Constraint Programming

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Topics

- Concepts
- Solvers
- Applications

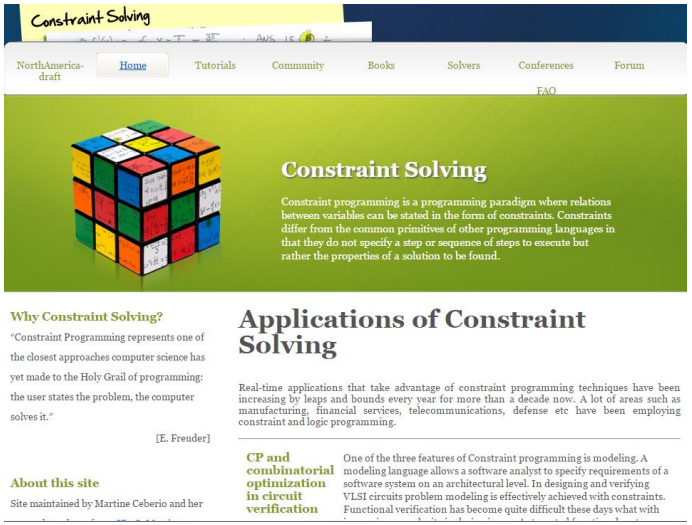
Concepts

1. Decision variables are represented by domains
2. Constraints
3. Objective function

→ **Similarities with Mathematical Programming!**

Special cases:

- **SAT Solving**
- **CP over finite domains**

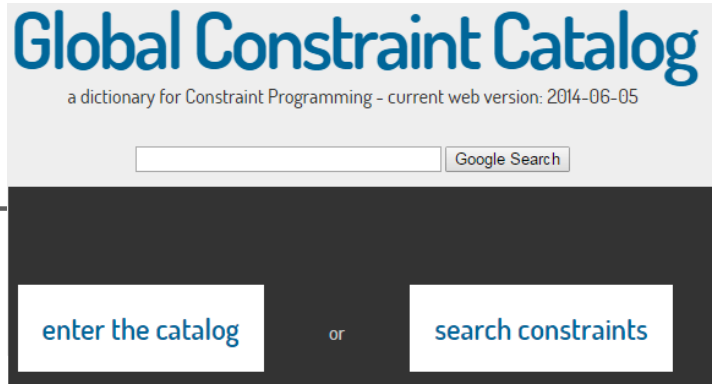


http://www.constraintsolving.com

Domains

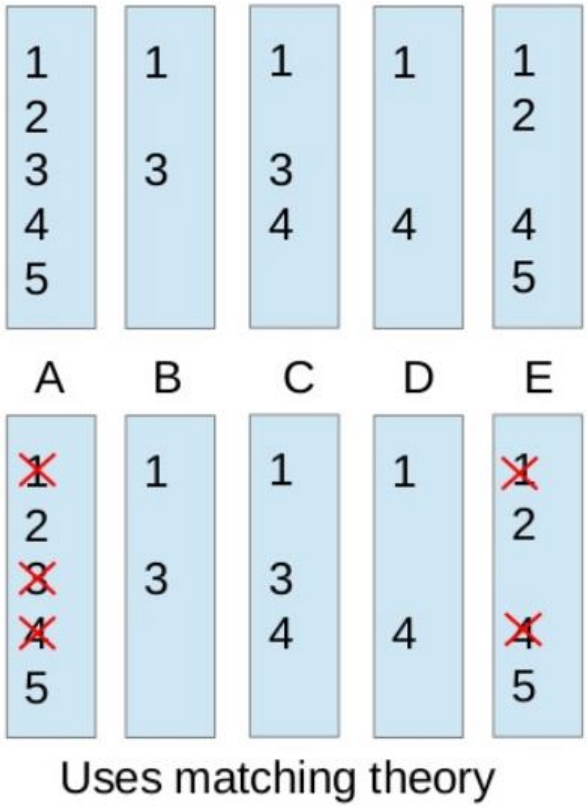
- **Domains [of variables] can be Boolean, integer, numerical.** (commonly finite domain)
- **Scheduling-oriented variables are provided** (e.g., Interval variable representing a job/task)
- **Finite domains are internally represented by intervals.** (e.g., x=[1,5]={1,2,3,4,5}) → *Leads to time-independent models!*
- **Explicit representation when "holes" appear.** (e.g., x'={1,3,4,10})

Constraints



- **Constraints link variables (domains) – not necessarily linearly.**
- **They can be defined individually…**
  (e.g., $x^2>2$)
- **and there are many pre-defined.**
  (e.g., ALL_DIFFERENT(x,x',x''))
- **A constraint needs a propagator, which identifies constraint implications on other domains.**
  (re-apply after domain change)

Constraints (cont.)



ALL_DIFFERENT(A,B,C,D,E)

Objectives

→ **Build any expression**
(e.g., use MAX/MIN, IF/THEN, POWER)

- **Access interval variable properties**
- **Some carefulness with expressions needed**

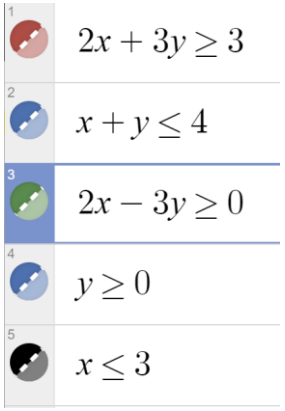
Branch & Bound

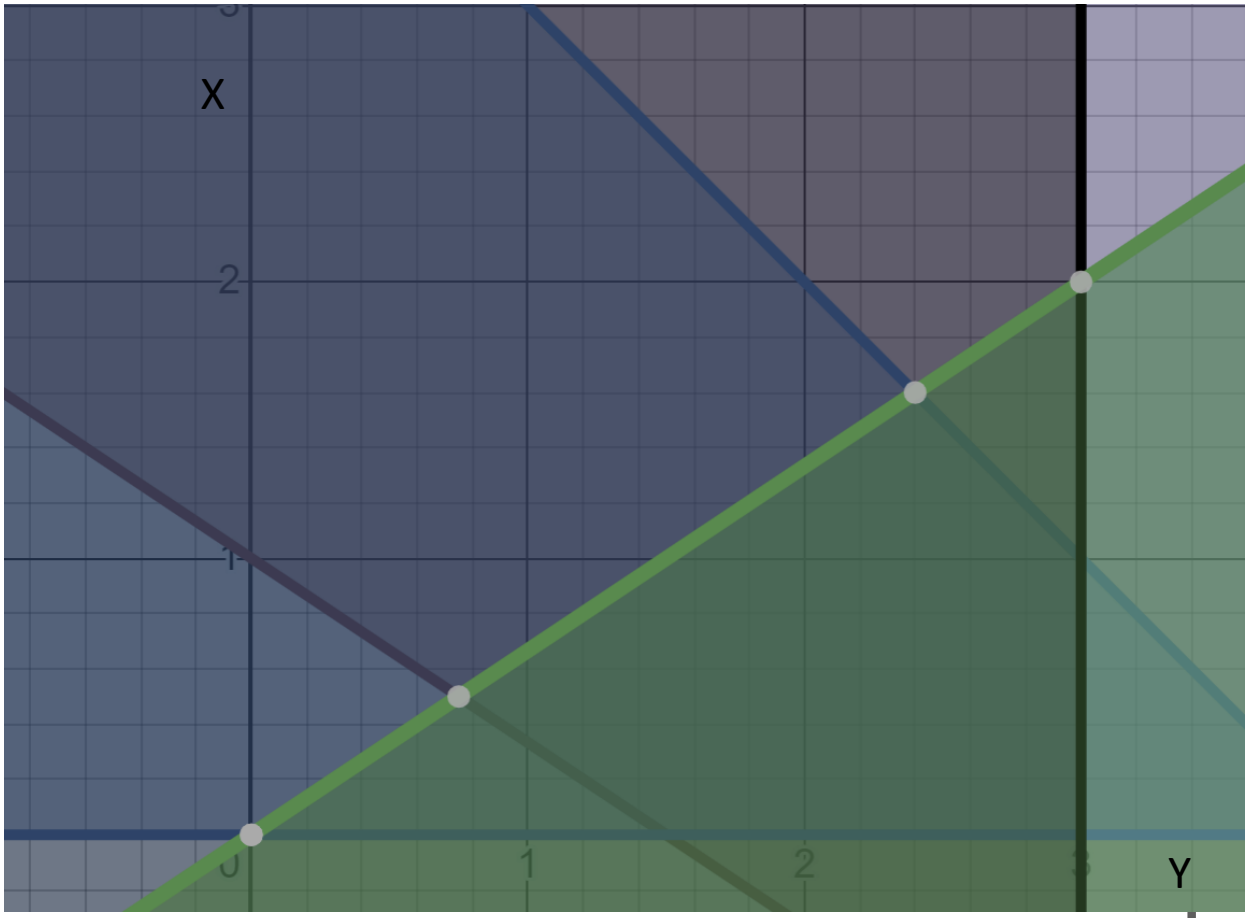
Min $2X + Y$

s.t.

$$2X + 3Y \geq 3$$
$$X + Y \leq 4$$
$$2X - 3Y \geq 0$$
$$X \leq 3$$
$$X \geq 0$$

X, Y intere





Modelling languages

FlatZinc: Standard low-level modelling language

MiniZinc: High-level modelling language

```
% Colouring Australia using nc colours
int: nc = 3;

var 1..nc: wa;   var 1..nc: nt;  var 1..nc: sa;   var 1..nc: q;
var 1..nc: nsw;  var 1..nc: v;   var 1..nc: t;

constraint wa != nt;
constraint wa != sa;
constraint nt != sa;
constraint nt != q;
constraint sa != q;
constraint sa != nsw;
constraint sa != v;
constraint q != nsw;
constraint nsw != v;
solve satisfy;

output ["wa=", show(wa), "\t nt=", show(nt),
        "\t sa=", show(sa), "\n", "q=", show(q),
        "\t nsw=", show(nsw), "\t v=", show(v), "\n",
         "t=", show(t),  "\n"];
```

Solvers

Key ingredients:

- Branch and bound
- Domain filtering (Propagation)

Also:

- Impact-based branching rules
  No-good learning
- Automated tuning
- Linear programming
- Neighborhood search
- Machine learning
- Probing

Competition: MiniZinc Challenges (yearly, 2008-today)

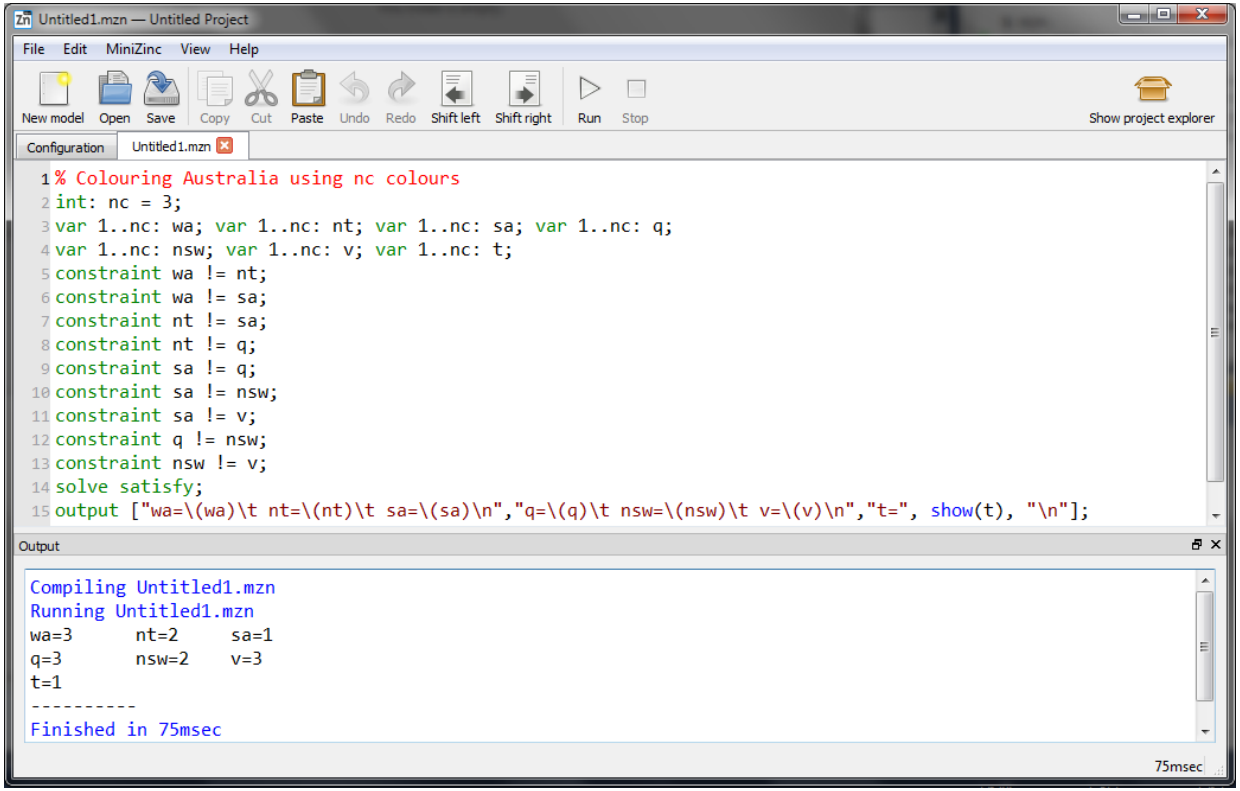
Commercial:

IBM ILOG CP Optimizer (research: free)

Free:

GECODE (www.gecode.org)
CHUFFED (SAT-hybrid)
CHOCO (java, open-source, www.choco-solver.org)
…

MiniZinc Tool



- **User interface**
- **Provides modelling language**
- **Translates model into FlatZinc**
- **Interfaces various solvers**
- **Free**

IBM ILOG CP Optimizer

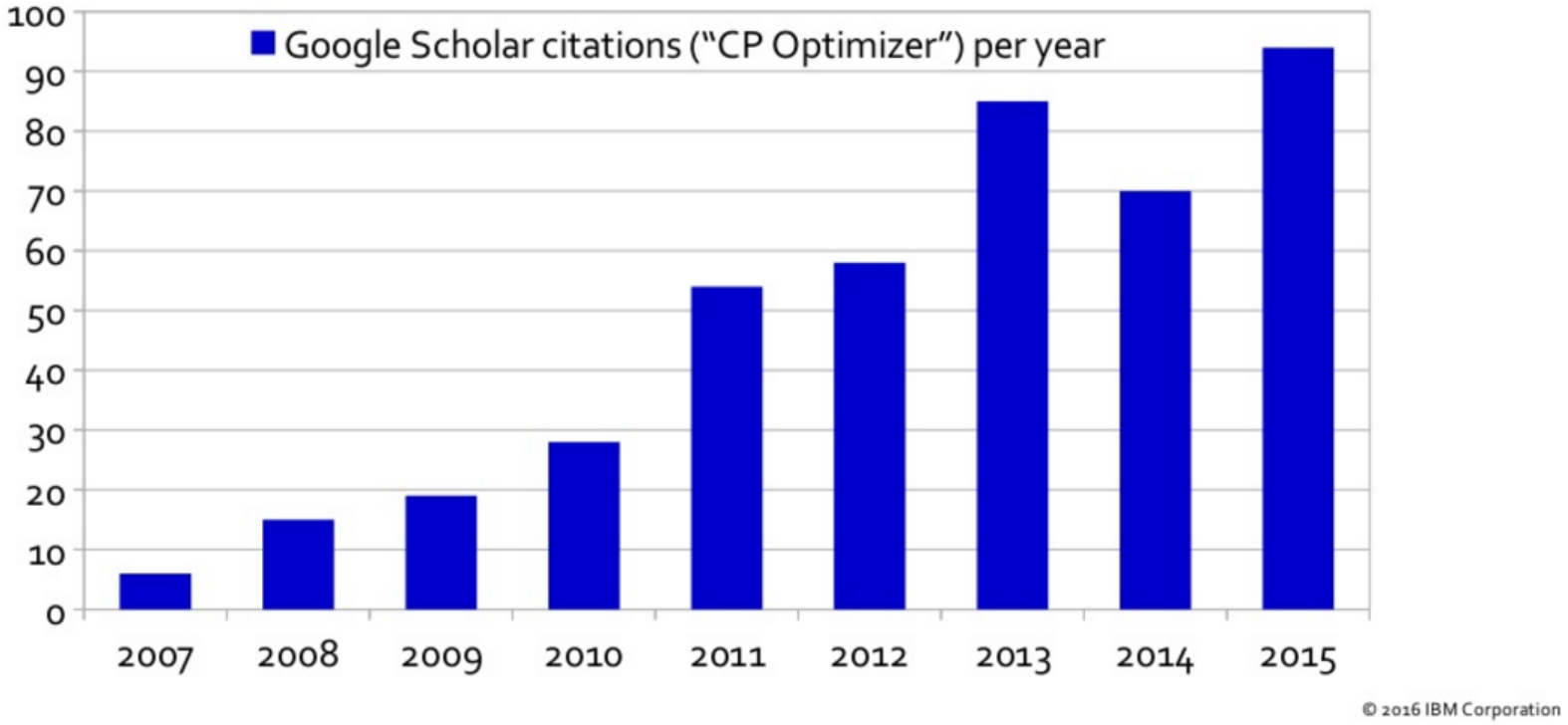
CP Optimizer model for RCPSP:

```
dvar interval a[i in Tasks] size i.pt;

cumulFunction usage[r in Resources] =
  sum (i in Tasks: i.qty[r]>0) pulse(a[i], i.qty[r]);

minimize max(i in Tasks) endOf(a[i]);
subject to {
  forall (r in Resources)
    usage[r] <= Capacity[r];
  forall (i in Tasks, j in i.succs)
    endBeforeStart(a[i], a[<j>]);
}
```

IBM

- **Interfaces:** C/C++, Java, Python, OPL (decent APIs)
- **Focus on**
  - **Integer models** (allows floating point)
  - **Scheduling**
- **Rich constraint library**
  (e.g., packing, assignments, resources, precedences, set-up costs/times)
- **Automated tuning**
- **Conflict analysis**

IBM ILOG CP Optimizer (cont.)



Successful applications

- **Scheduling**
- **Timetabling**
- **Product configuration**
- **…**

Bench index	Problem type	MRD	# Imp. UBs / # Instances
1	Trolley	−10.2%	15/15
2	Hybrid flow-shop	−11.3%	19/20
3	Job-shop w/ E/T	−6.2%	41/48
4	Air traffic management	−7.0%	1/1
5	Max. quality RCPSP	−2.7%	NA/3600
6	Flow-shop w/ E/T	−1.1%	5/12
7	RCPSP w/ E/T	−2.1%	16/60
8	Cumulative job-shop	−0.1%	15/86
9	Semiconductor testing	−0.3%	7/18
10	Single proc. tardiness	0.3%	0/20
11	Open-shop	0.3%	0/28
12	MaScLib single machine	0.6%	0/60
13	Shop w/ setup times	0.4%	3/15
14	RCPSP	1.2%	2/600
15	Air land	0.0%	0/8
16	Parallel machine w/ E/T	1.6%	4/52
17	Job-shop	1.9%	0/33
18	Flow-shop	0.9%	4/22
19	Flow-shop w/ buffers	3.9%	11/30
20	Single machine w/ E/T	7.4%	0/40
21	Aircraft assembly	8.7%	0/1
22	Common due-date	6.8%	4/20

CP: Pros

- Easy optimization **model setup** → **Model & Run**
- Modeling of **complex constraints**
- **Scheduling-oriented**
- Various strong **solvers** available
- **Efficient** for problems with complex combinatorial structure (up to a certain size)
- Finds **feasible solutions** fast
- Detects **infeasibility** fast

CP: Cons

- Only **little structural information** for hard large-scale problems
- No direct solution **quality guarantee**
- **Black box behavior**
- **Efficient problem formulation is difficult**

Some resources

- **IBM ILOG CP Optimizer for Detailed Scheduling Illustrated on Three Problems**, Philippe Laborie, 2009
- **CP Optimizer Walkthrough**, Paul Shaw, 2013
- **Modeling and Solving Scheduling Problems with CP Optimizer**, Philippe Laborie, 2014
- **An Introduction to CP Optimizer**, Philippe Laborie, 2016
- **Solver challenge:** www.minizinc.org/challenge.html
- **Global Constraint Catalogue:** http://sofdem.github.io/gccat
- www.constraintsolving.com
- **MiniZinc tool:** www.minizinc.org