Nonnegative Matrix Factorisation

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CSE 392 - Parallel Algorithms

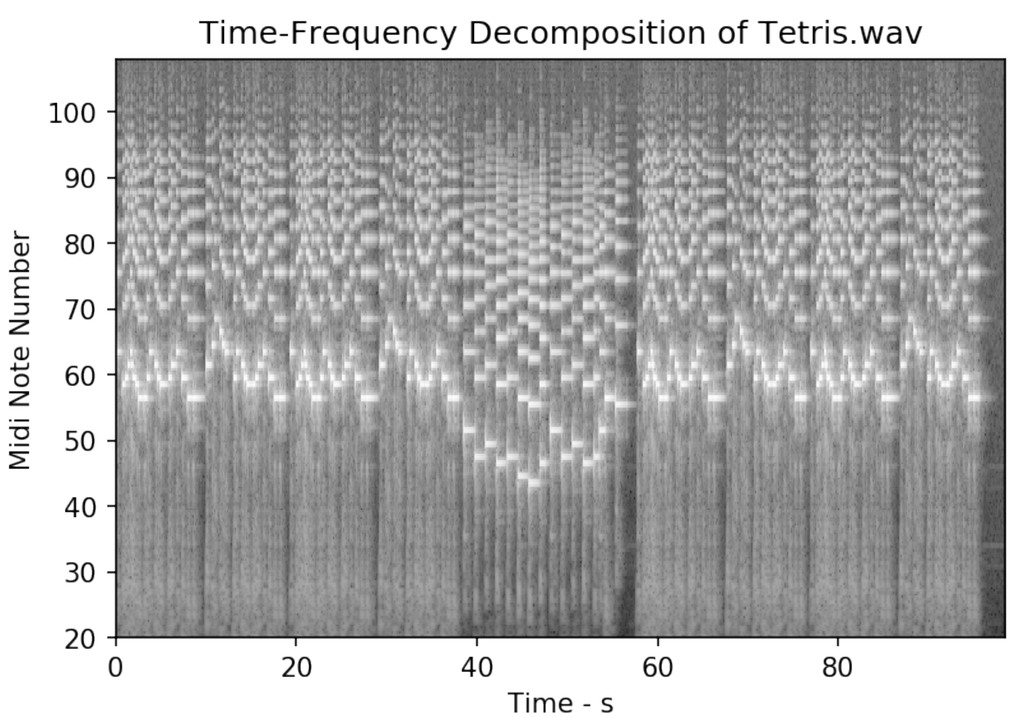
Project Proposal

Introduction

Suppose we have a matrix $\mathbf{V}$ containing nonnegative data; for example, the magnitude image of a time-frequency decomosition of an audio recording.

In [4]:

```
import numpy as np
import matplotlib.pyplot as plt
TFd = np.load("TFd.npy")
plt.figure(dpi = 150, figsize=(6,4))
extent=[0, 98.5, 20, 108]
plt.imshow(TFd, origin='lower', extent=extent, \
           interpolation='none', aspect='auto', cmap='gray')
plt.xlabel('Time - s')
plt.ylabel('Midi Note Number')
plt.title('Time-Frequency Decomposition of Tetris.wav');
```



The problem of nonnegative matrix factorisation (NMF) amounts to factorising $\mathbf{V}$ into two factors $\mathbf{W}$ and $\mathbf{H}$ which are also nonnegative. That is,

$$\mathbf{V} \approx \hat{\mathbf{V}} = \mathbf{W}\mathbf{H}$$

This technique can be used to learn recurring patterns in the data matrix. In this case, $\mathbf{W}$ represents a learned dictionary and the $\mathbf{H}$ represent represents the decomposition. When applied to the time-frequency decomposition of a music recording, $\mathbf{W}$ contain the learned spectral envelopes of each instrument in the recording and $\mathbf{H}$ contains a transcription of the music.

Many variations of NMF algorithms are well-established, and several libraries are available, such as `decomposition.nmf()` in scikit-learn. For the class project, We propose implementing parallel versions of these algorithms:

- To learn about NMF algorithms, which are currently an open field of research
- To gain experience and intuition for different parallel programming models by implimenting NMF using shared memory, message passing, and GPU programming
- To learn about implementation of iterative algorithms with high data parallelism
- To improve upon the performance and functionality of existing NMF libraries

Description of Algorithm

The most widely used algorithms for NMF employ a multiplicative weight update method based on the pioneering work of Lee and Sung [3]. The algorithm consists of the following steps:

- Initialize $\mathbf{W}$ and $\mathbf{H}$ with non-negative values
- Iteratively update $\mathbf{W}$ and $\mathbf{H}$ using the following rules: (n is the iteration)

$$\mathbf{H}^{n+1}_{[i,j]} \leftarrow \frac{\left((\mathbf{W}^n)^\top \mathbf{V}\right)_{[i,j]}}{\left((\mathbf{W}^n)^\top \mathbf{W}^n \mathbf{H}^n\right)_{[i,j]}}$$

$$\mathbf{W}^{n+1}_{[i,j]} \leftarrow \frac{\left(\mathbf{V}(\mathbf{H}^{n+1})^\top\right)_{[i,j]}}{\left(\mathbf{W}^n \mathbf{H}^{n+1} (\mathbf{H}^{n+1})^\top\right)_{[i,j]}}$$

Initial Performance Benchmarks

A test of the scikit-learn `decomposition.nmf()` implementation using the default parameters on a 90 second, single instrument audio recording provides a starting benchmarking for the algorithm. There are a few performance characterstics to note:

- On a workstation with twelve processors, one processor is fully utilized. The remaining processors are utilized at approximately 25%.
- The wall-clock run-time on the 90 second recording is approximately 2.5 hours.
- Each iteration takes about 7.0 seconds
- The number of iterations required scales rapidly as the converge tolerance is lowered.

In [5]:

```
import sklearn.decomposition
import time
model = sklearn.decomposition.NMF(n_components=264, max_iter=20, tol = 1e-4)
t1 = time.time()
W = model.fit_transform(TFd - np.min(TFd))
H = model.components_
t2 = time.time()
print(t2-t1)
```

```
13.925910472869873
```

Existing Parallel Implementations

GPU implementations are available, with most targeting bioinformatics applications

References

[1] S. Makino, Ed., Audio source separation. New York, NY: Springer Berlin Heidelberg, 2018.

[2] E. Vincent, T. Virtanen, and S. Gannot, Audio source separation and speech enhancement. 2018.