Introduction

Dan Calderone

djcal@uw.edu
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Post-doctoral scholar (Prof. Ratliff’s group)
University of Washington

PhD: Berkeley, (under Shankar Sastry, 2017)
PostDoc: in AA & EE at UW (Ratliff, Ackimese, 2018-2019)
Lecturer: AA & EE at UW (2019-2022)

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Game theory & optimization
applied to transportation networks

Personal Interests: Math visualization

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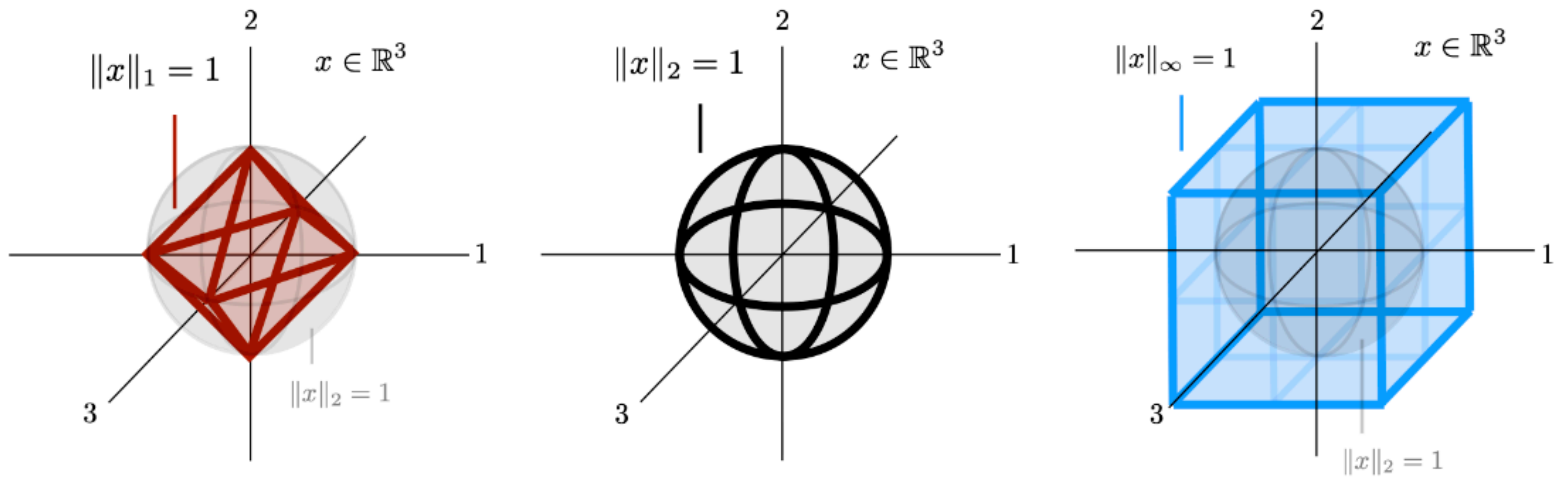

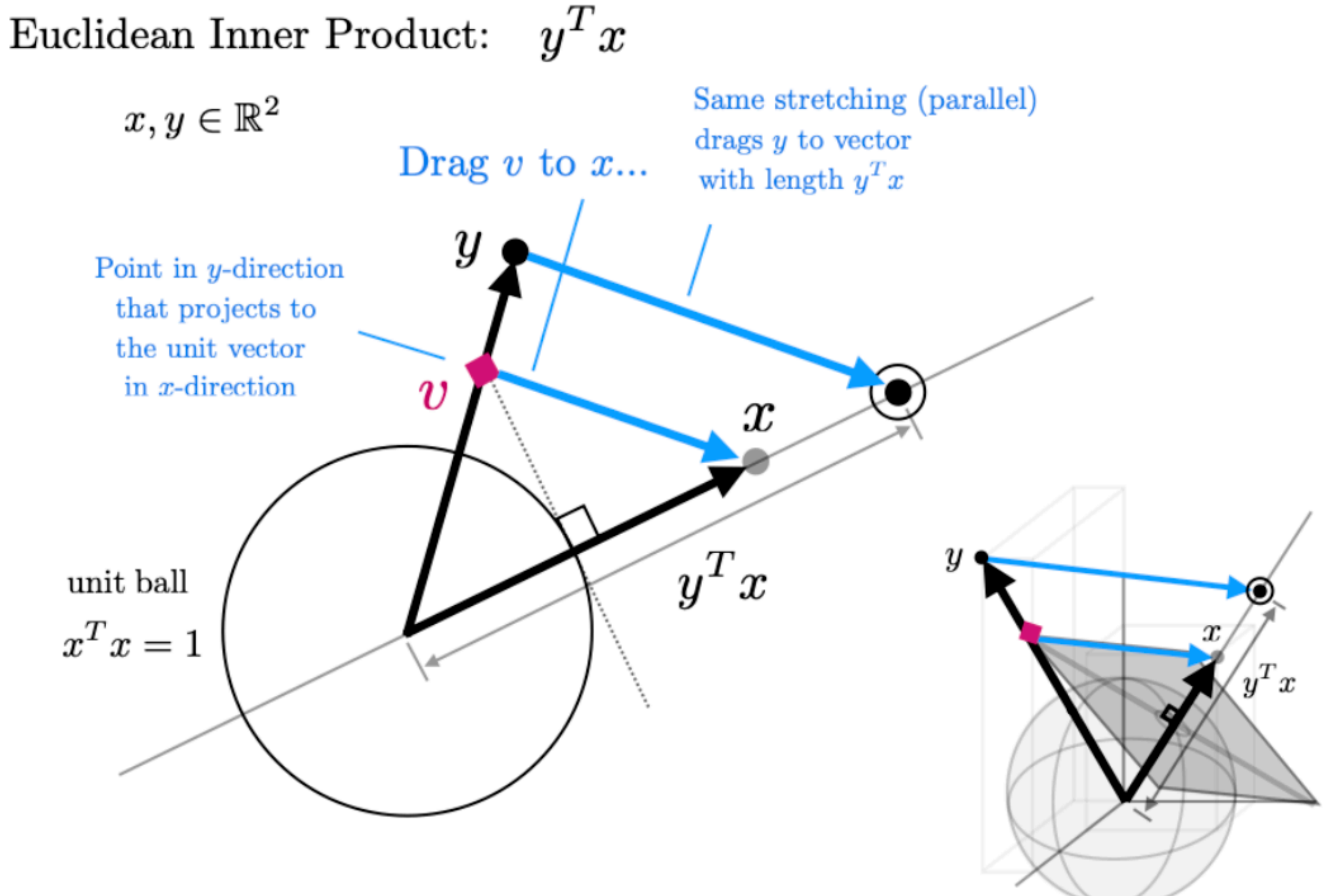

Note: for higher dimensions, the same picture works within the plane spanned by the two vectors.

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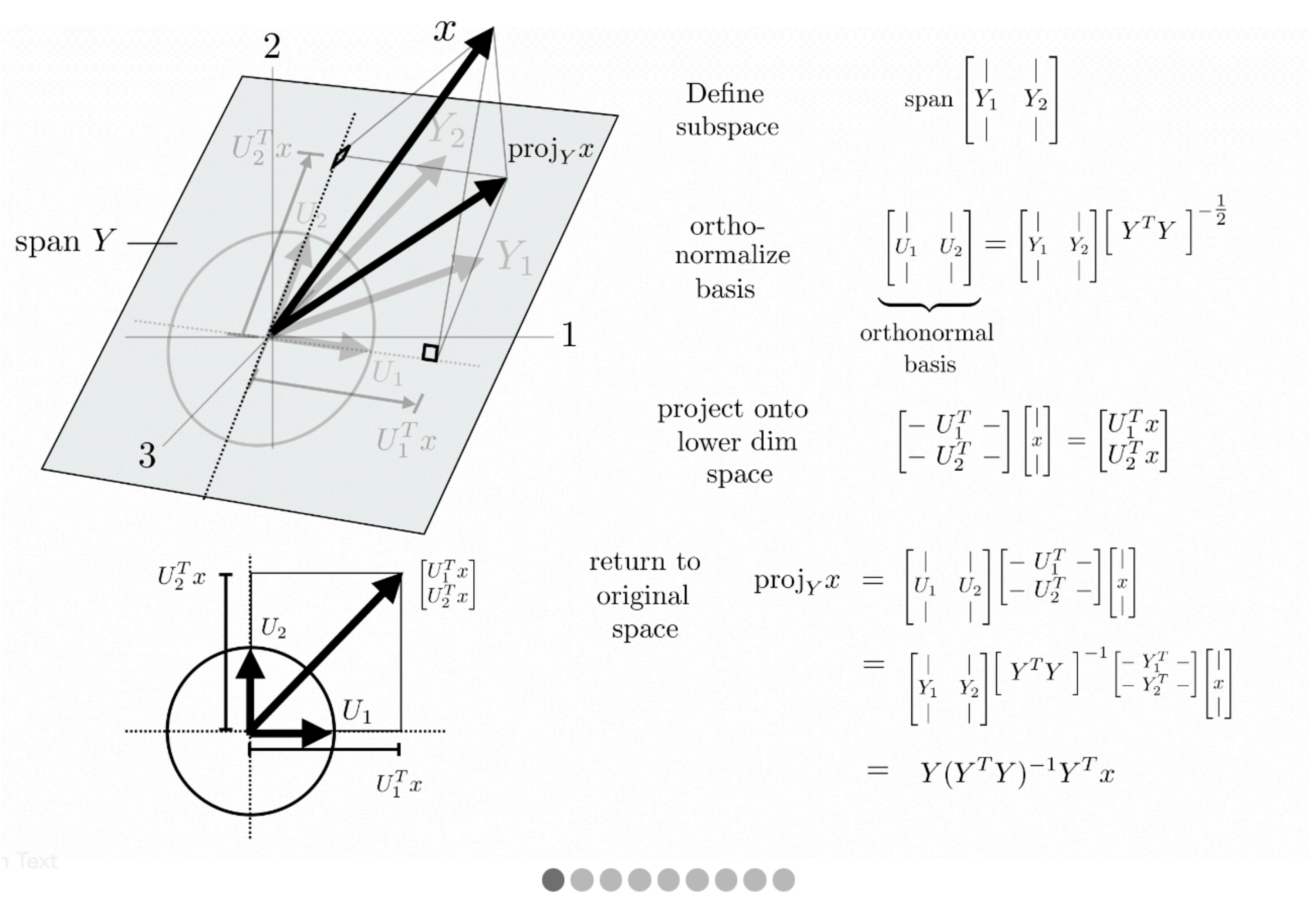

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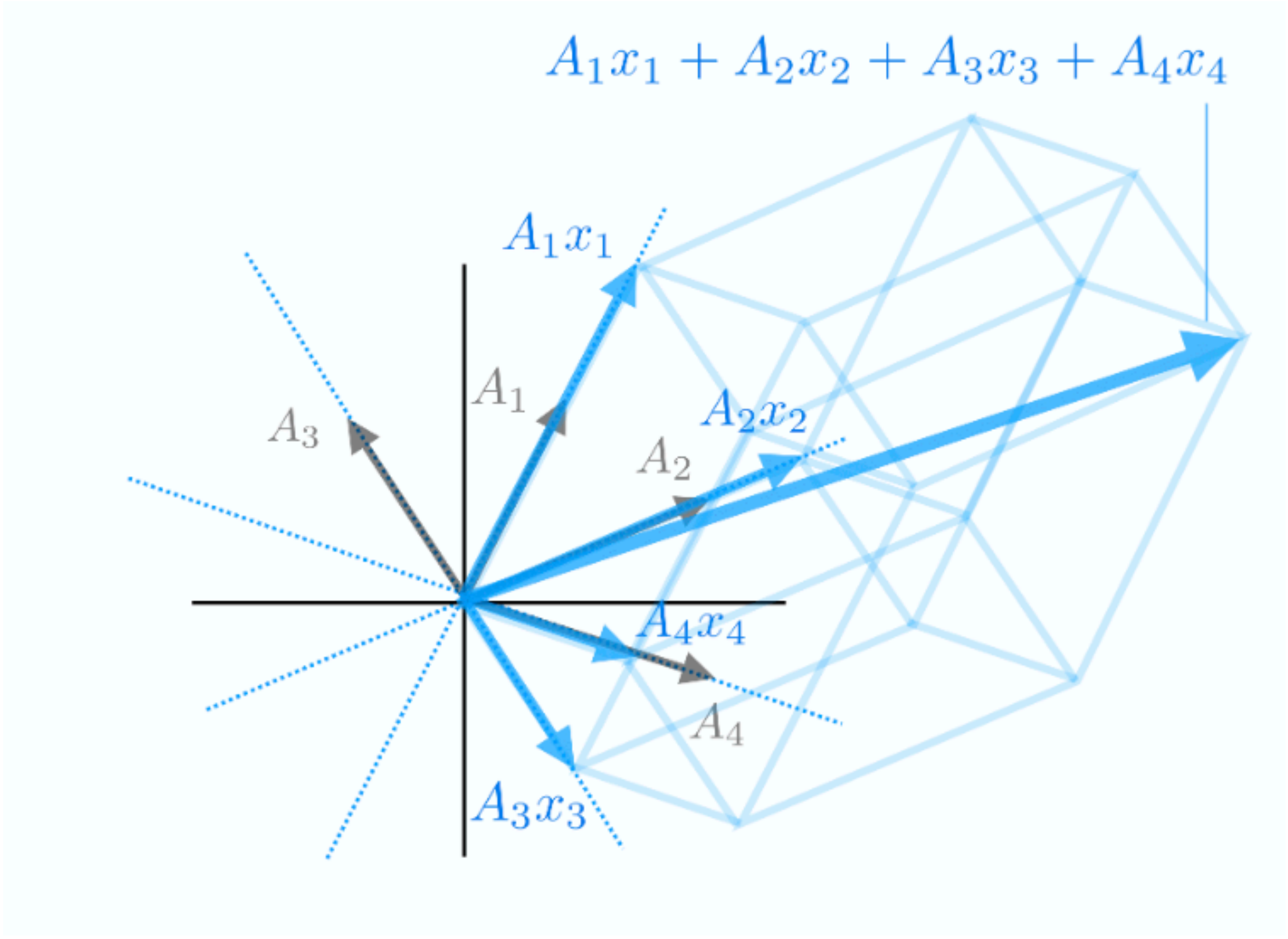

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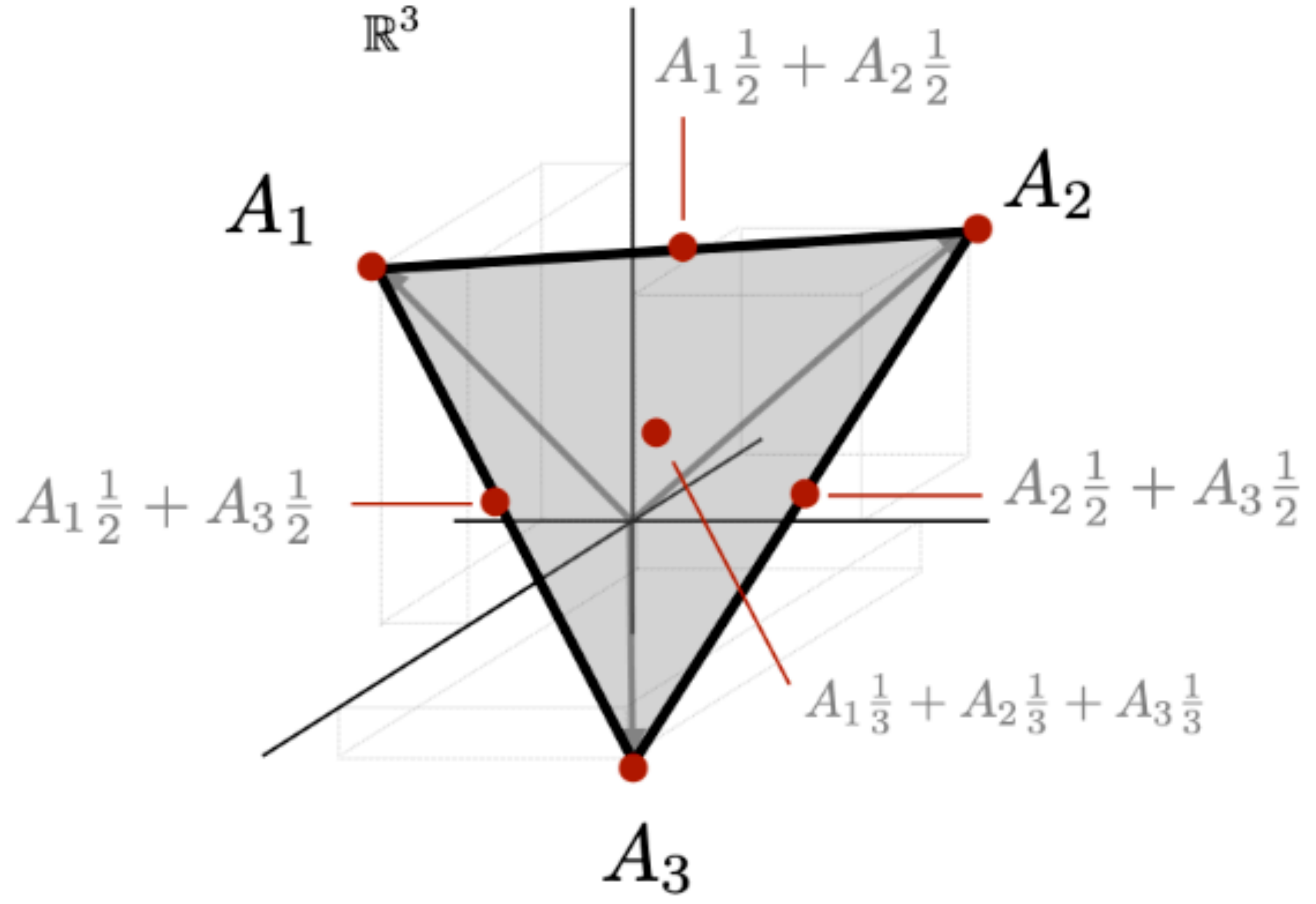

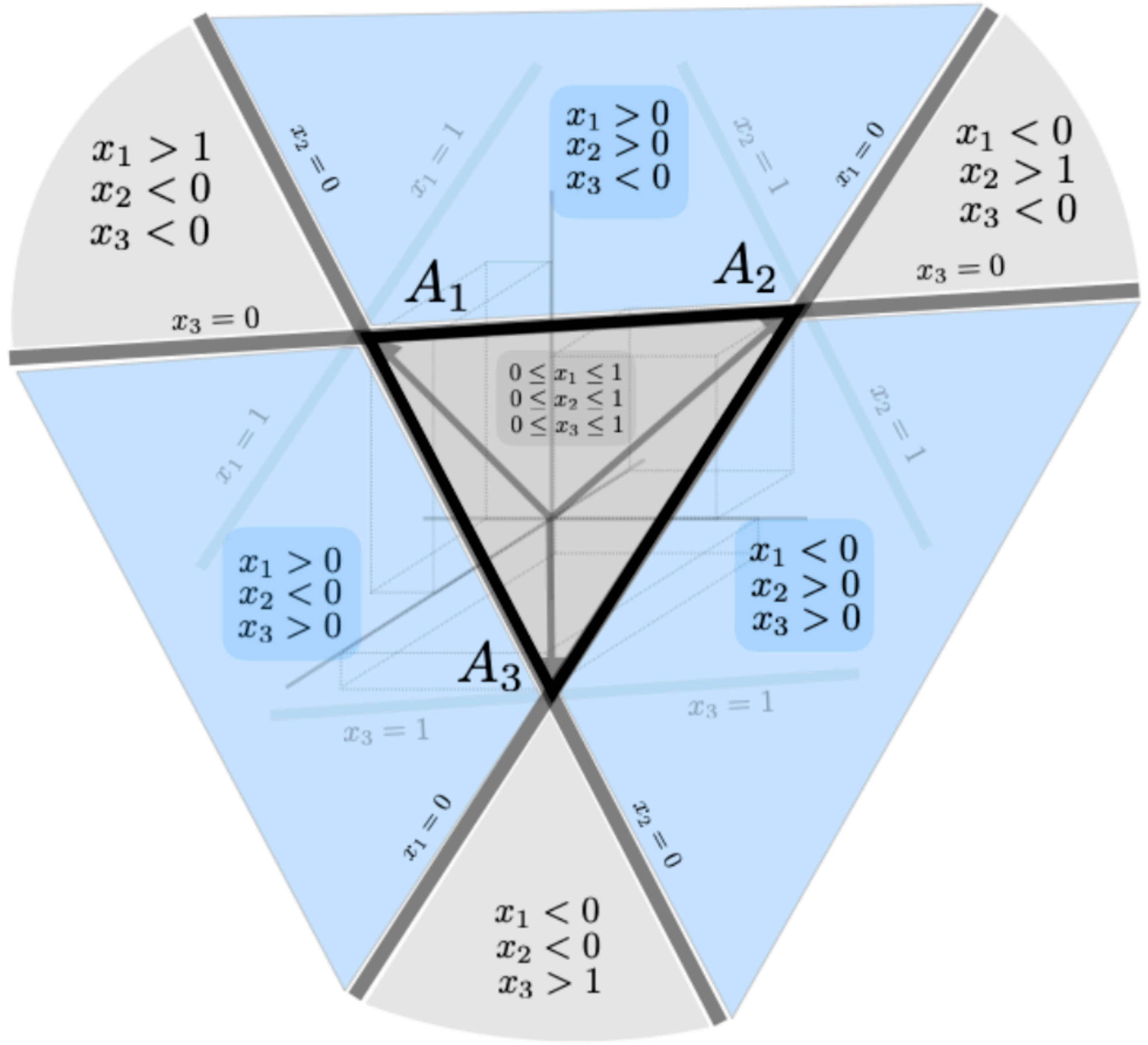

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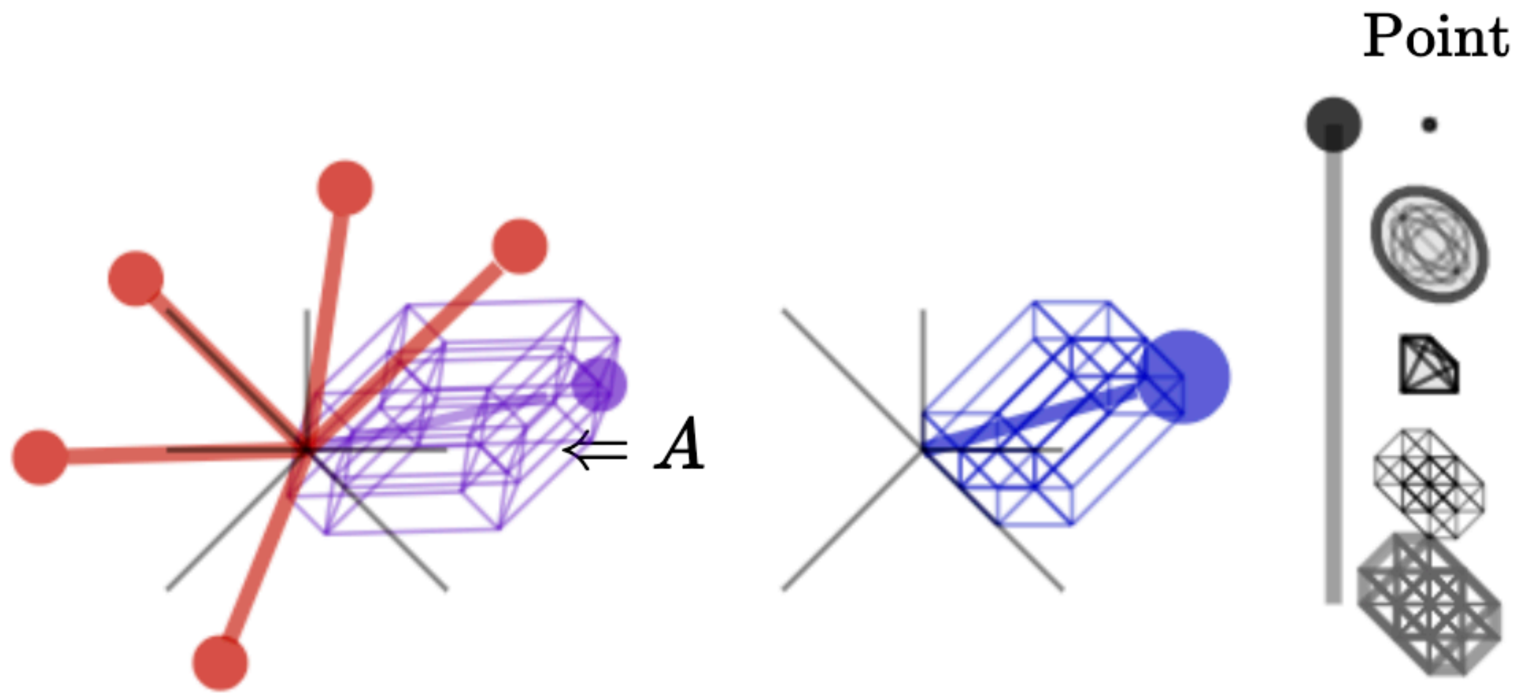

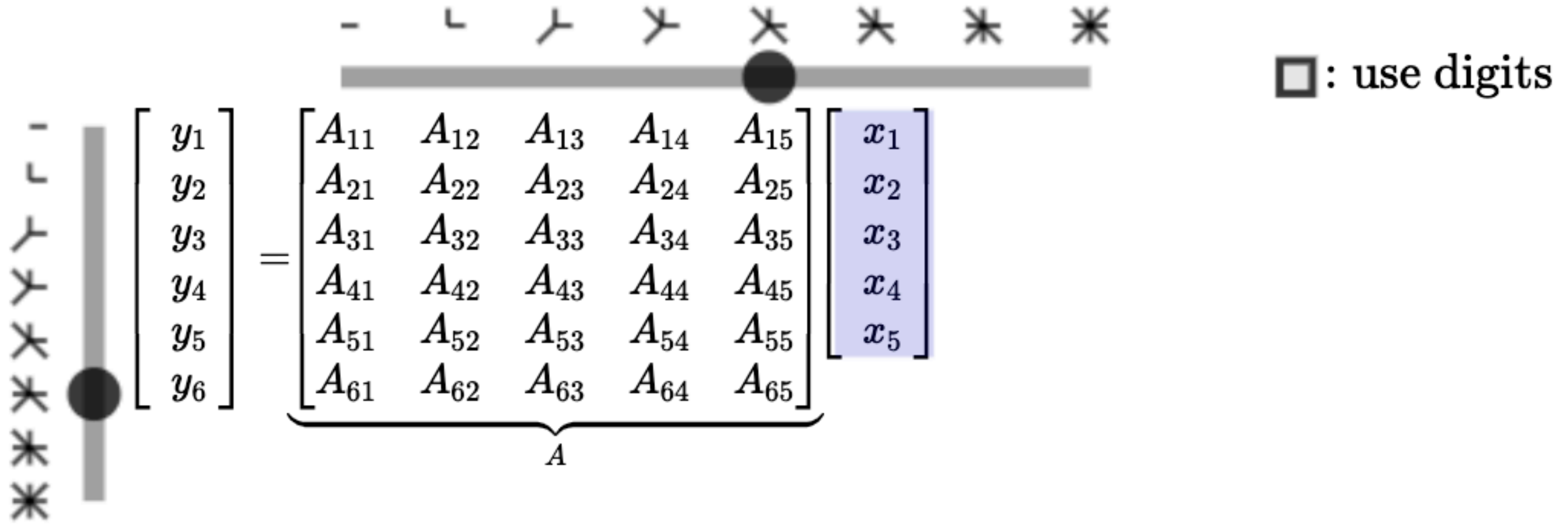

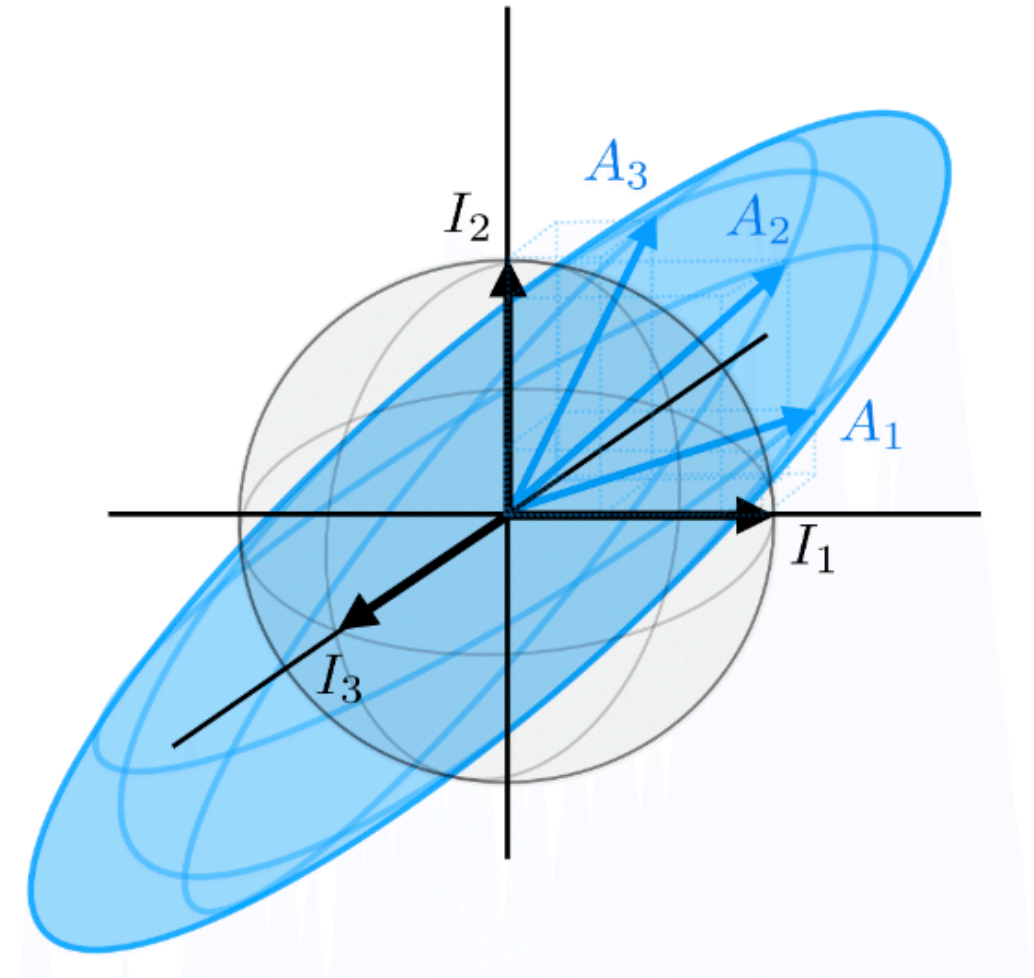

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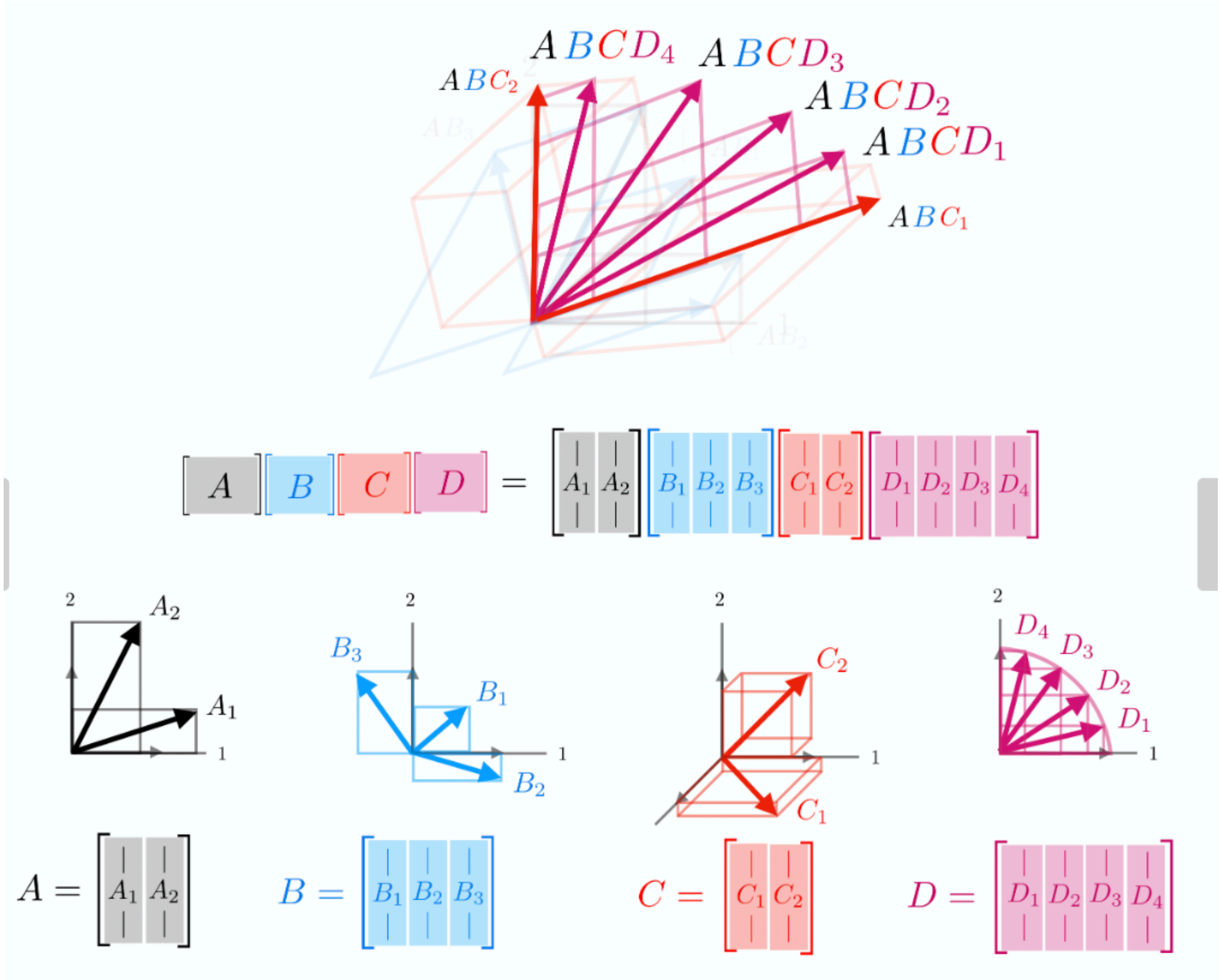

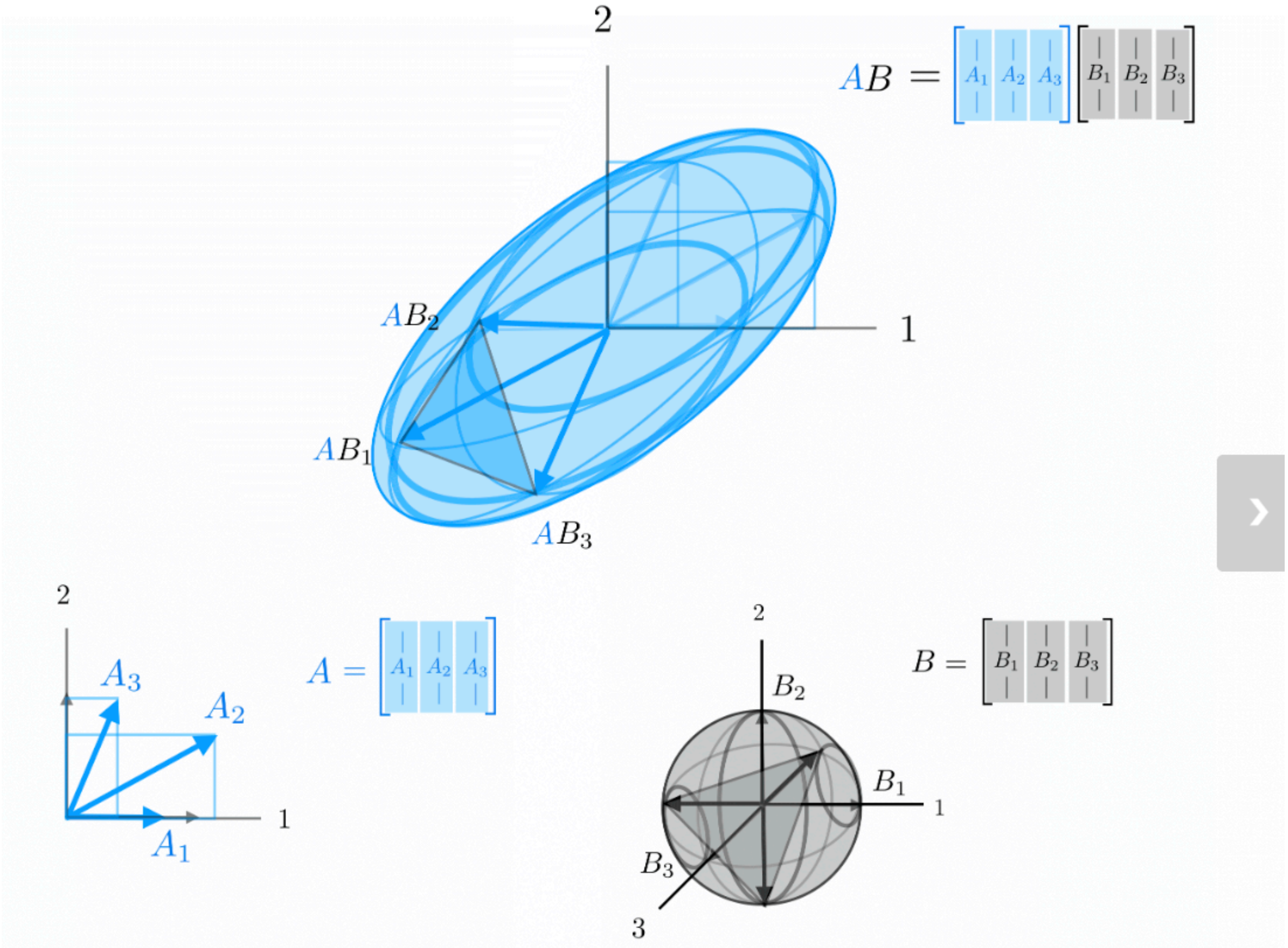

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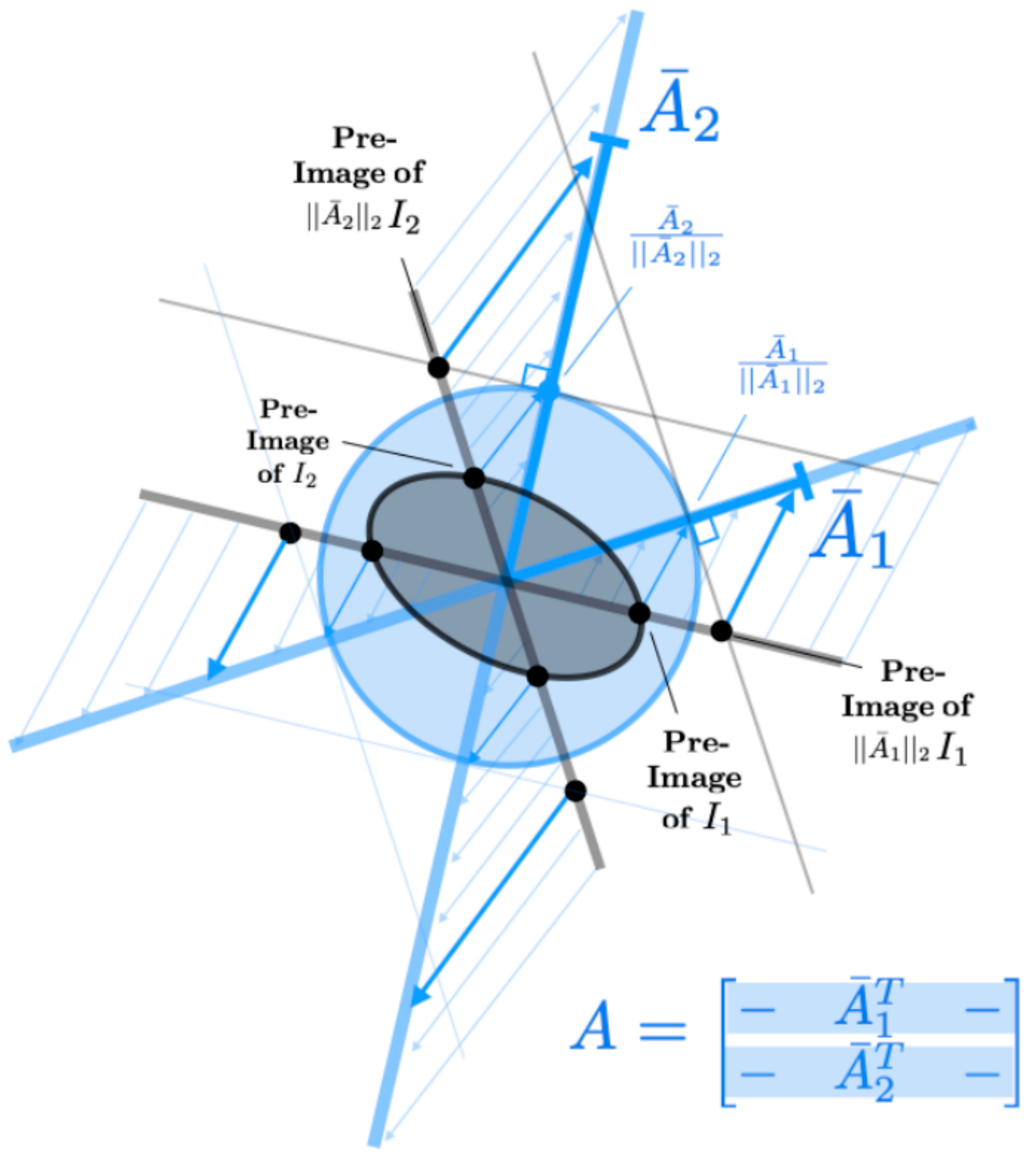

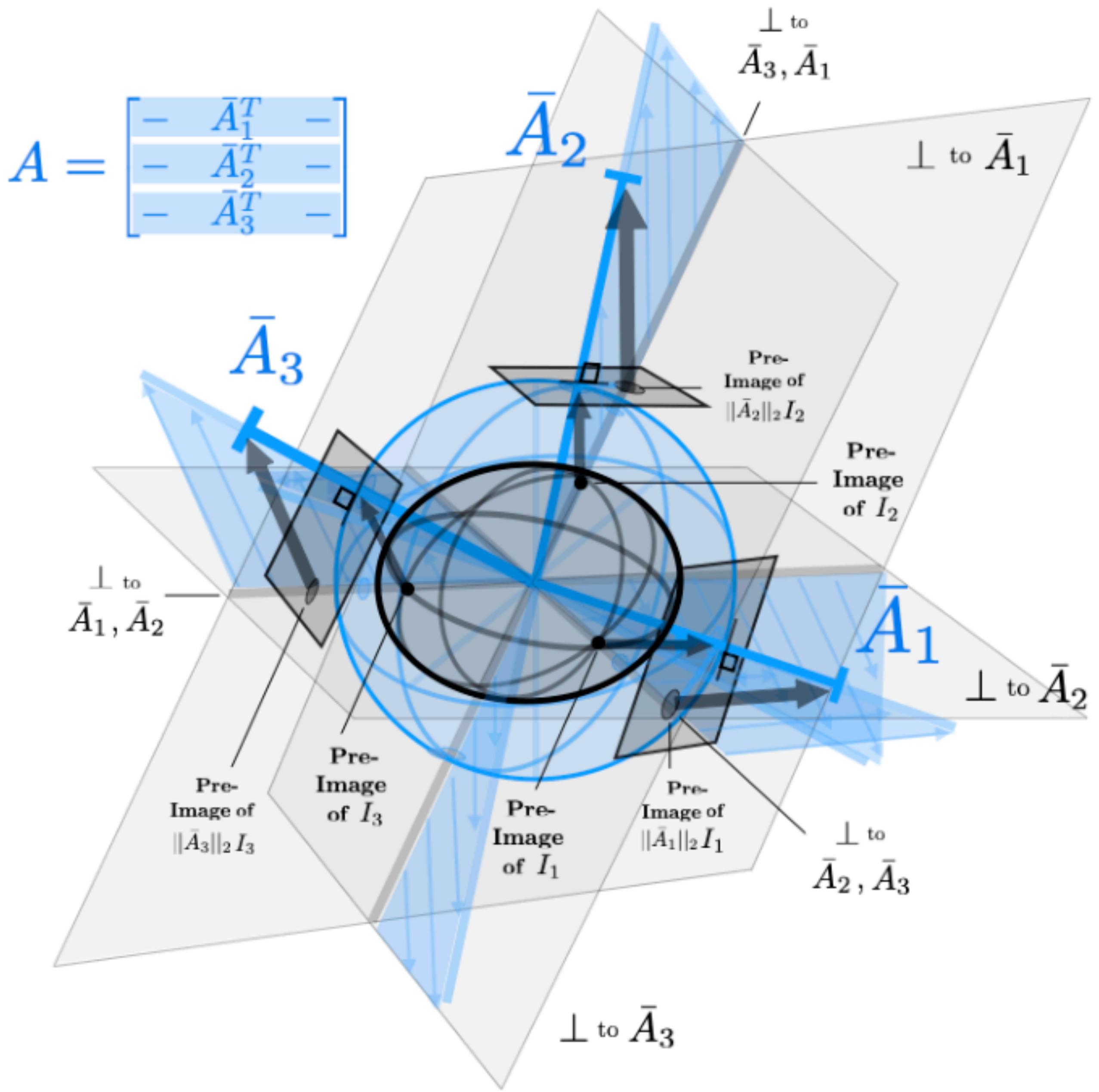

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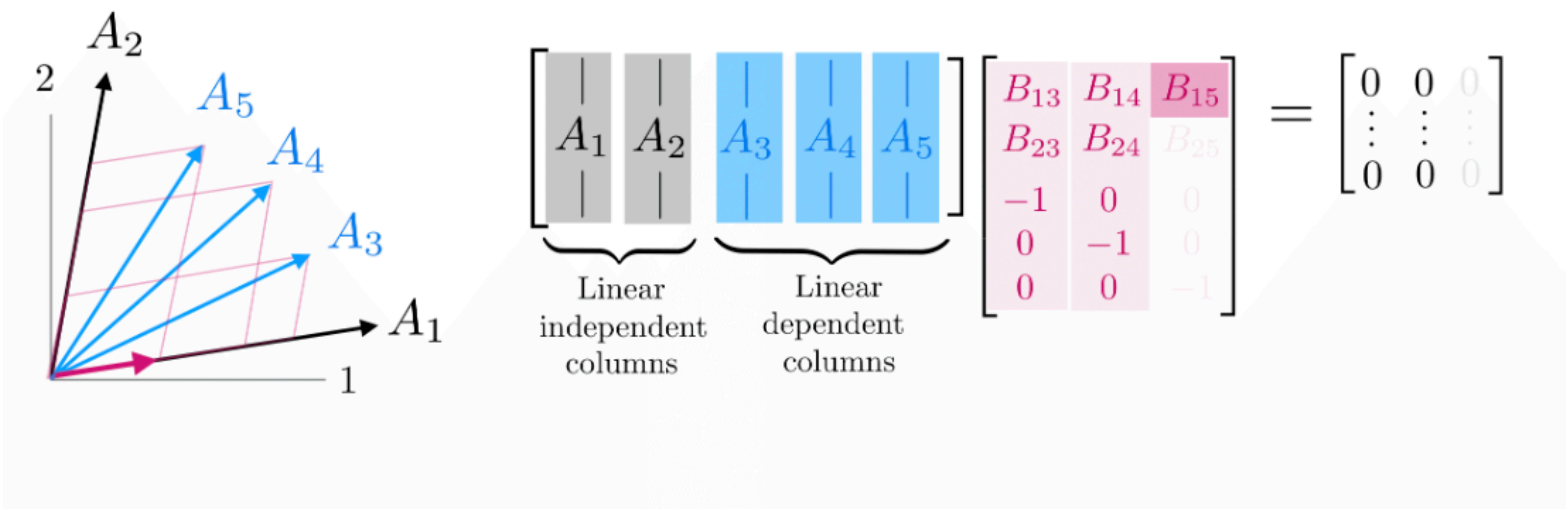

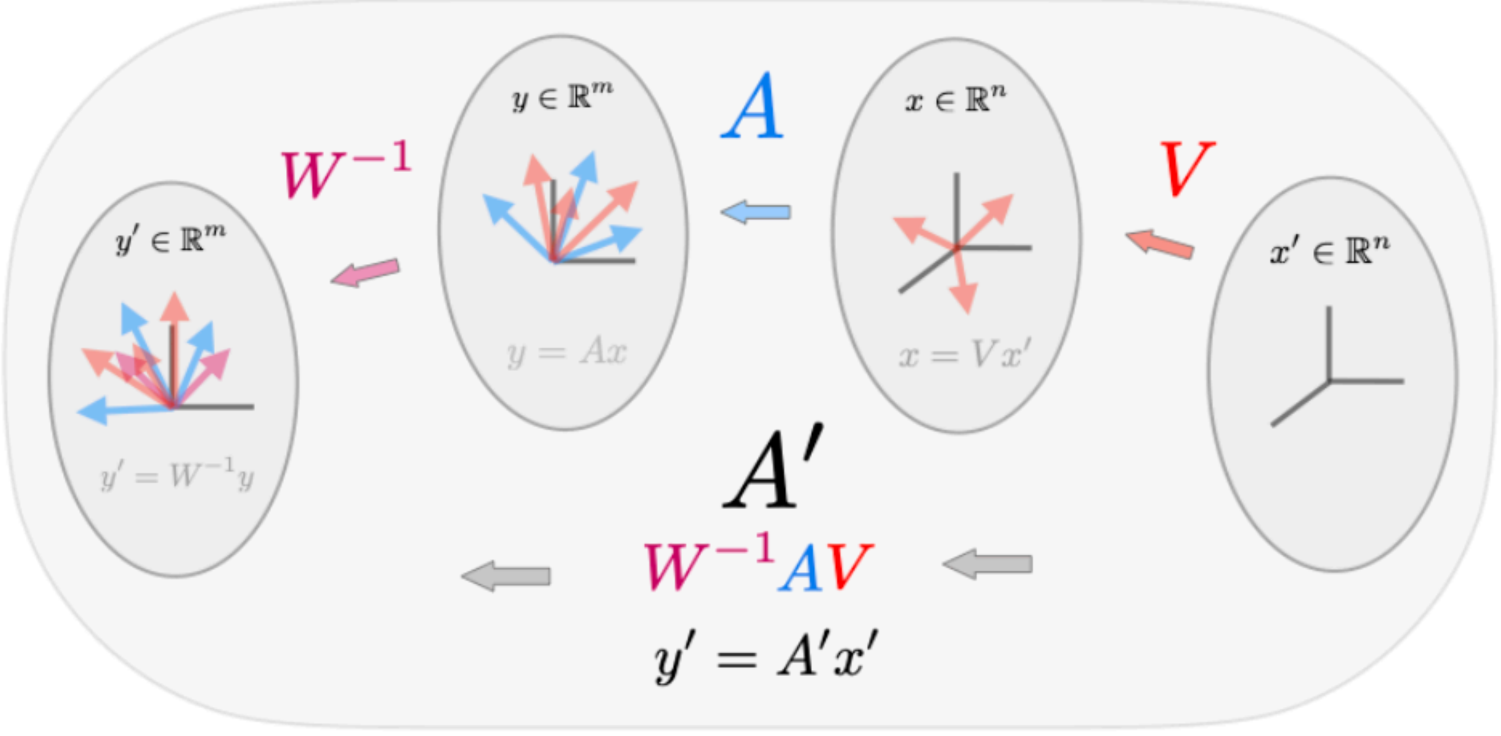

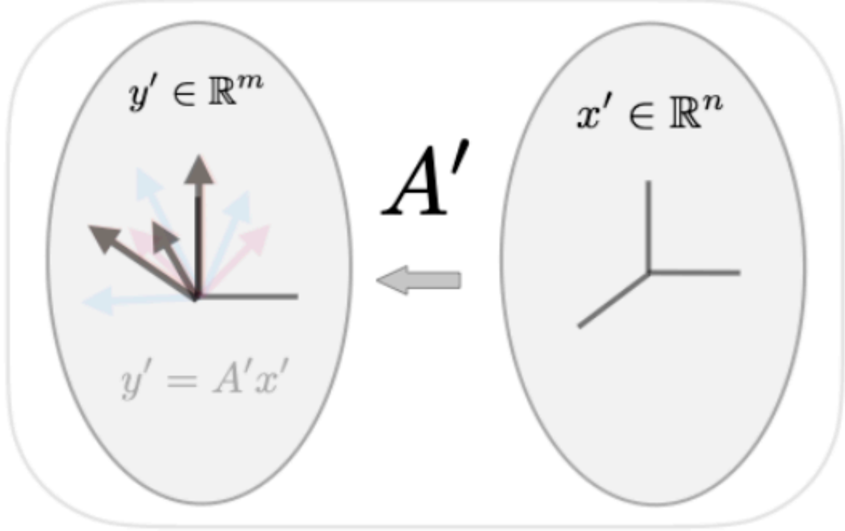

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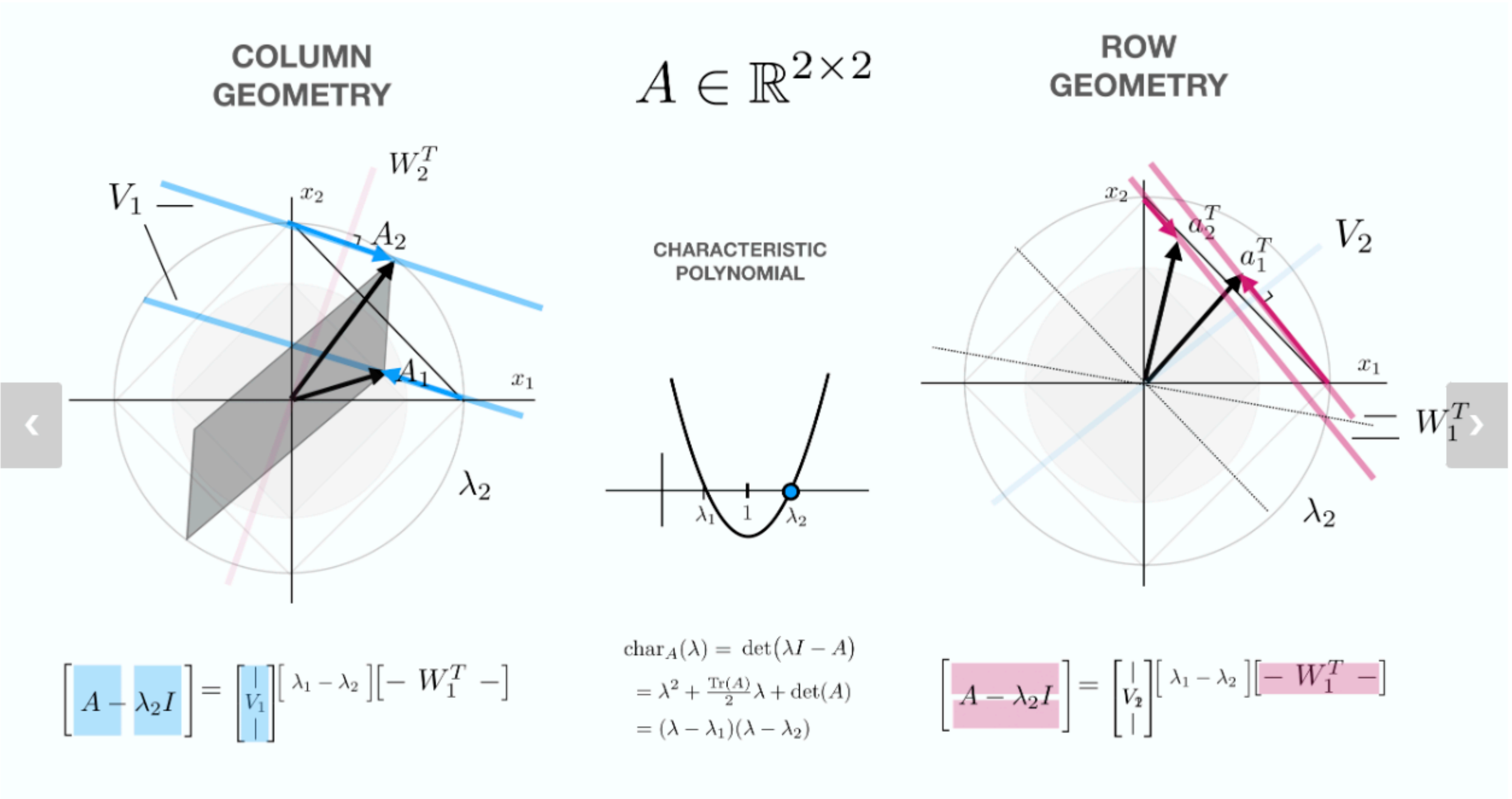

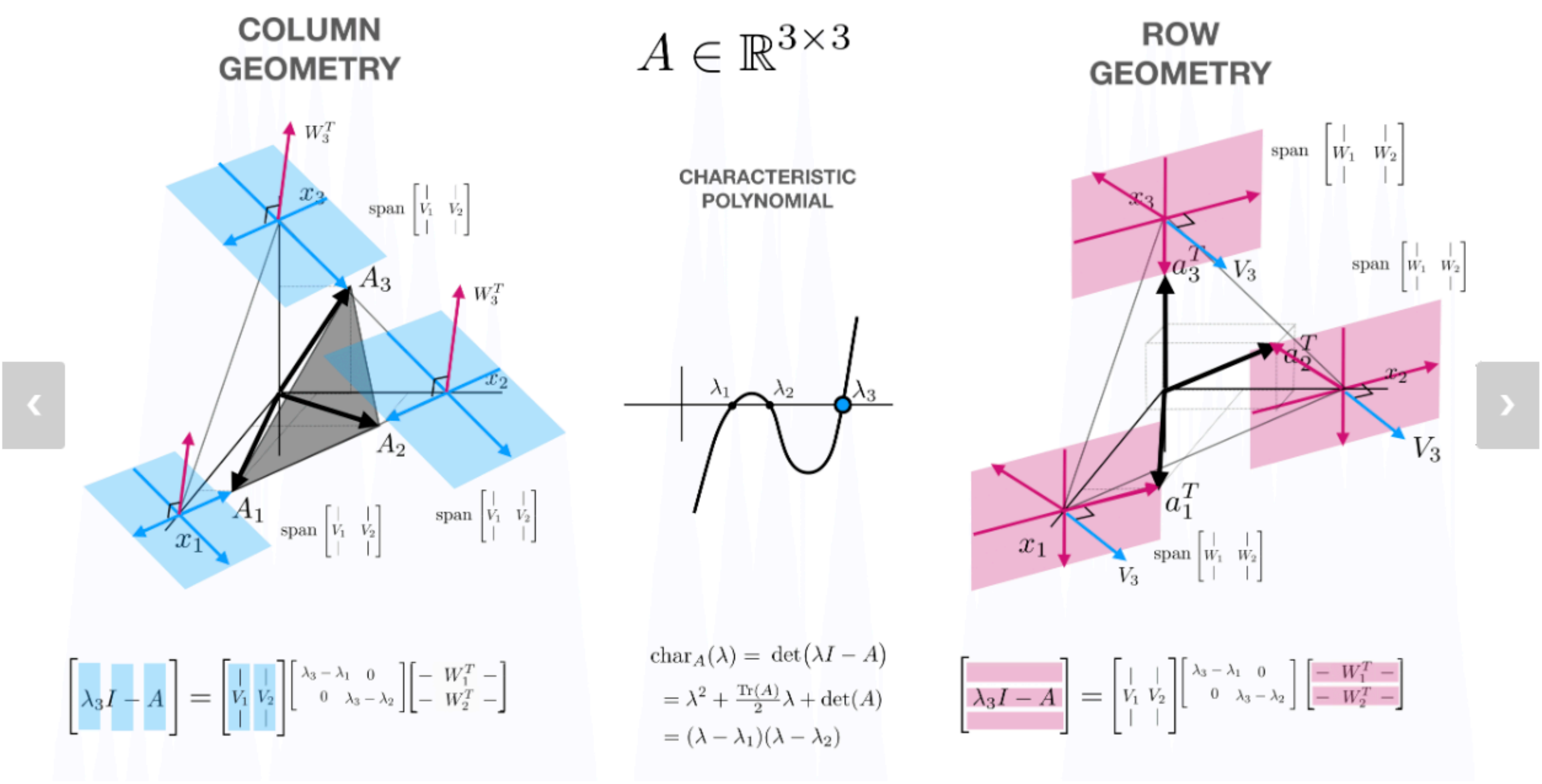

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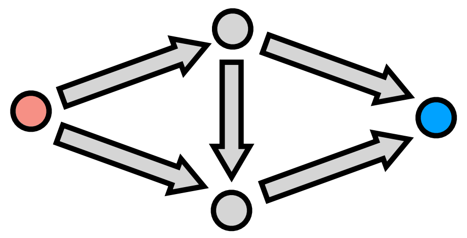
Potential Games

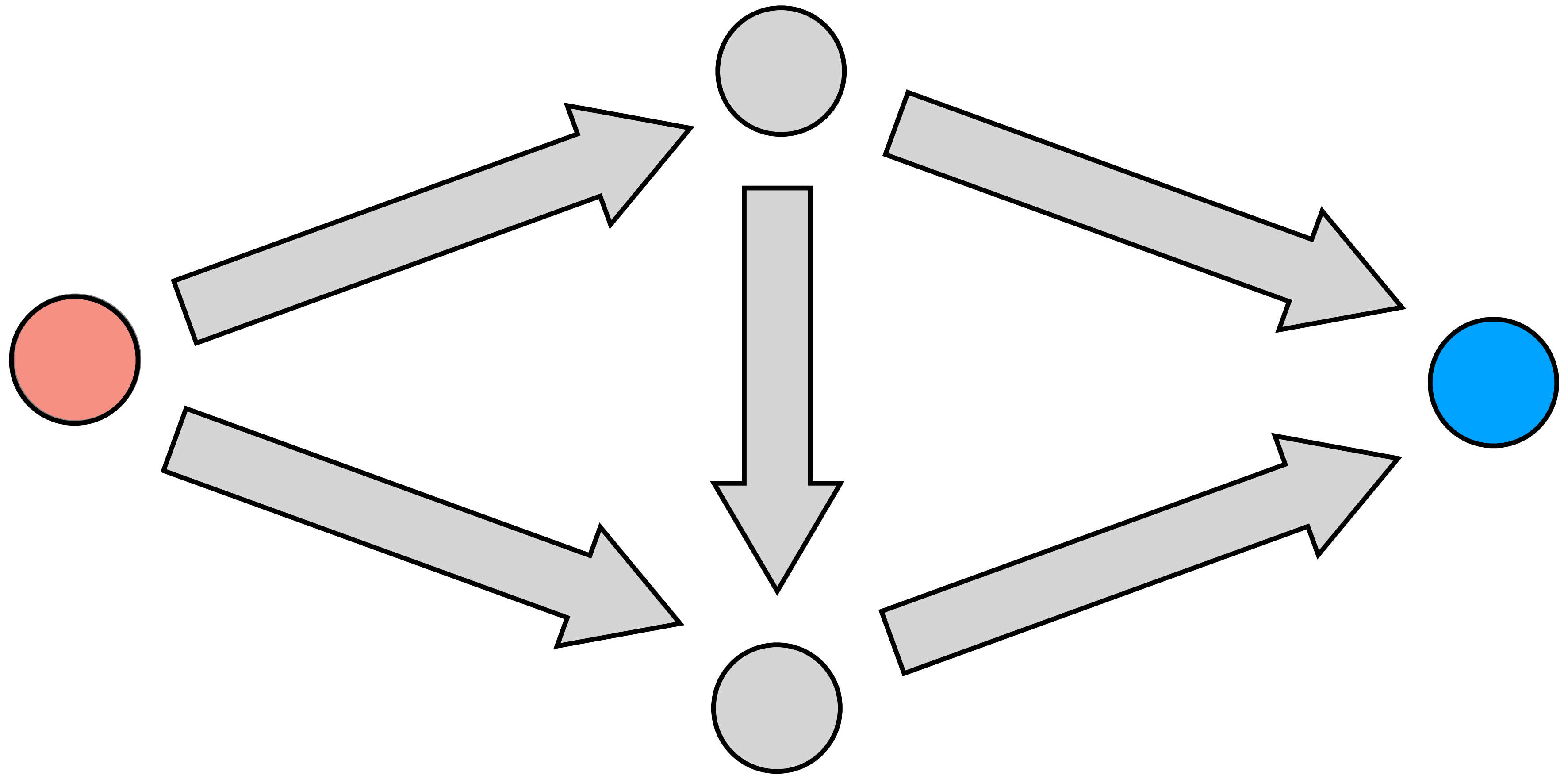
Routing Games



Routing Games

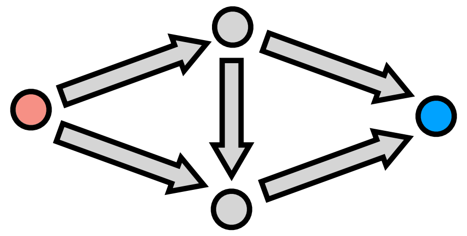
x : edge traffic

z : route traffic



Potential Games

Routing Games



Routing Games

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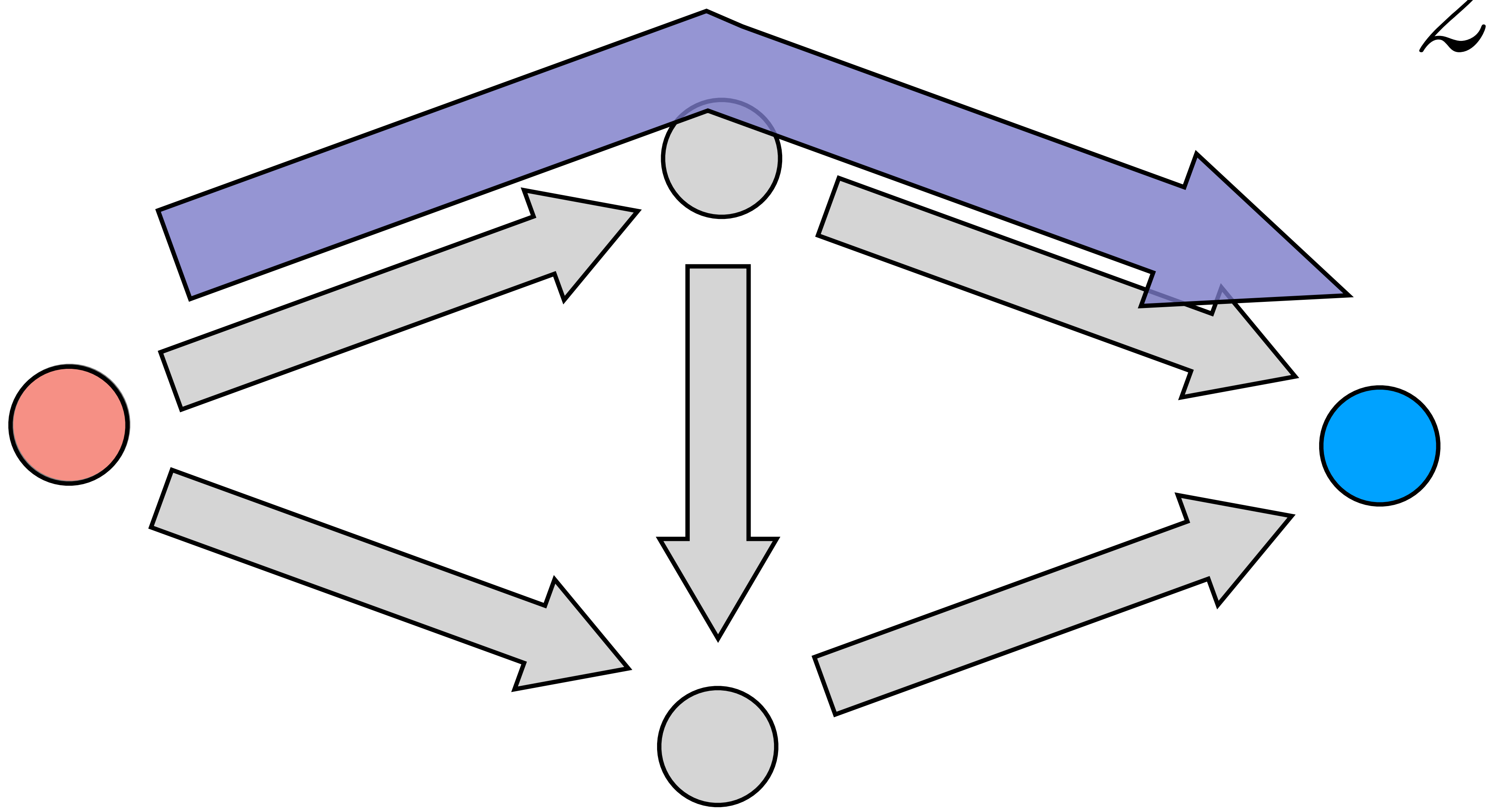
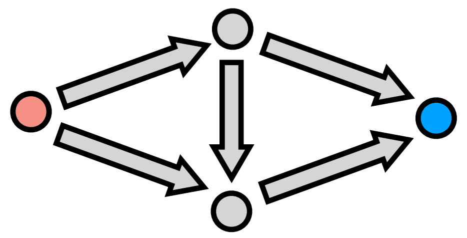


Potential Games

Routing
Games



Routing Games

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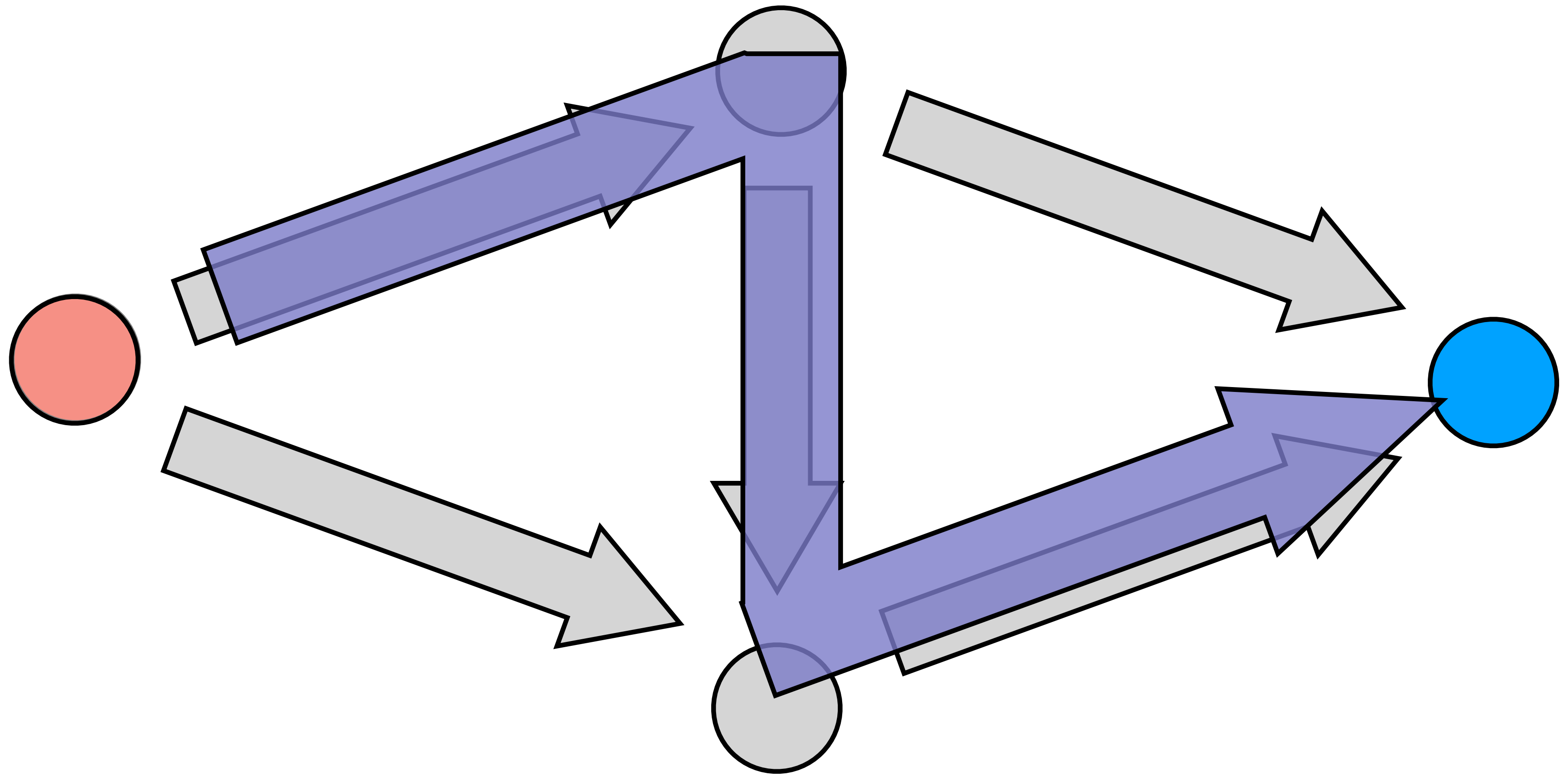
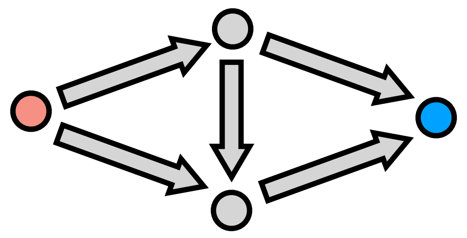


Potential Games

Routing Games



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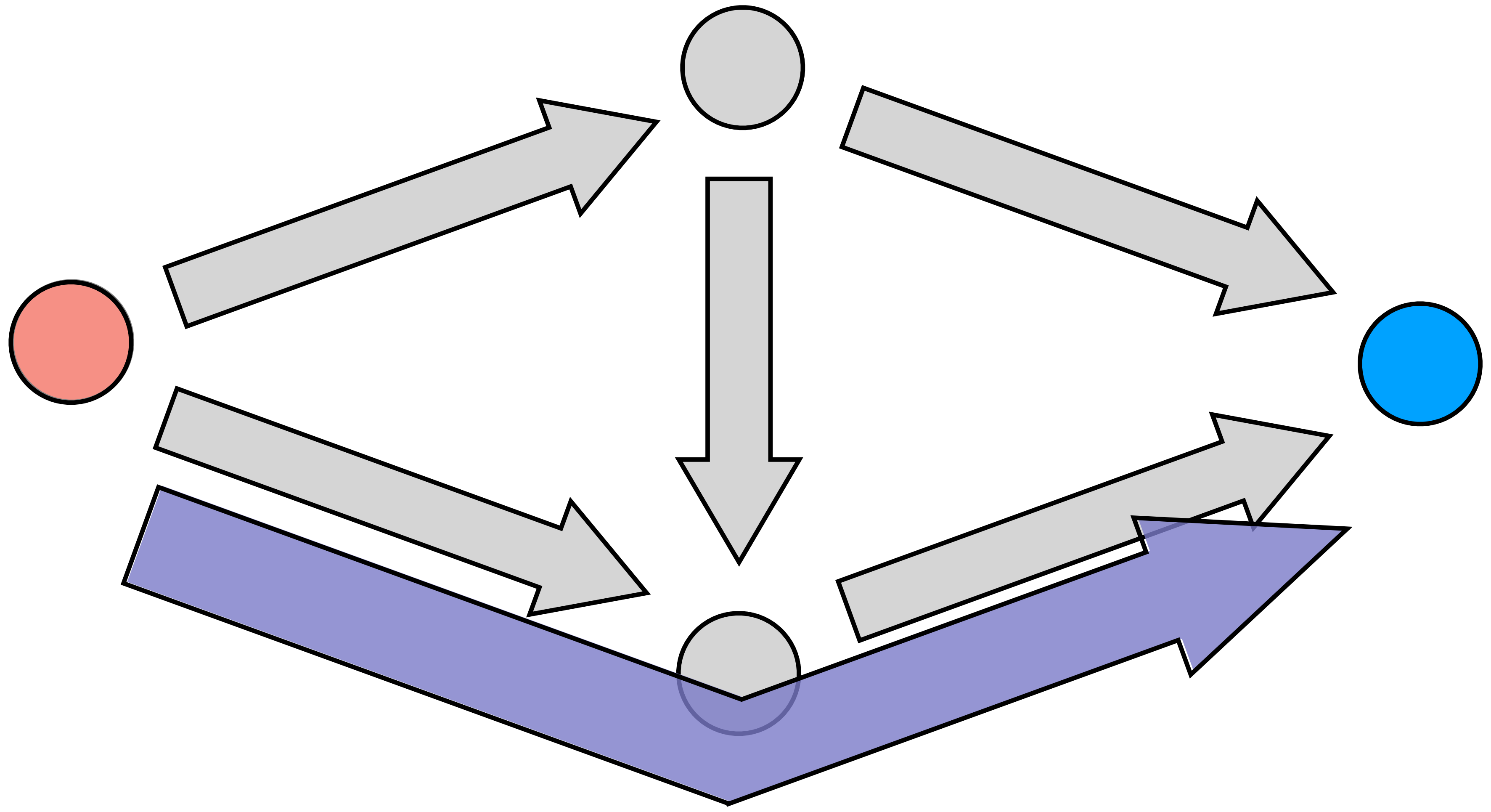
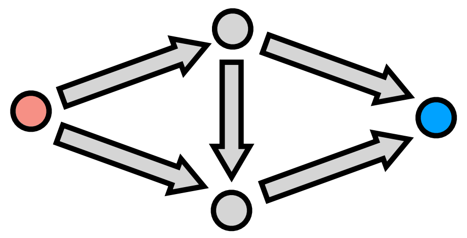


Potential Games

Routing Games



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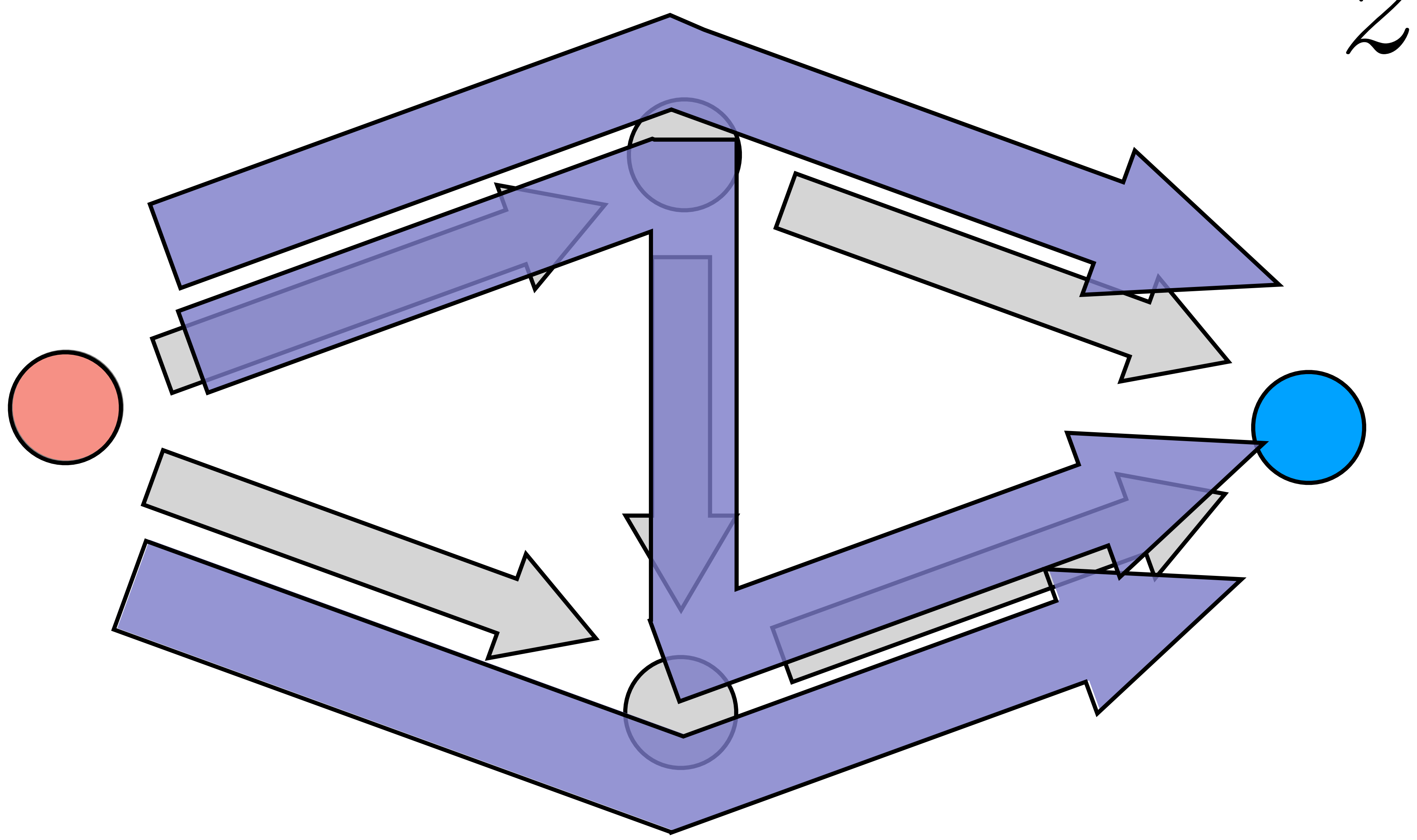
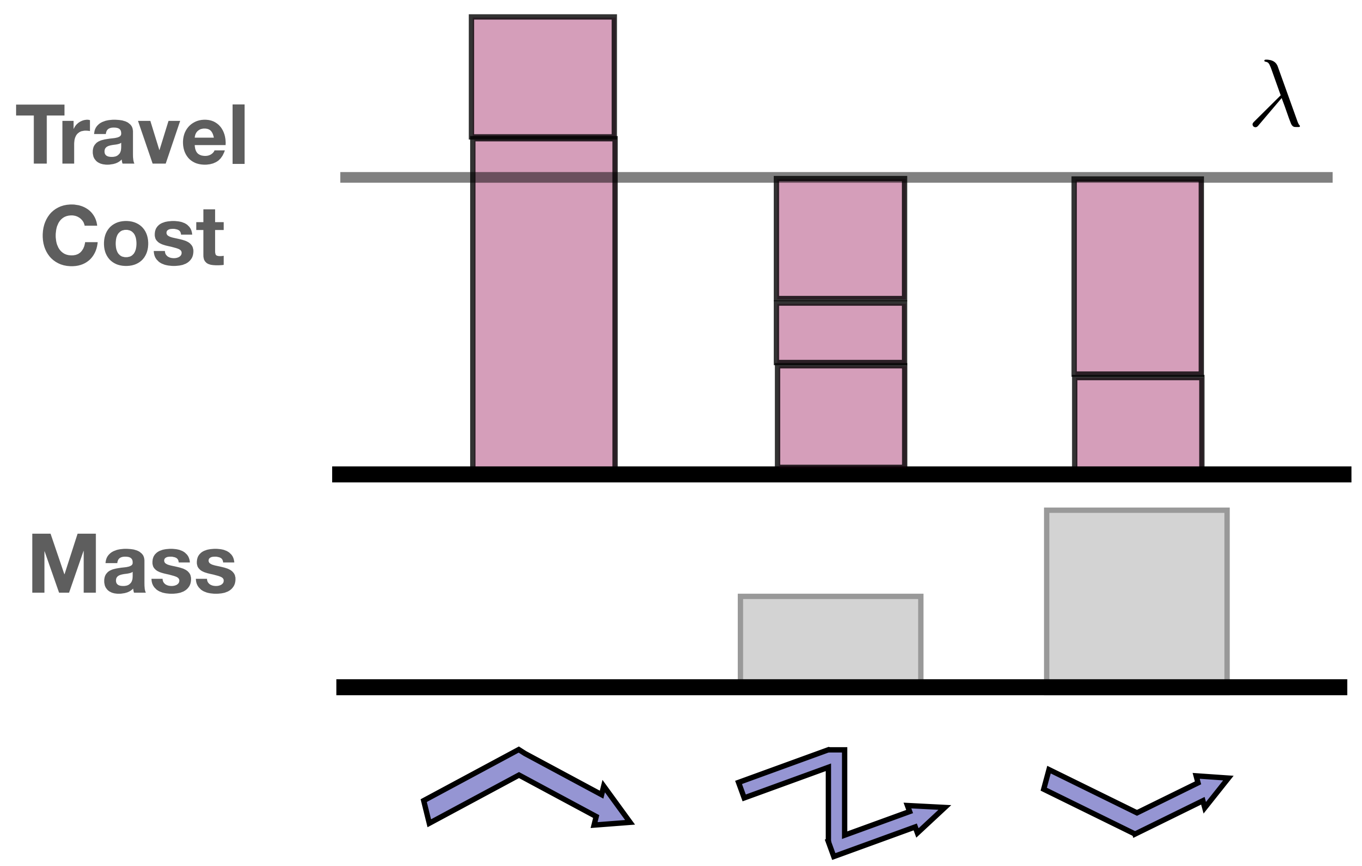
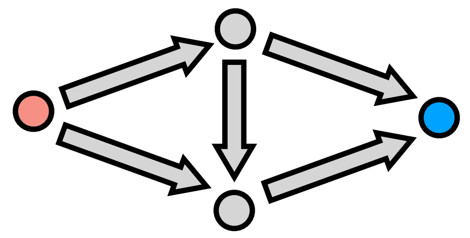


Wardrop Equilibrium



Potential Games

Routing Games



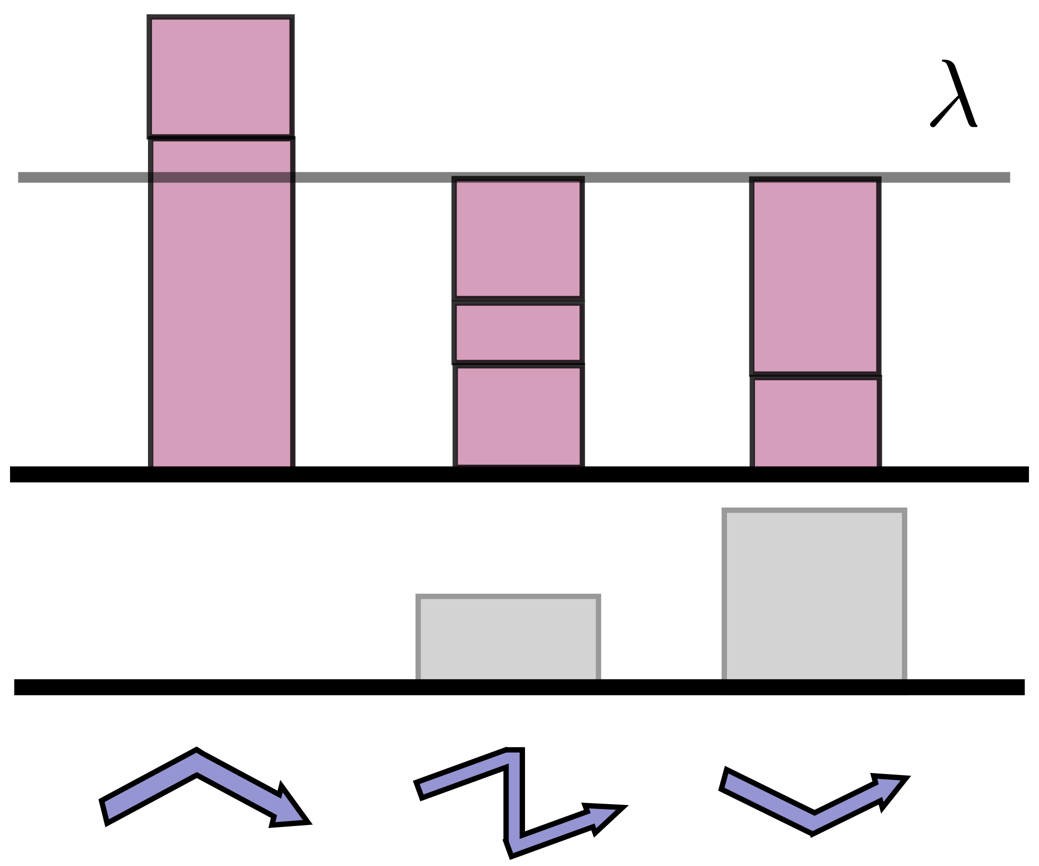
Potential Function

$$F(x)$$

Wardrop Equilibrium

Travel Cost

Mass



Routing Games

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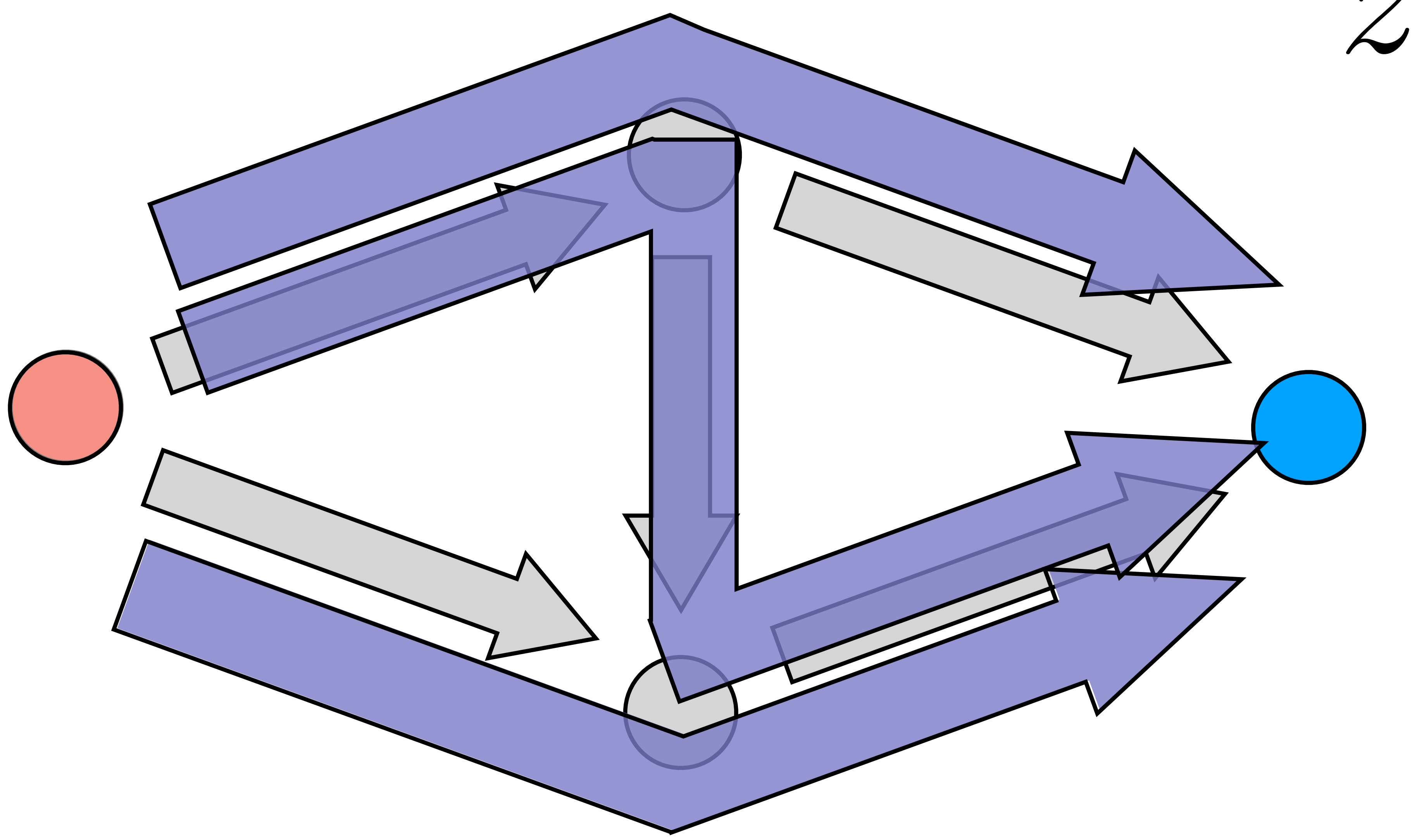
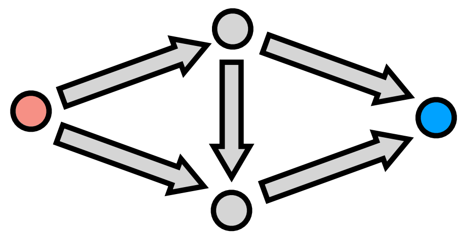


Potential Games

Routing Games



Potential Function $F(x)$

Routing Games

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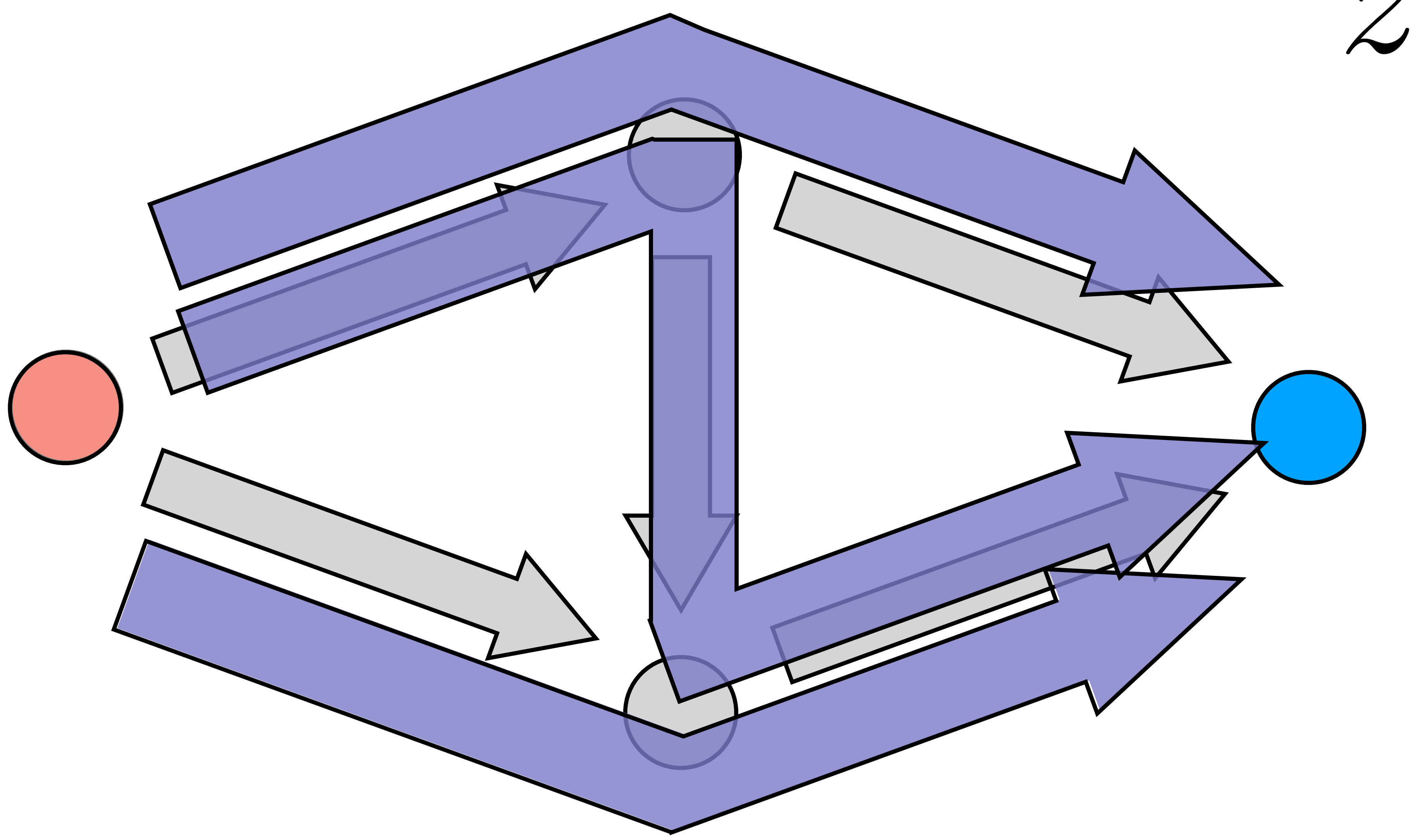

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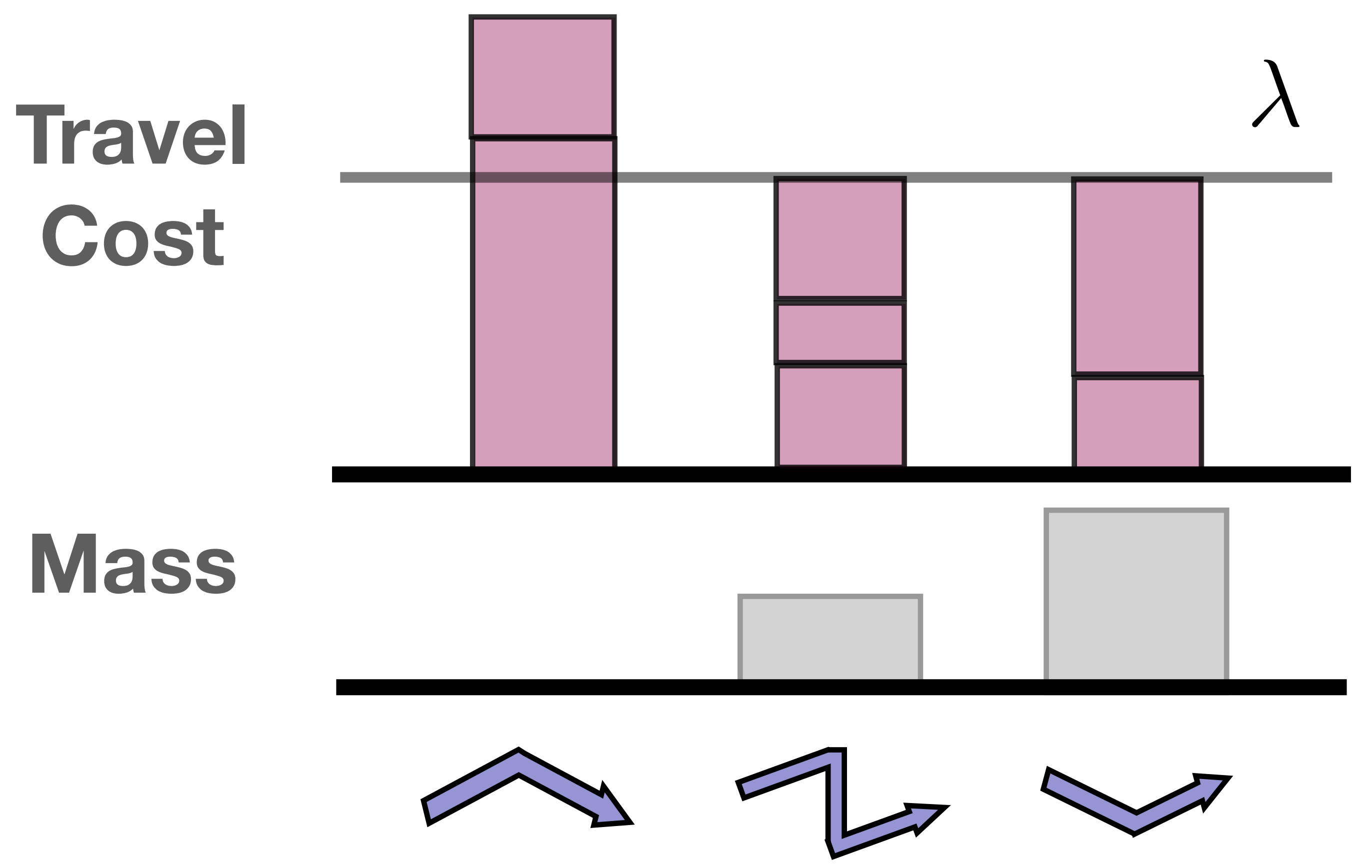


$$\min_{x,z} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T z = m \qquad z \geq 0$$

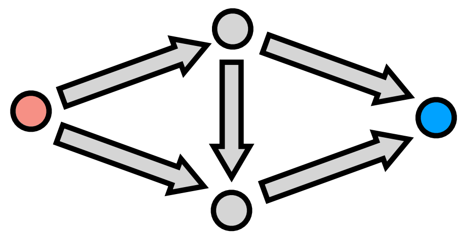
Mass conservation

$$x = \mathbf{R} z$$

Edges in routes

Potential Games

Routing Games



Potential Function $F(x)$

Routing Games

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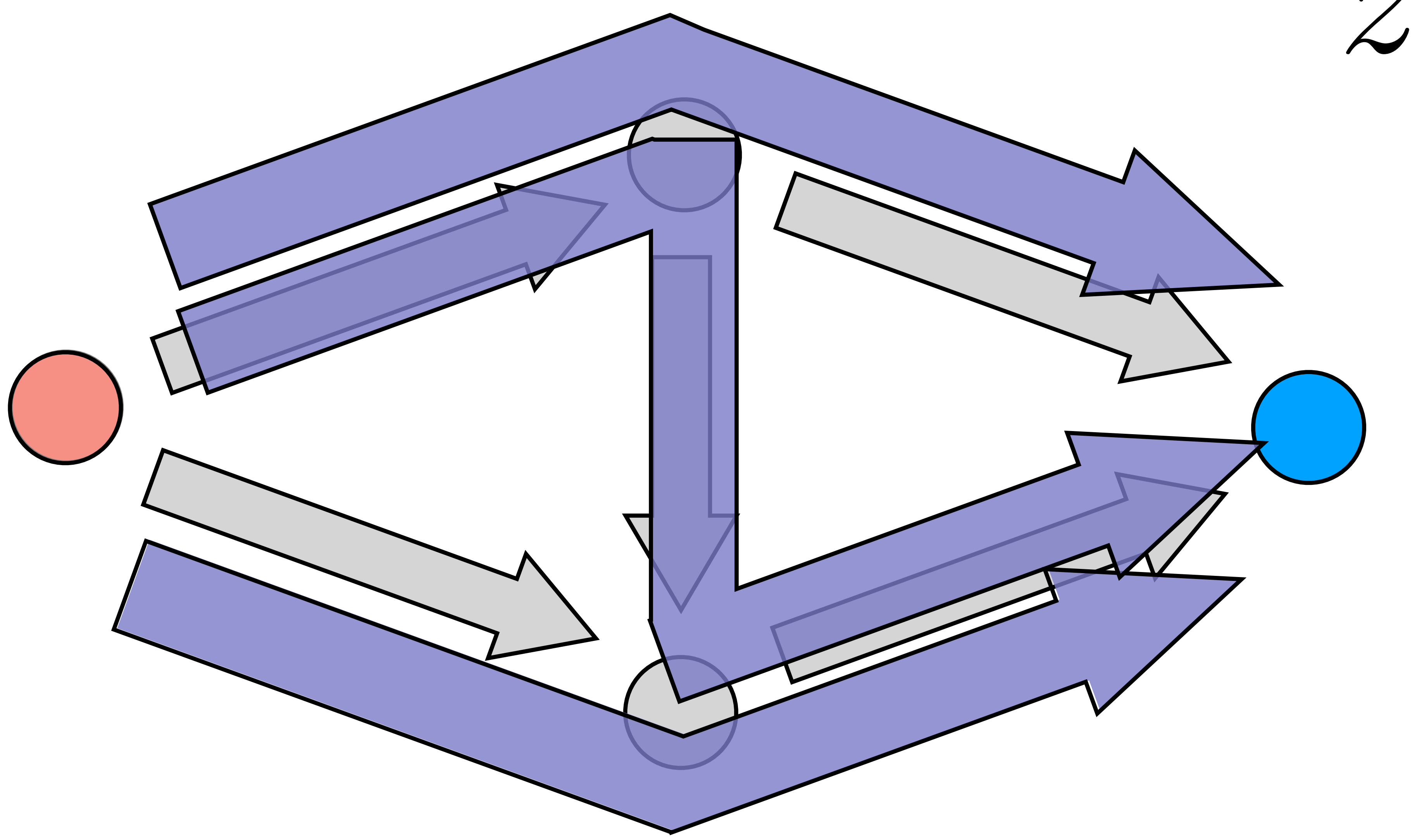
z : route traffic



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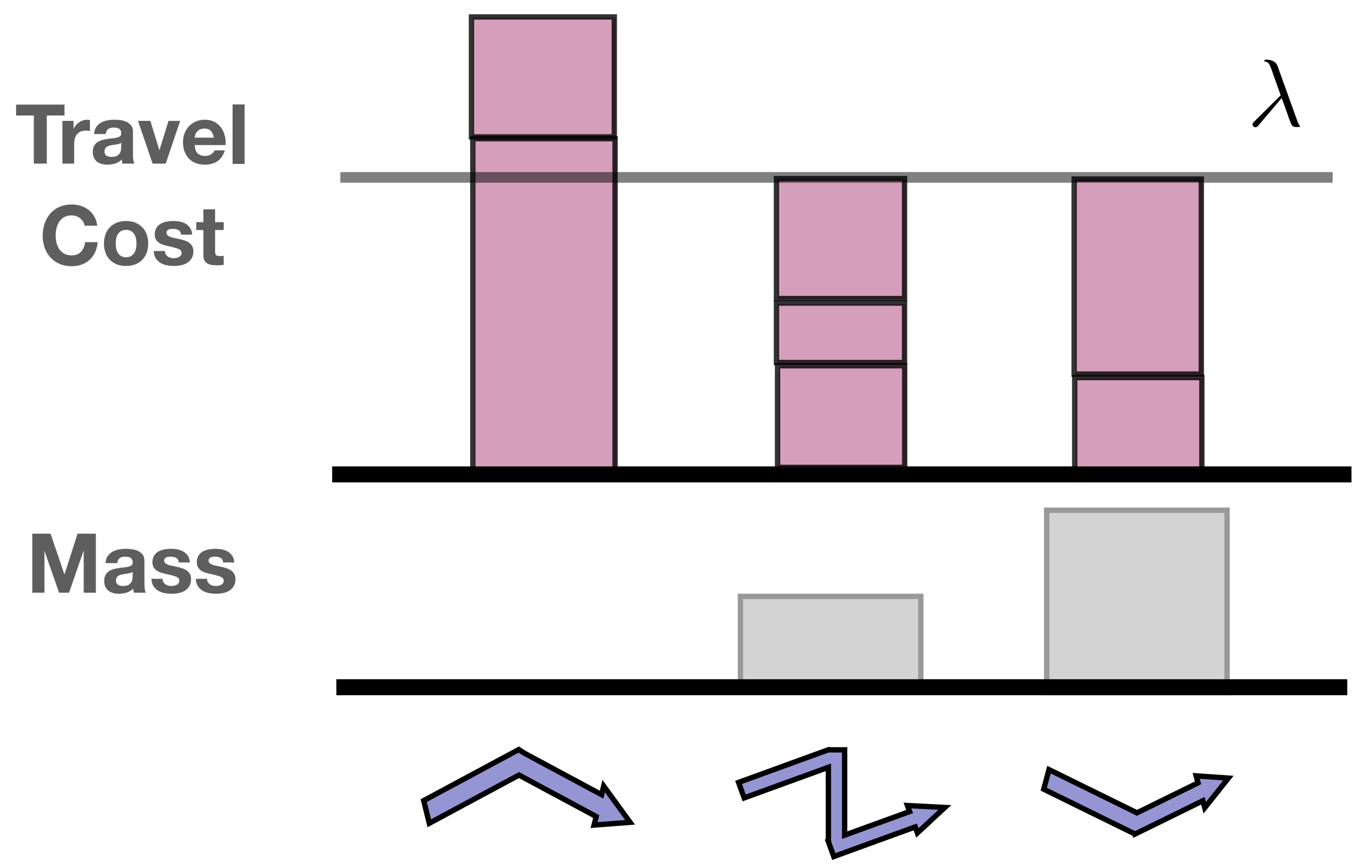

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Edges in routes

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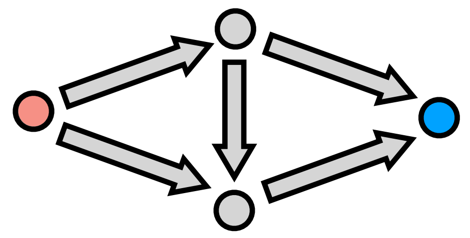
Routing Games



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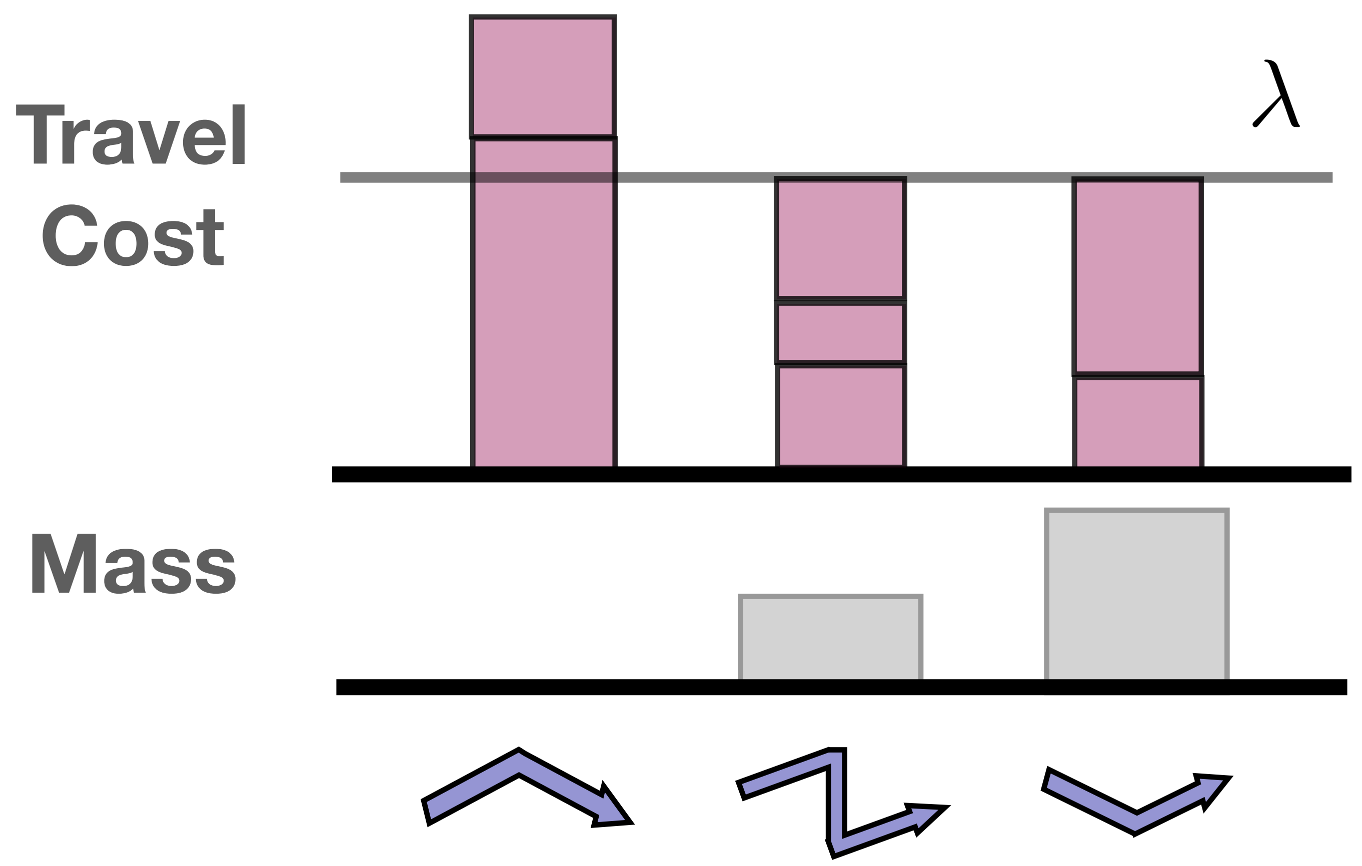

Routing Games

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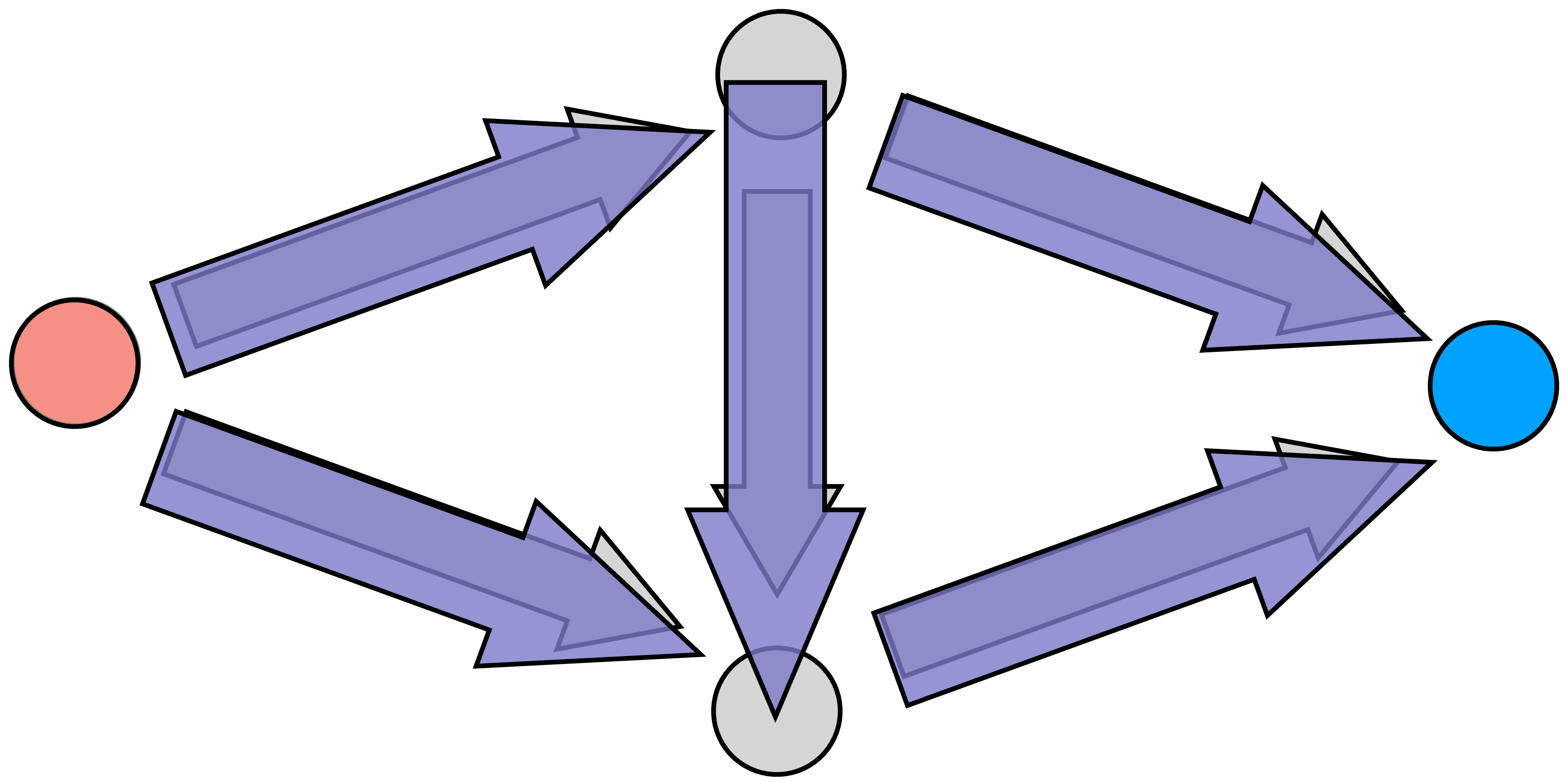
z : route traffic



$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad Ex = Sm, \quad x \geq 0$$

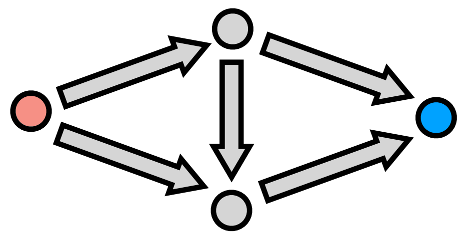
Mass conservation

Graph structure

Origin-destination

Potential Games

Routing Games

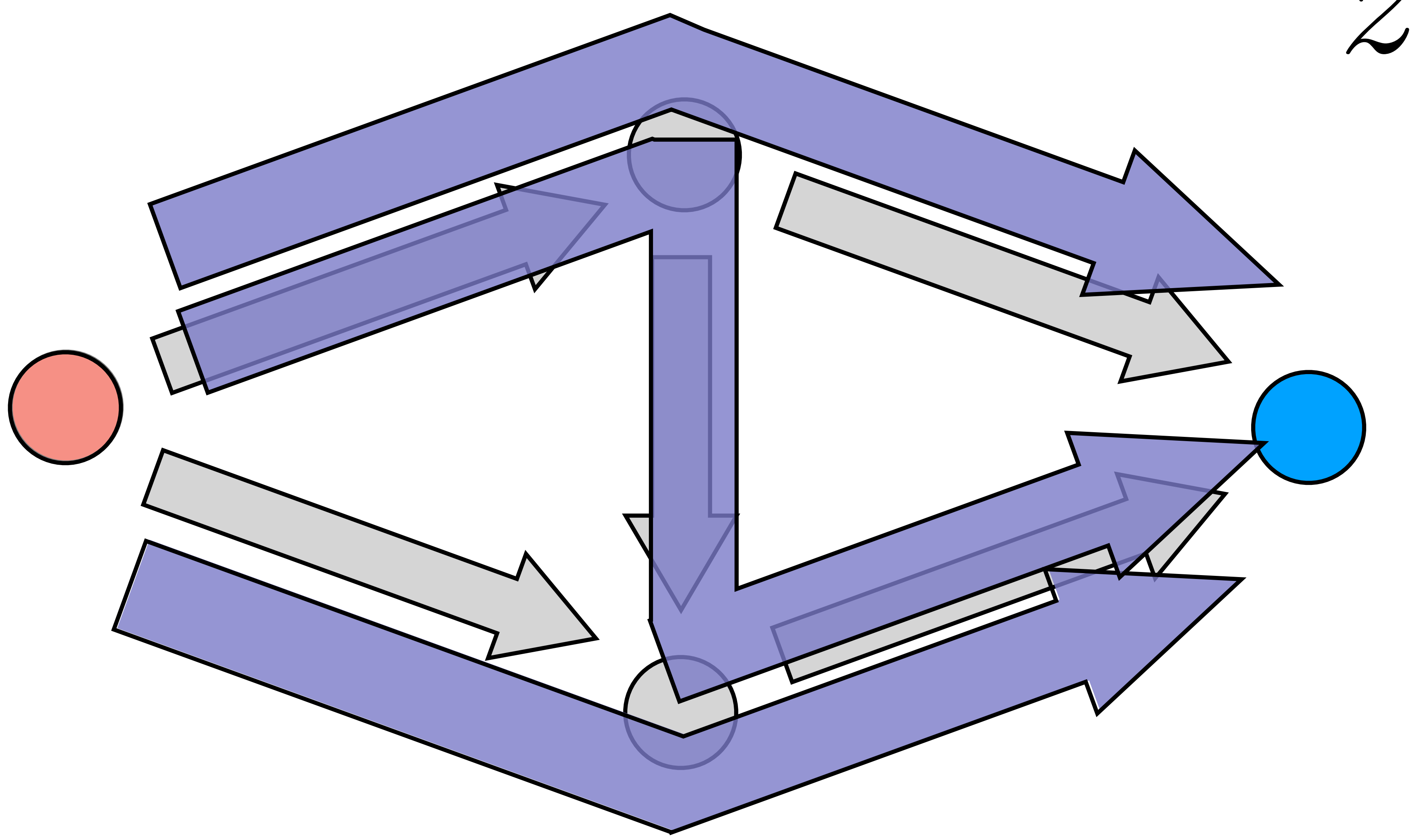


Potential Function $F(x)$

Routing Games

x : edge traffic

z : route traffic



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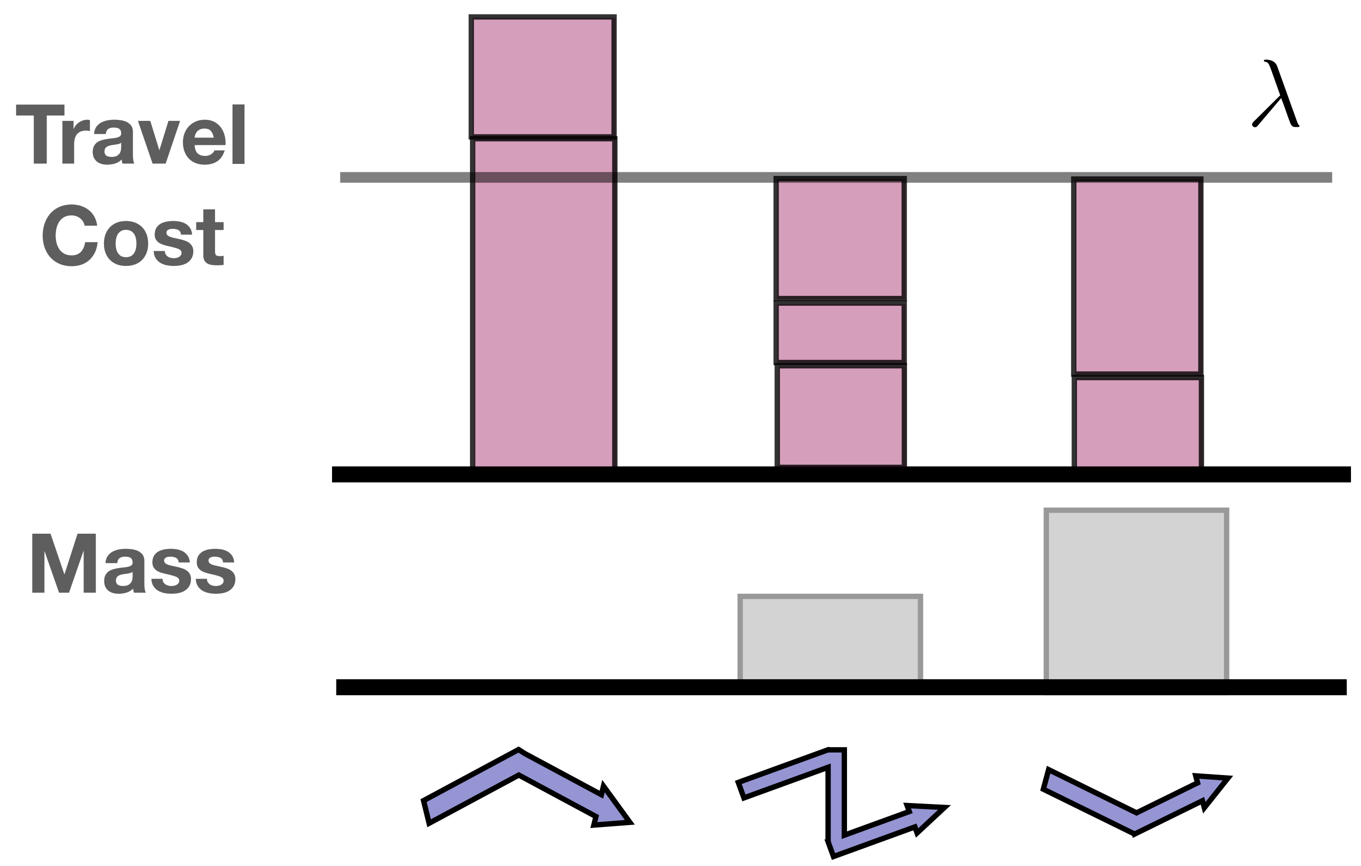


$$\min_{x,z} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T z = m \;\; (\lambda), \qquad z \geq 0 \;\; (\nu), \qquad x = \mathbf{R}z \;\; (w)$$

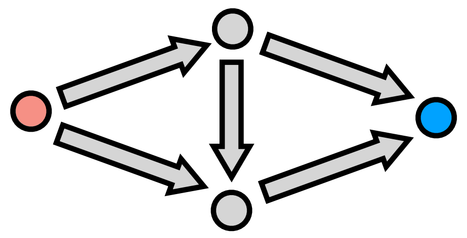
λ : travel cost

ν : route inefficiency

w : edge costs

Potential Games

Routing Games

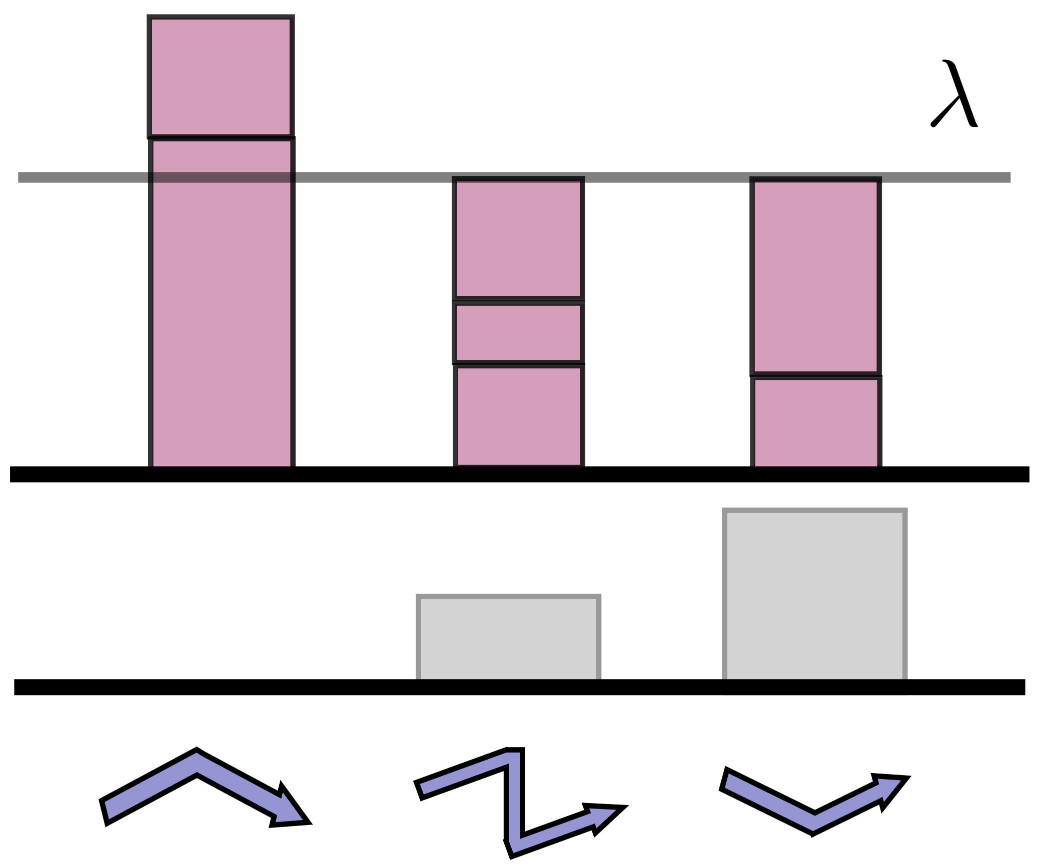


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

Mass



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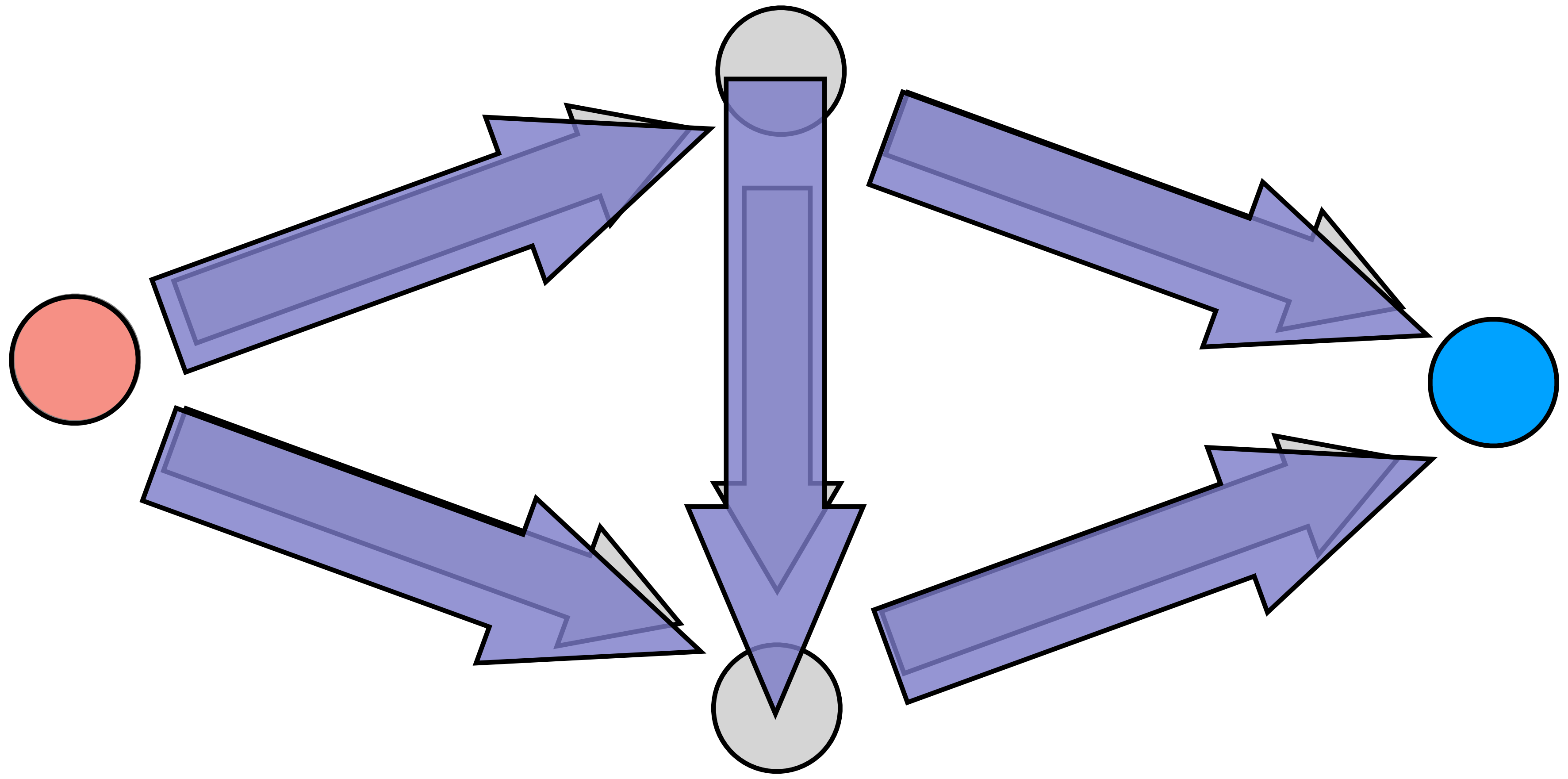


$$\min_{x} \quad F(x)$$

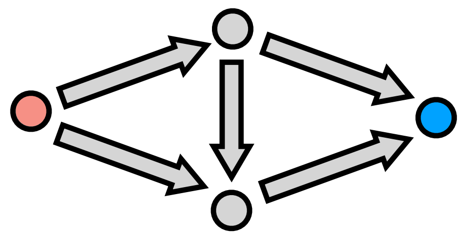
$$\text{s.t.} \quad \boxed{Ex = Sm, \ \boxed{v}} \quad \boxed{x \geq 0 \ \boxed{\mu}}$$

μ : edge inefficiency

v : value function

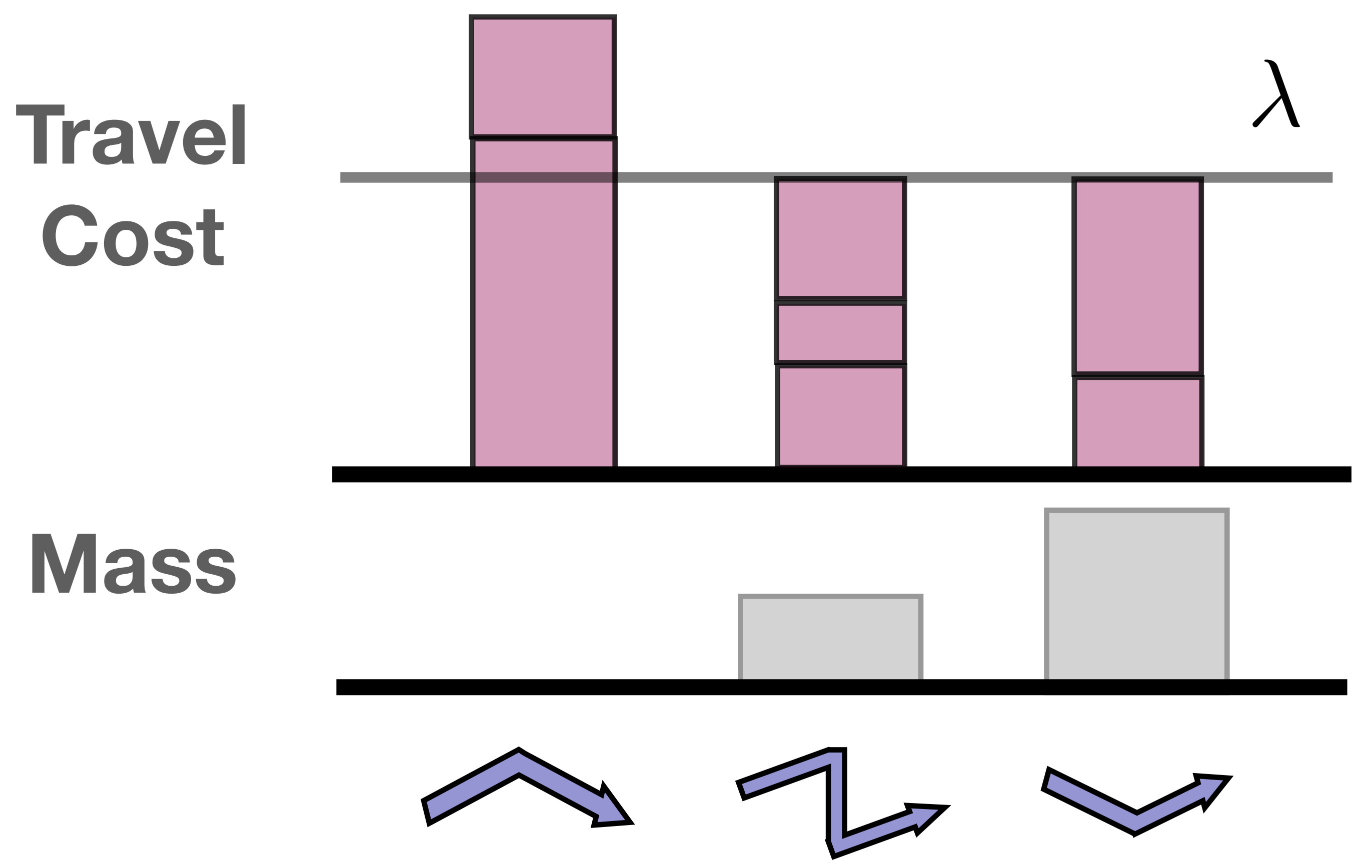
Potential Games

Routing Games



Potential Function $F(x)$

Wardrop Equilibrium



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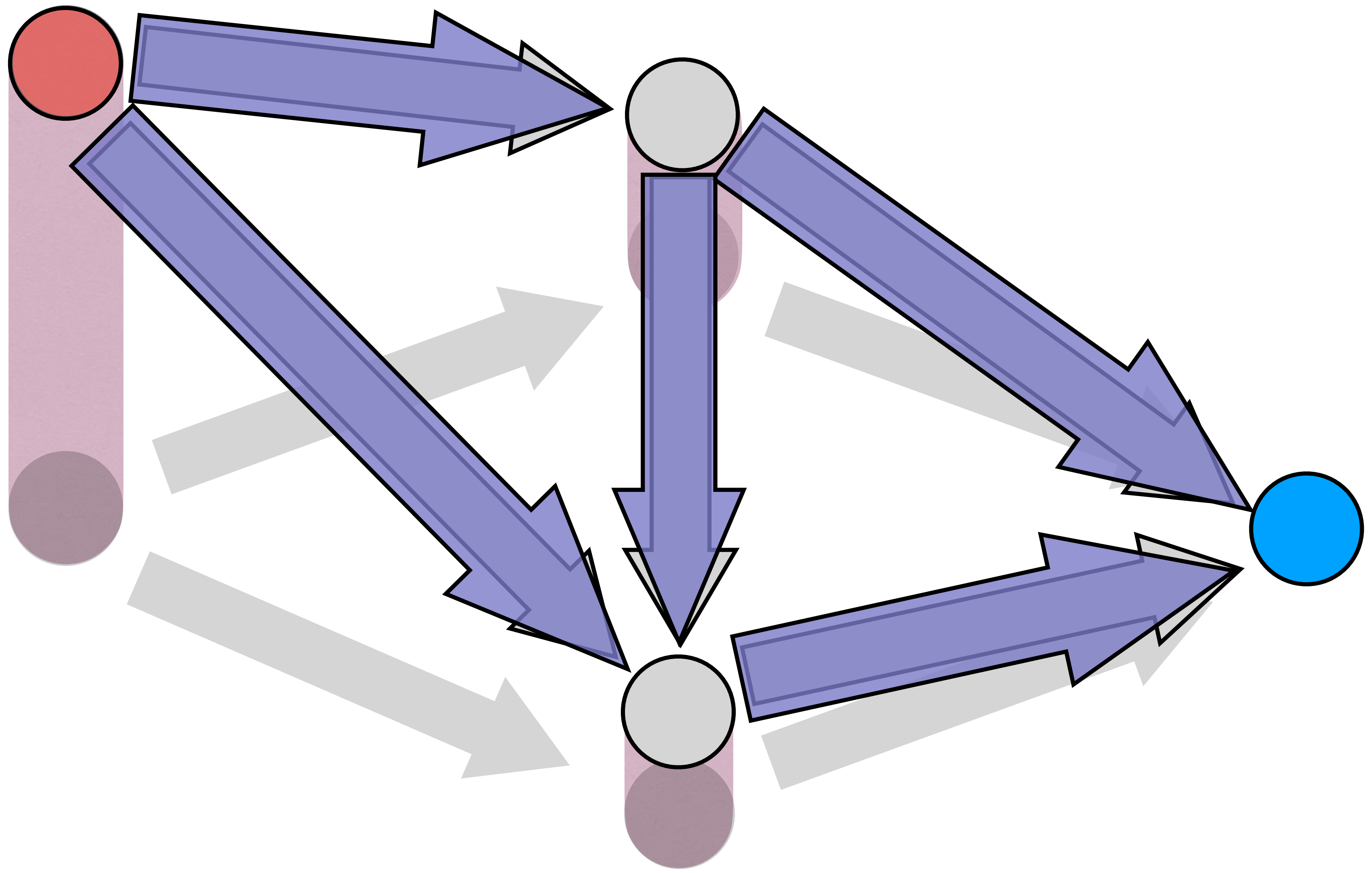

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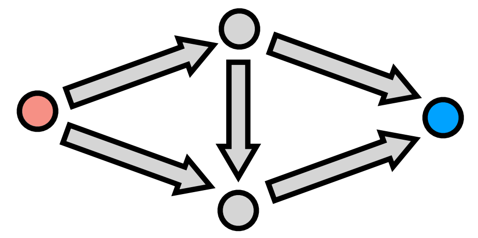
$$\text{s.t.} \quad \boxed{Ex = Sm, \ v} \quad \boxed{x \geq 0 \ \mu}$$

μ : edge inefficiency

v : value function

Potential Games

Routing Games

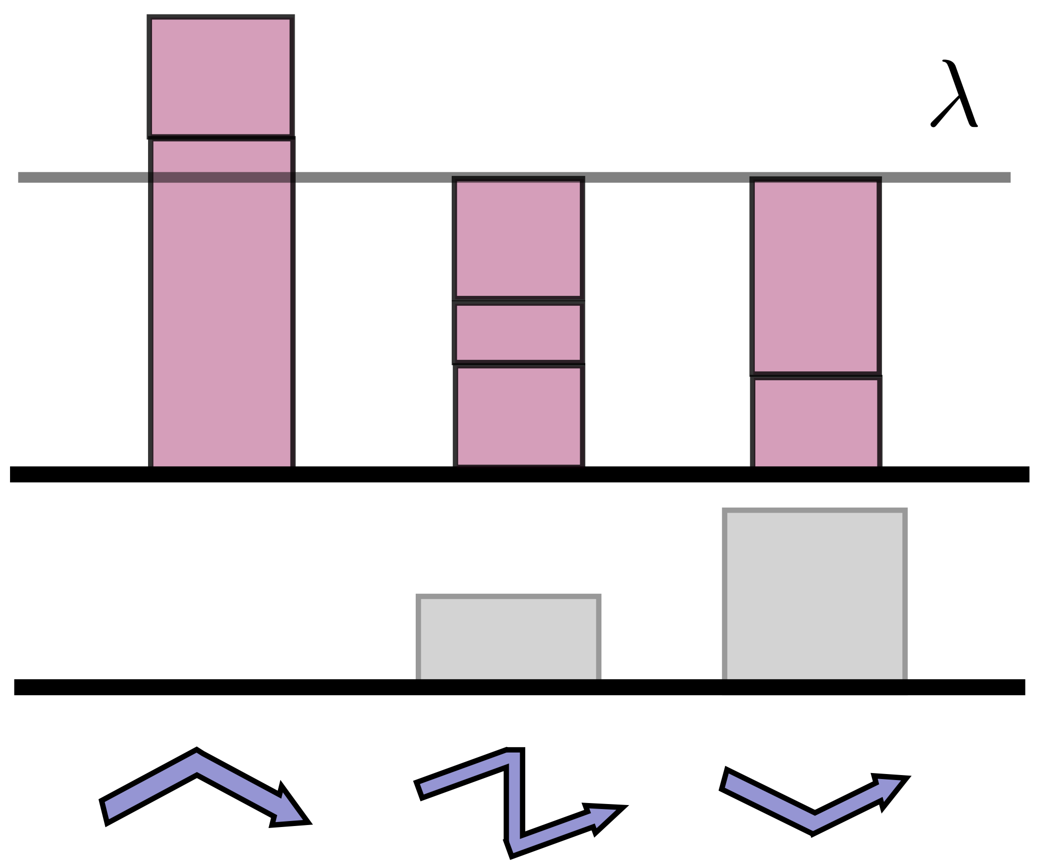


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

Mass



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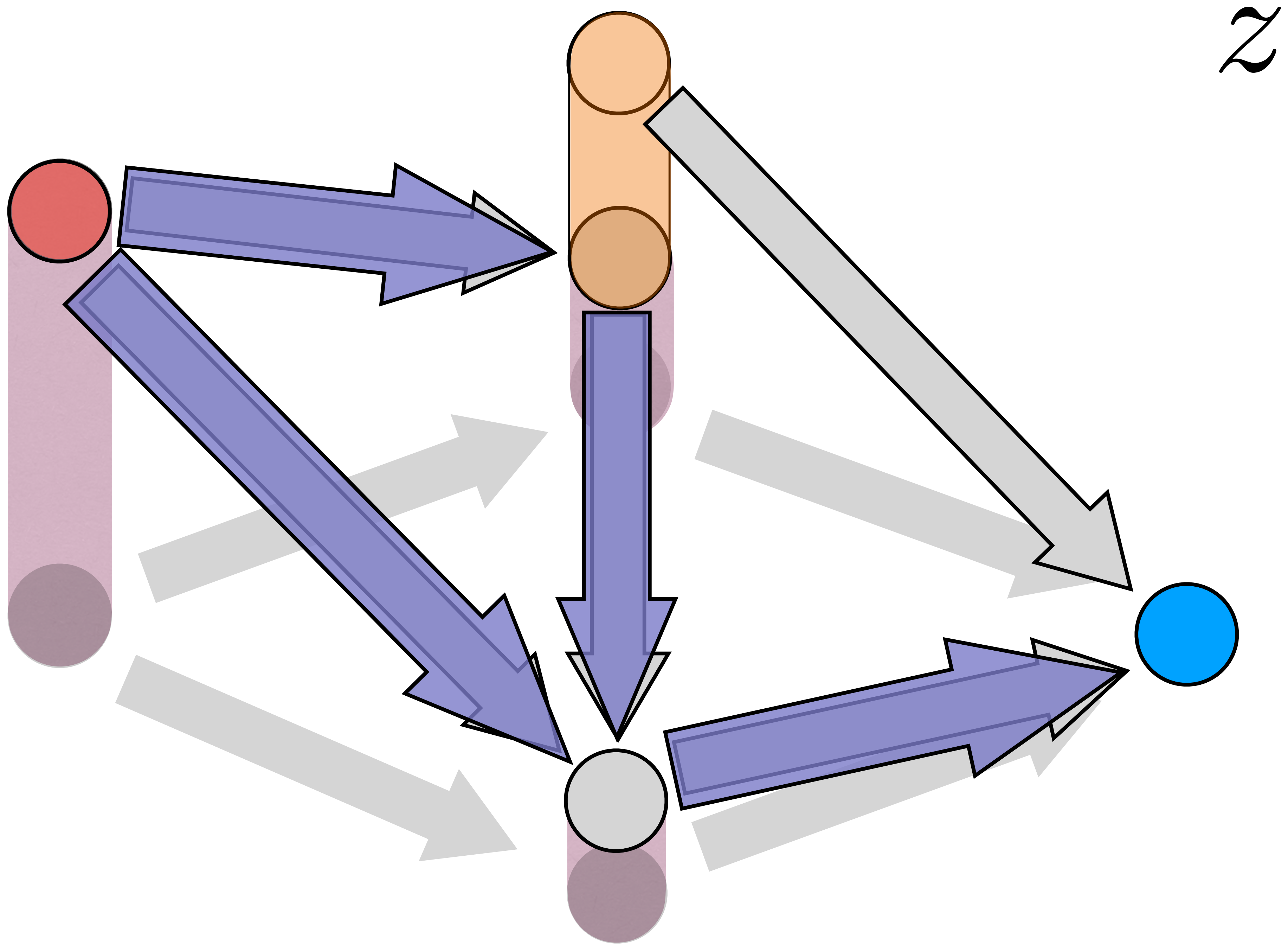

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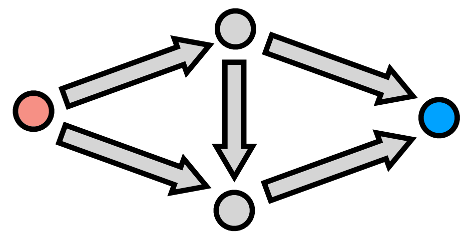
Potential Games

Routing Games

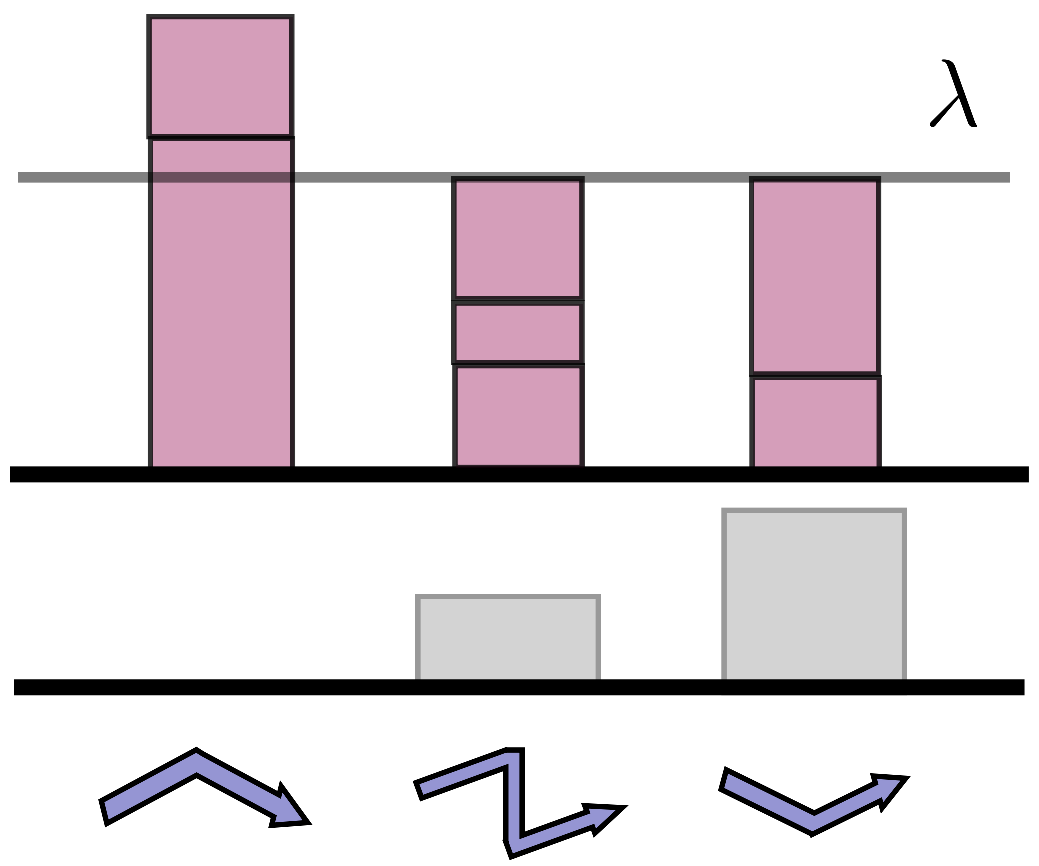


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

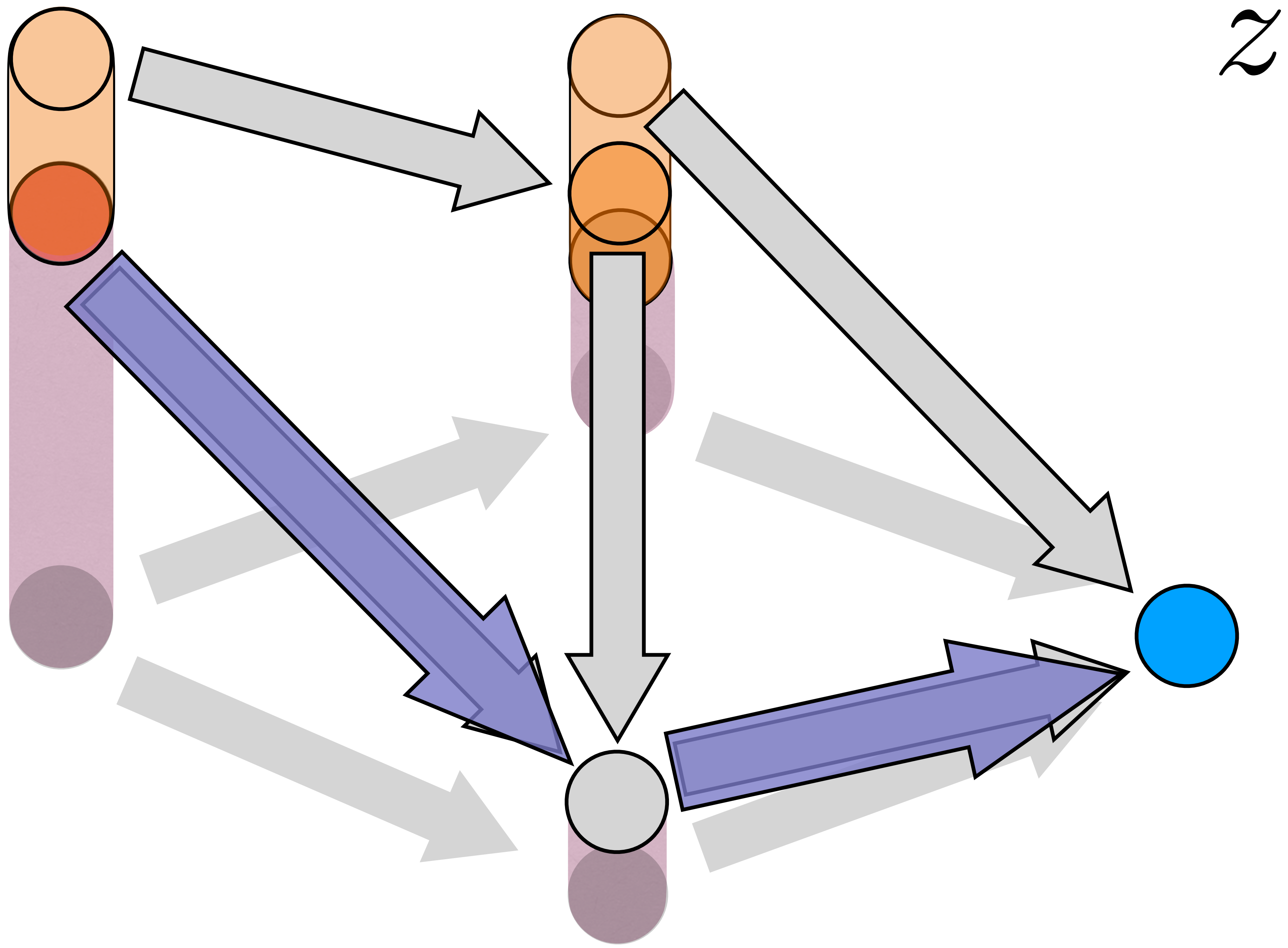
Mass



Routing Games

x : edge traffic

z : route traffic



$$\min_{x} \quad F(x)$$

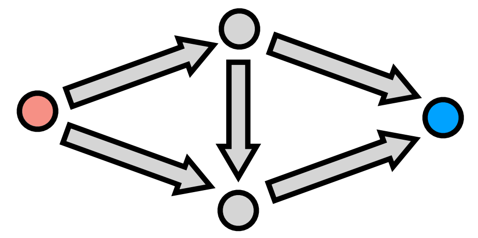
$$\text{s.t.} \quad \boxed{Ex = Sm, \; v} \quad \boxed{x \geq 0 \; \mu}$$

μ : edge inefficiency

v : value function

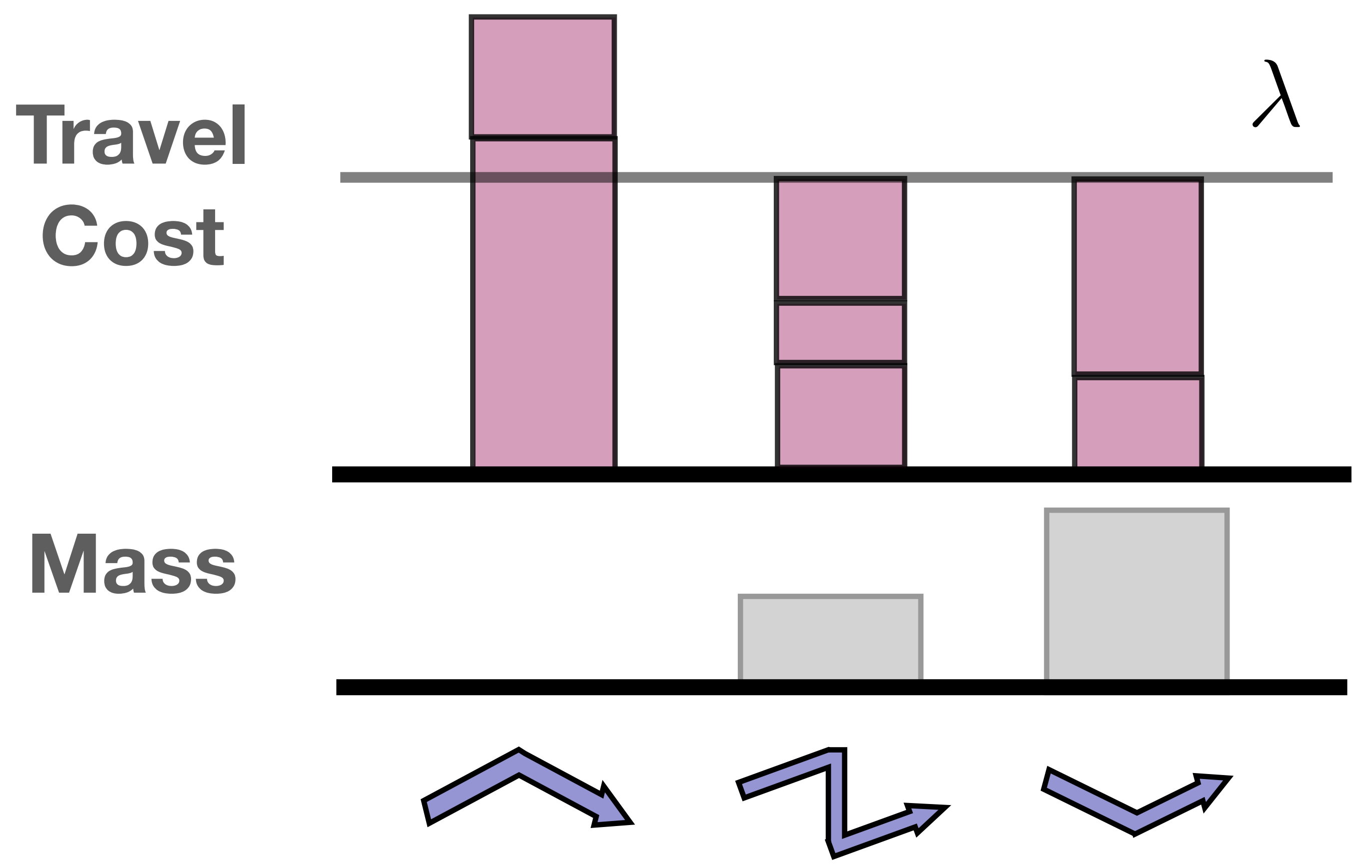
Potential Games

Routing Games



Potential Function $F(x)$

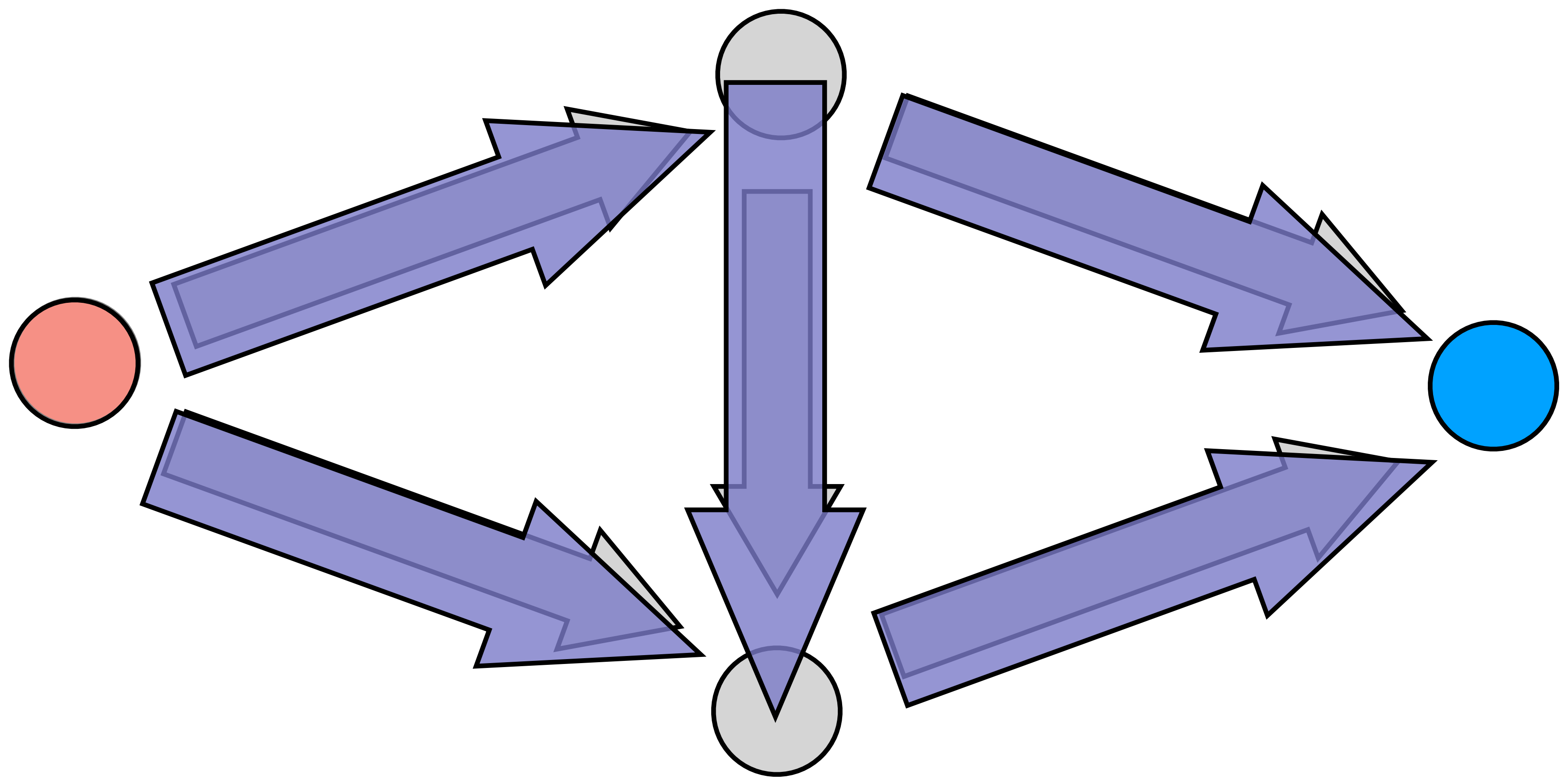
Wardrop Equilibrium



Routing Games

x : edge traffic

z : route traffic



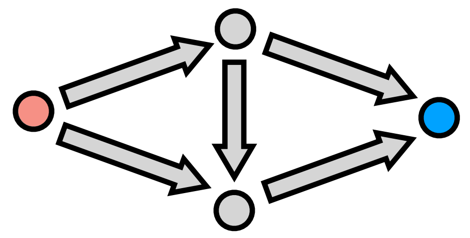
REFERENCES

- Some theoretical aspects of road traffic research **[Wardrop, 1952]**
- Studies in the economics of transportation **[Beckmann, McGuire, Winsten, 1956]**

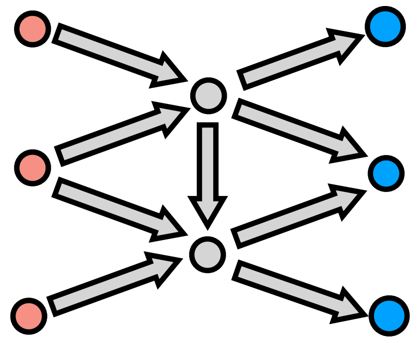
- The Traffic Assignment Problem: Models and Methods **[Patriksson, 2015]**

Potential Games

Routing Games



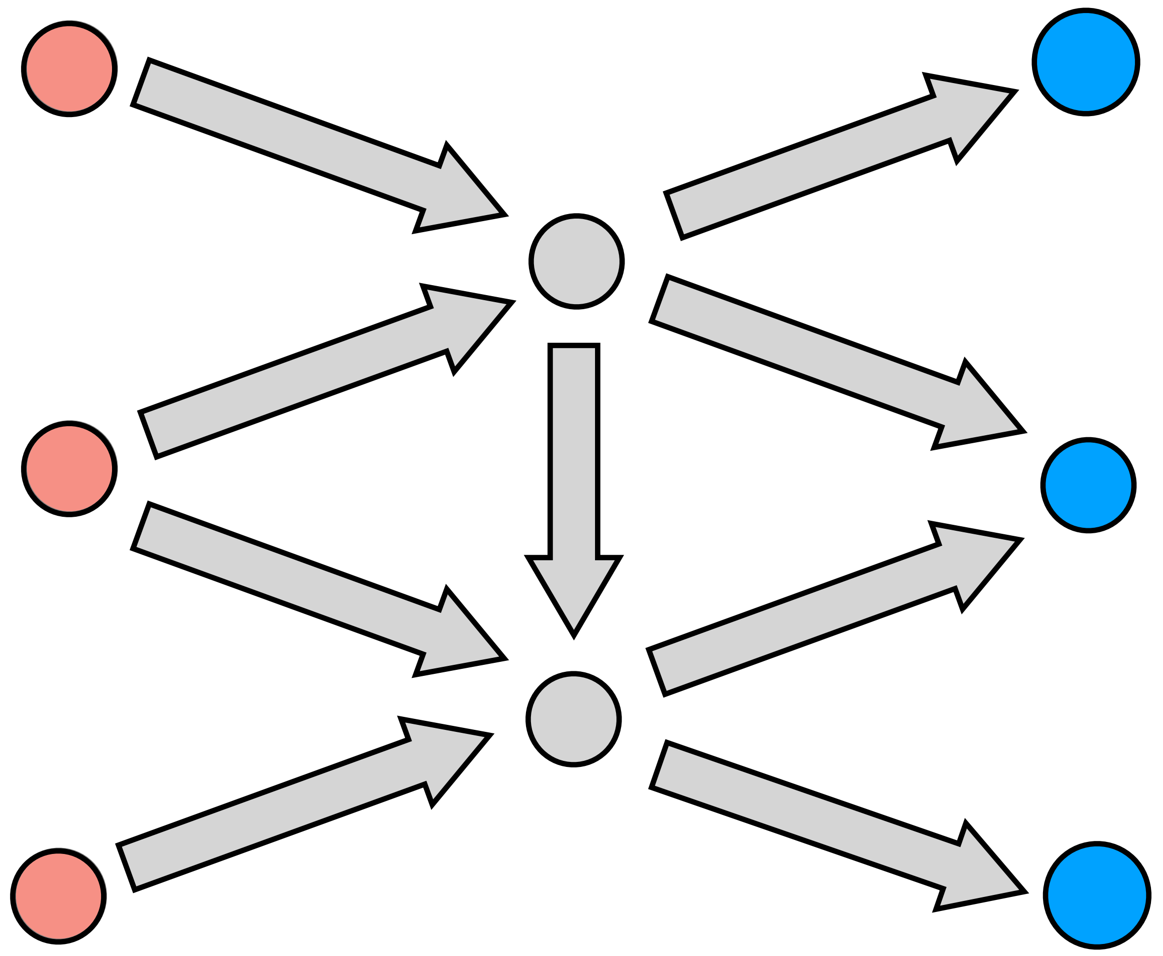
Multiple sources/ sinks



Multiple Source/Sinks

x : edge traffic

z : route traffic

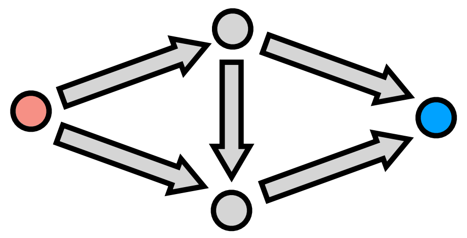


$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad E_i x_i = S_i m_i, \ \forall i \quad v_i \qquad x_i \geq 0, \ \forall i \quad \mu_i$$

Potential Games

Routing Games



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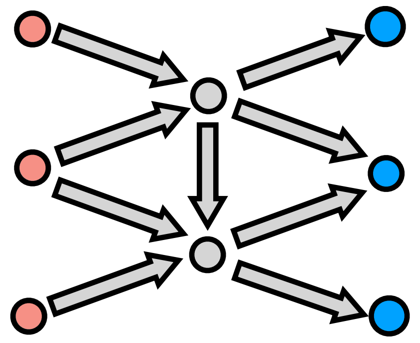
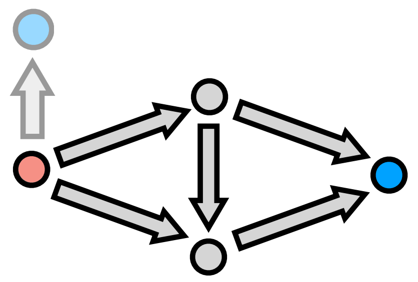


Variable Demand

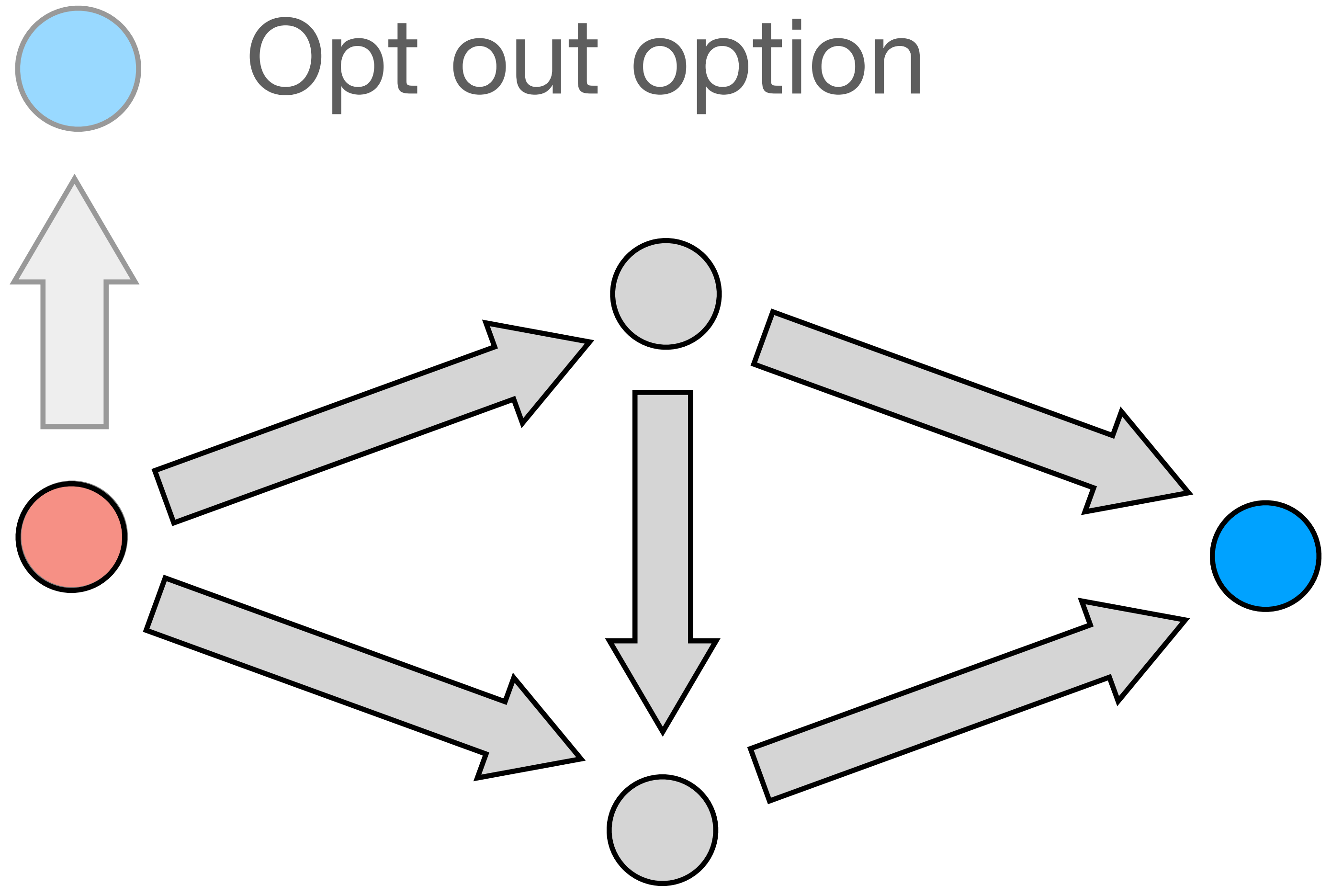


Variable Demand

x : edge traffic

z : route traffic

$m(\lambda)$

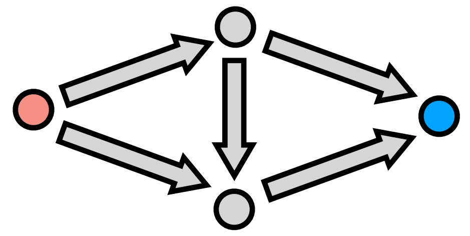


$$\min_x \quad F(x) + \int_0^m m^{-1}(u)\, du$$

$$\text{s.t.} \quad \boxed{Ex = Sm, \quad \boxed{v}} \quad \boxed{x \geq 0 \quad \boxed{\mu}}$$

Potential Games

Routing Games



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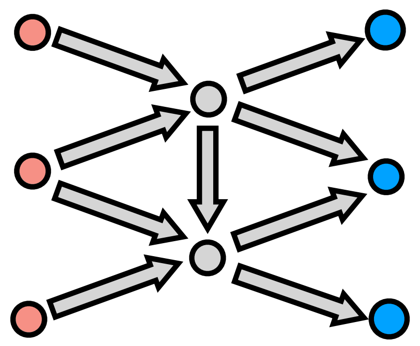
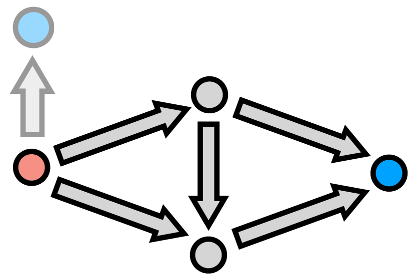
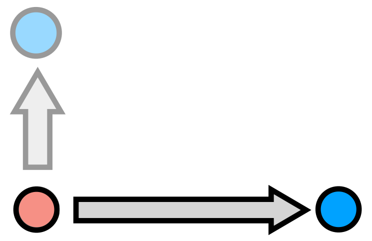


Variable Demand



Supply & Demand

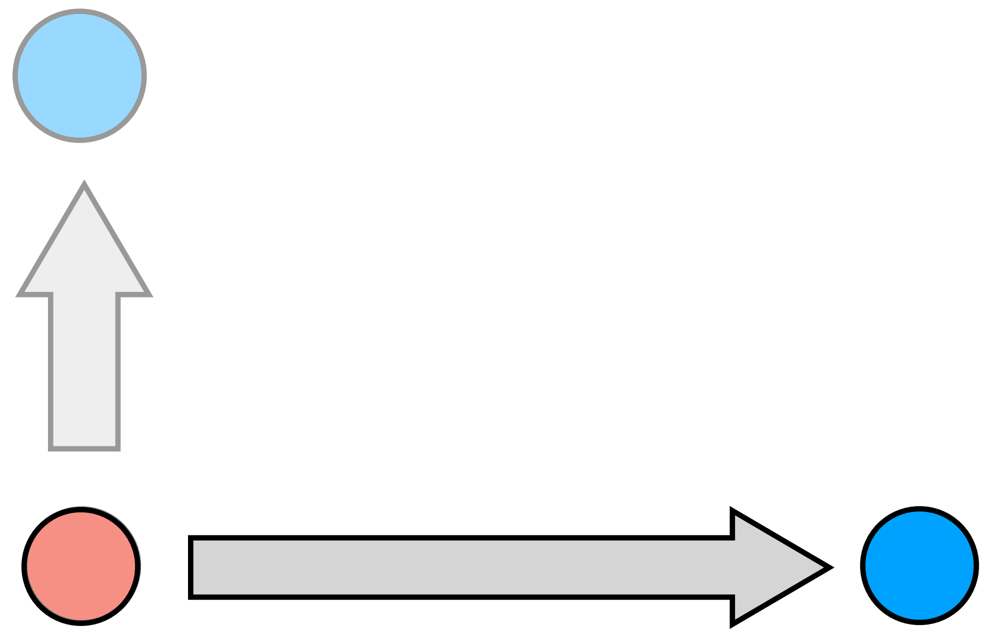


Supply & Demand

x : edge traffic

z : route traffic

$m(\lambda)$

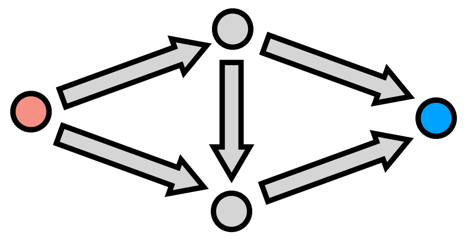


$$\min_{x} \quad F(x) + \int_0^m m^{-1}(u)\, du$$

$$\text{s.t.} \quad Ex = Sm, \;\; \nu \qquad x \geq 0 \;\; \mu$$

Potential Games

Routing Games



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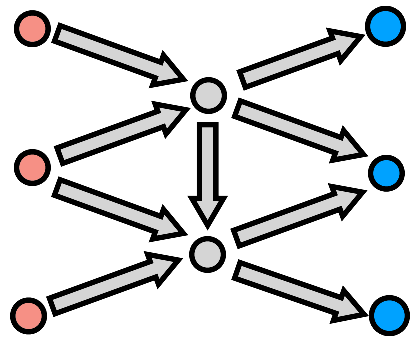
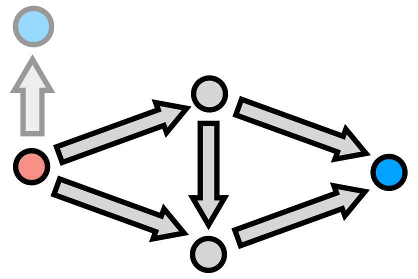
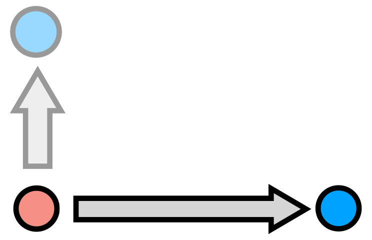


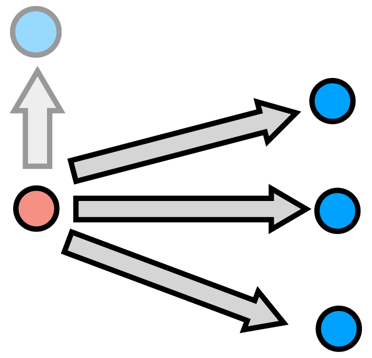
Variable Demand



Supply & Demand



Cournot Market



Cournot Market

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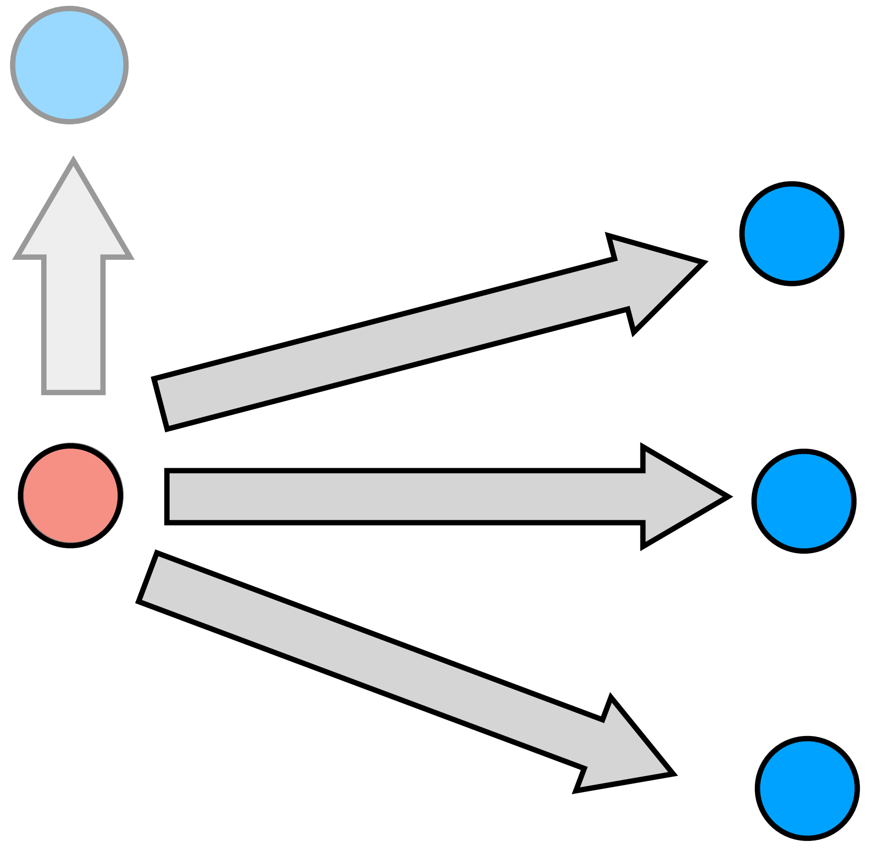
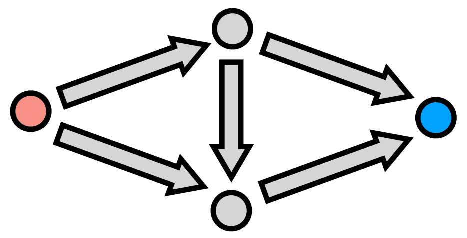


$$\min_{x} \quad F(x) + \int_0^m m^{-1}(u)\, du$$

$$\text{s.t.} \quad \boxed{Ex = Sm, \quad \nu} \quad \boxed{x \geq 0 \quad \mu}$$

Potential Games

Routing Games



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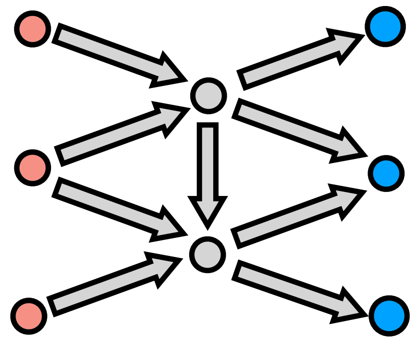
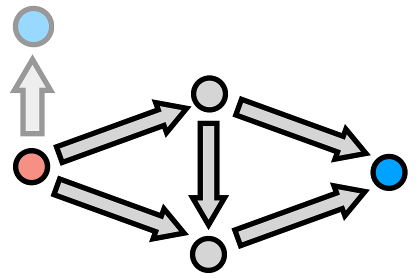


Variable Demand



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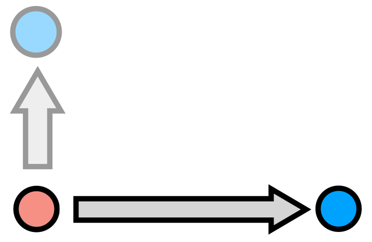
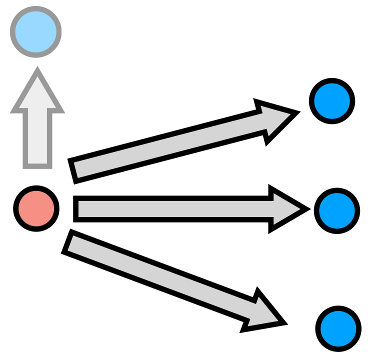
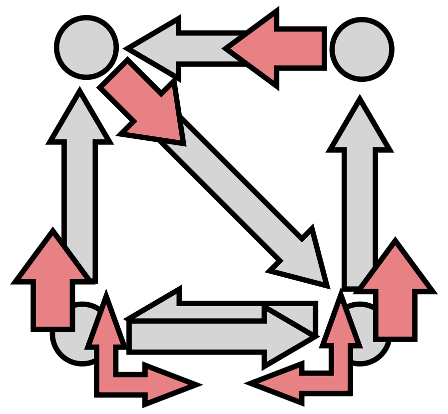


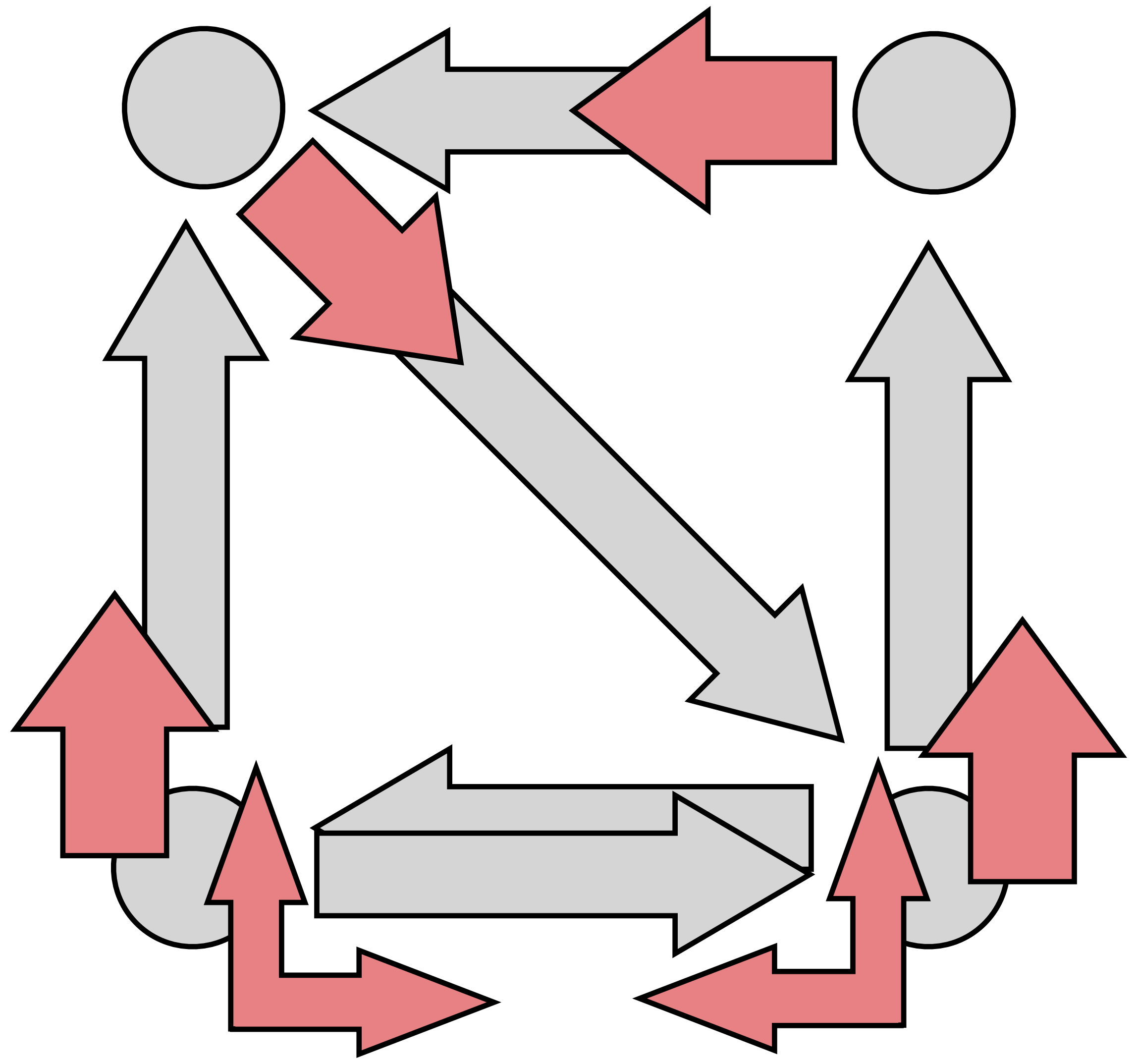
Cournot Market



MDP Congestion Game



Markov Decision Process Congestion Game



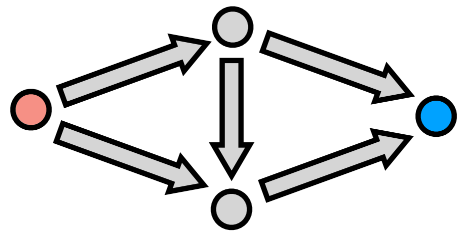
$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \quad \lambda \qquad x \geq 0 \quad \mu$$

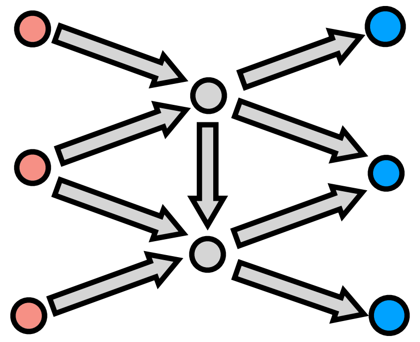
$$EWx = 0 \quad v$$

Potential Games

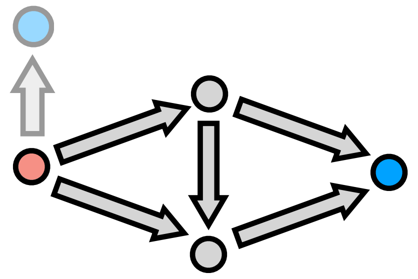
Routing Games



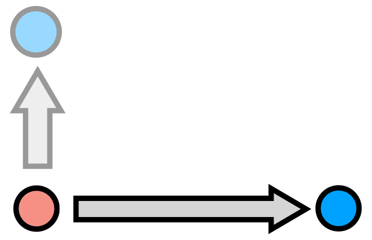
Multiple sources/ sinks



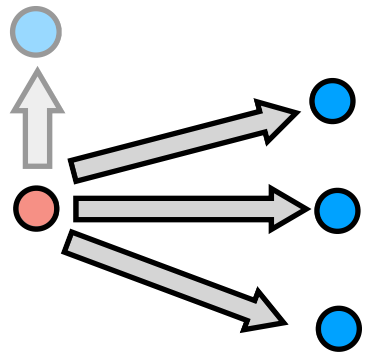
Variable Demand



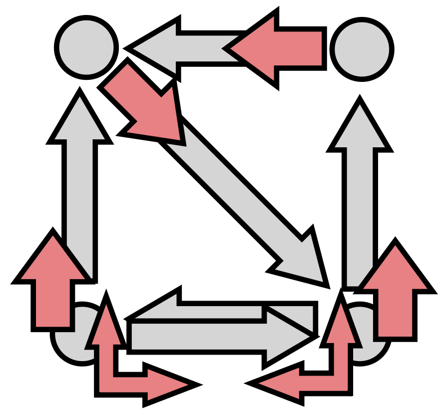
Supply & Demand



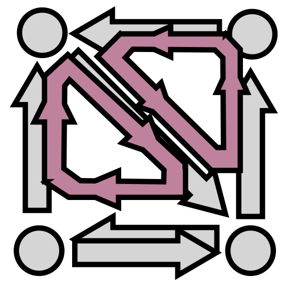
Cournot Market

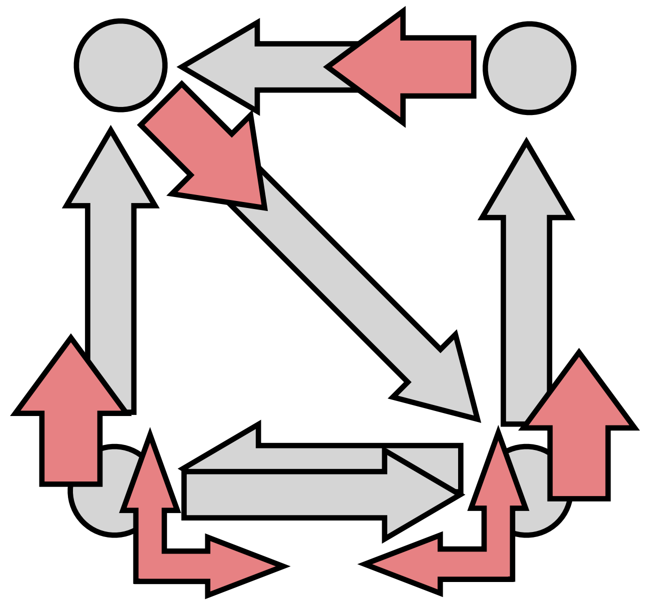


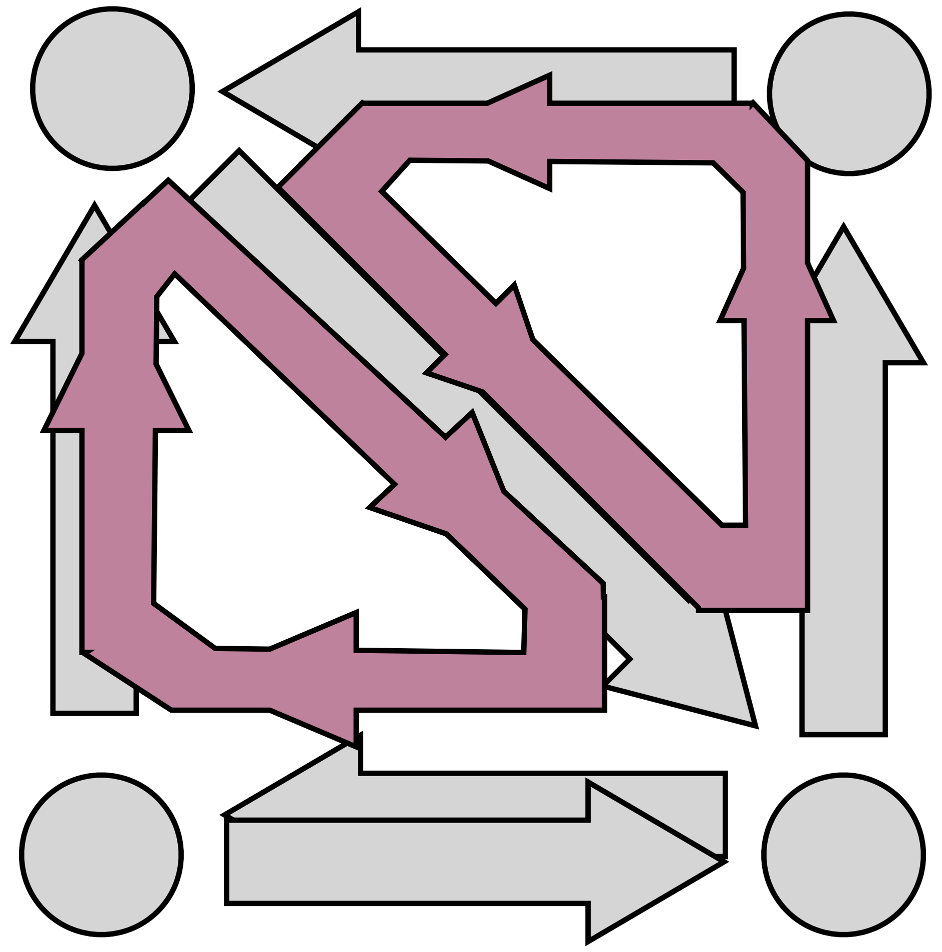
MDP Congestion Game



Markov Decision Process Congestion Game







$$\min_{x} \quad F(x)$$

s.t.

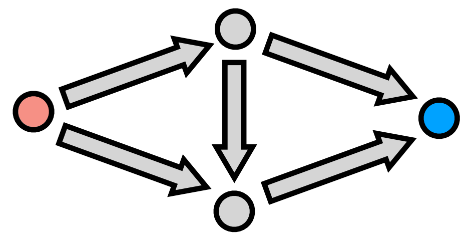
$\mathbf{1}^T x = m$ λ

$x \geq 0$ μ

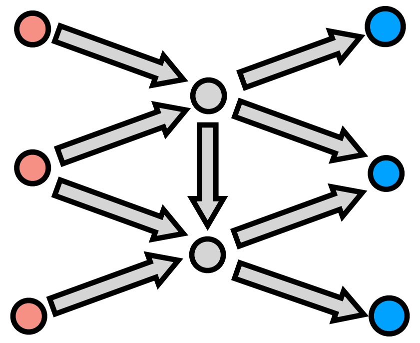
$EWx = 0$ v

Potential Games

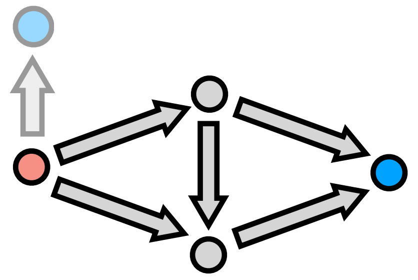
Routing Games



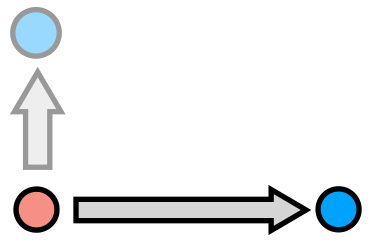
Multiple sources/ sinks



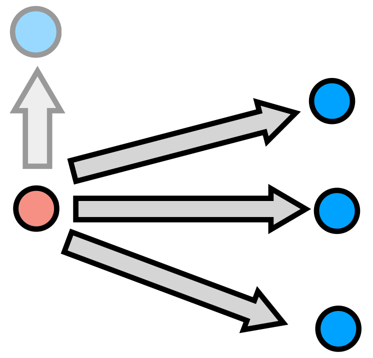
Variable Demand



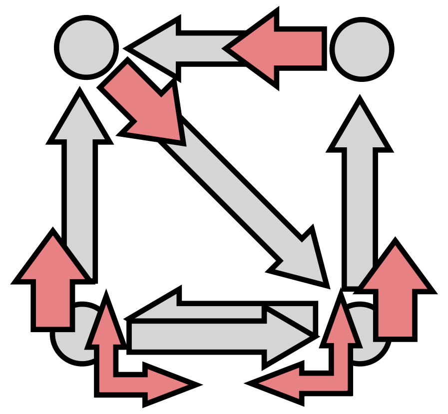
Supply & Demand



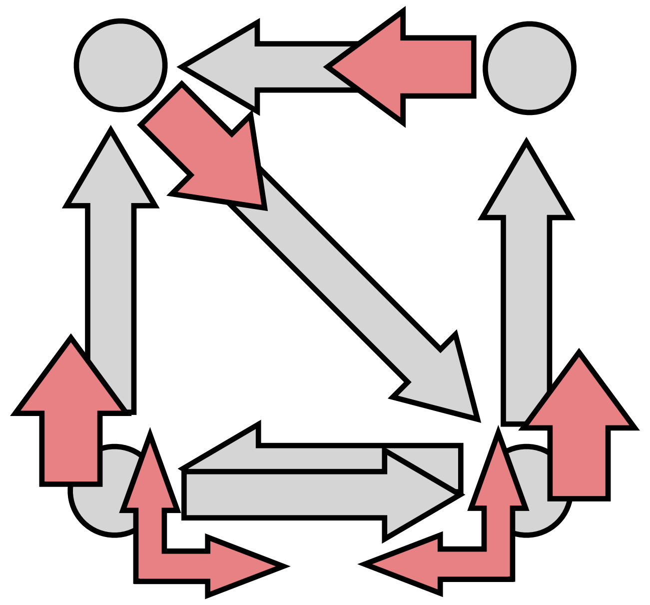
Cournot Market

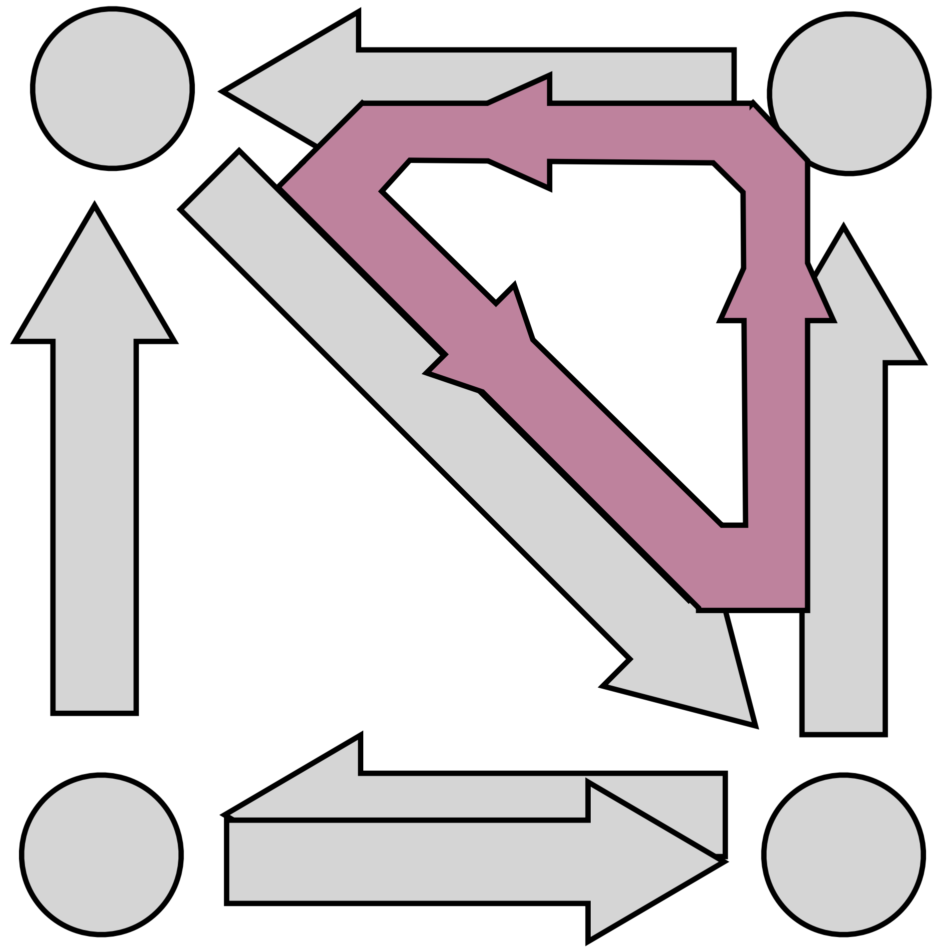


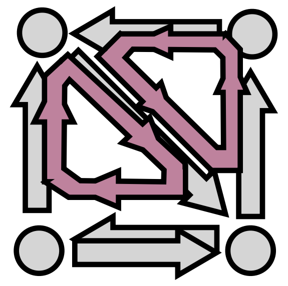
MDP Congestion Game

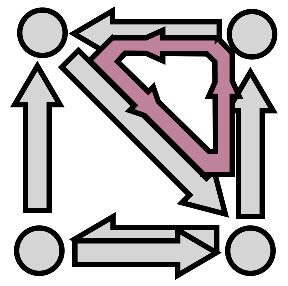


Markov Decision Process Congestion Game









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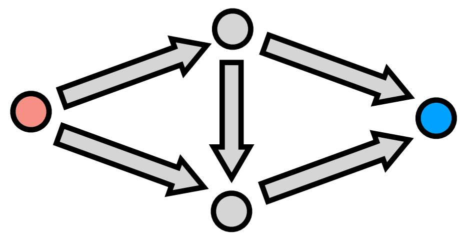
$$EWx = 0 \;\; v$$

Potential Games

Routing Games



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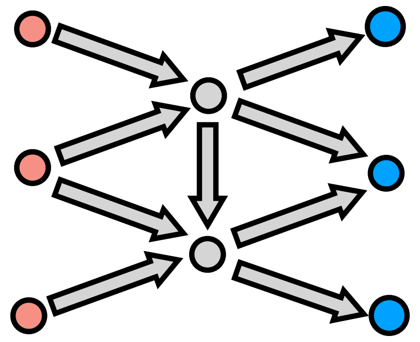
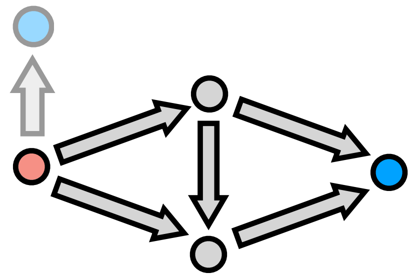
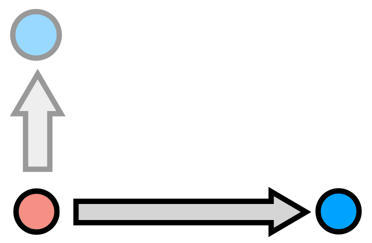


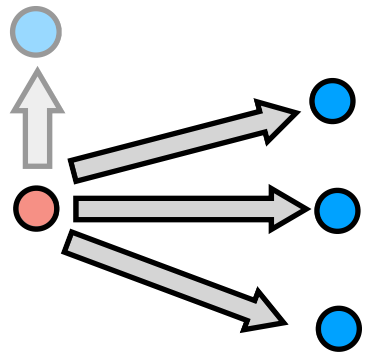
Variable Demand



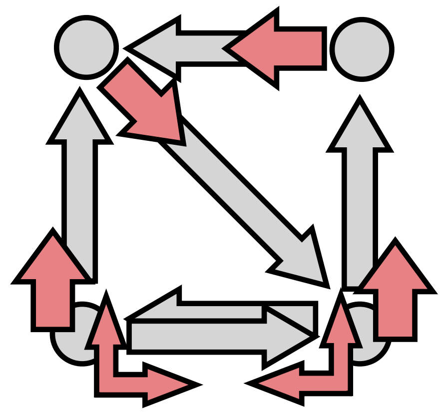
Supply & Demand



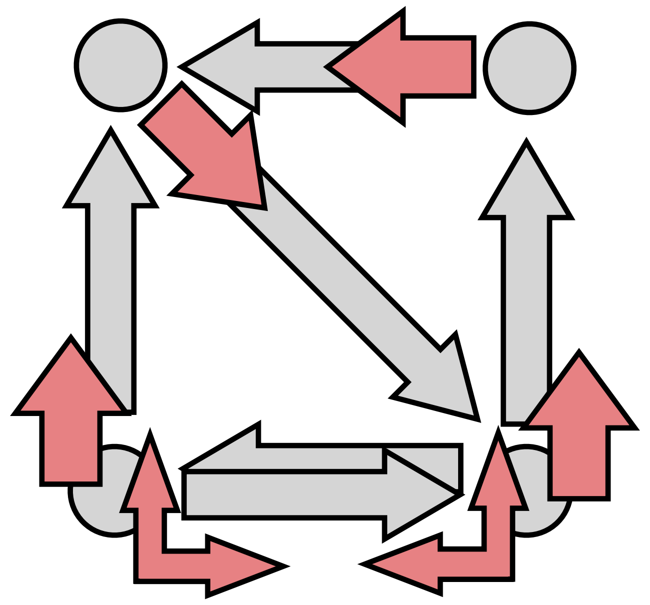
Cournot Market

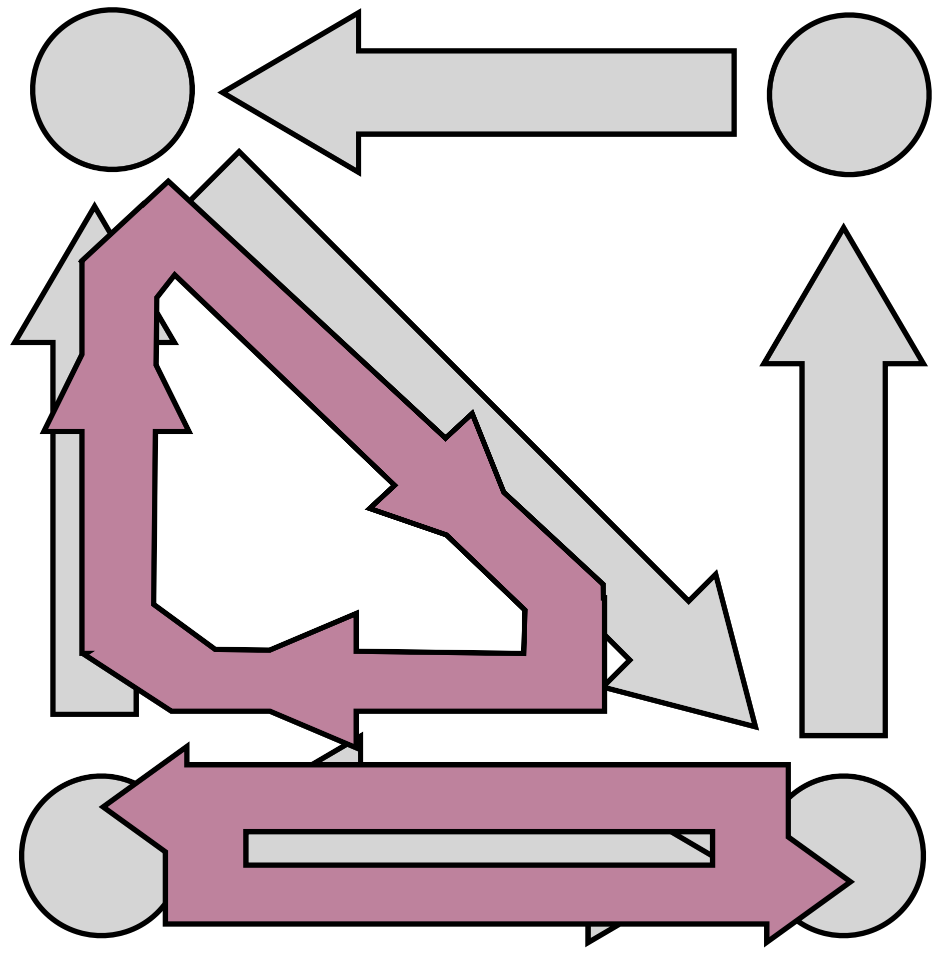


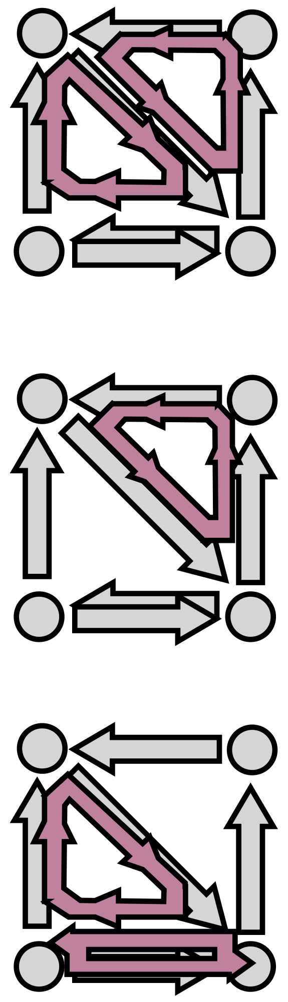
MDP Congestion Game



Markov Decision Process Congestion Game







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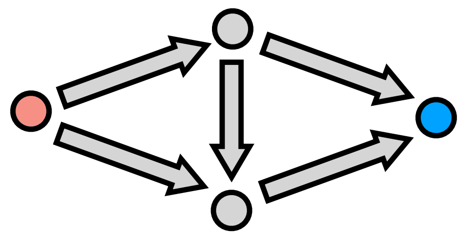
Potential Games

Routing Games



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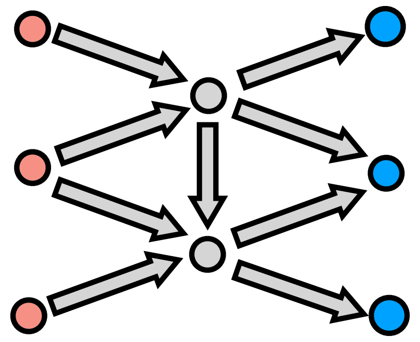
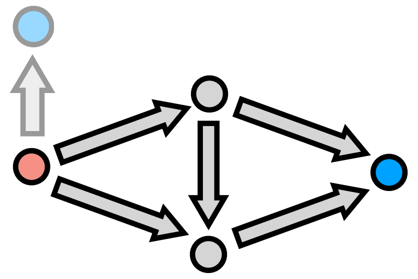
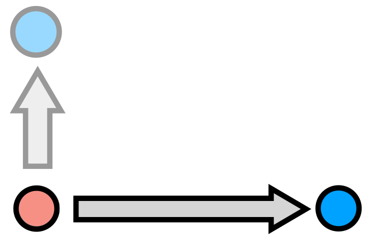


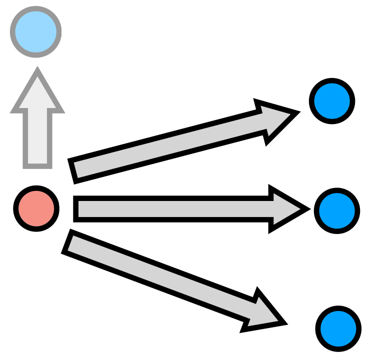
Variable Demand



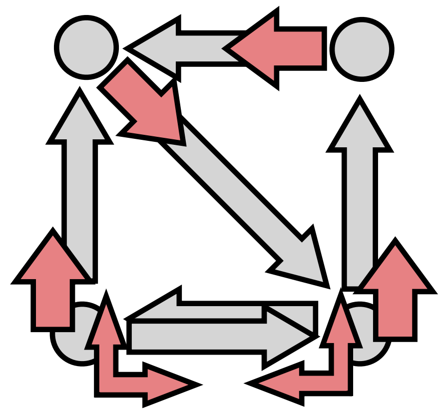
Supply & Demand



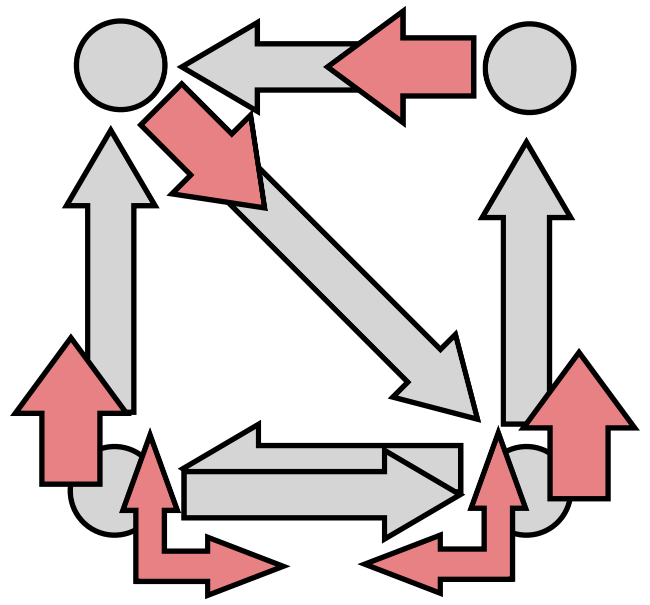
Cournot Market

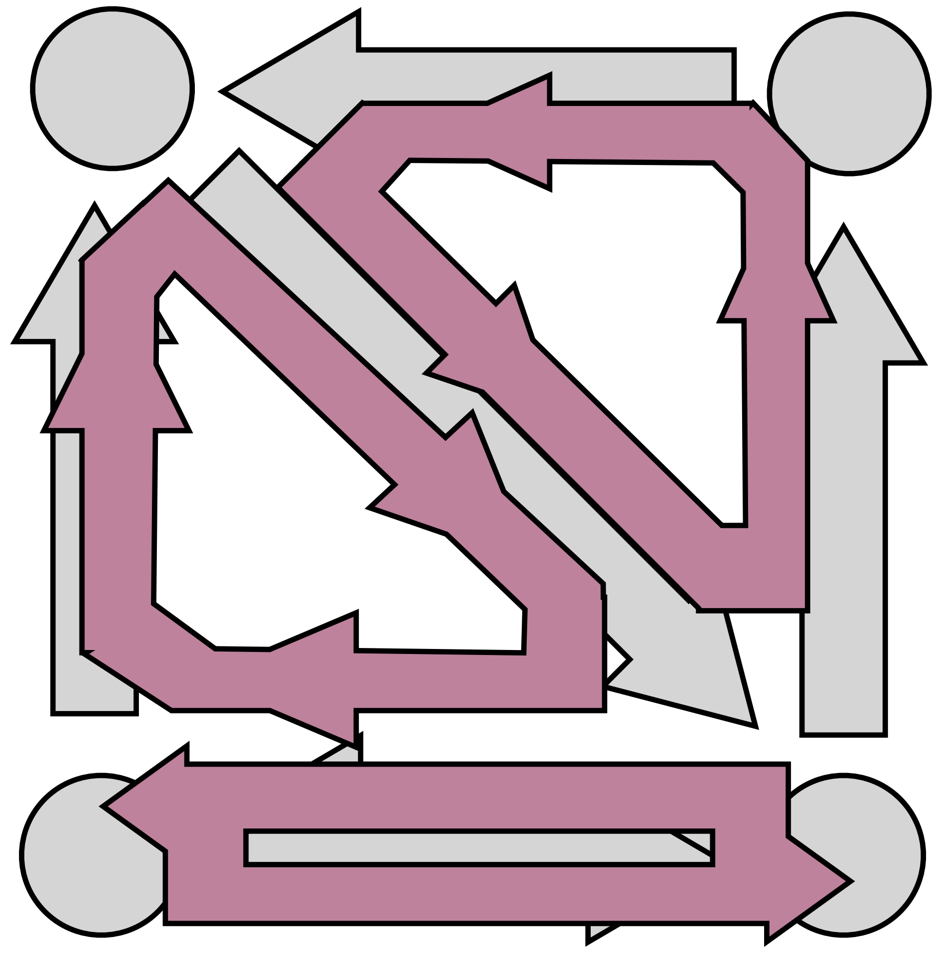


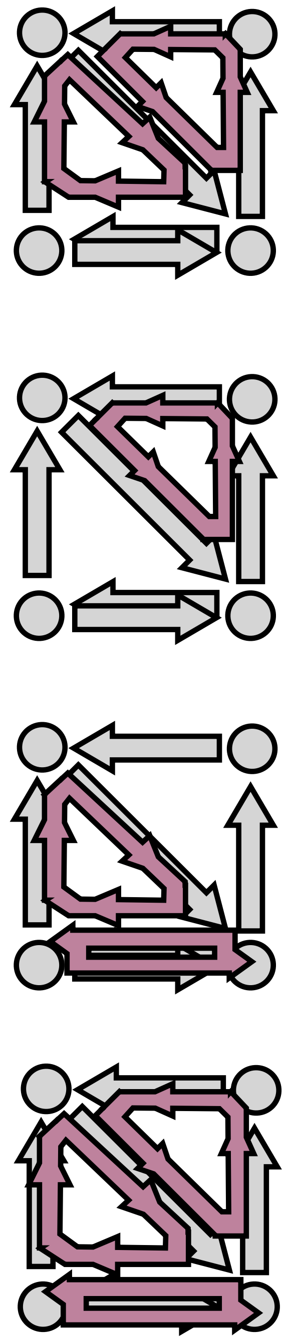
MDP Congestion Game



Markov Decision Process Congestion Game







$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \;\; (\lambda) \qquad x \geq 0 \;\; (\mu)$$

$$EWx = 0 \;\; (v)$$

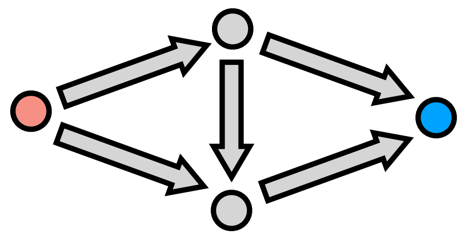
λ : average reward

μ : action inefficiency

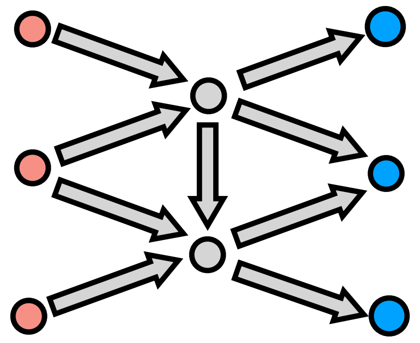
v : value function

Potential Games

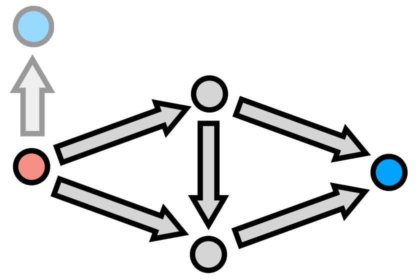
Routing Games



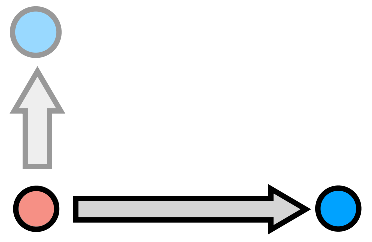
Multiple sources/ sinks



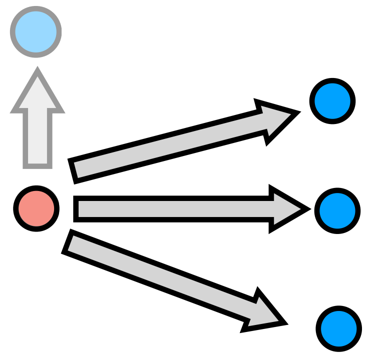
Variable Demand



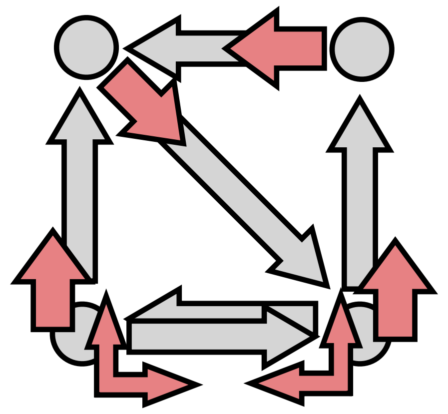
Supply & Demand



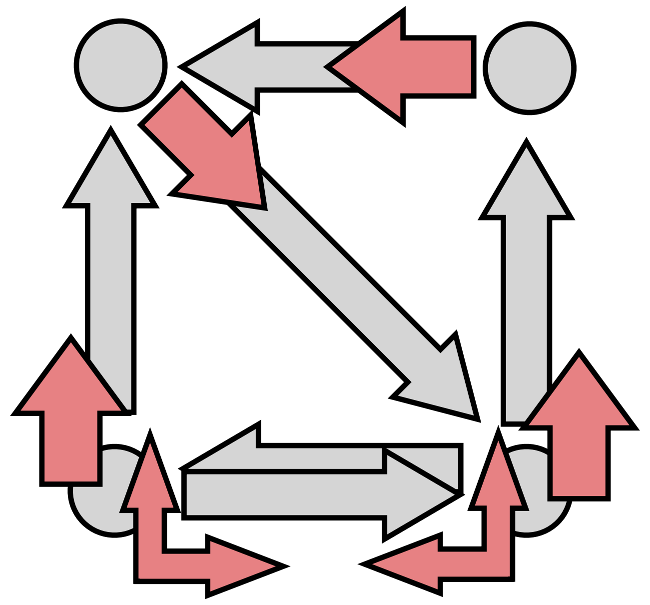
Cournot Market

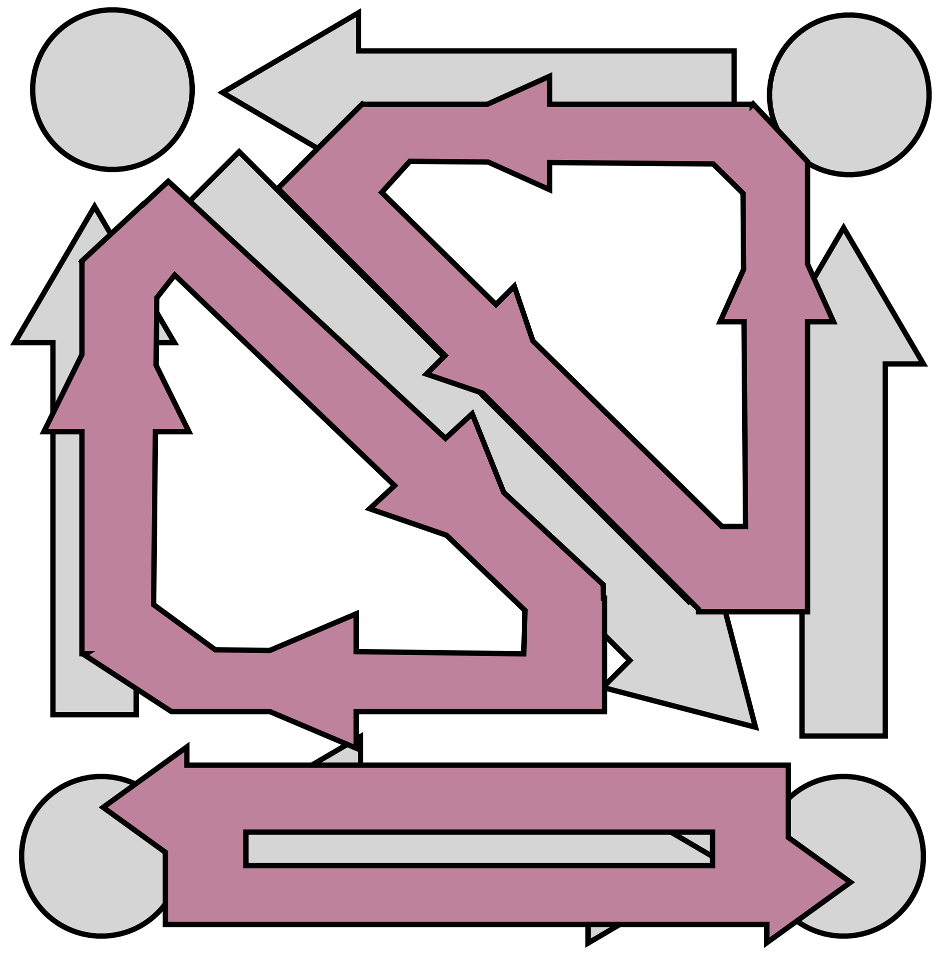


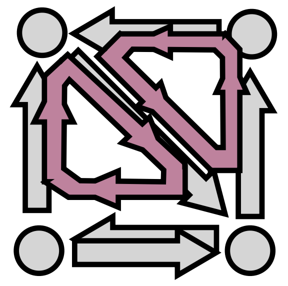
MDP Congestion Game

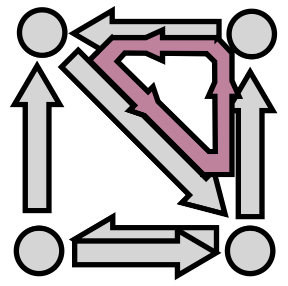


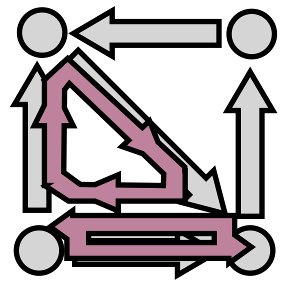
Markov Decision Process Congestion Game

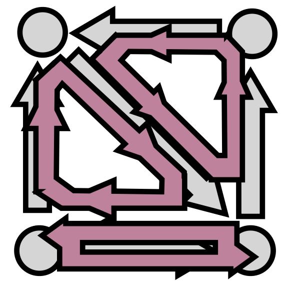












APPLICATIONS

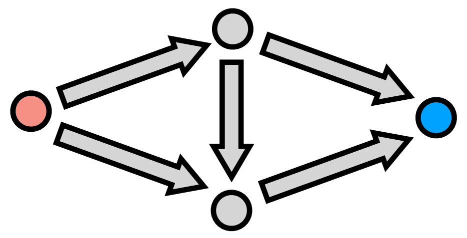
- Ride-sharing drivers planning routes
- Cars circling for street parking
- Air-traffic routing

PAPERS

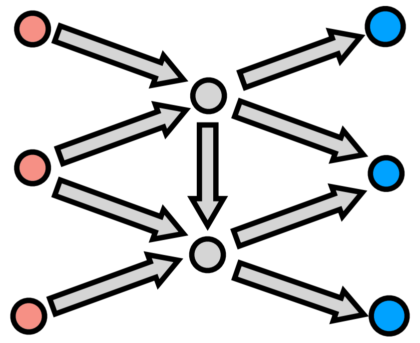
- Markov decision process routing games **[Calderone, Sastry, 2017]**
- Infinite horizon average cost Markov decision process routing games **[Calderone, Sastry, 2017]**
- Adaptive constraint satisfaction for Markov decision process congestion games: Applications to transportation networks **[Li, Calderone, Ratliff, et al. 2021]**
- Variable demand and multi-commodity flow in Markovian network equilibrium **[Yu, Calderone, Ratliff, et al. 2021]**

Potential Games

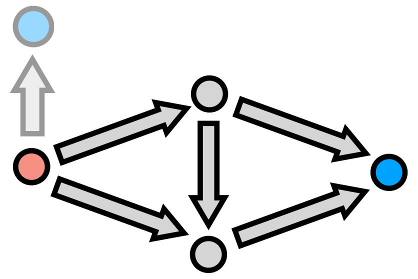
Routing Games



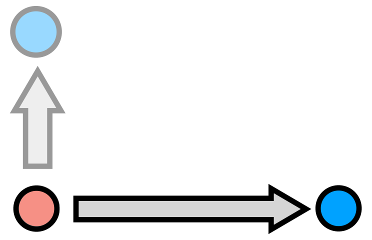
Multiple sources/ sinks



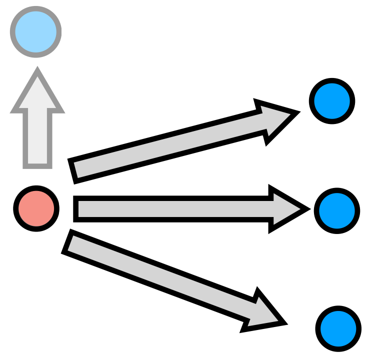
Variable Demand



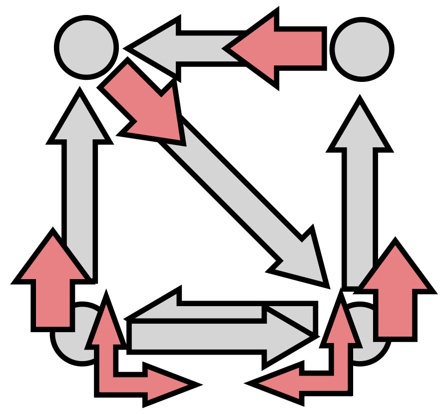
Supply & Demand



Cournot Market

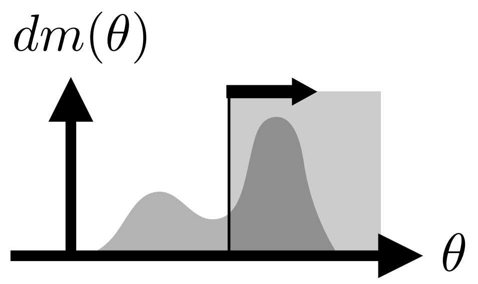


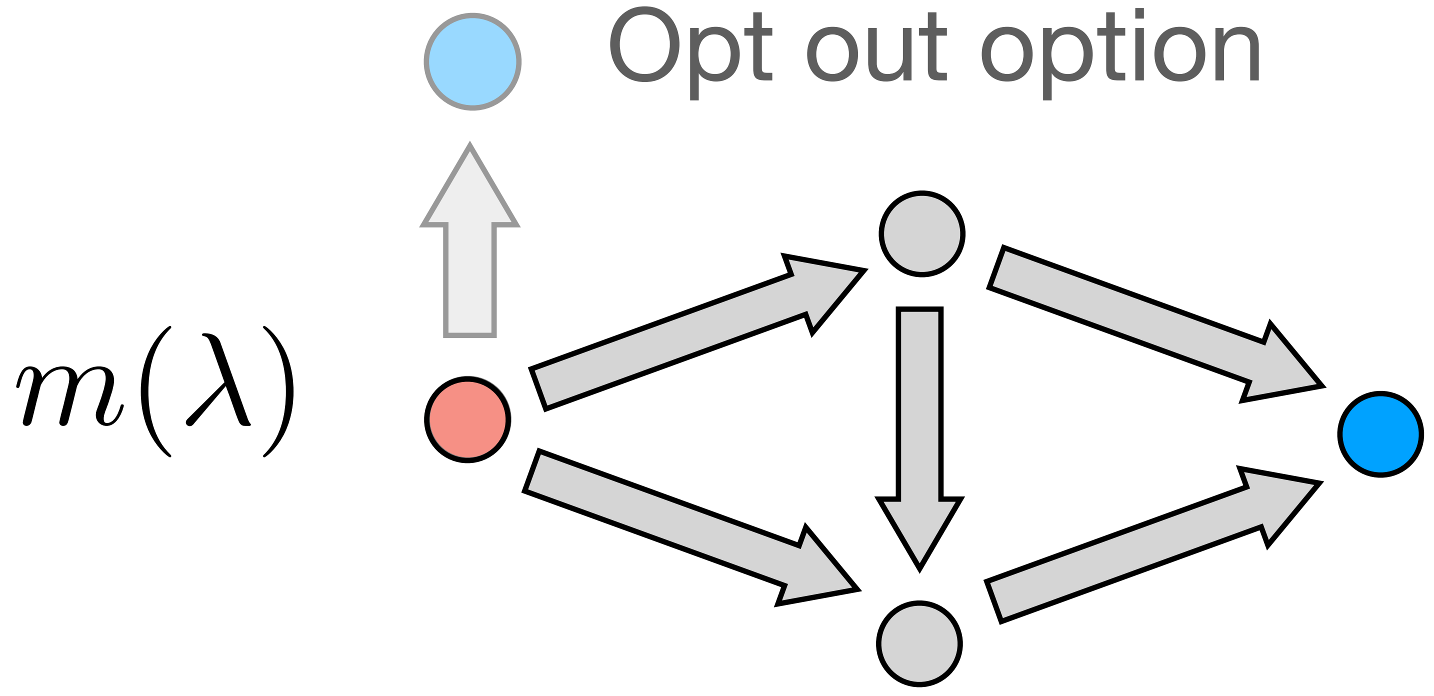
MDP Congestion Game

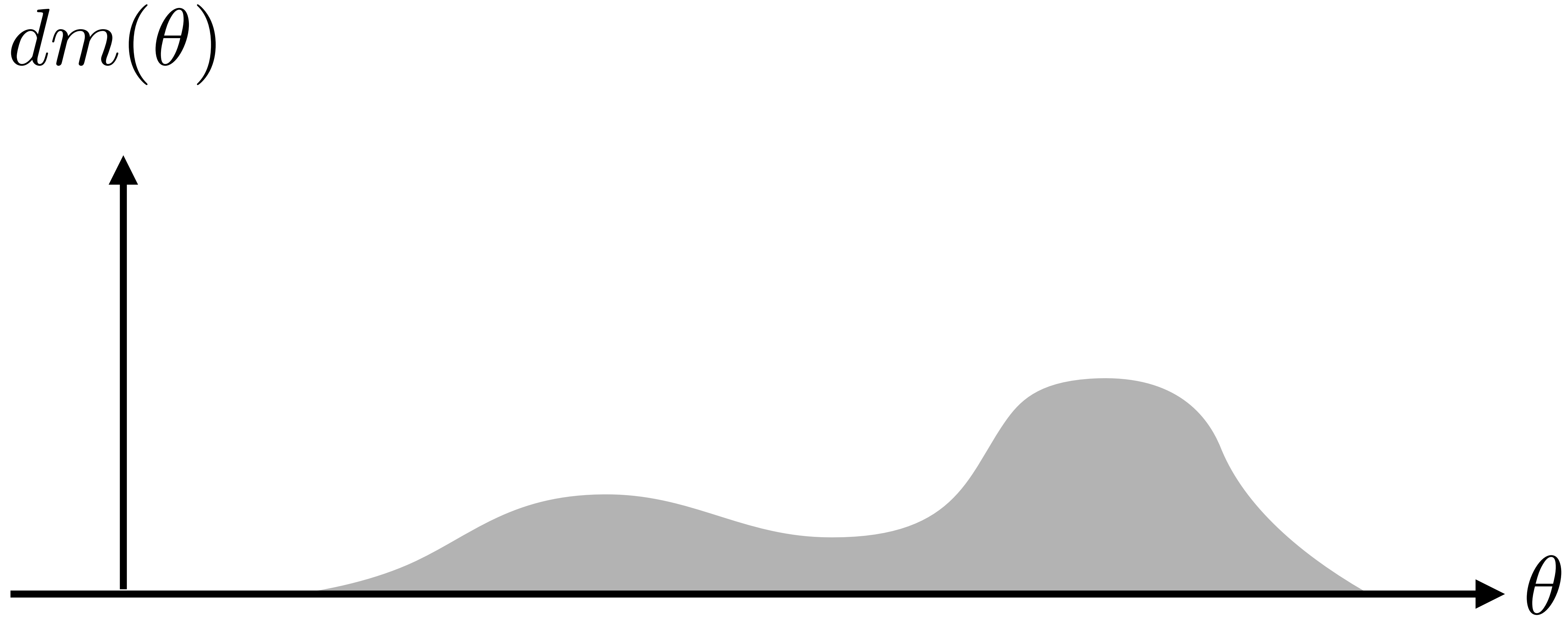


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences



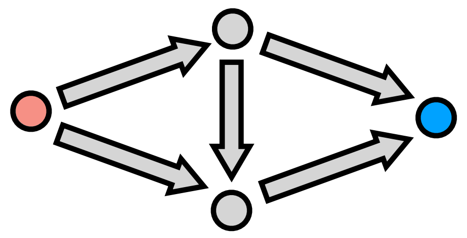




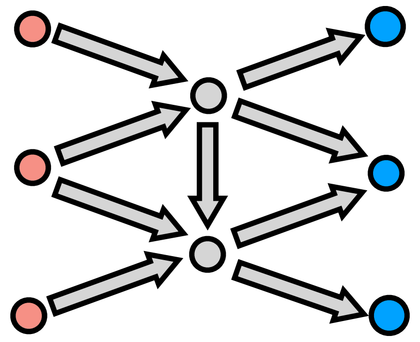
θ : Max cost $dm(\theta)$ will pay

Potential Games

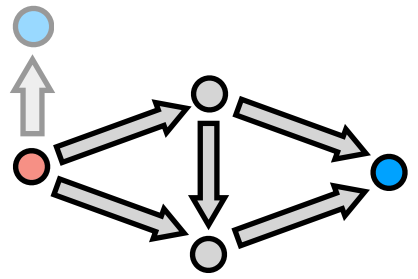
Routing Games



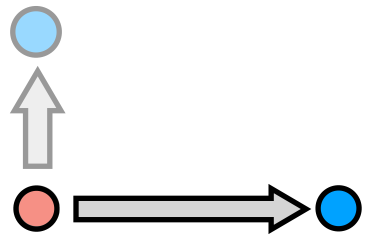
Multiple sources/ sinks



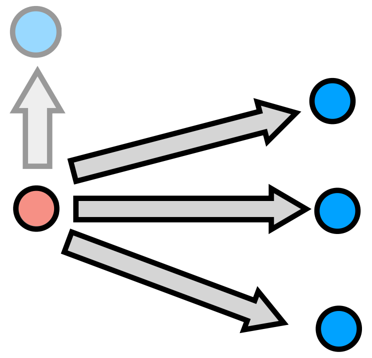
Variable Demand



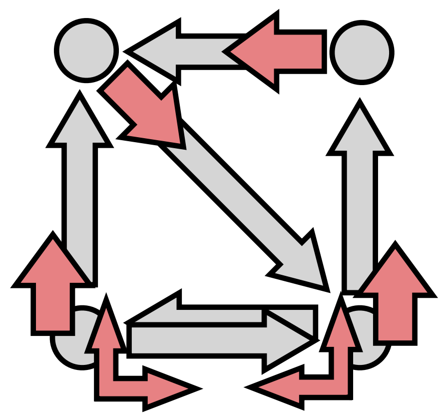
Supply & Demand



Cournot Market

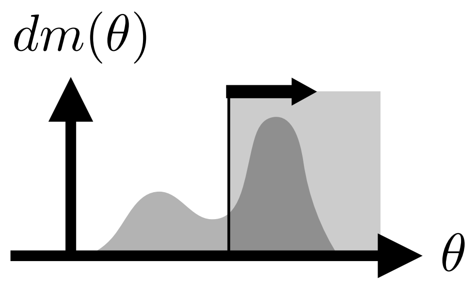


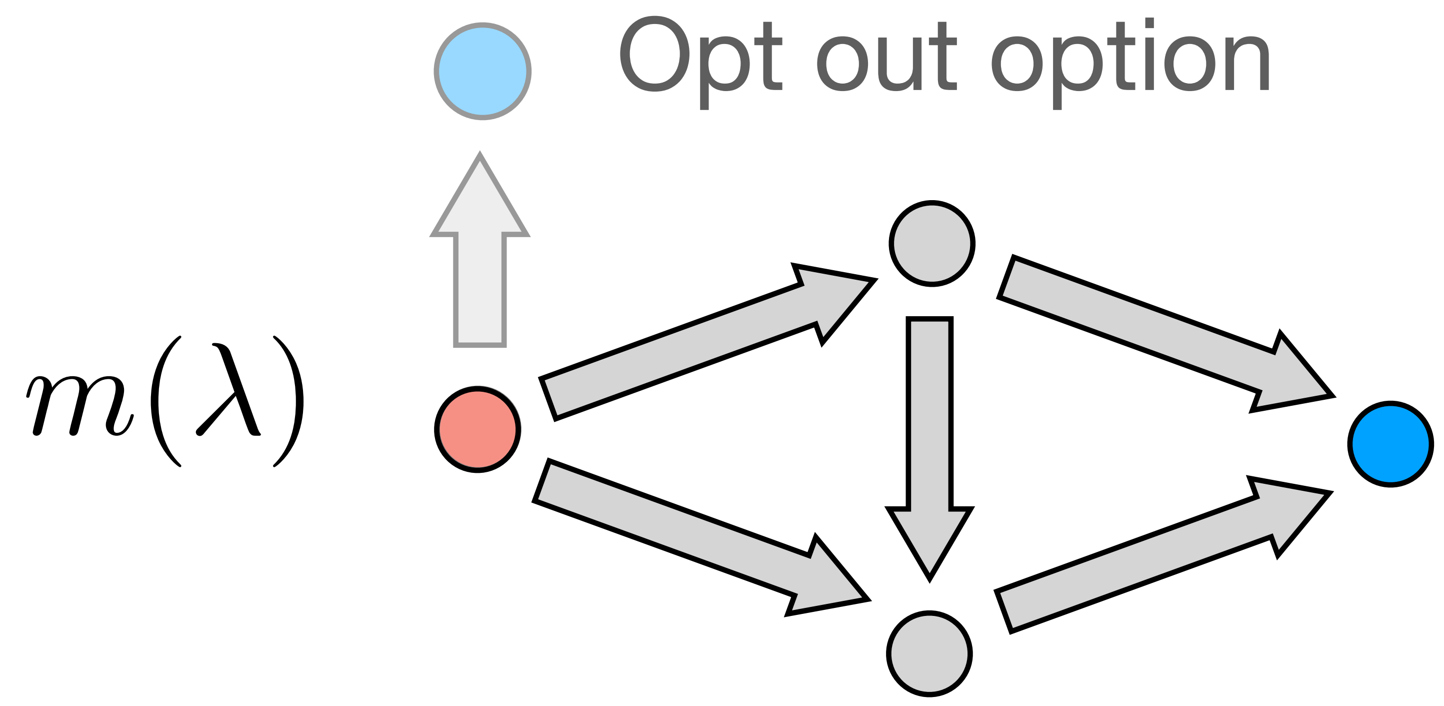
MDP Congestion Game

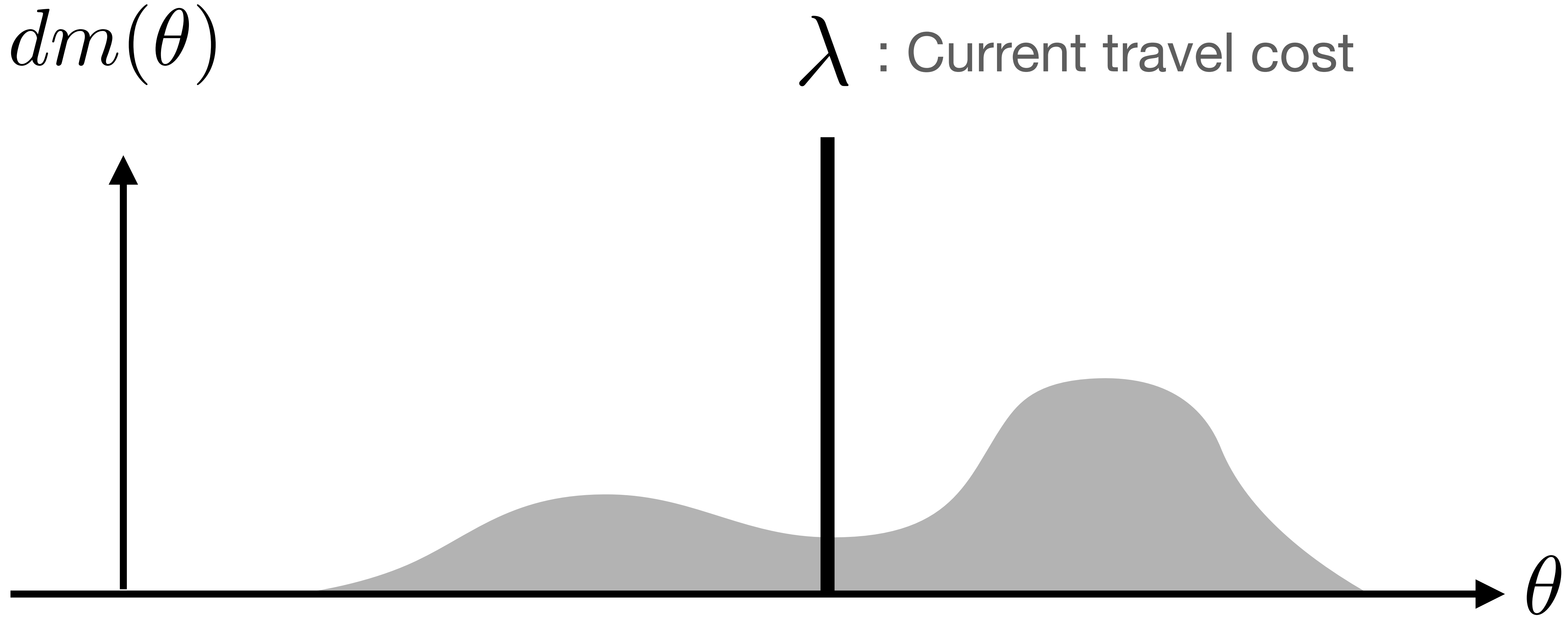


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences







θ : Max cost $dm(\theta)$ will pay

Potential Games

Routing Games

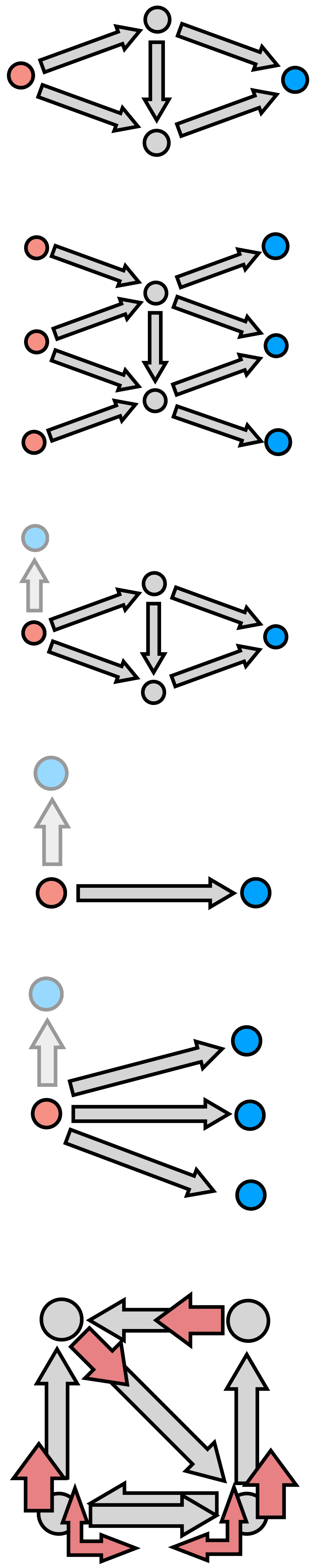
Multiple sources/ sinks

Variable Demand

Supply & Demand

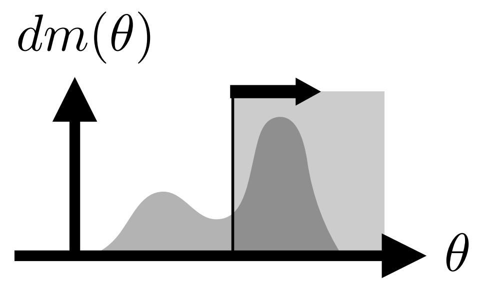
Cournot Market

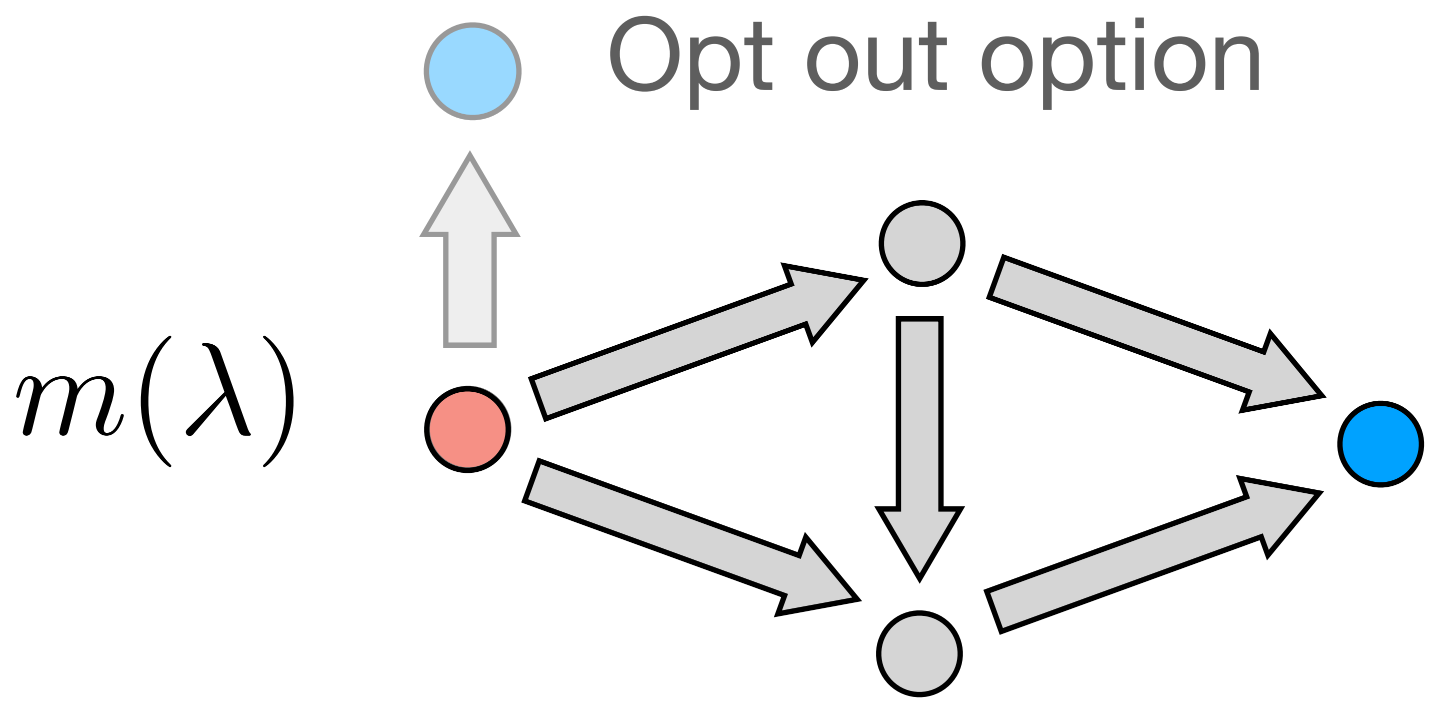
MDP Congestion Game

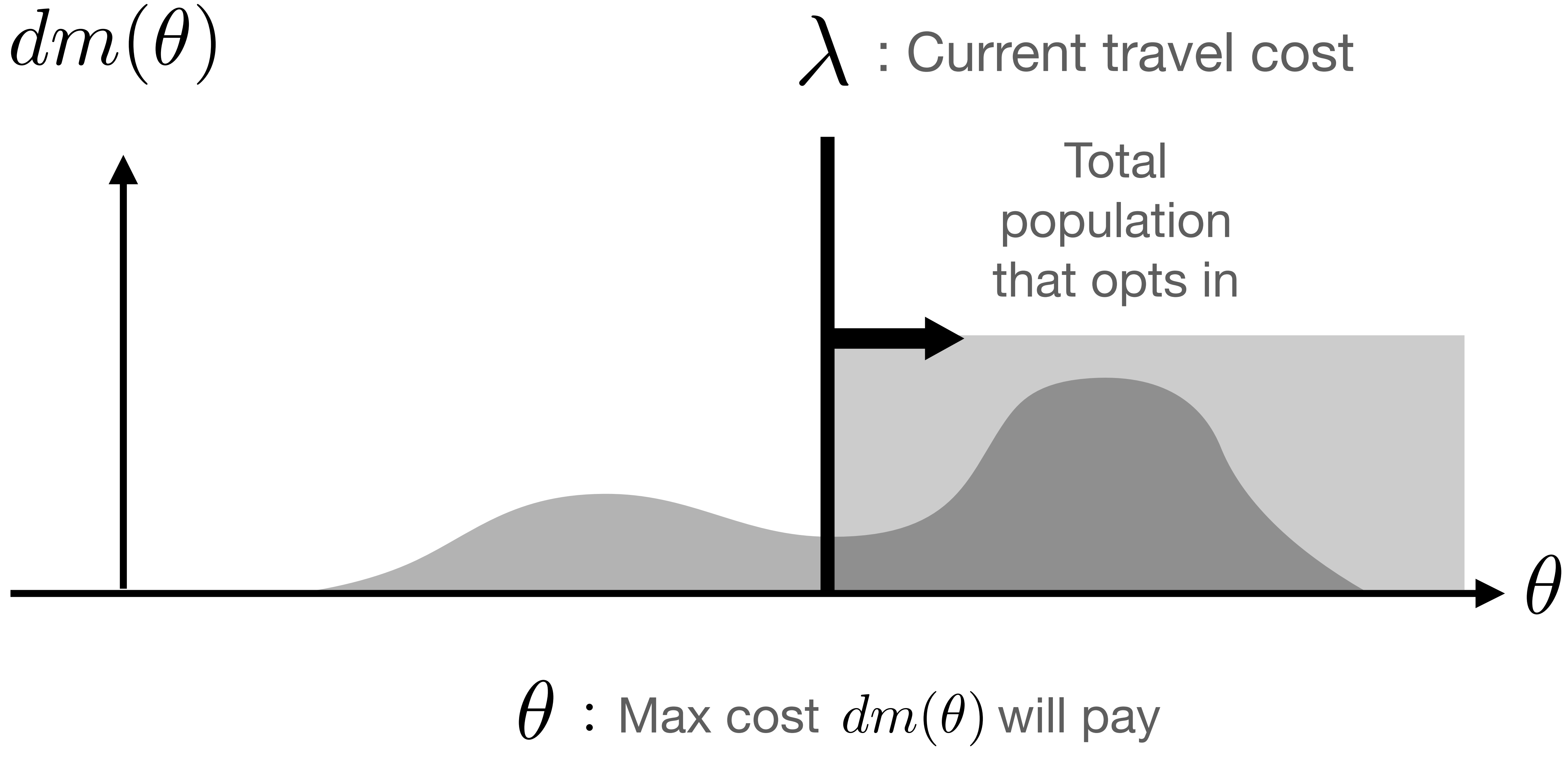


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences

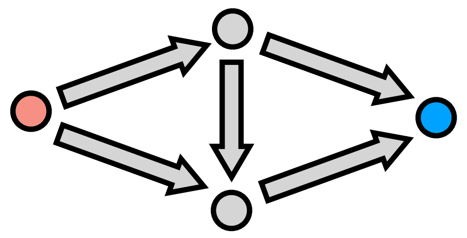






Potential Games

Routing Games



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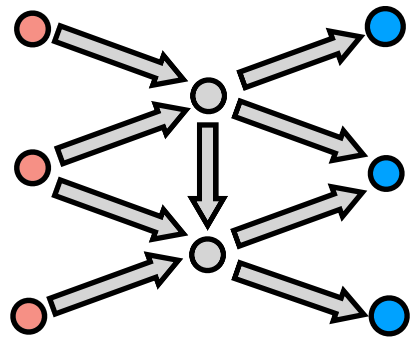
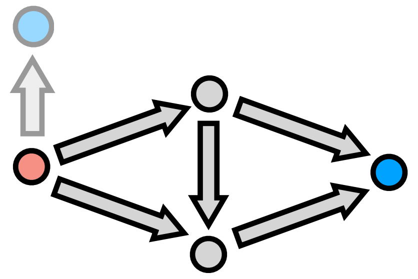
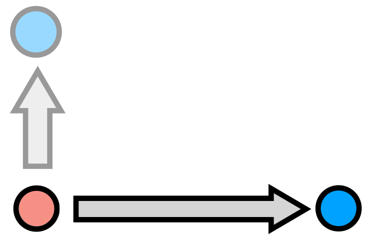


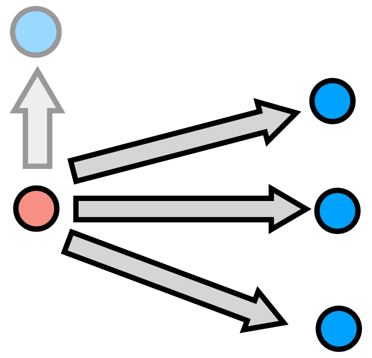
Variable Demand



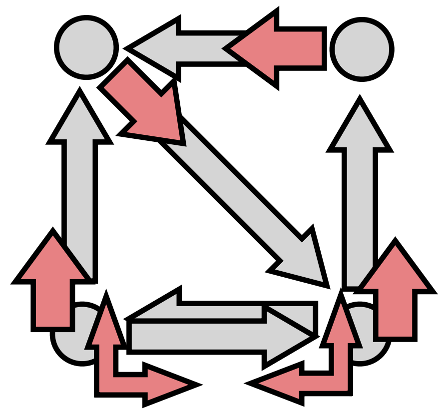
Supply & Demand



Cournot Market

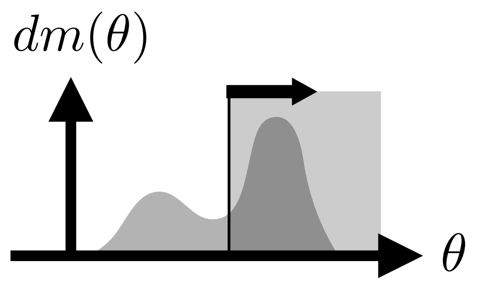


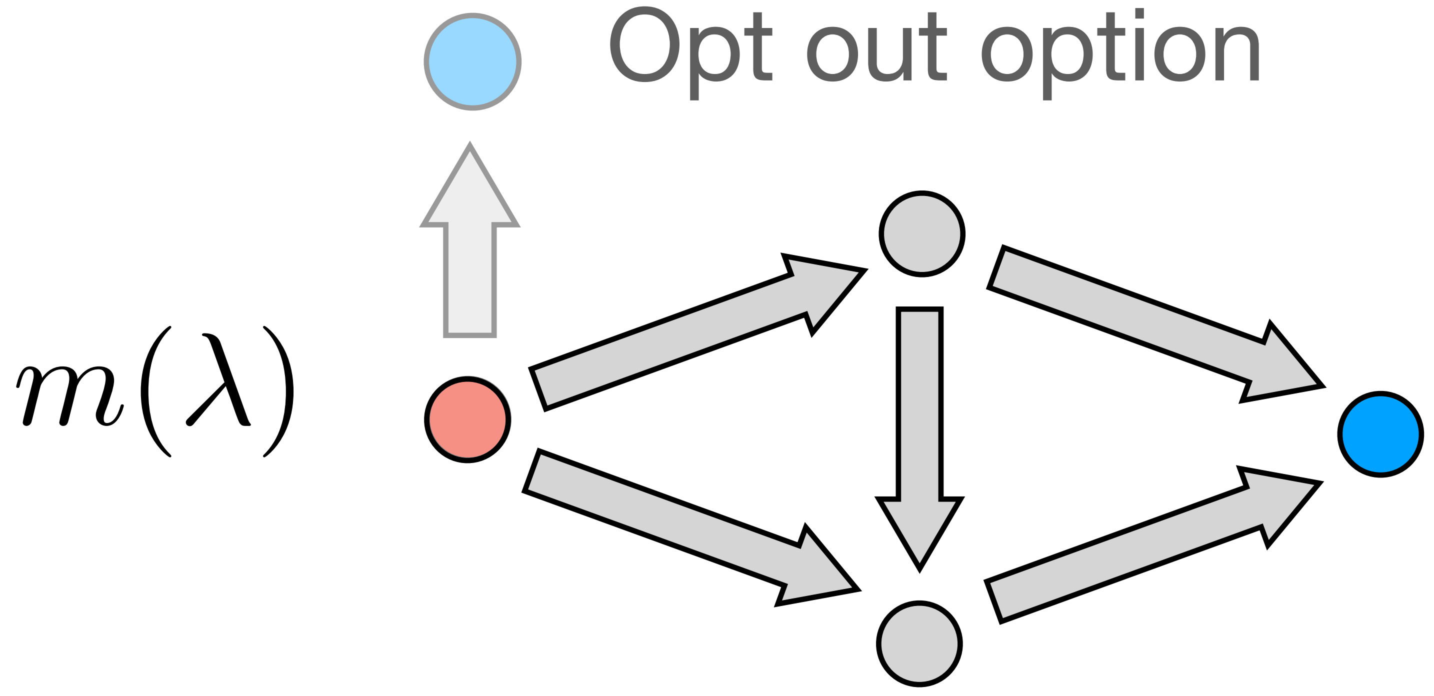
MDP Congestion Game

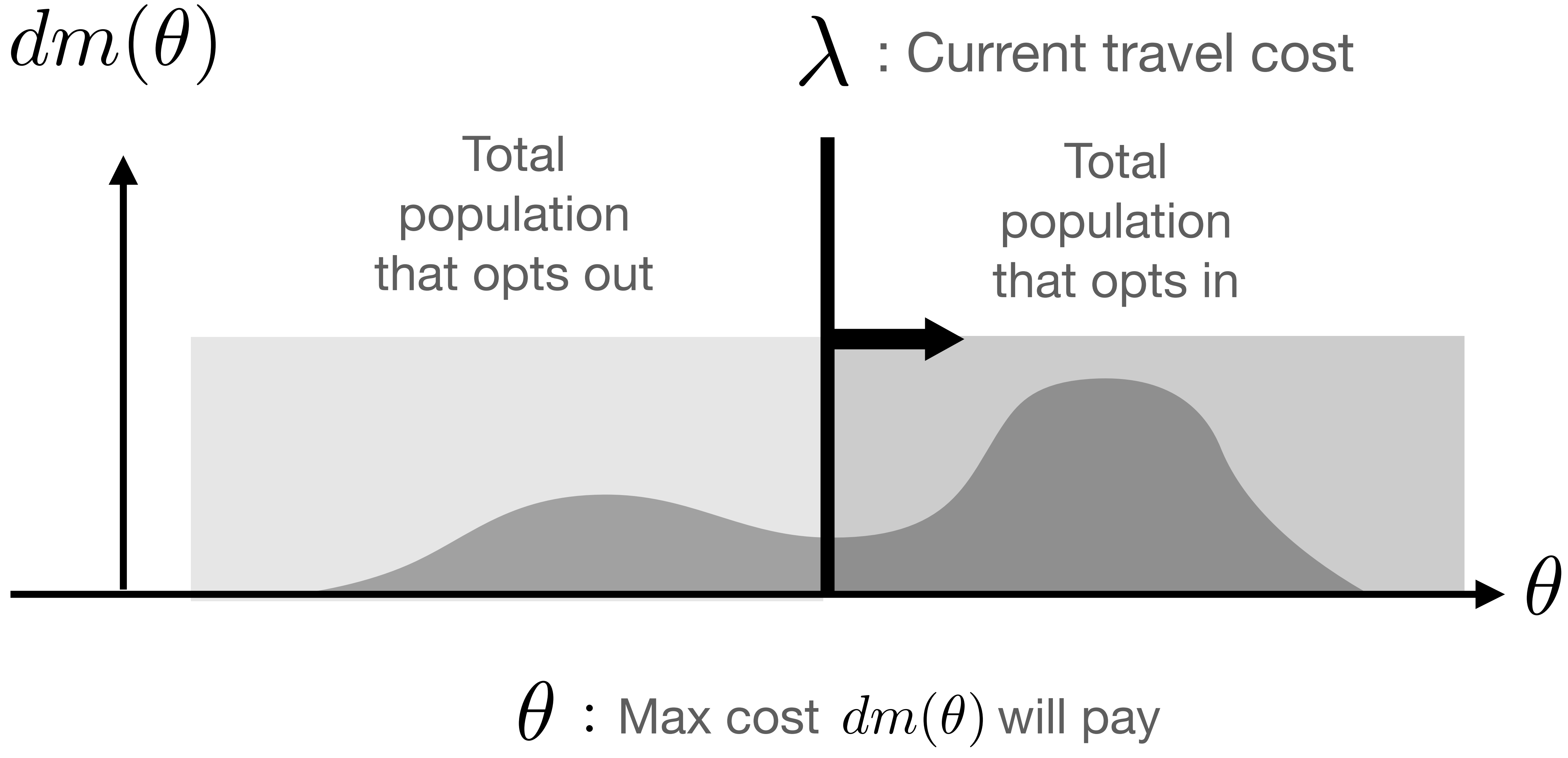


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences

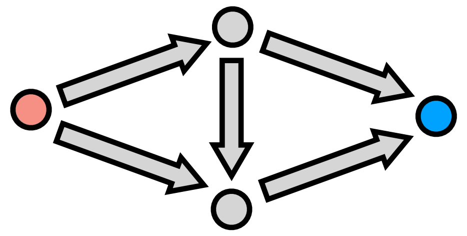




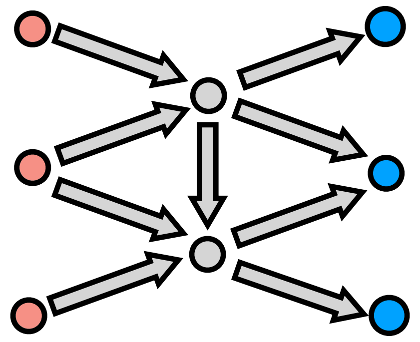


Potential Games

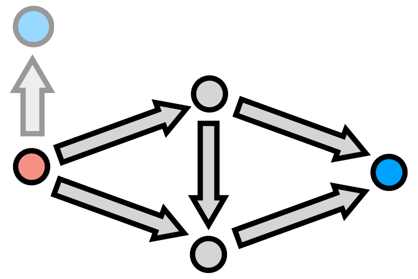
Routing Games



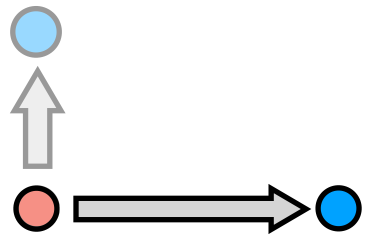
Multiple sources/ sinks



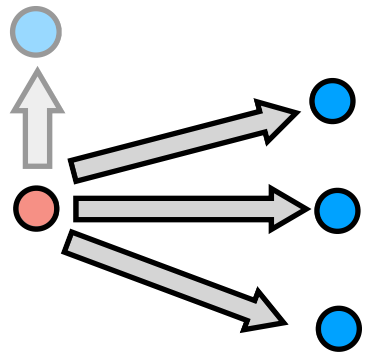
Variable Demand



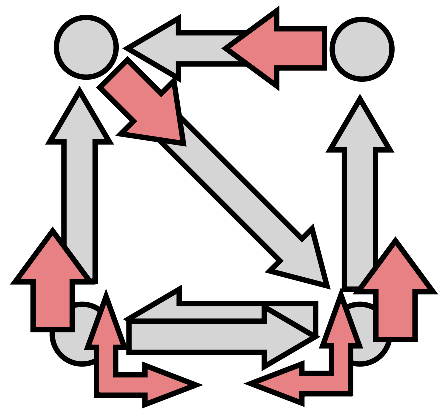
Supply & Demand



Cournot Market

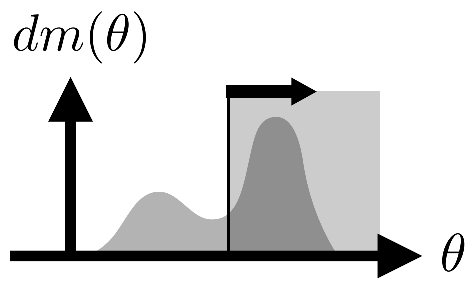


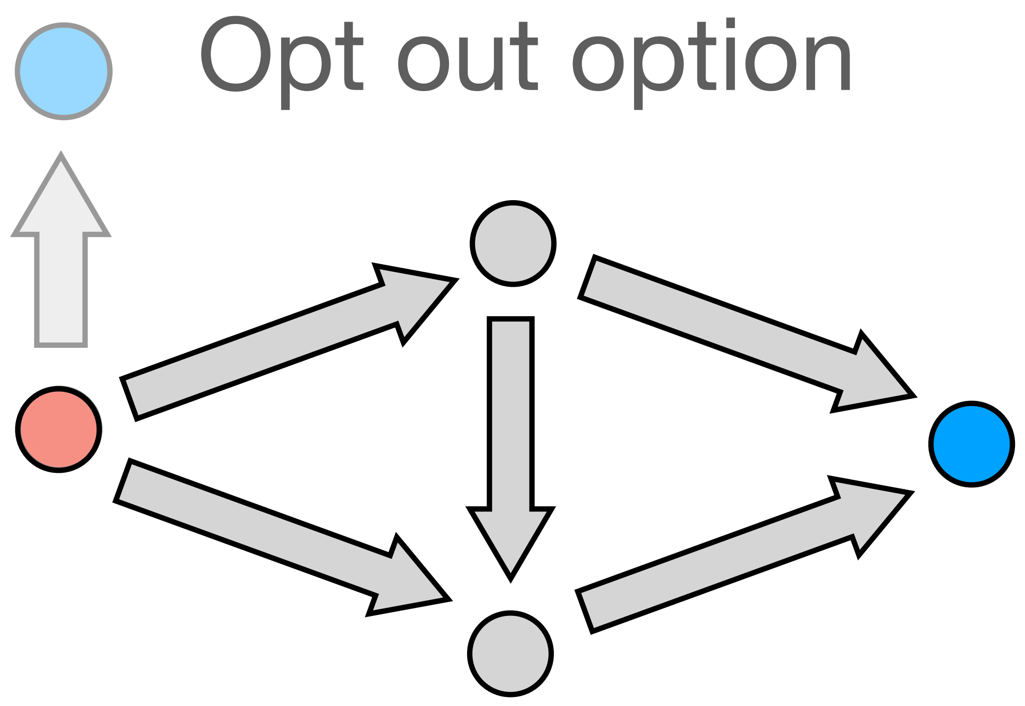
MDP Congestion Game



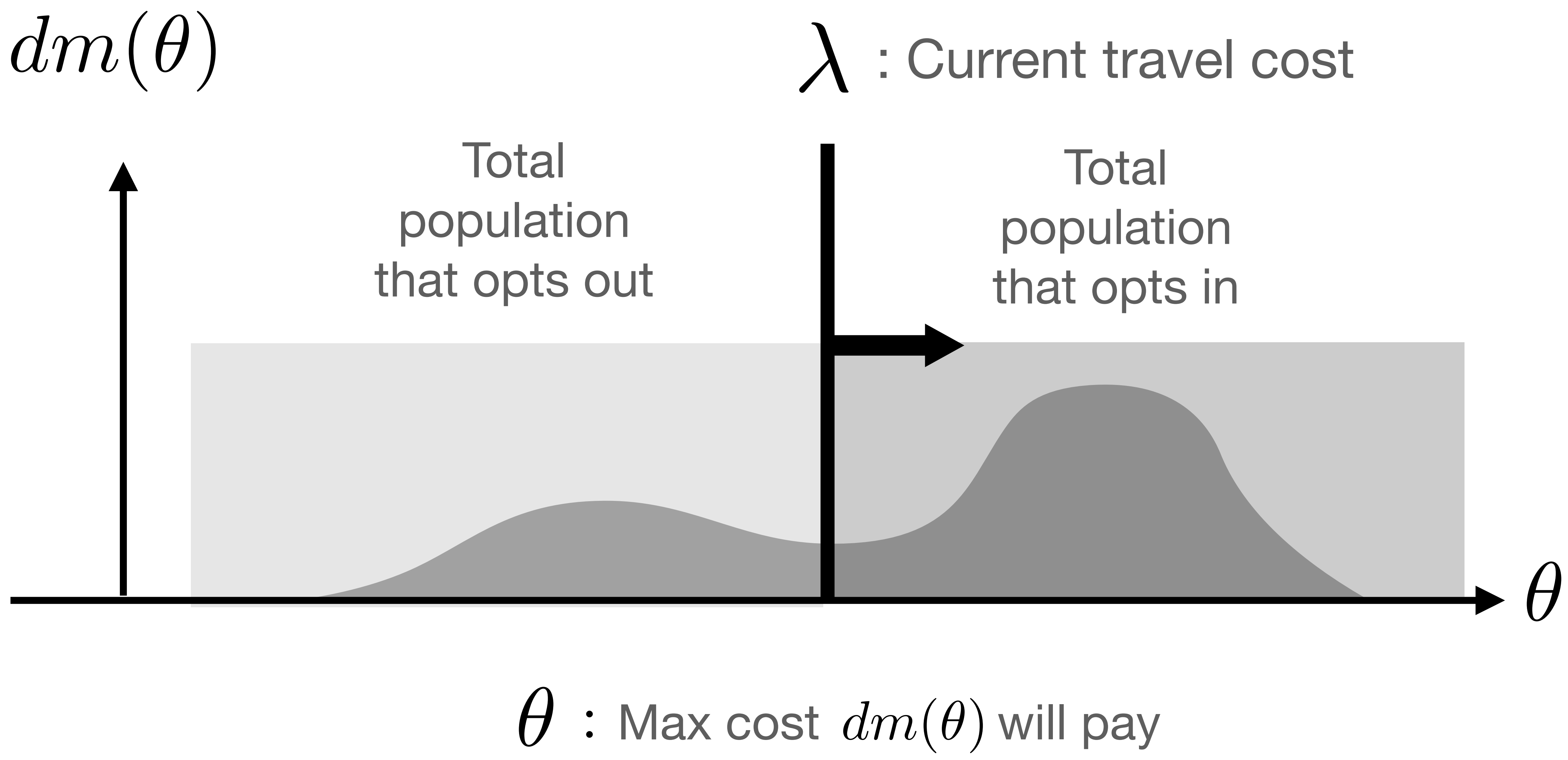
Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences



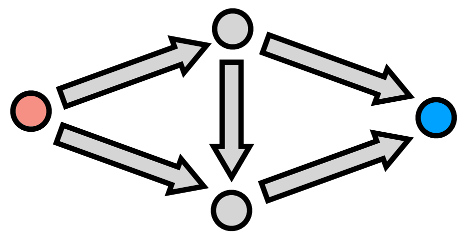


$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$



Potential Games

Routing Games



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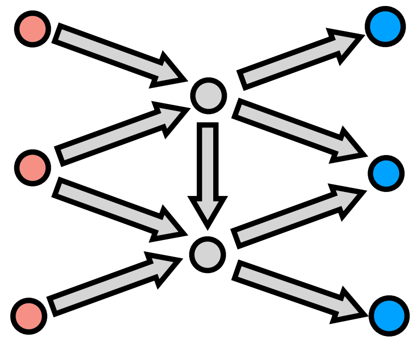
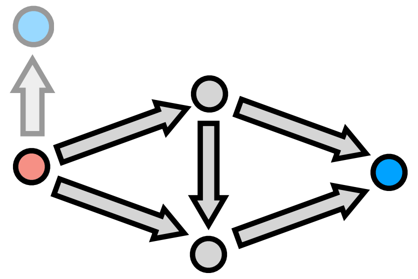
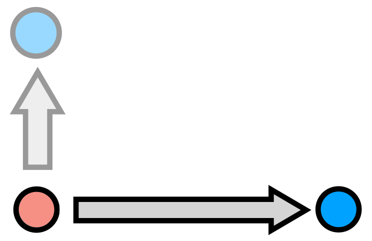


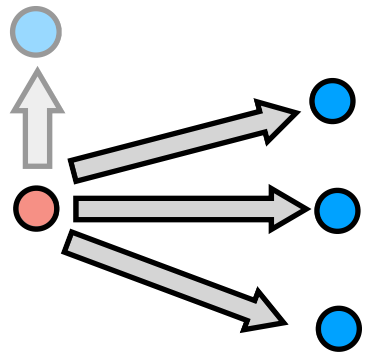
Variable Demand



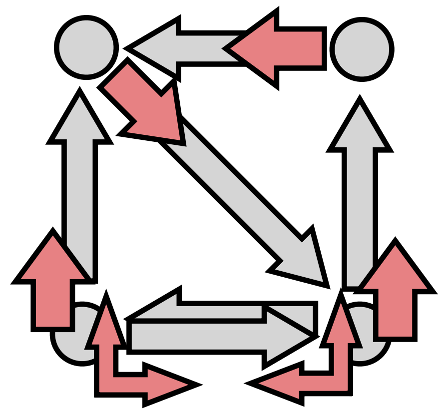
Supply & Demand



Cournot Market

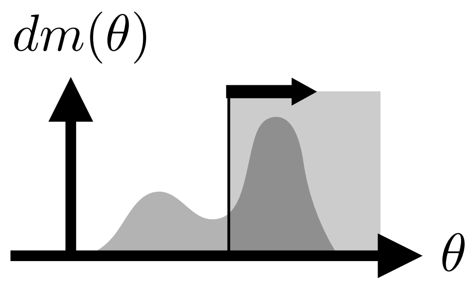


MDP Congestion Game



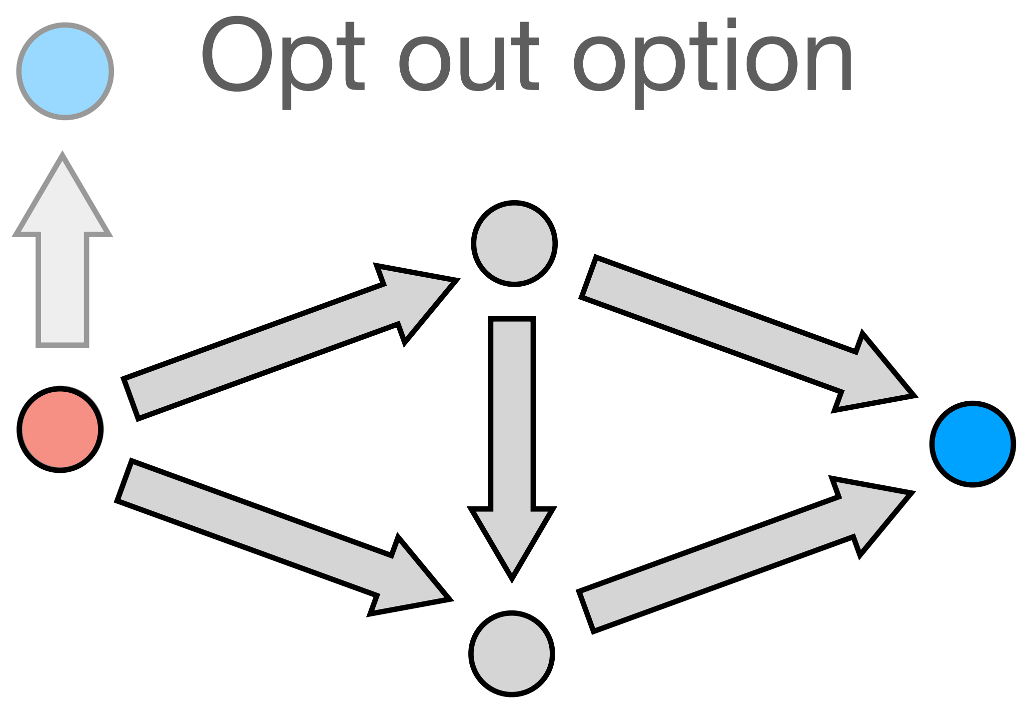
Variable Demand - Non-Homogeneous Preferences

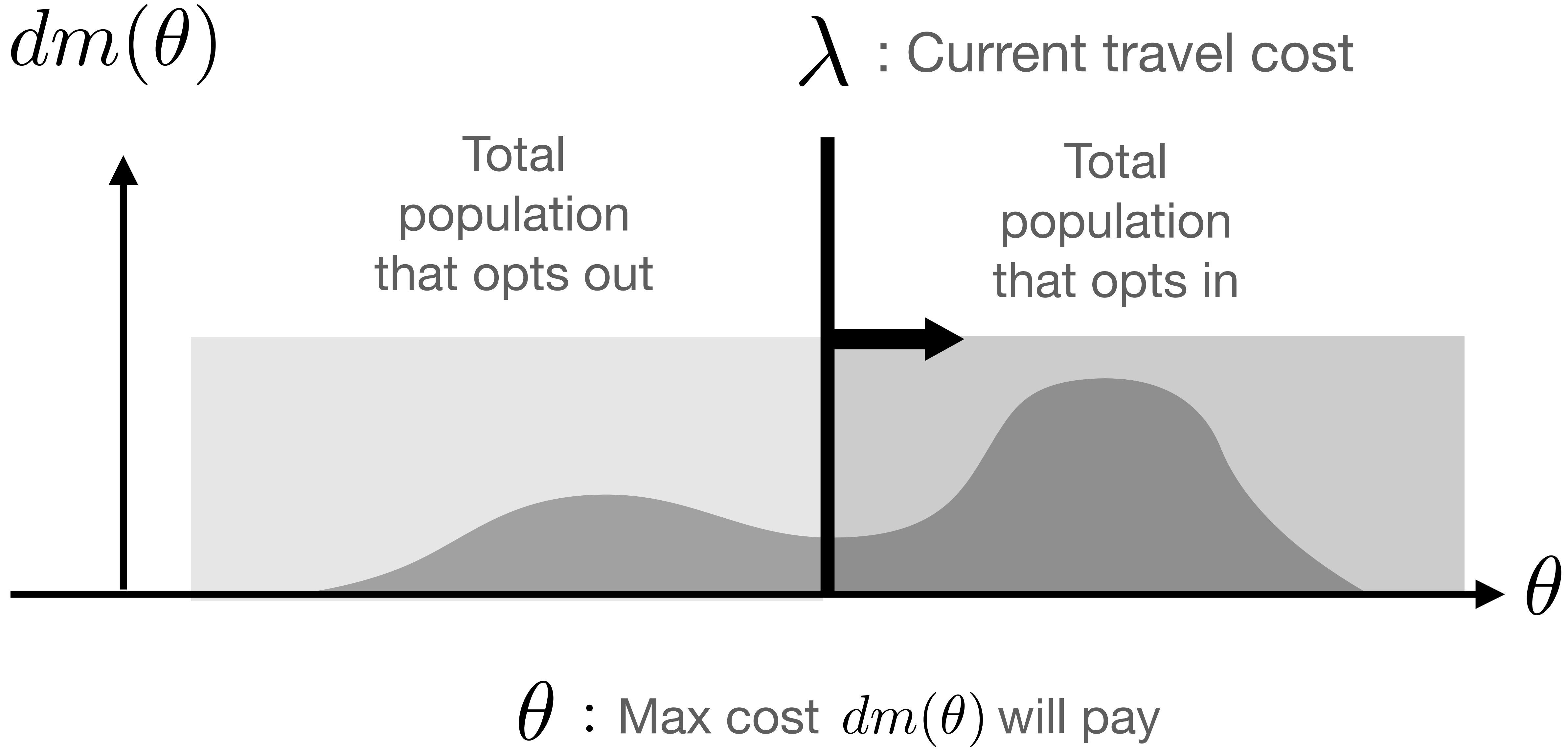
Non-homo-geneous prefences



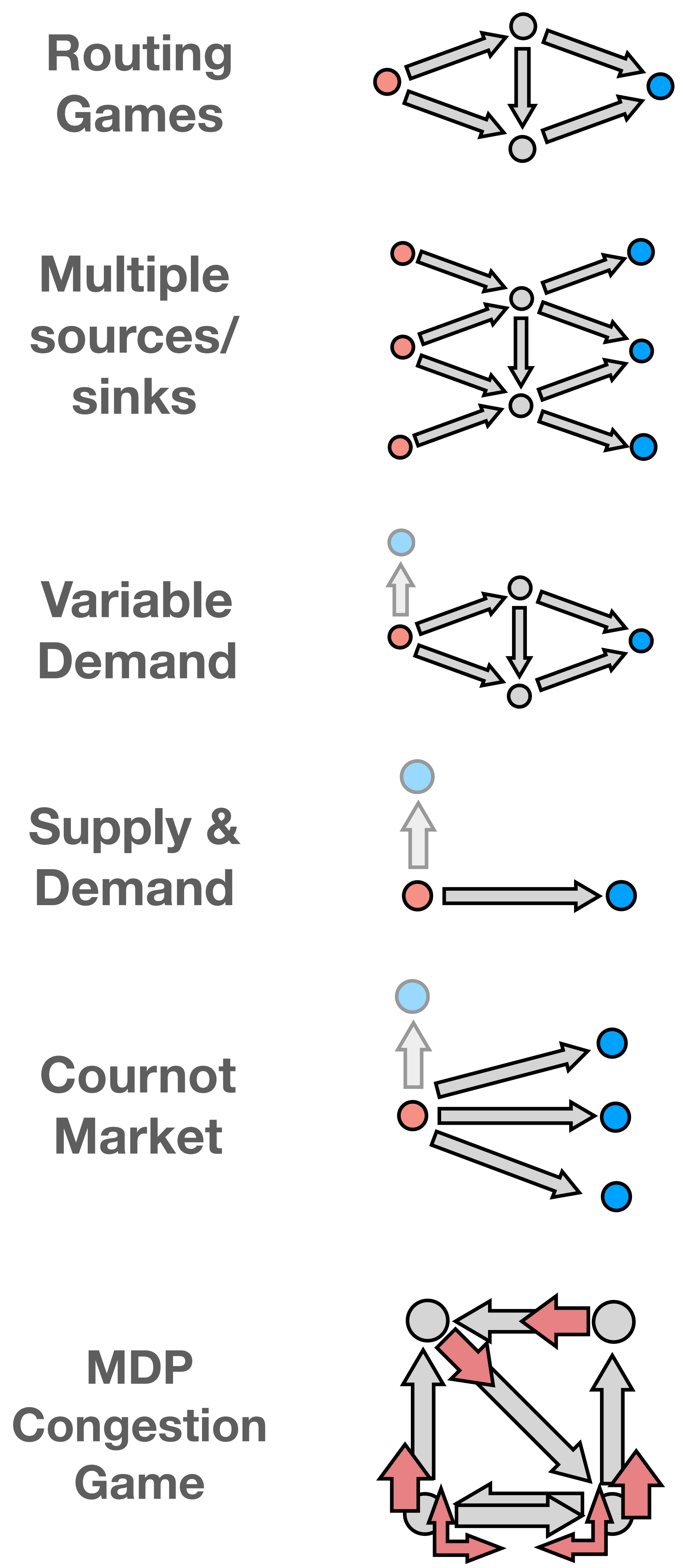
Multi-Variate Preferences

$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$

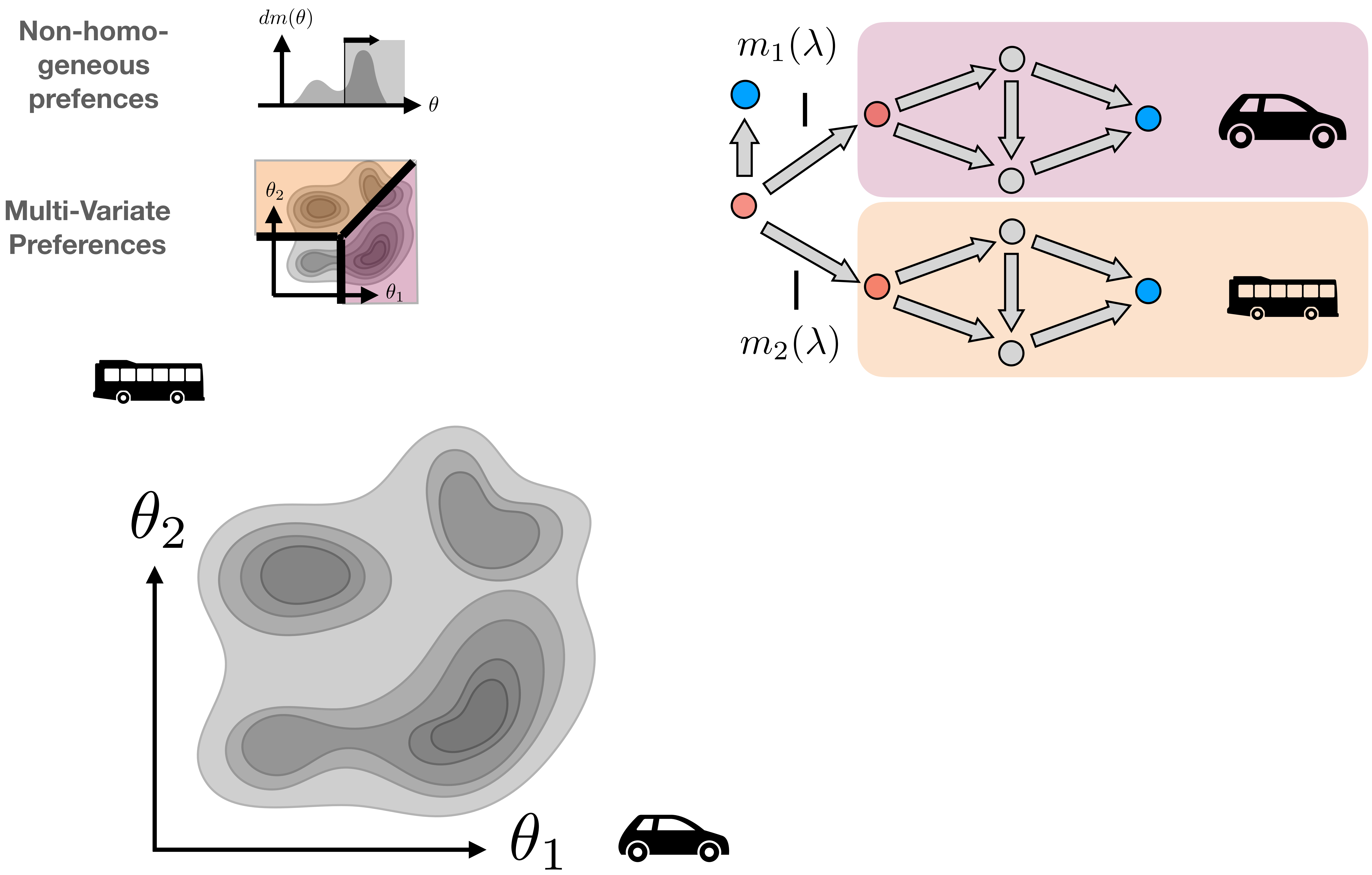




Potential Games



Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games

Routing Games

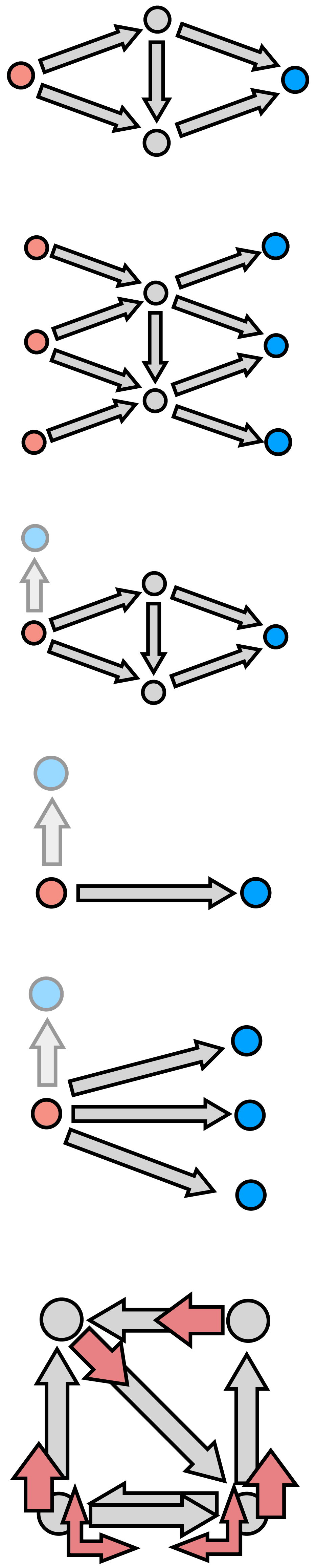
Multiple sources/ sinks

Variable Demand

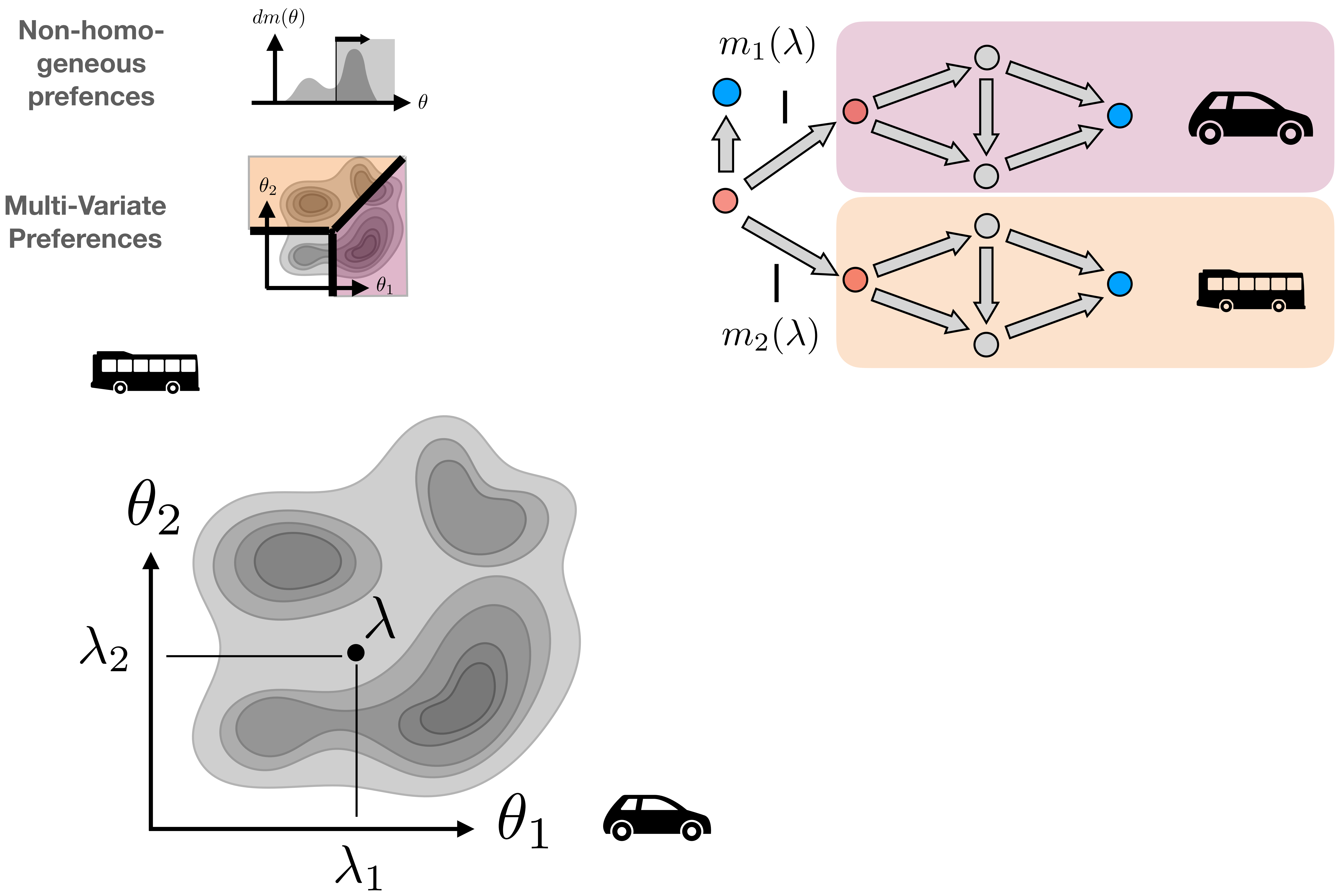
Supply & Demand

Cournot Market

MDP Congestion Game



Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games

Routing Games

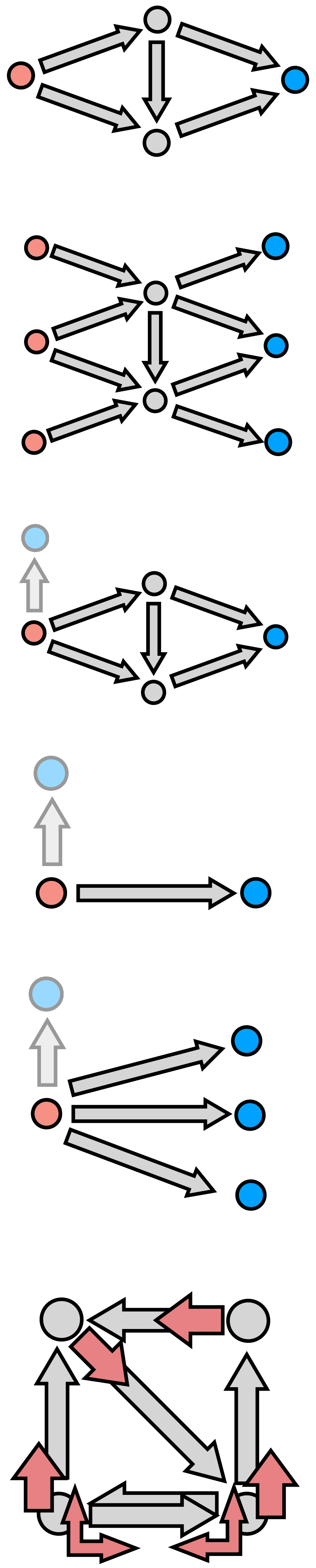
Multiple sources/ sinks

Variable Demand

Supply & Demand

Cournot Market

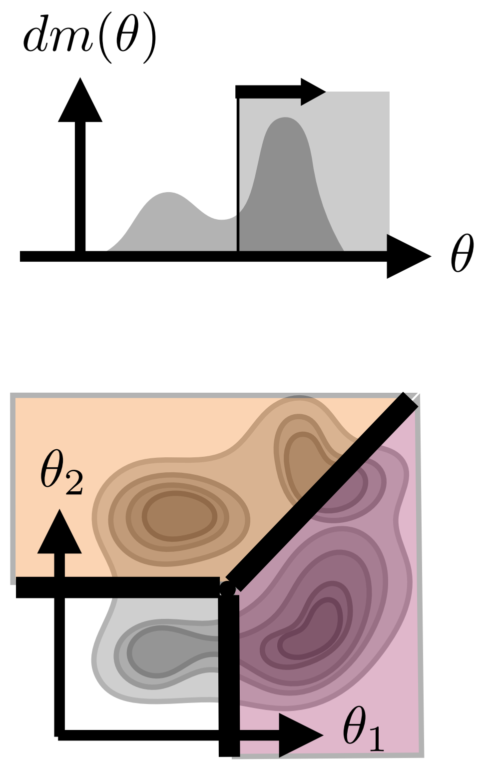
MDP Congestion Game

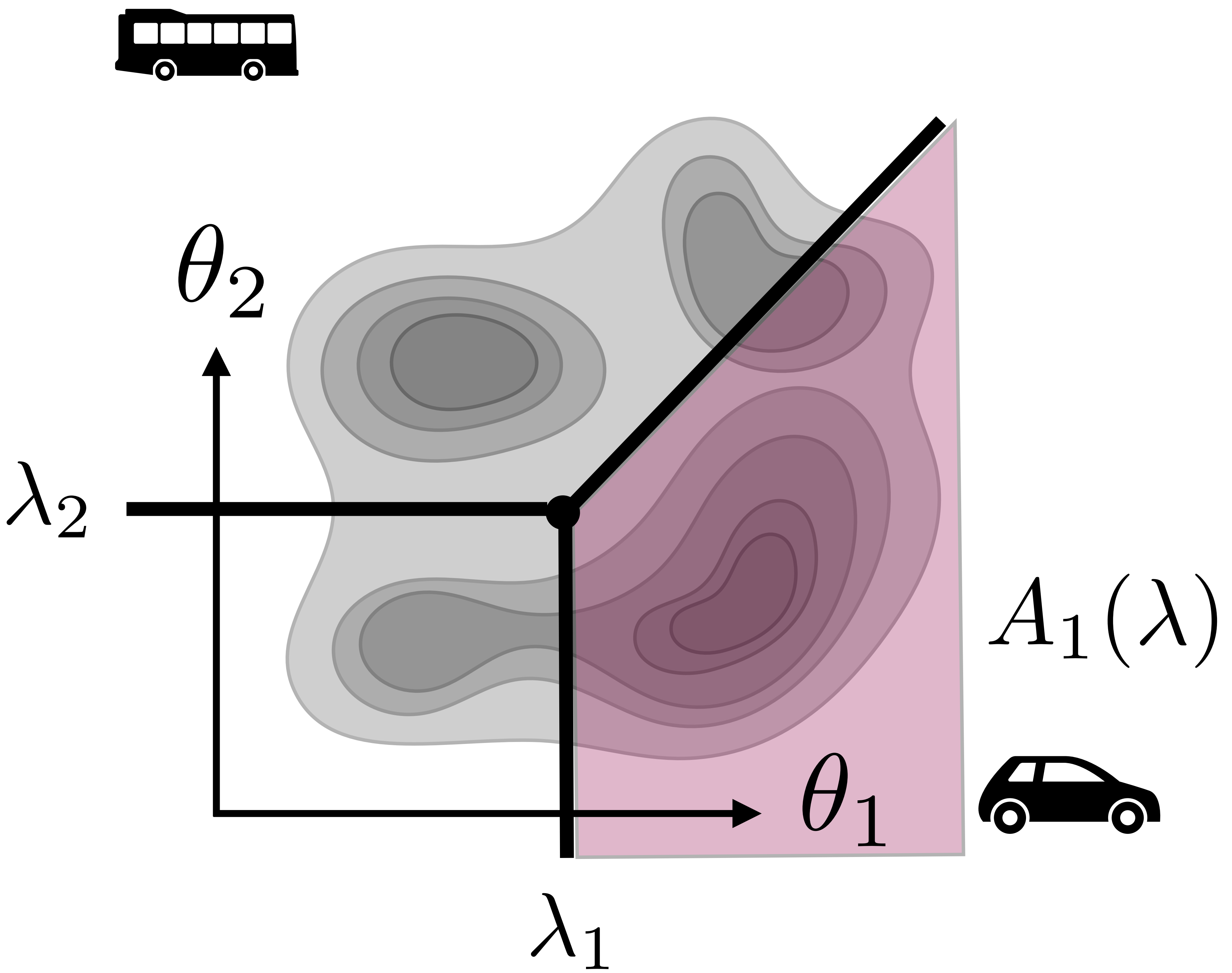


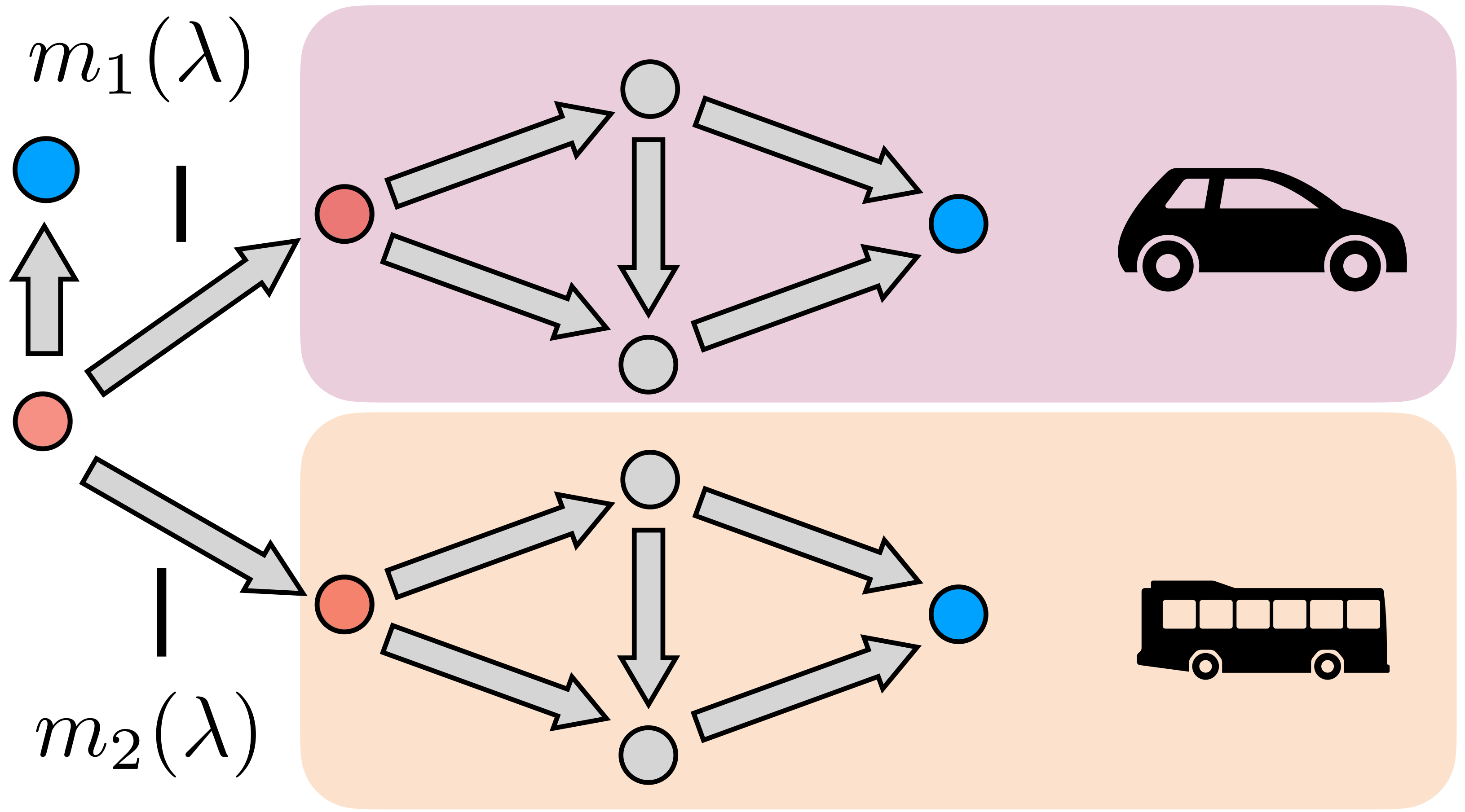
Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous prefences

Multi-Variate Preferences







$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

Potential Games

Routing Games

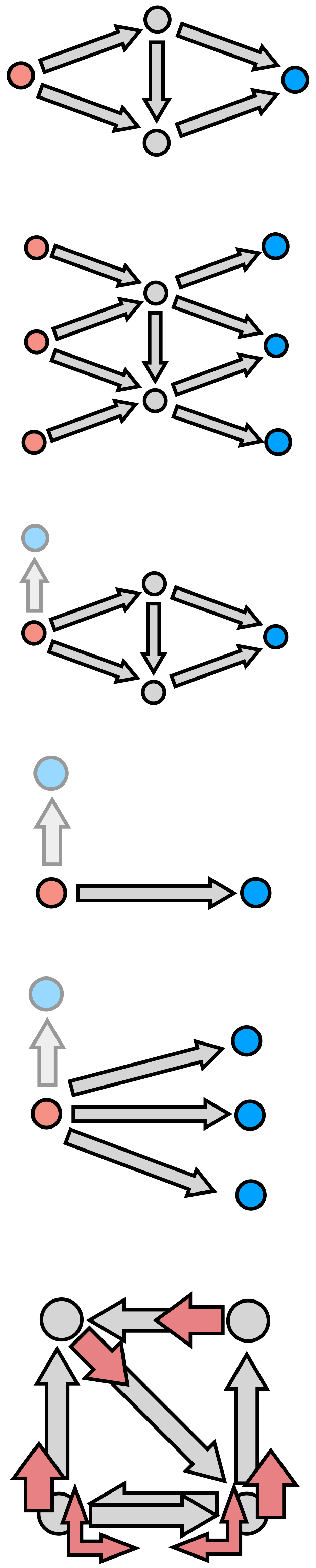
Multiple sources/ sinks

Variable Demand

Supply & Demand

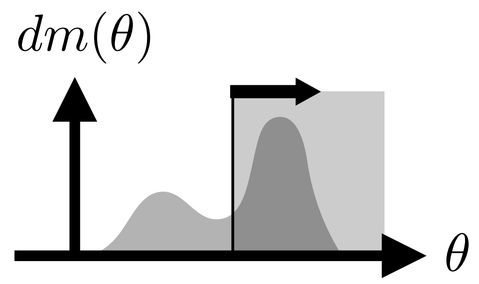
Cournot Market

MDP Congestion Game

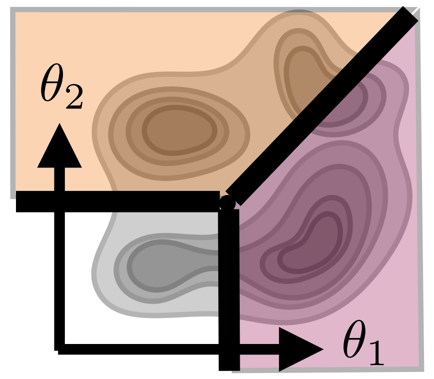


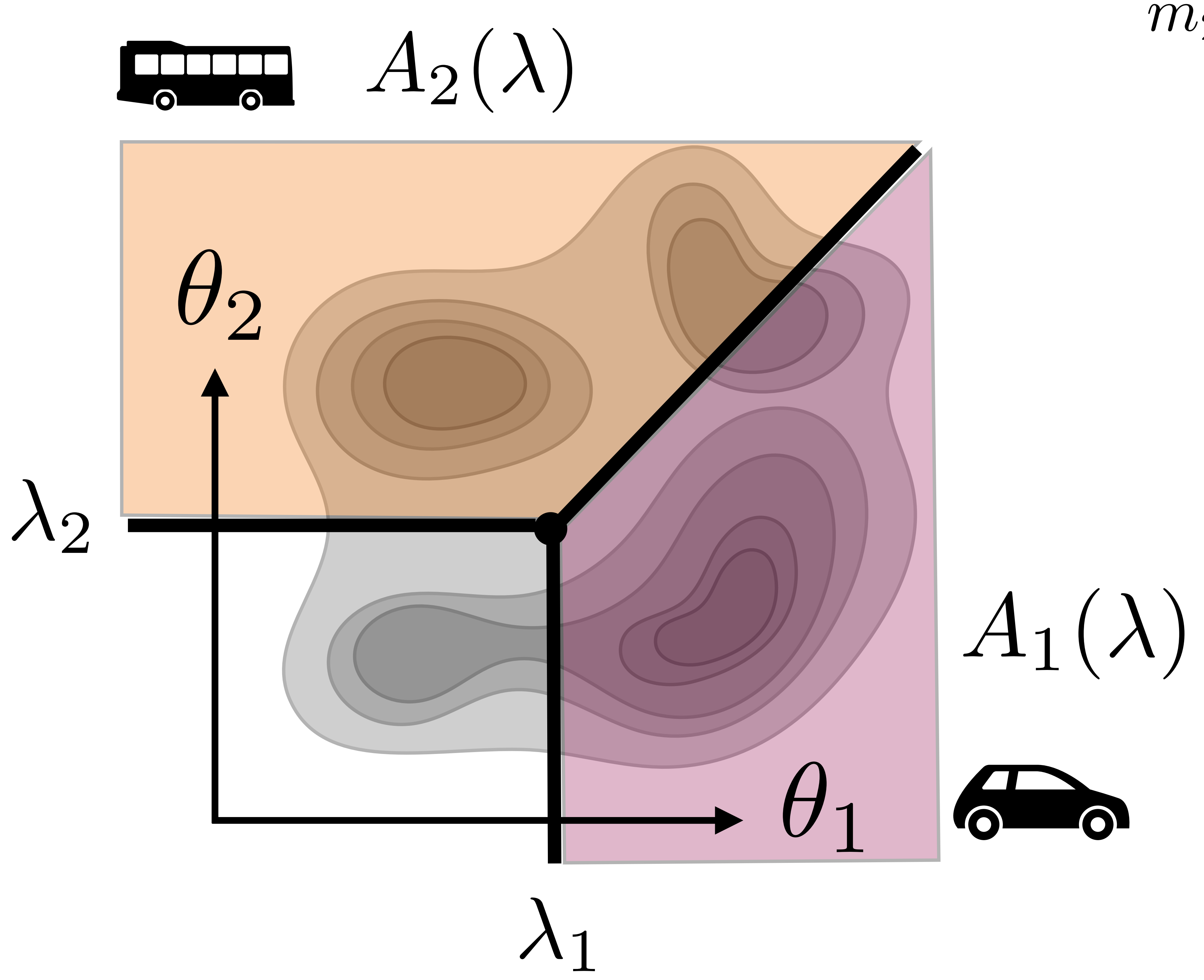
Variable Demand - Multi-Variate Non-Homogeneous Preferences

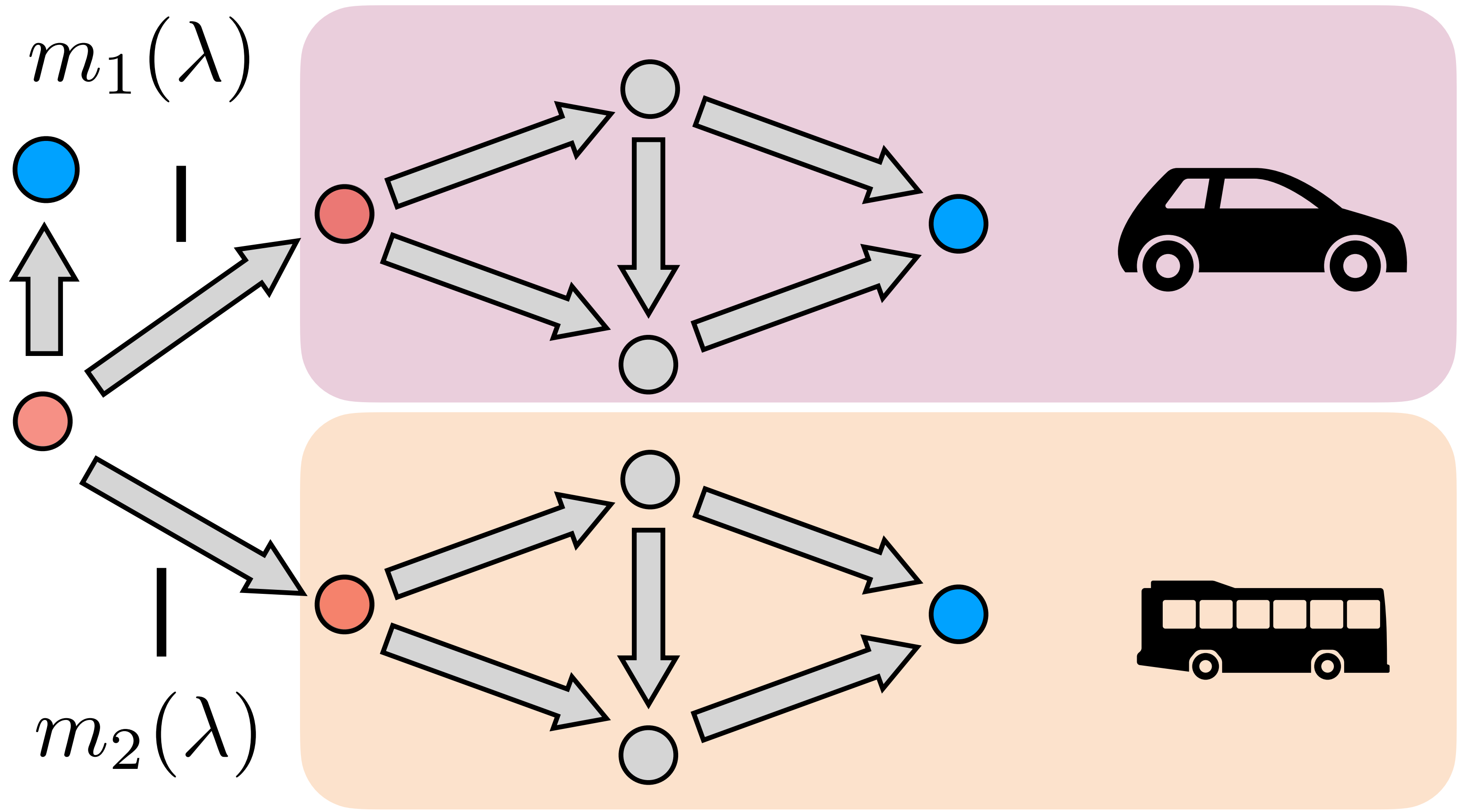
Non-homo-geneous prefences



Multi-Variate Preferences





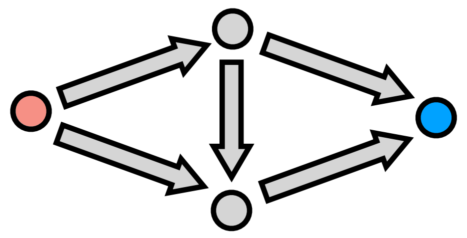


$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

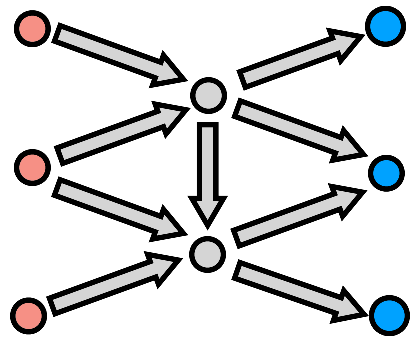
$$m_2(\lambda) = \int_{A_2(\lambda)} dm(\theta)$$

Potential Games

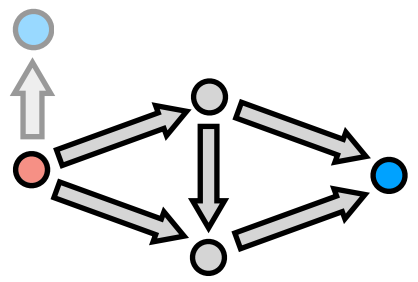
Routing Games



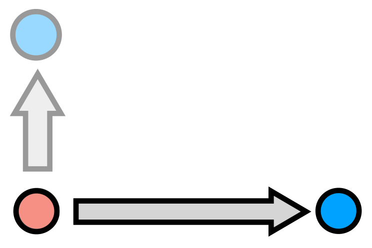
Multiple sources/ sinks



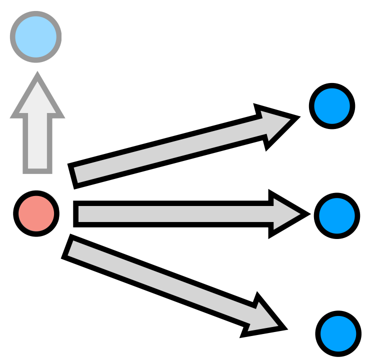
Variable Demand



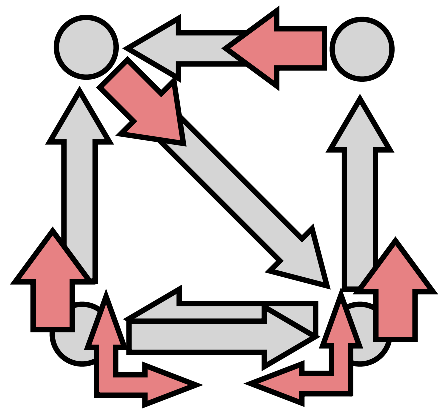
Supply & Demand



Cournot Market

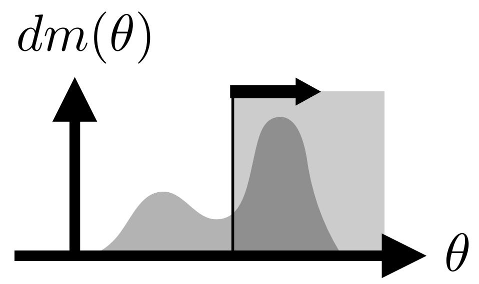


MDP Congestion Game

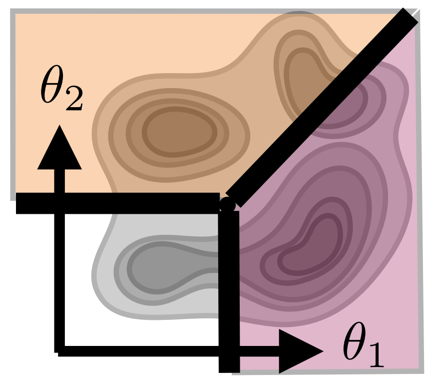


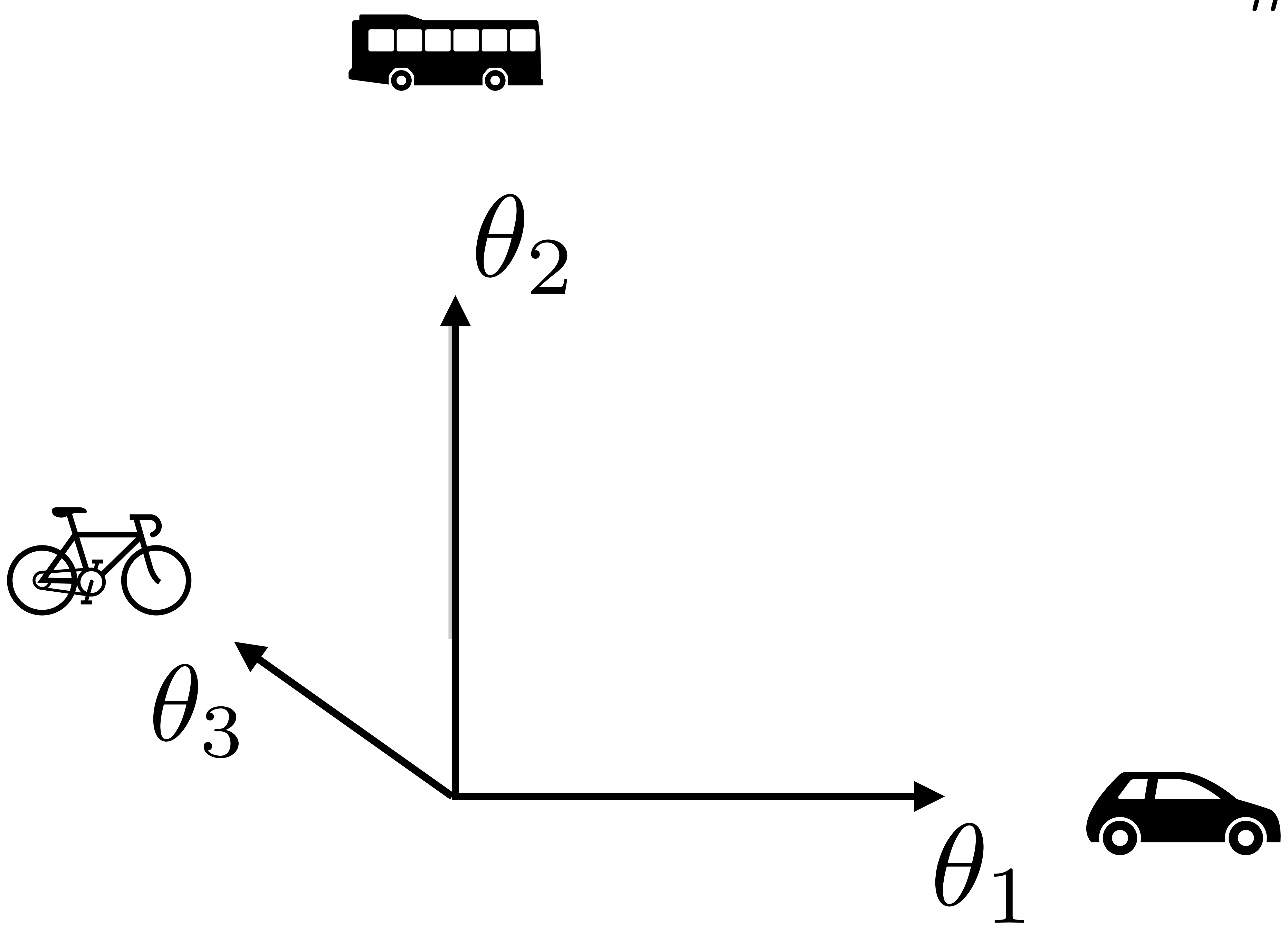
Variable Demand - Multi-Variate Non-Homogeneous Preferences

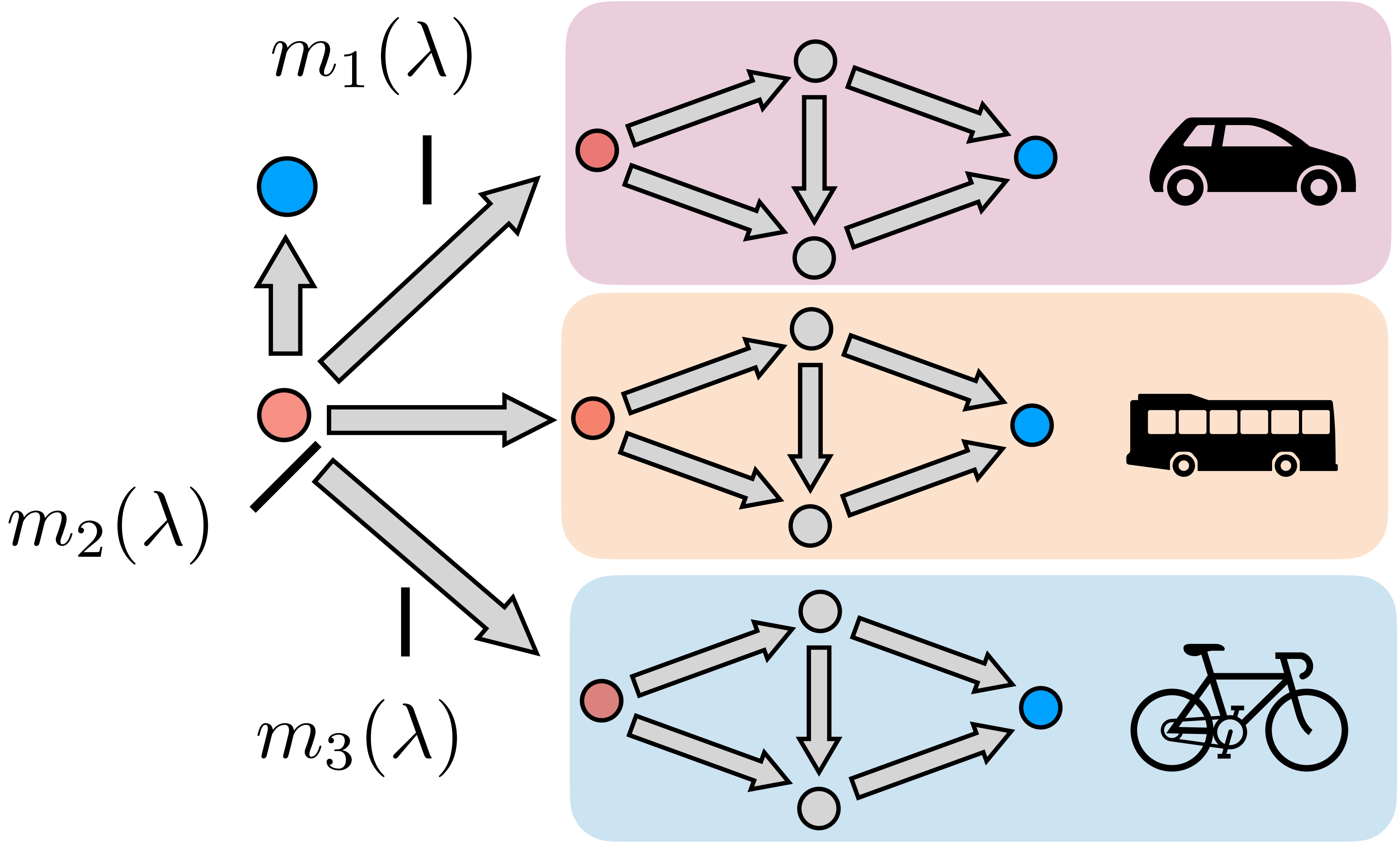
Non-homo-geneous prefences



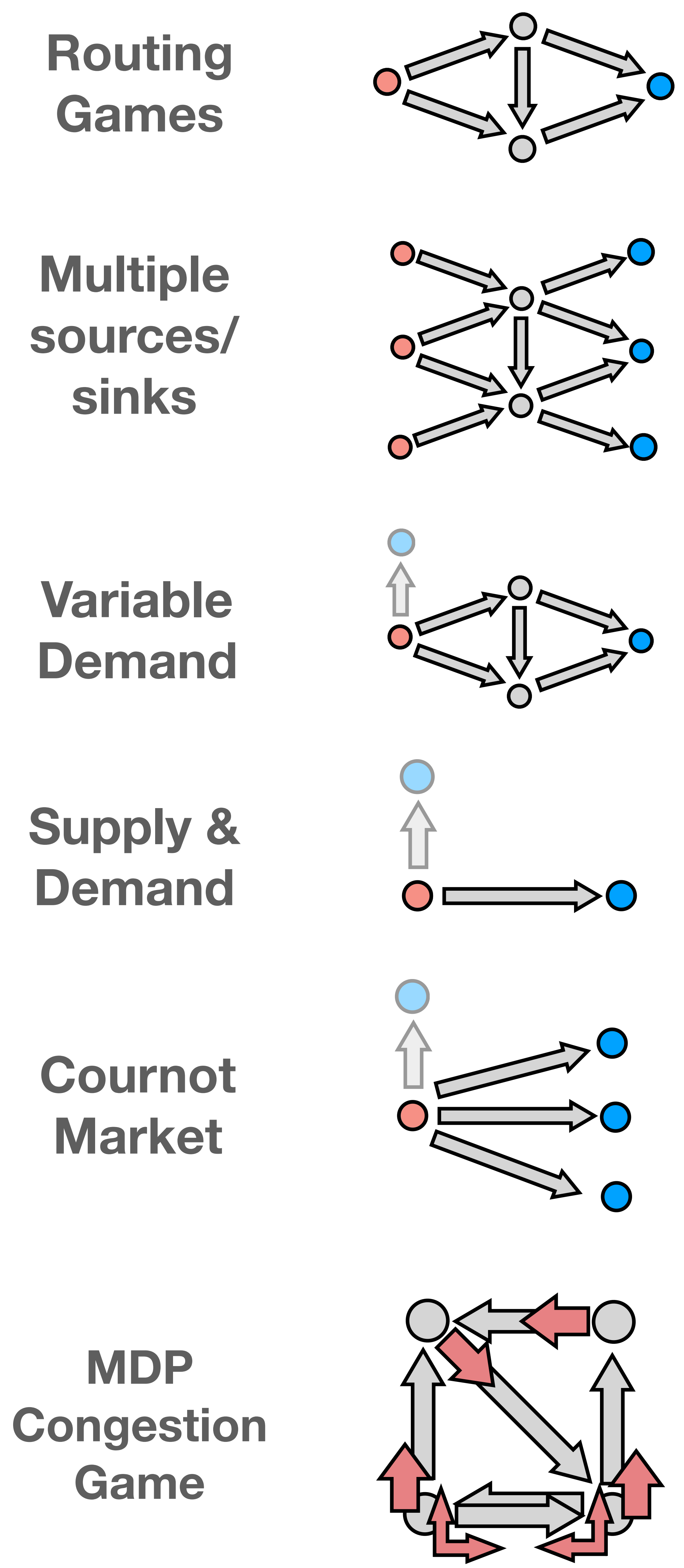
Multi-Variate Preferences



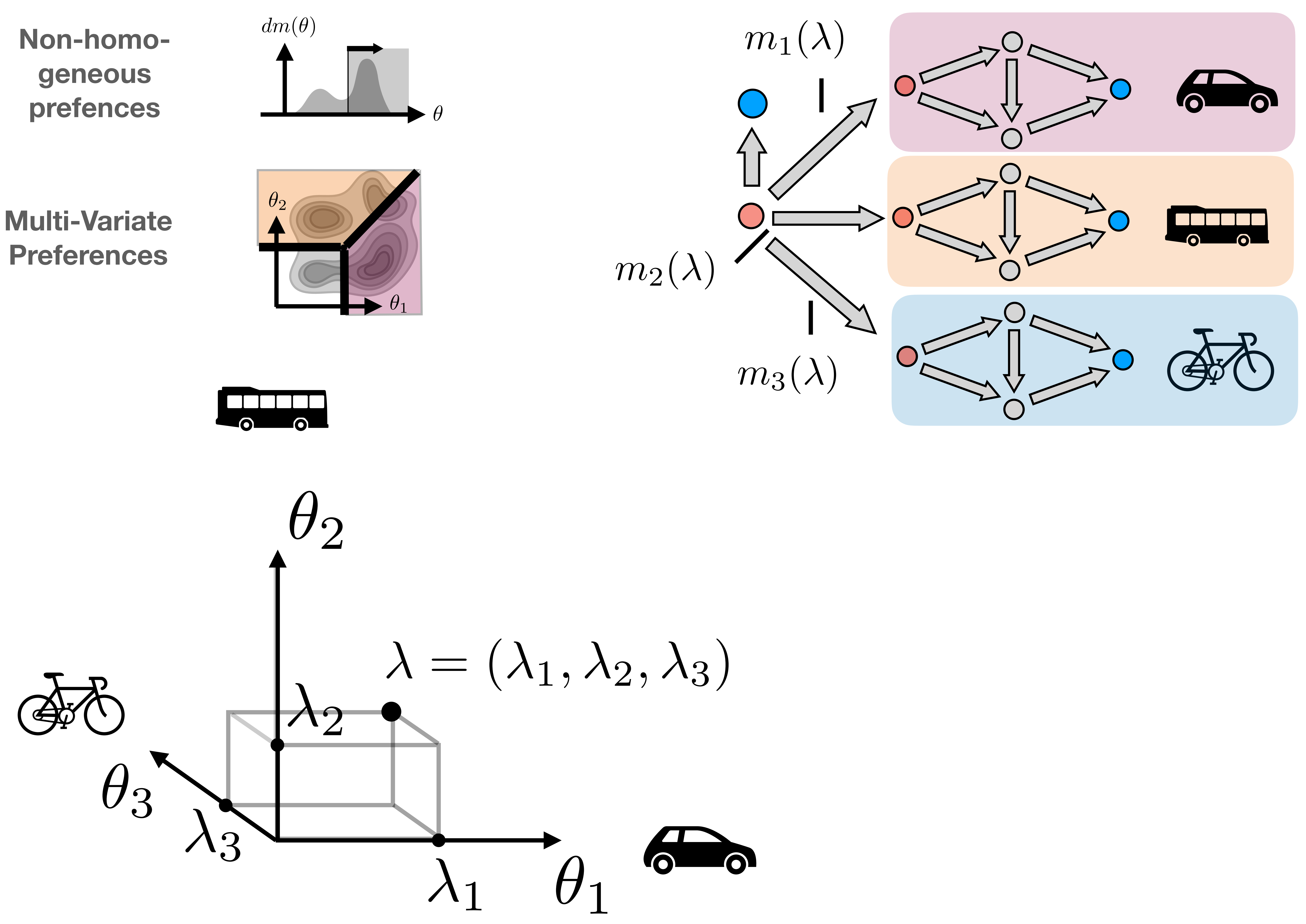




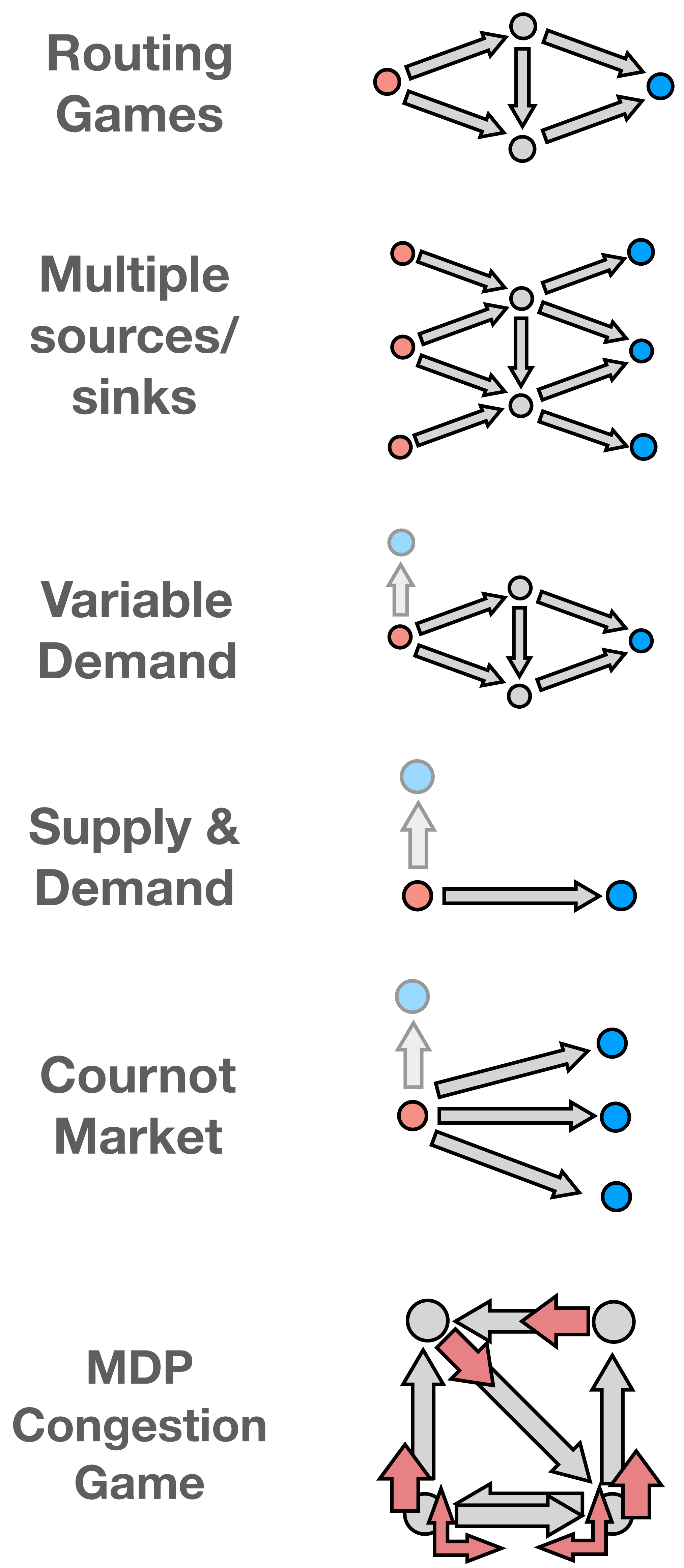
Potential Games



Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games



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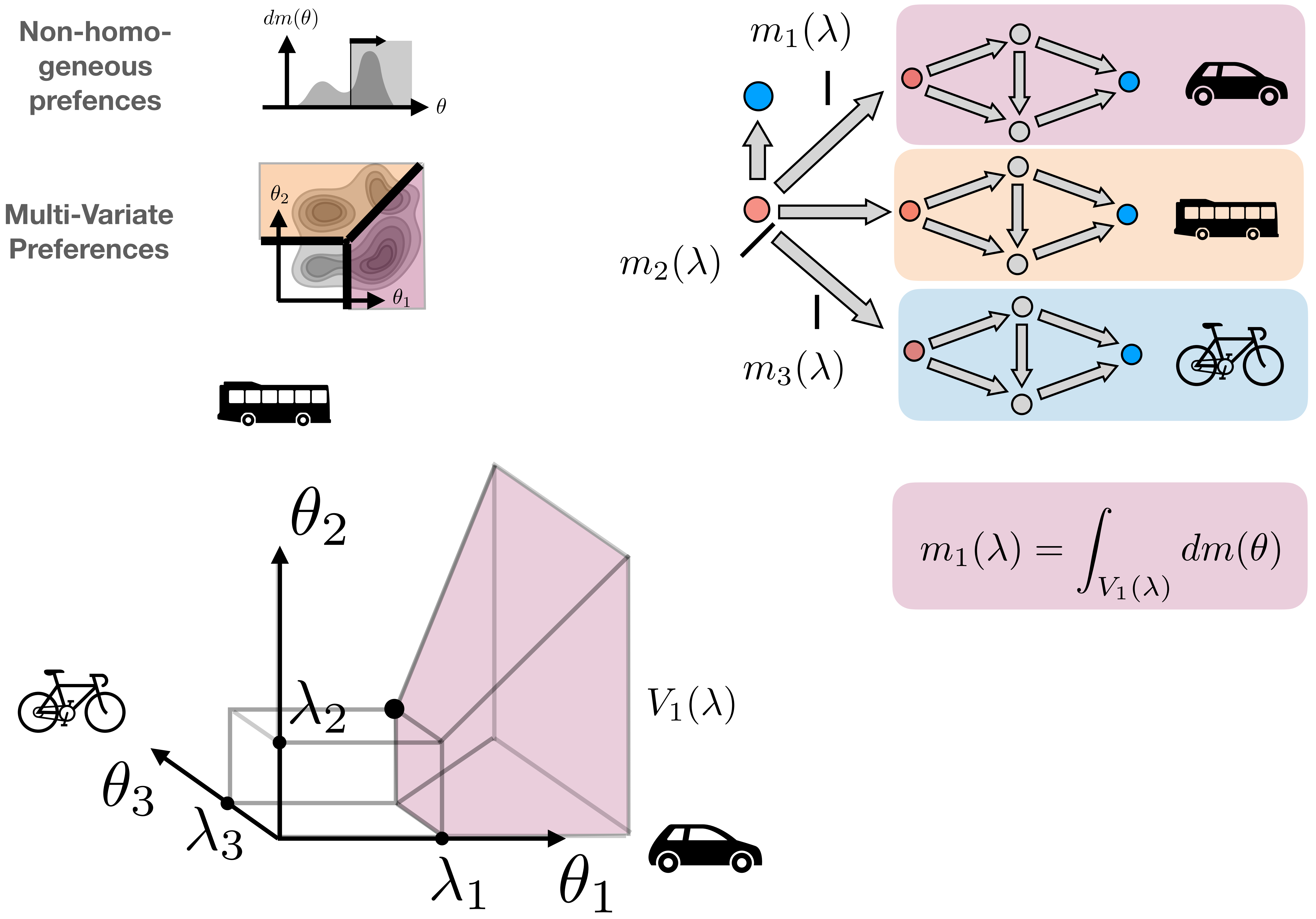
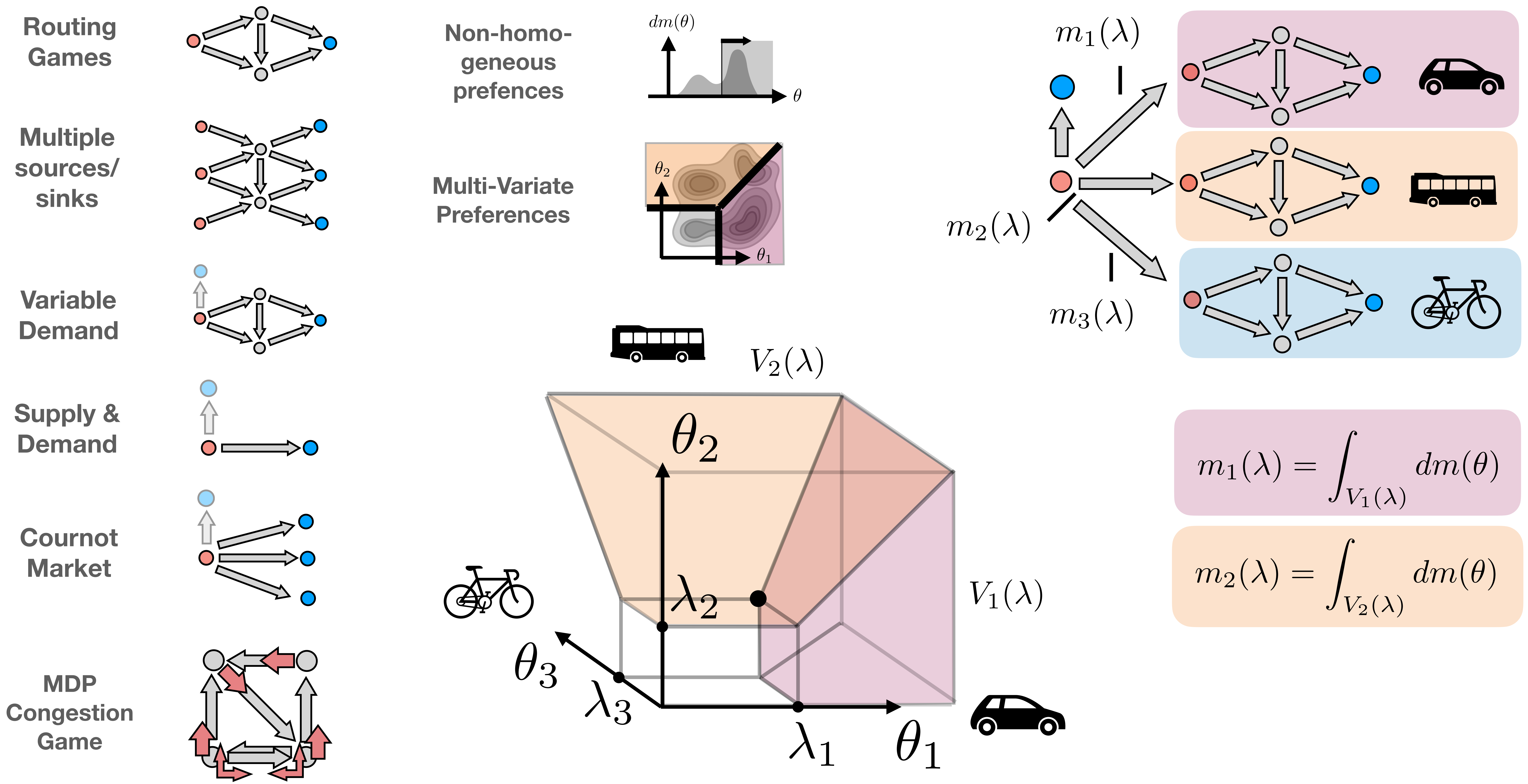
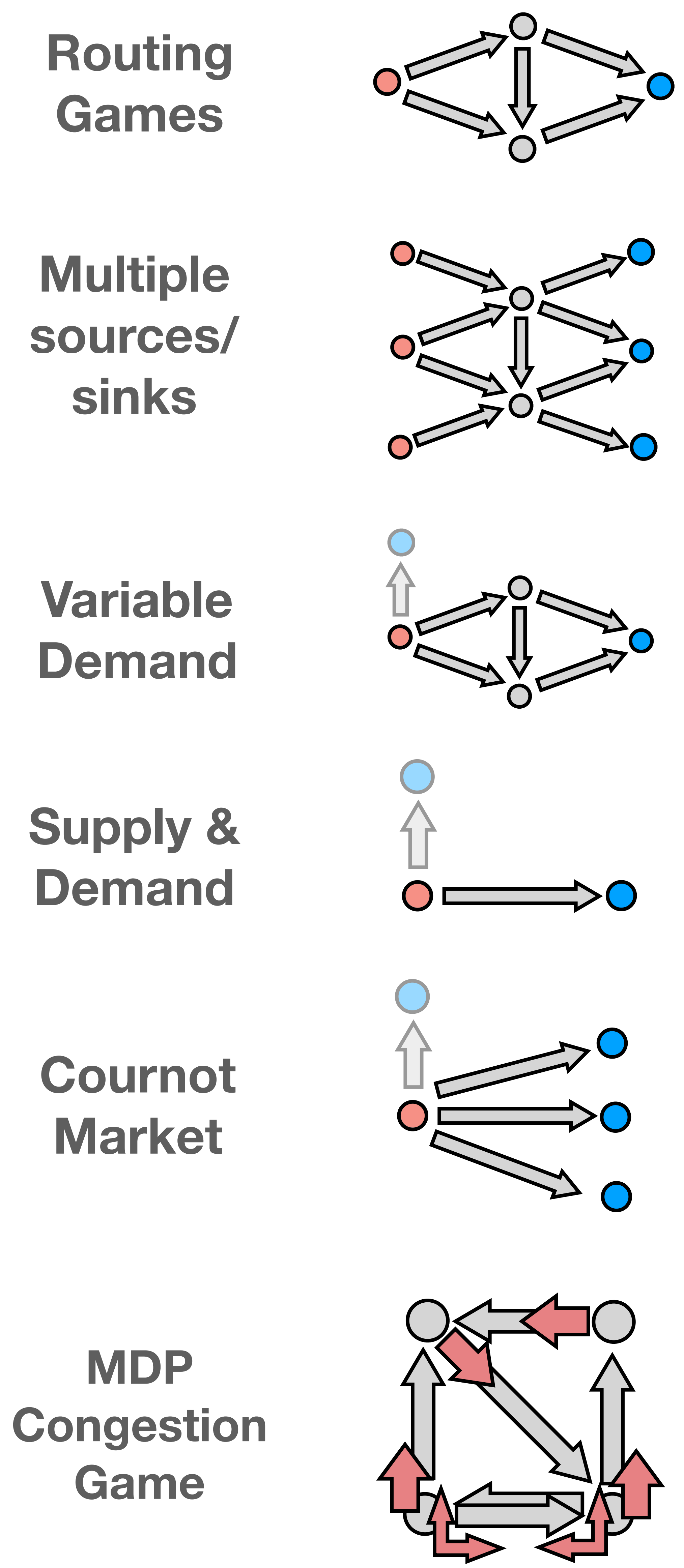


Potential Games

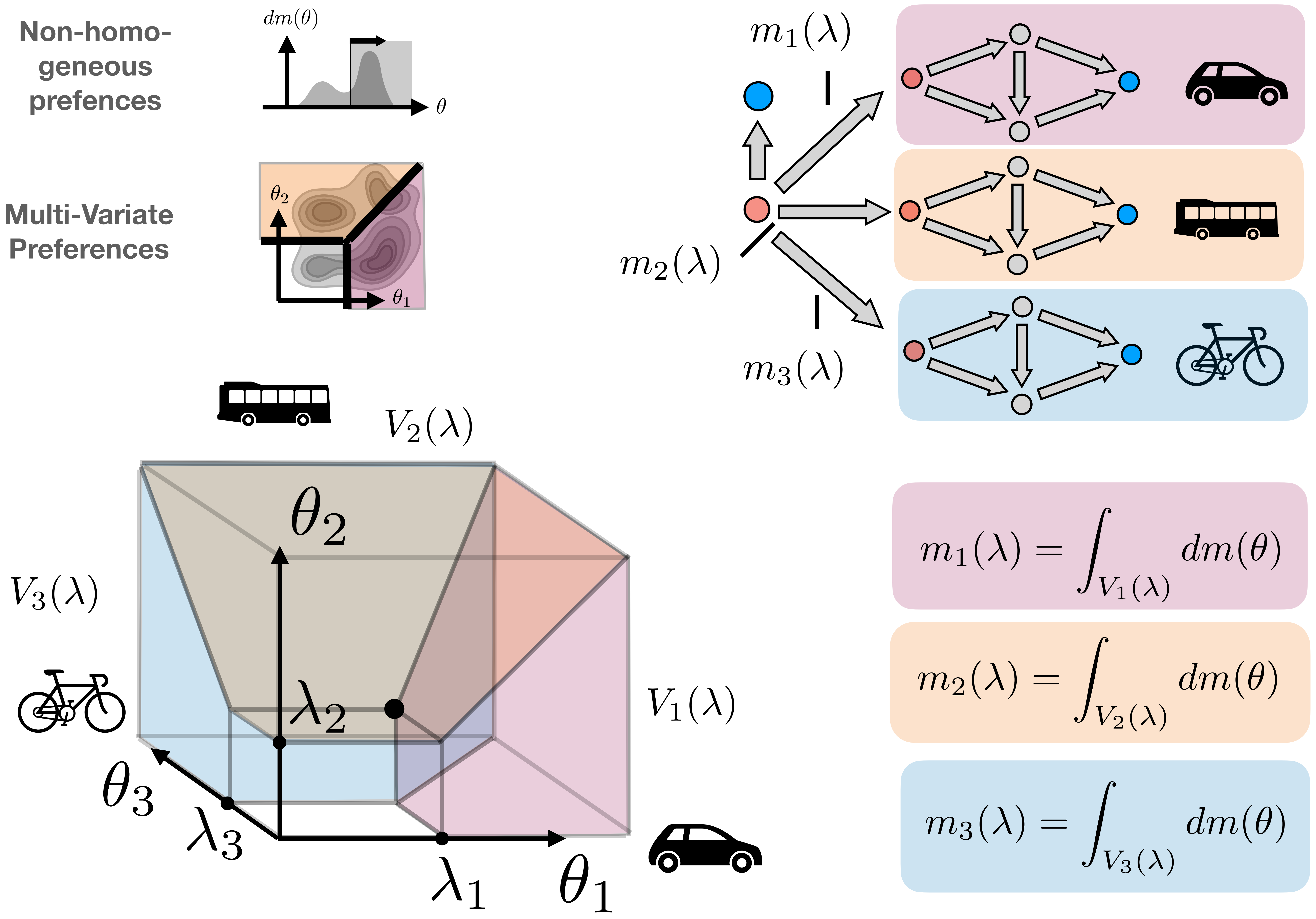
Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games



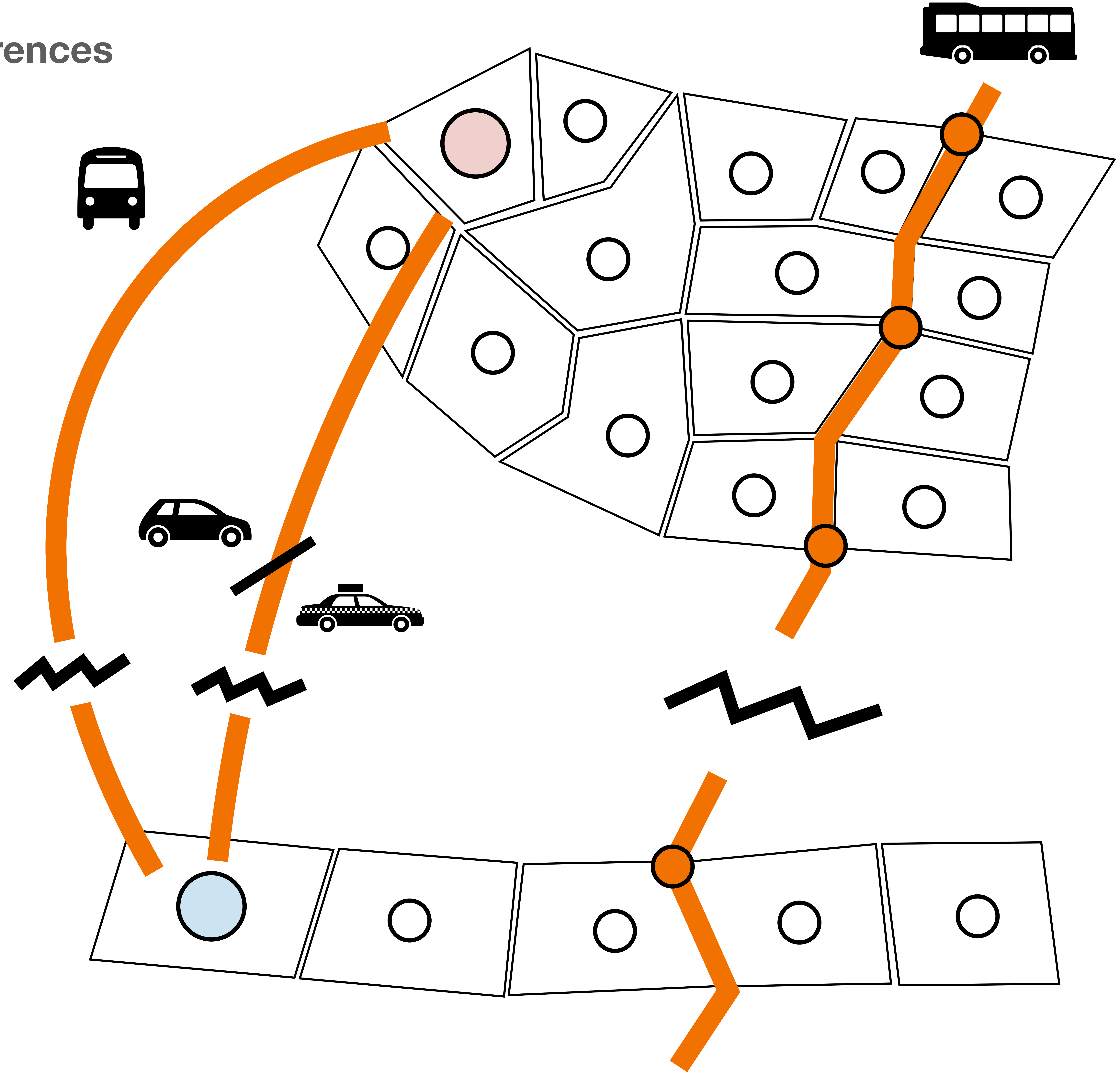
Variable Demand - Multi-Variate Non-Homogeneous Preferences



Multi-Variate Non-Homogeneous Preferences

Primary Modes

- Bus
- Dial-a-ride
- Drive



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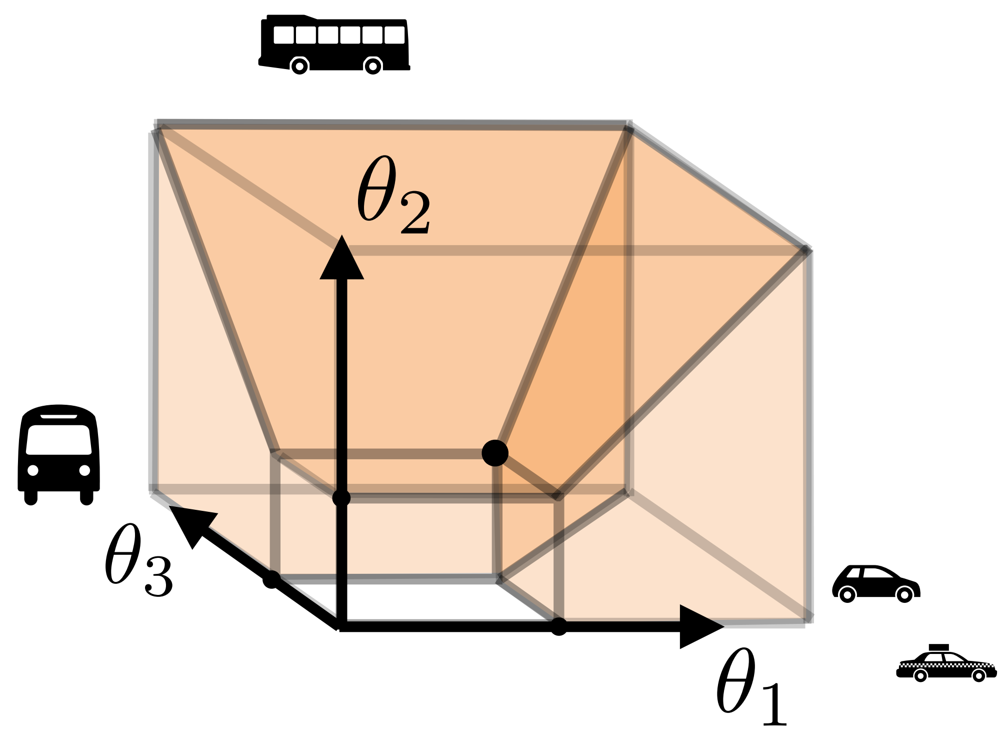

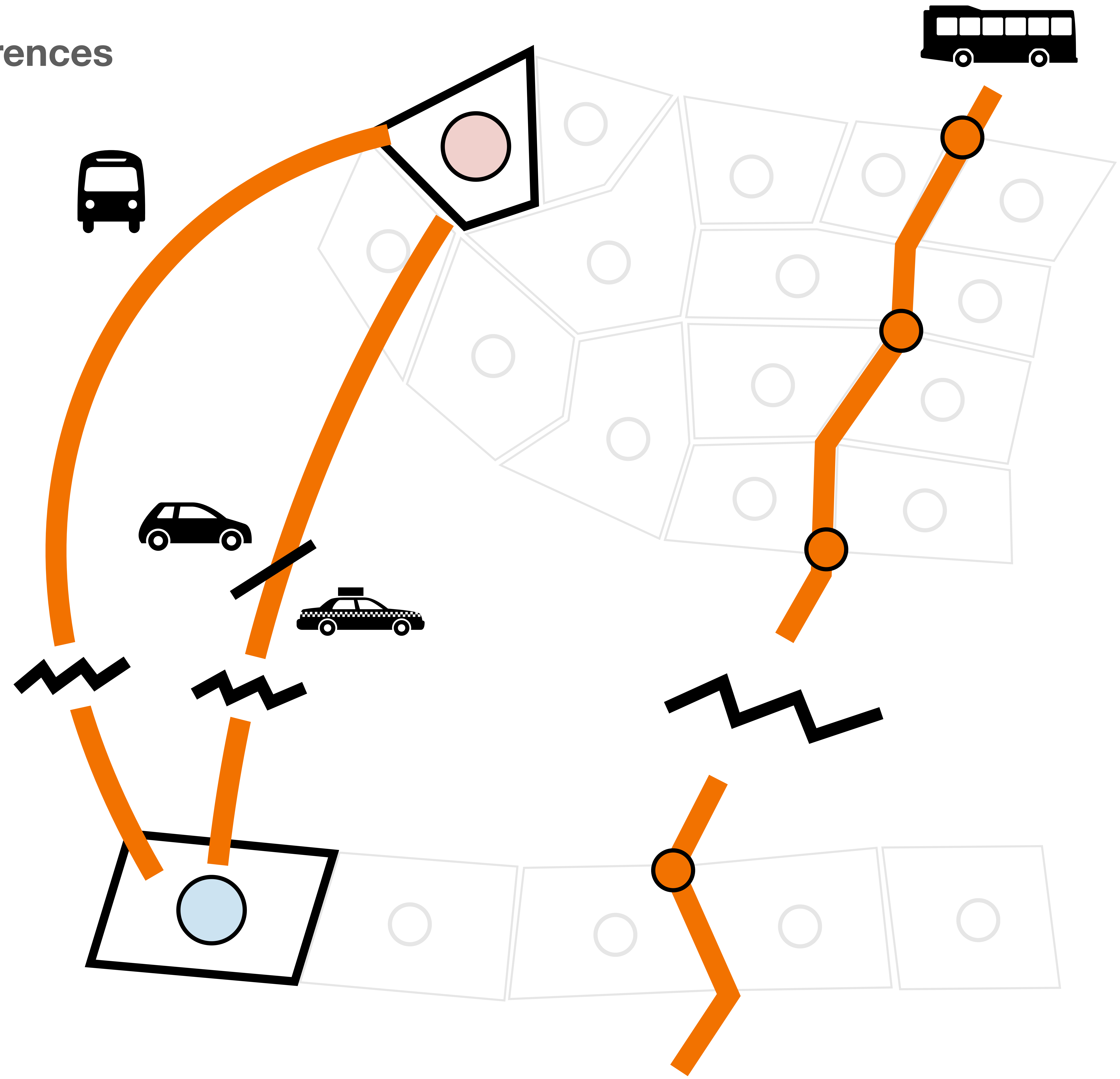

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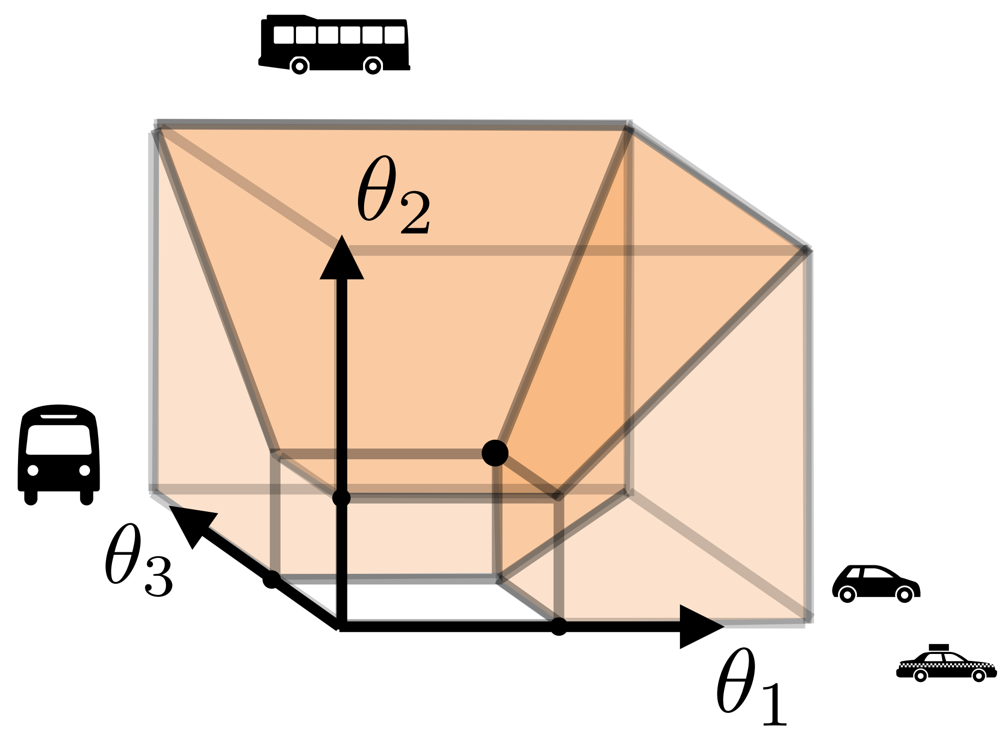
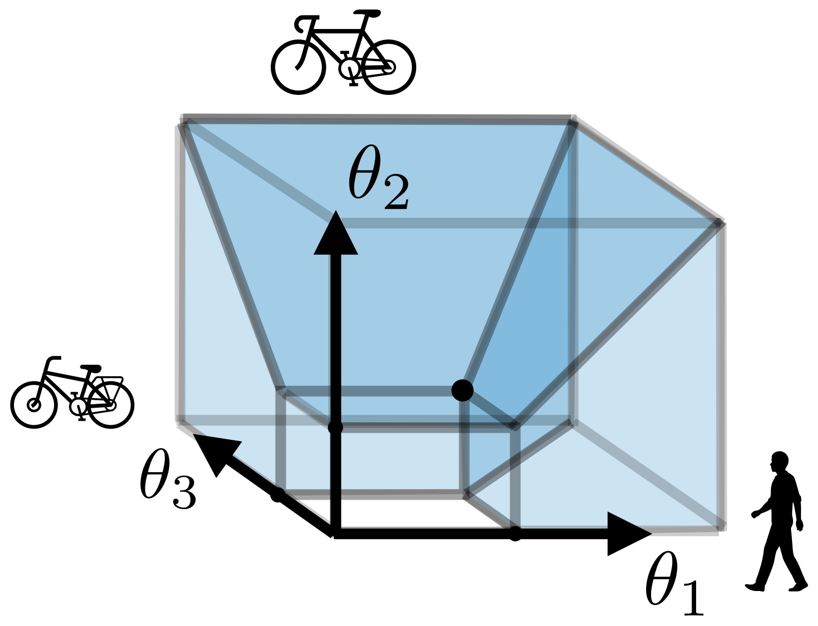
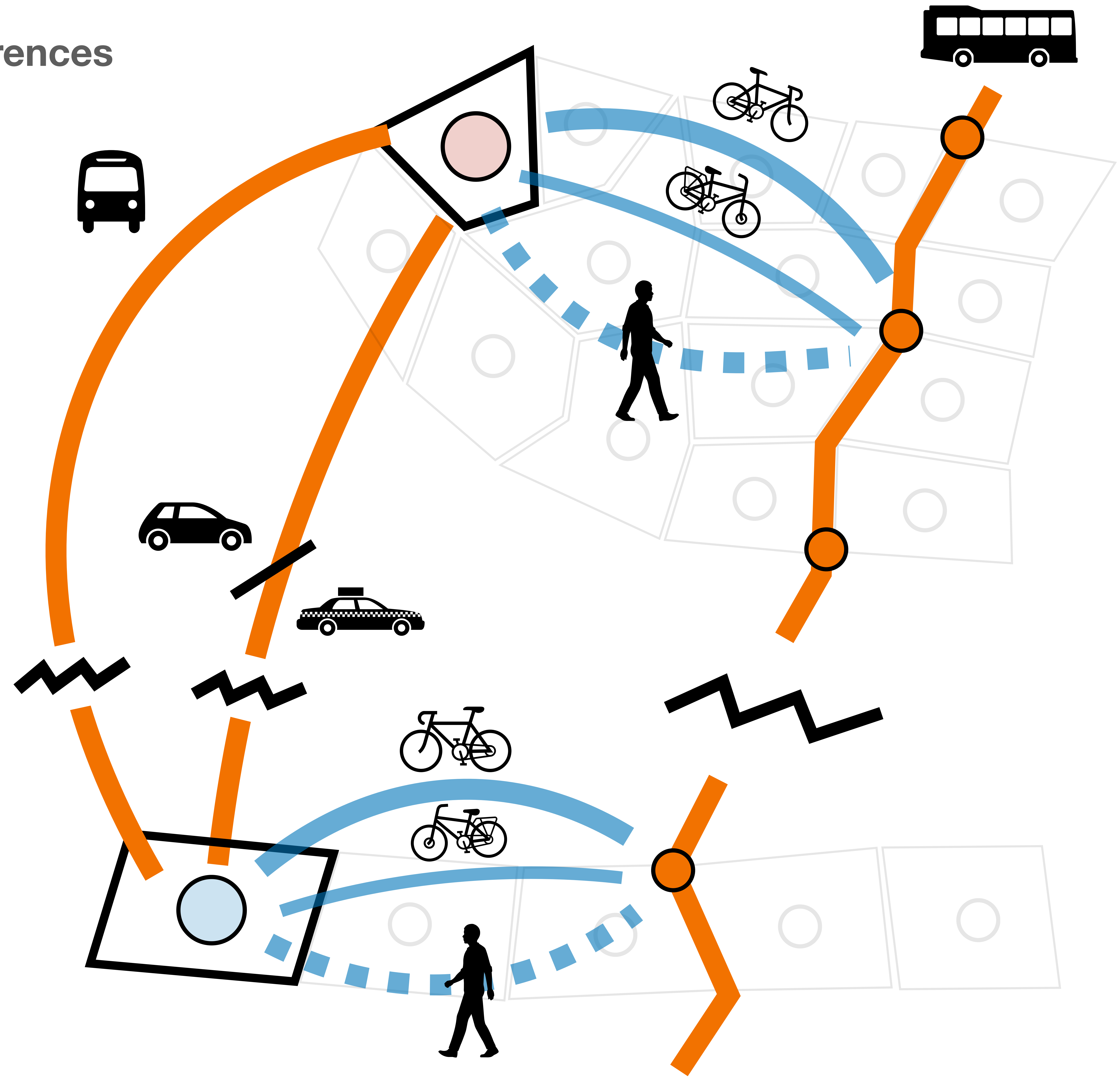


Secondary Modes

Bus…

- Walking
- Biking
- Bike share





Multi-Variate Non-Homogeneous Preferences

Primary Modes

- Bus
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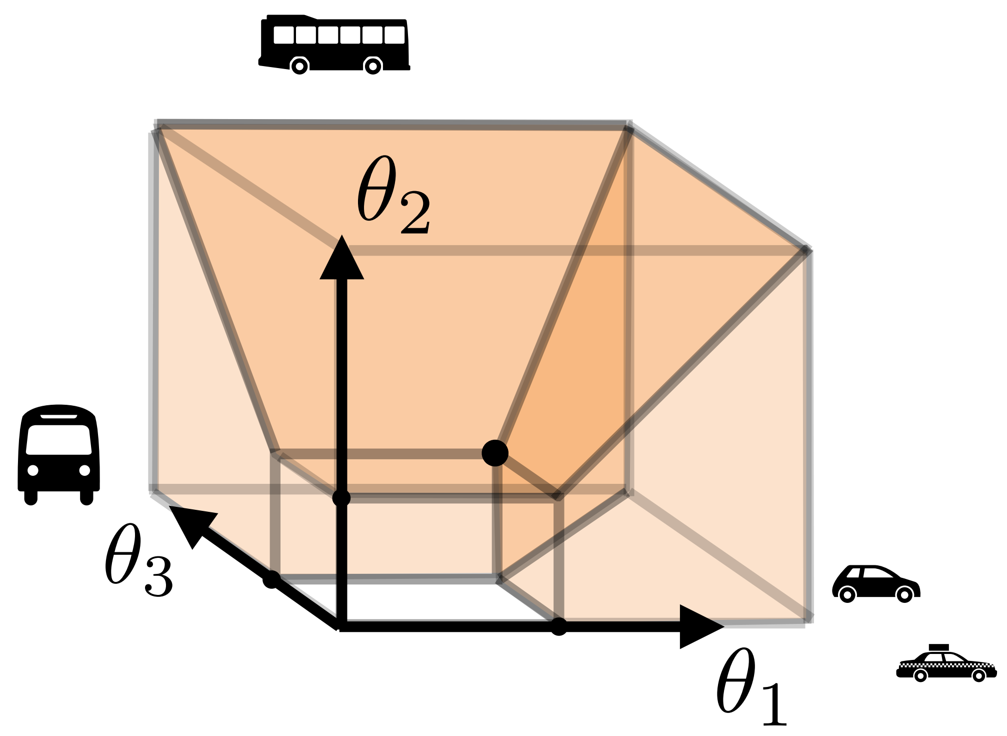

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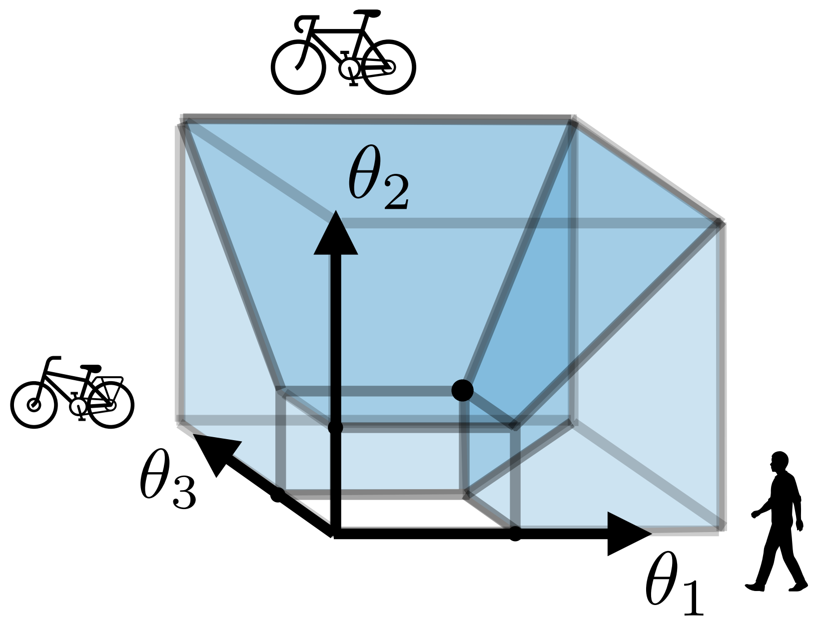
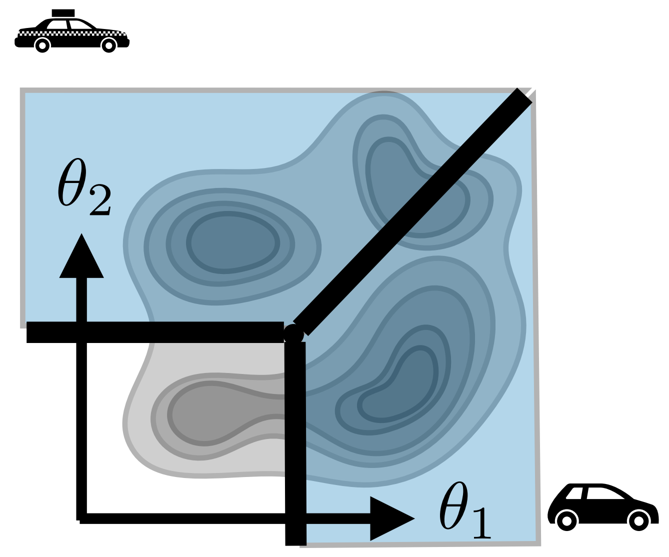
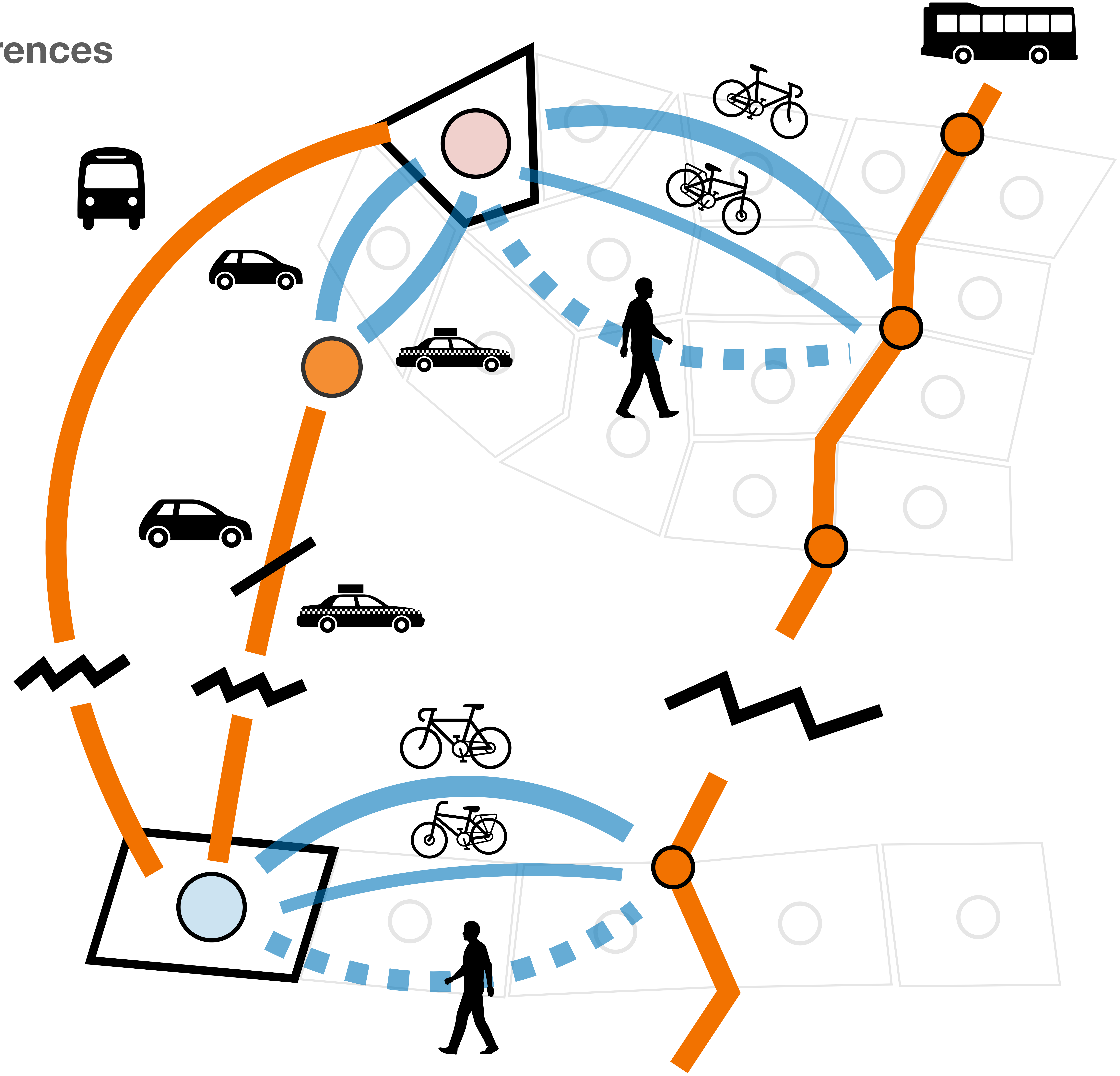


Drive…

- Personal car
- Ride-share

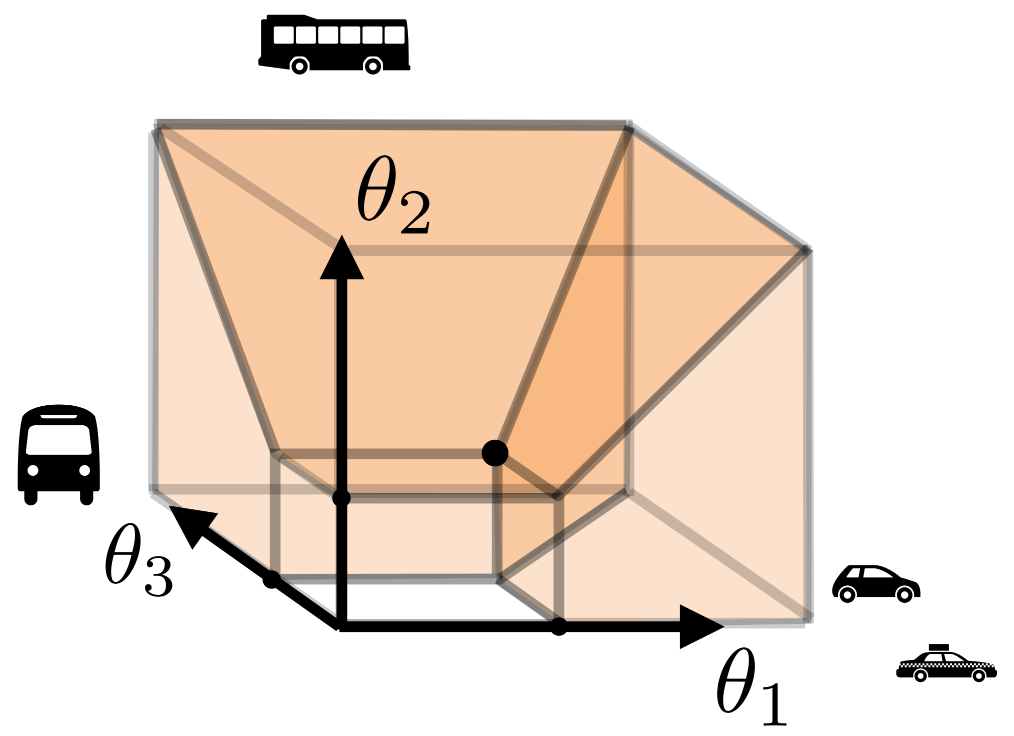




Multi-Variate Non-Homogeneous Preferences

Primary Modes

- Bus
- Dial-a-ride
- Drive



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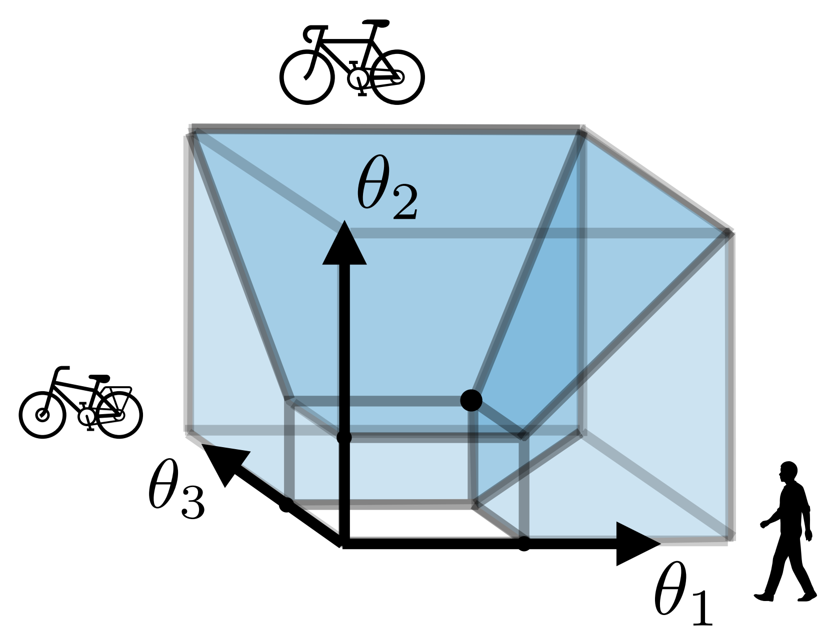

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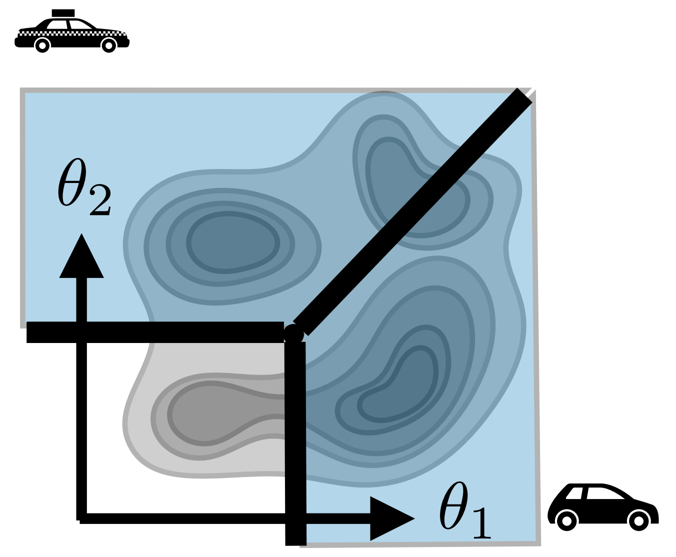
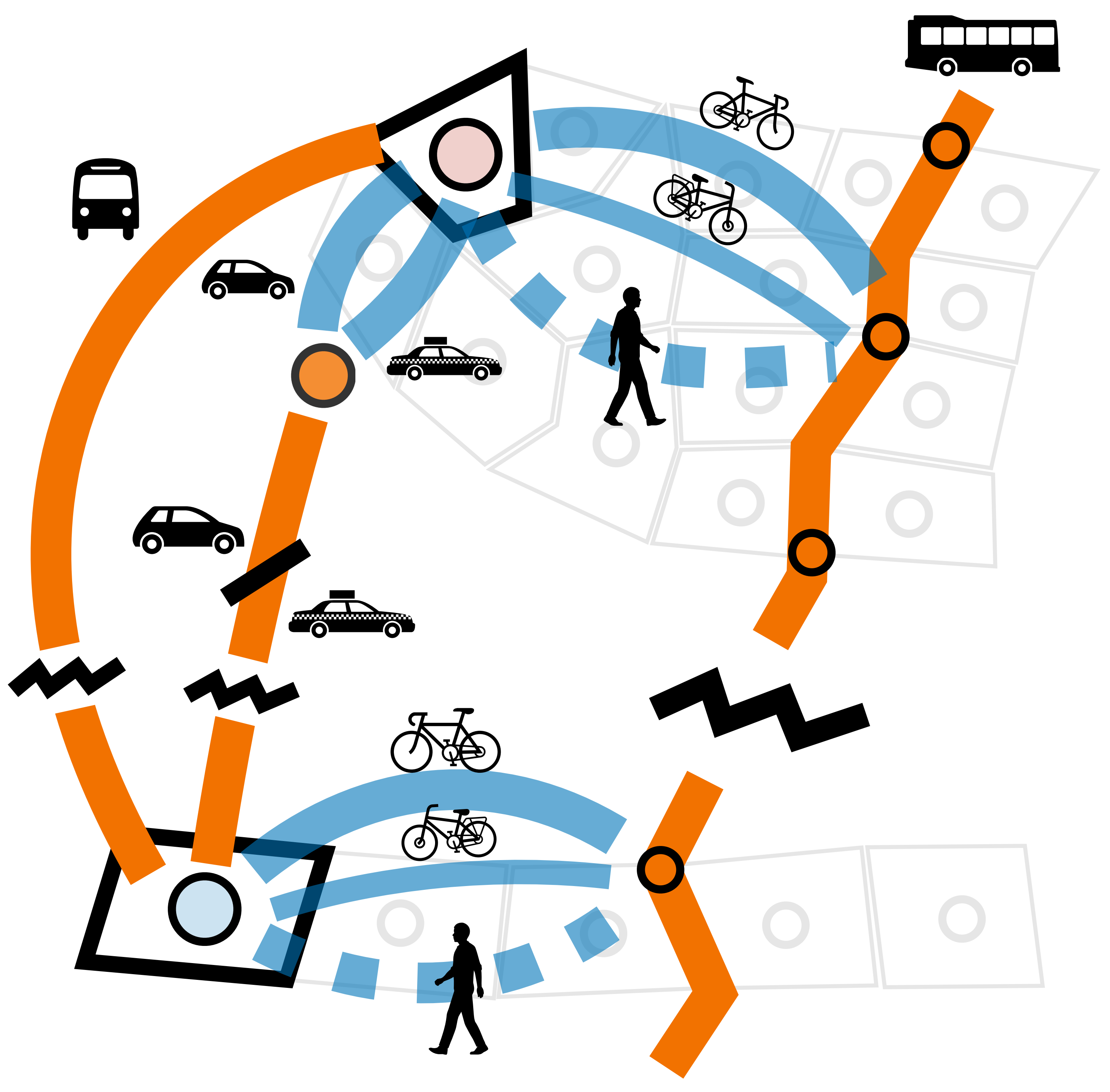




Individual's Cost

Primary Mode Preference + **Secondary Mode Preference** + **Travel Time**

Equilibrium

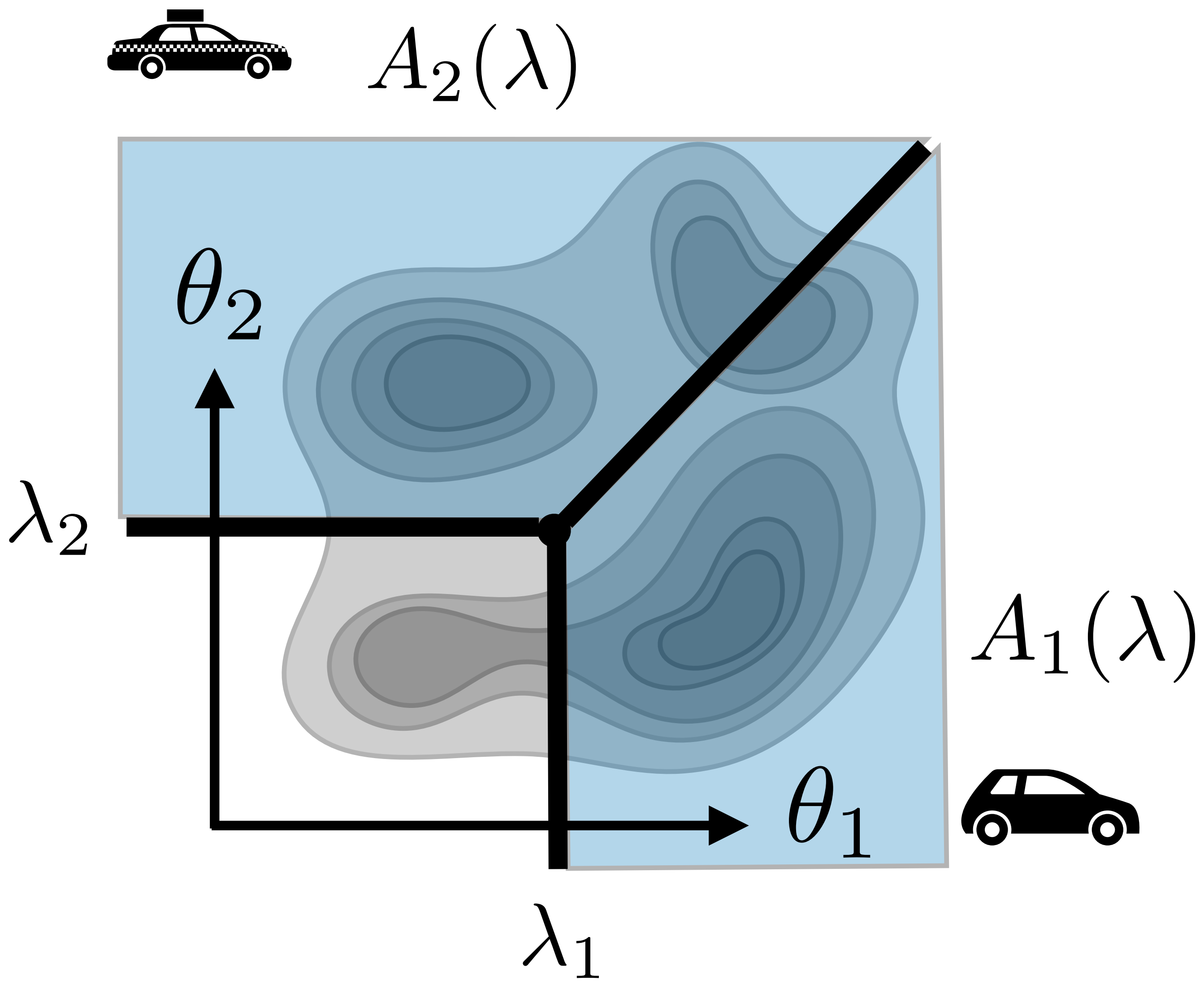
Population distribution over transport options

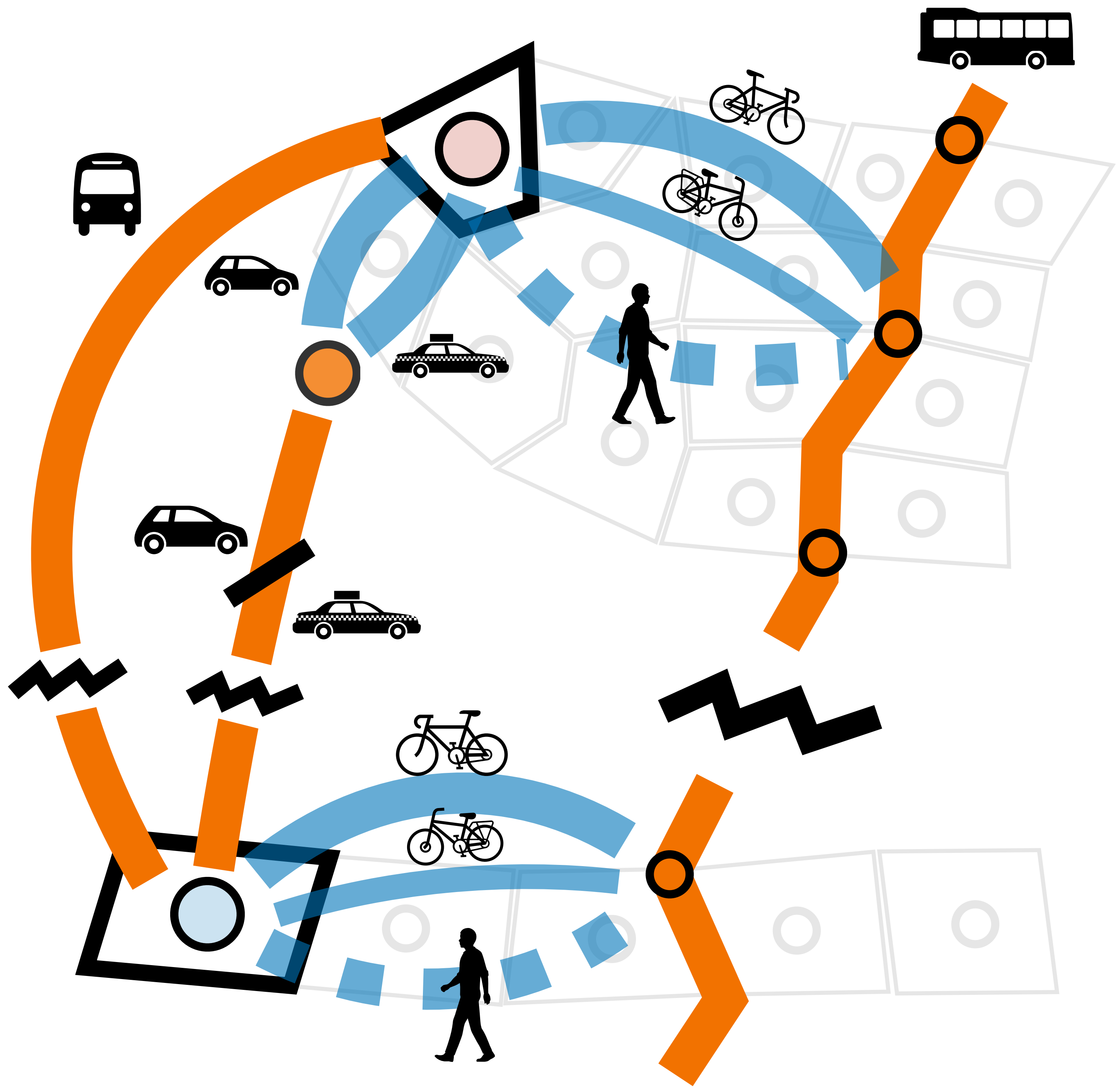
"No one can do any better"

Multi-Variate Non-Homogeneous Preferences

Computing mass distributions (at each origin point) → Survey (or guess)

Compare options -
Would need to standardize units in survey questions





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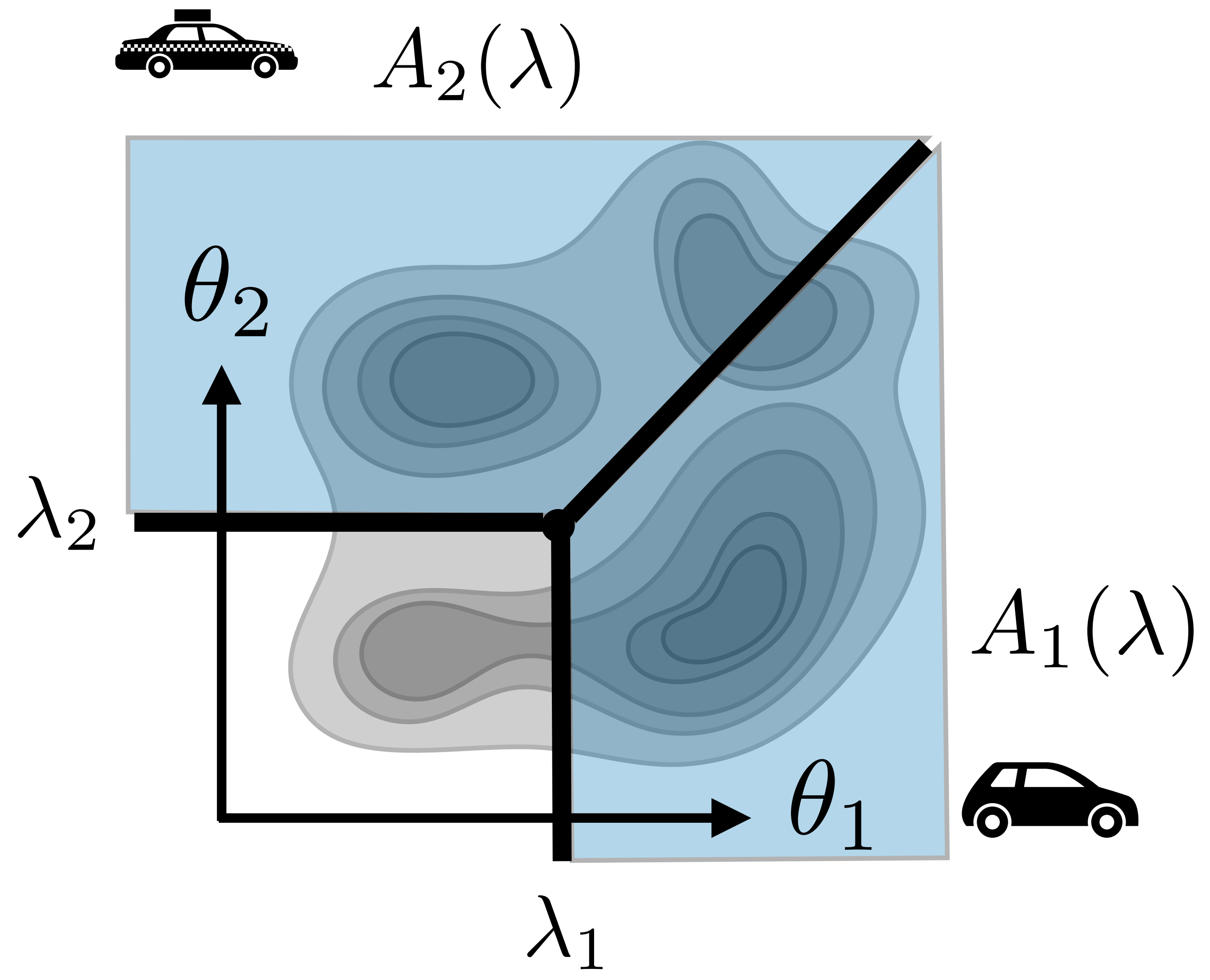

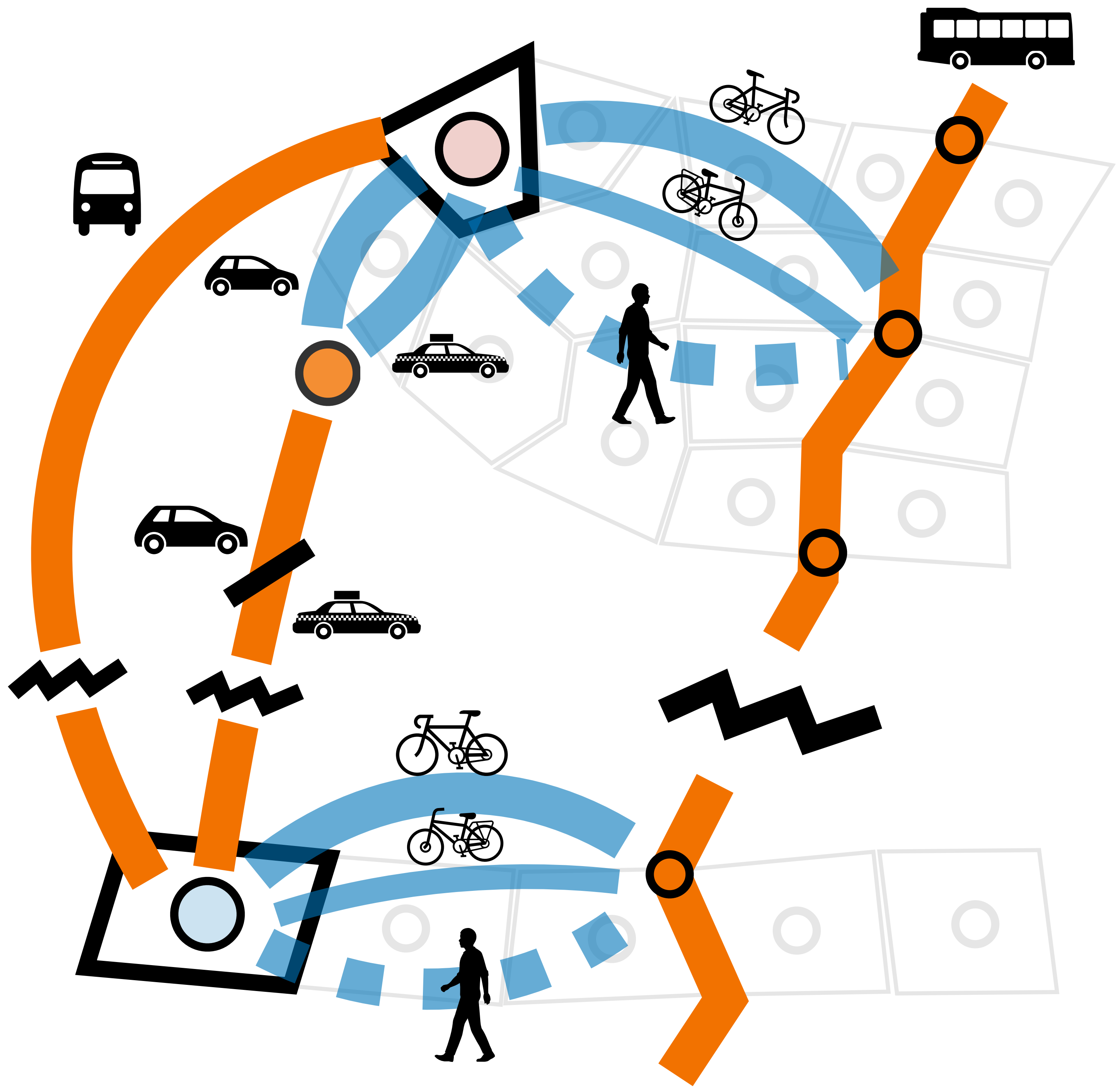

Computation:

Algorithm: **Frank-Wolfe**

LP for descent direction

Individual's Cost

Primary Mode Preference + **Secondary Mode Preference** + **Travel Time**

Equilibrium

Population distribution
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Computing mass distributions (at each origin point) → Survey (or guess)

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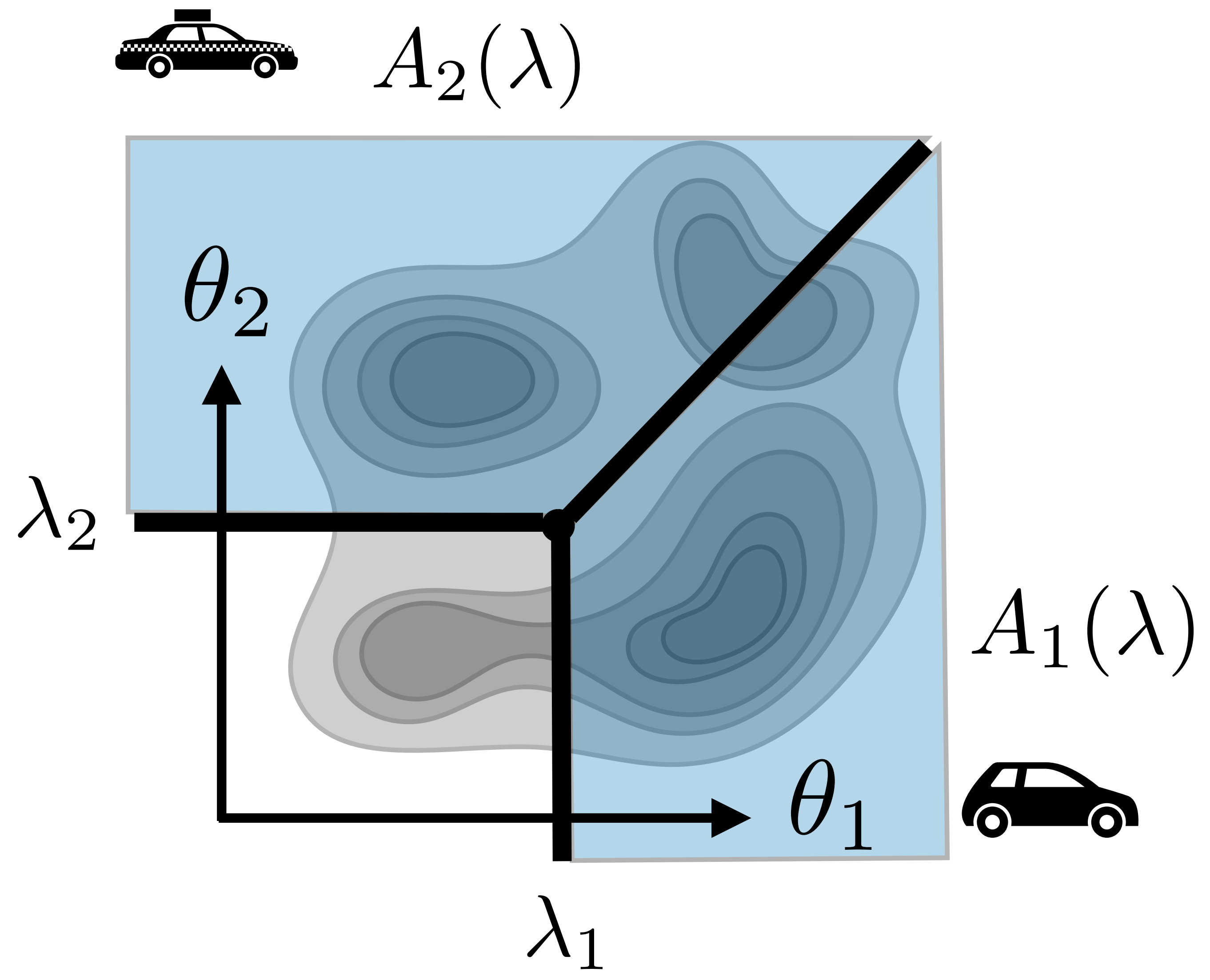

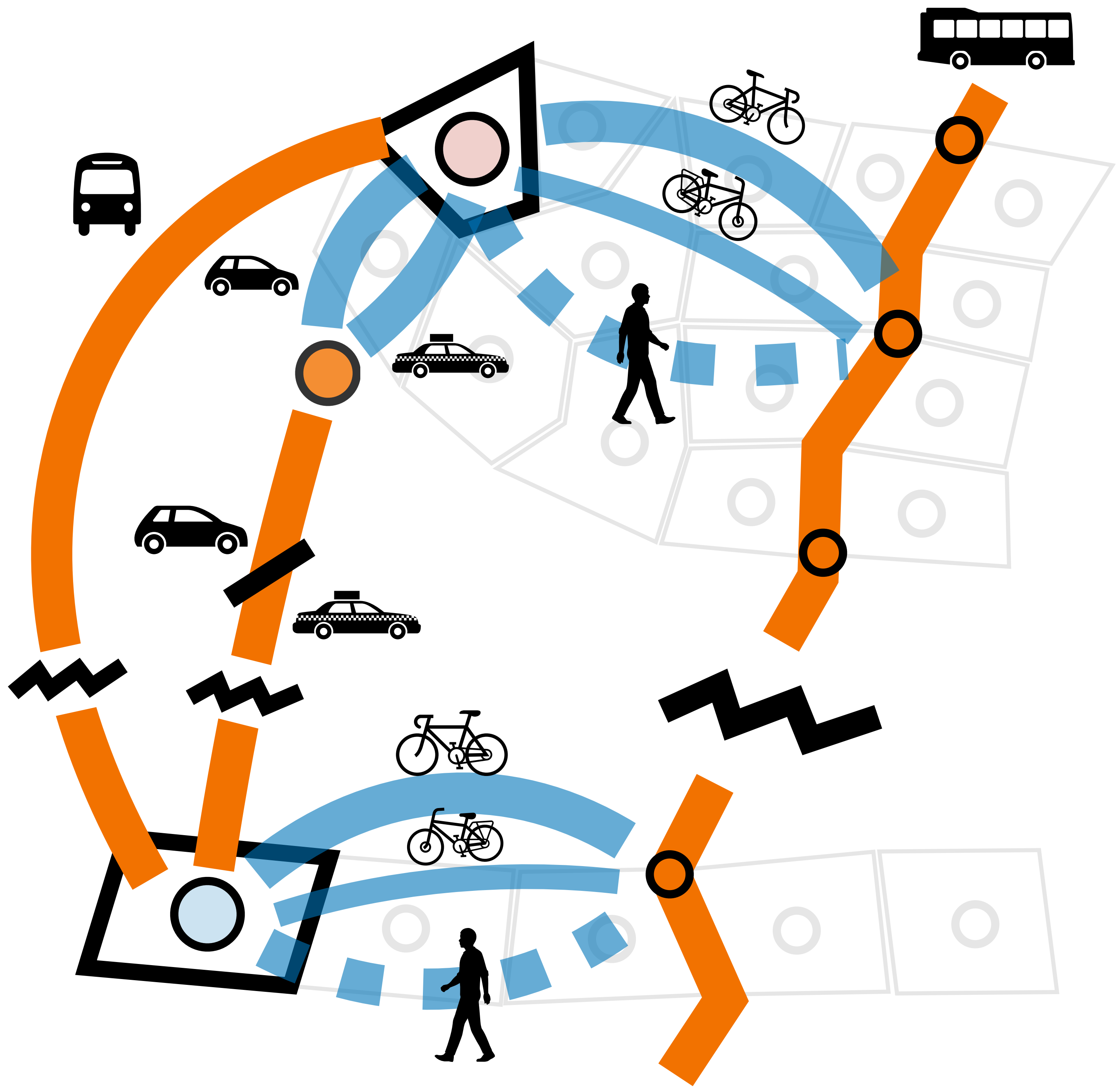

Computation:

Algorithm: **Frank-Wolfe**

LP for descent direction

Size: OD Pairs × # Primary Options × # Secondary Options

Individual's Cost

Primary Mode Preference + Secondary Mode Preference + Travel Time

Equilibrium

Population distribution
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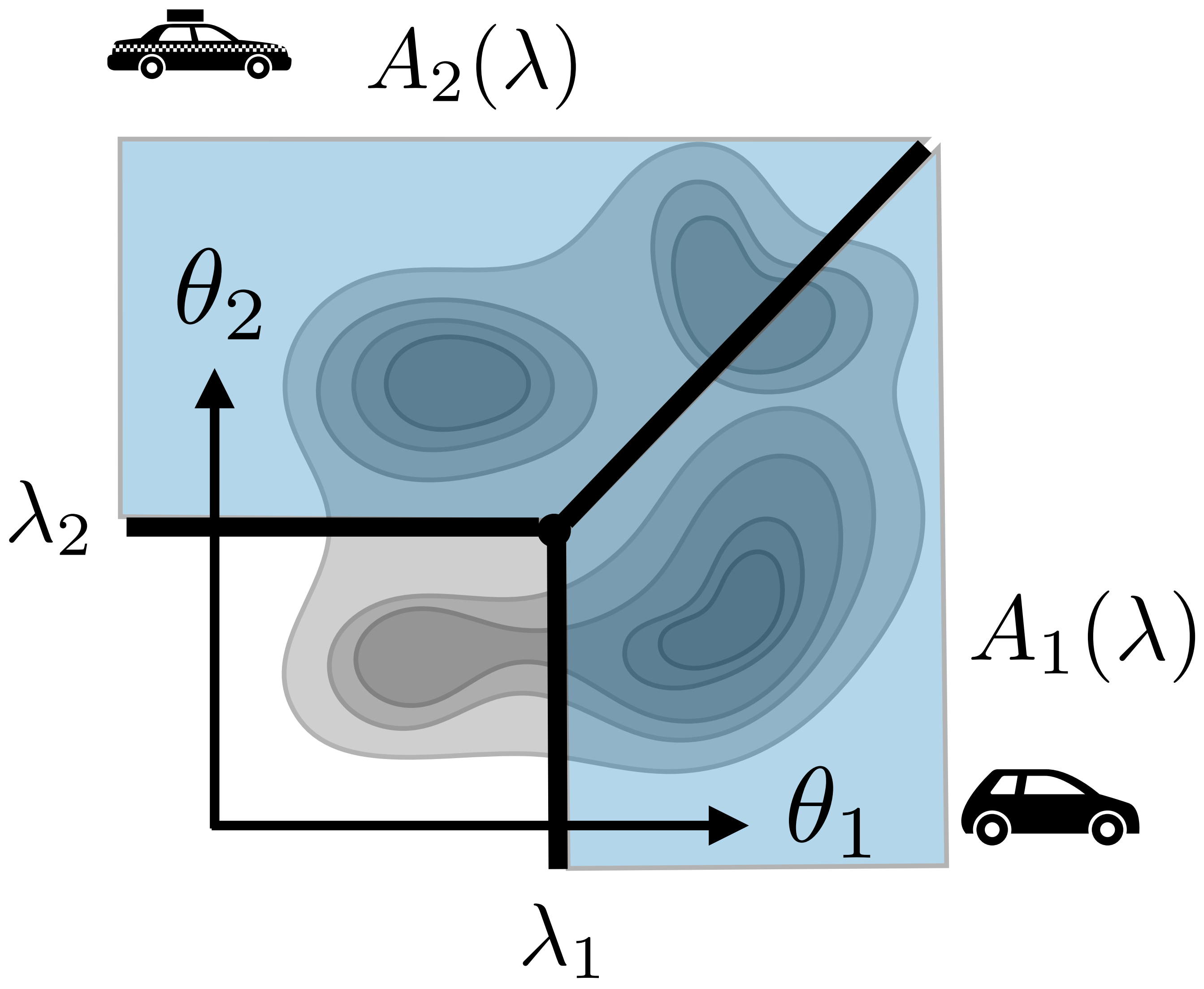
"No one can do any better"

Multi-Variate Non-Homogeneous Preferences

Computing mass distributions (at each origin point) → Survey (or guess)



Compare options -
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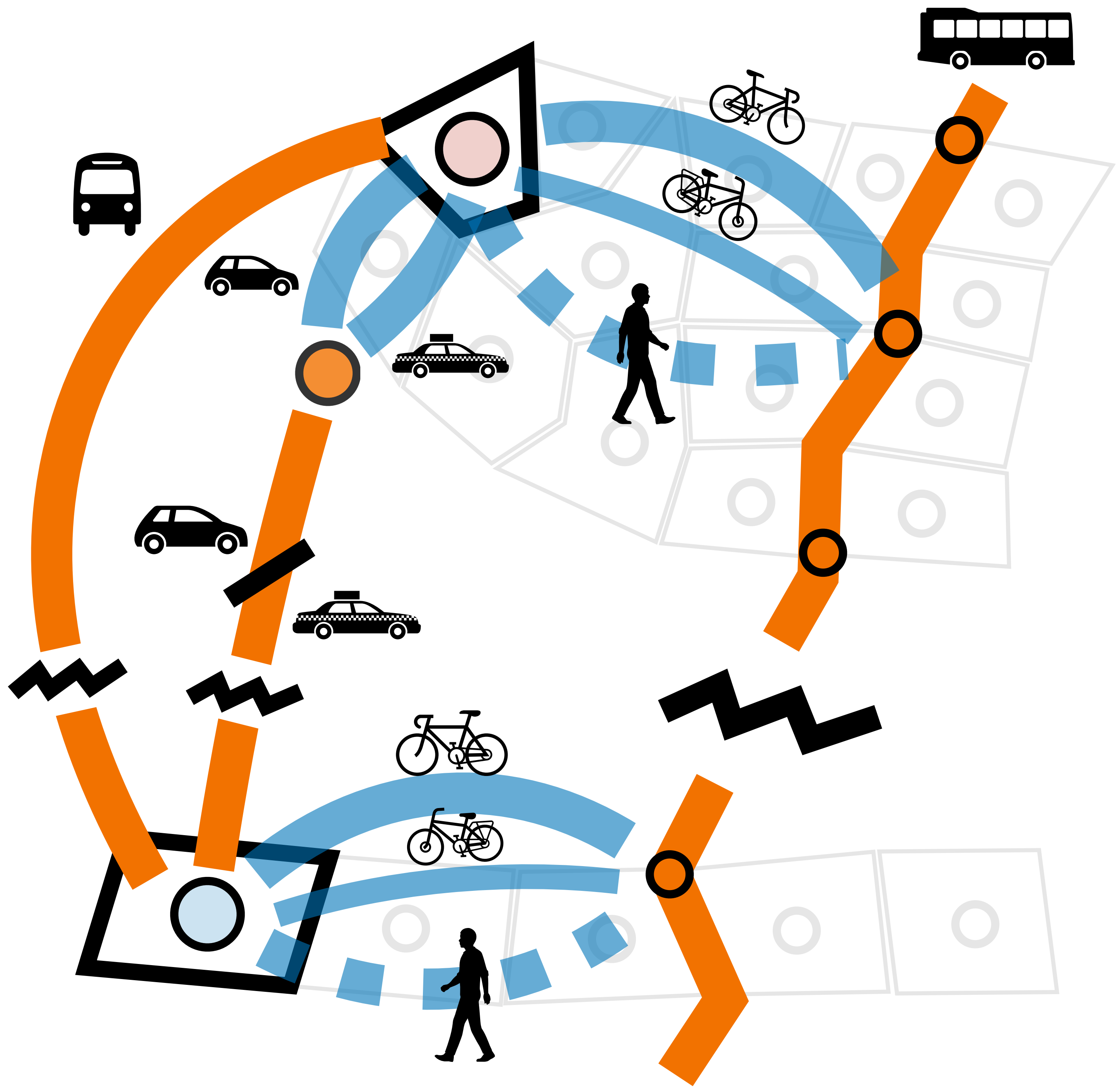


Computation:

Algorithm: **Frank-Wolfe**

LP for descent direction

Size: OD Pairs × # Primary Options × # Secondary Options × Primary Mode Sub-problem Size × Primary Mode Sub-problem Size

Individual's Cost

Primary Mode Preference + **Secondary Mode Preference** + **Travel Time**

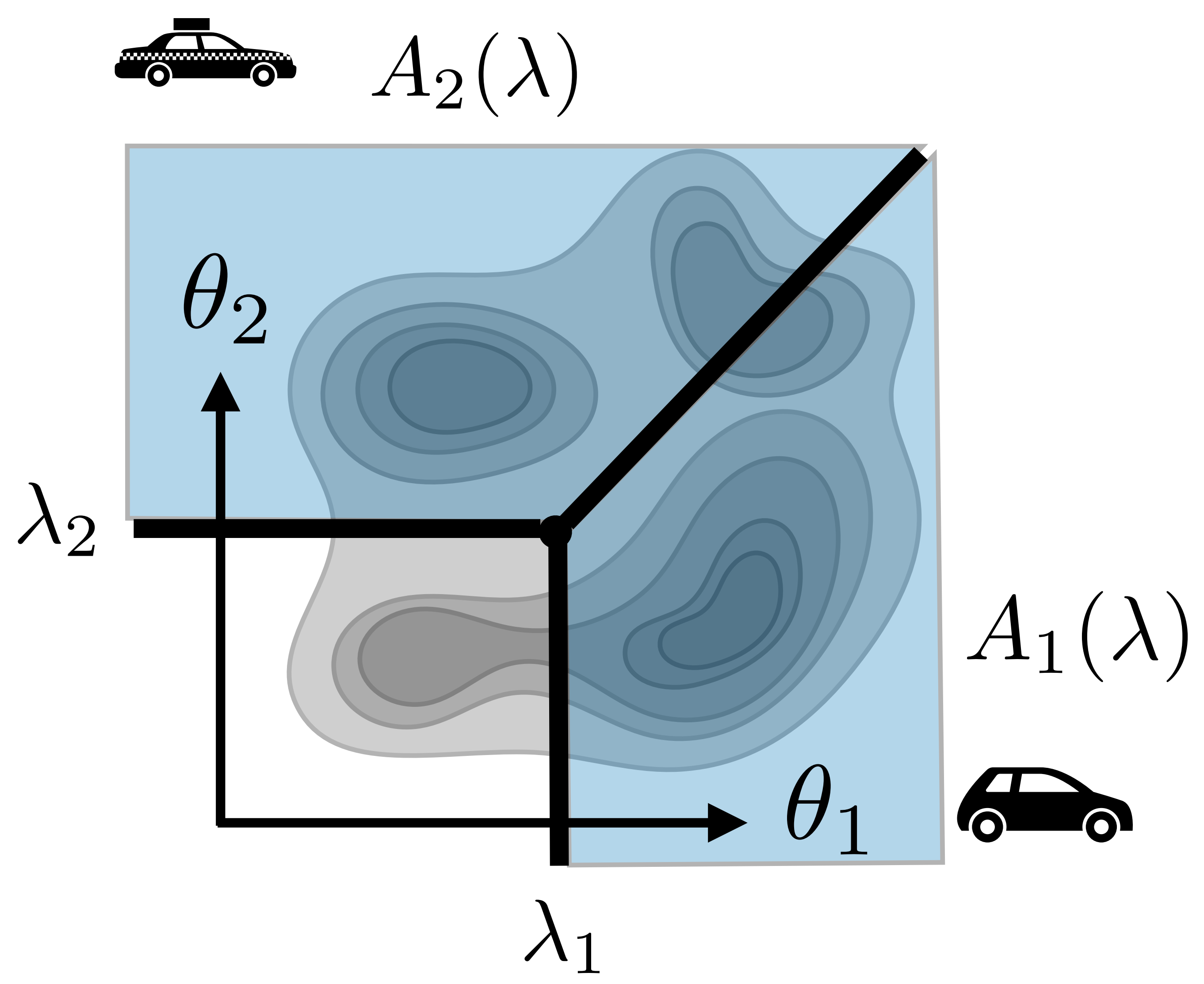
Equilibrium

Population distribution over transport options

"No one can do any better"

Multi-Variate Non-Homogeneous Preferences

Computing mass distributions (at each origin point) → Survey (or guess)



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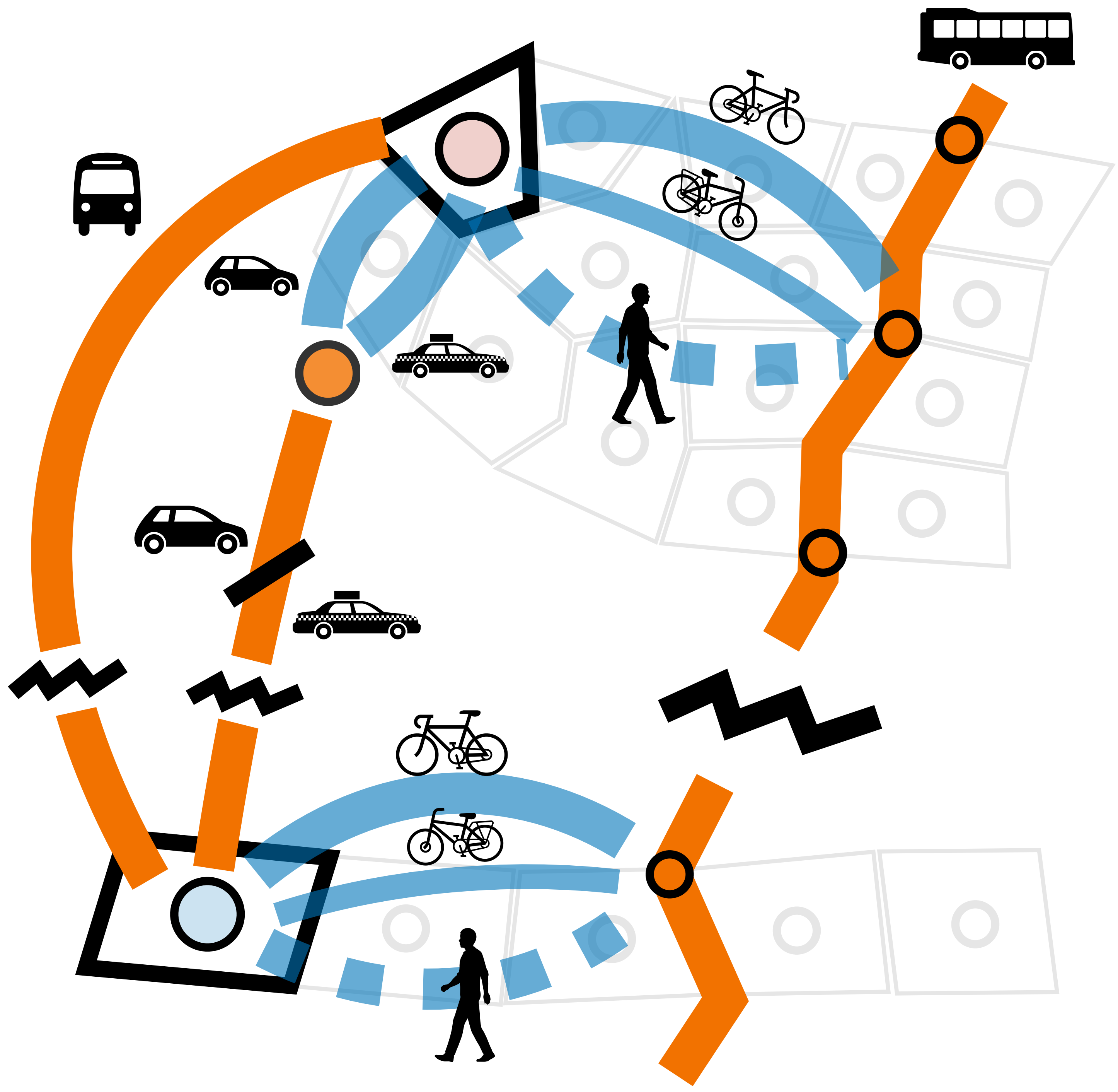


Computation:

Algorithm: **Frank-Wolfe**

LP for descent direction

Size: OD Pairs × # Primary Options × # Secondary Options

Individual's Cost

Primary Mode Preference + **Secondary Mode Preference** + **Travel Time**

Uses:

1. **Prediction**
2. **Incentive design -** parking prices, etc
3. **Route design — *difficult***
   (Monte Carlo search, probabilistic planning,…)

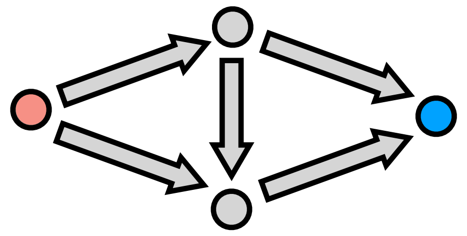
Equilibrium

Population distribution over transport options

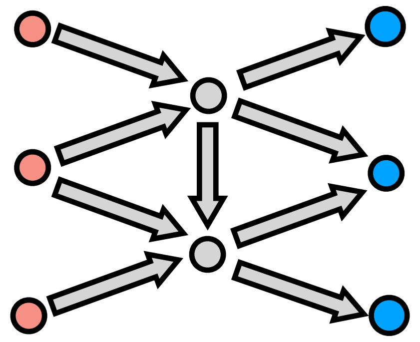
"No one can do any better"

Potential Games

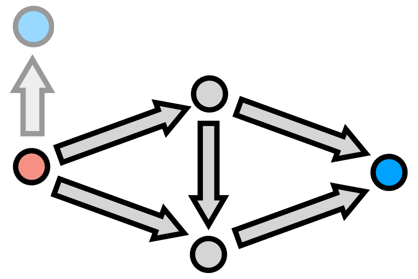
Routing Games



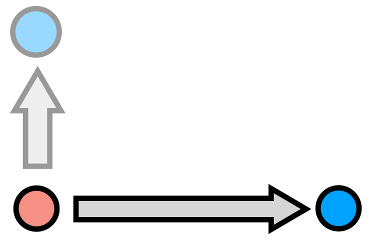
Multiple sources/ sinks



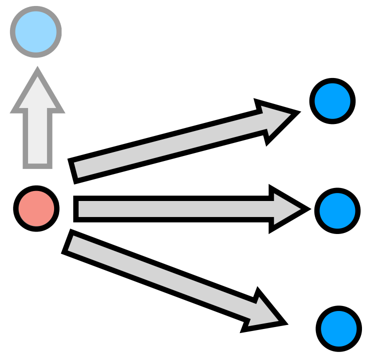
Variable Demand



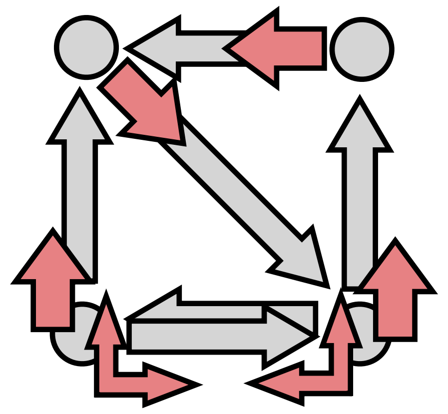
Supply & Demand



Cournot Market

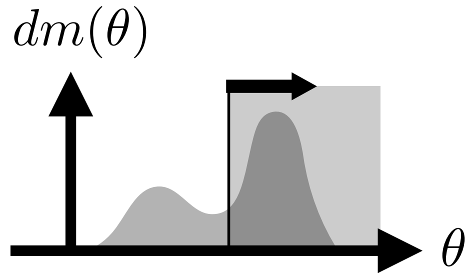


MDP Congestion Game

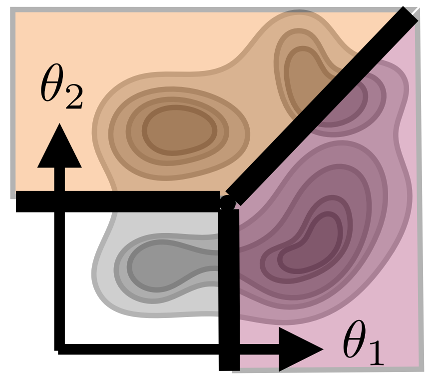


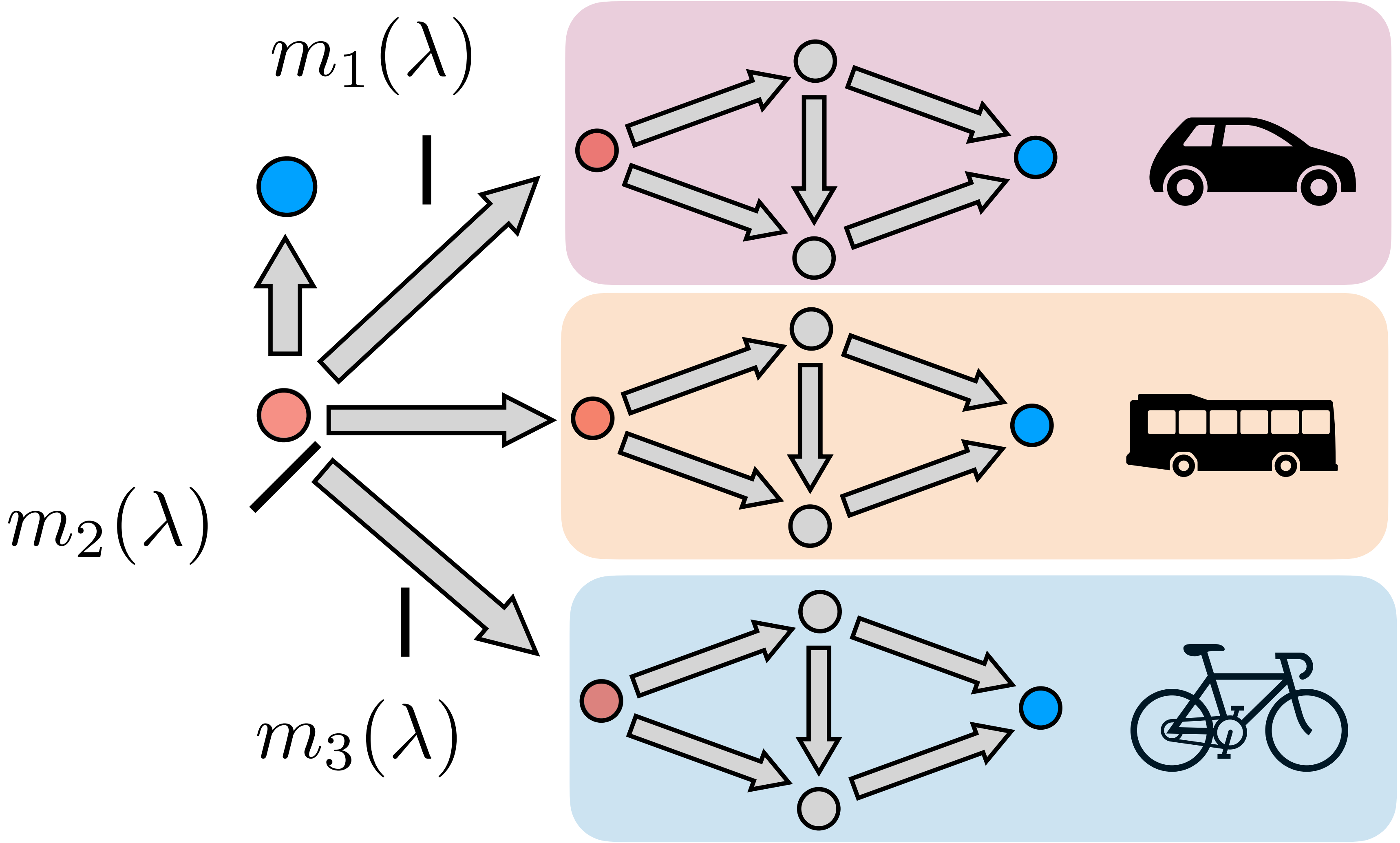
Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous prefences



Multi-Variate Preferences





APPLICATIONS

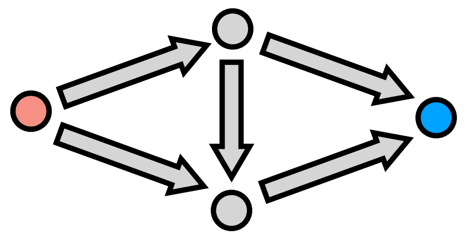
- Multi-modal transportation networks
- Non-homogeneous supply/demand

PAPERS

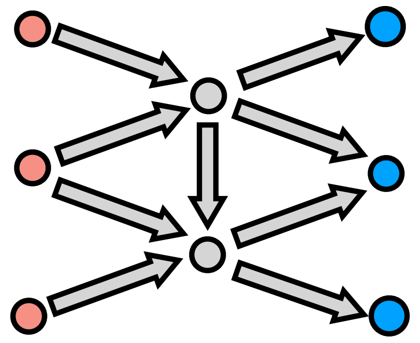
- External-cost continuous-type Wardrop equilibria in routing games
  [Calderone, Dong, Sastry, 2017]
- Multi-dimensional continuous type population potential games
  [Calderone, Ratliff, 2019]

Potential Games

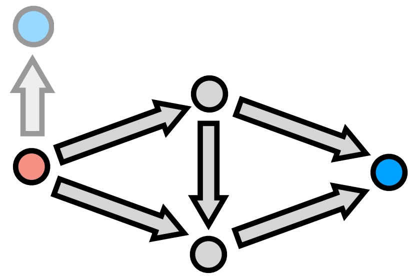
Routing Games



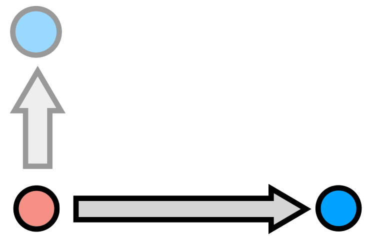
Multiple sources/ sinks



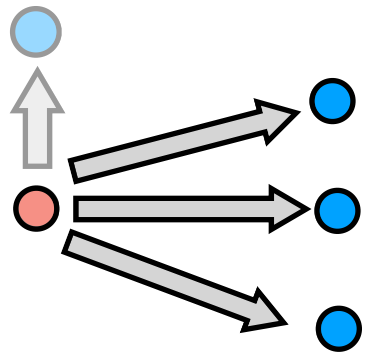
Variable Demand



Supply & Demand



Cournot Market



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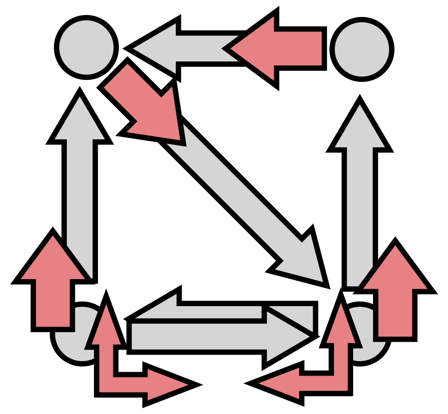
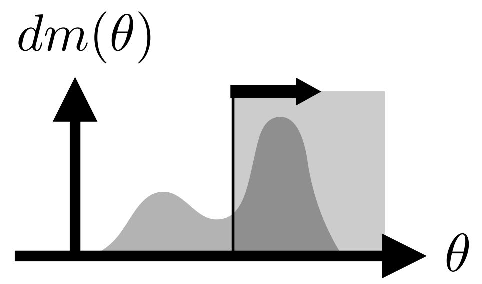
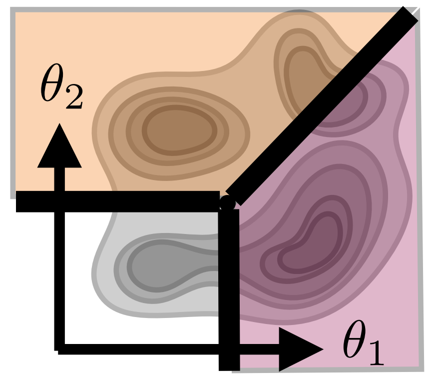


Braess Paradox

Non-homo-geneous prefences



Multi-Variate Preferences



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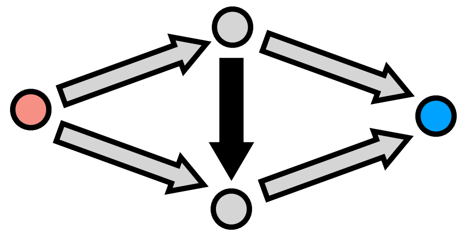

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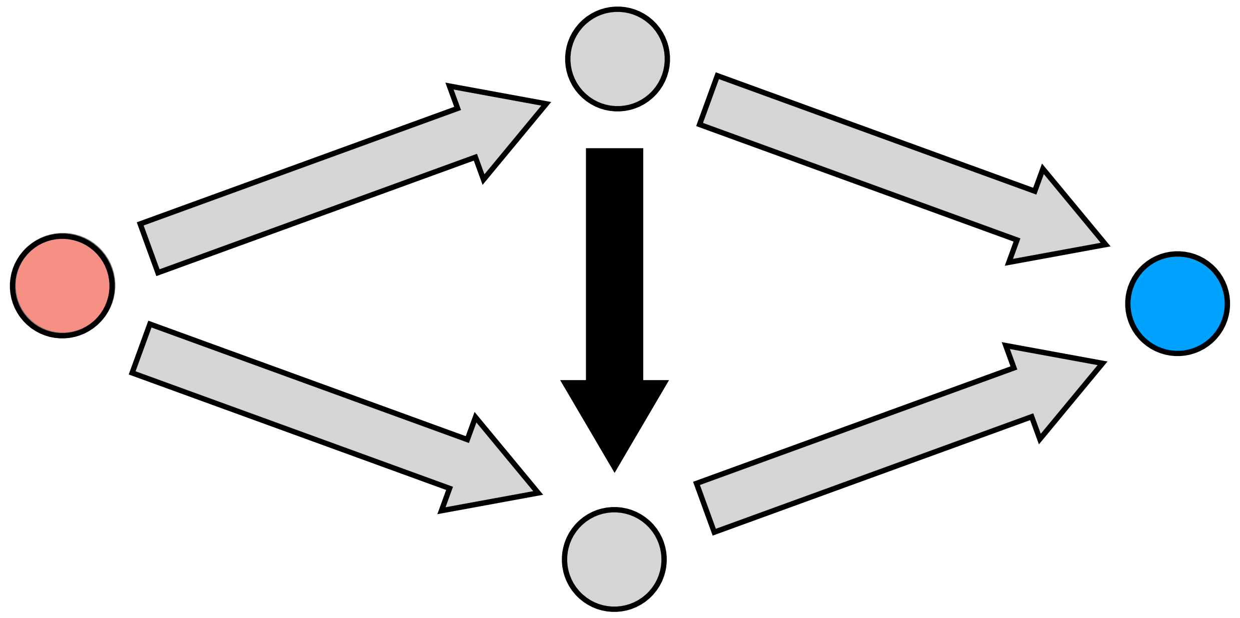
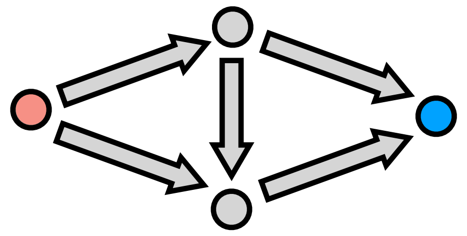


Potential Games

Braess Paradox

Routing Games



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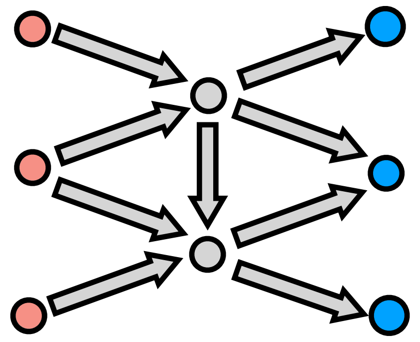
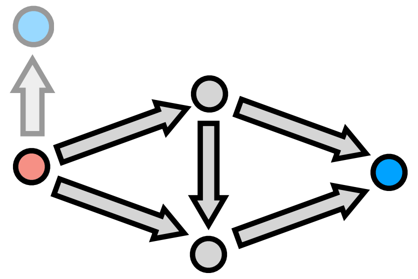
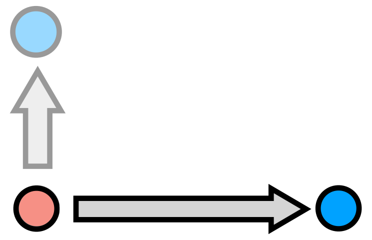


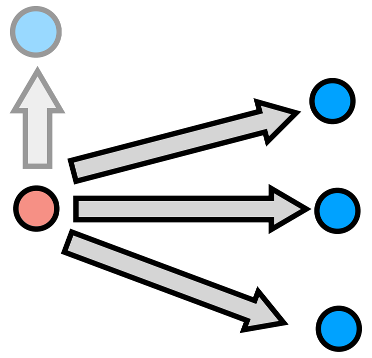
Variable Demand



Supply & Demand



Cournot Market



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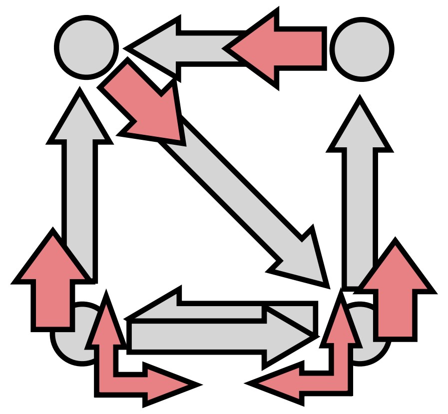

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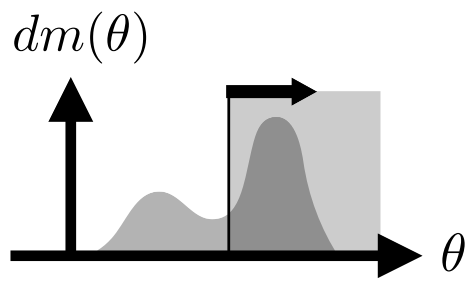

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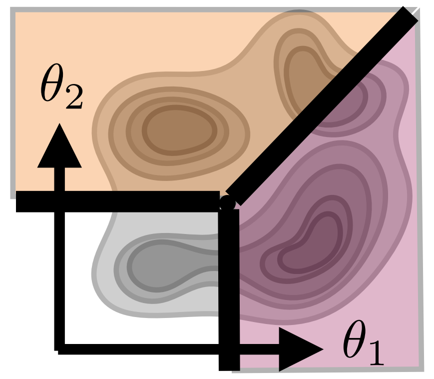

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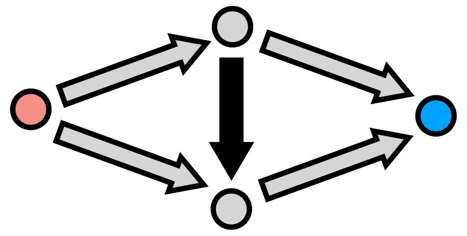

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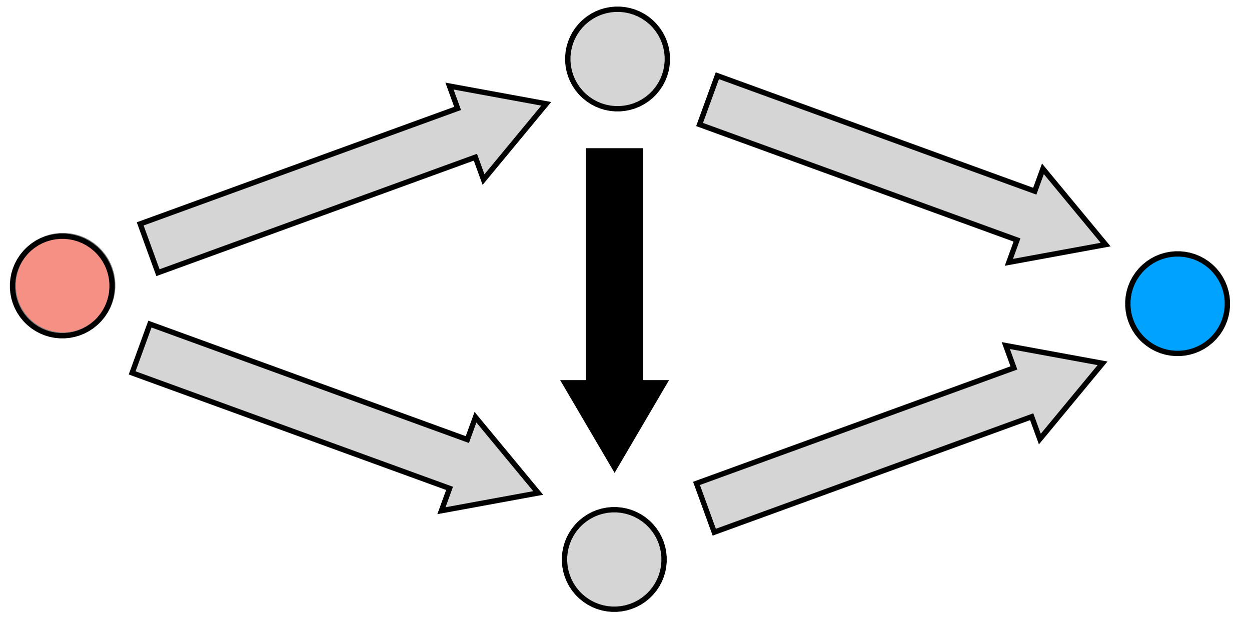
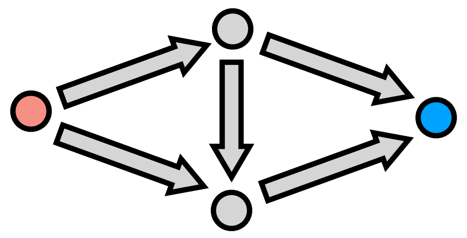


Adding center road can make traffic worse!

Potential Games

Routing Games



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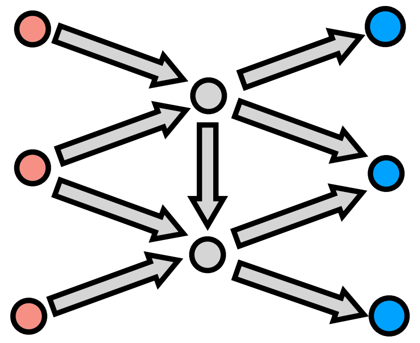

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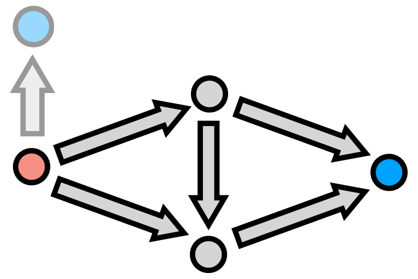

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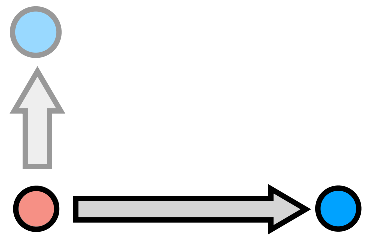

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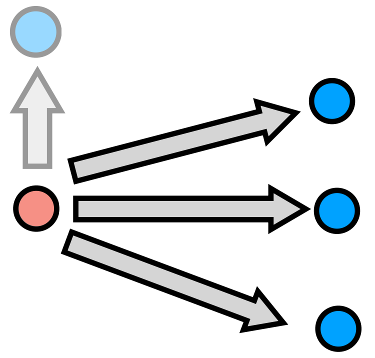

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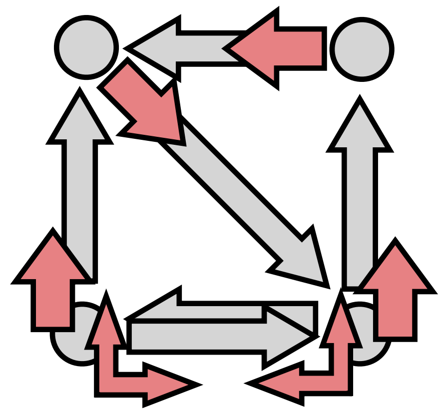

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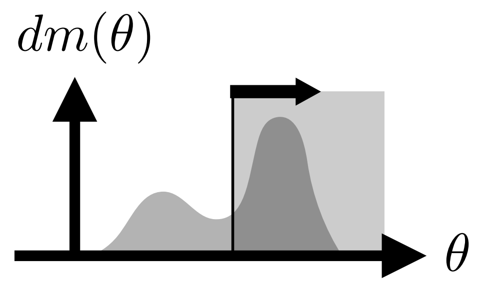

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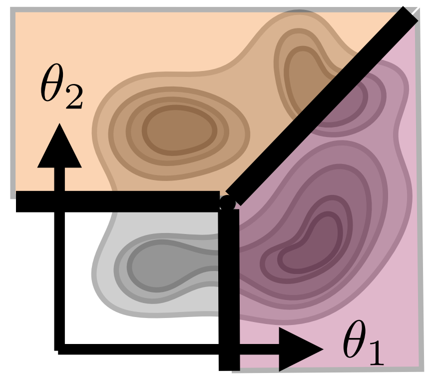

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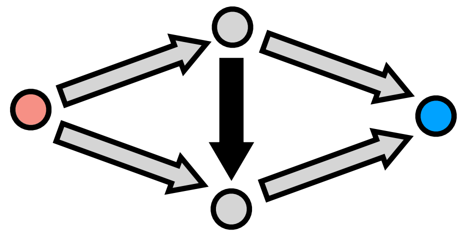

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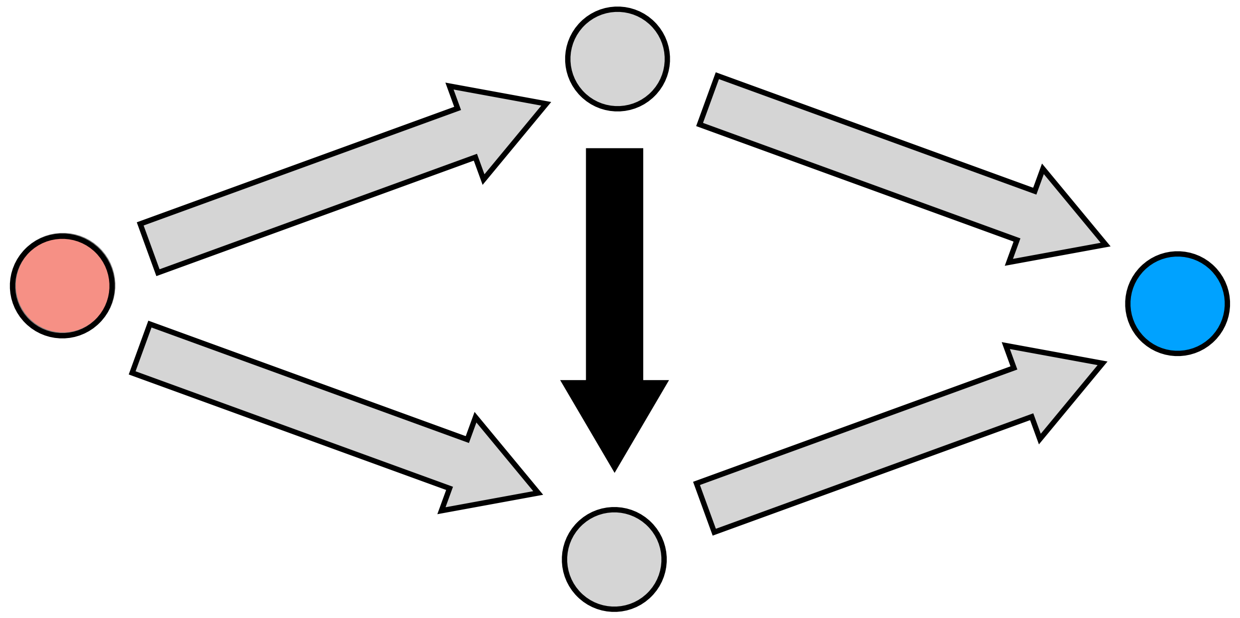
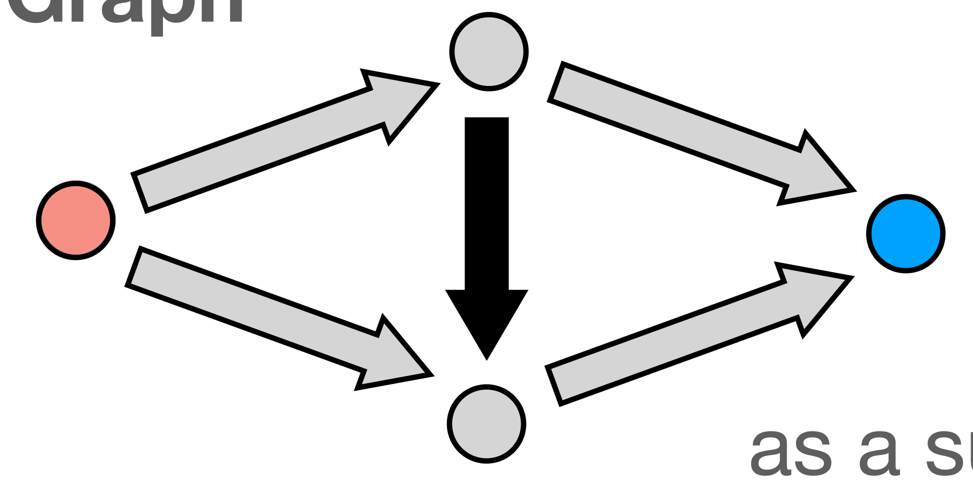


Adding center road can make traffic worse!

Network Characterization:

Every network has

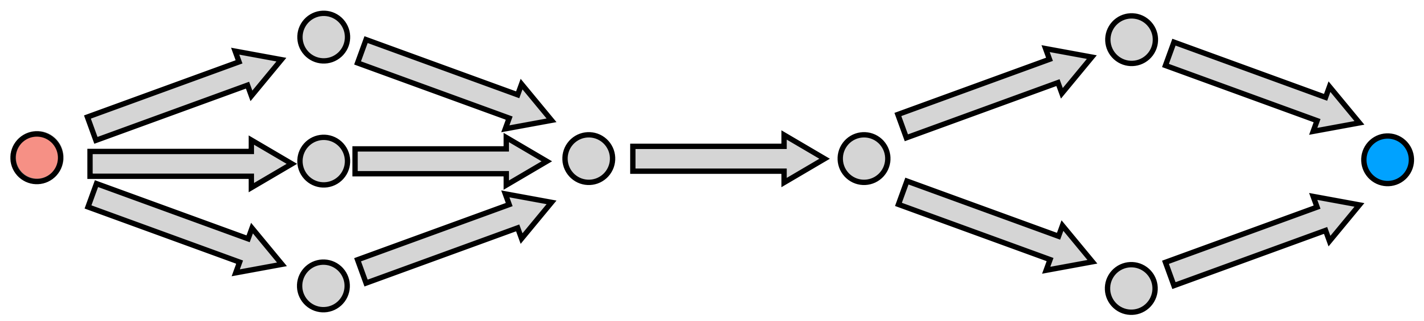
Braess Graph



as a subgraph

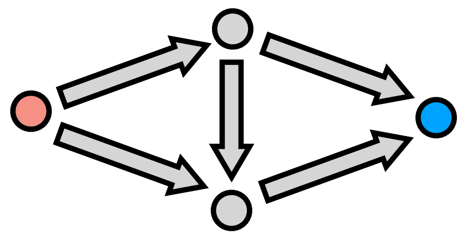
OR is

Series-Parallel Graph



Potential Games

Routing Games



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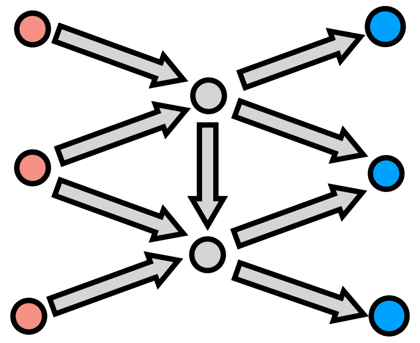

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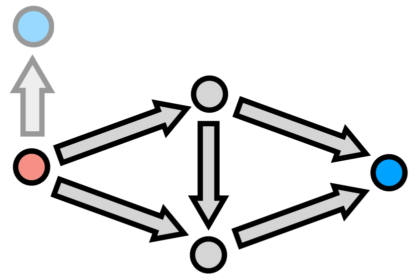

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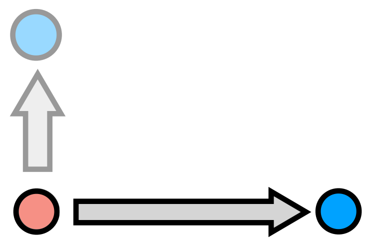

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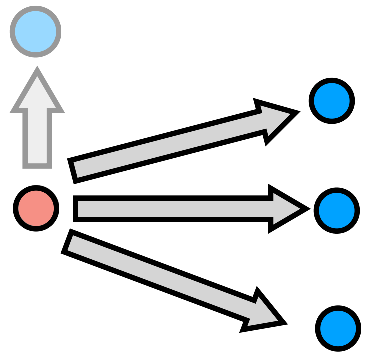

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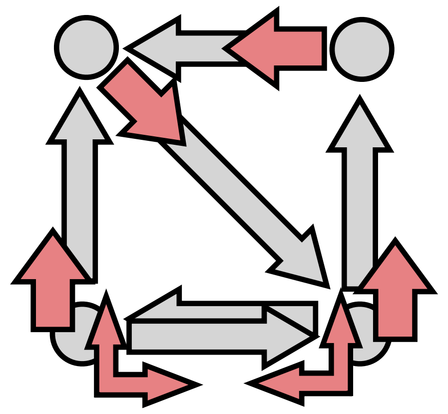

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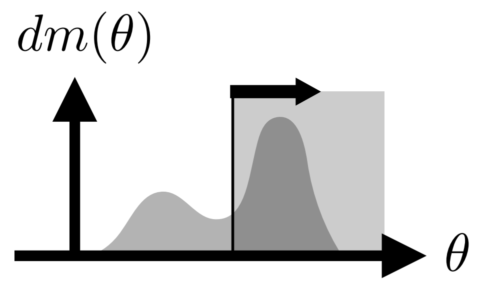

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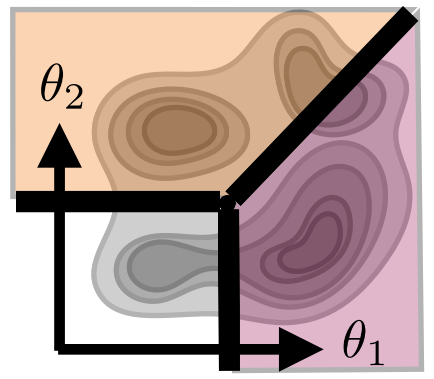

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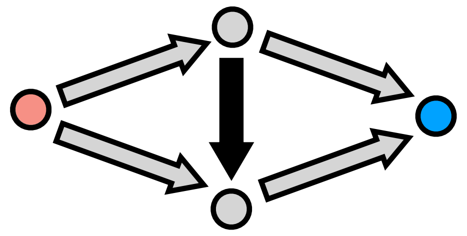

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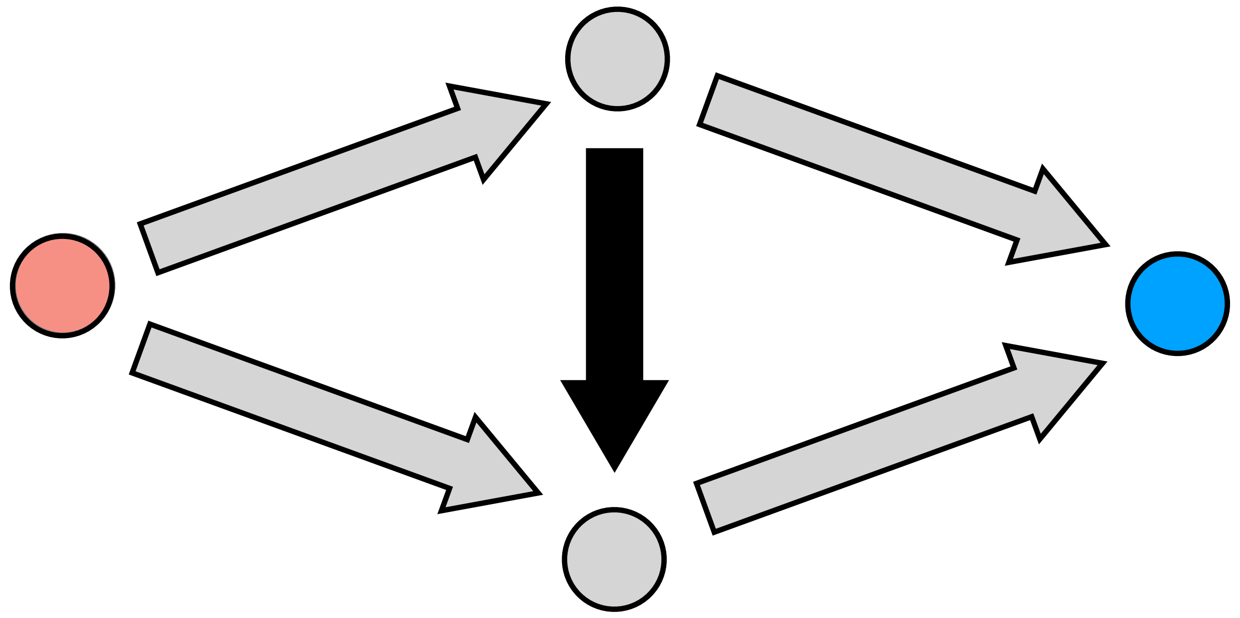

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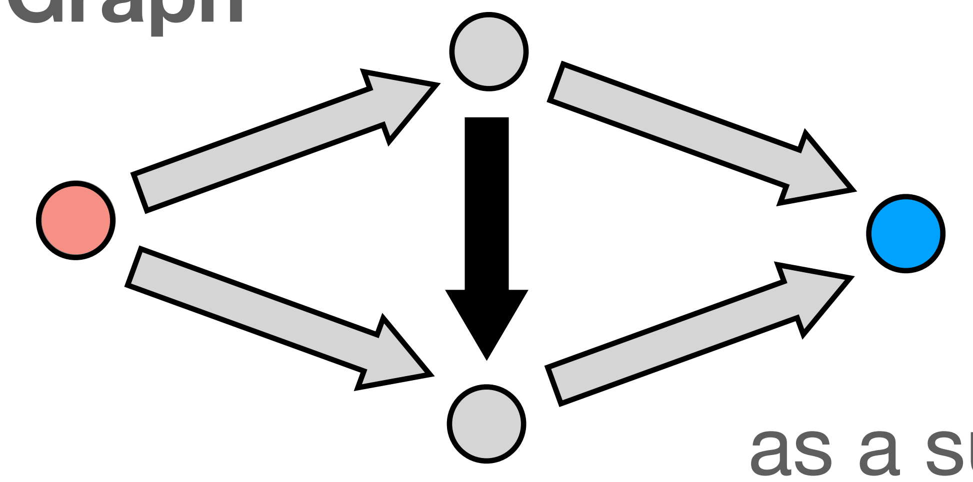

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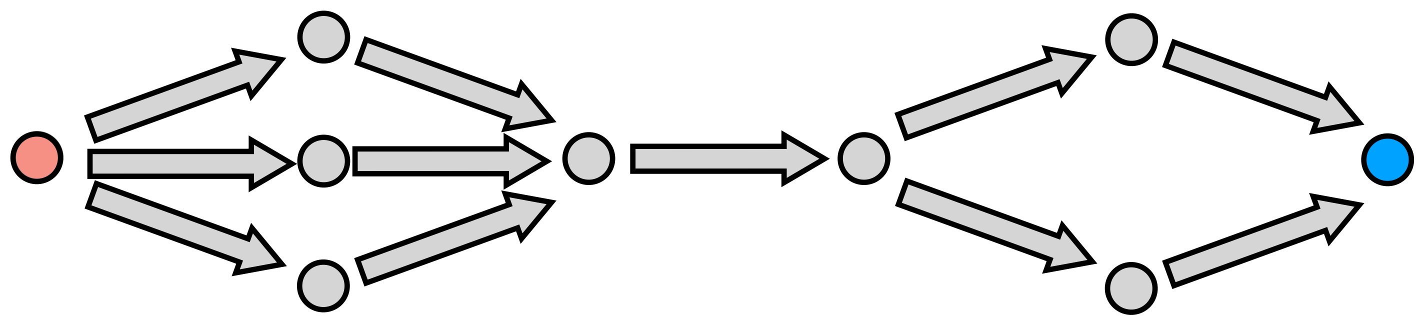
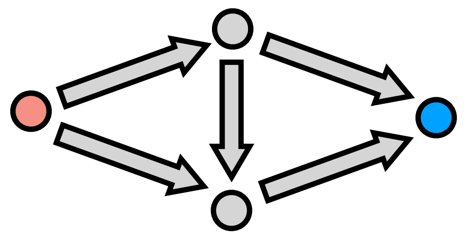


Series - Parallel graphs cannot suffer from Braess paradox ← **linear-algebraic characterization/proof**

Potential Games

Routing Games



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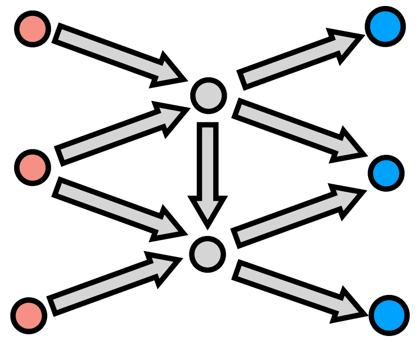

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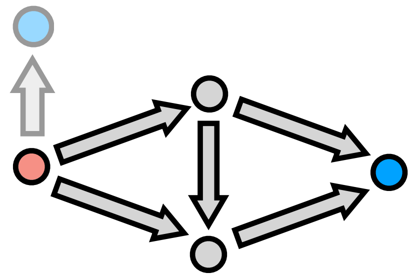

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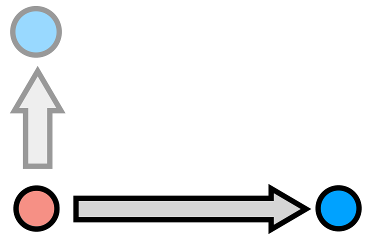

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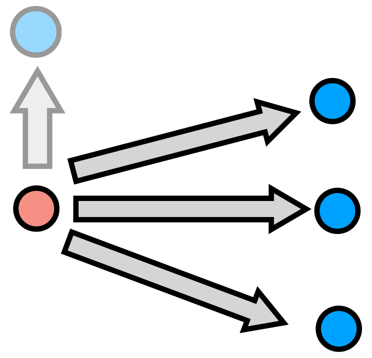

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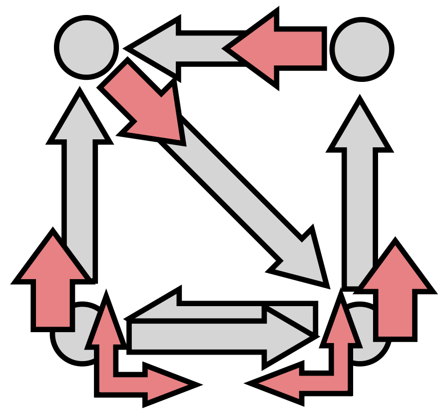

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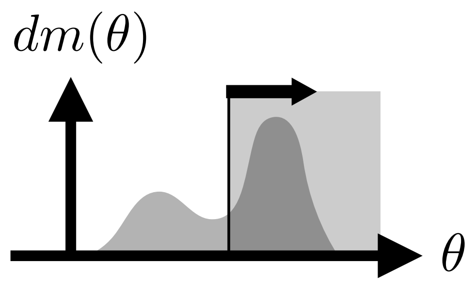

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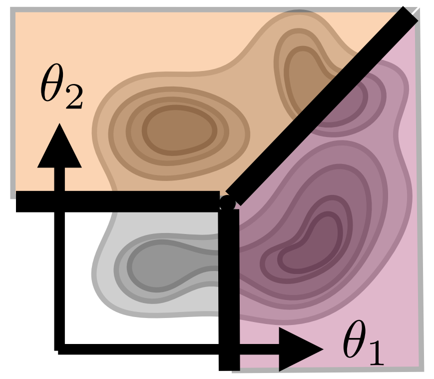

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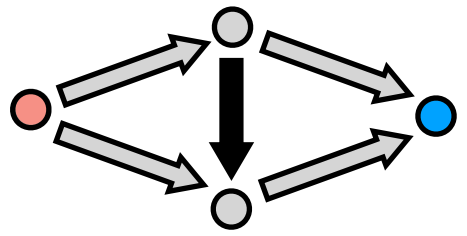

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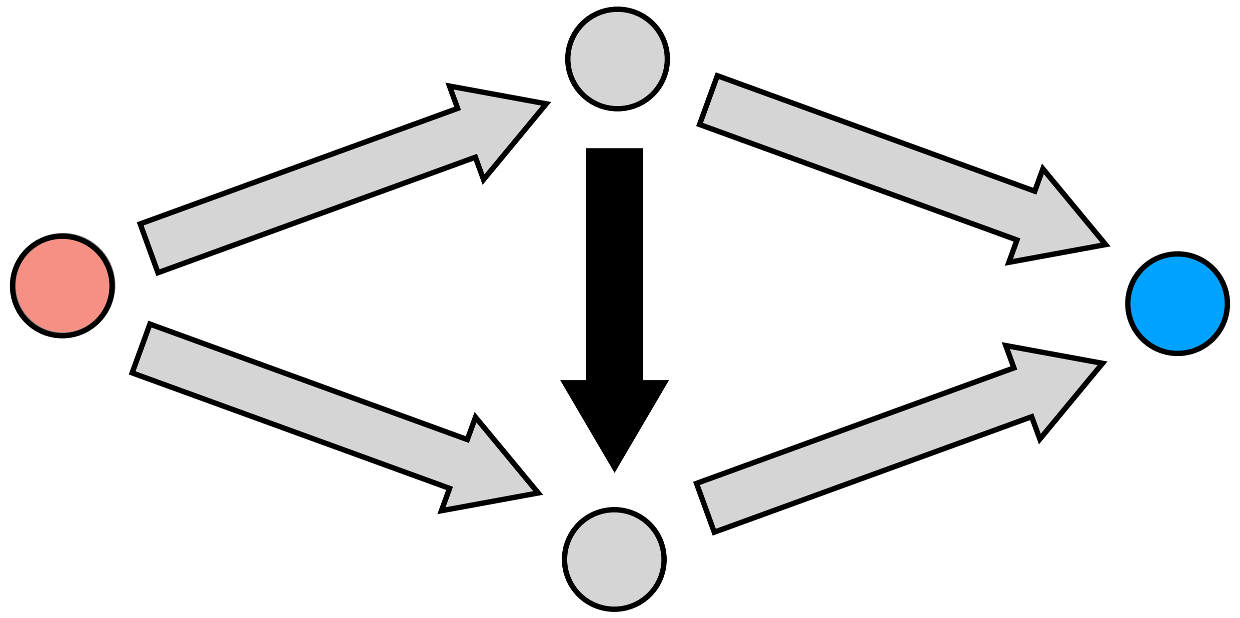

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REFERENCES

- Uber ein Paradoxon der verkehrsplanung **[Braess, 1969]**
- Topology of series-parallel networks **[Duffin, 1965]**
- Network topology and the efficiency of equilibrium **[Milchtaich, 2006]**

PAPERS

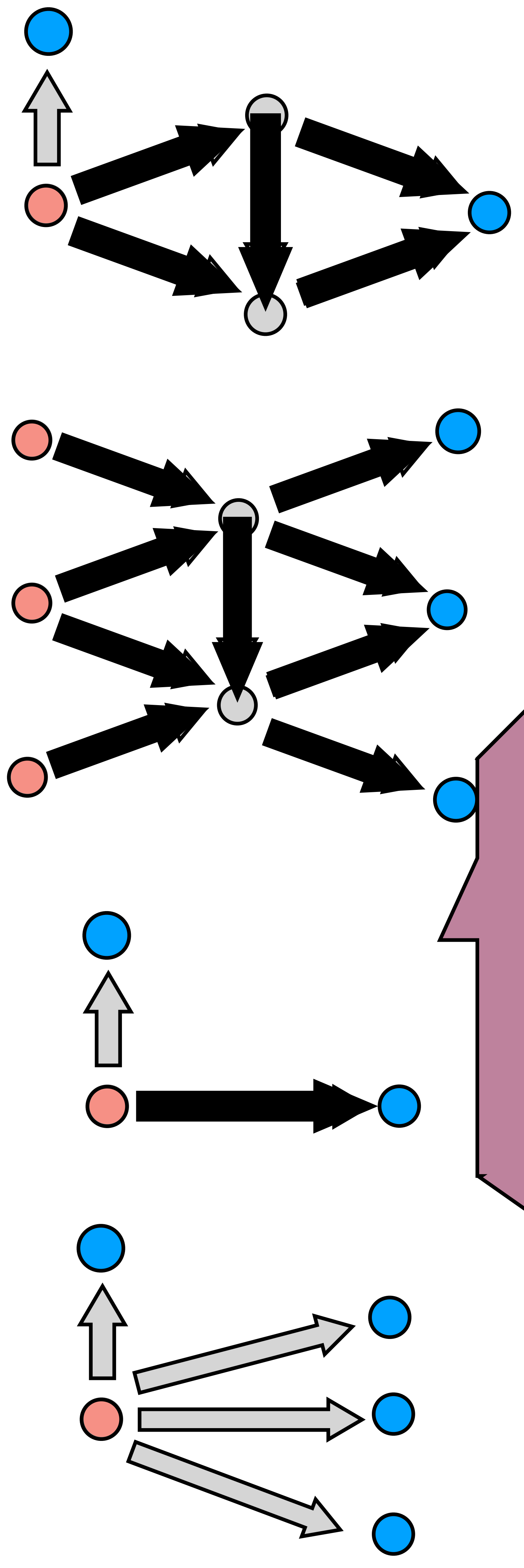
- Sensitivity analysis for Markov decision process congestion games **[Li, Calderone, Ratliff, 2019]**
- Algebraic characterization of Braess paradox: Network efficiency in series-parallel and Braess networks **[Calderone, Ratliff, in prep]**

Variable Demand

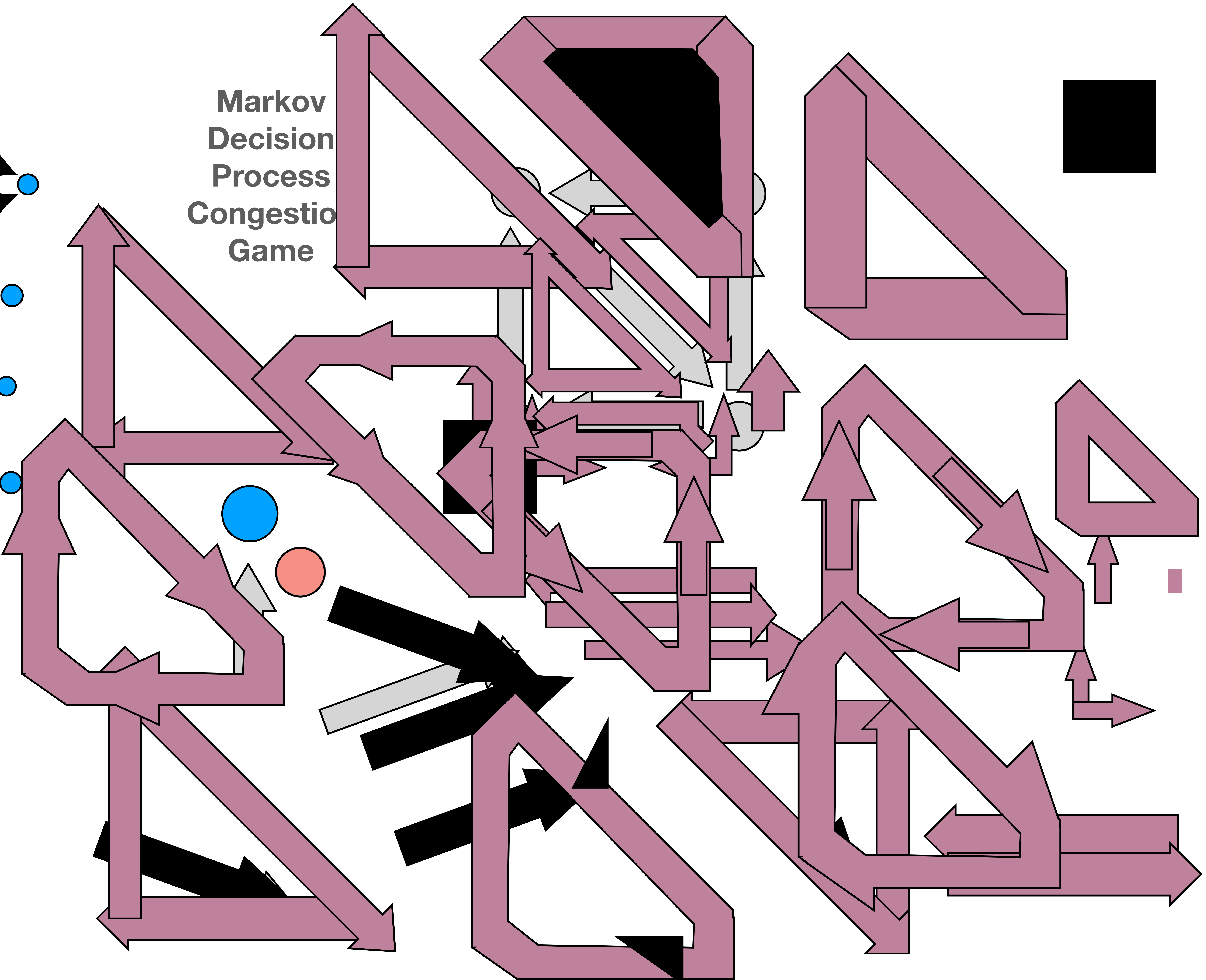
Multiple sources/ sinks

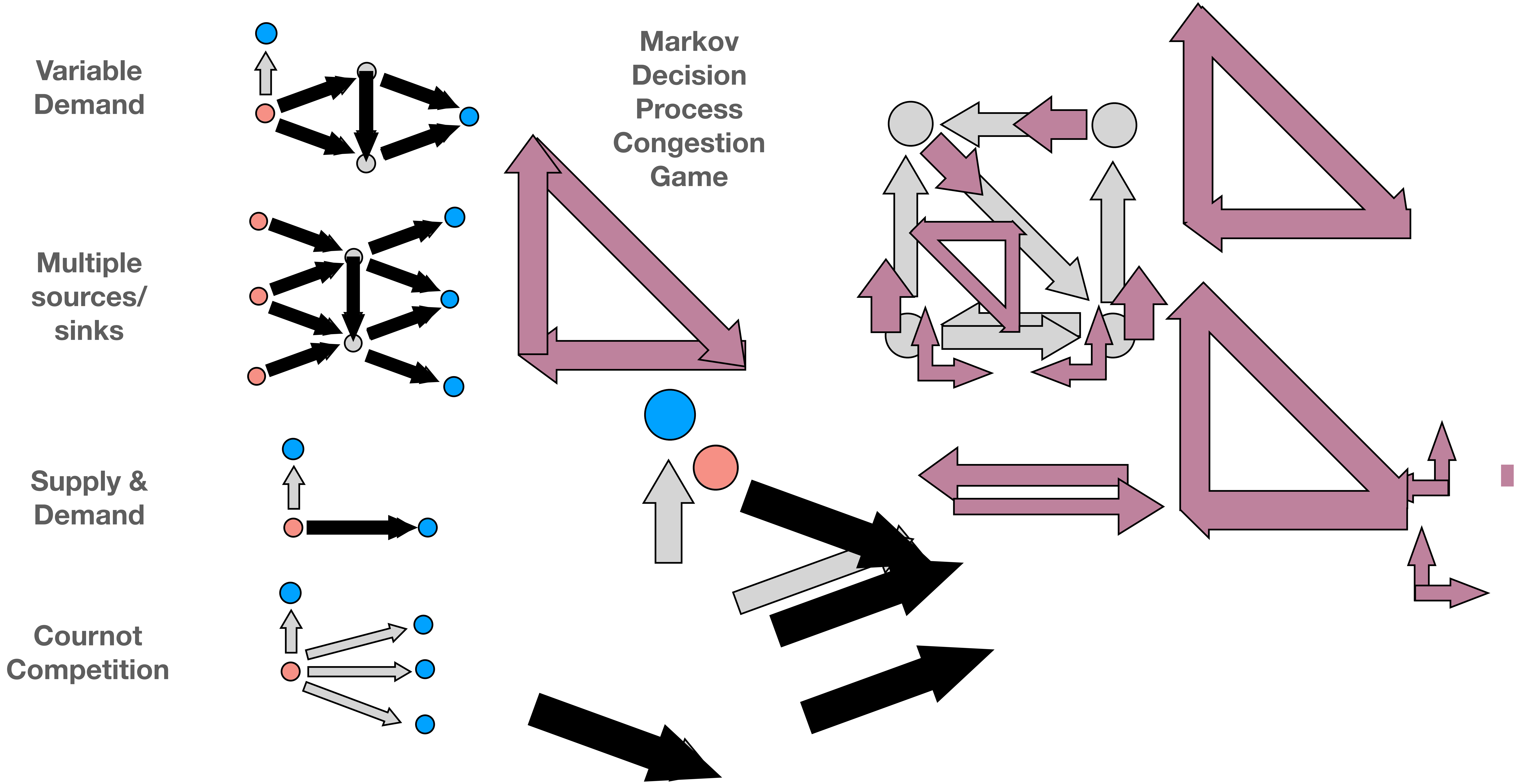
Supply & Demand

Cournot Competition



Markov Decision Process Congestio Game




Variable Demand
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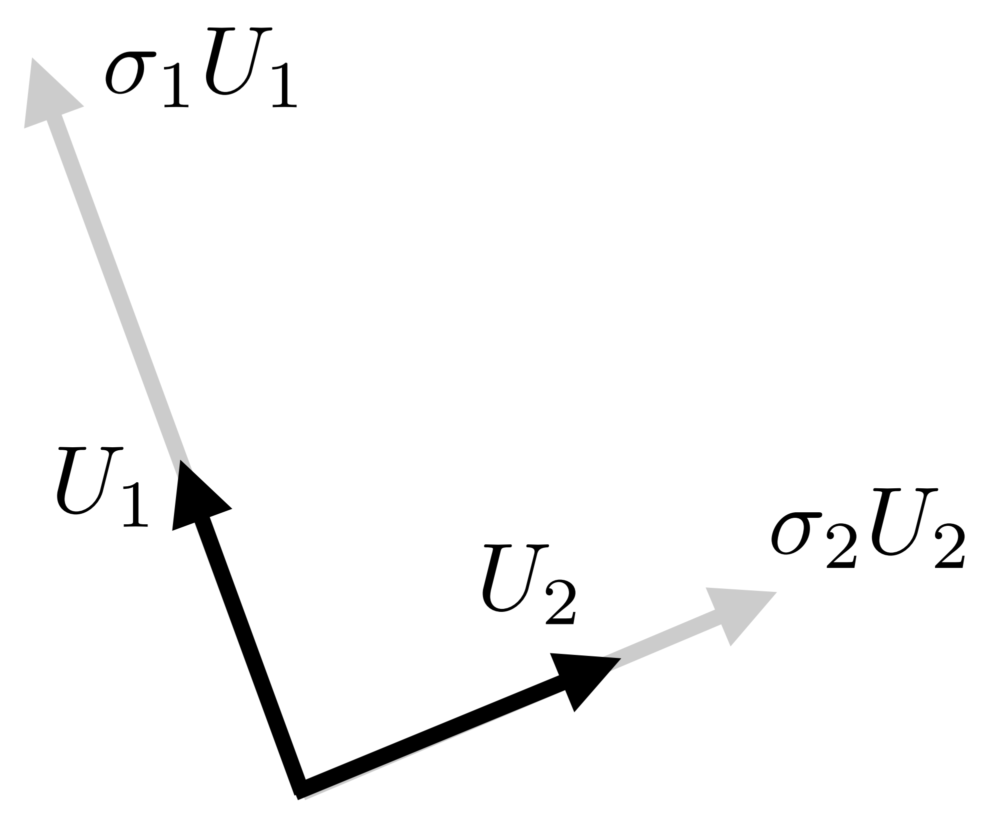
Velocity Space $\mathbb{R}^2$



directions of motion in velocity space

DECODER

$$D \in \mathbb{R}^{2\times 64}$$

$$D = U\,[\Sigma \quad 0]\,V^T$$

$$D = \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \\ & \vdots & \\ - & V_{64}^T & - \end{bmatrix}$$

Orthonormal basis for nullspace

$$D = \begin{bmatrix} | \\ U_1 \\ | \end{bmatrix} \sigma_1 \begin{bmatrix} - & V_1^T & - \end{bmatrix} + \begin{bmatrix} | \\ U_2 \\ | \end{bmatrix} \sigma_2 \begin{bmatrix} - & V_2^T & - \end{bmatrix}$$

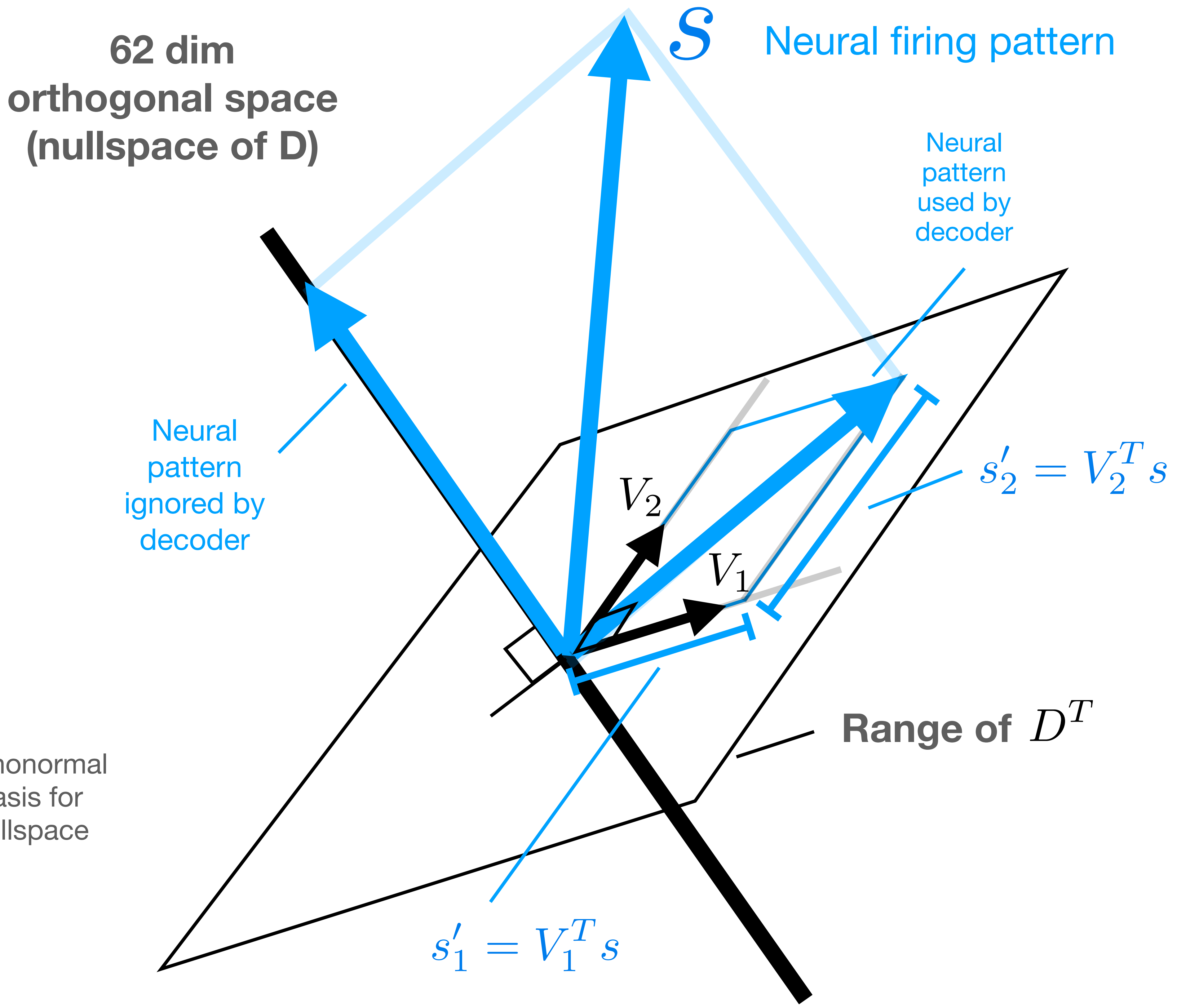
Correlations between neurons firing and motion in the U_1 direction

$$\begin{bmatrix} \sigma_1(V_1)_1 & \cdots & \sigma_1(V_1)_{64} \end{bmatrix}$$

... and the U_2 direction

$$\begin{bmatrix} \sigma_2(V_2)_1 & \cdots & \sigma_2(V_2)_{64} \end{bmatrix}$$

Neural Inputs: $\mathbb{R}^{64}$

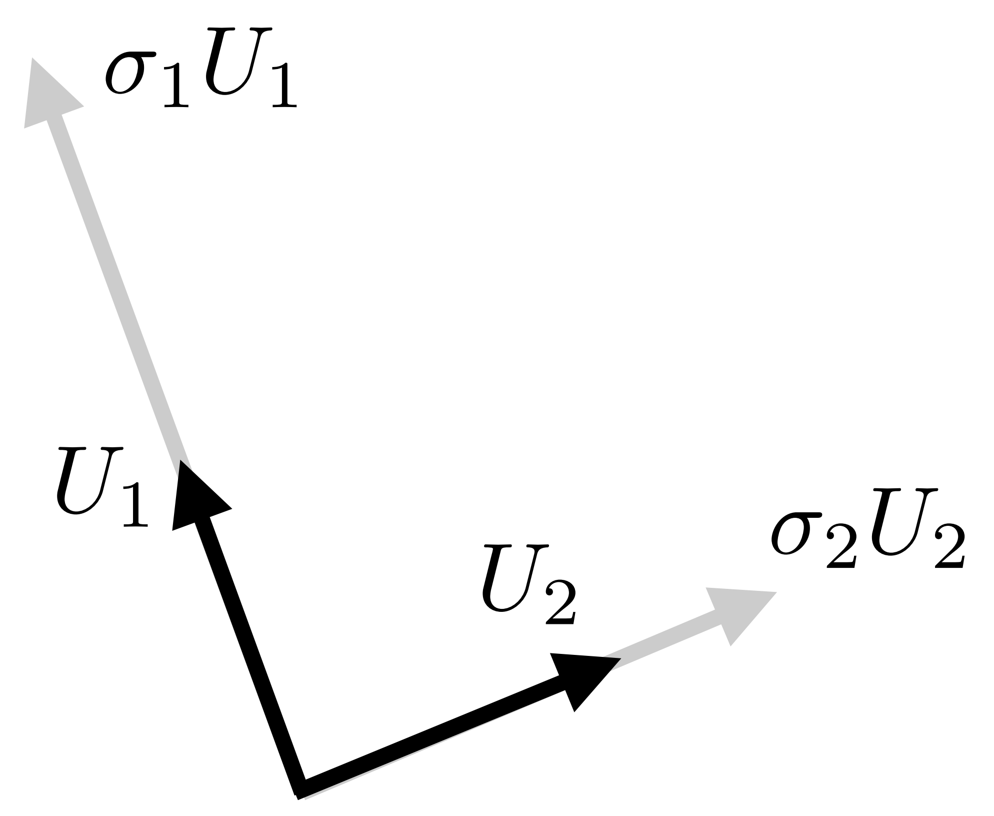


Coordinate Transform: $s' = V^T s$

$$\begin{bmatrix} s_1' \\ s_2' \\ \vdots \end{bmatrix} = \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ & \vdots & \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix} = \begin{bmatrix} V_1^T s \\ V_2^T s \\ \vdots \end{bmatrix}$$

Velocity Space $\mathbb{R}^2$

If sigma1 = sigma2,
then U1 and U2 are arbitrary
and only span of V1 and V2 matters



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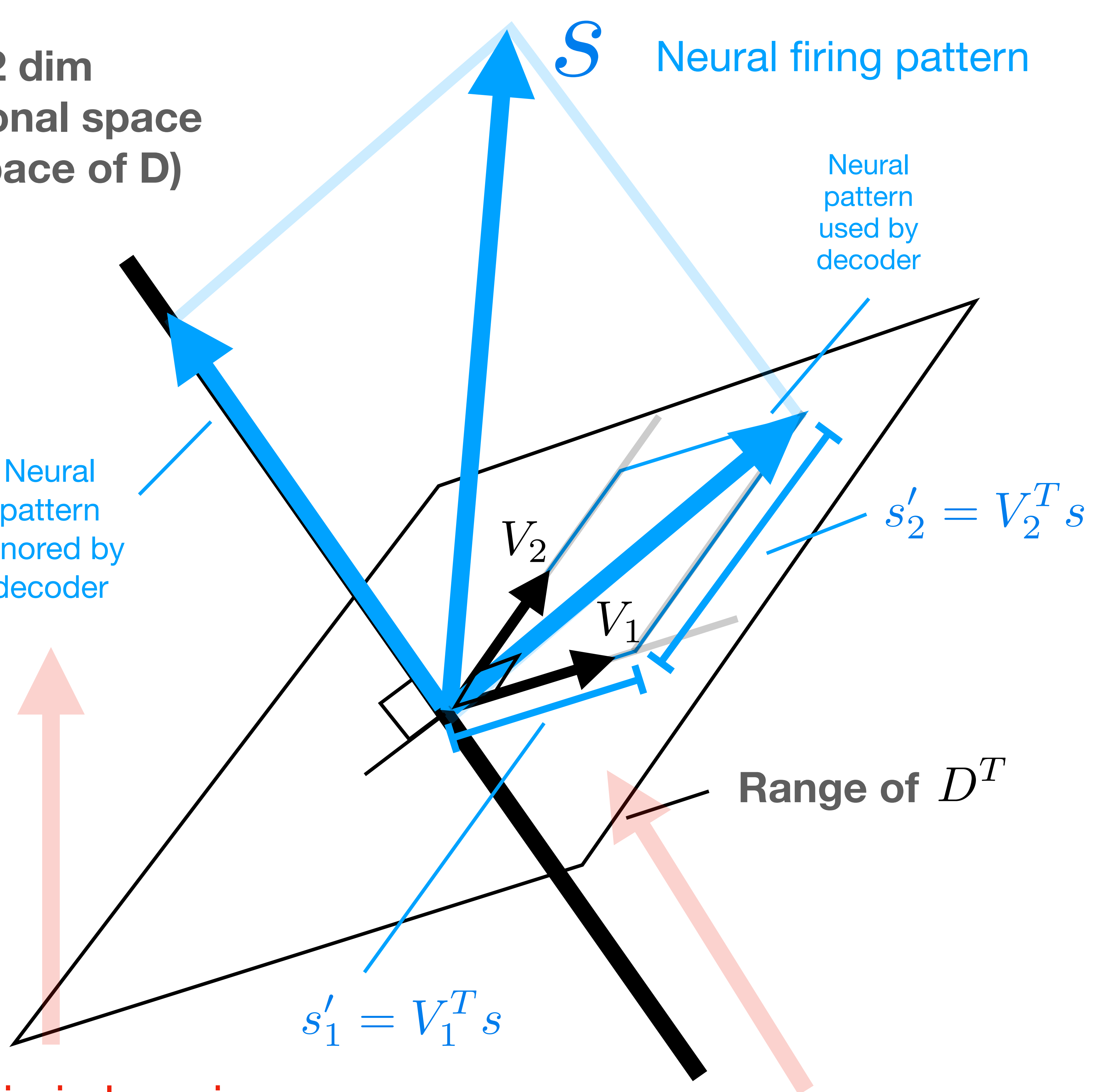
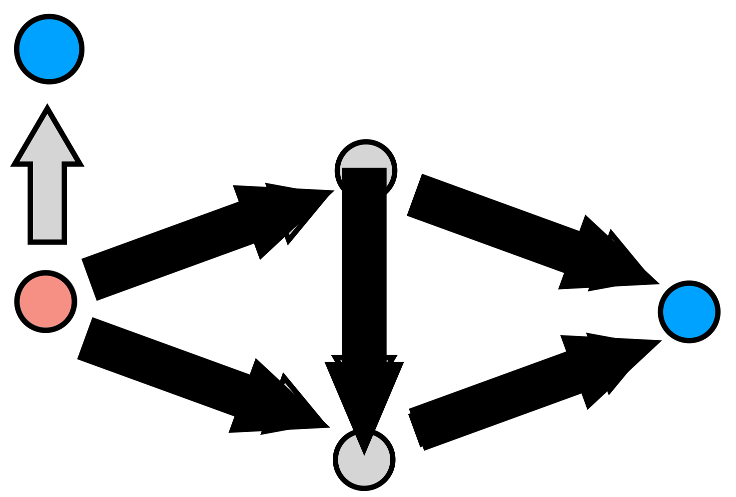


As brain is learning decoder, seems this component should shrink… (if not, brain is not halting firing of useless neurons.)

Would expect this subspace to converge and remain consistent over trials (for one person)

(span of V1 & V2 consistent)

Variable Demand



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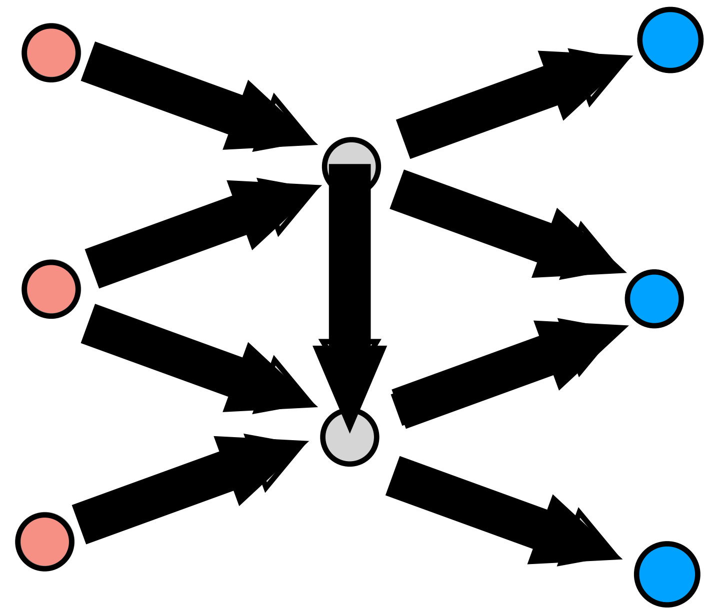

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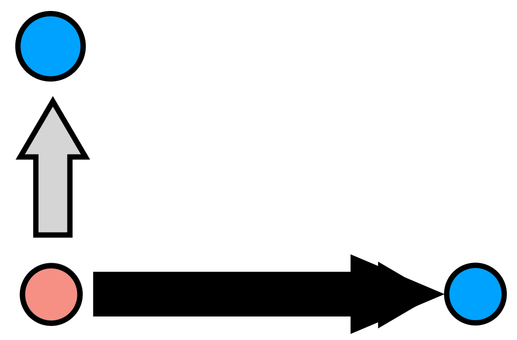

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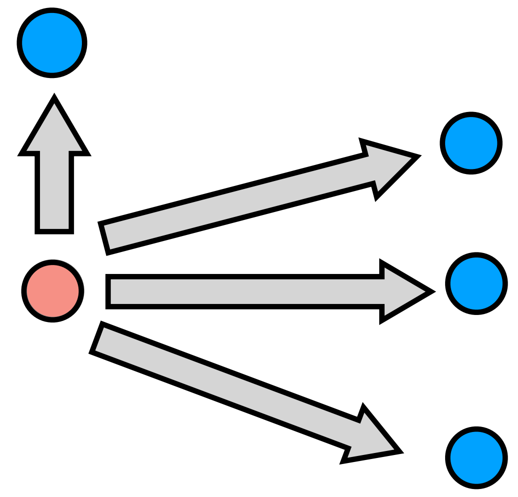

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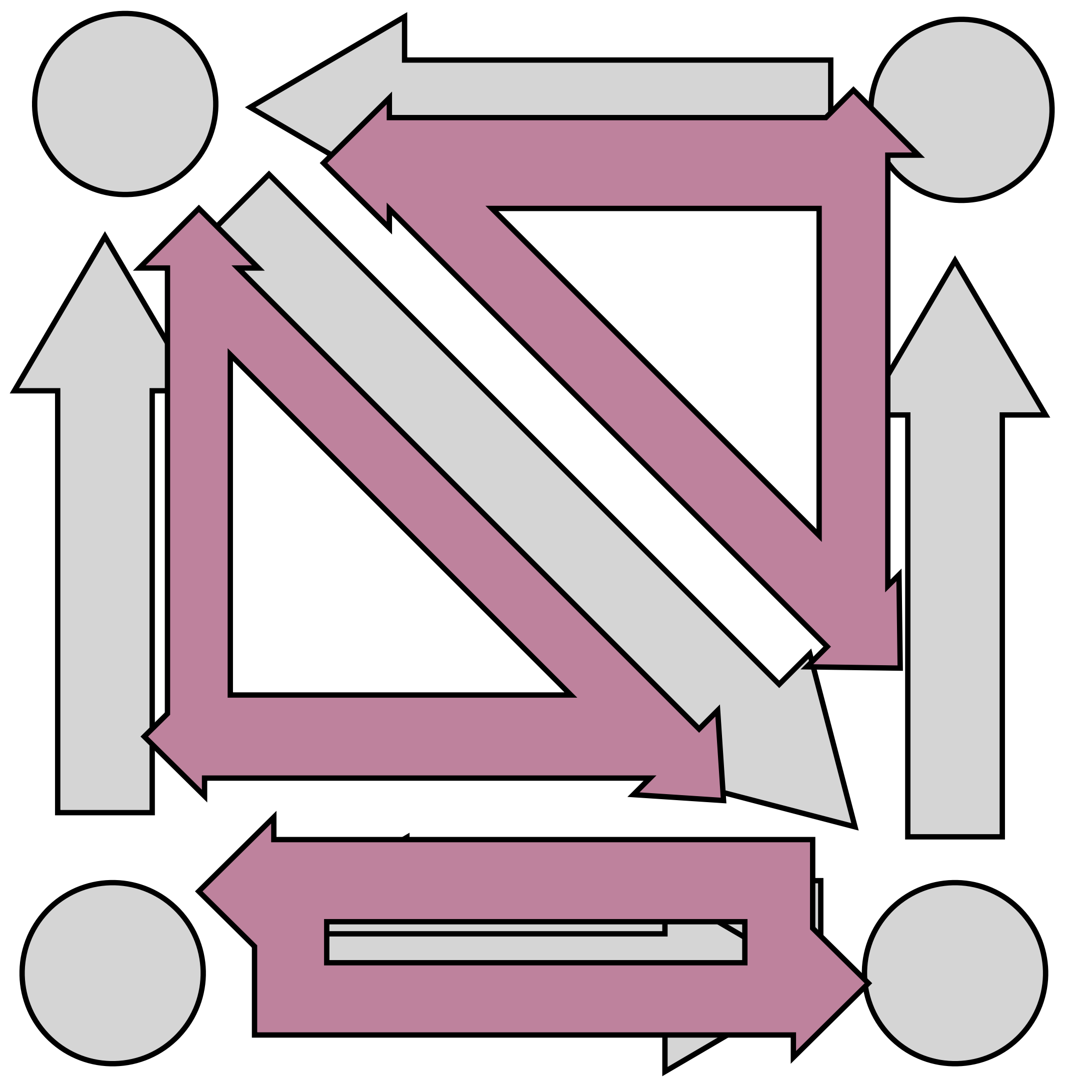
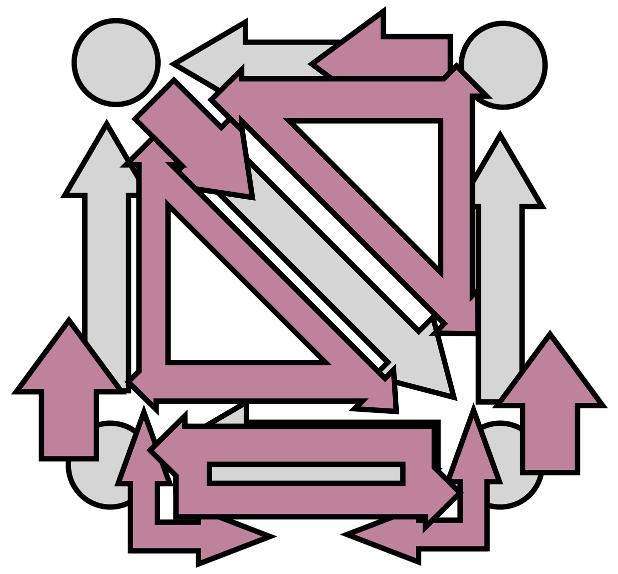




Finite or Infinite horizon

$$\min_x \quad F(x)$$

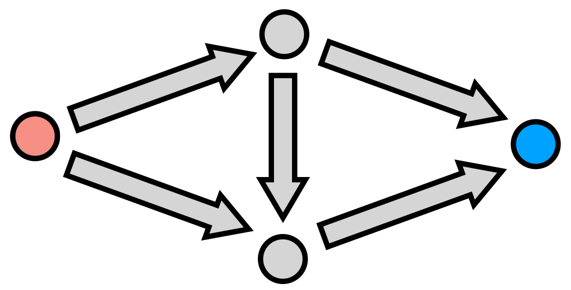
$$\text{s.t.} \quad EWx = 0 \qquad x \geq 0$$

$$\mathbf{1}^T x = m$$

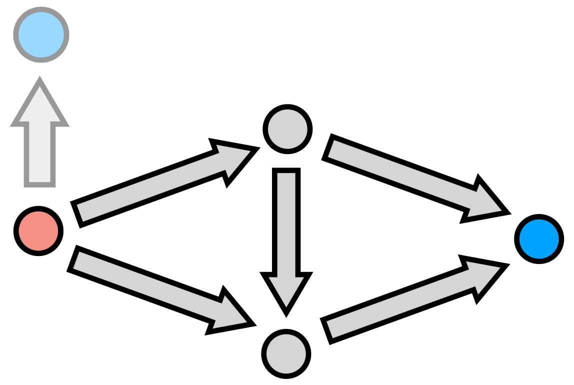
- Paper 1
- Paper 2
- Paper 3
- Paper 4

Potential Games

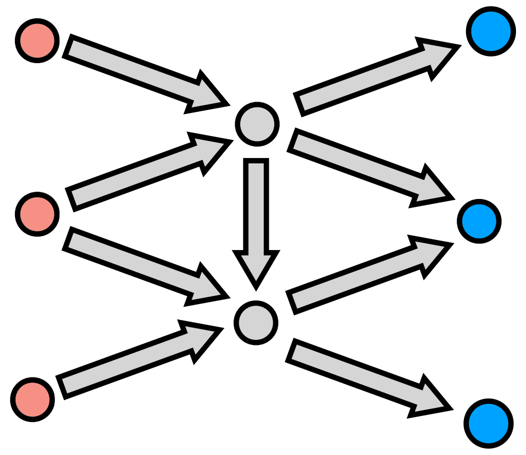
Routing Games



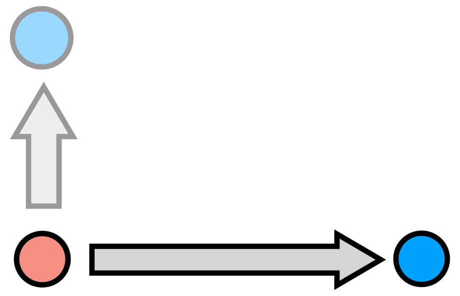
Variable Demand



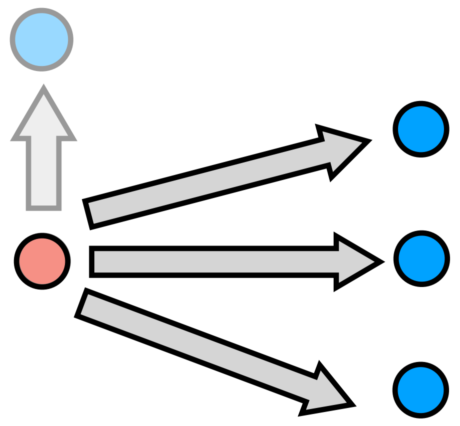
Multiple sources/ sinks



Supply & Demand

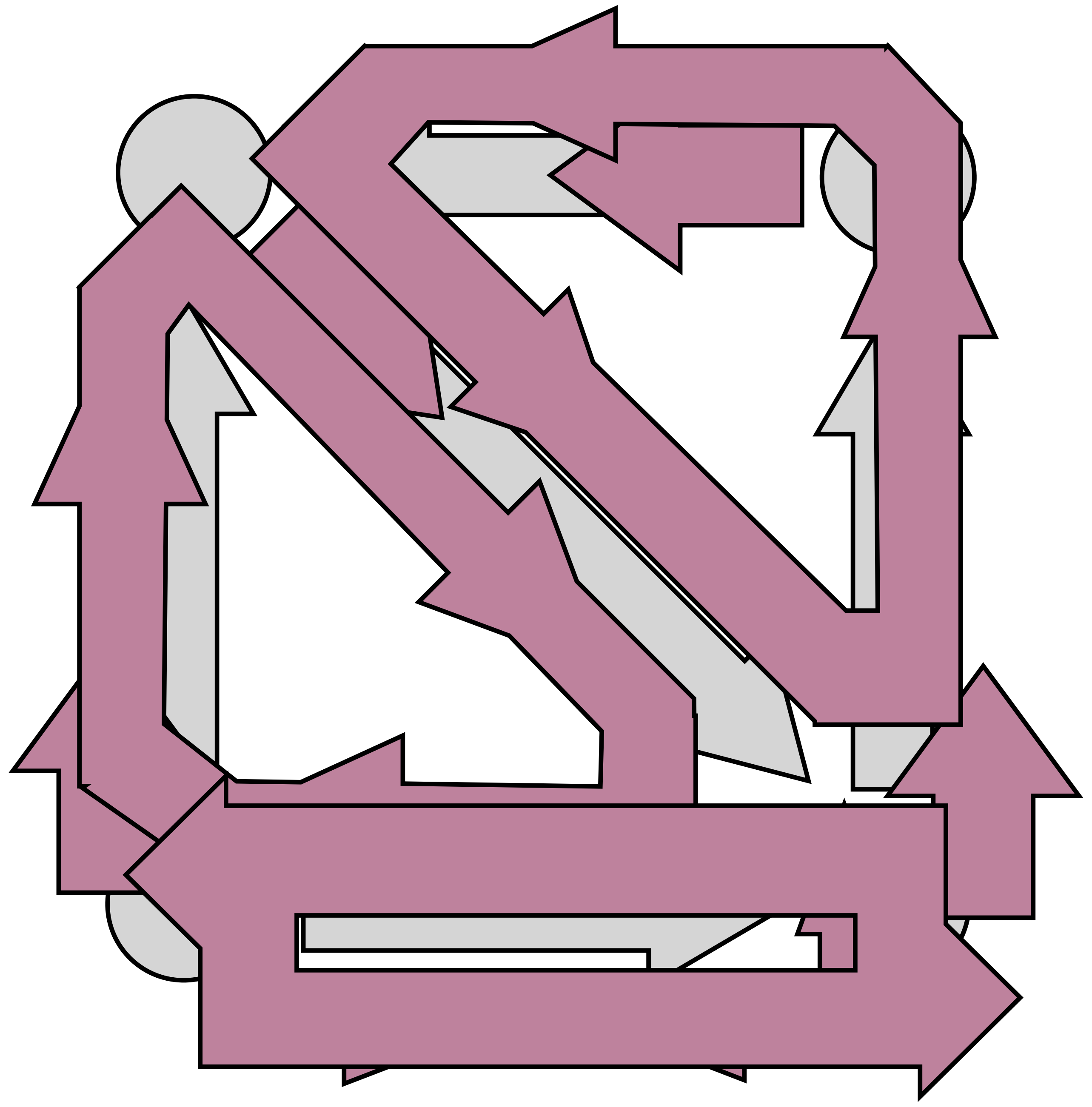


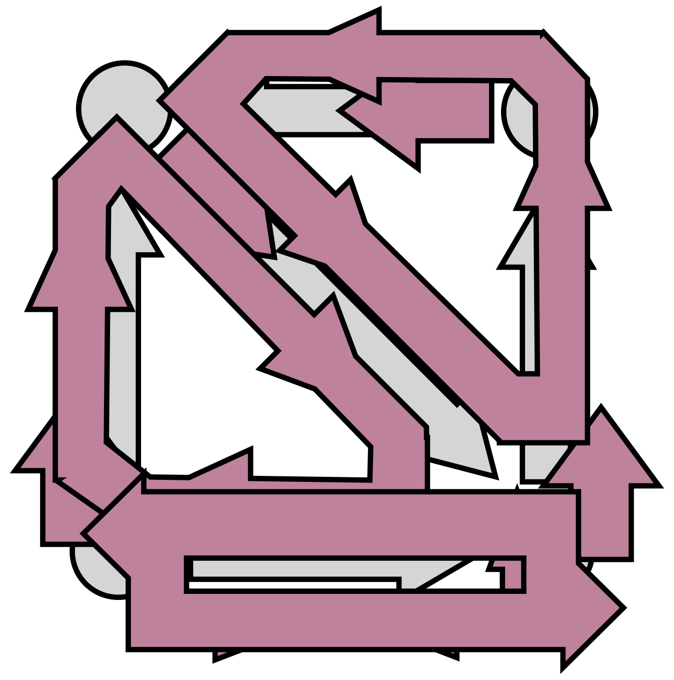
Market/ Cournot Competition



Markov Decision Process Congestion Game

Finite or Infinite Horizon





$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad EWx = 0 \qquad x \geq 0$$

$$\mathbf{1}^T x = m$$

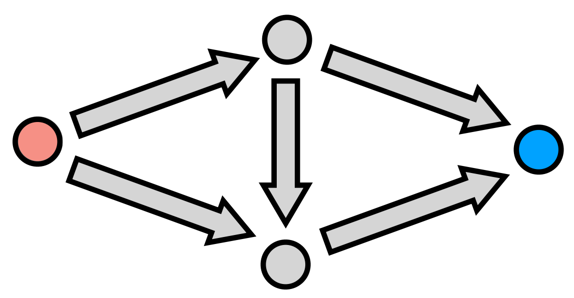
Potential Games

Routing Games

x : edge traffic

z : route traffic

Routing Games

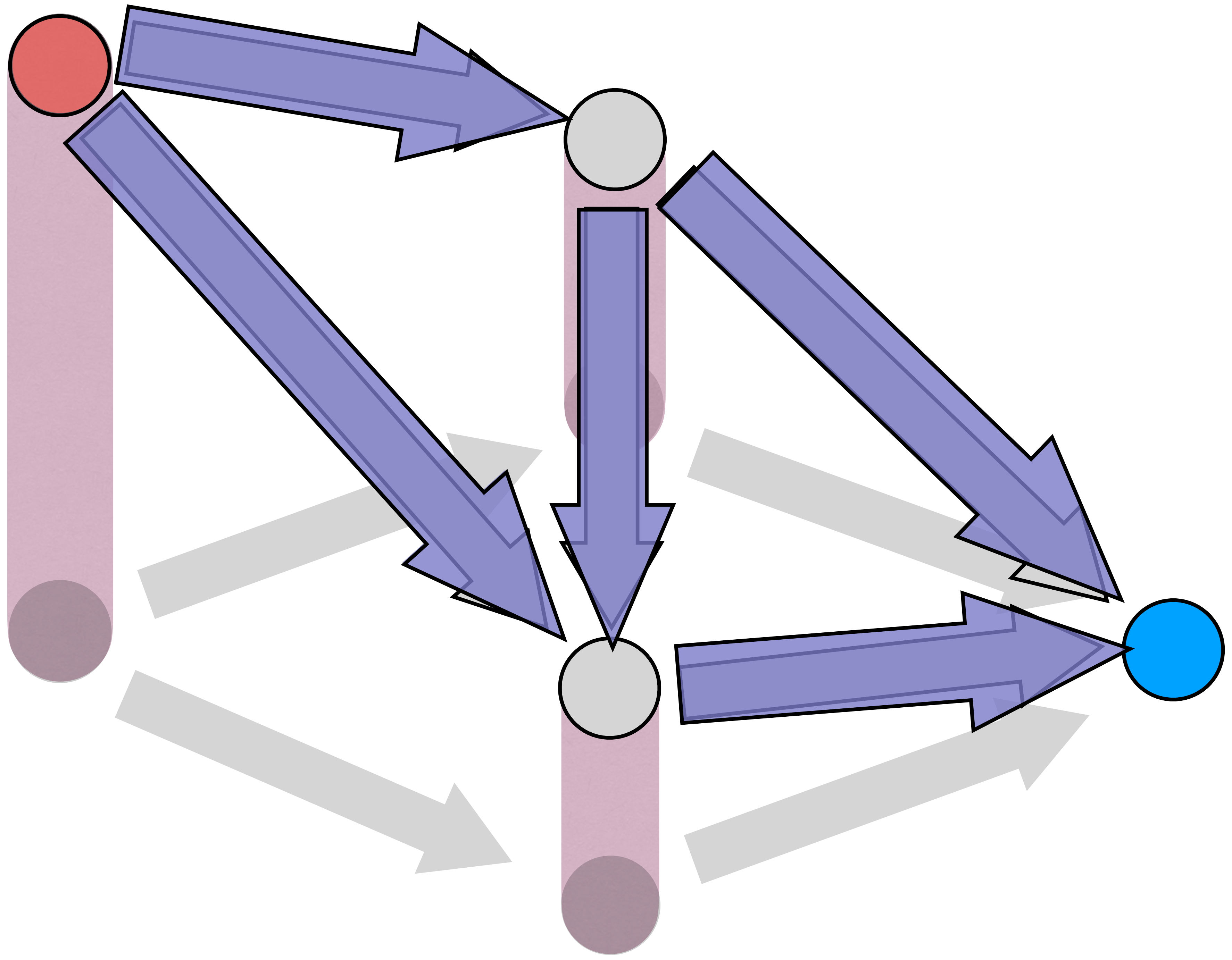


$F(x)$ **Potential Function**

$$\min_x \quad F(x)$$

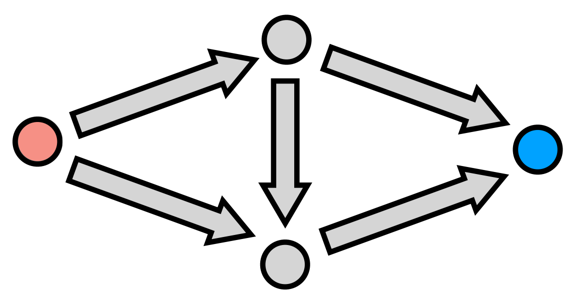
s.t. $\boxed{Ex = Sm, \quad v}$

$\boxed{x \geq 0 \quad \mu}$



Potential Games

Routing Games



$F(x)$ **Potential Function**

$$\min_x \quad F(x)$$

s.t. $Ex = Sm, \ v$

$x \geq 0 \ \mu$

Routing Games

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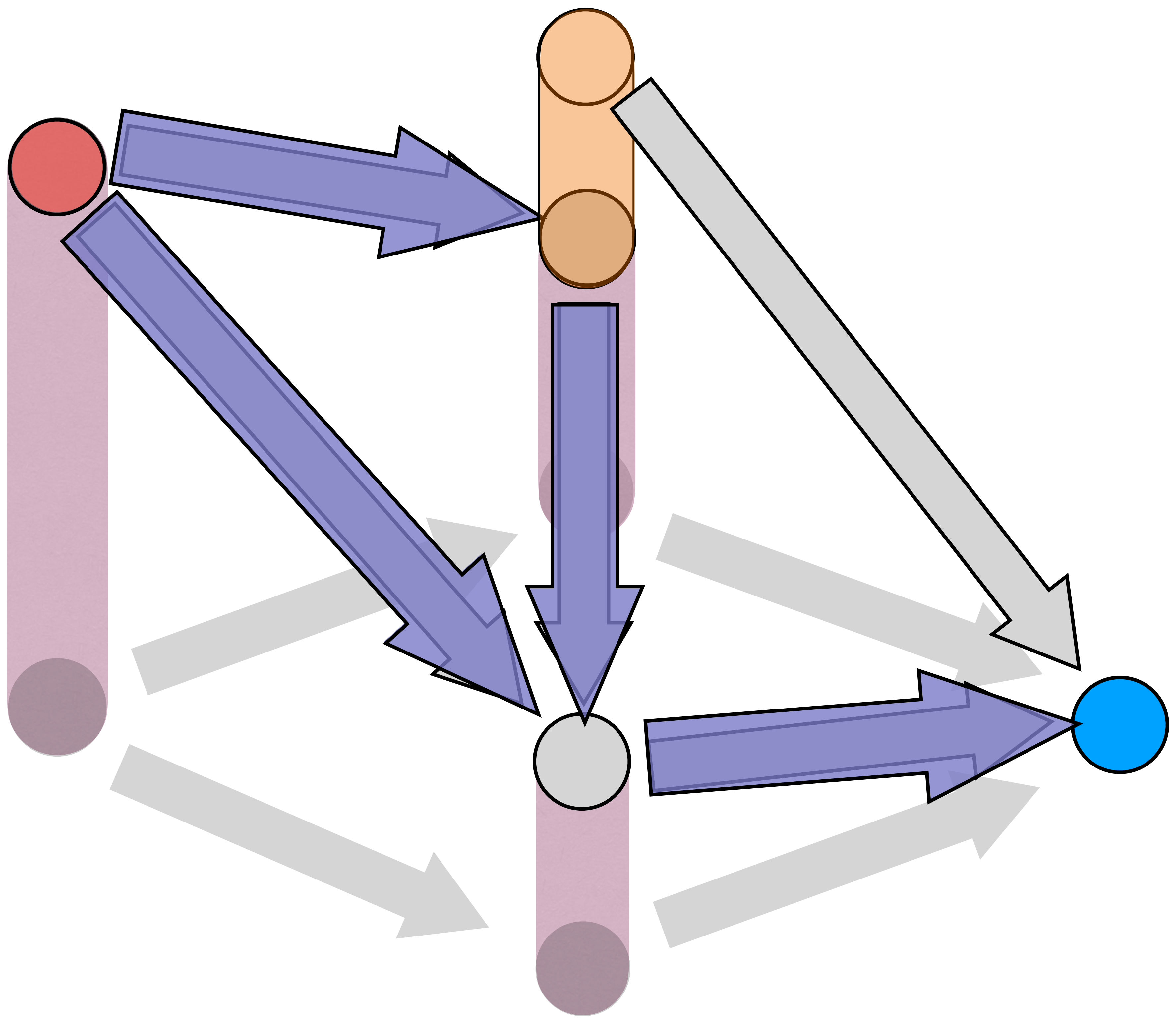

Potential Games

Routing Games

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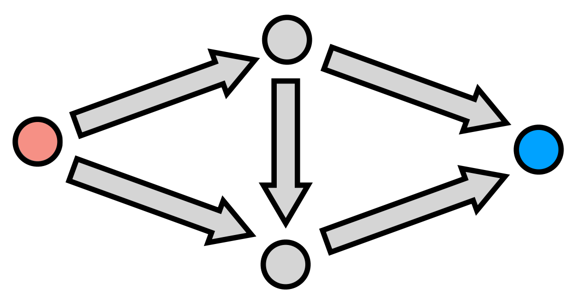
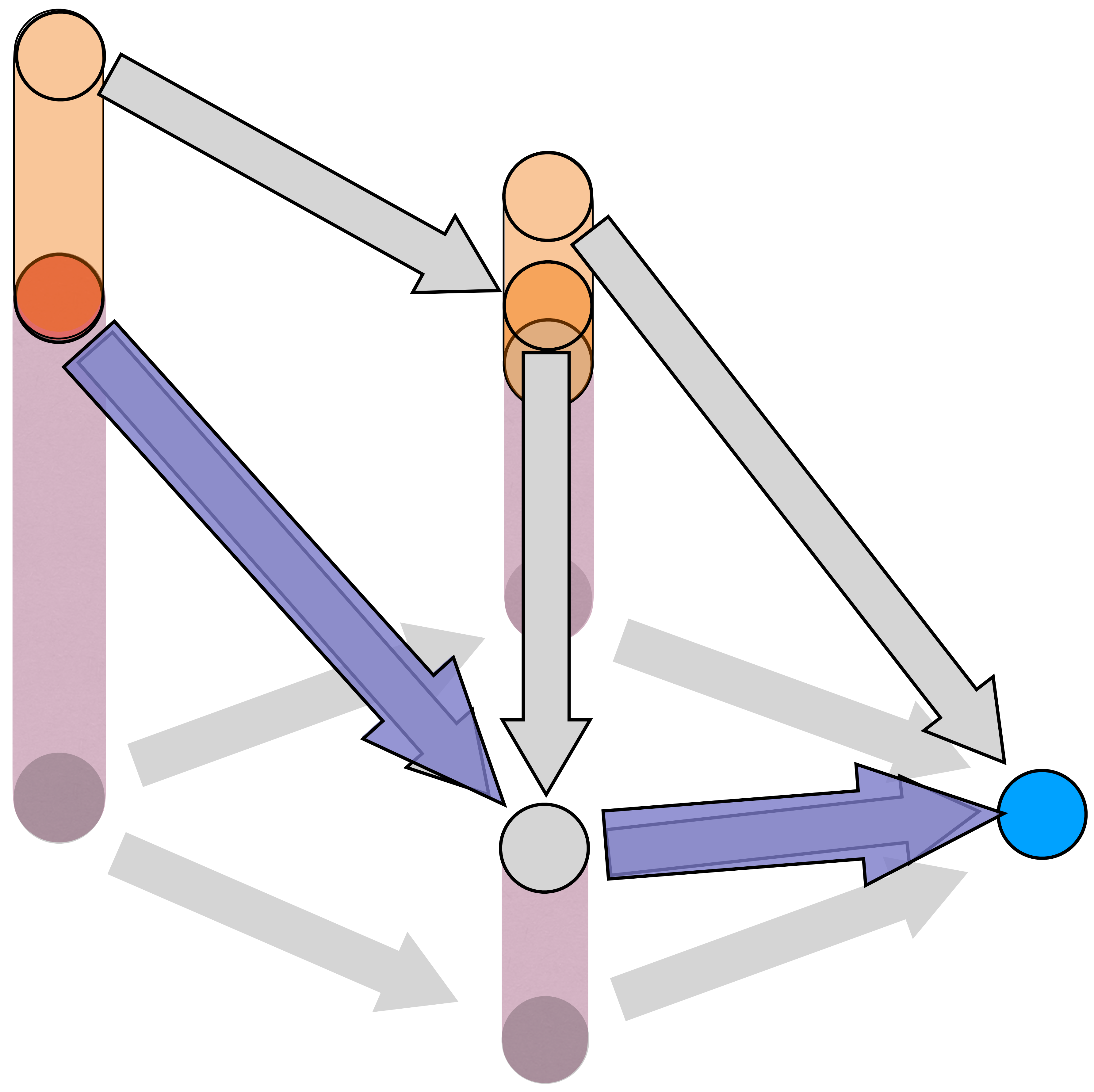
Routing Games





$F(x)$ **Potential Function**

$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \boxed{Ex = Sm, \ \boxed{v}}$$

$$\boxed{x \geq 0 \ \boxed{\mu}}$$