CS-2333
Danyal Khan
Assignment 1

QUESTION 1:-

a-) Let $f: \mathbb{R} \to \mathbb{R}$ be the f defined by $f(x) = \frac{x}{10} + 3$

(i) one-to-one and onto

b-) Let $g: \{0,1\}^* \to \{0,1\}^*$ be the function defined by
$g(x) = x11$

(ii) one-to-one, but not onto

c-) Let $S = \{a, b\}^*$ and let $T = \{a^i b^j : i, j \in \mathbb{Z}^{nonneg}\}$
$h: S \to T \quad h(w) = a^{|w|_a} b^{|w|_b}$

(iii) onto, but not one-to-one

QUESTION 2:-

$$R = \{(x,y) : x, y \in \{0,1\}^*, |x| > |y| - 2\}$$

a-) Yes, R is reflective because for any $a \in \{0,1\}^*$, we have $|a| > |a| - 2$, so $(a,a) \in R$.

b-) No, R is not symmetric because there exists strings x and y such that $(x,y) \in R$ but $(y,x) \notin R$.
Example: $(000, 0) \in R$ but $(0, 000) \notin R$.

c-) No, R is not transitive because when
$|x| = 5 \quad |y| = 6 \quad |z| = 7$
$(x,y) \in R$ because $|x| = 5 > 6 - 2$
$(y,z) \in R$ because $|y| = 6 > 7 - 2$
But $(x,z) \notin R$ because $|x| = 5 \not> 7 - 2$

Since $(x,y) \in R$ and $(y,z) \in R$, but $(x,z) \notin R$, the relation is not transitive.

QUESTION 3: $R = \{ (m, n) : m, n \in \mathbb{Z},$

a-) Yes

b-) Yes

c-) No

QUESTION 4: a-) This relation is **not** reflexive because no one is a parent of themselves. It is also **not** transitive because if person A is a parent of person B, and person B is a parent of person C, the A is a grandparent not a parent of C.

b-) This answer is correct.

c-) • This relation is **not** reflexive because everyone is **not** taller than themselves.

• This relation **is** transitive because if A is taller than B and B is taller than C. Then A is definitely taller than C.

QUESTION 5:

Class (0,0) = { 0000, 0010, 0100, 0110 }
Class (0,1) = { 0011, 0101, 0001, 0111 }
Class (1,0) = { 1000, 1010, 1100, 1110 }
Class (1,1) = { 1001, 1101, 1011, 1111 }

QUESTION 6:

Class 1 = { 0, 5, 10 }

Class 2 = { 1, 4, 6, 9, 11 }

Class 3 = { 2, 3, 7, 8 }

0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121

QUESTION 7: a-) $L_1 = \{ w \in \{0,1\}^* : |w| \leq 4, |w|_1 \text{ is odd} \}$

$L = \{ 1, 01, 10, 001, 100, 010, 111, 0001, 0010, 0100, 1000, 0111, 1110, 1011, 1101 \}$

b-) $L_2 = \{ w \in \{a,b,c\}^* : w \text{ is a substring of } abac \}$

$L = \{ \varepsilon, a, b, c, ab, ba, ac, aba, bac, abac \}$

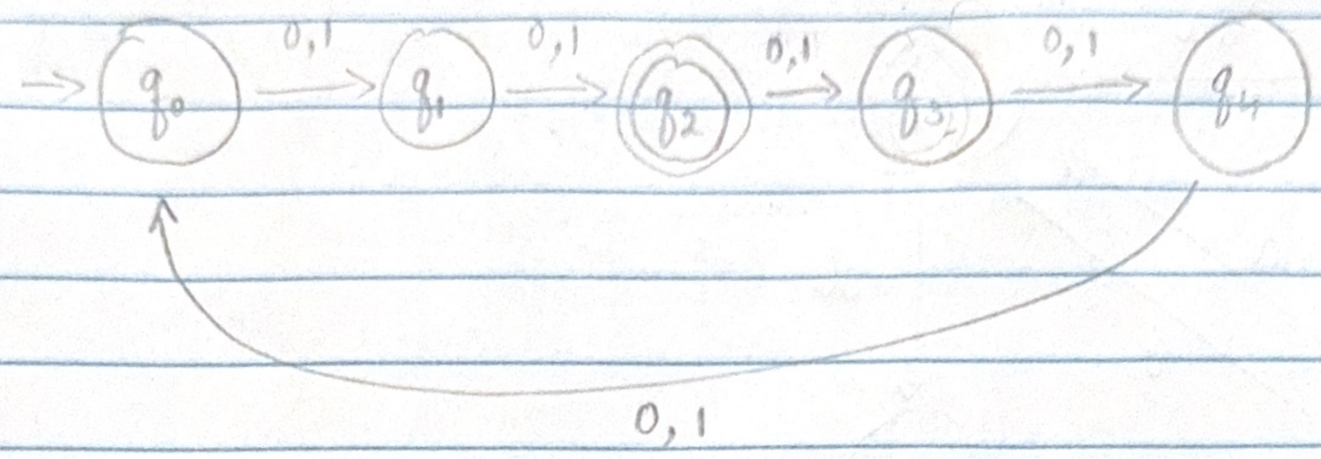
c-) $L_3 = \{ a^i b^j c^k : i,j,k \in \mathbb{Z}^{nonneg}, 0 \leq i \leq j \leq k \leq 2 \}$

$L = \{ \varepsilon, c, bc, abc, cc, bbcc, aabbcc, bcc, abcc, abbcc \}$

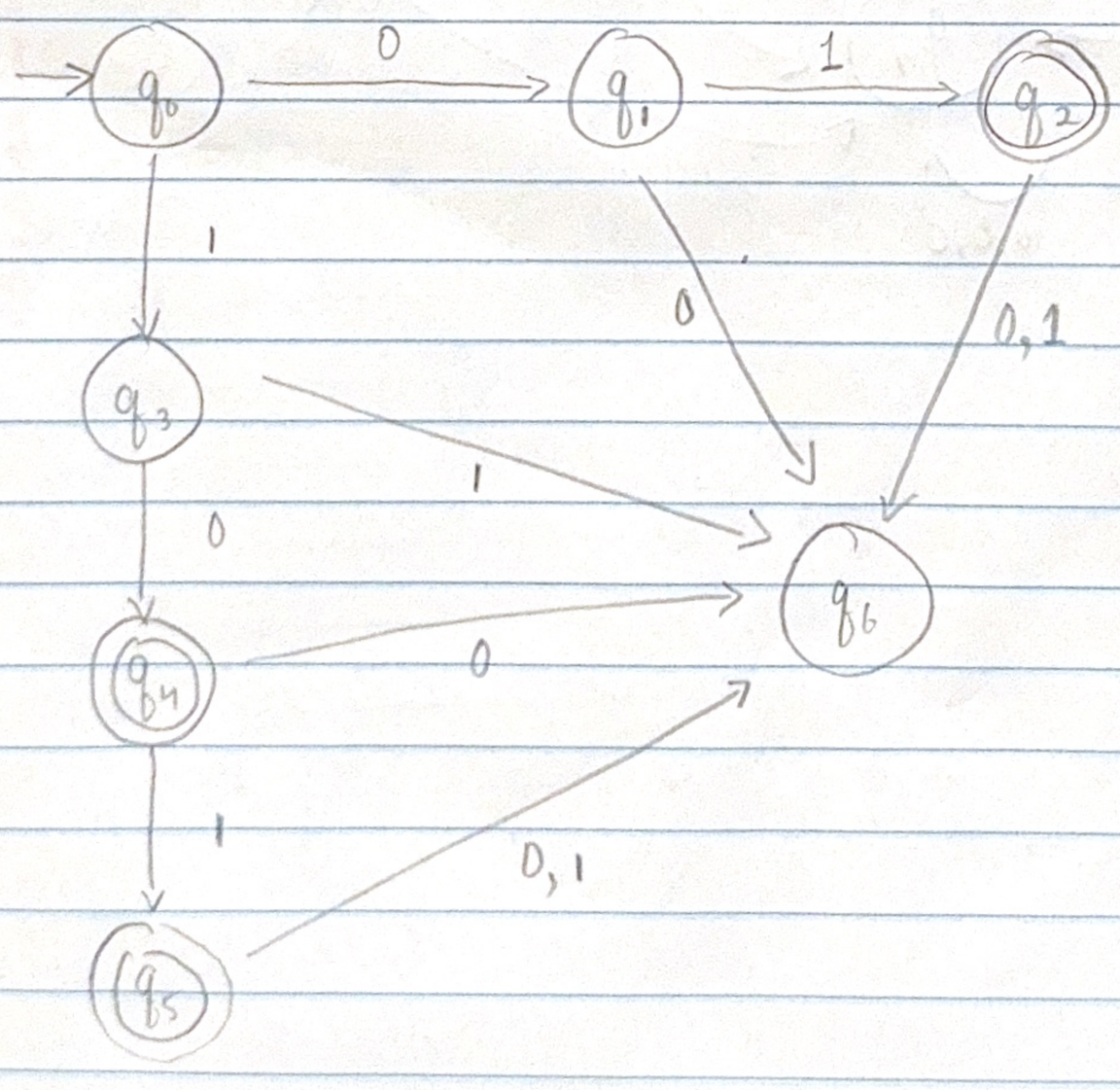
d-) $L_4 = \{ a^i b^j c^k : i,j,k \in \mathbb{Z}^{nonneg}, 3 \leq i+j+k \leq 5, i = jk \}$

$L = \{ abc, aabcc, aabbc, ccc, cccc, ccccc, bbb, bbbb, bbbbb \}$

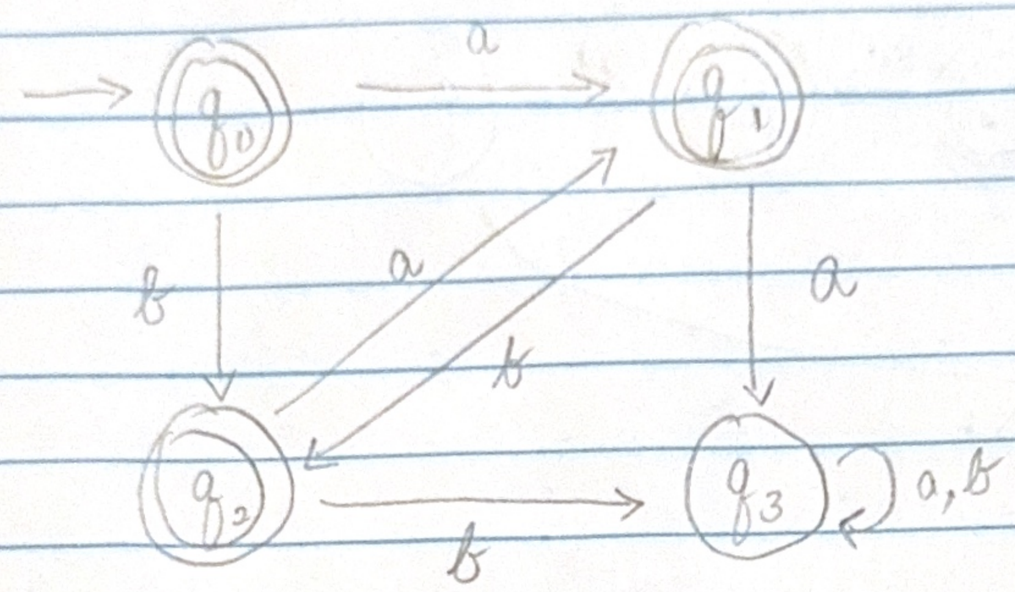
QUESTION 8: a.) $L_1 = \{ w \in \{0,1\}^* : |w| = 5k+2 \}$



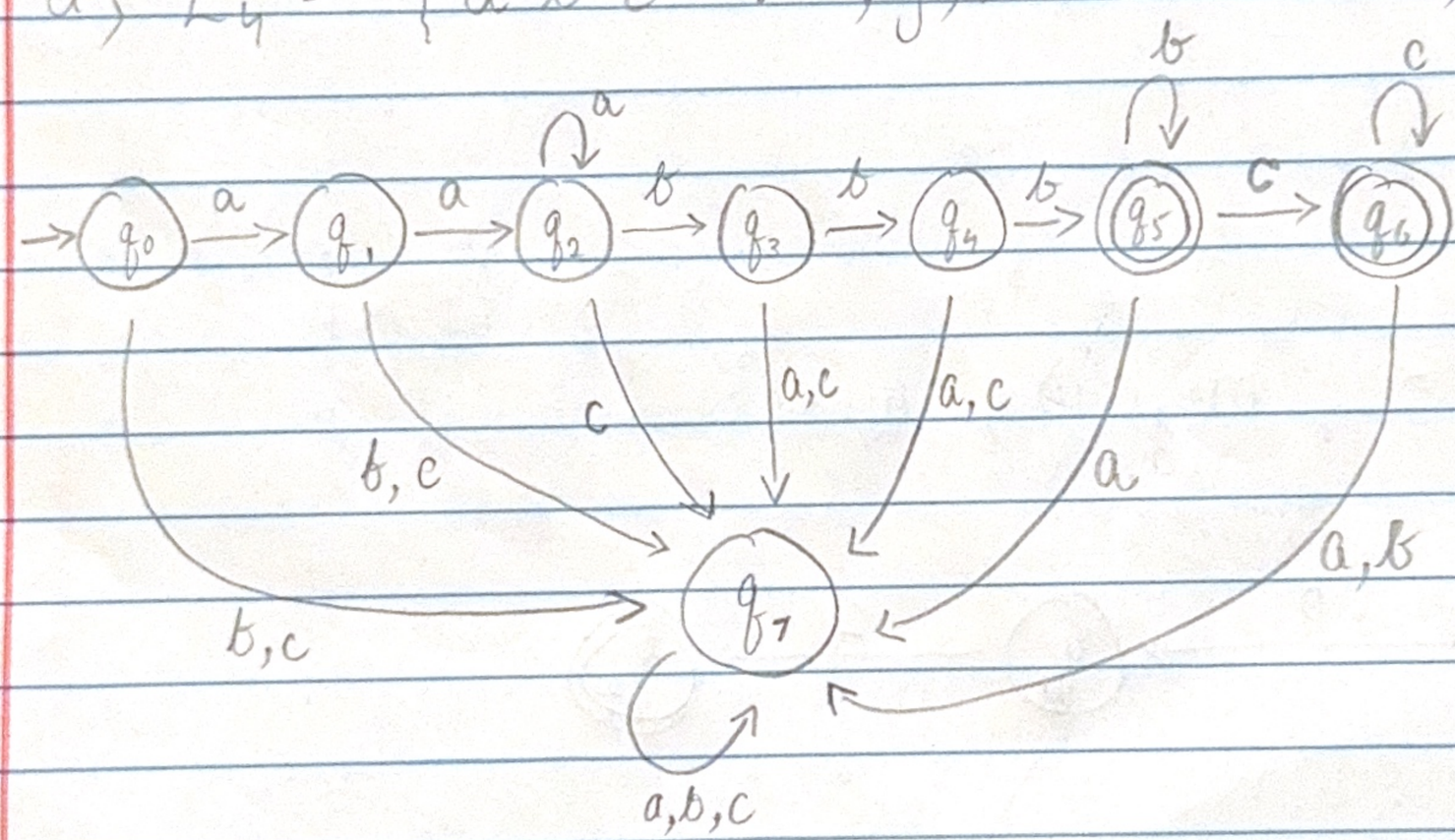
b.) $L_2 = \{ 01, 10, 101 \}$



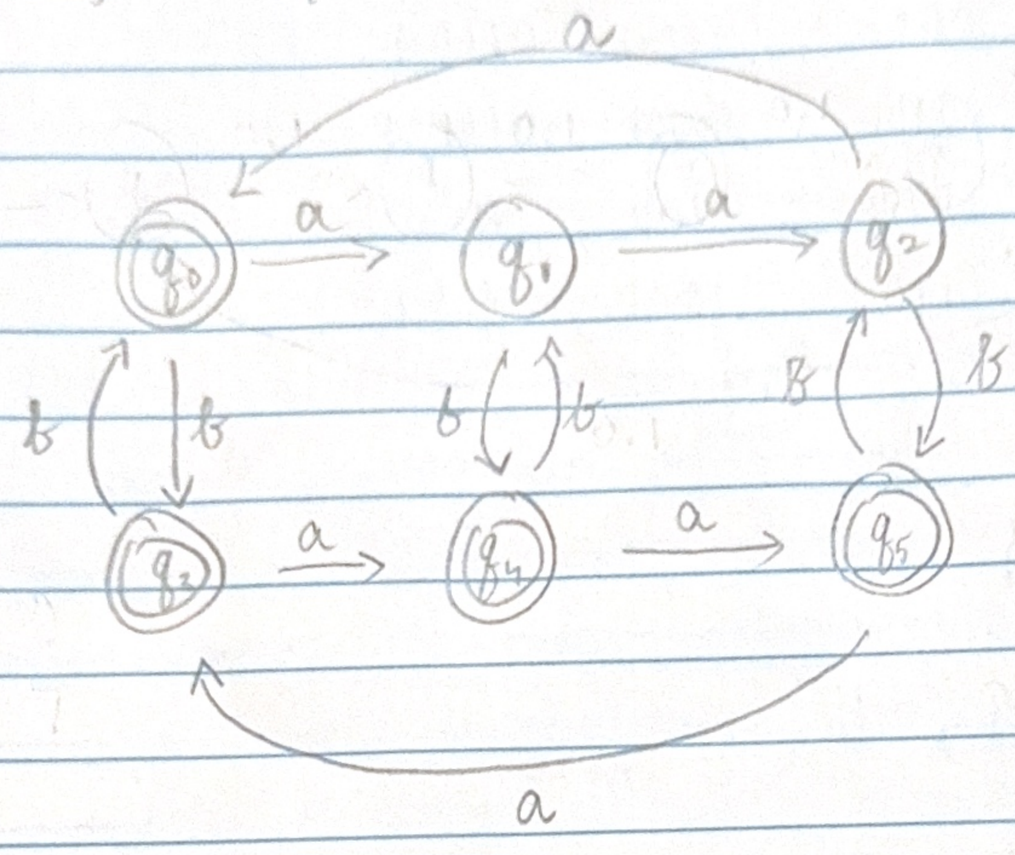
c.) $L_3 = \{ w \in \{a, b\}^* :$ w does not have any occurrences of the same symbol appearing twice or more in a row $\}$



d.) $L_4 = \{ a^i b^j c^k : i, j, k \in \mathbb{Z}^{nonneg}, i \geq 2, j \geq 3, k \geq 0 \}$



e-) $L_5 = \{ w \in \{a,b\}^* : |w|_a = 3k \text{ or } |w|_b \text{ is odd} \}$



QUESTION 9:

$Q = \{w, x, y, z\}$

$\Sigma = \{a, b, c\}$

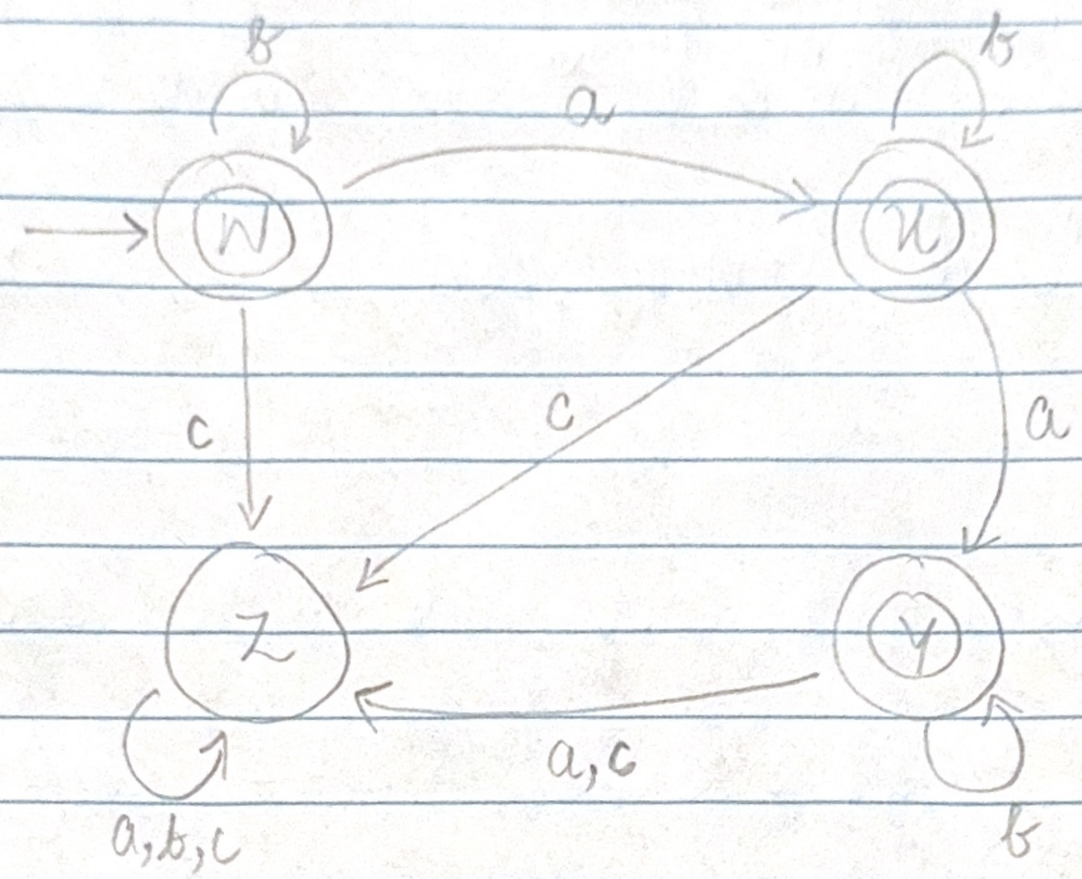
$q_0 = w$

$F = \{w, x, y\}$

$\delta : Q \times \Sigma \to Q$ is defined,

	a	b	c
w	x	w	z
x	y	x	z
y	z	y	z
z	z	z	z

a-)



B-) $L = \{w \in \{a, b, c\} : |w|_a \leq 2 \text{ and } |w|_c = 0\}$

QUESTION 10:

Every accepted string starts with two b's and ends with two or more consecutive a's, so;

$L = \{w \in \{a, b, c\} :$ starts with bb and ends with atleast two consecutive a's $\}$