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Name :

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Section :

BS Data Science

Course :

Analysis of Algorithm

Assignment : 2

Problems

1) Linear Search

```
int linearSearch(int arr[], int n, int target){  
    for(int i=0; i<n; i++){  
        if(arr[i] == target){  
            return i;  
        }  
    }  
    return -1; }
```

Explanation:

A single loop run from 0 to n-1.

Each iteration perform 1 comparison

Worst Case: All n elements checked

Best Case: find at index 1

Calculation:

$$T(n) = n$$

$$O(n)$$

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2) Binary Search

```
int binarySearch(int arr[], int left, int right, int target) {
```

```
    while (left <= right) {  
        int mid = left + (right - left) / 2;  
        if (arr[mid] == target) return mid;  
        if (arr[mid] < target) left = mid + 1;  
        else right = mid - 1;  
    }  
    return -1; }
```

Explanation:

Each iteration half the arr size

Max step = $\log n$

Each step does constant comparison

Best case: forced at first. mid.

Calculation:

Interval after k step = $n/2^k$

$$n/2^k \leq 1$$

$$T(n) = \lceil \log n \rceil$$

$$O(\log n)$$

3) Bubble Sort:

```
void bubbleSort(int arr[], int n) {  
    for (int i=0; i<n-1; i++) {  
        for (int j=0; j<n-i-1; j++) {  
            if (arr[j] > arr[j+1]) {  
                std::swap(arr[j], arr[j+1]);  
            }  
        }  
    }  
}
```

Explanation:

Two nested loops

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$$\text{Outer loop} = 1 + 2 + \dots + (n-1)$$

$$(n-1)n/2 = (n^2-n)/2$$

$$T(n) = n^2/2$$

$O(n^2)$

4) Selection Sort:

```
void selectionSort(int arr[], int n){  
    for (int i=0; i<n-1; i++) {  
        int minIndex = i;  
        for (int j=1; j<n; j++) {  
            if (arr[j] < arr[minIndex]) {  
                minIndex = j; } }  
        std::swap(arr[j], arr[minIndex]); } }
```

Explanation:

Outer loop runs $n-1$ times

Inner loop compare elements to find min.

Total comparisons $\approx (n-1)+(n-2)+\dots+1$

Swap = $n-1$ (negligible)

Calculation:

$$\text{Total comparison} = (n-1)n/2 = (n^2-n)/2$$

$$T(n) = n^2/2$$

$O(n^2)$

5) Matrix Multiplication:

```
void matrixMultiplication(int A[ ][N], int I[ ][N],  
                           int C[ ][N], int n) {
```

```
for (int i=0; i<n; i++) {
```

```
    for (int j=0; j<n; j++) {
```

```
        C[i][j] = 0;
```

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```
for (int k = 0; k < n; k++) {  
    C[i][j] += A[i][k] * B[k][j]; }  
}
```

Explanation:

Three nested loops so growth is cubic
each runs n times.

work per step = 1 multiplication + 1 add

Calculation:

$$\text{Iteration} = n \times n \times n = n^3$$

$$\text{Multiplication} = n^3$$

$$\text{Addition} = n^3 - n^2$$

$$T(n) = 2n^3$$

$$O(n^3)$$

1) Fibonacci (Recursive)

```
int fibonacci(int n) {  
    if (n <= 1) return n;  
    return fibonacci(n-1) + fibonacci(n-2); }
```

Explanation:

Each call generates 2 further call

Tree of recursive call

Height of tree = n

Growth is exponential

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Calculation:

Recurrence: $T(n) = T(n-1) + T(n-2) + 1$

Solution $\approx T(n) \approx \phi^n$, $\phi \approx 1.618$

upper bound: $O(2^n)$

$O(2^n)$

7) Fibonacci (Iterative)

```
int fibonacci (int n) {  
    if (n <= 1) return n;  
    int a = 0, b = 1, c;  
    for (int i = 2; i < n; i++) {  
        c = a + b;  
        a = b;  
        b = c;  
    }  
    return b;  
}
```

Explanation:

Single loop from 2 to n

Each iteration does constant work

Linear growth with n

Same cost in best/worst cases.

Calculation:

Iteration = n - 1

work per iteration = constant

$T(n) = n - 1$

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$O(n)$

1) Factorial (Recursive)

```
int factorial (int n): {  
    if (n == 0) return 1;  
    return n * factorial (n-1); }
```

Explanation :

each call reduces n by 1

Total calls = $n+1$ (including base)

each call does constant work
linear growth

Calculation :

$$\text{Recurrence: } T(n) = T(n-1) + 1$$

$$T(n) = n + 1$$

$O(n)$

2) Nested Loops

```
void nestedLoops(int n){  
    for (int i=0; i<n; i++) {  
        for (int j=0; j<n; j++) {  
            std::cout << i << " " << j << std::endl;  
        } } }
```

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Explanation:

Outer loop runs n times.

Inner loop also runs n times

Total iterations = $n \times n$

Growth is quadratic

Calculation:

Iteration = $n \times n = n^2$

Each iteration = constant point if

$$T(n) = n^2$$

$O(n^2)$

10) Recursive Power Function

```
int power(int x, int n) {  
    if (n == 0) return 1;  
    return x * power(x, n-1); }
```

Explanation:

Each call decreases n by 1

Total calls = $n+1$ (including base)

Each call does constant multiplication

linear growth

Calculation:

Recurrence: $T(n) = T(n-1) + 1$

$$T(n) = n+1$$

$O(n)$