_{i1}	0	2	1	3	2	1	2	3	1	1	1	0
b_{i2}	0	1	0	1	0	1	1	0	2	2	1	0

1. **Find an optimal schedule.**
   Option 1: Build the Excel IP model.
   Option 2: Use CP.
   Option 3: Use the solver of your choice.

2. **Visualize the schedule using a Gantt chart.**

3. **Visualize activity and resource profiles.**

4. **How much does the makespan increase if we reduce the availability of resource 1 to 5?**
   Verify your results visually.