London South Bank University

Analogue Computers

Introduction to Analogue Computers and Simulation of 1st and 2nd Order Systems

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1. Abstract

1.1 Aim

The aim of this report is to show an understanding by the author of the function of Analogue Computers and their application to simulating systems characterised by first order and second order differential equations.

1.2 Objectives

1.2.1 Acquaintance of the author with the component elements of an Analogue Computer

1.2.2 Develop understanding of the use of Summer and Summer-Integrator parts elements

1.2.3 Evaluate the capability of the Analogue Computer to Simulate a First Order and Second Order System

1.3 Summary

This investigation demonstrates that the BICC-Vero Analogue Computer is a suitable device for simulating the response of First and Second Order Systems; however it is not without its limitations in this application.

Through the process of undertaking this investigation, the author has developed a greater understanding for Analogue Computers and their component elements. It has also led the author to develop a greater appreciation of modern digital computers and software, and the power they possess to perform complex engineering calculations, facilitating the engineer's job.

2. Introduction

2.1 Background

Since the dawn of astronomy in the Ancient Greek era, Scientists and Mathematicians have strived to develop models to explain and predict the complex systems governing the movement of the planets and stars. This led to the earliest theoretical models, however it was realised that hand calculating numerous iterations of theoretical models was a laborious and time consuming endeavour.

Around 200-100BC an unknown inventor developed the Antikythera mechanism, a mechanical device which modelled the movement of the planets and moon around the earth. The Antikythera mechanism is attributed to be the first Analogue Computer, whereby a set of initial conditions can be set, and any eventuality of the system predicted with respect to a known variable.

Mechanical Analogue Computers were used for centuries to calculate and model numerous problems, until the 1930's when the development of suitable electrical components made the Electronic Digital Computer a possibility. During the Second World War the electronic computer became a vital tool for many purposes including code breaking, gun trajectory data computing and aircraft and ship control systems.

During the 1960's and 70's the Analogue computer was superseded by the advent of the Digital Computer, made possible the commercialisation of semiconductor transistors and advent of the microchip. Electronic Analogue Computers were still used up until the late 1980's for modelling the output of complex engineering systems characterised by differential equations with constant coefficients as they maintained speed advantages over numerical methods used by Digital Computers.

Many of the protocols and conventions of Analogue Computing were carried over and form the basis of Modern Digital Computing.

The BICC-Vero Analogue Computer *(see Figure 2.1.1)* was developed as an educational tool in the 1970's to teach students the fundamentals of computing, a then fledgling field.

2.2 Elements of Electronic Analogue Computing

2.2.1 The Operational Amplifier

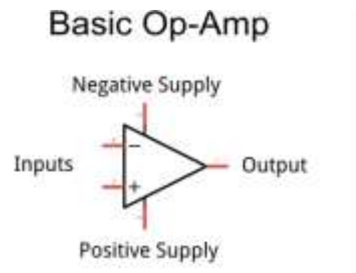

Figure 2.2.1 Schematic Diagram of a Basic Op-Amp

The Basic Operational Amplifier or Op-Amp is a "general-purpose, DC-coupled, high gain, inverting feedback amplifier" (Jung, W, 2006), it multiplies the input potential difference by the supply potential difference, outputting an amplified signal proportional to the input signal. In this way it can be used to perform mathematical operations.

A Basic Op-Amp can be used in conjunction with other electrical components to develop other types of Op-Amp that can be used to perform a variety of mathematical operations. Combining these Op-Amps in a Mathematical Analyser or Computer creates a powerful machine capable of performing calculations and modelling systems in minutes that would take many hundreds of hours by hand.

The BICC-Vero contains Op-Amps that can perform Summing, Multiplying, Differentiating and Integrating functions. For brevity this report will only present explanations of those Op-Amps concerned with the investigation.

2.2.2 The Inverting Summer

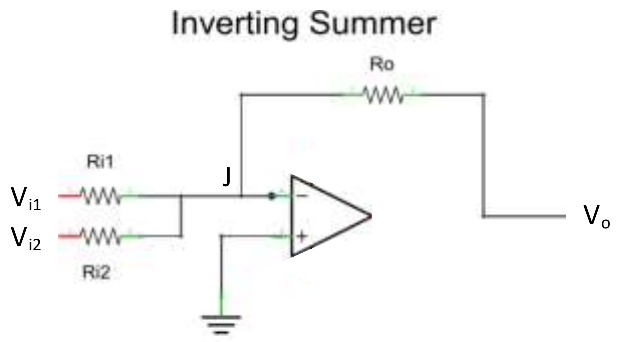

Figure 2.2.2 Schematic Diagram of an Inverting Summer Op-Amp

The following equations refer to *Figure 2.2.2*:

$$V = IR \qquad \textit{Equation 2.2.1}$$

$$and \quad I_o = I_{i1} + I_{i2} \qquad \textit{Equation 2.2.2}$$

$$\frac{-V_o}{R_o} = \frac{V_{i1}}{R_{i1}} + \frac{V_{i2}}{R_{i2}} \qquad \textit{Equation 2.2.3}$$

$$if \quad R_0 = R_{i1} = R_{i2} \qquad \textit{Equation 2.2.4}$$

$$then \quad -V_o = V_{i1} + V_{i2} \qquad \textit{Equation 2.2.5}$$

The output value is the negative feedback required to balance the current at Junction ***J*** in *Figure 2.2.2* according to *Equation 2.2.2,* therefore "the output is always the sign reversed sum of the scaled inputs" *(Unknown (LSBU), 2015).* This makes the assumption that the gain of the amplifier is infinite, in reality it will be finite but very large.

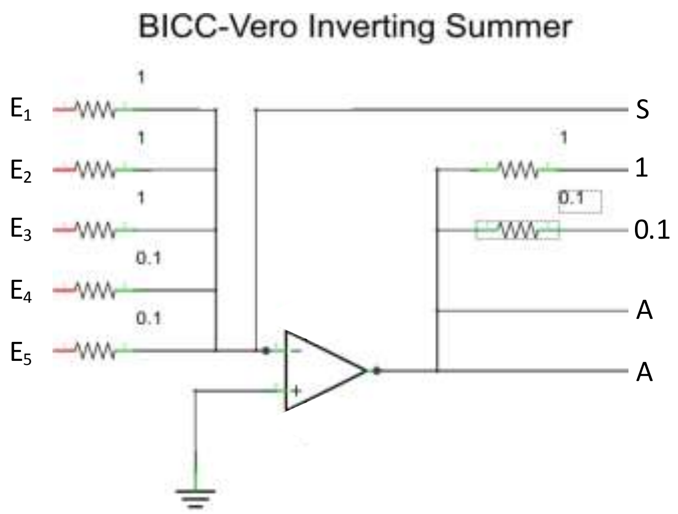

Figure 2.2.3 Schematic Diagram of a BICC-Vero Inverting Summer Op-Amp

The inverting summing amplifiers in the BICC-Vero have a number of inputs and the ability to multiply the output *S* by a Nose Gain value when an appropriate resistor is connected to the output pin. The BICC-Vero's Inverting Summers result in an output given by *Equation 2.2.6*.

$$V_o(t) = -[E_1(t) + E_2(t) + E_3(t) + 10E_4(t) + 10E_5(t)] \quad \textit{Equation 2.2.6}$$

Whereby it follows from the argument made in *Equations 2.2.1-2.2.5* that the potential difference across the input resistors results in a multiplication of inputs E_4 and E_5 by a factor of 10.

2.2.3 The Inverting Summing Integrator

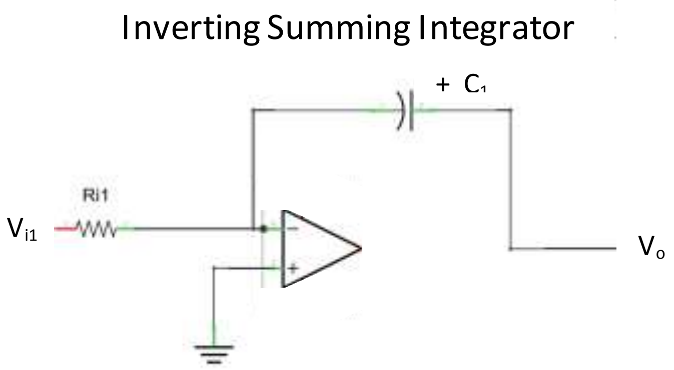

Figure 2.2.4 Schematic Diagram of an Inverting Integrator Op-Amp

The following equations refer to *Figure 2.2.2*:

$$I = -C_1 \frac{dV_o}{dt} \quad \textit{Equation 2.2.7}$$

$$as \quad V_{i1} \to 0 \quad \textit{Equation 2.2.8}$$

$$\frac{V_{i1}}{R_{i1}} = -C_1 \frac{dV_o}{dt} \quad \textit{Equation 2.2.9}$$

$$V_{i1} = -C_1 R_{i1} \frac{dV_o}{dt} \quad \textit{Equation 2.2.10}$$

$$\int V_{i1}\, dt = -C_1 R_{i1} \int \frac{dV_o}{dt}\, dt \quad \textit{Equation 2.2.11}$$

$$\int V_{i1}\, dt = -C_1 R_{i1} V_o \quad \textit{Equation 2.2.12}$$

$$-V_o = \frac{1}{C_1 R_{i1}} \int V_{i1}\, dt \quad \textit{Equation 2.2.12}$$

A summing Integrator places a capacitor in the negative feedback path. Resulting in a gain that ramps linearly at a rate dependant on the nose gain placed on the output.

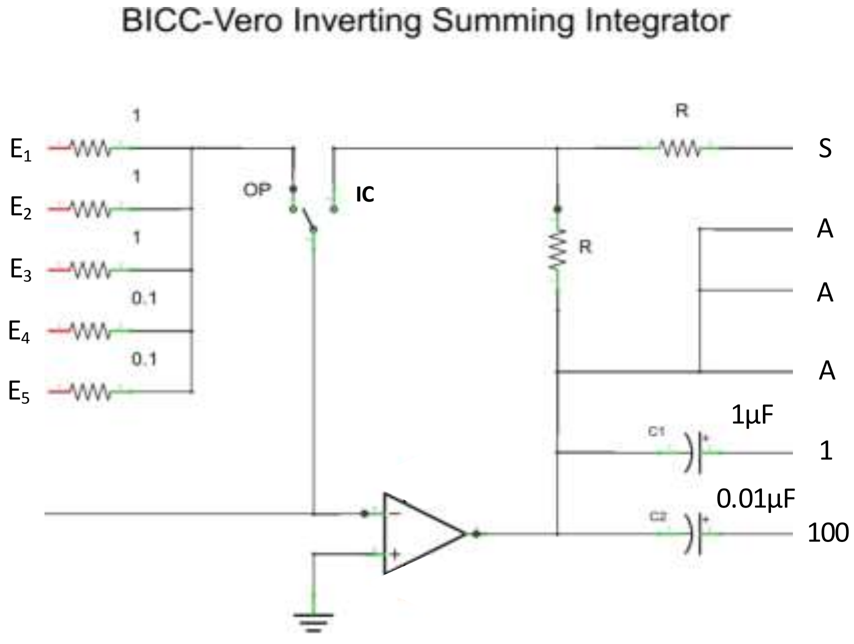

Figure 2.2.5 Schematic Diagram of a BICC-Vero Inverting Summing Integrator Op-Amp

The BICC-Vero's Inverting Summing Integrators result in an output dependent on *Equation 2.2.13*.

$$V_o(t) = -V[IC] - \frac{1}{NG}\int_0^t [E_1(t) + E_2(t) + E_3(t) + 10E_4(t) + 10E_5(t)]\,dt \qquad \textit{Equation 2.2.13}$$

Connecting an Input of 0.1 Machine Unit at E_1, (across a 1MΩ resistor), and linking the Output ***S*** to the 1µF capacitor will give a 1 second time constant , or nose gain of 1. This will cause the Integrator output to ramp up linearly at 0.1MU/s. Connecting the 0.01µF capacitor will result in the ramp rate increasing 100 fold, this provides the ability to study differing speed responses of a system by accelerating the timeframe of the test. The Integrator can also be placed in Initial Conditions ***IC*** Mode to pre-charge the capacitor to a desired initial value.

2.2.4 The Inverting Amplifier

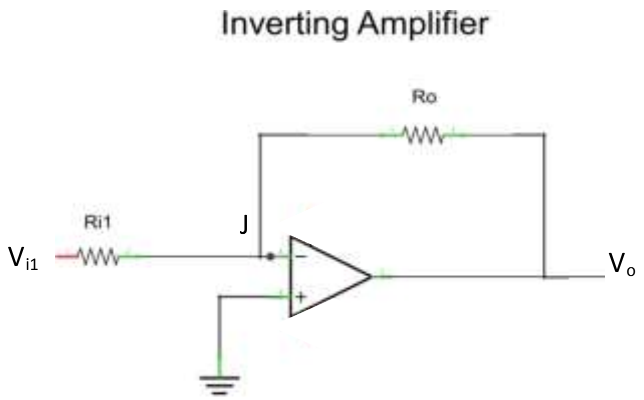

Figure 2.2.6 Schematic Diagram of an Inverting Op-Amp

The following equations refer to *Figure 2.2.6*:

$$V = IR$$ *Equation 2.2.14*

$$and \quad I_o = I_{i1}$$ *Equation 2.2.15*

$$\frac{-V_o}{R_o} = \frac{V_{i1}}{R_{i1}}$$ *Equation 2.2.16*

$$if \quad R_0 = R_{i1}$$ *Equation 2.2.17*

$$then \quad -V_o = V_{i1}$$ *Equation 2.2.18*

The Inverting Amplifier or Inverting Gain Amplifier is used to reverse the polarity of an output, especially useful when most Op-Amps invert. The output value is the negative feedback required to balance the current at Junction ***J*** in *Figure 2.2.6* according to *Equation 2.2.15,* therefore "the output is always the sign reversed sum of the scaled inputs" *(Unknown (LSBU), 2015).*

The Integrator Op-Amp from the BICC-Vero, *Figure 2.2.5* can be connected to produce an Inverting Amplifier. By connecting the Input to the Initial Conditions ***IC*** side of the Integrator at ***S*** and taking an output at ***A***.

2.2.5 The Multiplier

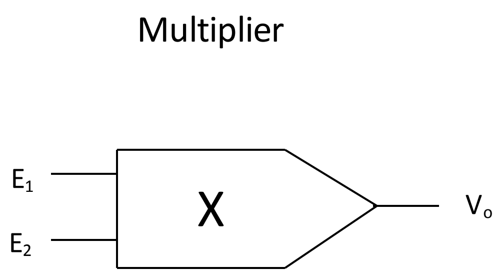

Figure 2.2.7 Schematic Diagram of a Multiplier

The following equation refers to *Figure 2.2.6*:

$$V_o = E_1E_2 = \frac{1}{4}(E_1 + E_2)^2 - (E_1 - E_2)^2$$ *Equation 2.2.19*

Multiplication is a complex function to achieve with analogue electronic components; most Analogue Computers use the multiplication technique shown in *Equation 2.2.19*, called parabolic multiplication using a number of time based diodes and an Op-Amp.

Connecting a Multiplier with one or more Op-Amps and diodes allows other mathematic operations to be executed, including Dividing, Squaring and Square Rooting.

2.2.4 The Potentiometer

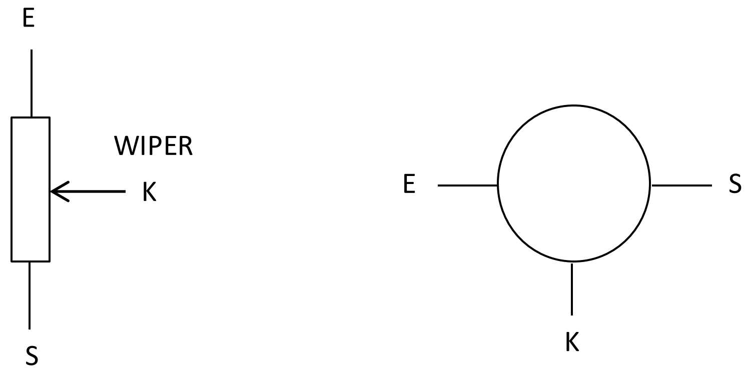

Figure 2.2.8 Conventional and Analogue Computing Schematic Diagrams of a Potentiometer

A Potentiometer or Pot is a potential difference divider; it is formed by a rheostat coil with a terminal at either end and a slider or wiper terminal between them. The end terminals are normally connected to the upper and lower voltage inputs; the wiper divides the resistance of the rheostat coil between the two end terminals in proportion to the length apportioned to each end, thus dividing the voltage correspondingly. This is very useful as it allows an input to be scaled to the desired value for making calculations using an Analogue Computer

In an Analogue Computer potentiometers are used in two ways. The first is as a Coefficient Pot, where the ***S*** terminal is connected to the signal ground point, creating a fixed reference variable. The other is as a Free Pot, where the ***S*** terminal can be connected back to the panel, creating a floating reference variable. Free Pots can be used with diodes in the feedback path of Summing Op-Amps to simulate Non-Linear Transfer Functions. This duality is analogous to fixed point and floating point calculations carried out by Digital Computers.

3. Apparatus

For this investigation the following components were used:

3.1 The BICC- Vero Analogue Computer

3.2 A Digital Personal Computer with Connection Interface and Data Logging Software for plotting the response of Simulated Systems

3.3 Colour Connection Wires

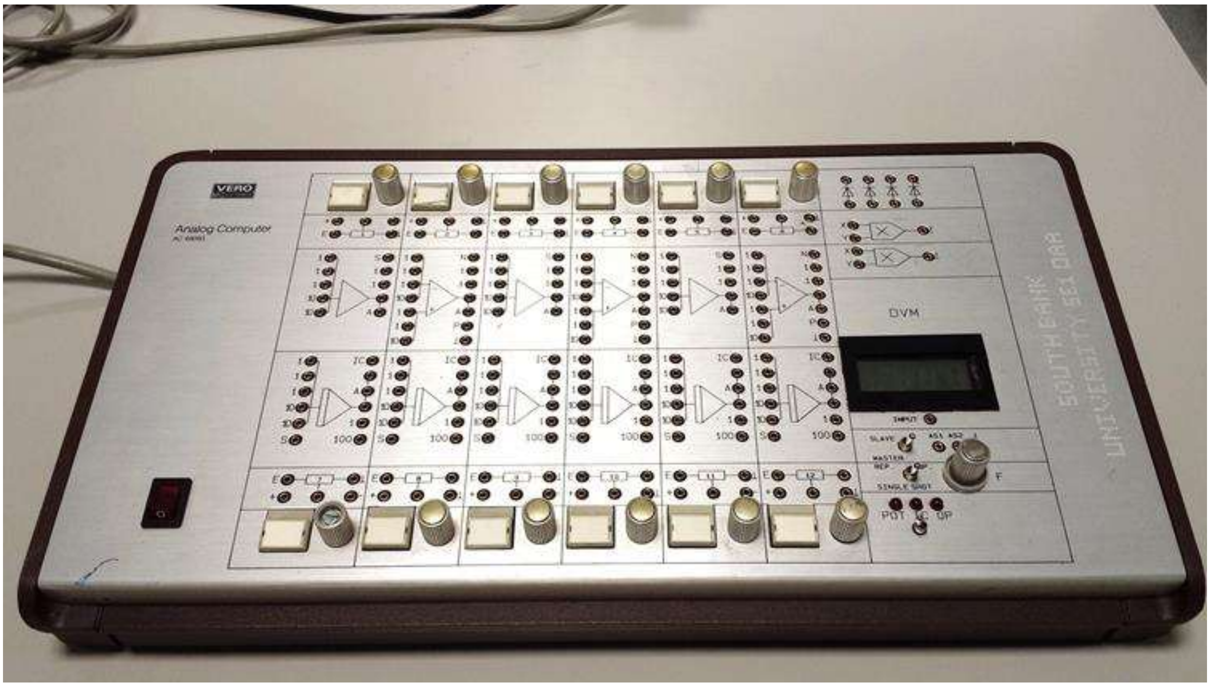

Figure 3.1.1 The BICC-Vero Analogue Computer (J.F.Goddings 12/12/2014)

4. Investigation

The procedure of this investigation is split into a sequence of tasks to familiarise the operator with the Analogue Computer's components and build understanding:

4.1 Investigation of a Summing Amplifier

4.1.1 Method

The BICC-Vero is wired in accordance with *Figure 4.1.1* while the machine is off.

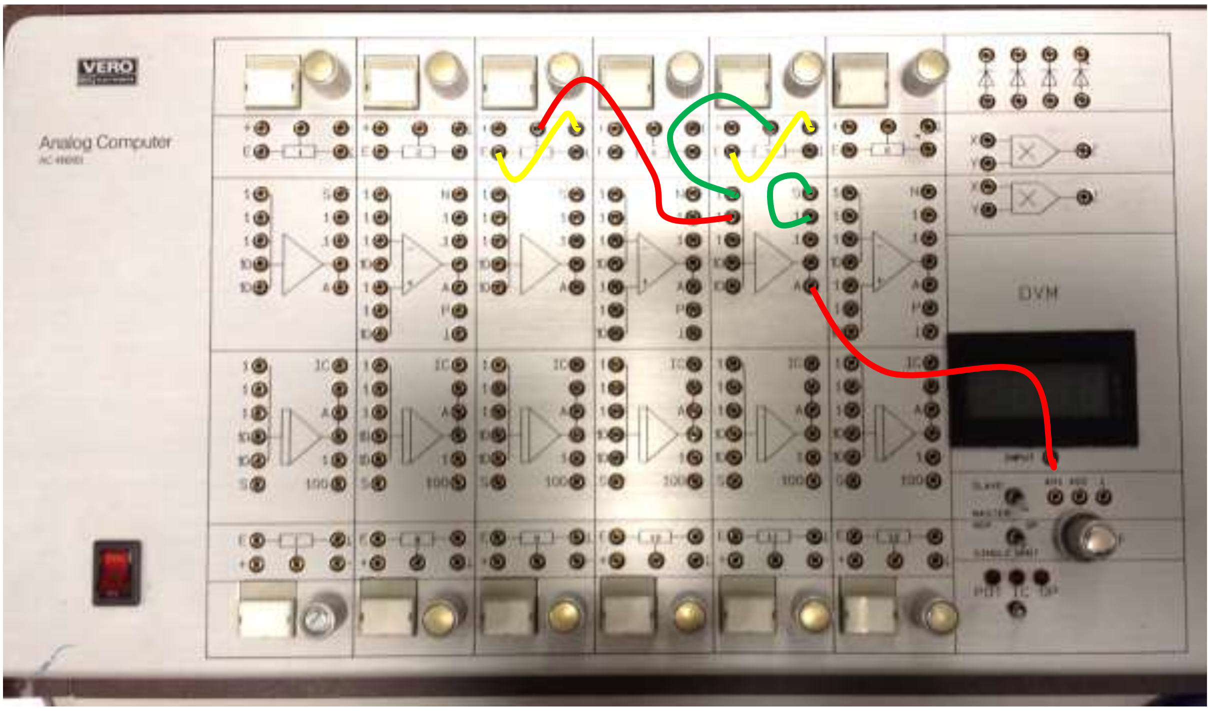

Figure 4.1.1 Diagram of Connections on the BICC-Vero for Investigation 4.1

The Pot on Station 3 is set to 0.500MU and the Pot on Station 5 set to 0.000 MU, this is done by setting the ***OP/IC/POT*** Toggle Switch to ***POT***, holding the button adjacent to the Pot dial, which connects the LCD display across the Pot and the Wiper, and adjusting the dial until the LCD display reads the correct value.

Ensure the Master/Slave Switch is set to master, switch On the BICC-Vero and then toggle the ***OP/IC/POT*** Switch to ***OP***. The sum of the two inputs from ***Pot 3 = E_1*** and ***Pot 5 = E_2*** *(see Figures 2.2.3 and 4.41)* are summed according to *Equation 2.2.6* and displayed on the LCD display. These two inputs were then adjusted and the results recorded in *Table 4.1.1*.

4.1.2 Results

E_1	E_2	Output/Sum	Error	% Error
-0.502	0	0.519	0.017	3.3
-0.701	0	0.720	0.019	2.6
-0.900	0	0.915	0.015	1.6
-0.500	-0.100	0.618	0.018	2.9
-0.500	-0.300	0.824	0.024	2.9
+0.501	-0.299	-2.04	0.004	2.0
+0.501	+0.299	-0.824	0.024	2.9

Table 4.1.1 Table of Values Resulting from Summing Amplifier Investigation

4.1.3 Observations

The summations are reversed in polarity, as is consistent with the Op-Amp being an Inverting Summer, as discussed in the Introduction.

The error in the summations is consistent at approximately 2-3%, this would be considered an acceptable error for the BICC-Vero. It is difficult to ascertain from the narrow range of values whether the error is produced by an individual input or during the summation of the two inputs.

4.2 Investigation of a Summing Integrator

4.2.1 Method

The BICC-Vero is wired in accordance with *Figure 4.2.1* while the machine is off.

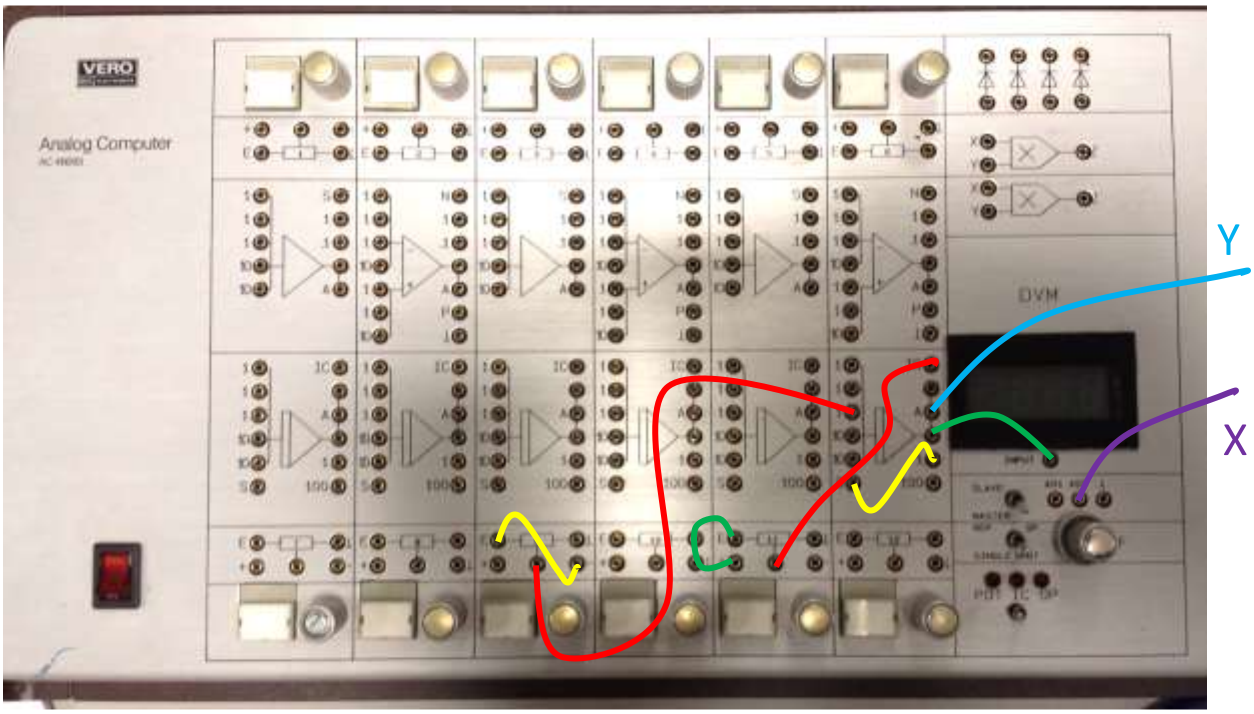

Figure 4.2.1 Diagram of Connections on the BICC-Vero for Investigation 4.2

The Pot on Station 9 is set to 0.100MU in the same manner as in section 4.1.1, and connected to E_3 of Station 12. The Pot on Station 11 which forms the Initial Conditions(IC) value is set to 0.000 MU and connected to ***IC*** of Station 12.

The Data Logging Interface for the PC is connected in accordance with *Figure 4.2.1* with the ***X*** terminal of the logger interface connected to ***AS1***, and the ***Y*** terminal of the logger interface connected to ***A*** on Station 12.

The data logging software is opened. The data logging software should start and finish automatically when the BICC-Vero is switched to OP mode.

4.2.2 Calculation

It is possible to hand calculate the expected results for this investigation as follows; these values are tabulated in Table 4.3.1. The first of these calculations is shown below as an example.

$$x(t) = \int_0^t 0.1$$ *Equation 4.2.1*

$$x(t) = [0.1x + c\,]_0^t$$ *Equation 4.2.2*

$$I.C.\;\; x(0) = 0,\;\; c = 0$$ *Equation 4.2.3*

$$x(t) = [0.1x\,]_0^t$$ *Equation 4.2.4*

$$x(t) = (0.1t - 0)$$ *Equation 4.2.5*

$$when\ x = 1$$ *Equation4.2.6*

$$t = \frac{1}{0.1} = 10s$$ *Equation 4.2.7*

4.2.3 Results

Test	E_3	IC	Time To Reach +1MU	Calculated Value	Error	% Error
A	0.1	0	9.66	10	0.44	4.6
B	0.2	0	4.78	5	0.22	4.6
C	0.1	0.2	8.43	8	0.43	5.3
D	0.1	-0.1	12.51	12	0.51	4.1

Table 4.3.1 Table of Values Resulting from Summing Integrator Amplifier Investigation

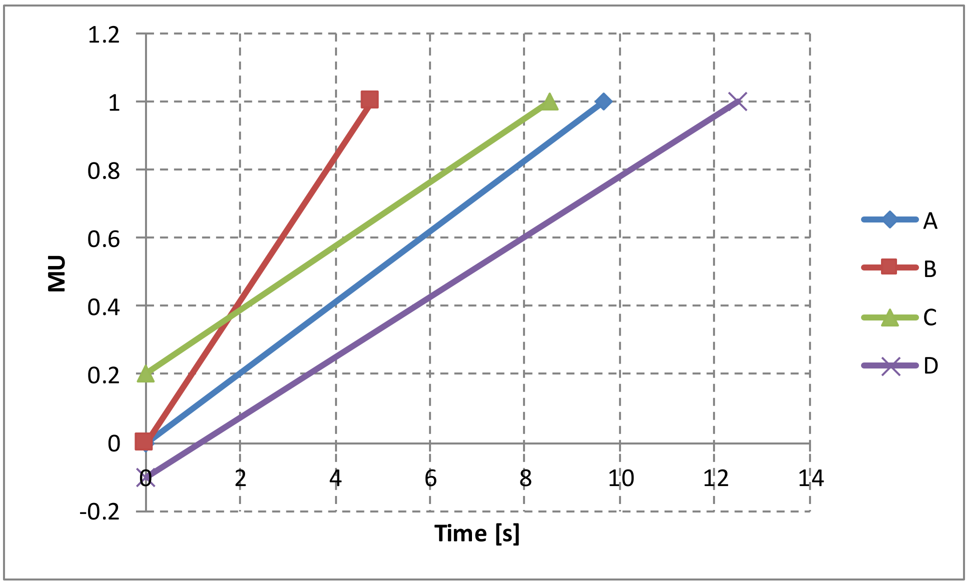

Chart 4.3.1 Chart of Summing Integrator Amplifier Investigation Results

4.2.3 Observations

The results from the BICC-Vero do correlate with the calculated data, showing that the Analogue Computer is capable of Integrating with sufficient accuracy to simulate a linear system.

It is made evident by *Chart4.2.1* that the gradient of the resulting function is equal to the input gain value E_3, set on Pot 9, and the ***IC*** value, set on Pot 11, provides a means of adding a constant offset initial condition. Therefore a function of the form:

$$x(t) = mx + c \qquad \textit{Equation 4.2.8}$$

Could be simulated, or solved using the BICC-Vero.

4.3 Simulation of a First Order System

The BICC-Vero is used to solve the system characterised by a first order differential equation:

$$\frac{dx}{dt} + \frac{3x}{2} + \frac{1}{2} = 0 \qquad \textit{Equation 4.3.1}$$

To achieve this using Integrators it is first preferable to rearrange *Equation 4.3.1* to isolate the highest derivative term.

$$\frac{dx}{dt} = -\left(\frac{3x}{2} + \frac{1}{2}\right) \qquad \textit{Equation 4.3.2}$$

This equation can then be modelled using the BICC-Vero following *Figure 4.3.1.*

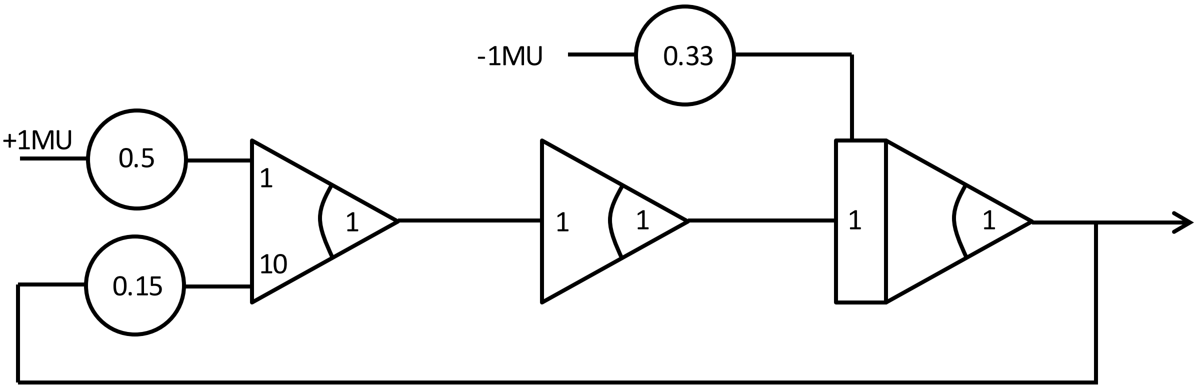

Figure 4.3.1 Program Flow Diagram for Investigation 4.3

4.3.1 Method

The BICC-Vero is wired in accordance with *Figure 4.3.2* while the machine is off.

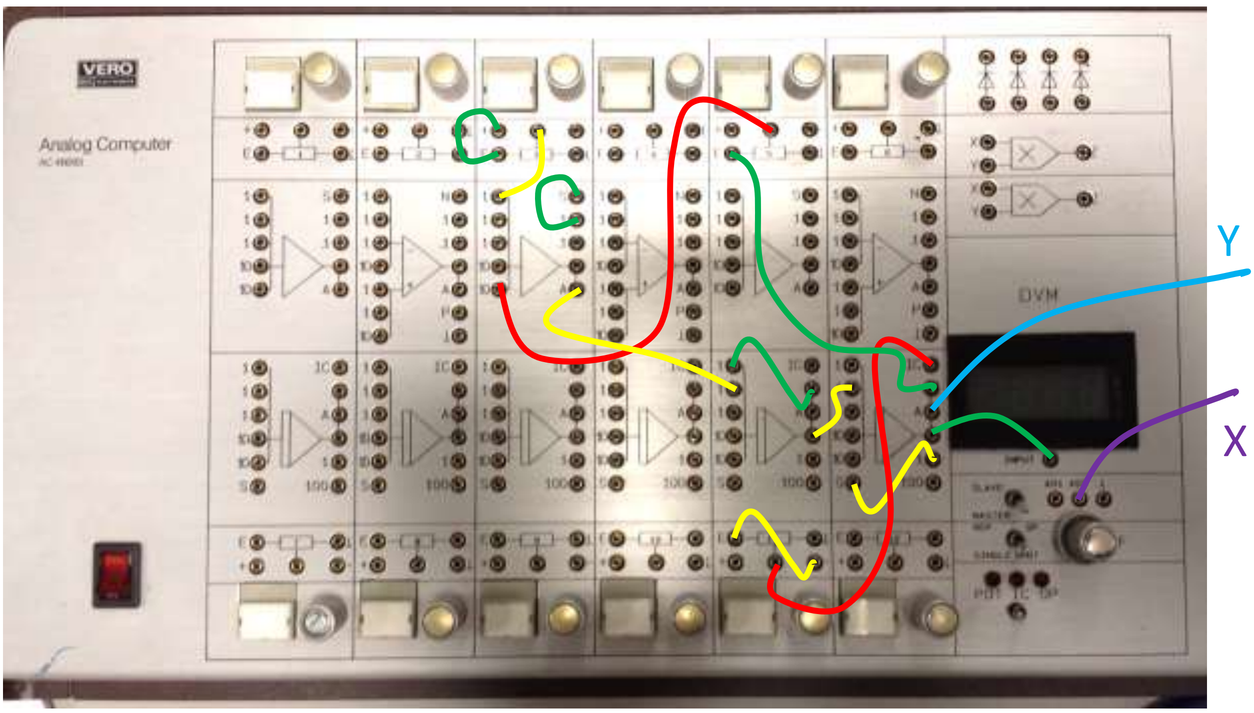

Figure 4.3.2 Diagram of Connections on the BICC-Vero for Investigation 4.3

The Pot on Station 3 is set to 0.500MU in the same manner as in section 4.1.1, and connected to the Summer to a 1X input.

The Pot on Station 5 is set to 0.150MU and the wiper connected to the Summer at Station 3 through one of the 10X inputs, creating the 1.5X factor.

The Pot on Station 11 which forms the Initial Conditions(IC) value is set to 0.333 MU and connected to ***IC*** of Station 12.

The Data Logging Interface for the PC is connected in accordance with *Figure 4.3.2* with the ***X*** terminal of the logger interface connected to ***AS1***, and the ***Y*** terminal of the logger interface connected to ***A*** on Station 12.

The data logging software is opened. The data logging software should start and finish automatically when the BICC-Vero is switched to OP mode.

4.3.2 Calculations

It is possible to solve *Equation 4.3.1* for *t* as follows:

$$Laplace\left\{\frac{dx}{dt}+\frac{3x}{2}+\frac{1}{2}\right\}= 0$$ *Equation 4.3.3*

$$sX(s)-x(0)+\frac{3X(s)}{2}+\frac{1}{2s}=0$$ *Equation 4.3.4*

$$\left(s+\frac{3}{2}\right)X(s)=\frac{1}{3}-\frac{1}{2s}$$ *Equation 4.3.5*

$$X(s)=\frac{\frac{1}{3}}{\left(s+\frac{3}{2}\right)}-\left[\frac{\frac{1}{2}}{s\left(s+\frac{3}{2}\right)}\right]$$ *Equation 4.3.6*

$$[\ldots]\rightarrow\frac{\frac{1}{2}}{s\left(s+\frac{3}{2}\right)}=\frac{A}{s}+\frac{B}{s+\frac{3}{2}}$$ *Equation 4.3.7*

$$[\ldots]\rightarrow\frac{1}{2}=A\left(s+\frac{3}{2}\right)+Bs$$ *Equation4.3.8*

$$s=0$$ *Equation 4.3.9*

$$[\ldots]\rightarrow\frac{1}{2}=\frac{3}{2}A\rightarrow A=\frac{1}{3}$$ *Equation 4.3.10*

$$s=-\frac{3}{2}$$ *Equation 4.3.11*

$$[\ldots]\rightarrow\frac{1}{2}=-\frac{3}{2}B\rightarrow B=-\frac{1}{3}$$ *Equation 4.3.12*

$$X(s)=\frac{\frac{1}{3}}{\left(s+\frac{3}{2}\right)}-\left[\frac{\frac{1}{3}}{s}-\frac{\frac{1}{3}}{\left(s+\frac{3}{2}\right)}\right]$$ *Equation 4.3.13*

$$x(t) = Laplace^{-1}\left\{\frac{\frac{1}{3}}{\left(s+\frac{3}{2}\right)} - \left[\frac{\frac{1}{3}}{s} - \frac{\frac{1}{3}}{\left(s+\frac{3}{2}\right)}\right]\right\}$$

Equation 4.3.14

$$x(t) = \frac{1}{3}e^{-\frac{3}{2}t} - \frac{1}{3} + \frac{1}{3}e^{-\frac{3}{2}t}$$

Equation 4.3.15

$$x(t) = \frac{2}{3}e^{-\frac{3}{2}t} - \frac{1}{3}$$

Equation 4.3.16

4.3.3 Results

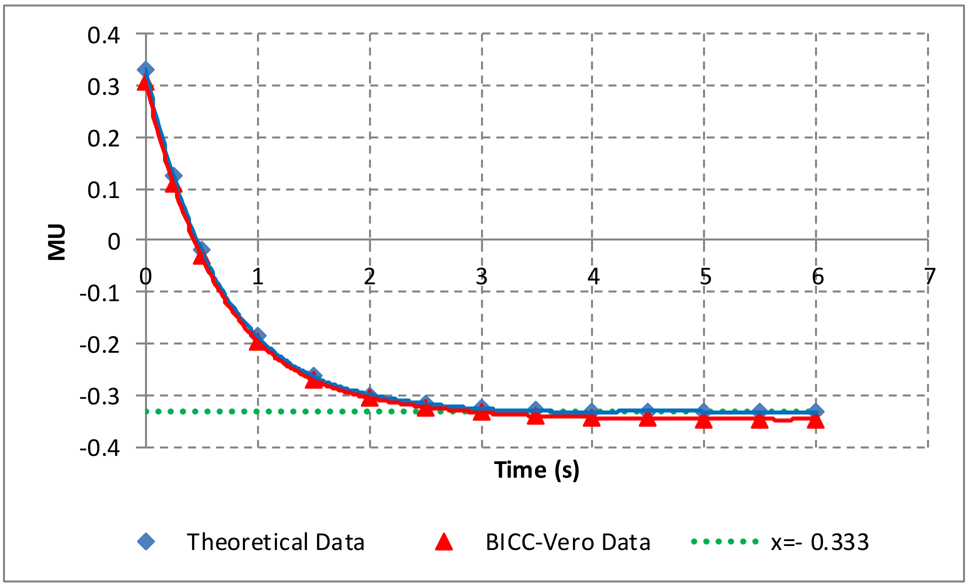

Chart 4.3.1 Chart of First Order System Simulation comparing the result from the BICC-Vero to Theoretical Calculations

4.3.4 Observations

The results from the BICC-Vero do correlate well with the calculated data, showing the BICC-Vero to be capable of modelling a First Order System.

There is an offset error throughout the simulation of approximately 0.01-0.02 MU which represents a 4-5% error, similar to that found with using the BICC-Vero Op-Amps previously.

4.4 Simulation of a Second Order System

The BICC-Vero is used to solve the system characterised by a second order differential equation:

$$\theta_i = \ddot{\theta}_o + 2\zeta\omega_n\dot{\theta}_o + \omega_n^2\theta_o$$

Equation 4.4.1

To achieve this using Integrators it is first preferable to rearrange *Equation 4.3.1* to isolate the highest derivative term.

$$\ddot{\theta}_o = -2\zeta\omega_n\dot{\theta}_o - \omega_n^2\theta_o + \theta_i$$

Equation 4.4.2

This equation can then be modelled using the BICC-Vero following *Figure 4.4.1.*

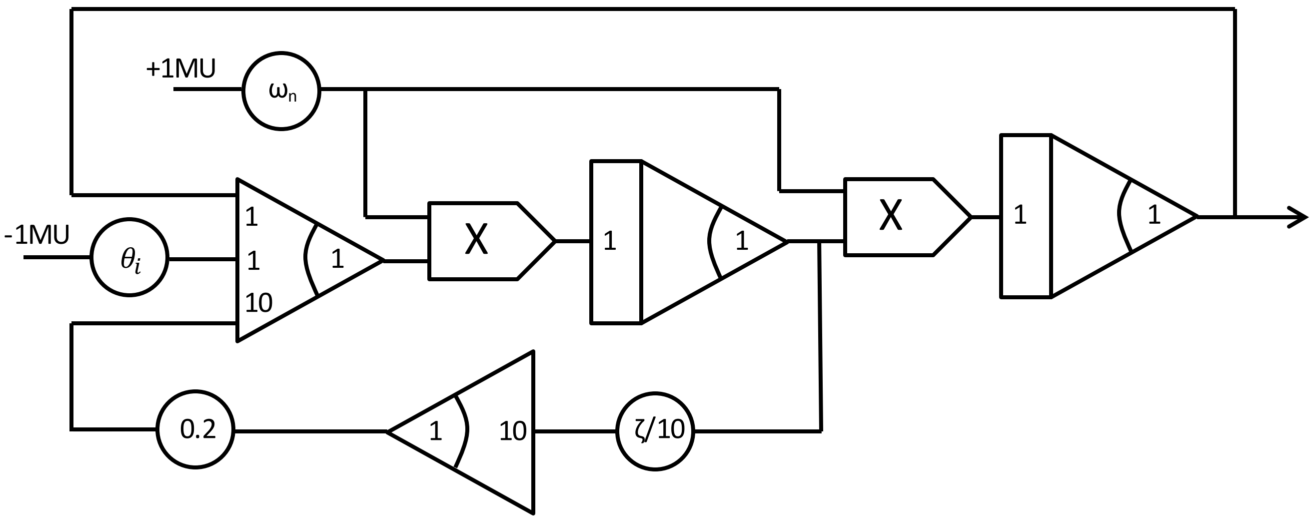

Figure 4.4.1 Program Flow Diagram for Investigation 4.4

4.4.1 Method

The BICC-Vero is wired in accordance with *Figure 4.4.2* while the machine is off.

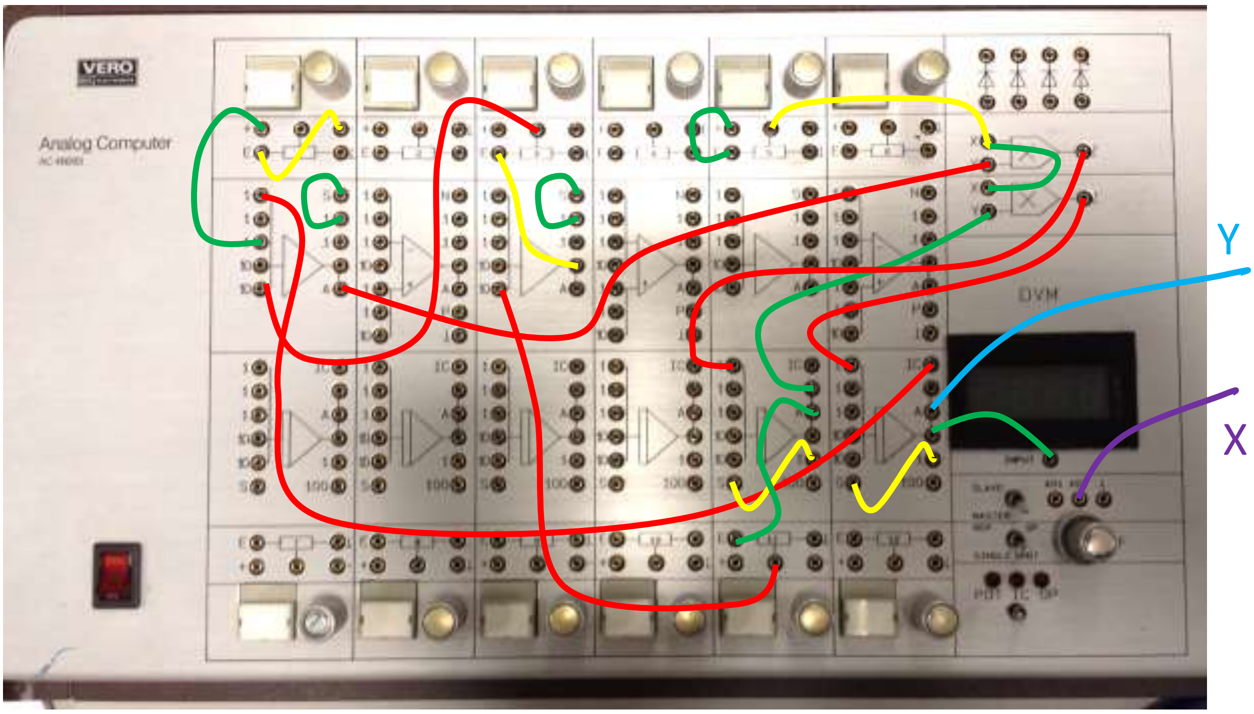

Figure 4.4.2 Diagram of Connections on the BICC-Vero for Investigation 4.4

The Pot on Station 1 is set to 0.500MU in the same manner as in section 4.1.1, and connected to the Summer to a 1X input.

The Pot on Station 5 is set to 1.000MU and connected to the X input terminal of both Multipliers.

The Pot on Station 11 is set to one tenth of the damping factor and connected to the inverting amplifier on Station 3 to a 10X input.

The output from Station 3 is connected to the Pot on Station 3 and the Pot on Station 3 is set to 0.200MU and connected to the Summer on Station 1 to a 10X input.

The output from Station 1 is connected to Y input the first Multiplier, the output of which is connected to the Integrator on Station 11 to a 1X input.

The Integrators at Stations 11 and 12 are both connected through a 1μF capacitor.

The output from Station 11 is connected to both the Pot on Station 11 and the Y input on the second Multiplier, the output of which is connected to the Integrator on Station 12 to a 1X input.

The output from Station 12 is connected to the Data Logging Interface.

The Data Logging Interface for the PC is connected in accordance with *Figure 4.4.2* with the ***X*** terminal of the logger interface connected to ***AS1***, and the ***Y*** terminal of the logger interface connected to ***A*** on Station 12.

The data logging software is opened. The data logging software should start and finish automatically when the BICC-Vero is switched to OP mode.

4.4.2 Calculations

By LaPlace Transforming Equation 4.3.1 it is possible to solve it for each separate set of initial conditions by hand, however this is time consuming, with a Digital Computer and MatLab it is possible to solve each iteration or a whole matrix of iterations very efficiently and accurately.

In the following charts MatLab is used to model the same functions as the BICC-Vero, resulting from varying the initial conditions of the system, and a chart is plotted with the traces compared.

4.4.3 Results

Chart 4.4.1 shows the results of the BICC-Vero Simulation and the MatLab Model for a system with the initial conditions and resulting transfer function that follows in *Equation 4.4.7:*

$$T(s) = \frac{\theta_O(s)}{\theta_I(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ *Equation 4.4.3*

$$\theta_I(s) = \frac{1}{s}$$ *Equation 4.4.4*

$$T(s) = \frac{\theta_O(s)}{\theta_I(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ *Equation 4.4.5*

$$K = 0.5, \qquad \omega_n = 1, \qquad \zeta = 0.2$$ *Equation 4.4.6*

$$T(s) = \frac{0.5}{s^2 + 0.4s + 1}$$ *Equation 4.4.7*

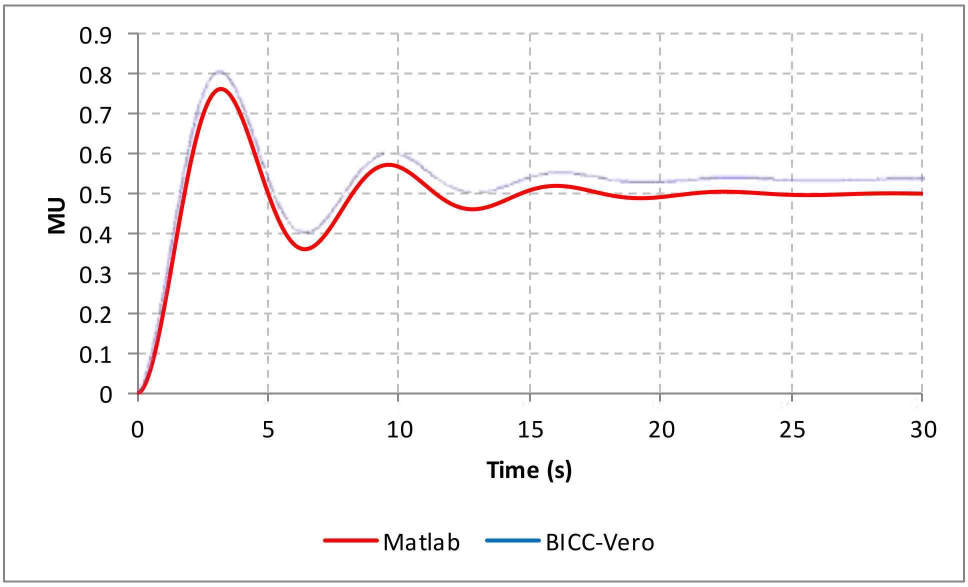

Chart 4.4.1 Chart of Second Order System Simulation comparing the result from the BICC-Vero to MatLab Model for Equation 4.4.7

Chart 4.4.2 shows the results of the BICC-Vero Simulation and the MatLab Model for a system with the initial conditions and resulting transfer function that follows in *Equation 4.4.9:*

$$K = 0.5, \qquad \omega_n = 1, \qquad \zeta = 0.4$$ *Equation 4.4.8*

$$T(s) = \frac{0.5}{s^2 + 0.8s + 1}$$ *Equation 4.4.9*

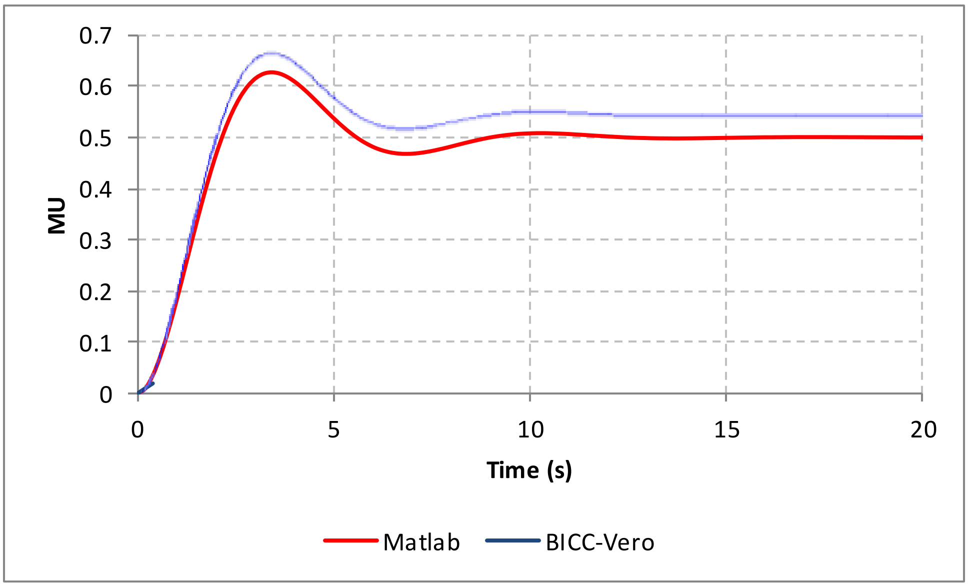

Chart 4.4.2 Chart of Second Order System Simulation comparing the result from the BICC-Vero to MatLab Model for Equation 4.4.9

Chart 4.4.3 shows the results of the BICC-Vero Simulation and the MatLab Model for a system with the initial conditions and resulting transfer function that follows in *Equation 4.4.11:*

$$K = 0.5, \qquad \omega_n = 1, \qquad \zeta = 0.6$$ *Equation 4.4.10*

$$T(s) = \frac{0.5}{s^2 + 1.2s + 1}$$ *Equation 4.4.11*

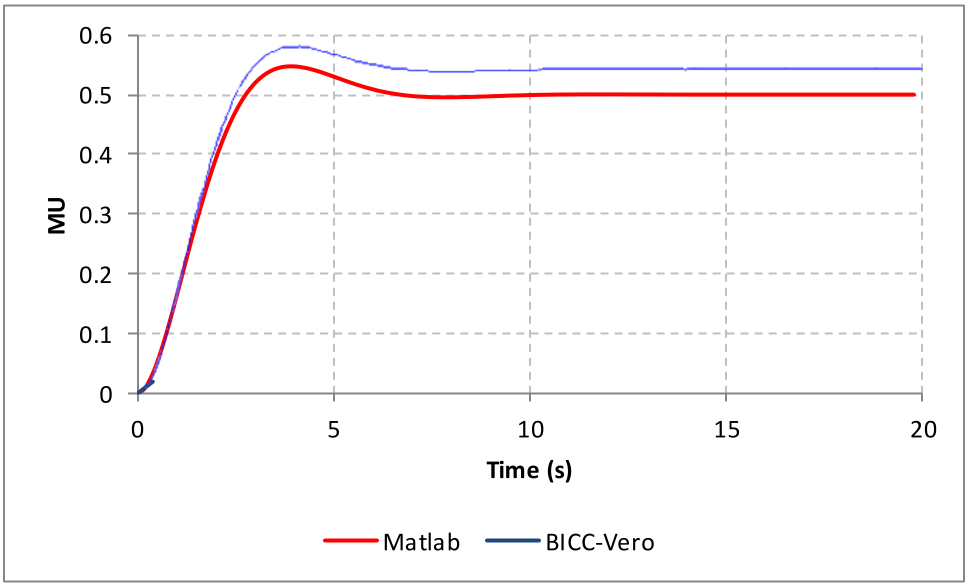

Chart 4.4.3 Chart of Second Order System Simulation comparing the result from the BICC-Vero to MatLab Model for Equation 4.4.11

Chart 4.4.4 shows the results of the BICC-Vero Simulation and the MatLab Model for a system with the initial conditions and resulting transfer function that follows in *Equation 4.4.13:*

$$K = 0.5, \qquad \omega_n = 1, \qquad \zeta = 1$$ *Equation 4.4.12*

$$T(s) = \frac{0.5}{s^2 + 2s + 1}$$ *Equation 4.4.13*

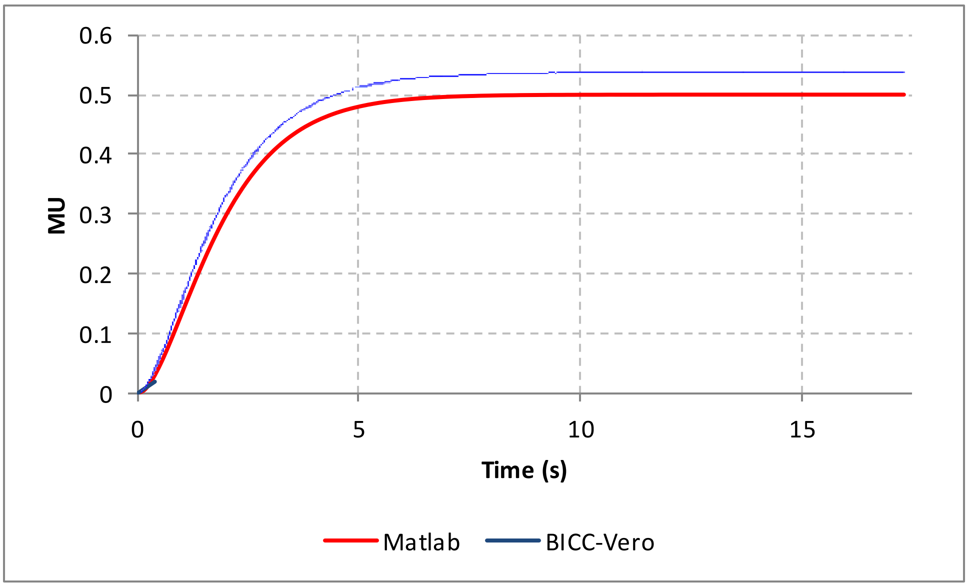

Chart 4.4.4 Chart of Second Order System Simulation comparing the result from the BICC-Vero to MatLab Model for Equation 4.4.13

Chart 4.4.5 shows the results of the BICC-Vero Simulation and the MatLab Model for a system with the initial conditions and resulting transfer function that follows in *Equation 4.4.15:*

$$K = 0.5, \qquad \omega_n = 1, \qquad \zeta = 2$$ *Equation 4.4.14*

$$T(s) = \frac{0.5}{s^2 + 4s + 1}$$ *Equation 4.4.15*

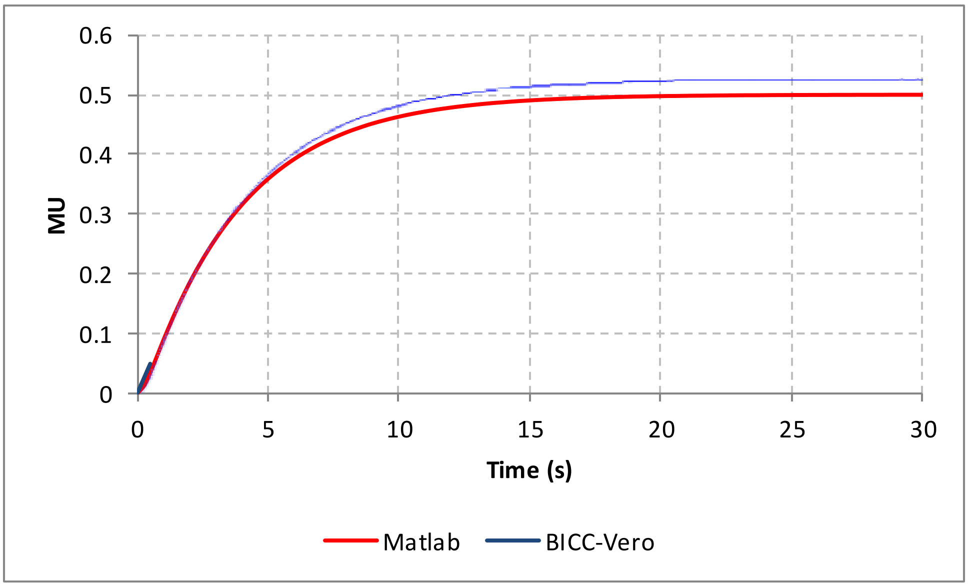

Chart 4.4.5 Chart of Second Order System Simulation comparing the result from the BICC-Vero to MatLab Model for Equation 4.4.15

4.4.4 Observations

The simulations of the functions resulting from the Second Order System explore a range of damping ratios that result in the system going from being underdamped in *Chart 4.4.1,* to critically damped in *Chart 4.4.4,* to overdamped in 4.4.5.

The results from the BICC-Vero do correlate sufficiently well with the MatLab Models to prove the BICC-Vero to be capable of simulating a Second Order System for a range of Initial Conditions, however there is a considerable steady state error of approximately 7%.

5. Discussion and Conclusion

Electronic Analogue Computers were an essential stepping stone from Mechanical Analogue Computers to Digital Computers, and for many years were a more powerful tool for certain applications than their digital counterparts.

This investigation has enabled the author to expand his knowledge and understanding of Analogue Computers and their constituent components.

Through use of the BICC-Vero in this investigation, it has been made evident that it is a useful tool for making engineering calculations, models and simulations. It is a competent tool for making calculations, and simulating complex systems characterised by differential equations.

The BICC-Vero can be used to solve systems characterised by differential equations up to the sixth order, however the results from this investigation show that the BICC-Vero introduces error to its calculations and simulations. In the case of this investigation the error is not significant, but is notable. The error is likely to be due to the Op-Amp components, which in theory are required to operate with infinite gain, however in practice they cannot and in the case of the BICC-Vero operate with a gain of 10^6. For short simulations this does not normally pose a major problem, but in simulations of duration longer than 10^4 cycles, this error can become significant due to accumulative multiplication.

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