External-Cost Continuous-Type Wardrop Equilibria in Routing Games

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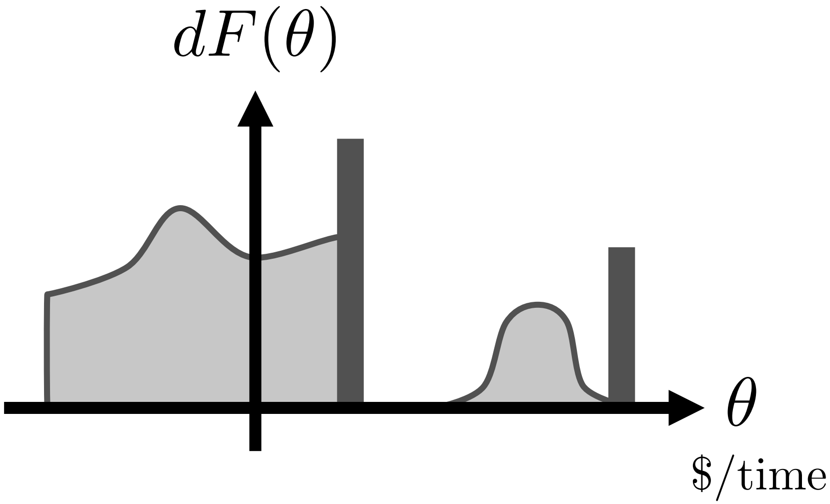
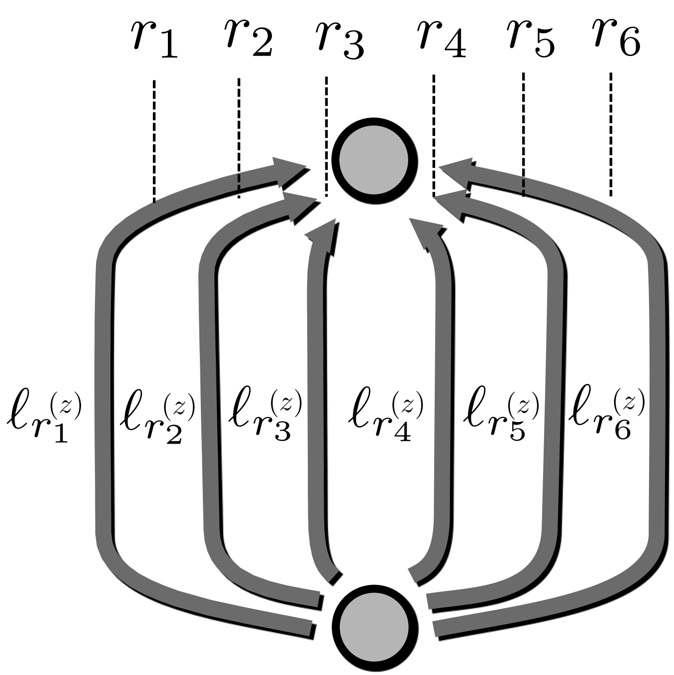
External Cost Equilibria

Outline

- Review: Non-atomic routing games
- External cost equilibria
- Equilibria intuition
- Applications



Classic Routing Game



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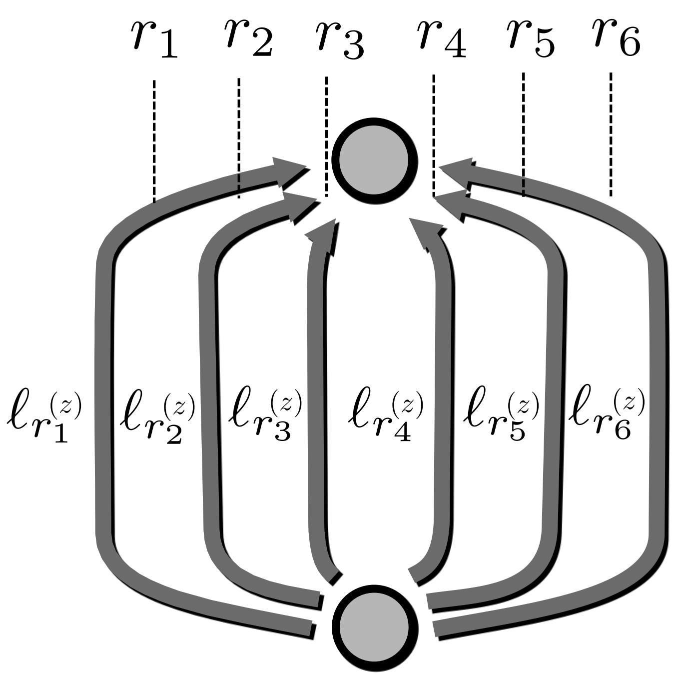

Wardrop Equilibrium

$$\ell_r(z) \le \ell_{r'}(z)$$

whenever some mass chooses route r

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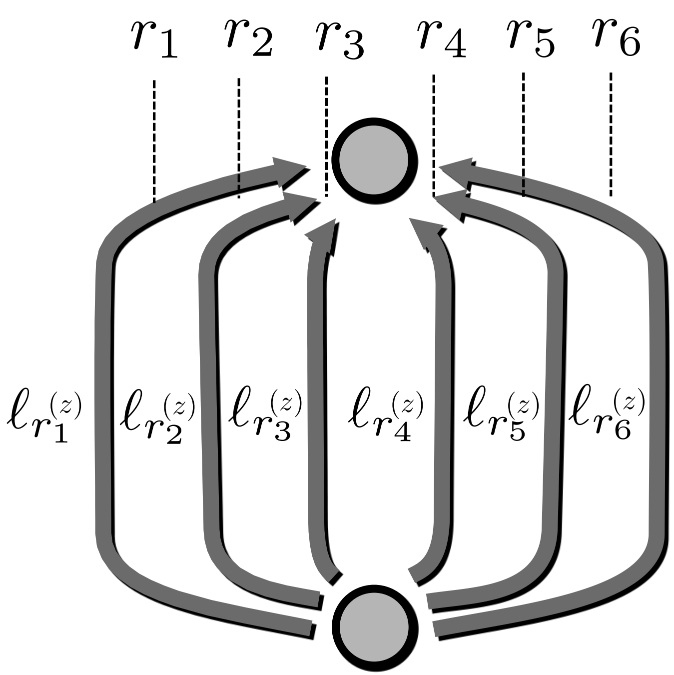

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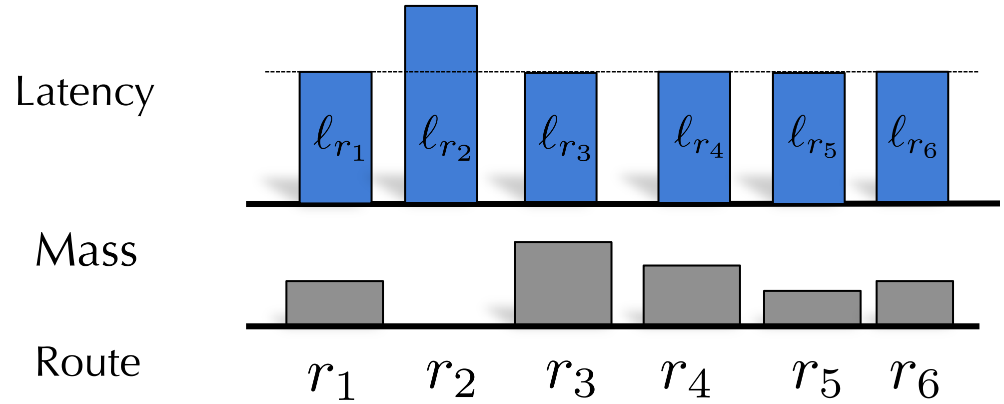

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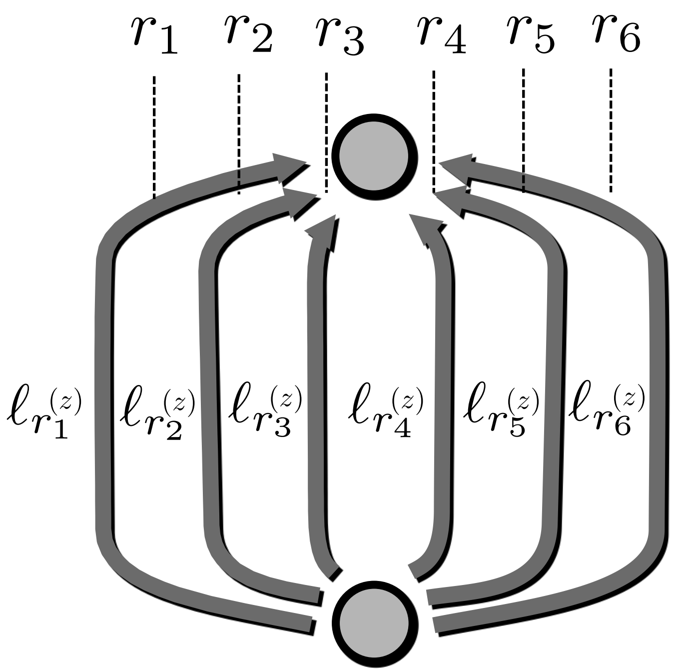

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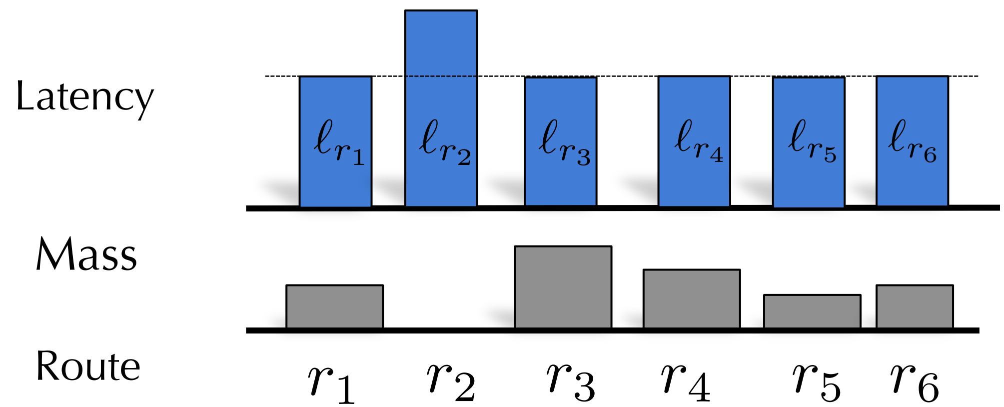
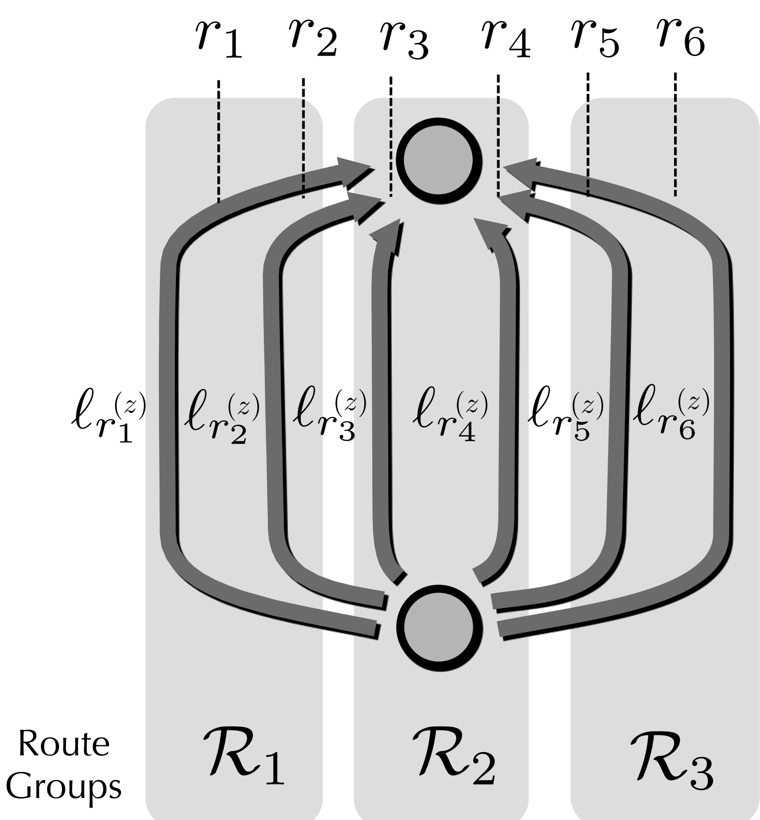


Potential Game

Potential Function $F(x) = \sum_e \int_0^{x_e} l_e(u)\, du$

External Cost Equilibria



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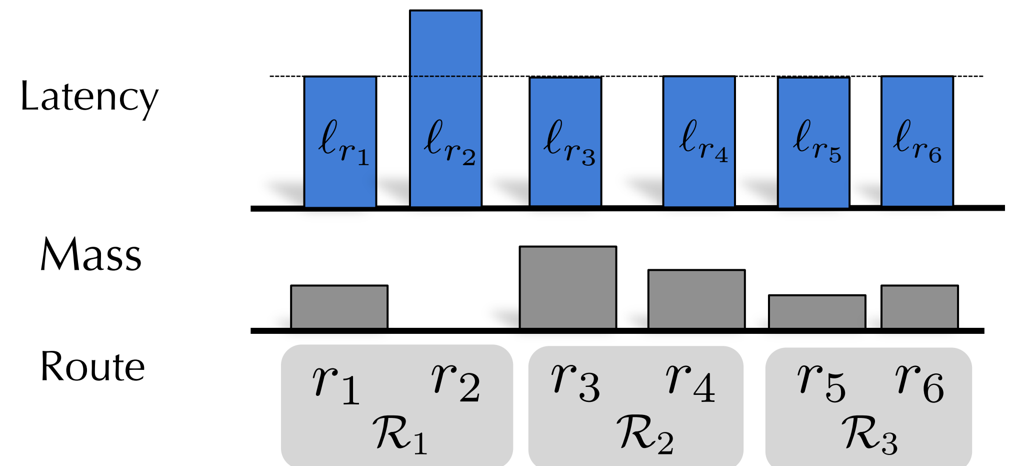

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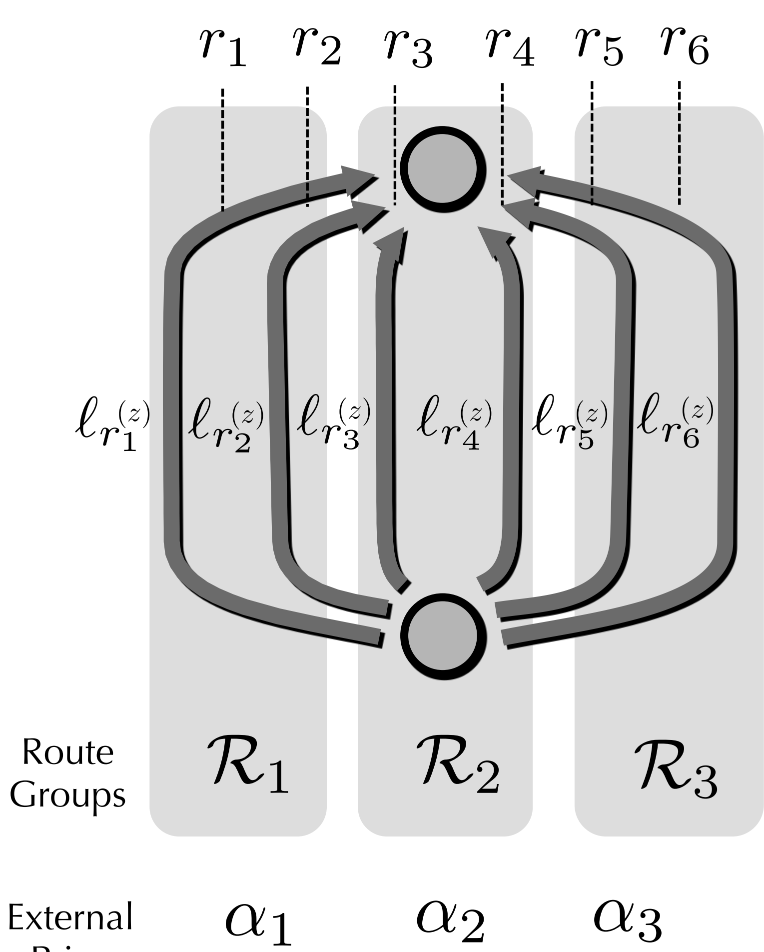

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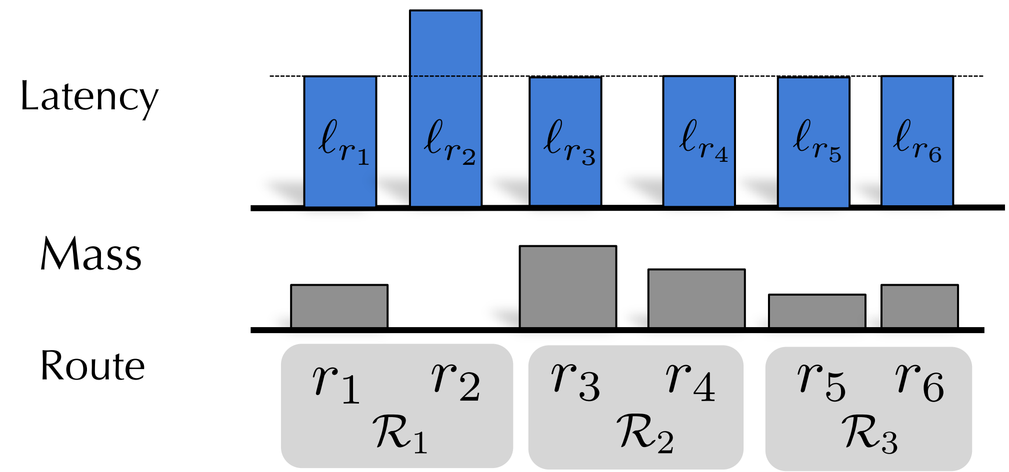

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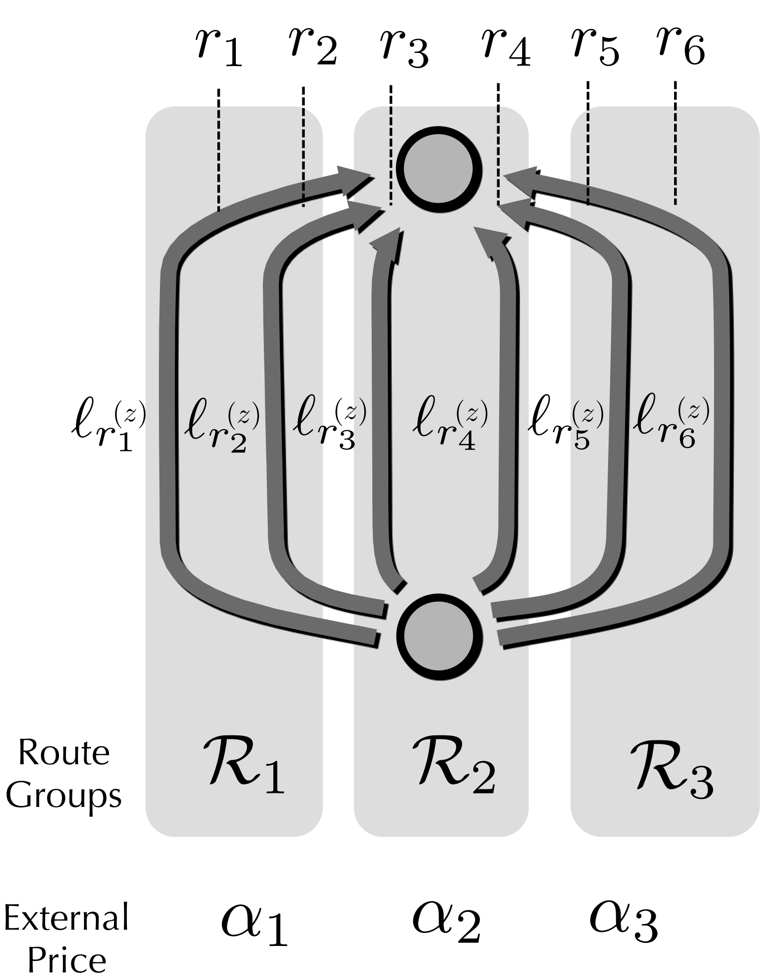
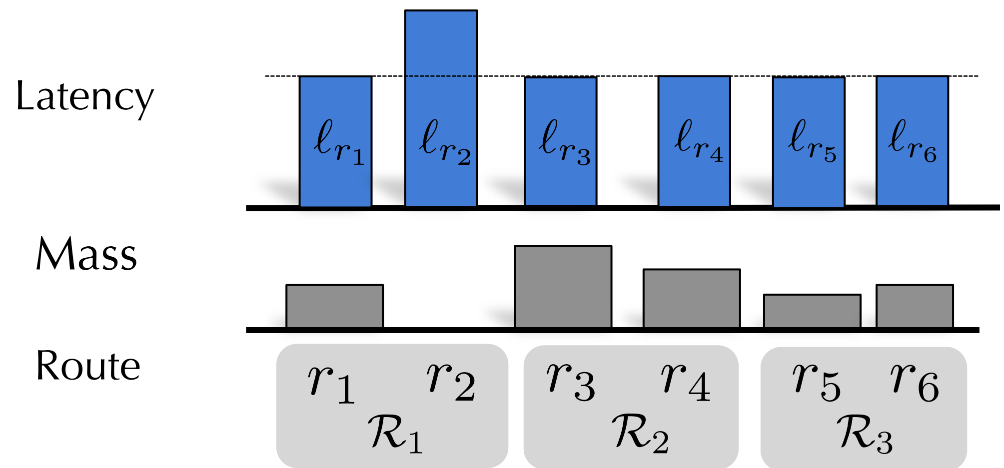


Wardrop Equilibrium

$$\ell_r(z) + \alpha_i\theta \leq \ell_{r'}(z) + \alpha_j\theta$$

whenever some users choose group $\mathcal{R}_i$ and route $r \in \mathcal{R}_i$



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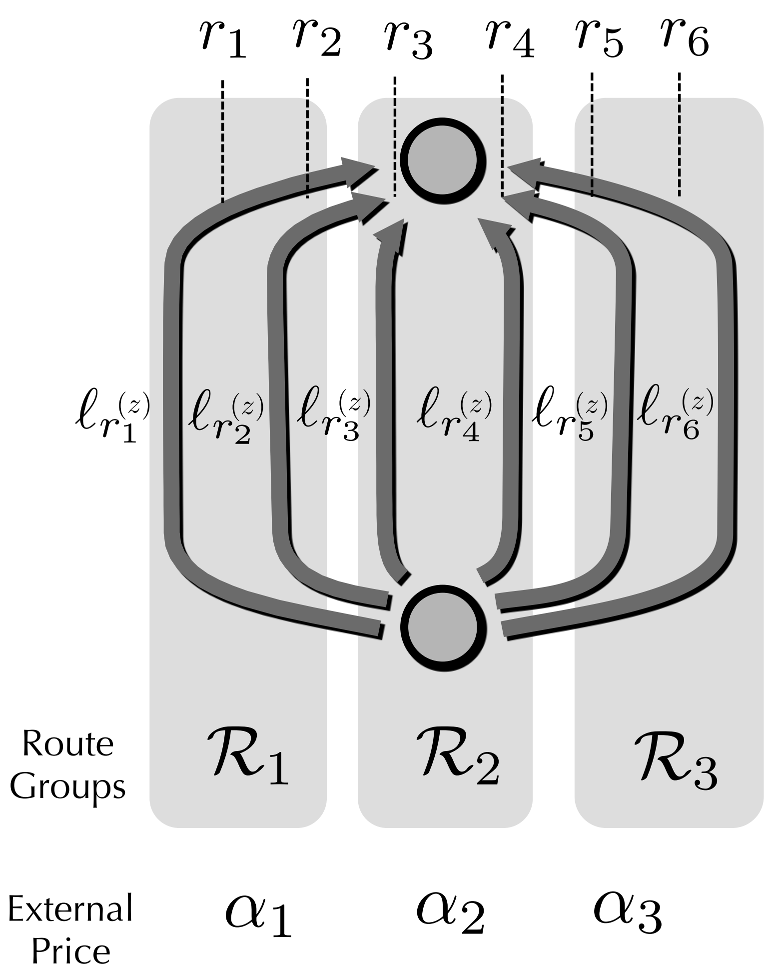

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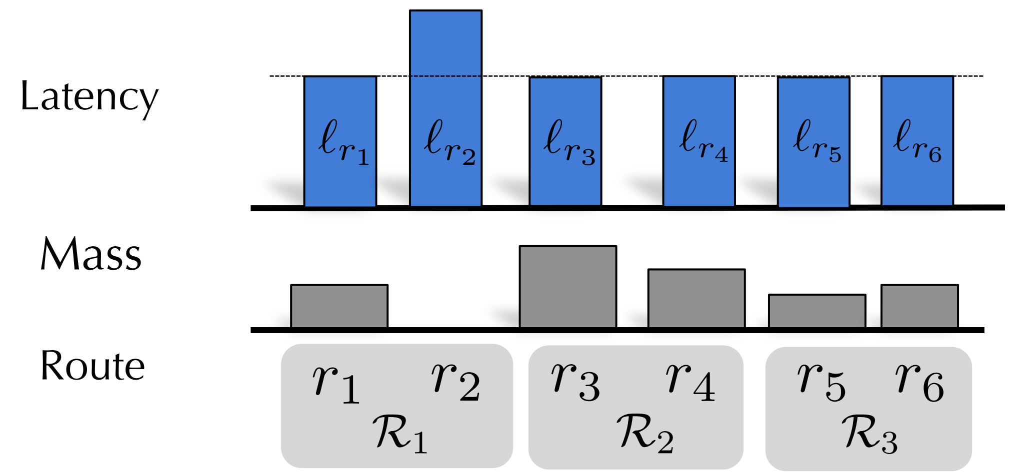
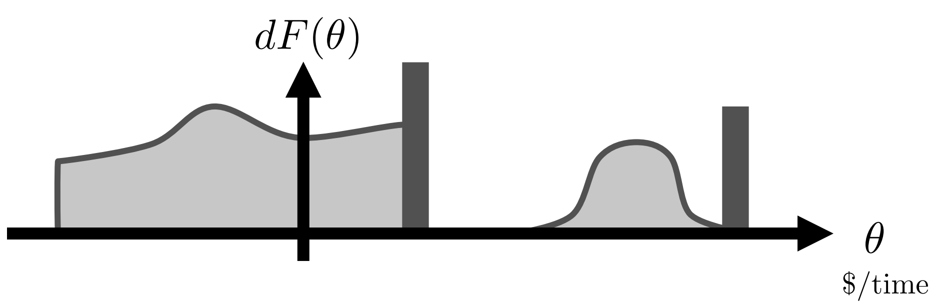


External Cost Preference Parameter



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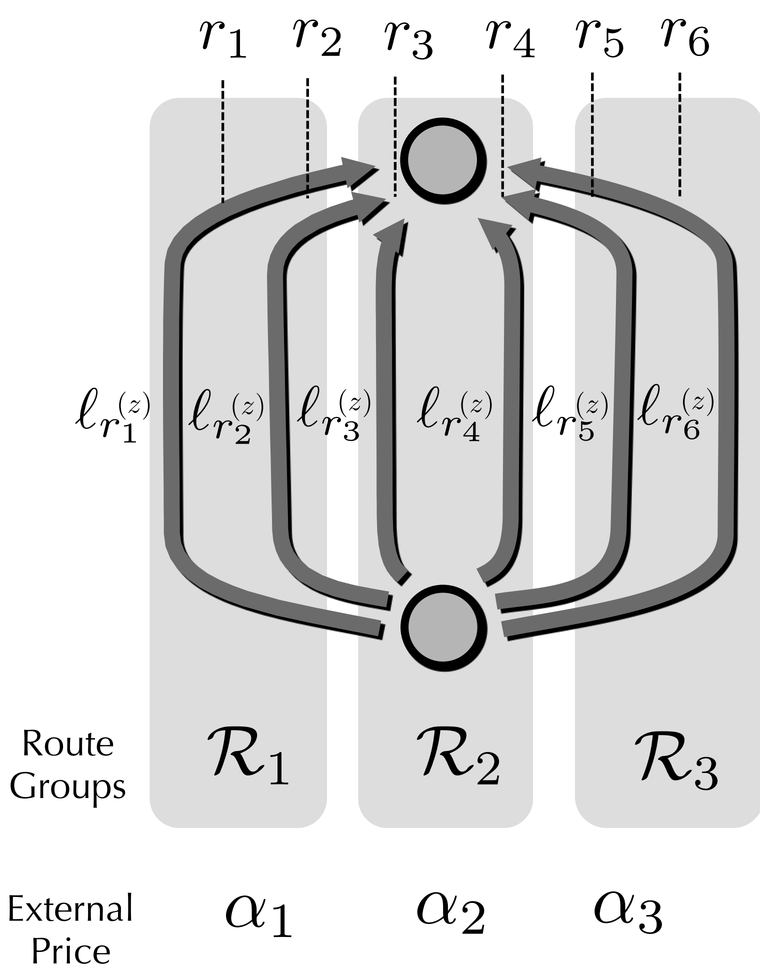

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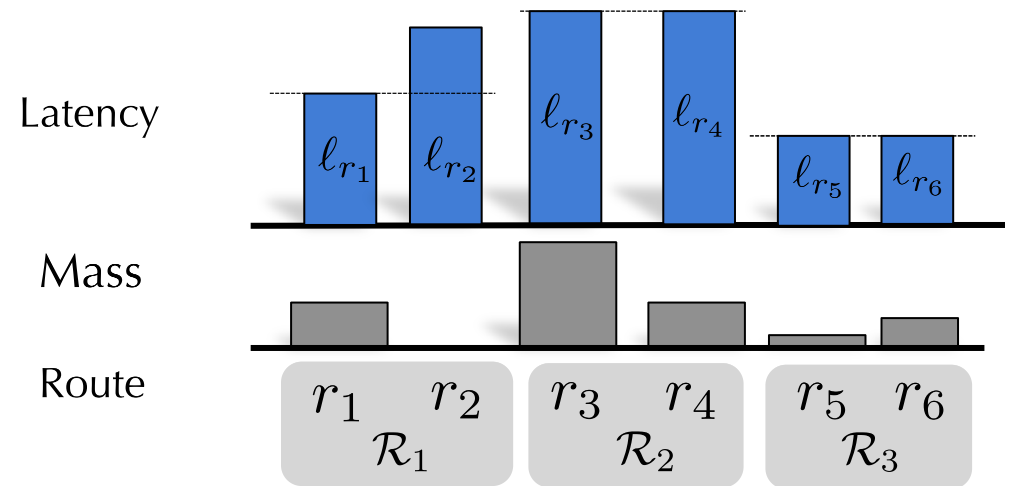

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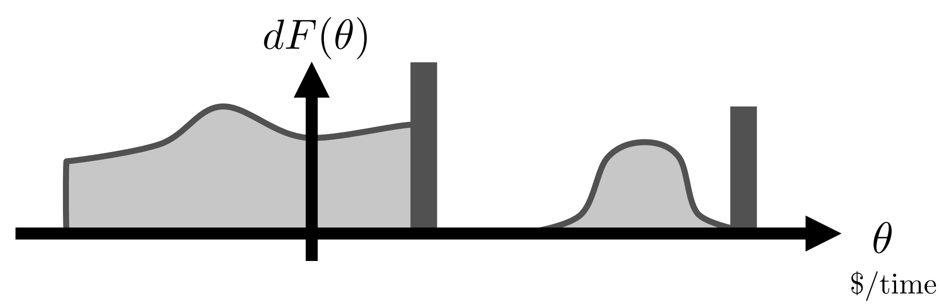

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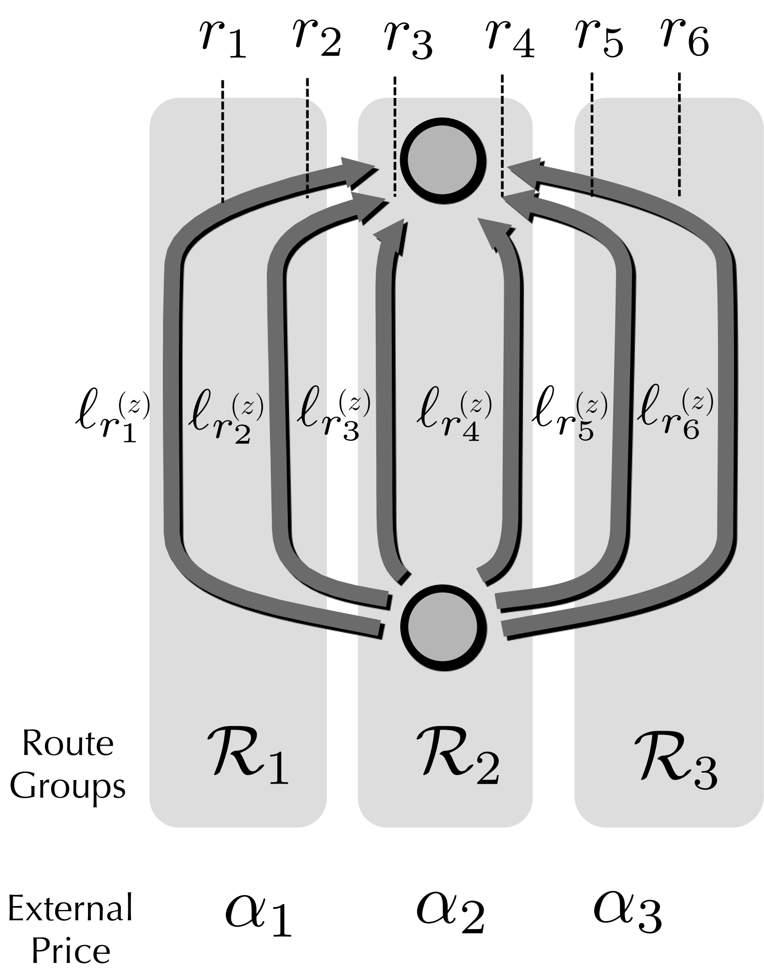

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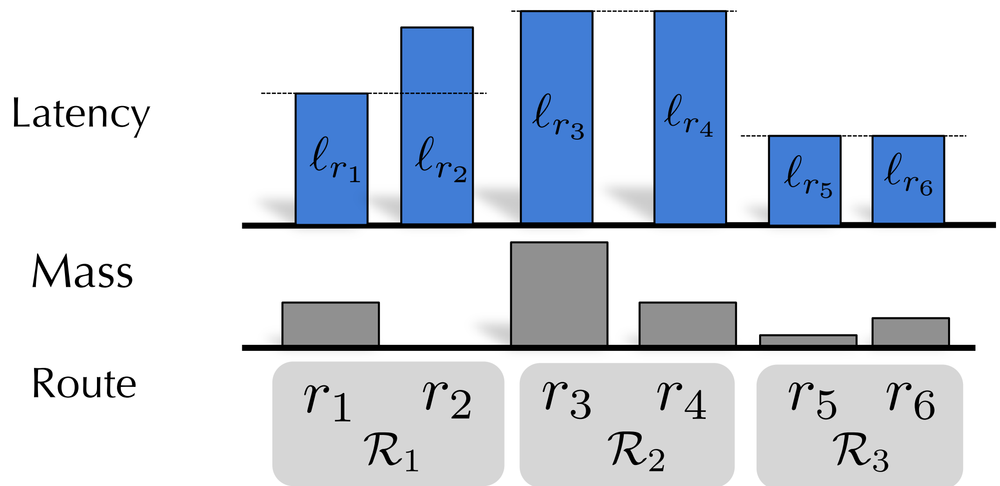
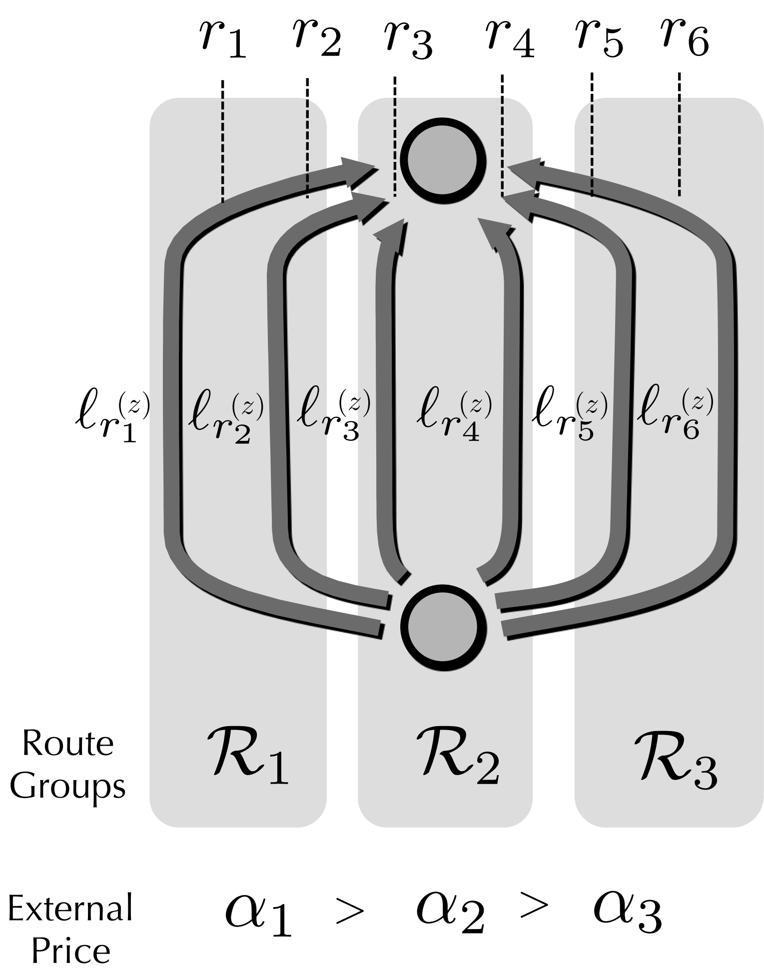


Literature – traditionally for tolling

- **F. Leurent**, "Cost versus time equilibrium over a network," 1993.
- **P. Marcotte and D.L. Zhu**,"Equilibria with infinitely many differentiated classes of customers," 1997.

External Cost Equilibria



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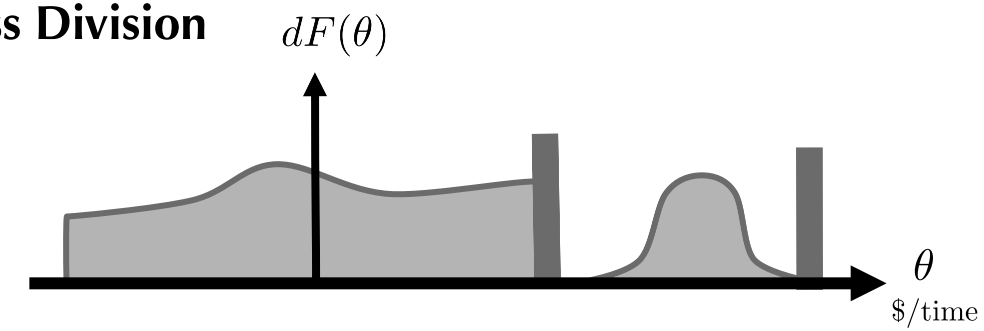
Mass Division



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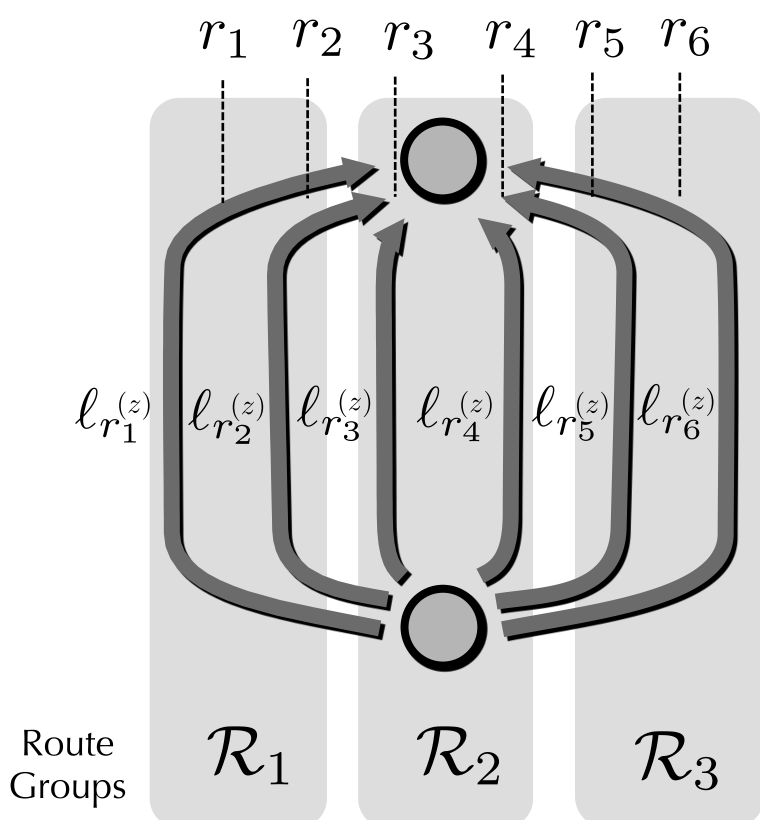

External Price $\alpha_1 > \alpha_2 > \alpha_3$

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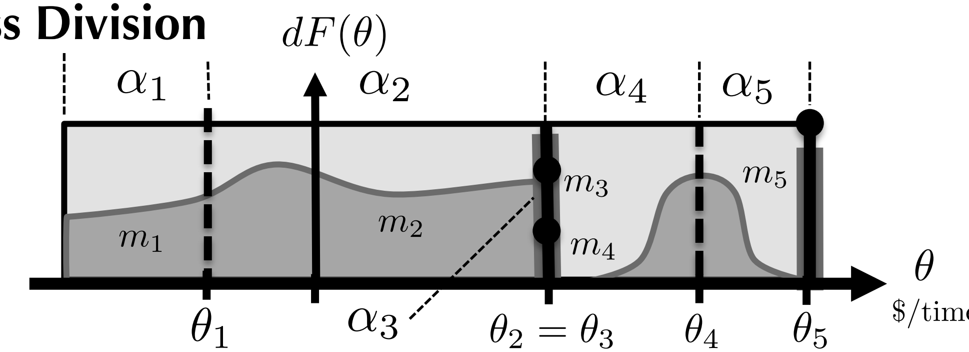

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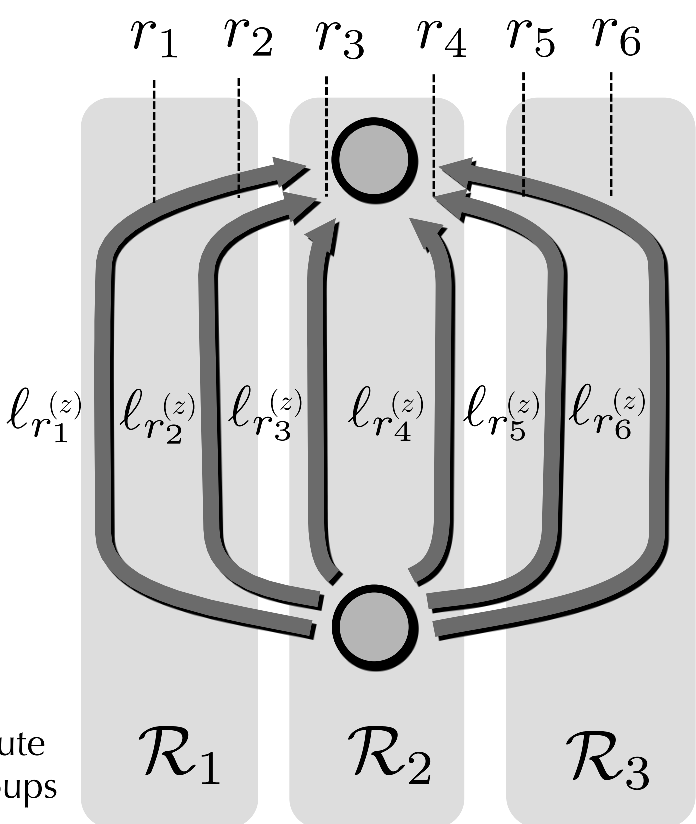

Route Groups

External Price $\alpha_1 > \alpha_2 > \alpha_3$

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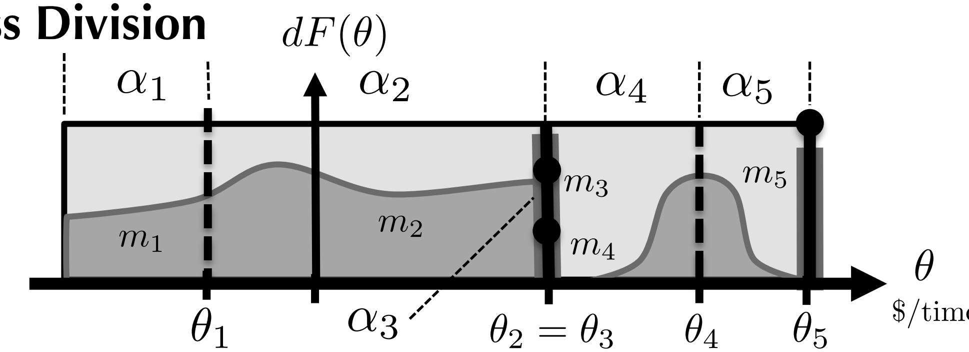
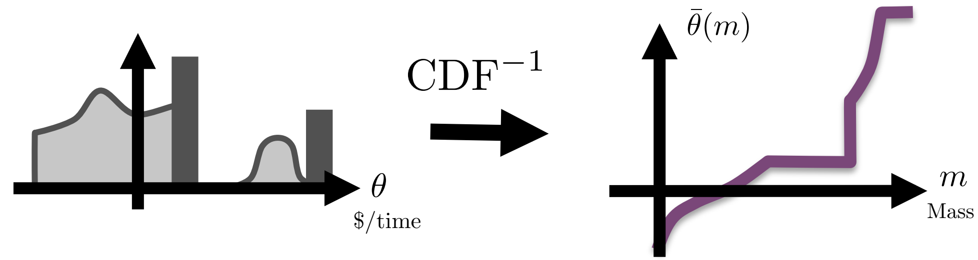


Inverse CDF function



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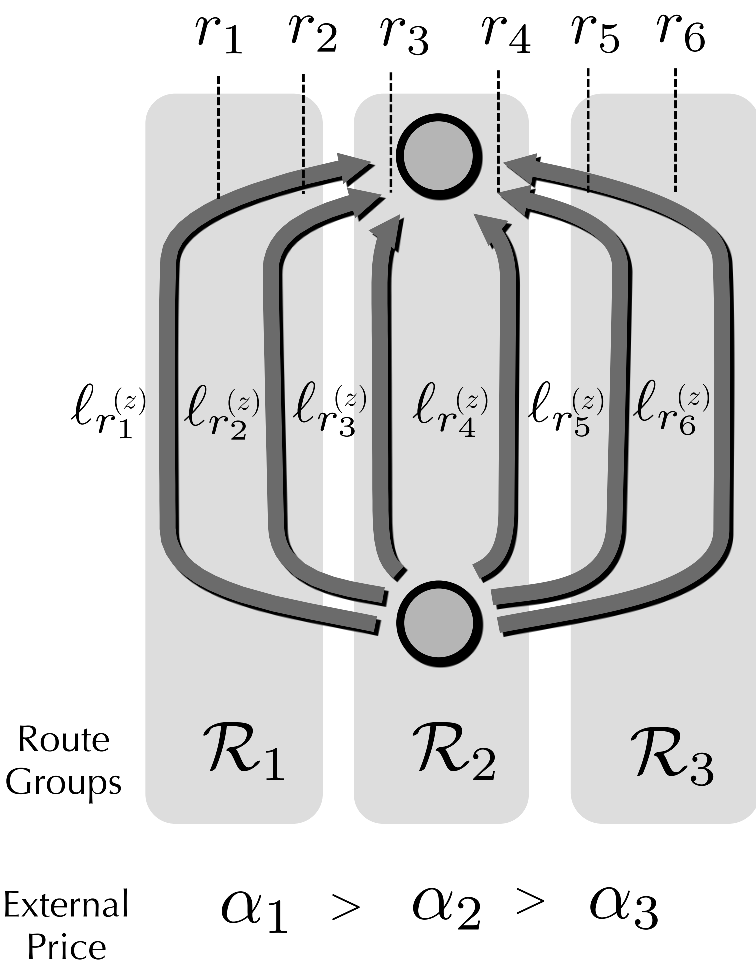

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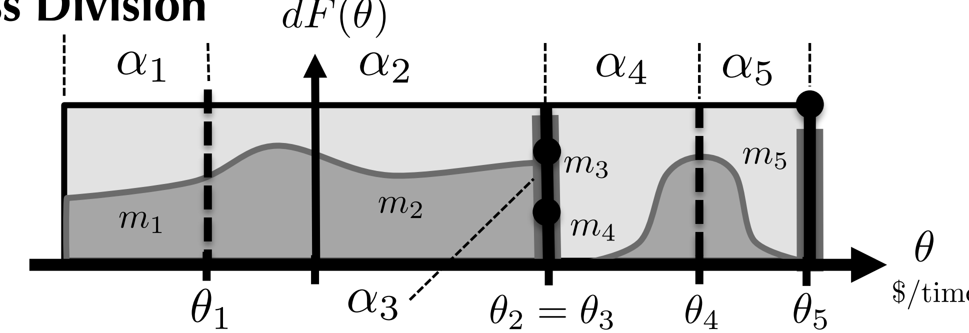

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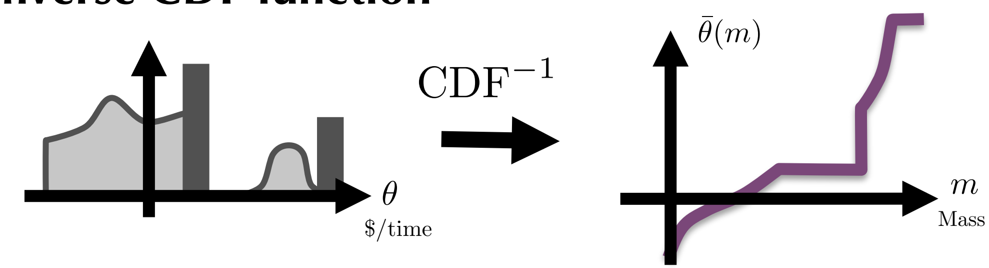
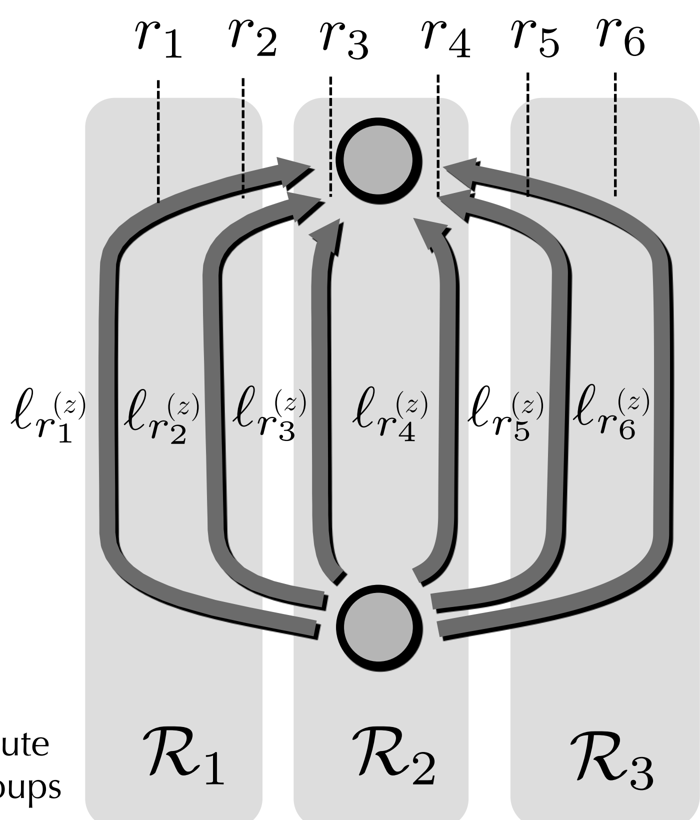


$$\theta_1 = \bar{\theta}(m_1) \qquad \theta_3 = \bar{\theta}(m_1 + m_2 + m_3)$$

$$\theta_2 = \bar{\theta}(m_1 + m_2) \qquad \vdots$$

External Cost Equilibria



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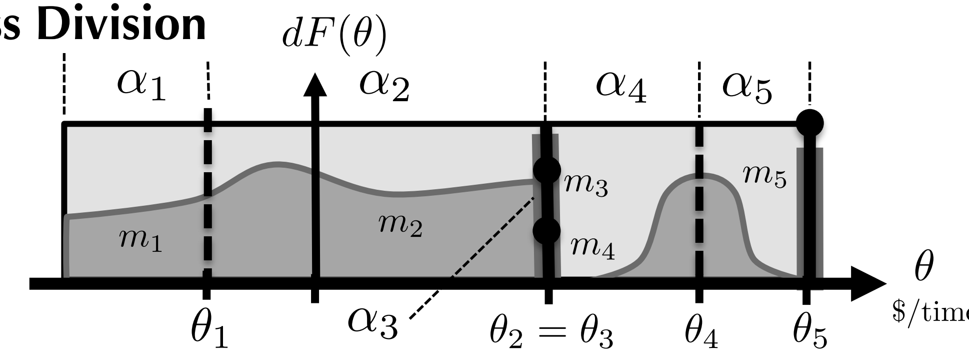

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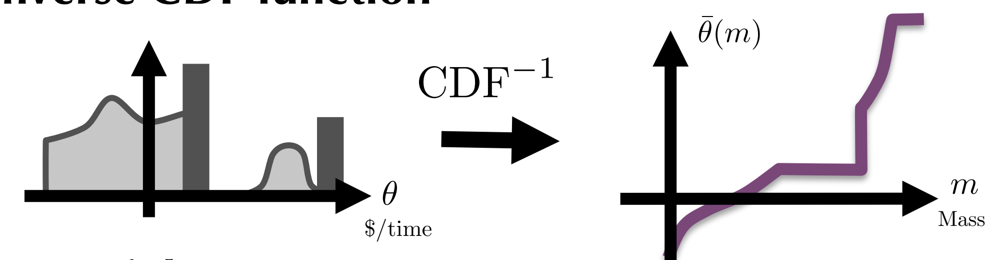


Potential Game

$$F(x) = \sum_e \int_0^{x_e} l_e(u)\ du + \sum_o \int_{\sum_{i<o} m_i}^{\sum_{i \leq o} m_i} \alpha_o \bar{\theta}(u)\ du$$

Intuition: $\alpha_i \geq 0,\ \theta \geq 0,$ uniform

Price $\quad \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 > \alpha_5 > 0$

External Cost

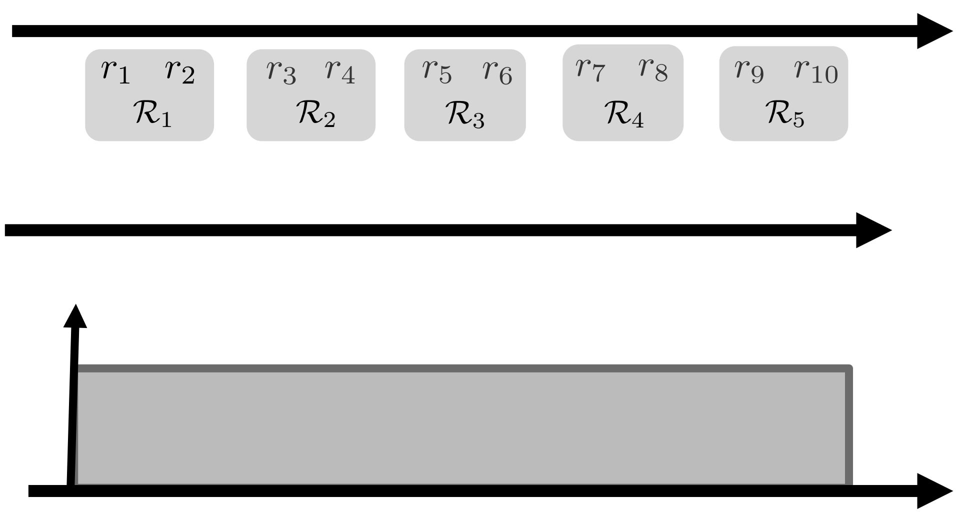
Travel Latency

Costs

Routes
Groups

Masses

$dF(\theta)$



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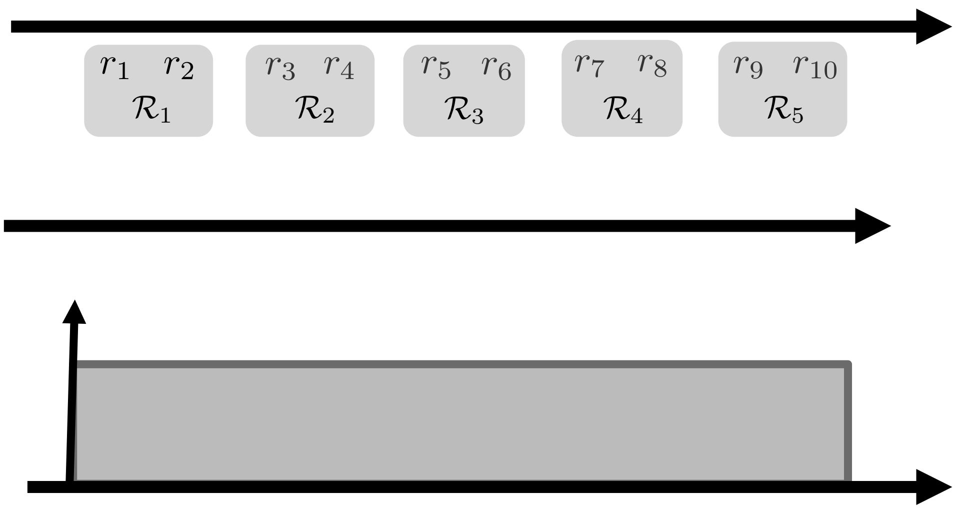
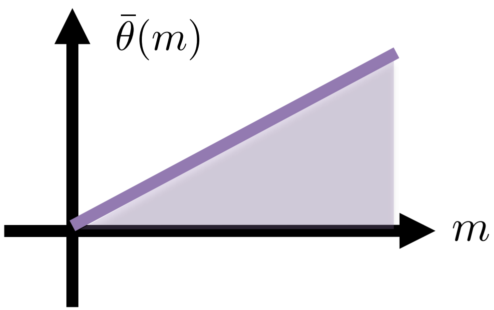


$\mathrm{CDF}^{-1}(m)$



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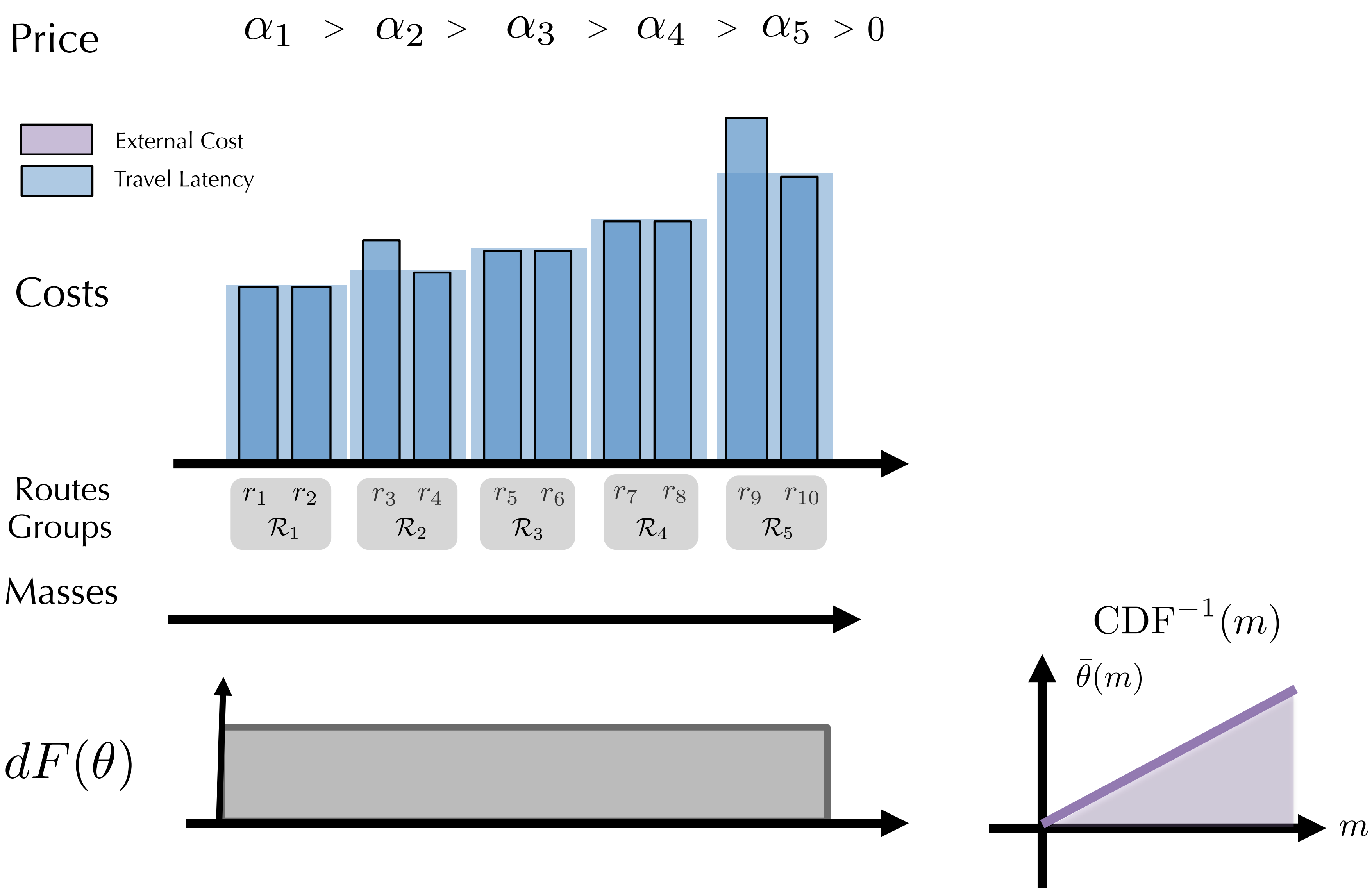

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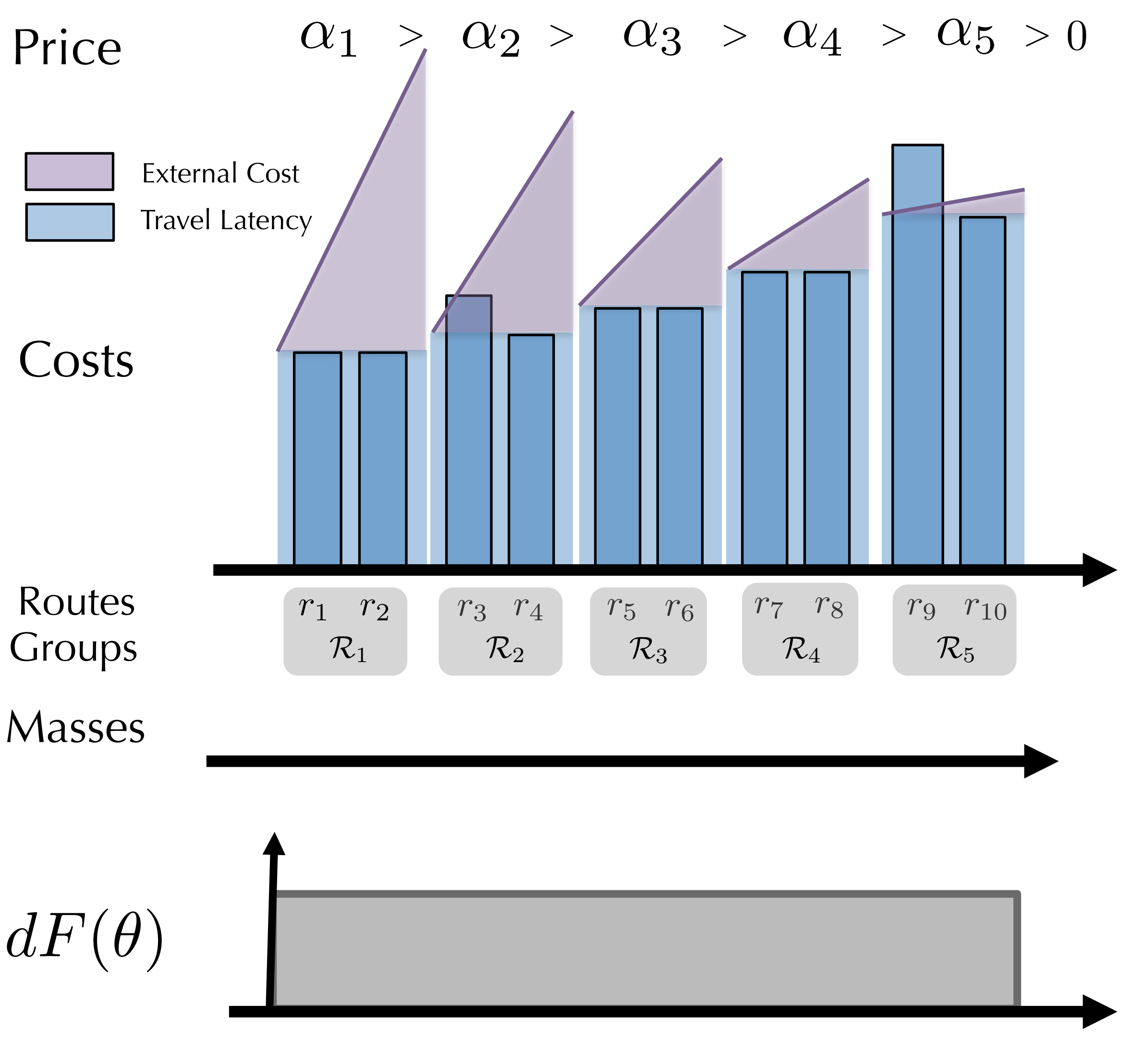

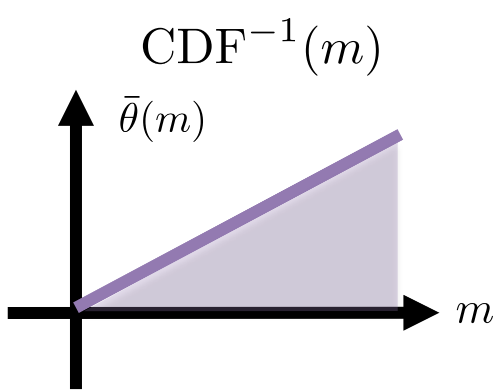

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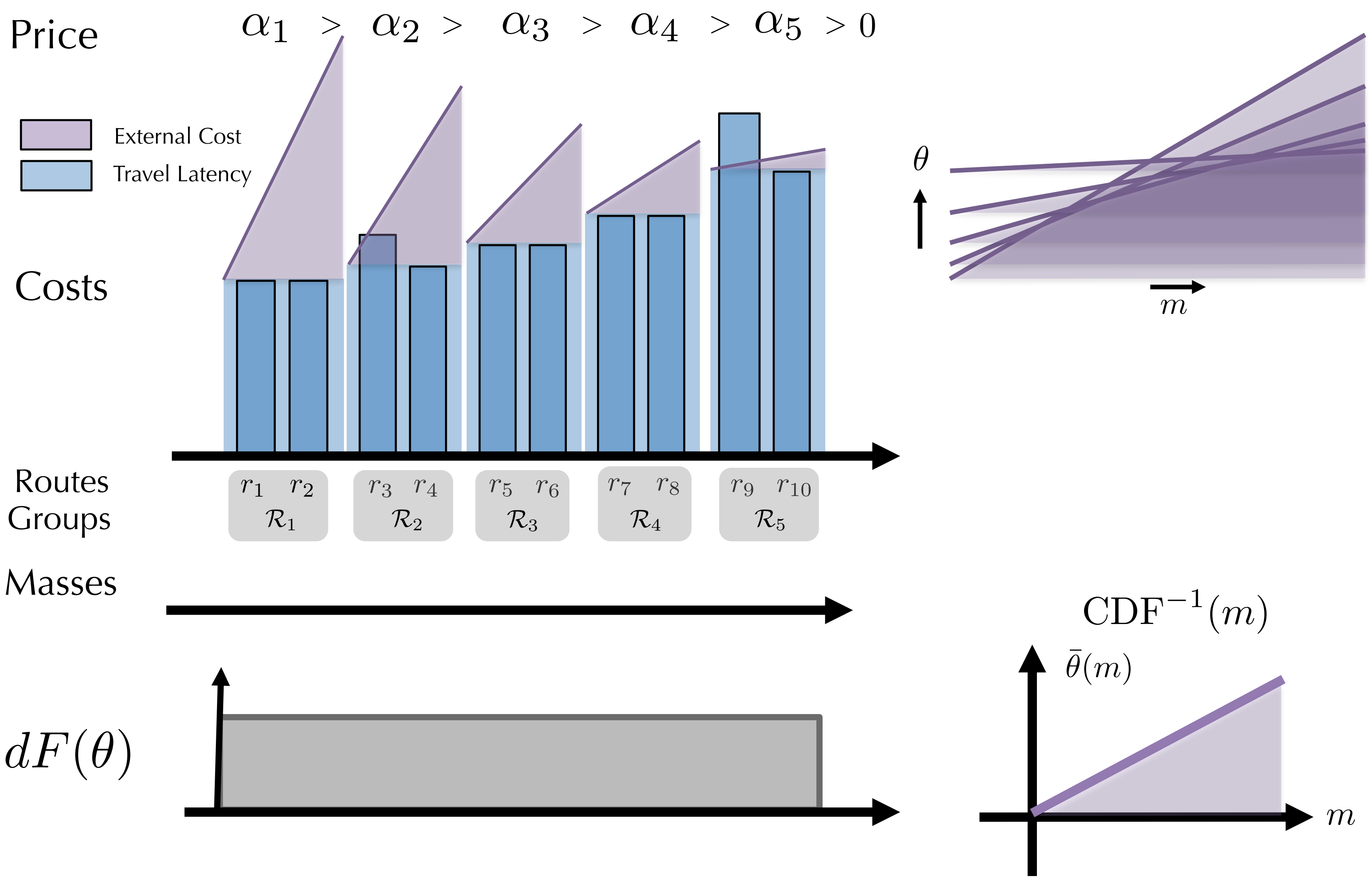

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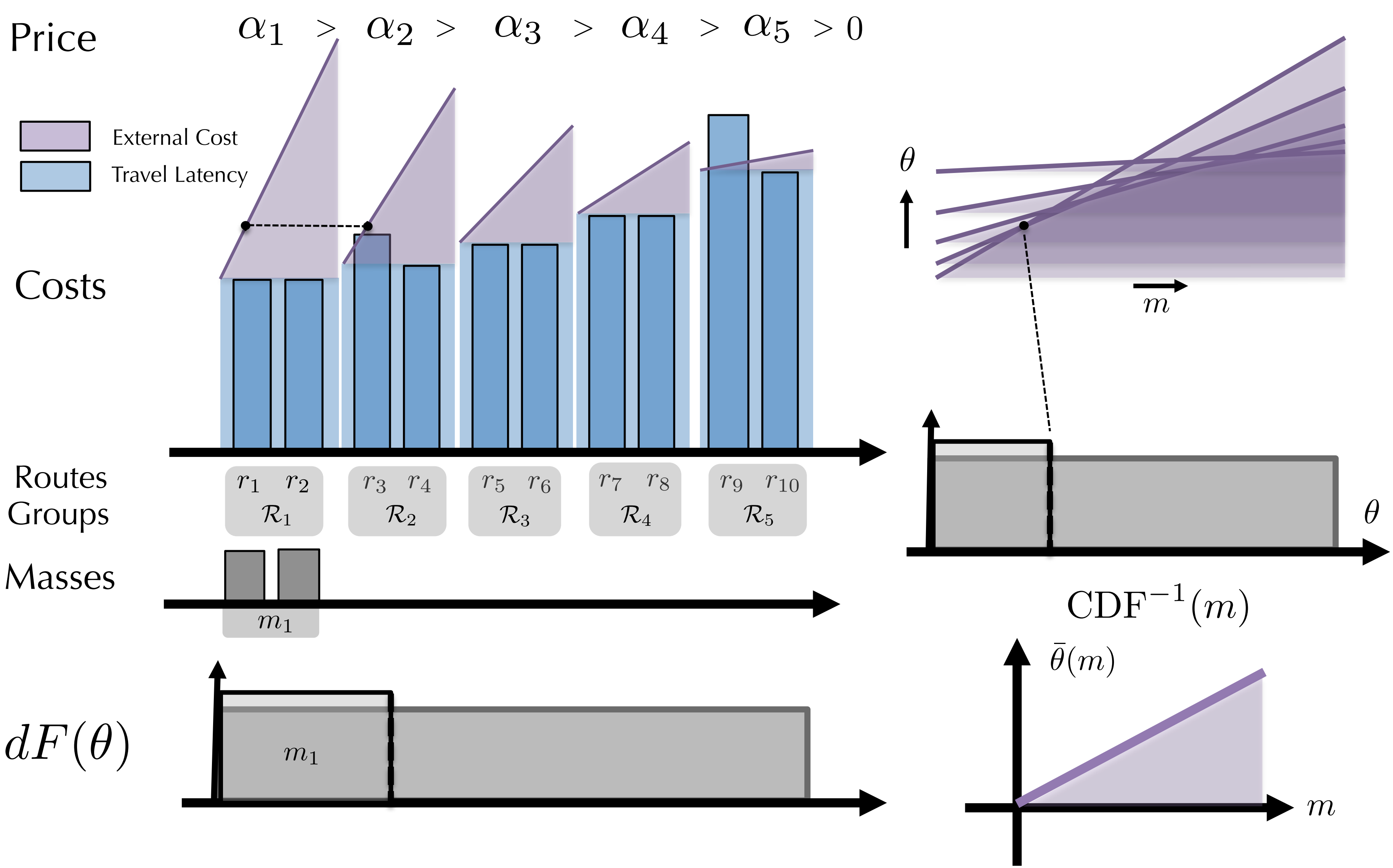

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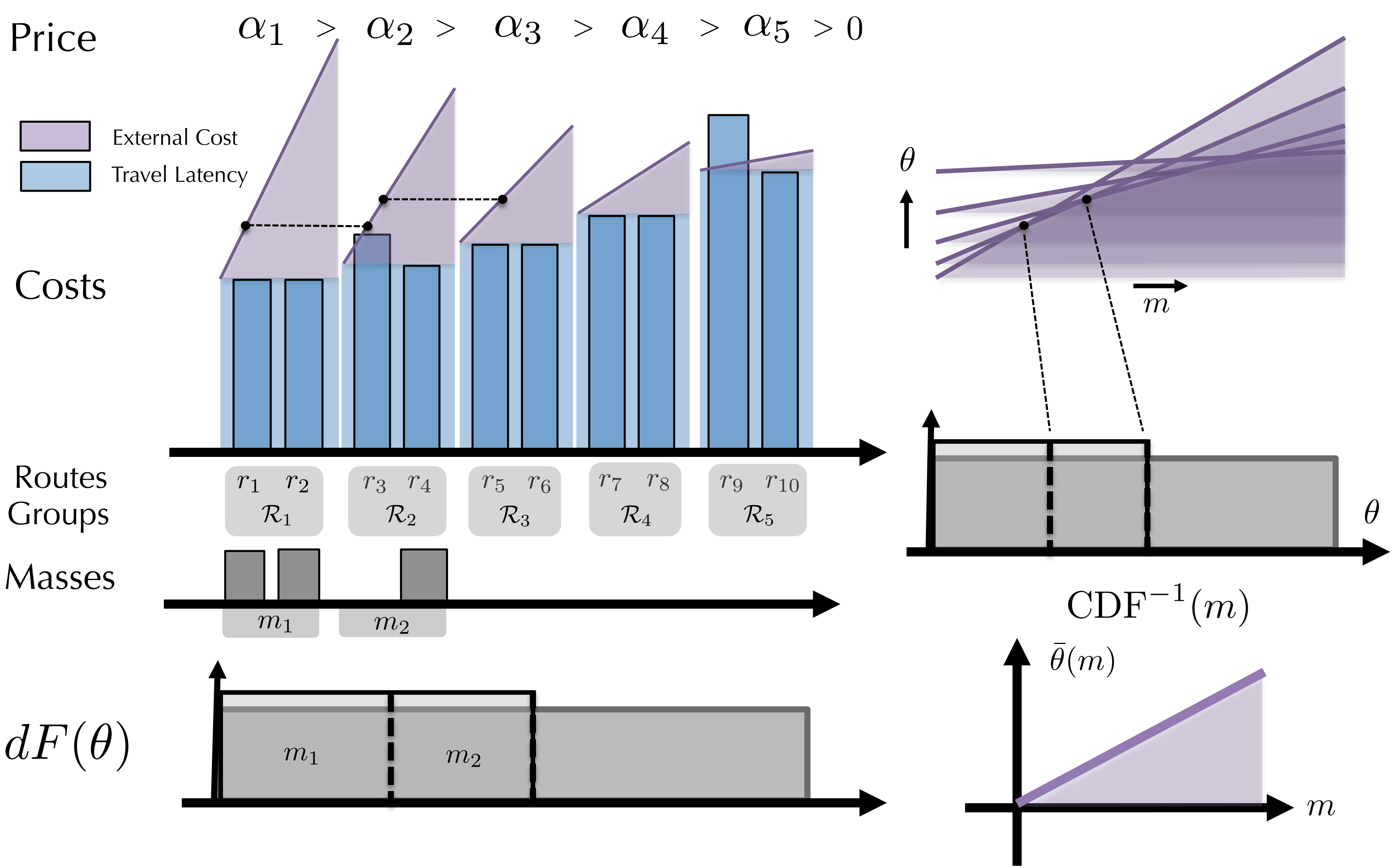

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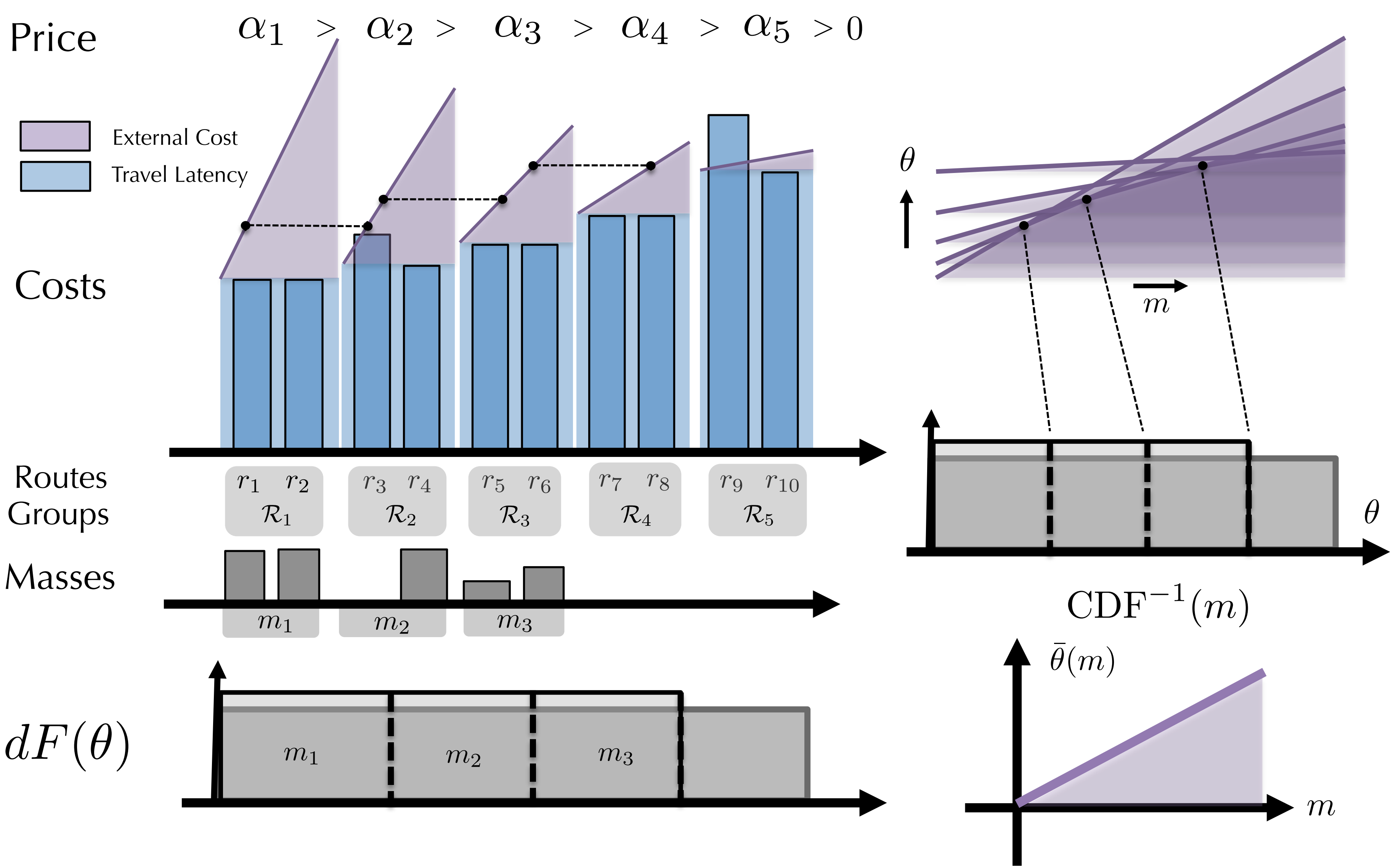

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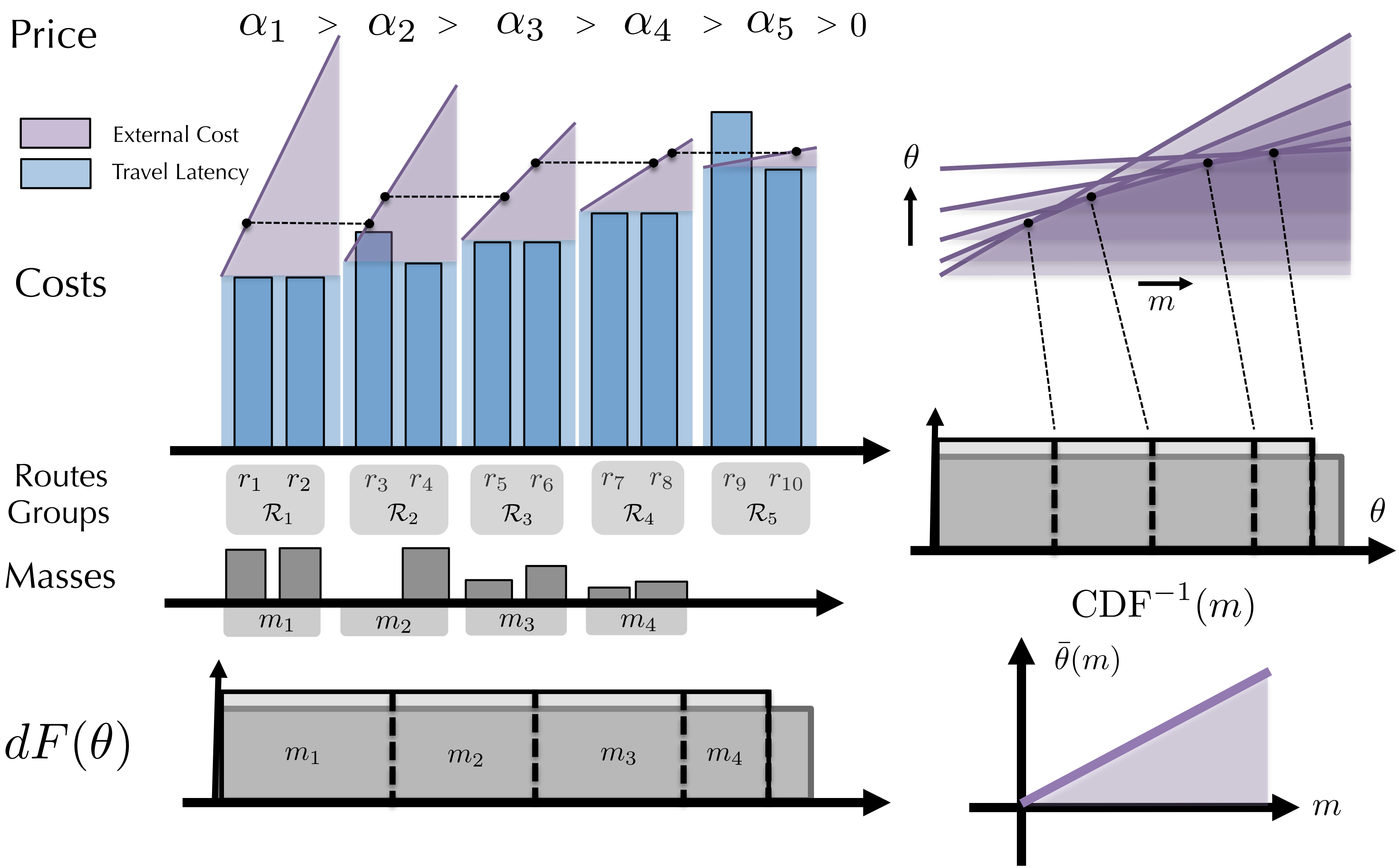

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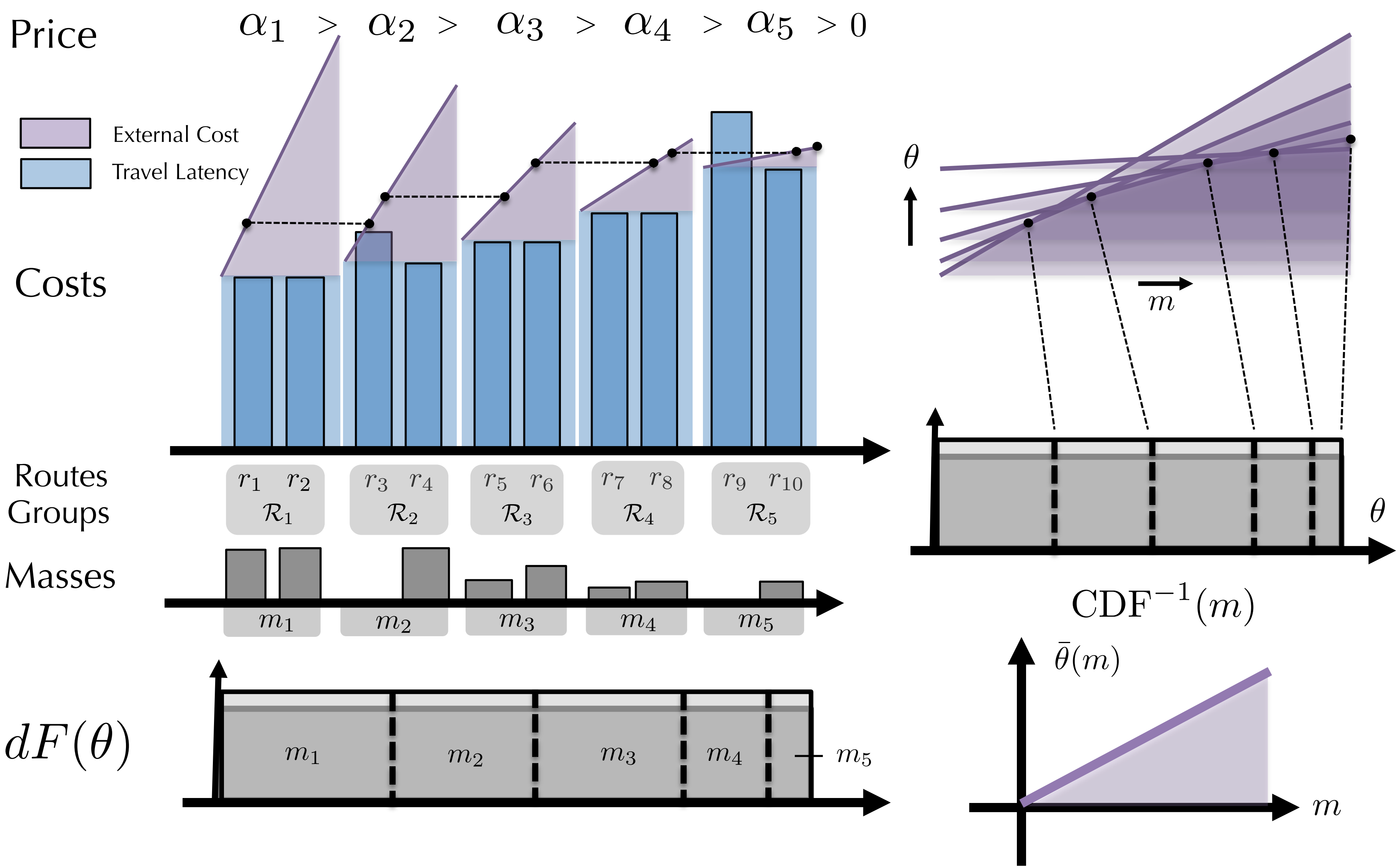

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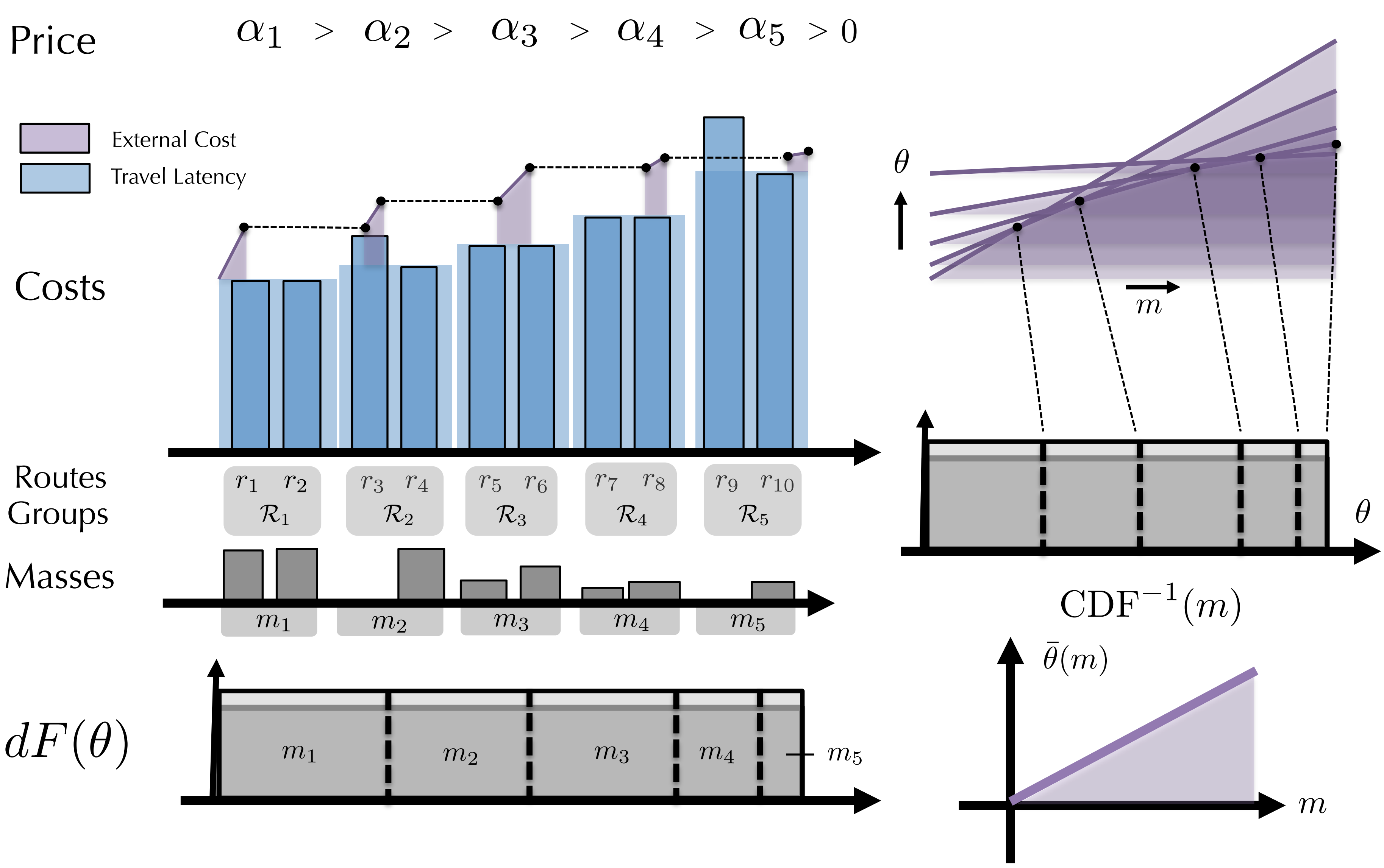

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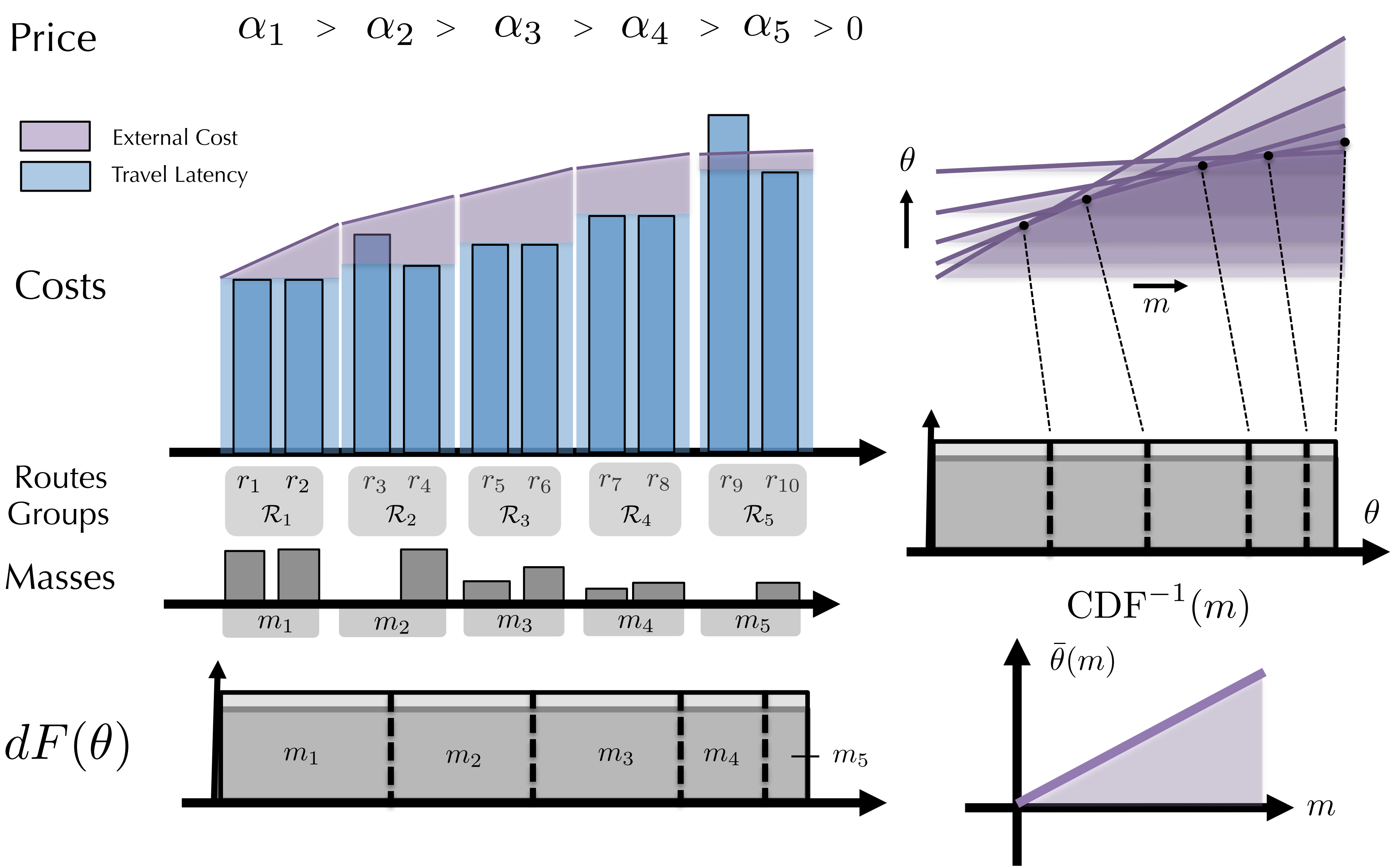

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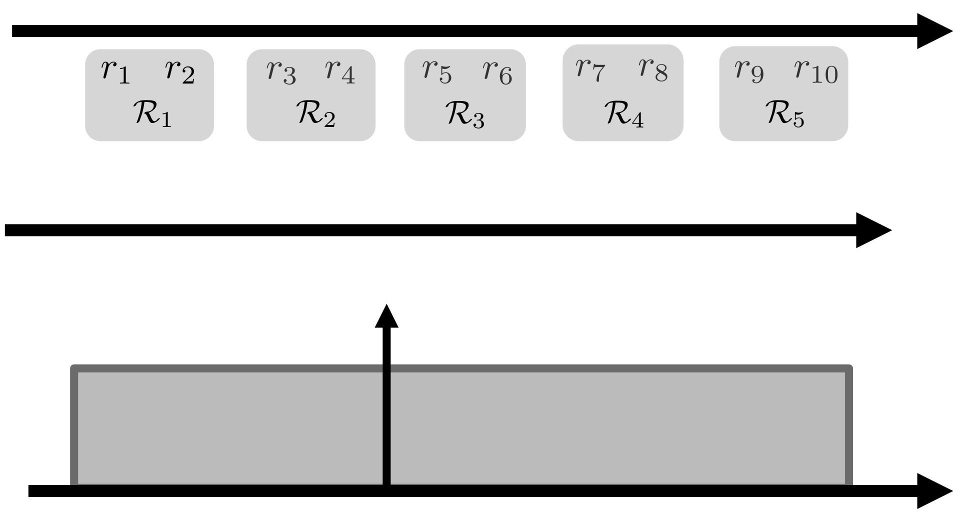

$dF(\theta)$

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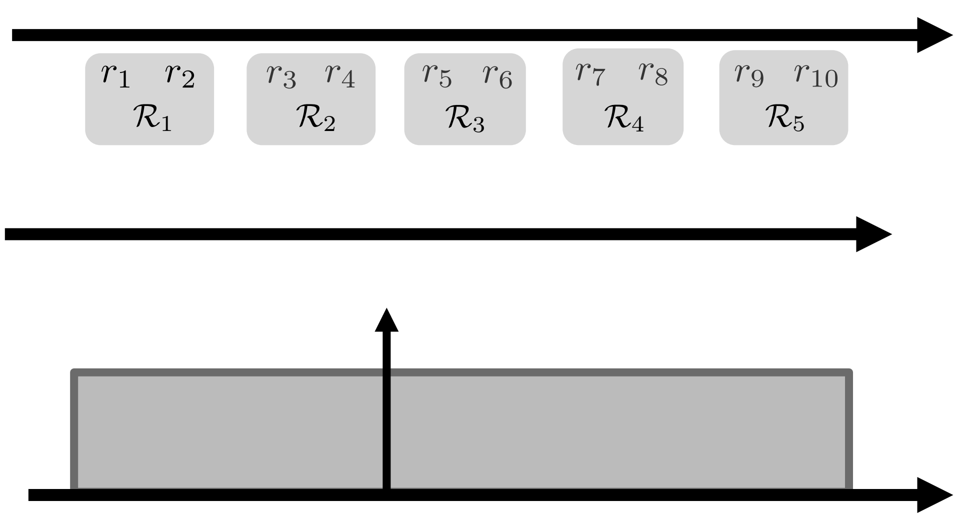

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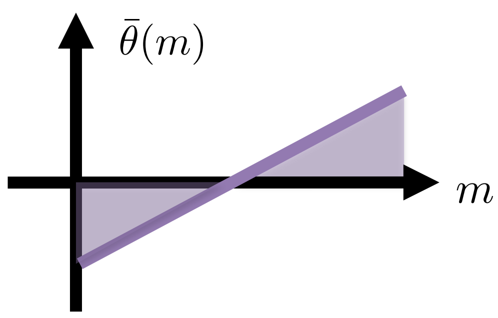

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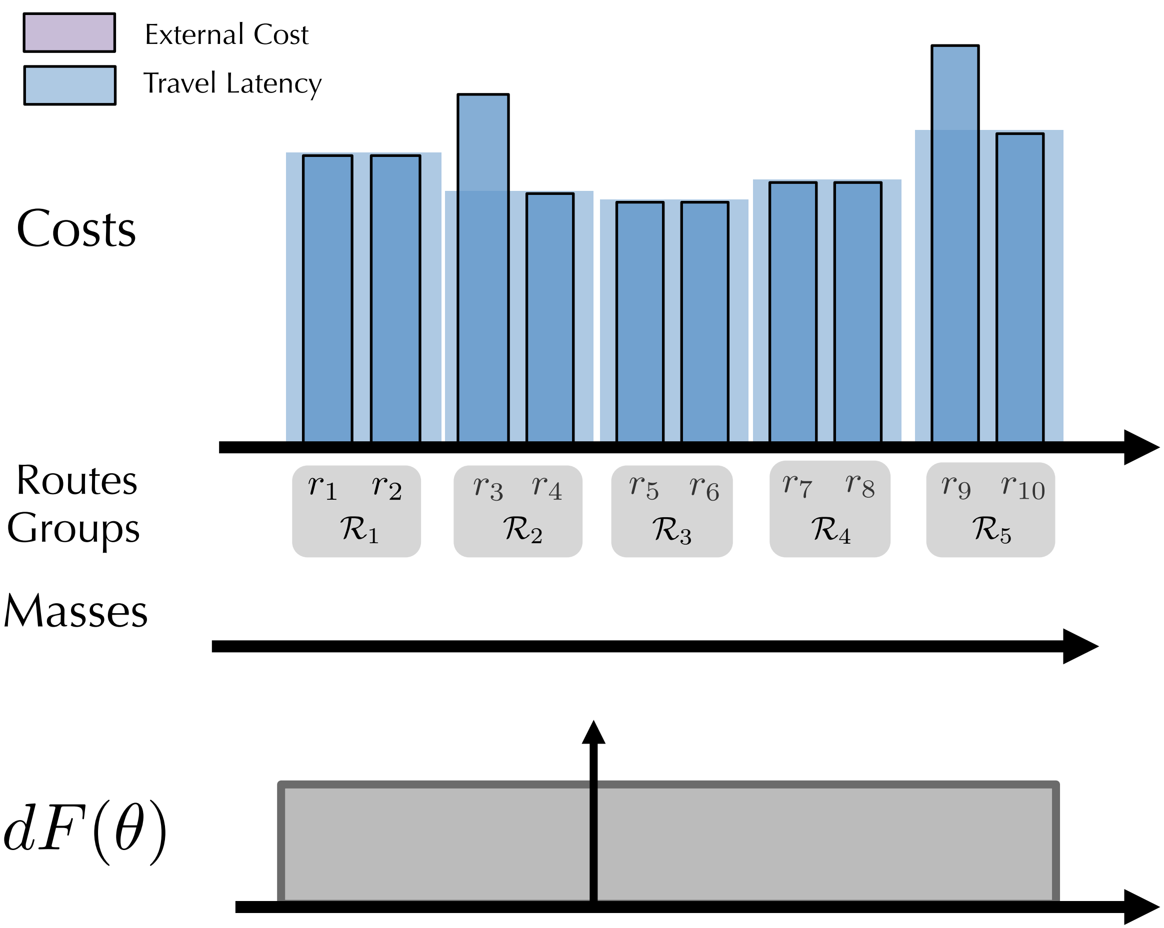
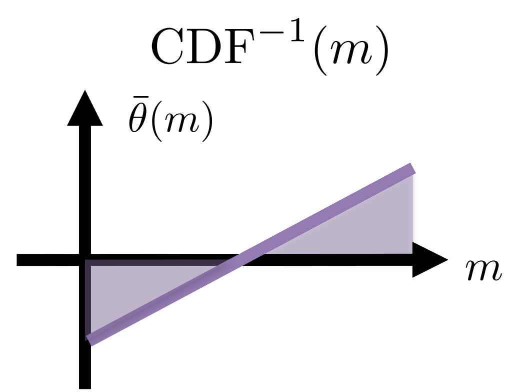
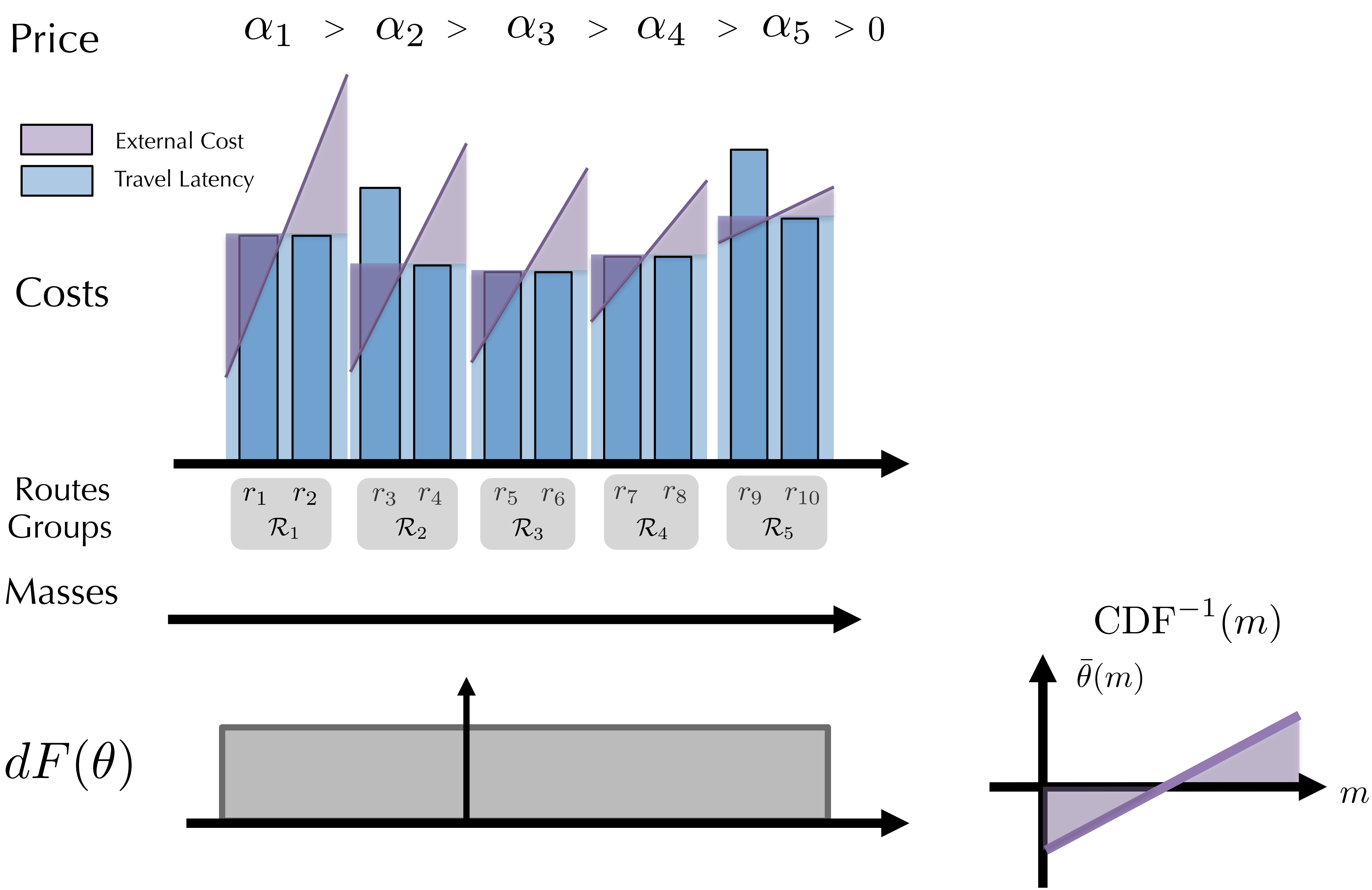




Intuition: $\alpha_i \geq 0$, θ uniform



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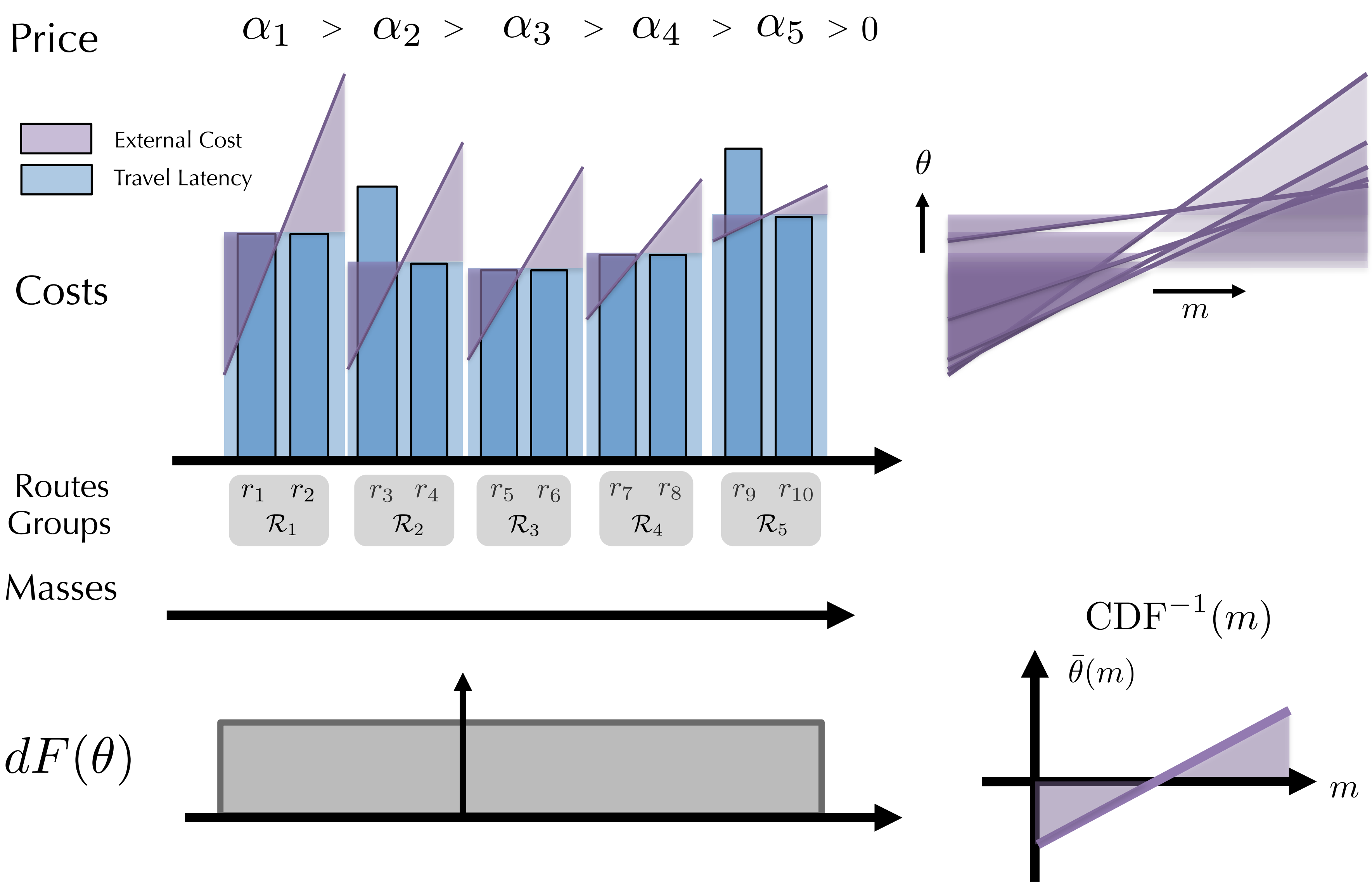

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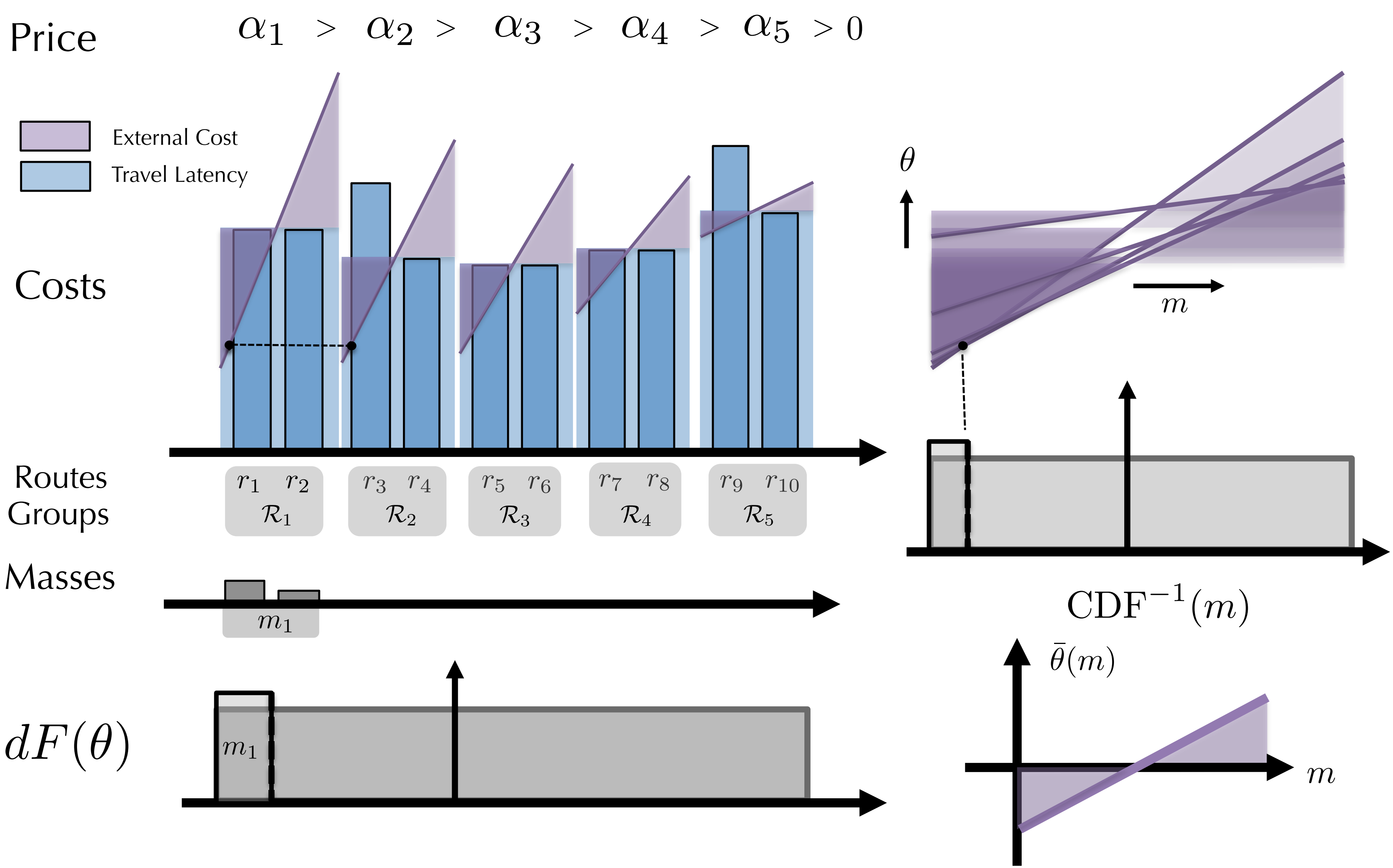

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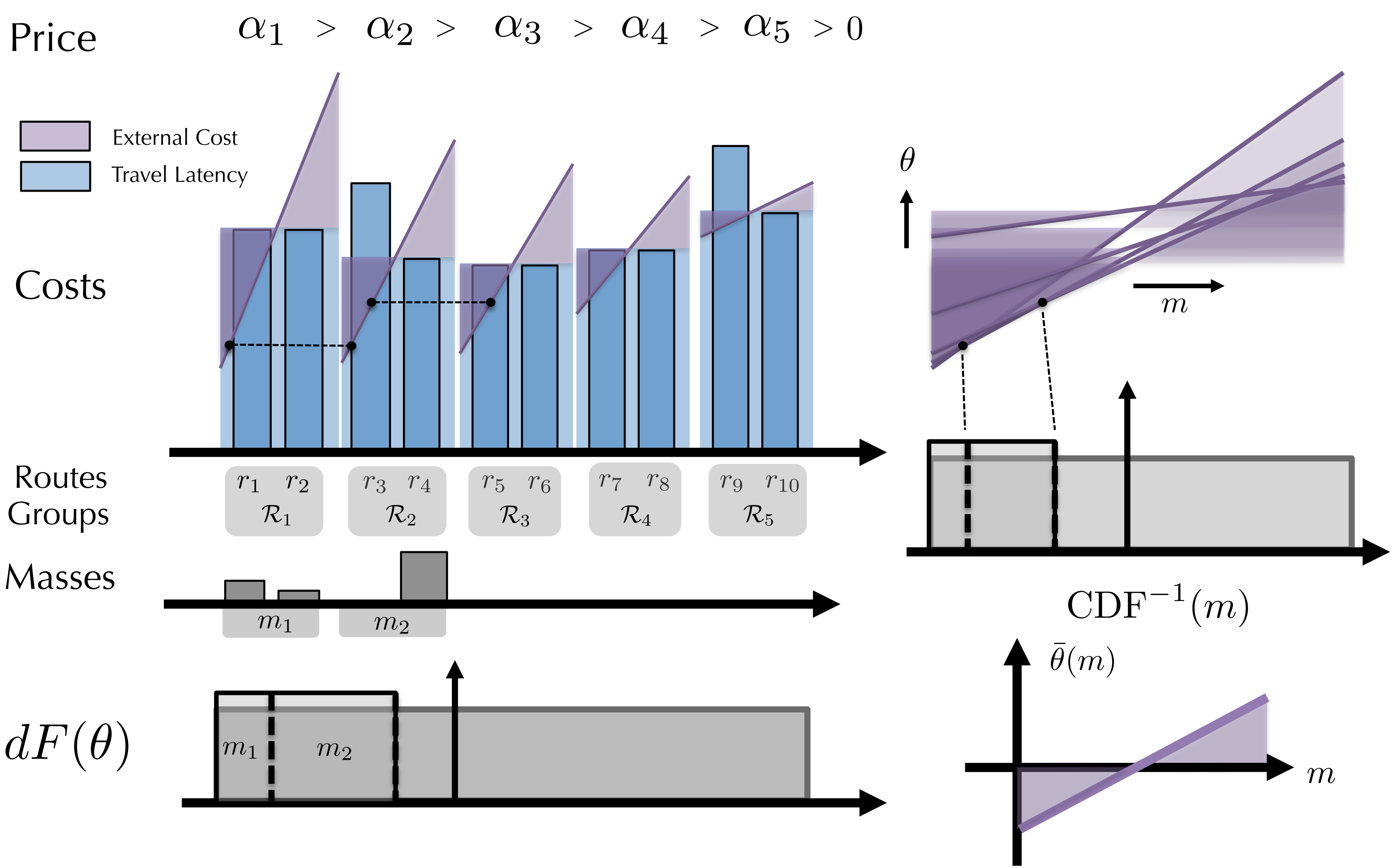

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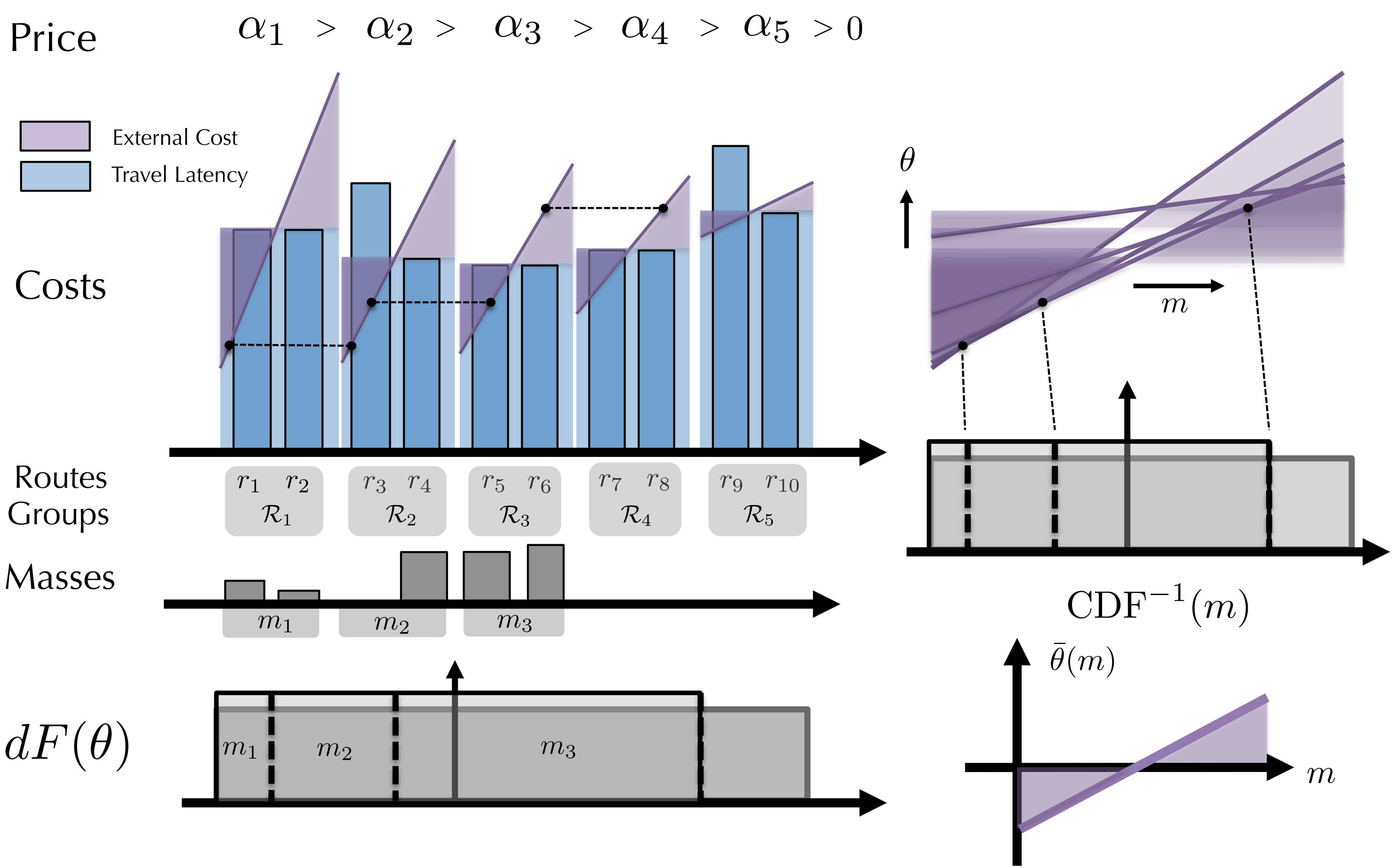
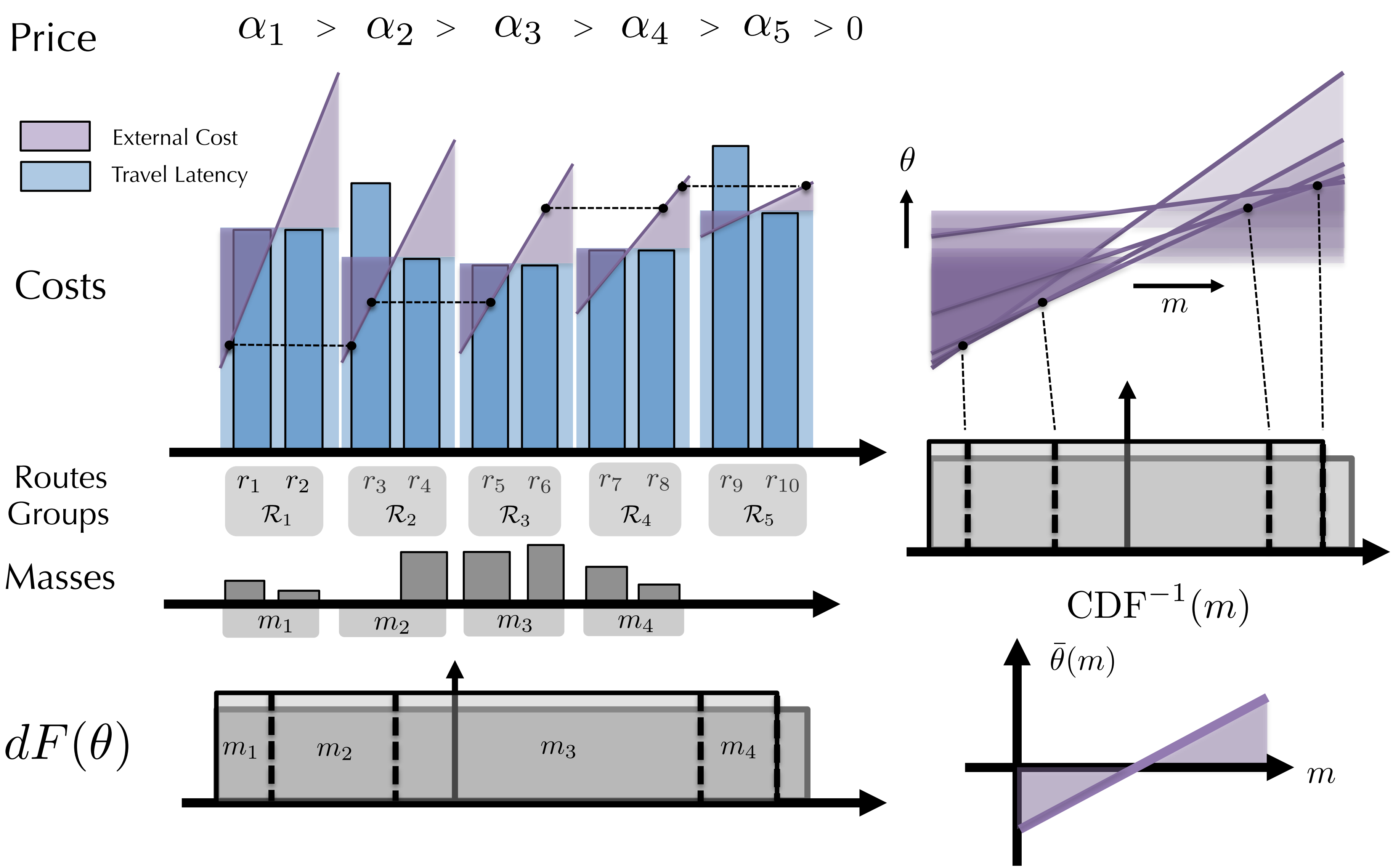
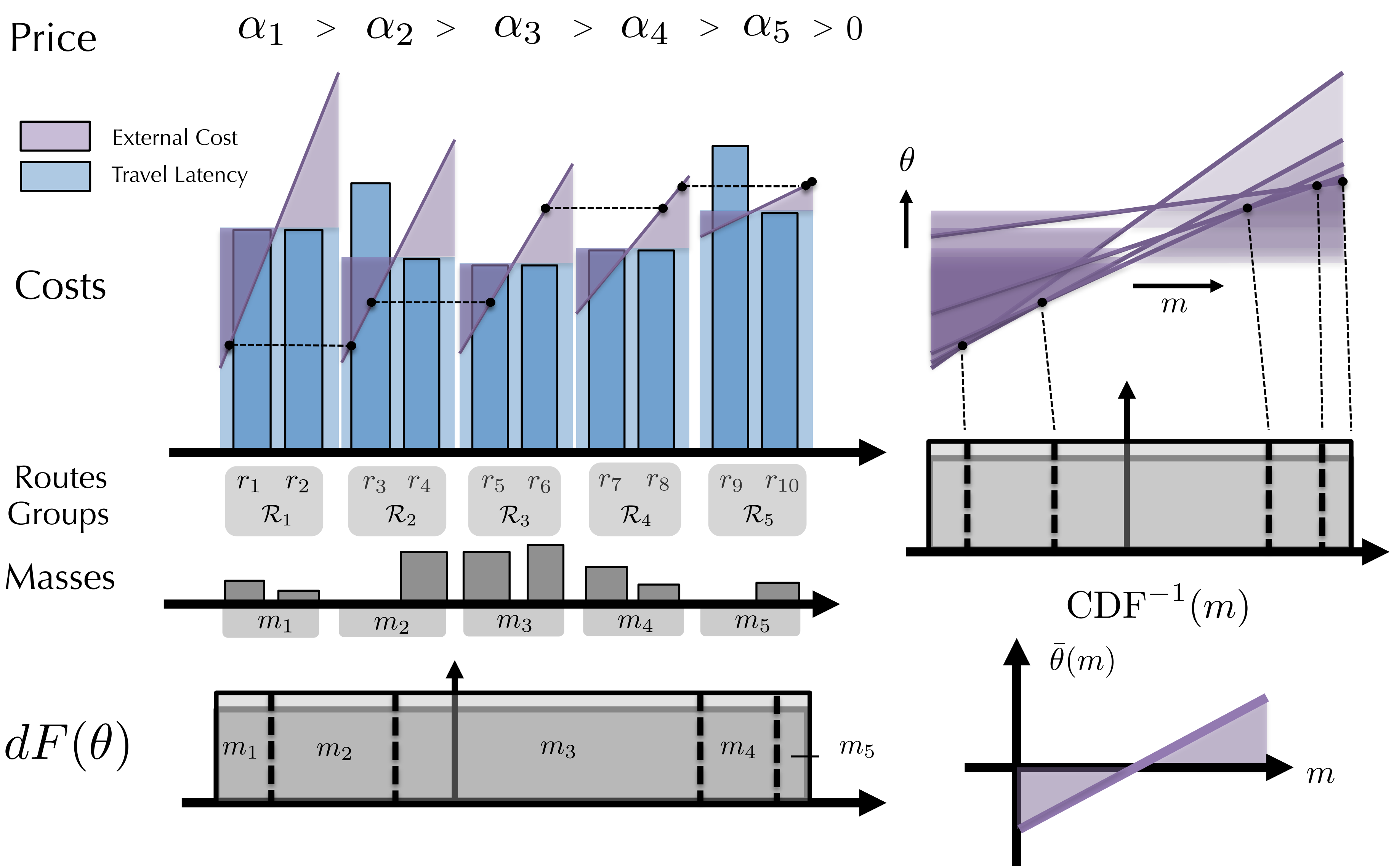


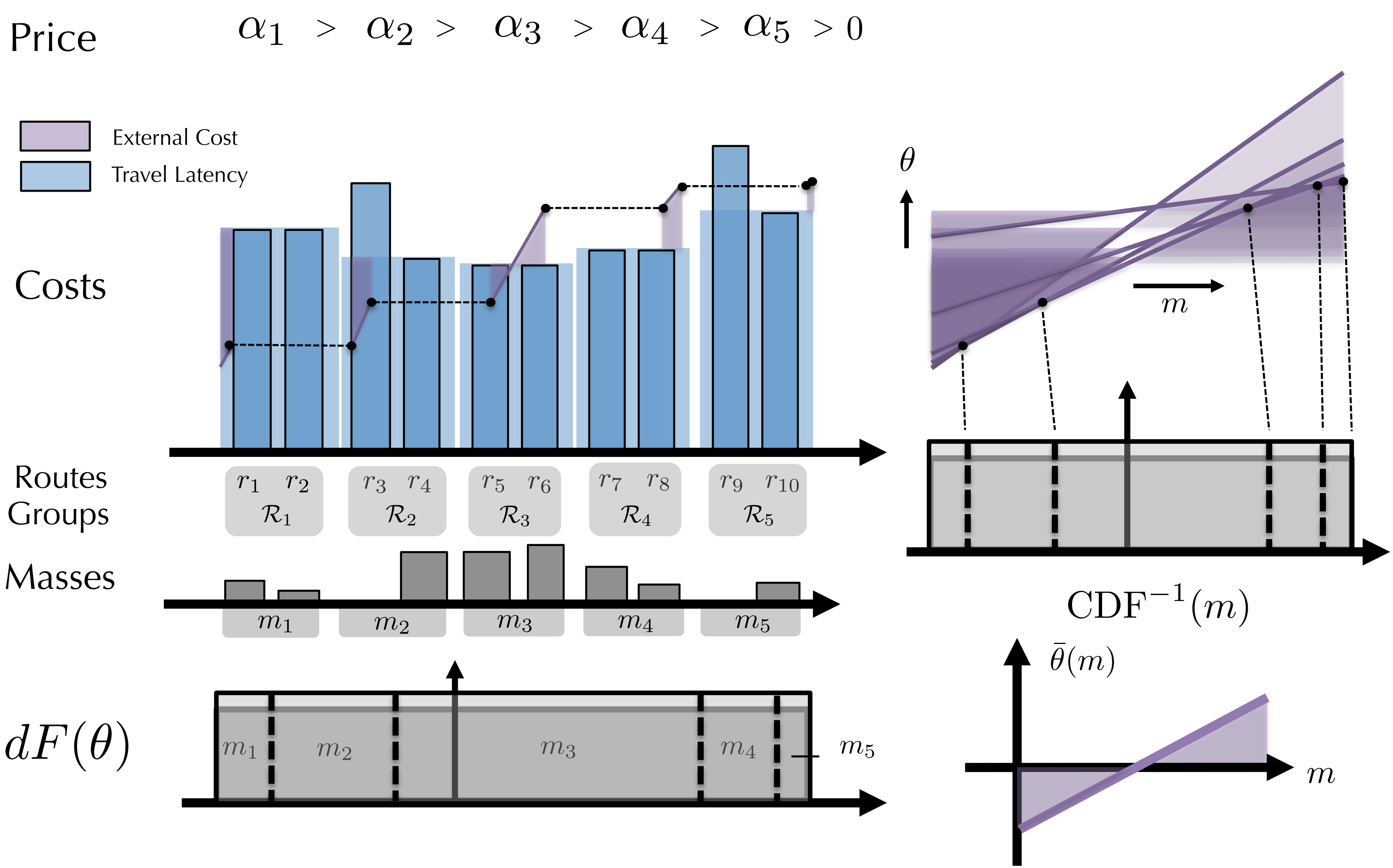
Intuition: $\alpha_i \geq 0$, θ uniform



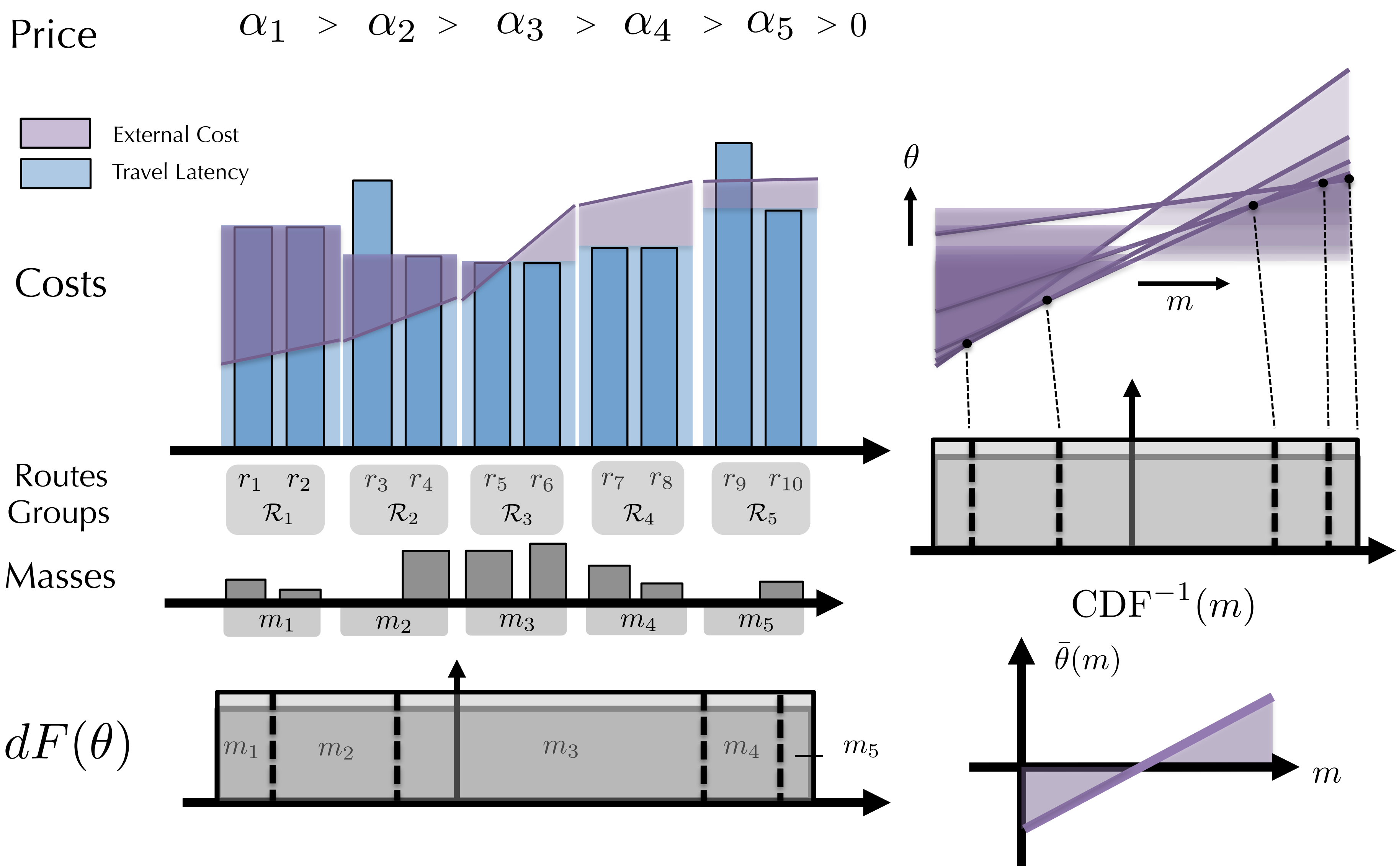
Intuition: $\alpha_i \geq 0$, θ uniform



Intuition: $\alpha_i \geq 0$, θ uniform



Intuition: $\alpha_i \geq 0,\ \theta$ uniform



Intuition: θ uniform

Price $\alpha_1 > \alpha_2 > \alpha_3 > 0 > \alpha_4 > \alpha_5$

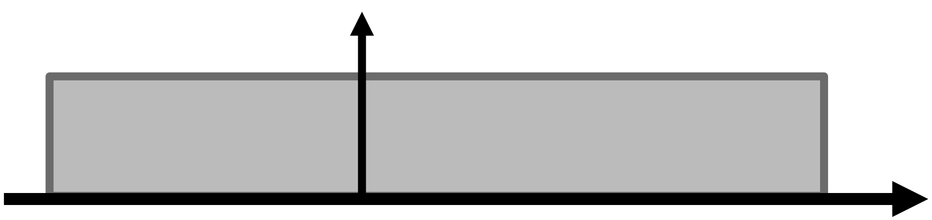
- External Cost
- Travel Latency

Costs

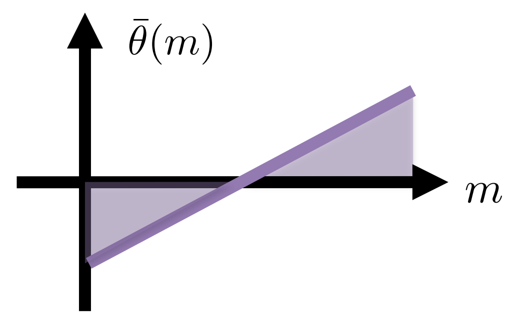
Routes	r_1 r_2	r_3 r_4	r_5 r_6	r_7 r_8	r_9 r_{10}
Groups	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$

Masses

$dF(\theta)$



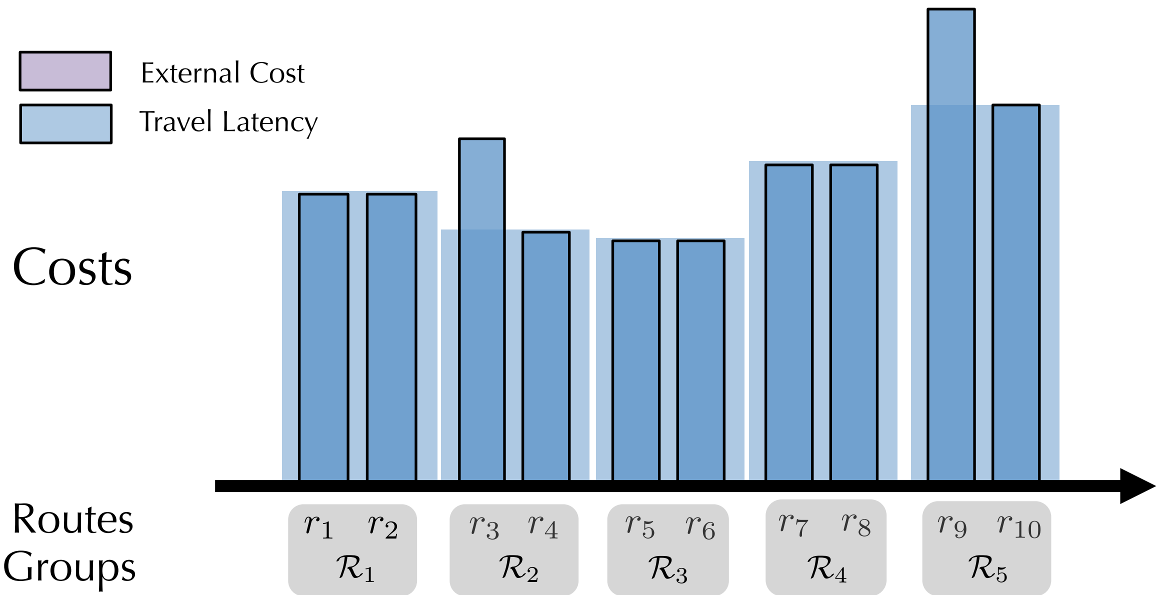
$\mathrm{CDF}^{-1}(m)$



Intuition: θ uniform

Price $\quad \alpha_1 > \alpha_2 > \alpha_3 > 0 > \alpha_4 > \alpha_5$

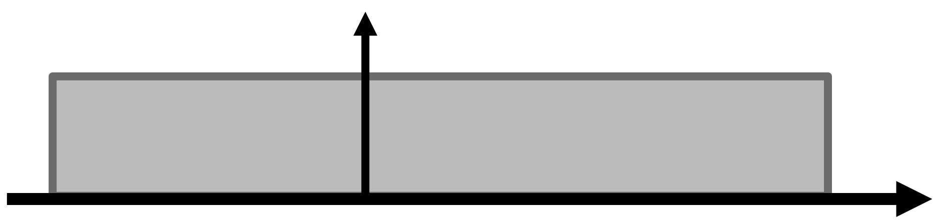
Costs



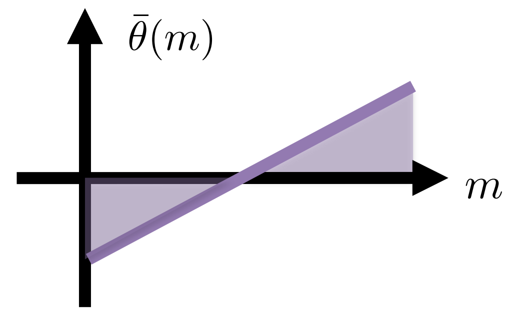
Routes
Groups

Masses

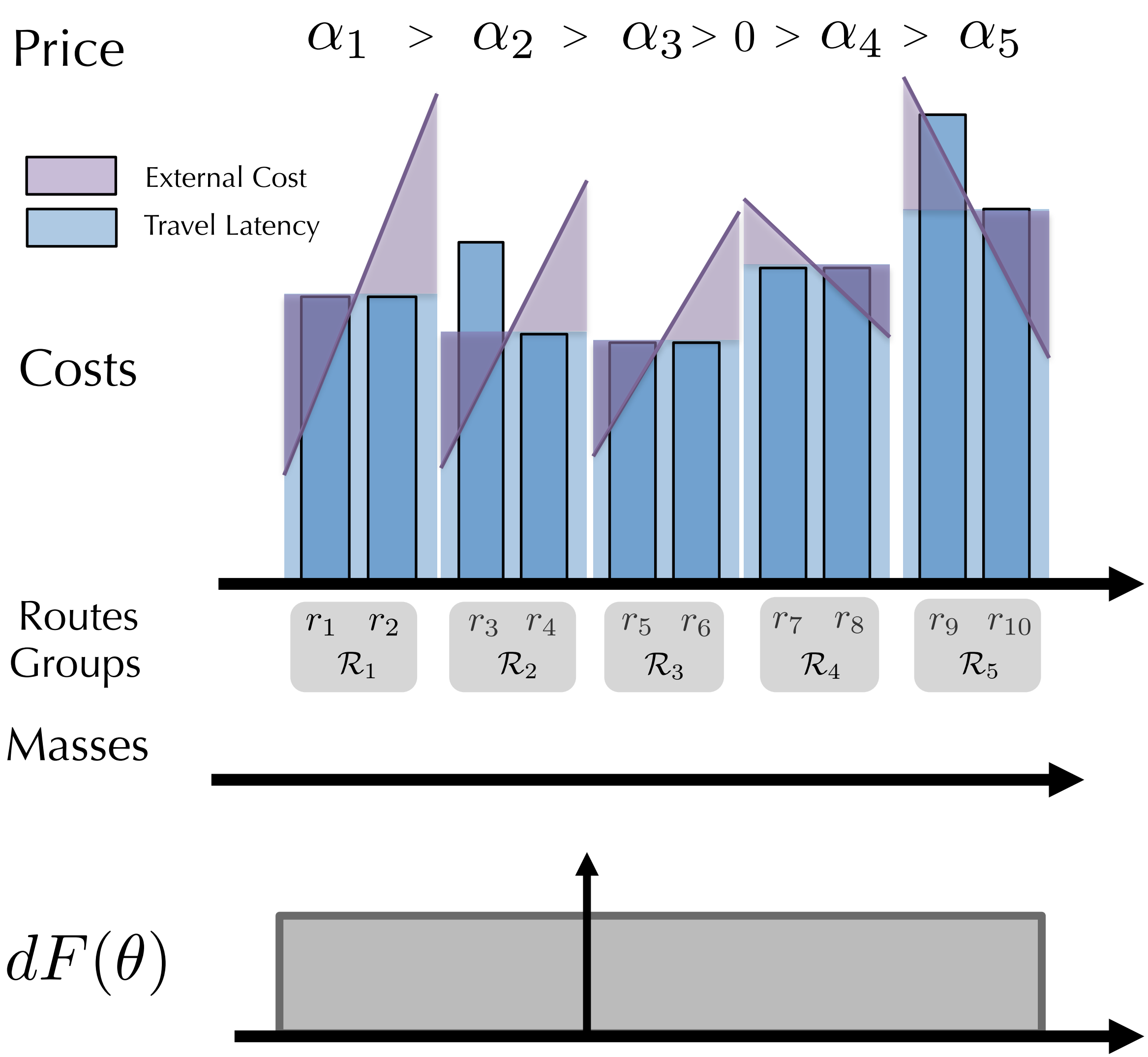
$dF(\theta)$



$\mathrm{CDF}^{-1}(m)$



Intuition: θ uniform



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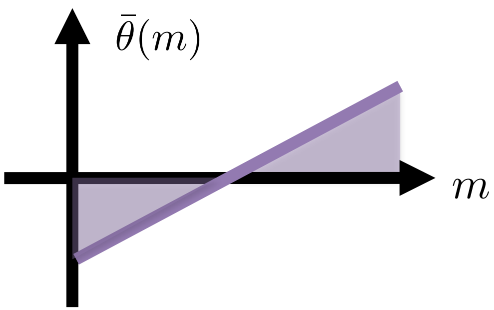
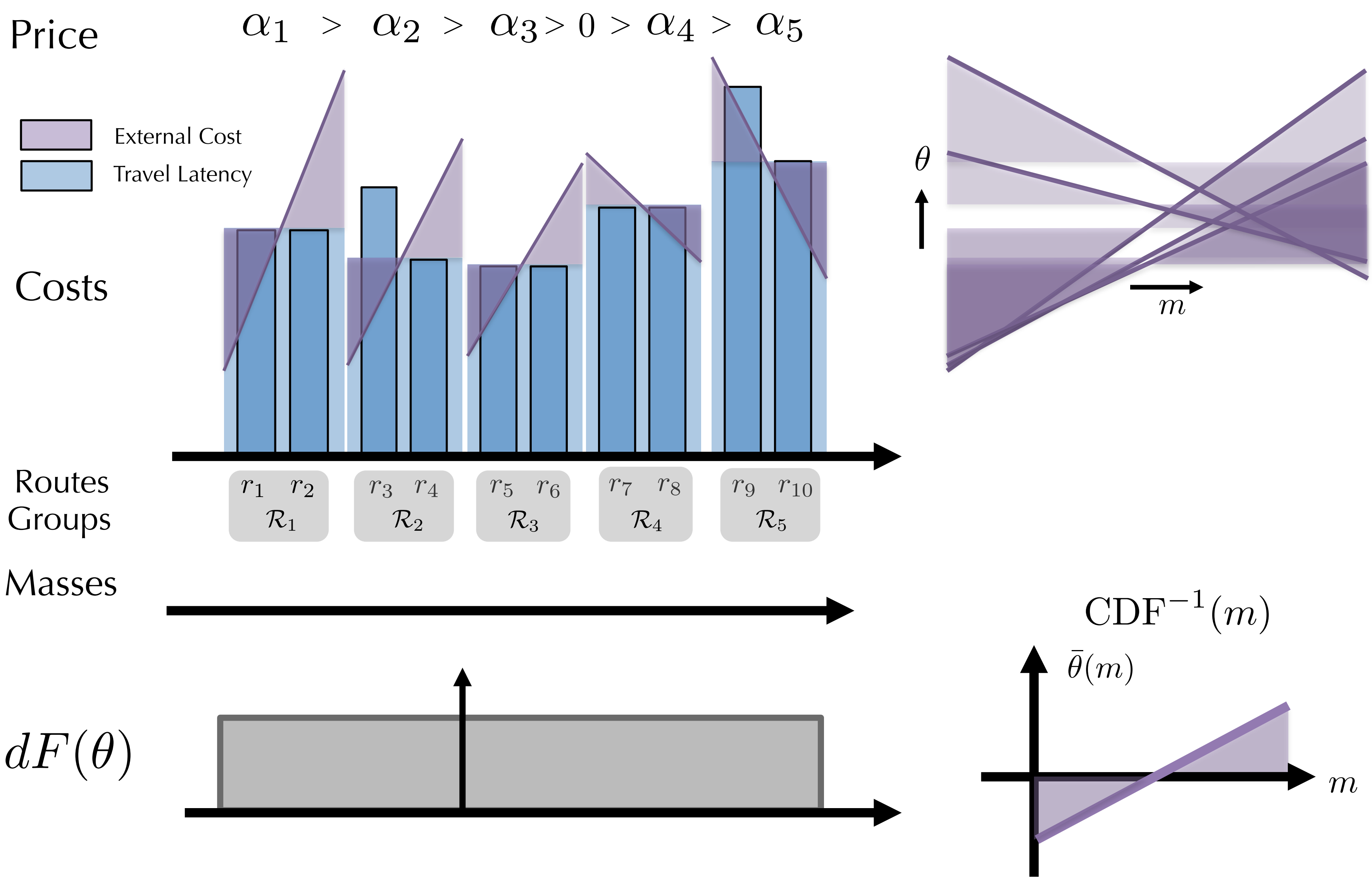
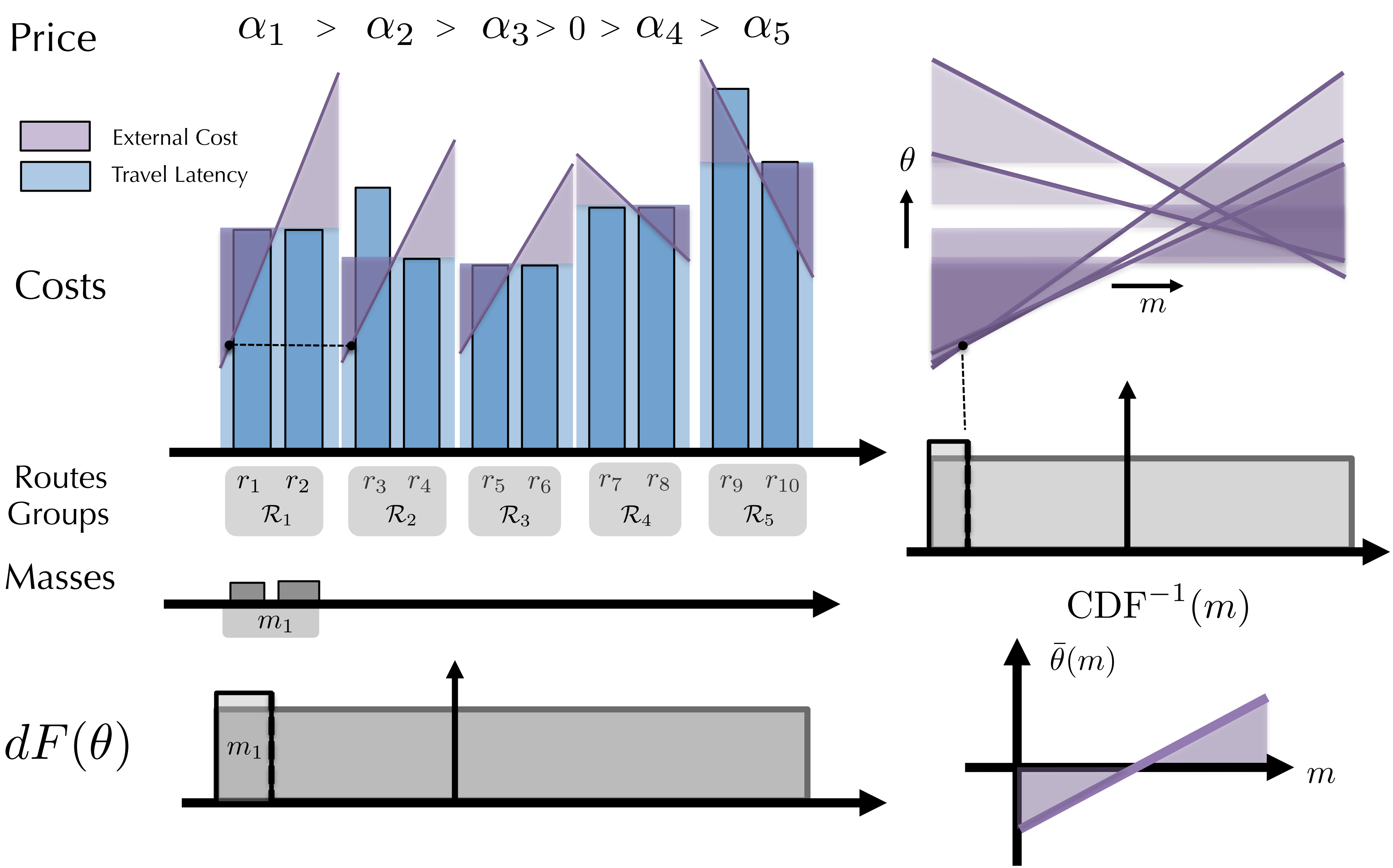


Intuition: θ uniform



Intuition: θ uniform



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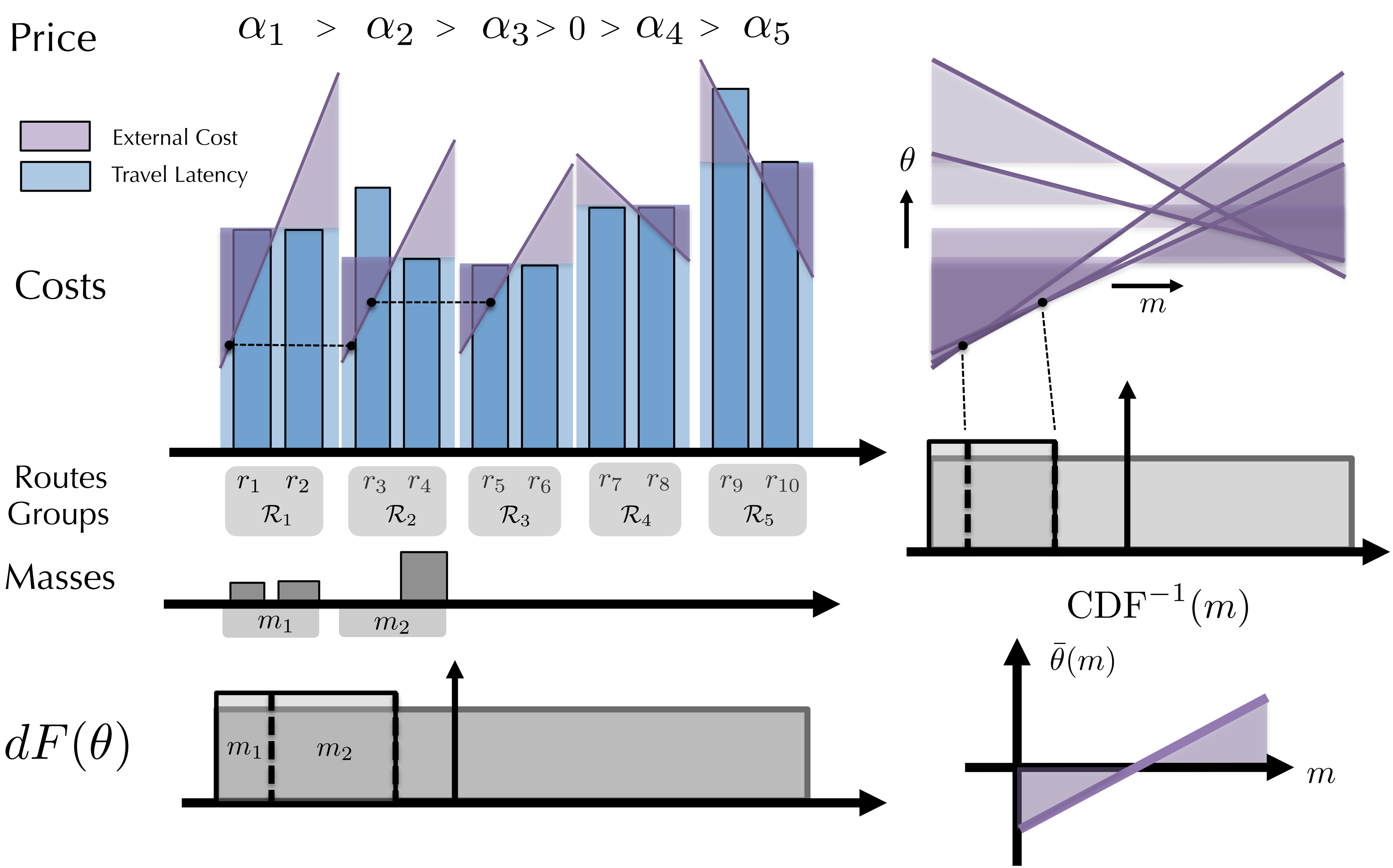

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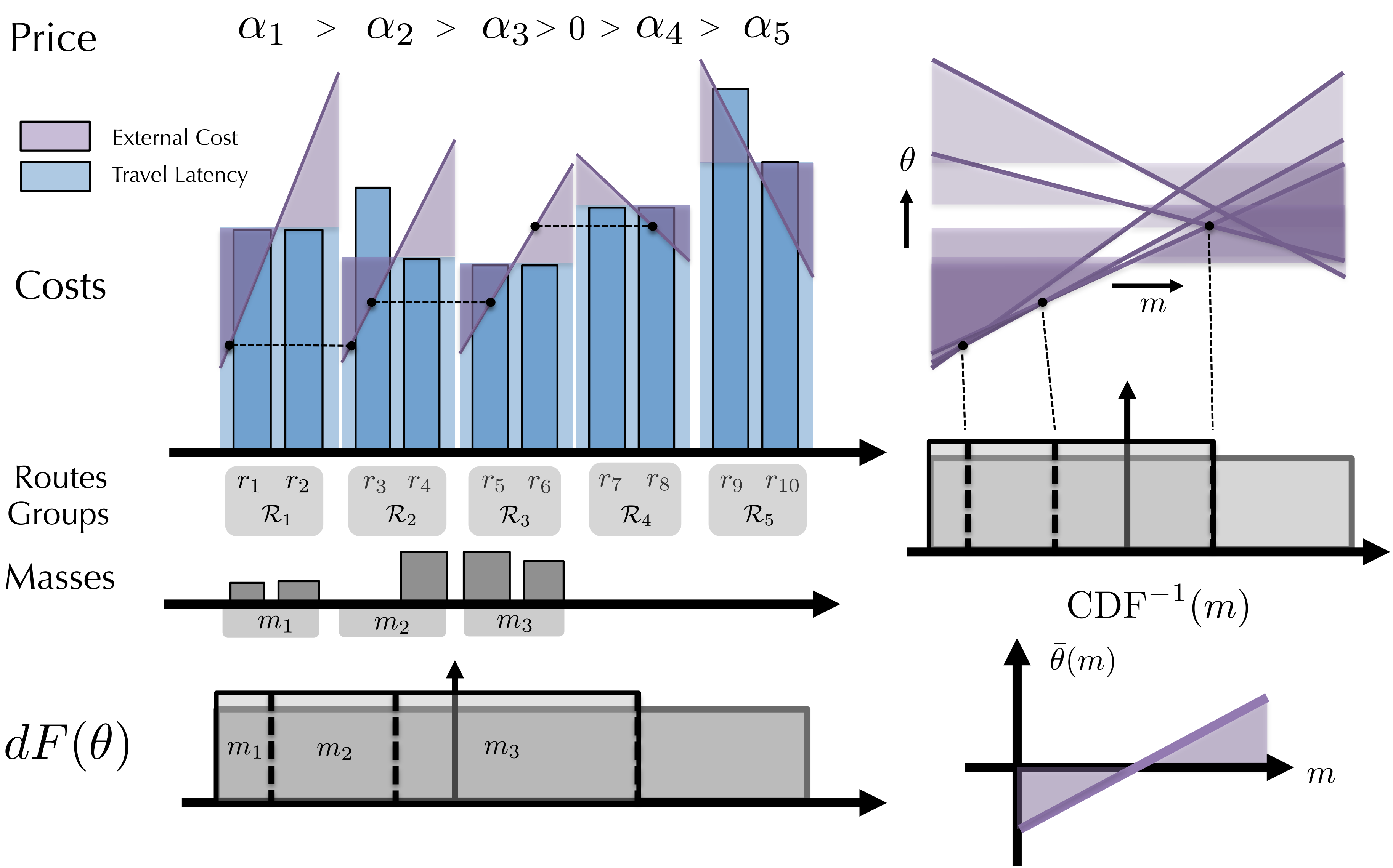

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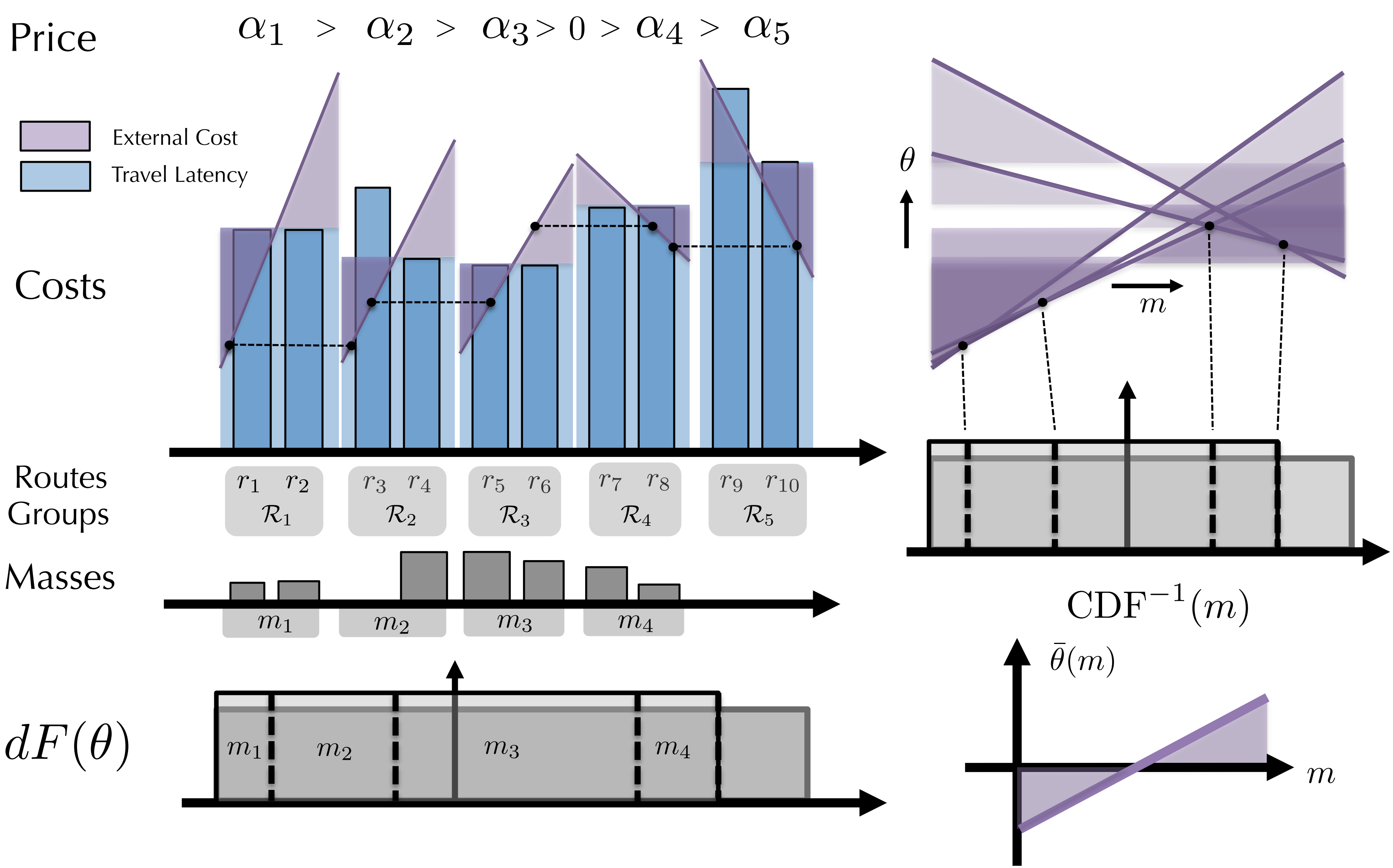
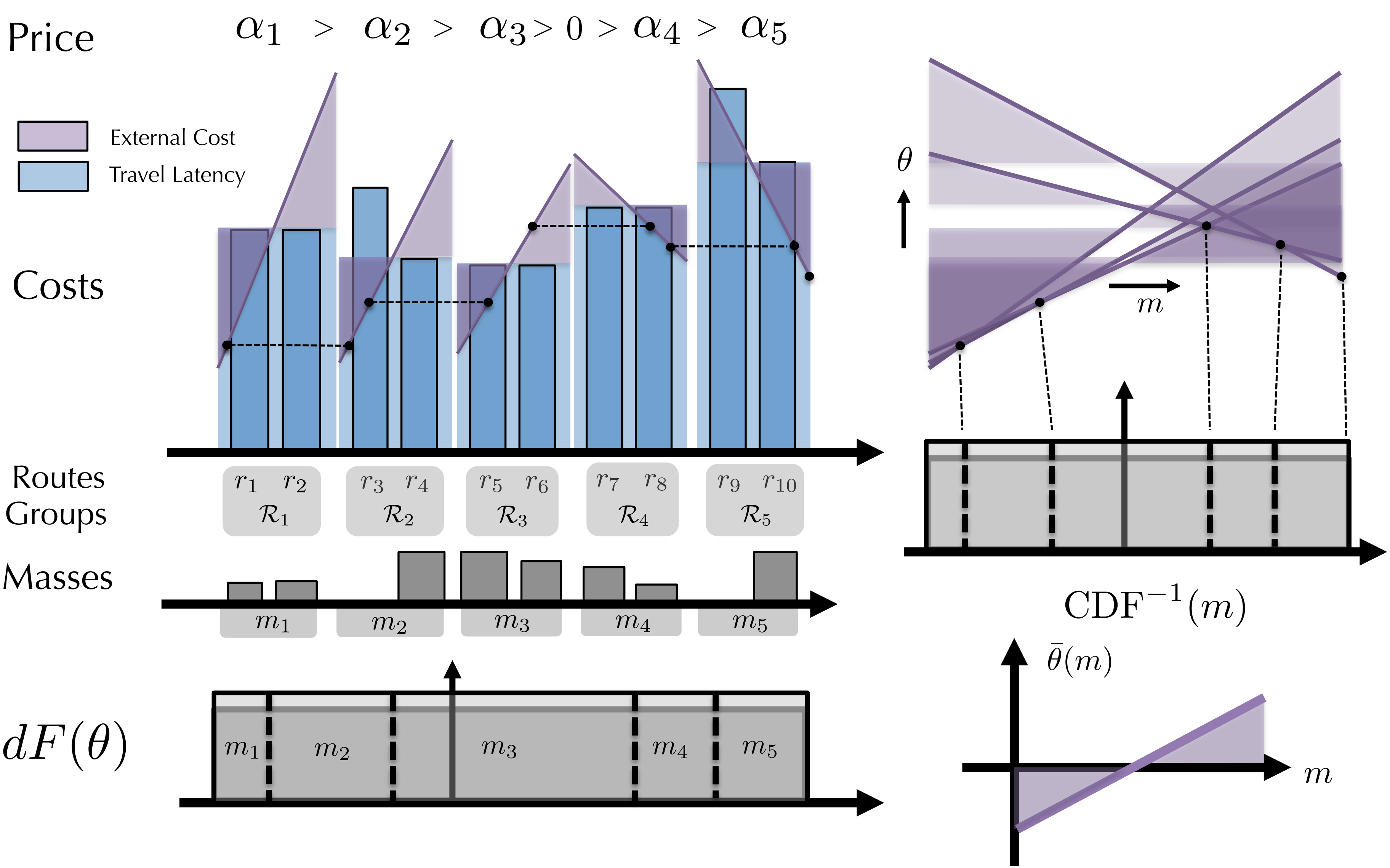
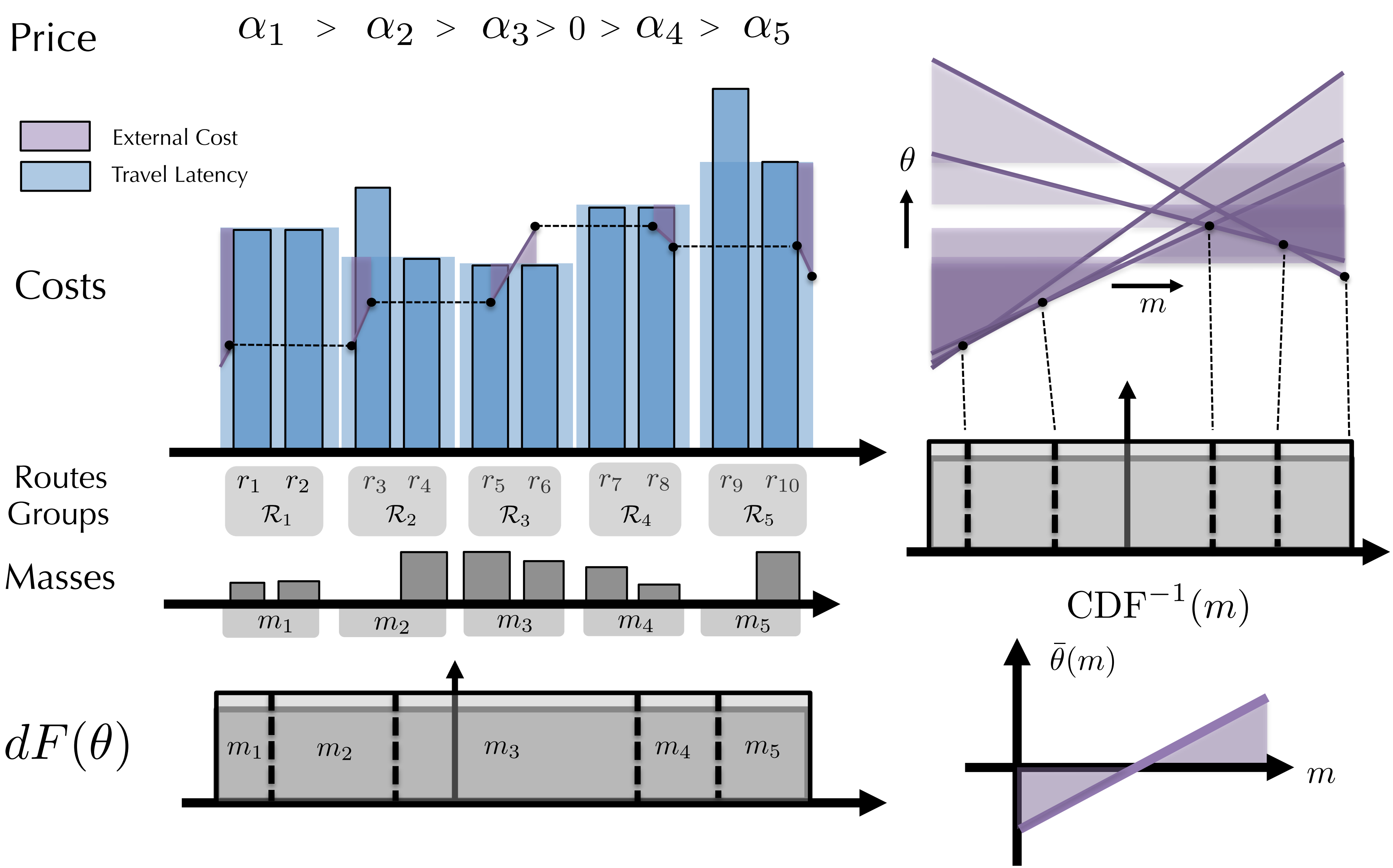


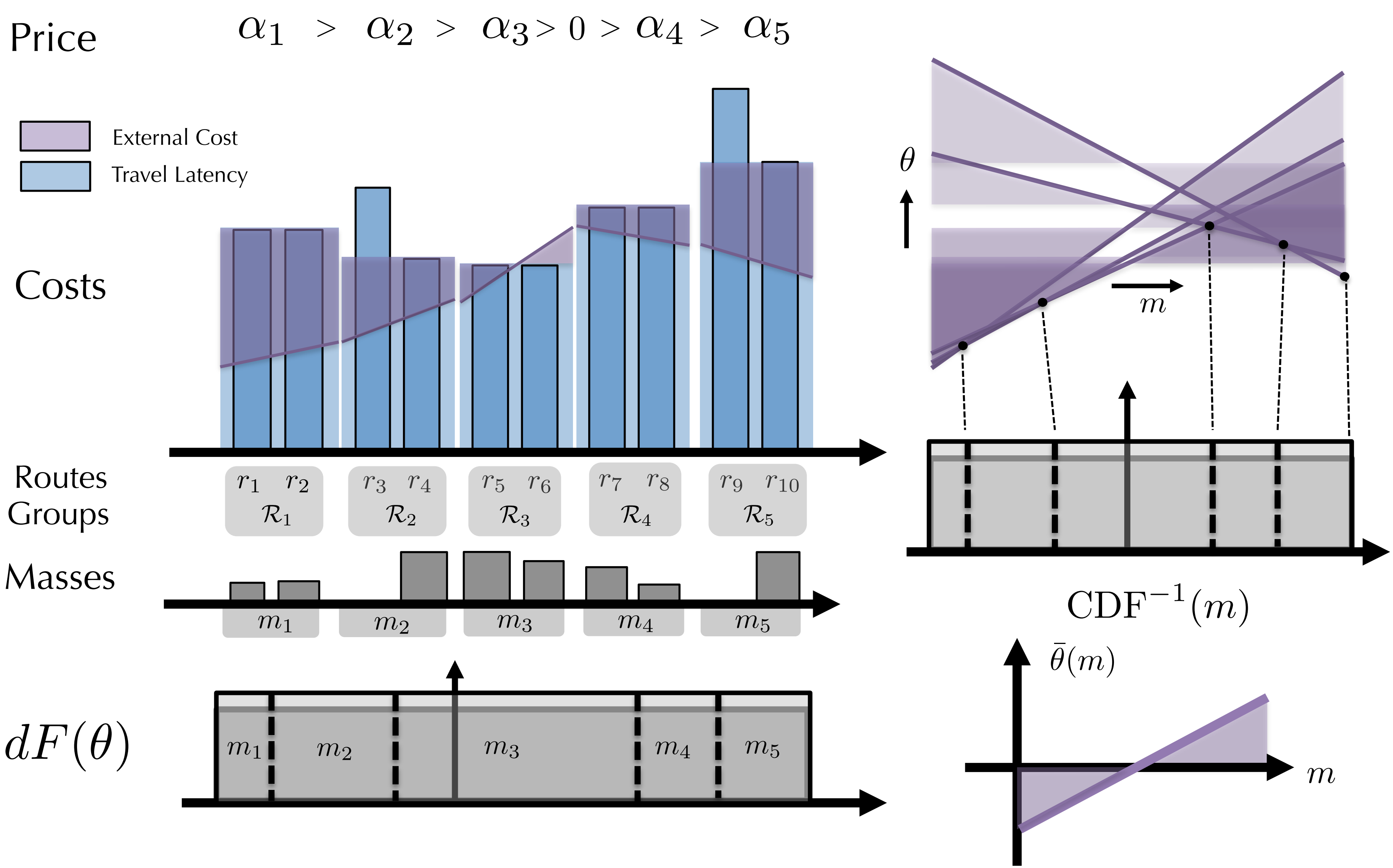
Intuition: θ uniform



Intuition: θ uniform



Intuition: θ uniform



Intuition: θ general

Price $\alpha_1 > \alpha_2 > \alpha_3 > 0 > \alpha_4 > \alpha_5$

External Cost

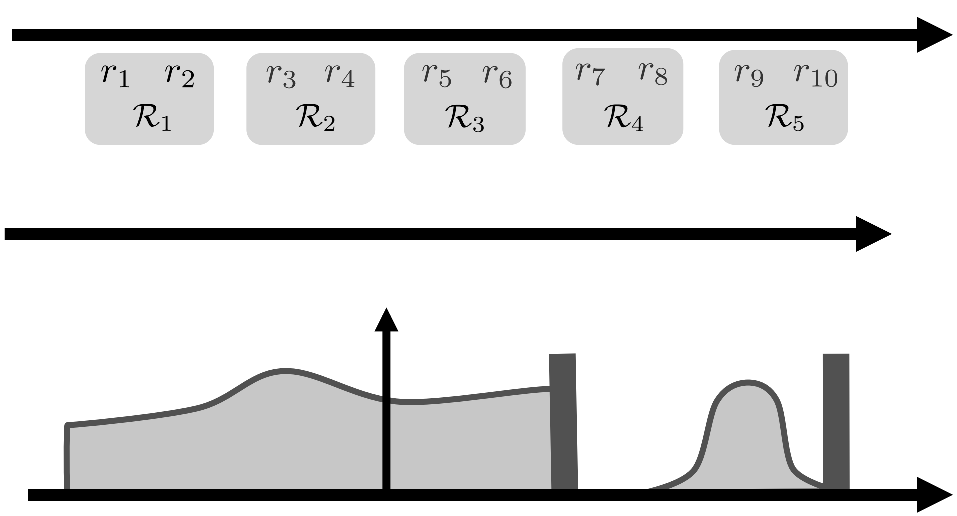
Travel Latency

Costs

Routes

Groups

Masses



$dF(\theta)$

Intuition: θ general

Price $\quad \alpha_1 > \alpha_2 > \alpha_3 > 0 > \alpha_4 > \alpha_5$

External Cost

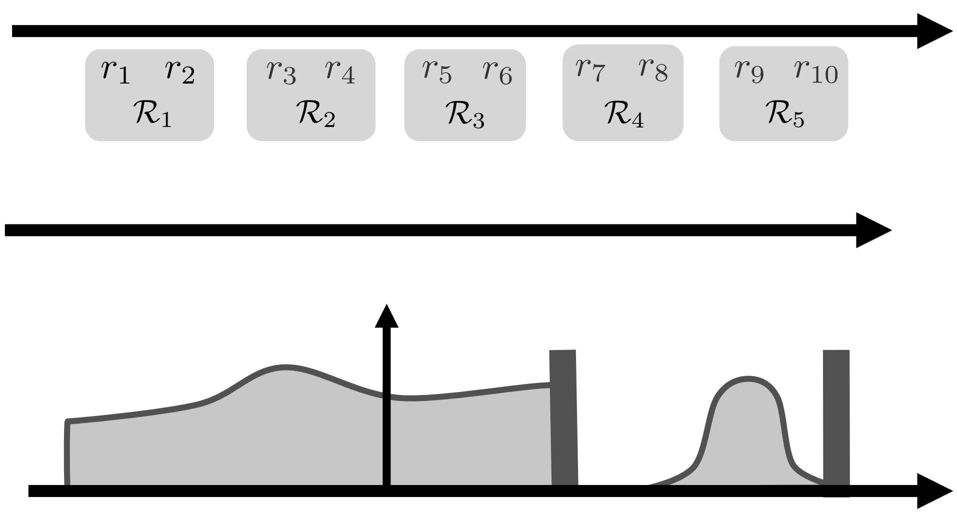
Travel Latency

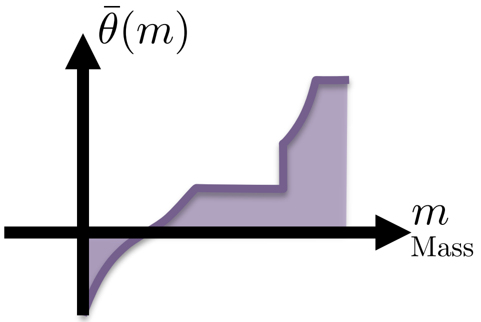
Costs

Routes
Groups

Masses

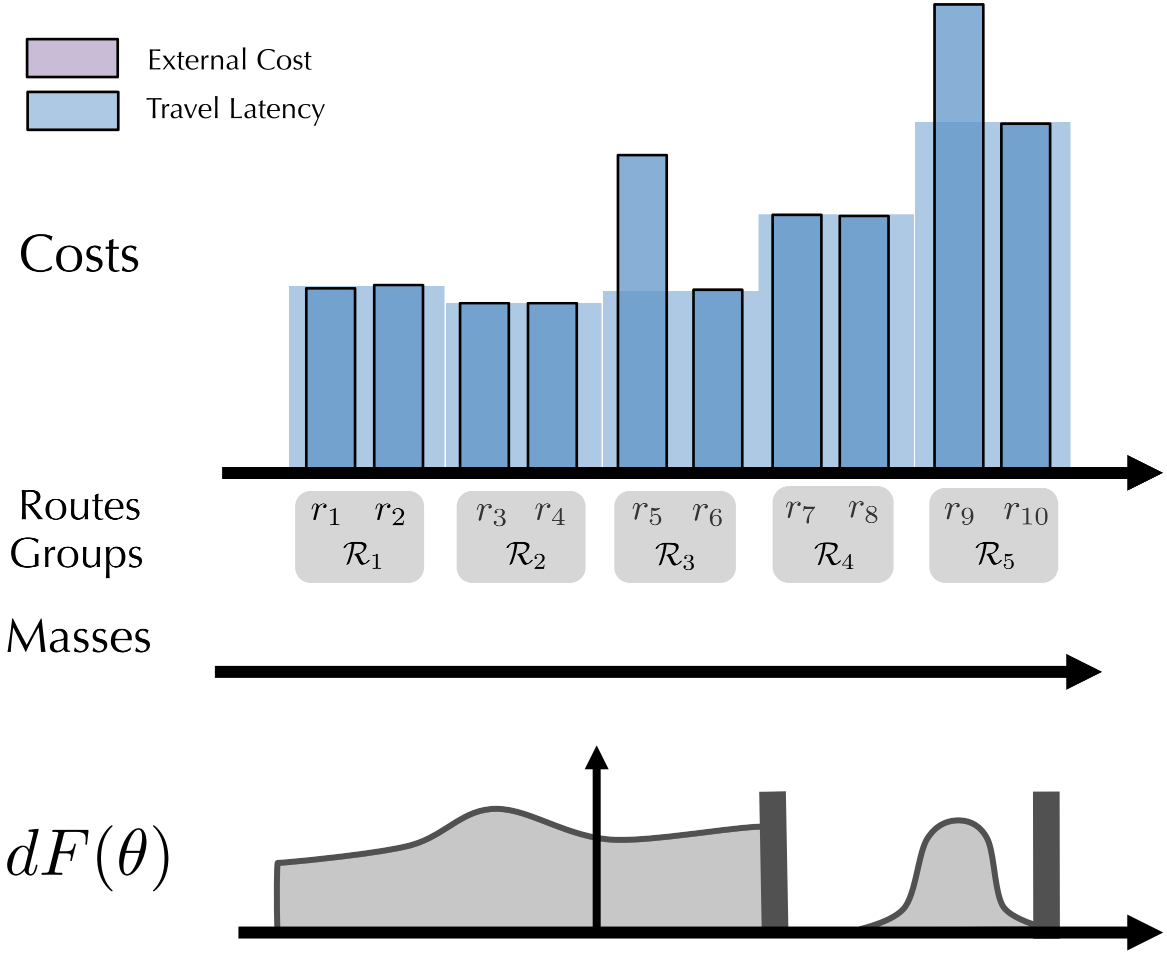
$dF(\theta)$

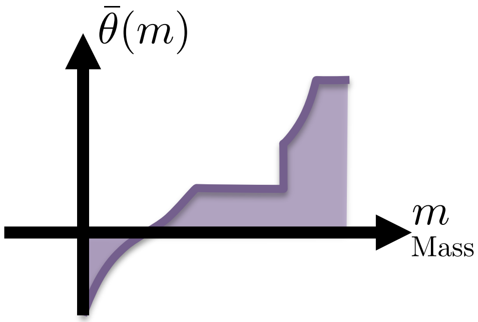




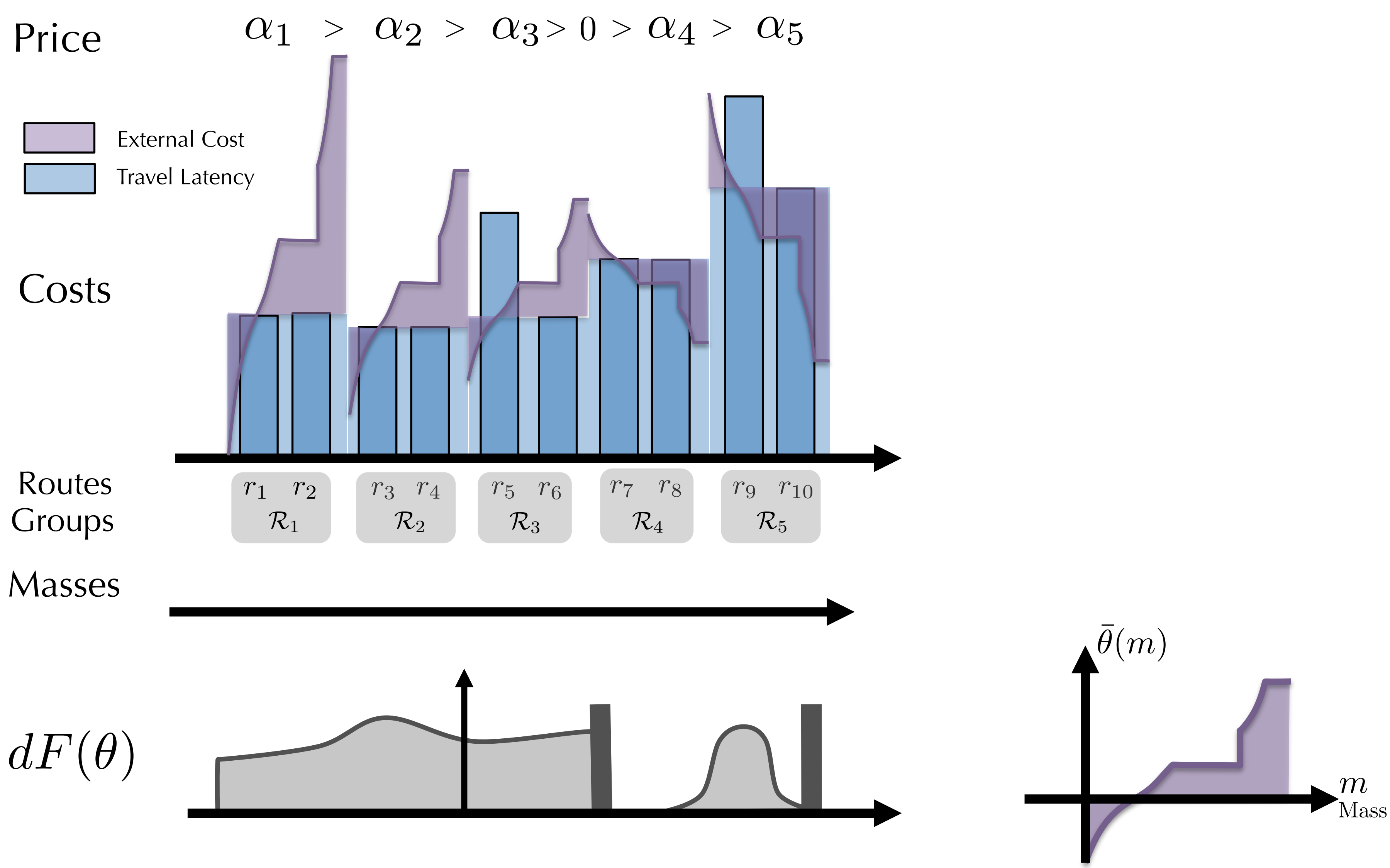
Intuition: θ general

Price $\alpha_1 > \alpha_2 > \alpha_3 > 0 > \alpha_4 > \alpha_5$

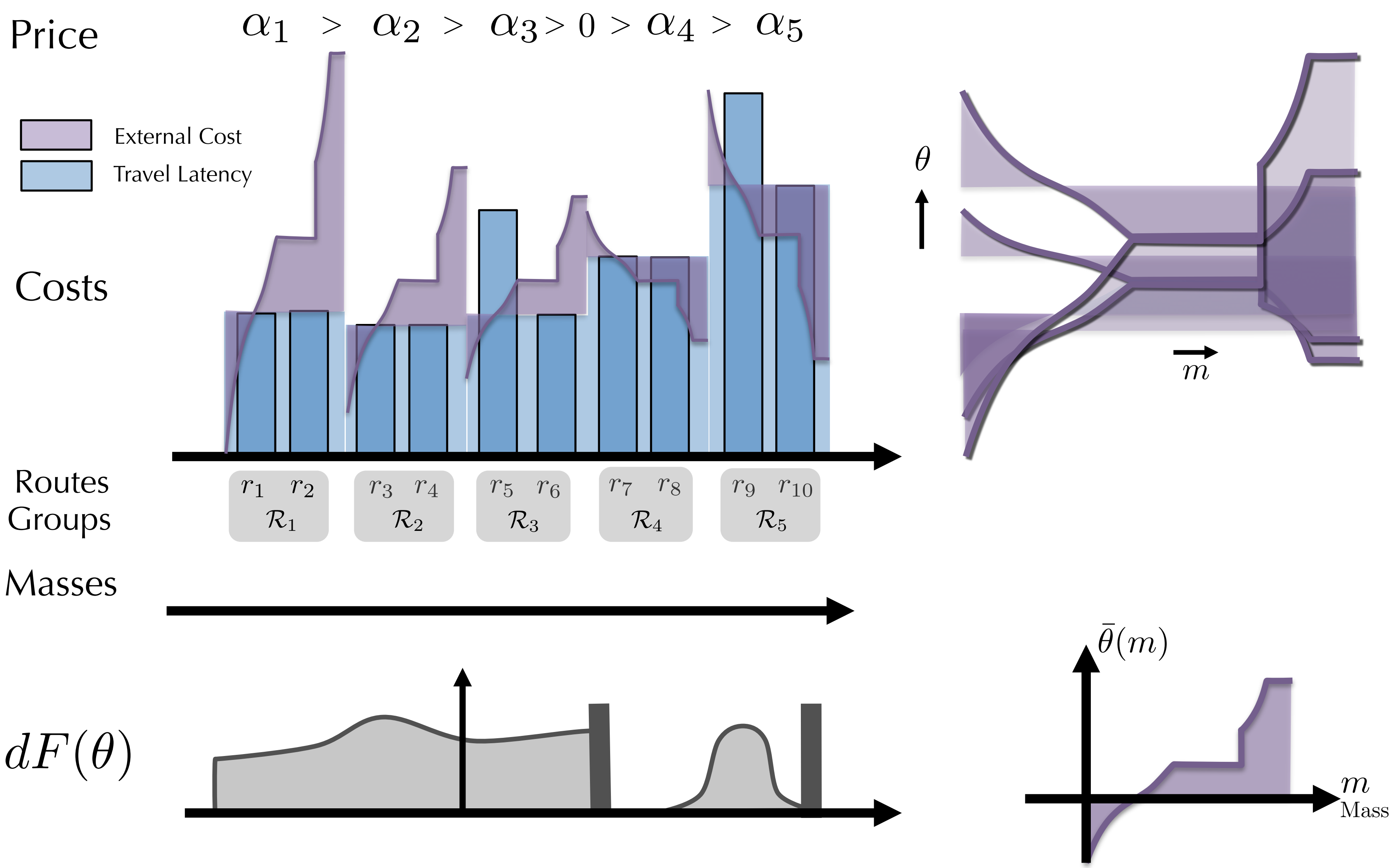




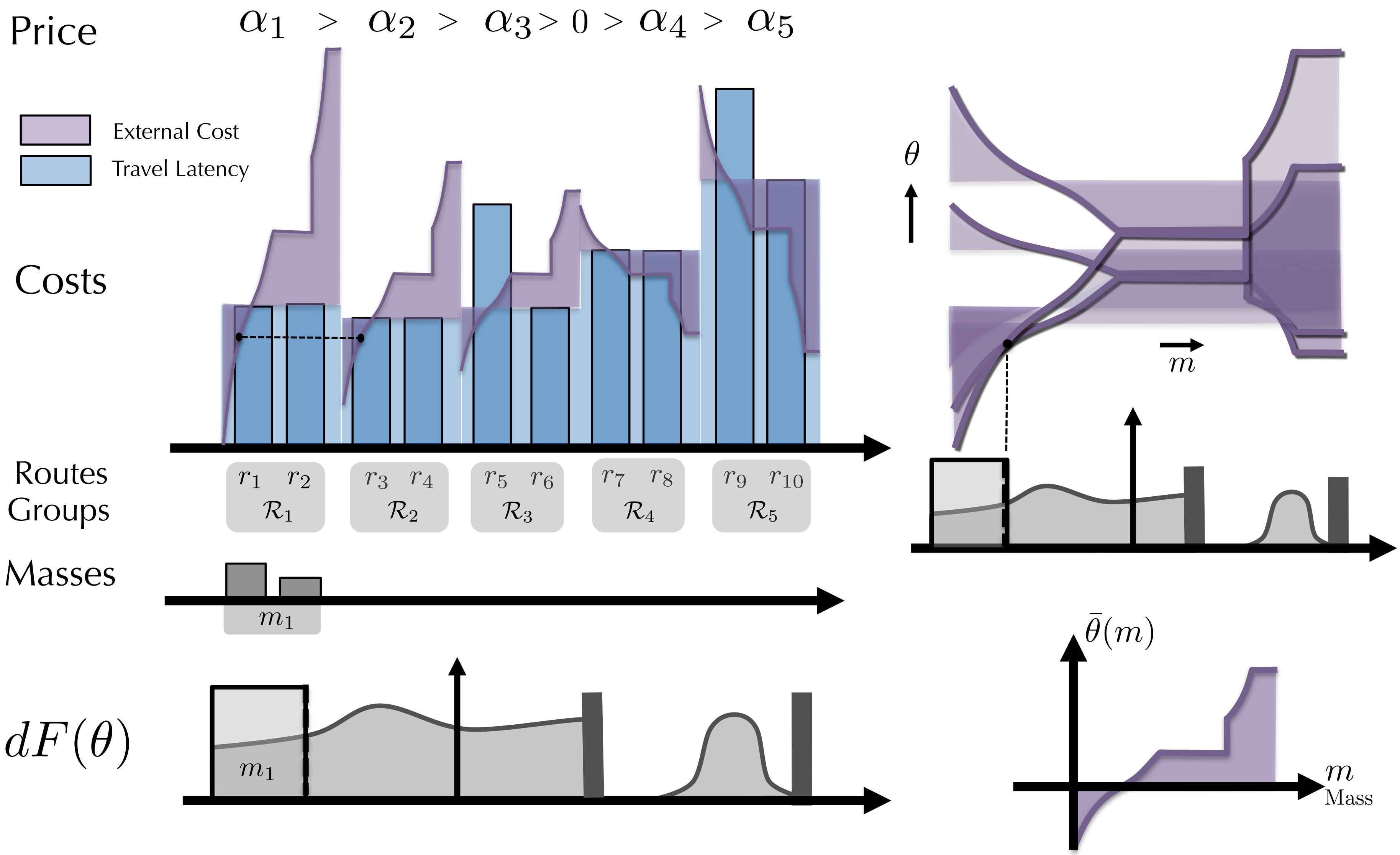
Intuition: θ general



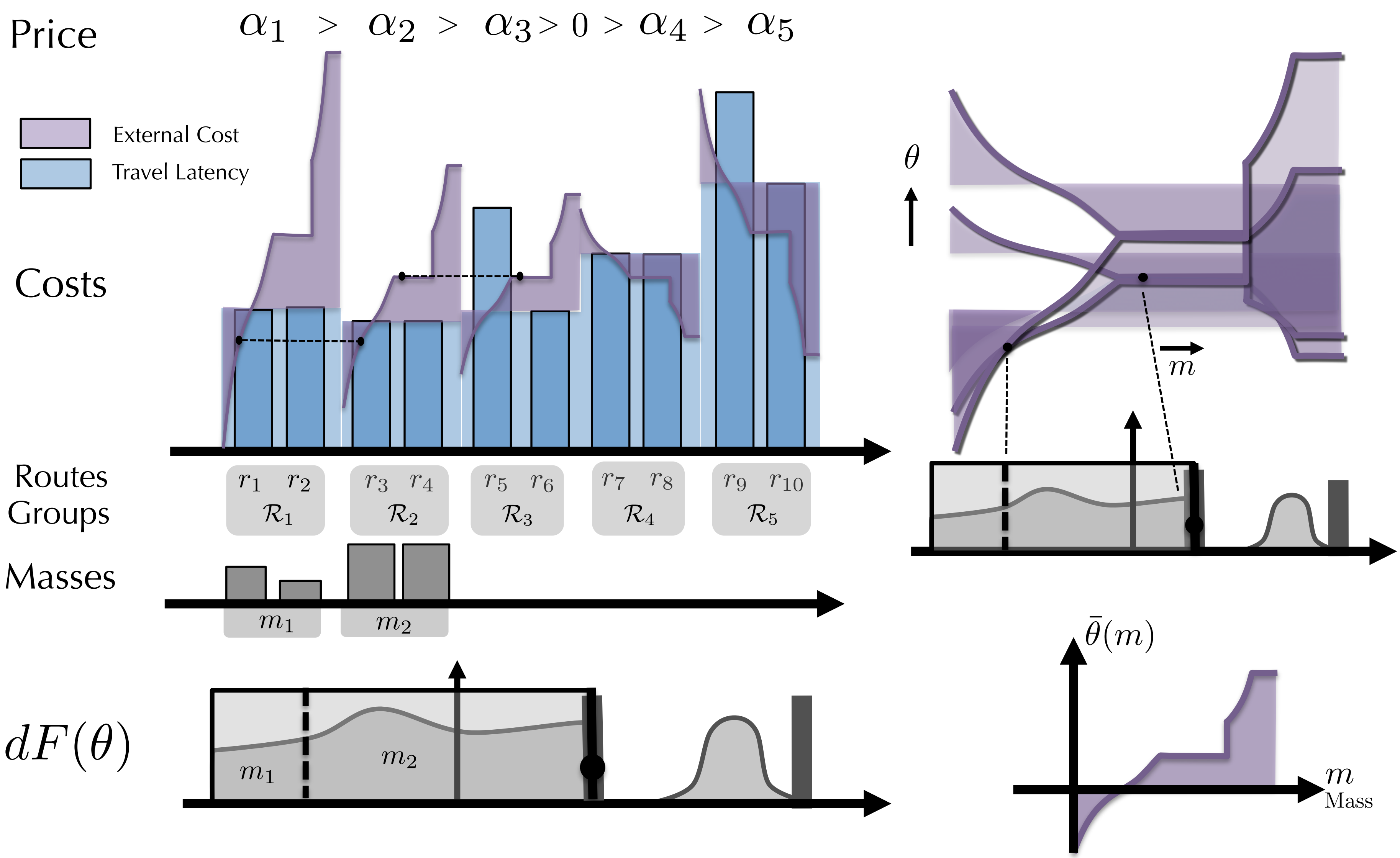
Intuition: θ general



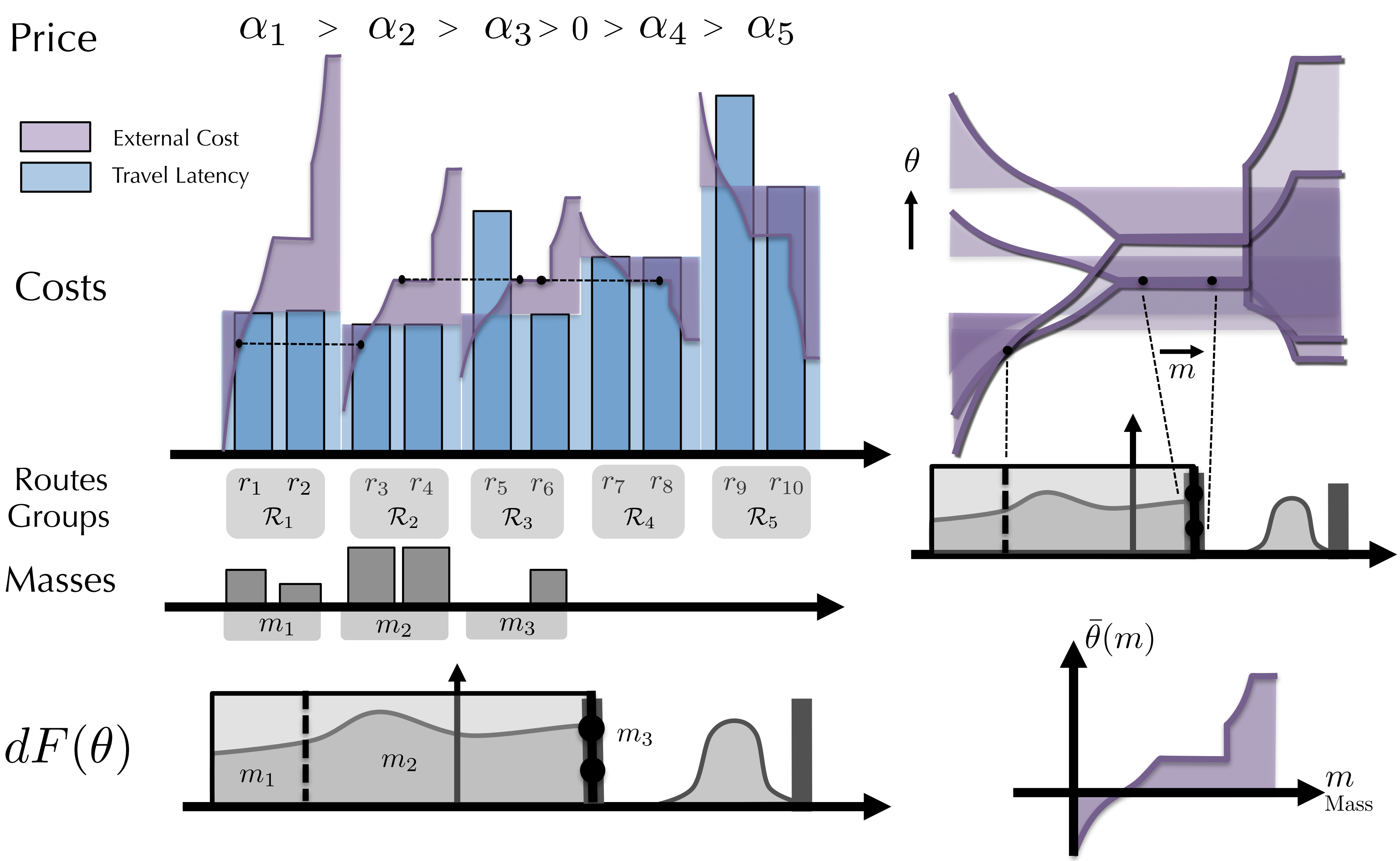
Intuition: θ general



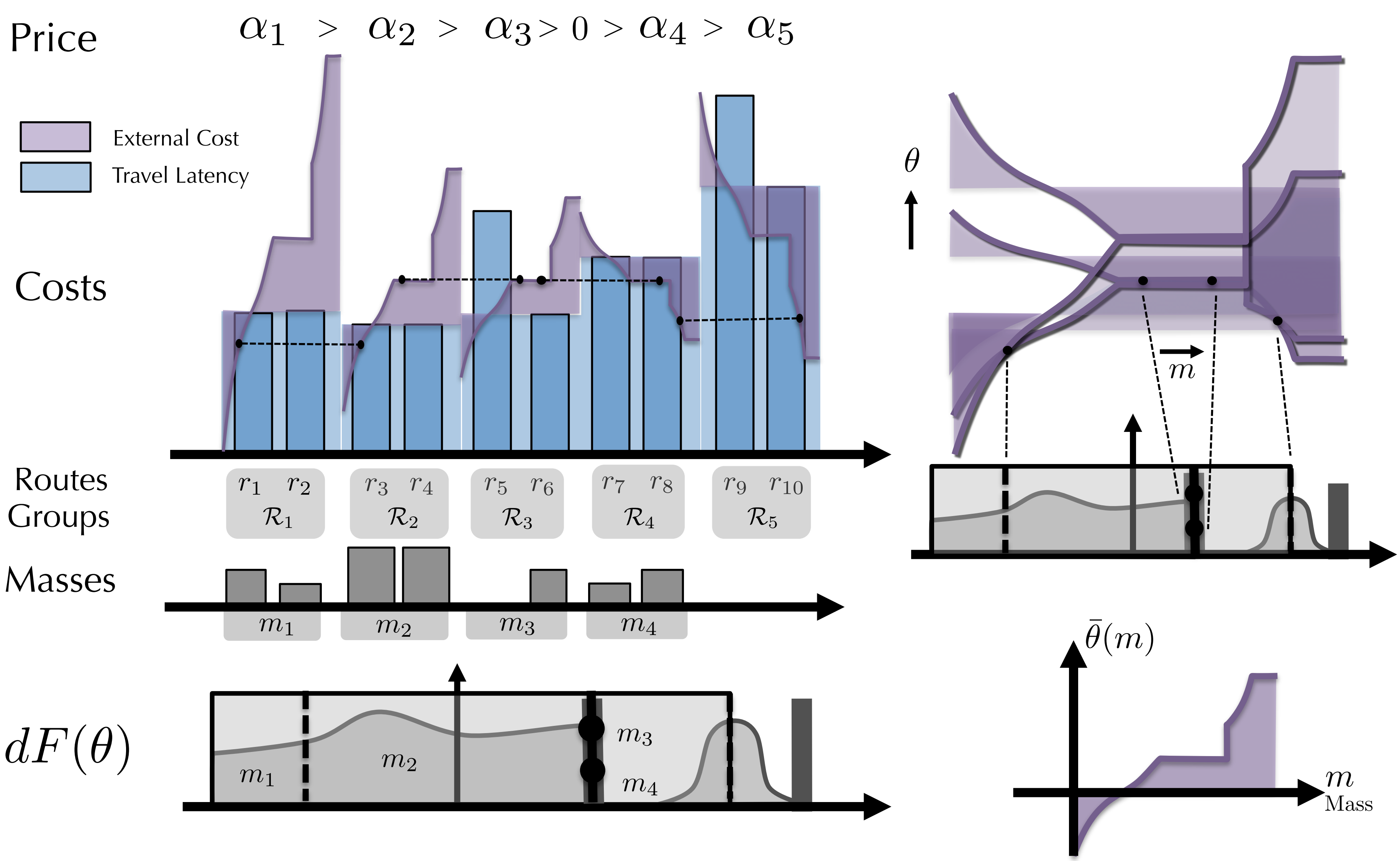
Intuition: θ general



Intuition: θ general



Intuition: θ general



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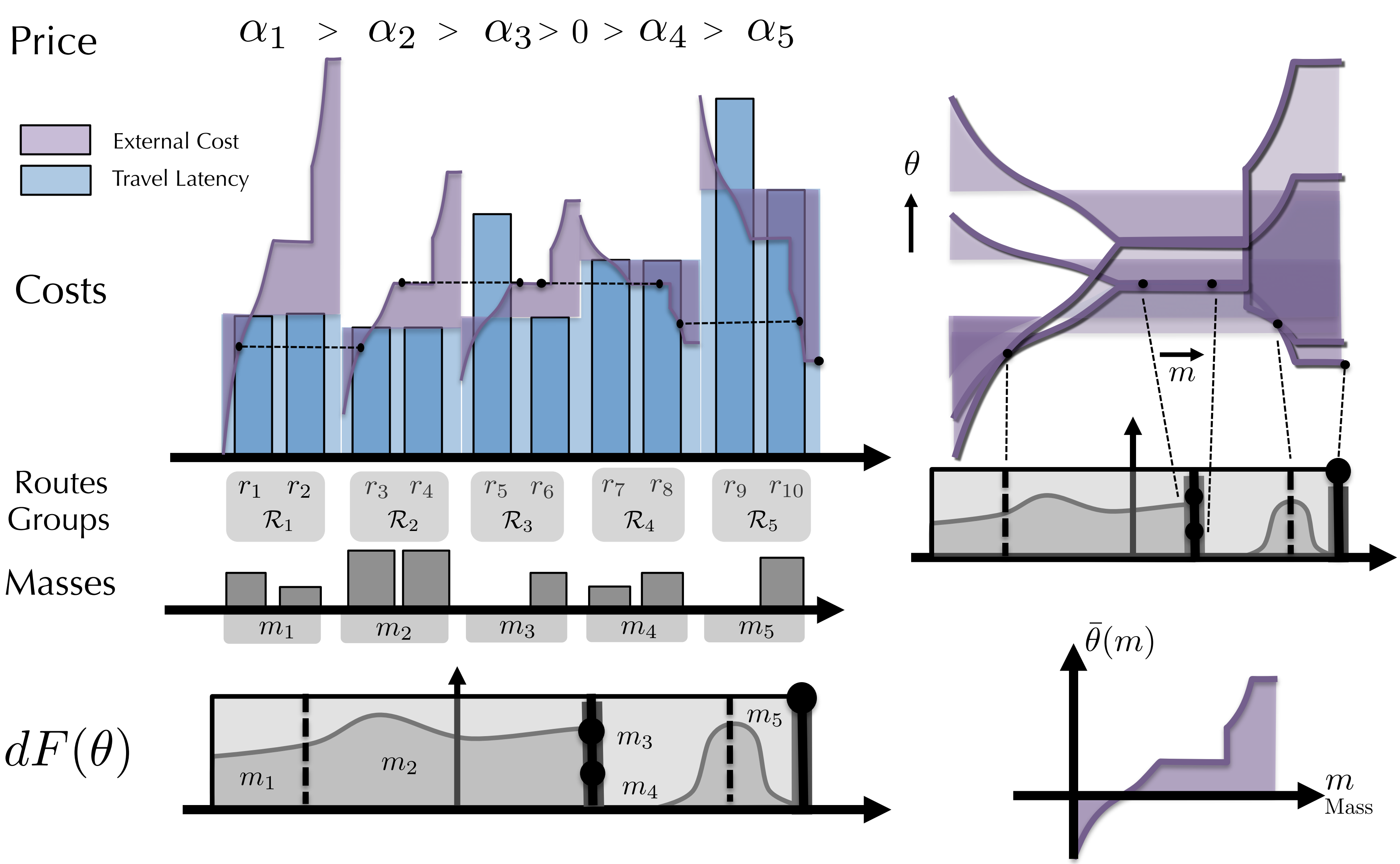

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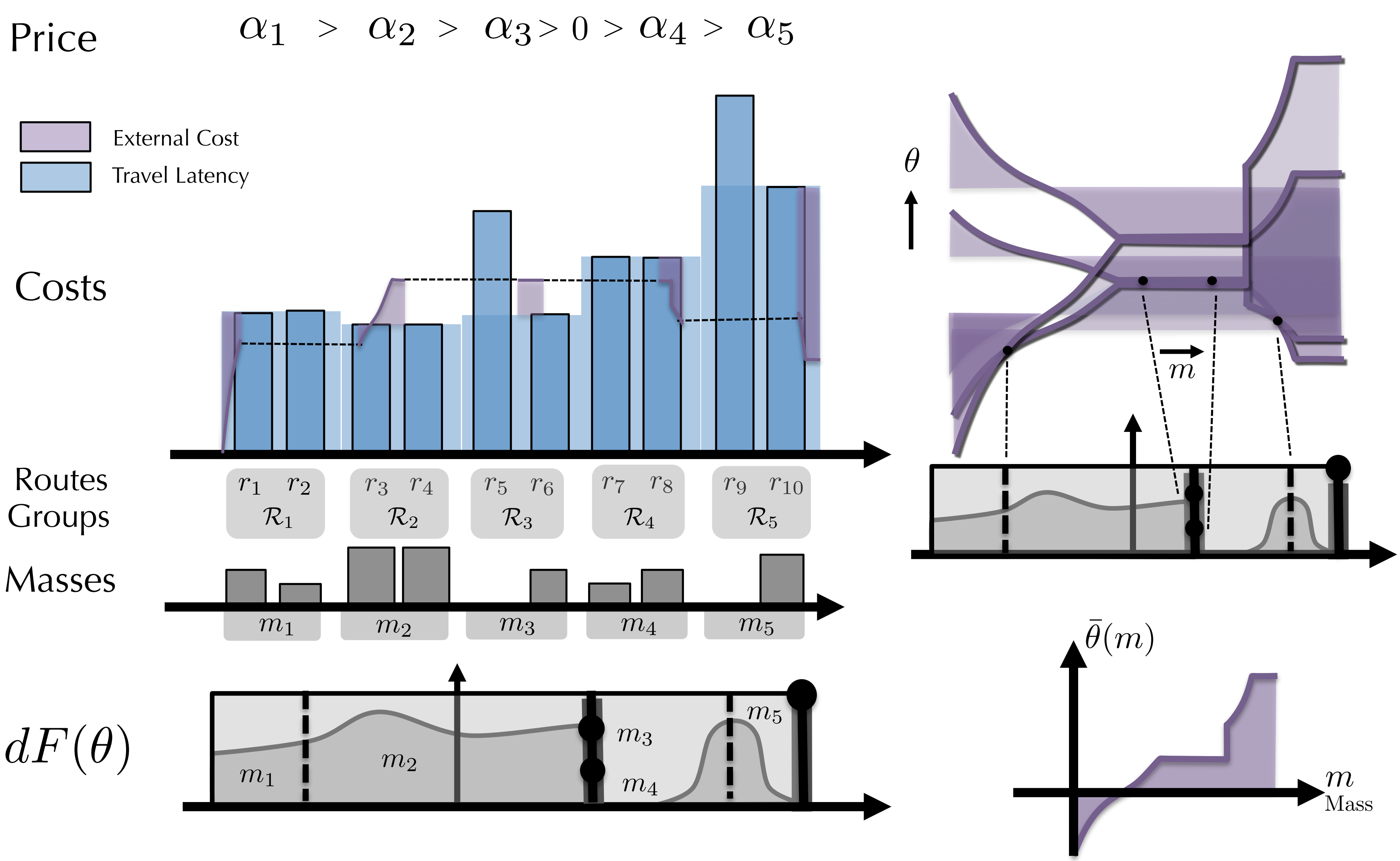

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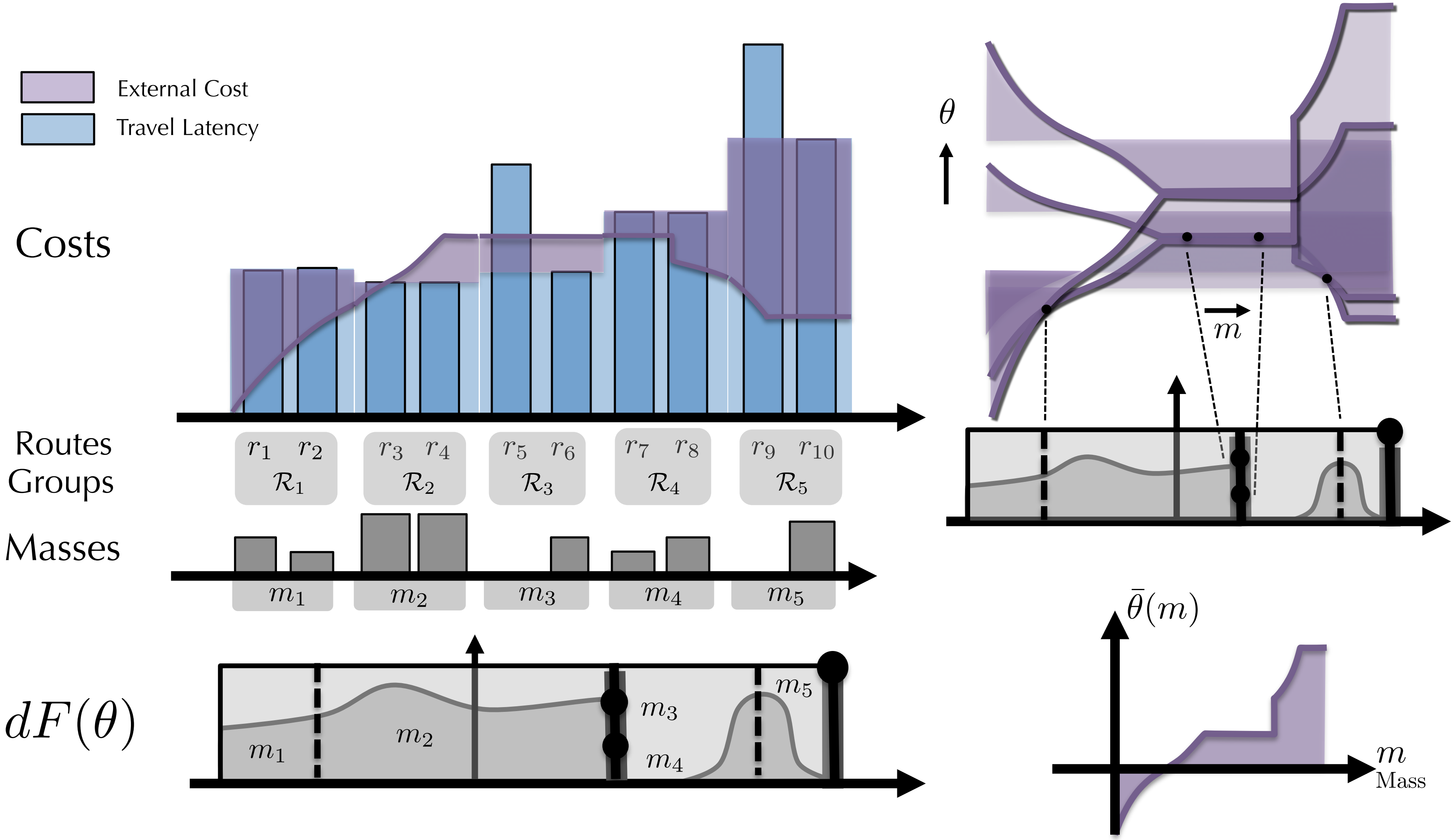
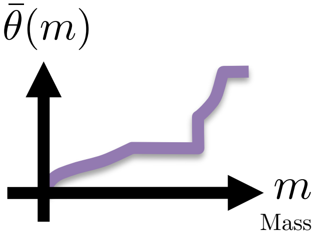


Applications

Classic routing game

Variable demand case

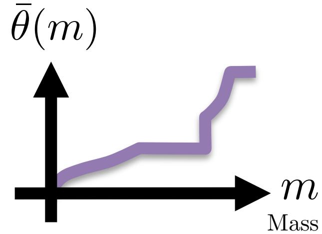


$=$ Inverse Demand Function

Applications

Classic routing game

Variable demand case

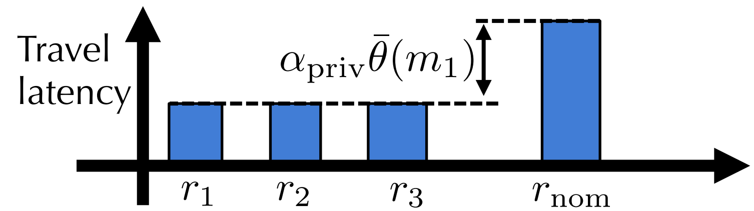


$=$ Inverse Demand Function

Privacy

$\mathcal{R}_1 = \mathcal{R}$ $\quad \mathcal{R}_2 = \{r_{\text{nom}}\}$

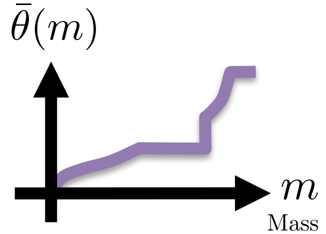
$\alpha_1 = \alpha_{\text{priv}}$ $\quad \alpha_2 = 0$



Applications

Classic routing game

Variable demand case



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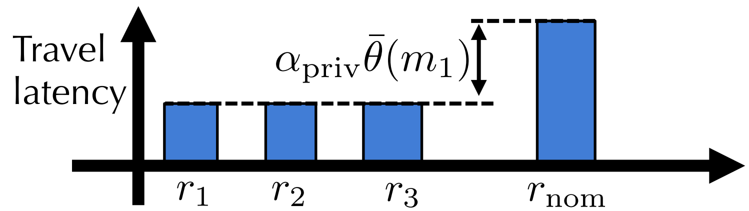
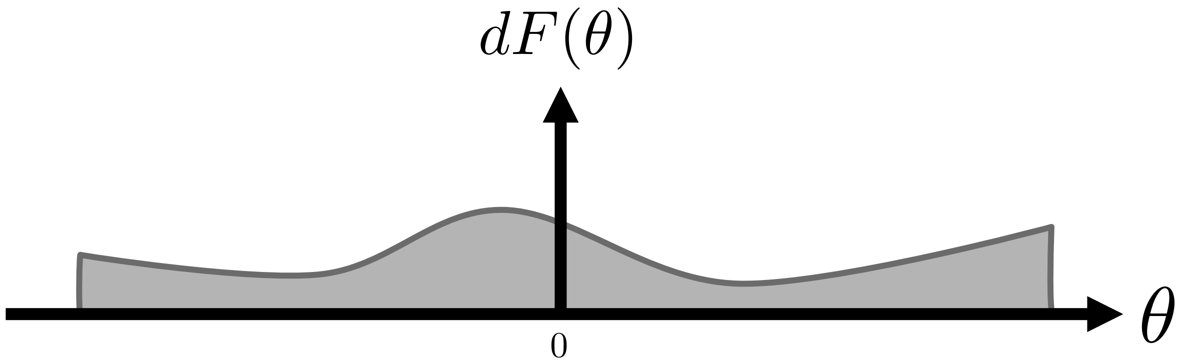


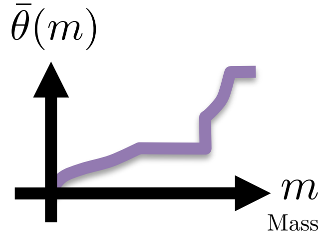
Multi-modal routing – ex. cars vs. trains



Applications

Classic routing game

Variable demand case



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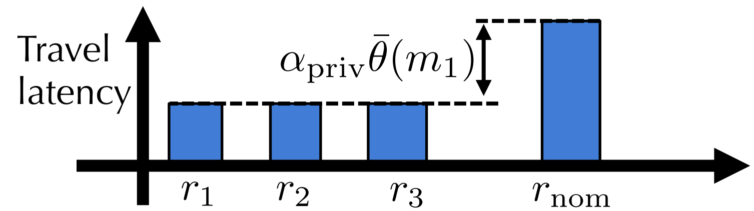

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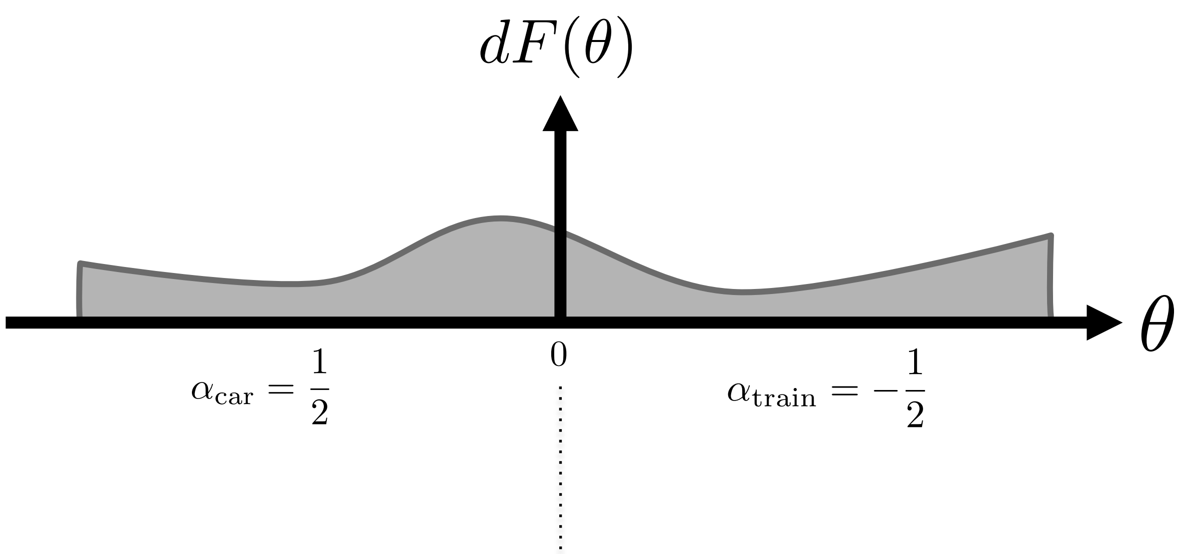
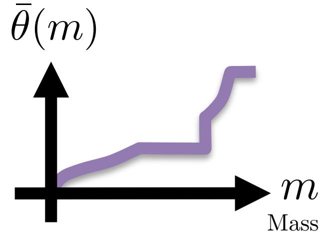


Applications

Classic routing game

Variable demand case



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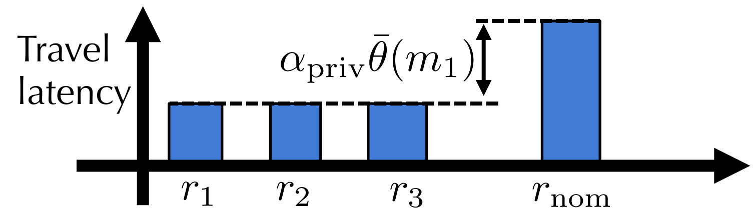

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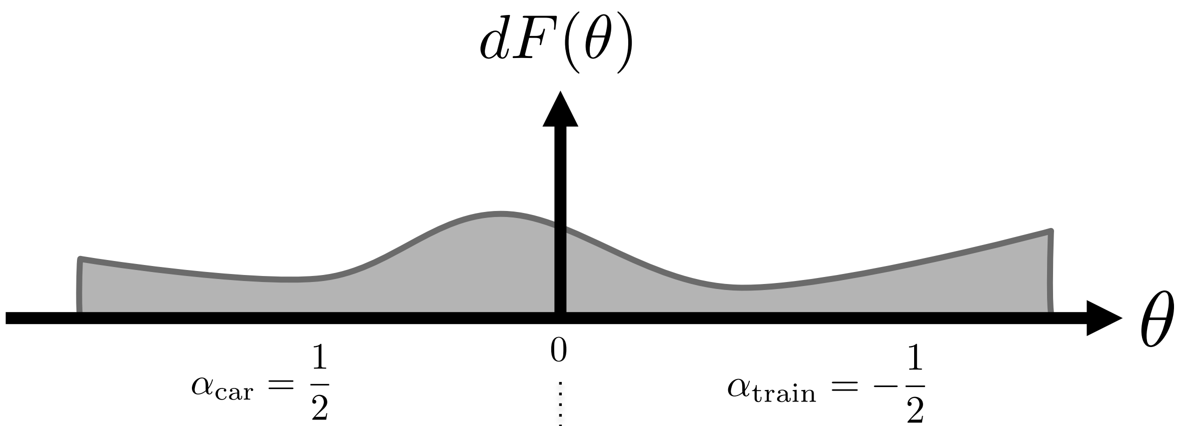


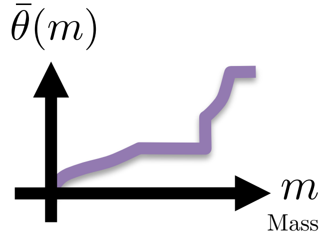
Population that **prefers cars**

Population that **prefers trains**

Applications

Classic routing game

Variable demand case



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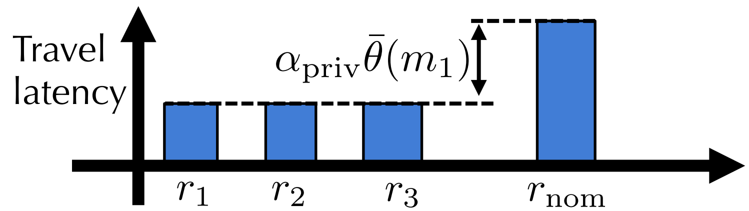
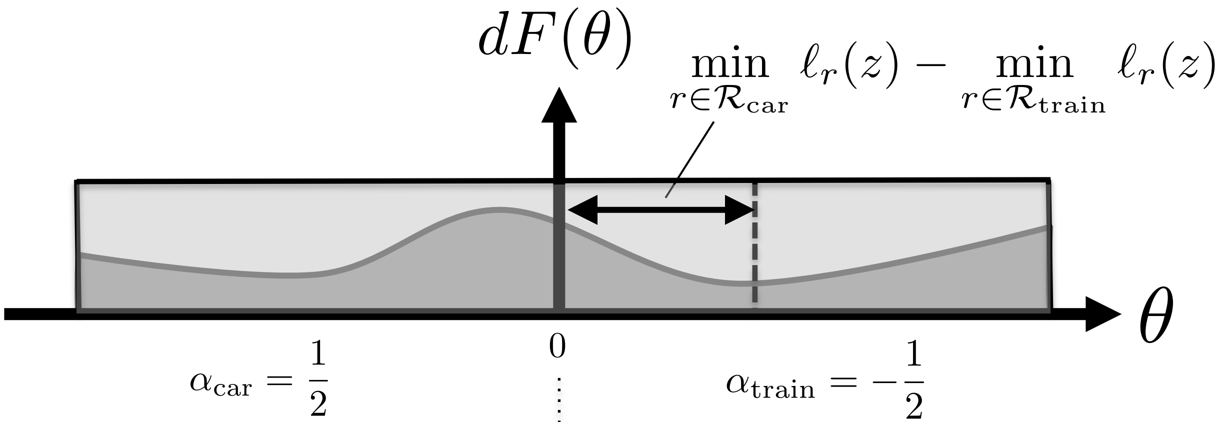


Multi-modal routing – ex. cars vs. trains



Population that **prefers cars**

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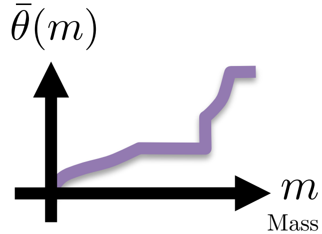
Applications

Classic routing game

Variable demand case



= Inverse Demand Function

Privacy

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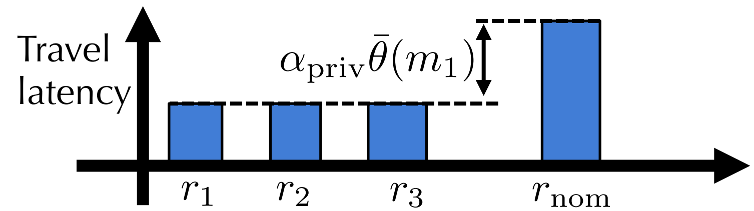
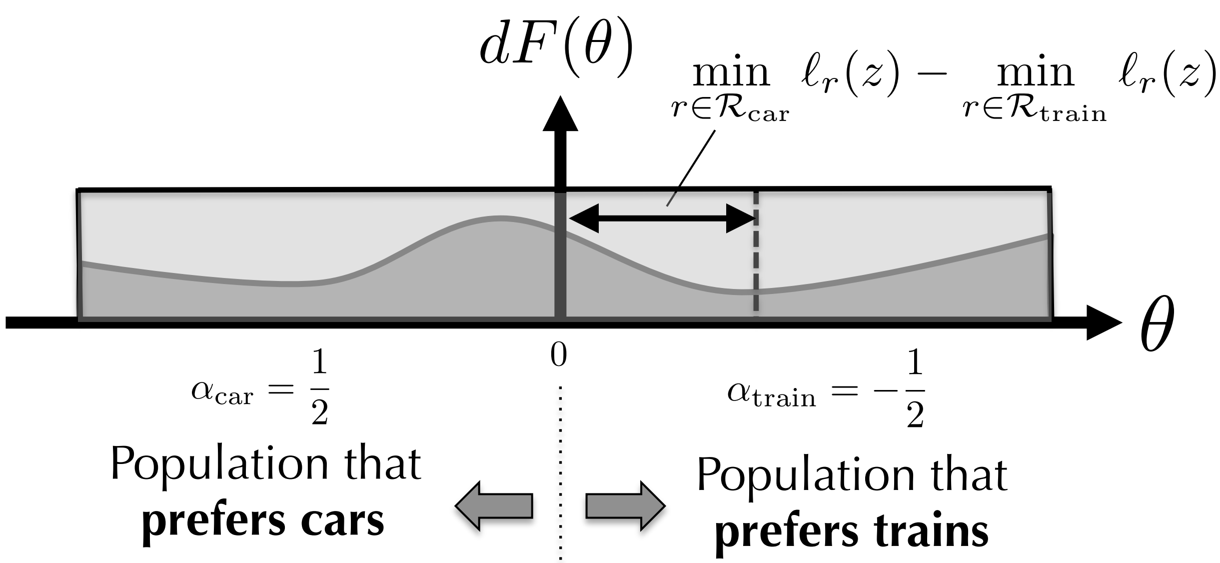


Multi-modal routing – ex. cars vs. trains

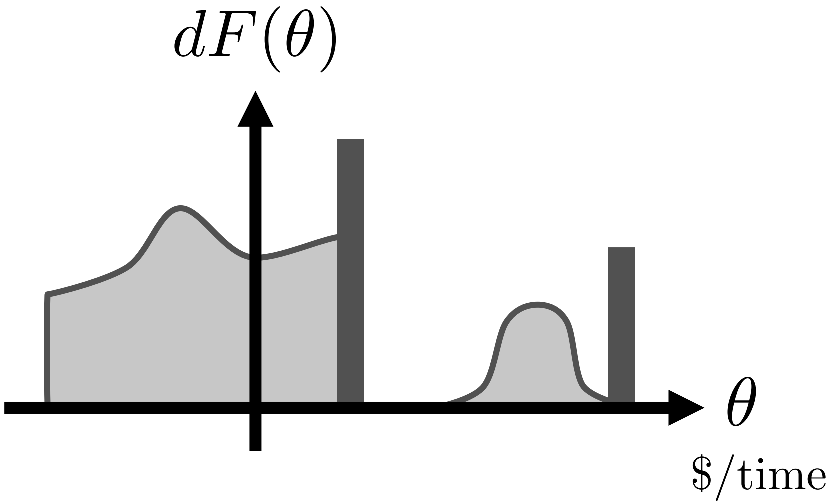


θ for each population member is how much faster cars have to be than trains before they will switch to driving.

Conclusion – Future Work

Future work

- Multi-dimensional θ



Thanks!