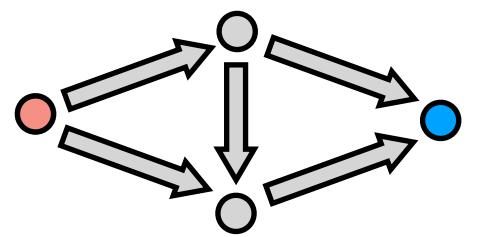
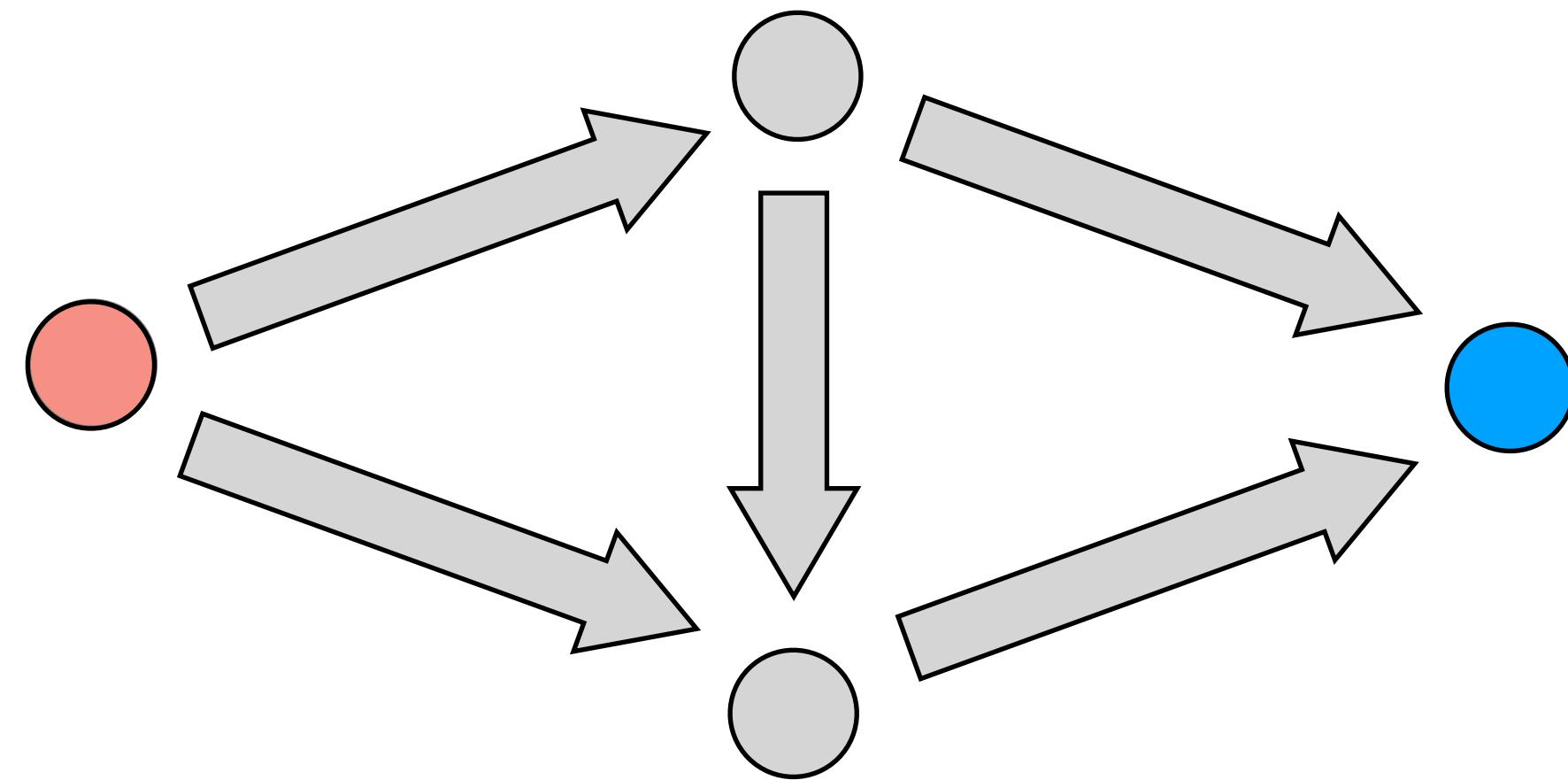


Potential Games

Routing
Games



Routing Games

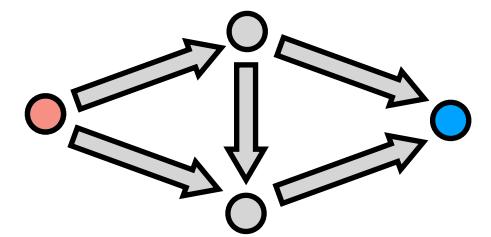


x : edge traffic

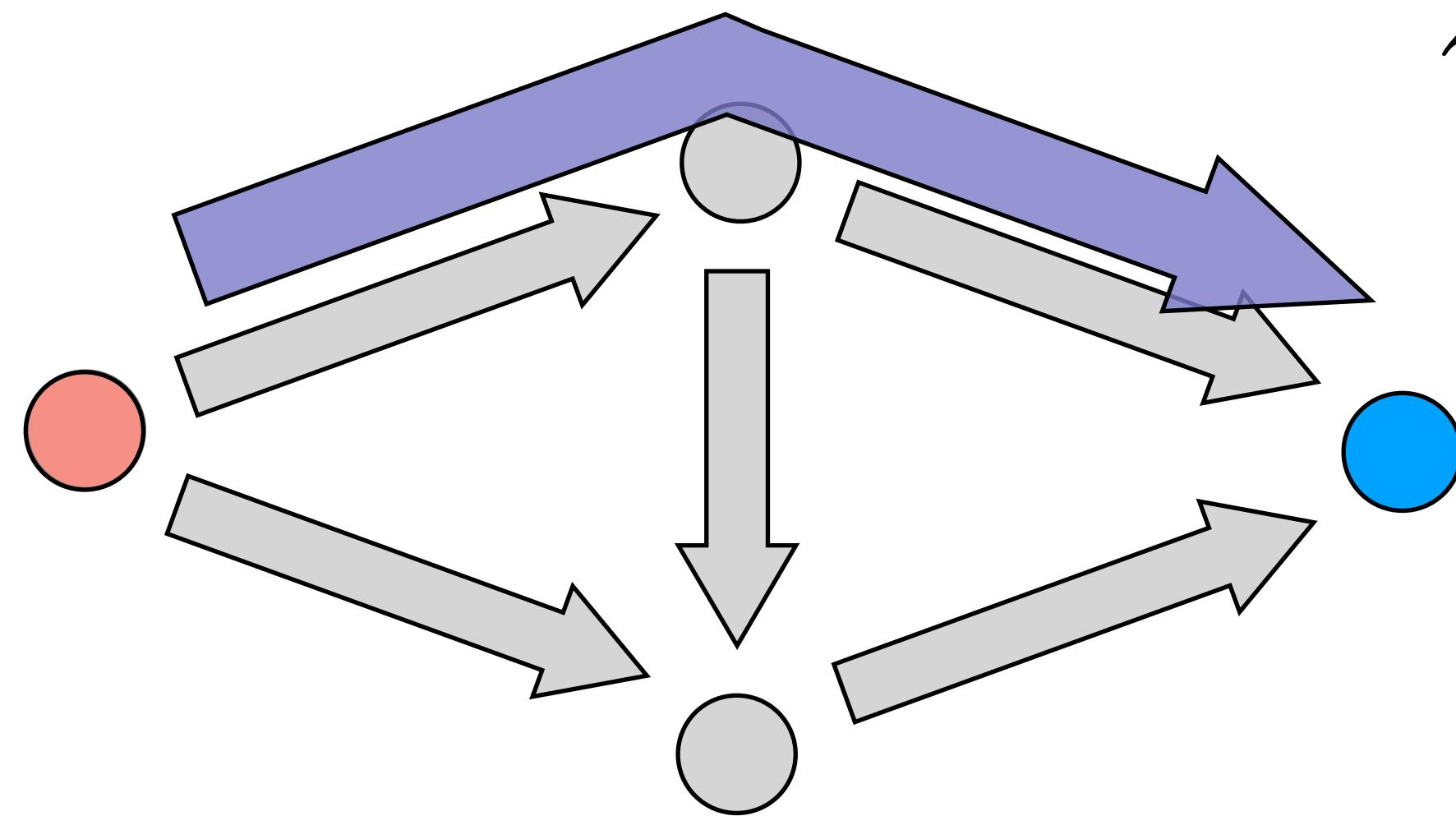
z : route traffic

Potential Games

Routing
Games



Routing Games

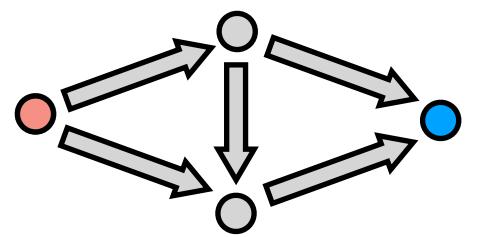


x : edge traffic

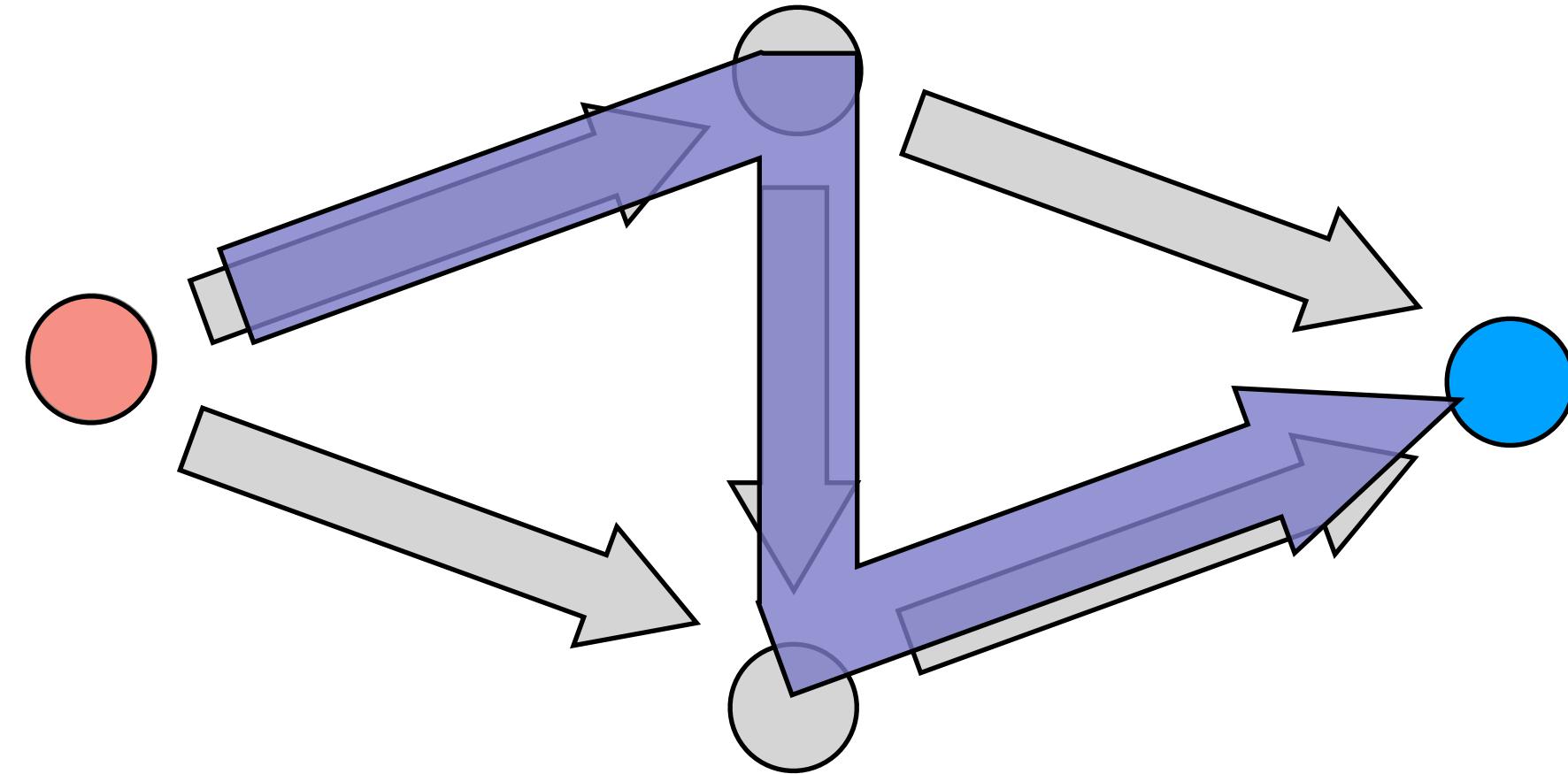
z : route traffic

Potential Games

Routing
Games



Routing Games

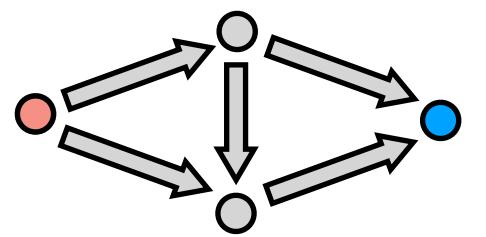


x : edge traffic

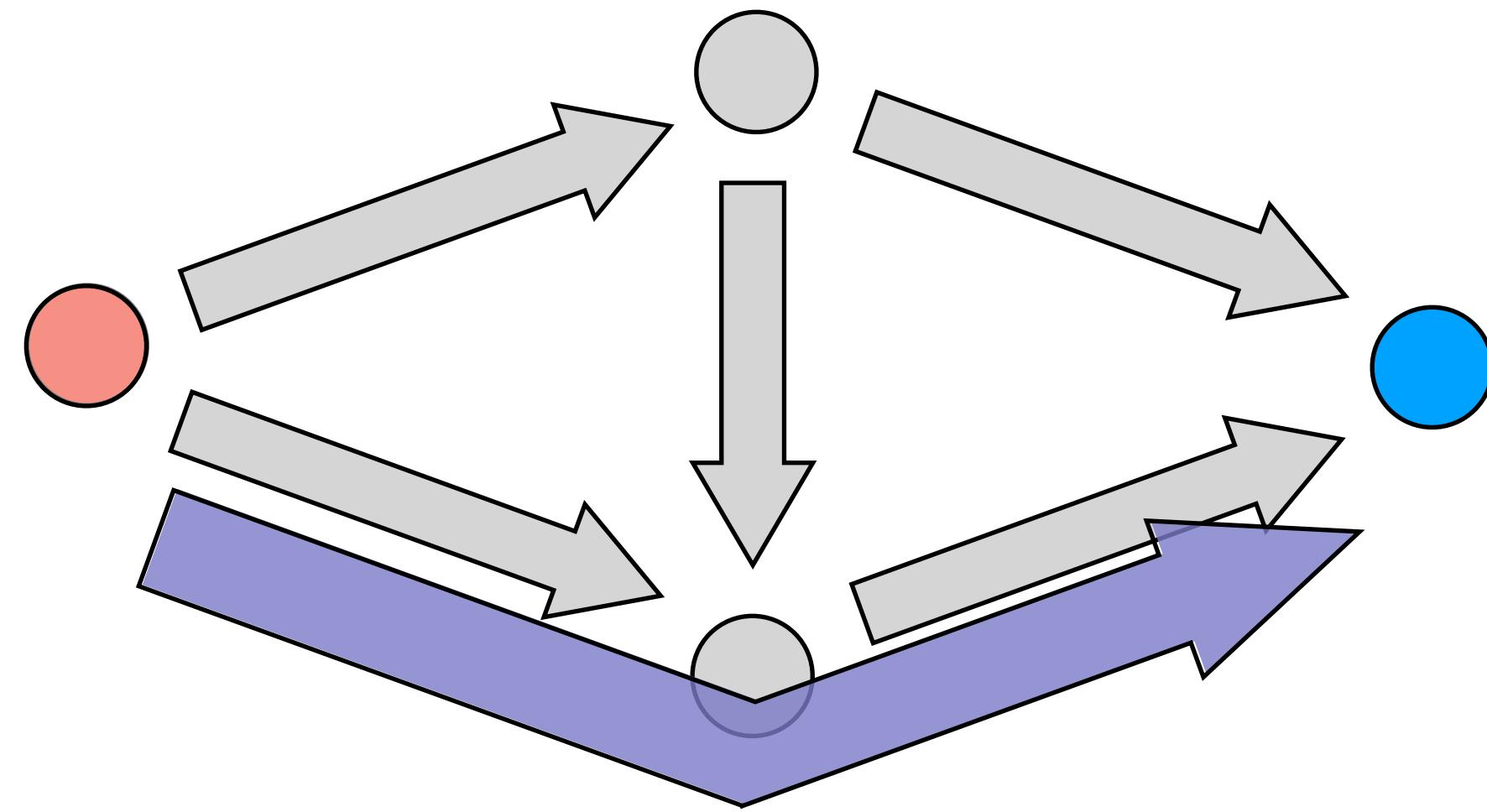
z : route traffic

Potential Games

Routing
Games



Routing Games

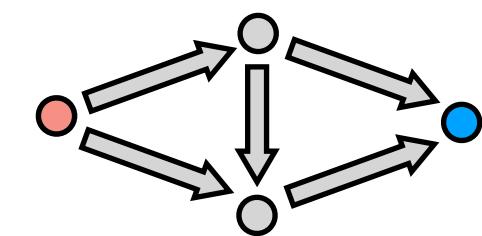


x : edge traffic

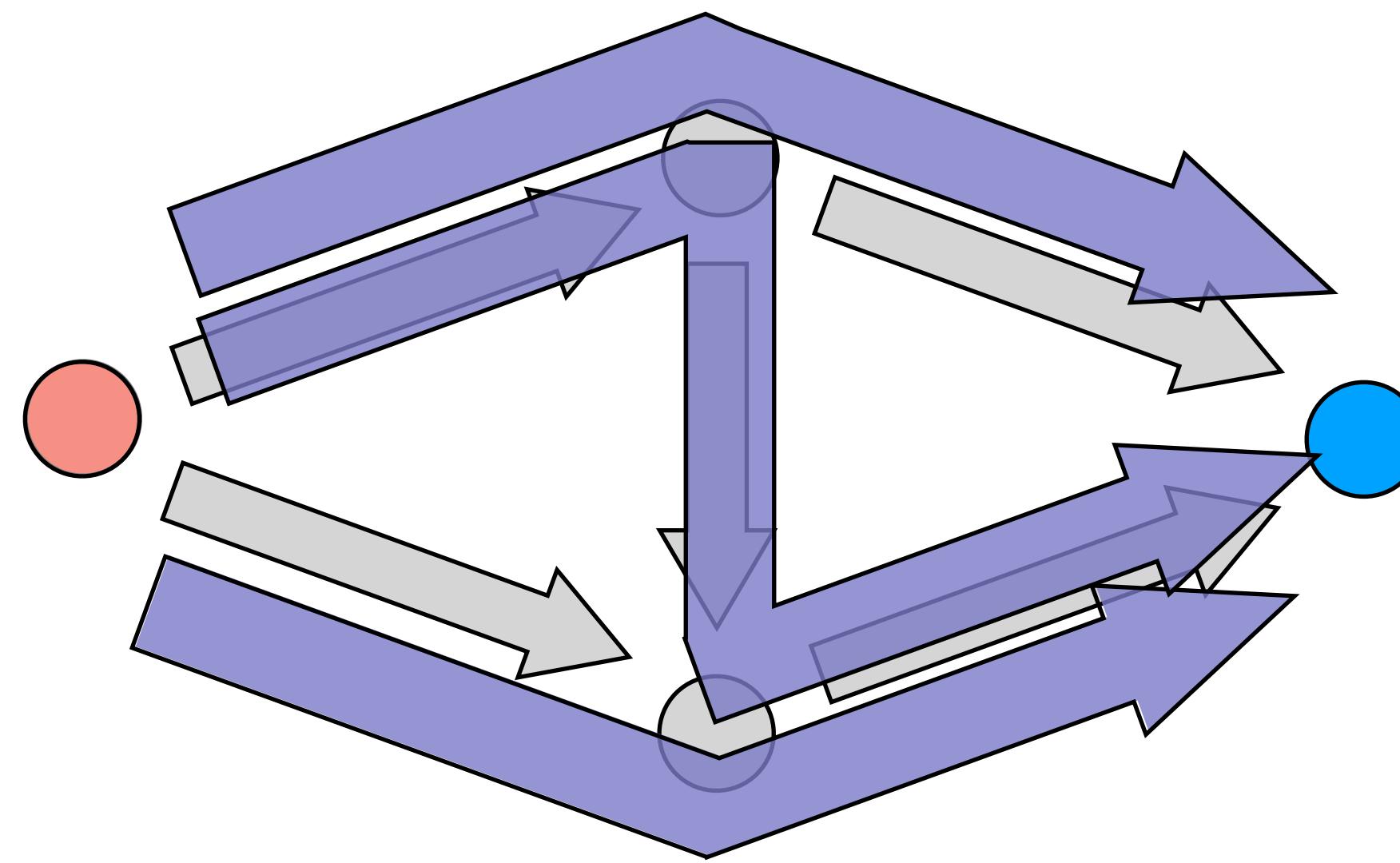
z : route traffic

Potential Games

Routing
Games



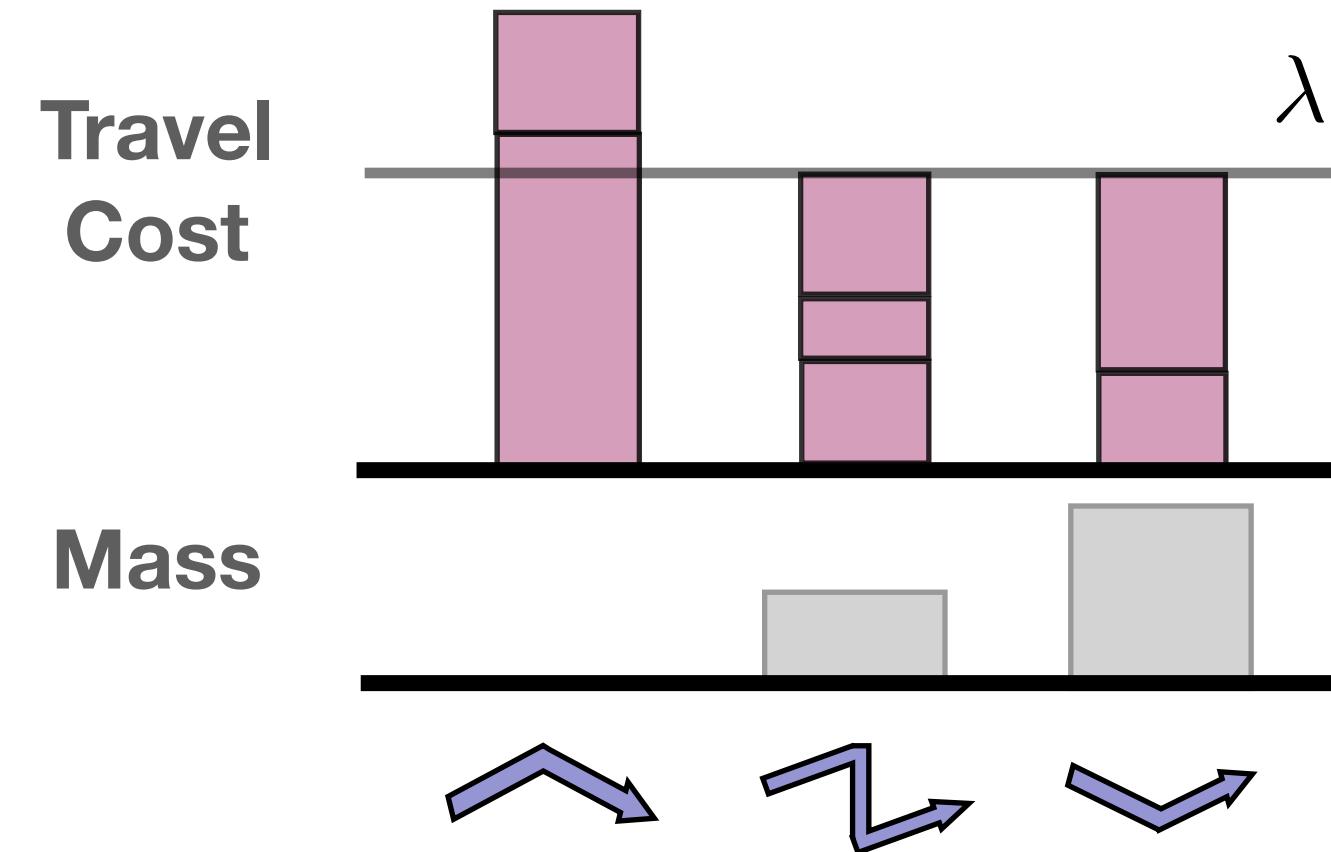
Routing Games



x : edge traffic

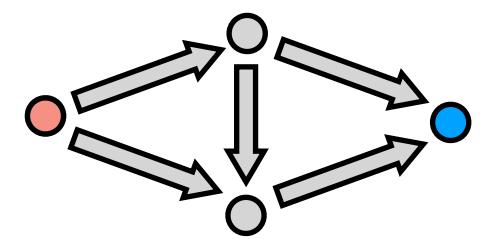
z : route traffic

Wardrop Equilibrium



Potential Games

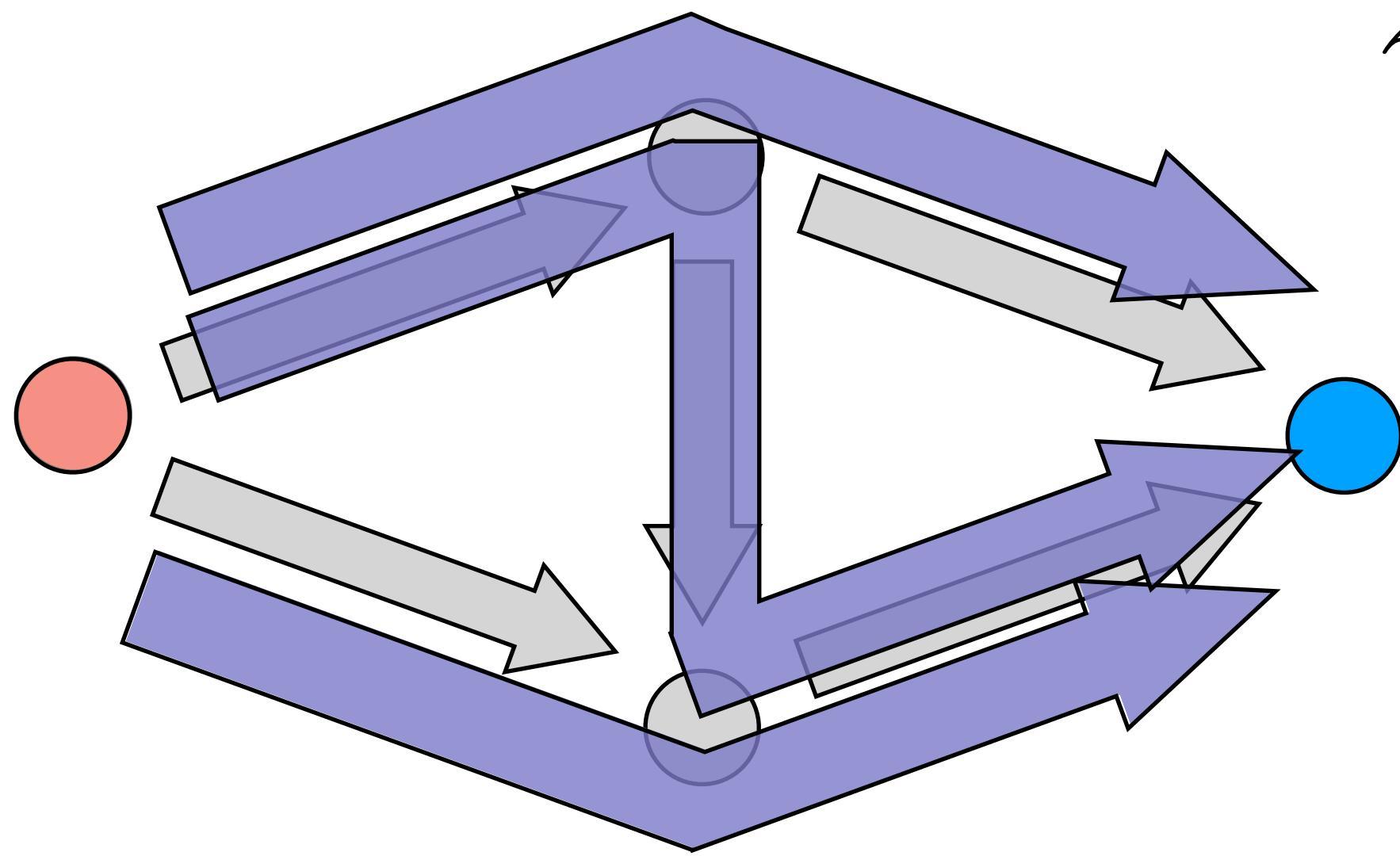
Routing
Games



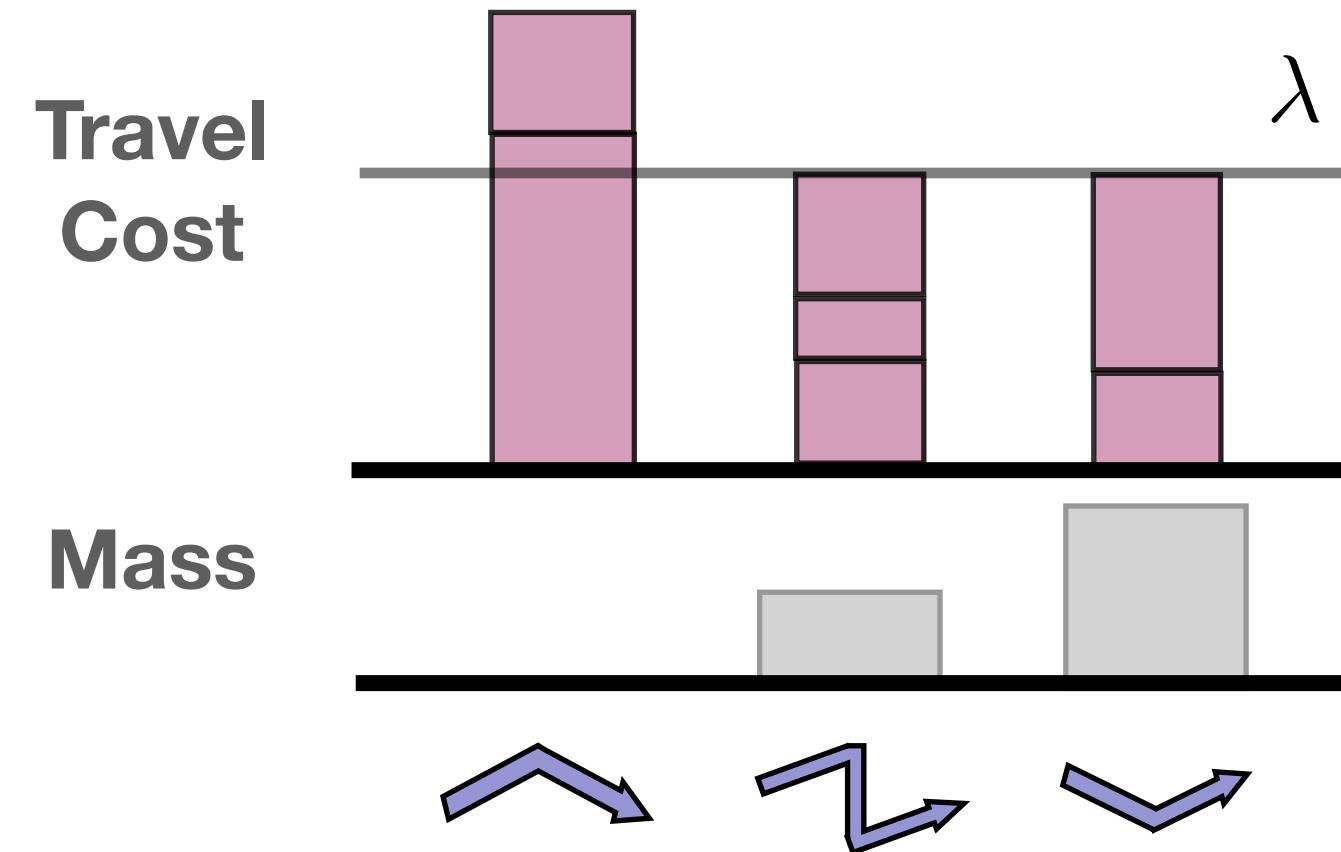
Potential
Function

$$F(x)$$

Routing Games



Wardrop Equilibrium

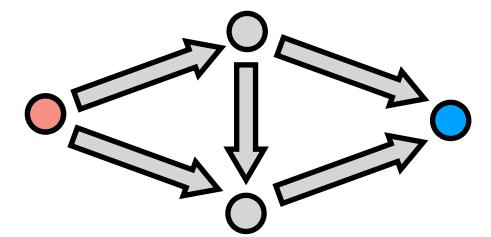


x : edge traffic

z : route traffic

Potential Games

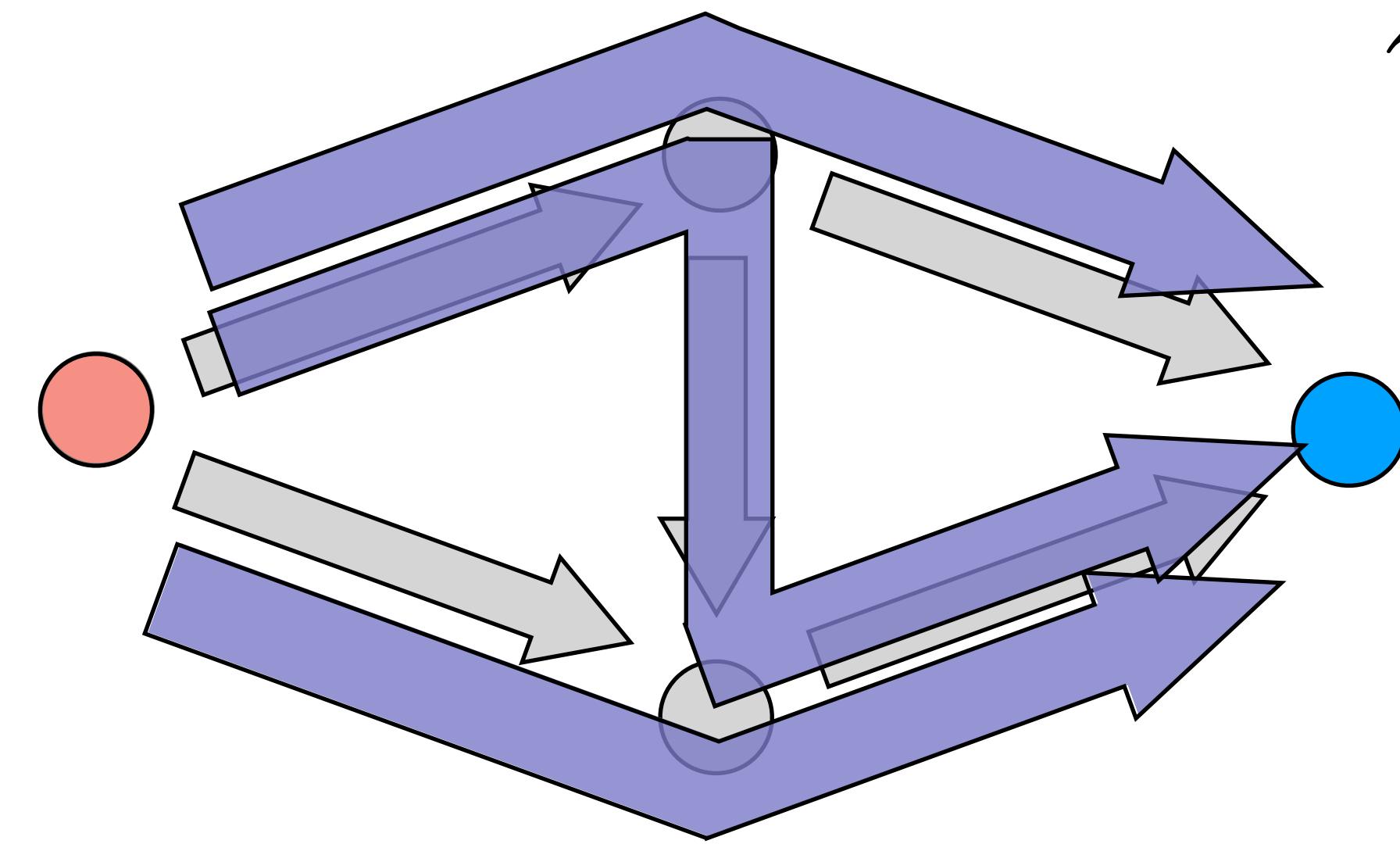
Routing
Games



Potential
Function

$$F(x)$$

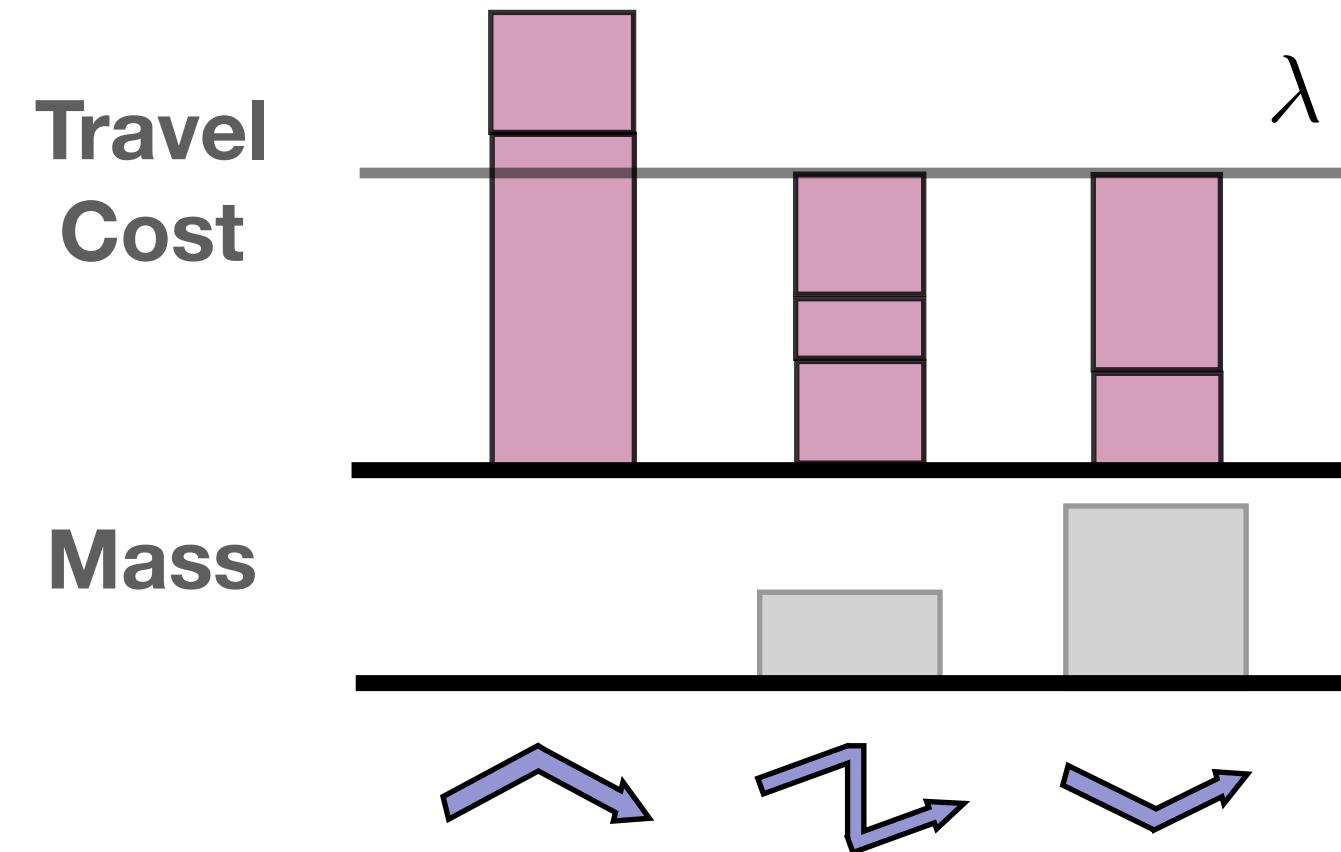
Routing Games



x : edge traffic

z : route traffic

Wardrop Equilibrium



$$\min_{x,z}$$

$$F(x)$$

$$\text{s.t.} \quad 1^T z = m \quad z \geq 0$$

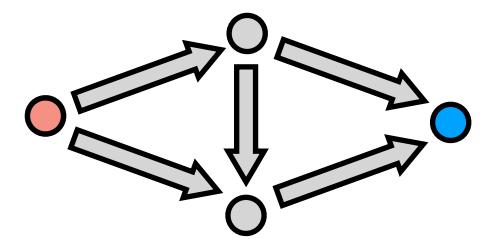
$$x = Rz$$

Mass conservation

Edges in routes

Potential Games

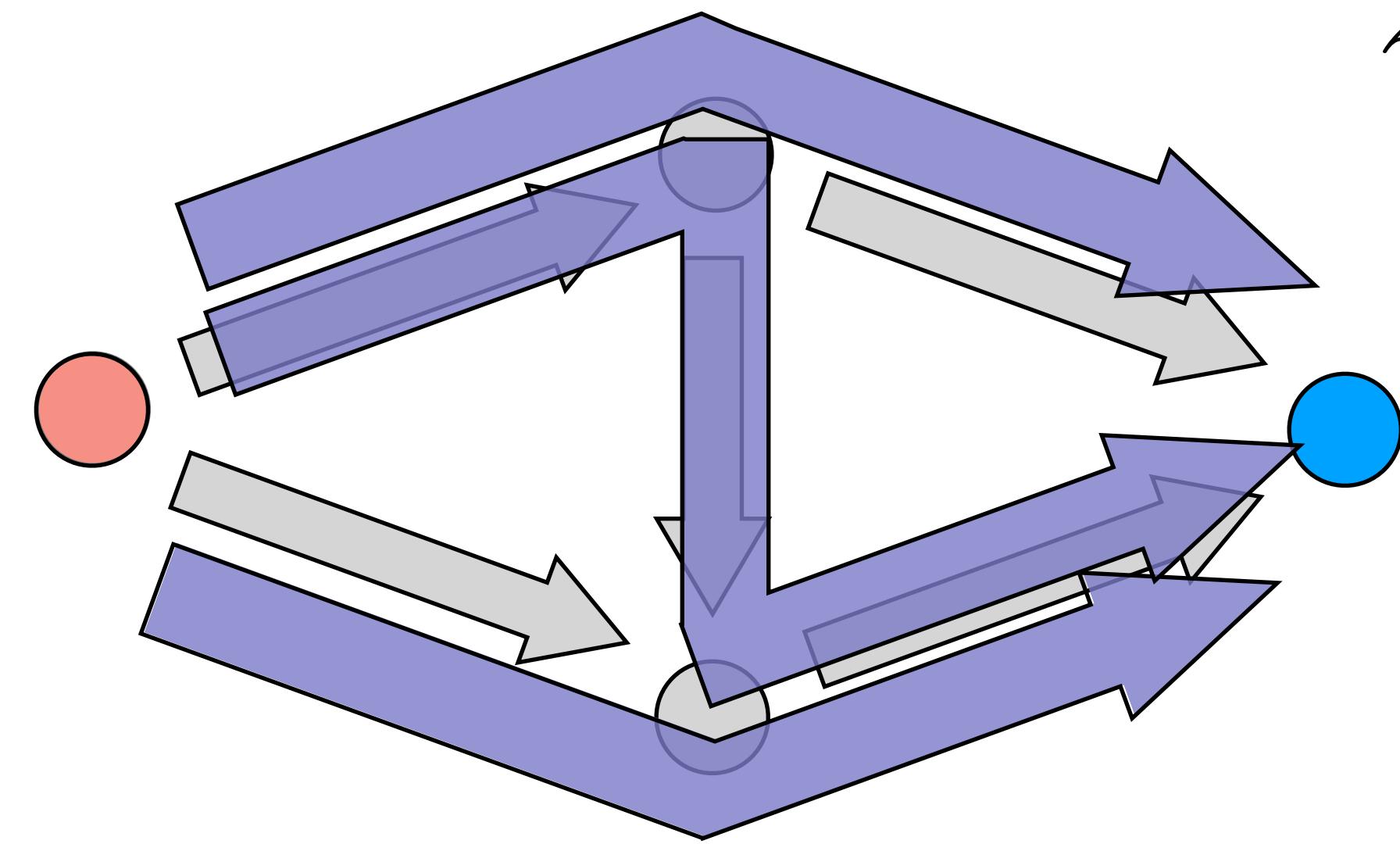
Routing
Games



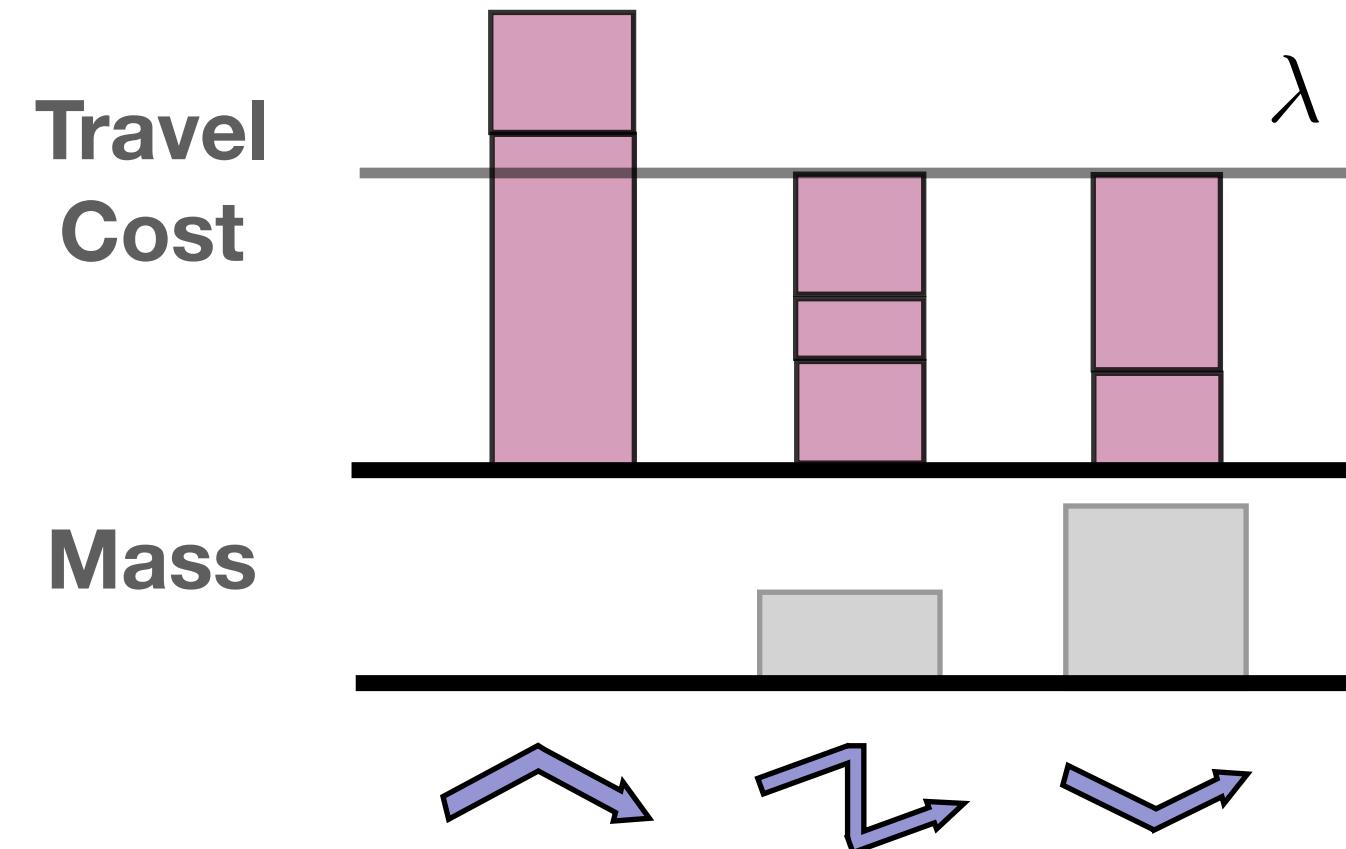
Potential
Function

$$F(x)$$

Routing Games



Wardrop Equilibrium



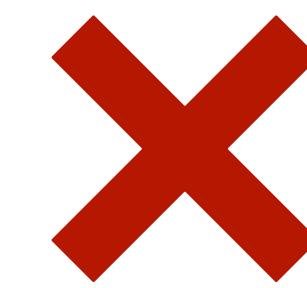
$$\min_{x,z}$$

$$F(x)$$

s.t.

$$1^T z = m \quad z \geq 0$$

$$x = Rz$$



Mass conservation

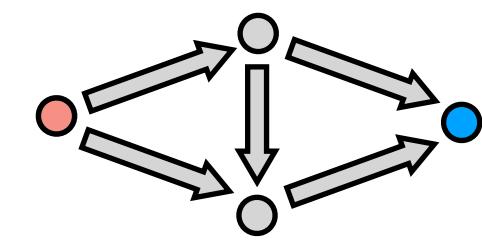
Edges in routes

x : edge traffic

z : route traffic

Potential Games

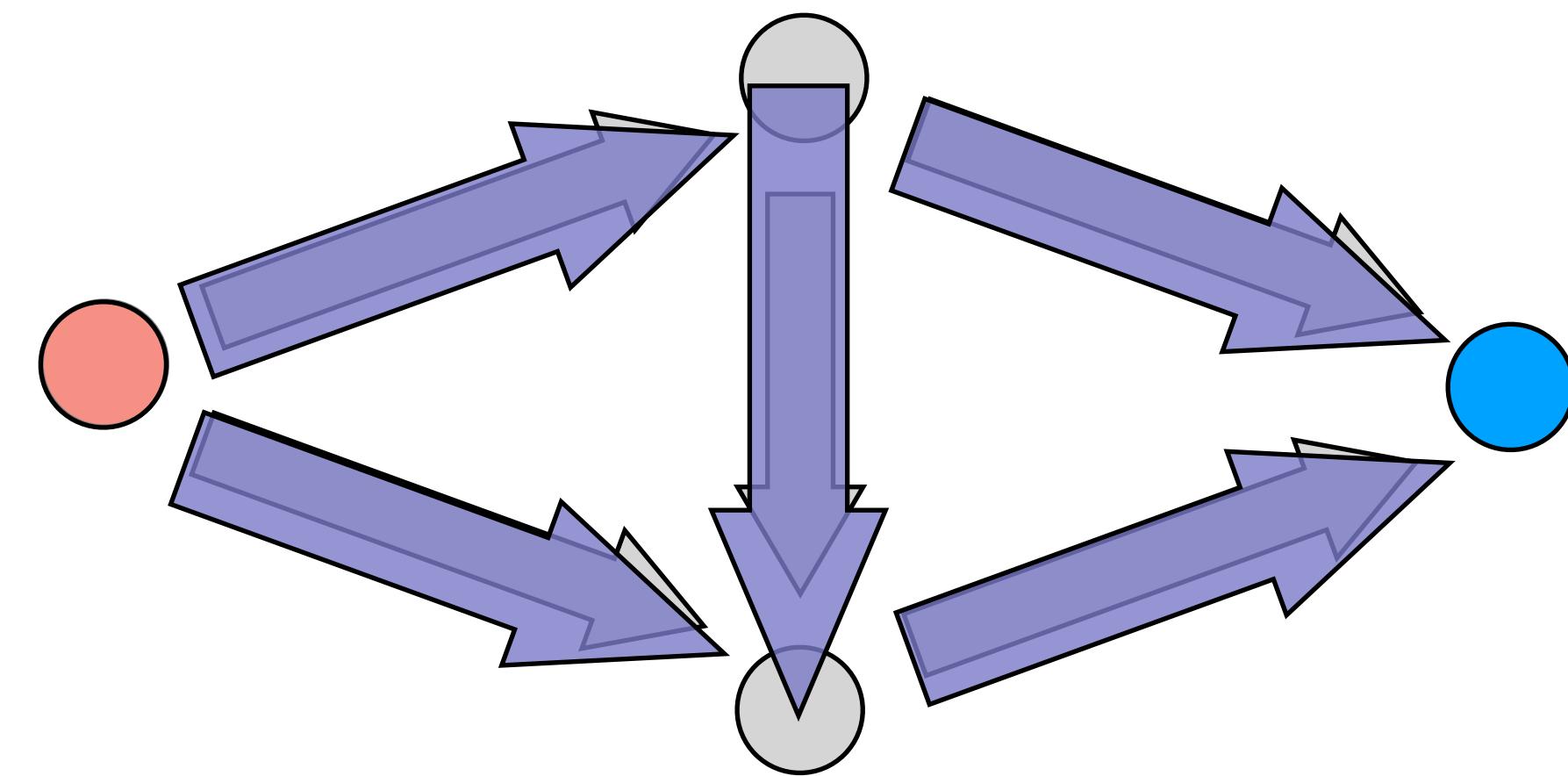
Routing
Games



Potential
Function

$$F(x)$$

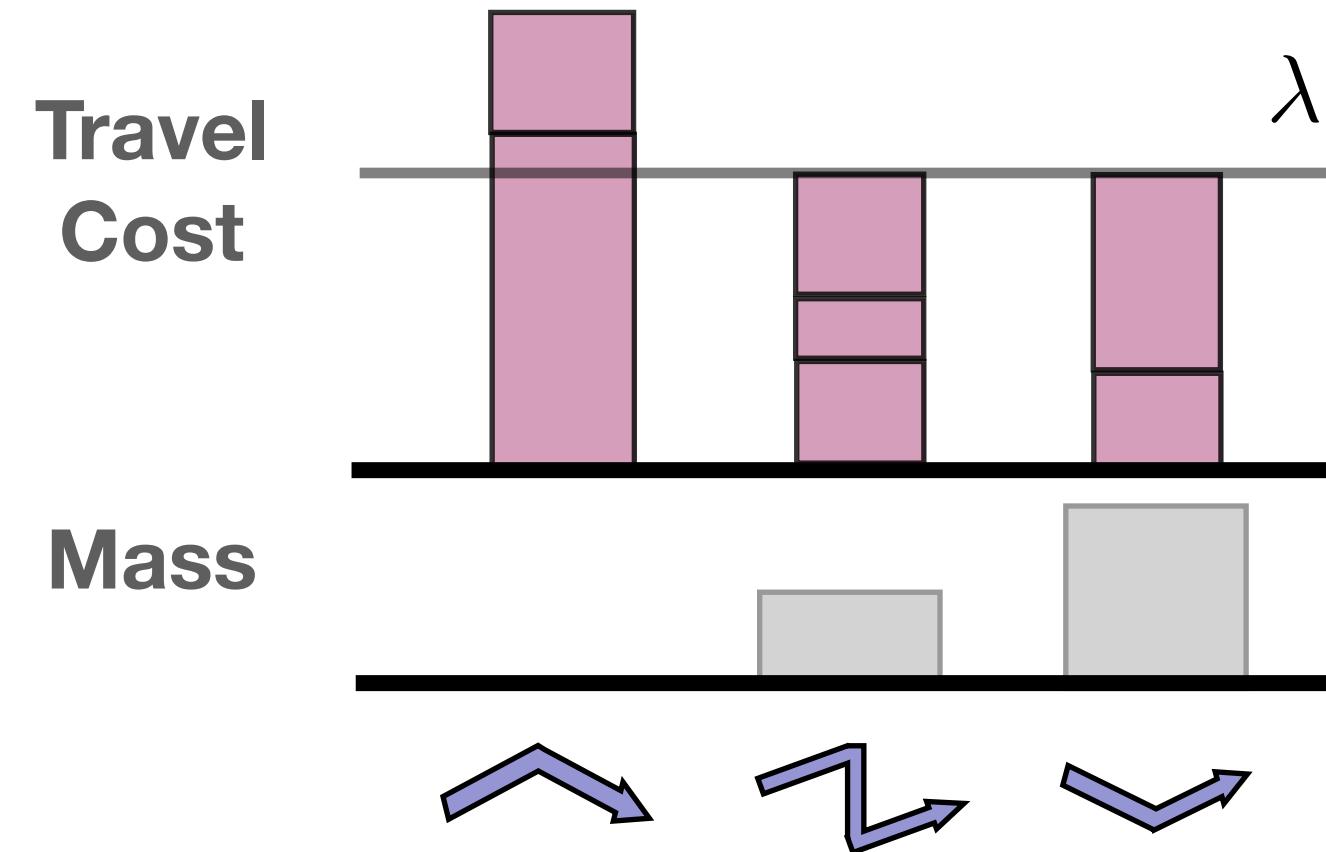
Routing Games



x : edge traffic

z : route traffic

Wardrop Equilibrium



$$\min_x$$

$$F(x)$$

s.t.

$$Ex = Sm, \quad x \geq 0$$

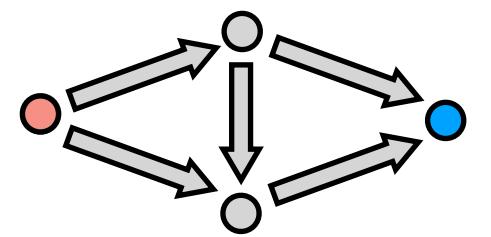
Mass conservation

**Graph
structure**

**Origin-
destination**

Potential Games

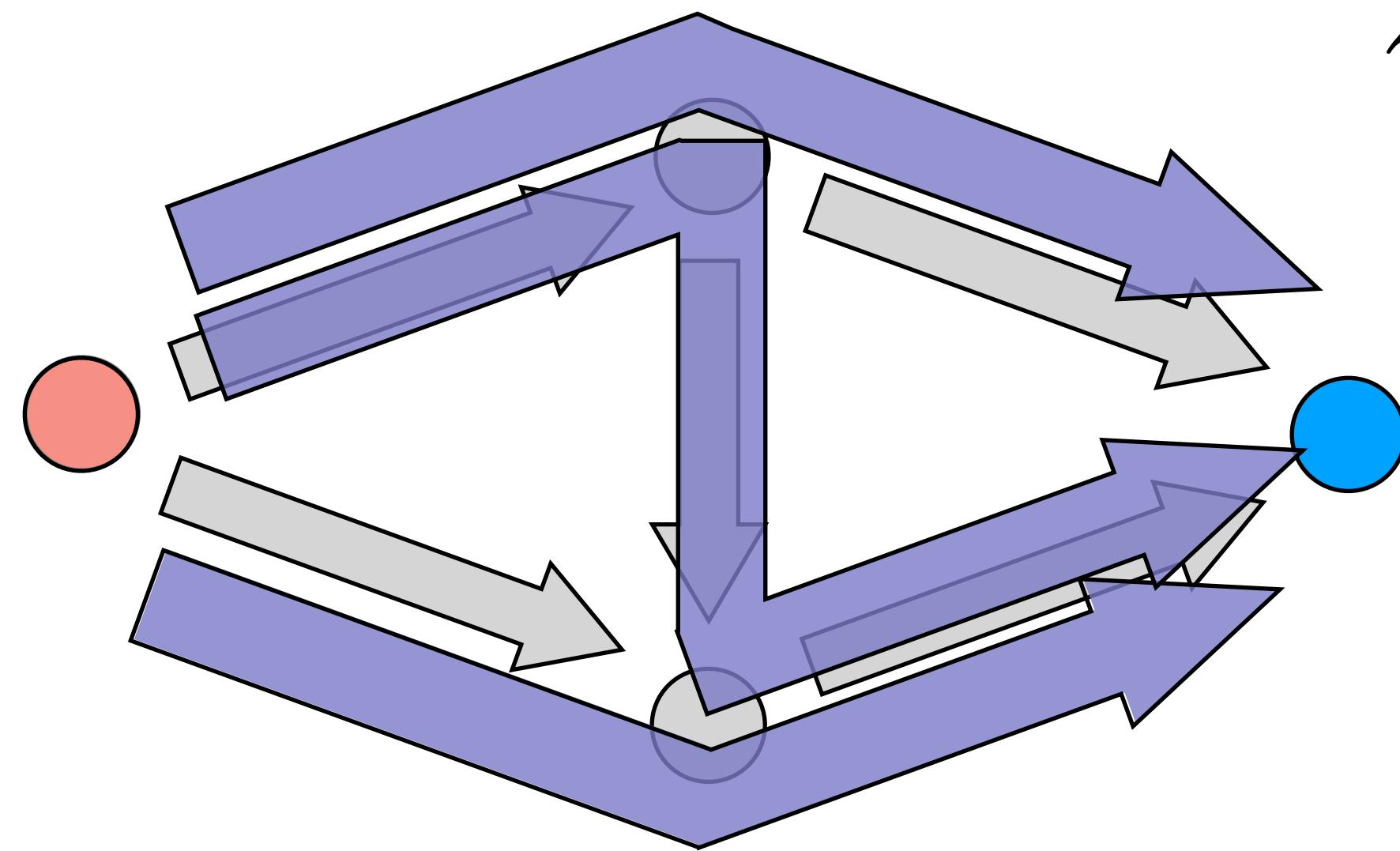
Routing
Games



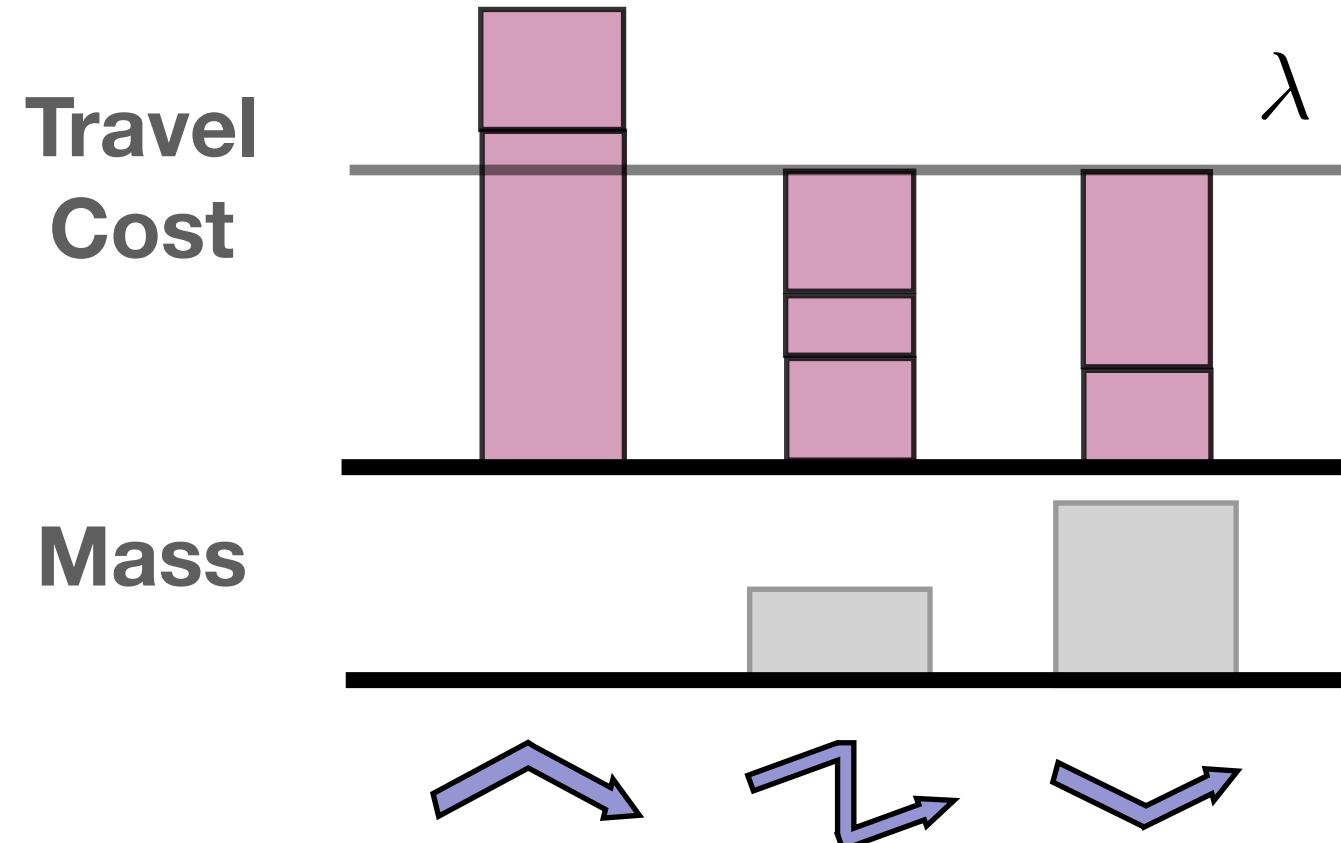
Potential
Function

$$F(x)$$

Routing Games



Wardrop Equilibrium



$$\min_{x,z}$$

$$F(x)$$

s.t.

$$1^T z = m \quad \boxed{\lambda}$$

$$z \geq 0 \quad \boxed{\nu}$$

$$x = \mathbf{R}z \quad \boxed{w}$$

x : edge traffic

z : route traffic

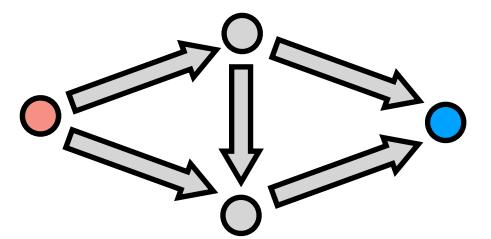
λ : travel cost

ν : route inefficiency

w : edge costs

Potential Games

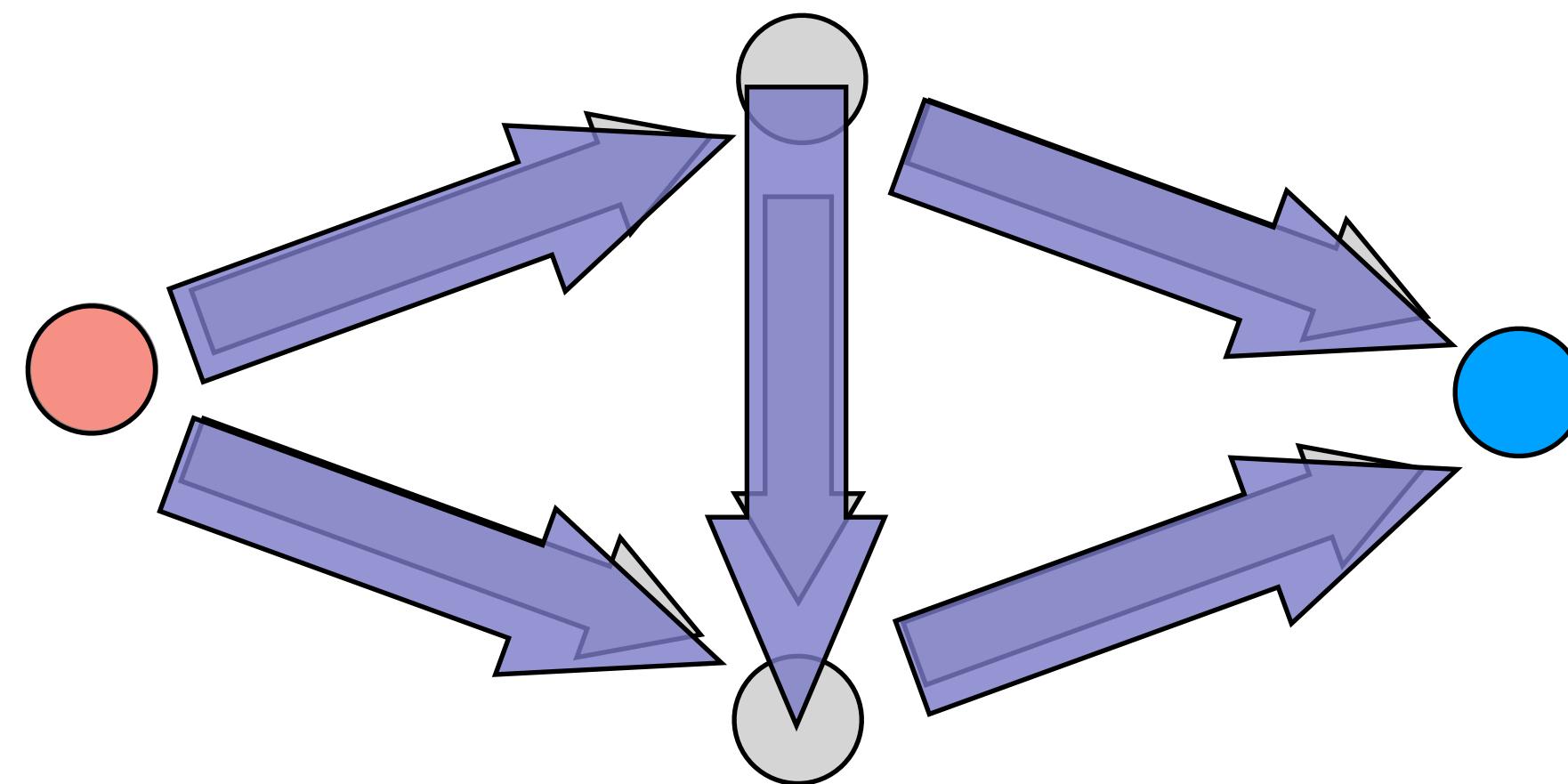
Routing
Games



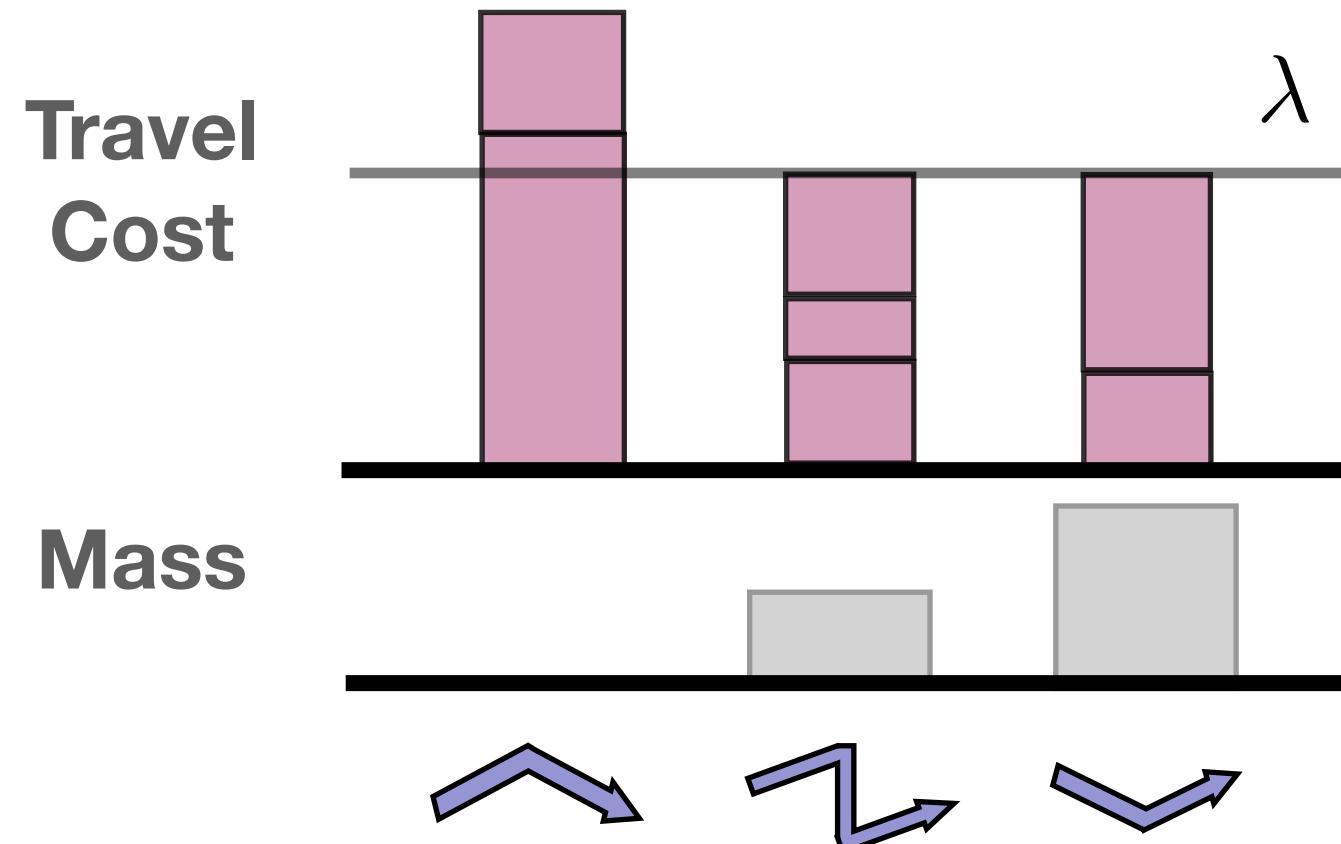
Potential
Function

$$F(x)$$

Routing Games



Wardrop Equilibrium



$$\min_x F(x)$$

s.t.

$$Ex = Sm, \quad v$$

$$x \geq 0 \quad \mu$$

x : edge traffic

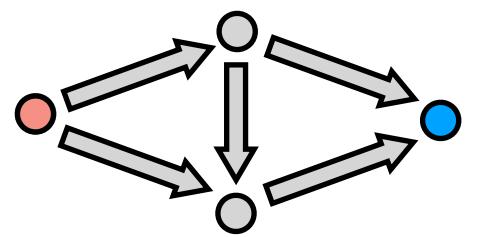
z : route traffic

μ : edge inefficiency

v : value function

Potential Games

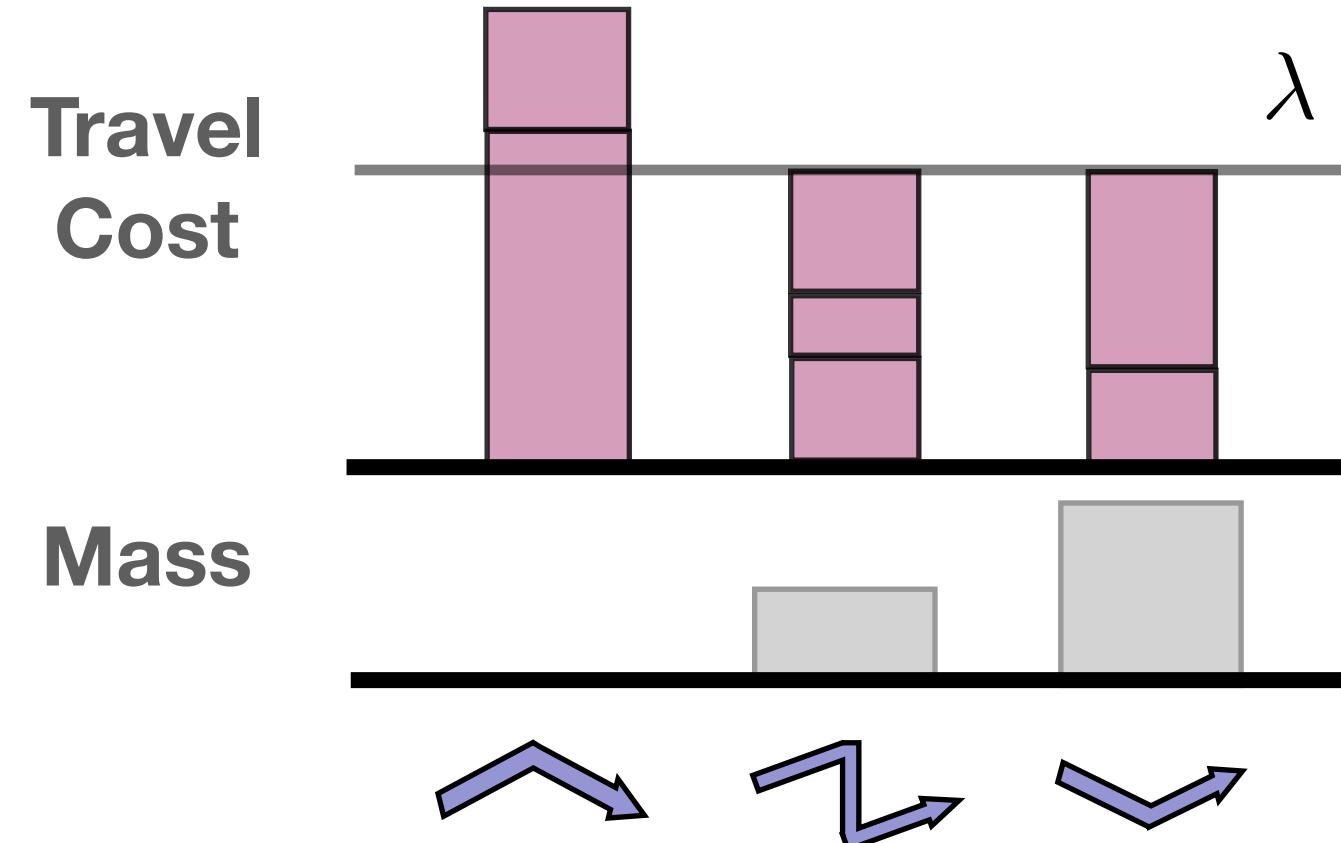
Routing
Games



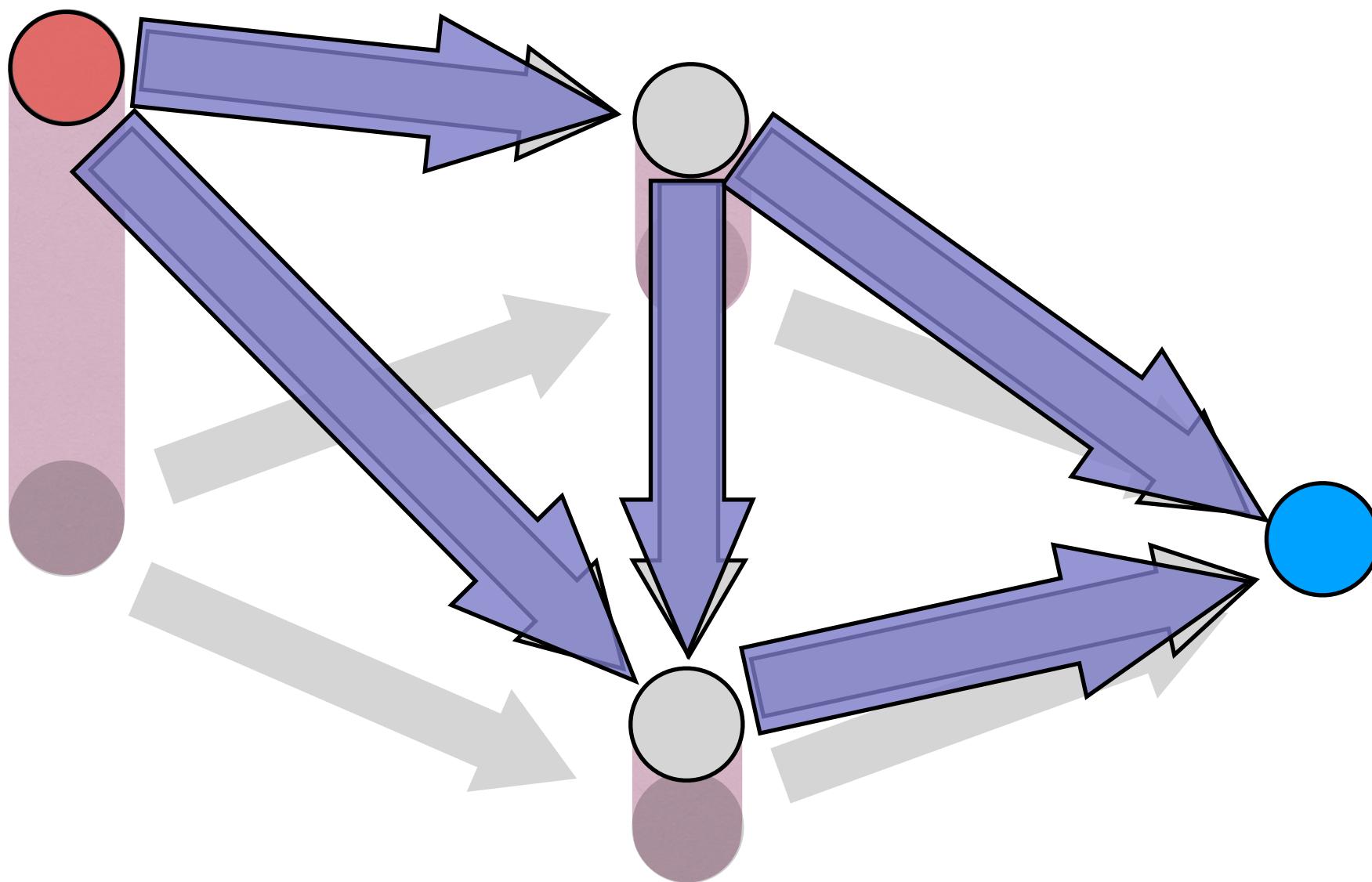
Potential
Function

$$F(x)$$

Wardrop Equilibrium



Routing Games



$$\min_x F(x)$$

s.t.

$$Ex = Sm, \quad v$$

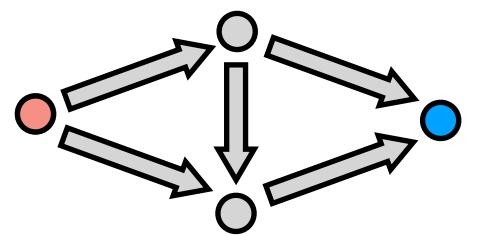
$$x \geq 0 \quad \mu$$

μ : edge inefficiency

v : value function

Potential Games

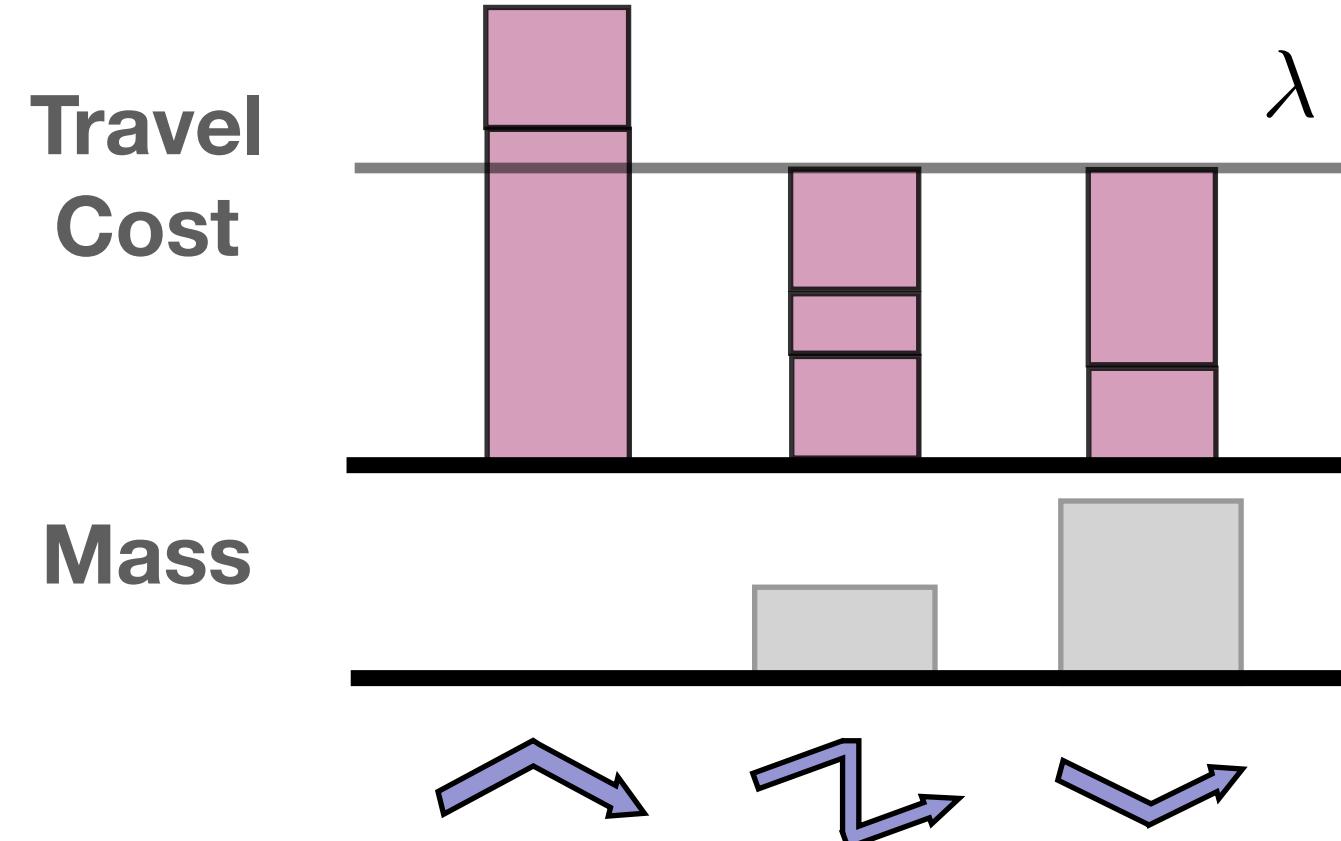
Routing
Games



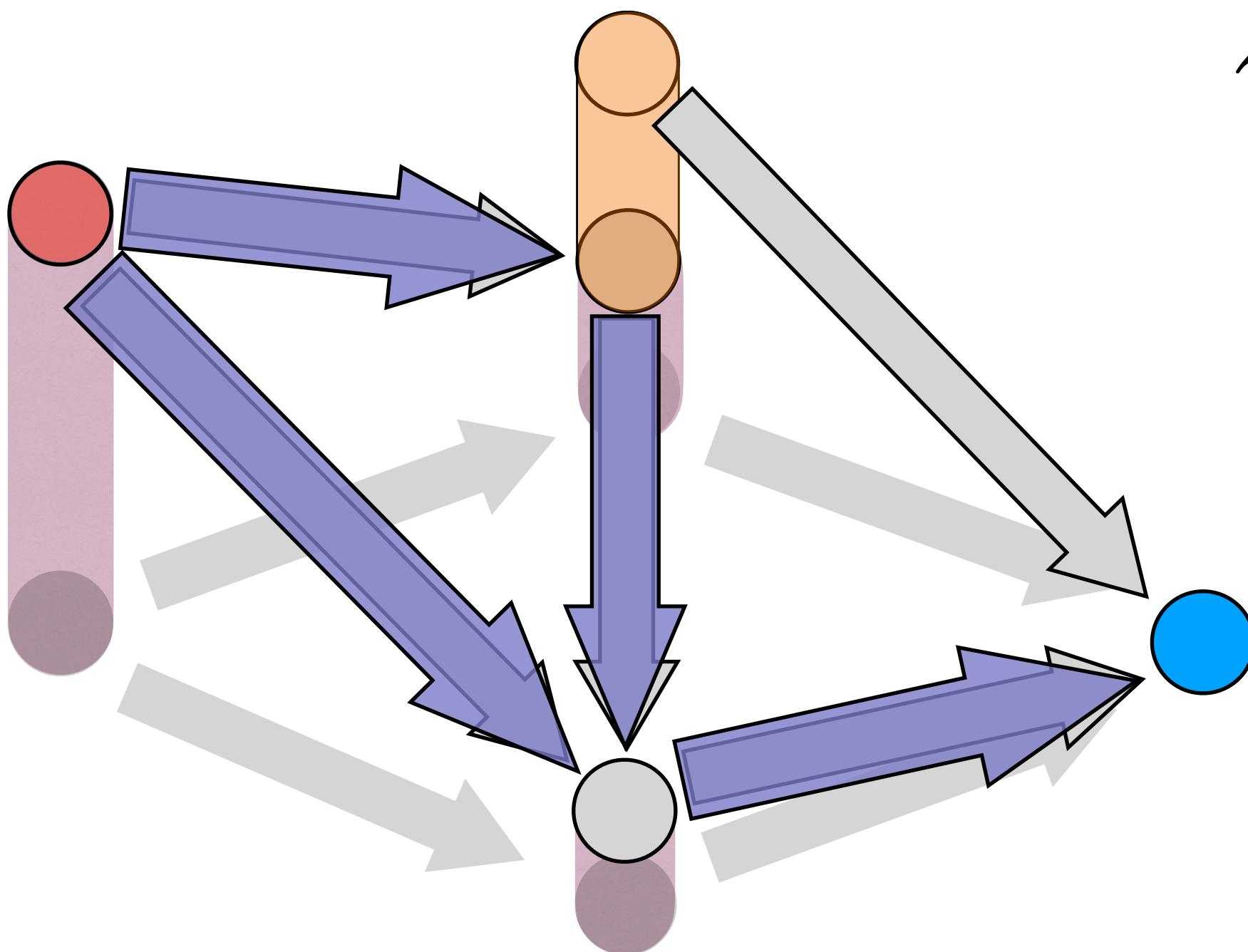
Potential
Function

$$F(x)$$

Wardrop Equilibrium



Routing Games



$$\min_x F(x)$$

s.t.

$$Ex = Sm, \quad v$$

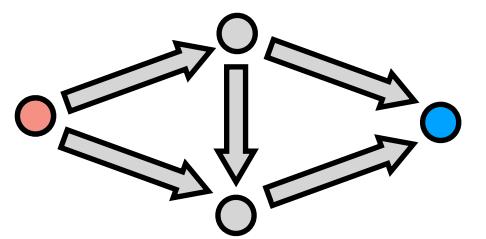
$$x \geq 0 \quad \mu$$

μ : edge inefficiency

v : value function

Potential Games

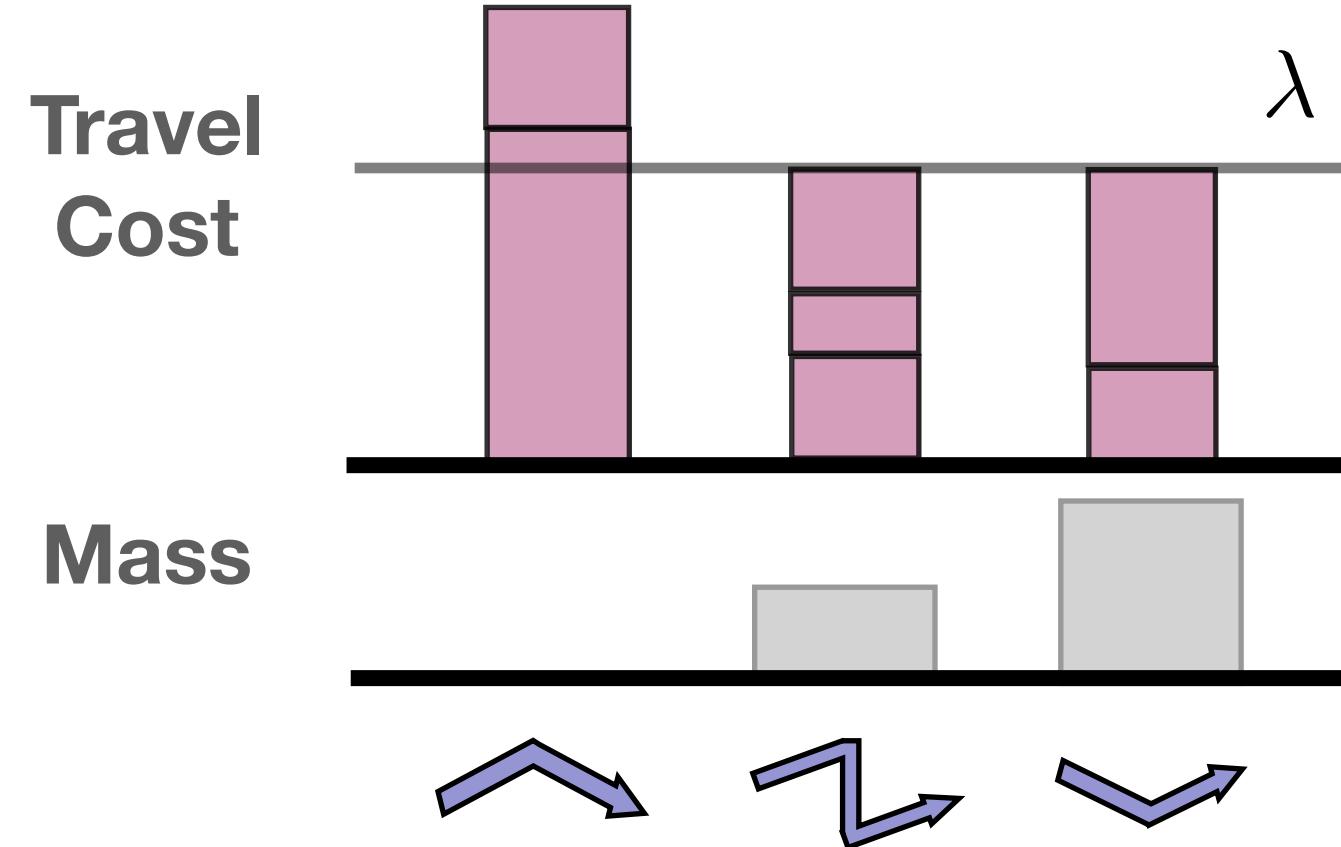
Routing
Games



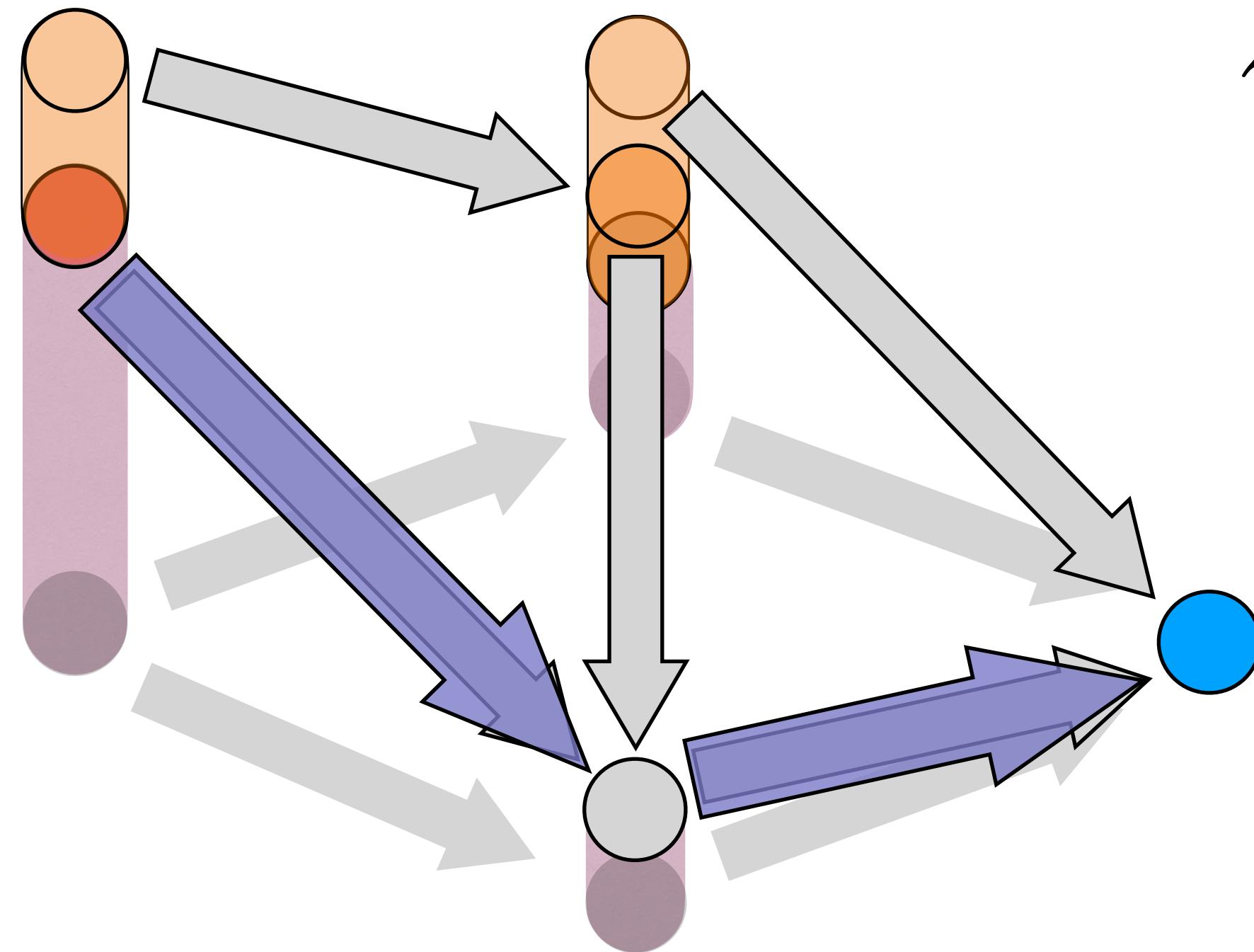
Potential Function

$$F(x)$$

Wardrop Equilibrium



Routing Games



$$\min_x F(x)$$

s.t.

$$Ex = Sm, \quad v$$

$$x \geq 0 \quad \mu$$

μ : edge inefficiency

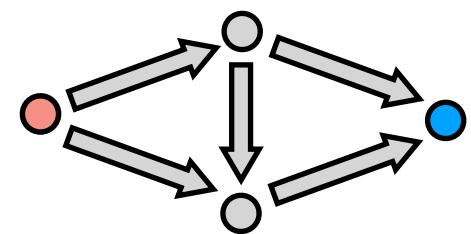
v : value function

x : edge traffic

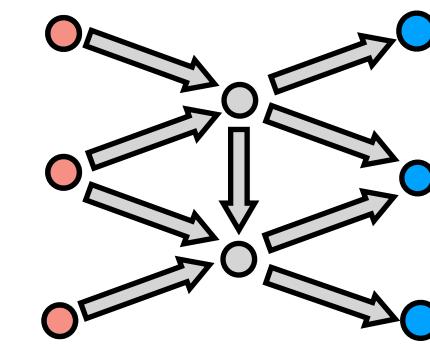
z : route traffic

Potential Games

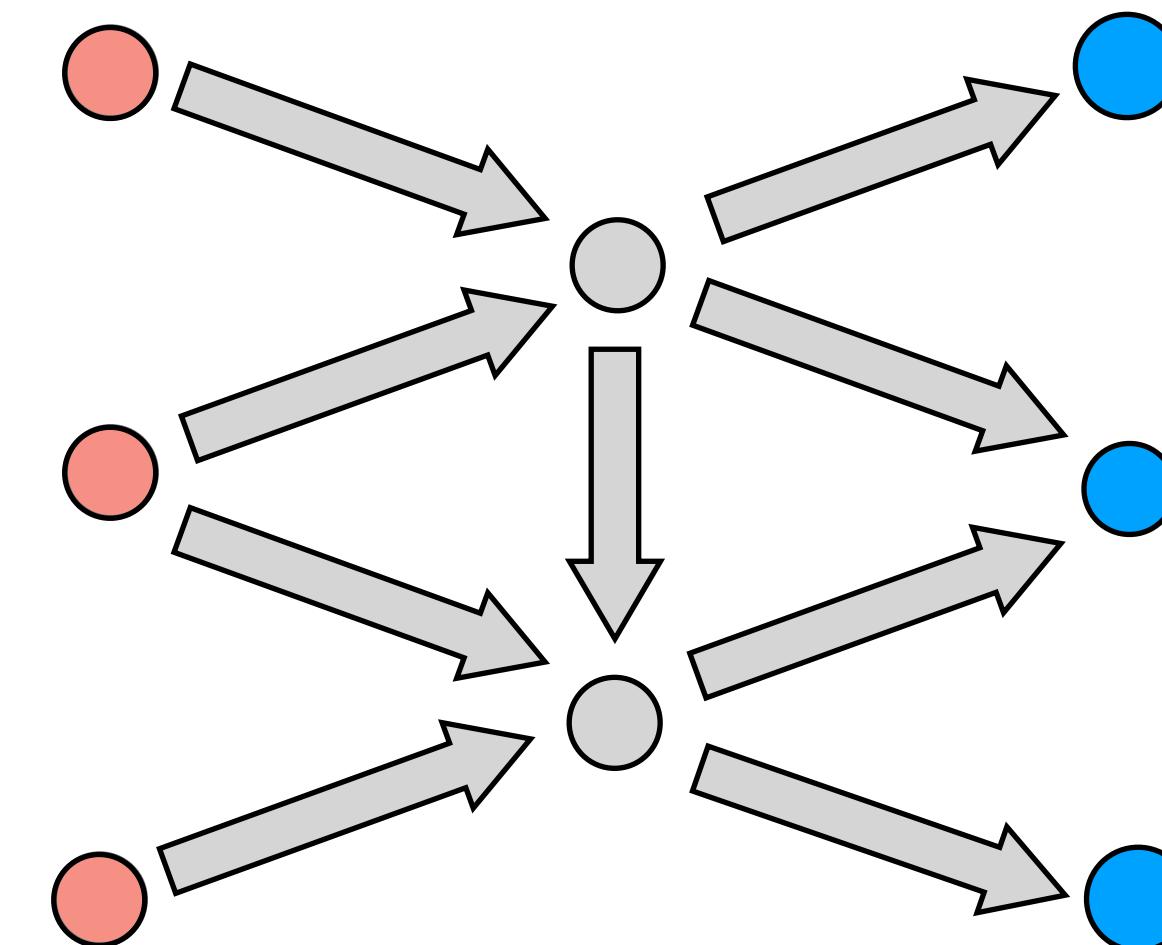
Routing
Games



Multiple
sources/
sinks



Multiple Source/Sinks



x : edge traffic

z : route traffic

$$\min_x F(x)$$

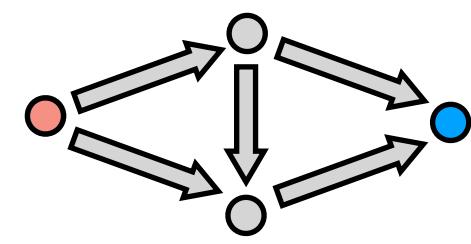
s.t.

$$E_i x_i = S_i m_i, \quad \forall i \quad \boxed{v_i}$$

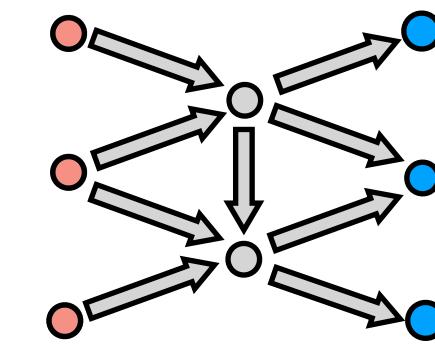
$$x_i \geq 0, \quad \forall i \quad \boxed{\mu_i}$$

Potential Games

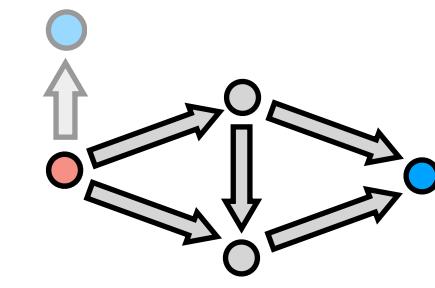
Routing Games



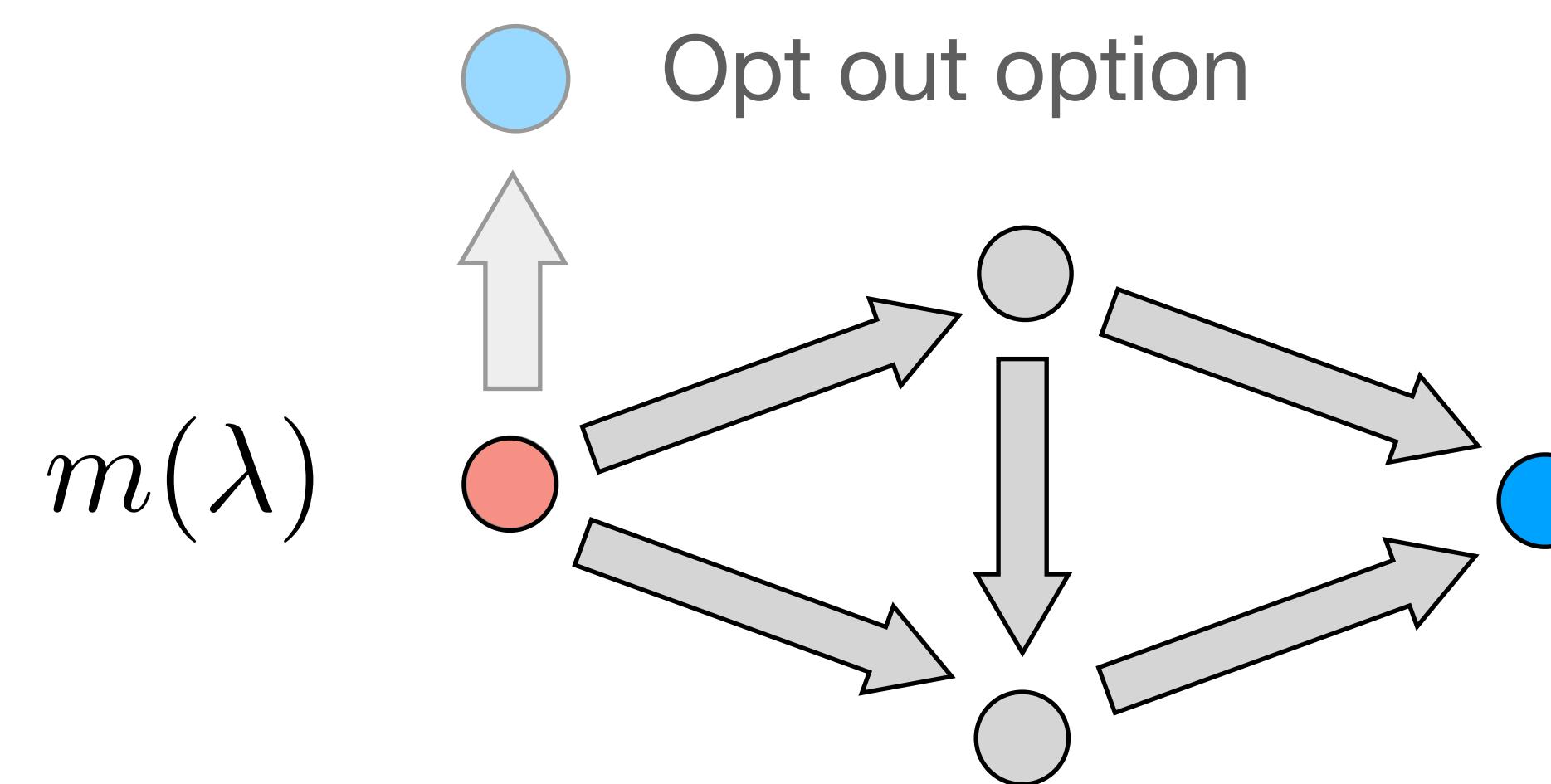
Multiple sources/
sinks



Variable Demand



Variable Demand



$$\min_x \quad F(x) + \int_0^m m^{-1}(u) du$$

s.t.

$$Ex = Sm, \quad \boxed{v}$$

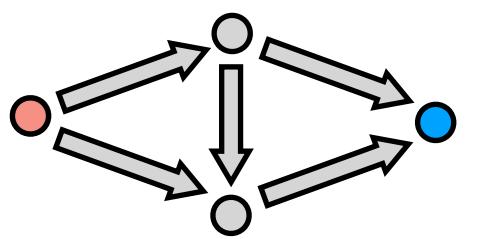
$$x \geq 0 \quad \boxed{\mu}$$

x : edge traffic

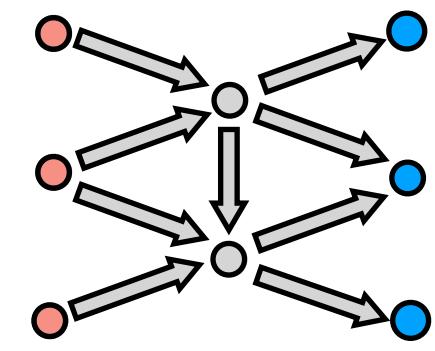
z : route traffic

Potential Games

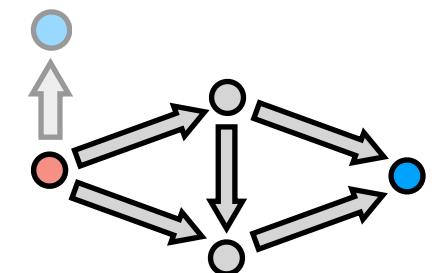
Routing Games



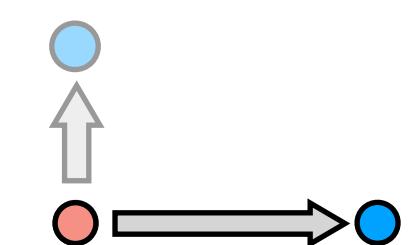
Multiple sources/
sinks



Variable Demand

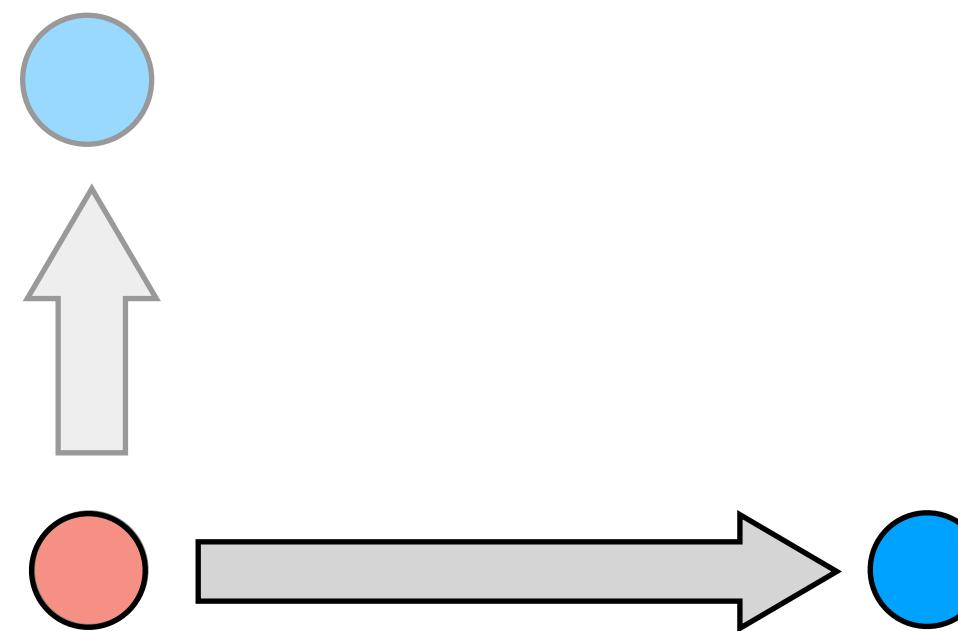


Supply &
Demand



Supply & Demand

$$m(\lambda)$$



$$\min_x \quad F(x) + \int_0^m m^{-1}(u) du$$

s.t.

$$Ex = Sm, \quad \boxed{v}$$

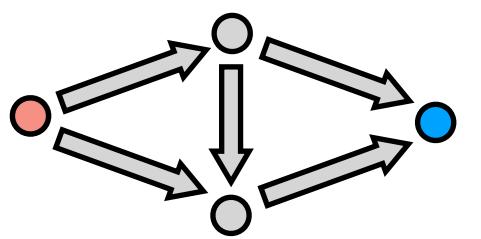
$$x \geq 0 \quad \boxed{\mu}$$

x : edge traffic

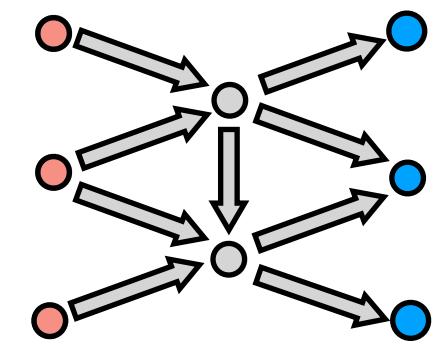
z : route traffic

Potential Games

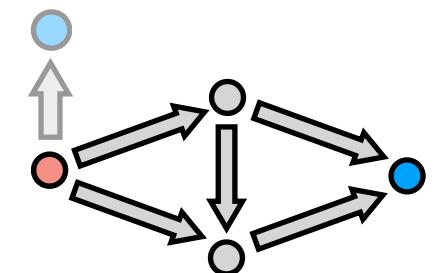
Routing Games



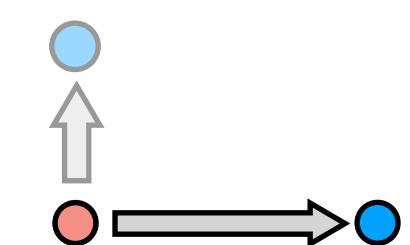
Multiple sources/
sinks



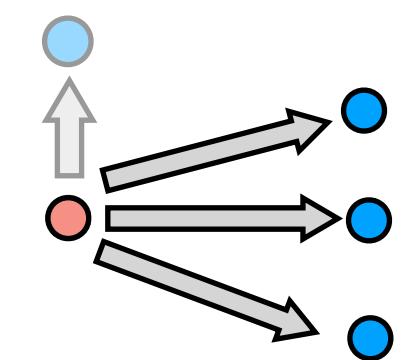
Variable Demand



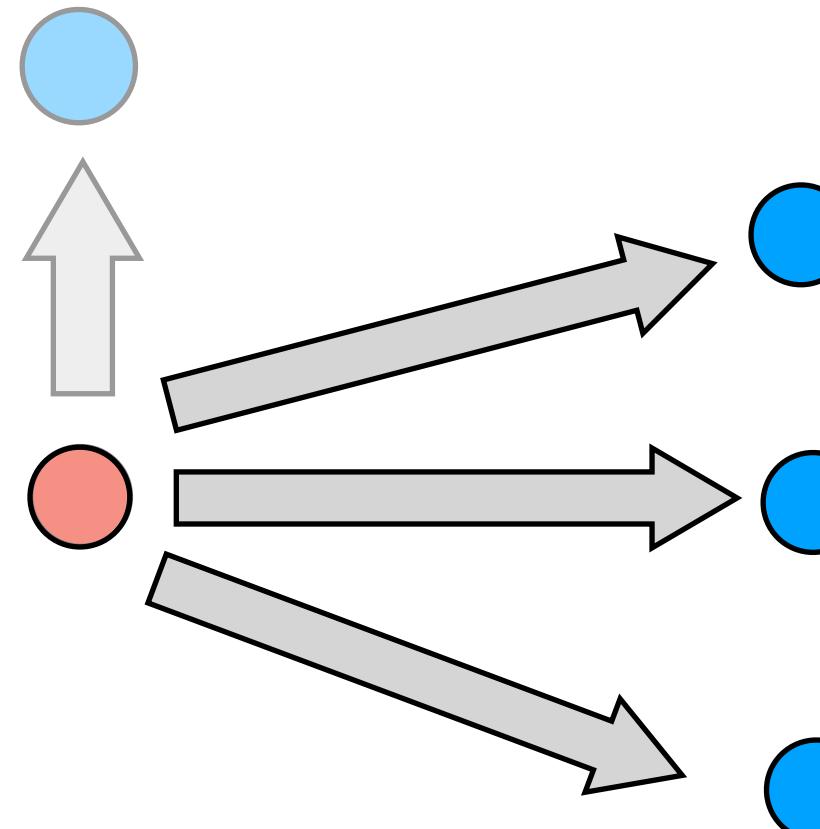
Supply &
Demand



Cournot Market



Cournot Market



\min_x

$$F(x) + \int_0^m m^{-1}(u) du$$

s.t.

$$Ex = Sm, \quad v$$

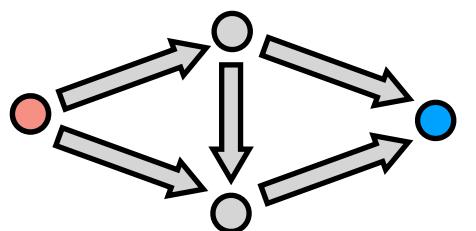
$$x \geq 0 \quad \mu$$

x : edge traffic

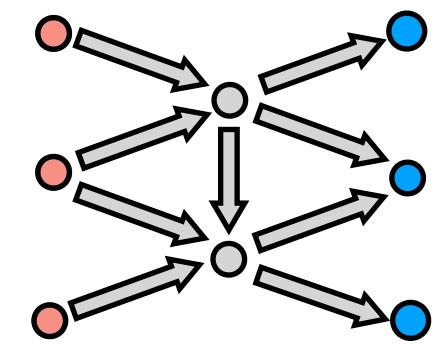
z : route traffic

Potential Games

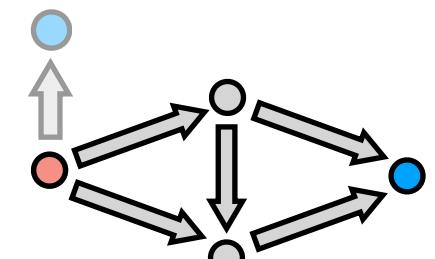
Routing Games



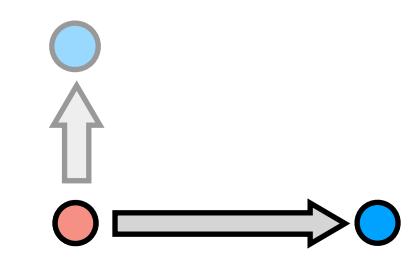
Multiple sources/
sinks



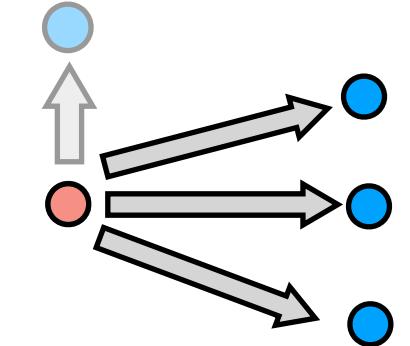
Variable Demand



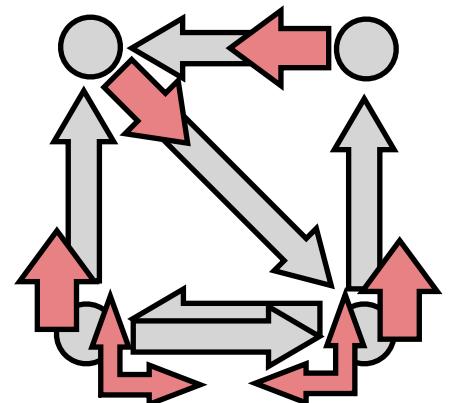
Supply &
Demand



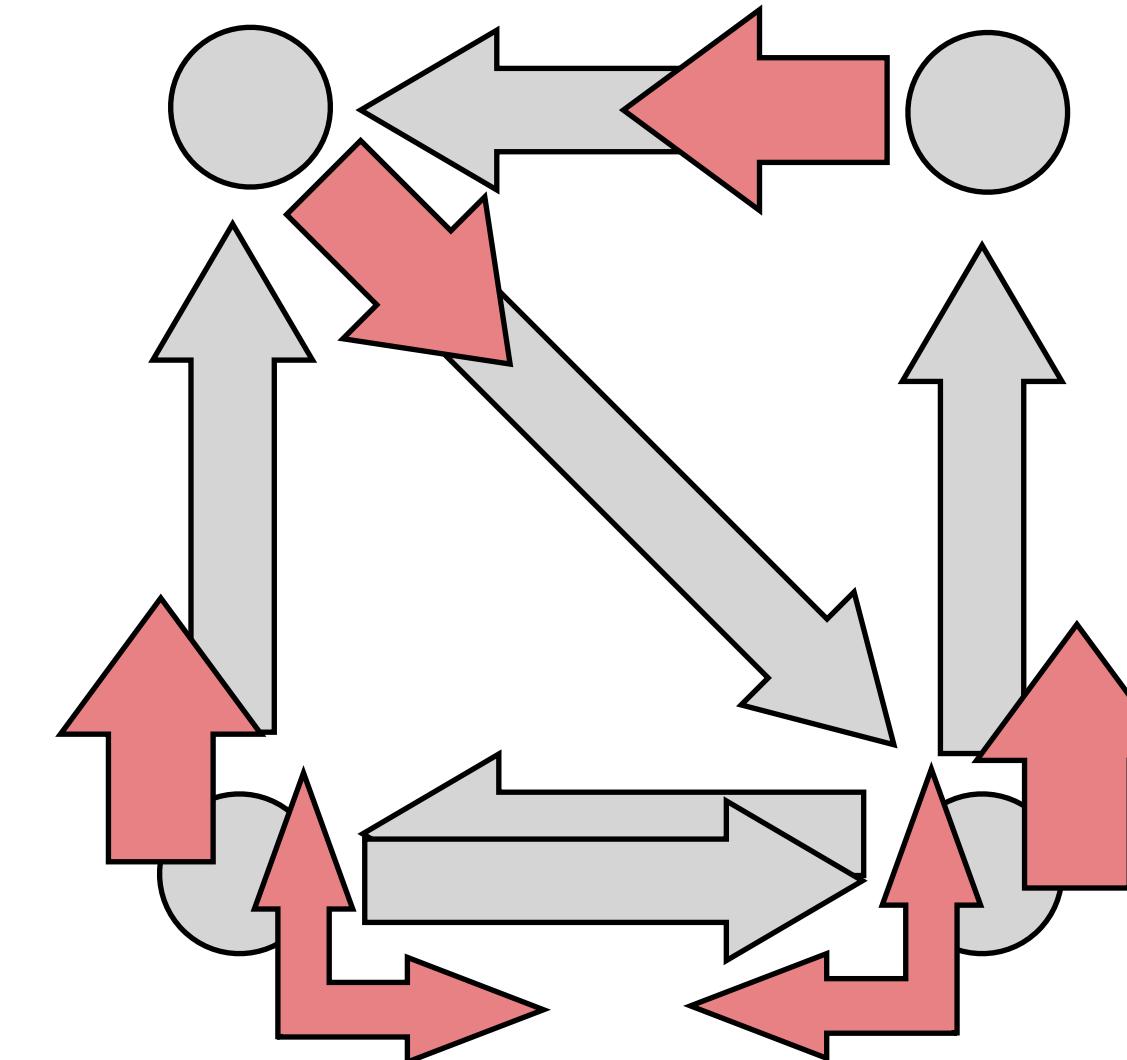
Cournot Market



MDP
Congestion Game



Markov Decision Process Congestion Game



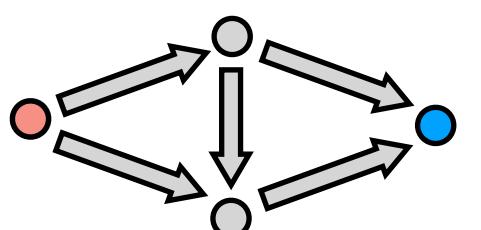
$$\min_x \quad F(x)$$

s.t.

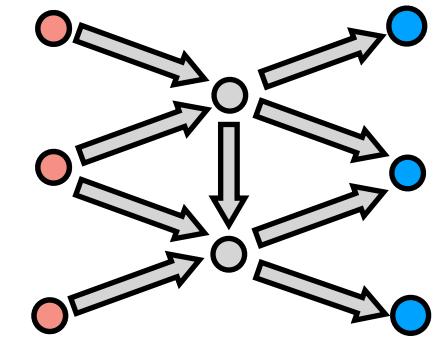
$$\begin{array}{ll} 1^T x = m & (\lambda) \\ x \geq 0 & (\mu) \\ EWx = 0 & (v) \end{array}$$

Potential Games

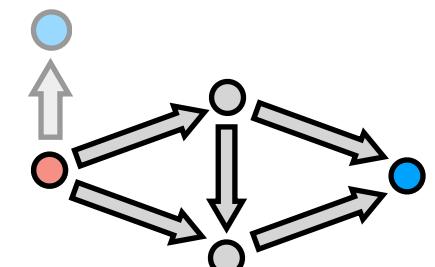
Routing Games



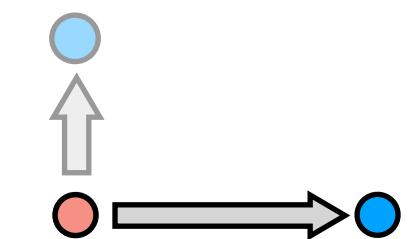
Multiple sources/
sinks



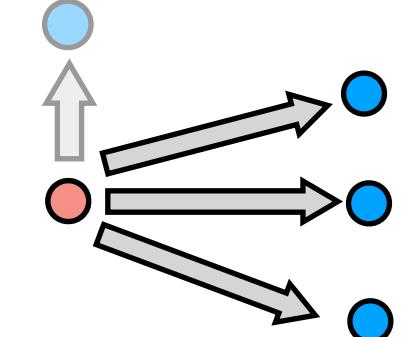
Variable Demand



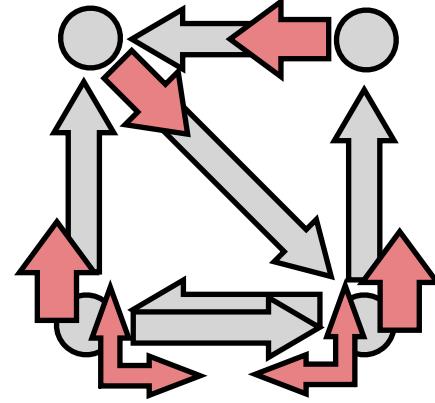
Supply &
Demand



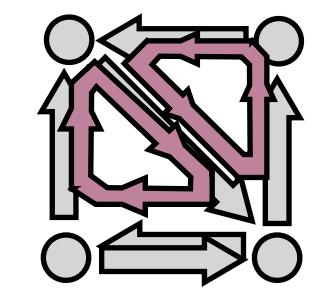
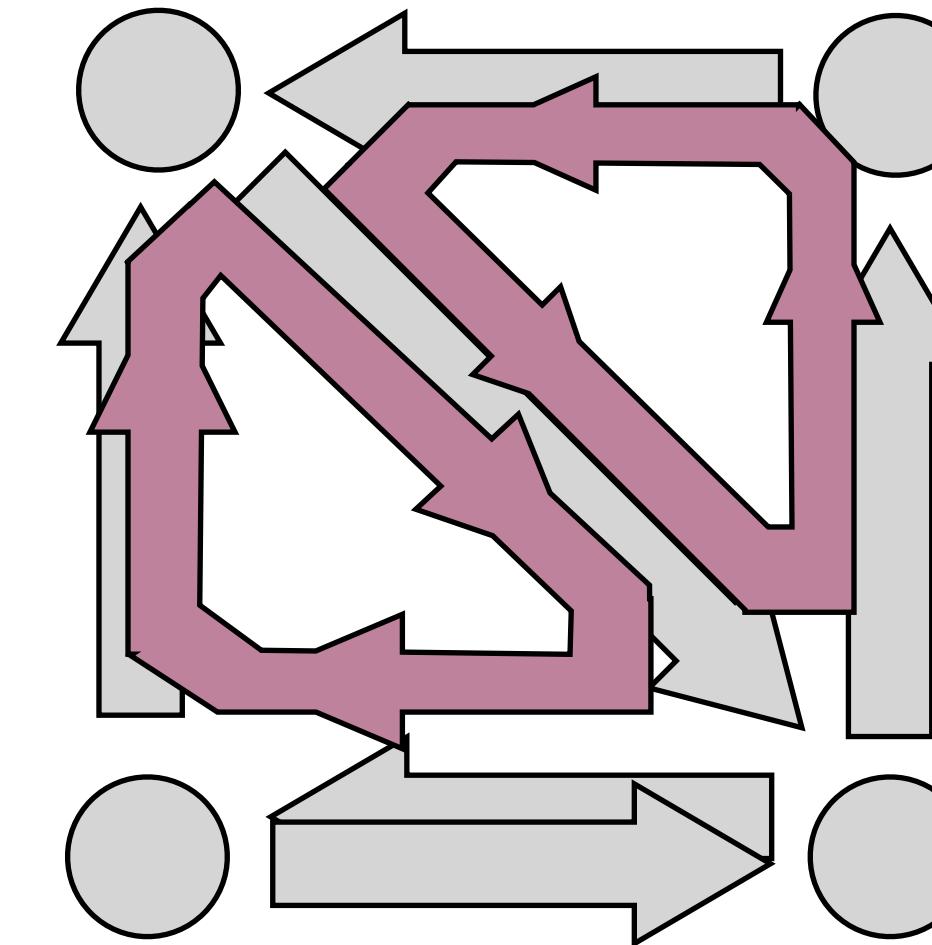
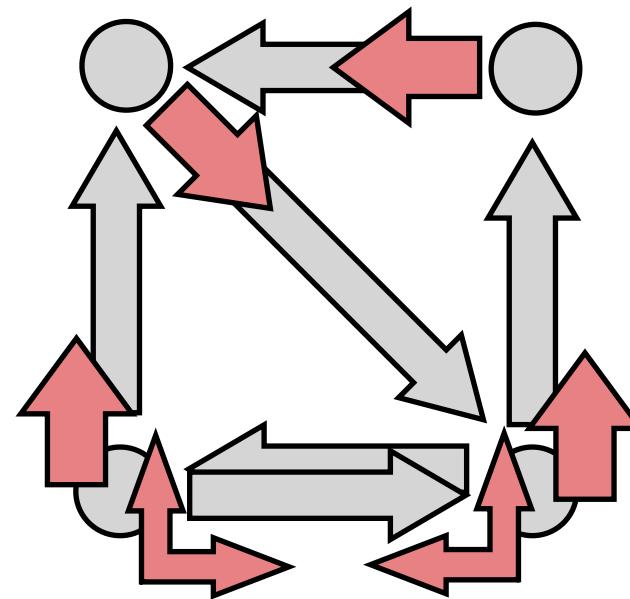
Cournot Market



MDP
Congestion Game



Markov Decision Process Congestion Game



$$\min_x \quad F(x)$$

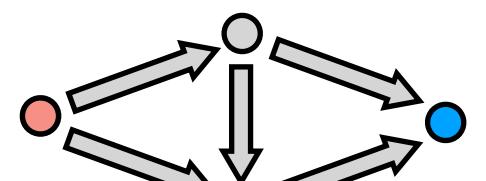
$$\text{s.t.} \quad 1^T x = m \quad (\lambda)$$

$$EWx = 0 \quad (\nu)$$

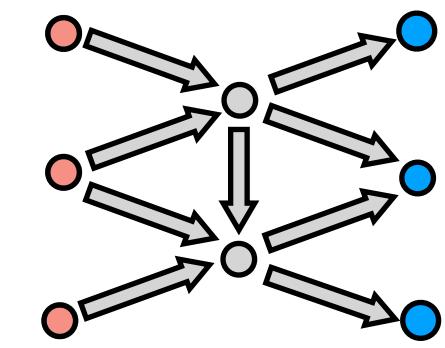
$$x \geq 0 \quad (\mu)$$

Potential Games

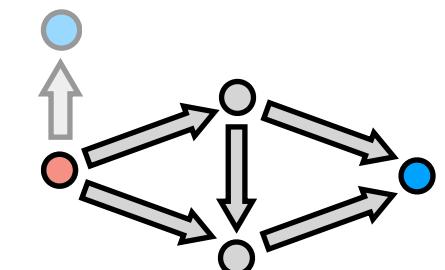
Routing Games



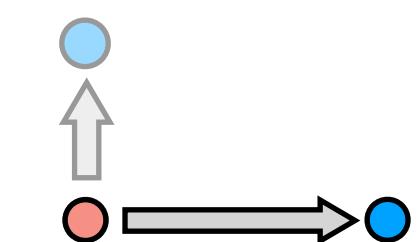
Multiple sources/
sinks



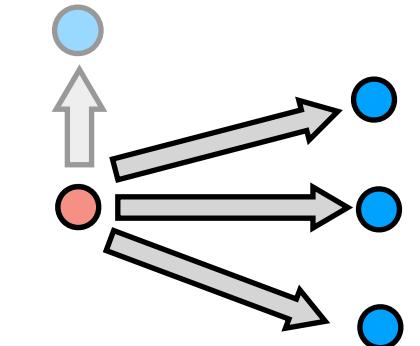
Variable Demand



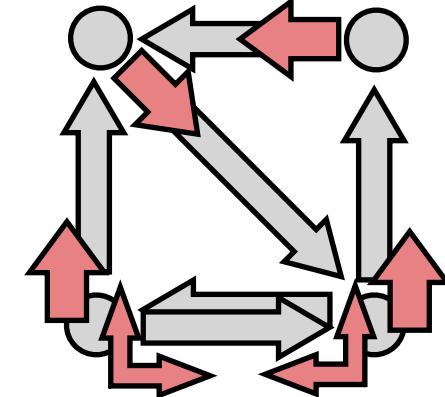
Supply &
Demand



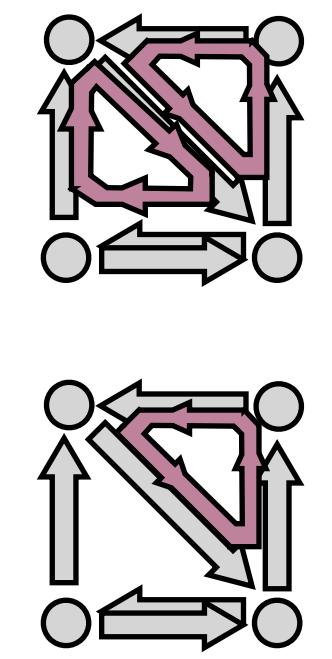
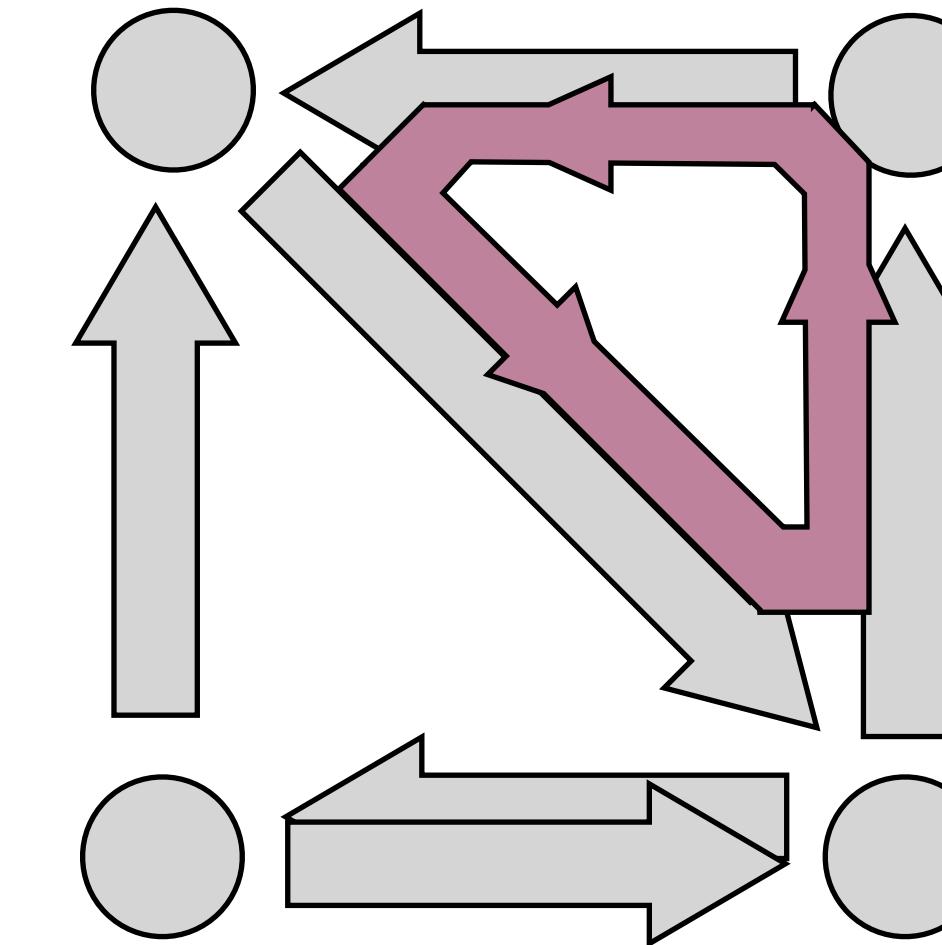
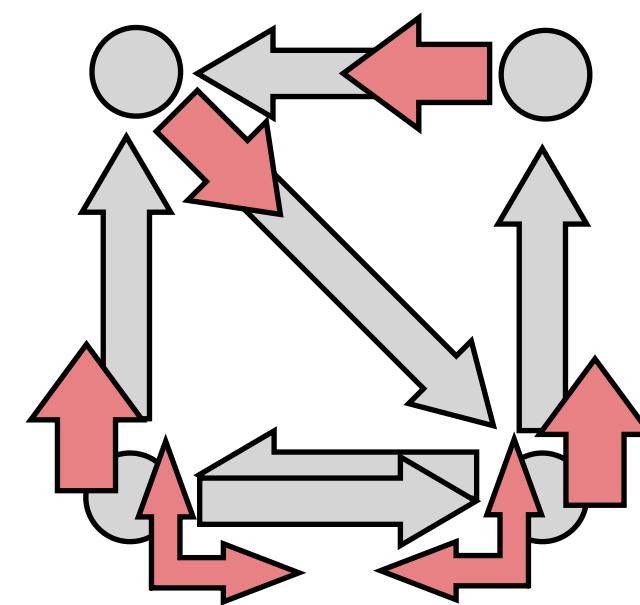
Cournot Market



MDP
Congestion Game



Markov Decision Process Congestion Game



$$\min_x \quad F(x)$$

s.t.

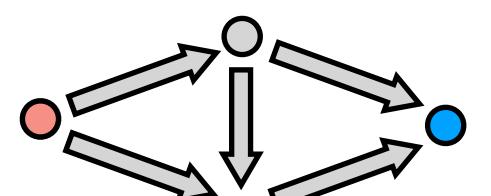
$$1^T x = m \quad (\lambda)$$

$$x \geq 0 \quad (\mu)$$

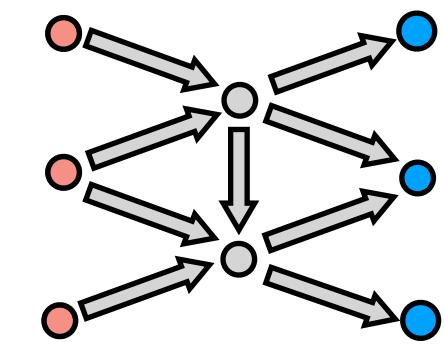
$$EWx = 0 \quad (v)$$

Potential Games

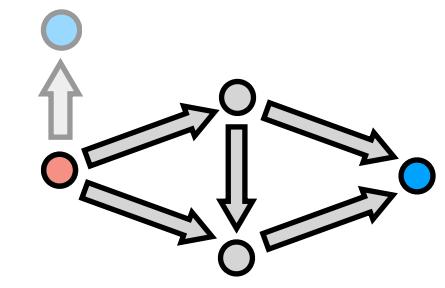
Routing Games



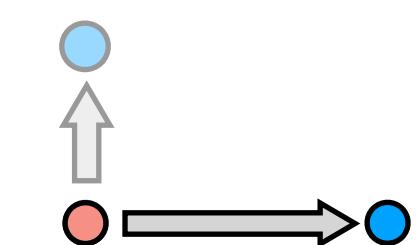
Multiple sources/
sinks



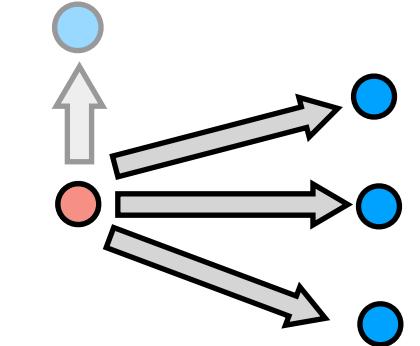
Variable Demand



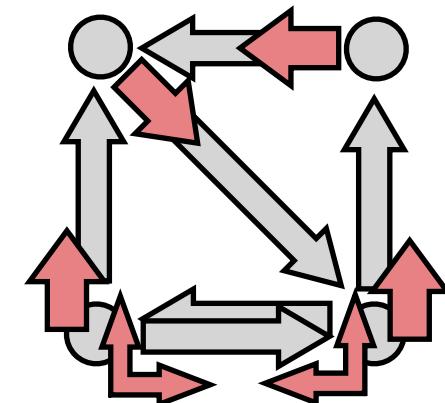
Supply &
Demand



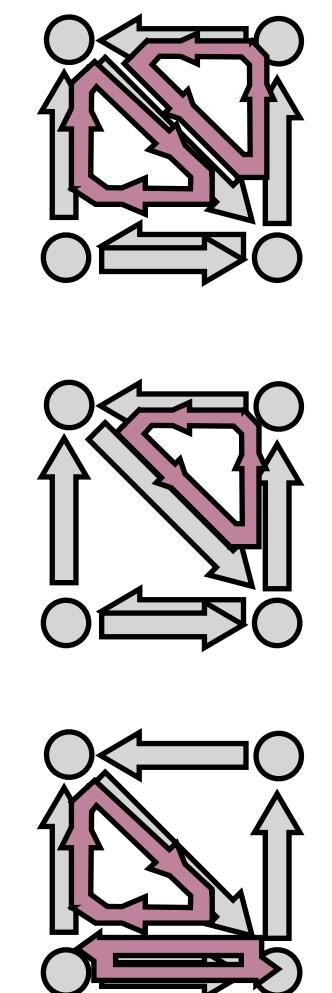
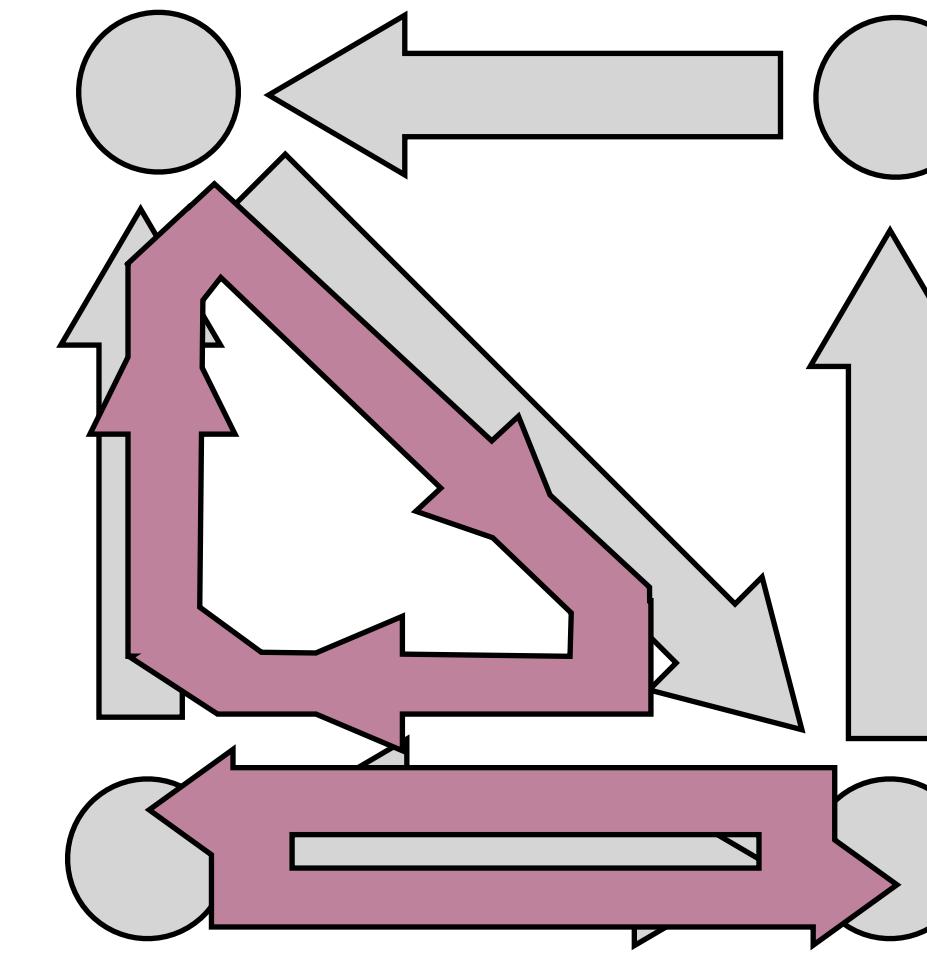
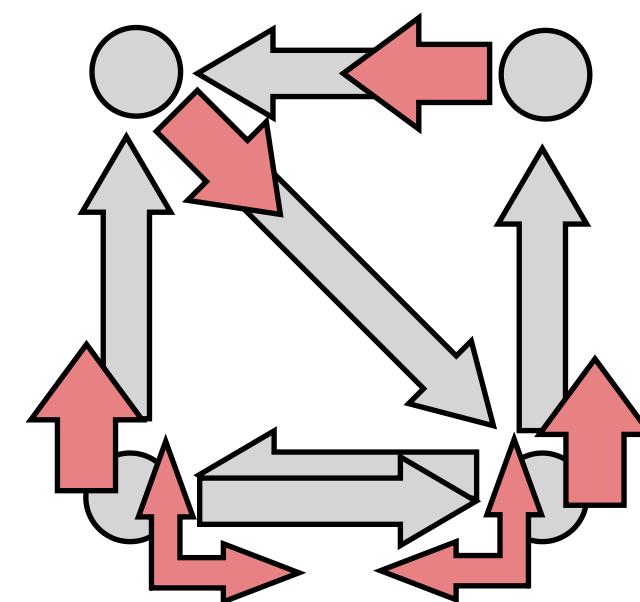
Cournot Market



MDP
Congestion Game



Markov Decision Process Congestion Game



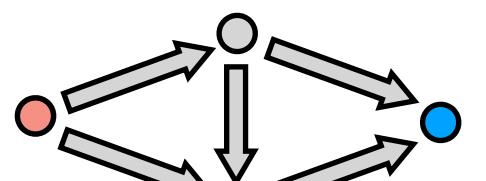
$$\min_x \quad F(x)$$

$$\text{s.t.} \quad \begin{array}{l} 1^T x = m \quad (\lambda) \\ x \geq 0 \quad (\mu) \end{array}$$

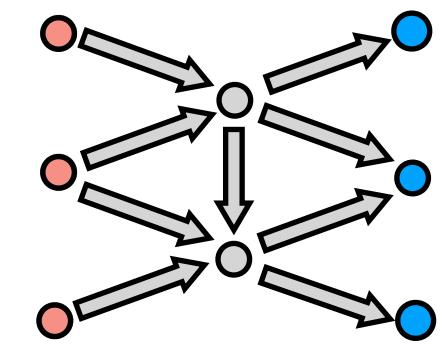
$$EWx = 0 \quad (v)$$

Potential Games

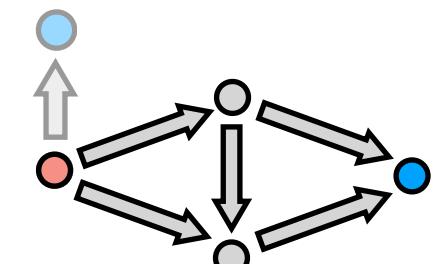
Routing Games



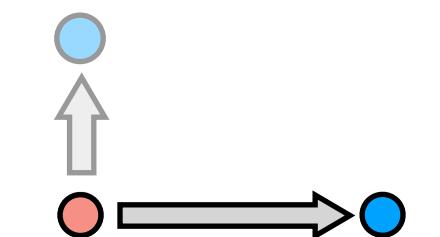
Multiple sources/sinks



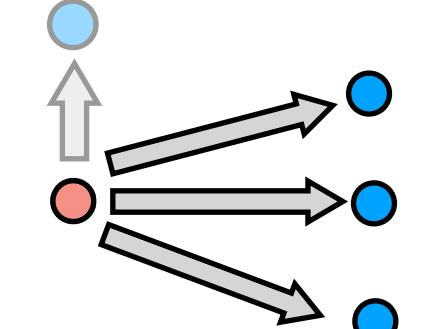
Variable Demand



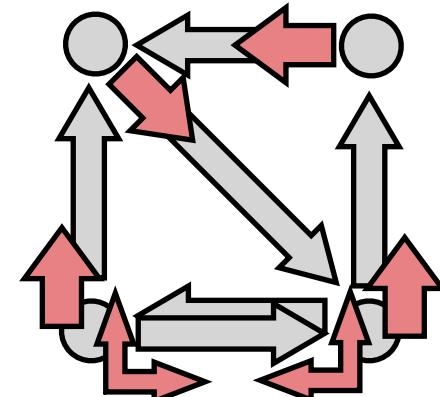
Supply & Demand



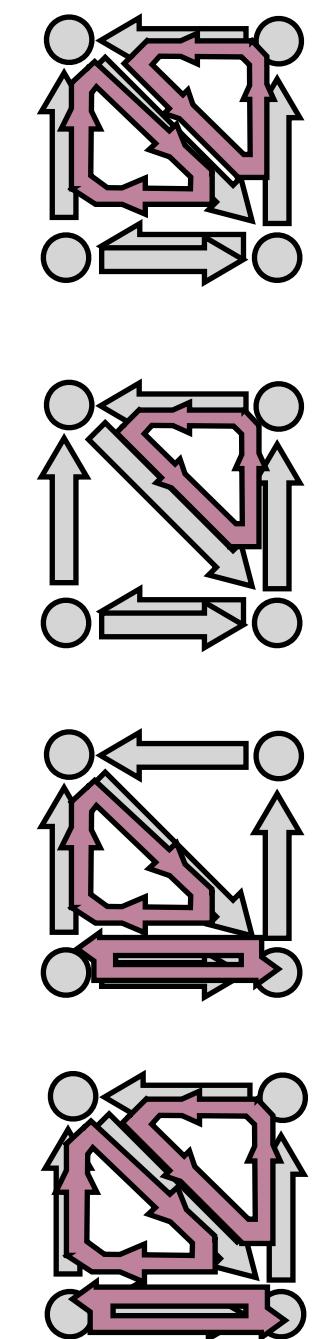
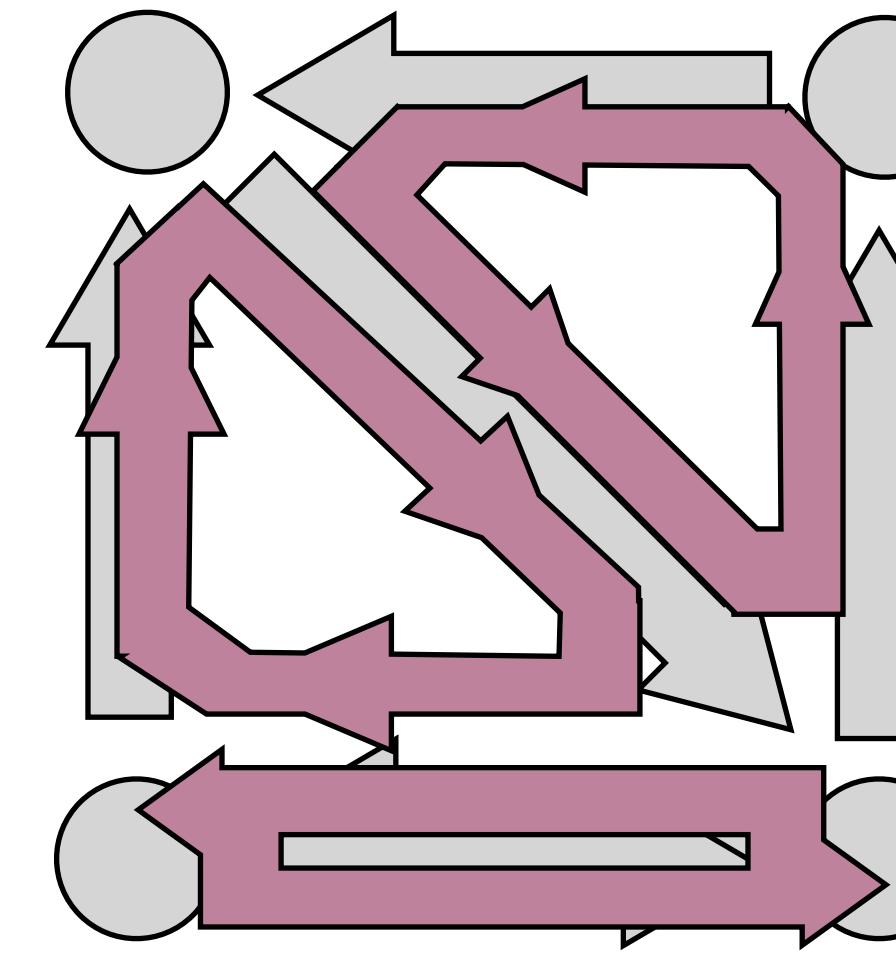
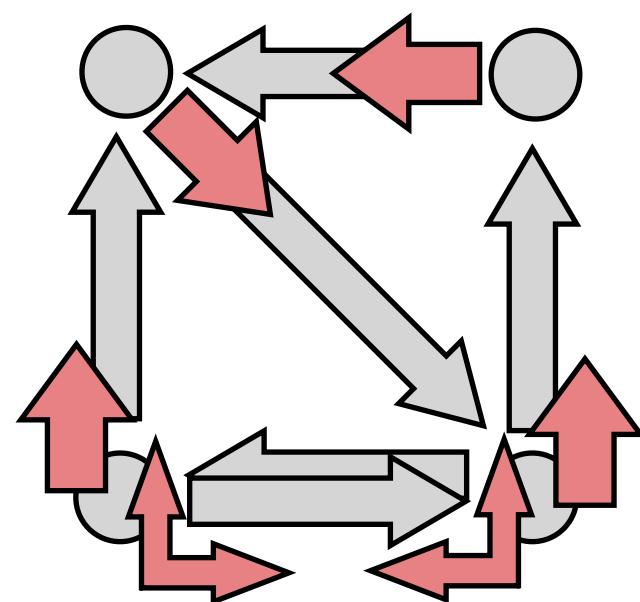
Cournot Market



MDP Congestion Game



Markov Decision Process Congestion Game



$$\min_x F(x)$$

s.t.

$$1^T x = m \quad \lambda$$

$$EWx = 0 \quad v$$

$$x \geq 0 \quad \mu$$

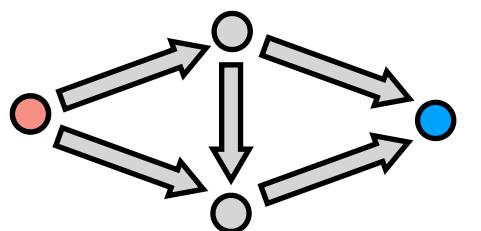
λ : average reward

μ : action inefficiency

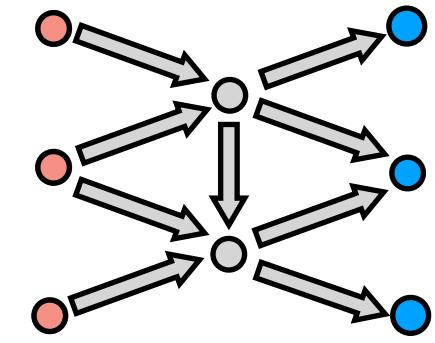
v : value function

Potential Games

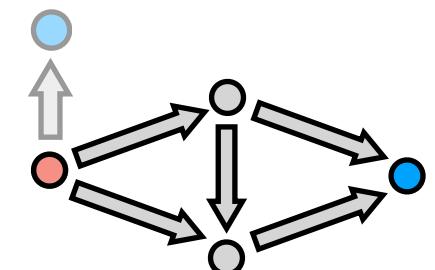
Routing Games



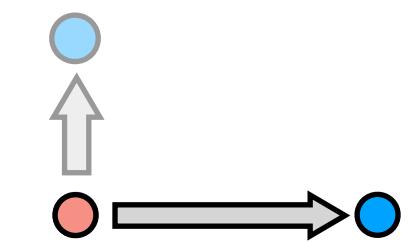
Multiple sources/sinks



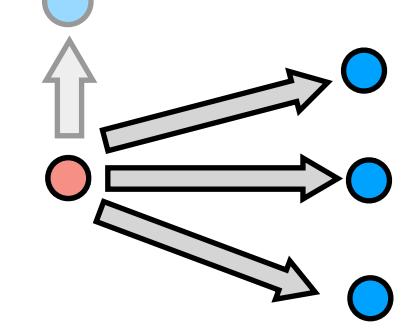
Variable Demand



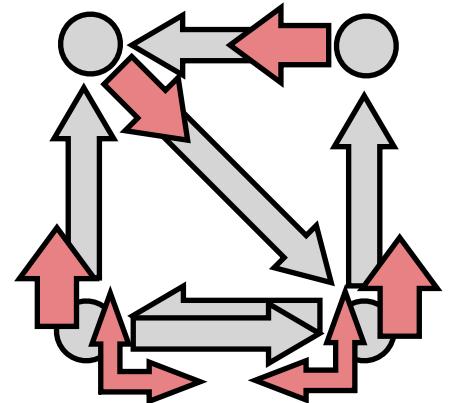
Supply & Demand



Cournot Market

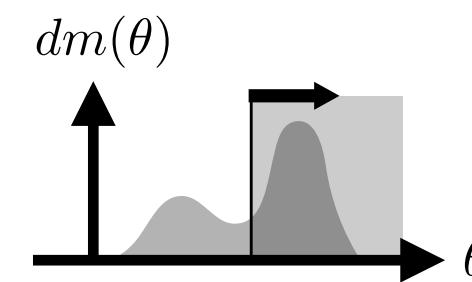


MDP Congestion Game

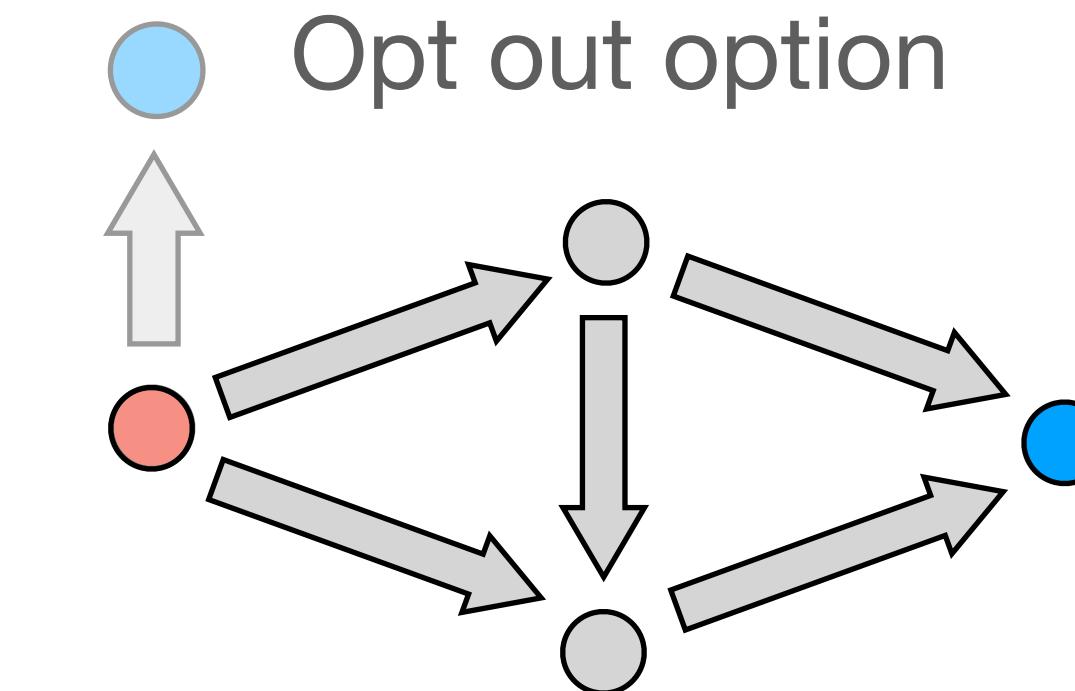


Variable Demand - Non-Homogeneous Preferences

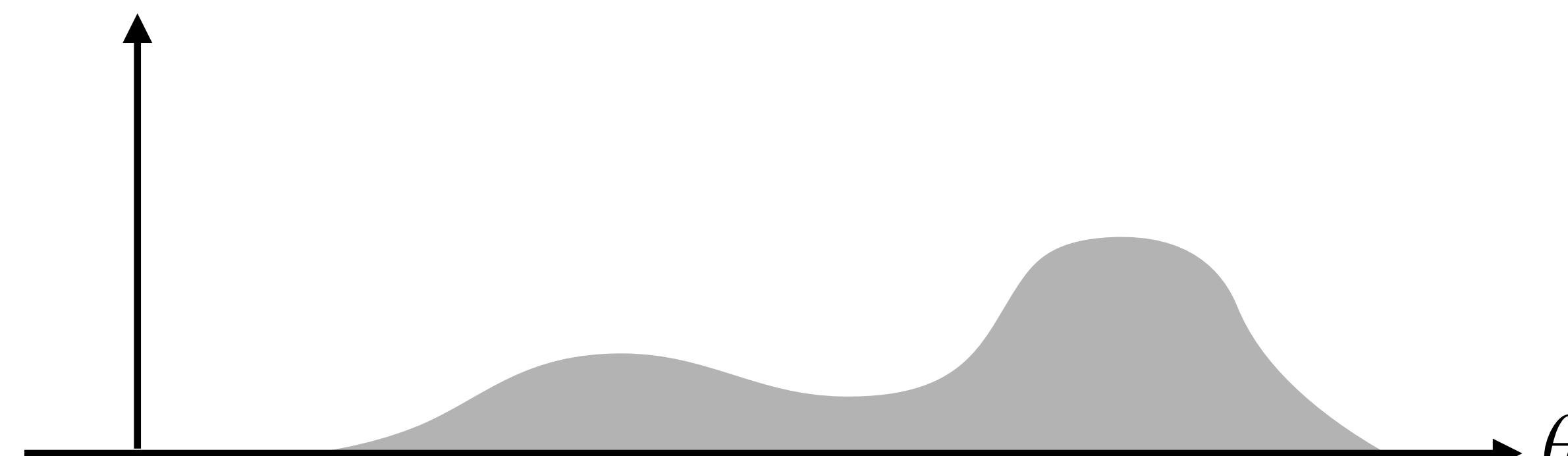
Non-homo-geneous preferences



$$m(\lambda)$$



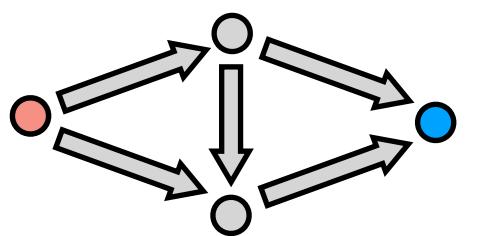
$$dm(\theta)$$



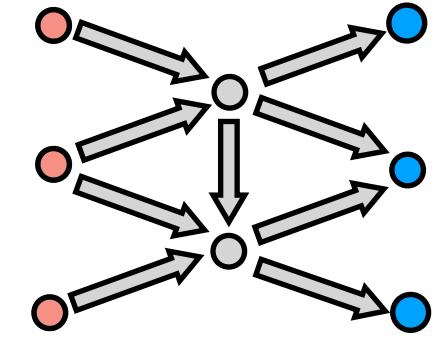
θ : Max cost $dm(\theta)$ will pay

Potential Games

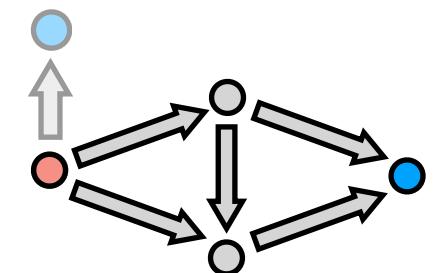
Routing Games



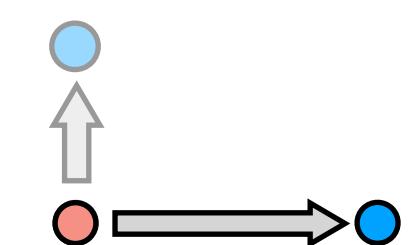
Multiple sources/sinks



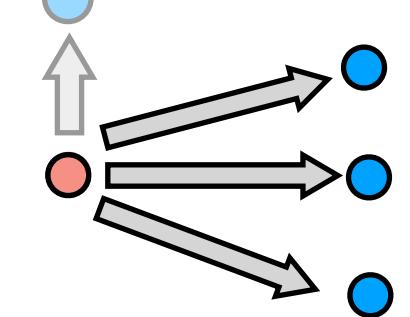
Variable Demand



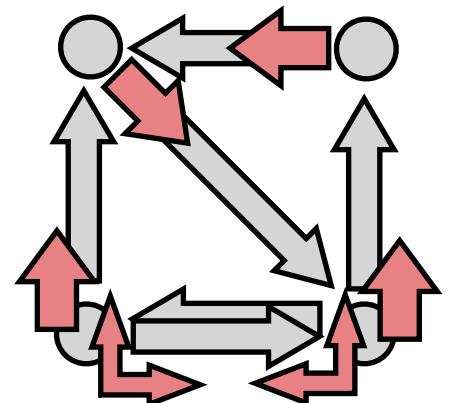
Supply & Demand



Cournot Market

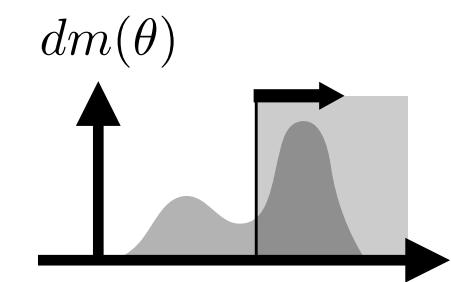


MDP Congestion Game

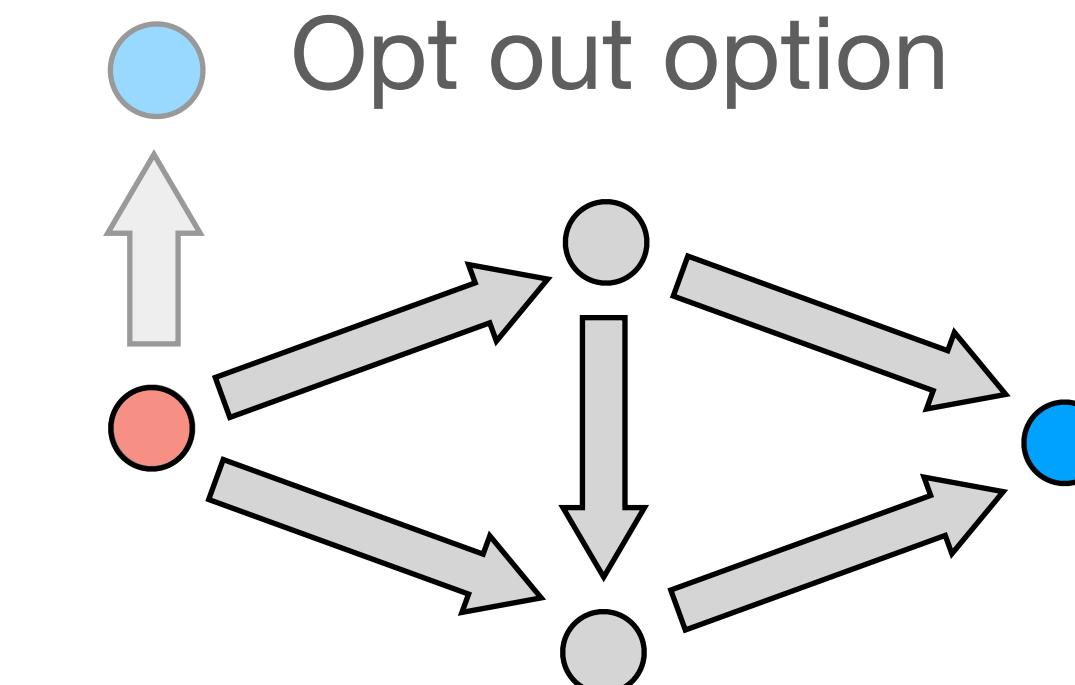


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous preferences



$$m(\lambda)$$



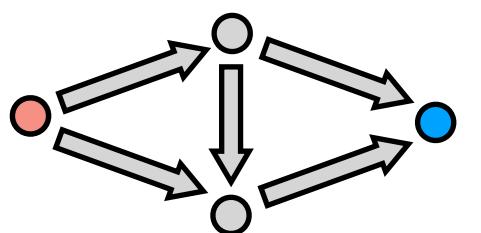
λ : Current travel cost

$$dm(\theta)$$

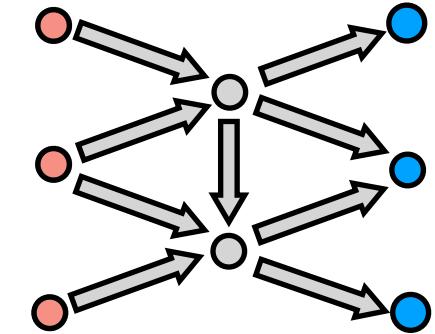
θ : Max cost $dm(\theta)$ will pay

Potential Games

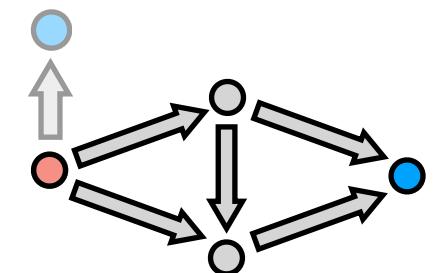
Routing Games



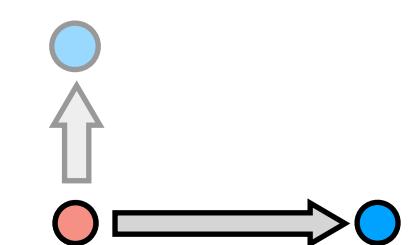
Multiple sources/sinks



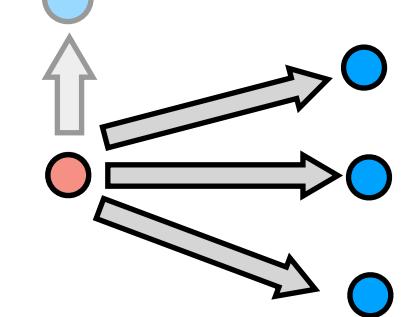
Variable Demand



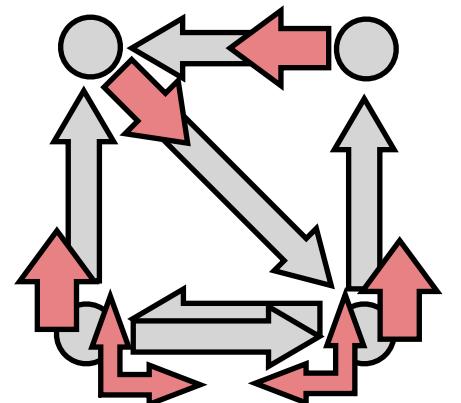
Supply & Demand



Cournot Market

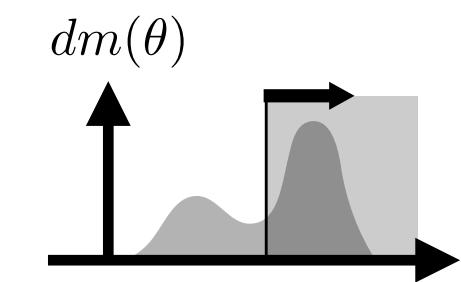


MDP Congestion Game

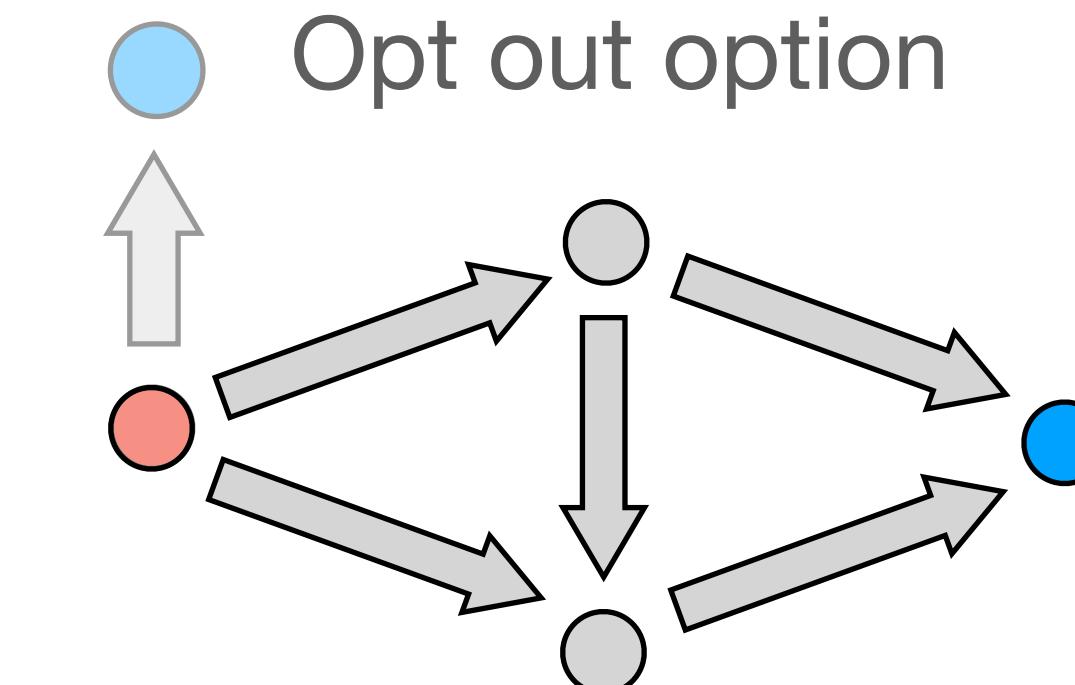


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous preferences

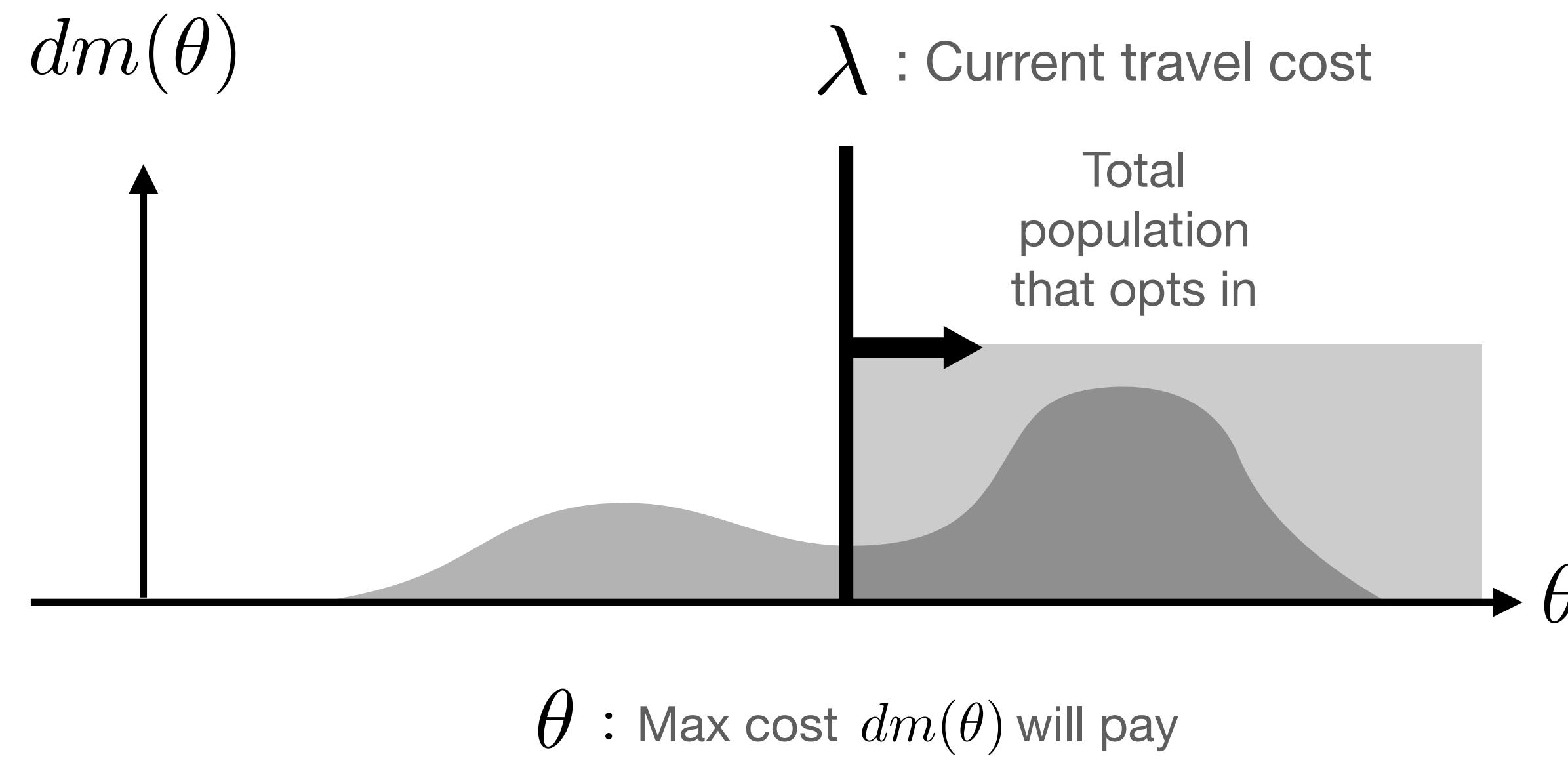


$$m(\lambda)$$



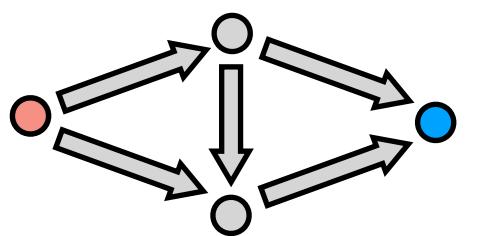
λ : Current travel cost

Total population that opts in

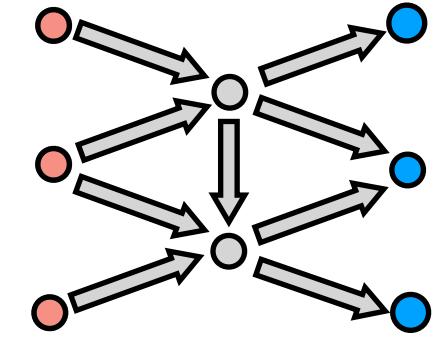


Potential Games

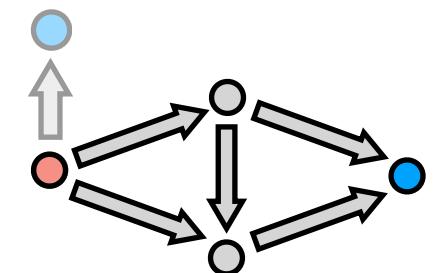
Routing Games



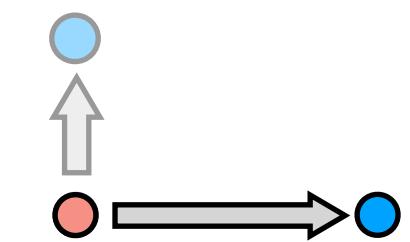
Multiple sources/sinks



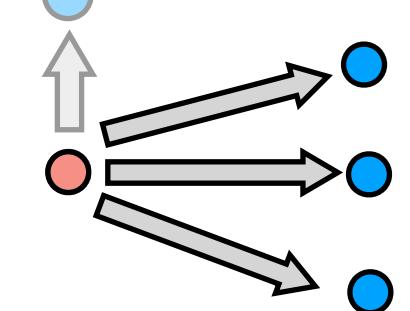
Variable Demand



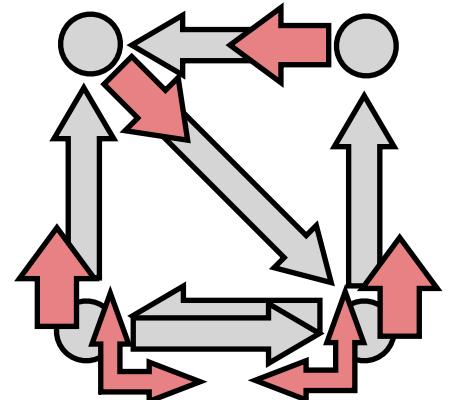
Supply & Demand



Cournot Market

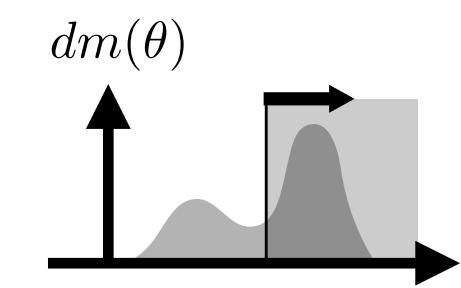


MDP Congestion Game

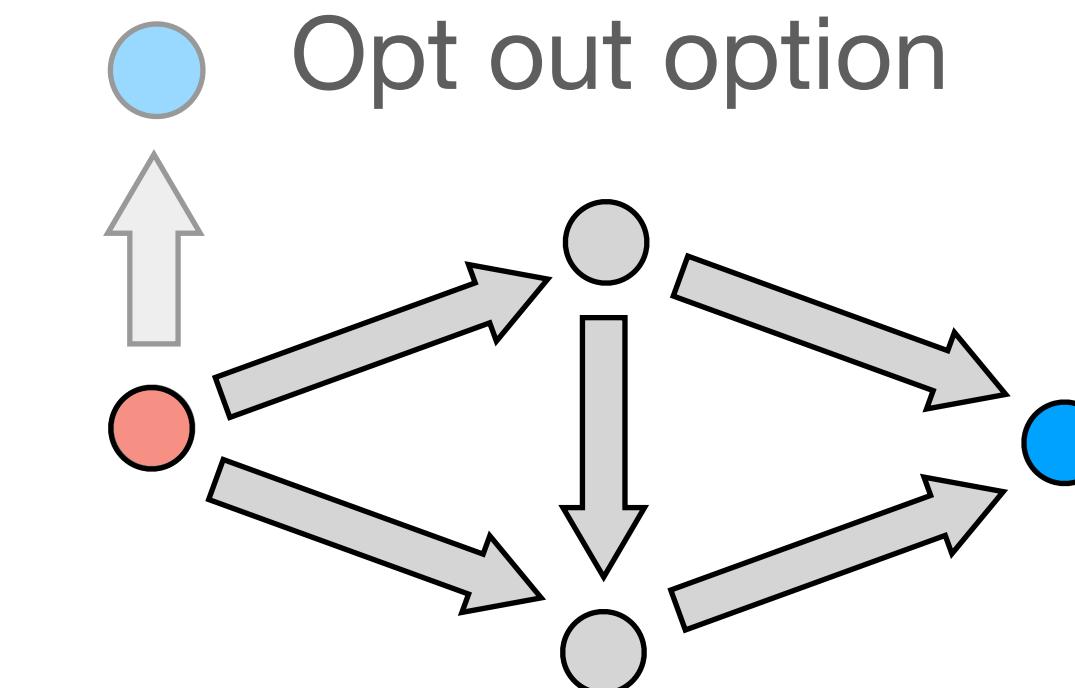


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous preferences



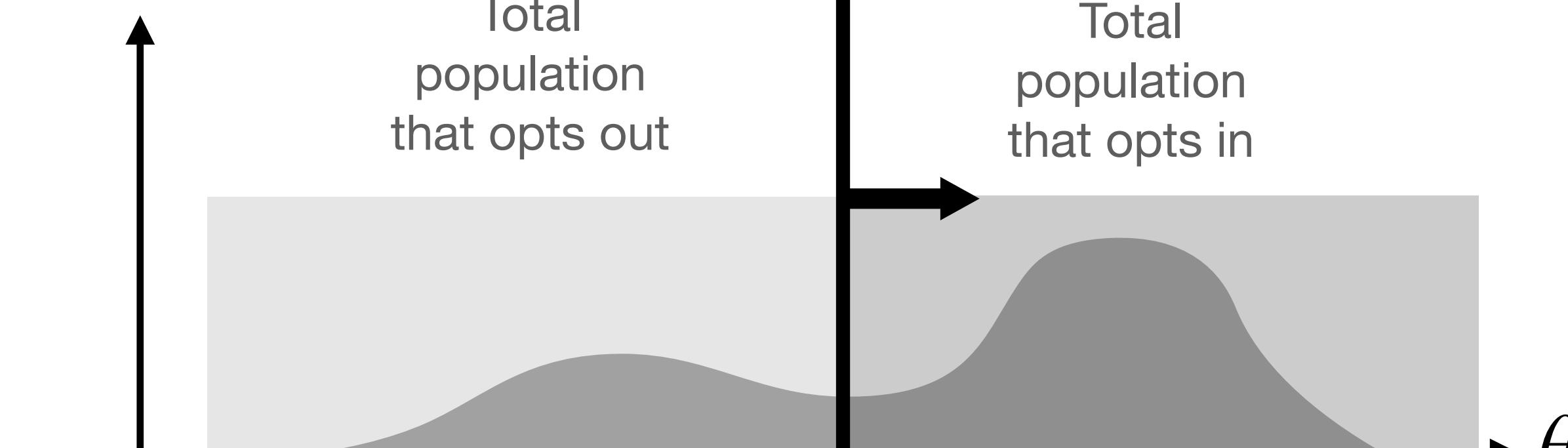
$$m(\lambda)$$



λ : Current travel cost

$$dm(\theta)$$

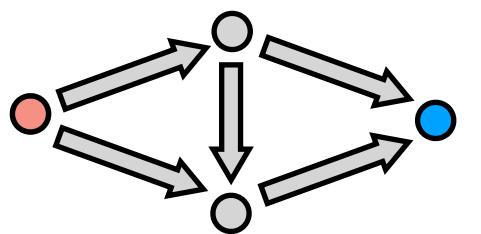
Total population that opts out



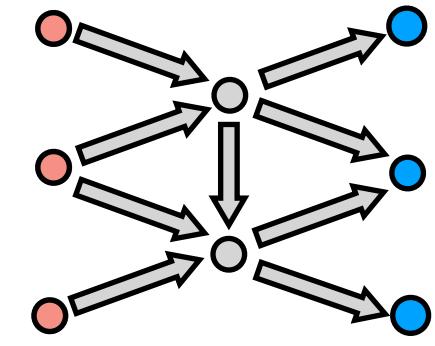
θ : Max cost $dm(\theta)$ will pay

Potential Games

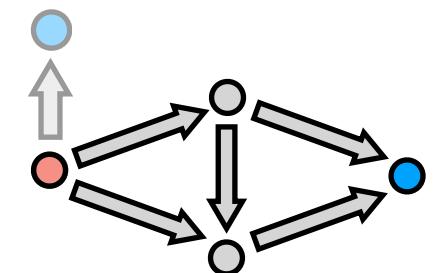
Routing Games



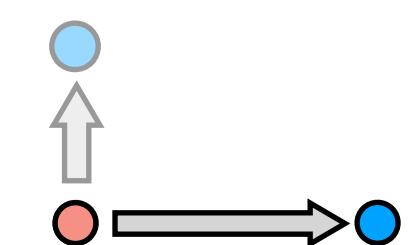
Multiple sources/sinks



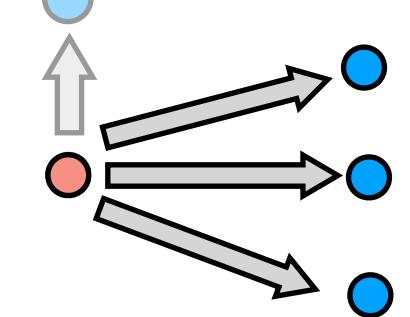
Variable Demand



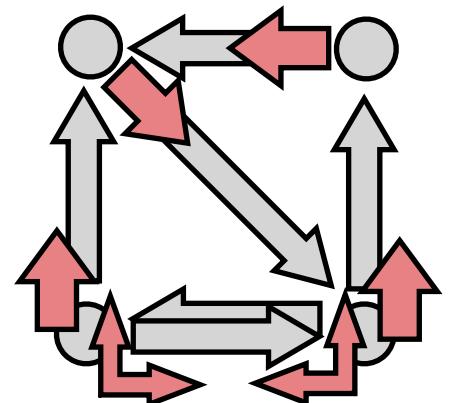
Supply & Demand



Cournot Market

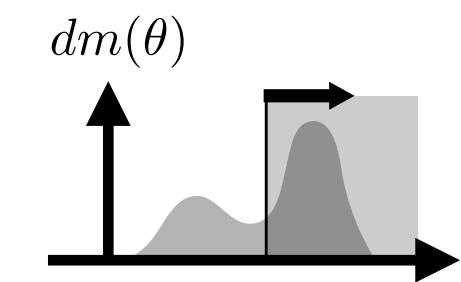


MDP Congestion Game

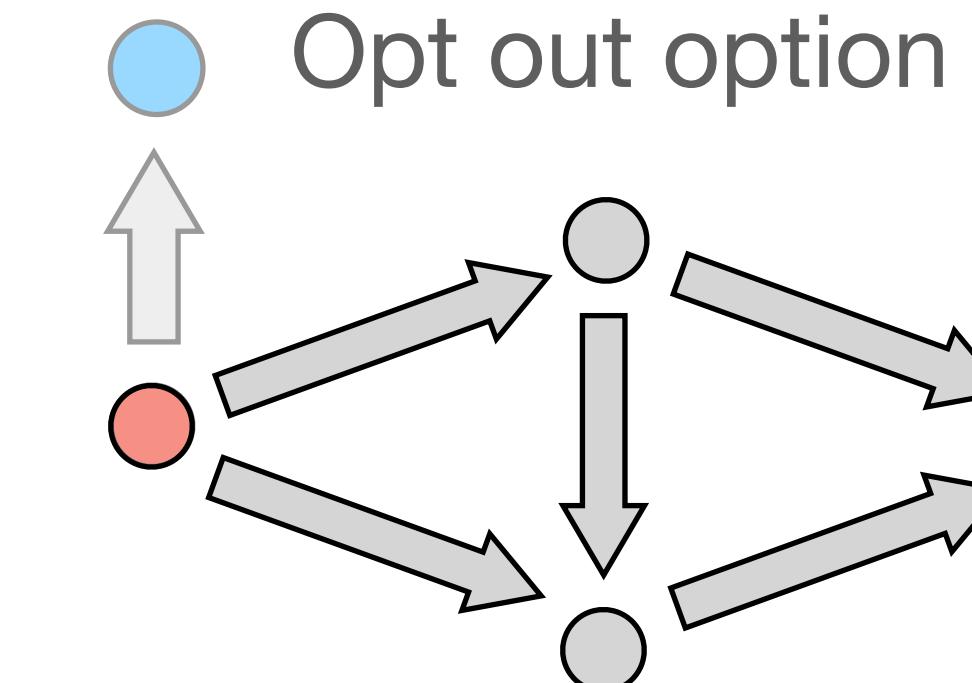


Variable Demand - Non-Homogeneous Preferences

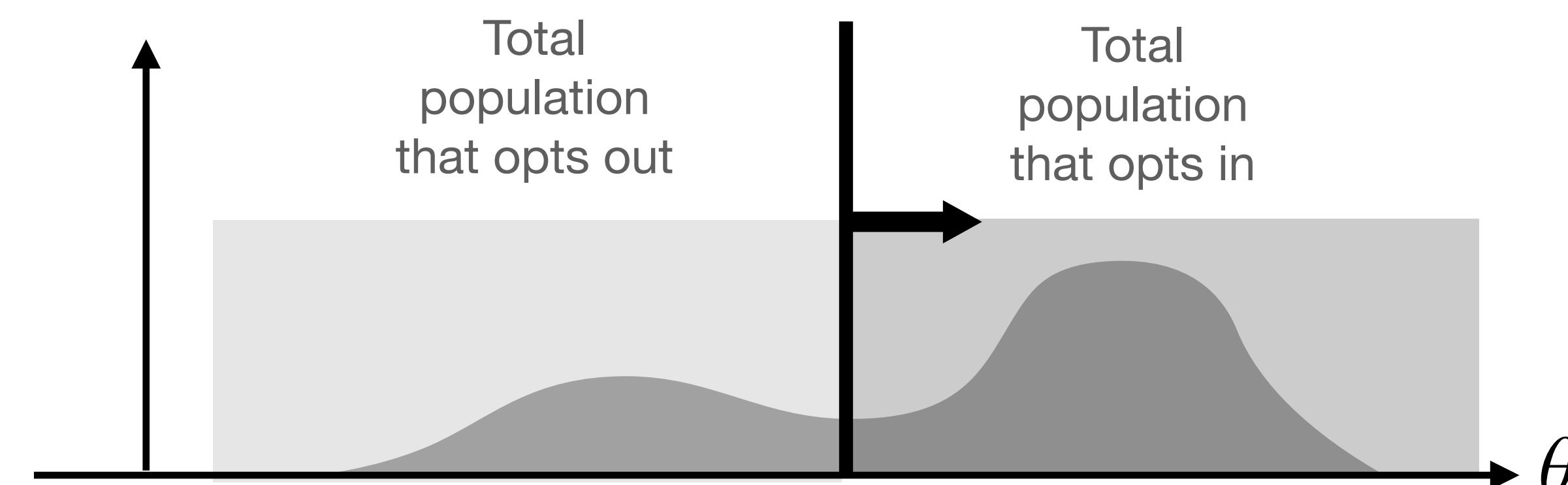
Non-homo-geneous preferences



$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$



$$dm(\theta)$$

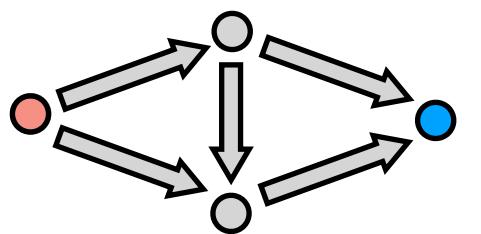


θ : Max cost $dm(\theta)$ will pay

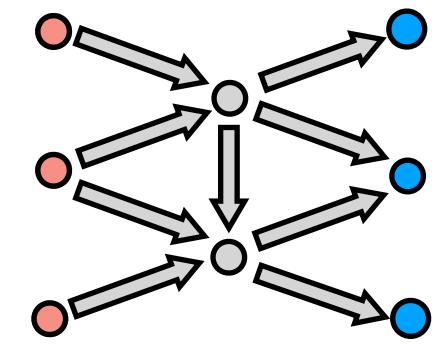
λ : Current travel cost

Potential Games

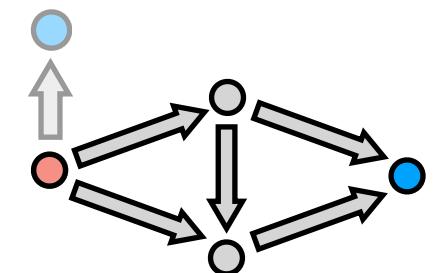
Routing Games



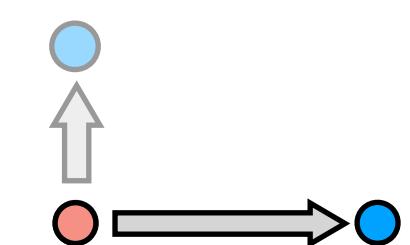
Multiple sources/sinks



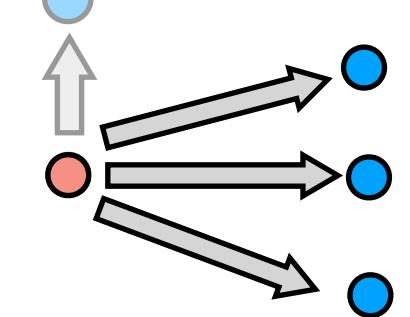
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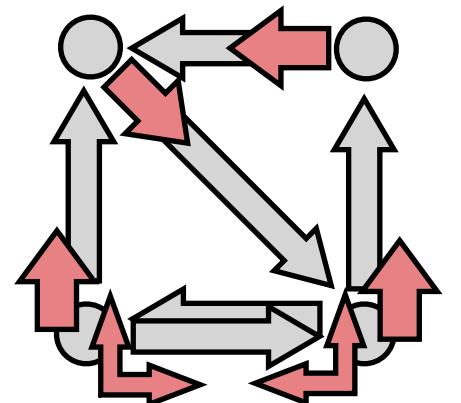
Supply & Demand



Cournot Market

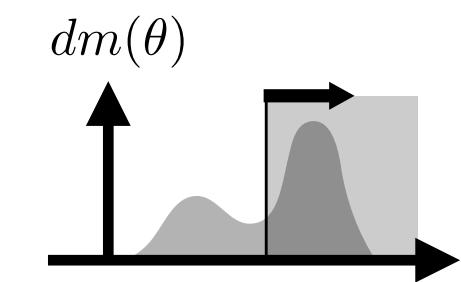


MDP Congestion Game



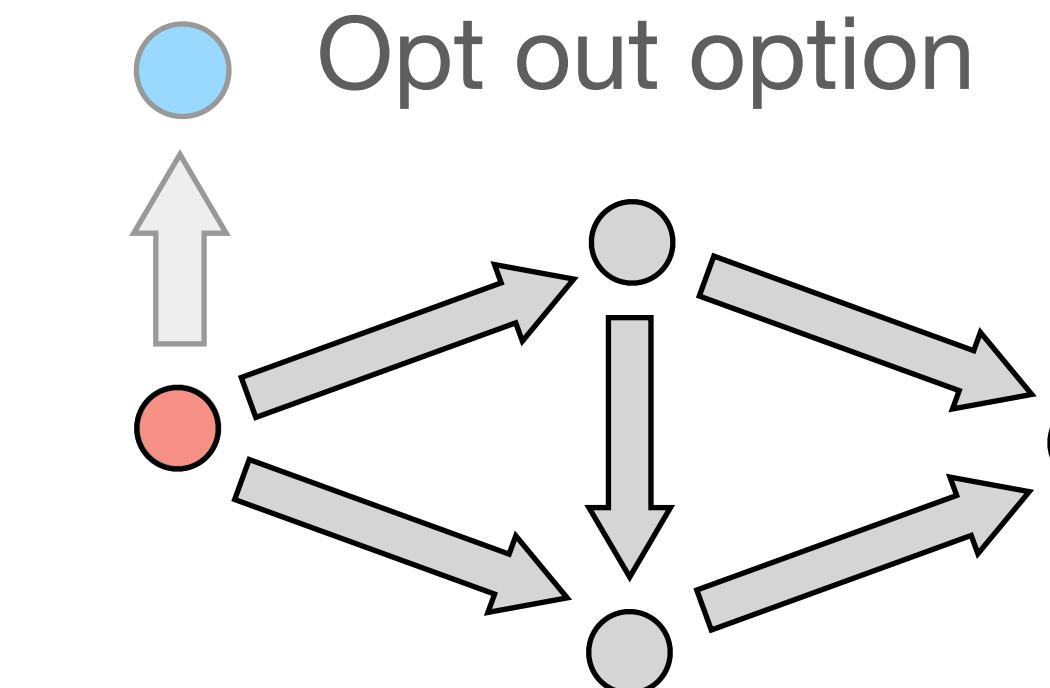
Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous preferences



Multi-Variate Preferences

$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$



$$dm(\theta)$$

λ : Current travel cost

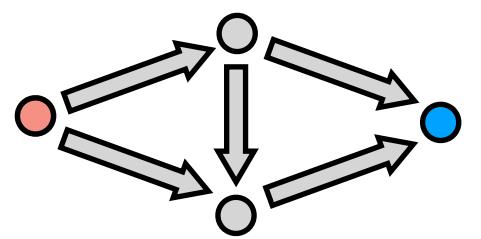
Total population that opts out

Total population that opts in

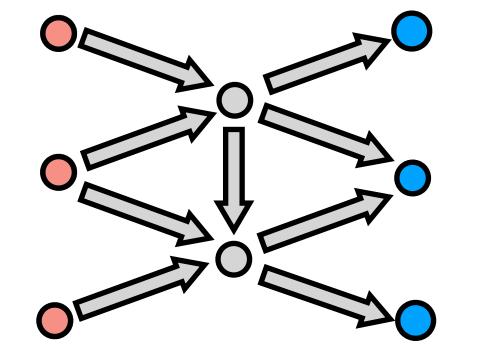
θ : Max cost $dm(\theta)$ will pay

Potential Games

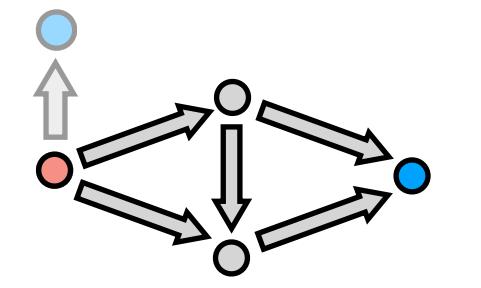
Routing Games



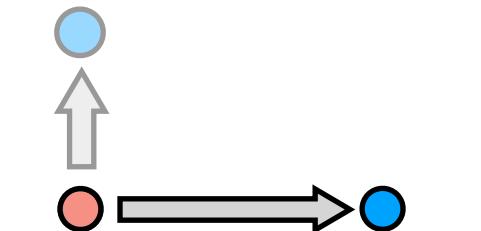
Multiple sources/sinks



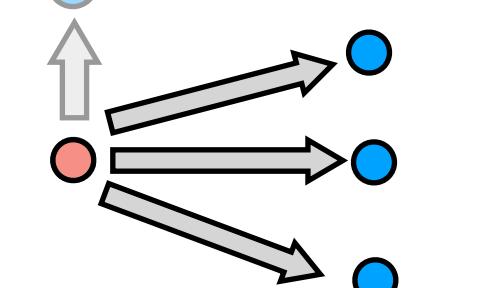
Variable Demand



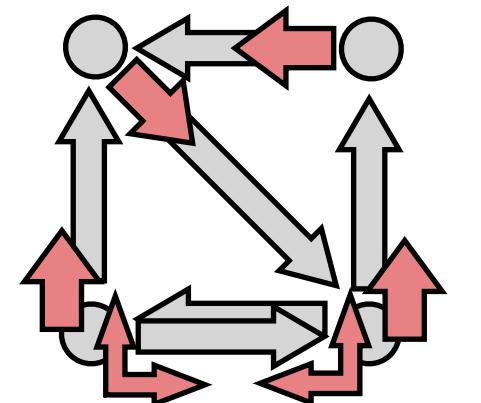
Supply & Demand



Cournot Market

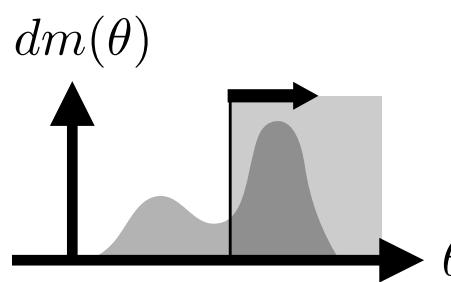


MDP Congestion Game

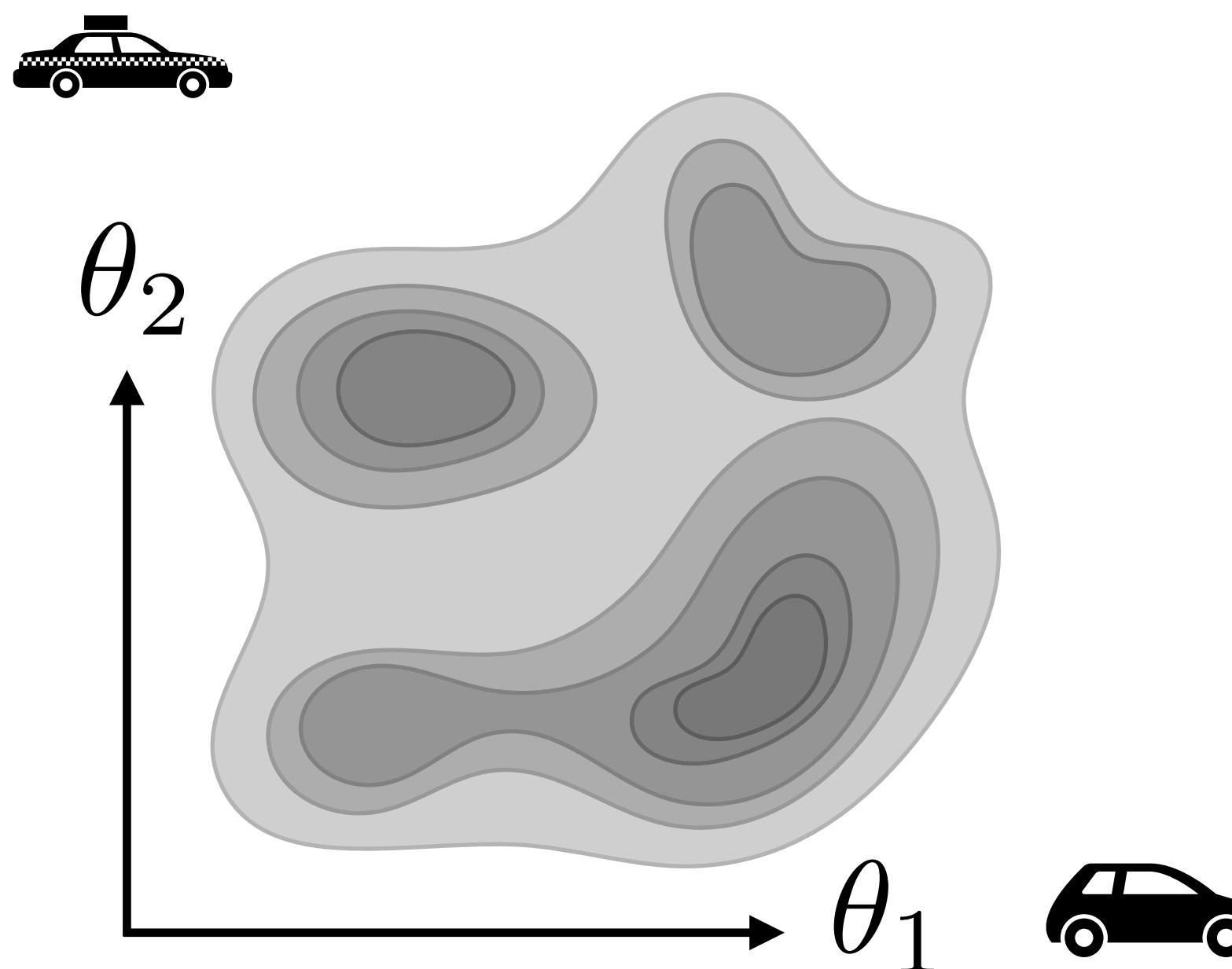
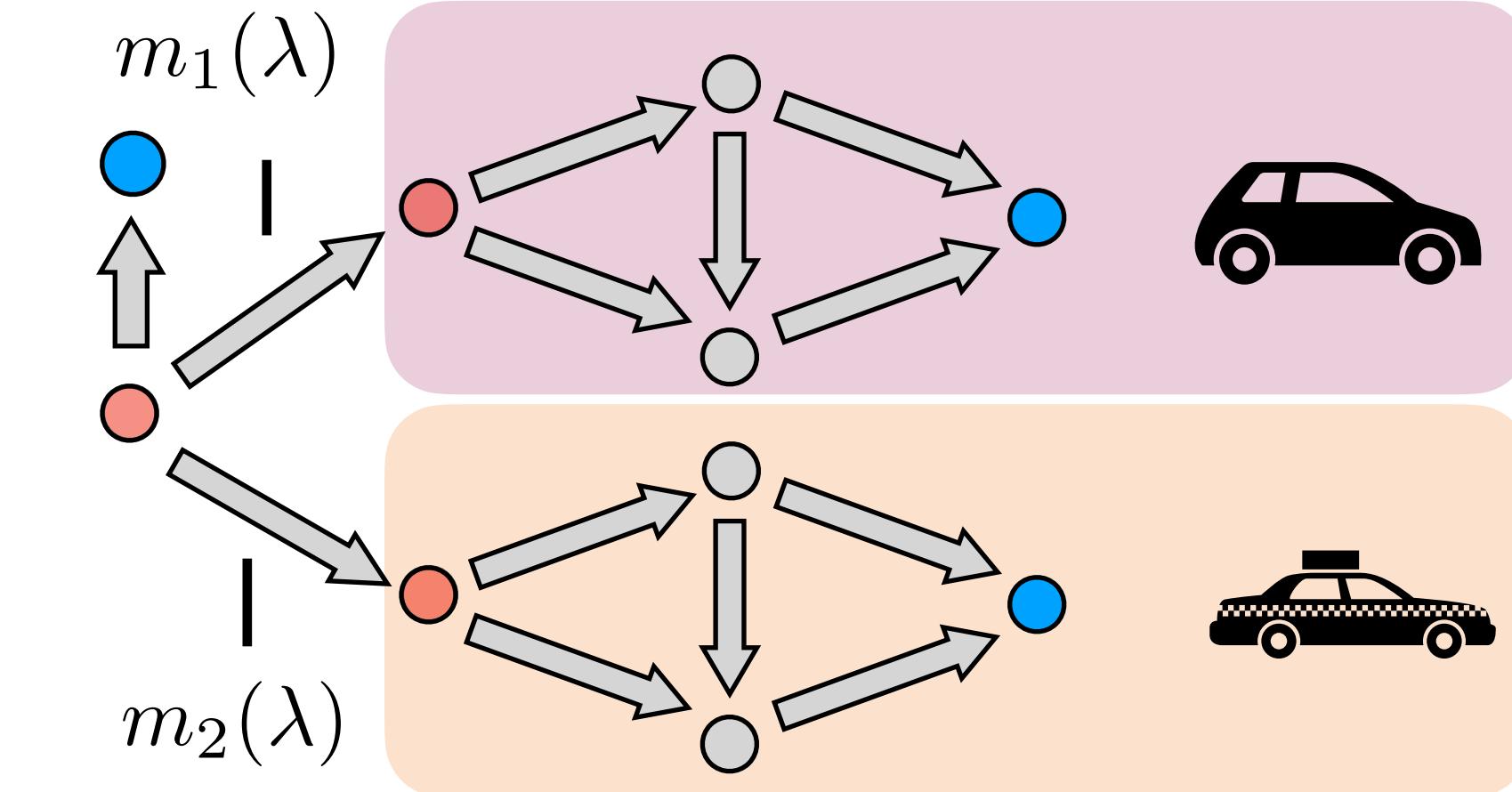
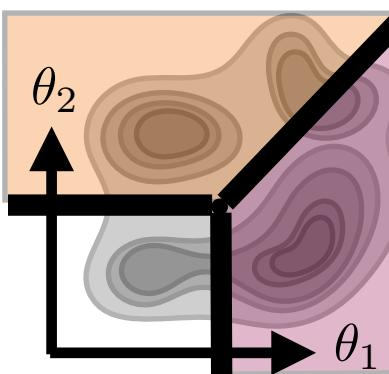


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences

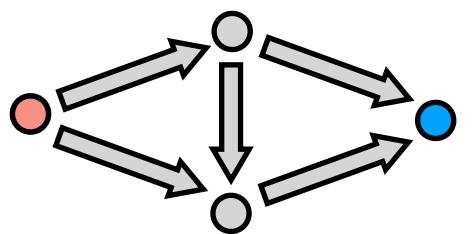


Multi-Variate Preferences

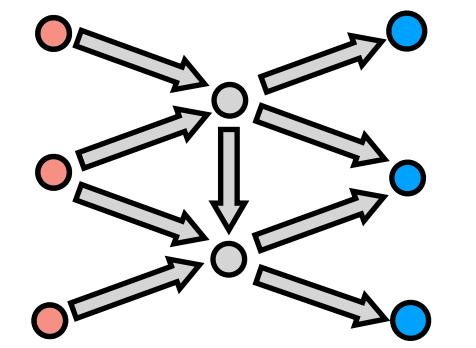


Potential Games

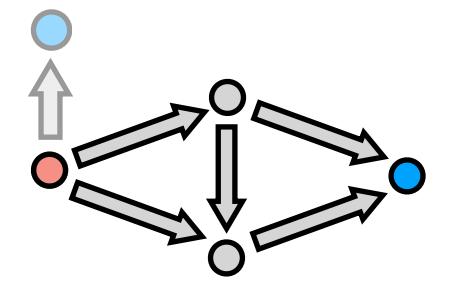
Routing Games



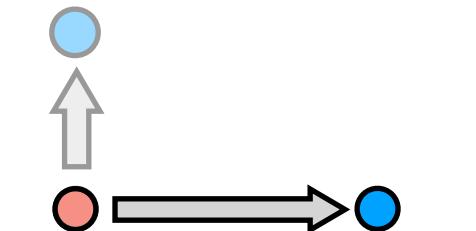
Multiple sources/sinks



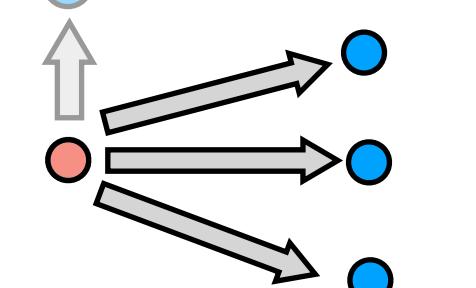
Variable Demand



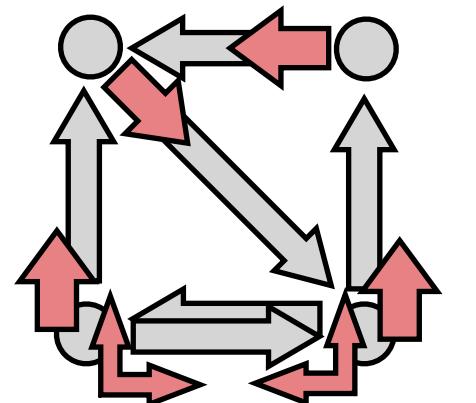
Supply & Demand



Cournot Market

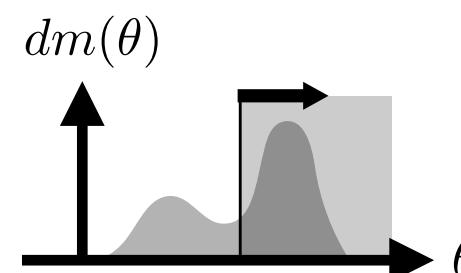


MDP Congestion Game

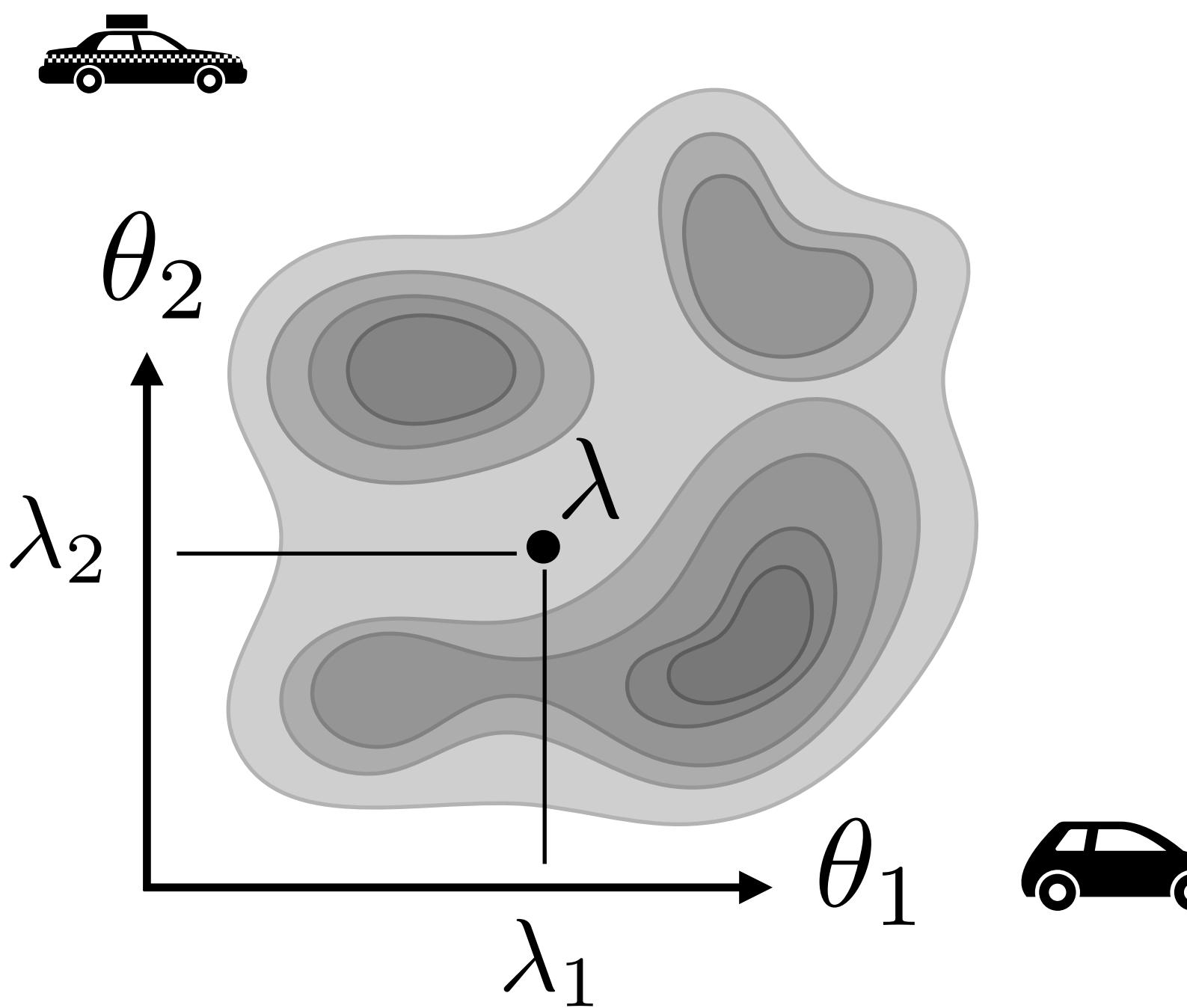
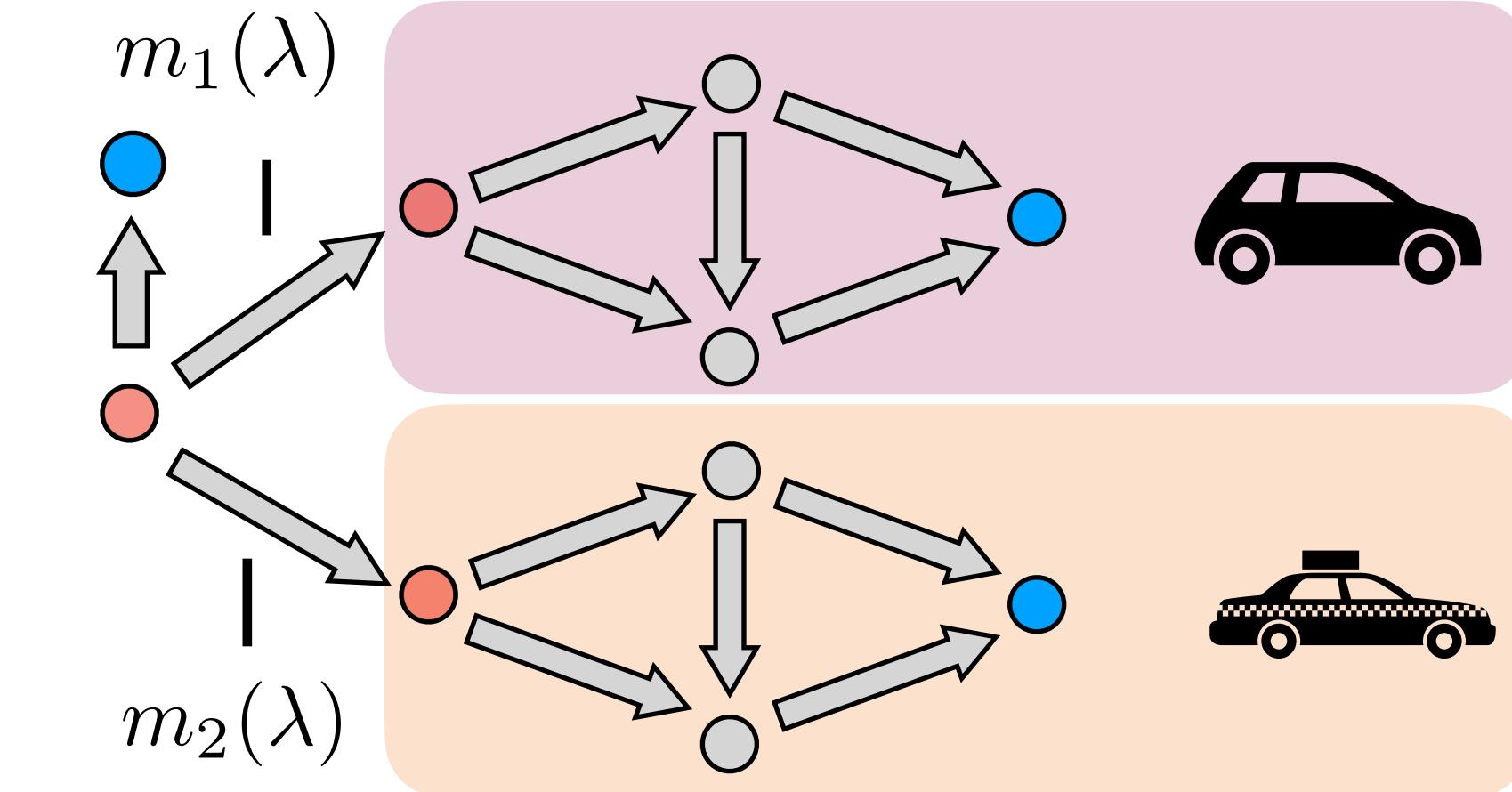
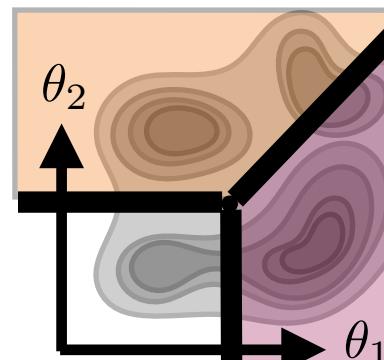


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences

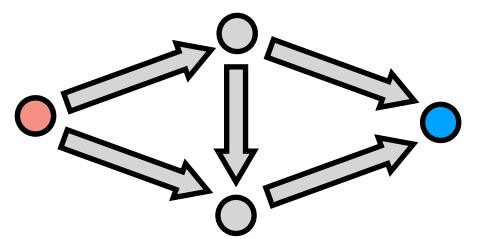


Multi-Variate Preferences

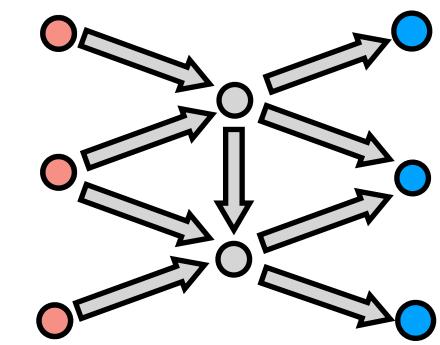


Potential Games

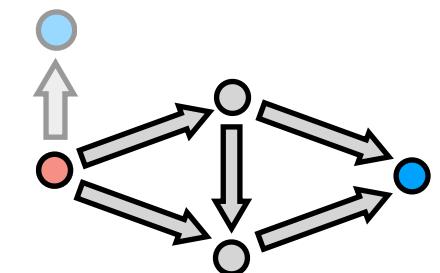
Routing Games



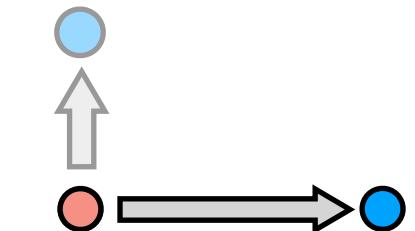
Multiple sources/sinks



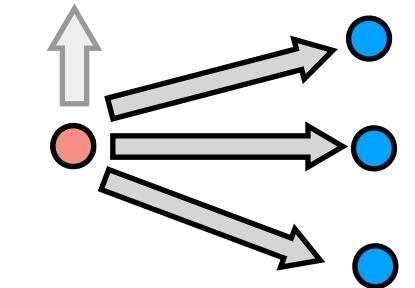
Variable Demand



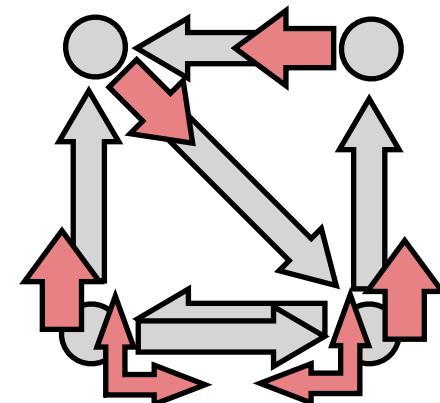
Supply & Demand



Cournot Market

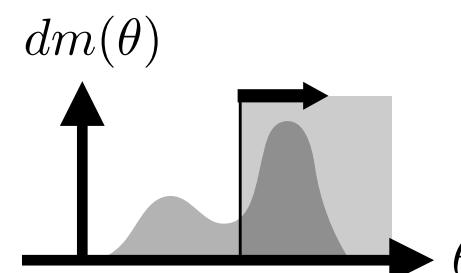


MDP Congestion Game

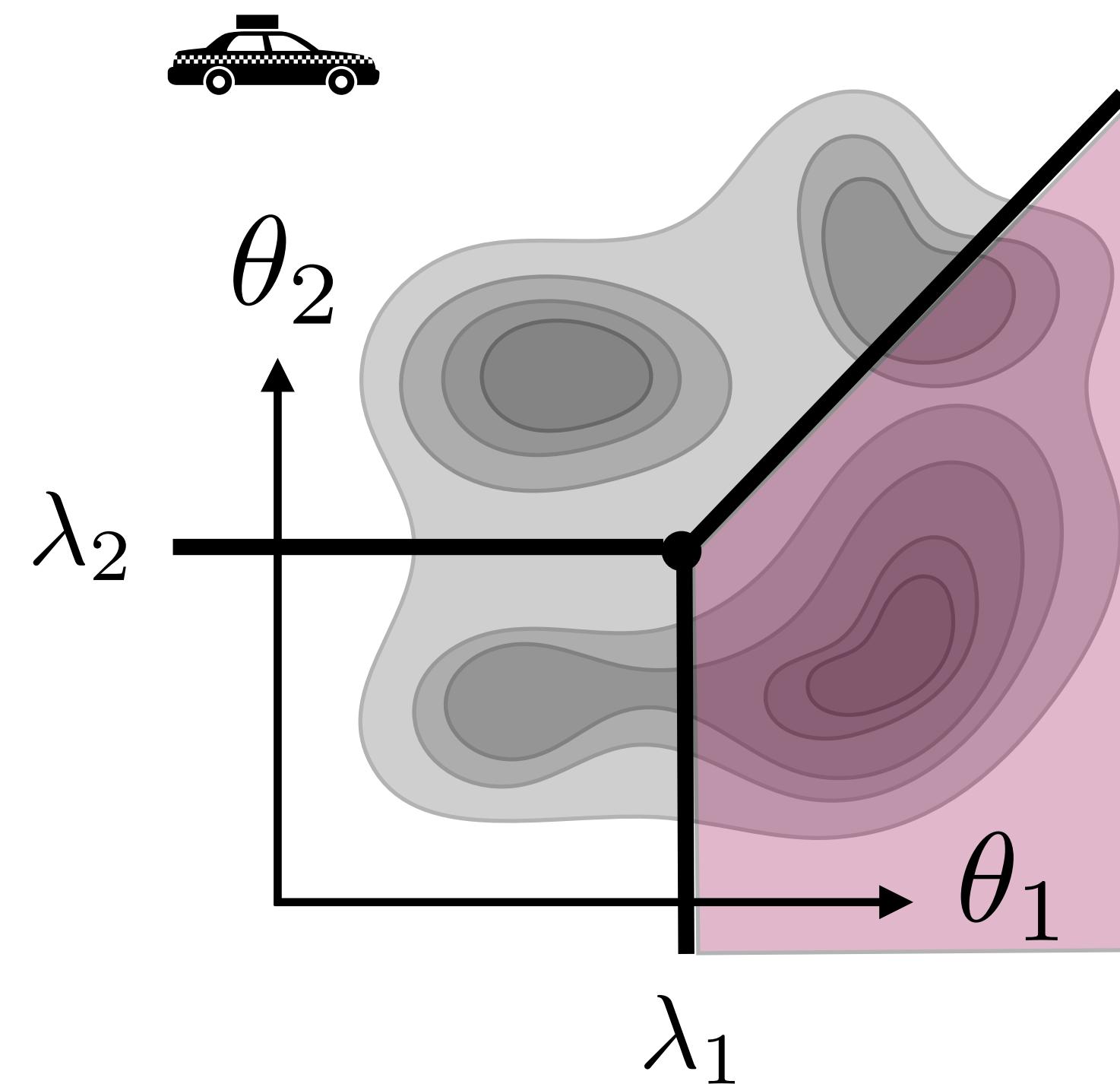
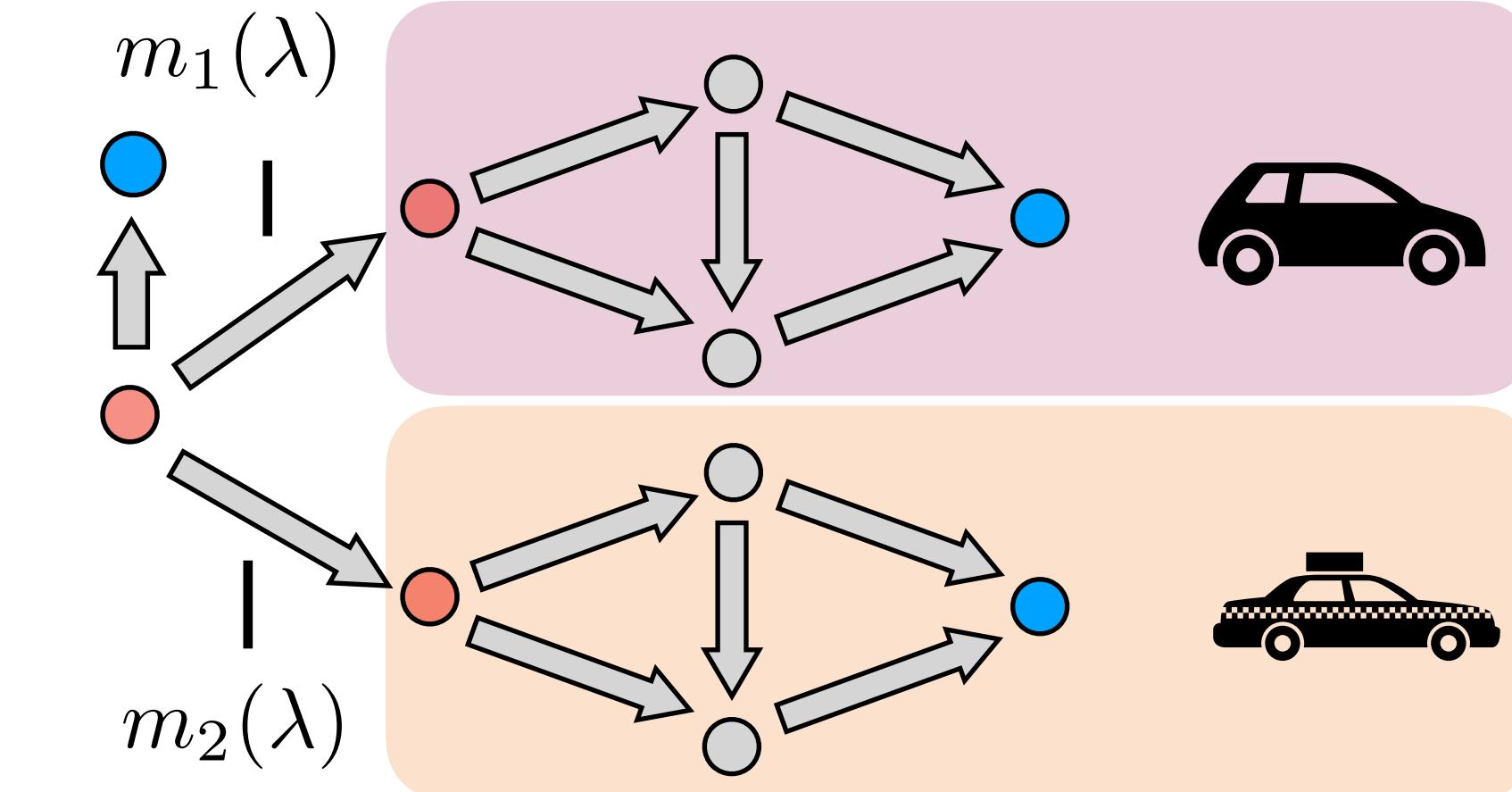
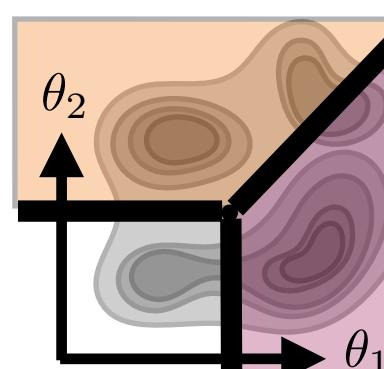


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences



Multi-Variate Preferences

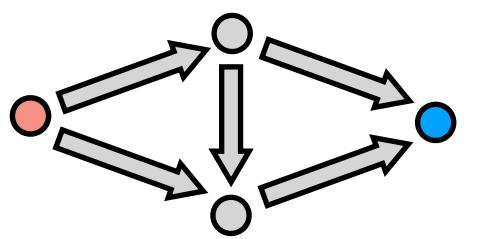


$A_1(\lambda)$

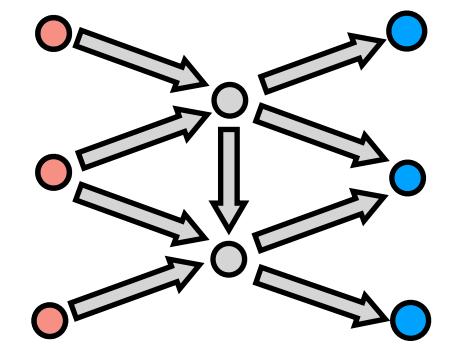
$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

Potential Games

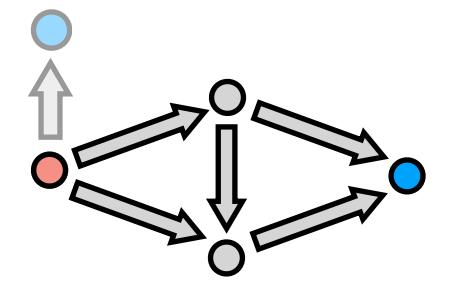
Routing Games



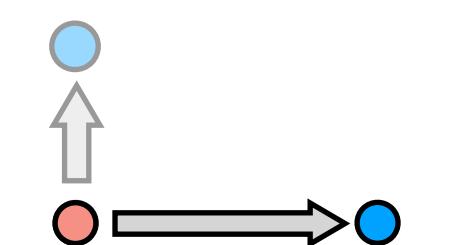
Multiple sources/sinks



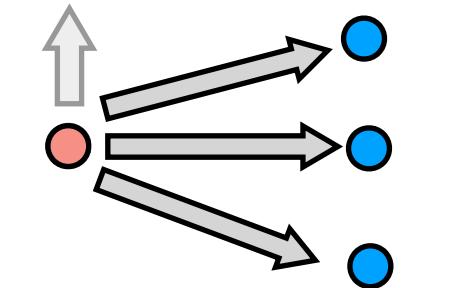
Variable Demand



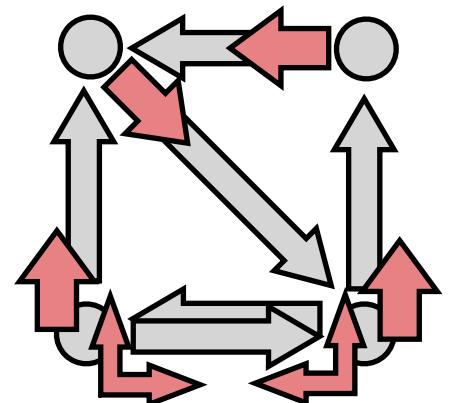
Supply & Demand



Cournot Market

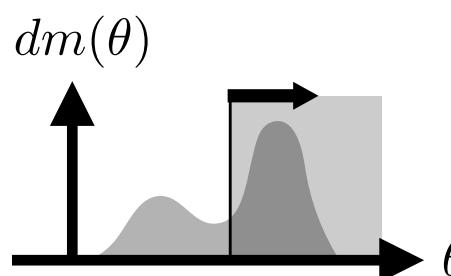


MDP Congestion Game

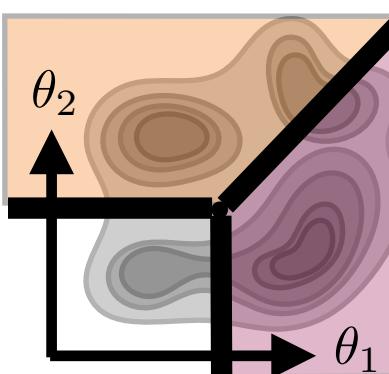


Variable Demand - Multi-Variate Non-Homogeneous Preferences

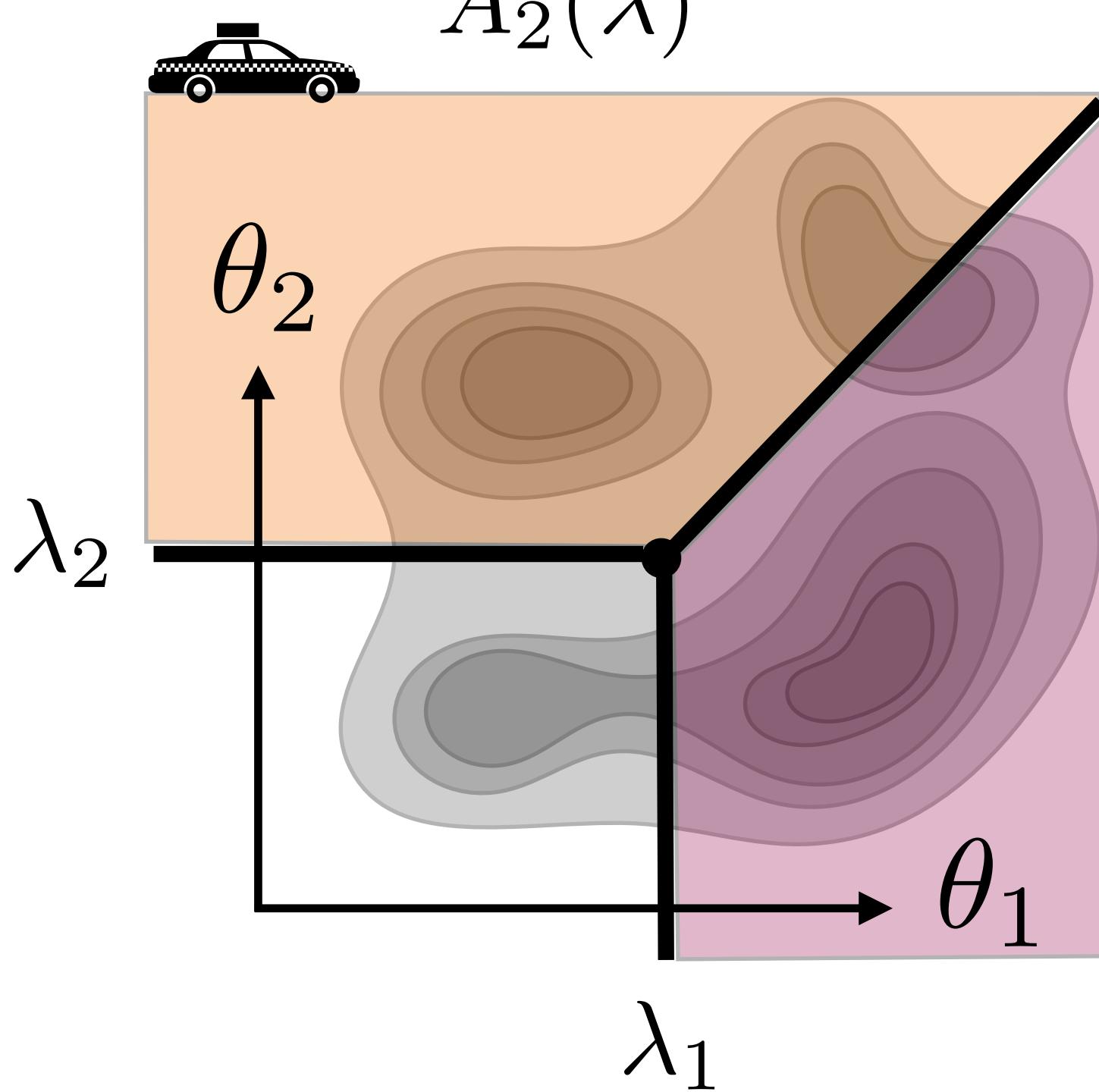
Non-homo-geneous preferences



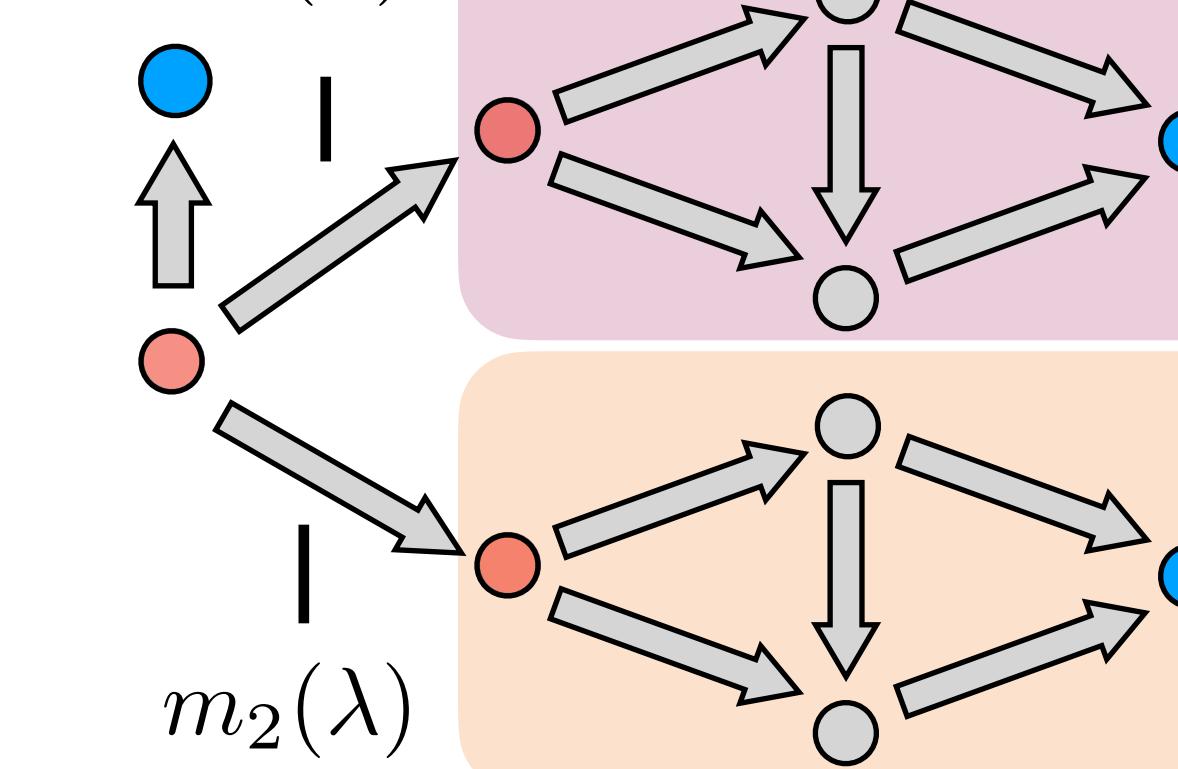
Multi-Variate Preferences



$$A_2(\lambda)$$



$$m_1(\lambda)$$



$$m_2(\lambda)$$



$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

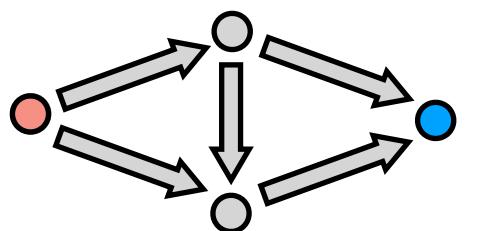
$$A_1(\lambda)$$



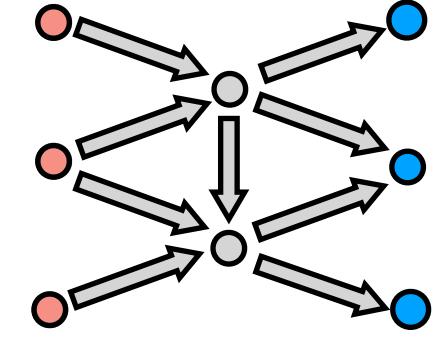
$$m_2(\lambda) = \int_{A_2(\lambda)} dm(\theta)$$

Potential Games

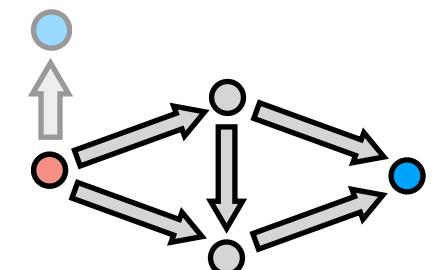
Routing Games



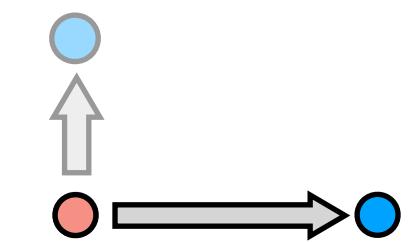
Multiple sources/sinks



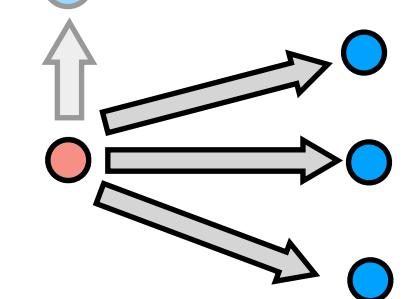
Variable Demand



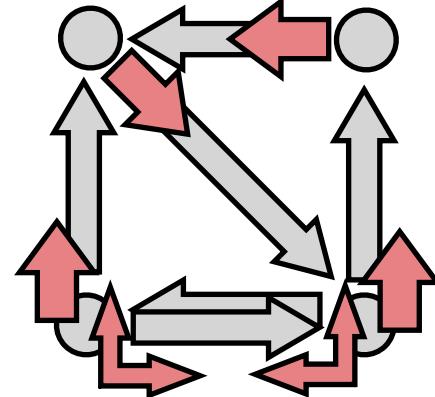
Supply & Demand



Cournot Market

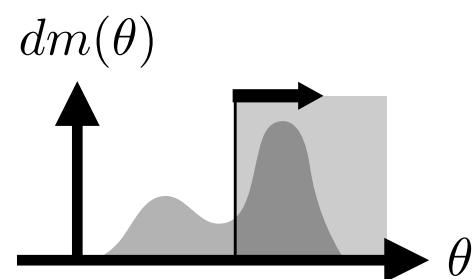


MDP Congestion Game

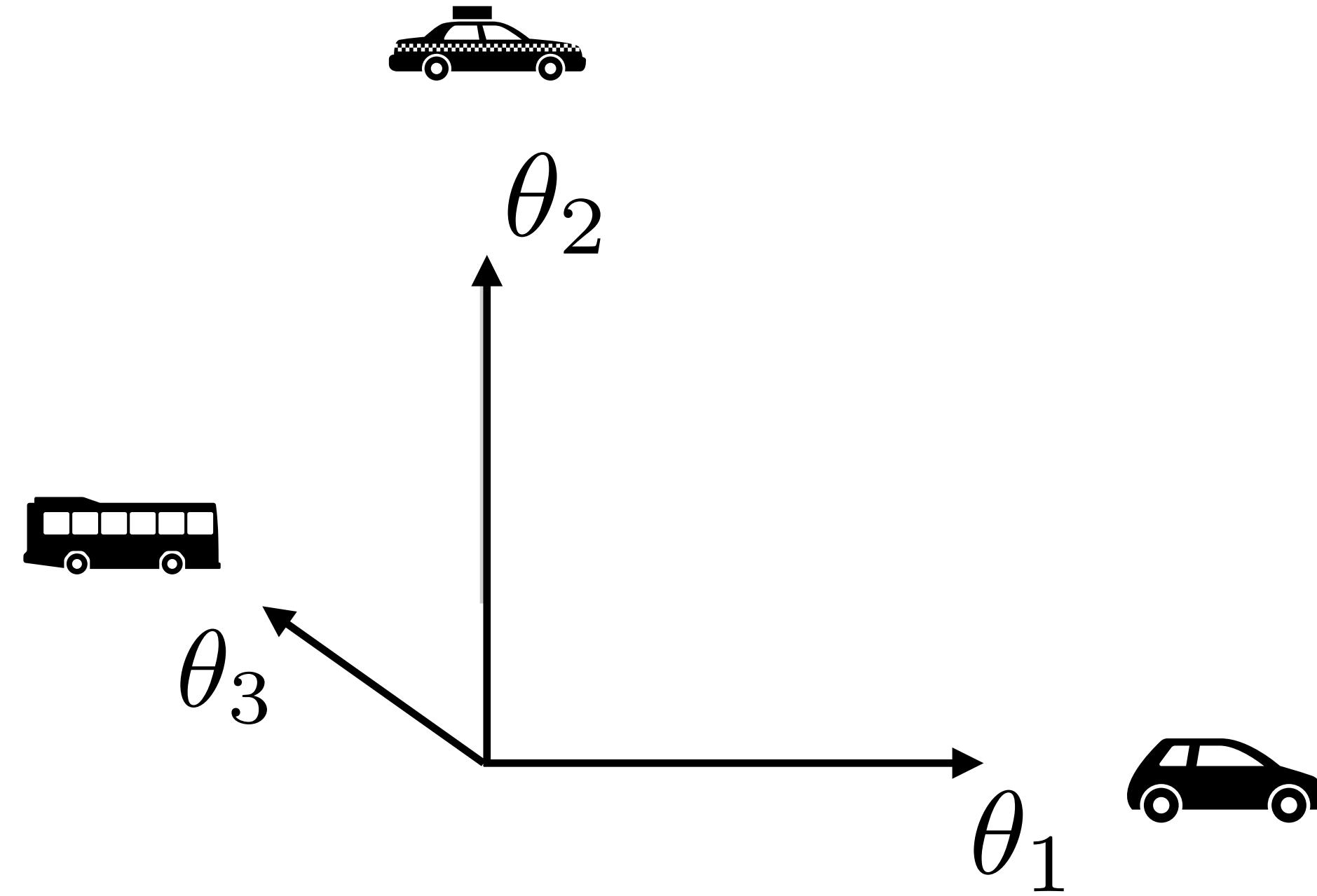
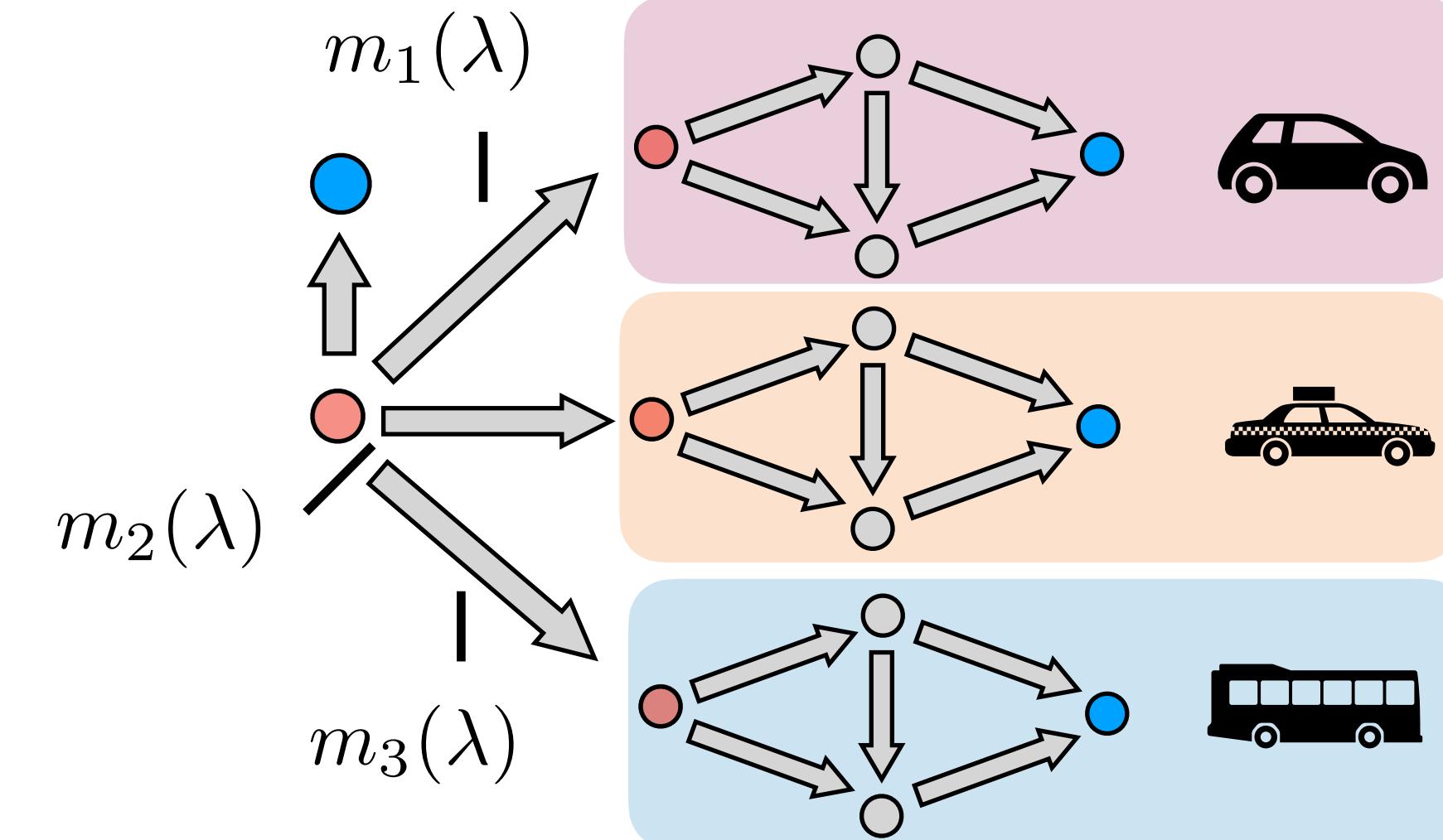
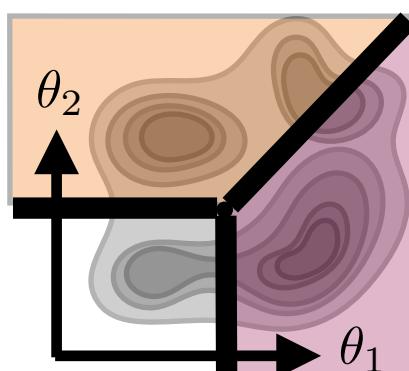


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences

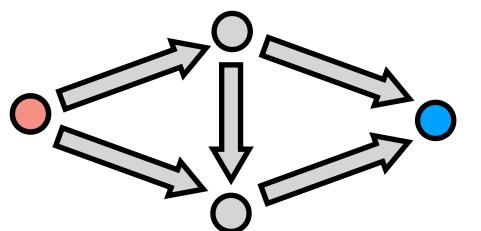


Multi-Variate Preferences

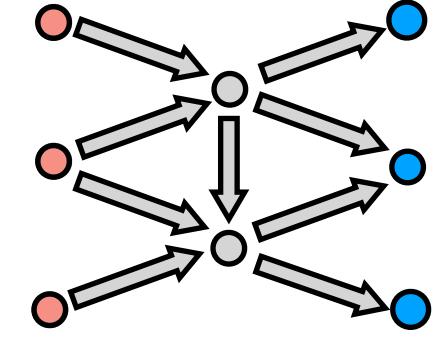


Potential Games

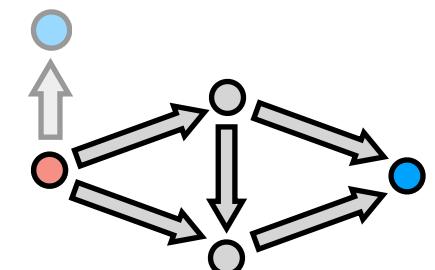
Routing Games



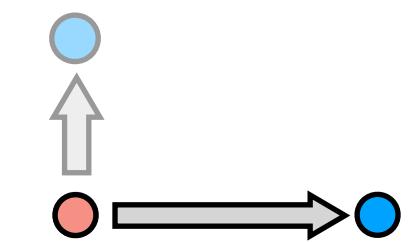
Multiple sources/sinks



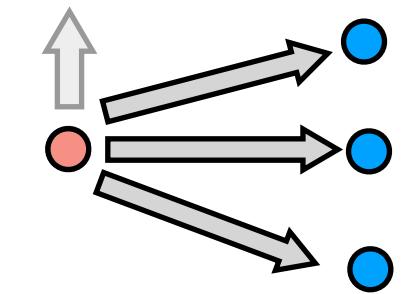
Variable Demand



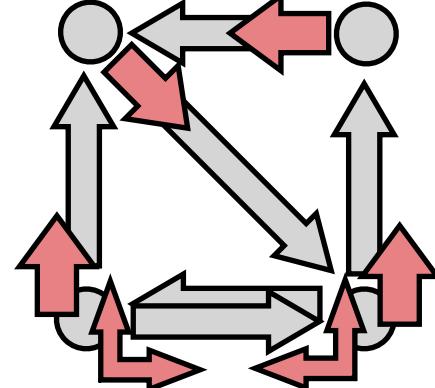
Supply & Demand



Cournot Market

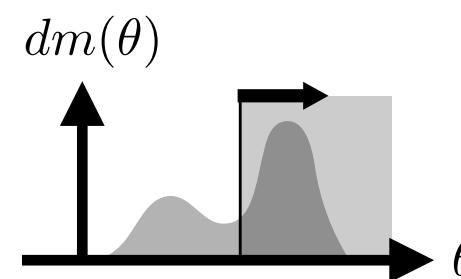


MDP Congestion Game

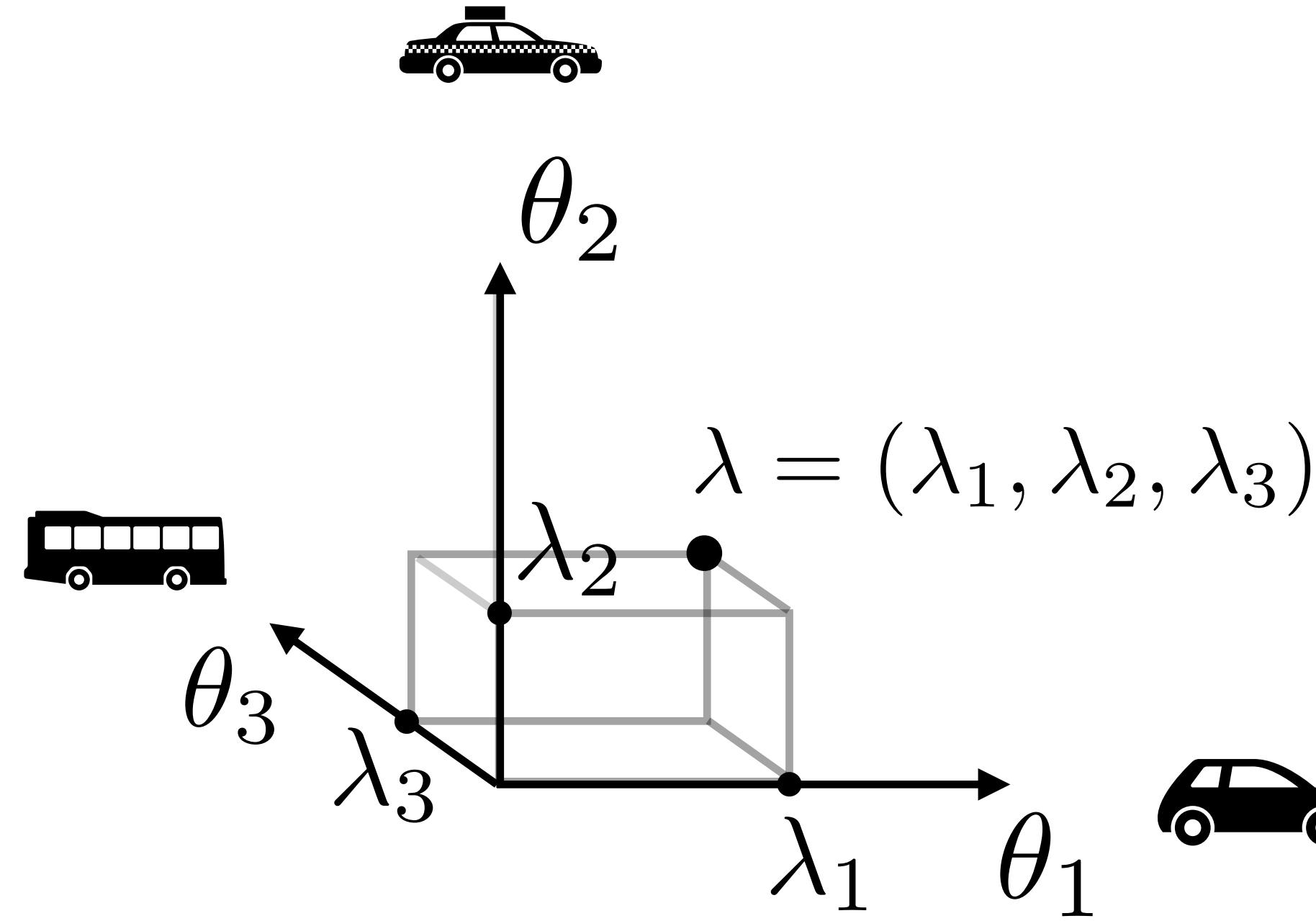
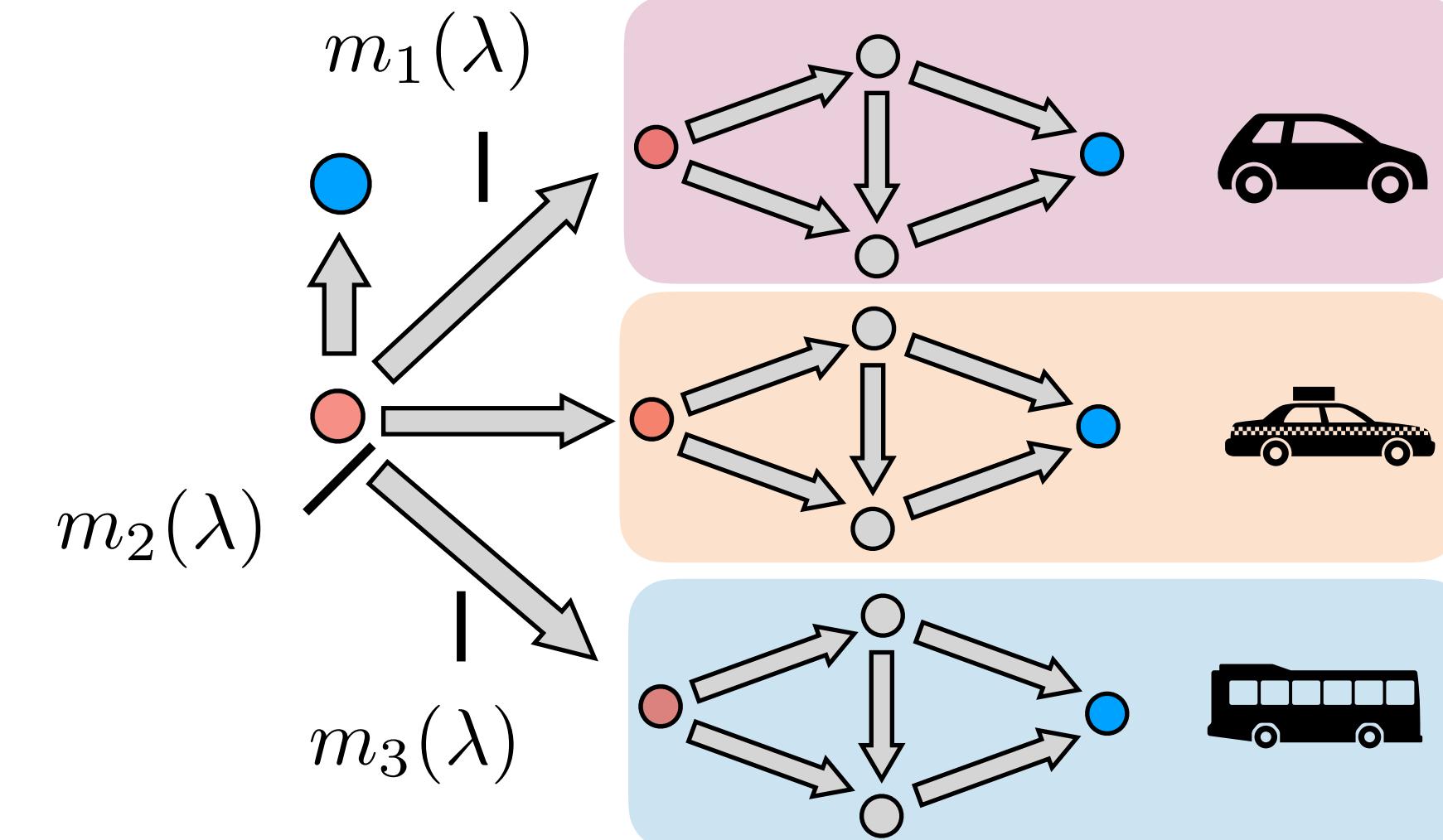
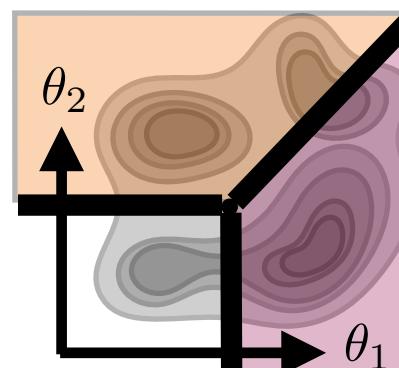


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences

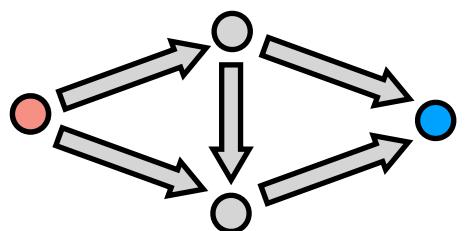


Multi-Variate Preferences

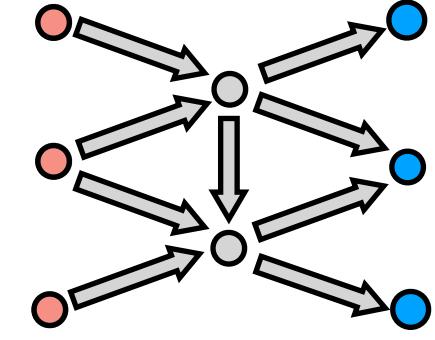


Potential Games

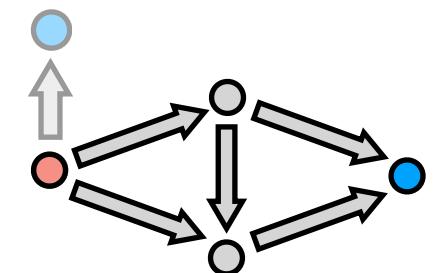
Routing Games



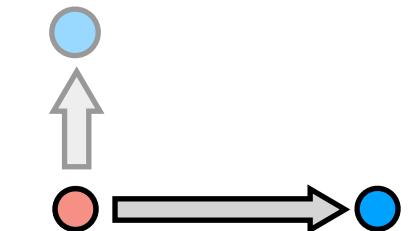
Multiple sources/sinks



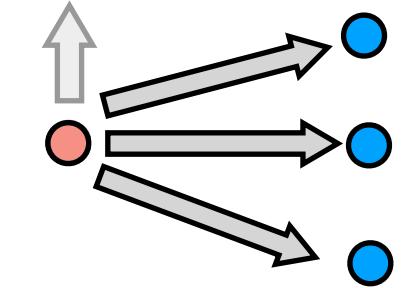
Variable Demand



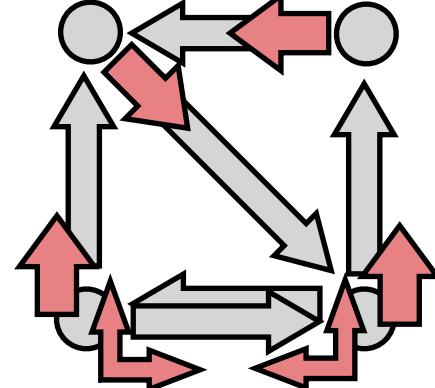
Supply & Demand



Cournot Market

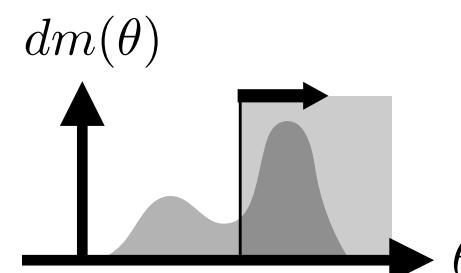


MDP Congestion Game

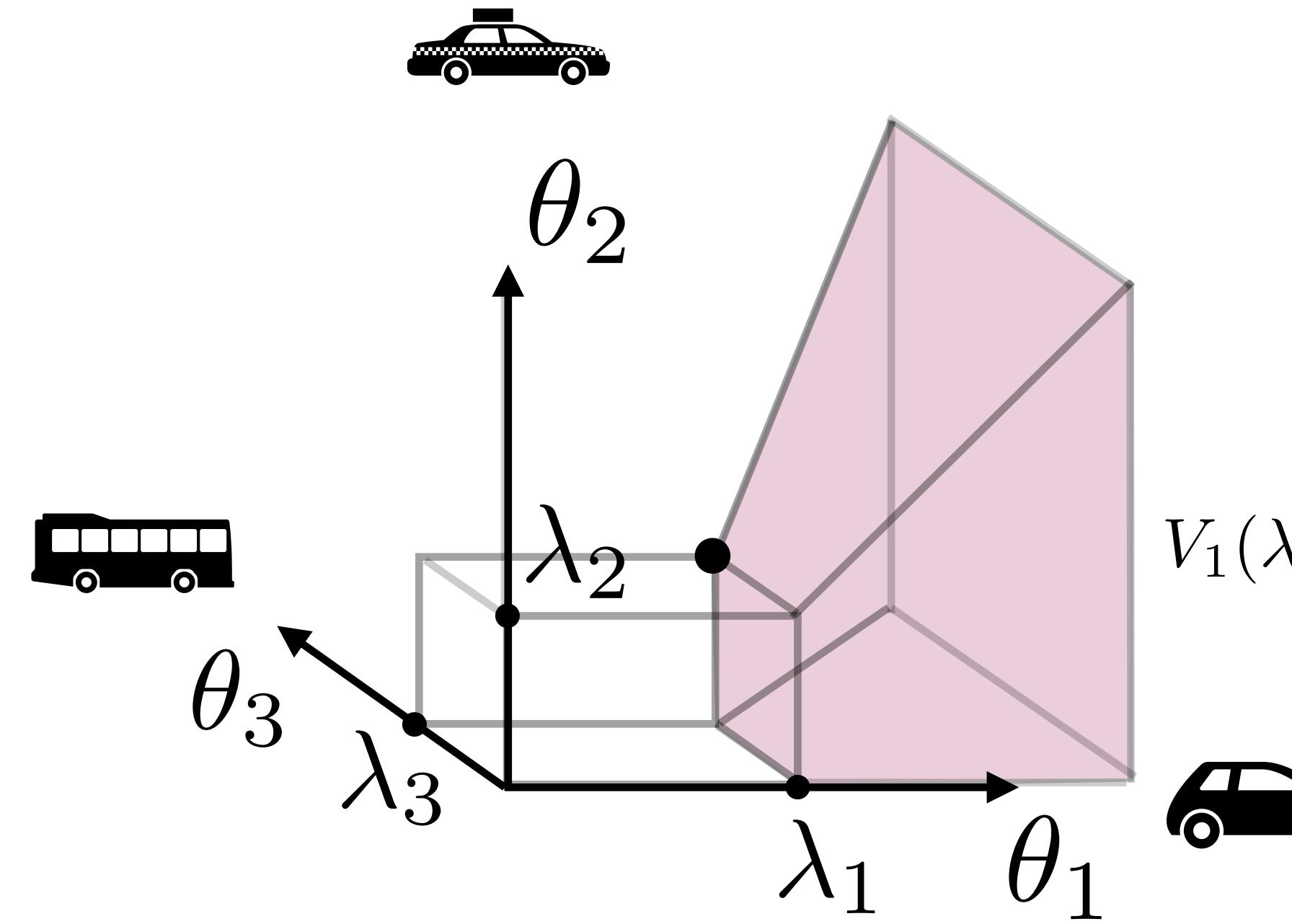
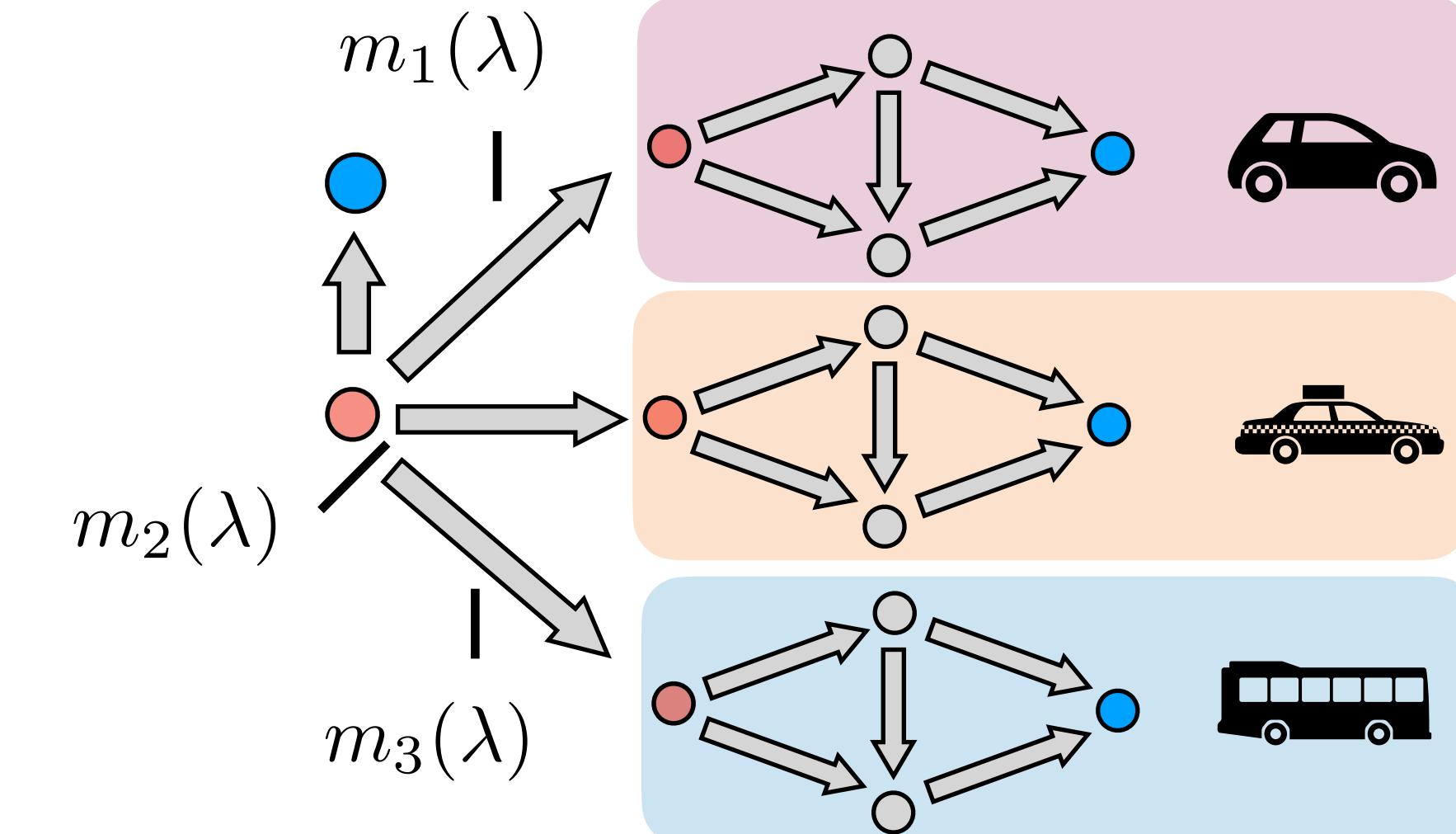
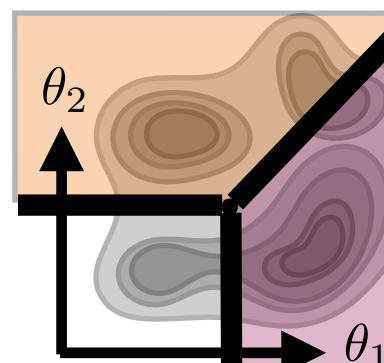


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences



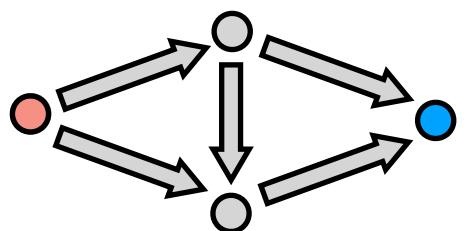
Multi-Variate Preferences



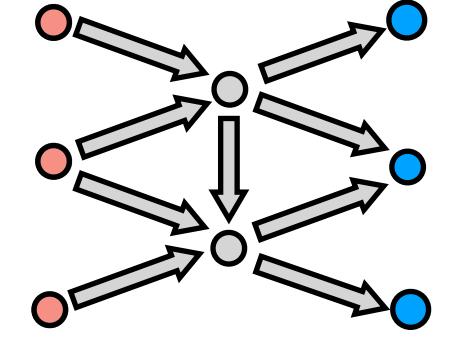
$$m_1(\lambda) = \int_{V_1(\lambda)} dm(\theta)$$

Potential Games

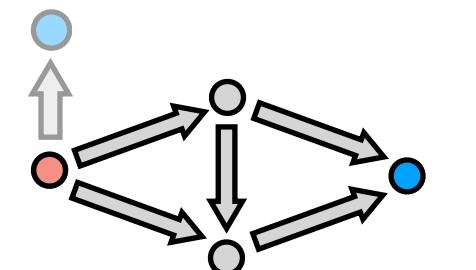
Routing Games



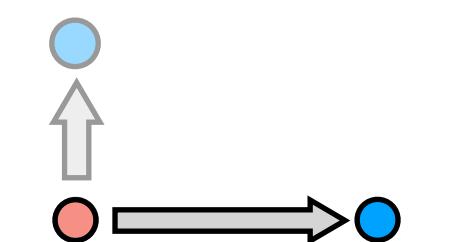
Multiple sources/sinks



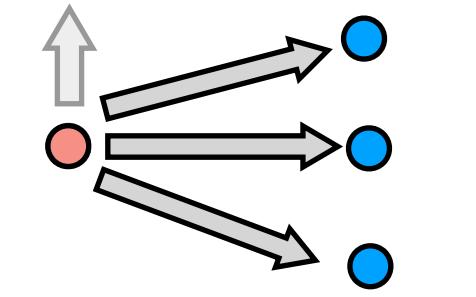
Variable Demand



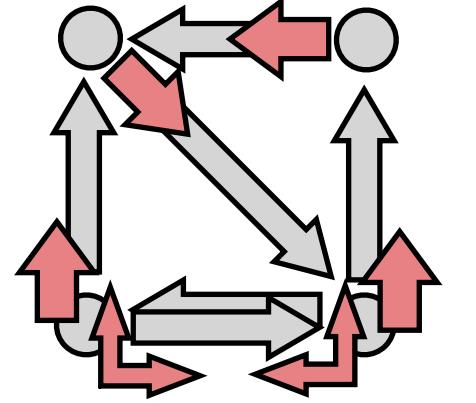
Supply & Demand



Cournot Market

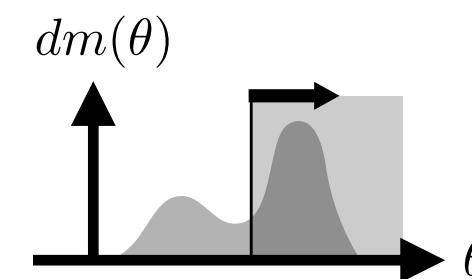


MDP Congestion Game

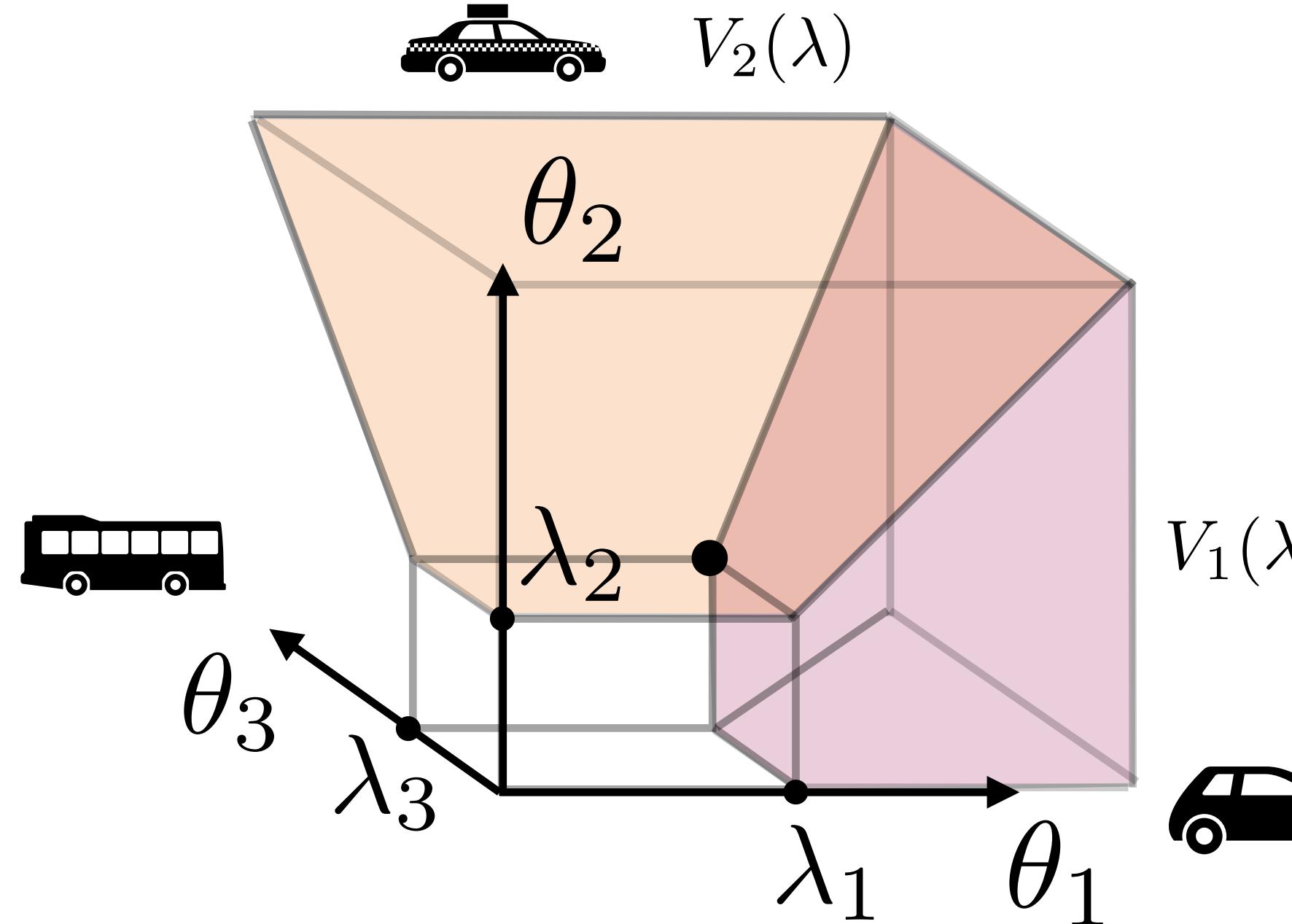
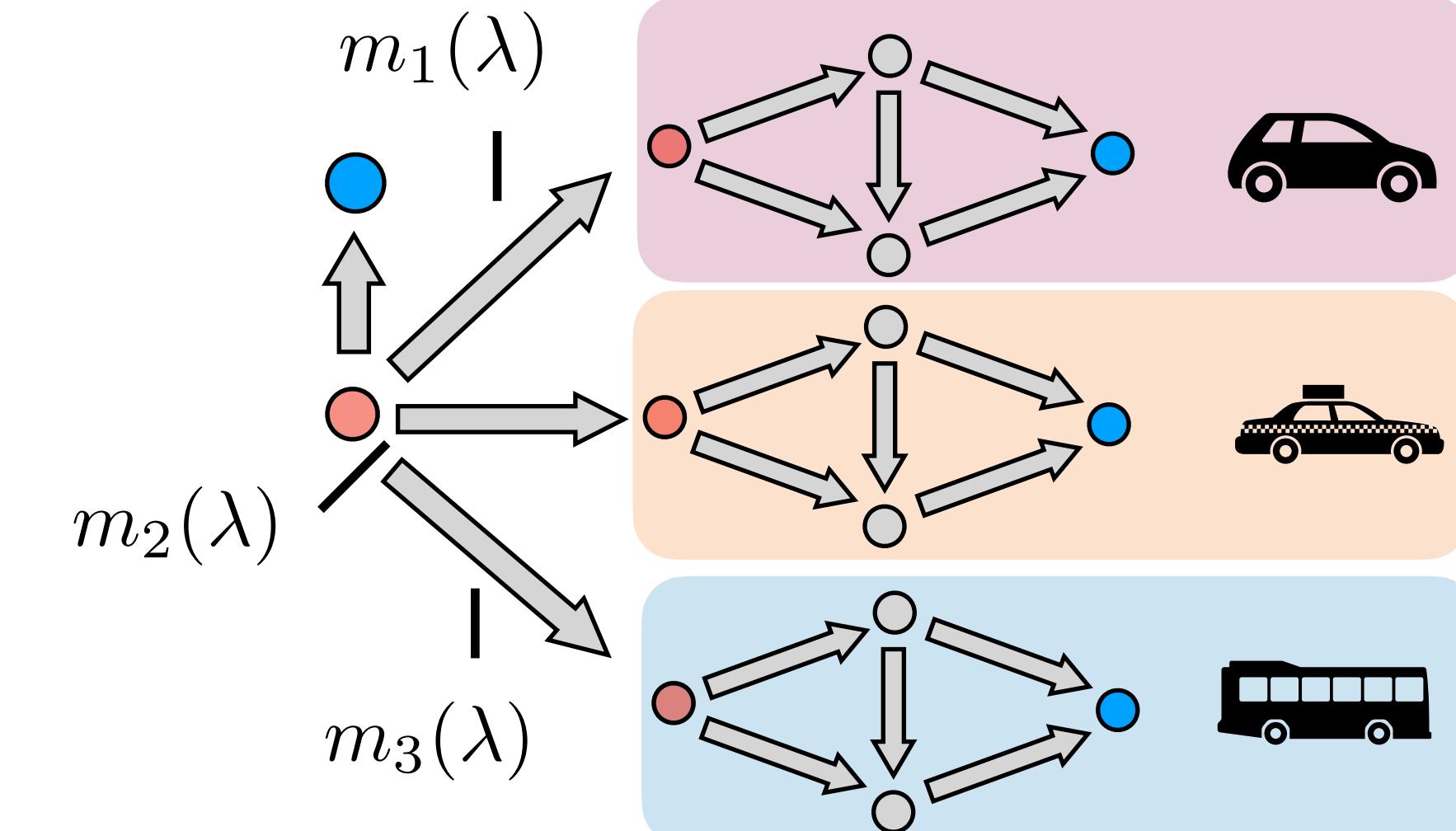
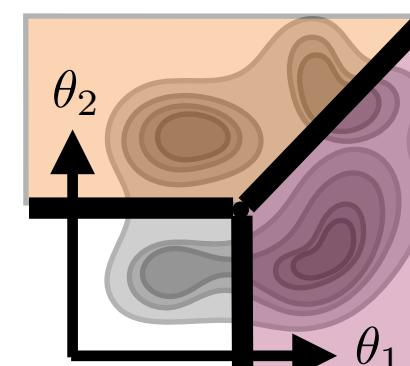


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences



Multi-Variate Preferences



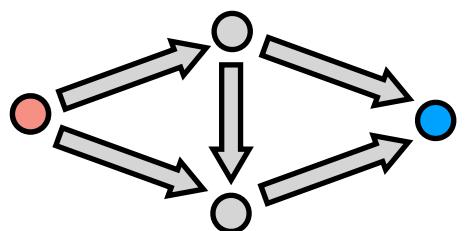
$$m_1(\lambda) = \int_{V_1(\lambda)} dm(\theta)$$

$$m_2(\lambda) = \int_{V_2(\lambda)} dm(\theta)$$

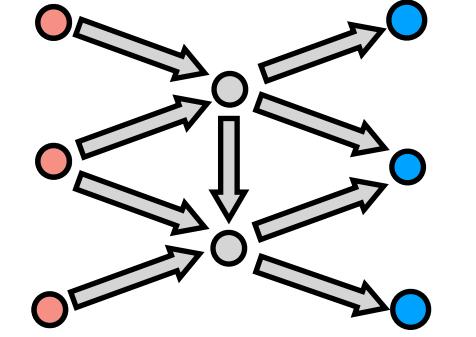
$$m_3(\lambda) = \int_{V_3(\lambda)} dm(\theta)$$

Potential Games

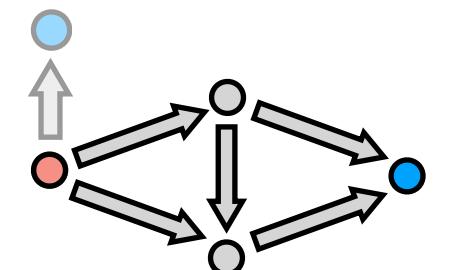
Routing Games



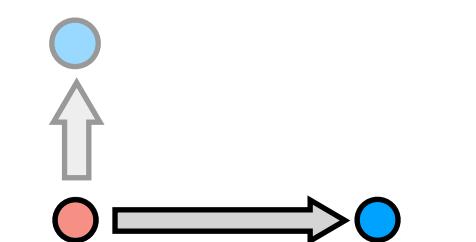
Multiple sources/sinks



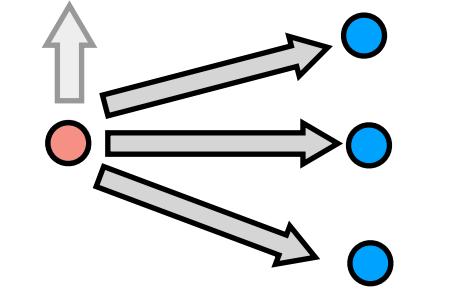
Variable Demand



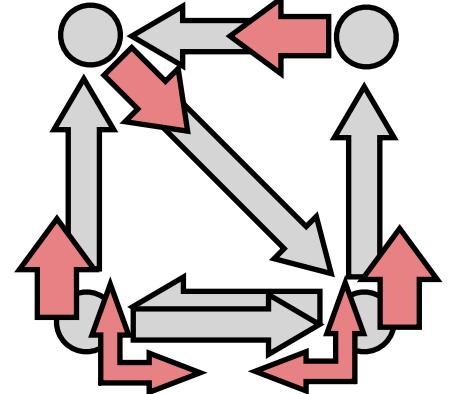
Supply & Demand



Cournot Market

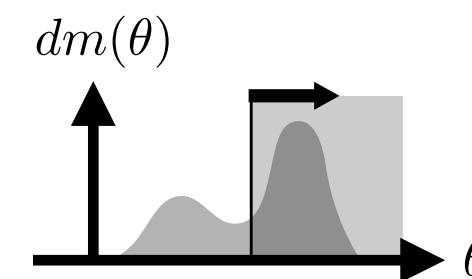


MDP Congestion Game

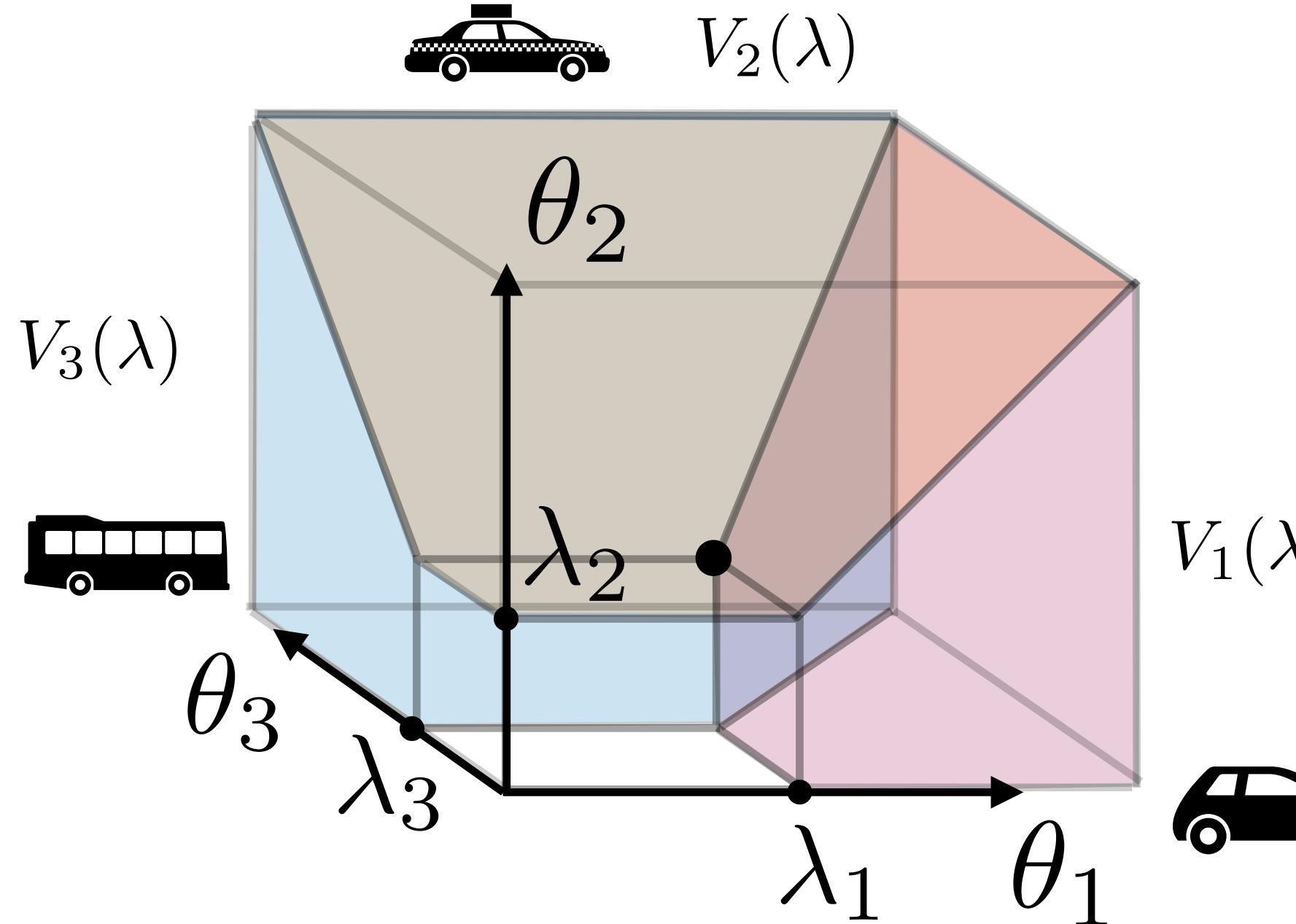
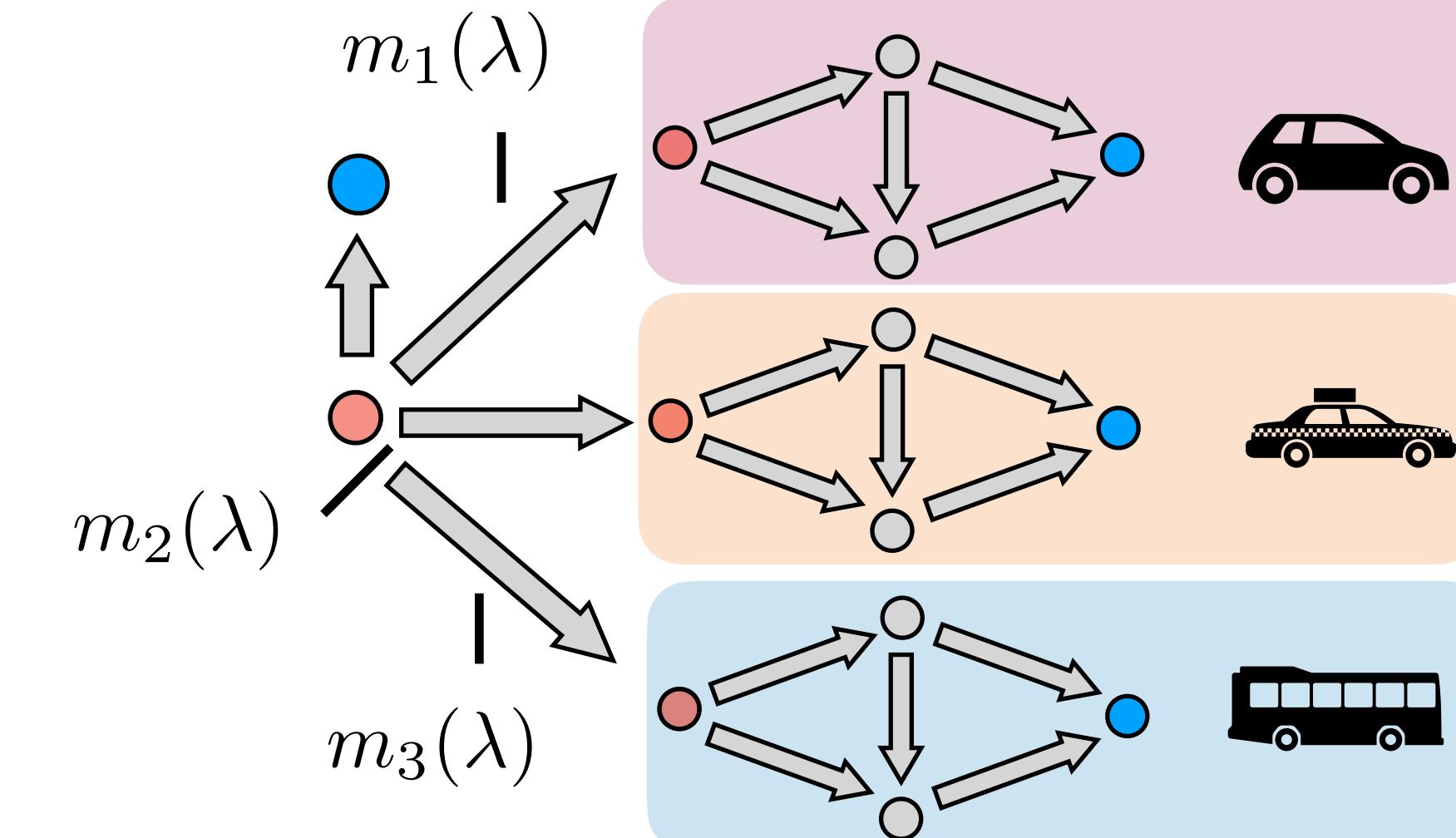
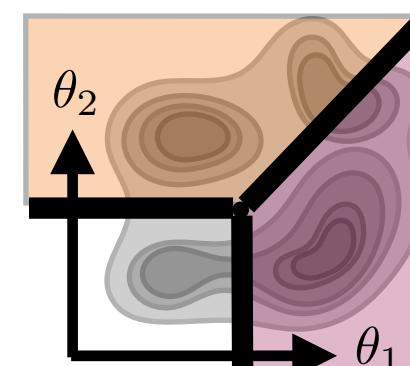


Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous preferences



Multi-Variate Preferences



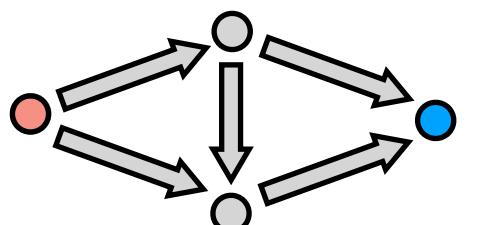
$$m_1(\lambda) = \int_{V_1(\lambda)} dm(\theta)$$

$$m_2(\lambda) = \int_{V_2(\lambda)} dm(\theta)$$

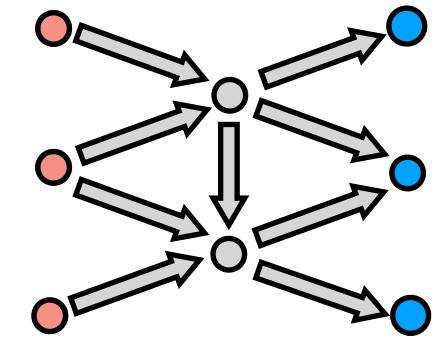
$$m_3(\lambda) = \int_{V_3(\lambda)} dm(\theta)$$

Potential Games

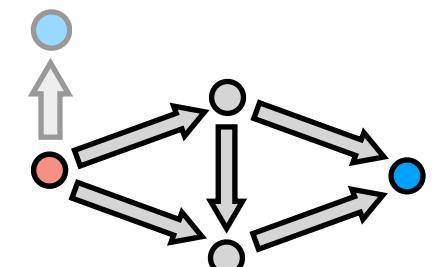
Routing Games



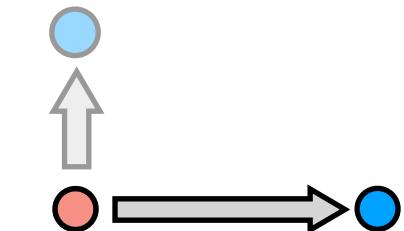
Multiple sources/sinks



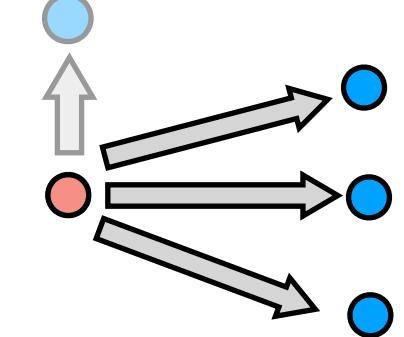
Variable Demand



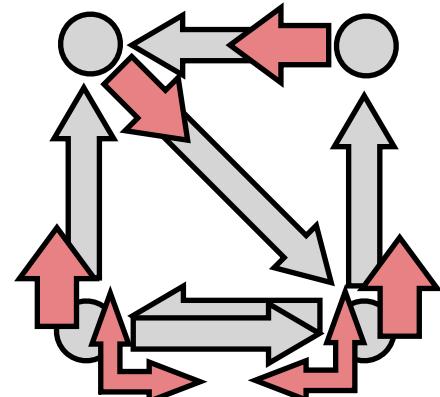
Supply & Demand



Cournot Market

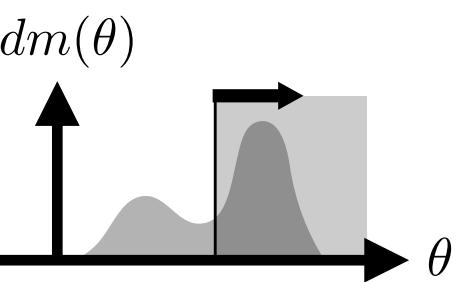


MDP Congestion Game

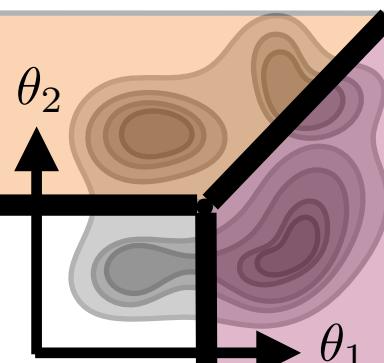


Variable Demand - Multi-Variate Non-Homogeneous Preferences

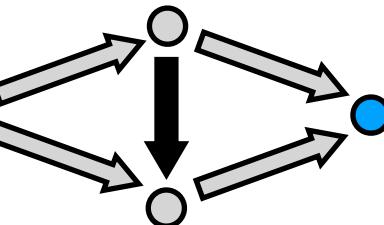
Non-homo-geneous preferences



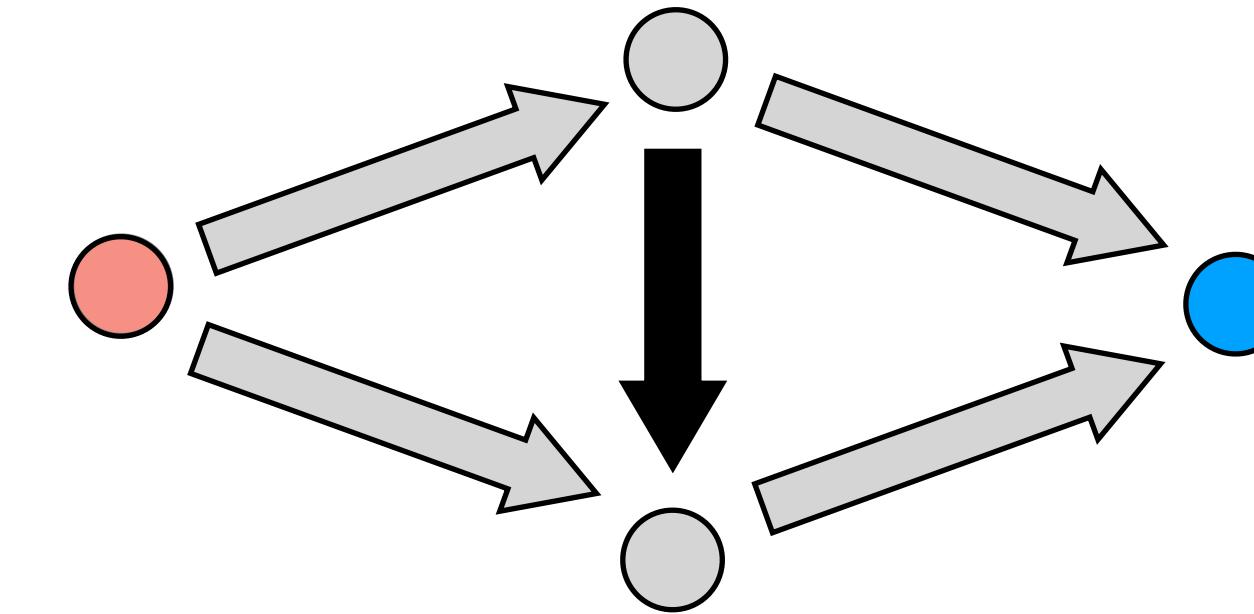
Multi-Variate Preferences



Braess Paradox

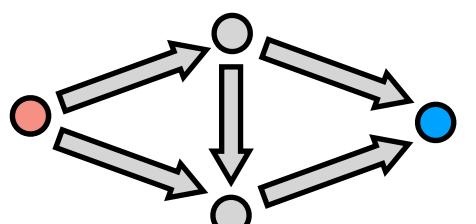


Braess Paradox

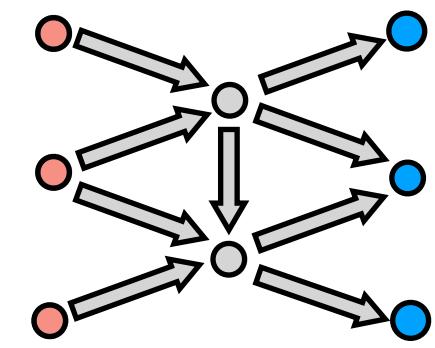


Potential Games

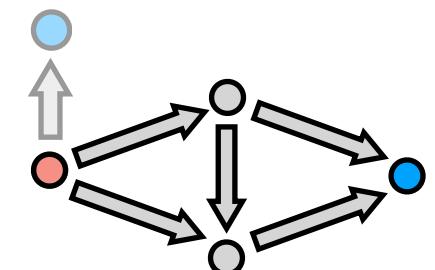
Routing Games



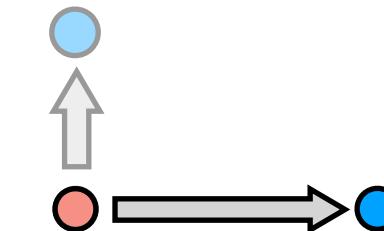
Multiple sources/sinks



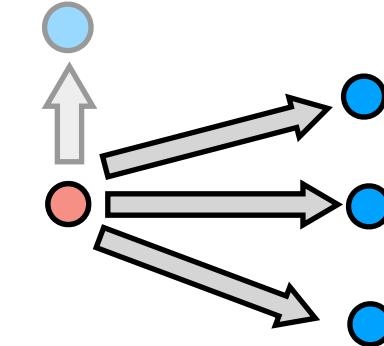
Variable Demand



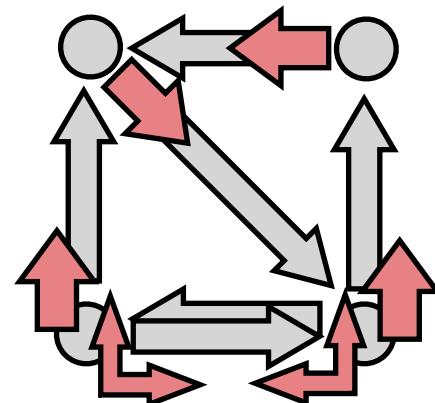
Supply & Demand



Cournot Market

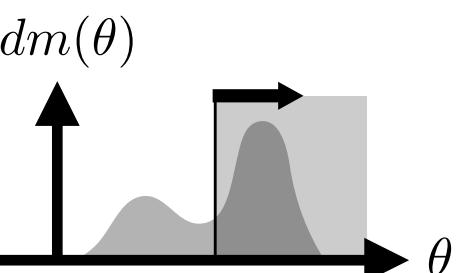


MDP Congestion Game

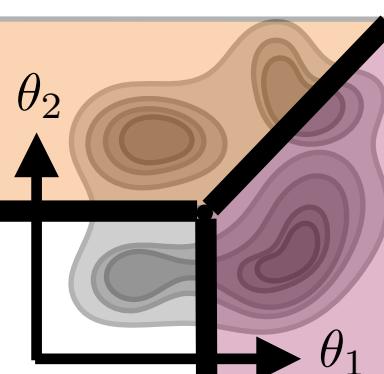


Variable Demand - Multi-Variate Non-Homogeneous Preferences

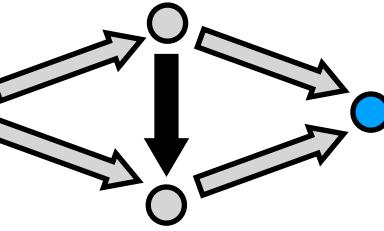
Non-homo-geneous preferences



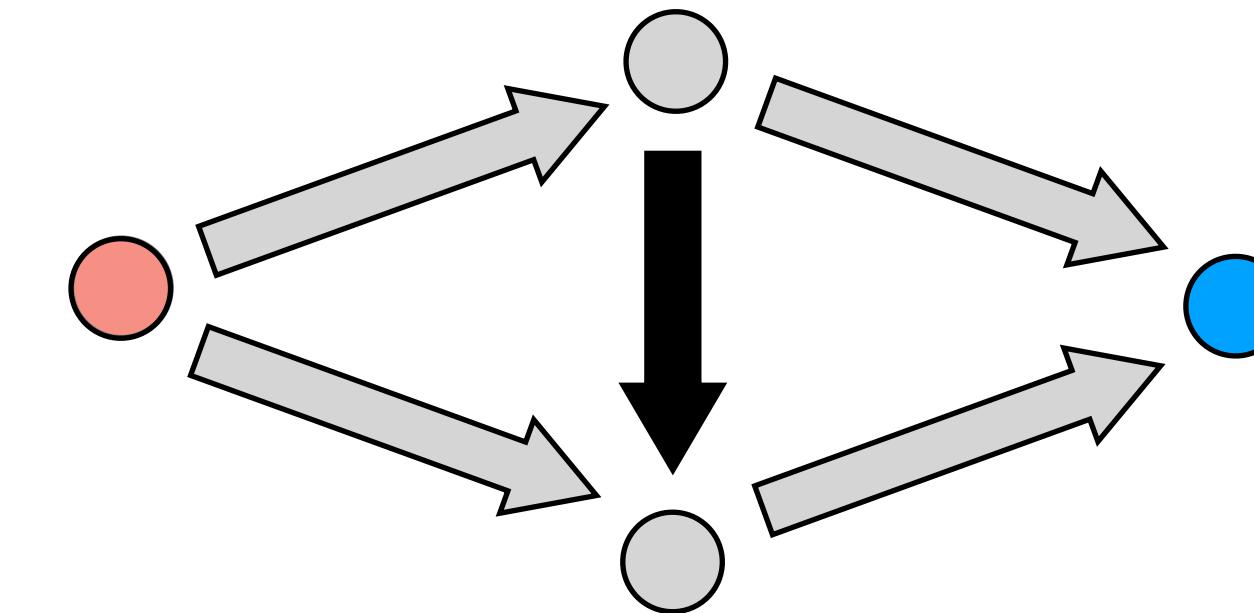
Multi-Variate Preferences



Braess Paradox



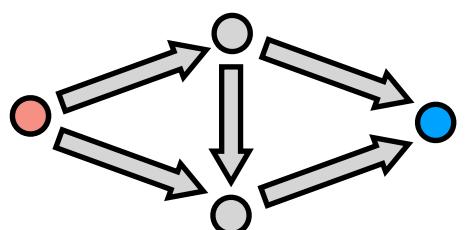
Braess Paradox



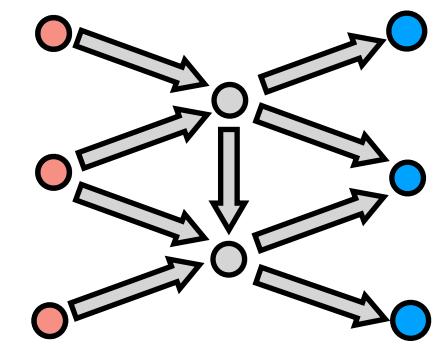
Adding center road can make traffic worse!

Potential Games

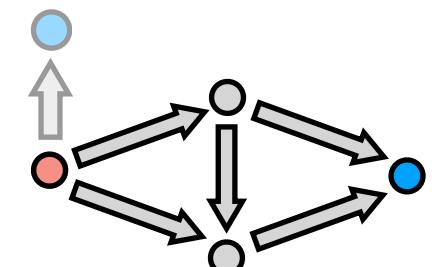
Routing Games



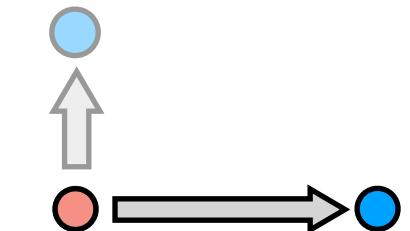
Multiple sources/sinks



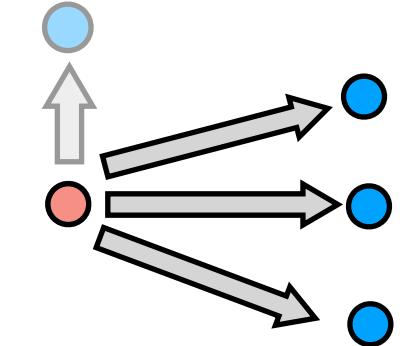
Variable Demand



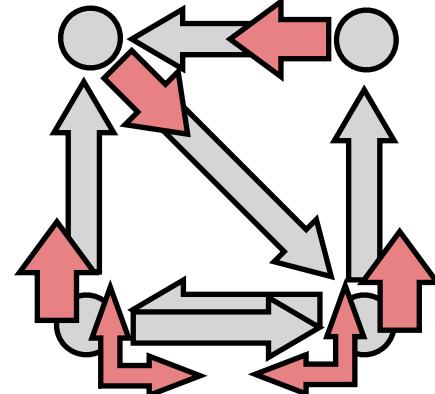
Supply & Demand



Cournot Market

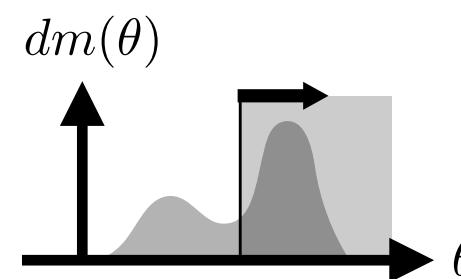


MDP Congestion Game

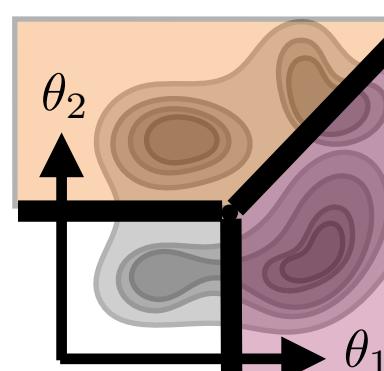


Variable Demand - Multi-Variate Non-Homogeneous Preferences

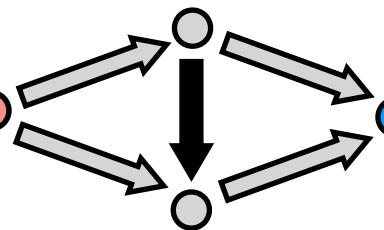
Non-homo-geneous preferences



Multi-Variate Preferences



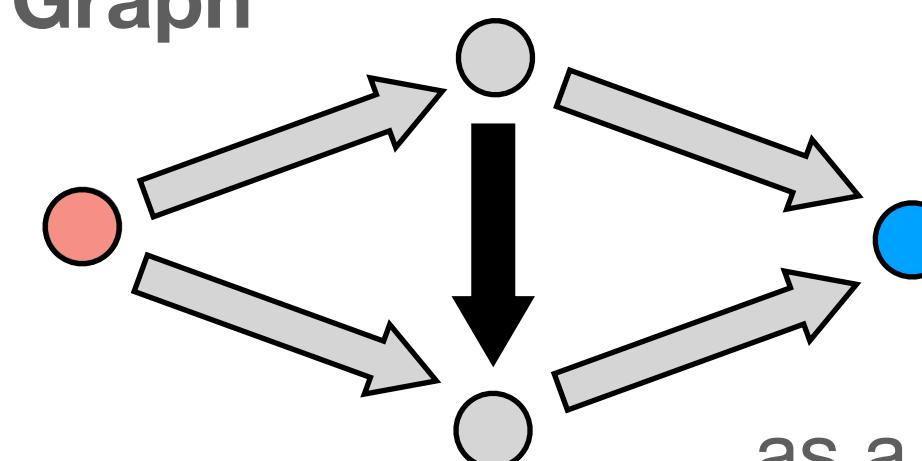
Braess Paradox



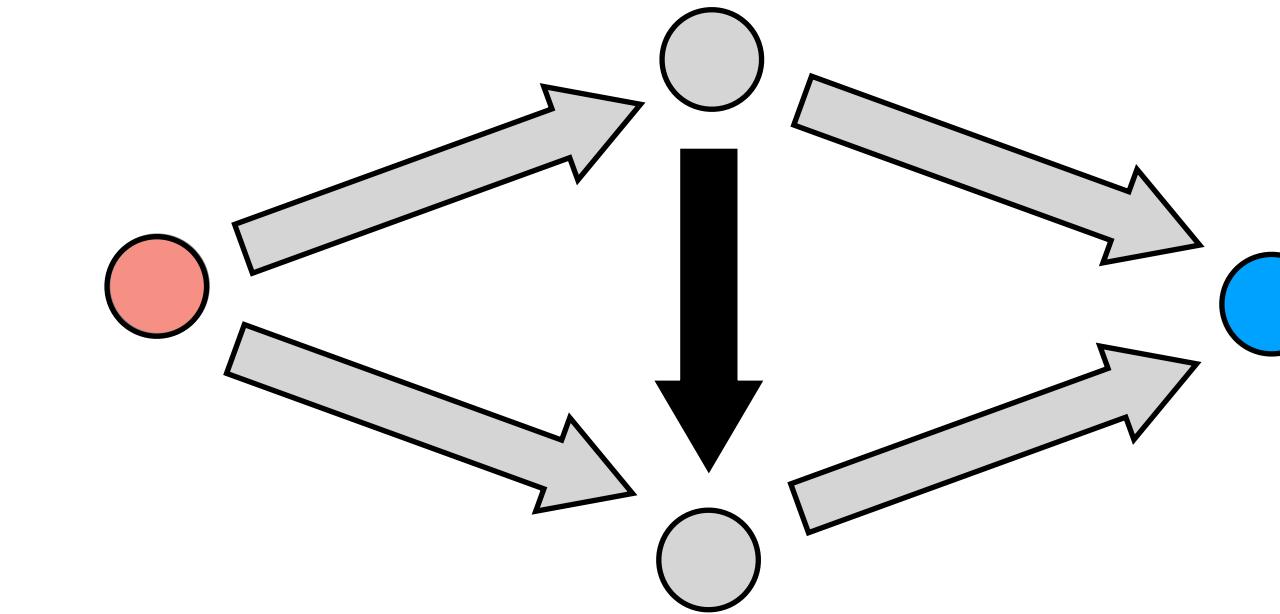
Network Characterization:

Every network has

Braess Graph



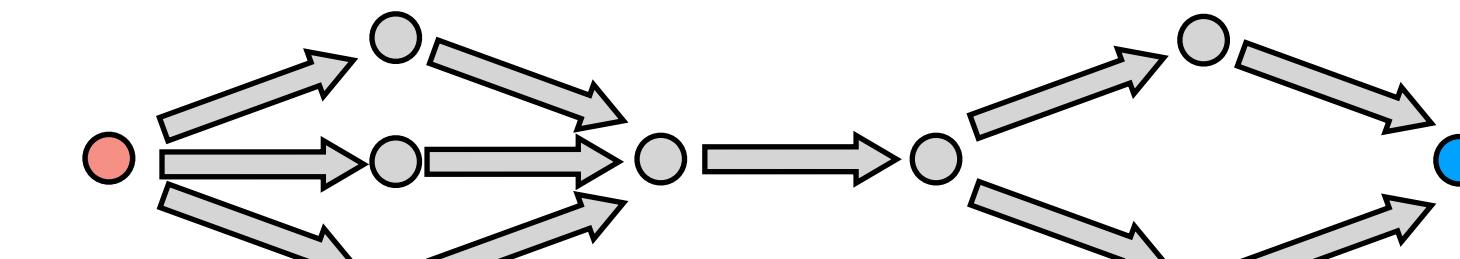
Braess Paradox



Adding center road can make traffic worse!

OR is

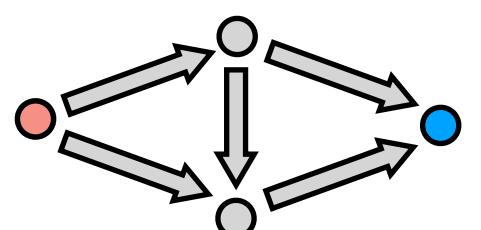
Series-Parallel Graph



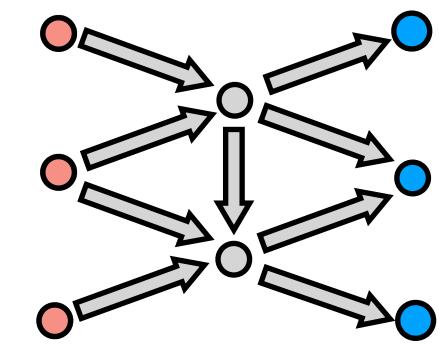
as a subgraph

Potential Games

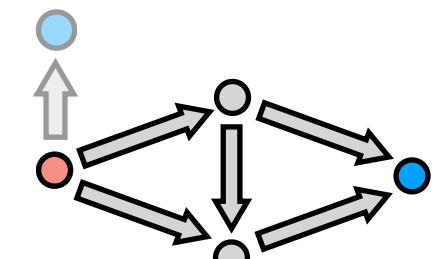
Routing Games



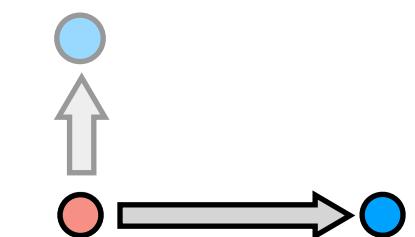
Multiple sources/sinks



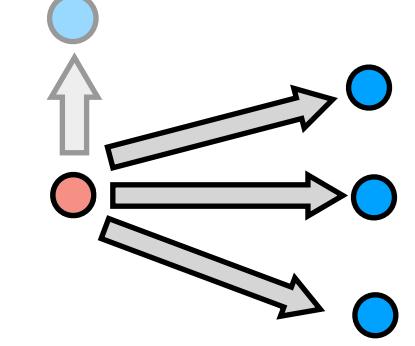
Variable Demand



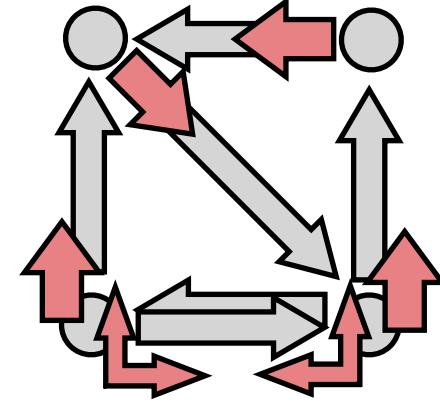
Supply & Demand



Cournot Market

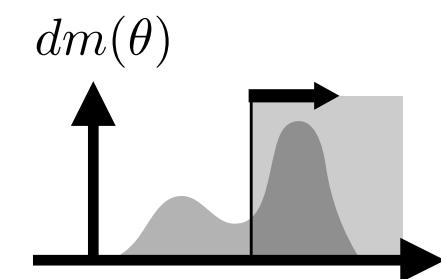


MDP Congestion Game

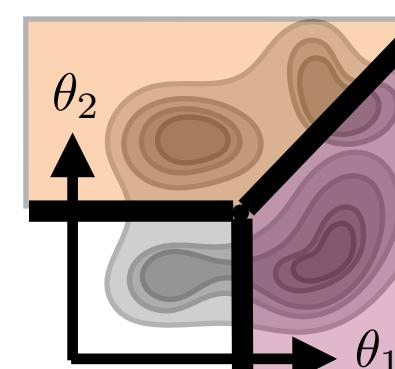


Variable Demand - Multi-Variate Non-Homogeneous Preferences

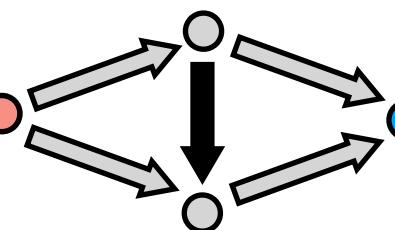
Non-homo-geneous preferences



Multi-Variate Preferences



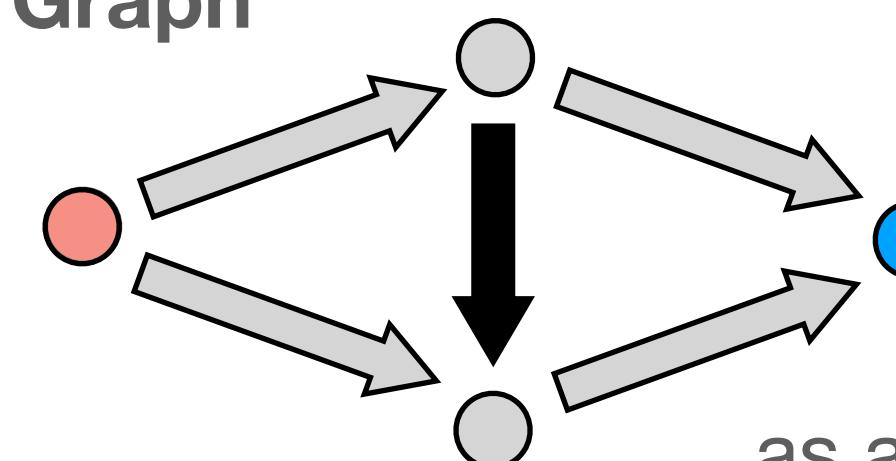
Braess Paradox



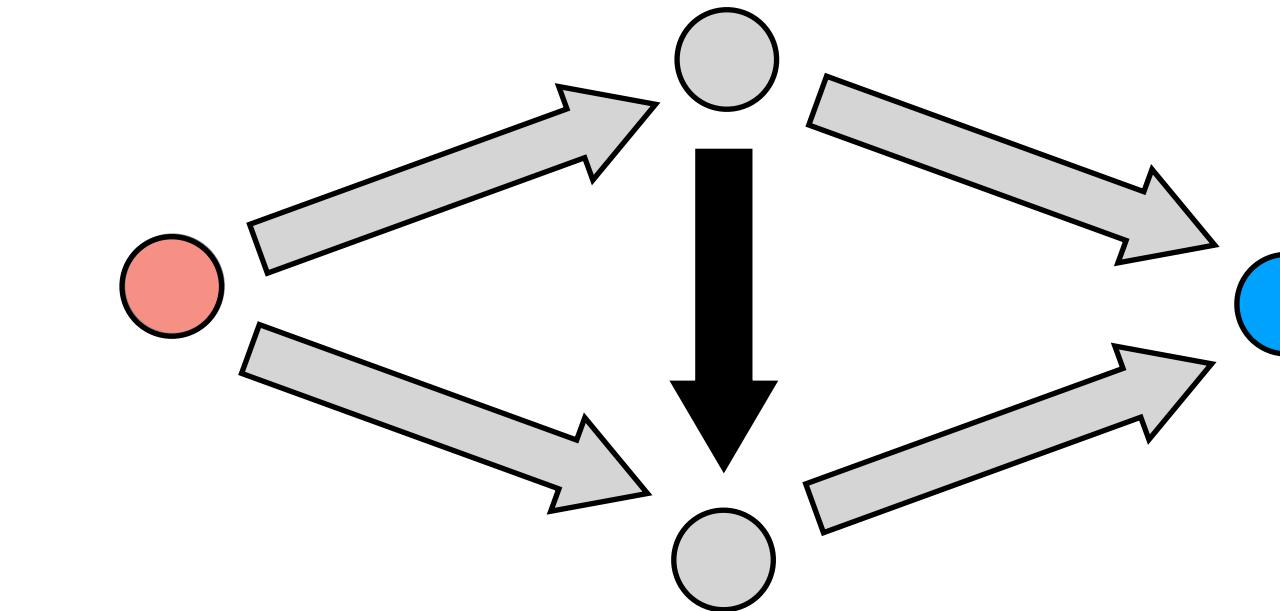
Network Characterization:

Every network has

Braess Graph



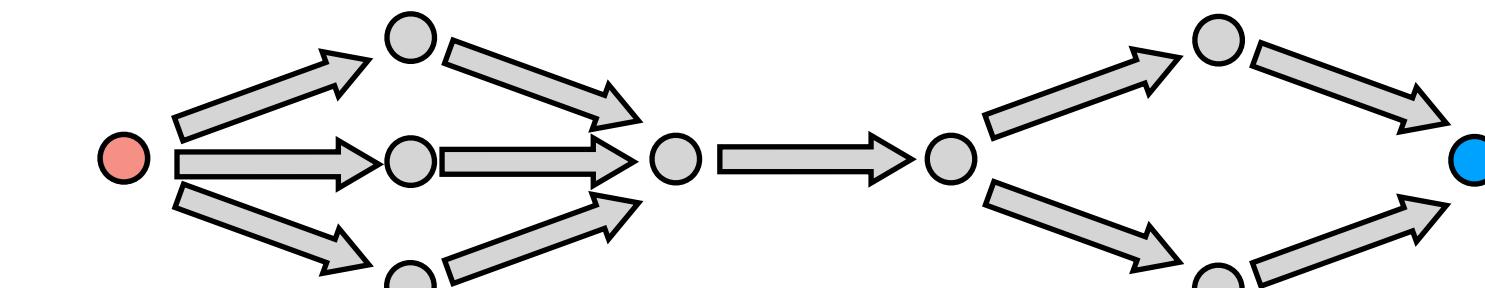
Braess Paradox



Adding center road can make traffic worse!

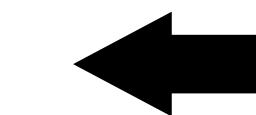
OR is

Series-Parallel Graph



as a subgraph

Series - Parallel graphs cannot suffer from Braess paradox



linear-algebraic characterization/proof