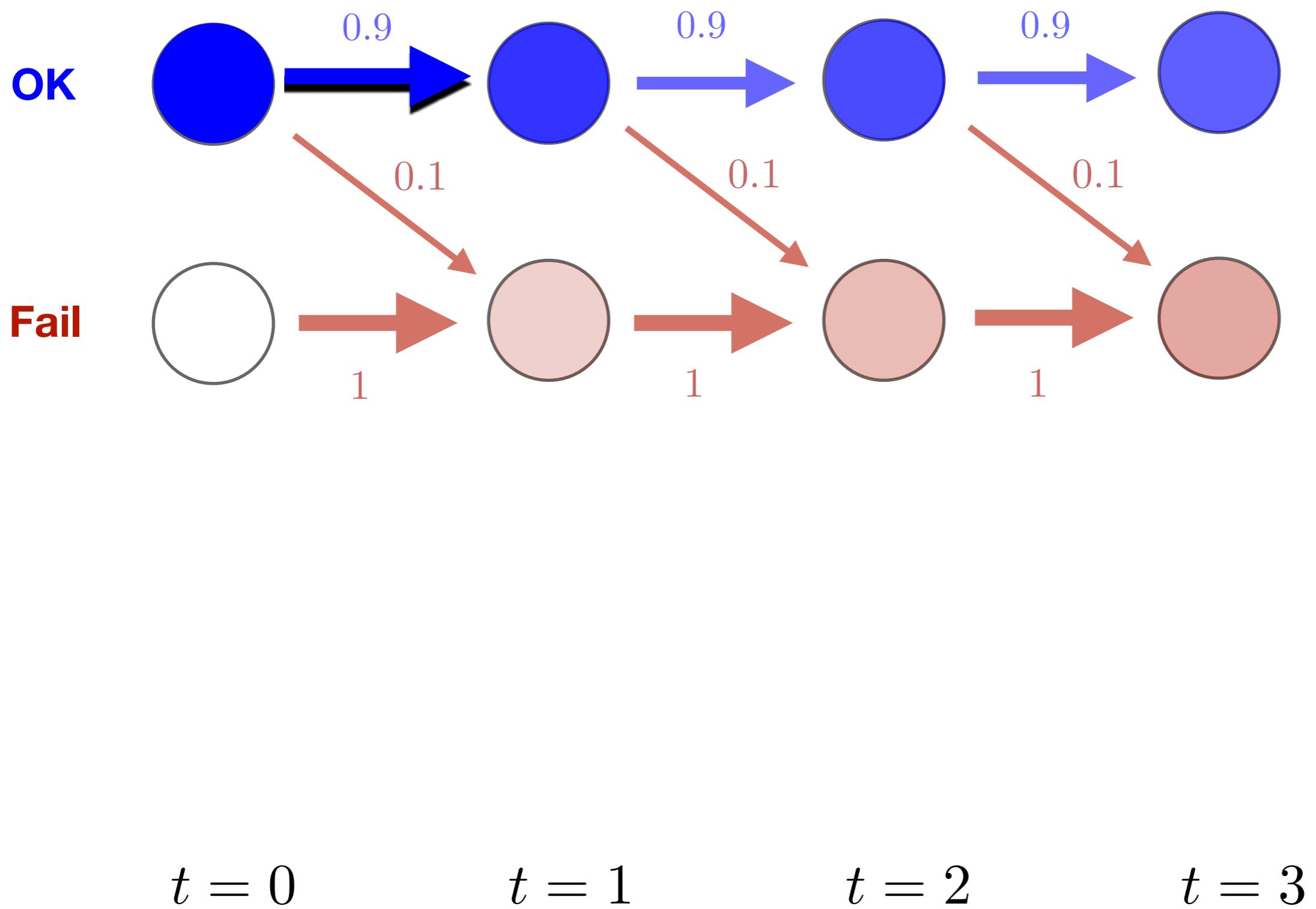
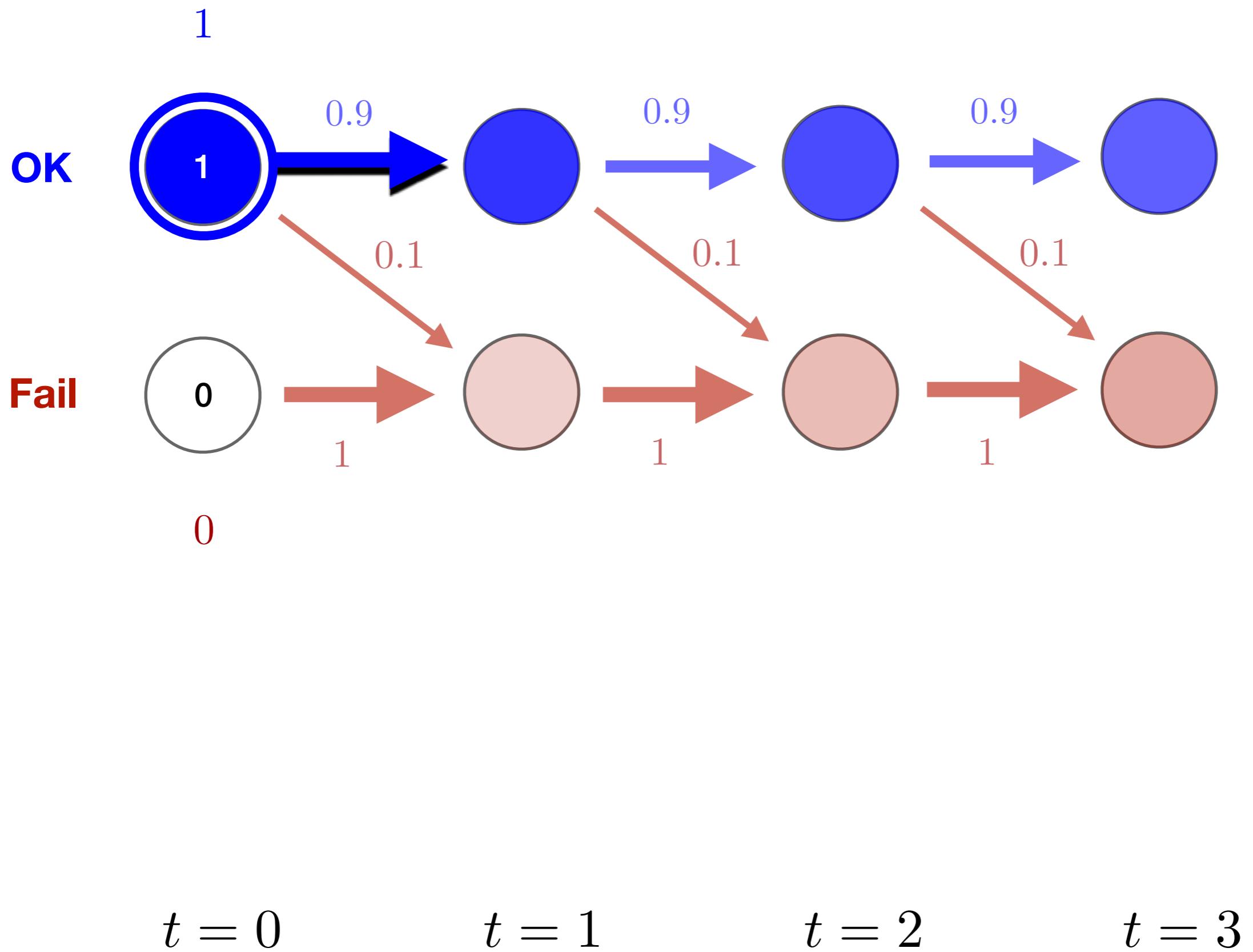


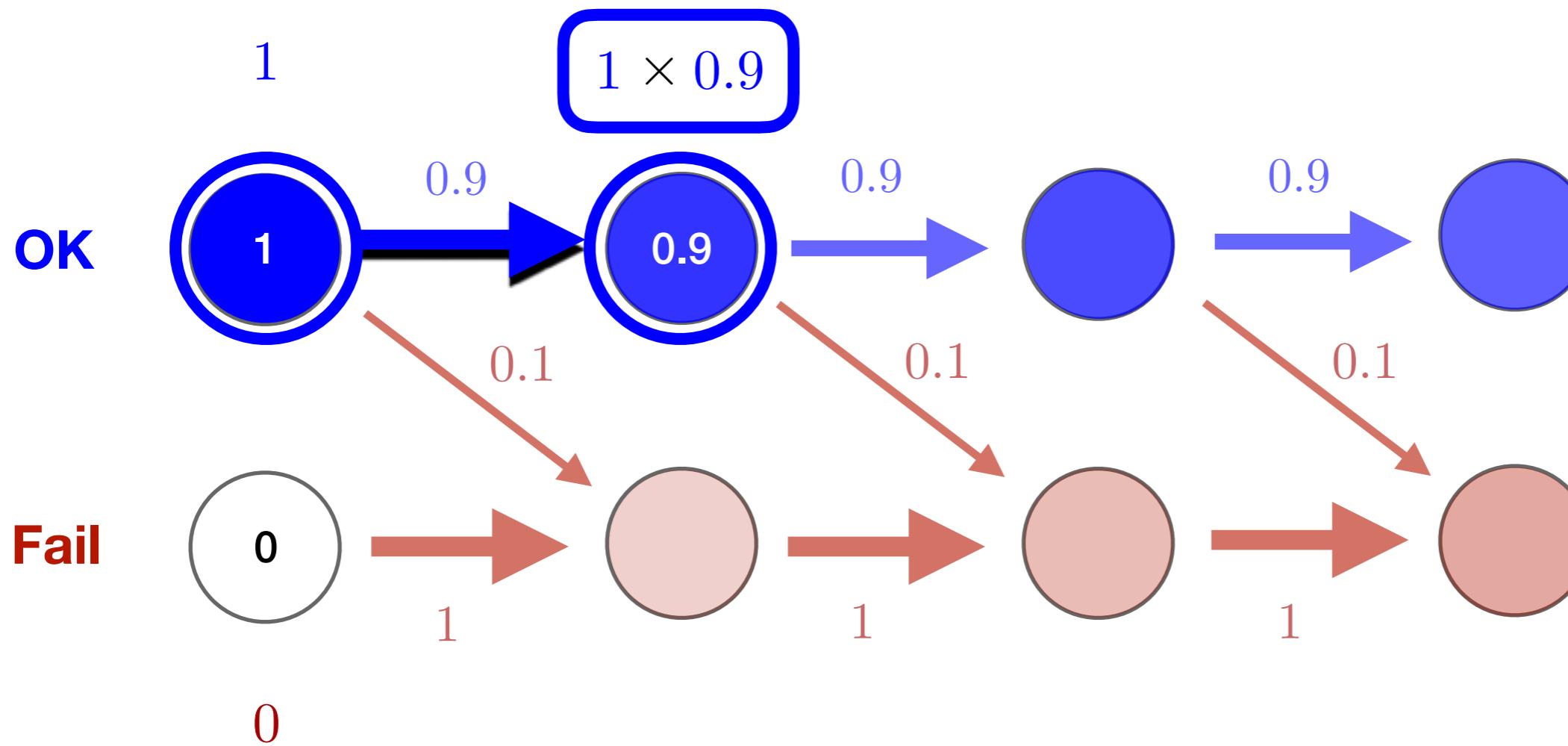
Markov Chains

Stochastic Processes

Major sources:





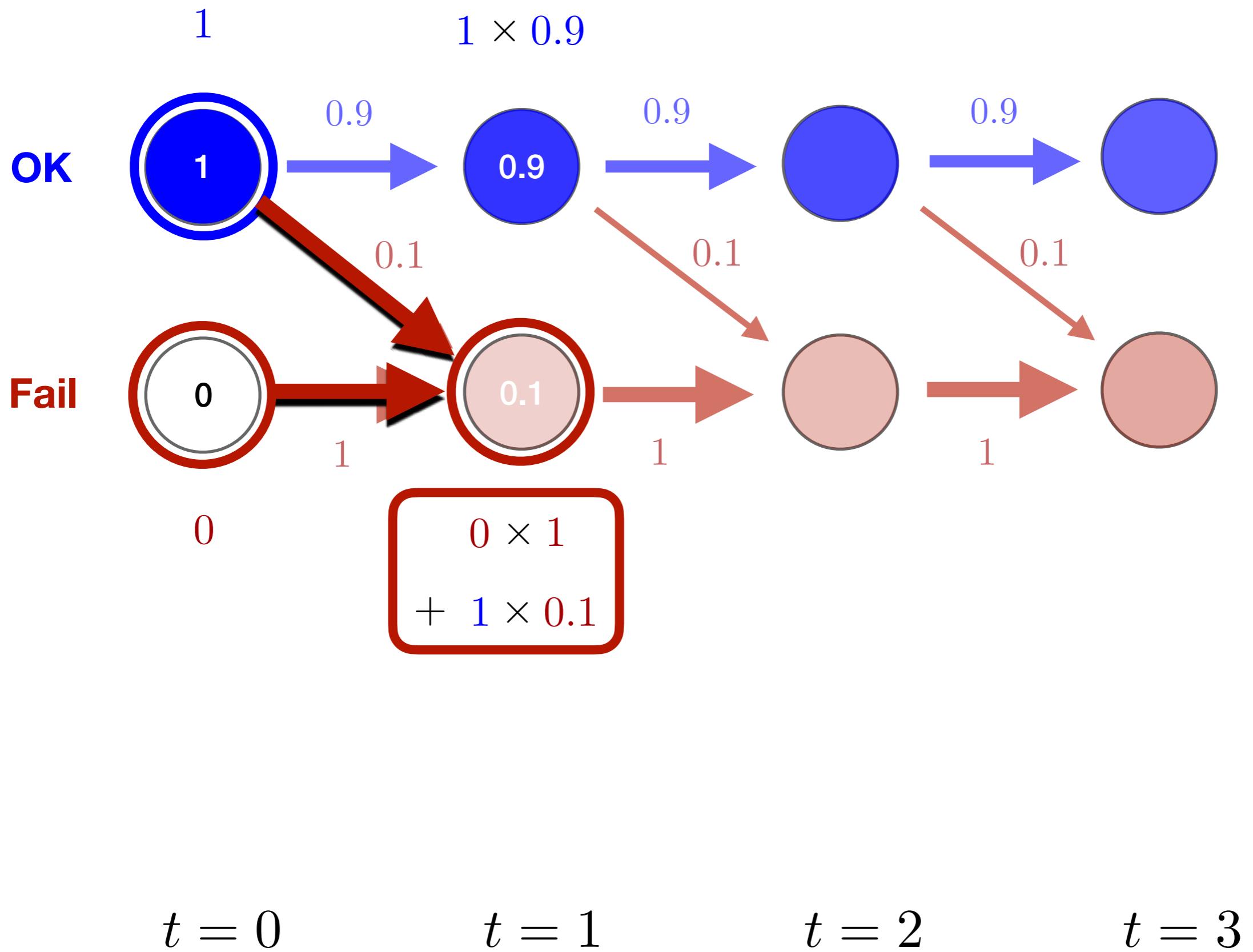


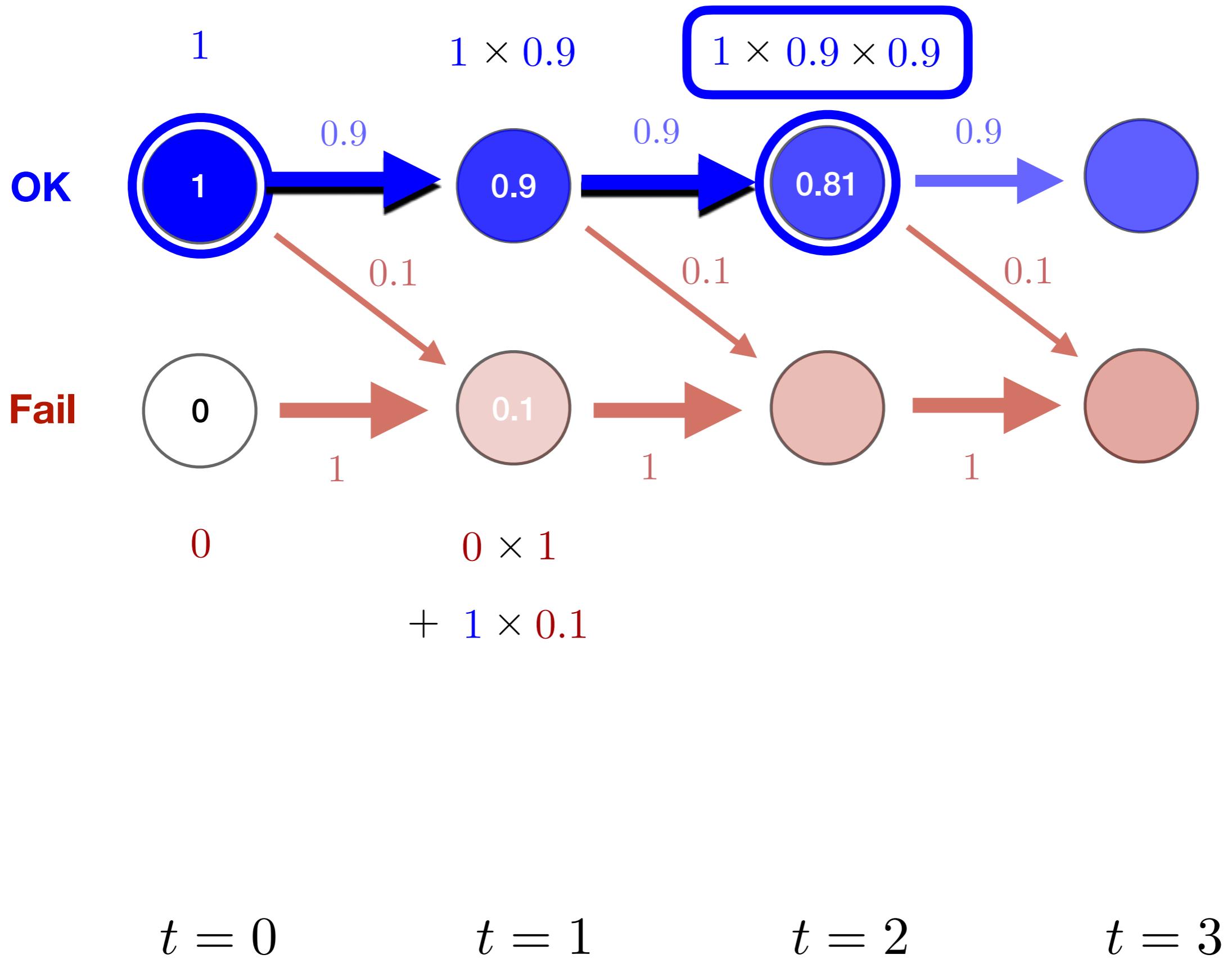
$$t = 0$$

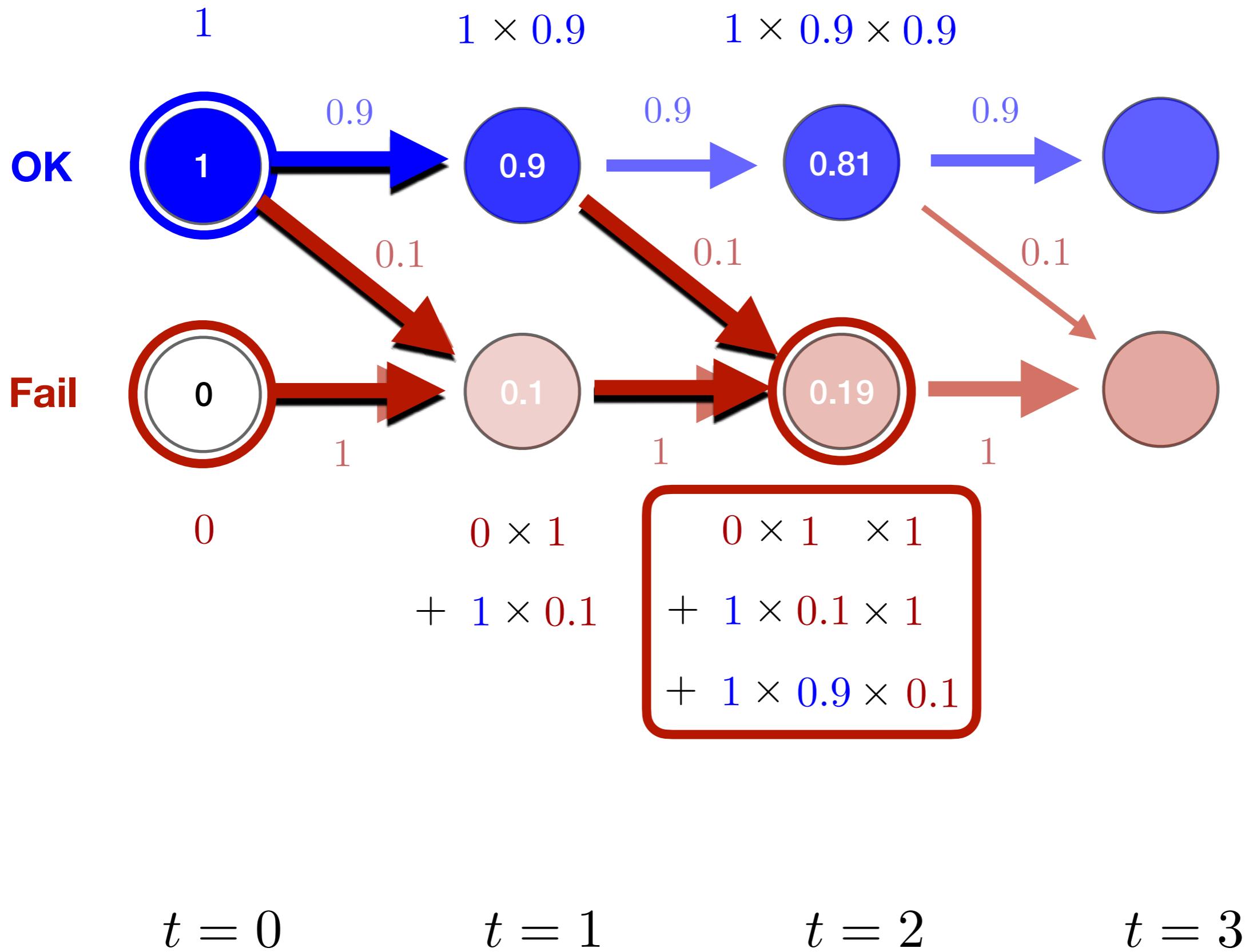
$$t = 1$$

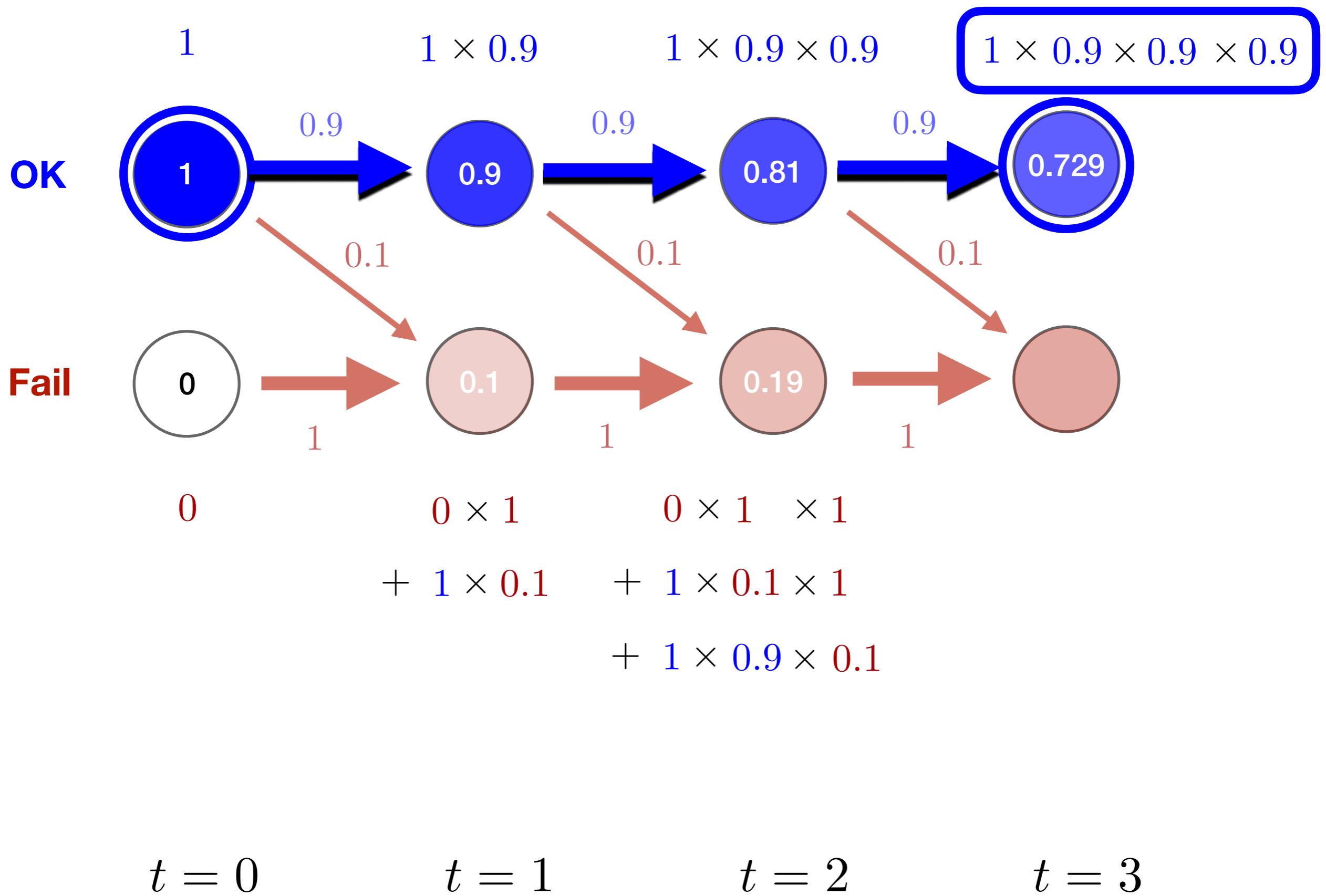
$$t = 2$$

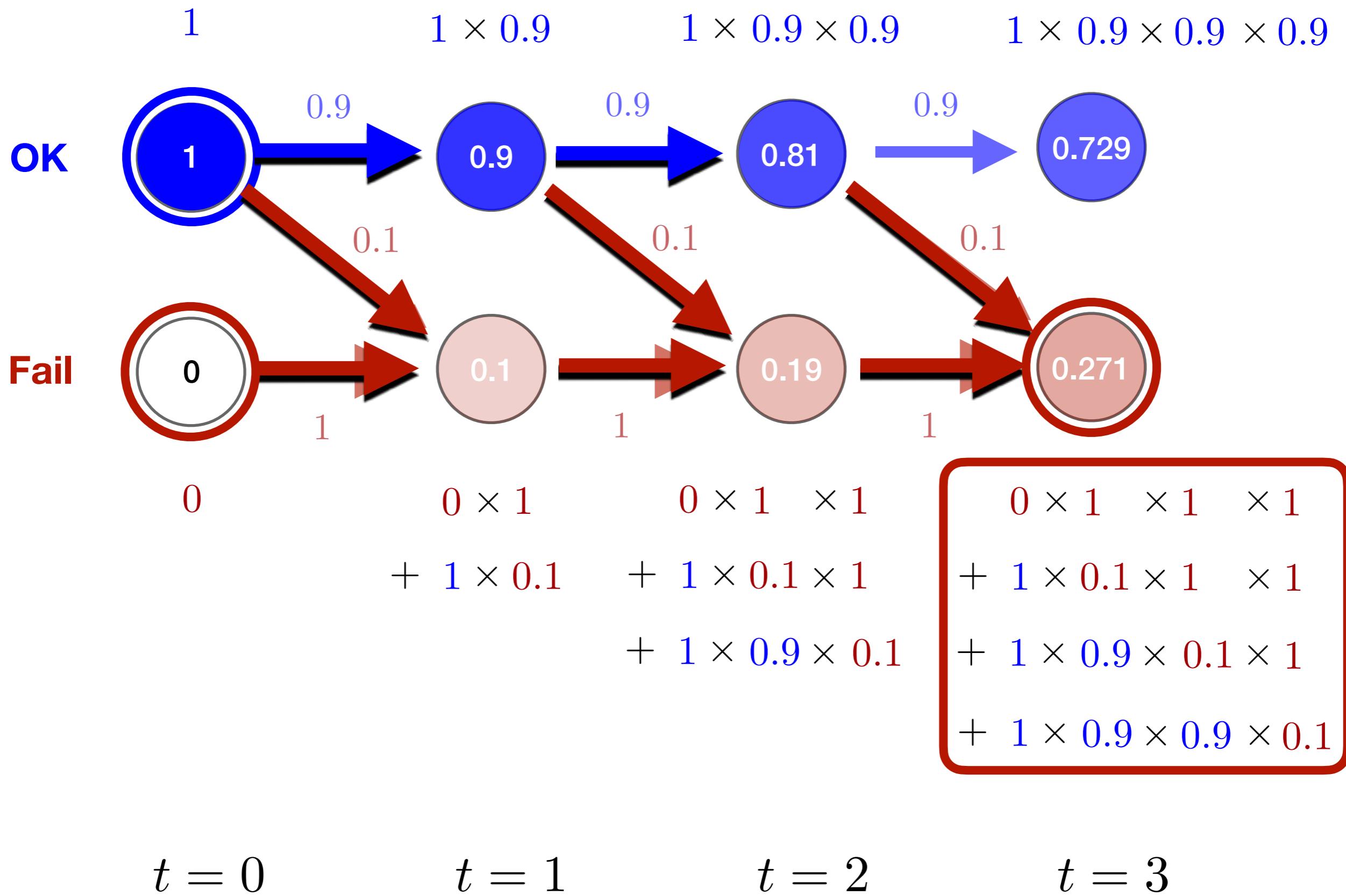
$$t = 3$$

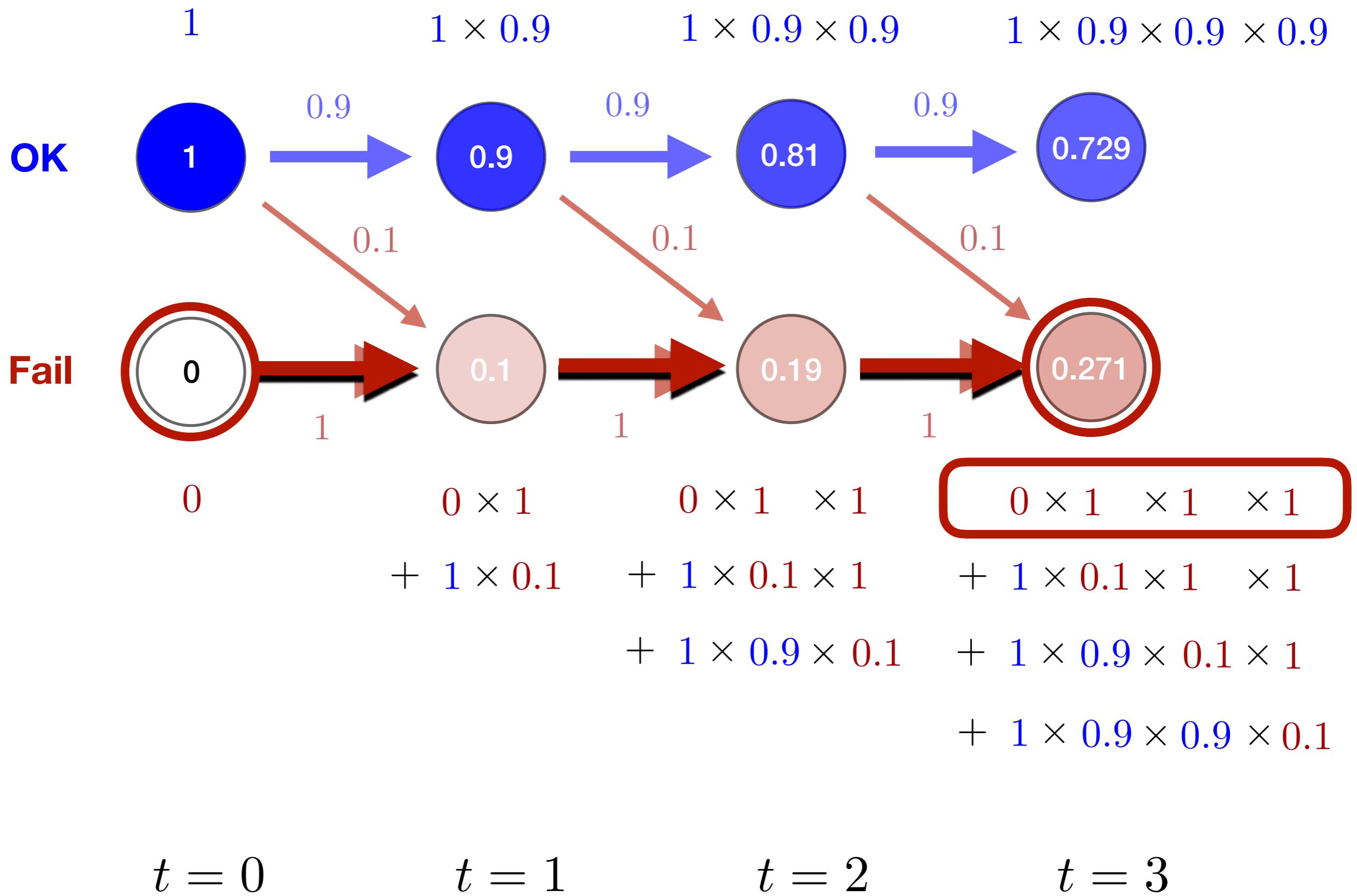


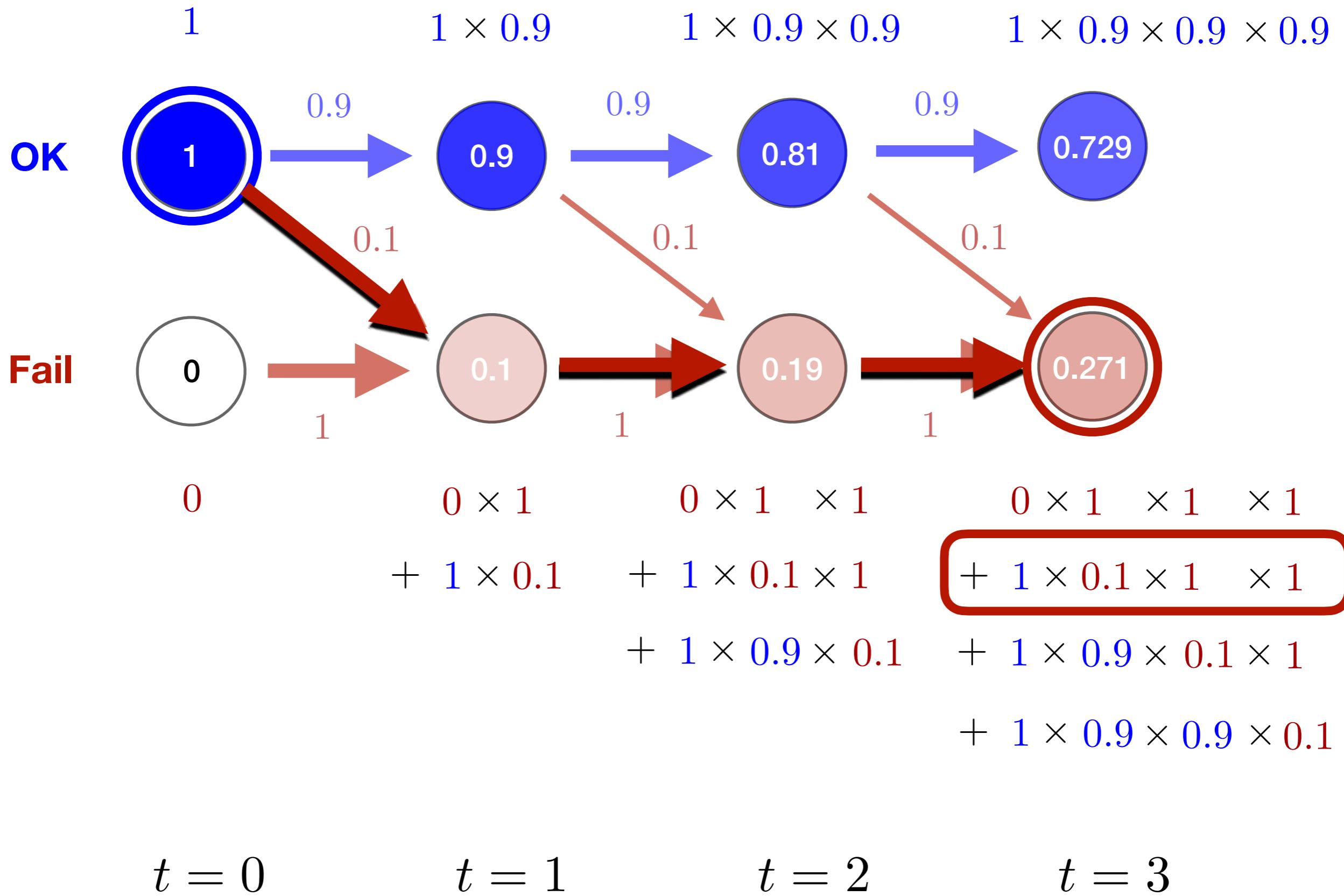


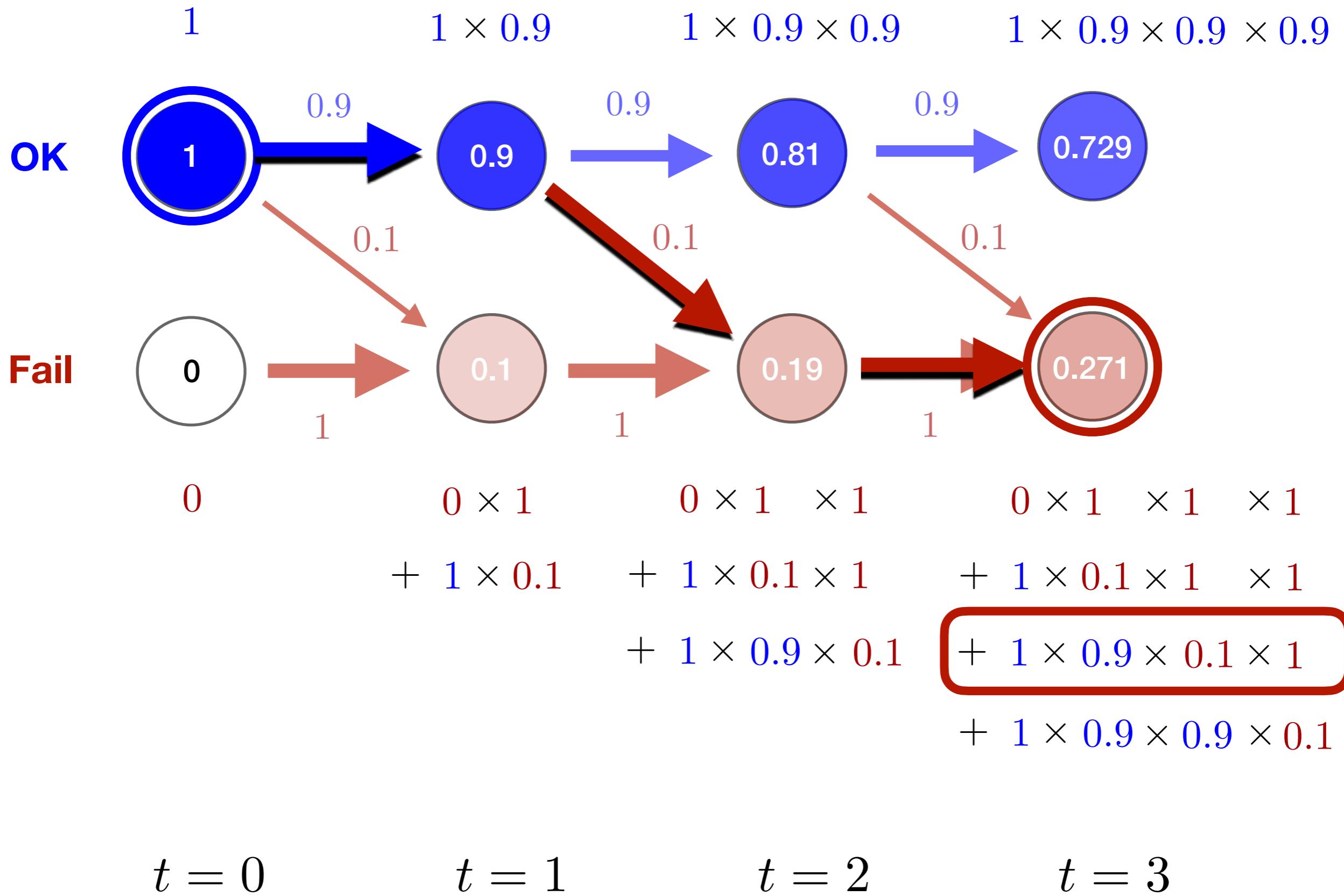


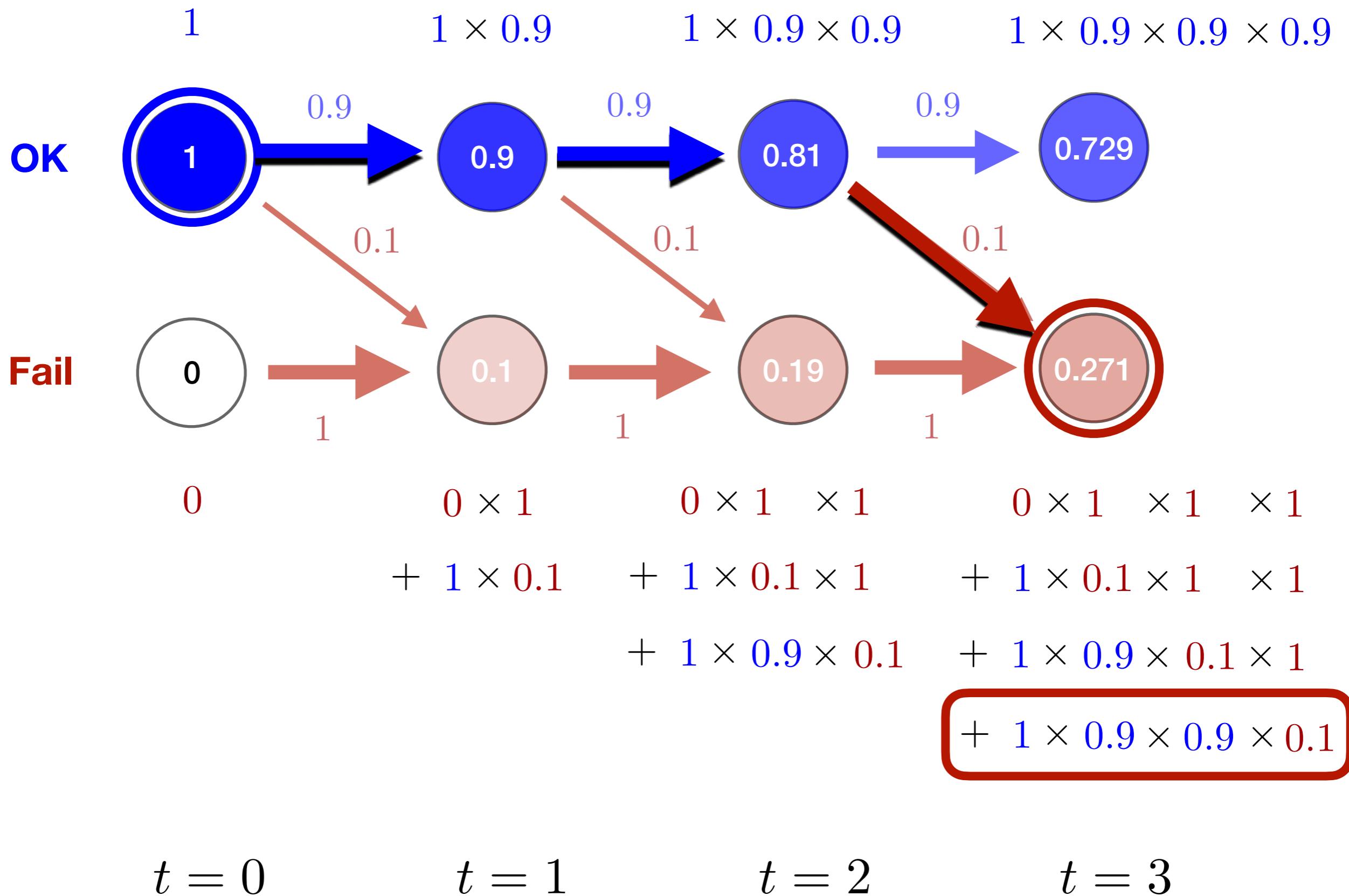


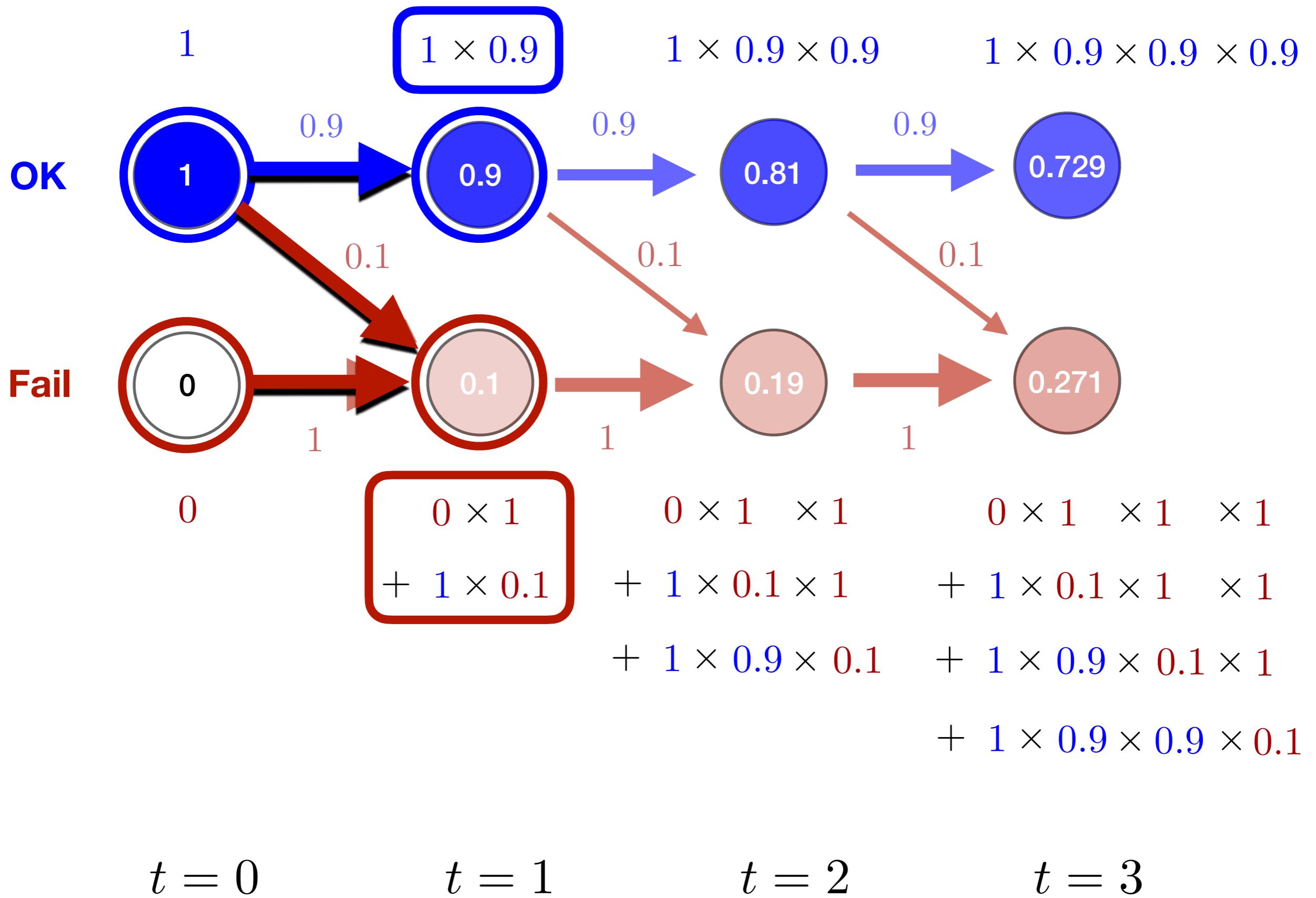


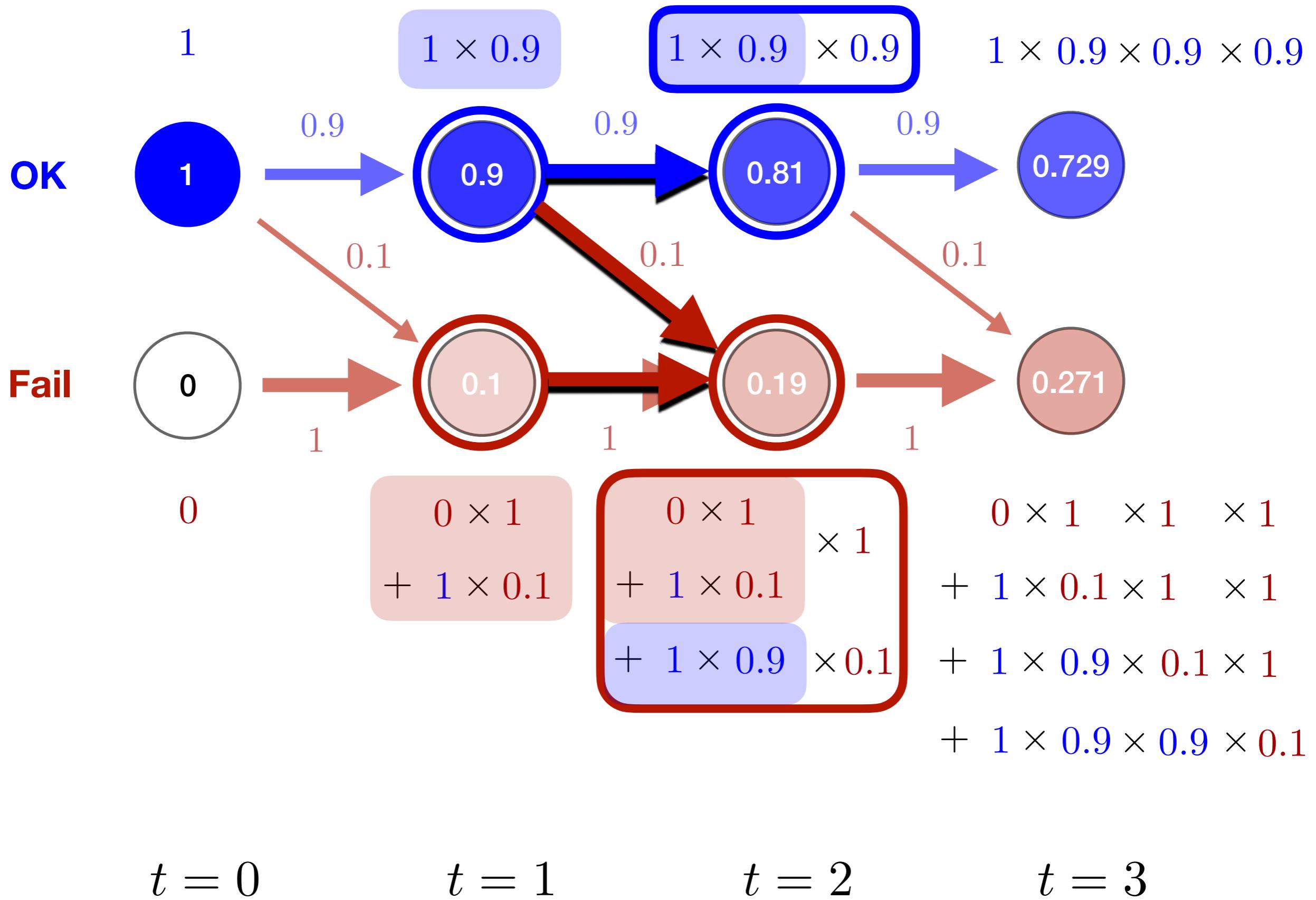


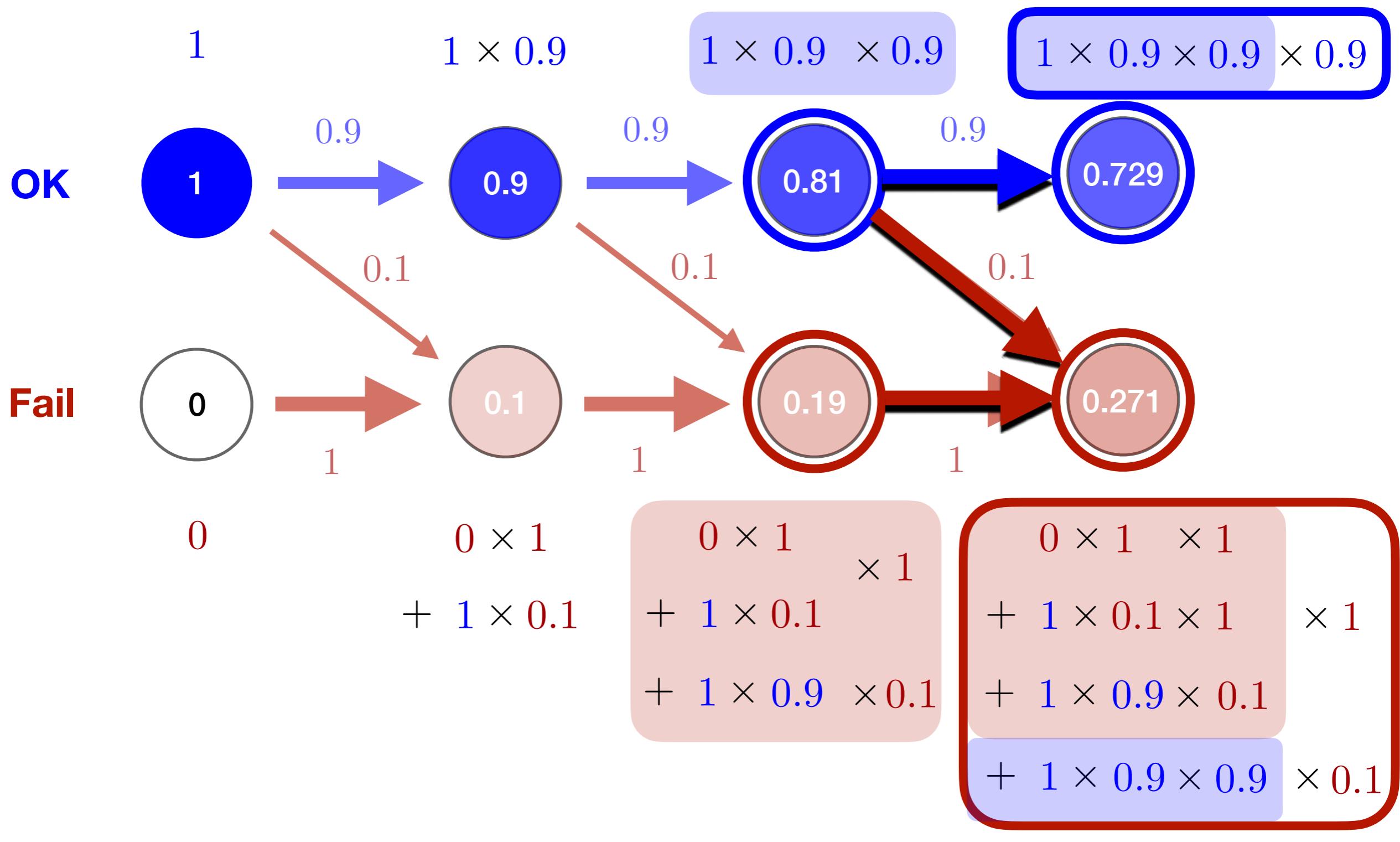


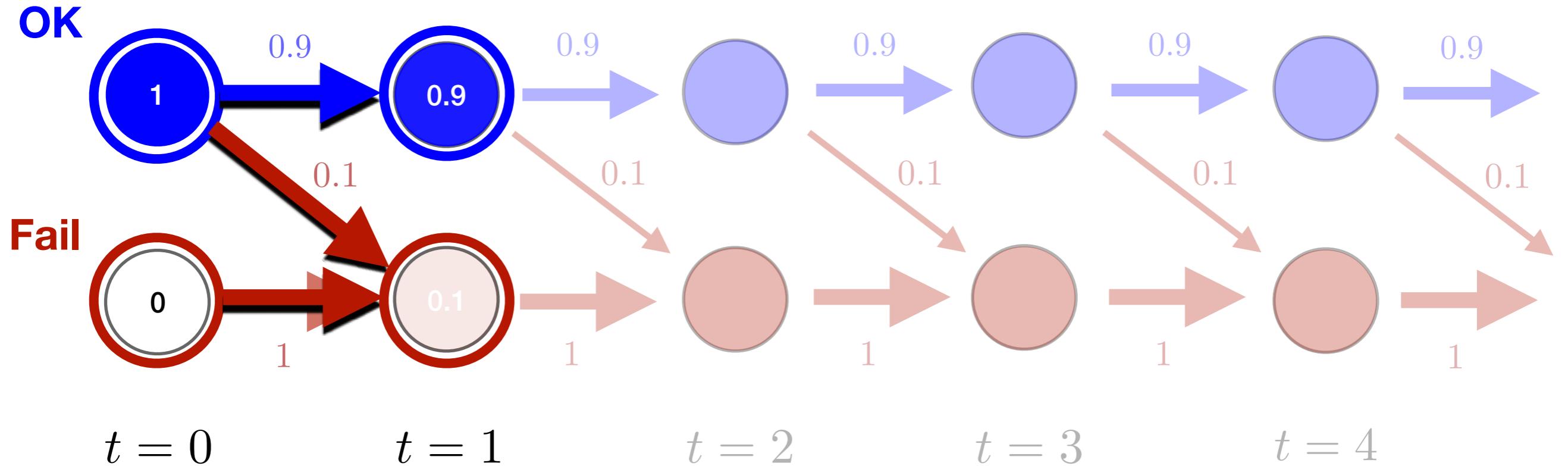




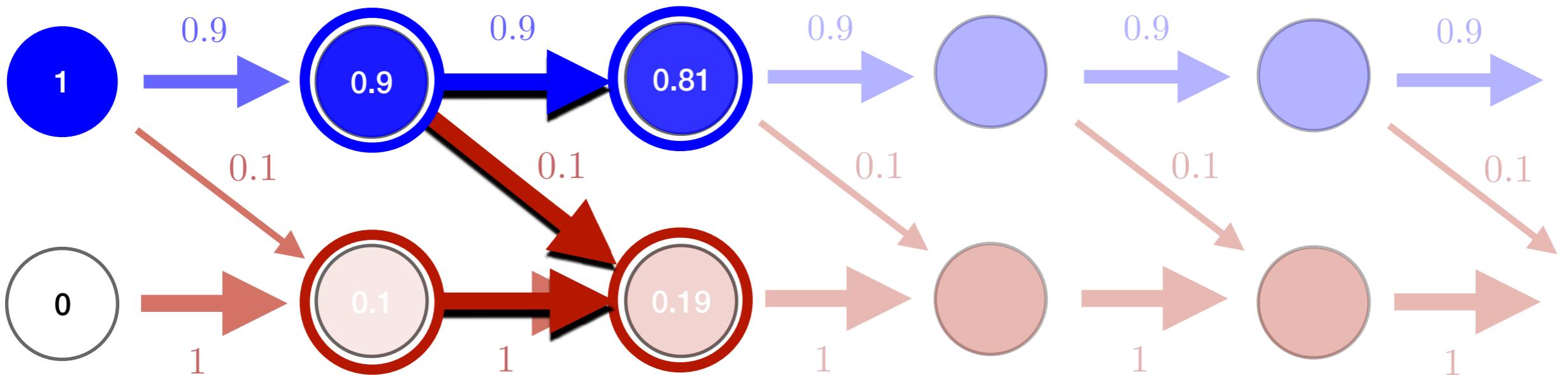








OK



$t = 0$

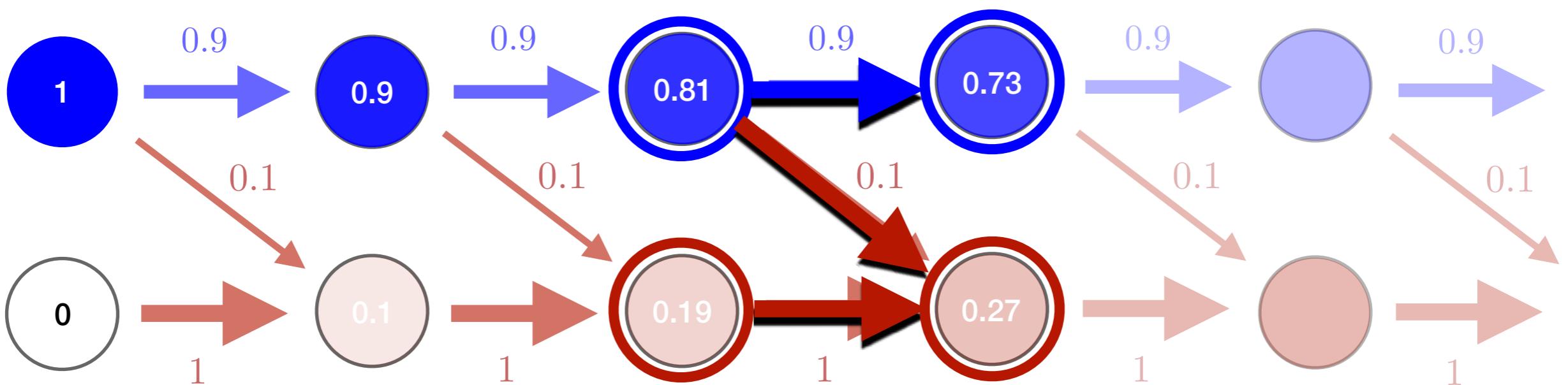
$t = 1$

$t = 2$

$t = 3$

$t = 4$

OK



$t = 0$

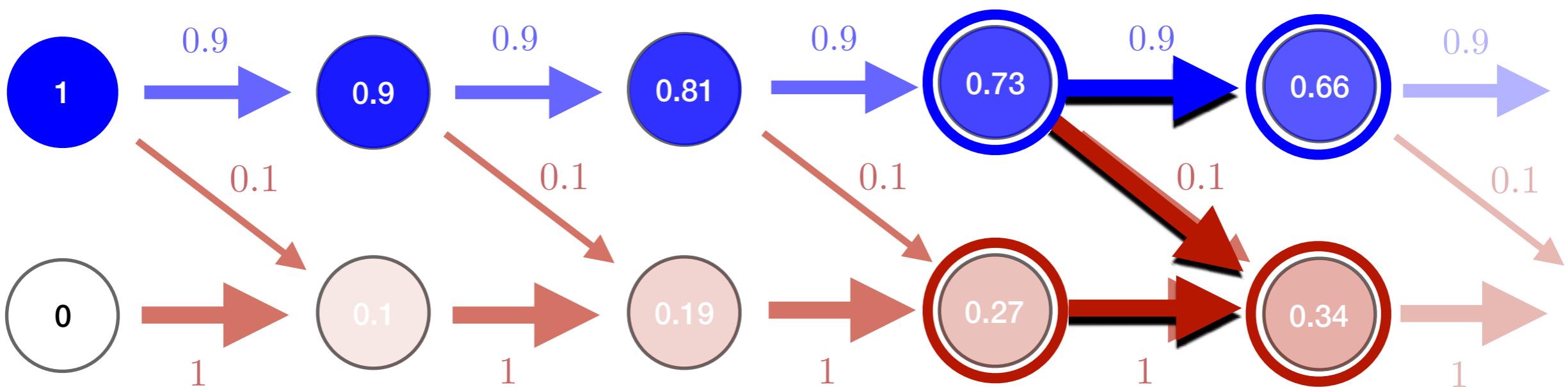
$t = 1$

$t = 2$

$t = 3$

$t = 4$

OK



$t = 0$

$t = 1$

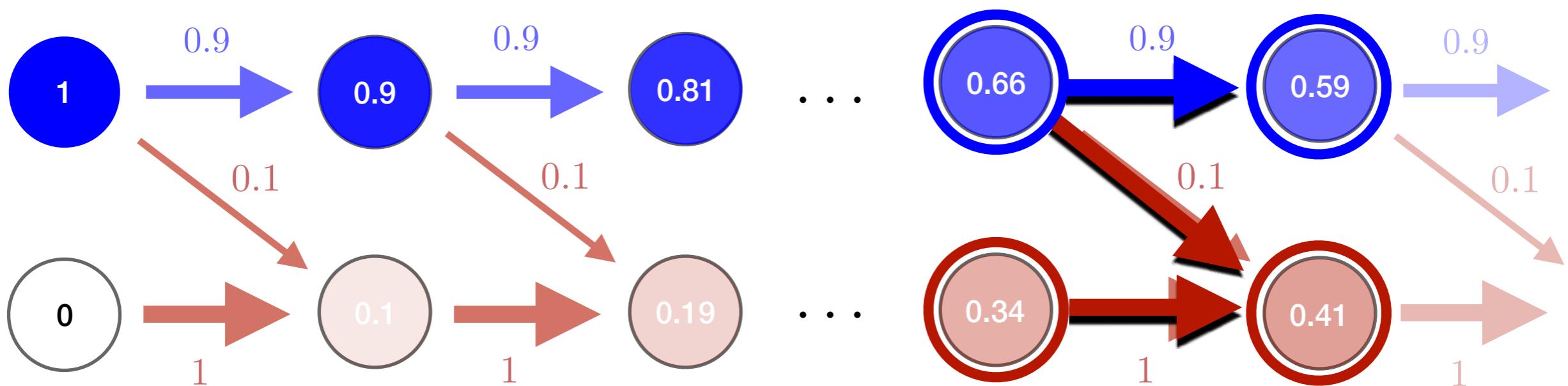
$t = 2$

$t = 3$

$t = 4$

Fail

OK



$t = 0$

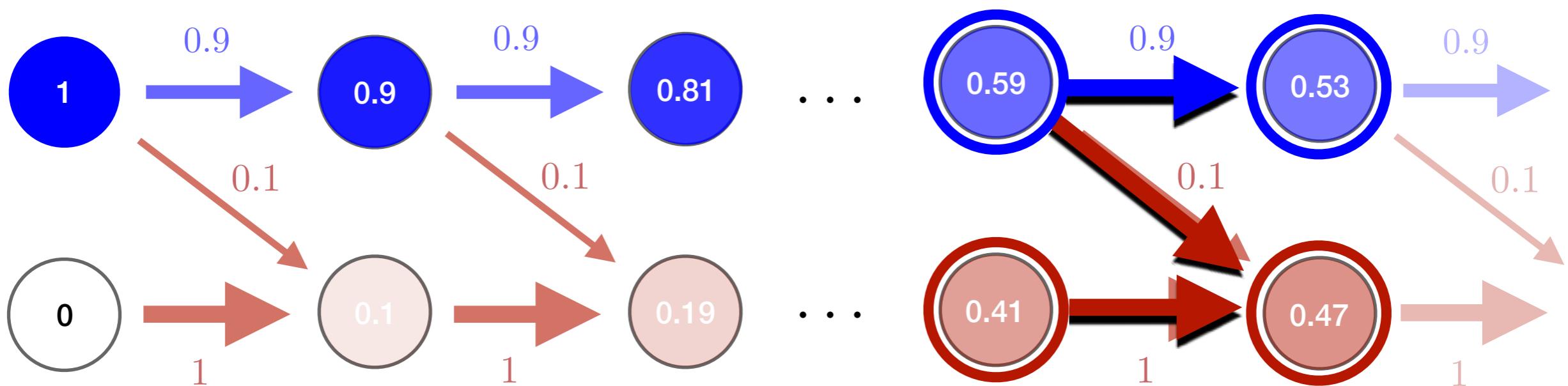
$t = 1$

$t = 2$

$t = 4$

$t = 5$

OK



$t = 0$

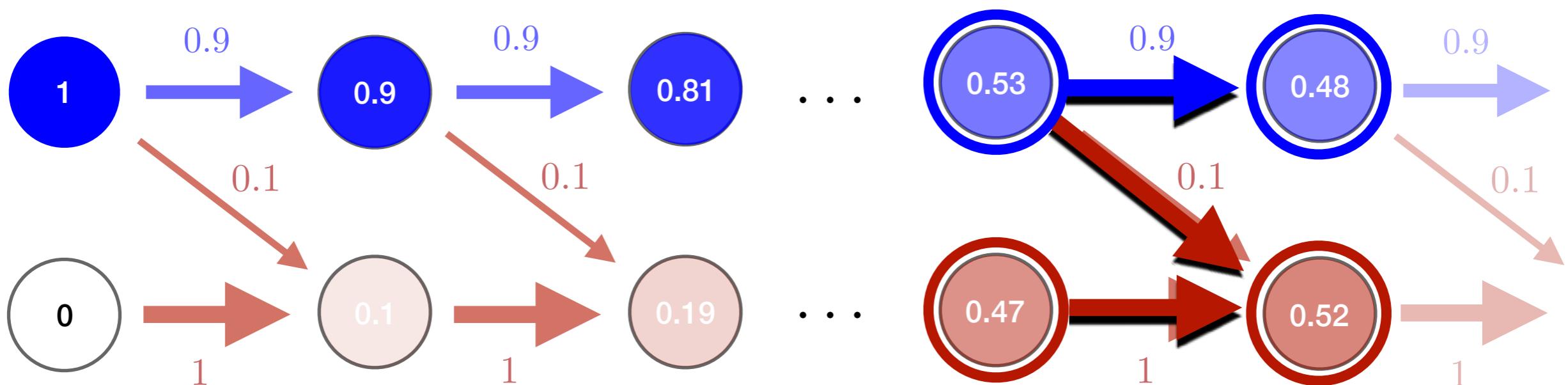
$t = 1$

$t = 2$

$t = 5$

$t = 6$

OK



$t = 0$

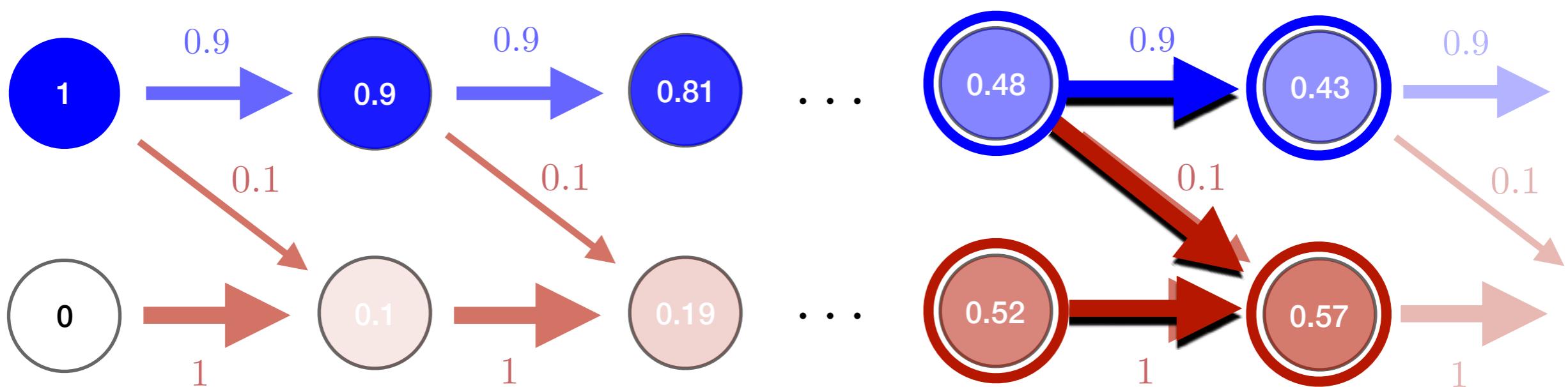
$t = 1$

$t = 2$

$t = 6$

$t = 7$

OK



$t = 0$

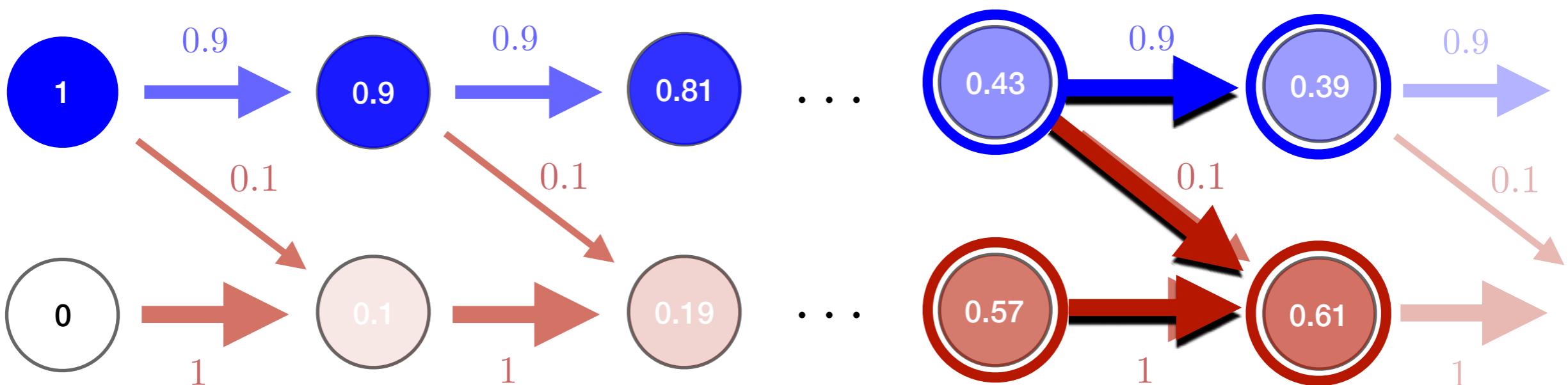
$t = 1$

$t = 2$

$t = 7$

$t = 8$

OK



$t = 0$

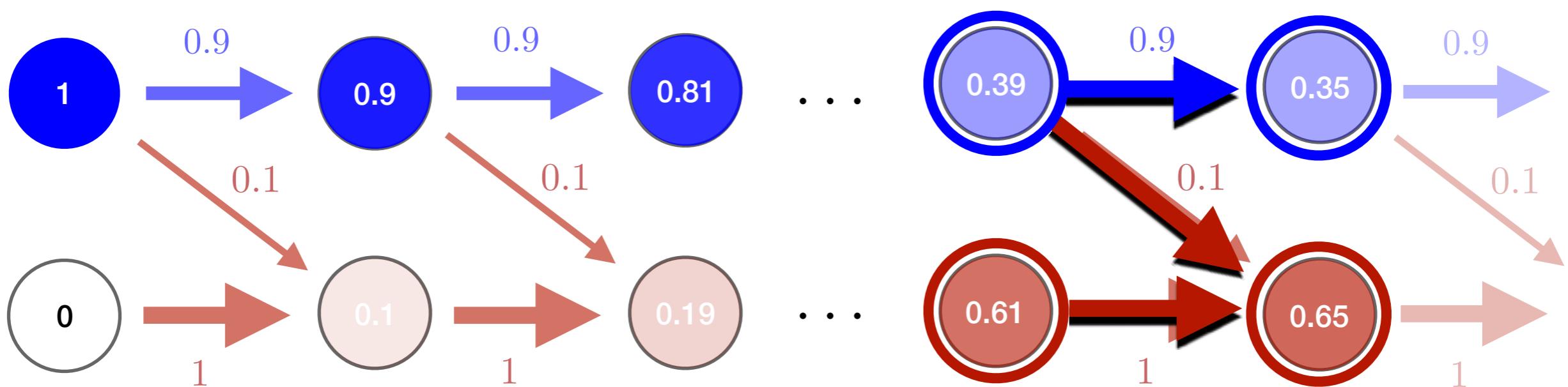
$t = 1$

$t = 2$

$t = 8$

$t = 9$

OK



$t = 0$

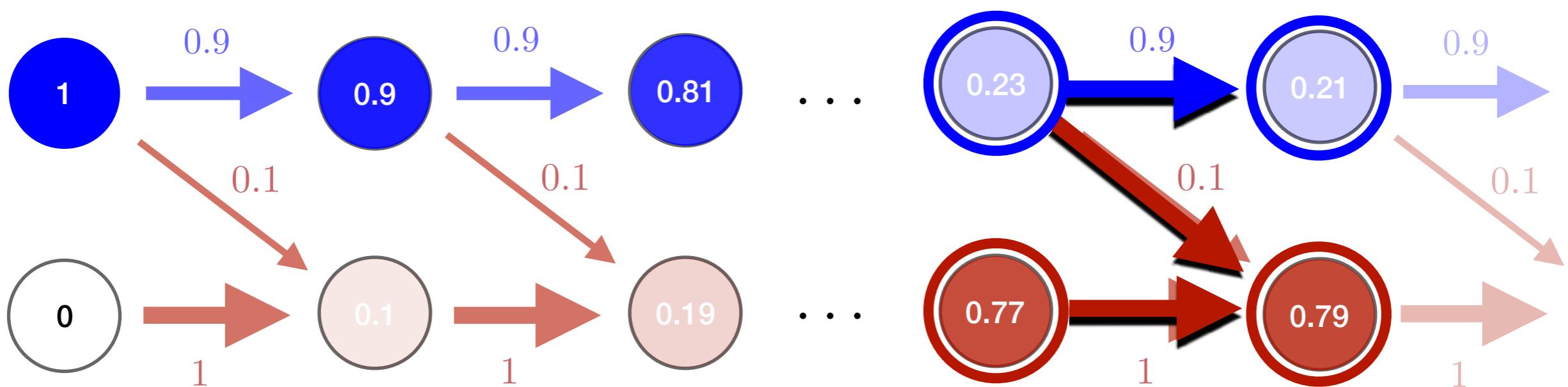
$t = 1$

$t = 2$

$t = 9$

$t = 10$

OK



$t = 0$

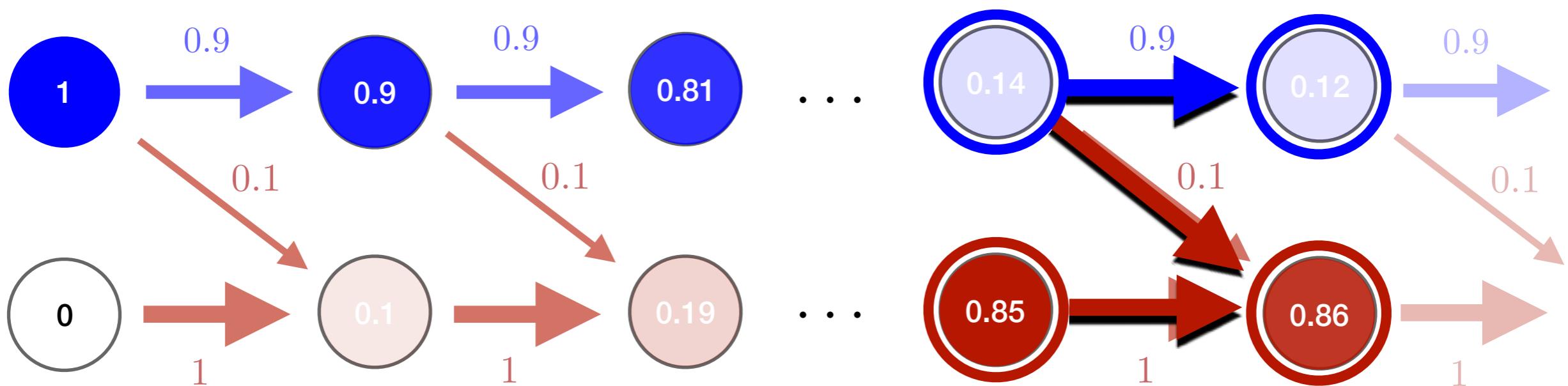
$t = 1$

$t = 2$

$t = 14$

$t = 15$

OK



$t = 0$

$t = 1$

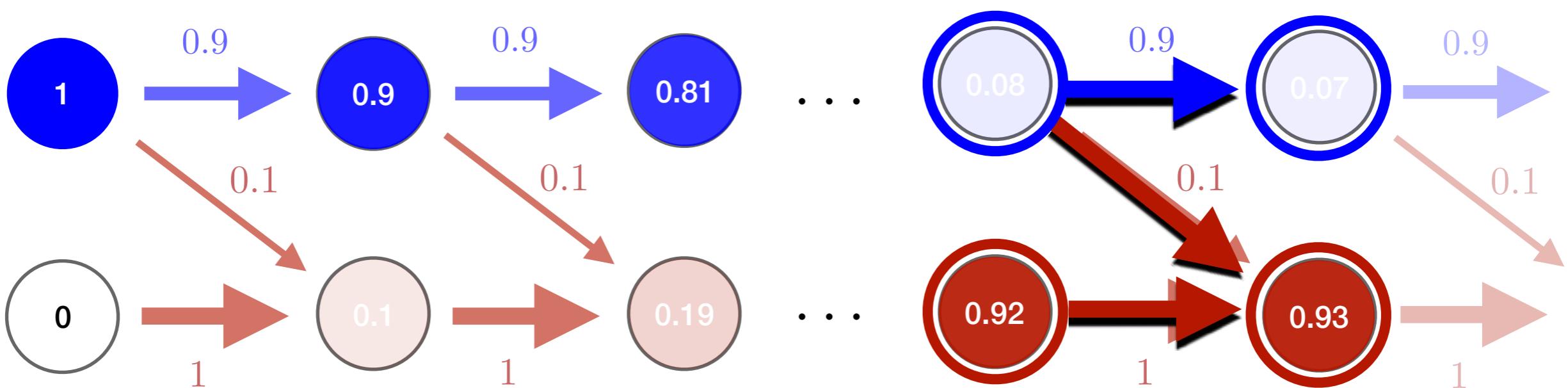
$t = 2$

$t = 19$

$t = 20$

Fail

OK



$t = 0$

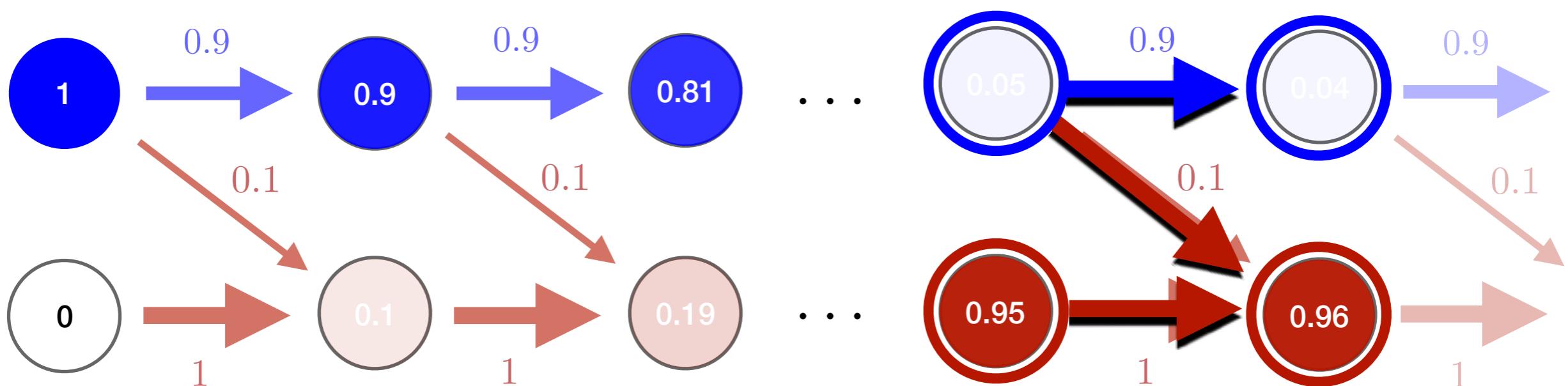
$t = 1$

$t = 2$

$t = 24$

$t = 25$

OK



$t = 0$

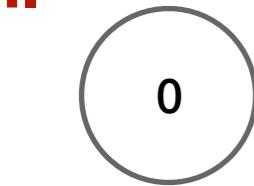
$t = 1$

$t = 2$

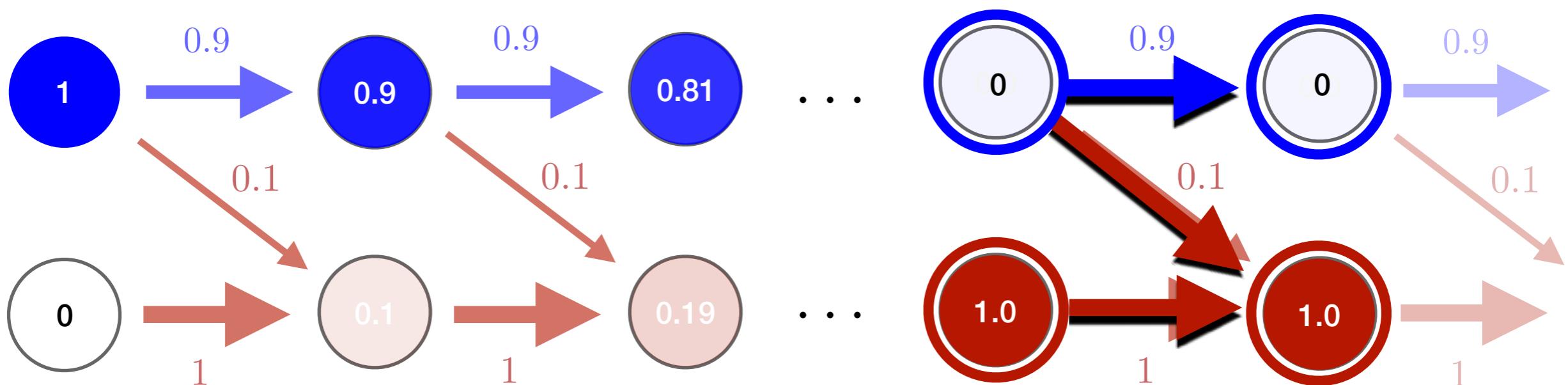
$t = 29$

$t = 30$

Fail



OK



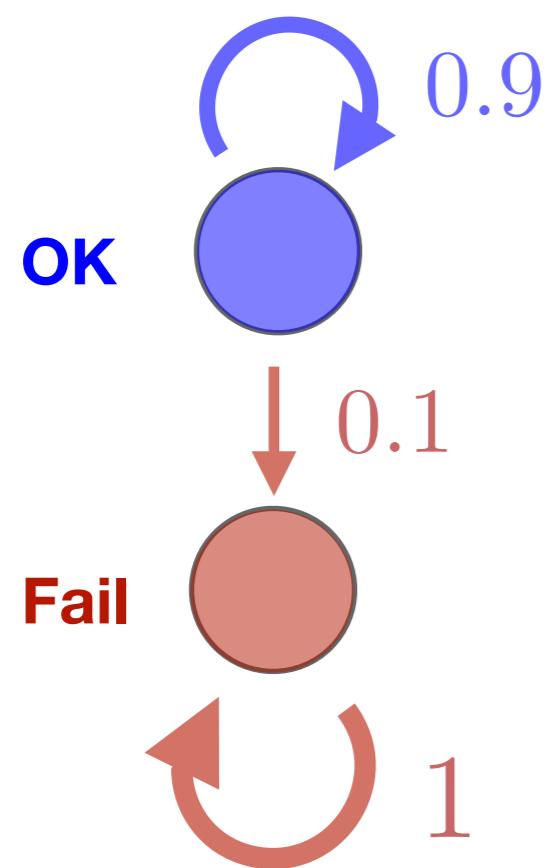
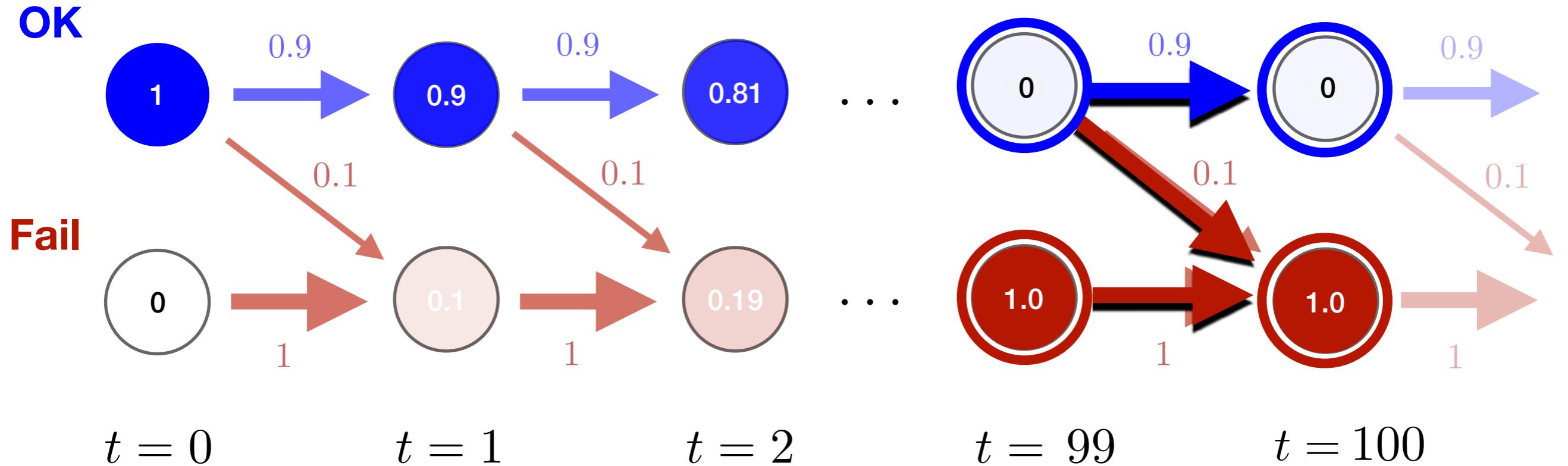
$t = 0$

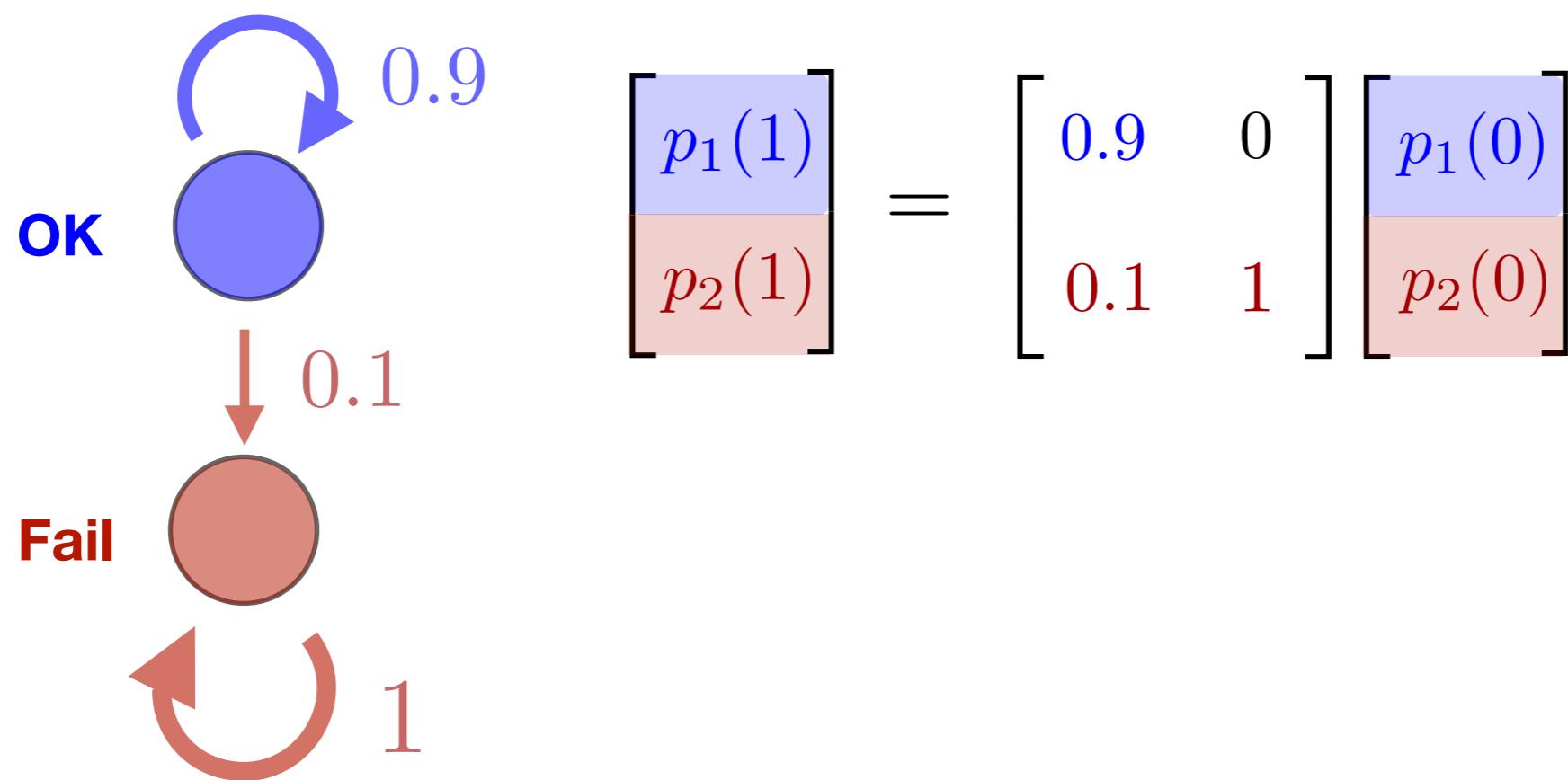
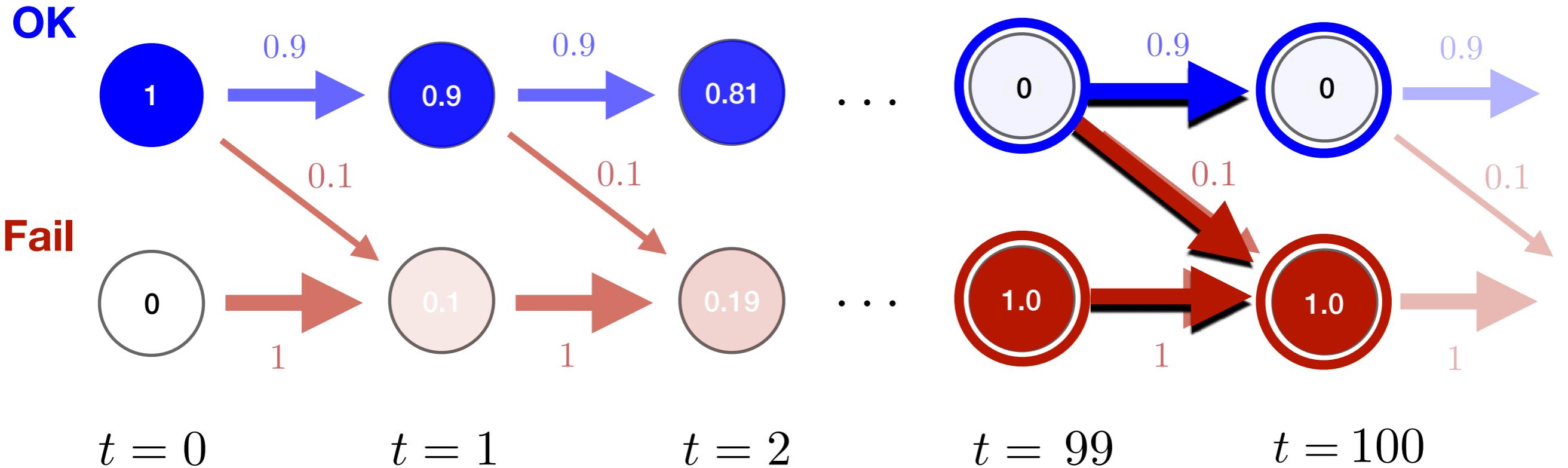
$t = 1$

$t = 2$

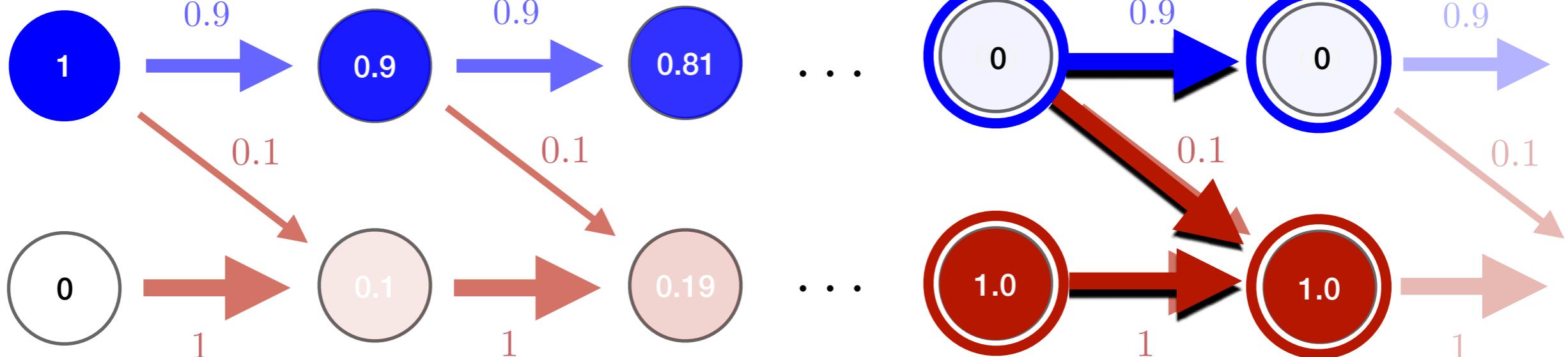
$t = 99$

$t = 100$





OK



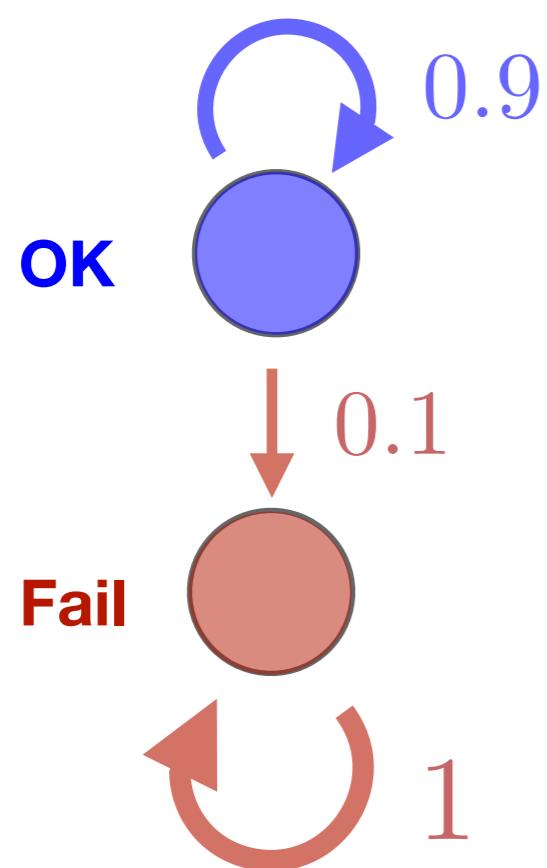
$t = 0$

$t = 1$

$t = 2$

$t = 99$

$t = 100$

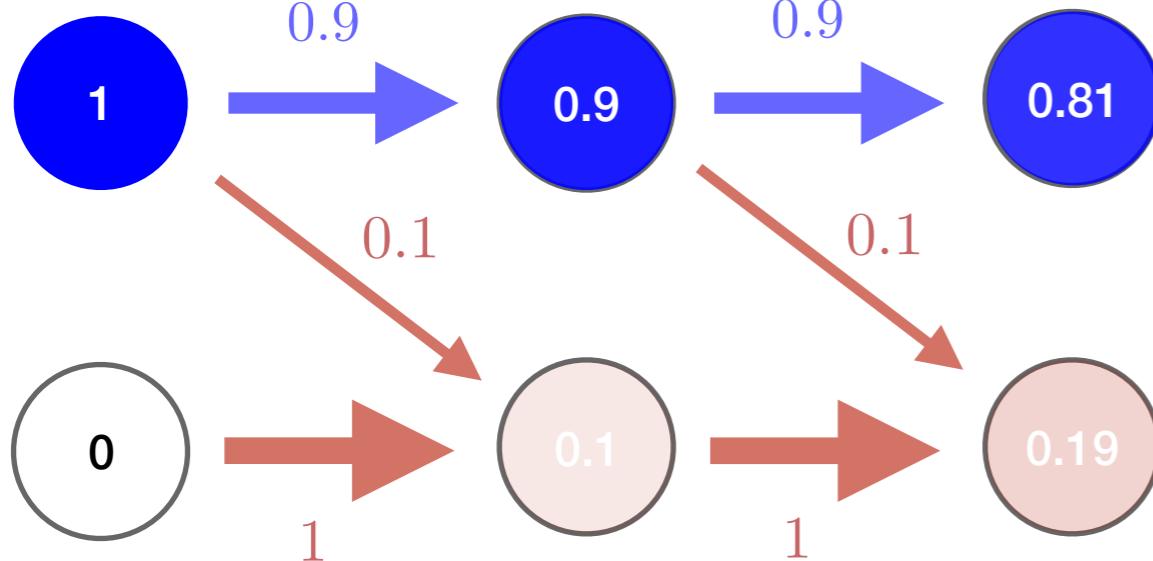


$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$p_1(1) = 0.9 \times p_1(0) + 0 \times p_2(0)$$

$$p_2(1) = 0.1 \times p_1(0) + 1 \times p_2(0)$$

OK

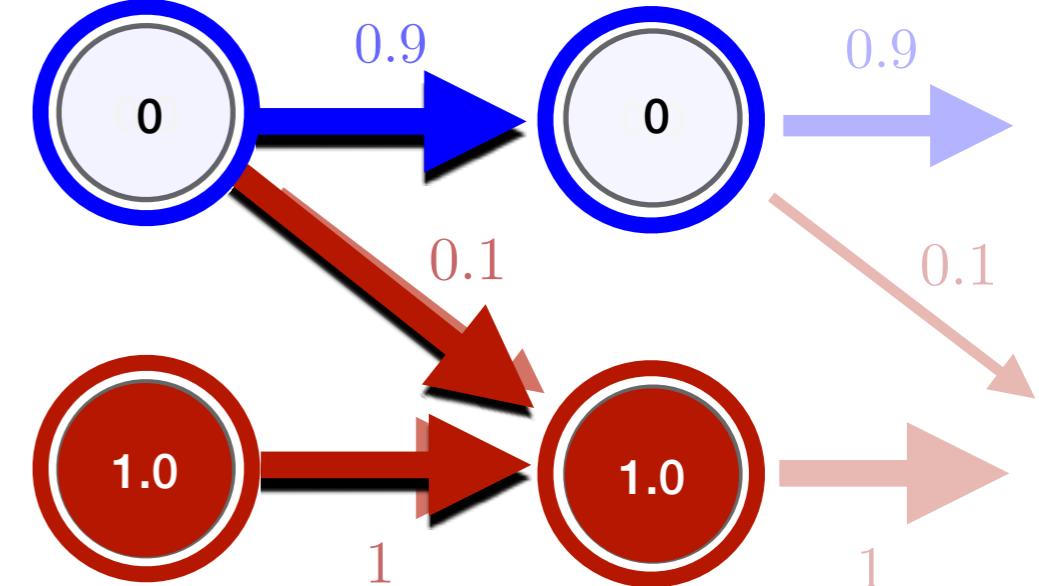


$t = 0$

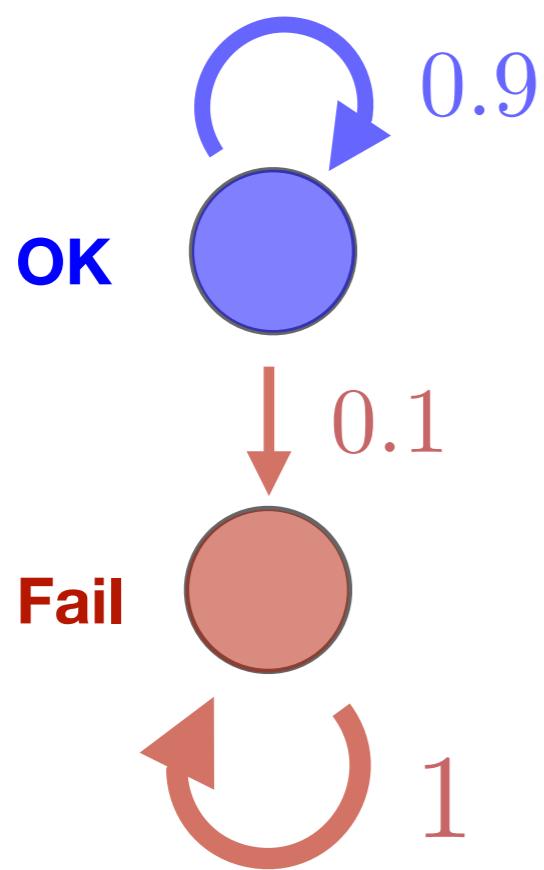
$t = 1$

$t = 2$

...



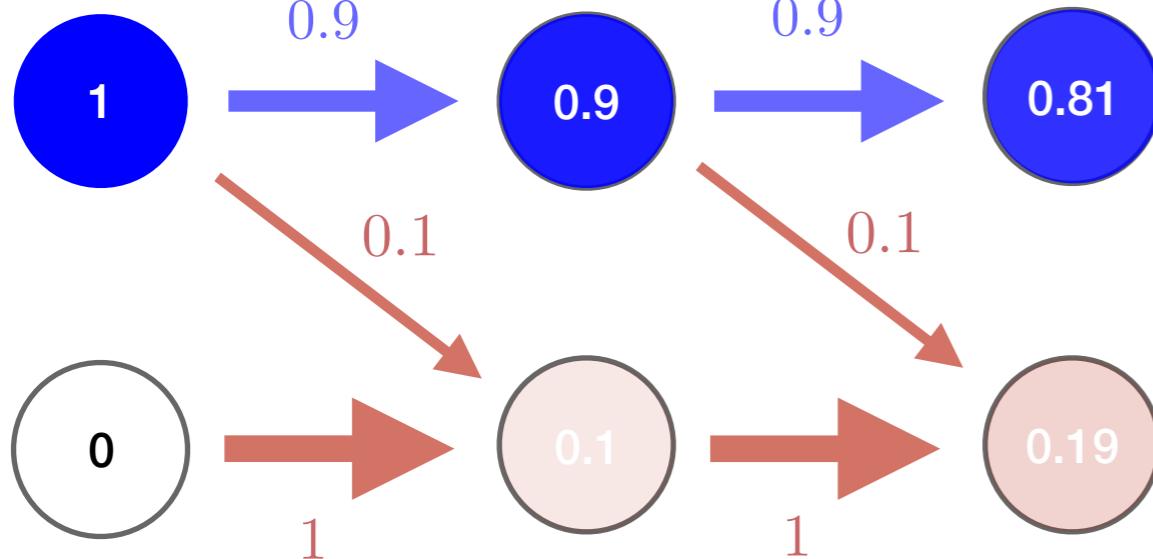
$t = 100$



$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\begin{bmatrix} p_1(2) \\ p_2(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix}$$

OK



$t = 0$

$t = 1$

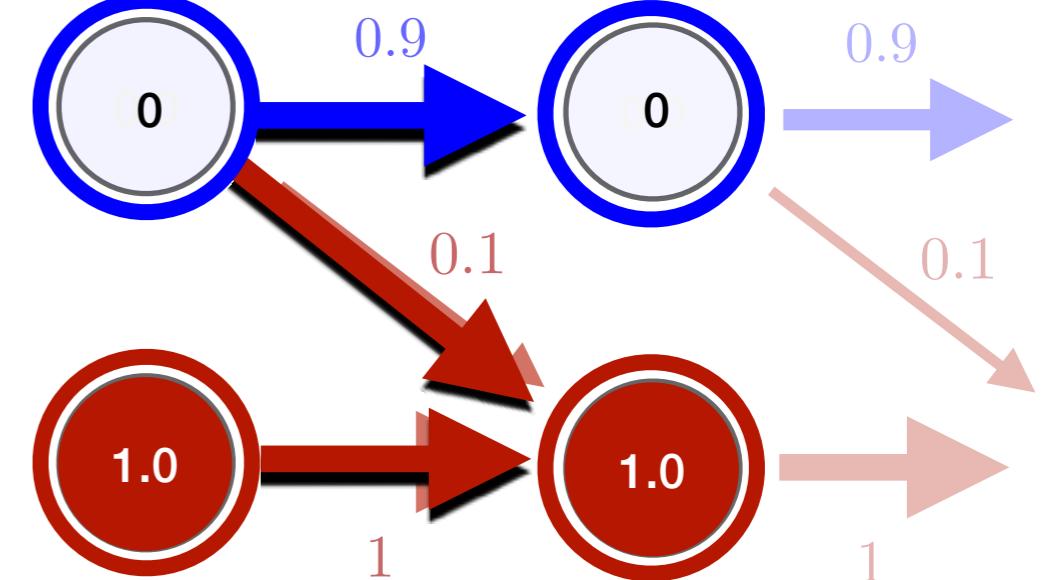
$t = 2$

$t = 99$

$t = 100$

Fail

...



0.1

0.1

1

1

...

1

1



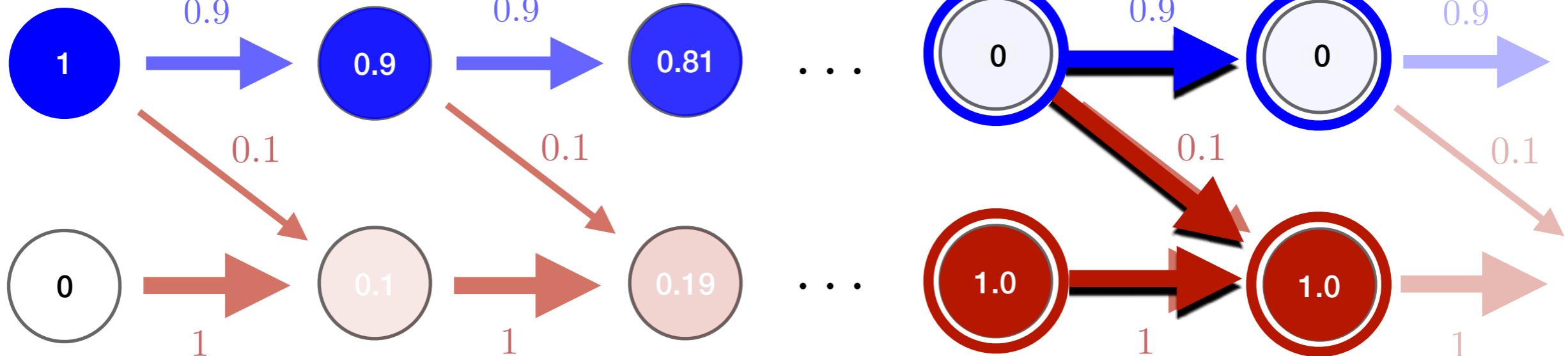
Fail



$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\begin{bmatrix} p_1(2) \\ p_2(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

OK



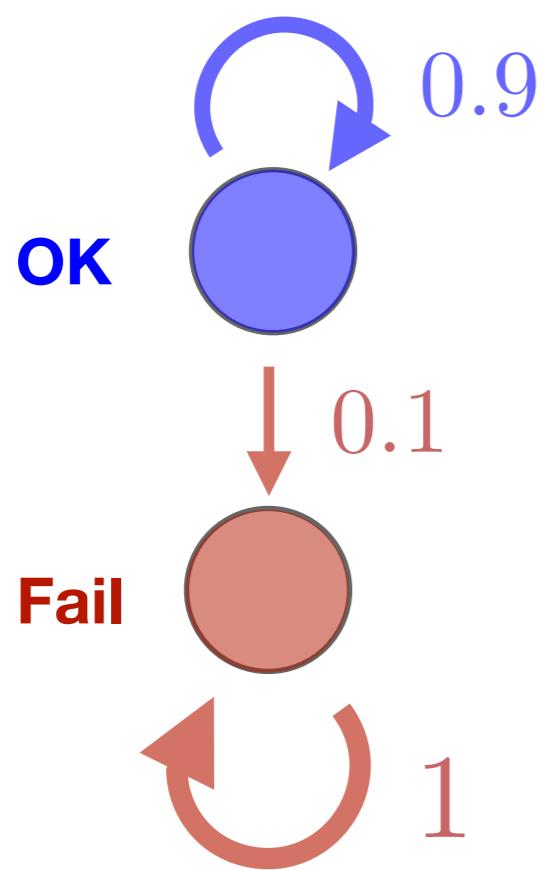
$t = 0$

$t = 1$

$t = 2$

$t = 99$

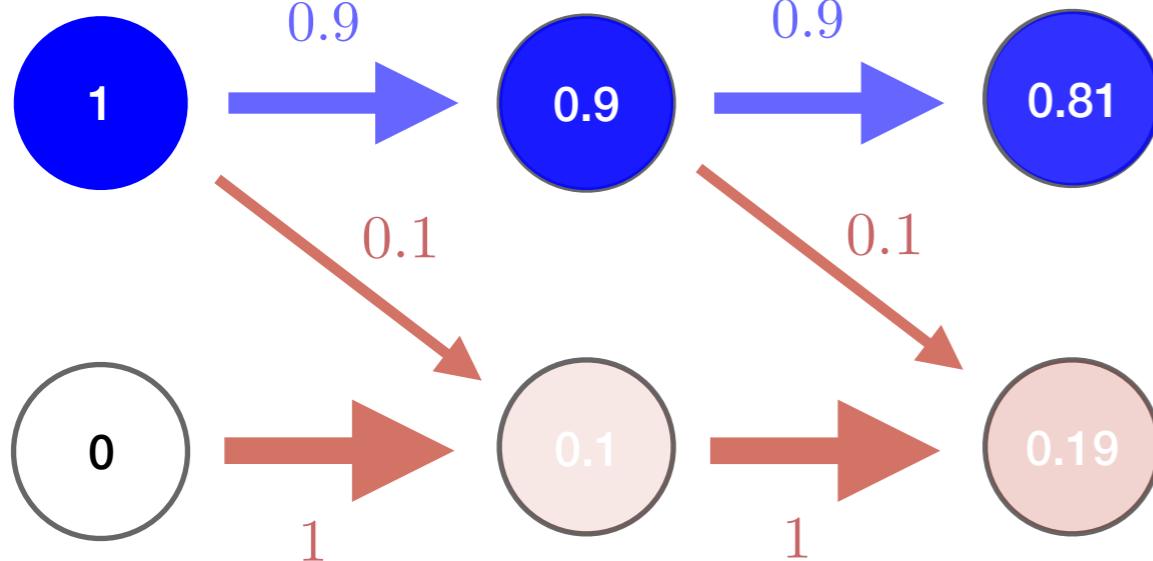
$t = 100$



$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\begin{bmatrix} p_1(2) \\ p_2(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix}^2 \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

OK

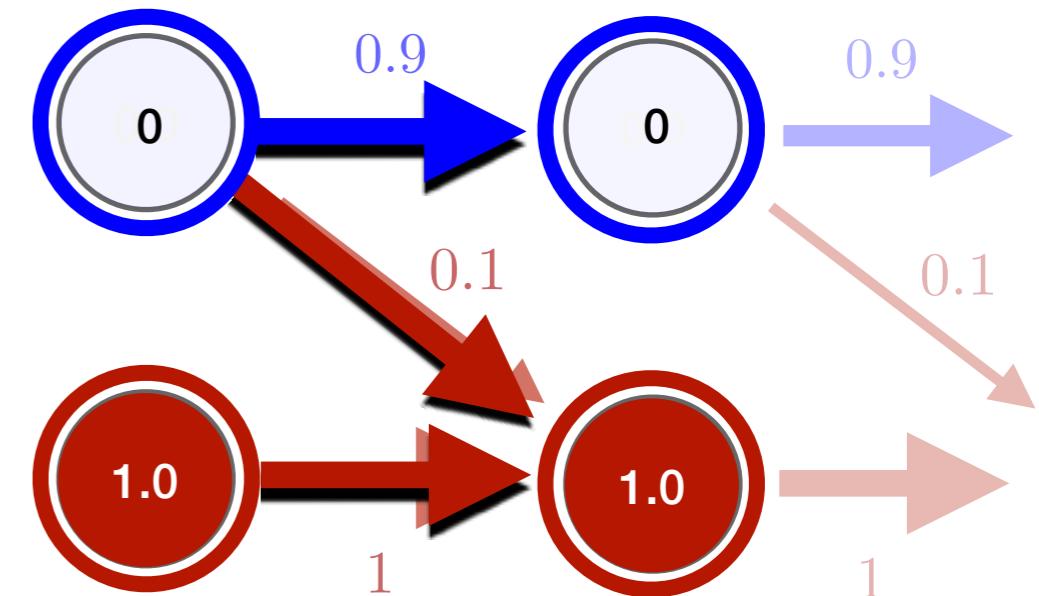


$t = 0$

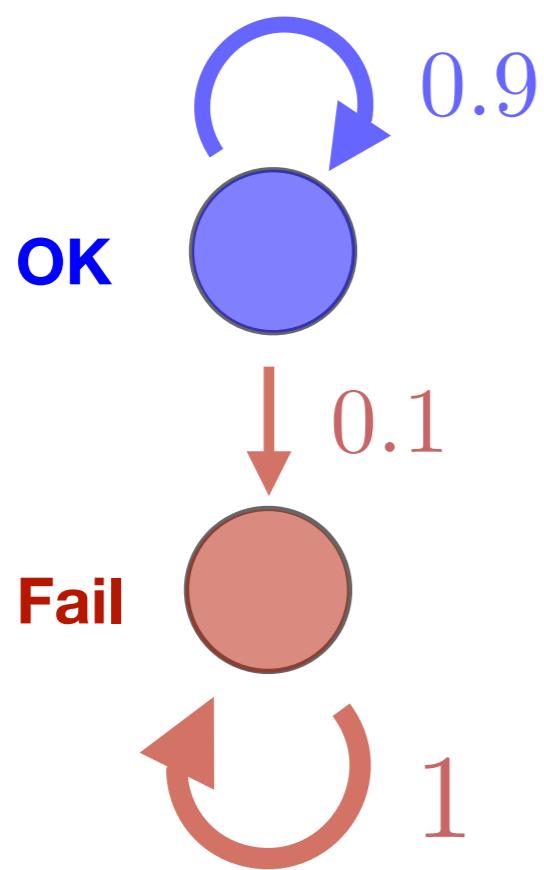
$t = 1$

$t = 2$

...

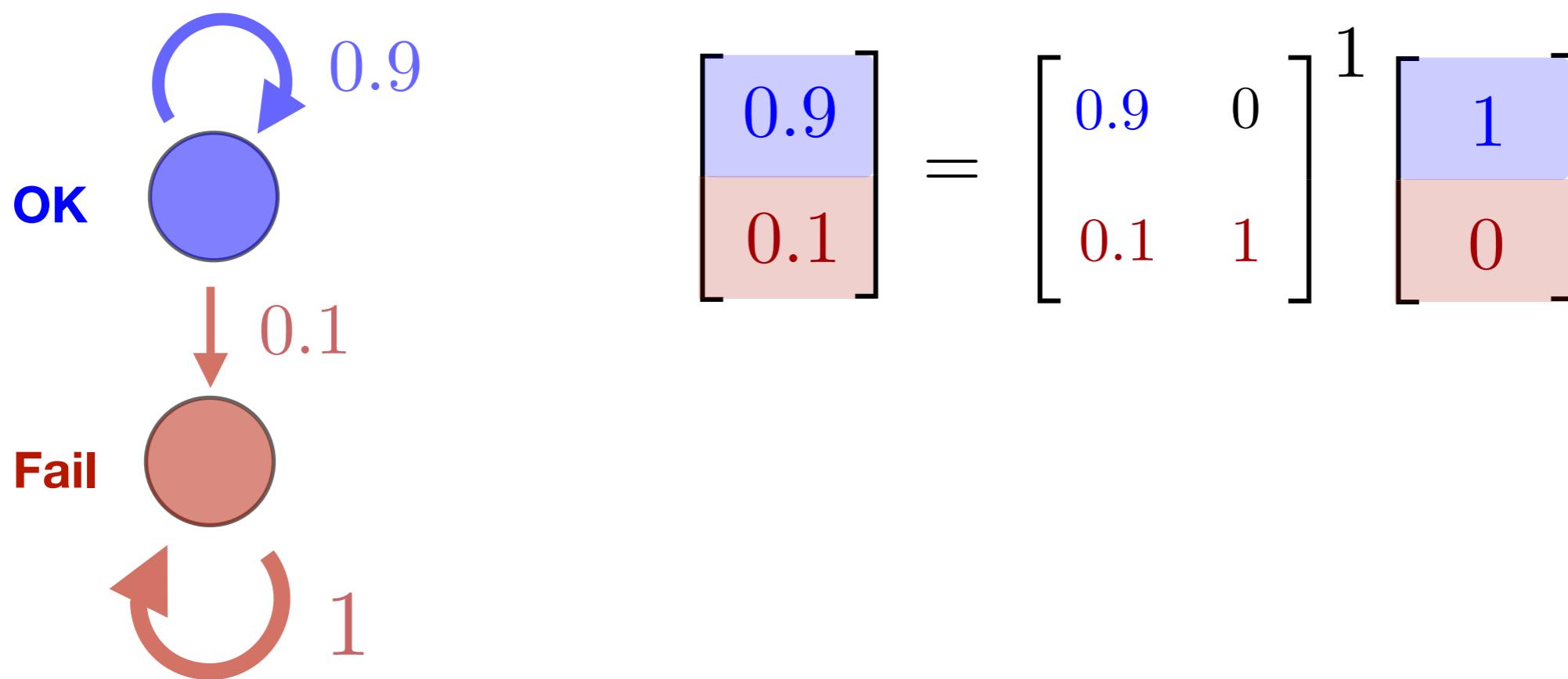
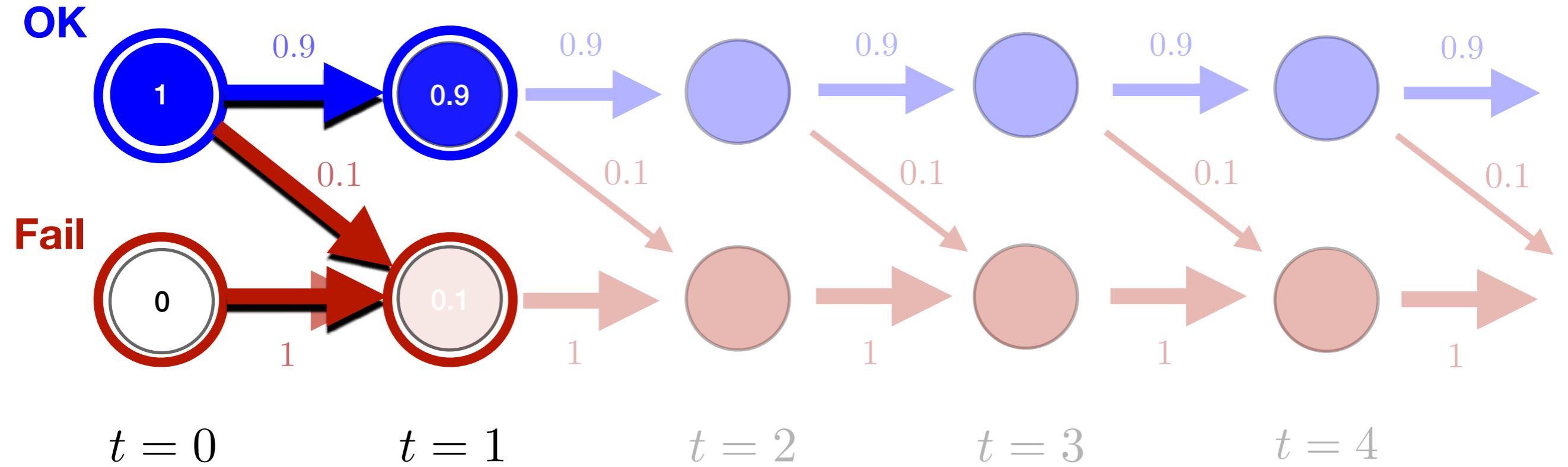


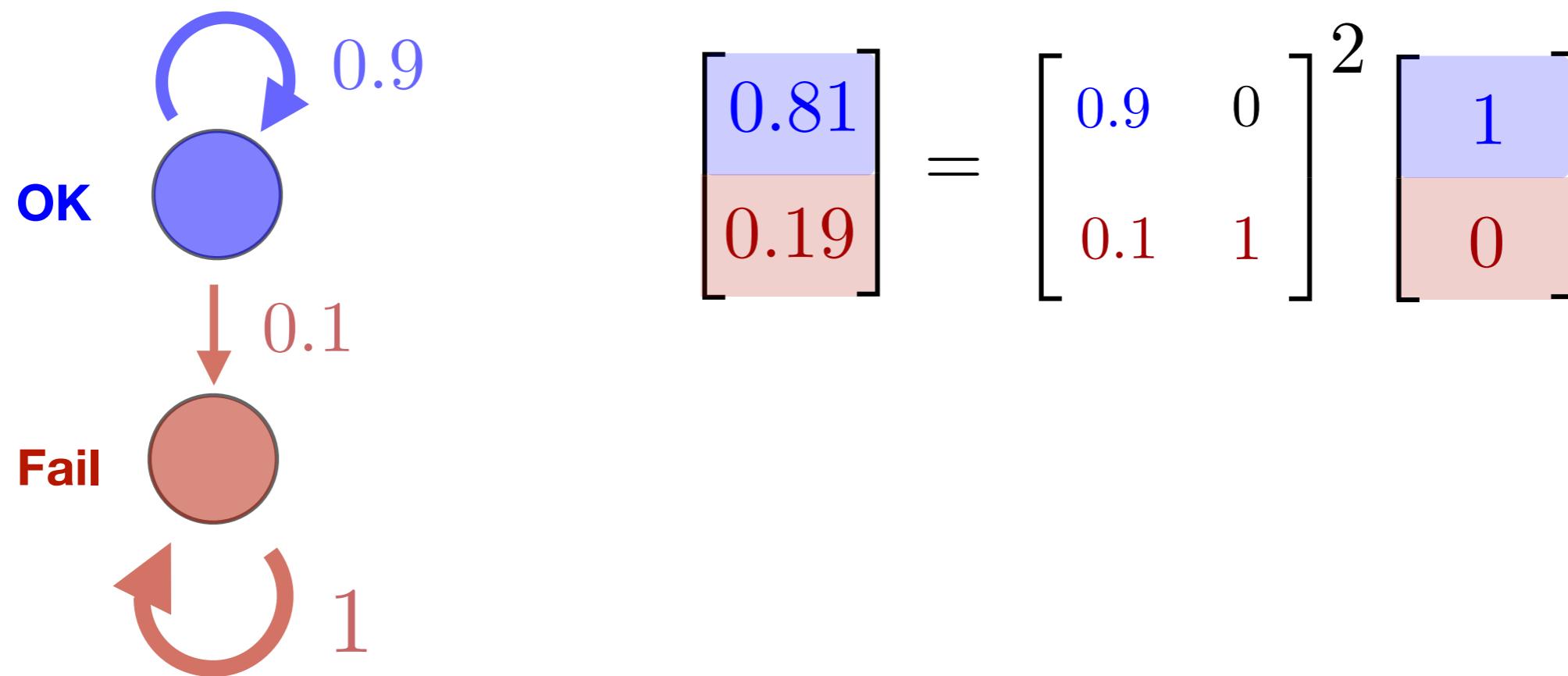
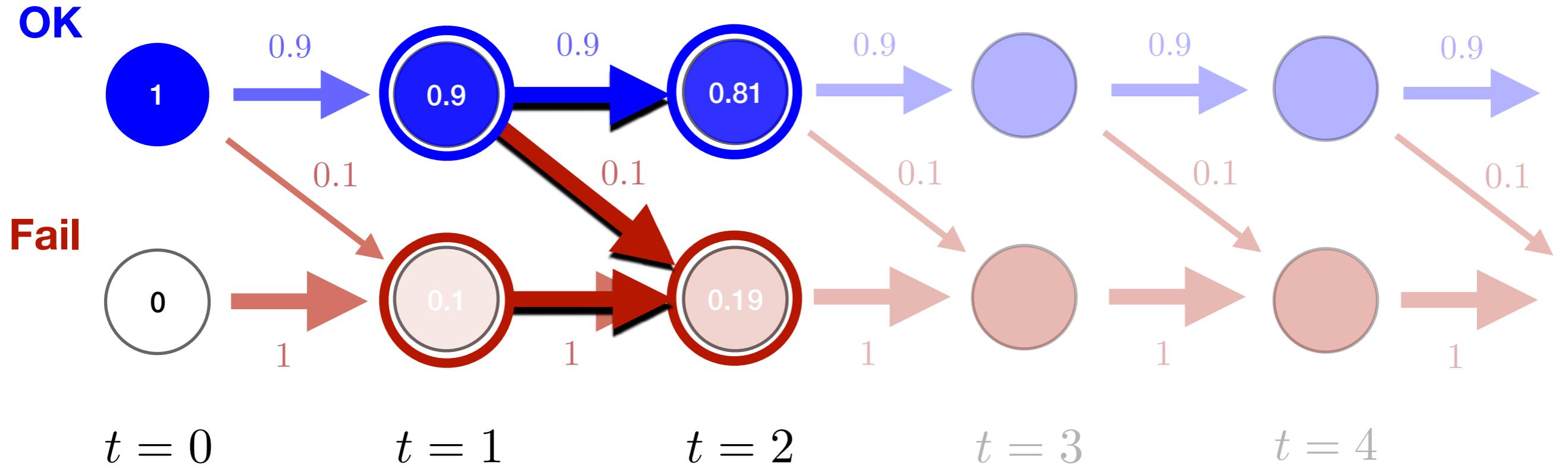
$t = 100$

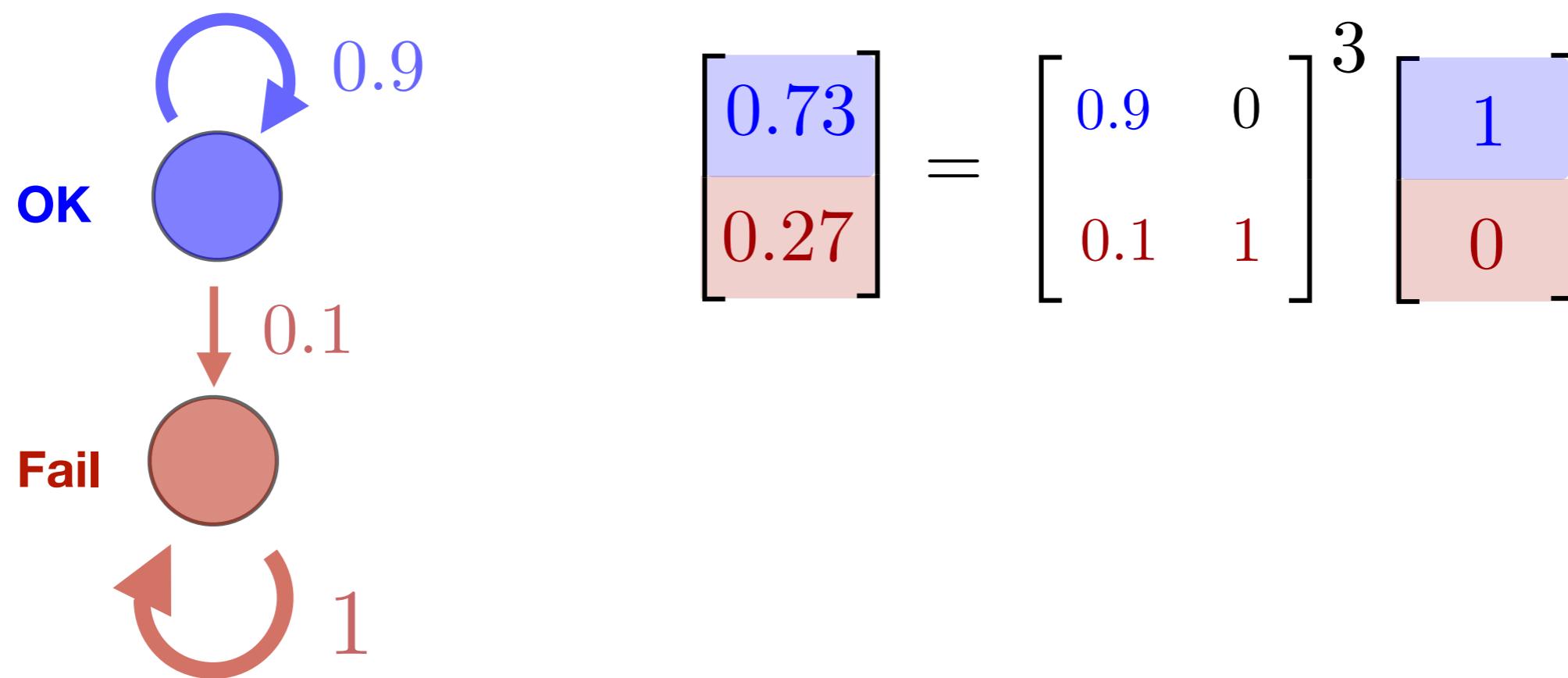
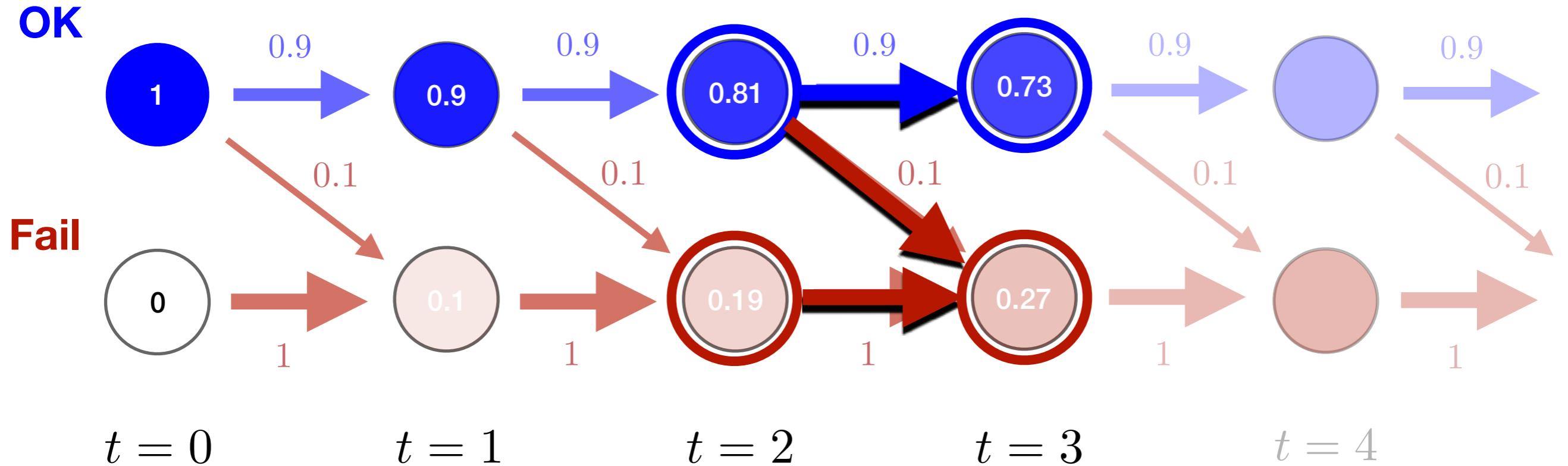


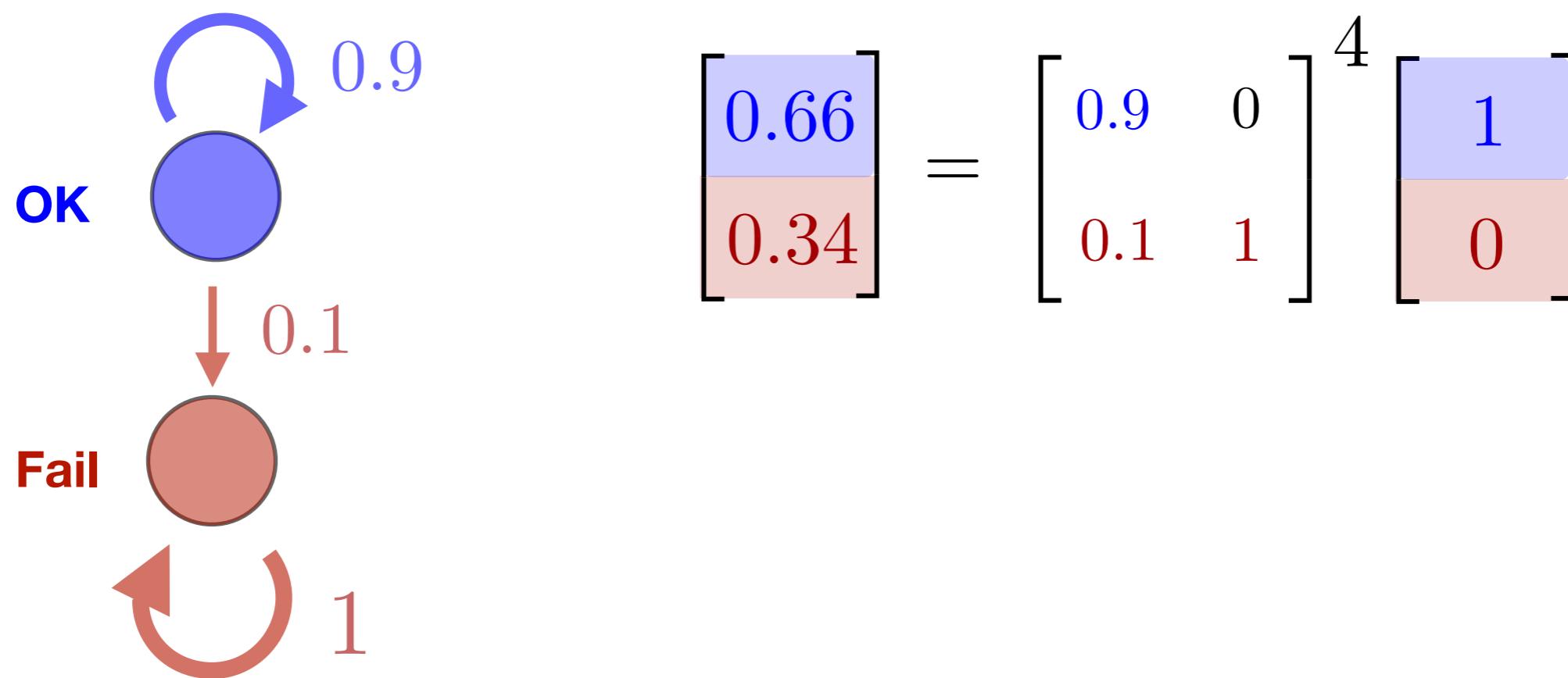
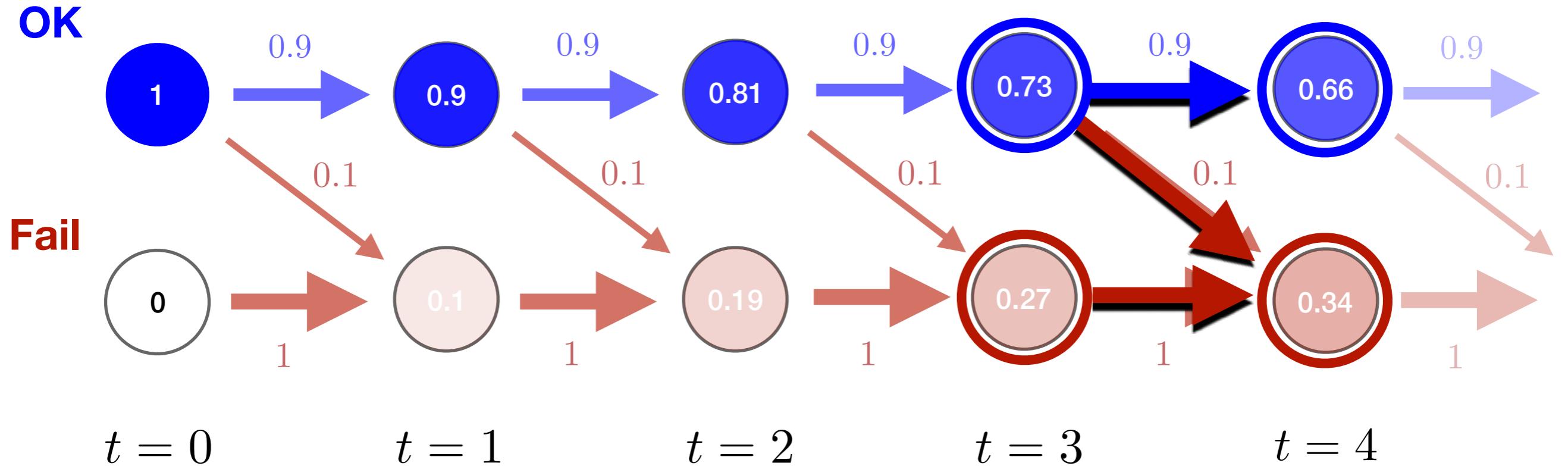
$$\begin{bmatrix} p_1(1) \\ p_2(1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

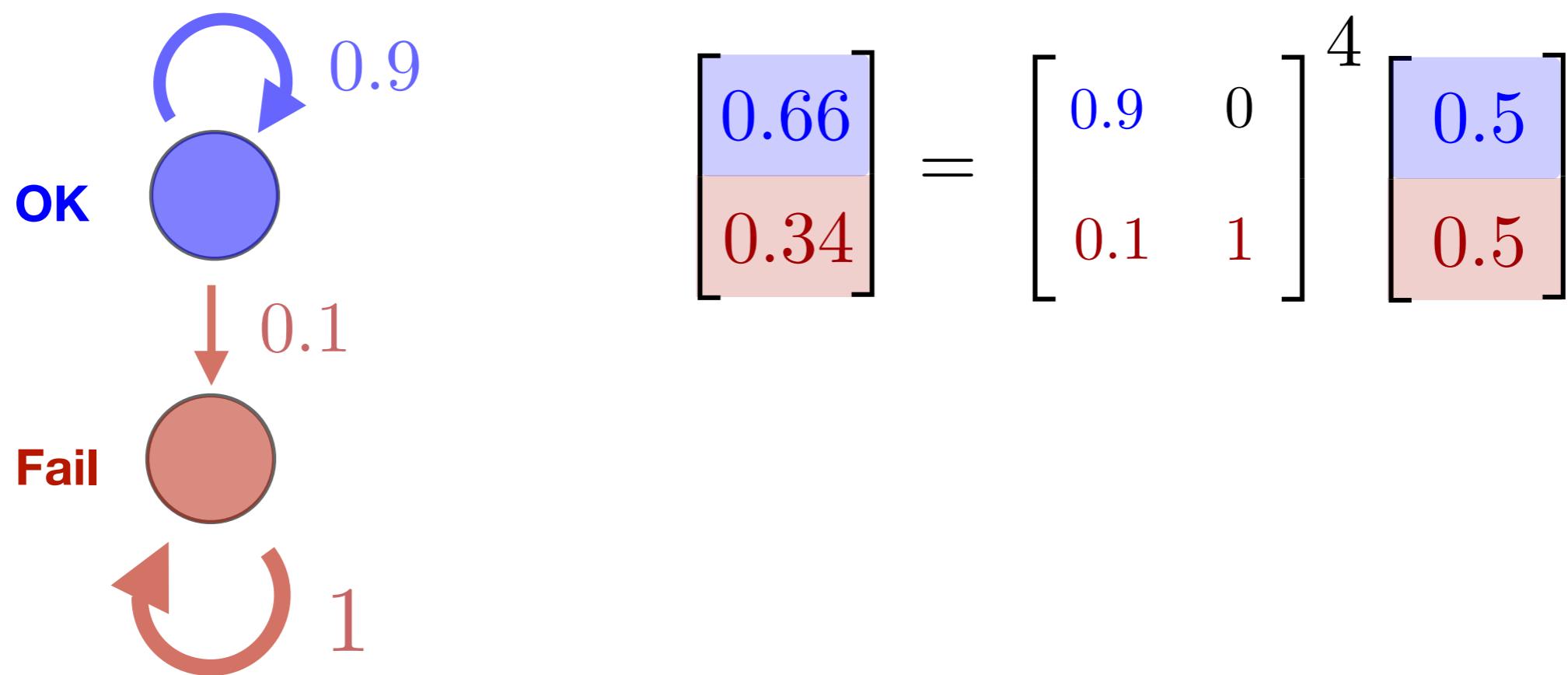
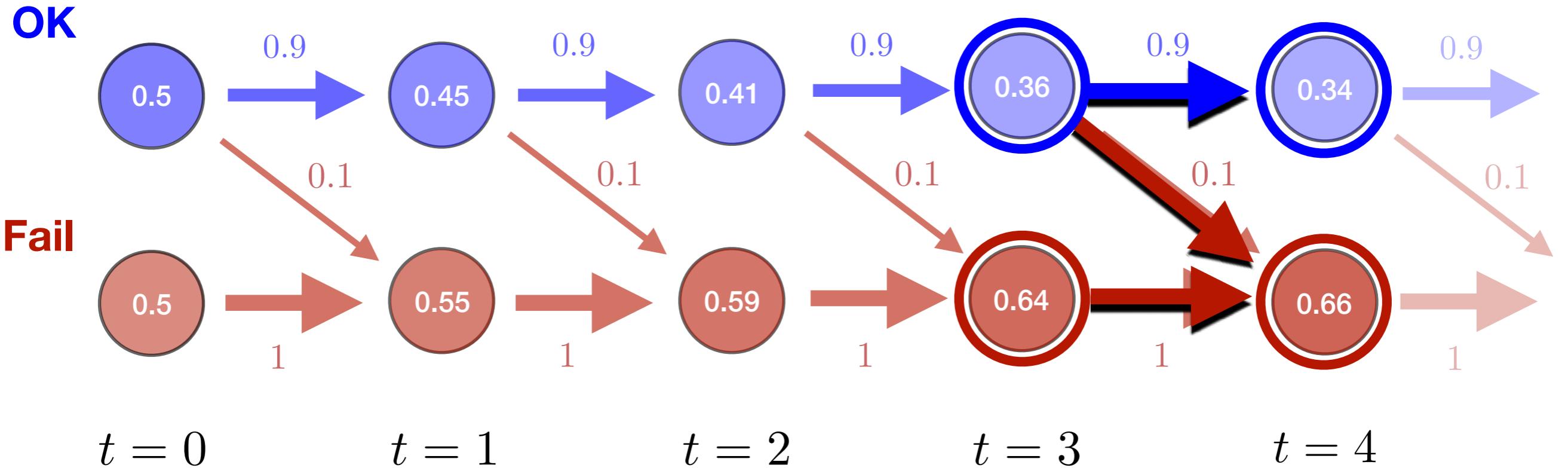
$$\begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 1 \end{bmatrix}^k \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

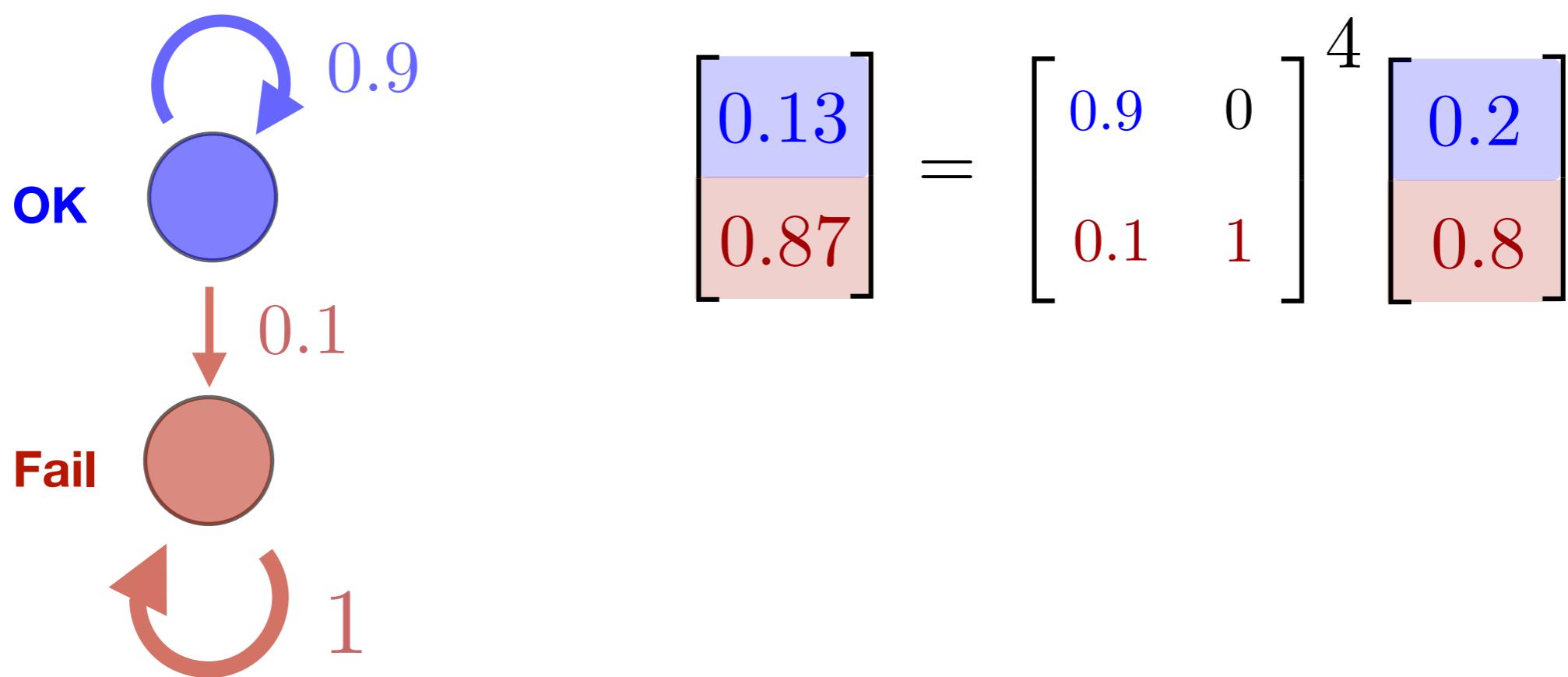
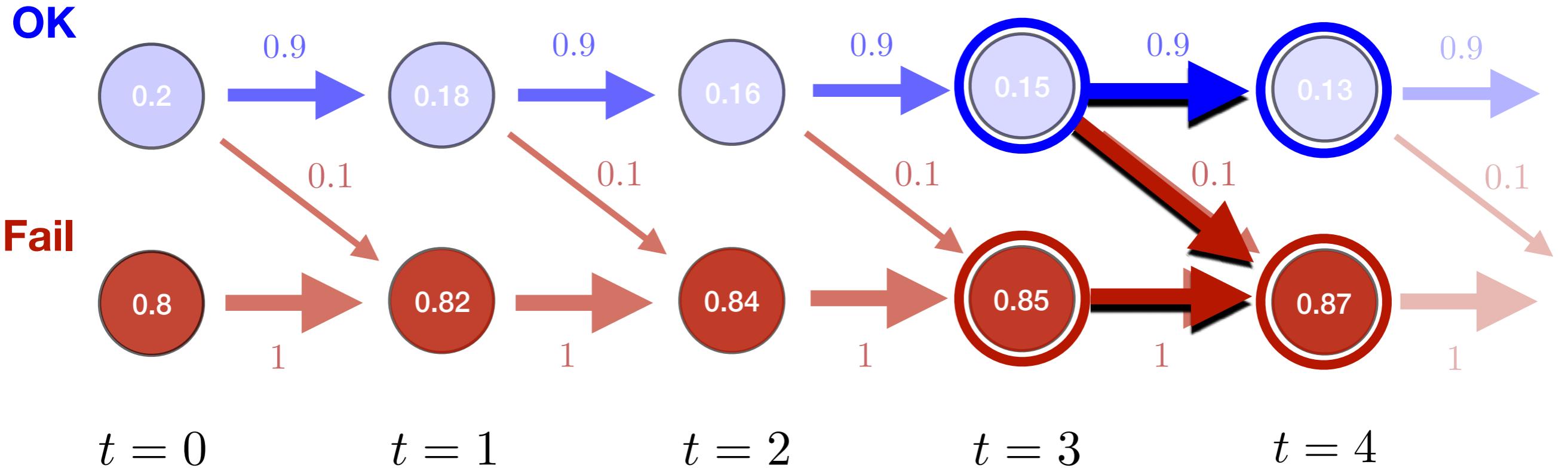


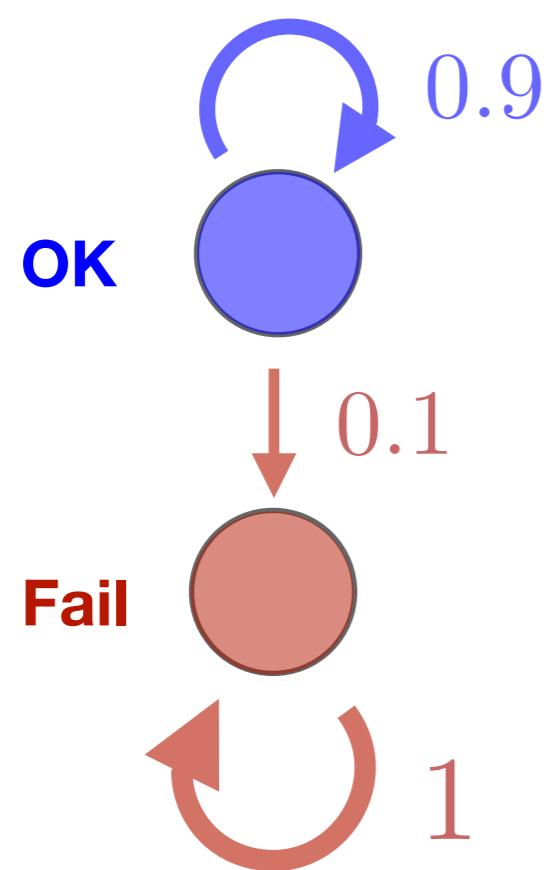
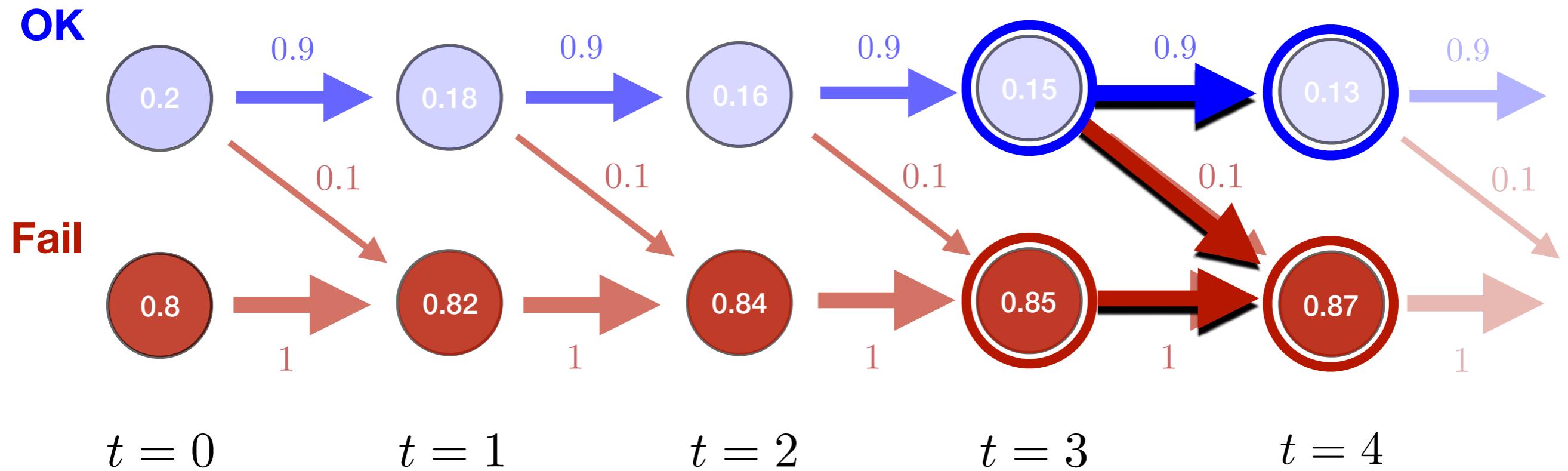




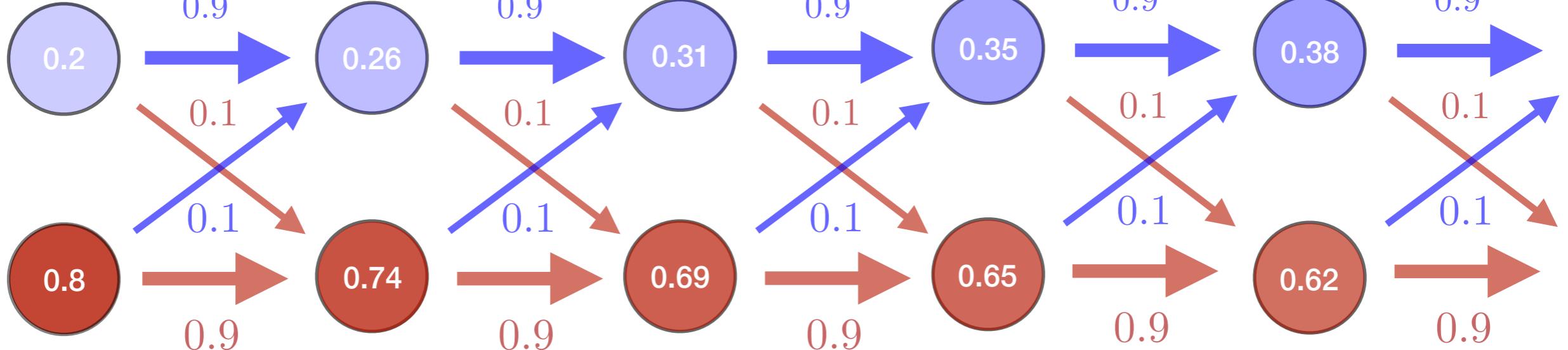
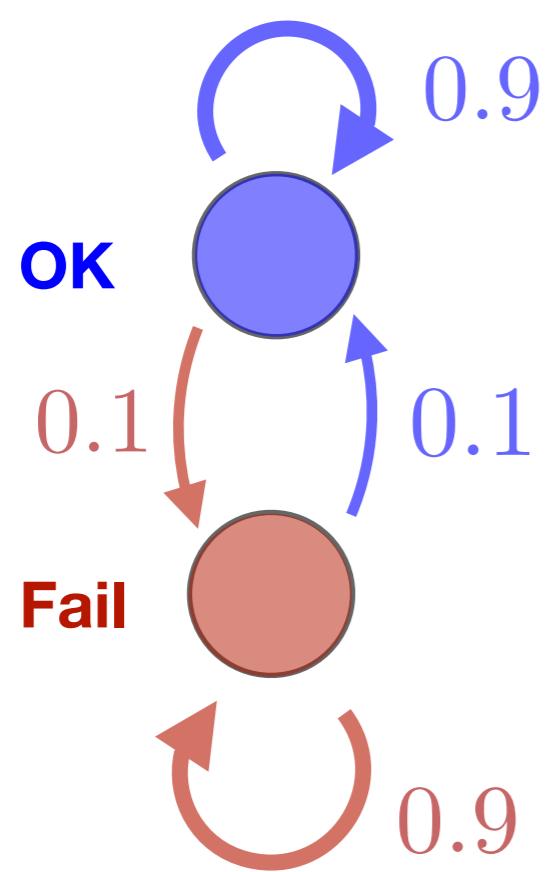








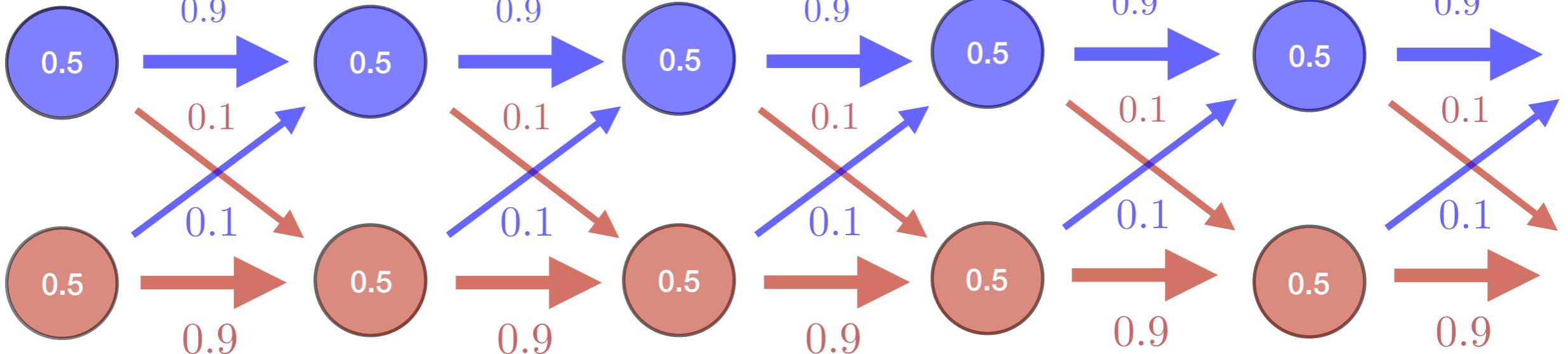
$$\lim_{k \rightarrow \infty} \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} p_1(\infty) \\ p_2(\infty) \end{bmatrix} = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}$$

OK $t = 0$ $t = 1$ $t = 2$ $t = 3$ $t = 4$ 

$$\begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}^k \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\lim_{k \rightarrow \infty} \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} p_1(\infty) \\ p_2(\infty) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

OK



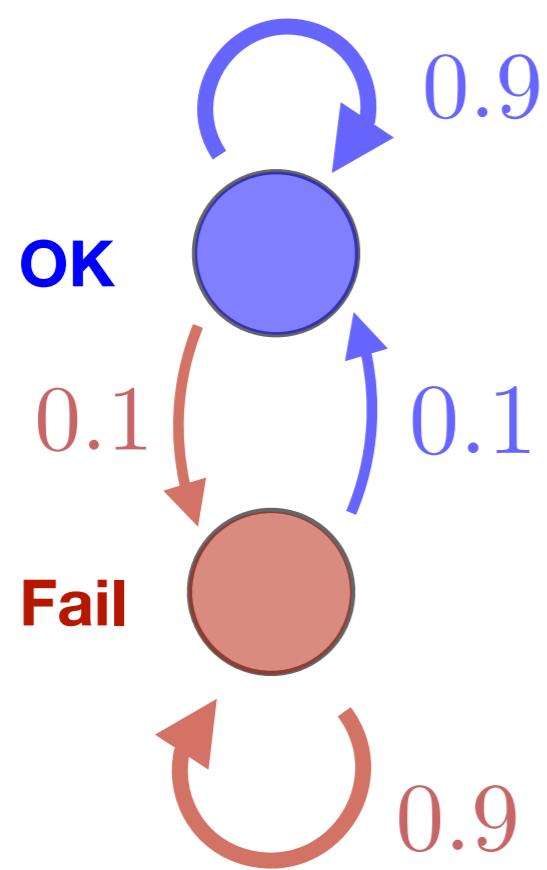
$t = 0$

$t = 1$

$t = 2$

$t = 3$

$t = 4$

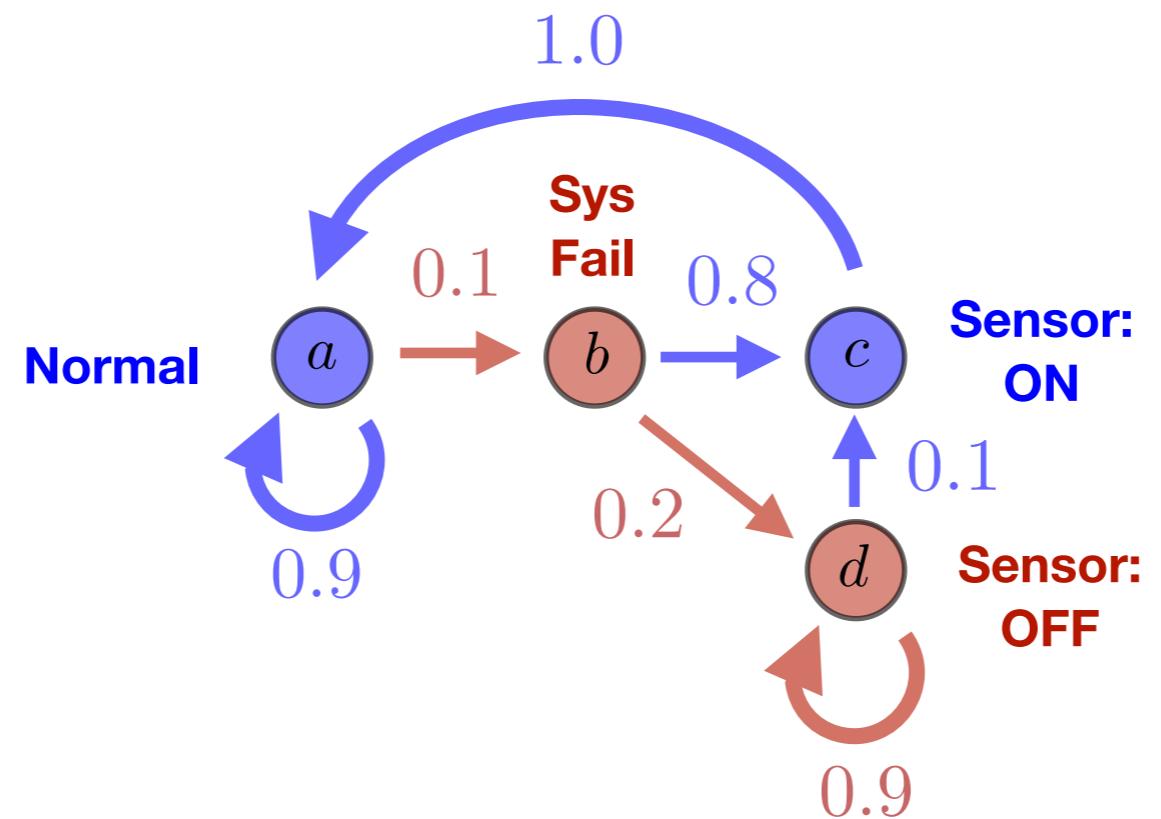


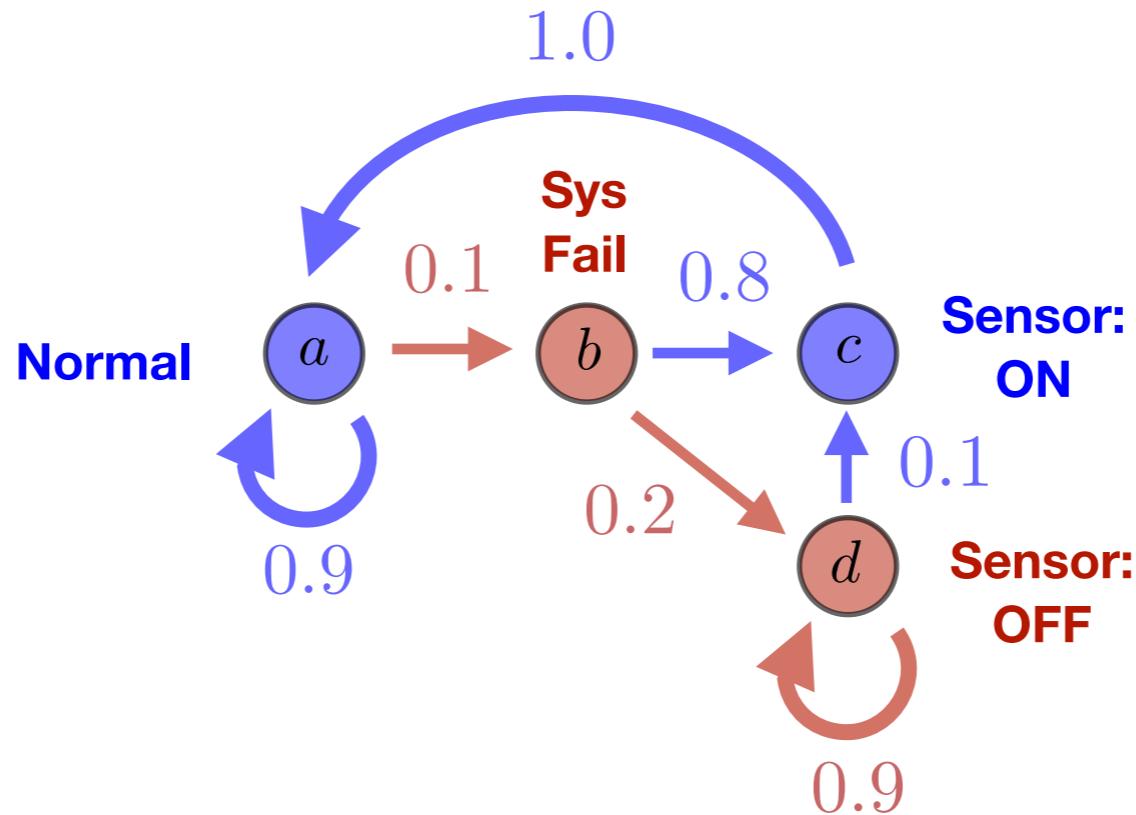
$\lim_{k \rightarrow \infty}$

$$\begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}^k \begin{bmatrix} p_1(0) \\ p_2(0) \end{bmatrix}$$

$$\lim_{k \rightarrow \infty} \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} = \begin{bmatrix} p_1(\infty) \\ p_2(\infty) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

examples



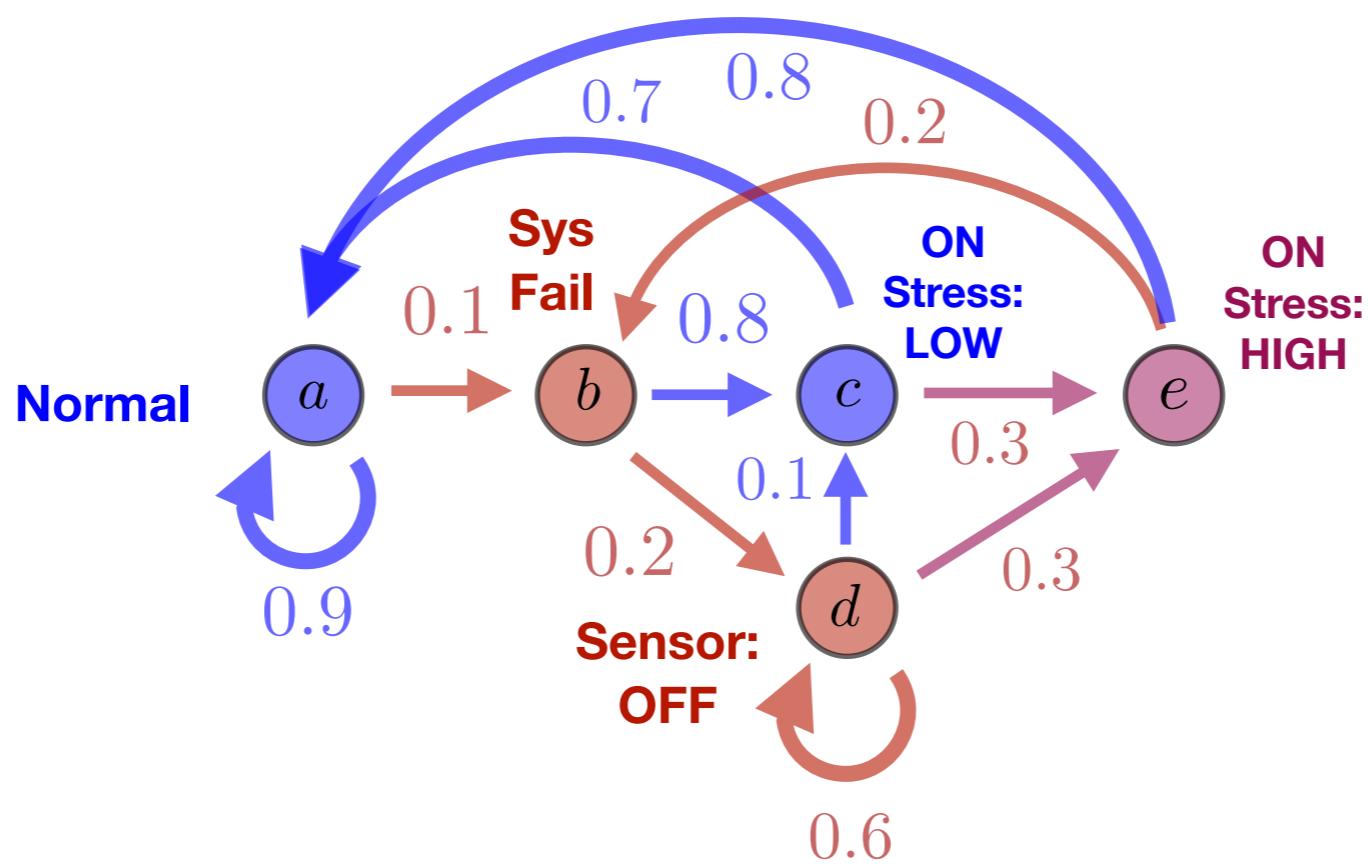


Update Equation:

$$\begin{bmatrix} p_a(k) \\ p_b(k) \\ p_c(k) \\ p_d(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 1.0 & 0 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0.1 \\ 0 & 0.2 & 0 & 0.9 \end{bmatrix}^k \begin{bmatrix} p_a(0) \\ p_b(0) \\ p_c(0) \\ p_d(0) \end{bmatrix}$$

Steady State Dist.

$$\begin{bmatrix} p_a(\infty) \\ p_b(\infty) \\ p_c(\infty) \\ p_d(\infty) \end{bmatrix} = \begin{bmatrix} 0.71 \\ 0.07 \\ 0.07 \\ 0.14 \end{bmatrix}$$



Update Equation:

Steady State Dist.

$$\begin{bmatrix} p_a(k) \\ p_b(k) \\ p_c(k) \\ p_d(k) \\ p_e(k) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0.7 & 0 & 0.8 \\ 0.1 & 0 & 0 & 0 & 0.2 \\ 0 & 0.8 & 0 & 0.1 & 0 \\ 0 & 0.2 & 0 & 0.6 & 0 \\ 0 & 0 & 0.3 & 0.3 & 0 \end{bmatrix}^k \begin{bmatrix} p_a(0) \\ p_b(0) \\ p_c(0) \\ p_d(0) \\ p_e(0) \end{bmatrix} = \begin{bmatrix} p_a(\infty) \\ p_b(\infty) \\ p_c(\infty) \\ p_d(\infty) \\ p_e(\infty) \end{bmatrix} = \begin{bmatrix} 0.77 \\ 0.08 \\ 0.07 \\ 0.04 \\ 0.03 \end{bmatrix}$$

