Nonnegative Matrix Factorisation for Audio Applications

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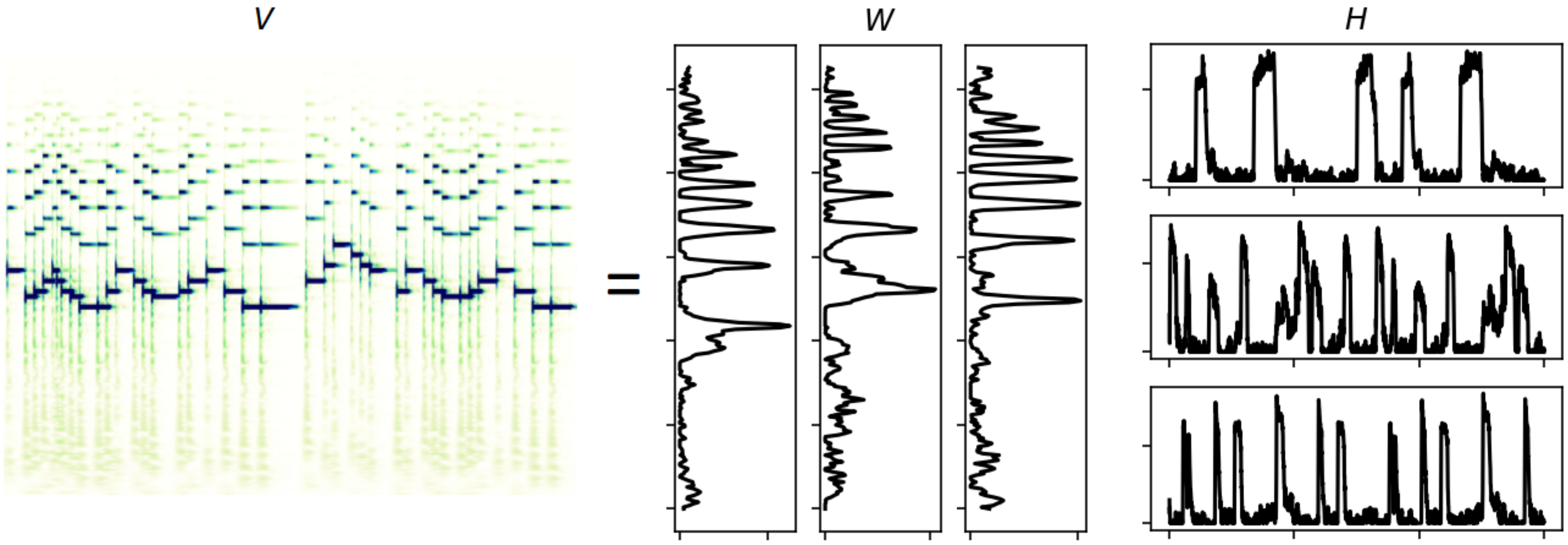


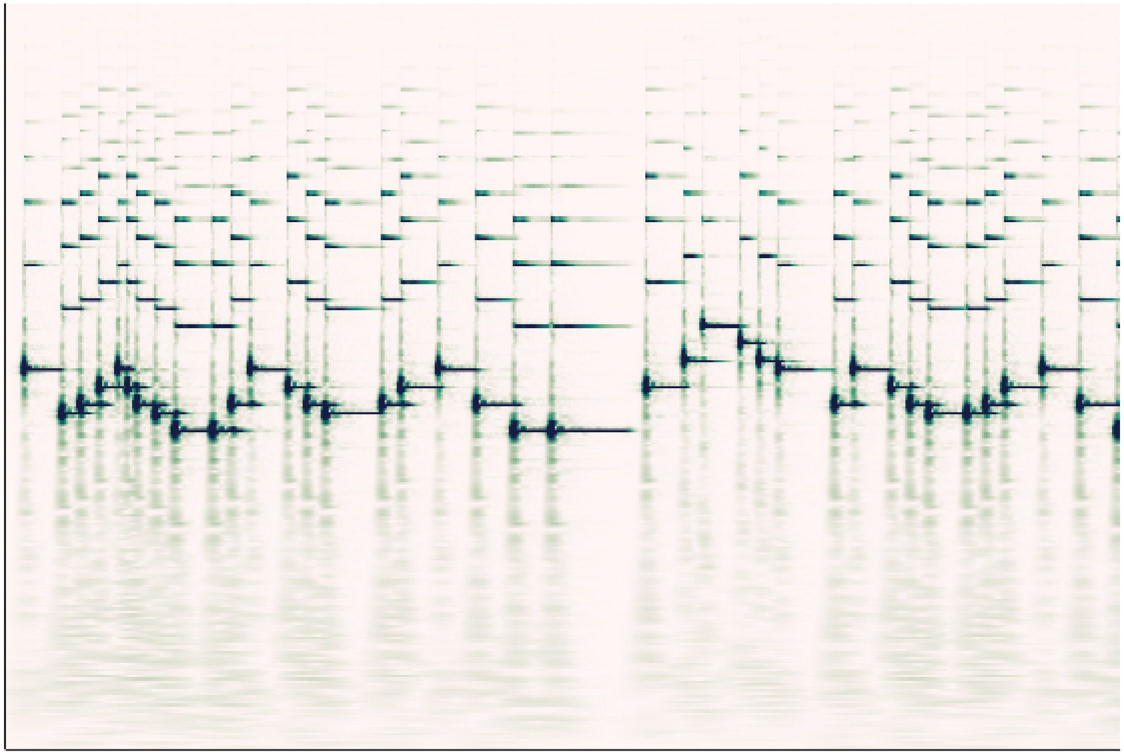
Nonnegative Matrix Factorisation

$$\mathbf{V} \approx \hat{\mathbf{V}} = \mathbf{WH}$$
$$\mathbf{W} \in \mathbb{R}^{\mathbf{m} \times \mathbf{k}} \quad \mathbf{W} \geq \mathbf{0}$$
$$\mathbf{H} \in \mathbb{R}^{\mathbf{k} \times \mathbf{n}} \quad \mathbf{H} \geq \mathbf{0}$$

- Unsupervised learning (think SVD)
- Number of components k is a parameter
- $\mathbf{V}$ is $m \times n$
- Setting $k < m$ is a form of lossy compression

Audio Example

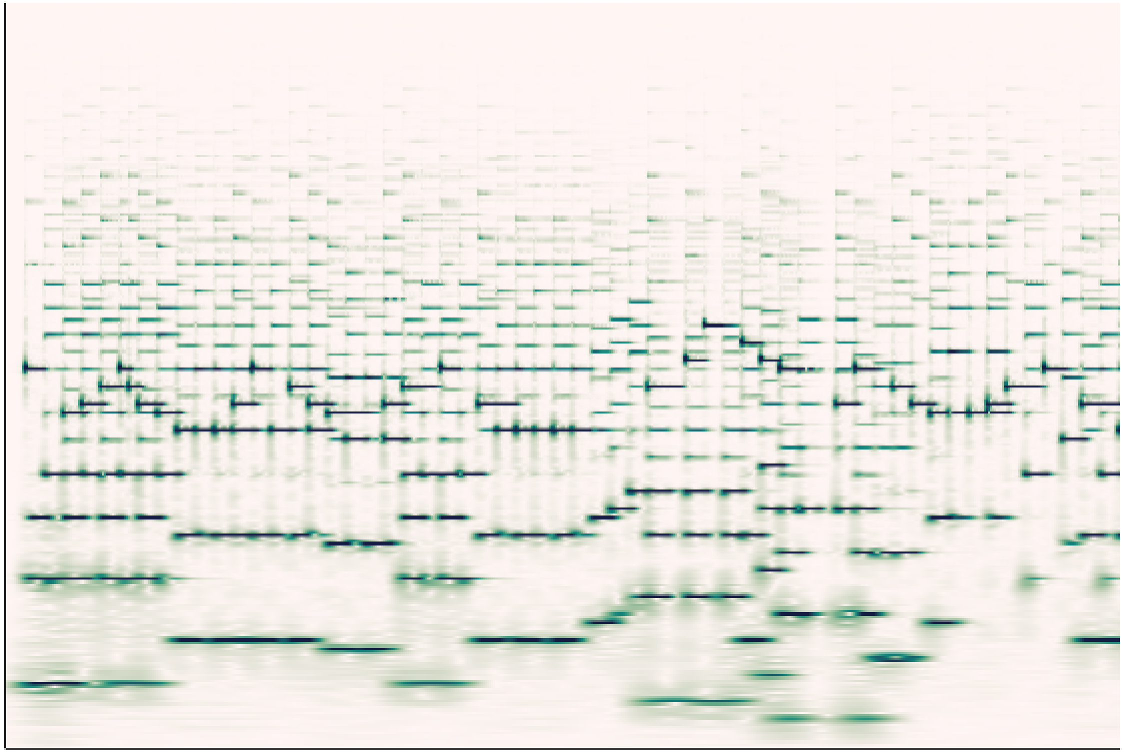
$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



Time-frequency representation of 'Korobeiniki' played on piano. The frequency is on a logarithmic scale.

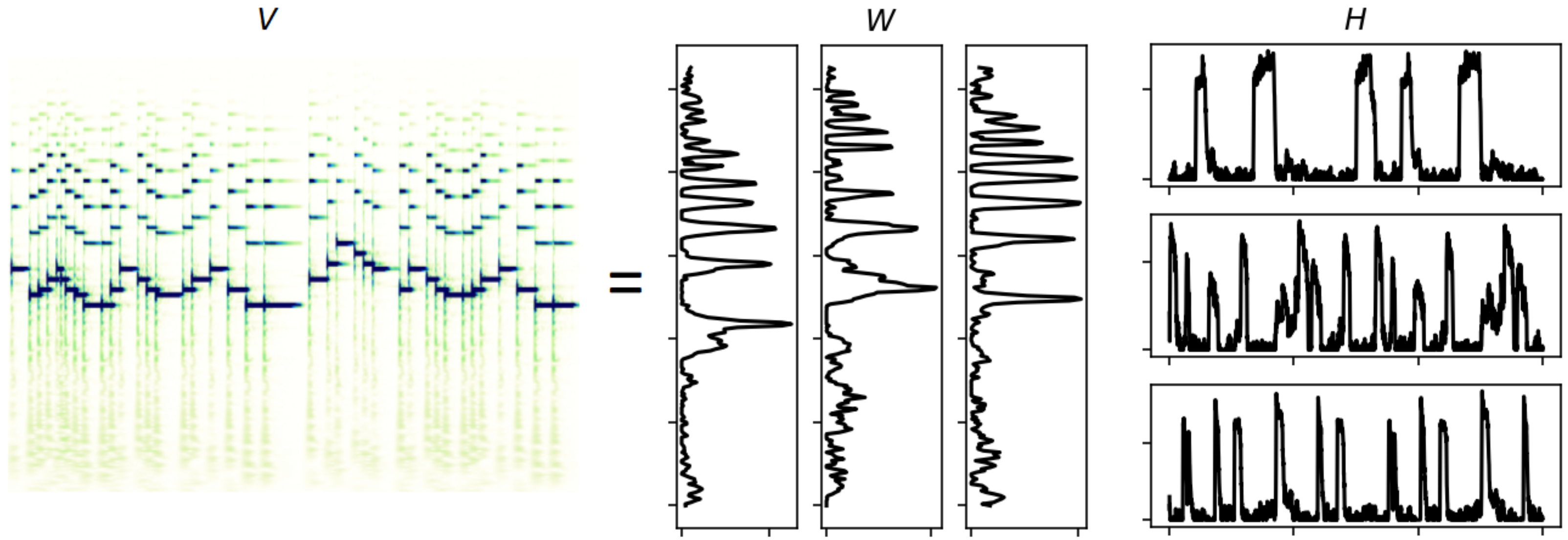
Audio Example

$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



Time-frequency representation of 'Korobeiniki' (polyphonic).

Audio Example



Algorithm: Multiplicative Update

- Initialize $\mathbf{W}$ and $\mathbf{H}$ with non-negative values
- Iteratively update $\mathbf{W}$ and $\mathbf{H}$ using the following rules: (n here is the iteration)

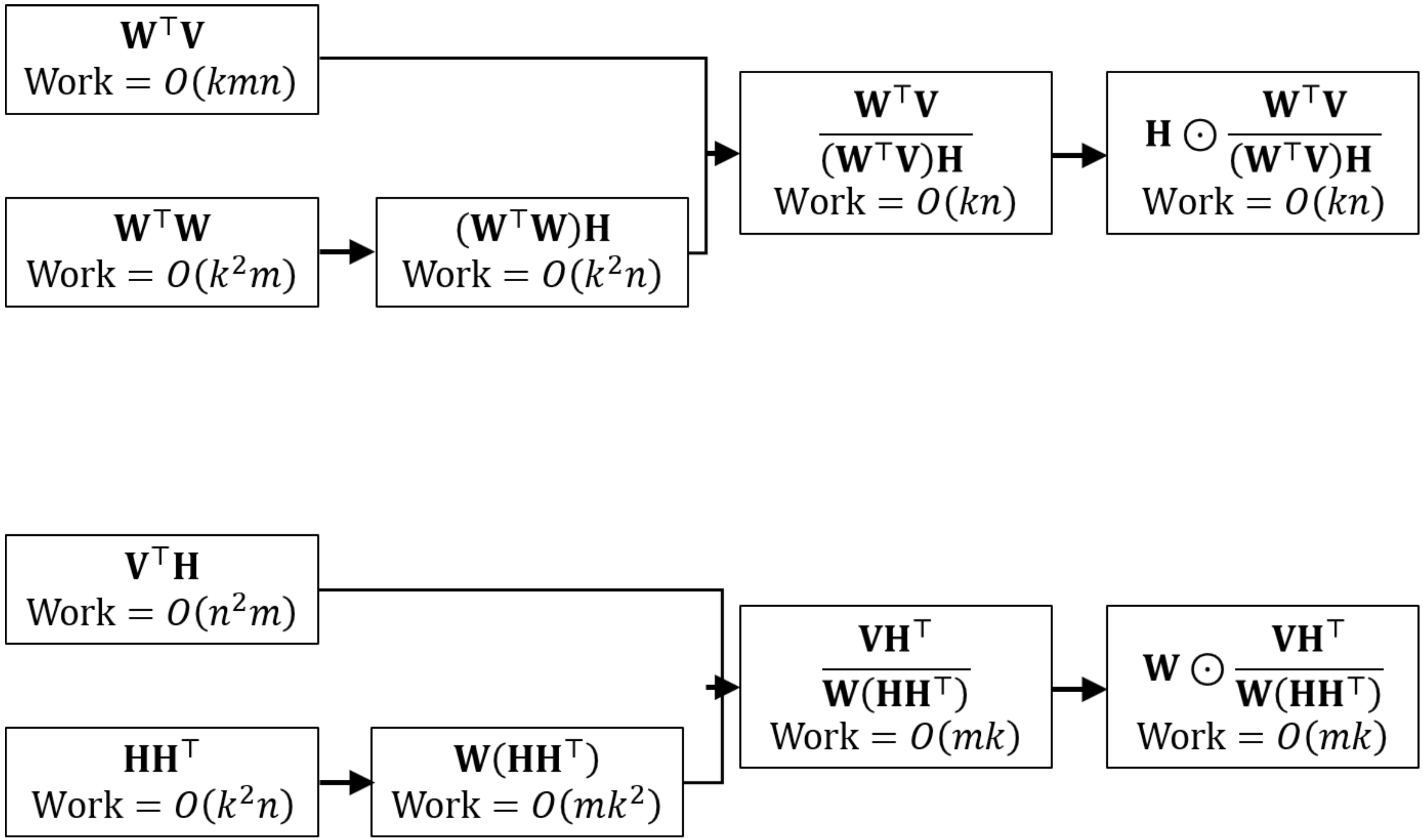
$$\mathbf{H}^{n+1}_{[i,j]} \leftarrow \mathbf{H}^{n}_{[i,j]} \odot \frac{\left((\mathbf{W}^n)^\top \mathbf{V}\right)_{[i,j]}}{((\mathbf{W}^n)^\top \mathbf{W}^n \mathbf{H}^n)_{[i,j]}}$$

$$\mathbf{W}^{n+1}_{[i,j]} \leftarrow \mathbf{W}^{n}_{[i,j]} \odot \frac{\left(\mathbf{V}(\mathbf{H}^{n+1})^\top\right)_{[i,j]}}{\left(\mathbf{W}^n \mathbf{H}^{n+1} (\mathbf{H}^{n+1})^\top\right)_{[i,j]}}$$

$\odot$ and division are element-wise.

ref. Lee, D.D., Seung, H.S., 2001. Algorithms for Non-negative Matrix Factorization, in: Advances in Neural Information Processing Systems 13. MIT Press, pp. 556–562.

Algorithm: Multiplicative Update



Parallelization on GPU : Motivation

- Matricies remain stationary in memory
- Matrix multiplies have high computational intensity compared to memory
- Single precision is sufficient
- Would like to use consumer hardware

Name	Clock(MHz)	GFLOPS(FP32)
Adreno 616	750	384
Adreno 630	710	727
Adreno 640	585	899

GPU Implementation

- Used Julia bindings to CUDA (CURAND, CUBLAS)
- Compiler allows easily mapping high level syntax to GPU
- Compiler finds best way to send data back and forth between device and host

```
a = CuArray([1., 2., 3.])
function apply(op, a)
   i = threadIdx().x
   a[i] = op(a[i])
return
end
@cuda threads=length(a) apply(x->x^2, a)


julia> a
3-element CuArray{Float32,1}:
1.0
4.0
9.0
```

```
@device_code_ptx @cuda apply(x->x^2, a)
apply(.param .b8 a[16])
{
        ld.param.u64     %rd1, [a+8];
        mov.u32         %r1, %tid.x;

        // index calculation
        mul.wide.u32     %rd2, %r1, 4;
        add.s64         %rd3, %rd1, %rd2;
        cvta.to.global.u64      %rd4, %rd3;

        ld.global.f32    %f1, [%rd4];
        mul.f32         %f2, %f1, %f1;
        st.global.f32    [%rd4], %f2;

        ret;
}
```

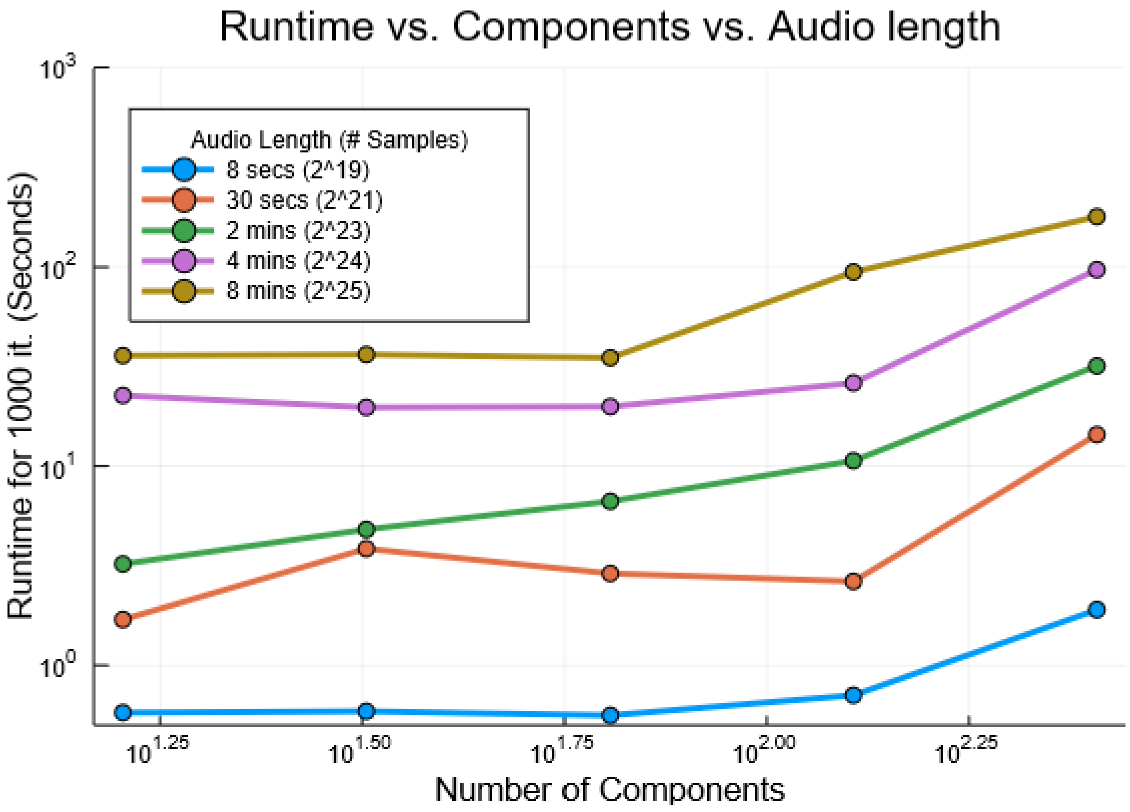
ref. https://github.com/JuliaGPU/CUDAnative.jl
(https://github.com/JuliaGPU/CUDAnative.jl)

GPU Implementation

- Must be careful and explicit with types and constants
- Otherwise, compiler will think you want to move all of the data off of the GPU and back

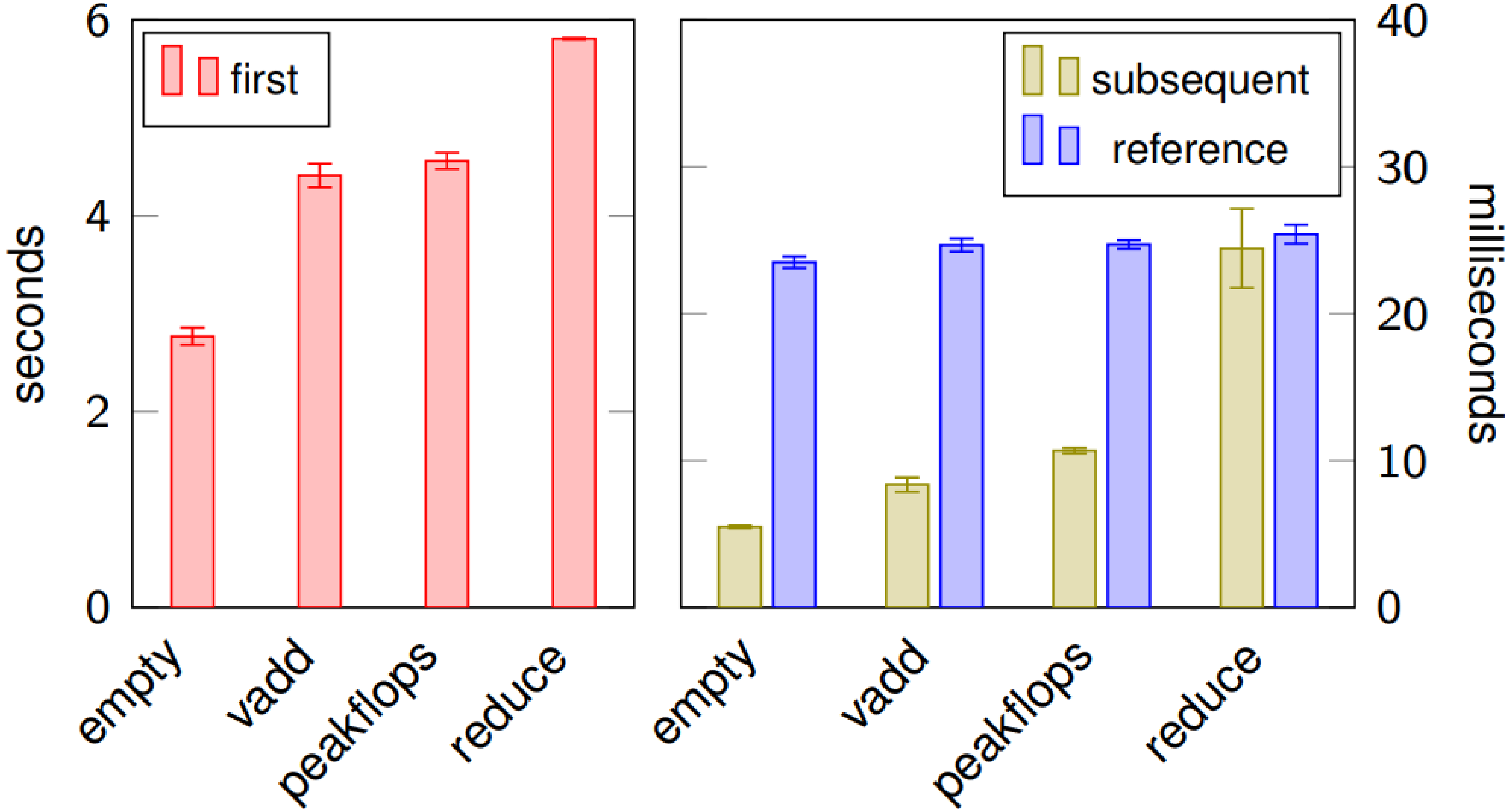
```
API calls:   91.92% 11.2821s    40 282.05ms  26.070us 1.18052s  cuMemcpyDtoH
              2.23% 273.60ms     3 91.198ms  692ns     273.59ms  cuDevicePrimaryCtxRelease
              2.16% 264.86ms     6 44.143ms  682ns     185.30ms  cudaFree
              0.91% 111.74ms   172 649.66us  2.9950us 9.1952ms  cuModuleUnload
              0.86% 105.85ms     1 105.85ms  105.85ms 105.85ms  cuDevicePrimaryCtxRetain
              0.74% 90.441ms 1346  67.192us 3.4970us 460.51us  cuMemAlloc
              0.25% 30.511ms 3856  7.9120us 5.2800us 246.57us  cudaLaunchKernel
```

GPU Performance



- Runtime is relatively flat as the number of components increases up to a point
- We can't fit everying in GPU memory after the number of components reaches ~128
- After this point, we are forced to send data between CPU and GPU *every* iteration
- Fortunately, the behavior as a function of the audio length is well-behaved

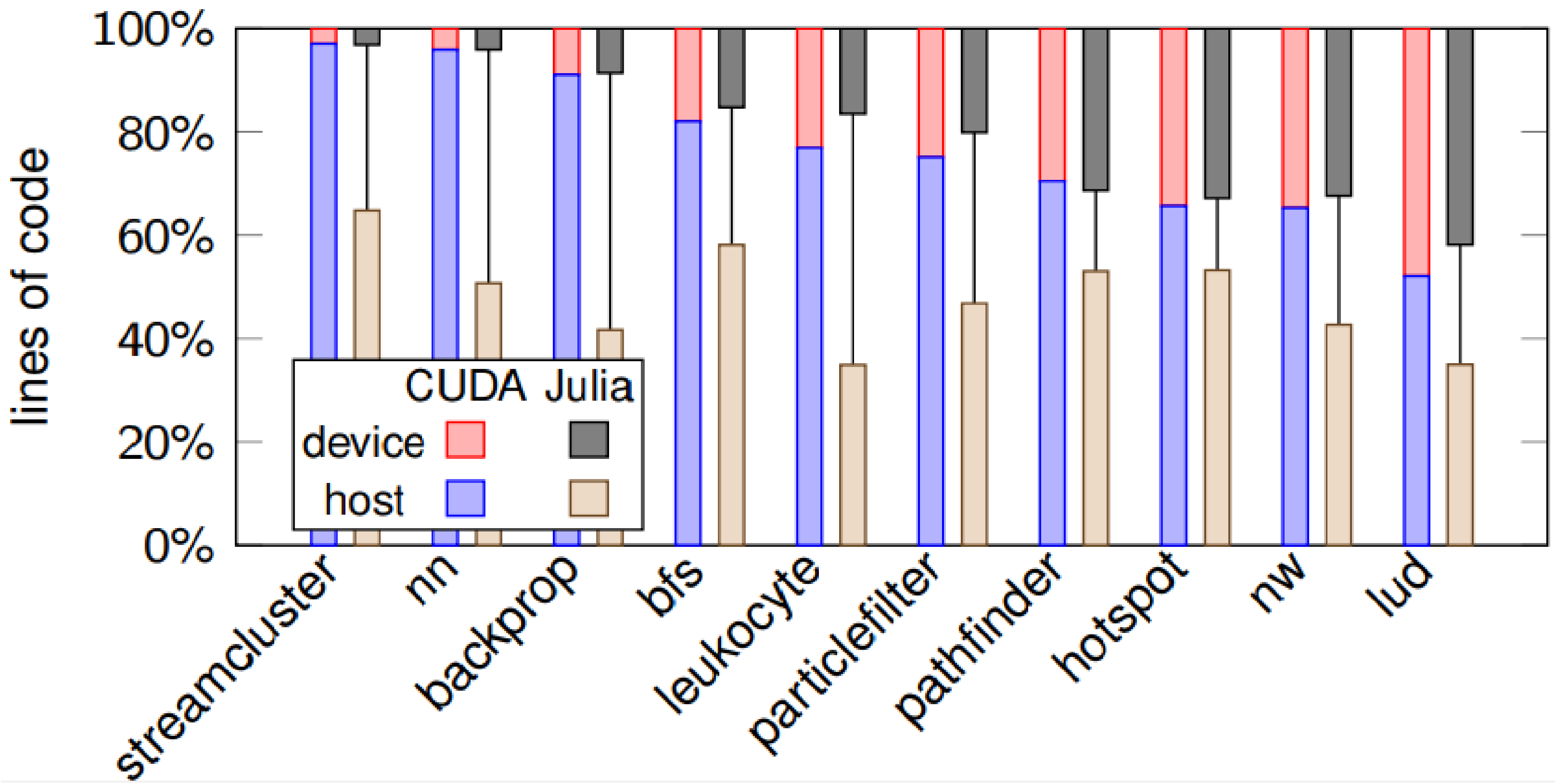
Compilation Performance



- Left figure shows orders of magnitude slowdown on the initial compilation of code for a variety of commonly used functions
- Right figure shows speedup for subsequent compilations of code

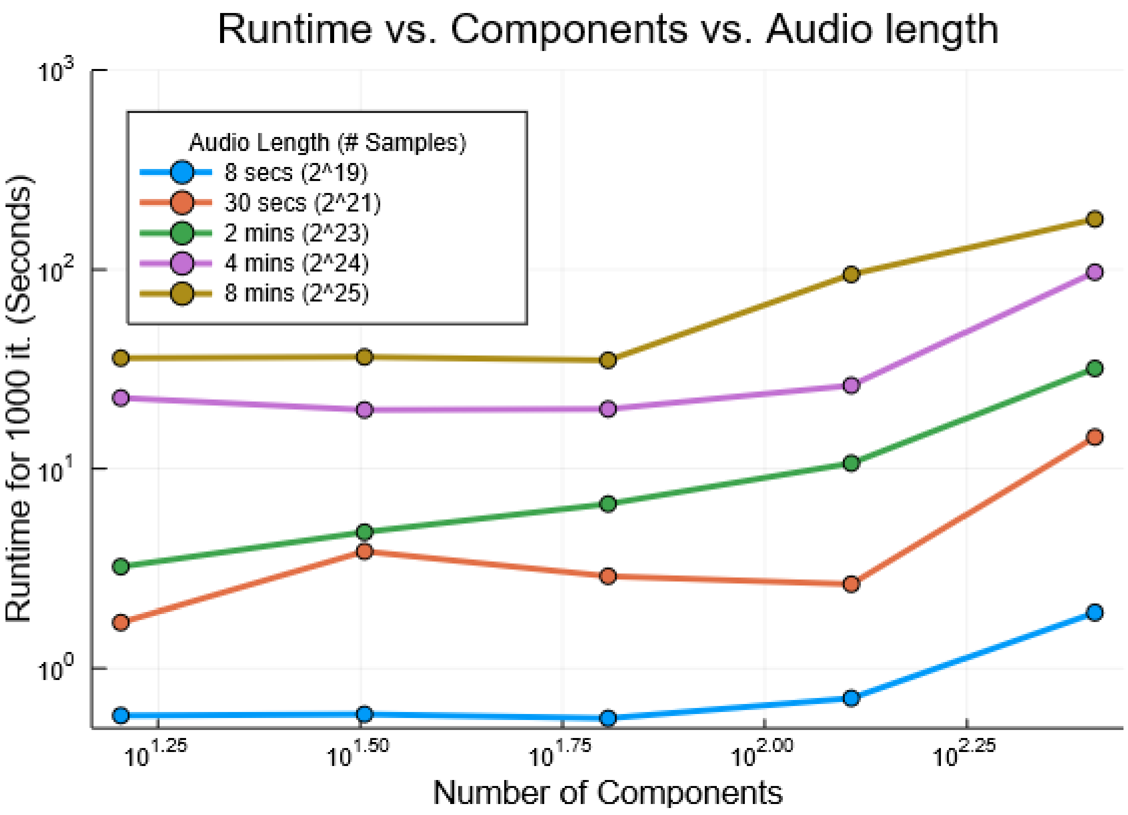
ref. Besard et. al. *Effective Extensible Programming: Unleashing Julia on GPUs*.

Lines of Code



- Lines of host and device code for Rodinia benchmarks. Comparison between CUDA C and Julia implementations.
- On average, device code reduced by 8 percent and host code by 38 percent.

GPU Performance



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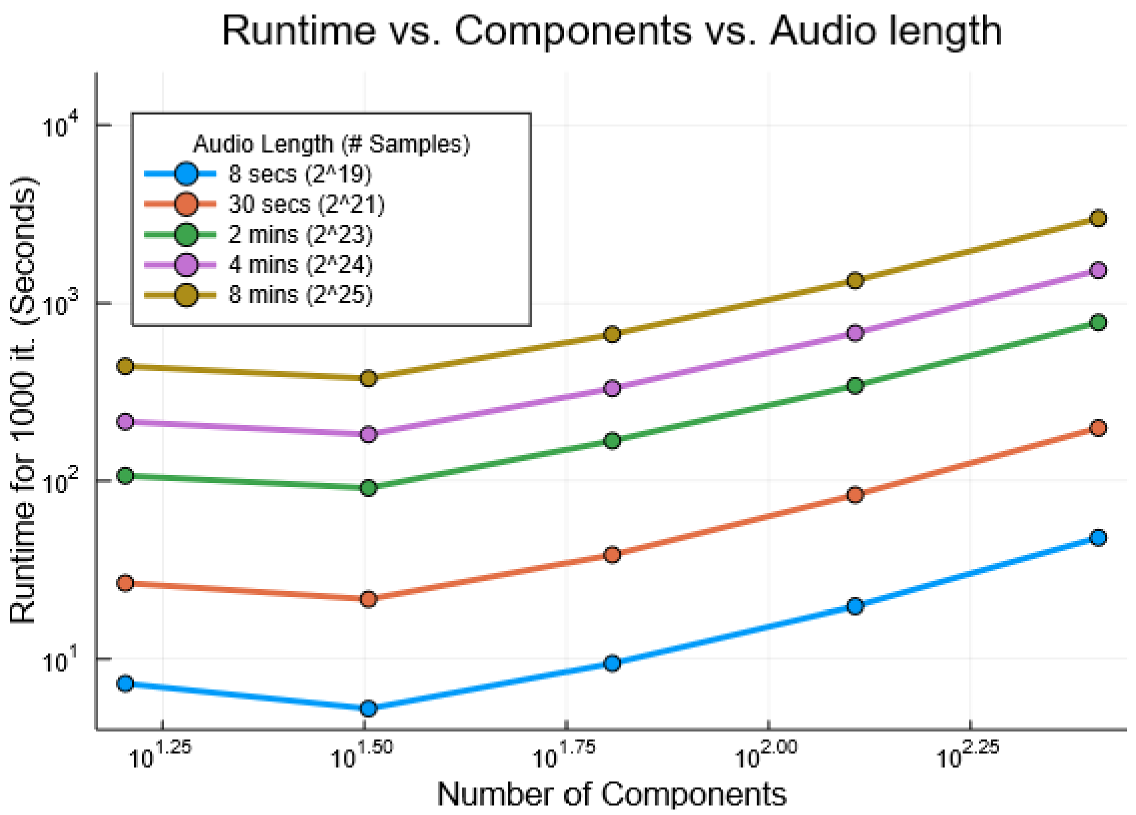
CPU Performance



- Scikit-learn implementation would require many CPU cores to match GPU runtime

GPU

$k \backslash n$	8 secs	30 secs	2 mins	4 mins	8 mins
16	7.27467	26.5809	106.867	215.206	441.376
32	5.25291	21.563	91.1168	182.27	376.61
64	9.44327	38.2992	167.939	331.227	667.127
128	19.7414	83.2758	343.023	678.67	1339.14
256	47.8782	198.339	778.588	1531.27	2997.77

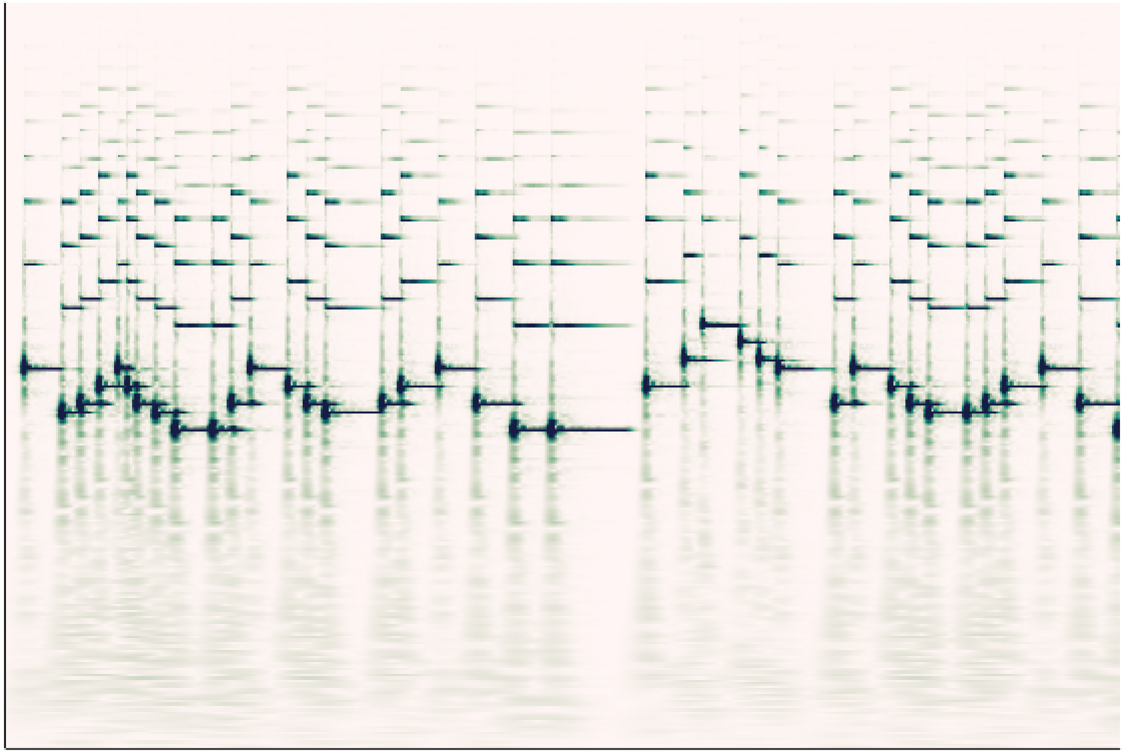
Scikit-learn (8 Cores, OpenBLAS)

$k \backslash n$	8 secs	30 secs	2 mins	4 mins	8 mins
16	0.57850	1.69655	3.21816	22.6694	35.9361
32	0.58758	3.84272	4.80095	19.733	36.3876
64	0.56073	2.88254	6.65211	19.9168	35.0325
128	0.70719	2.63012	10.6433	26.1736	94.5093
256	1.90019	4.4077	31.8833	97.0318	179.238

Runtime (in seconds) vs number of components (k) vs length of processed audio (n)

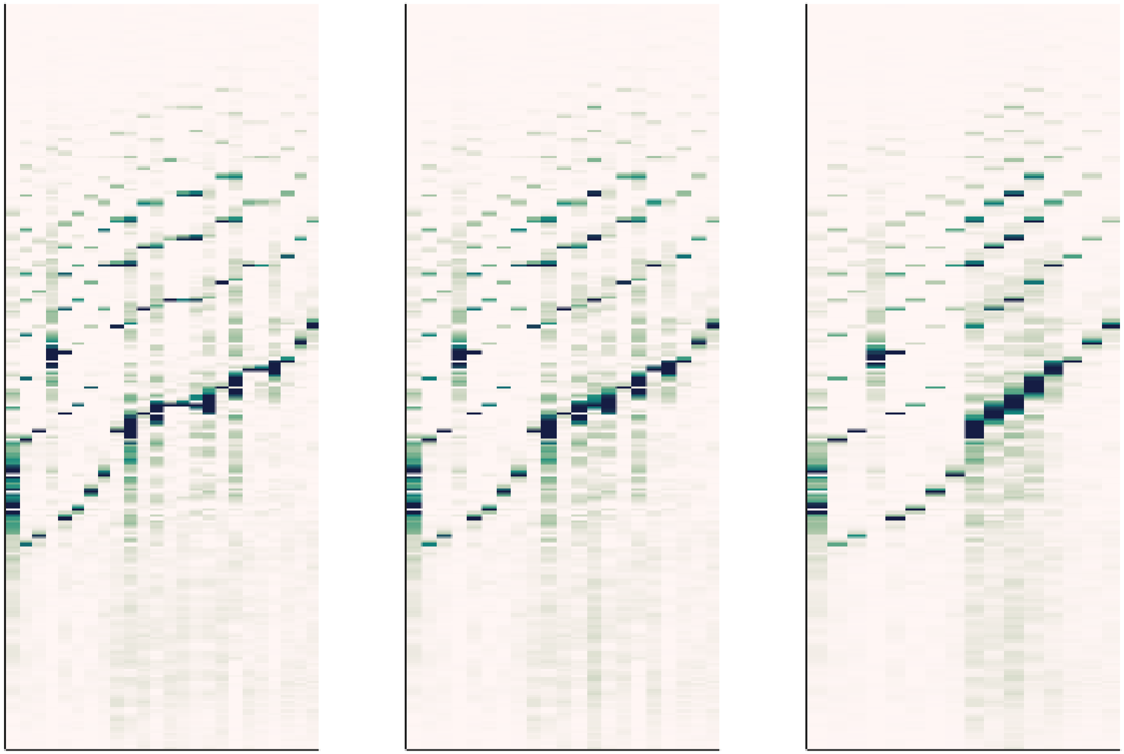
Demonstration

$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



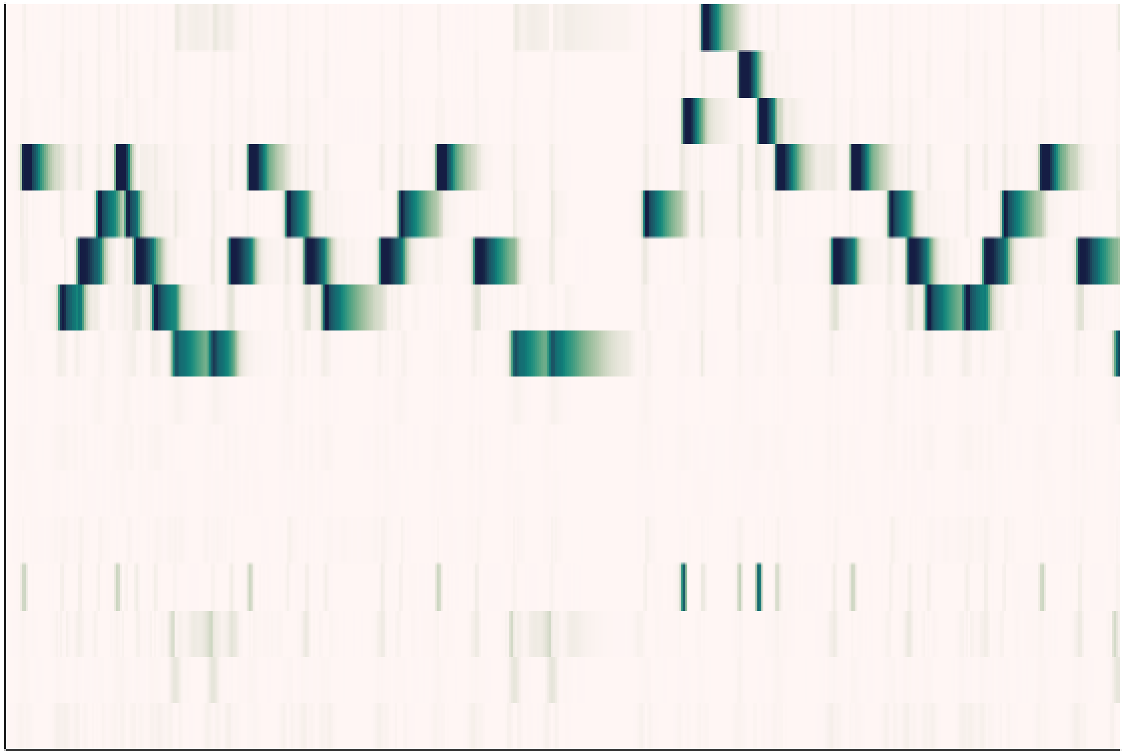
Time-frequency representation of 'Korobeiniki' (monophonic).

Demonstration



$\boldsymbol{W}$ *matrix arranged by fundamental frequency*

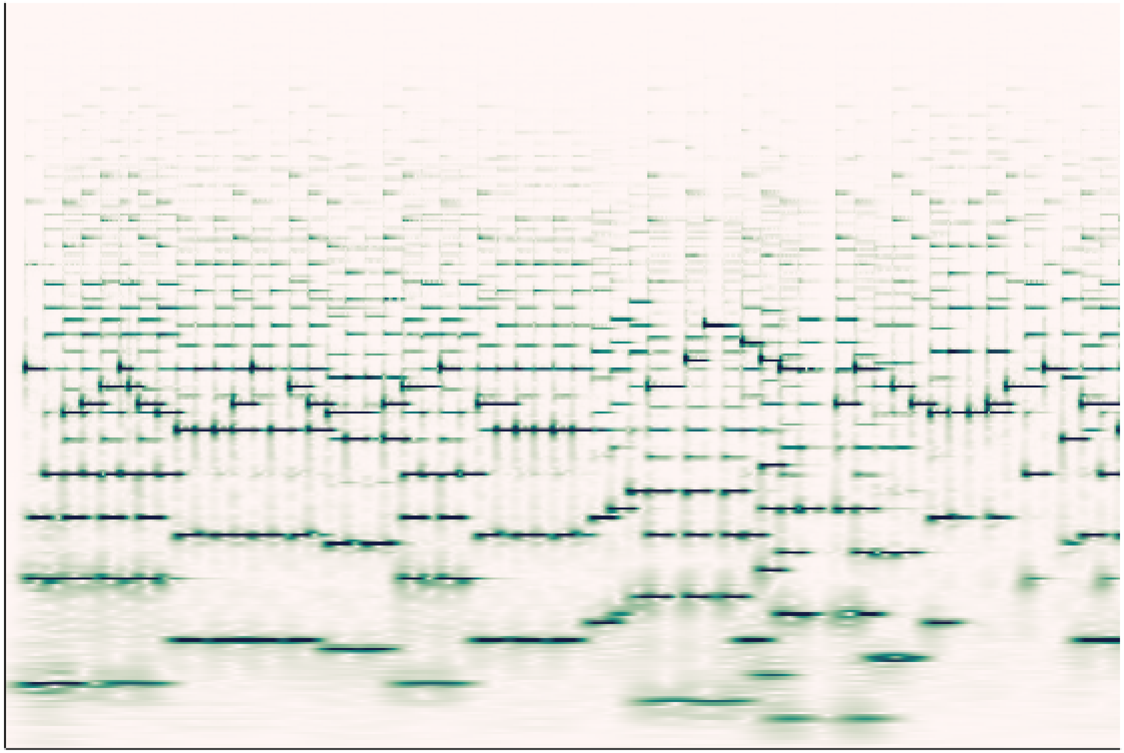
Demonstration



$\boldsymbol{H}$ *matrix collapsed into midi note bins*

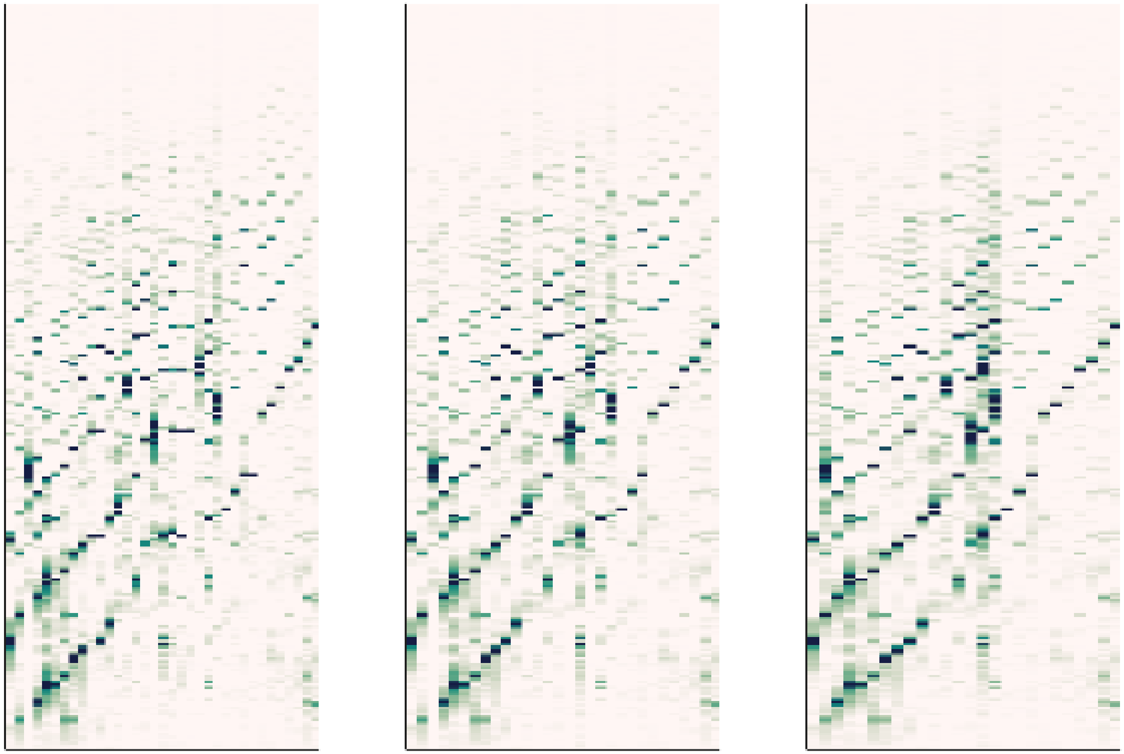
Demonstration

$$\mathbf{V} = |\mathbf{CQT}(\mathbf{x})| =$$



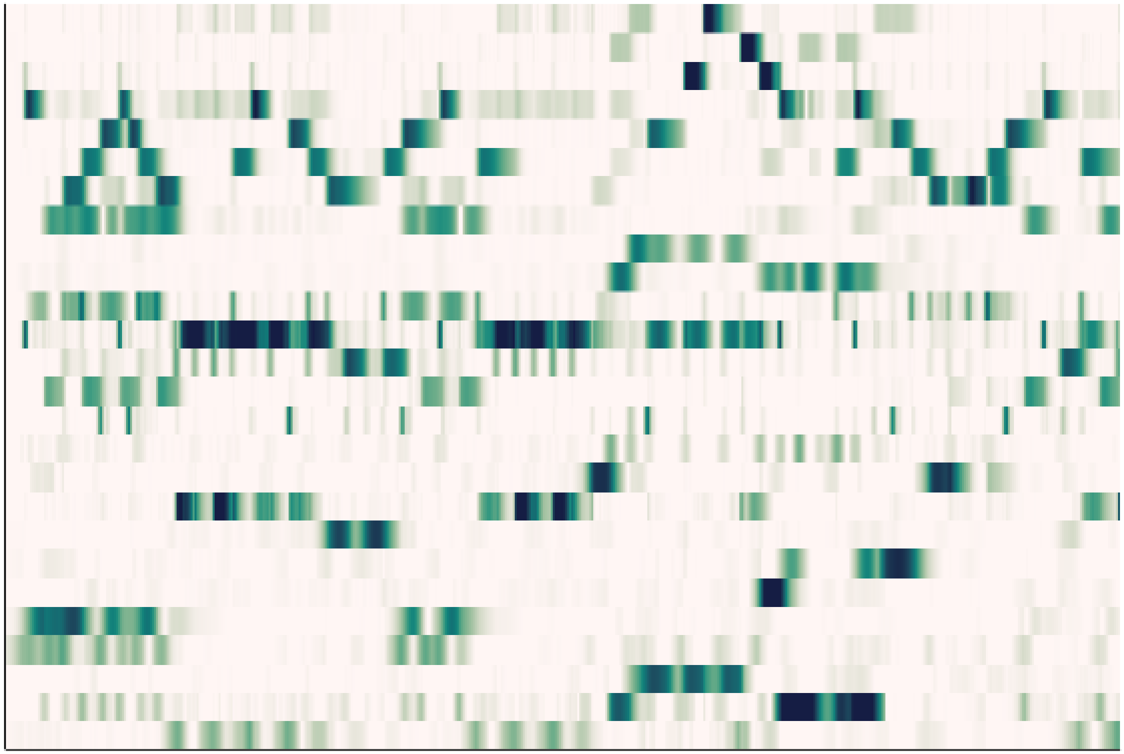
Time-frequency representation of 'Korobeiniki' (polyphonic).

Demonstration



$\boldsymbol{W}$ matrix arranged by fundamental frequency

Demonstration



$\boldsymbol{H}$ *matrix collapsed into midi note bins*

Backup