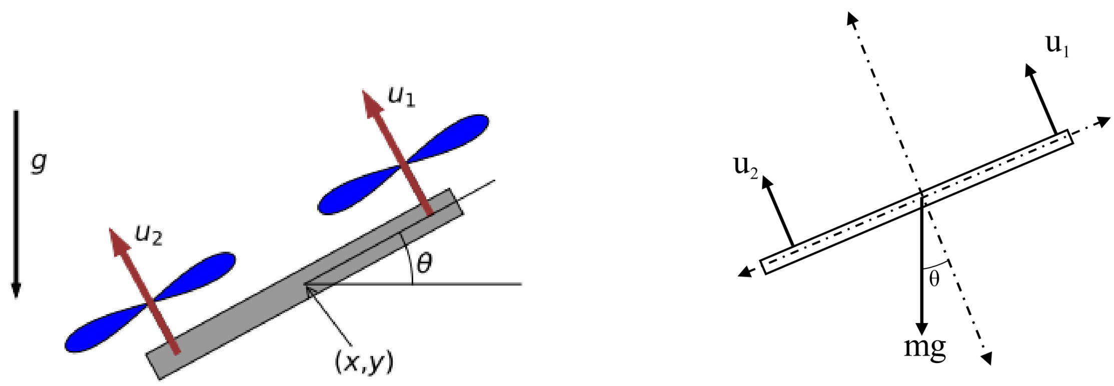
Planar Drone

The planar quadrotor is a simplified model of a quadrotor drone that operates within a two-dimensional plane, making it a valuable tool in control theory and robotics for studying dynamics, control strategies, and navigation algorithms. This model features a frame with four rotors positioned at the corners, each generating lift and capable of independent control. The quadrotor possesses two degrees of freedom in position, allowing movement along the X and Y axes, and one degree of freedom in orientation, enabling rotation about the vertical axis (yaw).



(a) Planar Drone

(b) Planar drone FBD

Figure 1: Shock Absorber System

Control inputs consist of thrust produced by each rotor, with differential thrust utilized to manage rotation and directional movement. The dynamics of the planar quadrotor can be described by equations derived from Newton's laws or Lagrangian mechanics, incorporating forces from thrust and moments from rotor speeds. These equations govern its position, velocity, orientation, and angular velocity, accounting for factors such as mass, acceleration due to gravity, and moment of inertia.

The planar quadrotor finds applications in various fields, including aerial robotics research, flight dynamics simulation, and the development of control algorithms. Its simplicity allows for easier analysis and the creation of control strategies that can be adapted for more complex three-dimensional systems, making it an essential model for understanding the principles of flight control in aerial vehicles.

The equations of motion are almost trivial, since it is only a single rigid body, and certainly fit into our standard manipulator equations:

$$m\ddot{x} = -mg\sin\theta \tag{1}$$

$$m\ddot{y} = (u_1 + u_2) - mg\cos\theta \tag{2}$$

$$I\ddot{\theta} = r(u_1 - u_2) \tag{3}$$

To express the given equations in state-space representation let $x_1 = x, \quad x_2 = \dot{x}, \quad x_3 = y, \quad x_4 = \dot{y}, \quad x_5 = \theta, \quad x_6 = \dot{\theta}$

$$\dot{x}_1 = x_2 \tag{4}$$
$$\dot{x}_2 = -g \sin x_5 \tag{5}$$
$$\dot{x}_3 = x_4 \tag{6}$$
$$\dot{x}_4 = \frac{(u_1 + u_2)}{m} - g \cos x_5 \tag{7}$$
$$\dot{x}_5 = x_6 \tag{8}$$
$$\dot{x}_6 = \frac{r}{I}(u_1 - u_2) \tag{9}$$

To linearize the given state-space representation using the small angle approximation, we assume that the angle θ or x_5 is small enough such that: $\cos x_5 \approx 1$ and $\sin x_5 \approx x_5$. Using these approximations, we can rewrite the equations as follows:

$$\dot{x}_1 = x_2 \tag{10}$$
$$\dot{x}_2 = -g x_5 \tag{11}$$
$$\dot{x}_3 = x_4 \tag{12}$$
$$\dot{x}_4 = \frac{(u_1 + u_2)}{m} - g \tag{13}$$
$$\dot{x}_5 = x_6 \tag{14}$$
$$\dot{x}_6 = \frac{r}{I}(u_1 - u_2) \tag{15}$$

The state-space equation is given by: $\dot{x} = Ax + Bu$. The matrices A and B are defined as:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & \frac{1}{m} \\ 0 & 0 \\ \frac{r}{I} & -\frac{r}{I} \end{bmatrix} u - \begin{bmatrix} 0 \\ 0 \\ 0 \\ g \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

The output equation is given by: $y = Cx + Du$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u \tag{17}$$

The transfer function of the system is

$$\begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} \frac{-gr}{Is^4} & \frac{gr}{Is^4} \\ \frac{1}{ms^2} & \frac{-1}{ms^2} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{s^2} \end{bmatrix} g \tag{18}$$