CS2333 - Summer 2025

Assignment # 1

Due: Wednesday, May 14, by 11:00 pm

Submission Instructions:

- Your answers should be submitted through the assignment dropbox on Desire2Learn. Please either submit as a single file or with one file per question.

  If you are submitting in a single file, name it `CS2333-A1.`$\langle extn \rangle$. Your answers should be in the same order as the questions.

  If you are submitting with one file per question, your files should be named `CS2333-A1Q1.`$\langle extn \rangle$, `CS2333-A1Q2.`$\langle extn \rangle$, and so on for each of the questions.

  In both cases, $\langle extn \rangle$ should be the appropriate extension for your file type. D2L supports a variety of file types for submission and marker annotation, including `.pdf`, `.docx`, `.png`, and `.jpg`.

  Contact your instructor if you have any questions.

- All answers you submit must be your own work. You may discuss general approaches to assignment problems with your classmates. However, these must be general and cannot include things such as detailed steps of an algorithm or a proof. Please see the course syllabus for more details.

- Late assignment submissions will be considered only for medical reasons or in other exceptional circumstances, and normally only if the instructor is contacted before the assignment deadline.

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1. **(6 marks)** For each of the functions below, indicate whether the function is:

   (i) one-to-one and onto

   (ii) one-to-one, but not onto

   (iii) onto, but not one-to-one

   (iv) neither one-to-one nor onto

   You do not have to provide any proofs or explanations.

   (a) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = \frac{x}{10} + 3$

   For example, $f(26) = \frac{26}{10} + 3 = 2.6 + 3 = 5.6$

   (b) Let $g : \{0,1\}^* \to \{0,1\}^*$ be the function defined by $g(x) = x11$ (the concatenation of x and 11)

   For example, $g(00) = 0011$, $g(1010) = 101011$, and $g(\varepsilon) = 11$

   (c) Let $S = \{\mathtt{a}, \mathtt{b}\}^*$ and let $T = \{\mathtt{a}^i\mathtt{b}^j \ : \ i, j \in \mathbb{Z}^{nonneg}\}$

   Let $h : S \to T$ be the function defined by $h(w) = \mathtt{a}^{|w|_\mathtt{a}}\ \mathtt{b}^{|w|_\mathtt{b}}$

   For example, $h(\mathtt{babaa}) = \mathtt{aaabb}$, $h(\mathtt{bba}) = \mathtt{abb}$, $h(\mathtt{aaa}) = \mathtt{aaa}$, and $h(\varepsilon) = \varepsilon$

2. **(6 marks)** Consider the relation R defined on the set $\{0,1\}^*$ as follows:

$$R = \{(x, y) \ : \ x, y \in \{0,1\}^*, \ |x| > |y| - 2\}$$

For example, the pair $(010101, 111)$ is in R because $|010101| = 6$, $|111| = 3$, and 6 is greater than $3 - 2$.

(a) Is R reflexive? Write a short proof (one or two sentences) to show that it is or isn't.
(b) Is R symmetric? Write a short proof (one or two sentences) to show that it is or isn't.
(c) Is R transitive? Write a short proof (one or two sentences) to show that it is or isn't.

3. **(3 marks)** Consider the relation R defined on the set $\mathbb{Z}$ as follows:

$$R = \{(m, n) \ : \ m, n \in \mathbb{Z}, \ mn \geq 0\}$$

For example, the pair $(8, 3)$ is in R because $(8)(3) = 24$, which is greater than or equal to 0.

(a) Is R reflexive? Just answer 'yes' or 'no'; you do not have to provide a proof.
(b) Is R symmetric? Just answer 'yes' or 'no'; you do not have to provide a proof.
(c) Is R transitive? Just answer 'yes' or 'no'; you do not have to provide a proof.

4. **(6 marks)** Consider the following actual exchanges with ChatGPT. In some cases, the answers might be correct, but in other cases, there might be one or more errors. For each one, you should either say "This answer is correct" or you should point out any errors that have been made (for example, "The relation actually is reflexive" or "The relation actually isn't symmetric").

   (a)

   You: Give me an example of a relation that is reflexive and transitive, but not symmetric.

   ChatGPT: Sure, here's an example:

   Consider the relation "is a parent of" on the set of people. This relation is reflexive because everyone is a parent of themselves (every person is their own parent). It is also transitive because if person A is a parent of person B, and person B is a parent of person C, then person A is also a parent of person C.

   However, this relation is not symmetric because if person A is a parent of person B, it doesn't necessarily mean that person B is a parent of person A. For example, if Alice is the parent of Bob, it doesn't mean that Bob is the parent of Alice.

   (b)

   You: Give an example of a relation that is transitive, but neither reflexive nor symmetric.

   ChatGPT: Certainly! Here's an example:

   Consider the relation "is an ancestor of" on the set of people.

   Transitive: This relation is transitive because if person A is an ancestor of person B, and person B is an ancestor of person C, then person A is also an ancestor of person C.

   Not reflexive: This relation is not reflexive because a person is not considered their own ancestor.

   Not symmetric: This relation is not symmetric because if person A is an ancestor of person B, it doesn't necessarily mean that person B is an ancestor of person A. For example, if Alice is an ancestor of Bob, it doesn't imply that Bob is an ancestor of Alice.

   (c)

   You: Give me an example of a relation that is reflexive, but neither symmetric nor transitive.

   ChatGPT: Sure, here's an example:

   Consider the relation "is taller than" on the set of people.

   Reflexive: This relation is reflexive because everyone is taller than themselves.

   Not symmetric: This relation is not symmetric because if person A is taller than person B, it doesn't necessarily mean that person B is taller than person A. For example, if Alice is taller than Bob, it doesn't mean that Bob is taller than Alice.

   Not transitive: This relation is also not transitive because even if person A is taller than person B, and person B is taller than person C, it doesn't guarantee that person A is taller than person C. For instance, Alice might be taller than Bob, and Bob might be taller than Charlie, but Alice might not be taller than Charlie; they could be the same height or Alice could be shorter than Charlie.

5. **(4 marks)**
   Let T be the set $\{w \in \{0,1\}^* \ : \ |w| = 4\}$.
   Let R be the equivalence relation defined on T as follows:

   $$R = \{(x,y) \ : \ x \in T, \ y \in T, \ x \text{ and } y \text{ have the same first symbol as each other, and } x \text{ and } y \text{ have the same last symbol as each other}\}$$

   As discussed in class, any equivalence relation will divide the underlying set (in this case, T) into subsets called equivalence classes.
   Every element in the set will appear in exactly one equivalence class and will be related to all elements in its class and not related to any elements outside of its class.

   What are the equivalence classes of T induced by the relation R?
   (For each equivalence class, list all elements that are in that class.)

   [Note: This topic is not covered in the book, but it was discussed in class.]

6. **(4 marks)** Let $A = \{0,1,2,3,4,5,6,7,8,9,10,11\}$.
   Let R be the equivalence relation defined on A as follows:

   $$R = \{(m,n) \ : \ m \in A, \ n \in A, \ m^2 - n^2 \text{ is a multiple of } 5\}$$

   [For example, $(8,2)$ is in R because $8^2 - 2^2 = 64 - 4 = 60$, which is a multiple of 5.]

   What are the equivalence classes of A induced by the relation R?
   (For each equivalence class, list all elements that are in that class.)

   [Hint: Make a list for yourself of all elements in A and their squares. Use that list to help you to determine which elements are related to each other according to relation R.]

7. **(8 marks)** Based on our discussion of strings and languages in class, list <u>all</u> of the strings in each of the languages below. A first example is provided for you.

   - $L = \{0^m 1^n \ : \ m,n \in \mathbb{Z}^{nonneg}, \ m+n = 5\}$
   - Answer: $L = \{00000, 00001, 00011, 00111, 01111, 11111\}$

   (a) $L_1 = \{w \in \{0,1\}^* \ : \ |w| \leq 4, \ |w|_1 \text{ is odd}\}$

   (Reminder: $|w|_1$ represents the number of ones in string w.)

   (b) $L_2 = \{w \in \{\texttt{a},\texttt{b},\texttt{c}\}^* \ : \ w \text{ is a substring of } \texttt{abac}\}$ (Only list each string in L_2 once.)

   (c) $L_3 = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k \ : \ i,j,k \in \mathbb{Z}^{nonneg}, \ 0 \leq i \leq j \leq k \leq 2\}$

   (d) $L_4 = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k \ : \ i,j,k \in \mathbb{Z}^{nonneg}, \ 3 \leq i+j+k \leq 5, \ i = jk\}$

8. **(20 marks)**

   For each of the following languages, draw the state diagram of a deterministic finite automaton that accepts the language.

   (**Note:** This means that your finite automaton should accept every string in the language and should not accept any strings that are not in the language.)

   (**Reminder:** *Deterministic* means that there is exactly one transition defined for each input symbol in each state.)

   (a) $L_1 = \{w \in \{0,1\}^* \ : \ |w| = 5k + 2 \text{ for some } k \in \mathbb{Z}\}$

   (b) $L_2 = \{01, 10, 101\}$

   (c) $L_3 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \ : \ w$ does not have any occurrences of the same symbol appearing twice (or more) in a row$\}$

   (d) $L_4 = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \ : \ i, j, k \in \mathbb{Z}^{nonneg}, \ i \geq 2, \ j \geq 3, \ k \geq 0\}$

   (e) $L_5 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \ : \ |w|_{\mathtt{a}} = 3k$ for some integer k, or $|w|_{\mathtt{b}}$ is odd$\}$

   (**Note:** This is an inclusive-or. We want to accept strings that satisfy one or both of the conditions listed.)

9. **(5 marks)** Given the formal finite automaton description below:

   - $Q = \{W, X, Y, Z\}$
   - $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$
   - $q_0 = W$
   - $F = \{W, X, Y\}$
   - $\delta : Q \times \Sigma \to Q$ is defined in the following table:

	a	b	c
W	X	W	Z
X	Y	X	Z
Y	Z	Y	Z
Z	Z	Z	Z

   (a) Draw the corresponding state diagram.

   (b) Identify the language that is accepted by this finite automaton.

10. **(3 marks)** For the finite automaton below, identify the language accepted by the FA. Try to describe the language in the simplest terms possible.