Resource-Constrained Project Scheduling: Heuristics

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Topological Sorting Algorithm for the Project Scheduling Problem (PSP, no resources)

INPUT: Jobs J, Precedence Network N

1. Initialize:
   - Empty schedule S
   - Auxiliary precedence network N':=N

2. Repeat the following until all jobs are scheduled:
   a) Pick a job j that has not been scheduled yet and has NO predecessors in the CURRENT precedence network N'.
   b) Remove j from the CURRENT precedence network N'.
   c) Schedule j in S to begin at the earliest possible start time. This is the latest end time in S of all predecessors of j in N.

This is an exact algorithm for the Project Scheduling Problem without resources.

Alternative description:
- Find a topological ordering Σ of the nodes in N.
- Schedule jobs in the order given by Σ at the earliest possible times.

Topological Sorting Heuristic for the RCPSP

TopoSort

INPUT: Jobs J, Precedence Network N, Resources R

1. Initialize:
   - Empty schedule S
   - Auxiliary precedence network N'

2. Repeat the following until all jobs are scheduled:
   a) Pick a job j that has not been scheduled yet and has NO predecessors in the CURRENT precedence network N'.
   b) Remove j from the CURRENT precedence network N'.
   c) Schedule j to begin at the earliest possible start time. This is at least the latest end time of all predecessors of j in N. In general, it is later due to resources limitations in the CURRENT schedule.

This is a heuristic method for the RCPSP (PSP with resources).

α - Point Heuristic for the RCPSP

INPUT: Jobs J, Precedences A, Resources R, Scheduling Threshold α

1. Solve the linear relaxation (LP) for an RCPSP formulation (IP).
   This gives you a fractional value $x_{j,t}$ for each potential start time* t for each job j.

2. Initialize empty job ordering Σ.

3. For potential job start time* t form 0 to MAX(T) do:
   - For each job that is NOT in Σ, calculate the start potential $a_{j,t}$ at time t: $a_{j,t} = \sum_{t'=0}^{t} x_{j,t}$
   - Let candidate set C_t contain all jobs that are not in Σ and for which $a_{j,t} \geq \alpha$.
   - Append jobs in C_t that are "precedence-feasible" to Σ.**

4. Schedule jobs according to the ordering Σ.

*Can also be done using job end times.
**Try appending in decreasing order of $a_{j,t}$. All predecessors of the job must already be in Σ.

The α – point (in time) for a job j is the time t when $a_{j,t} \geq \alpha$.

Strategy: Run this heuristic for various values of α and return the best schedule.
Example: $\alpha \in \{0.1, 0.2, \ldots, 1.0\}$

For very large RCPSPs, solving the LP can be time consuming!

Es. Capitol Construction (2a. Re-revisited)

La Capitol Construction Company deve completare la ristrutturazione del suo attuale ufficio il più rapidamente possibile…

Compito	Simbolo	Precedenca	Durata	Persone	Costi (in 1000)
Preparare opzioni di finanziamento	A	-	2	**3**	**3**
Preparare schizzi preliminari	B	-	3	**2**	**1**
Delineare le specifiche	C	-	1	**1**	**3**
Preparare disegni	D	A	4	**3**	**4**
Scrivere le specifiche	E	C, D	5	**3**	**1**
Eseguire le stampe	F	B	1	**1**	**1**

Resource availability: $q_{Persone} = 4$, $q_{Costi} = 5$

a) Trova una soluzione con il algoritmo TopoSort
b) Trova una soluzione con il algoritmo α - Point

Exercise 1)

In Table 1.1, a RCPSP instance is given with $n = 10$ real activities and $|\mathcal{R}| = 2$ resources with availabilities $B_1 = 7$ and $B_2 = 4$.

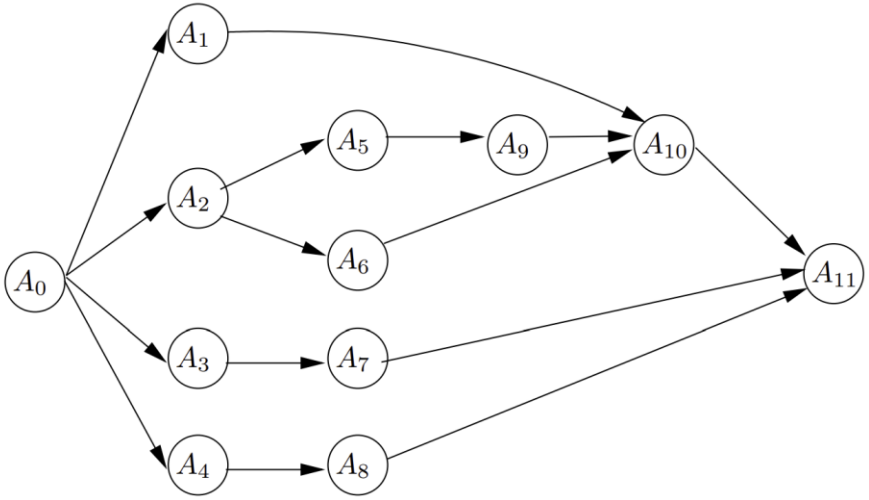
A_i	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
p_i	0	6	1	1	2	3	5	6	3	2	4	0
b_{i1}	0	2	1	3	2	1	2	3	1	1	1	0
b_{i2}	0	1	0	1	0	1	1	0	2	2	1	0

1. Find a schedule using the TopoSort Heuristic

2. Find a schedule using the α - Point Heuristic

3. Find an optimal schedule using the IP or CP

3. Visualize all schedules using Gantt charts
Verify your results visually.
Compare results.



Exercise 2)

1. **Implement an RCPSP Heuristic**
   Option 1: TopoSort Heuristic (TopoSort)Build the Excel IP model.
   Option 2: α –Point Heuristic.
   Option 3: Both heuristics.
   Use the language/system of your choice.

2. **Run your algorithm on larger test instances**
   Data on Virtuale.
   Careful: Different data formats!
   Feel free to use other large RCPSP instances.

3. **Can you visualize your results?**

4. **How good are your results compared to an exact method?**
   Use an exact IP Solver or MiniZinc?