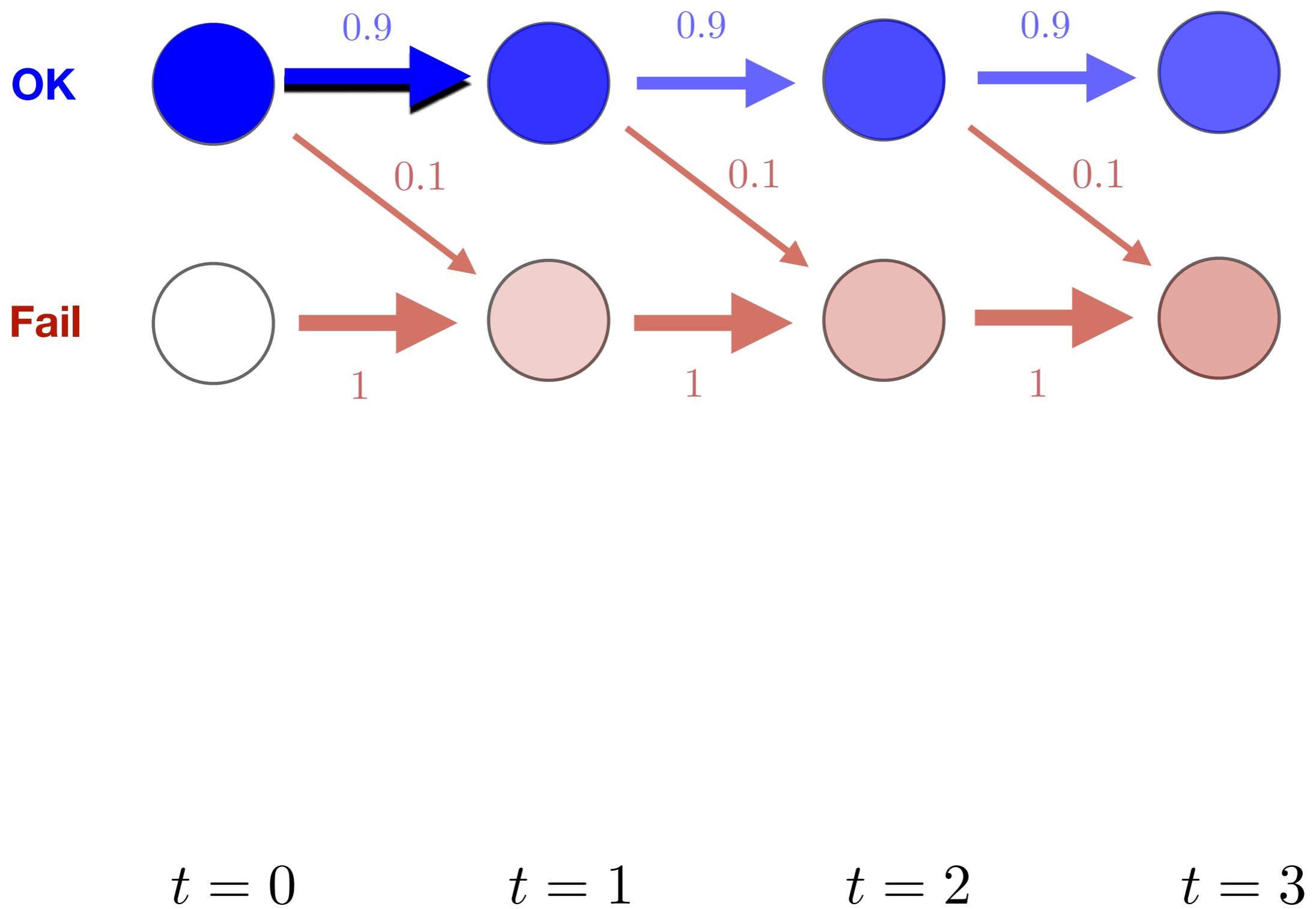
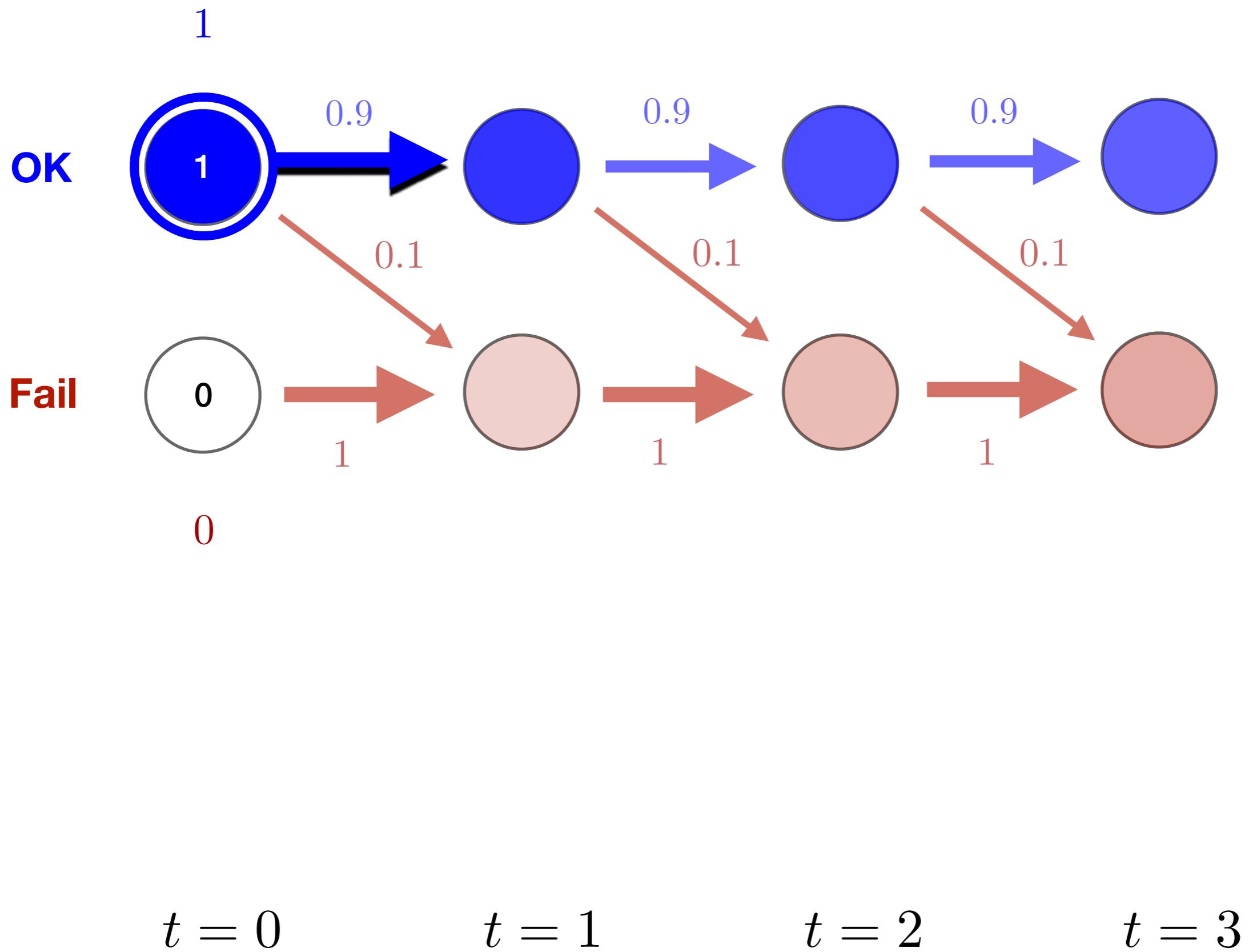


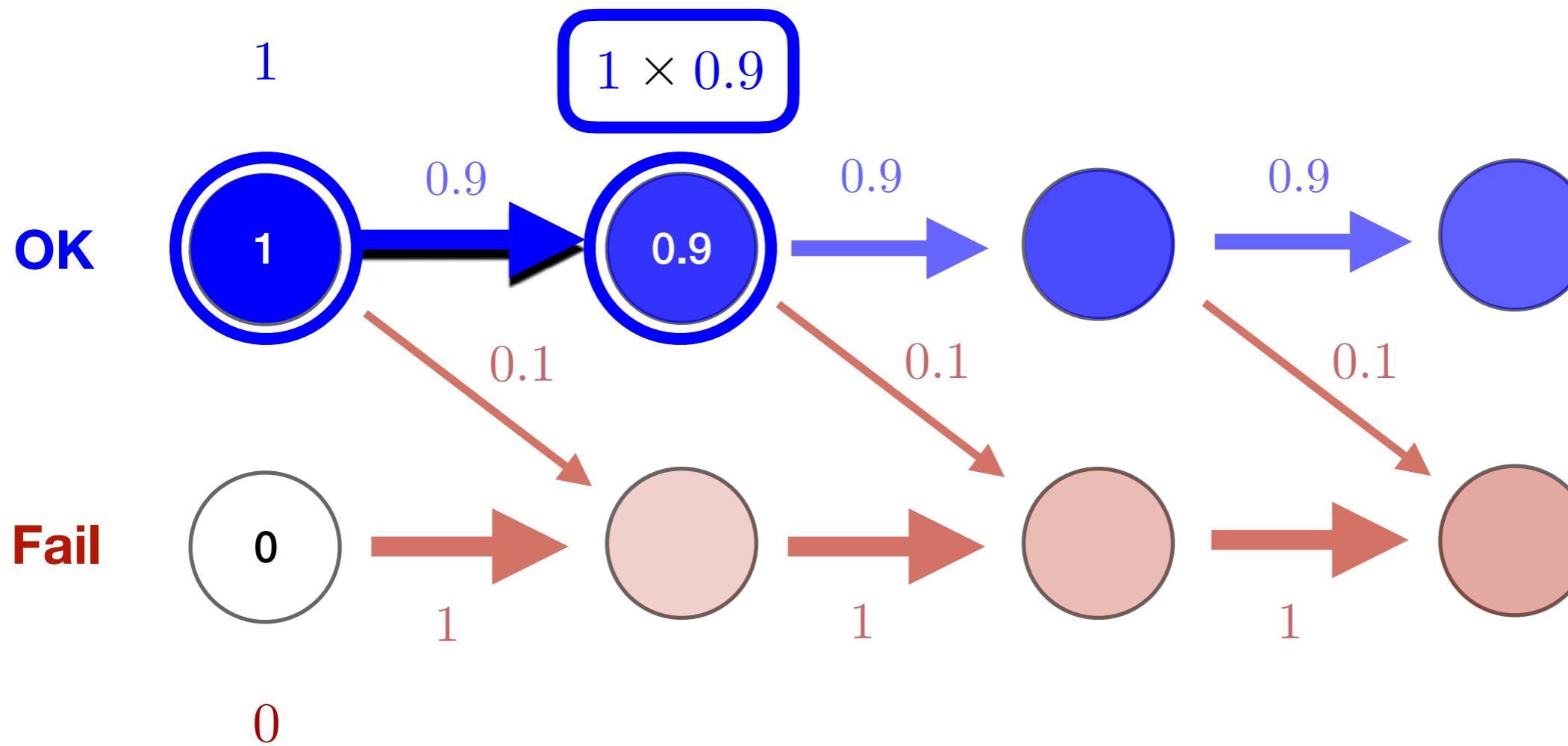
# **Markov Chains**

## **Stochastic Processes**

**Major sources:**





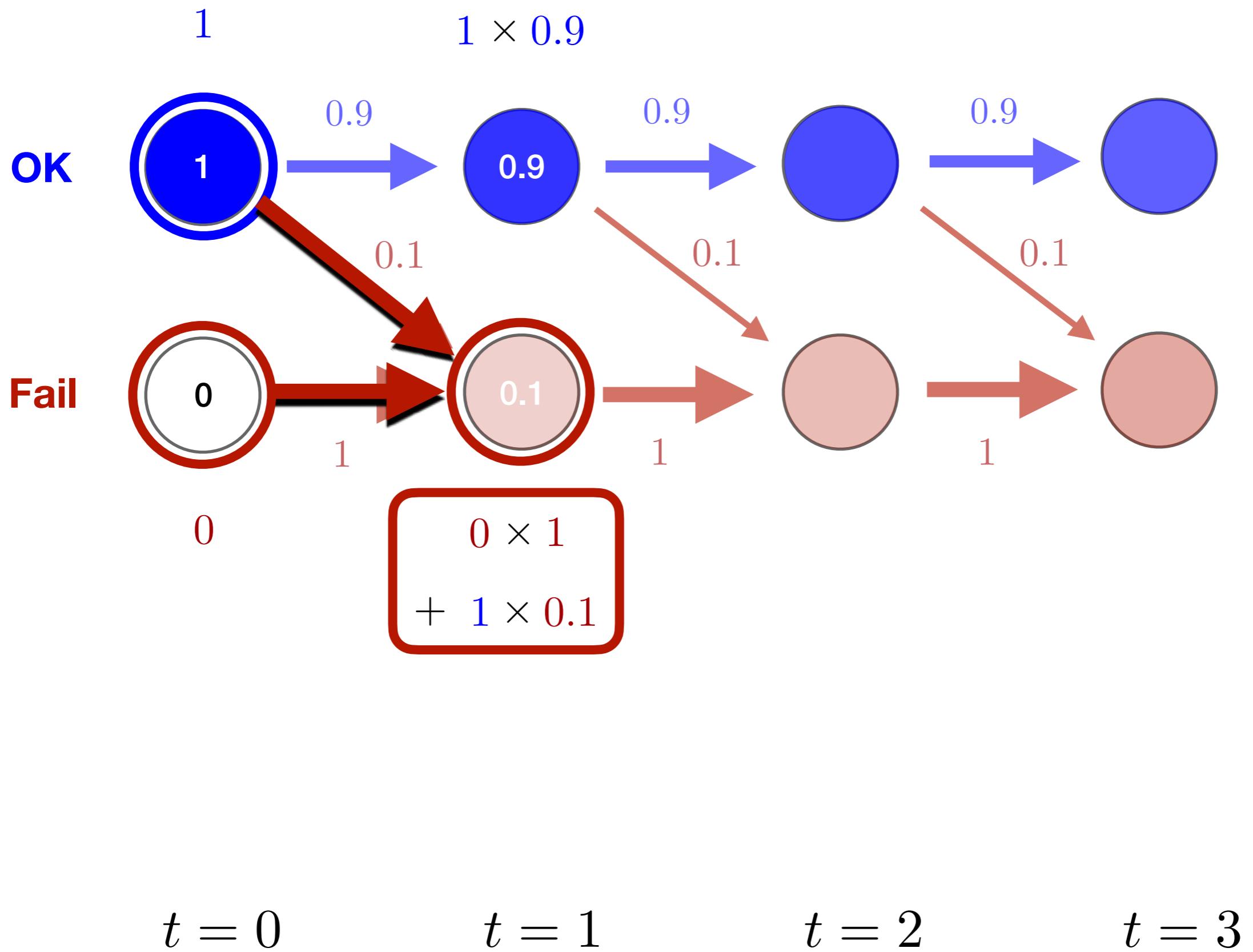


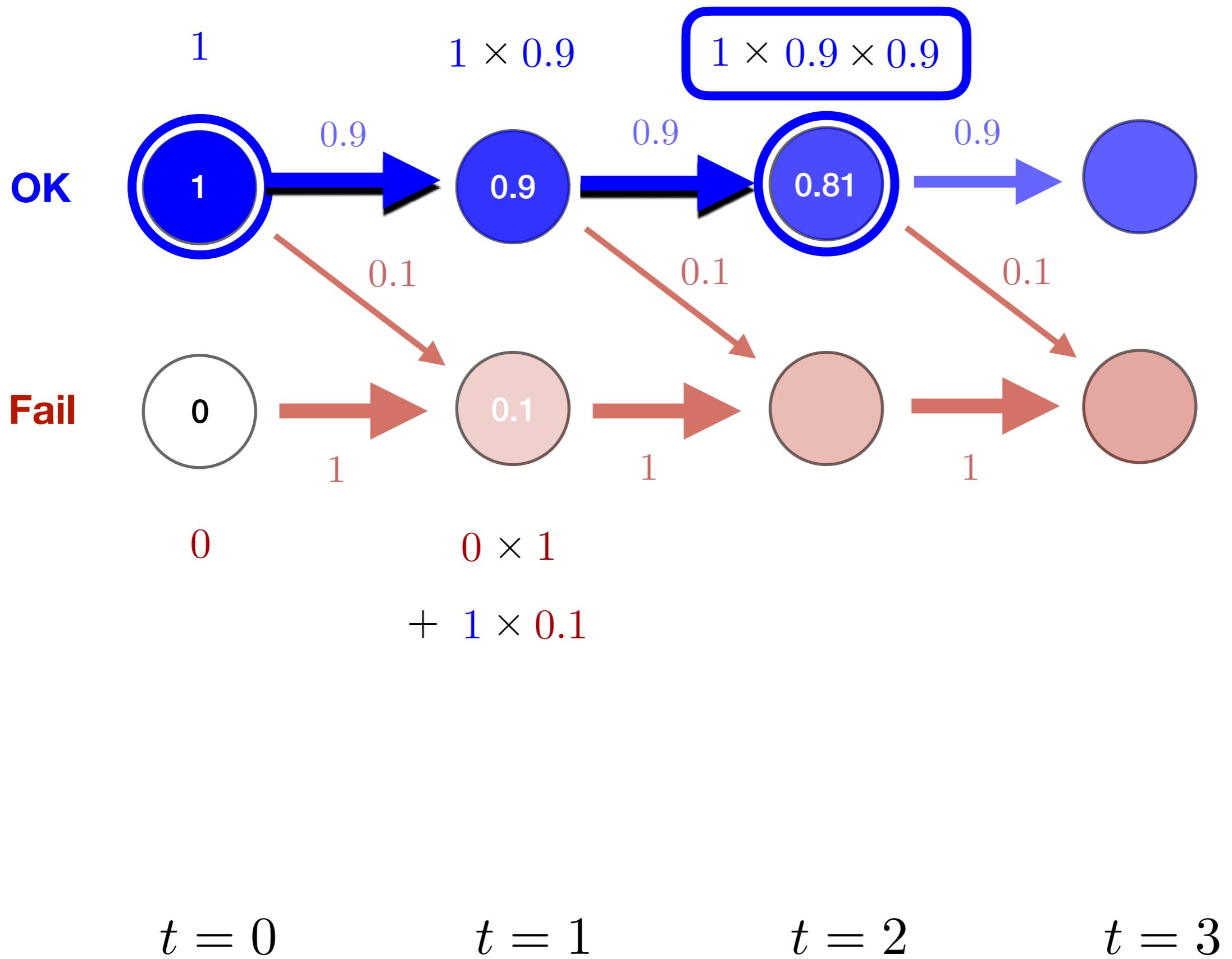
$$t = 0$$

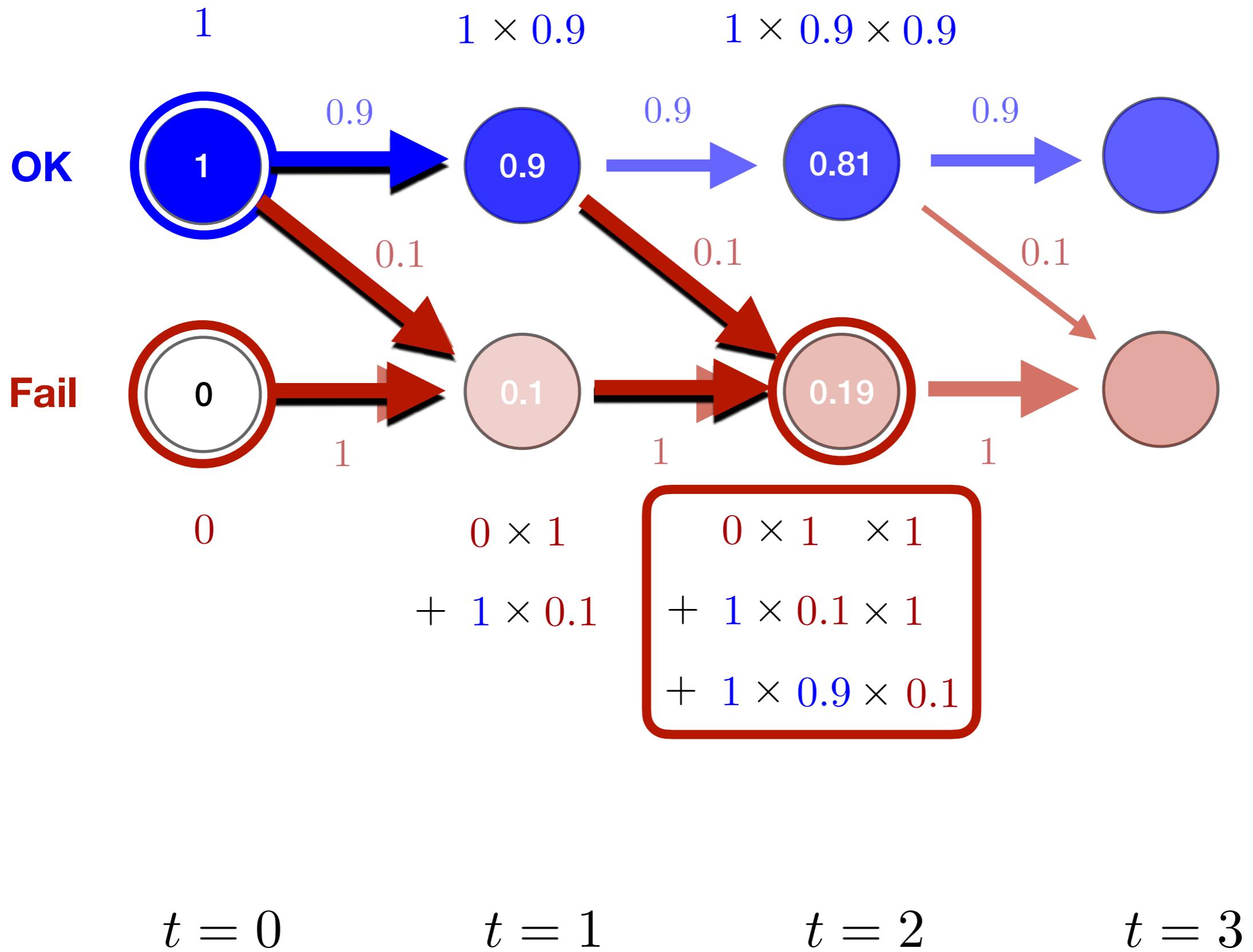
$$t = 1$$

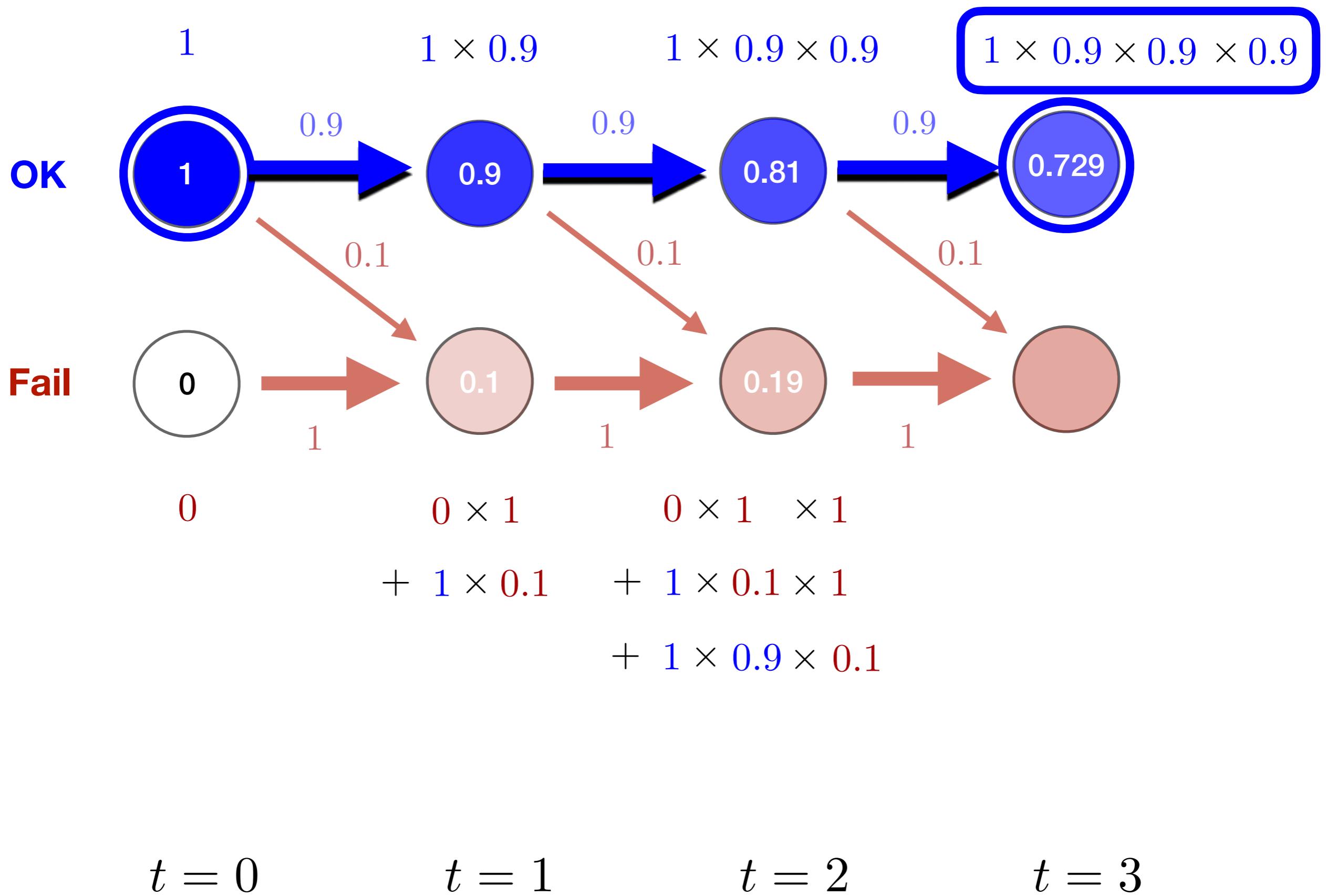
$$t = 2$$

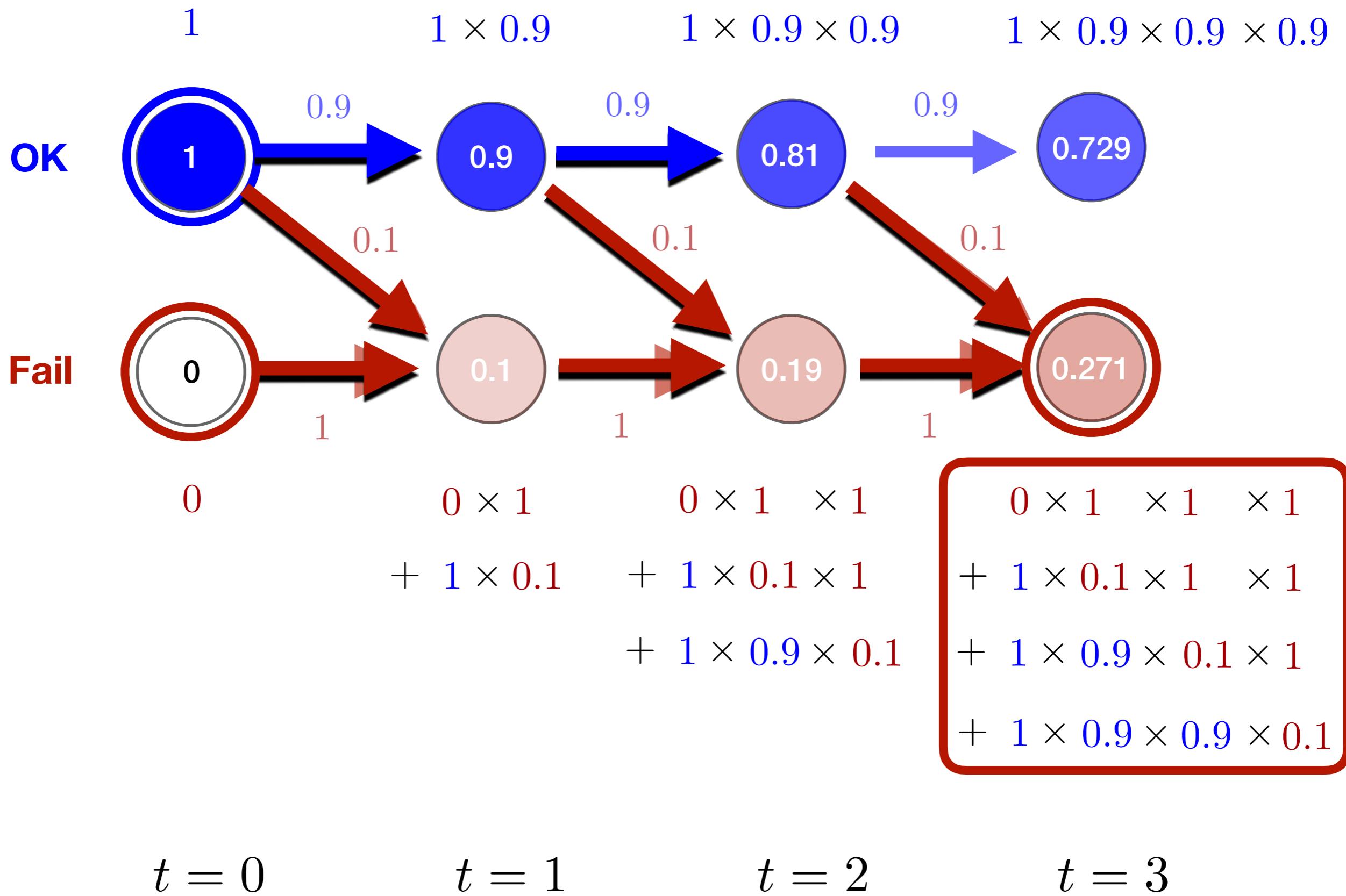
$$t = 3$$

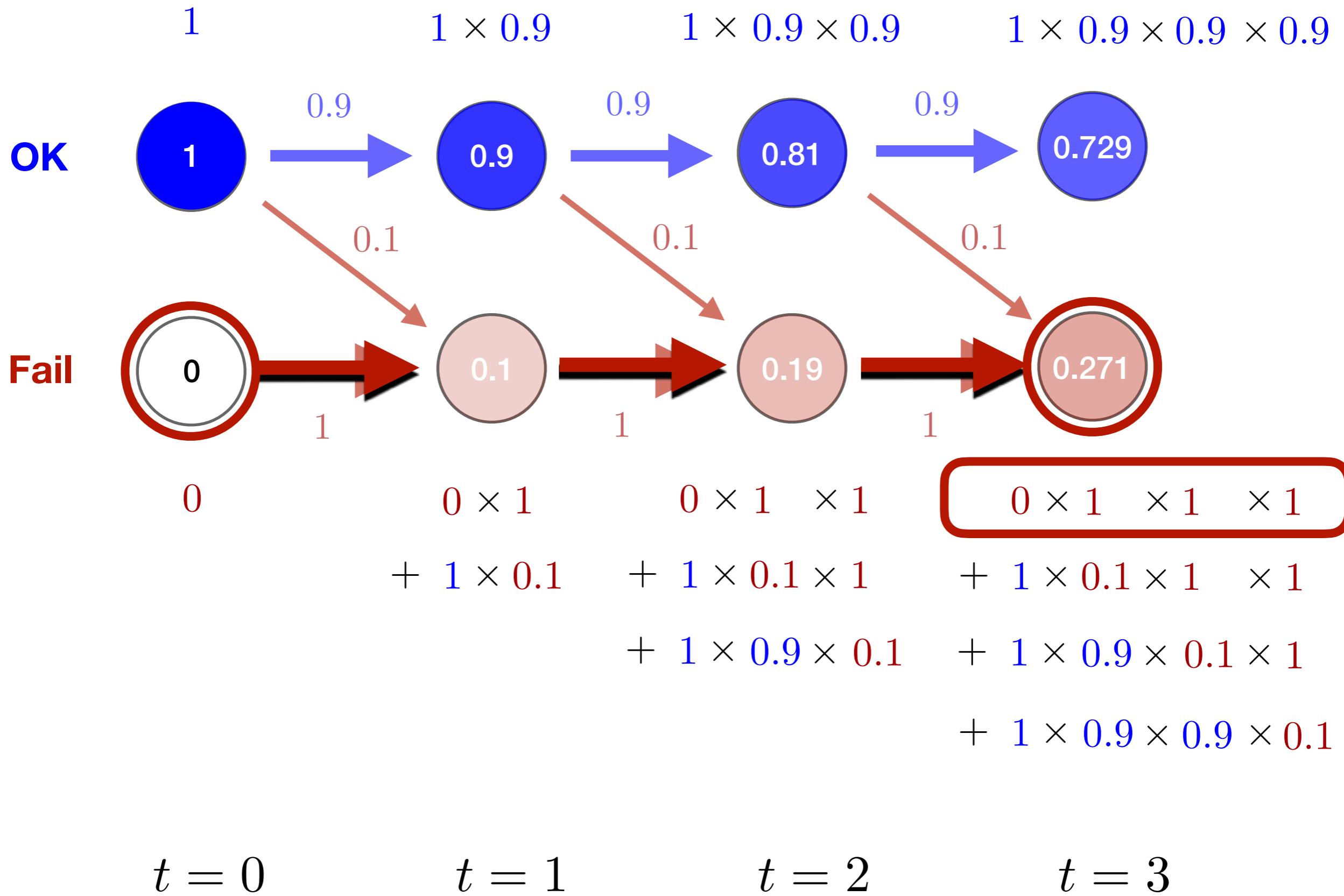


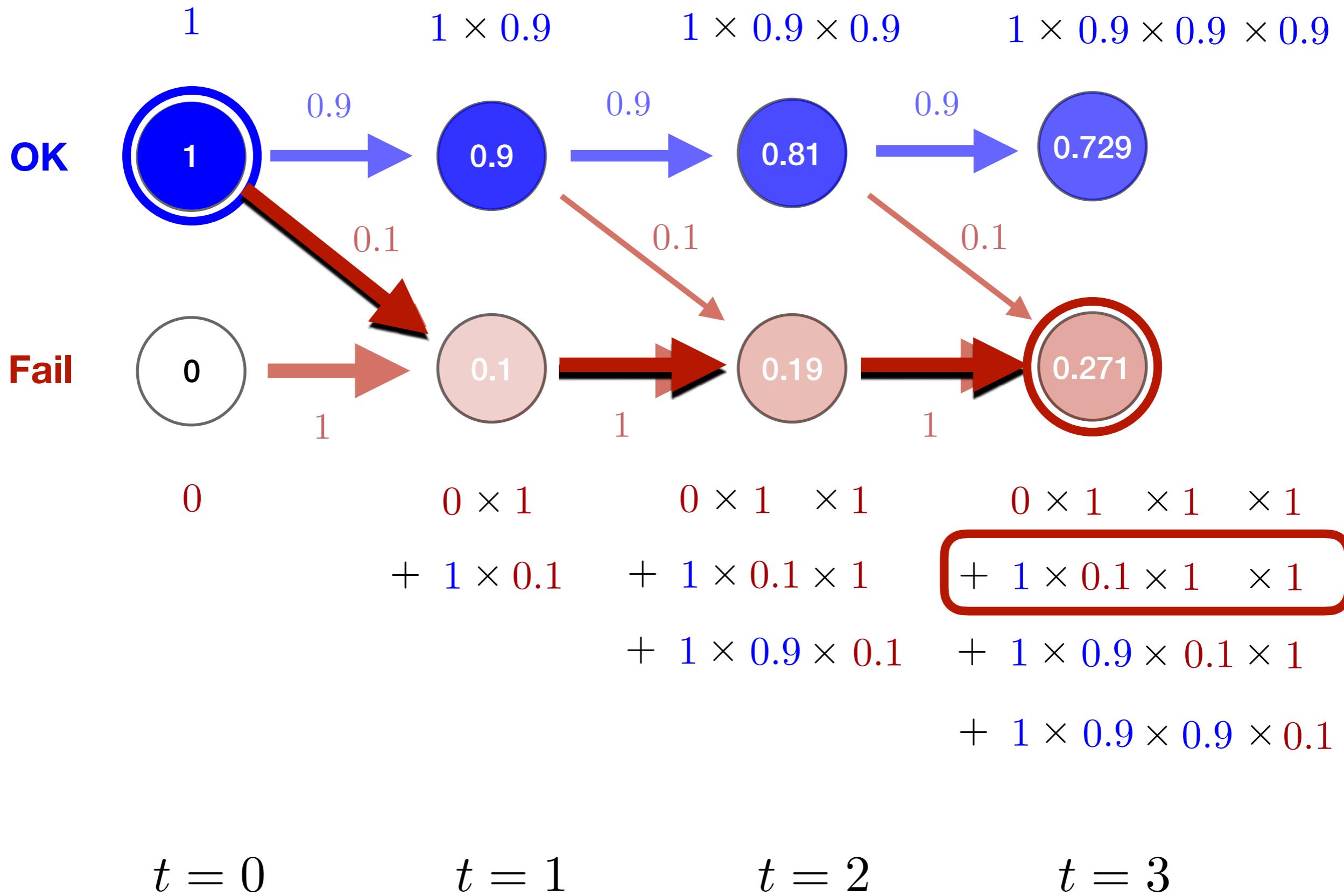


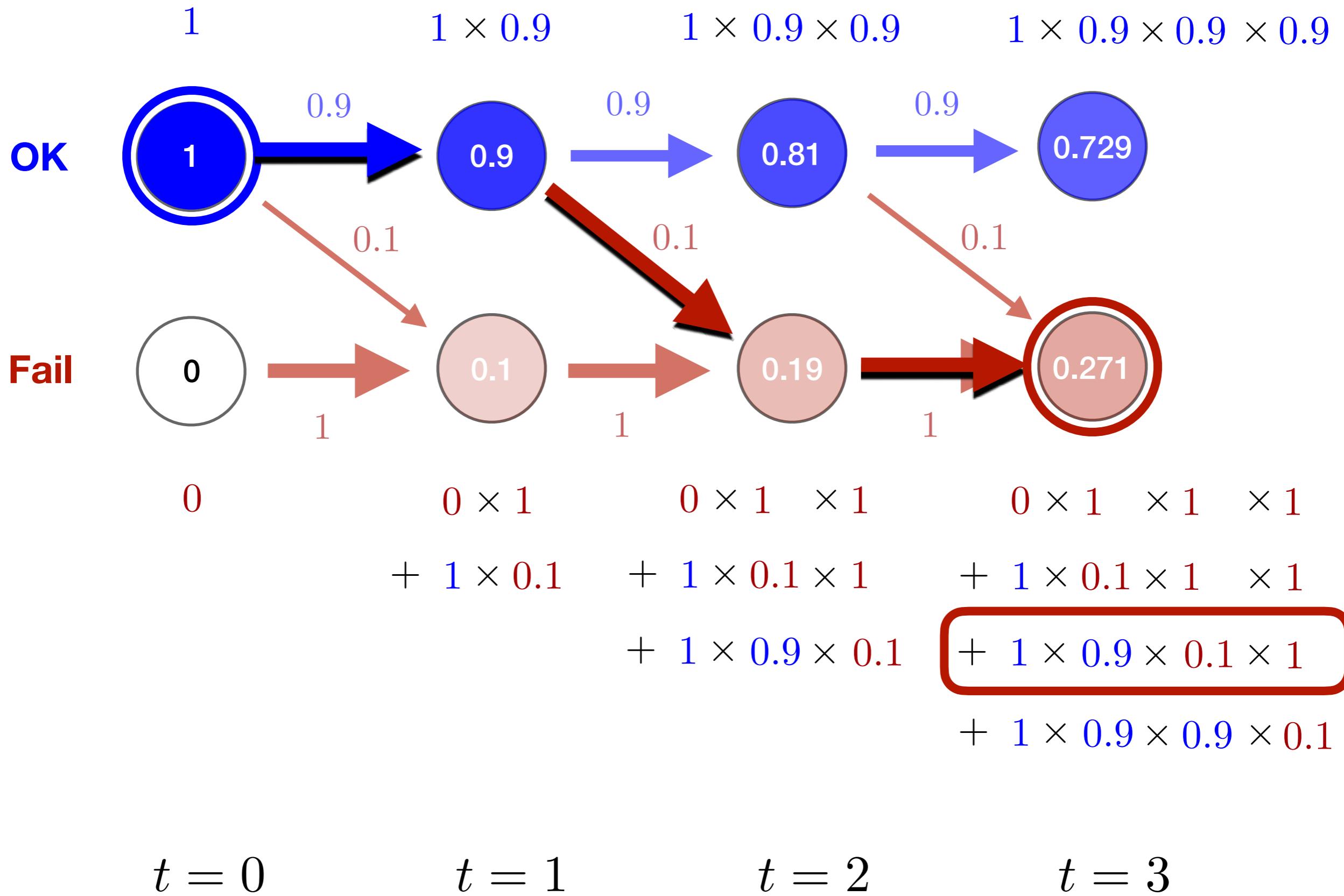


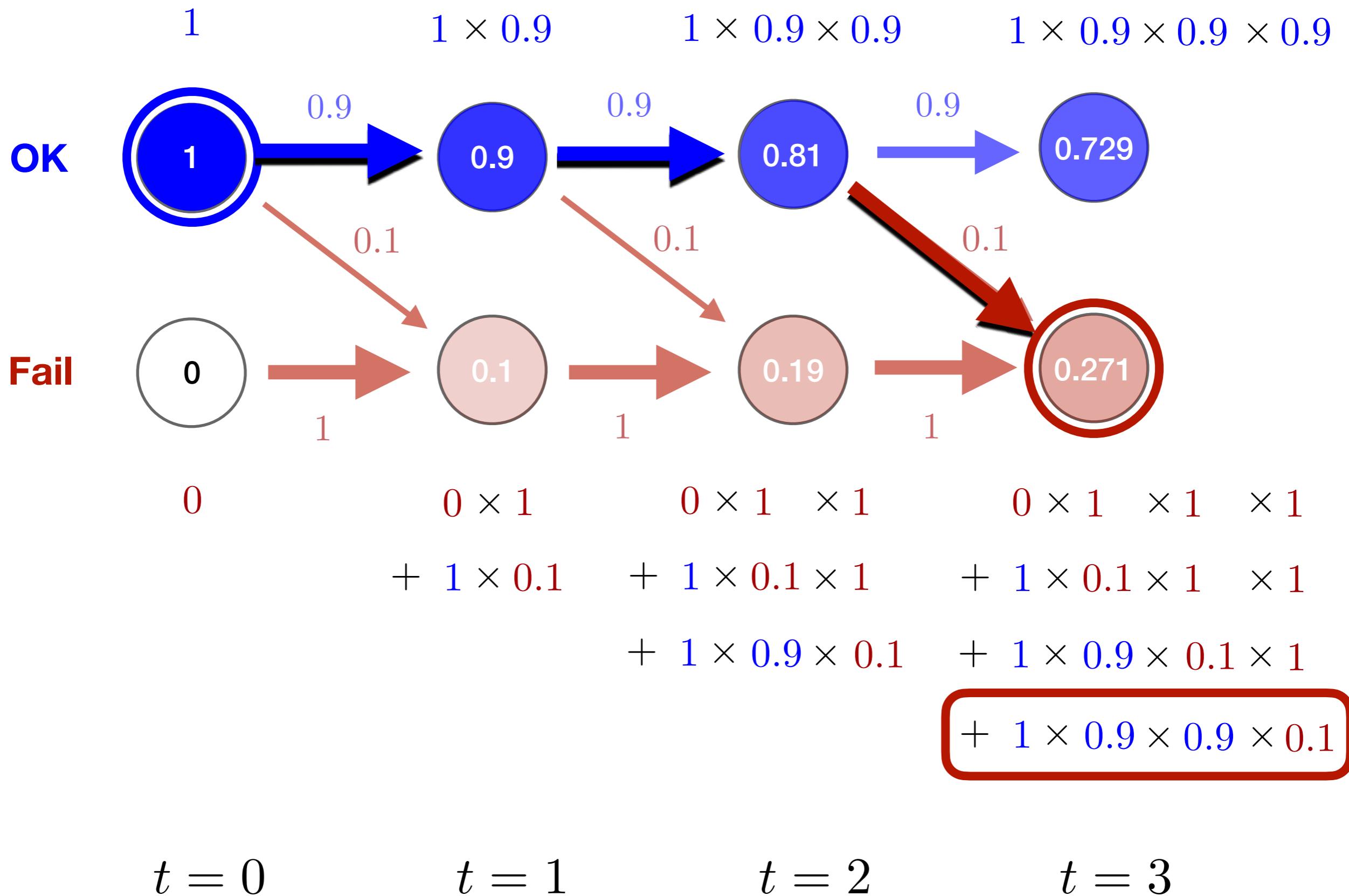


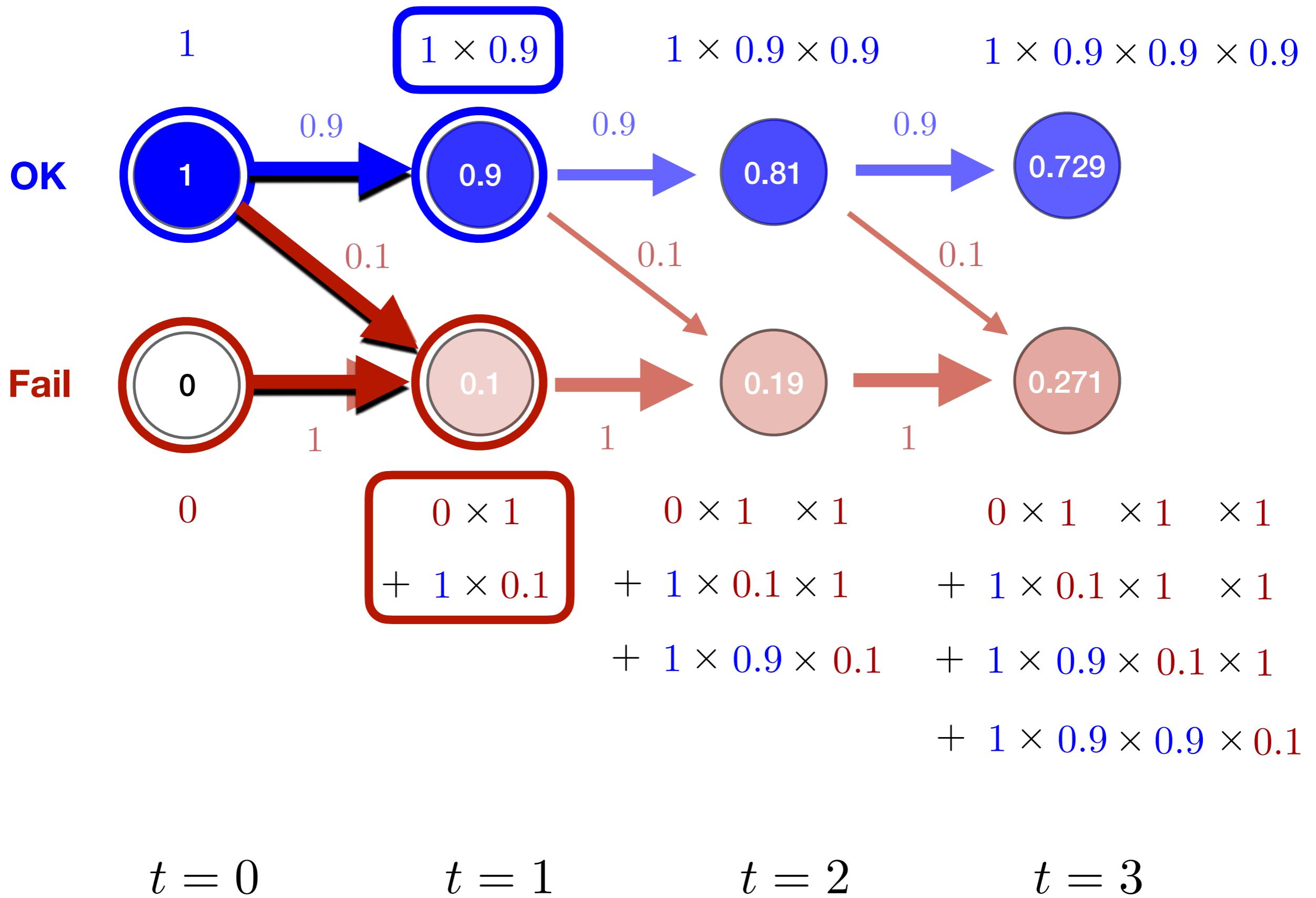


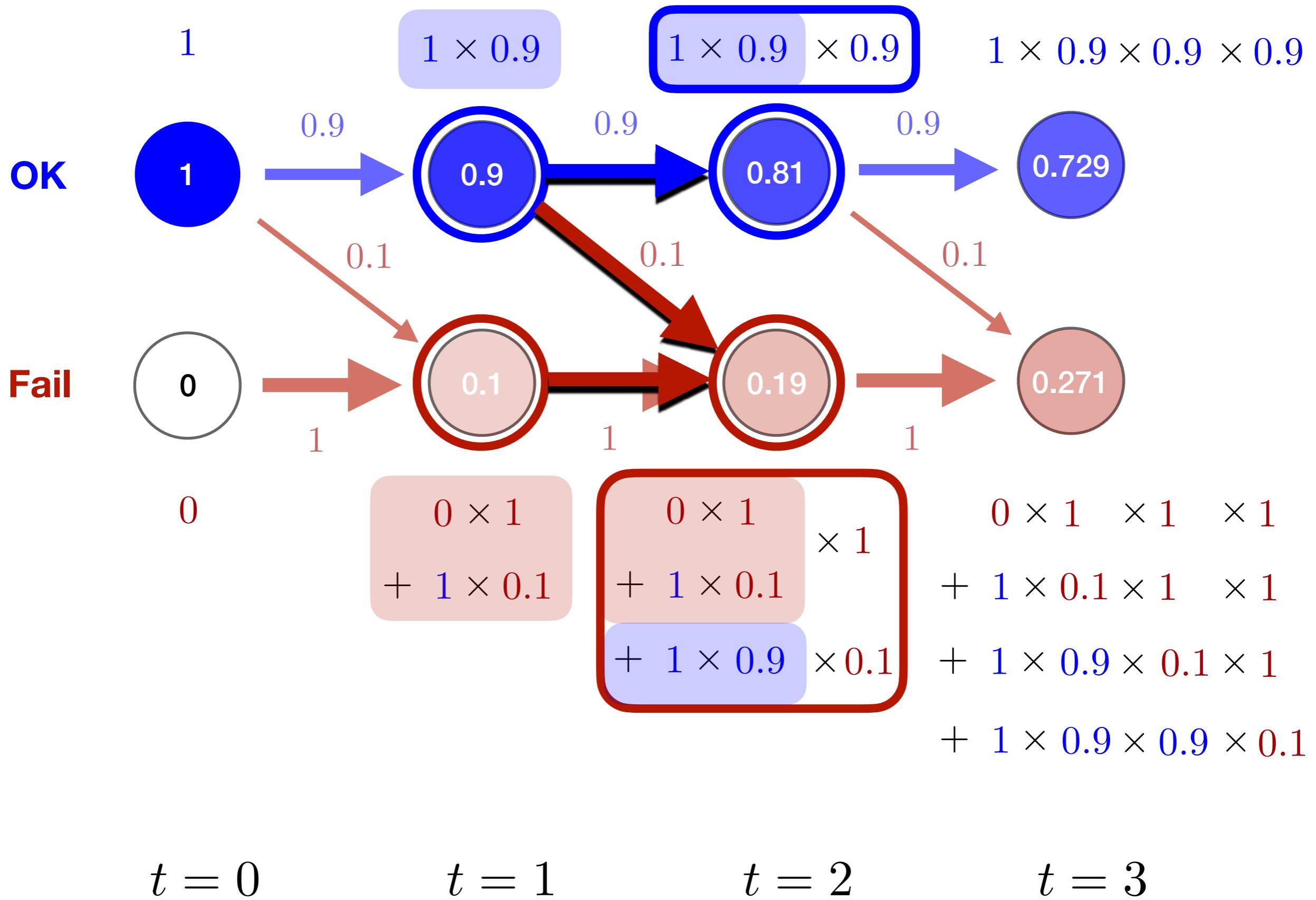


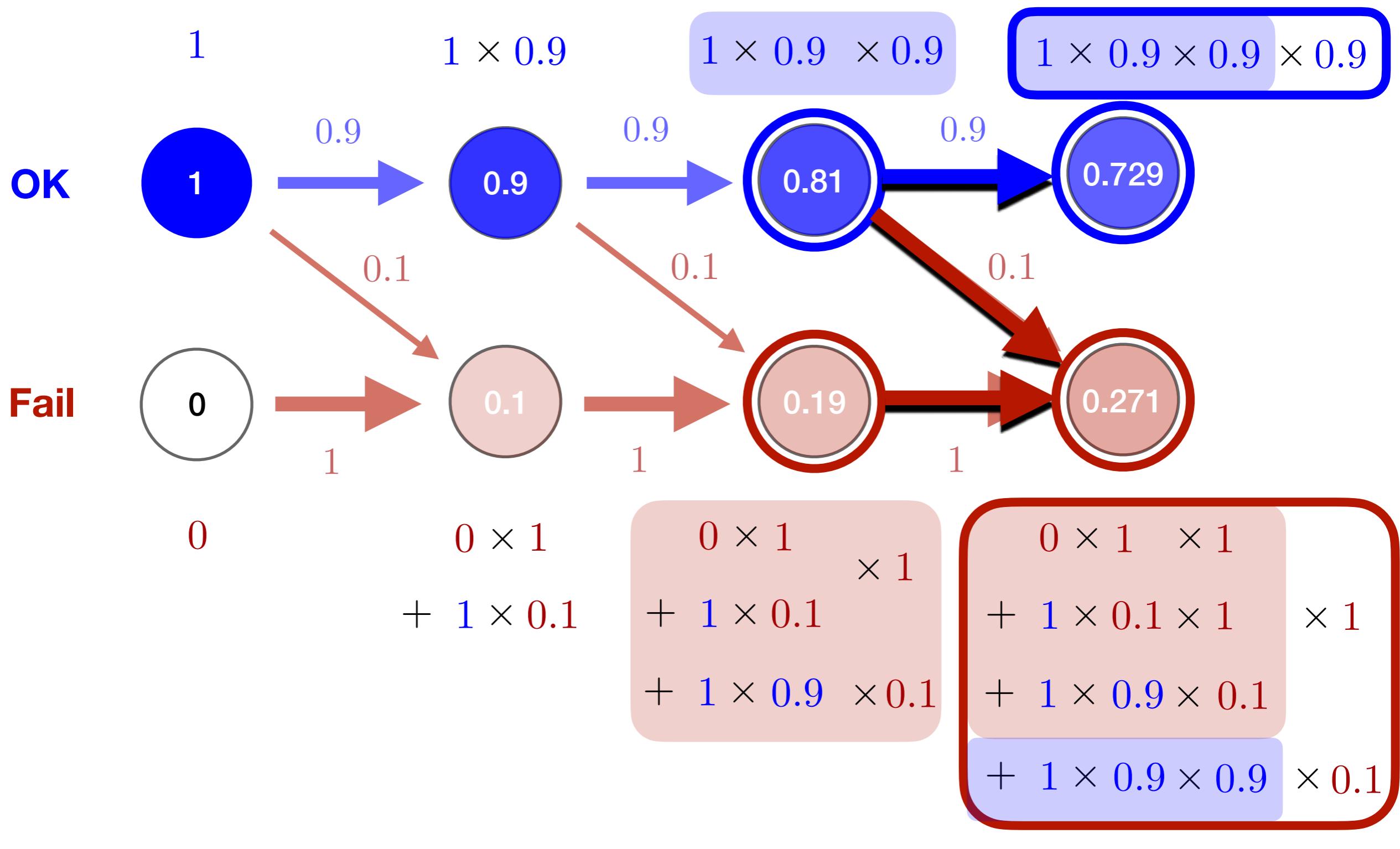


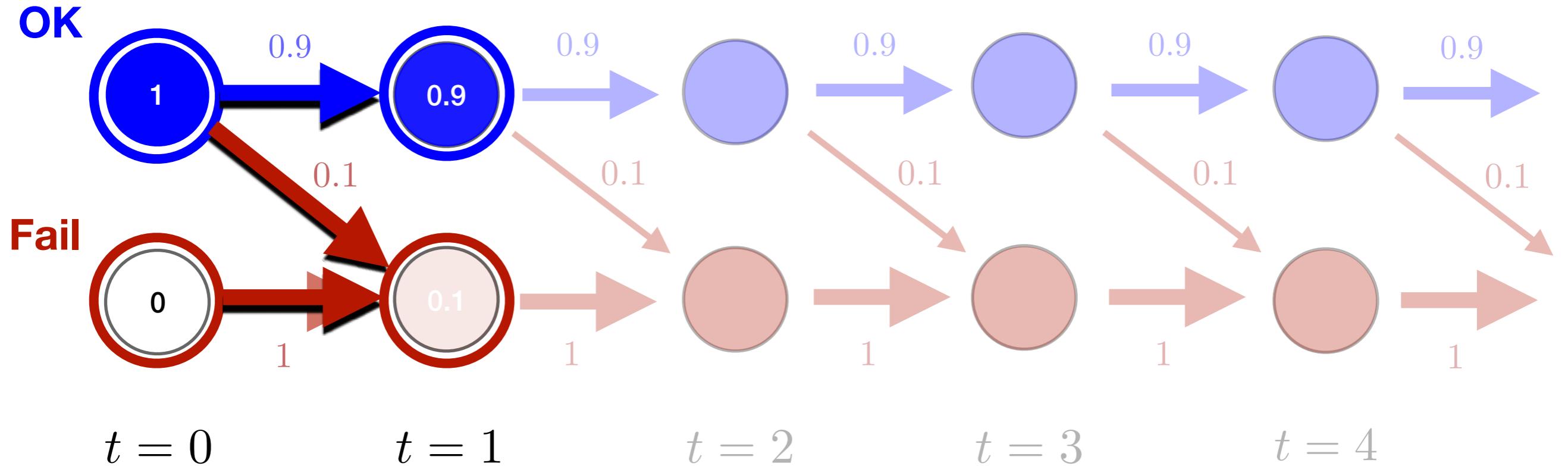




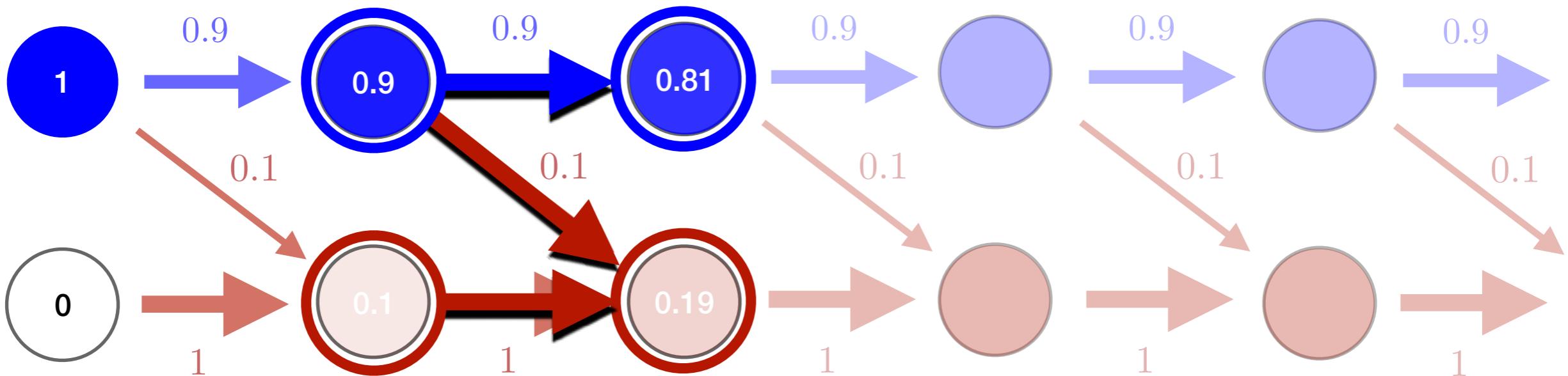








**OK**



$t = 0$

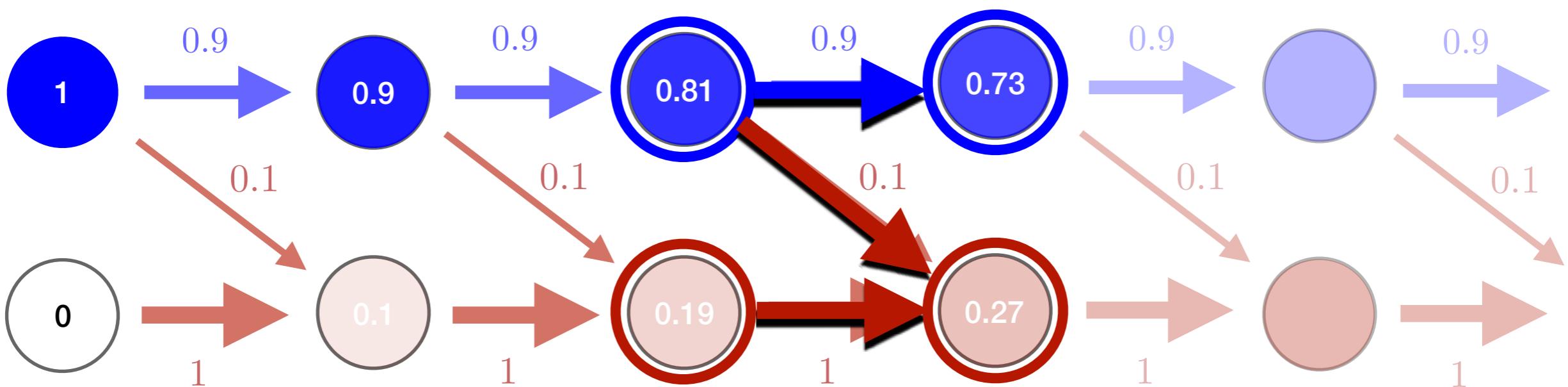
$t = 1$

$t = 2$

$t = 3$

$t = 4$

**OK**



$t = 0$

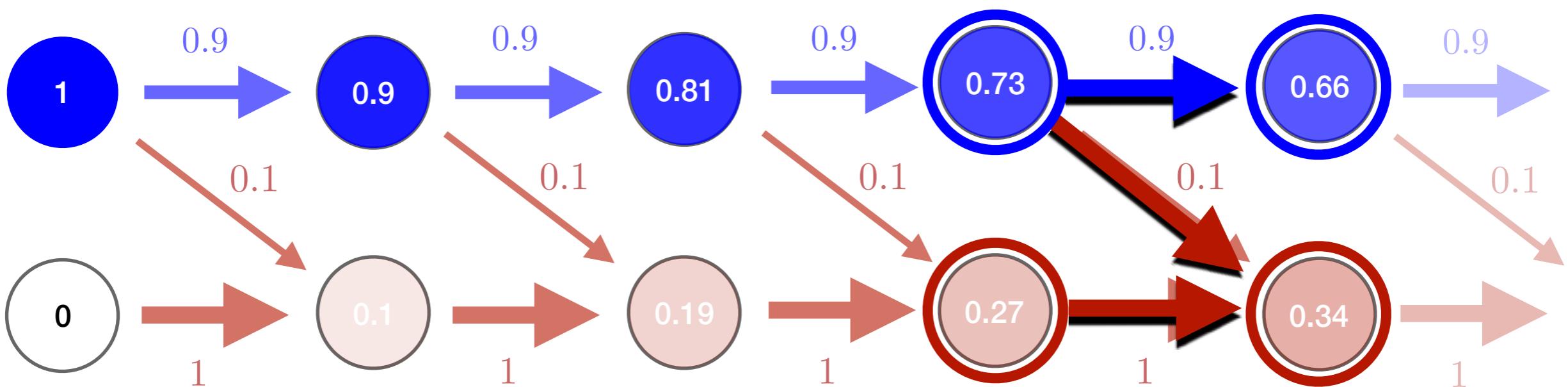
$t = 1$

$t = 2$

$t = 3$

$t = 4$

**OK**



$t = 0$

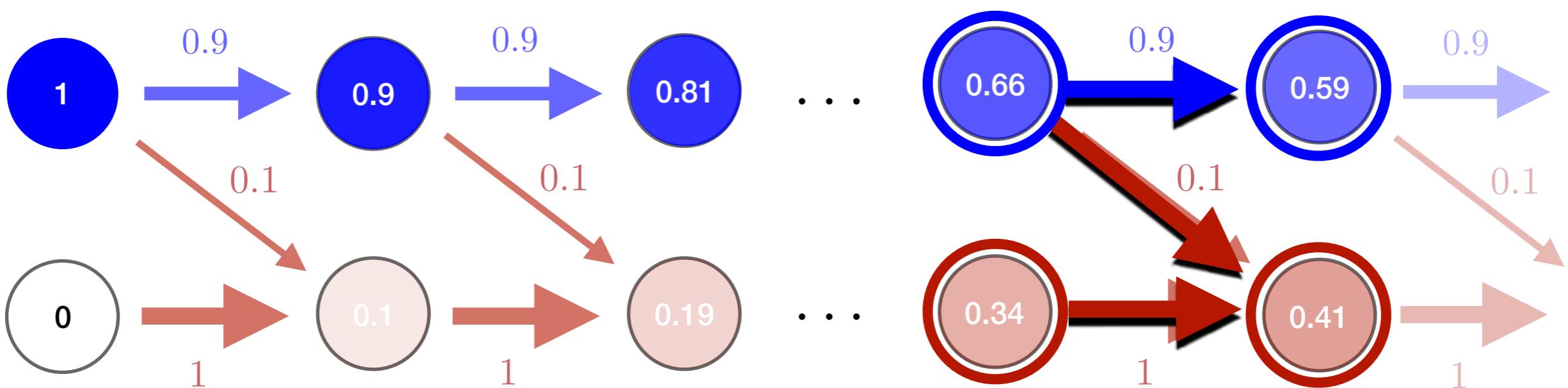
$t = 1$

$t = 2$

$t = 3$

$t = 4$

**OK**



$t = 0$

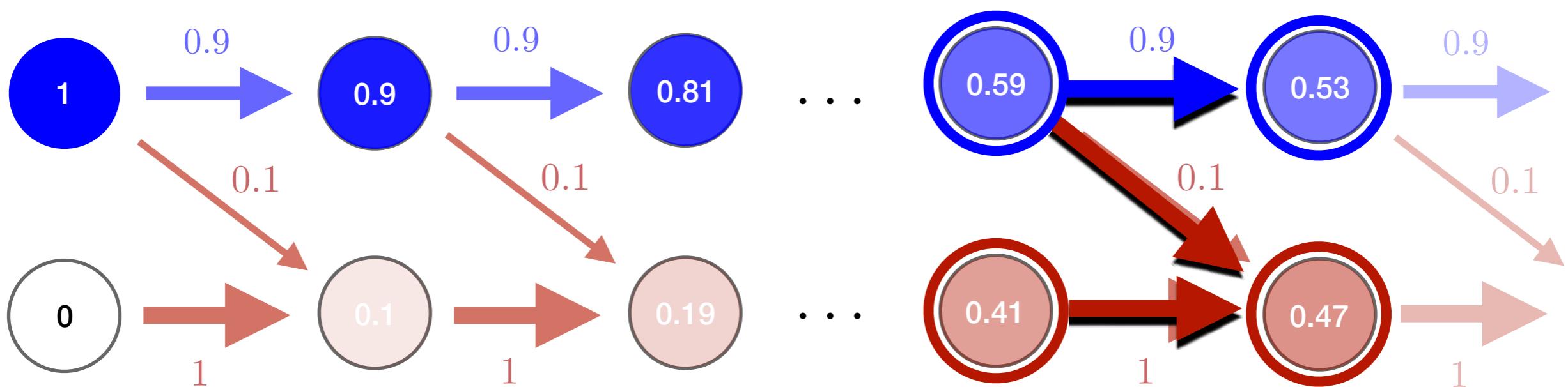
$t = 1$

$t = 2$

$t = 4$

$t = 5$

**OK**



$t = 0$

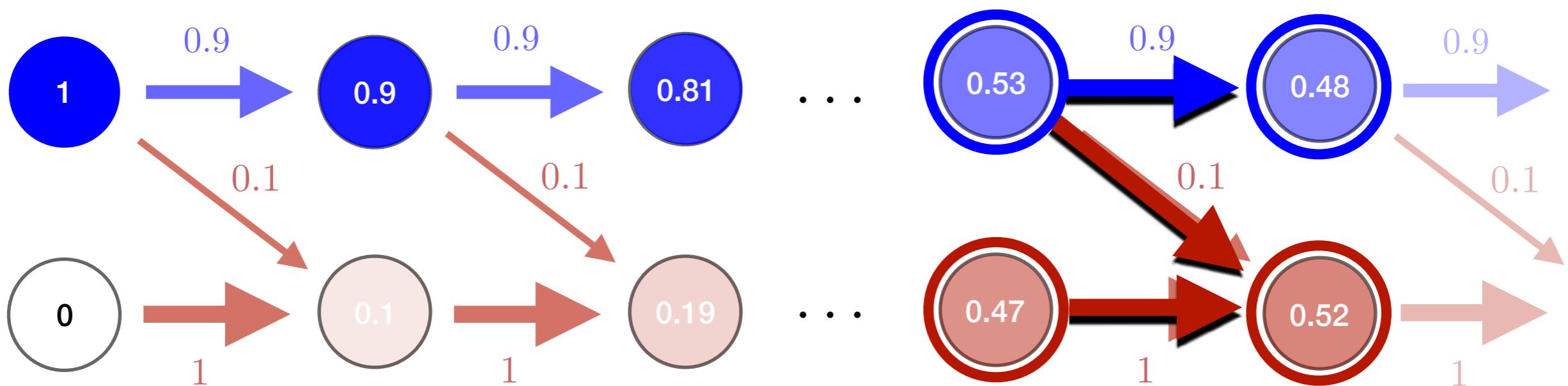
$t = 1$

$t = 2$

$t = 5$

$t = 6$

**OK**



$t = 0$

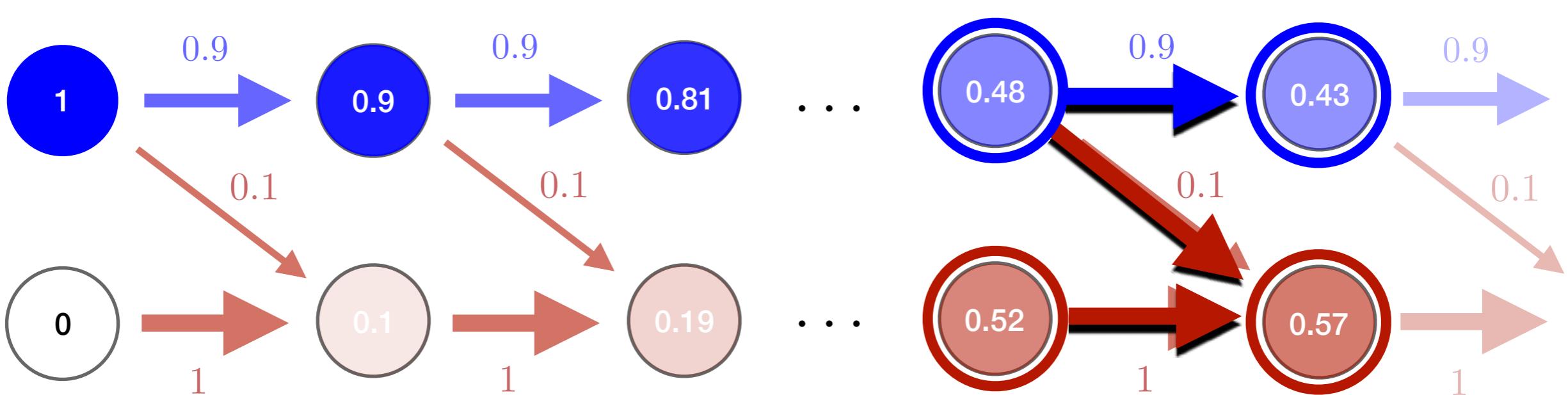
$t = 1$

$t = 2$

$t = 6$

$t = 7$

**OK**



$t = 0$

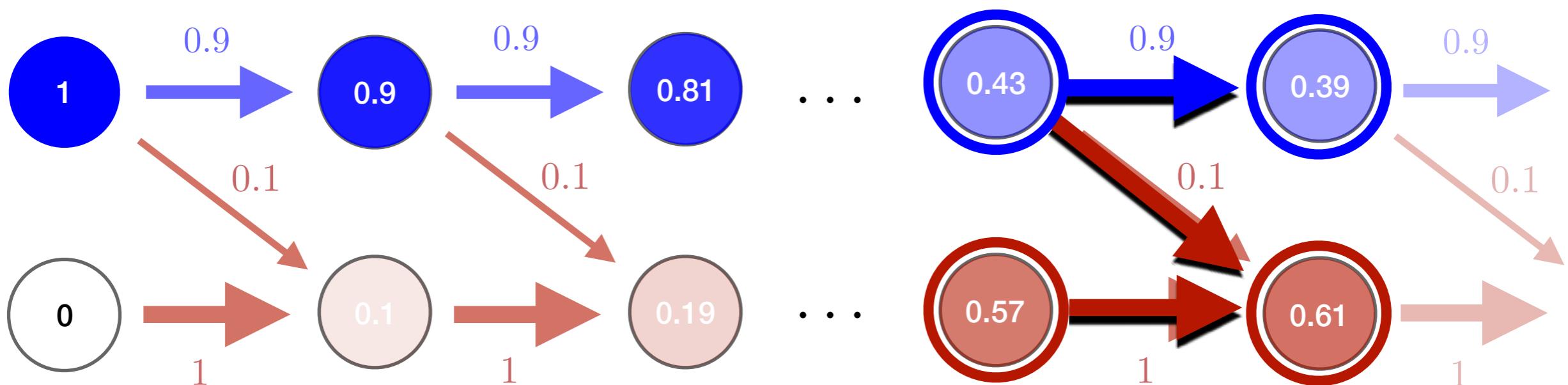
$t = 1$

$t = 2$

$t = 7$

$t = 8$

**OK**



$t = 0$

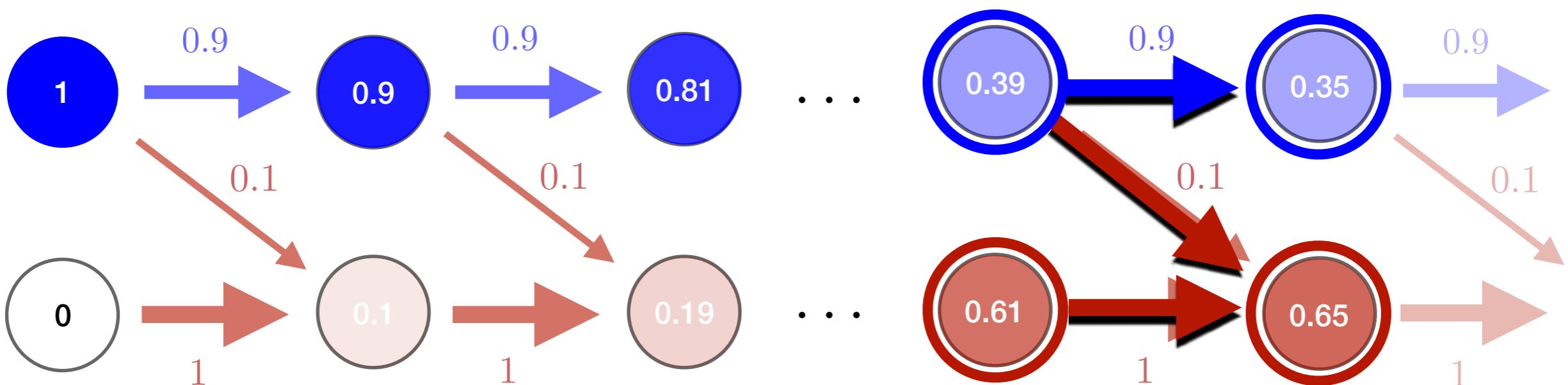
$t = 1$

$t = 2$

$t = 8$

$t = 9$

**OK**



$t = 0$

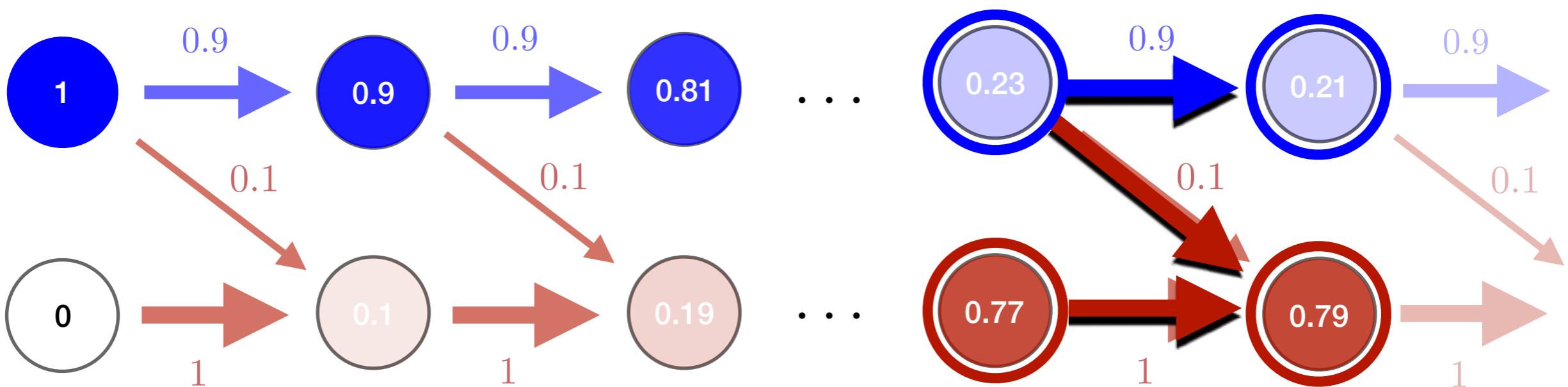
$t = 1$

$t = 2$

$t = 9$

$t = 10$

**OK**



$t = 0$

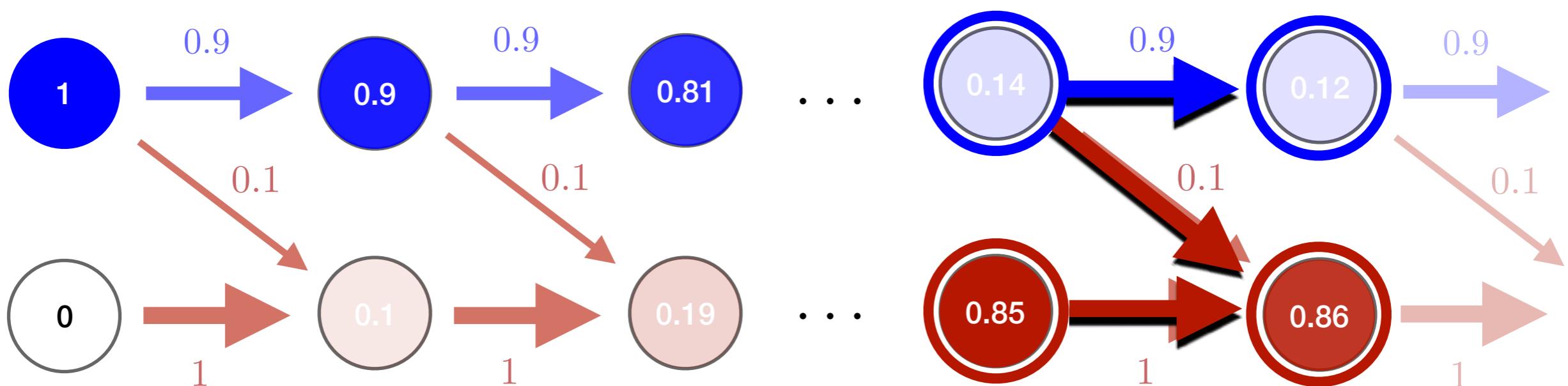
$t = 1$

$t = 2$

$t = 14$

$t = 15$

**OK**



$t = 0$

$t = 1$

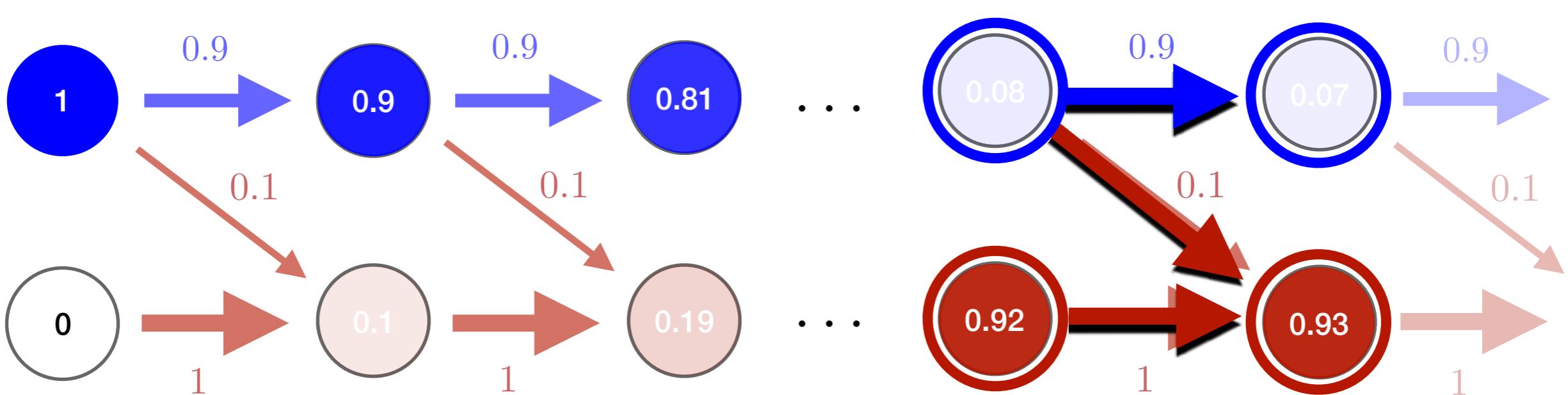
$t = 2$

$t = 19$

$t = 20$

**Fail**

**OK**



$t = 0$

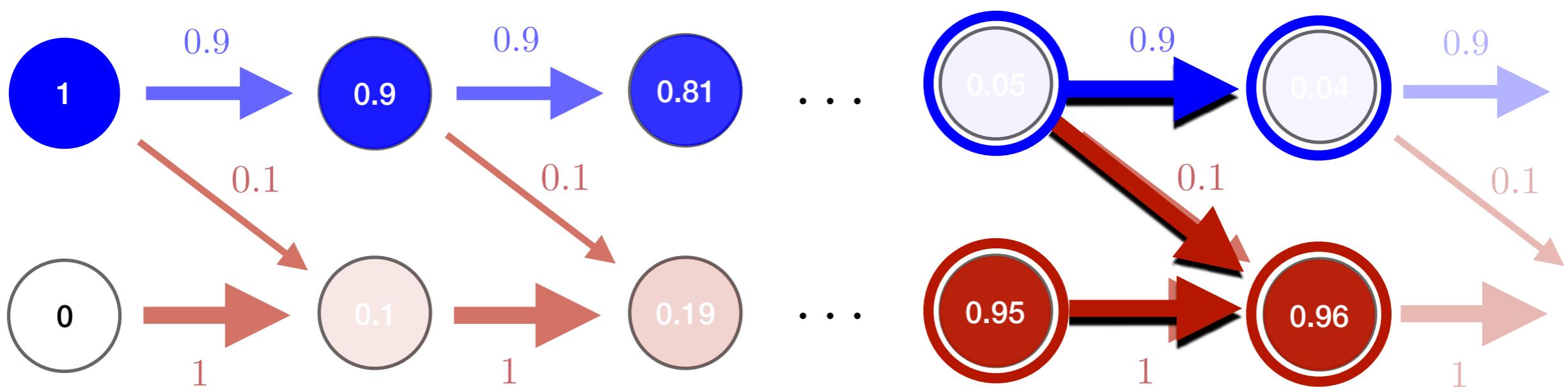
$t = 1$

$t = 2$

$t = 24$

$t = 25$

**OK**



$t = 0$

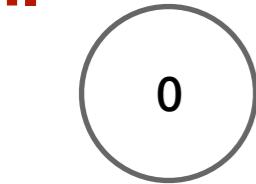
$t = 1$

$t = 2$

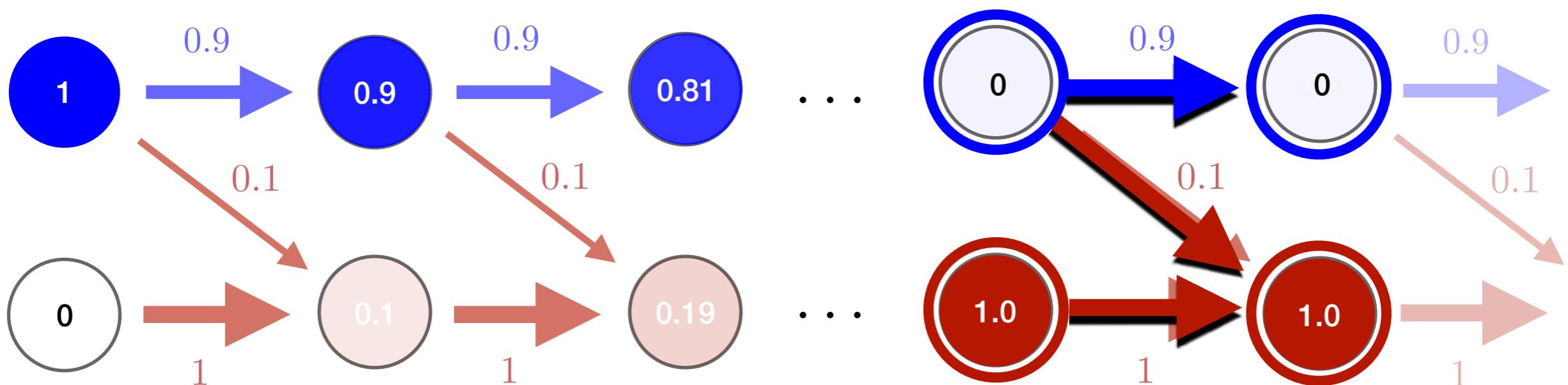
$t = 29$

$t = 30$

**Fail**



**OK**



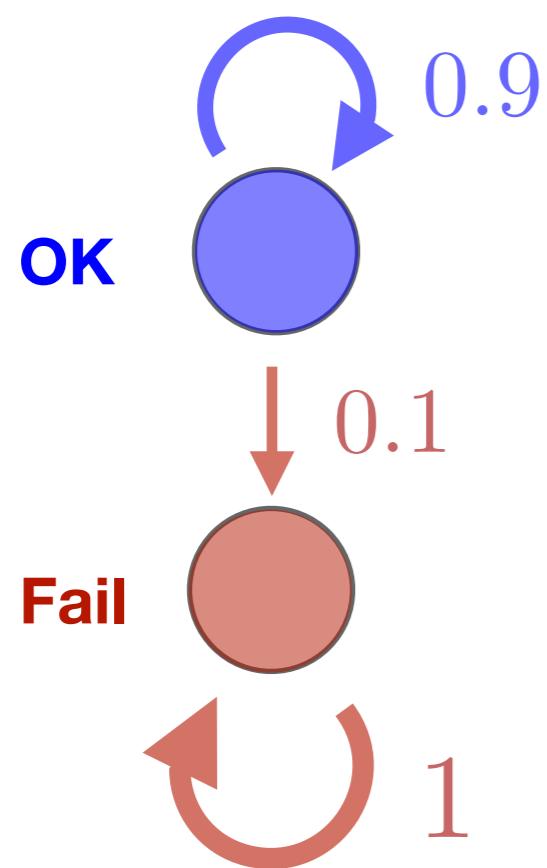
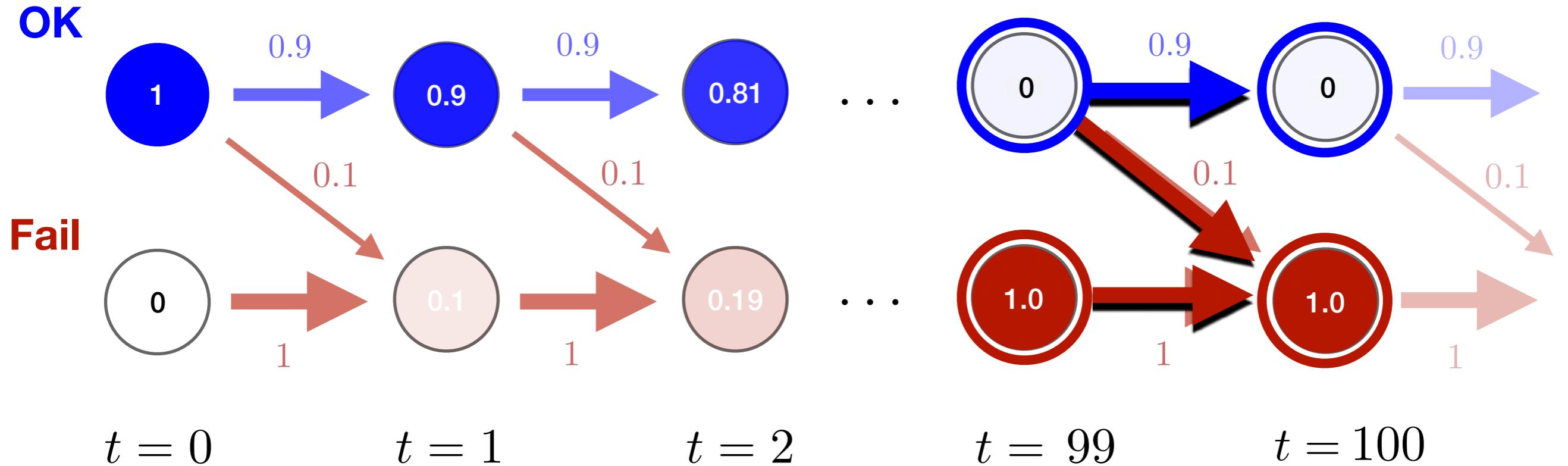
$t = 0$

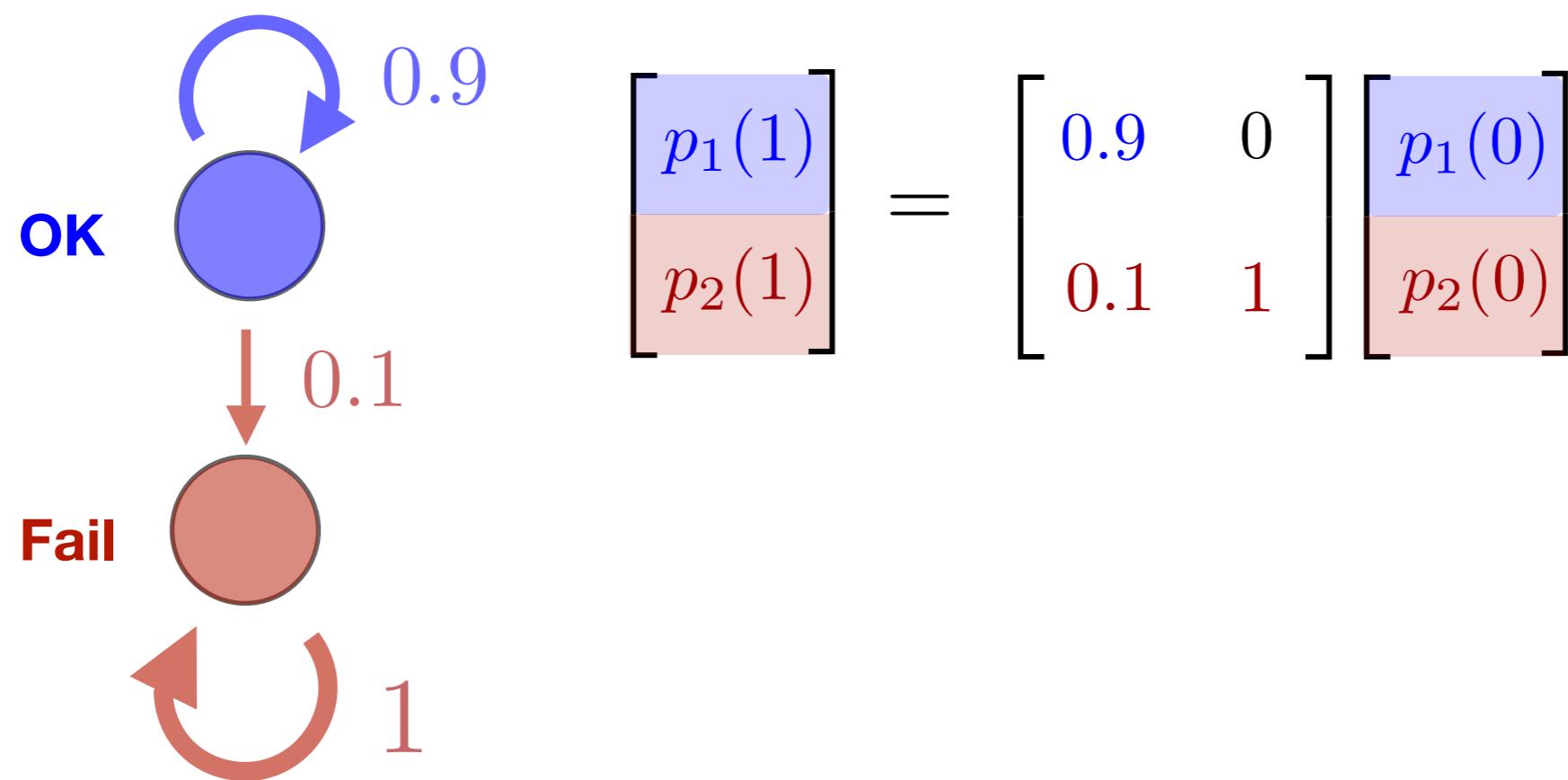
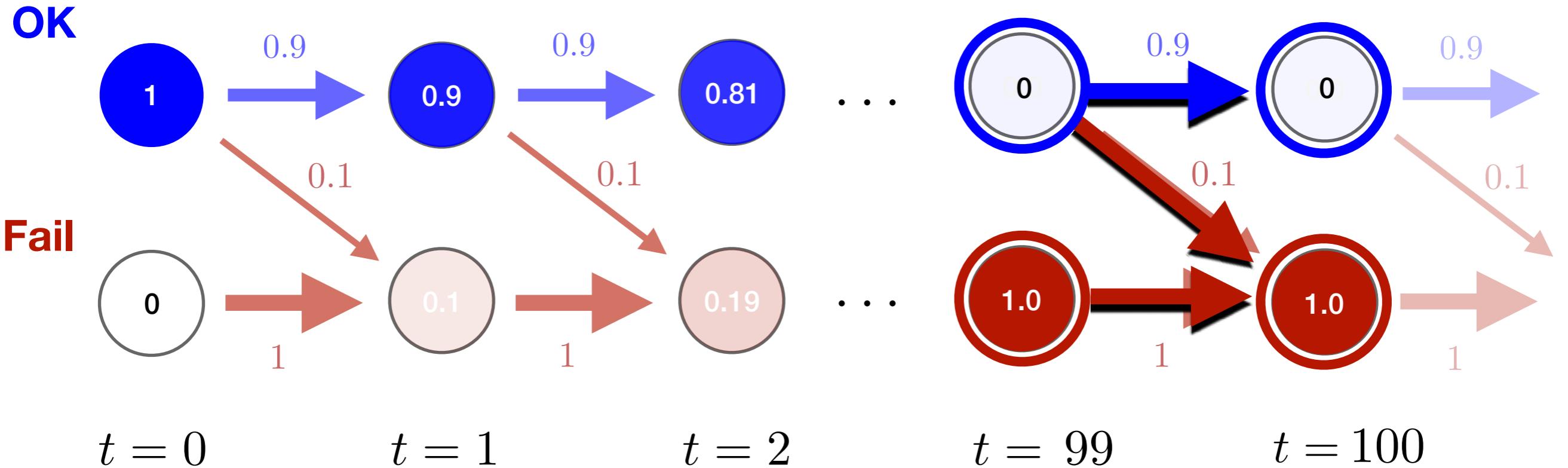
$t = 1$

$t = 2$

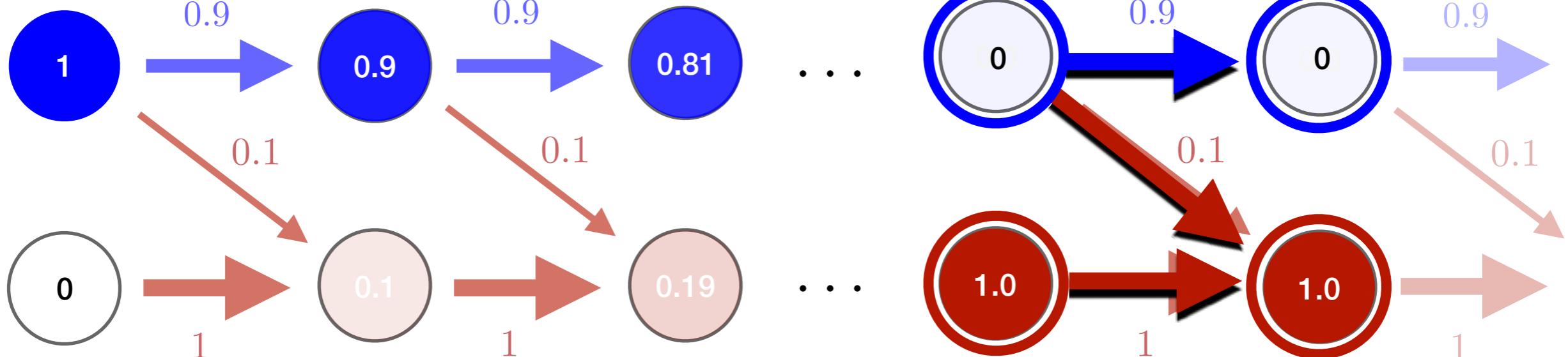
$t = 99$

$t = 100$





**OK**



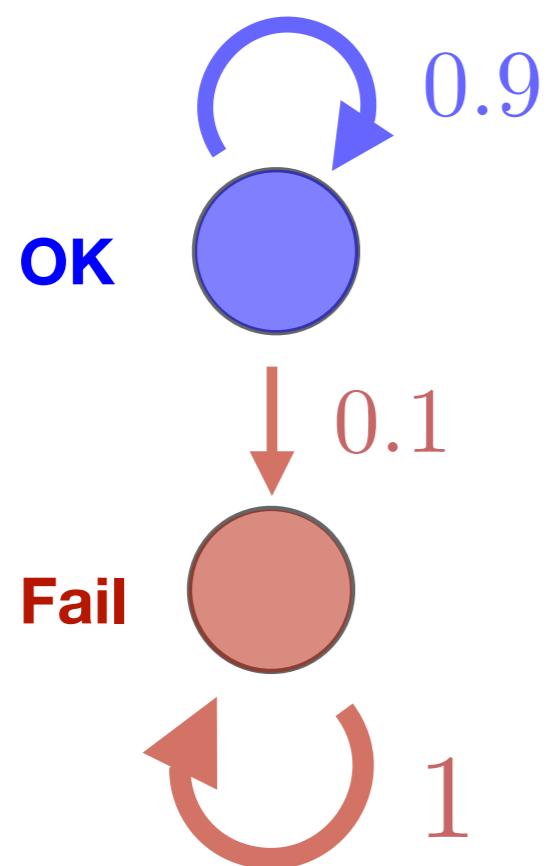
$t = 0$

$t = 1$

$t = 2$

$t = 99$

$t = 100$

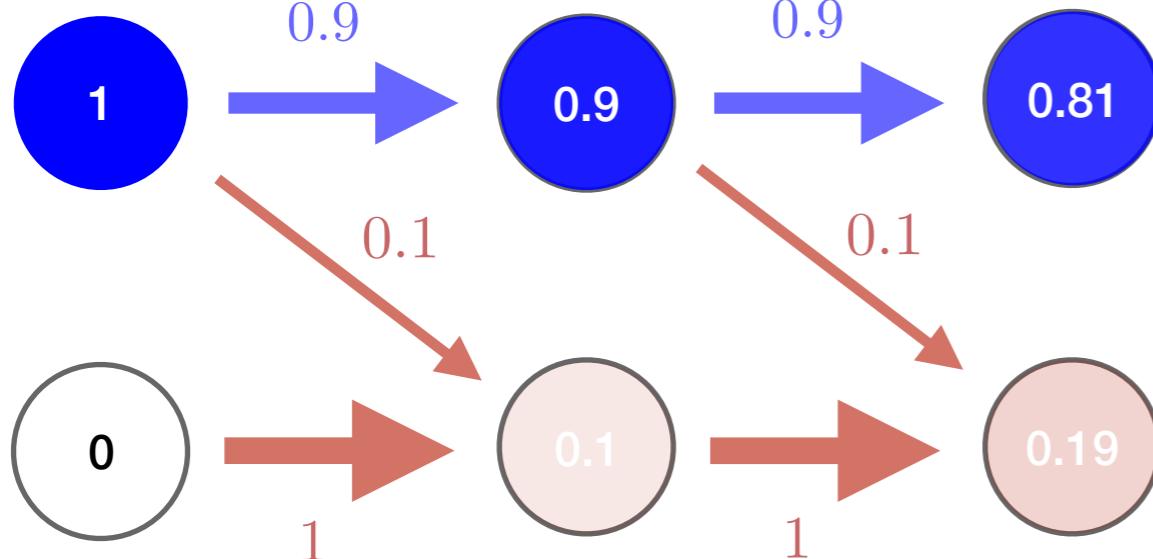


$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$p_1(1) = 0.9 \times p_1(0) + 0 \times p_2(0)$$

$$p_2(1) = 0.1 \times p_1(0) + 1 \times p_2(0)$$

**OK**



$t = 0$

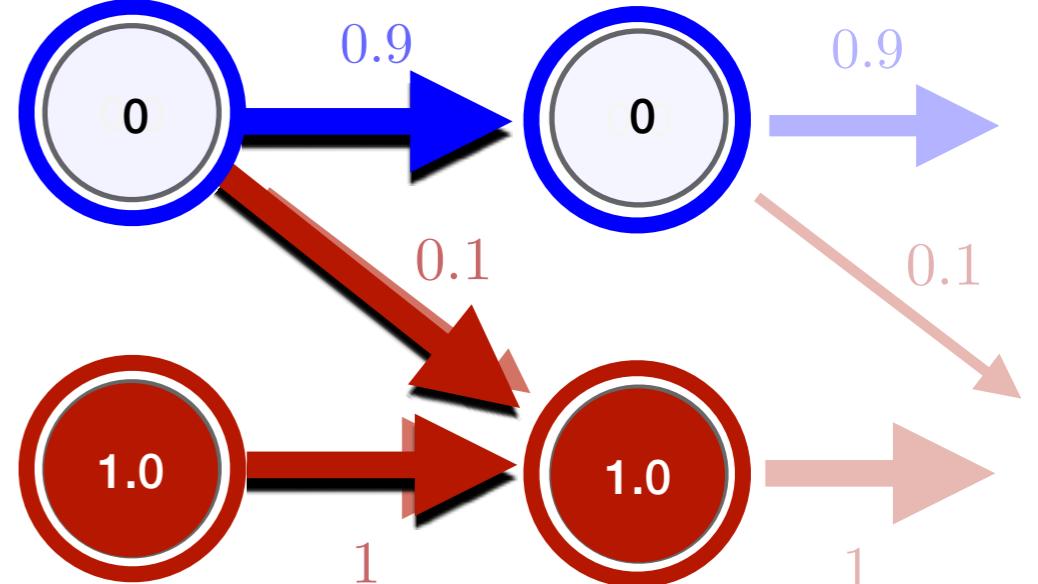
$t = 1$

$t = 2$

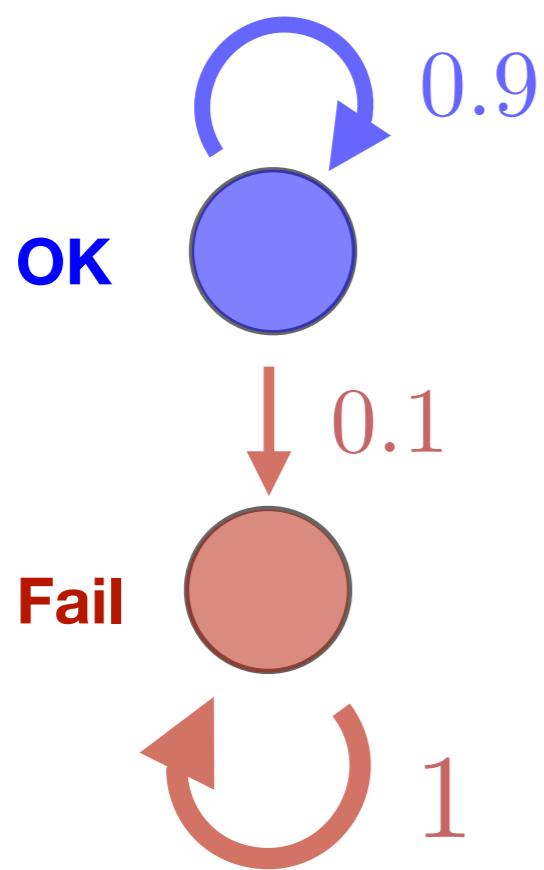
$t = 99$

$t = 100$

...



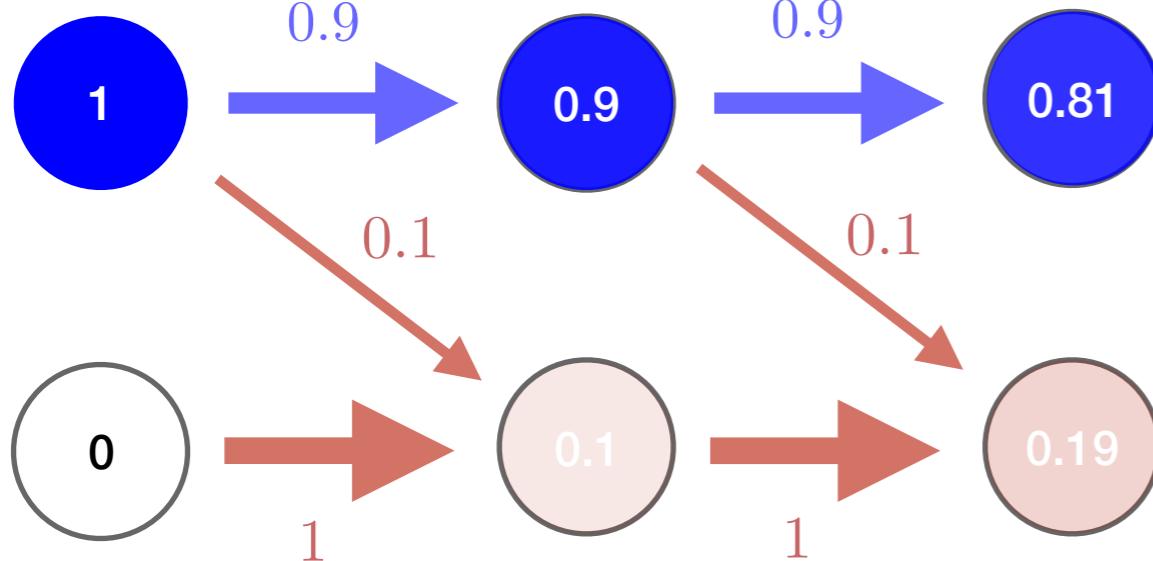
...



$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\begin{bmatrix} p_1(2) \\ p_2(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix}$$

**OK**

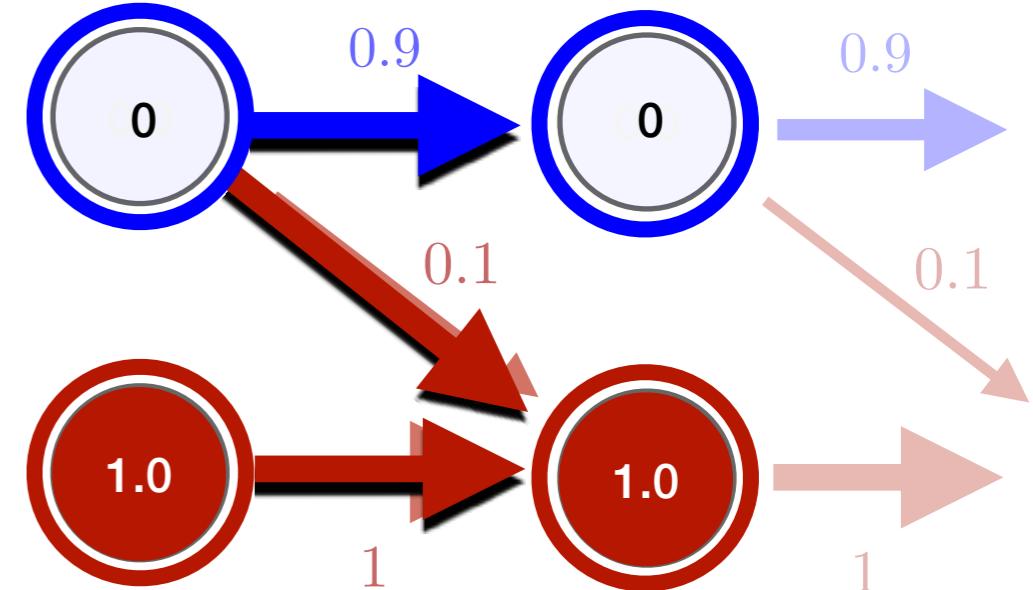


$t = 0$

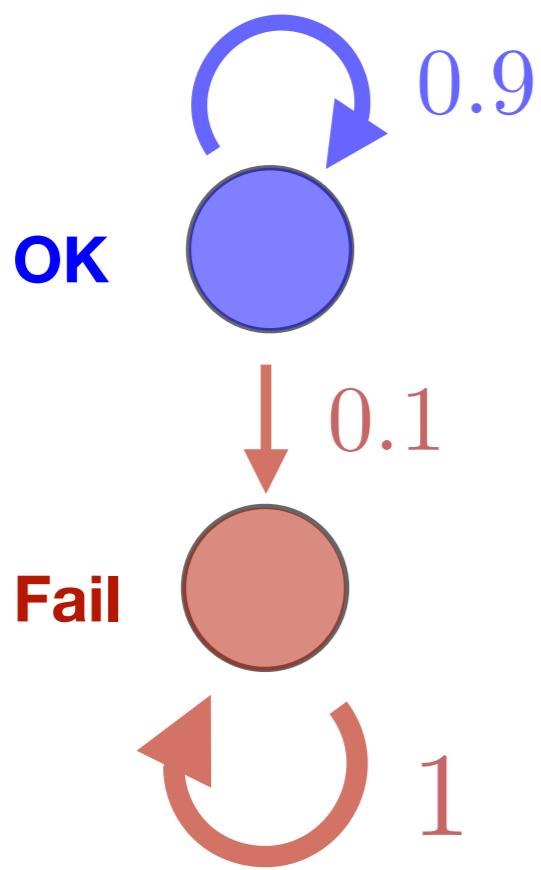
$t = 1$

$t = 2$

...



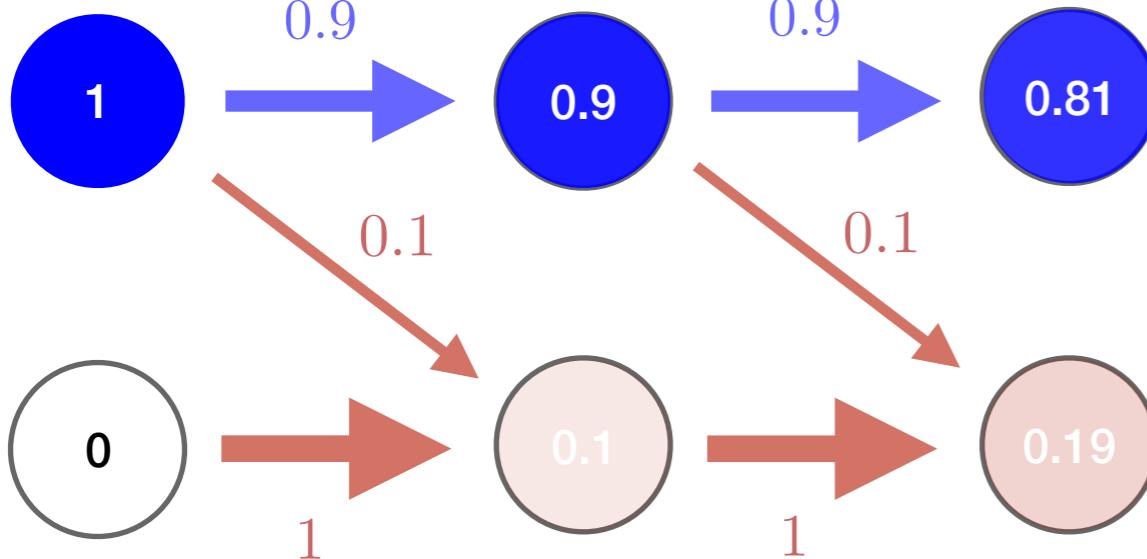
$t = 100$



$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\begin{bmatrix} p_1(2) \\ p_2(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

**OK**



$t = 0$

$t = 1$

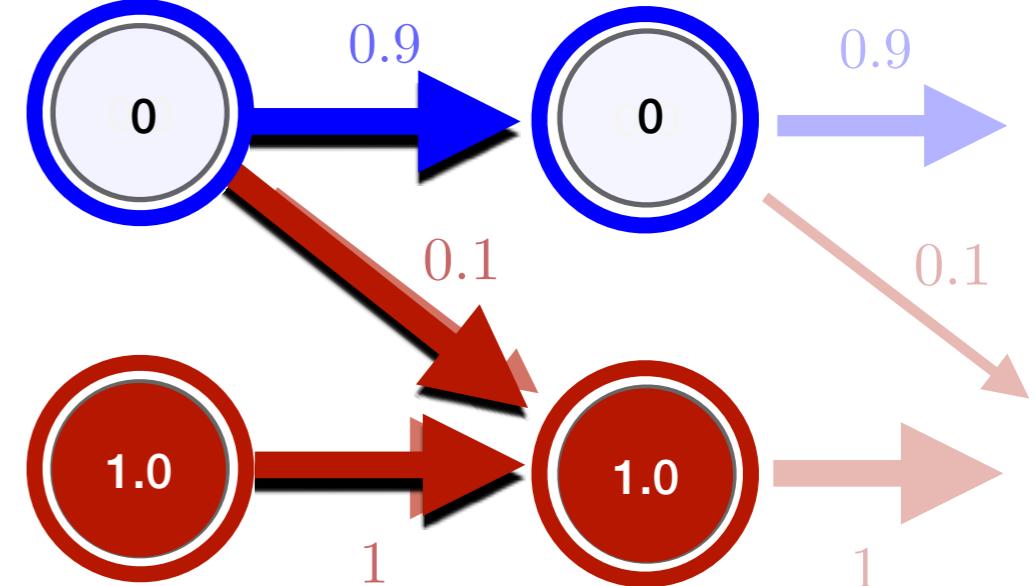
$t = 2$

$t = 99$

$t = 100$

**Fail**

...



0.1

0.1

1

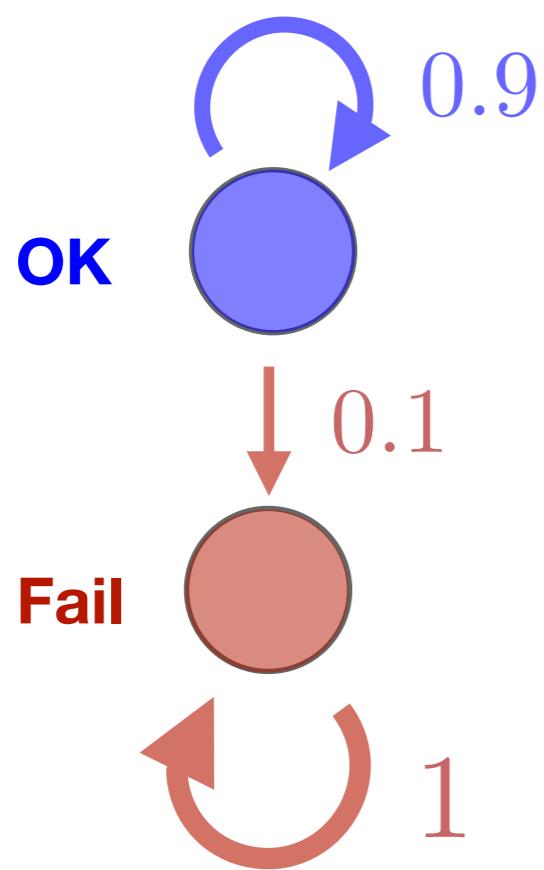
1

0.1

1

1

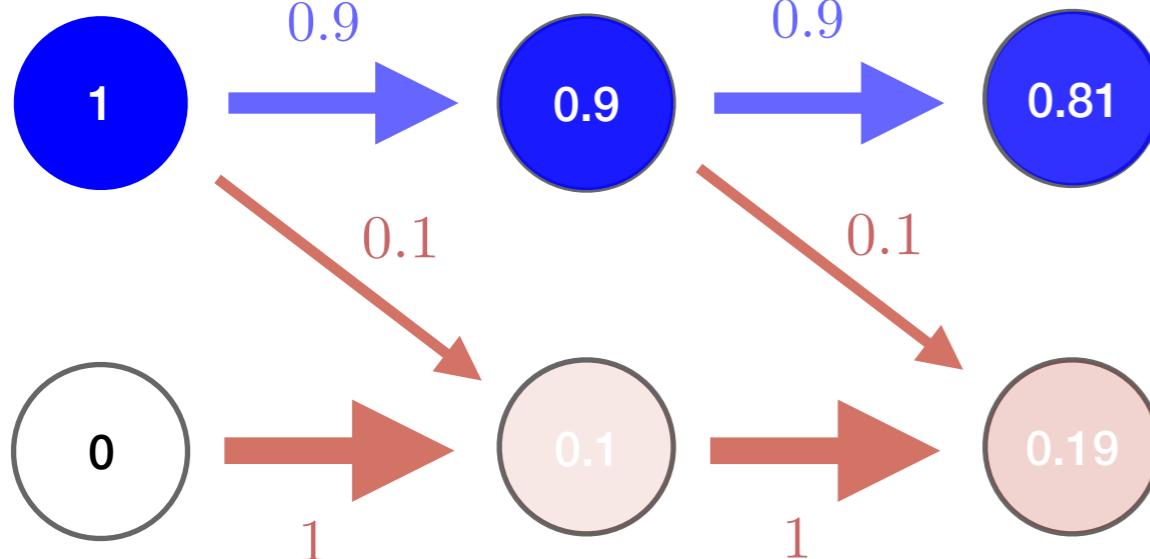
1



$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\begin{bmatrix} p_1(2) \\ p_2(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix}^2 \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

**OK**



$t = 0$

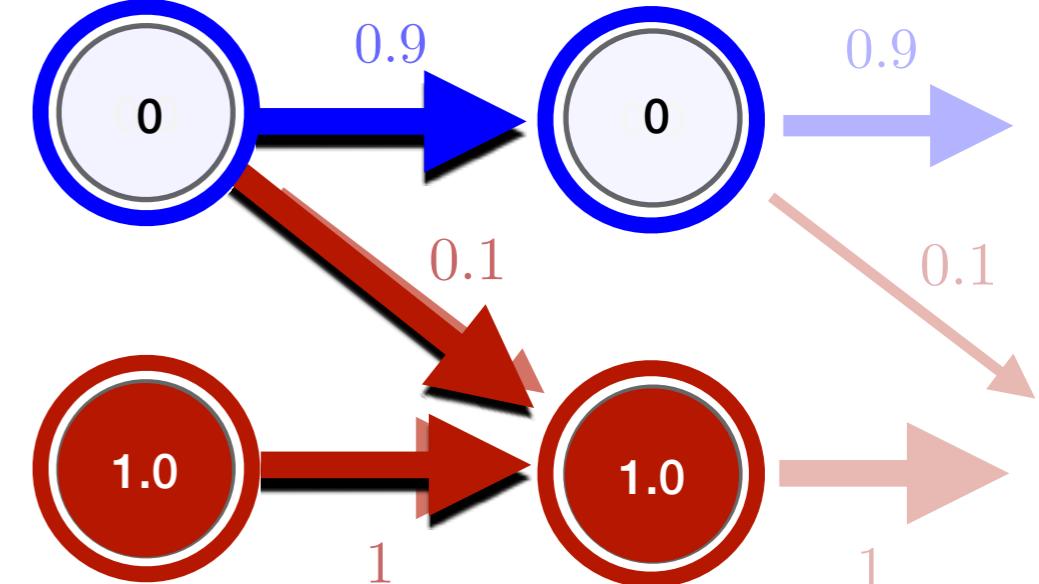
$t = 1$

$t = 2$

$t = 99$

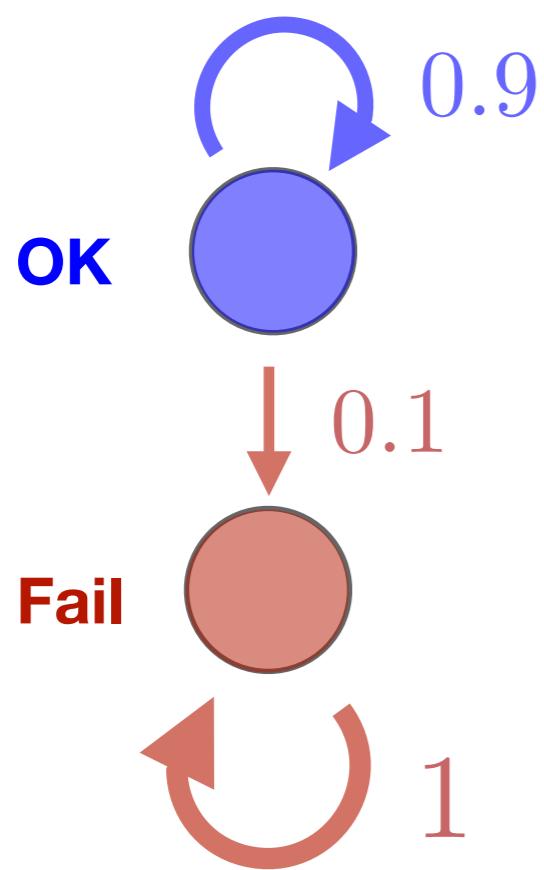
$t = 100$

...



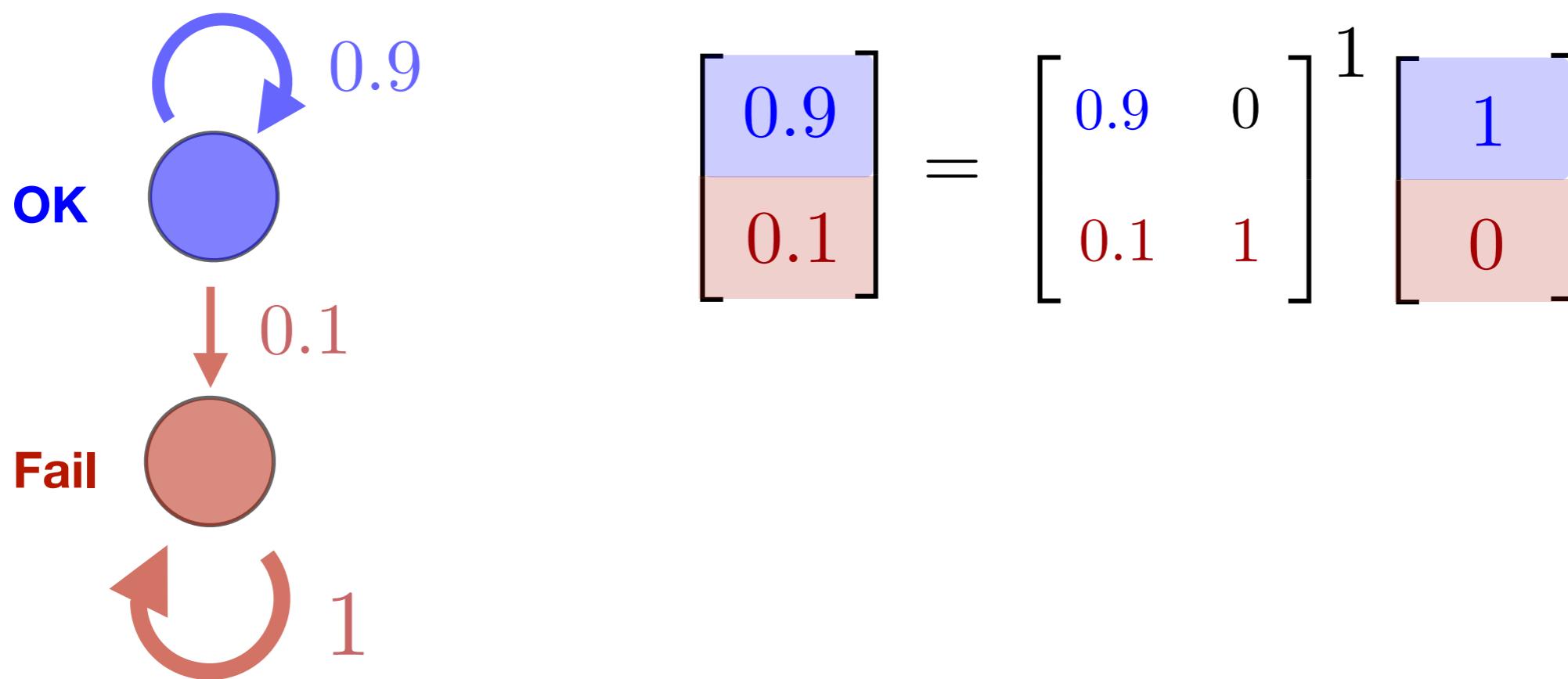
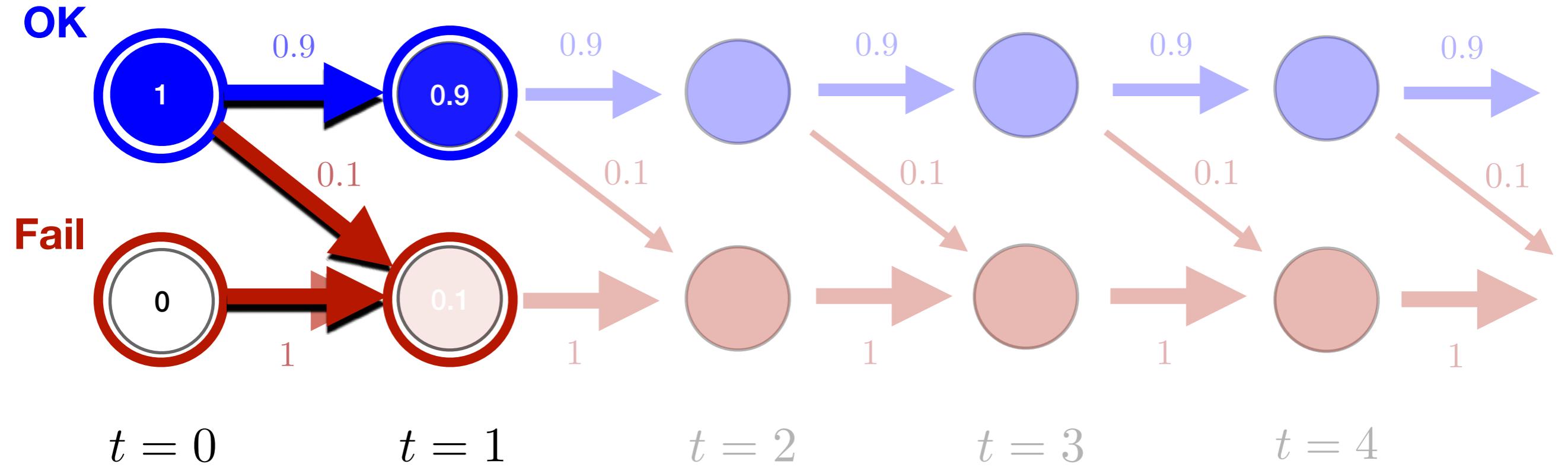
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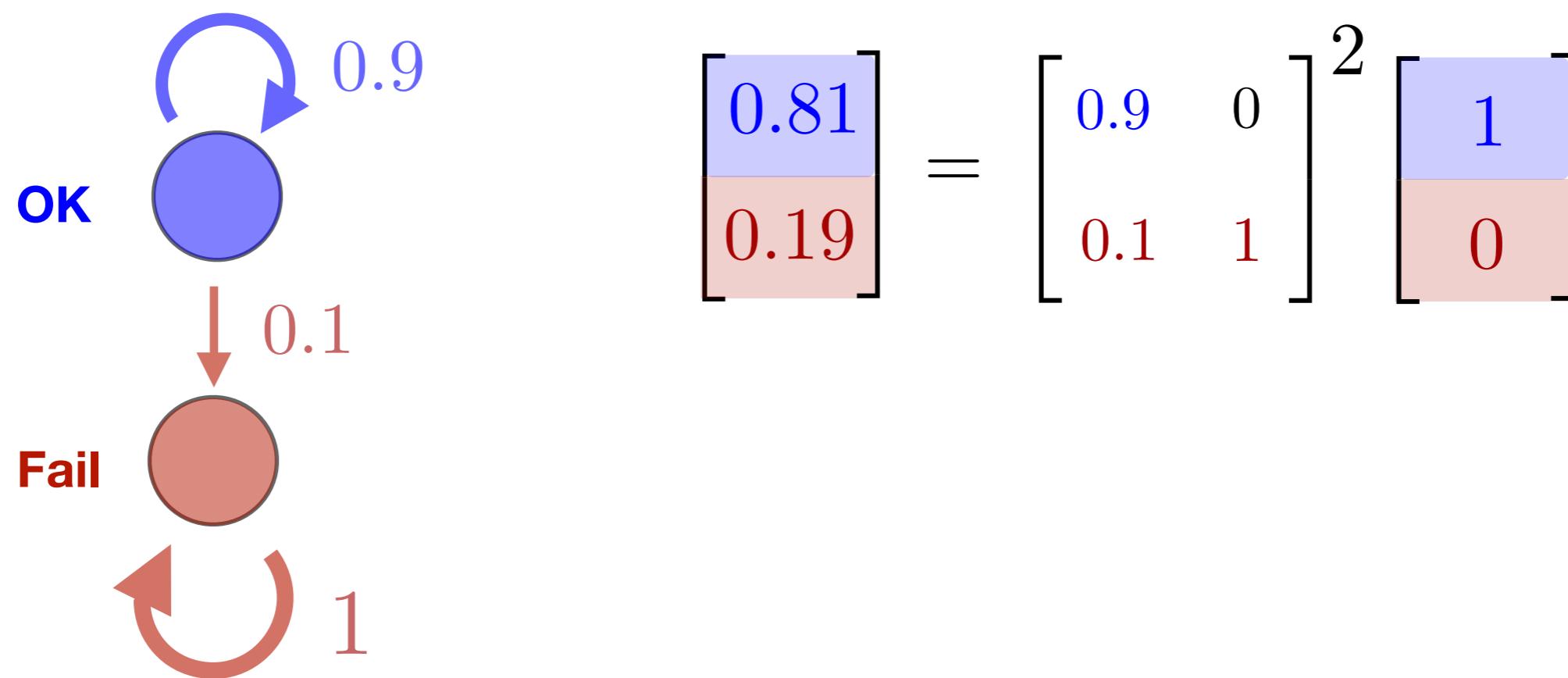
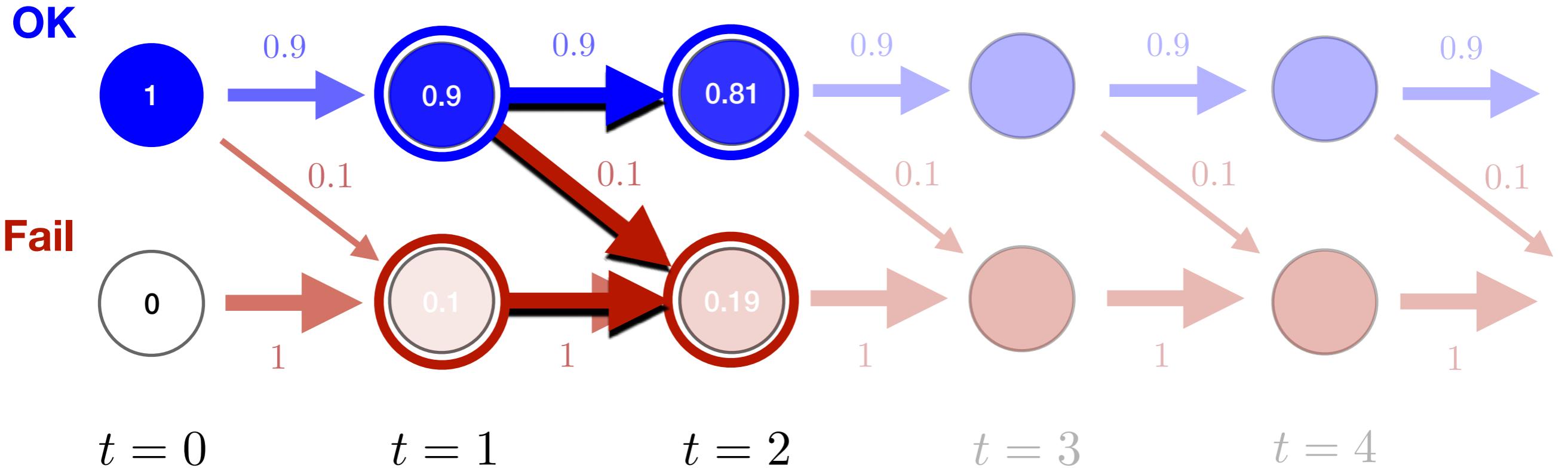
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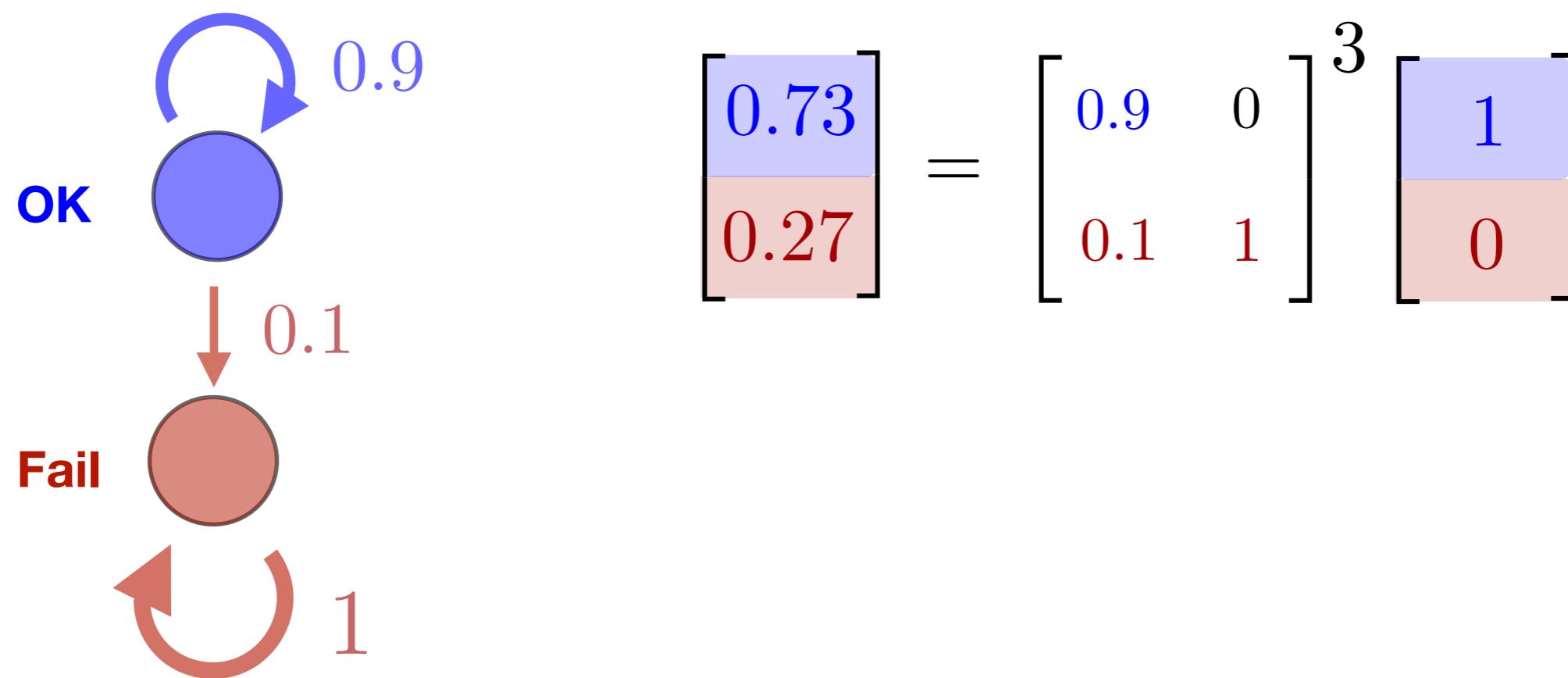
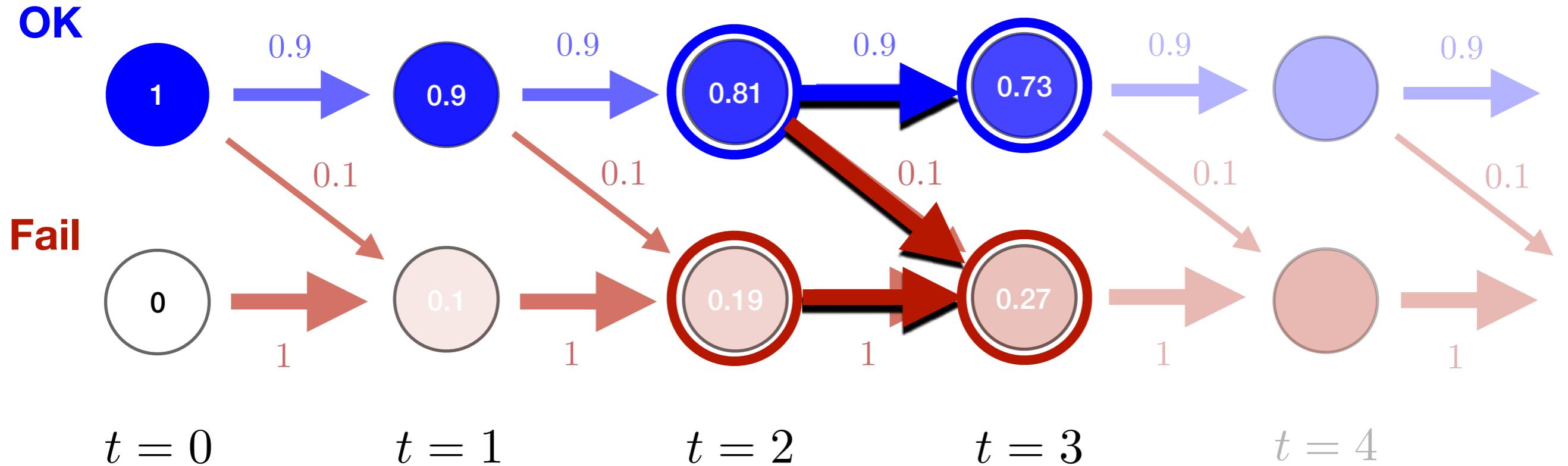


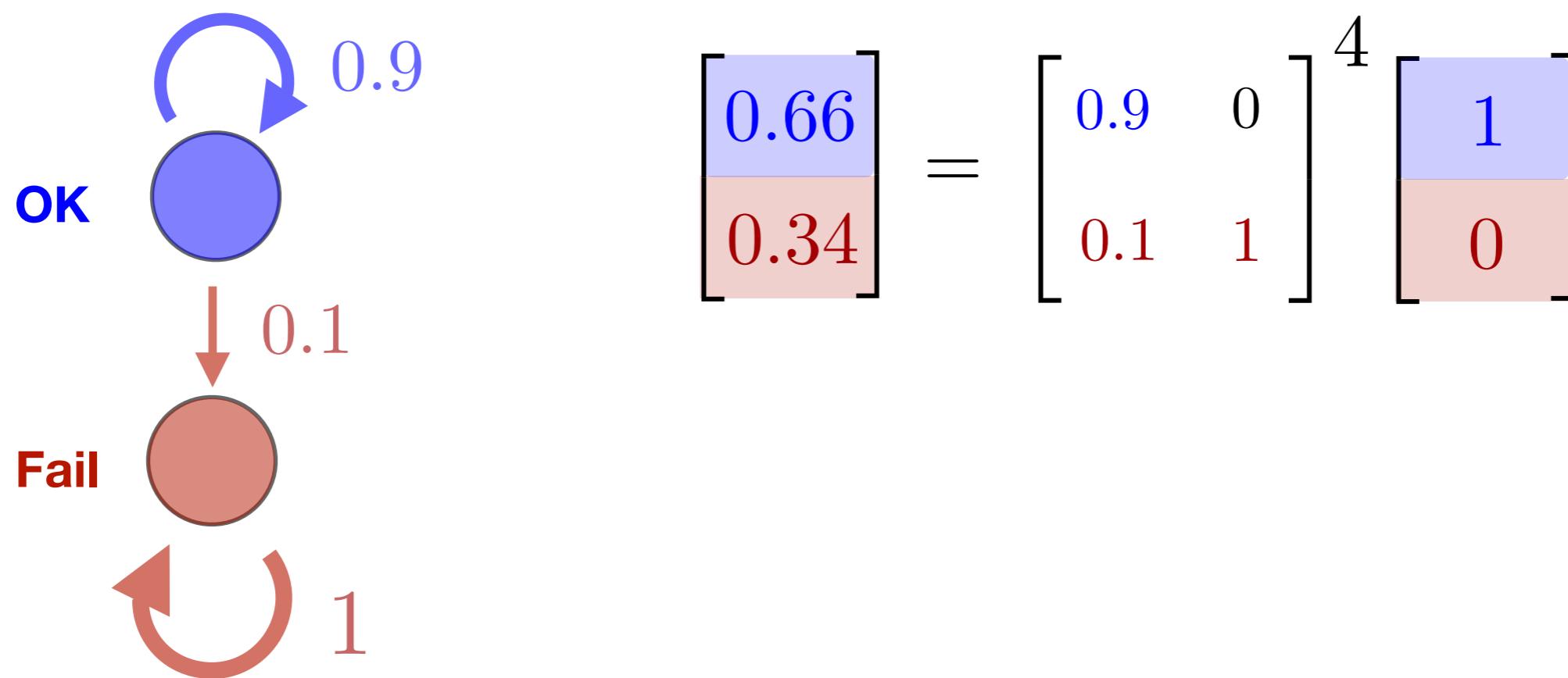
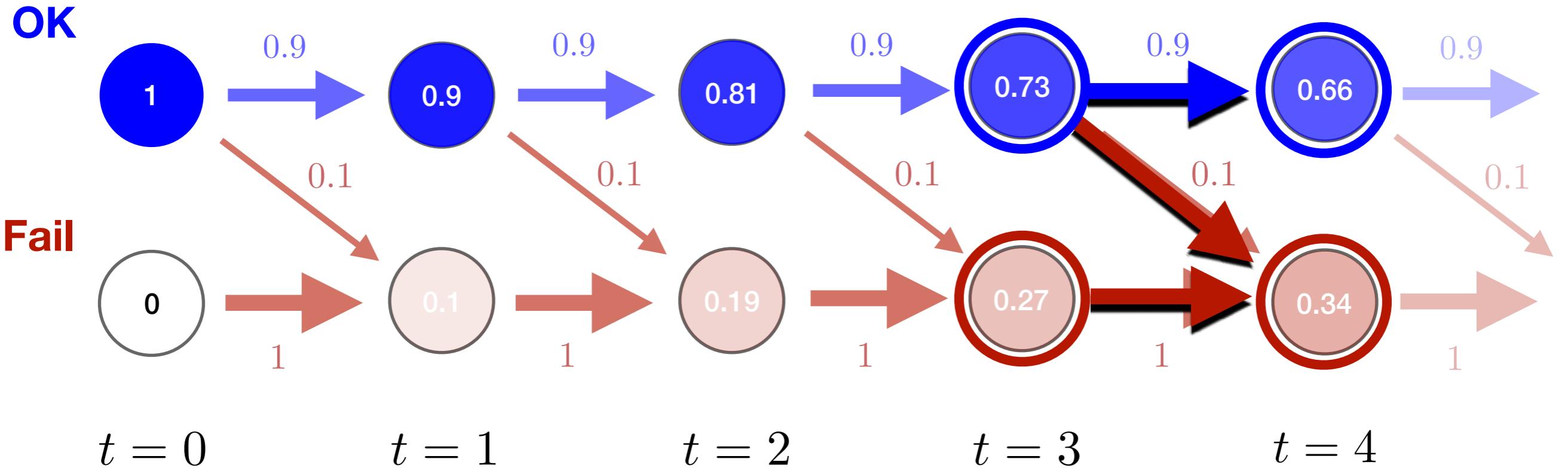
$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

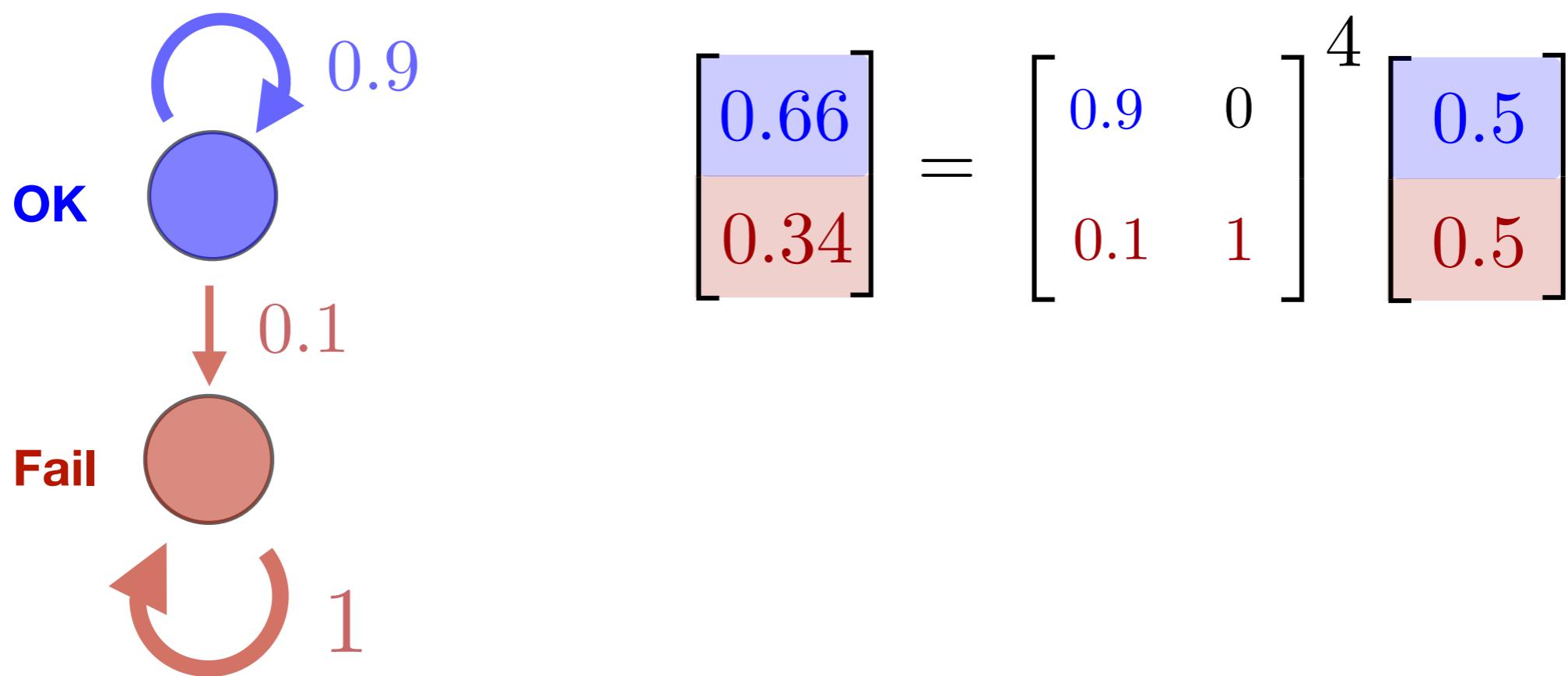
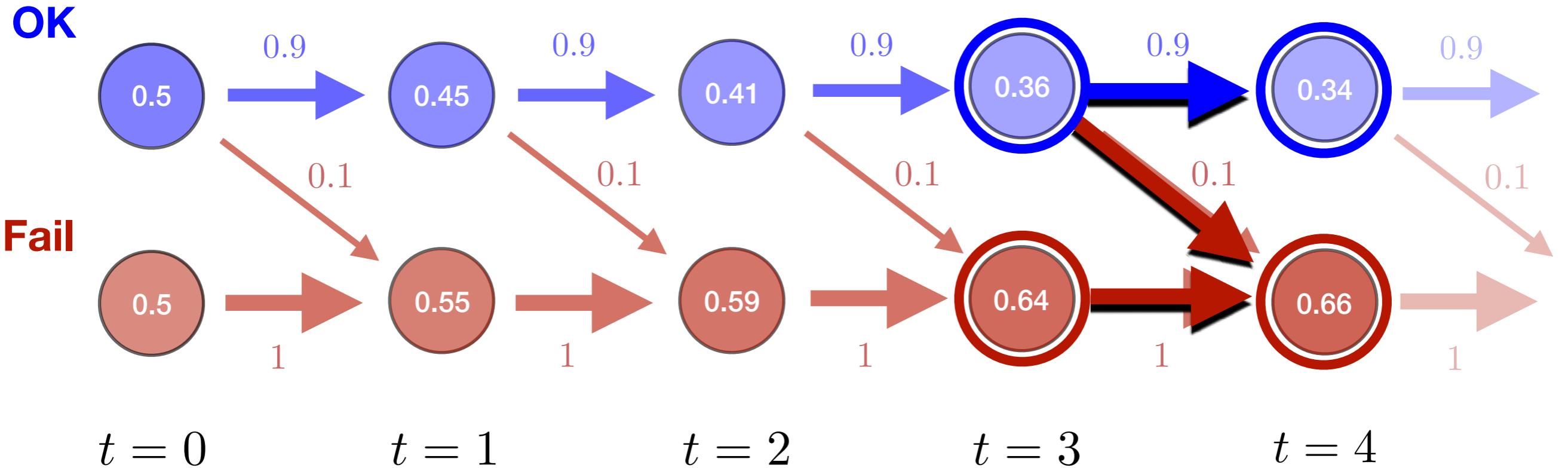
$$\begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix}^k \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

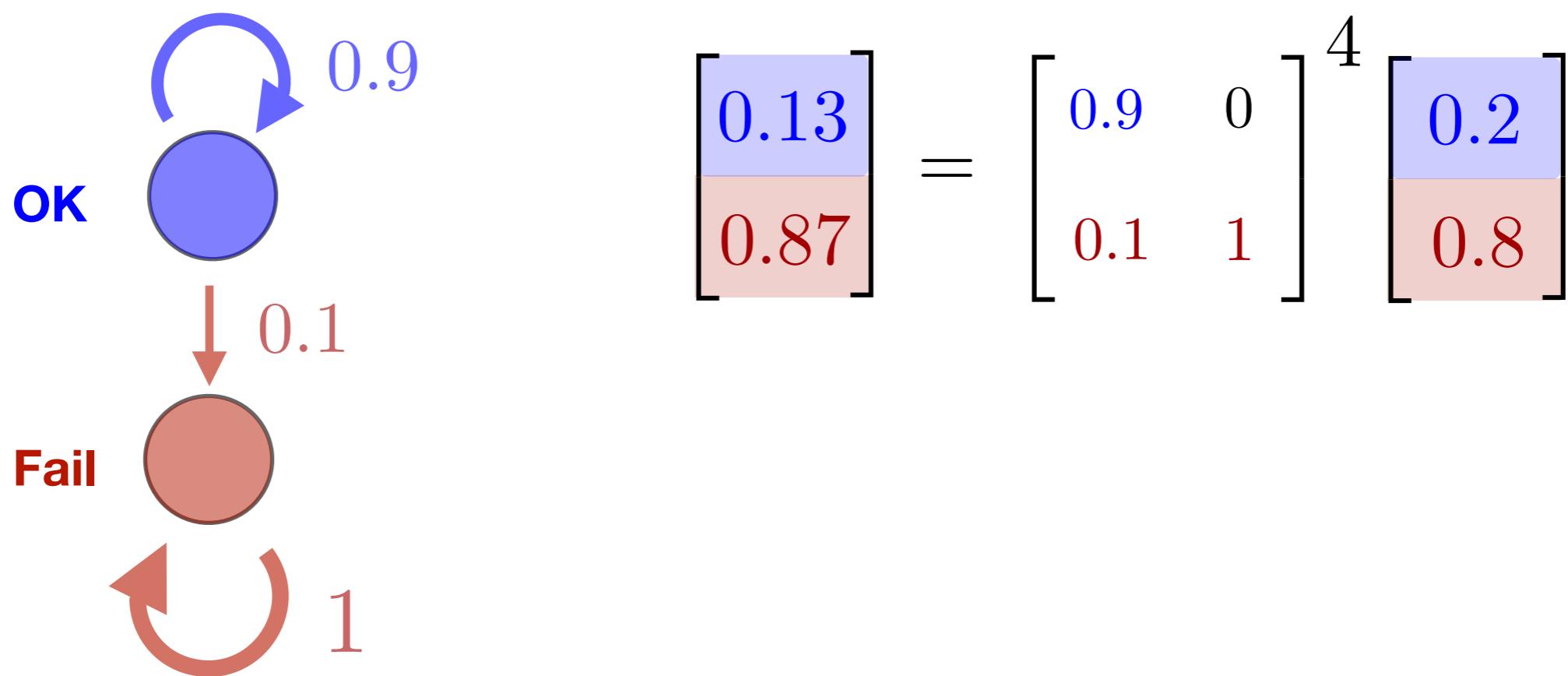
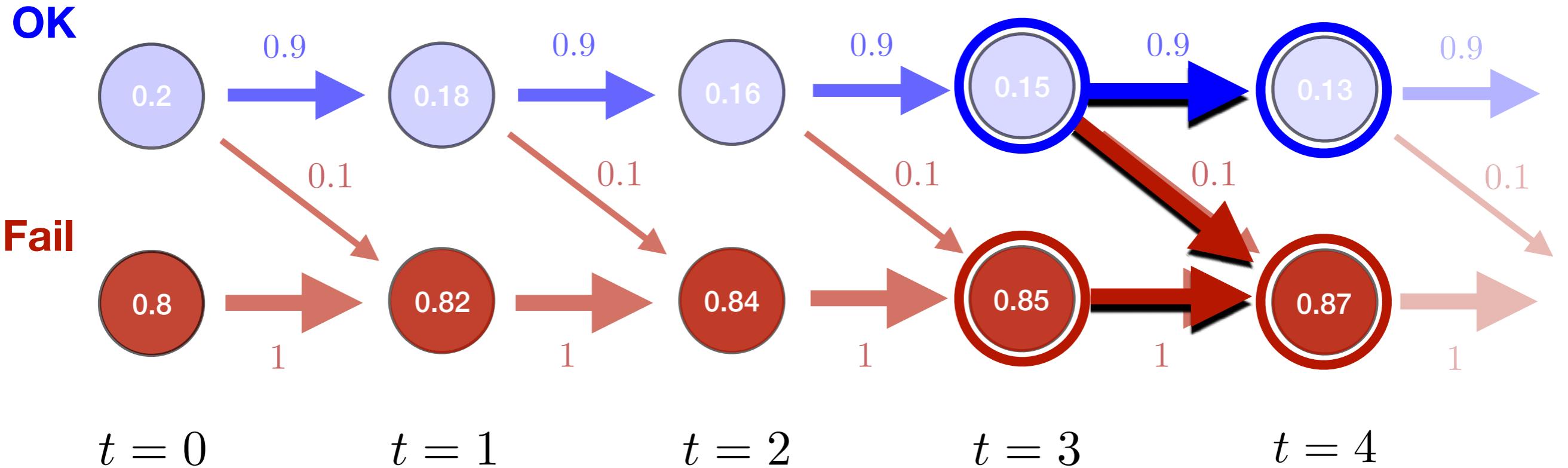


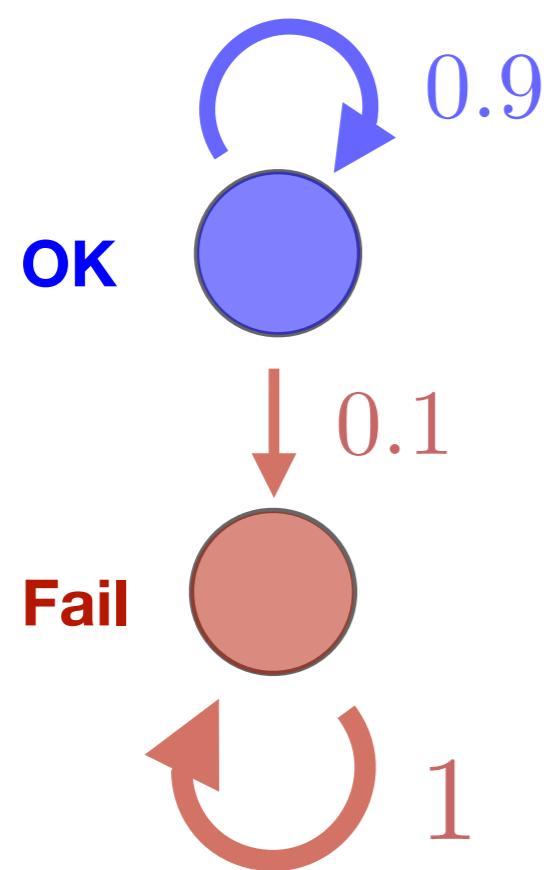
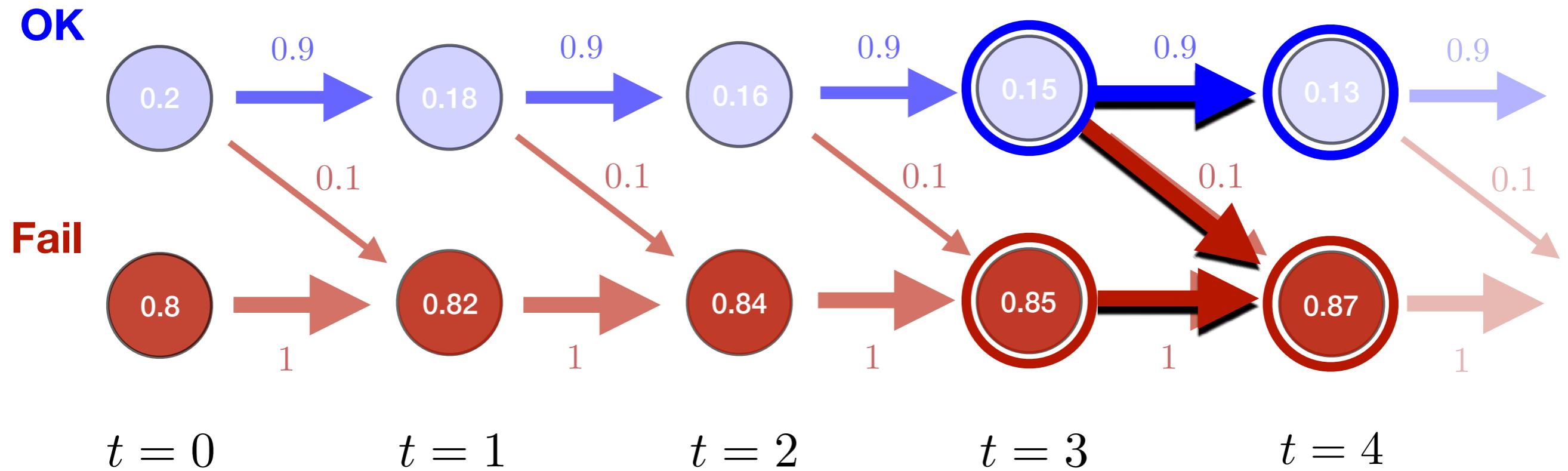




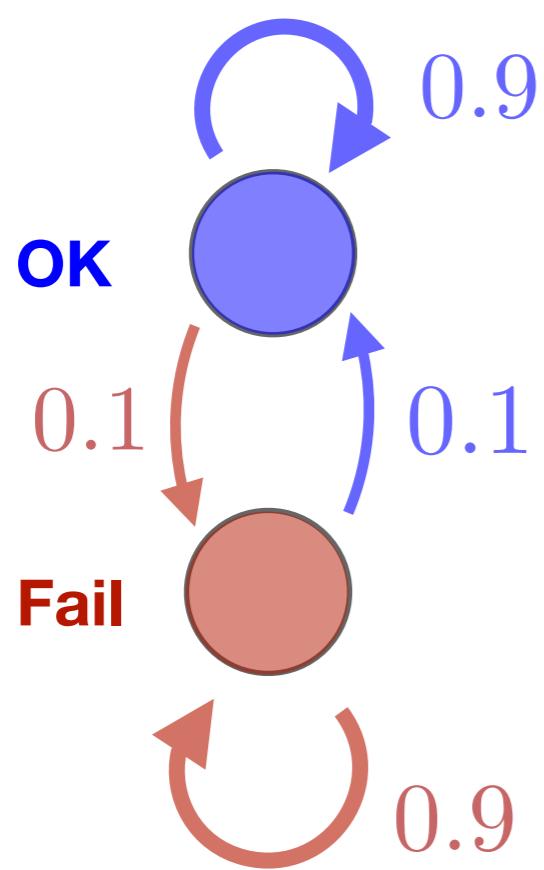
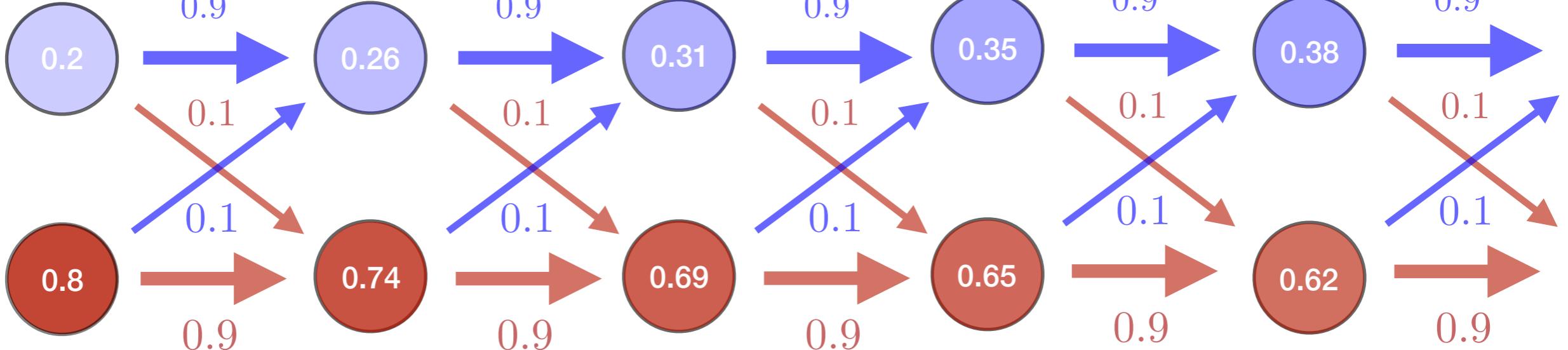








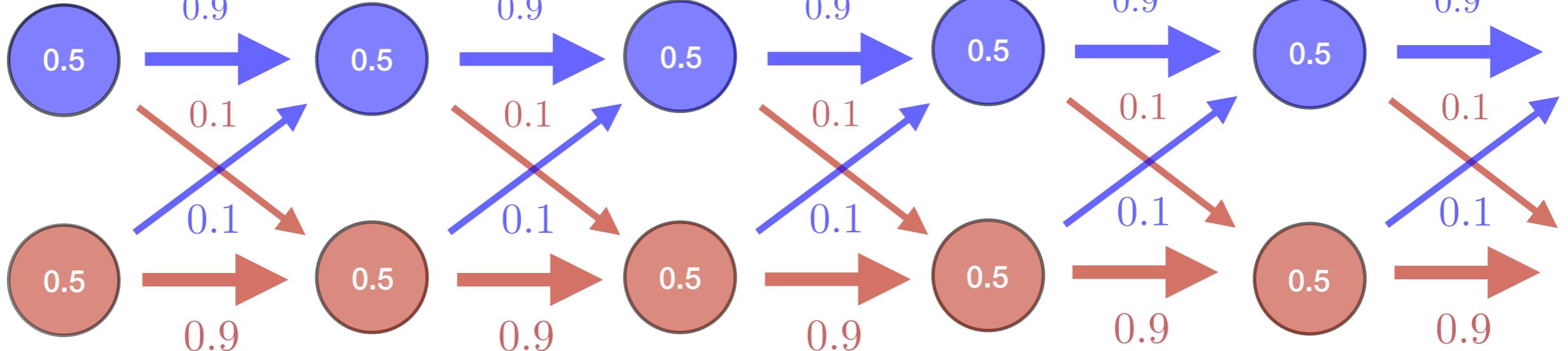
$$\lim_{k \rightarrow \infty} \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} p_1(\infty) \\ p_2(\infty) \end{bmatrix} = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}$$

**OK**

$$\begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}^k \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\lim_{k \rightarrow \infty} \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} p_1(\infty) \\ p_2(\infty) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

**OK**



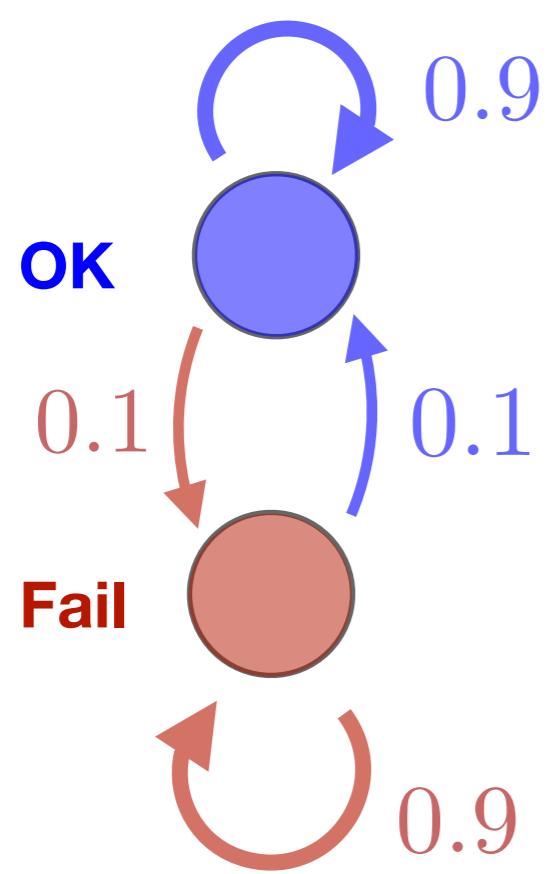
$t = 0$

$t = 1$

$t = 2$

$t = 3$

$t = 4$



$\lim_{k \rightarrow \infty}$

$$\begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}^k \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\lim_{k \rightarrow \infty} \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} p_1(\infty) \\ p_2(\infty) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$