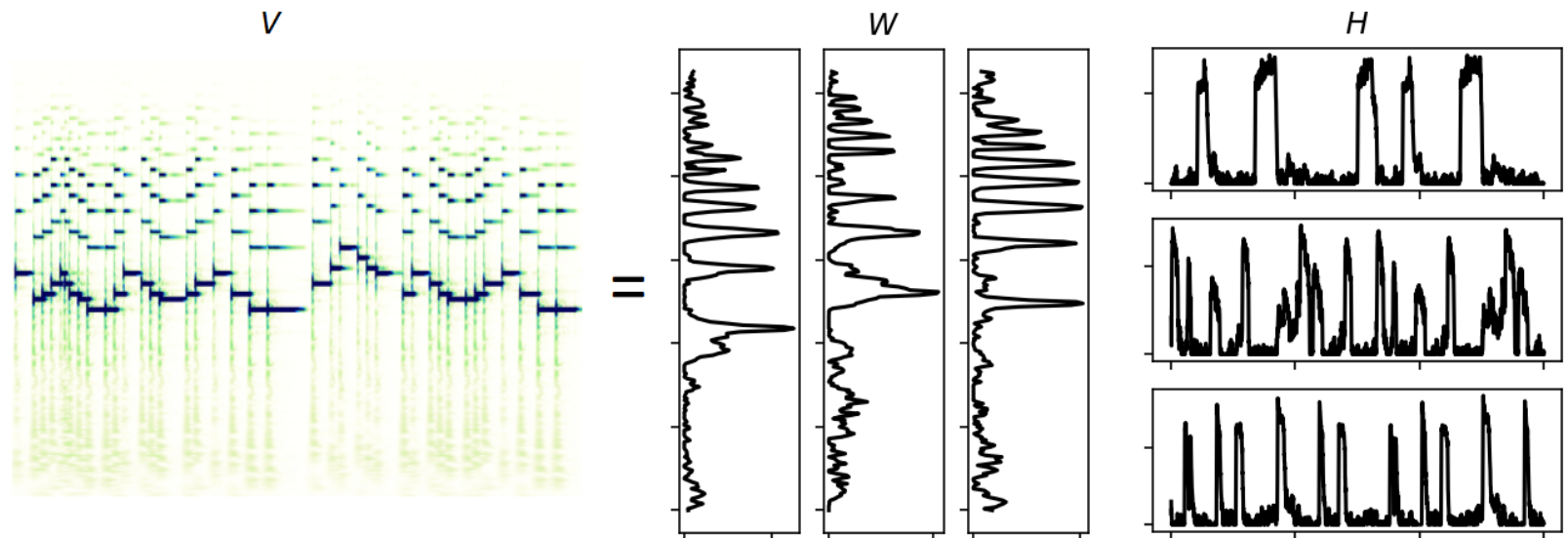


# Nonnegative Matrix Factorisation for Audio Applications

Dan Jacobellis, Tyler Masthay



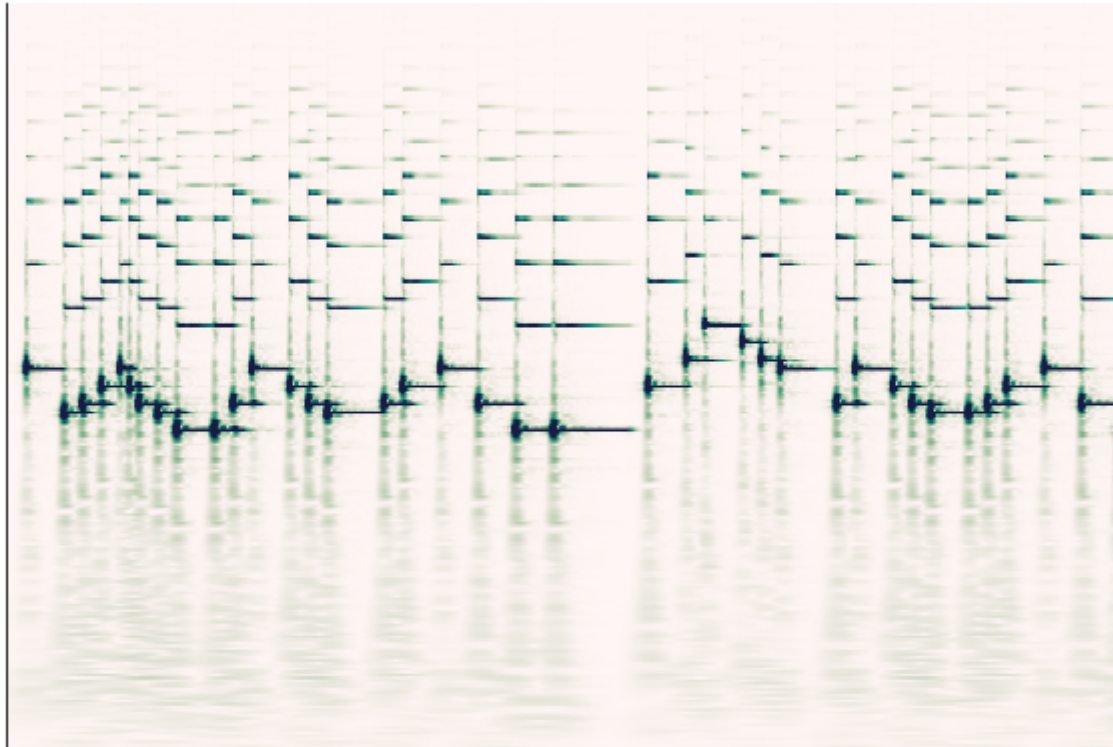
# Nonnegative Matrix Factorisation

$$\begin{aligned}\mathbf{V} &\approx \hat{\mathbf{V}} = \mathbf{WH} \\ \mathbf{W} &\in \mathbb{R}^{\mathbf{m} \times \mathbf{k}} \quad \mathbf{W} \geq \mathbf{0} \\ \mathbf{H} &\in \mathbb{R}^{\mathbf{k} \times \mathbf{n}} \quad \mathbf{H} \geq \mathbf{0}\end{aligned}$$

- Unsupervised learning (think SVD)
- Number of components  $k$  is a parameter
- $\mathbf{V}$  is  $m \times n$
- Setting  $k < m$  is a form of lossy compression

## Audio Example

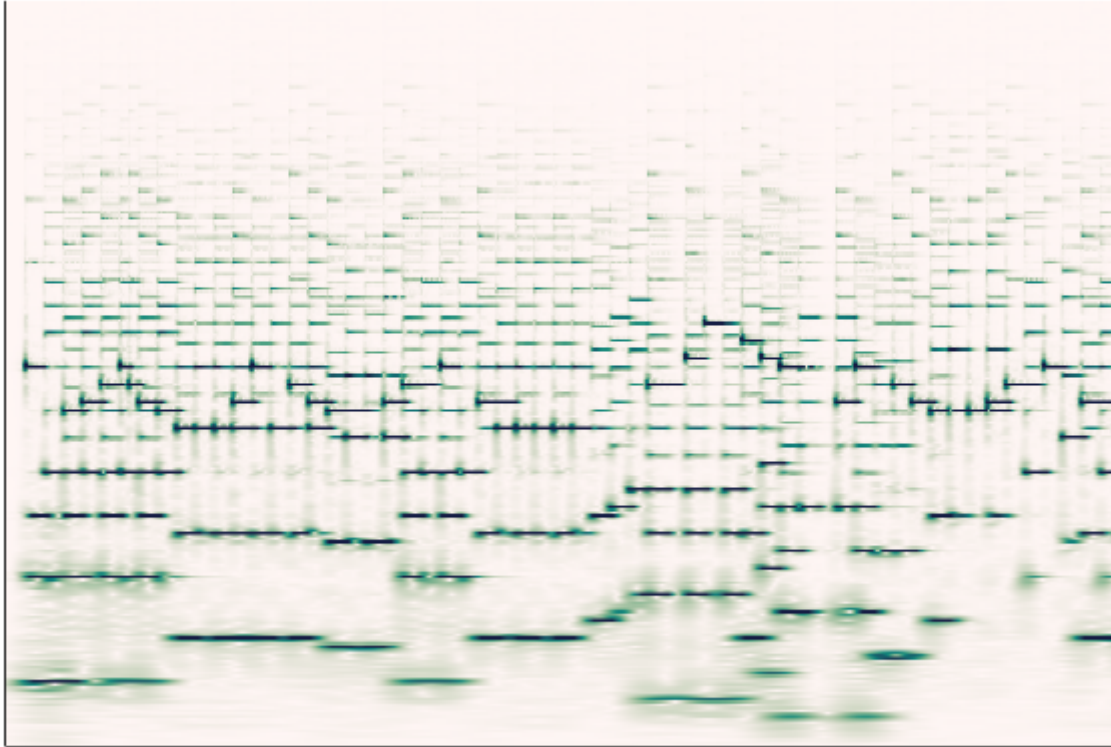
$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



*Time-frequency representation of 'Korobeiniki' played on piano. The frequency is on a logarithmic scale.*

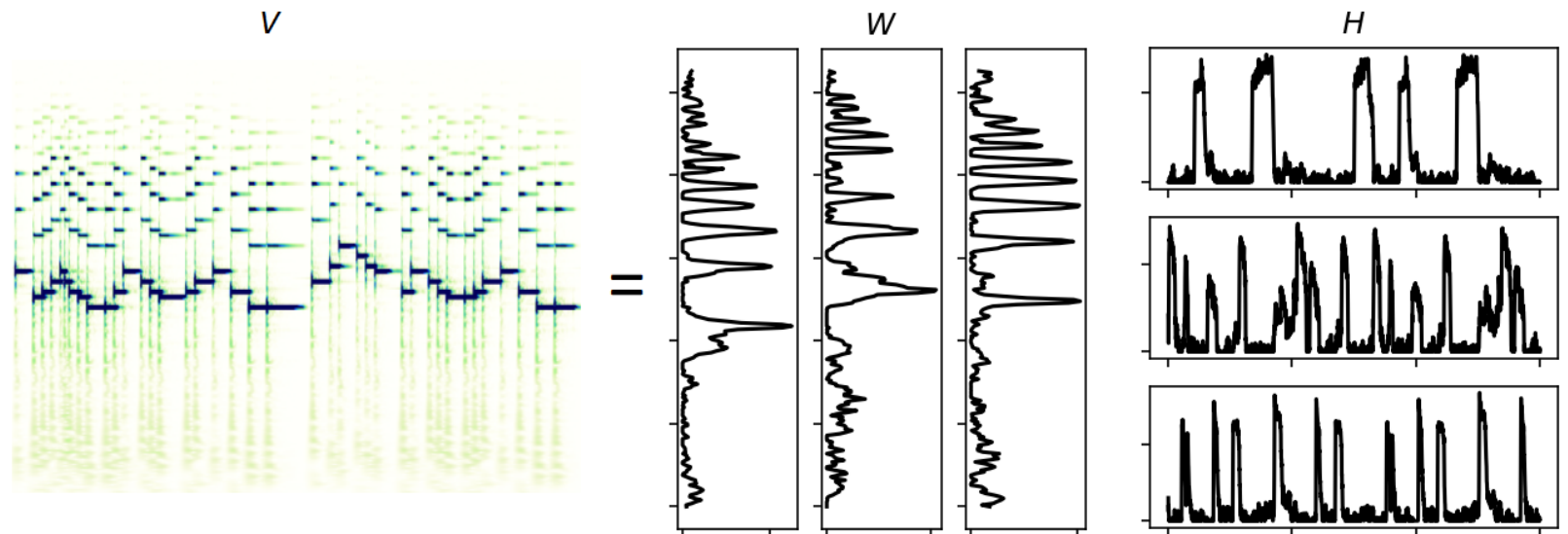
## Audio Example

$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



*Time-frequency representation of 'Korobeiniki' (polyphonic).*

# Audio Example



# Algorithm: Multiplicative Update

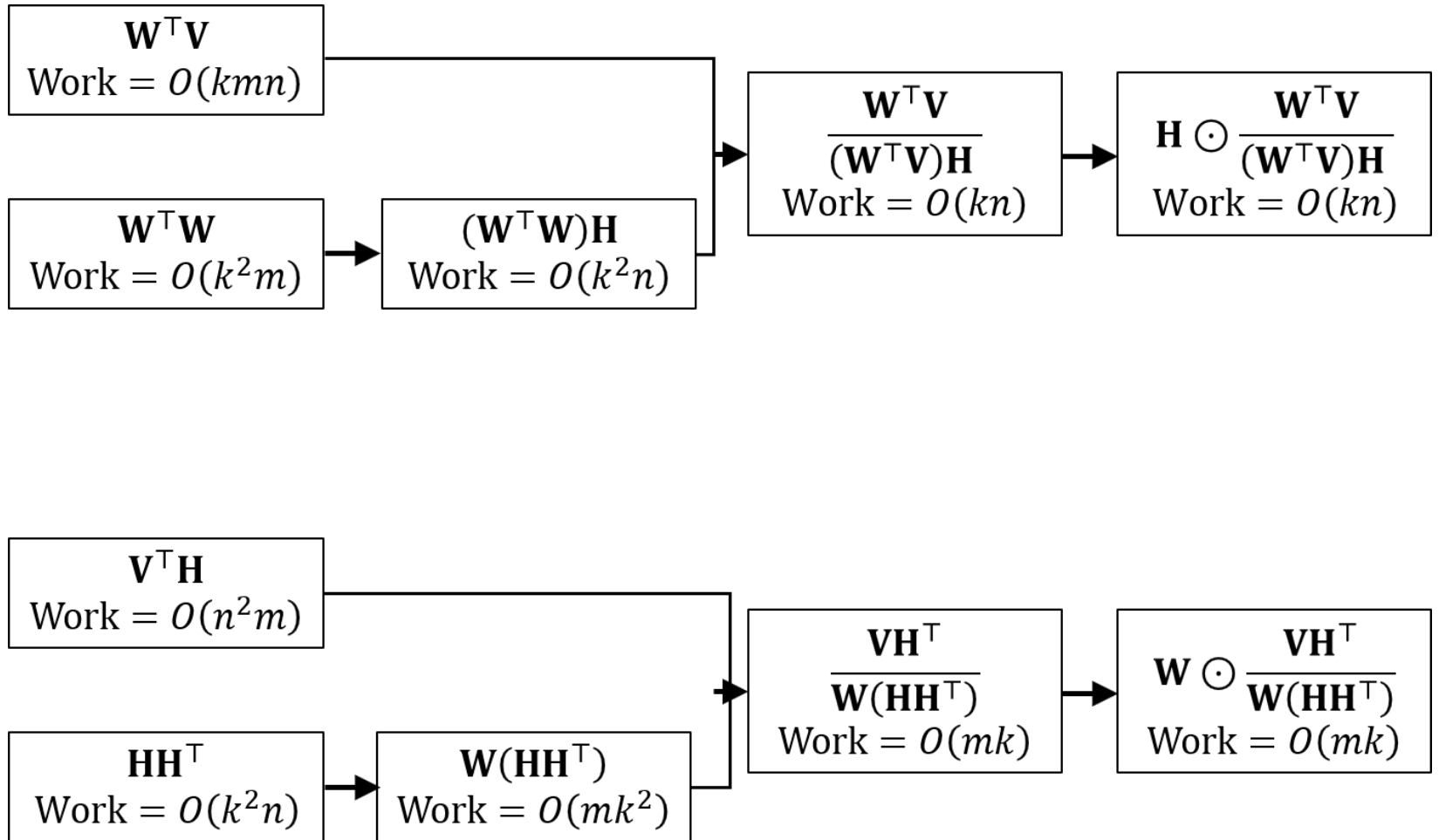
- Initialize  $\mathbf{W}$  and  $\mathbf{H}$  with non-negative values
- Iteratively update  $\mathbf{W}$  and  $\mathbf{H}$  using the following rules: ( $n$  here is the iteration)

$$\mathbf{H}_{[i,j]}^{n+1} \leftarrow \mathbf{H}_{[i,j]}^n \odot \frac{((\mathbf{W}^n)^\top \mathbf{V})_{[i,j]}}{((\mathbf{W}^n)^\top \mathbf{W}^n \mathbf{H}^n)_{[i,j]}}$$
$$\mathbf{W}_{[i,j]}^{n+1} \leftarrow \mathbf{W}_{[i,j]}^n \odot \frac{(\mathbf{V}(\mathbf{H}^{n+1})^\top)_{[i,j]}}{(\mathbf{W}^n \mathbf{H}^{n+1} (\mathbf{H}^{n+1})^\top)_{[i,j]}}$$

$\odot$  and division are element-wise.

ref. Lee, D.D., Seung, H.S., 2001. Algorithms for Non-negative Matrix Factorization, in: Advances in Neural Information Processing Systems 13. MIT Press, pp. 556–562.

# Algorithm: Multiplicative Update



## Parallelization on GPU : Motivation

- Matrices remain stationary in memory
- Matrix multiplies have high computational intensity compared to memory
- Single precision is sufficient
- Would like to use consumer hardware

Name	Clock(MHz)	<u>GFLOPS</u> (FP32)
Adreno 616	750	384
Adreno 630	710	727
Adreno 640	585	899



# GPU Implementation

- Used Julia bindings to CUDA (CURAND, CUBLAS)
- Compiler allows easily mapping high level syntax to GPU
- Compiler finds best way to send data back and forth between device and host

```
a = CuArray([1., 2., 3.])
function apply(op, a)
    i = threadIdx().x
    a[i] = op(a[i])
return
end
@cuda threads=length(a) apply(x->x^2, a)

julia> a
3-element CuArray{Float32,1}:
 1.0
 4.0
 9.0
```

```
@device_code_ptx @cuda apply(x->x^2, a)
apply(.param .b8 a[16])
{
    ld.param.u64    %rd1, [a+8];
    mov.u32        %r1, %tid.x;

    // index calculation
    mul.wide.u32    %rd2, %r1, 4;
    add.s64        %rd3, %rd1, %rd2;
    cvta.to.global.u64    %rd4, %rd3;

    ld.global.f32   %f1, [%rd4];
    mul.f32         %f2, %f1, %f1;
    st.global.f32   [%rd4], %f2;

    ret;
}
```

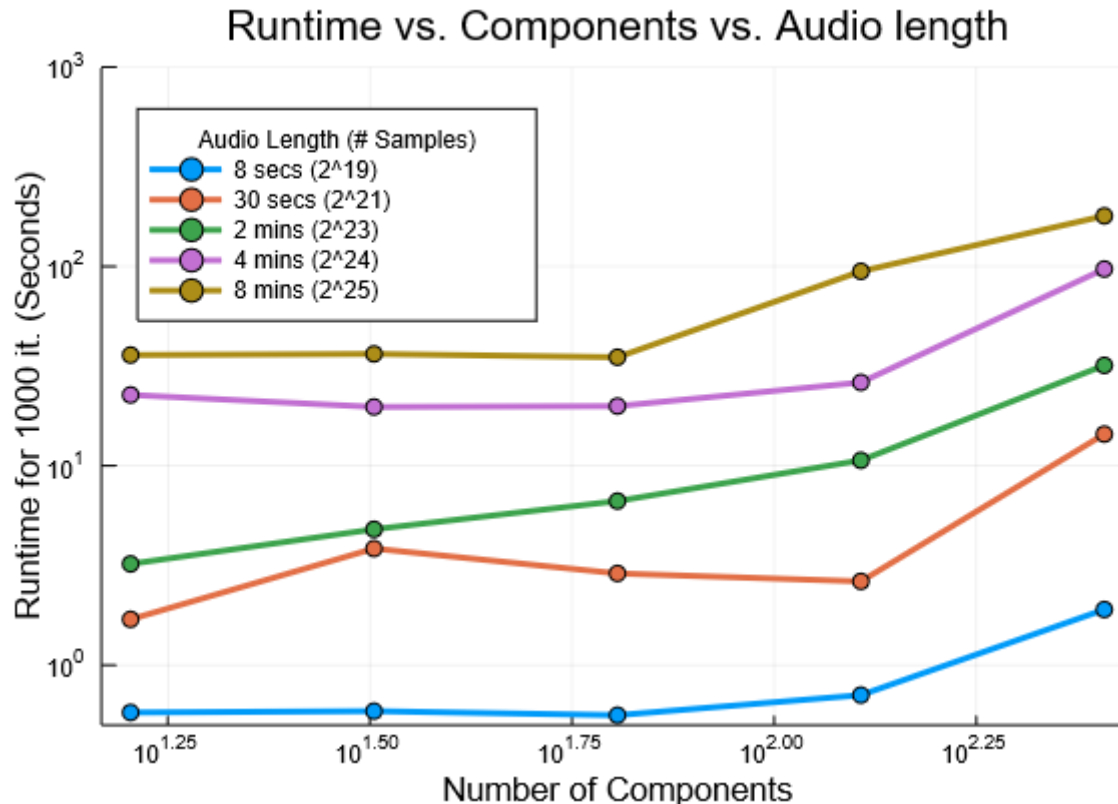
ref. <https://github.com/JuliaGPU/CUDAnative.jl>  
(<https://github.com/JuliaGPU/CUDAnative.jl>).

# GPU Implementation

- Must be careful and explicit with types and constants
- Otherwise, compiler will think you want to move all of the data off of the GPU and back

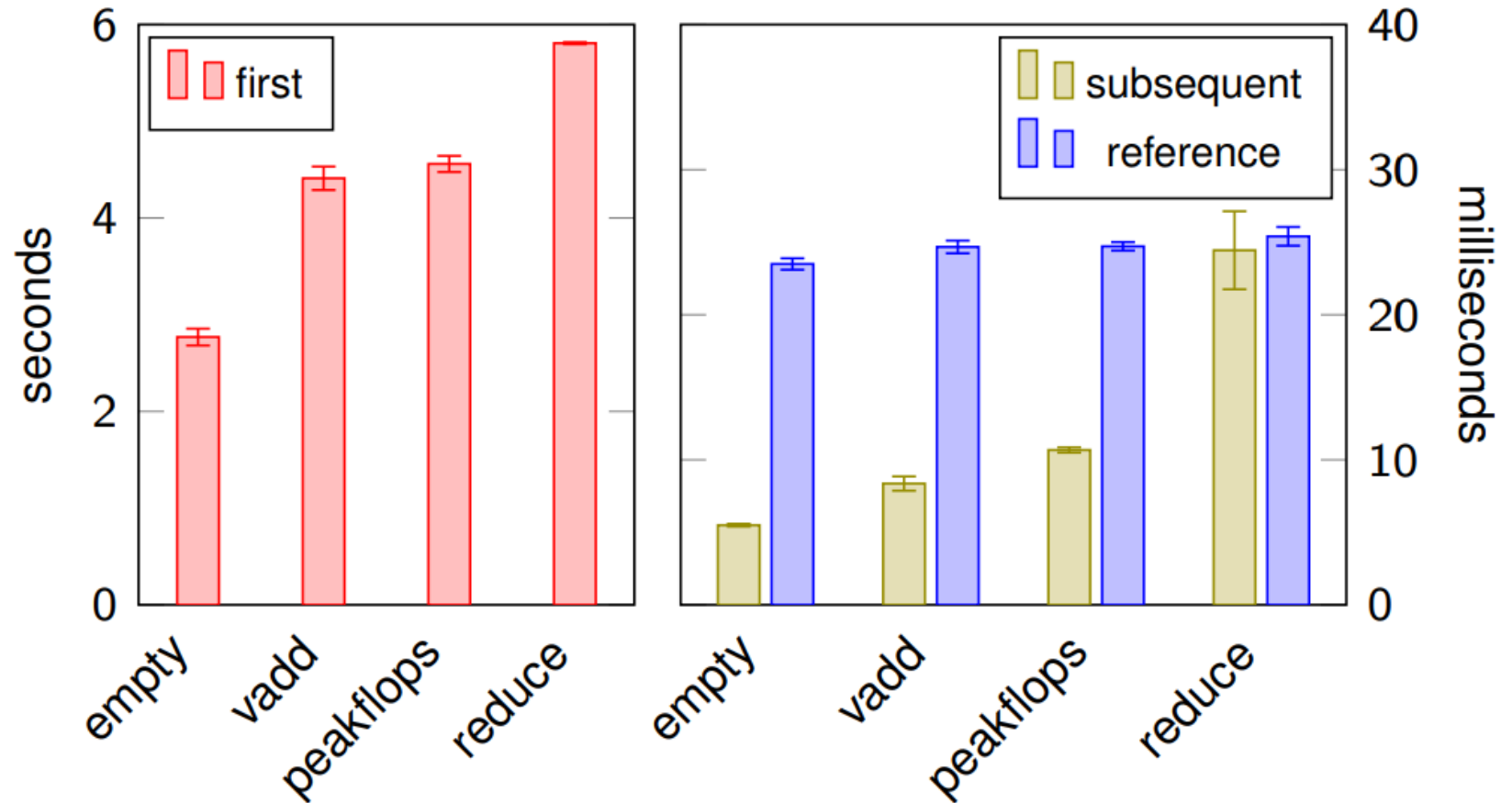
API calls:	91.92%	11.2821s	40	282.05ms	26.070us	1.18052s	cuMemcpyDtoH
	2.23%	273.60ms	3	91.198ms	692ns	273.59ms	cuDevicePrimaryCtxRelease
	2.16%	264.86ms	6	44.143ms	682ns	185.30ms	cudaFree
	0.91%	111.74ms	172	649.66us	2.9950us	9.1952ms	cuModuleUnload
	0.86%	105.85ms	1	105.85ms	105.85ms	105.85ms	cuDevicePrimaryCtxRetain
	0.74%	90.441ms	1346	67.192us	3.4970us	460.51us	cuMemAlloc
	0.25%	30.511ms	3856	7.9120us	5.2800us	246.57us	cudaLaunchKernel

# GPU Performance



- Runtime is relatively flat as the number of components increases up to a point
- We can't fit everything in GPU memory after the number of components reaches ~128
- After this point, we are forced to send data between CPU and GPU every iteration
- Fortunately, the behavior as a function of the audio length is well-behaved

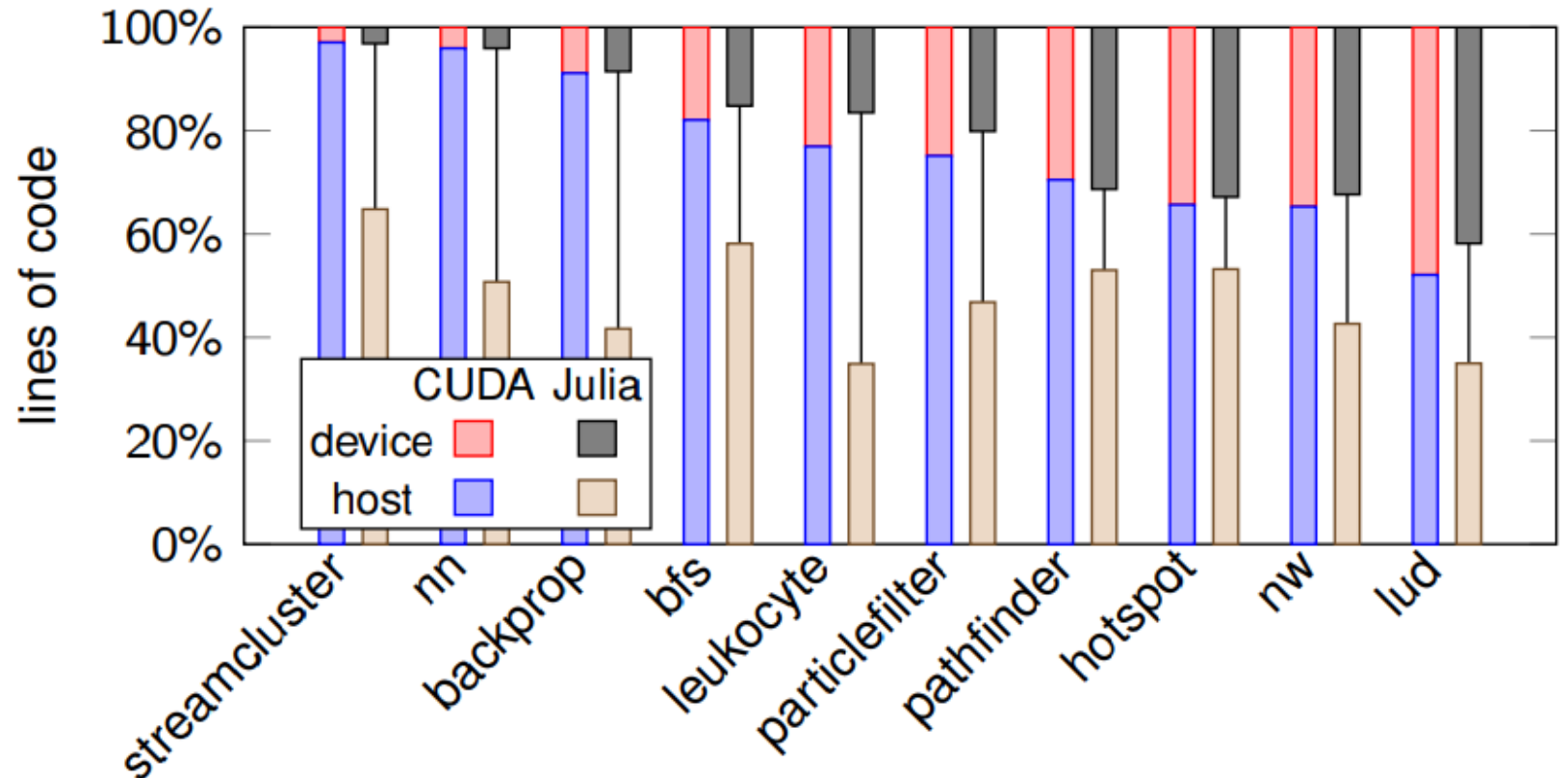
# Compilation Performance



- Left figure shows orders of magnitude slowdown on the initial compilation of code for a variety of commonly used functions
- Right figure shows speedup for subsequent compilations of code

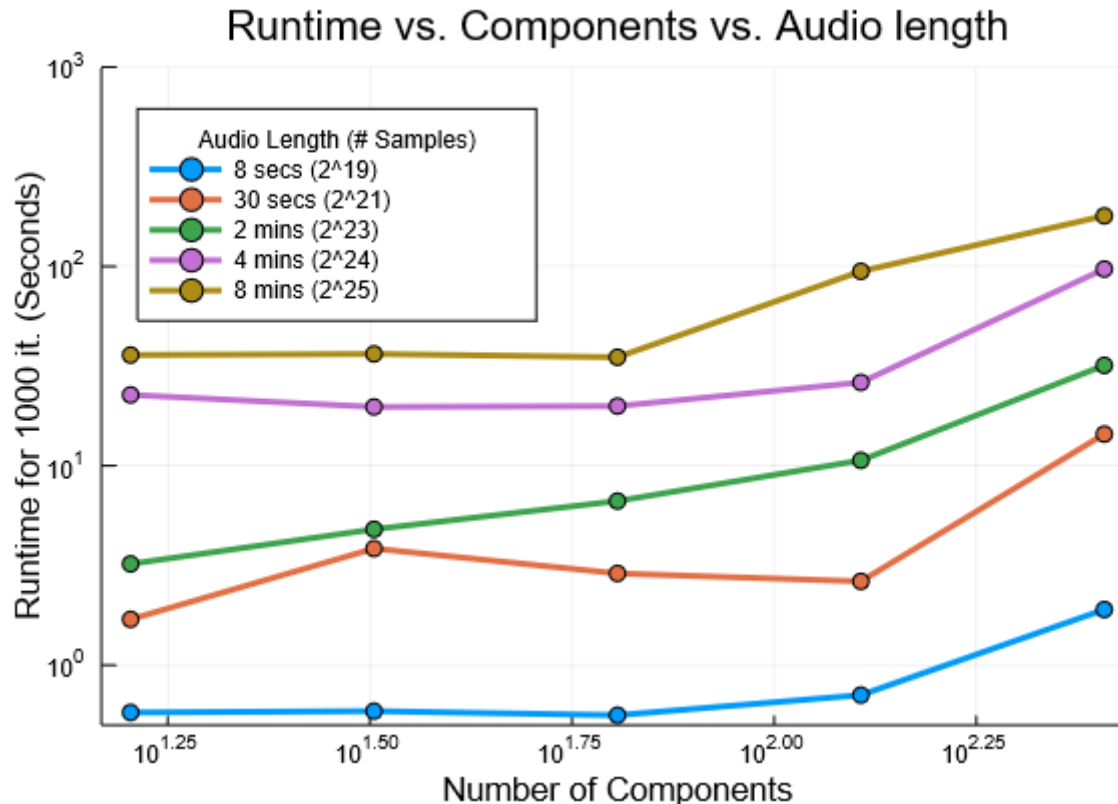
ref. Besard et. al. *Effective Extensible Programming: Unleashing Julia on GPUs*.

# Lines of Code



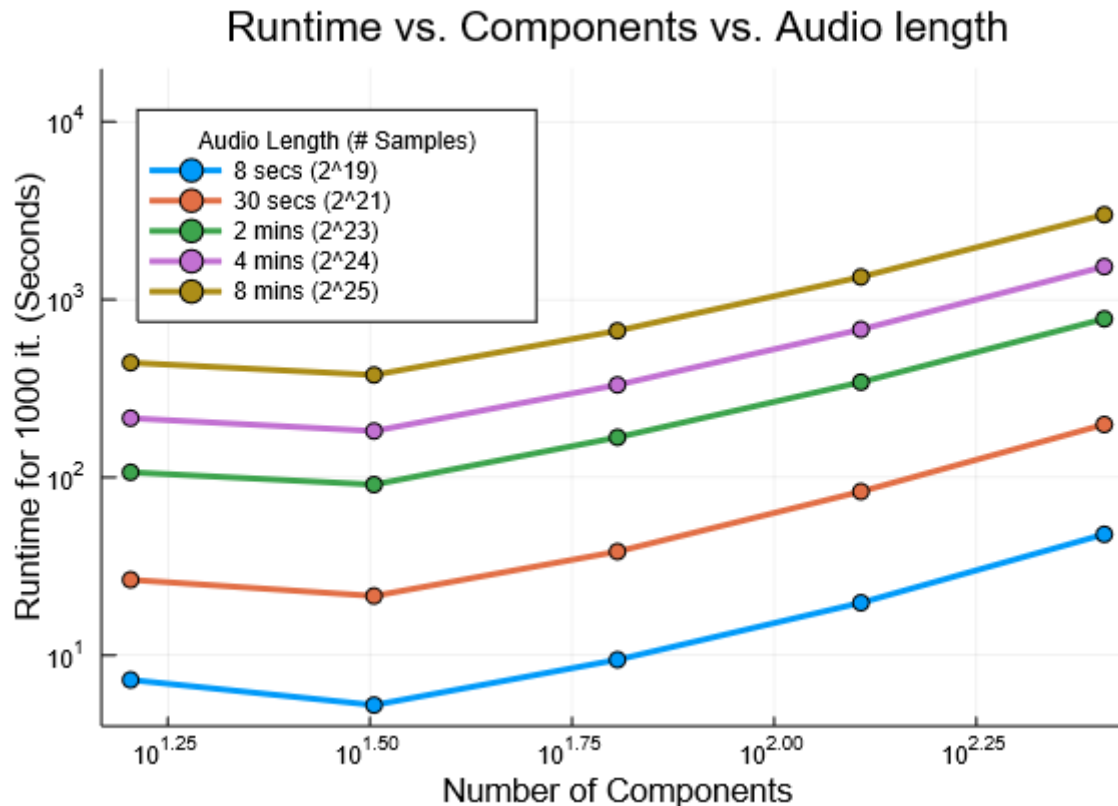
- Lines of host and device code for Rodinia benchmarks. Comparison between CUDA C and Julia implementations.
- On average, device code reduced by 8 percent and host code by 38 percent.

# GPU Performance



- Runtime is relatively flat as the number of components increases up to a point
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- After this point, we are forced to send data between CPU and GPU every iteration
- Fortunately, the behavior as a function of the audio length is well-behaved

# CPU Performance



- Scikit-learn implementation would require many CPU cores to match GPU runtime



### GPU

$k \backslash n$	8 secs	30 secs	2 mins	4 mins	8 mins
16	7.27467	26.5809	106.867	215.206	441.376
32	5.25291	21.563	91.1168	182.27	376.61
64	9.44327	38.2992	167.939	331.227	667.127
128	19.7414	83.2758	343.023	678.67	1339.14
256	47.8782	198.339	778.588	1531.27	2997.77

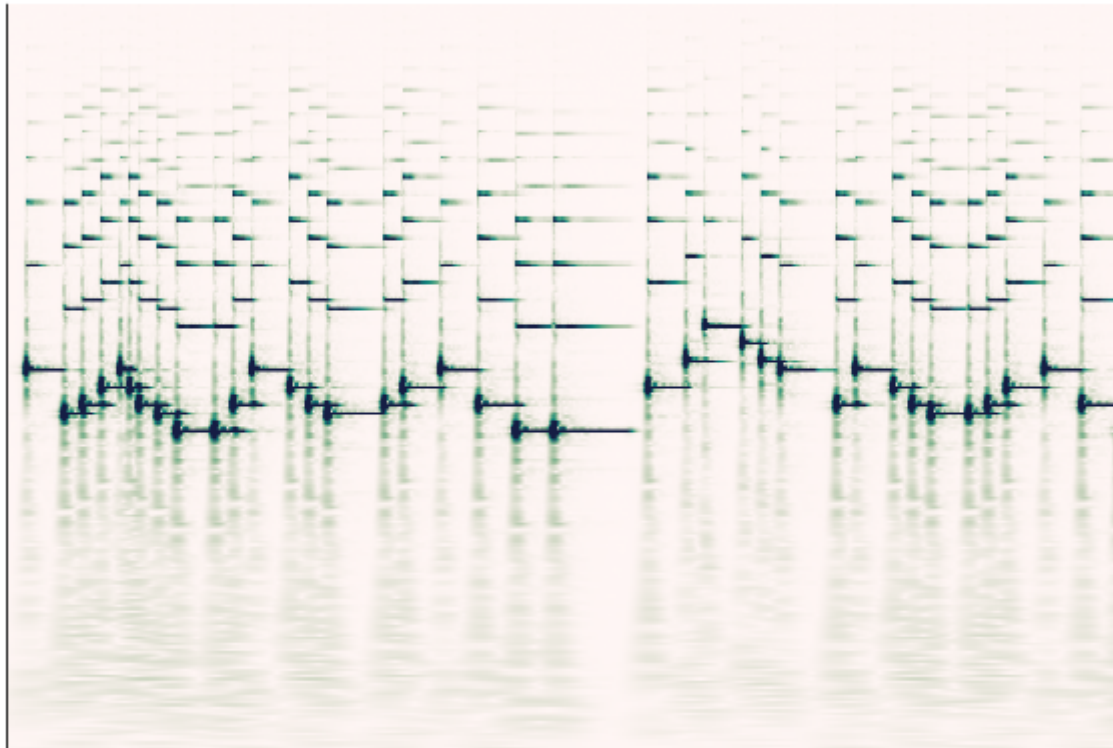
### Scikit-learn (8 Cores, OpenBLAS)

$k \backslash n$	8 secs	30 secs	2 mins	4 mins	8 mins
16	0.57850	1.69655	3.21816	22.6694	35.9361
32	0.58758	3.84272	4.80095	19.733	36.3876
64	0.56073	2.88254	6.65211	19.9168	35.0325
128	0.70719	2.63012	10.6433	26.1736	94.5093
256	1.90019	4.4077	31.8833	97.0318	179.238

Runtime (in seconds) vs number of components ( $k$ ) vs length of processed audio ( $n$ )

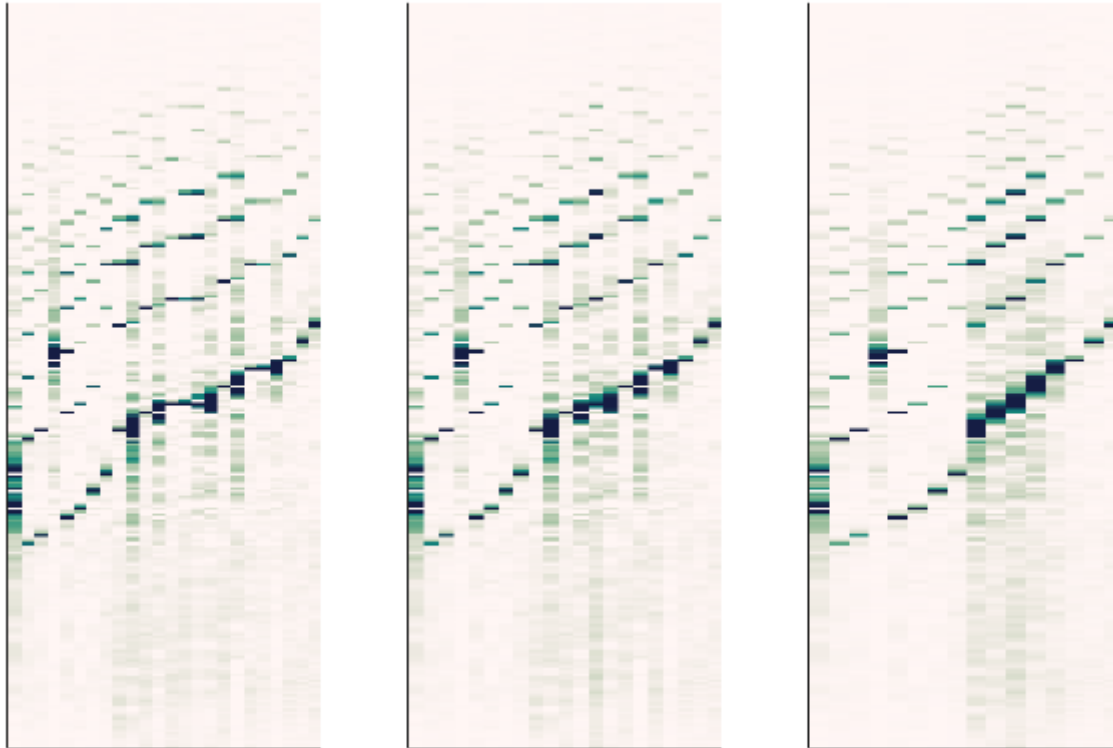
## Demonstration

$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



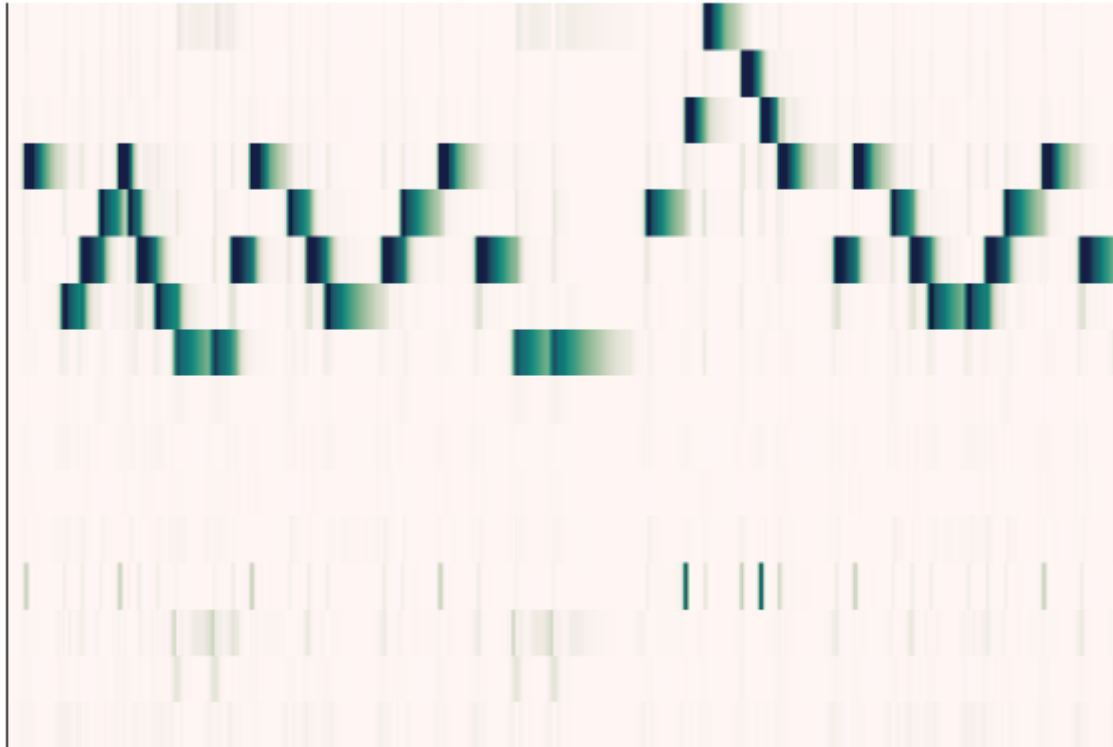
*Time-frequency representation of 'Korobeiniki' (monophonic).*

# Demonstration



*$W$  matrix arranged by fundamental frequency*

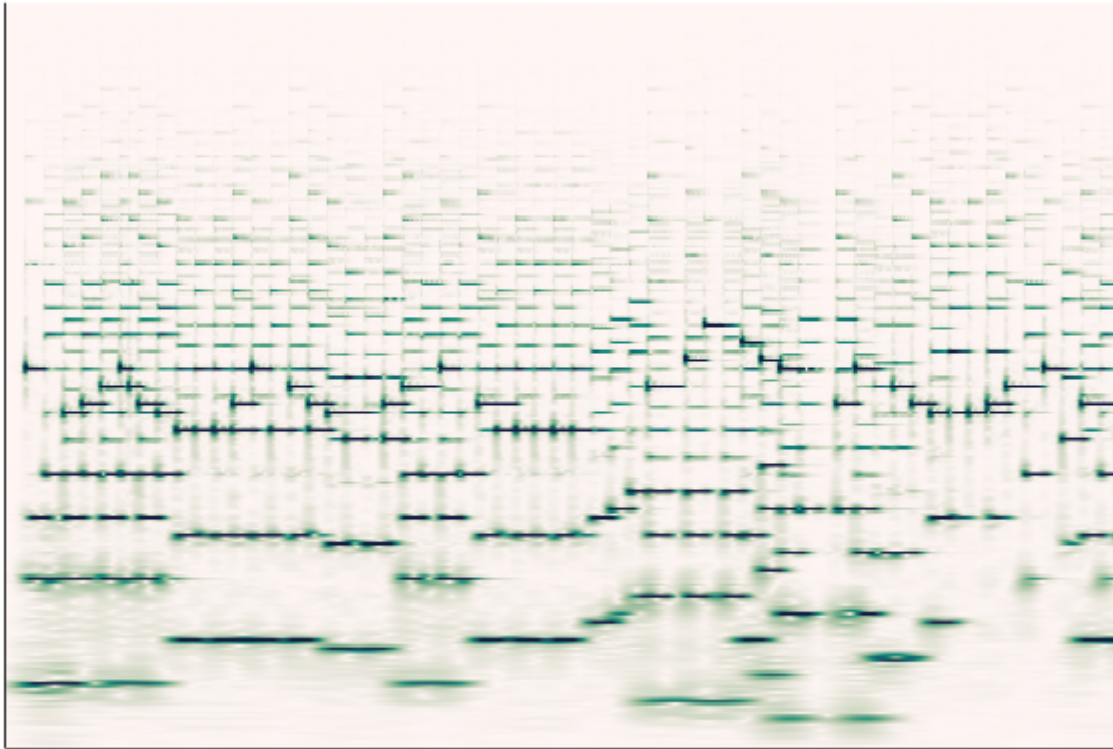
# Demonstration



*$H$  matrix collapsed into midi note bins*

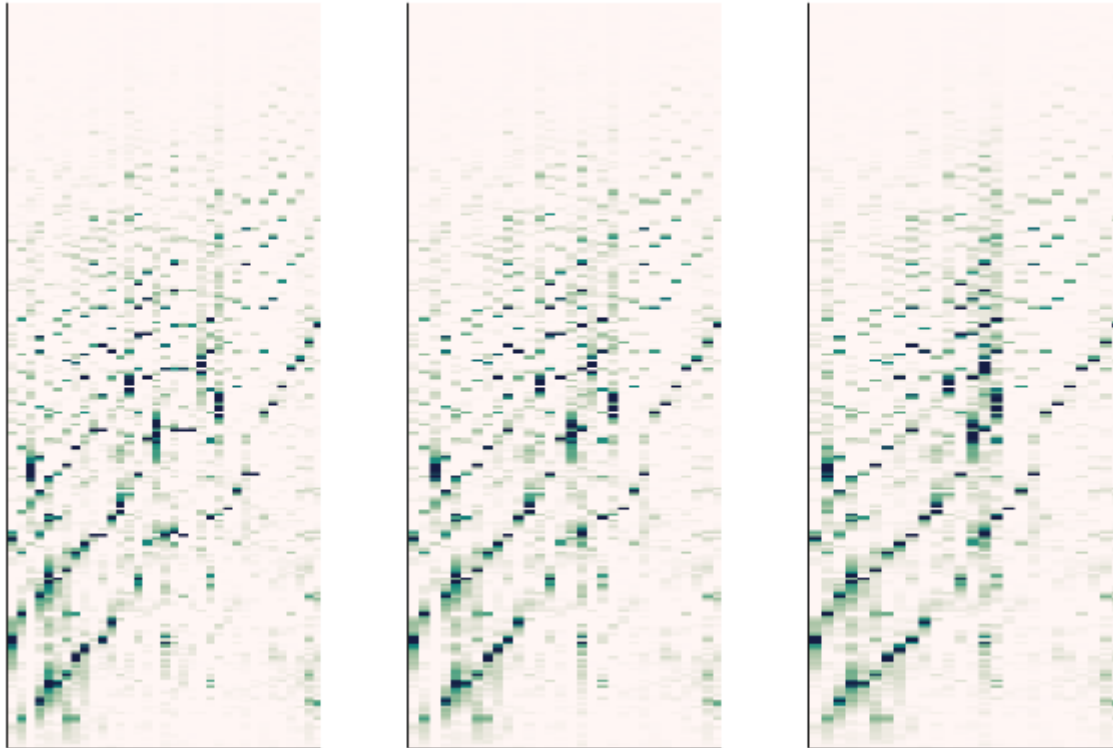
## Demonstration

$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



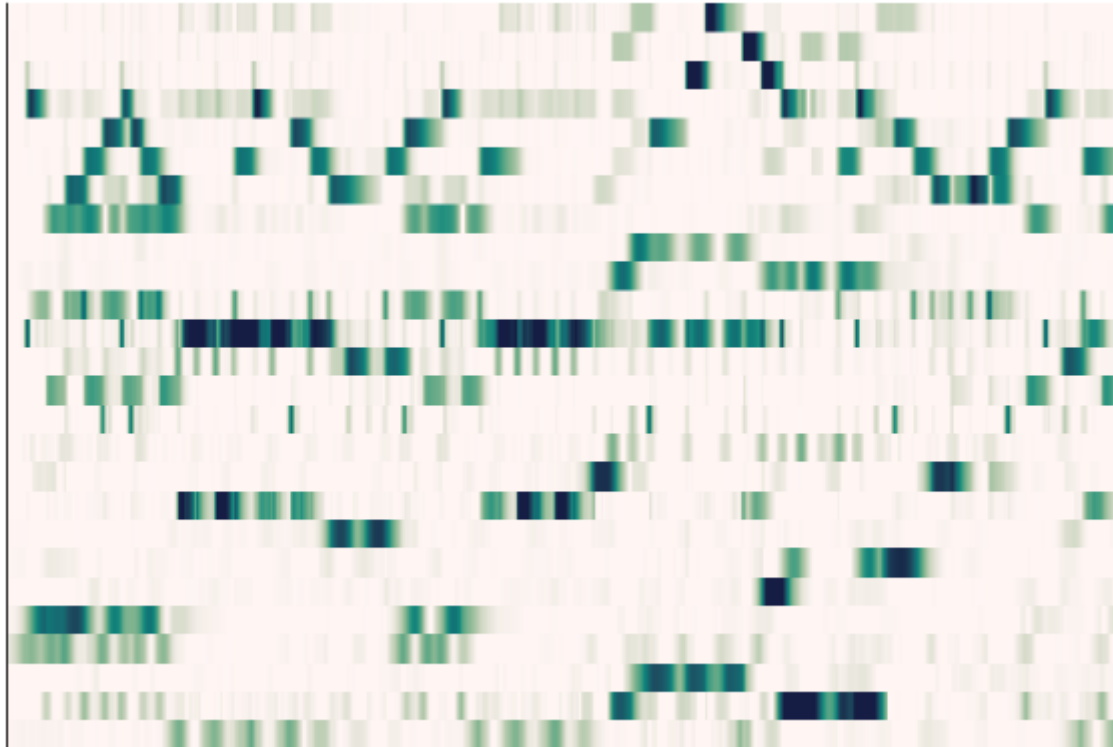
*Time-frequency representation of 'Korobeiniki' (polyphonic).*

# Demonstration



*$W$  matrix arranged by fundamental frequency*

## Demonstration



*$H$  matrix collapsed into midi note bins*

# Backup

