Introduction

Dan Calderone

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danjcalderone.github.io

Post-doctoral scholar (Prof. Ratliff's group)
University of Washington

PhD: Berkeley, (under Shankar Sastry, 2017)
PostDoc: in AA & EE at UW (Ratliff, Ackimese, 2018-2019)
Lecturer: AA & EE at UW (2019-2022)

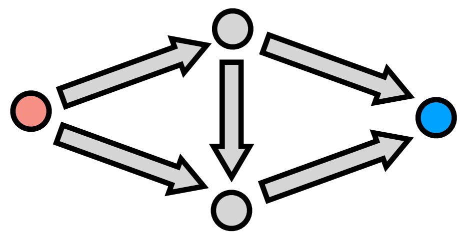
Research Interests:
Game theory & optimization
applied to transportation networks

Personal Interests: Math visualization

danjcalderone.github.io/teaching
danjcalderone.github.io/dcmath

Potential Games

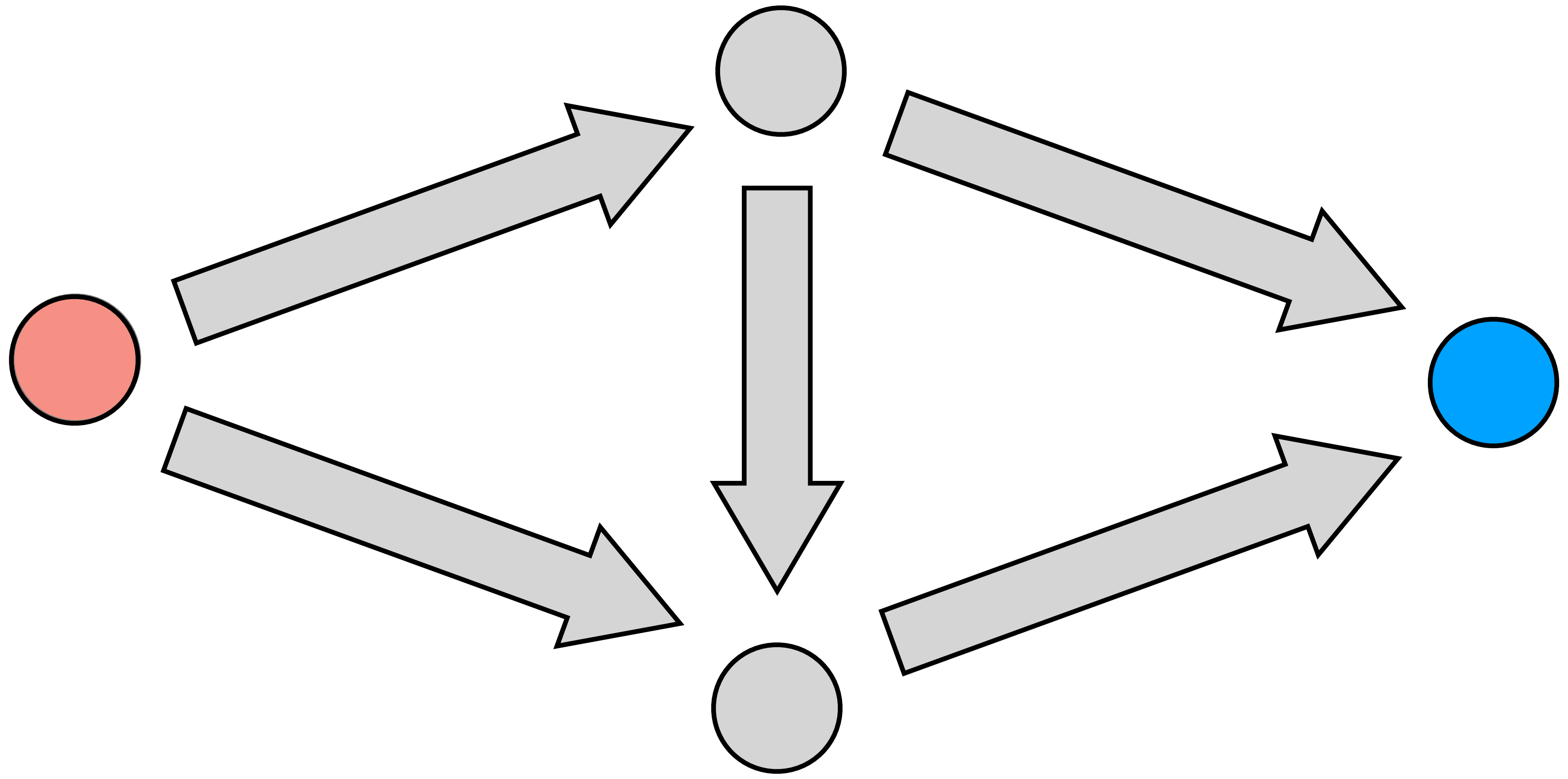
Routing Games



Routing Games

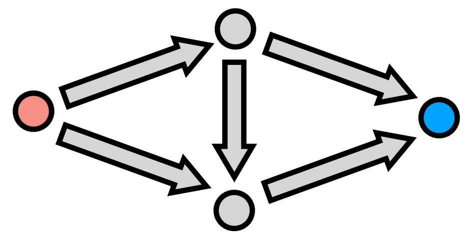
x : edge traffic

z : route traffic



Potential Games

Routing Games



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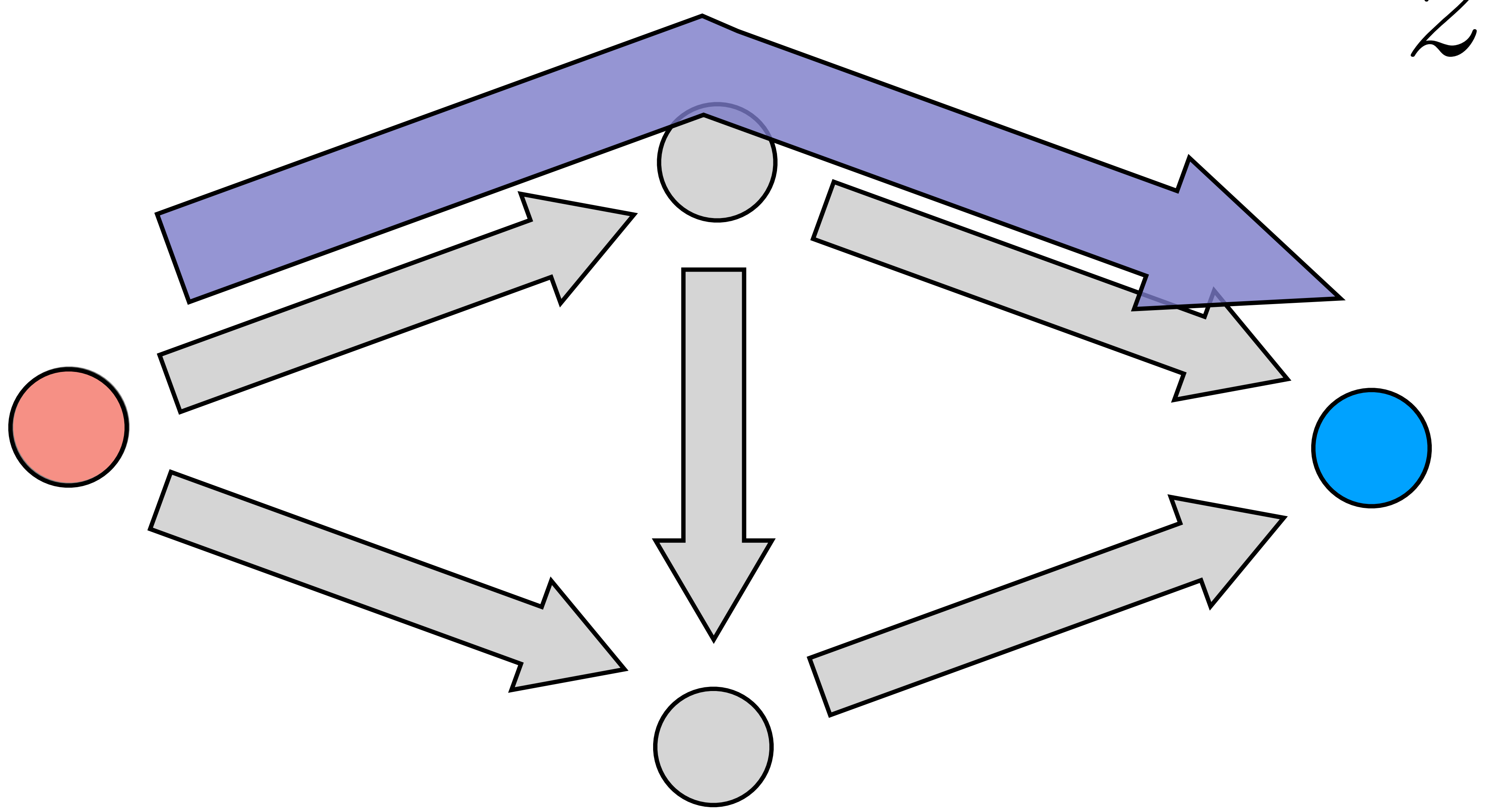
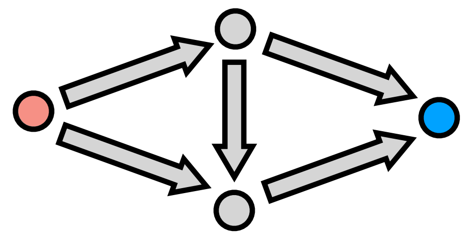


Potential Games

Routing Games



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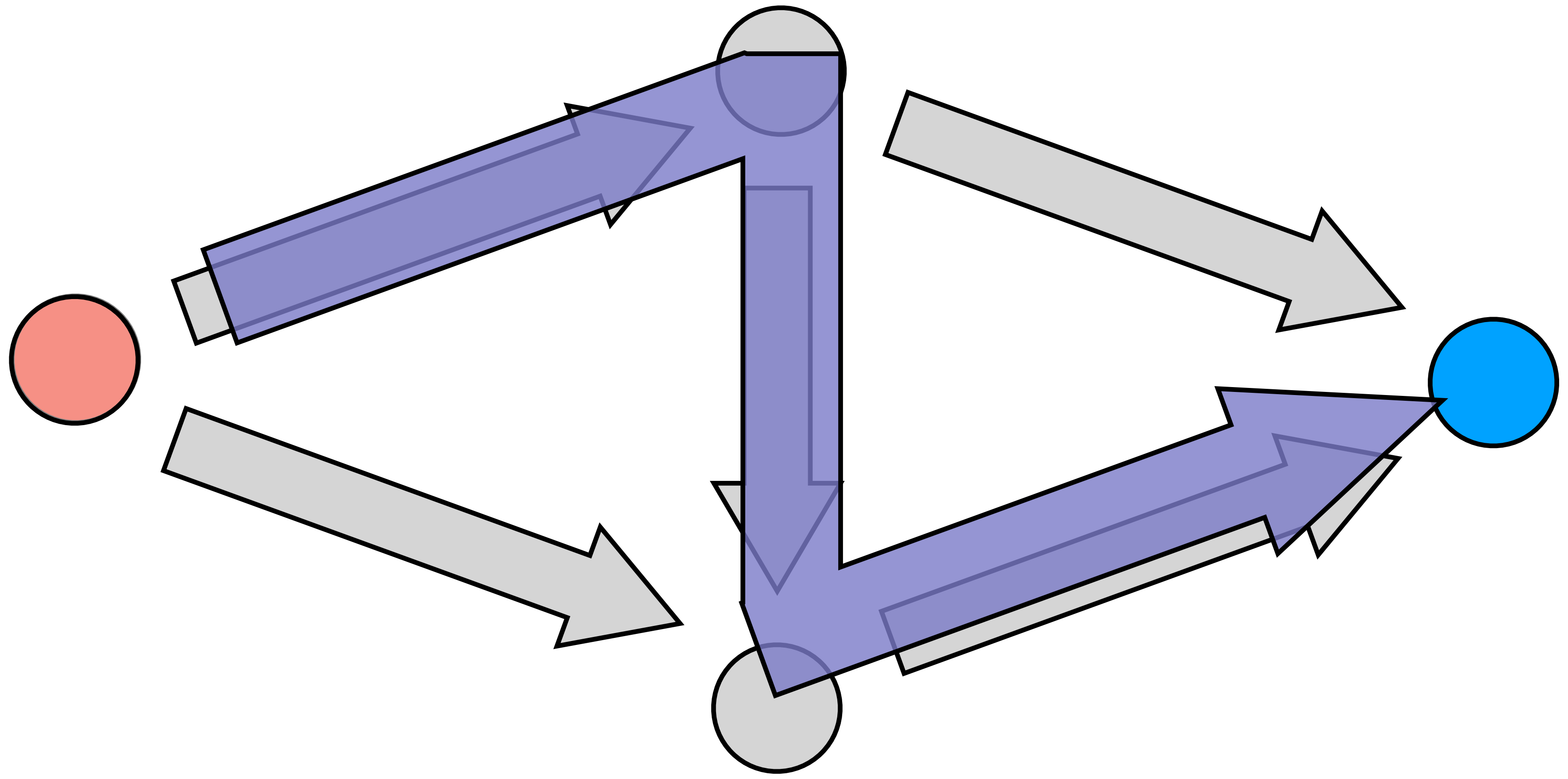
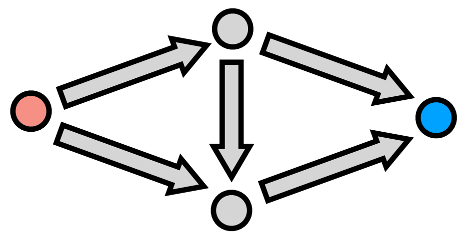


Potential Games

Routing Games



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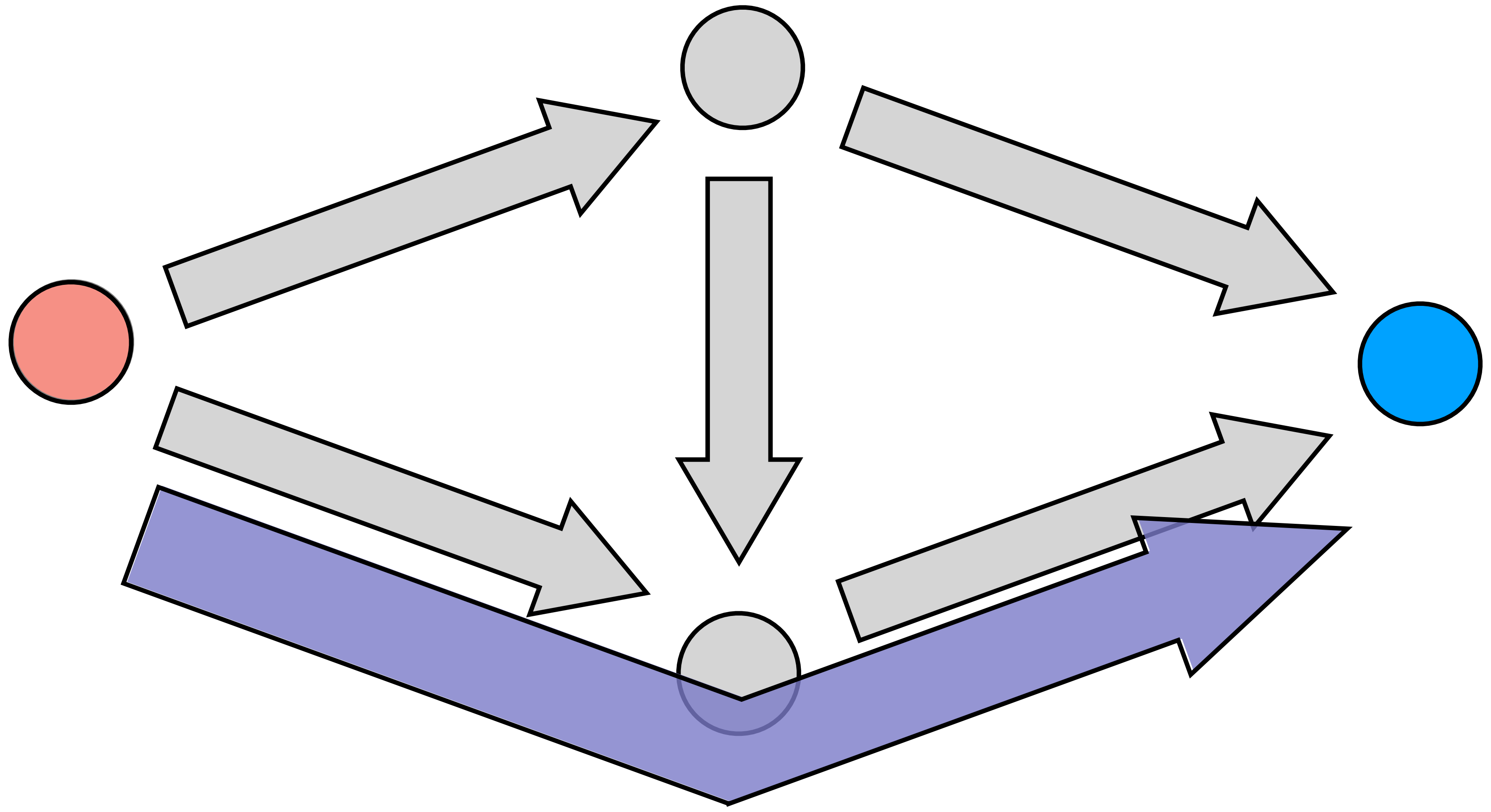
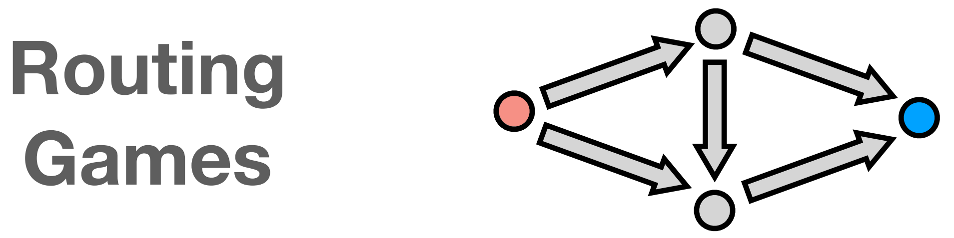


Potential Games



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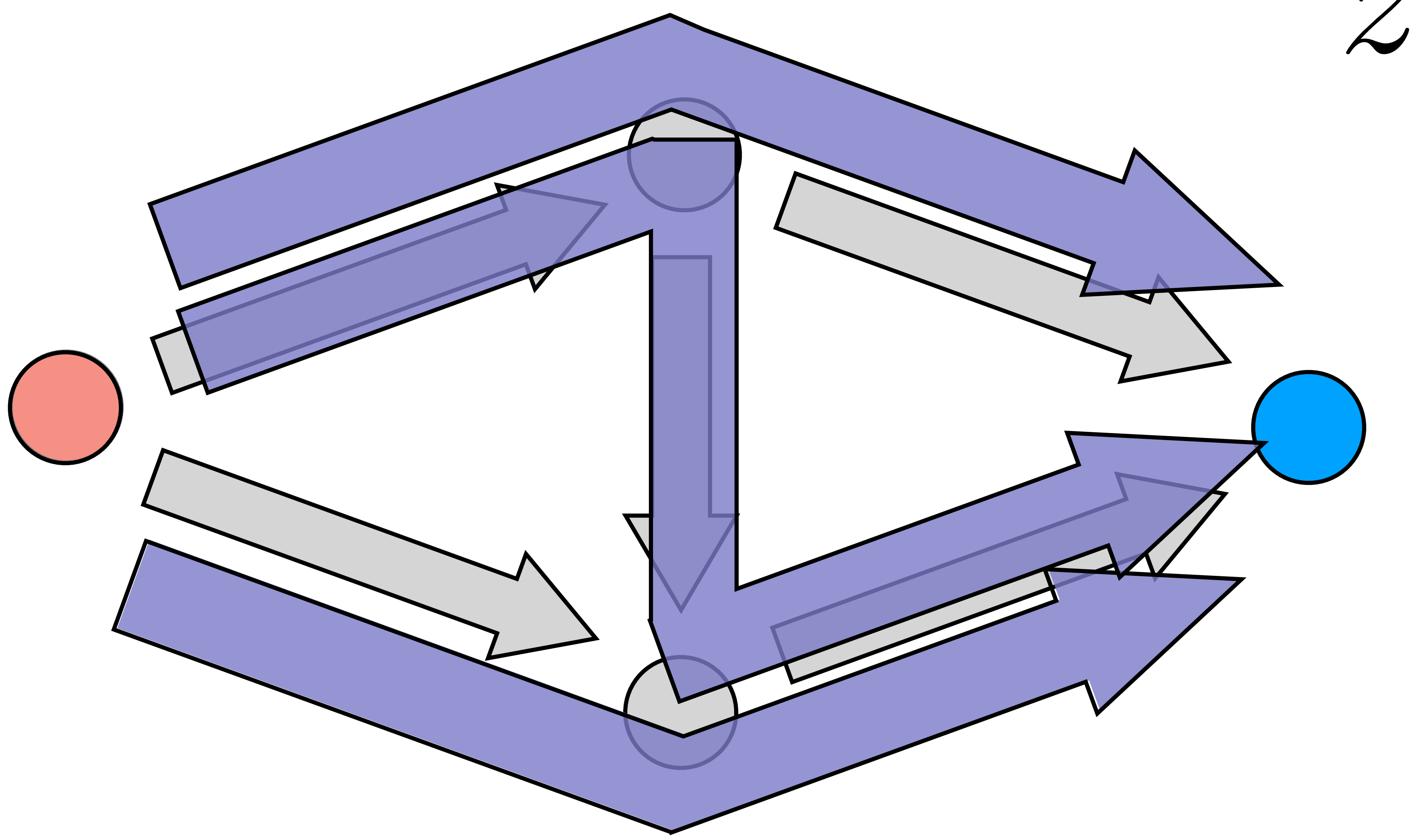
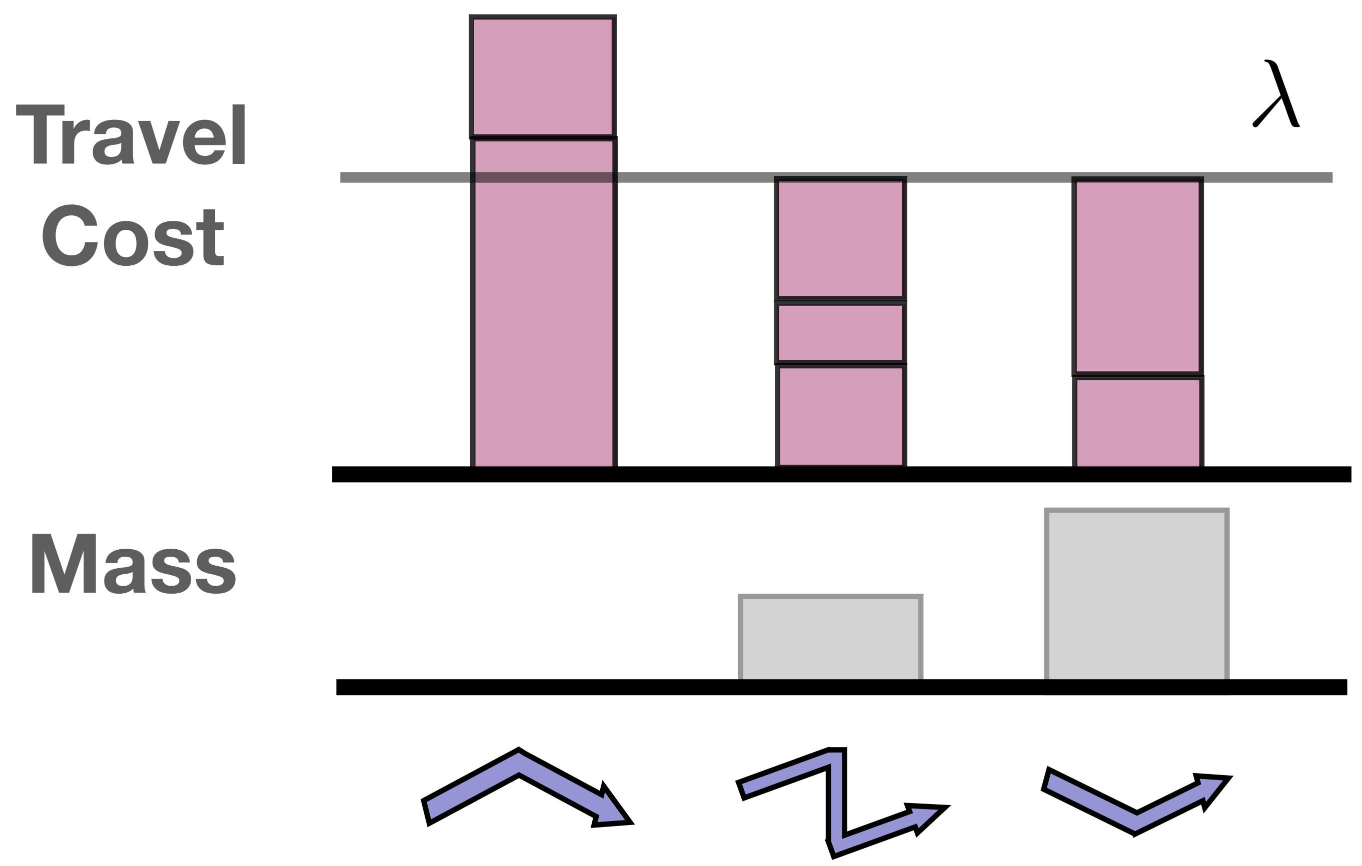
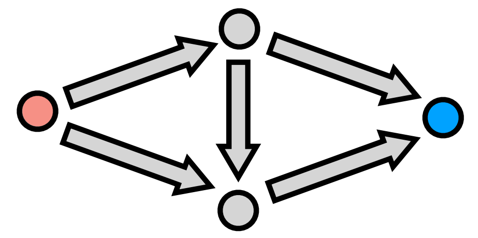


Wardrop Equilibrium



Potential Games

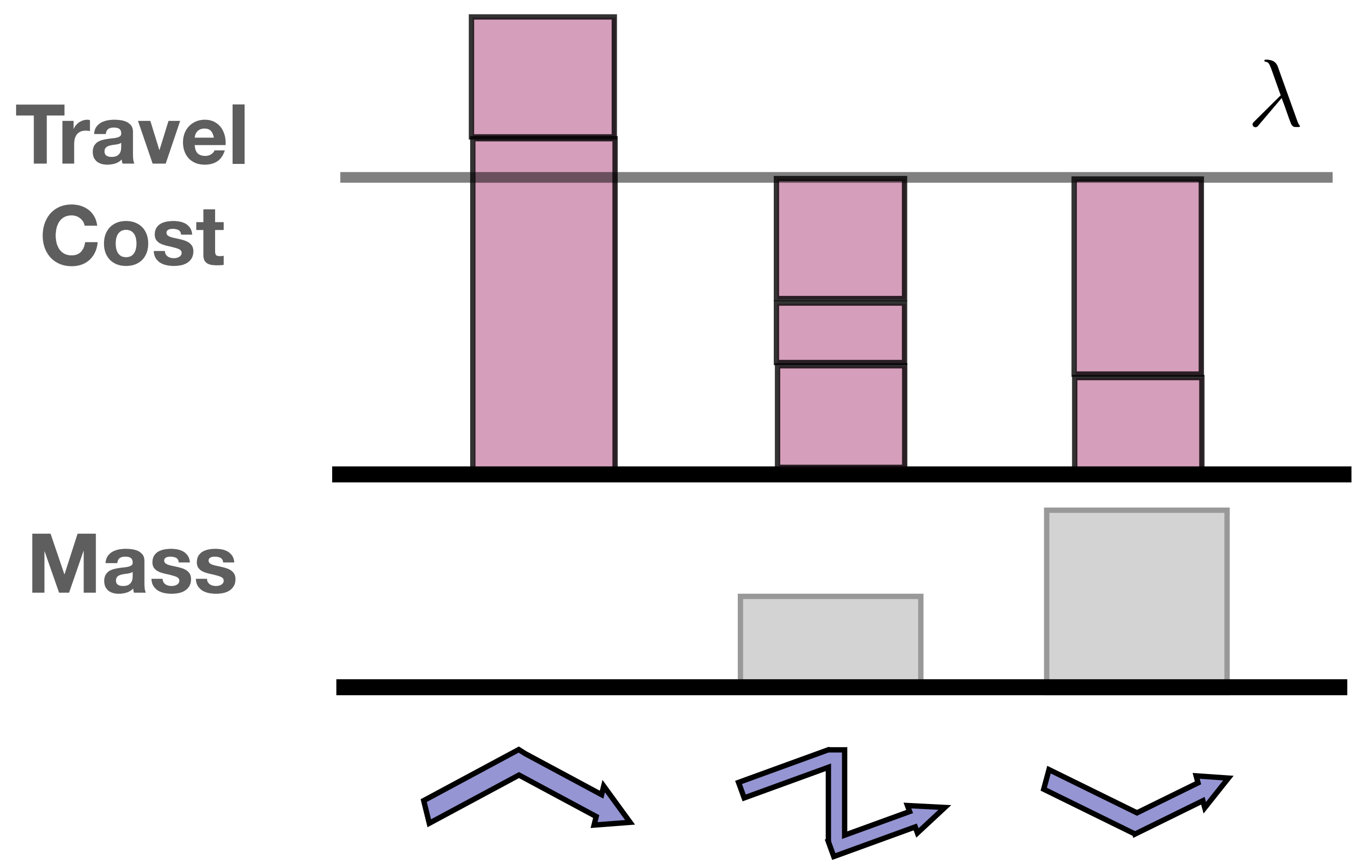
Routing Games



Potential Function

$$F(x)$$

Wardrop Equilibrium



Routing Games

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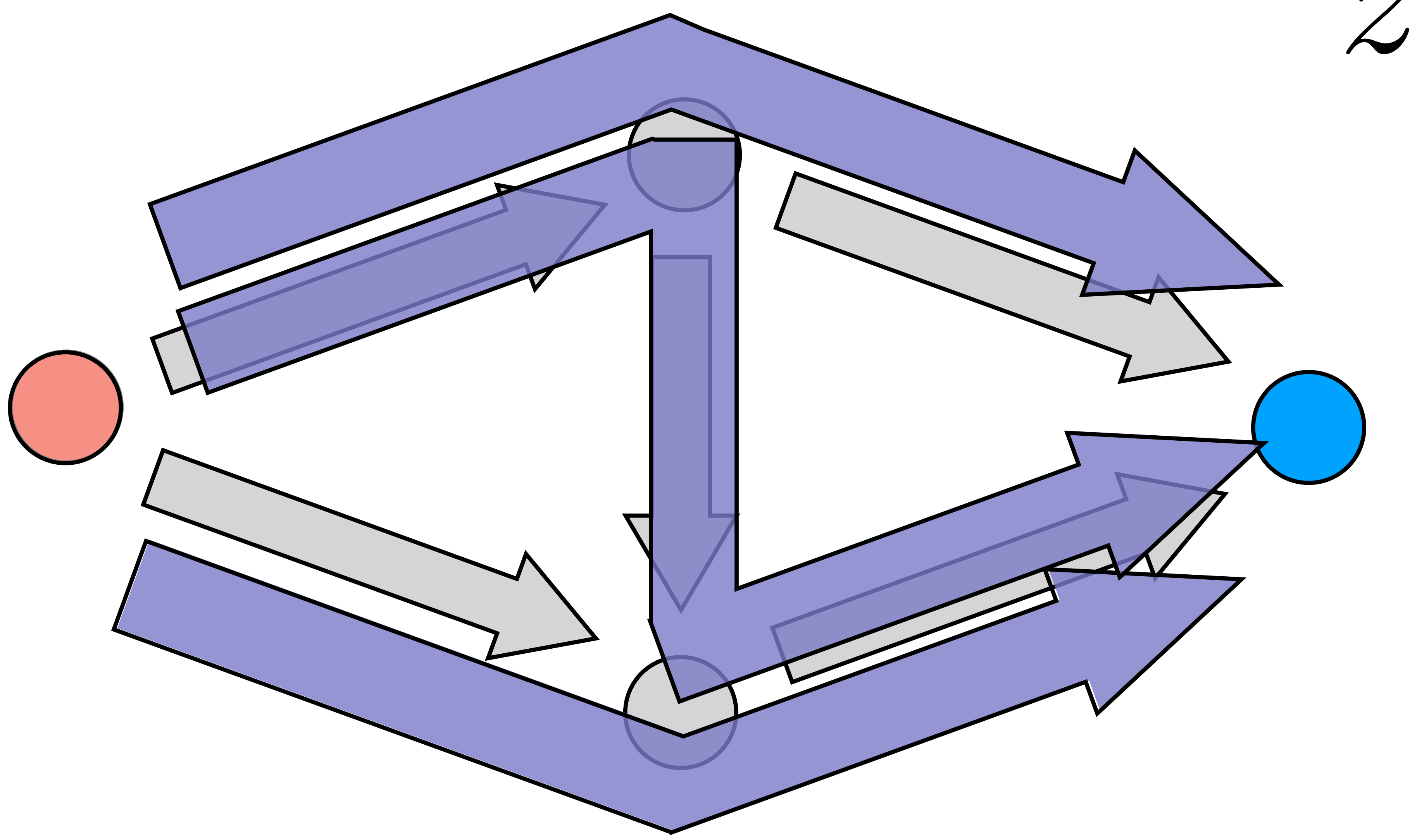
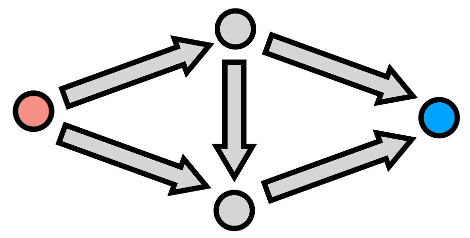


Potential Games

Routing Games



Potential Function $F(x)$

Routing Games

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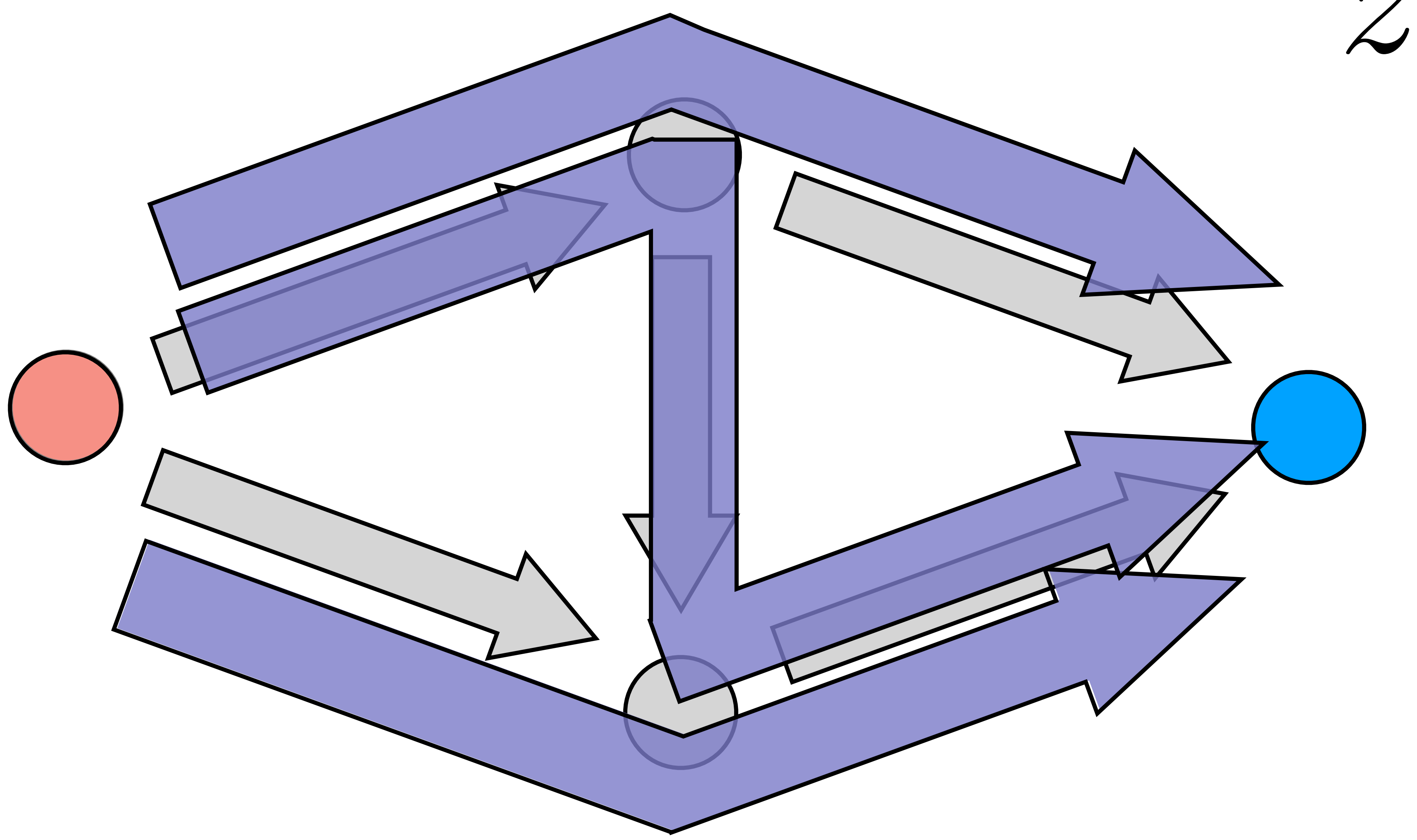
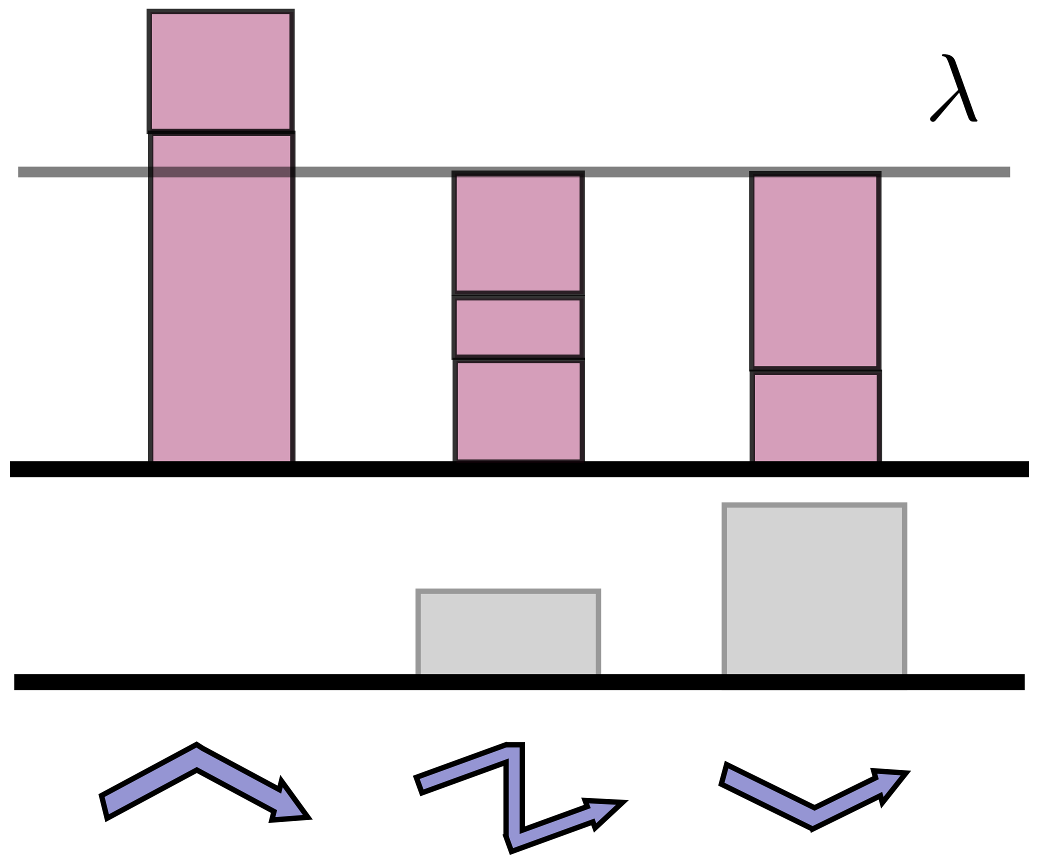


Wardrop Equilibrium



$$\min_{x,z} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T z = m \qquad z \geq 0$$

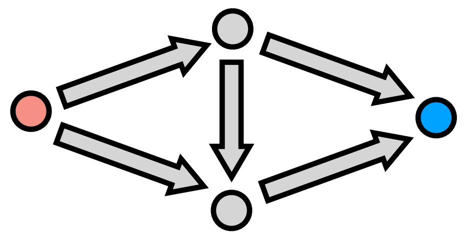
Mass conservation

$$x = \mathbf{R} z$$

Edges in routes

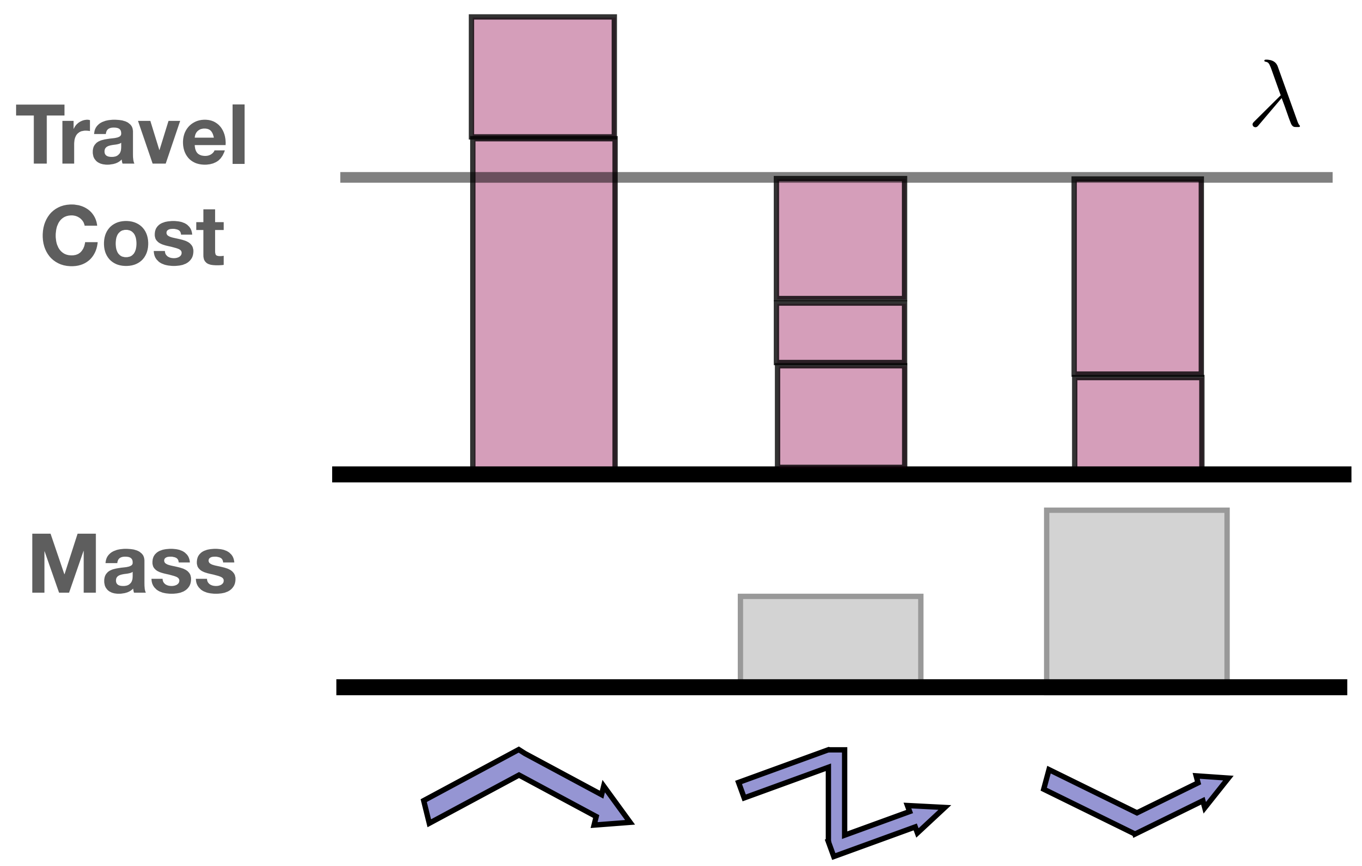
Potential Games

Routing Games



Potential Function $F(x)$

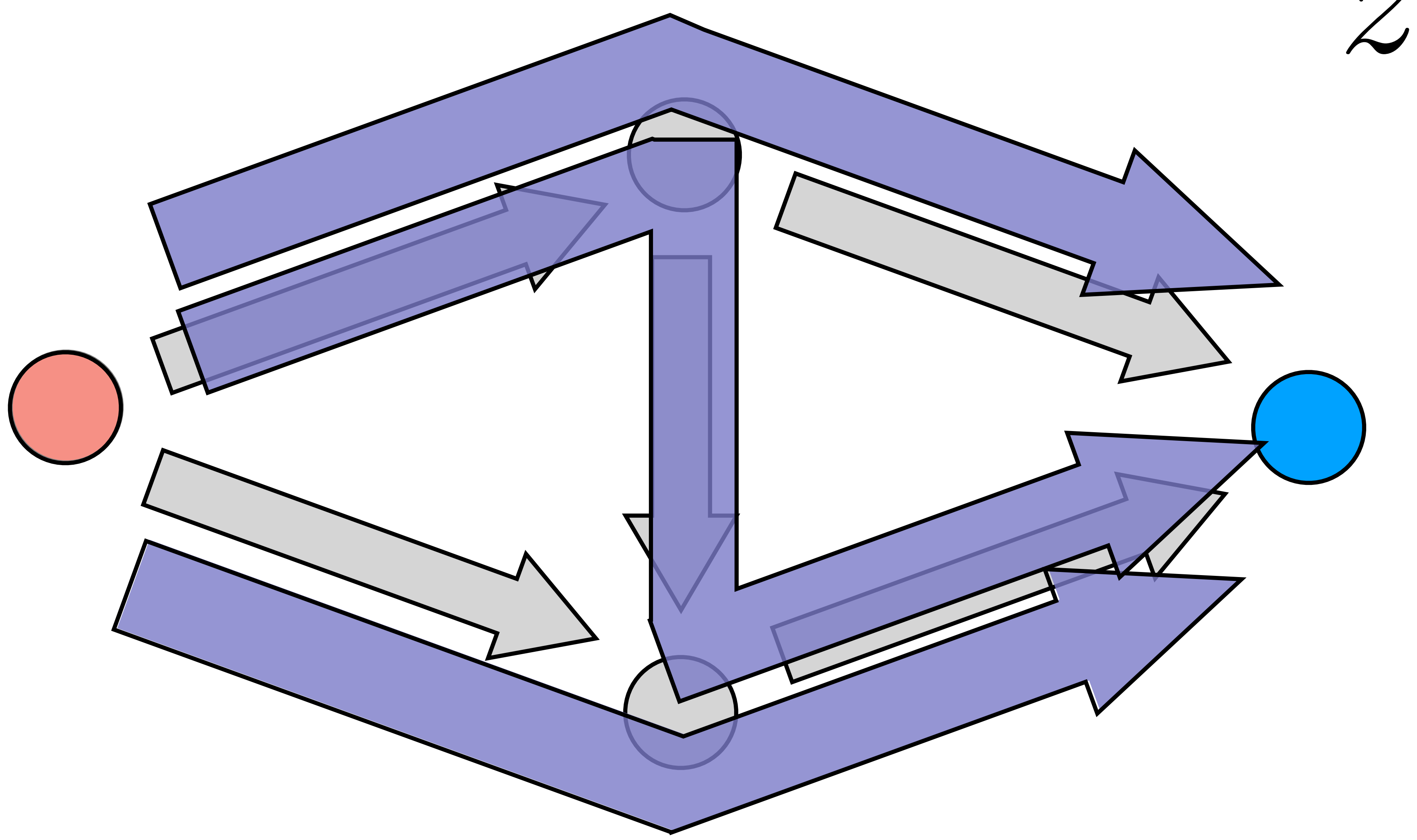
Wardrop Equilibrium



Routing Games

x : edge traffic

z : route traffic



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Mass conservation

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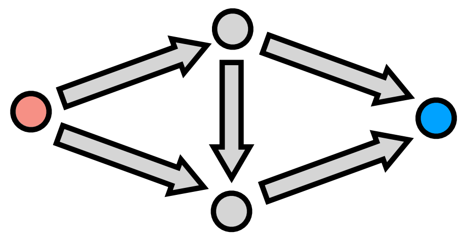
Edges in routes

Potential Games

Routing Games



Potential Function $F(x)$

Routing Games

x : edge traffic

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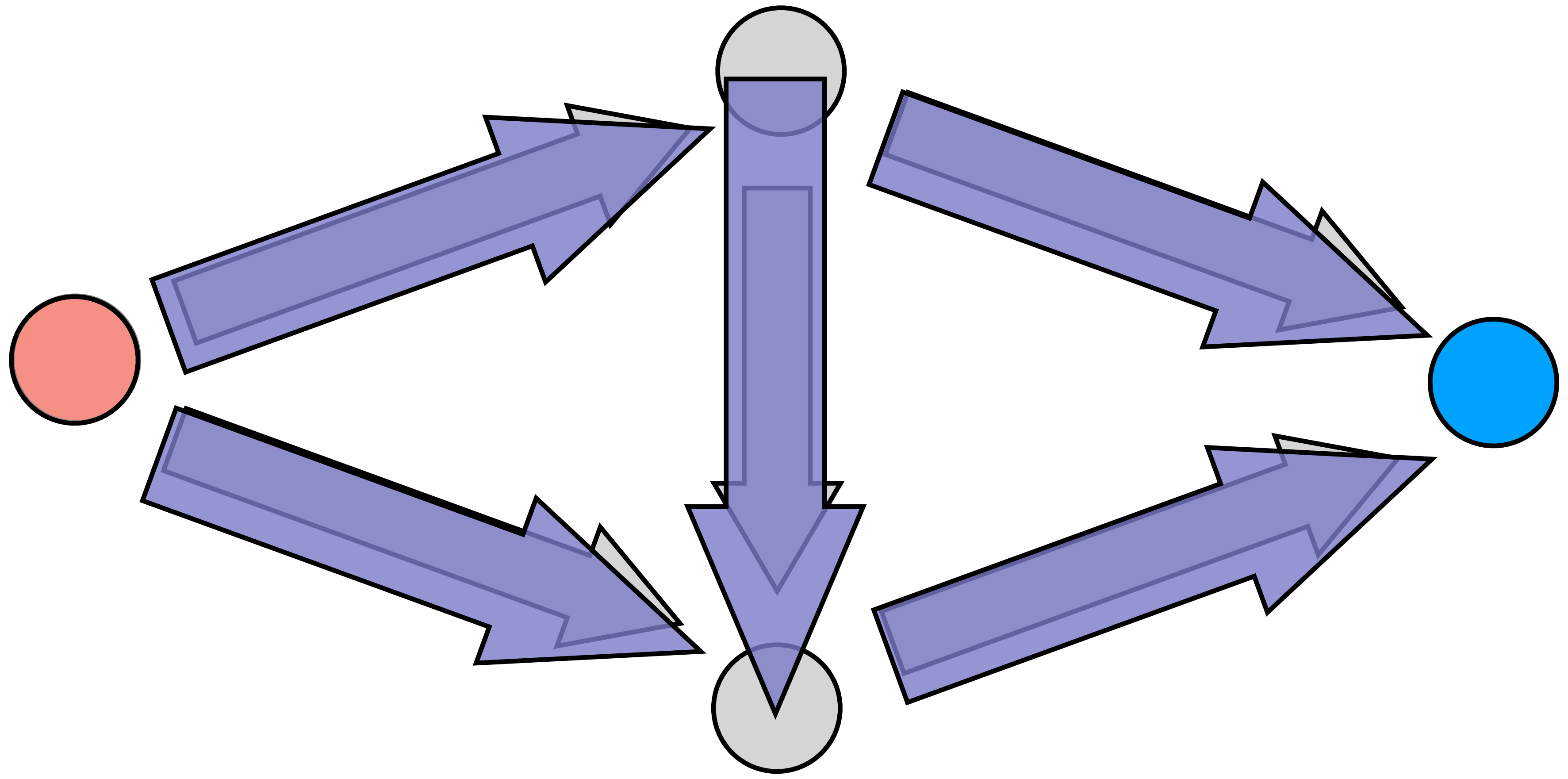
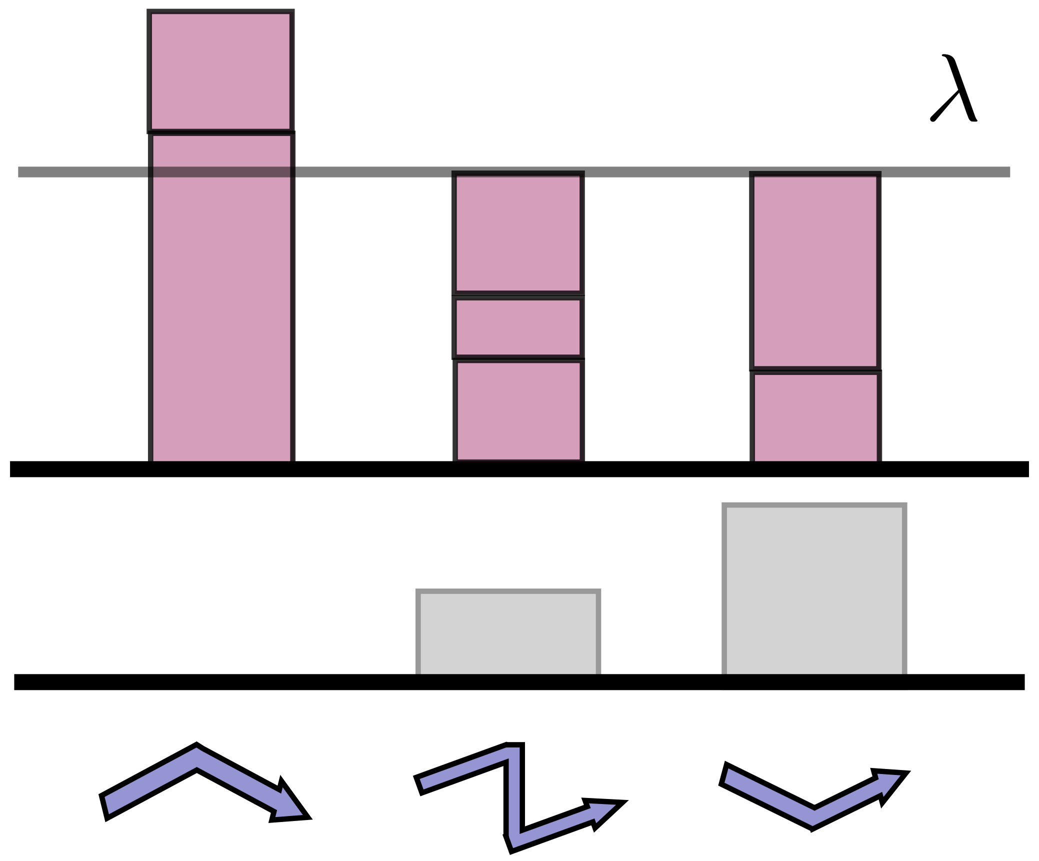


Wardrop Equilibrium

Travel Cost

Mass



$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad Ex = Sm, \quad x \geq 0$$

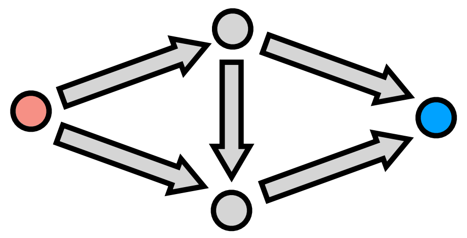
Mass conservation

Graph structure

Origin-destination

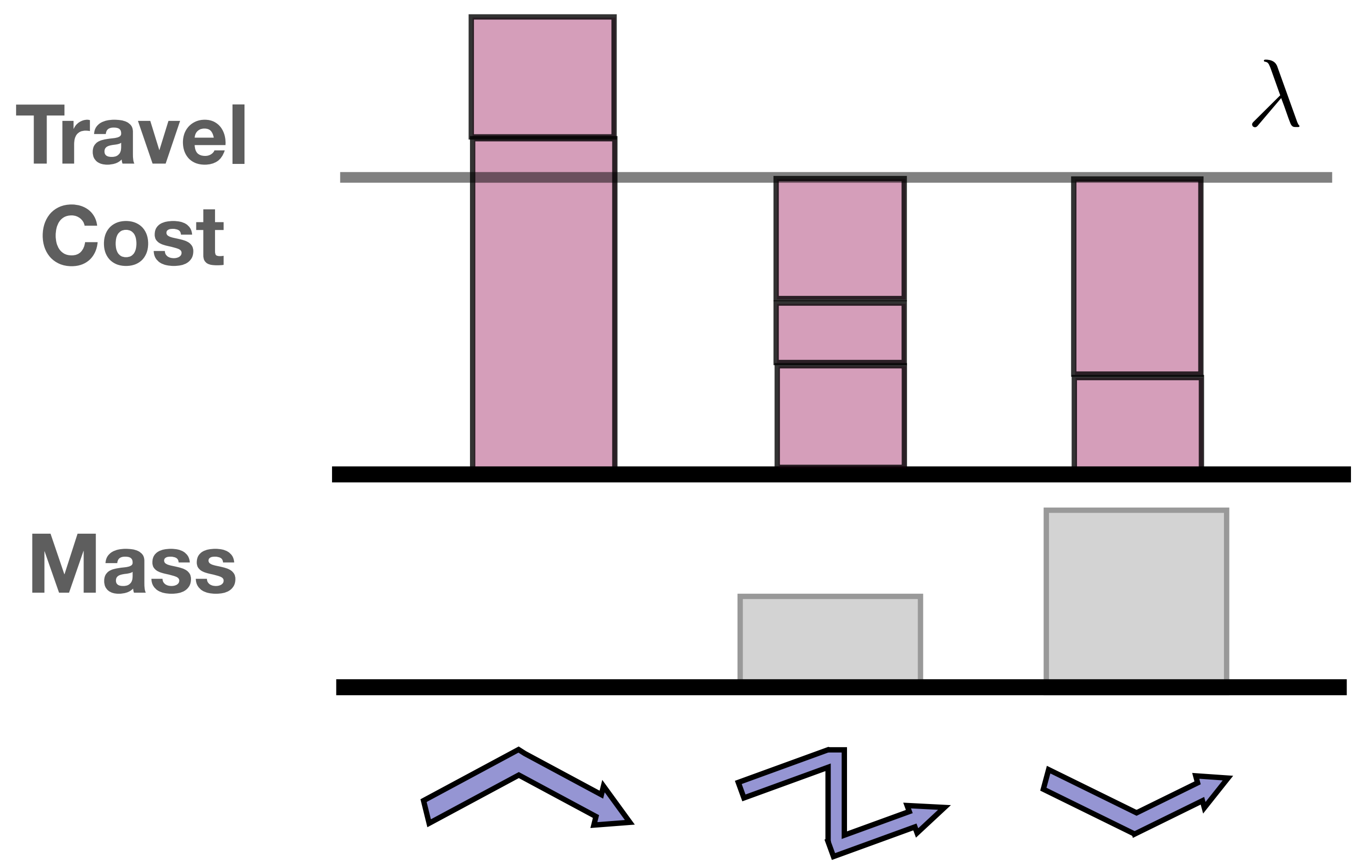
Potential Games

Routing Games



Potential Function $F(x)$

Wardrop Equilibrium



Routing Games

x : edge traffic

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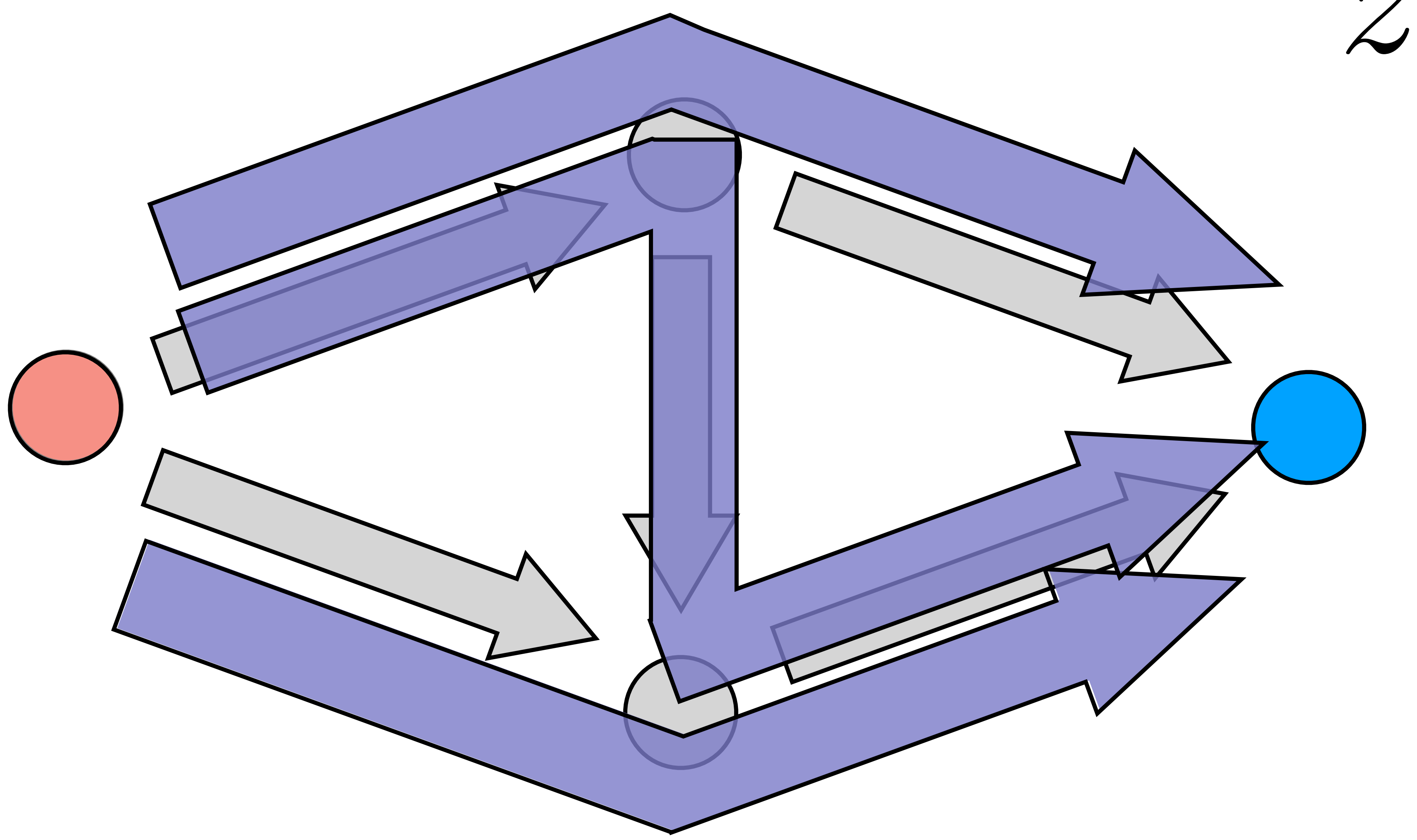


$$\min_{x,z} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T z = m \;\; (\lambda) \qquad z \geq 0 \;\; (\nu)$$

$$x = \mathbf{R} z \;\; (w)$$

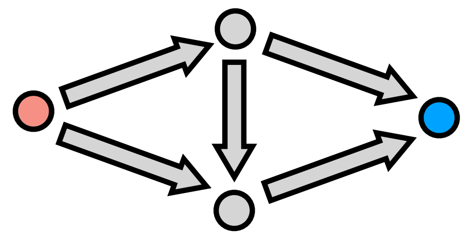
λ : travel cost

ν : route inefficiency

w : edge costs

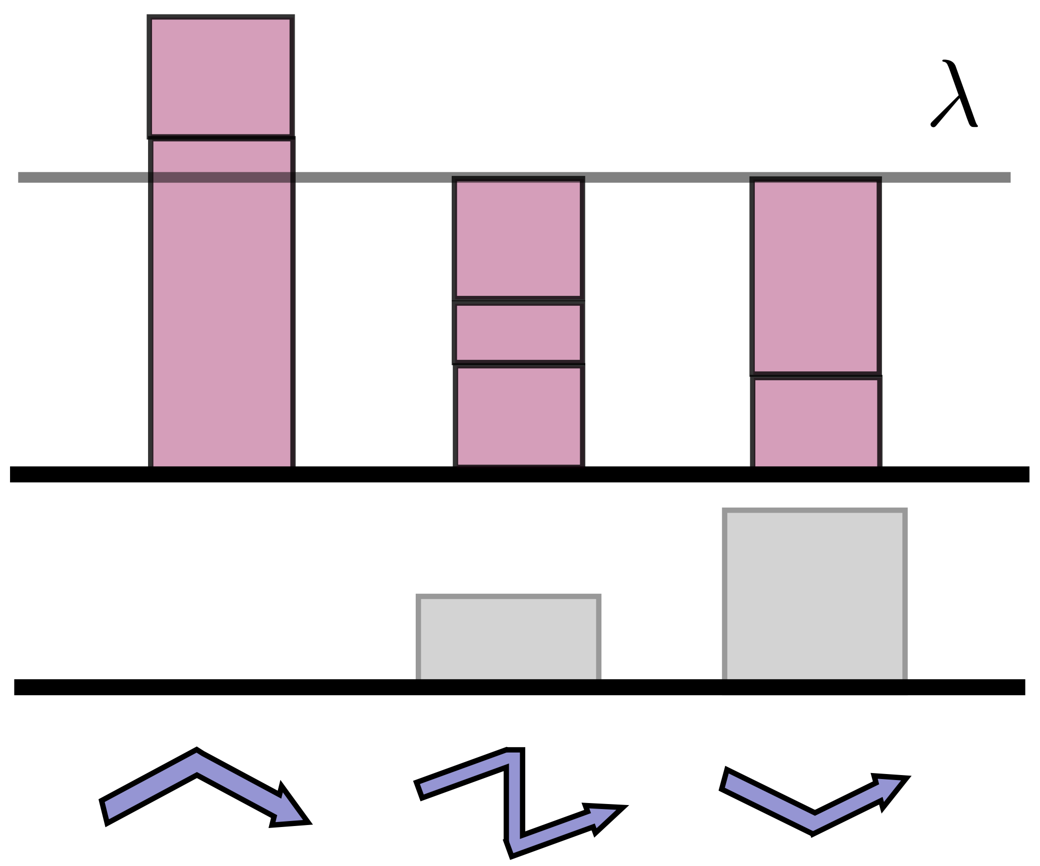
Potential Games

Routing Games



Potential Function $F(x)$

Wardrop Equilibrium



Routing Games

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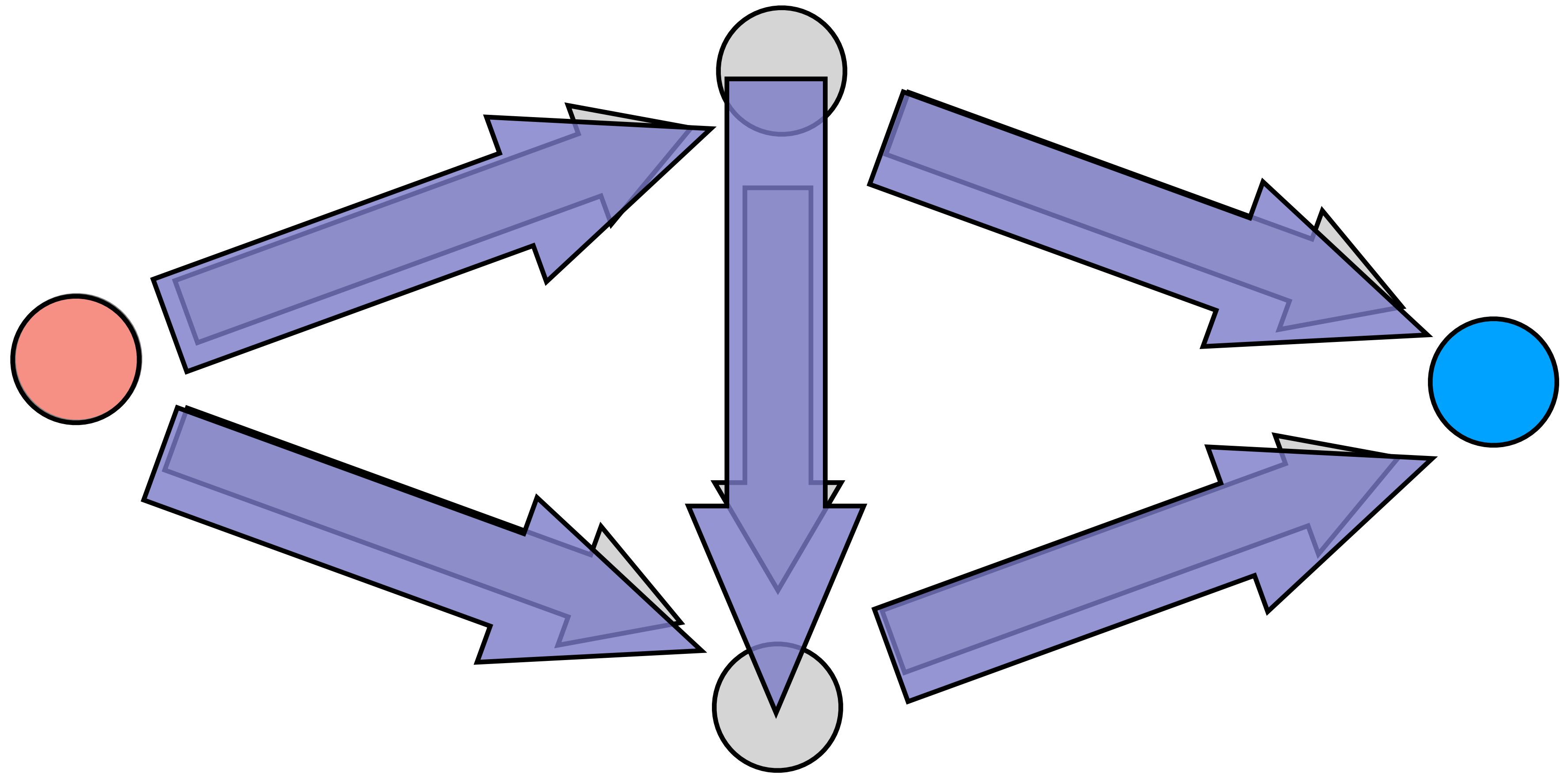


$$\min_{x} \quad F(x)$$

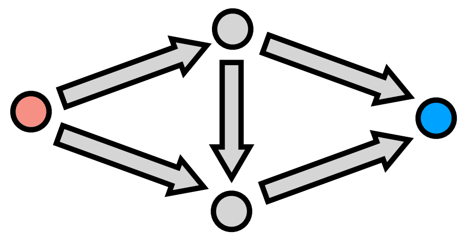
$$\text{s.t.} \quad Ex = Sm, \; v \qquad x \geq 0 \; \mu$$

μ : edge inefficiency

v : value function

Potential Games

Routing Games

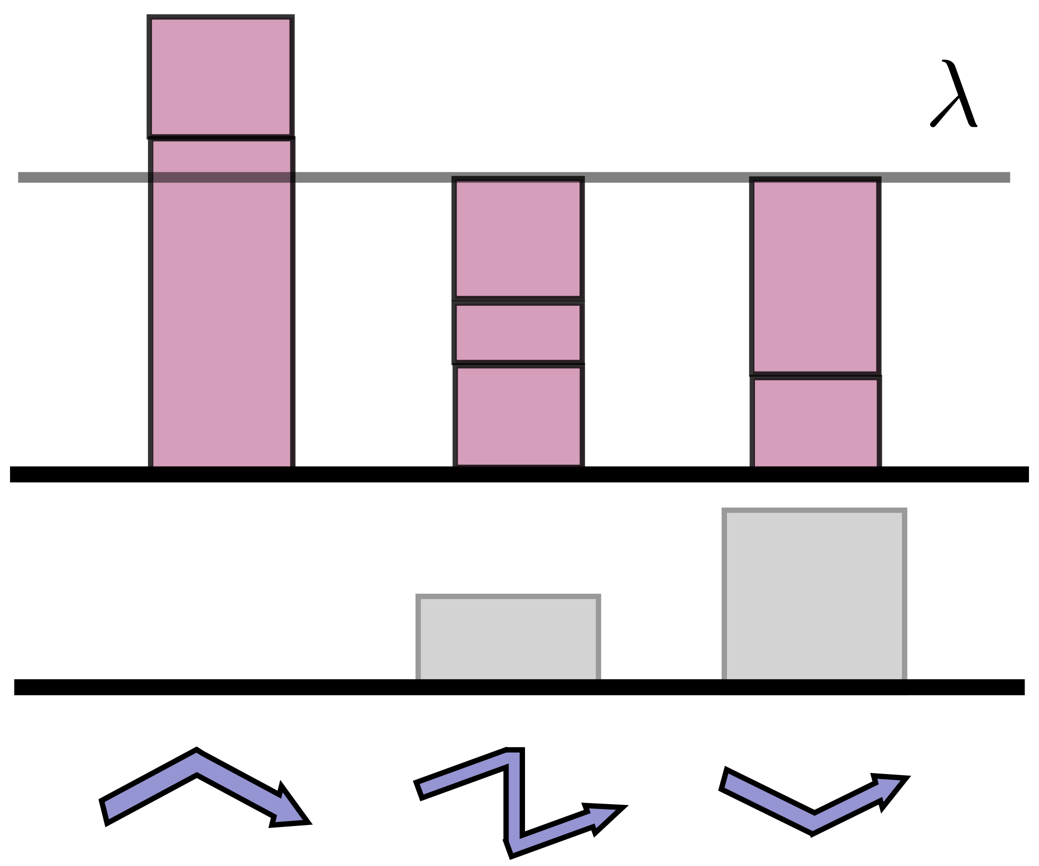


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

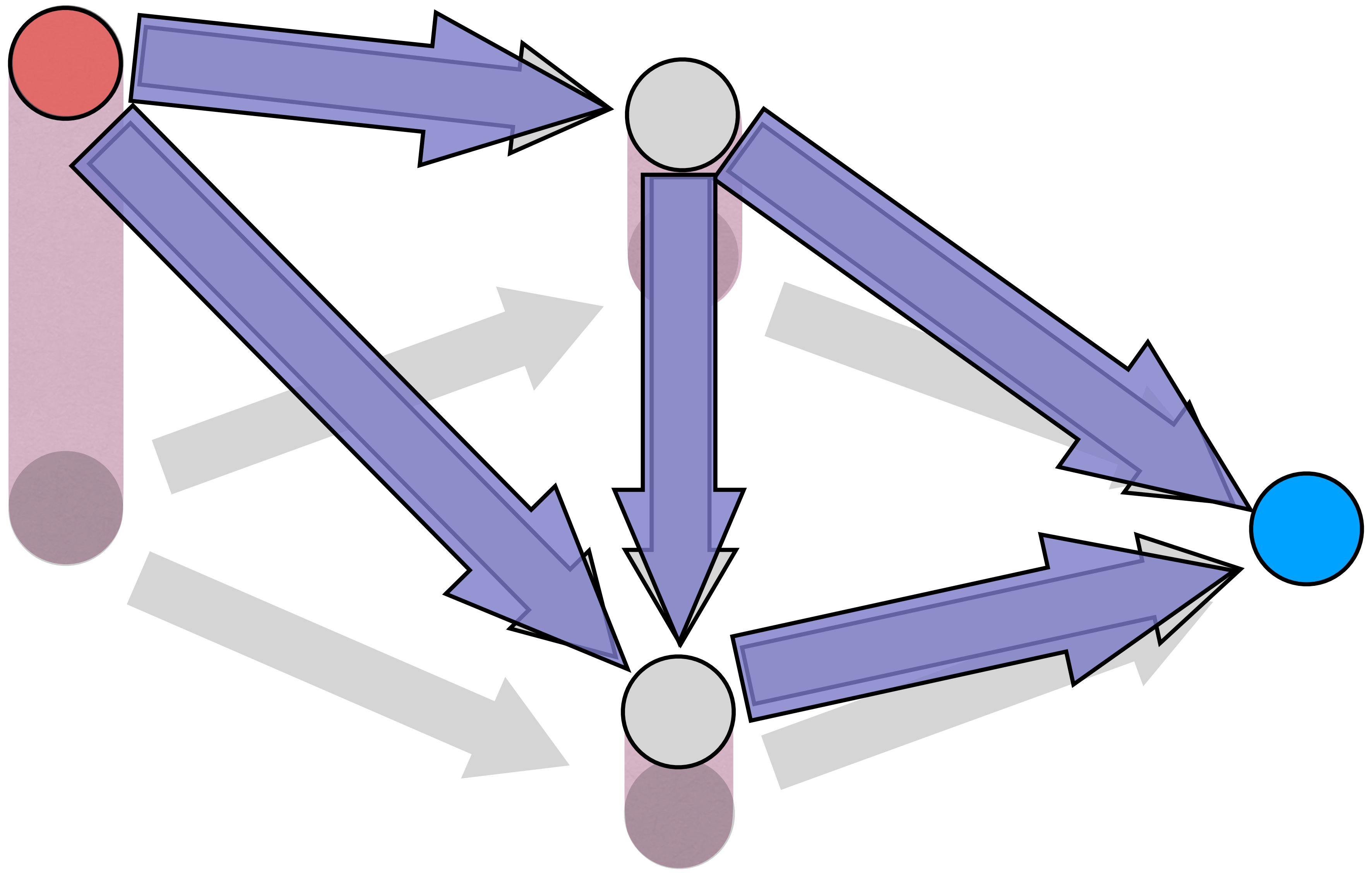
Mass



Routing Games

x : edge traffic

z : route traffic



$$\min_{x} \quad F(x)$$

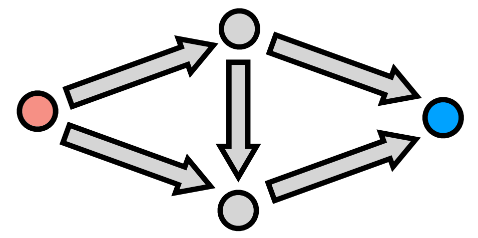
$$\text{s.t.} \quad Ex = Sm, \; (v) \qquad x \geq 0 \; (\mu)$$

μ : edge inefficiency

v : value function

Potential Games

Routing Games

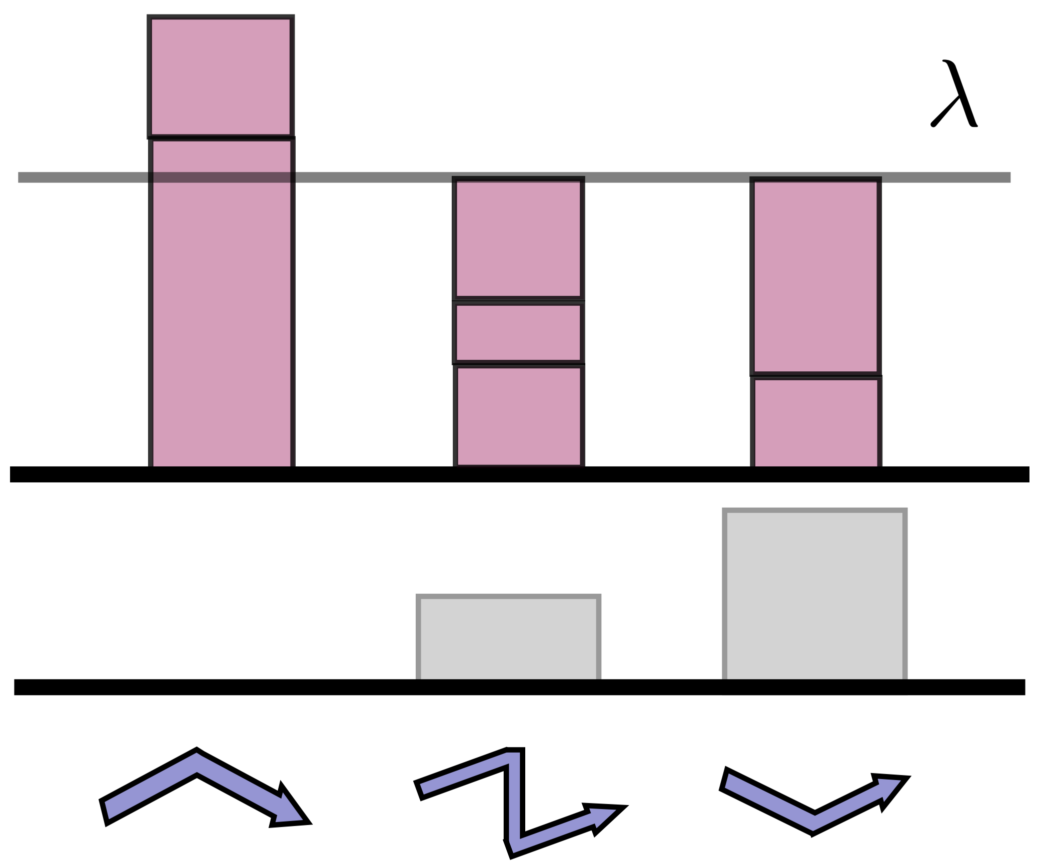


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

Mass



Routing Games

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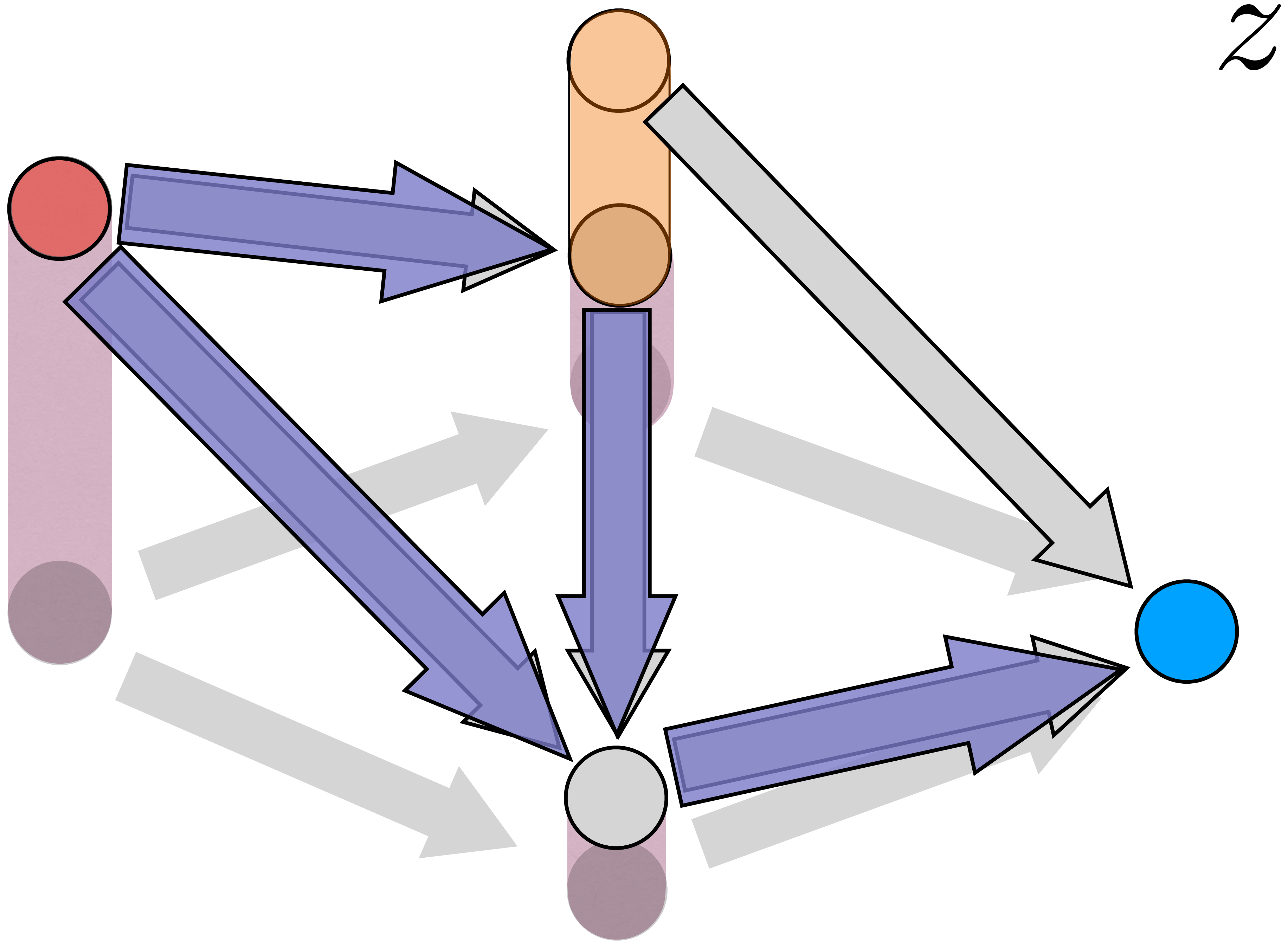

$$\min_{x} \quad F(x)$$

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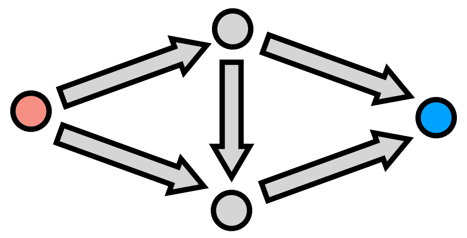
Potential Games

Routing Games



Potential Function

$$F(x)$$

Wardrop Equilibrium

Travel Cost

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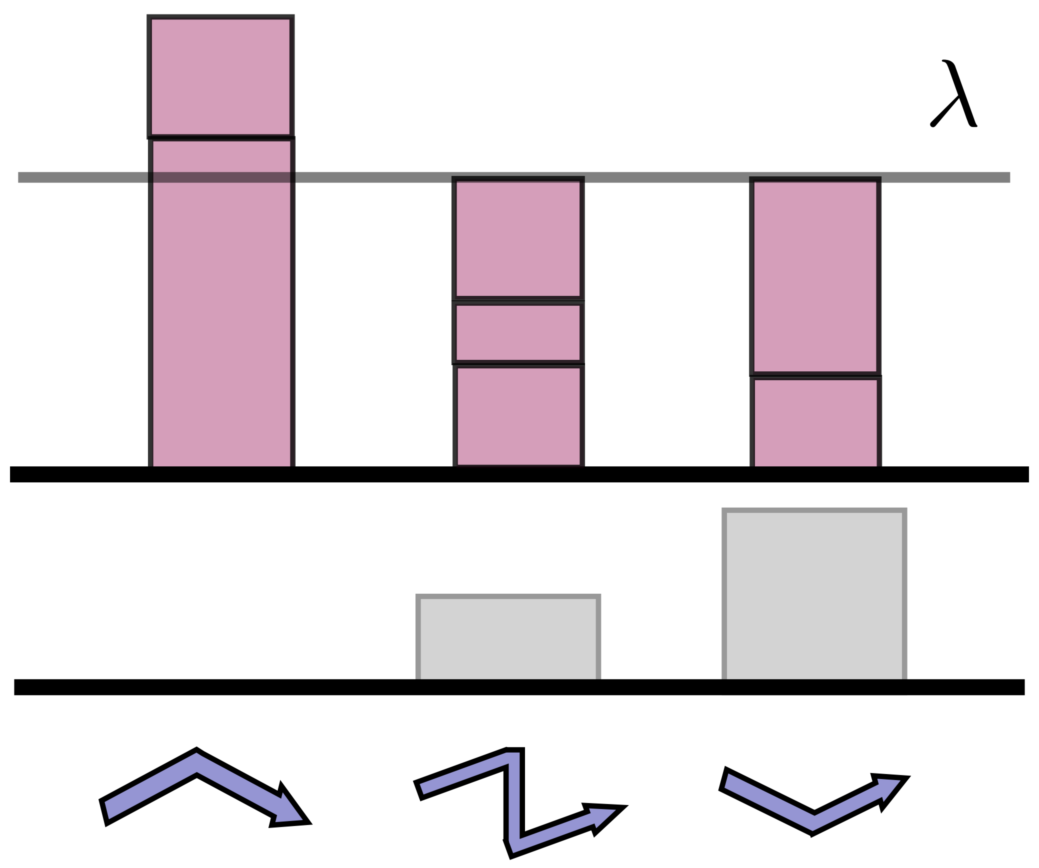

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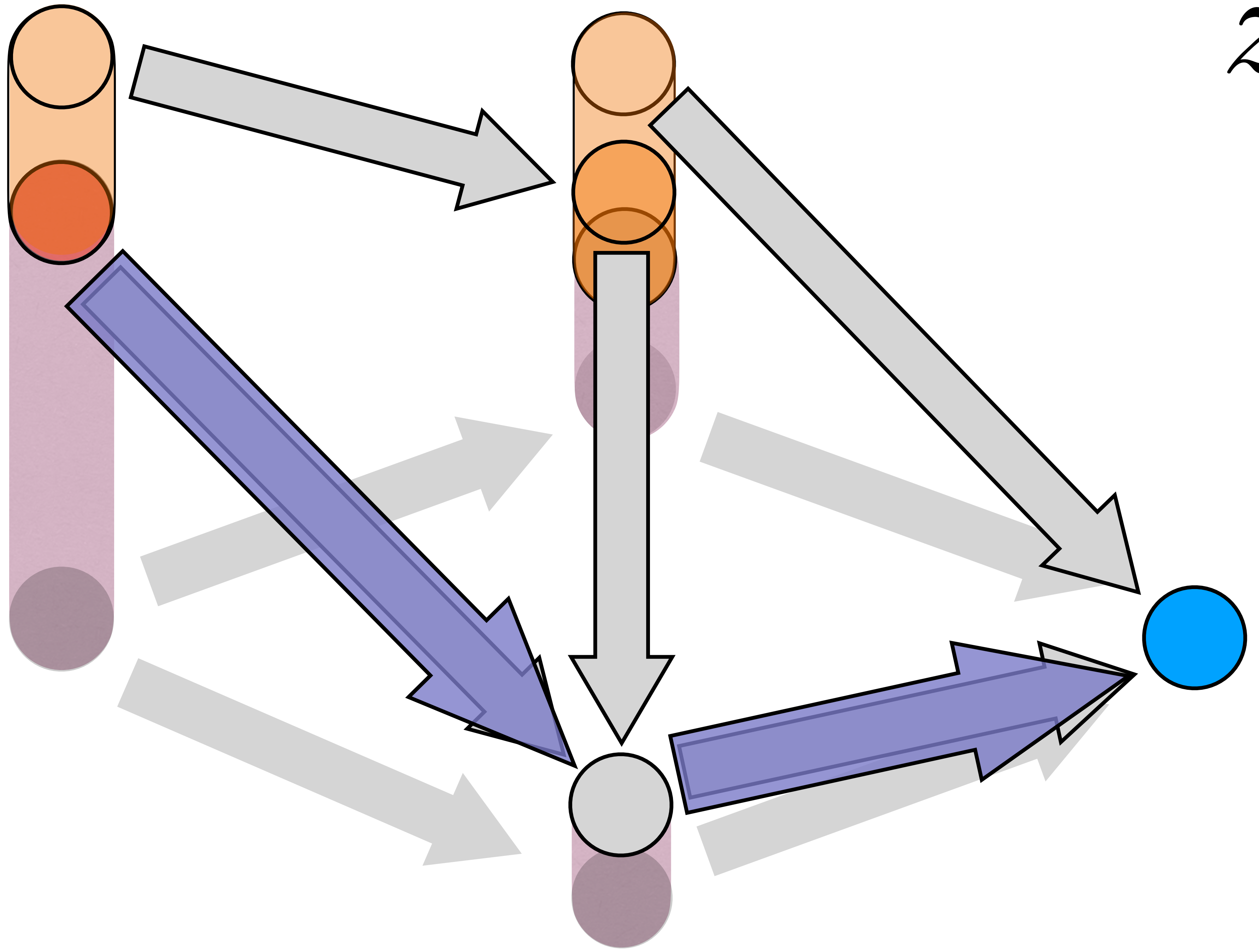

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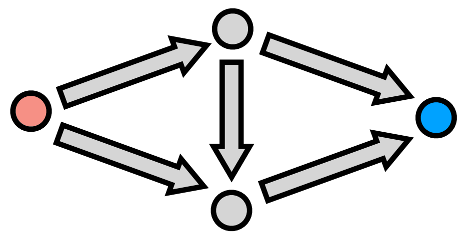
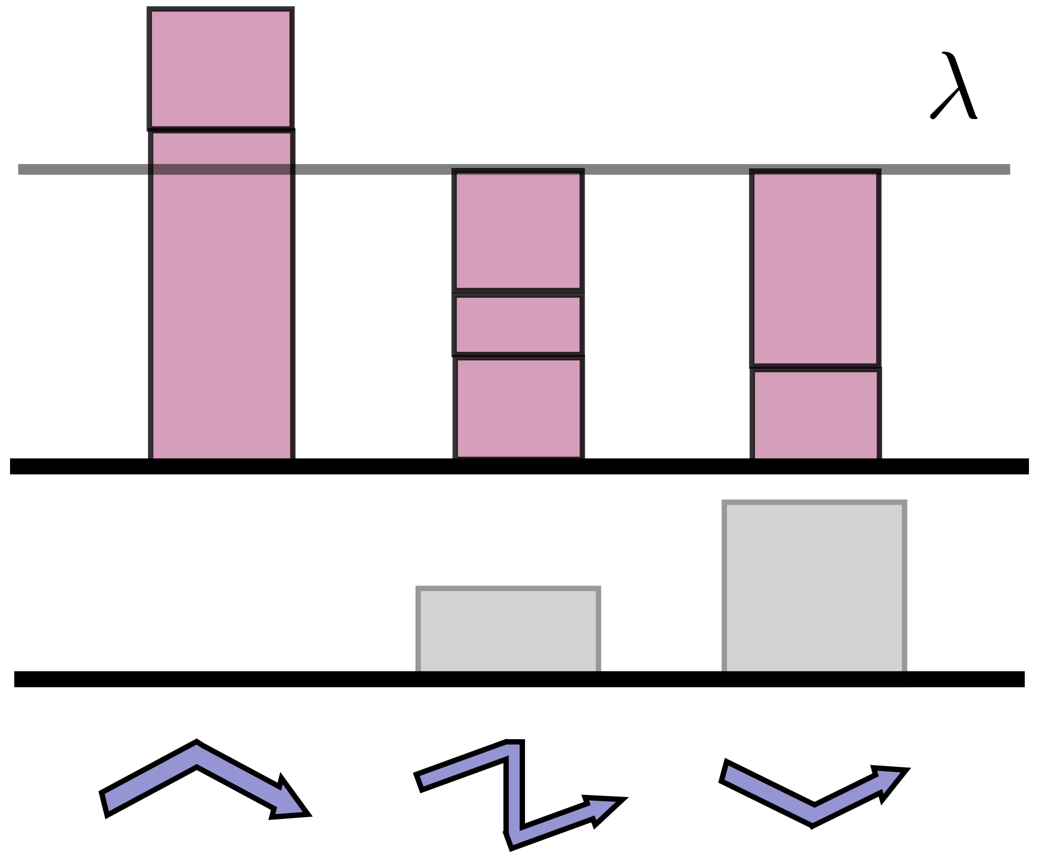
Potential Games

Routing Games



Potential Function $F(x)$

Wardrop Equilibrium



Routing Games

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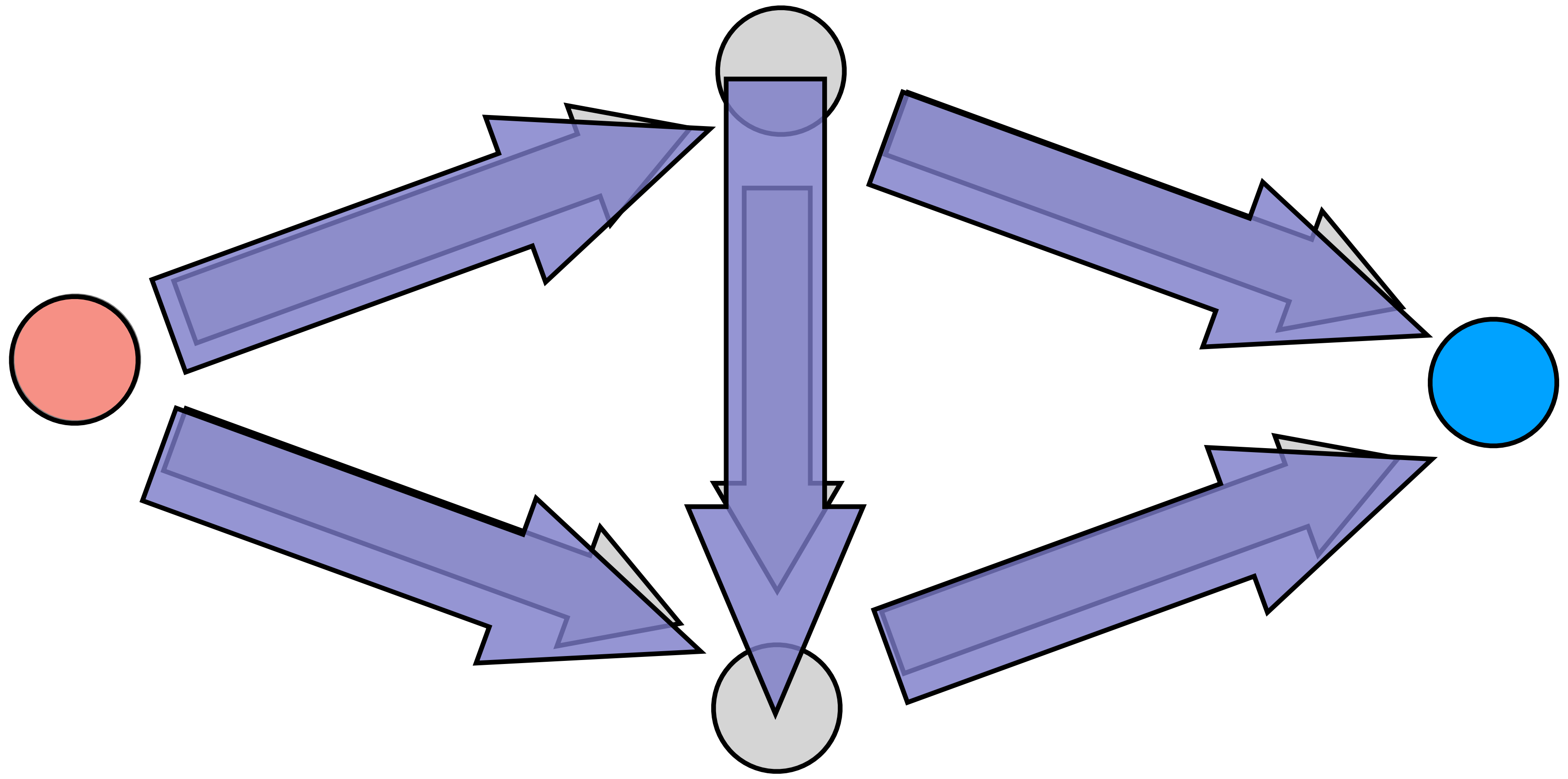


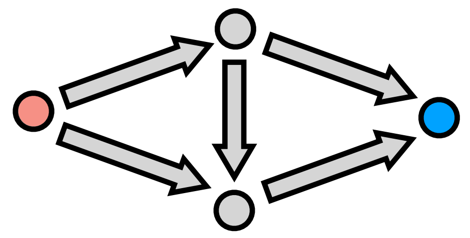
REFERENCES

- Some theoretical aspects of road traffic research **[Wardrop, 1952]**
- Studies in the economics of transportation **[Beckmann, McGuire, Winsten, 1956]**

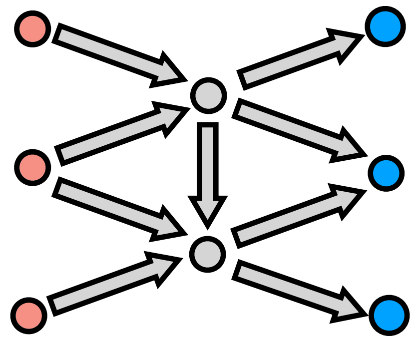
- The Traffic Assignment Problem: Models and Methods **[Patriksson, 2015]**

Potential Games

Routing Games



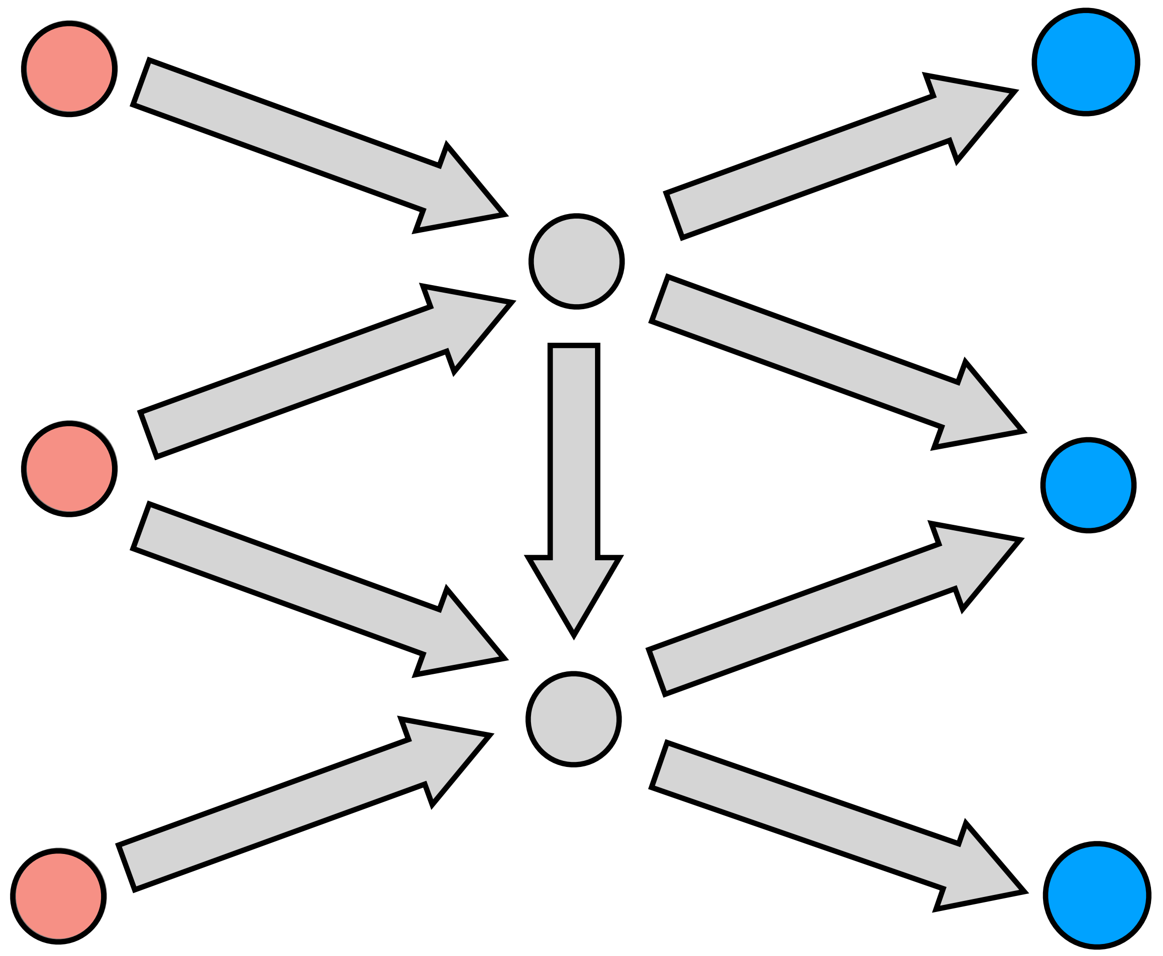
Multiple sources/ sinks



Multiple Source/Sinks

x : edge traffic

z : route traffic

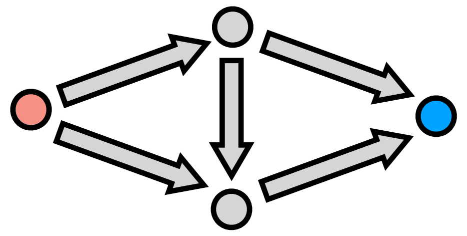


$$\min_x \quad F(x)$$

$$\text{s.t.} \quad \boxed{E_i x_i = S_i m_i, \ \forall i \quad v_i} \quad \boxed{x_i \geq 0, \ \forall i \quad \mu_i}$$

Potential Games

Routing Games



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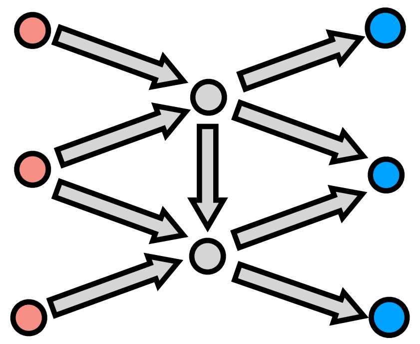
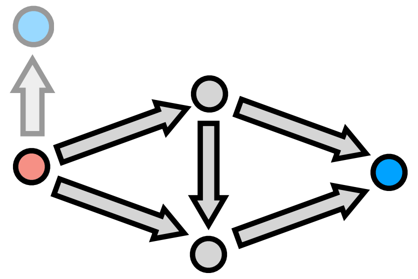


Variable Demand



Variable Demand

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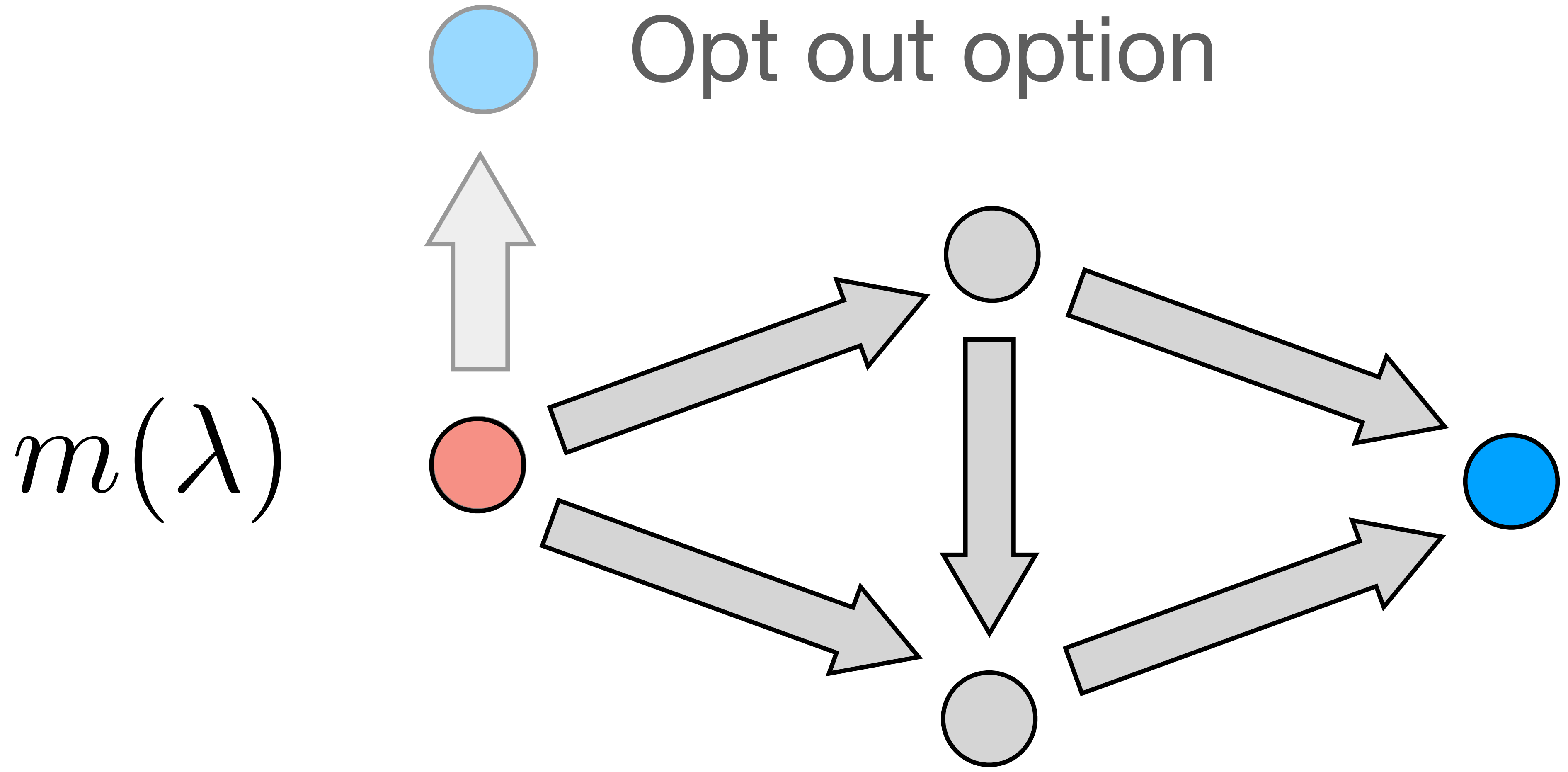
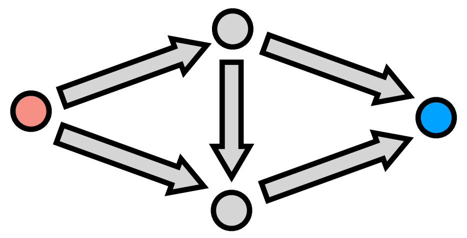


$$\min_{x} \quad F(x) + \int_0^m m^{-1}(u)\, du$$

$$\text{s.t.} \quad \boxed{Ex = Sm, \quad \nu} \quad \boxed{x \geq 0 \quad \mu}$$

Potential Games

Routing Games



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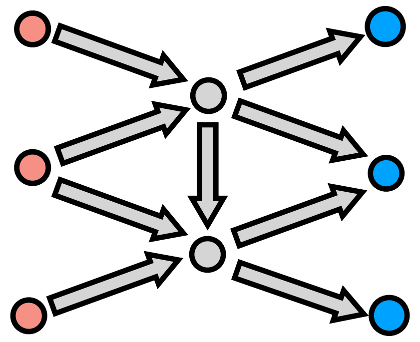
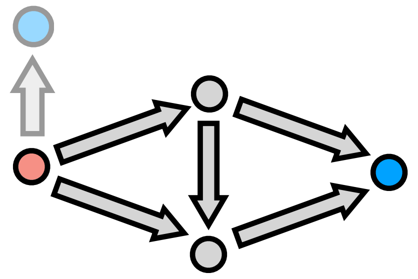
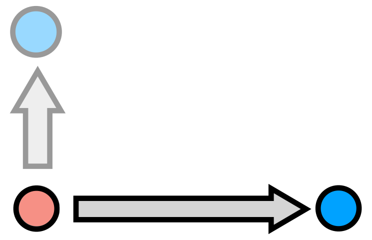


Variable Demand



Supply & Demand

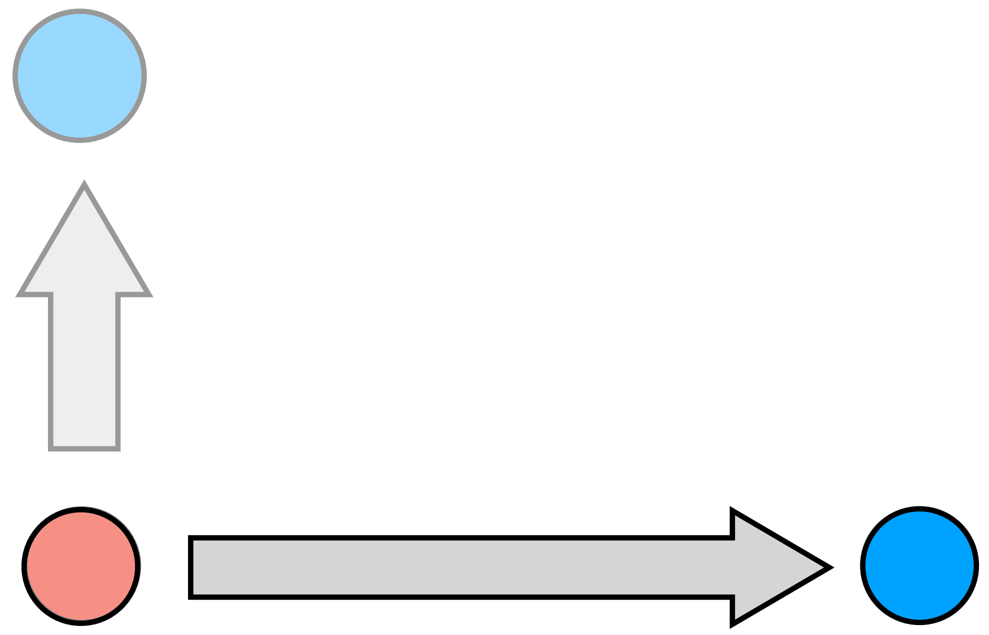


Supply & Demand

x : edge traffic

z : route traffic

$m(\lambda)$



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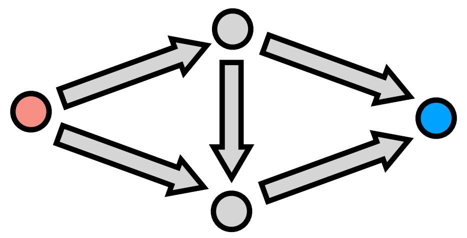
Potential Games

Routing Games



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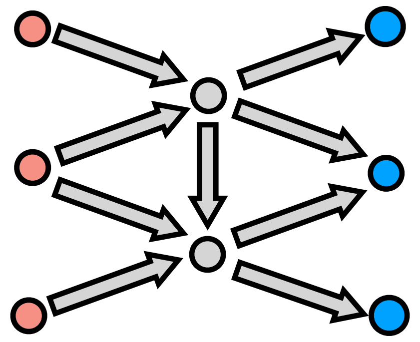
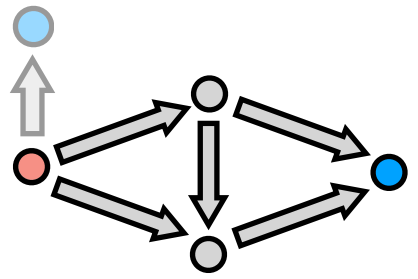


Variable Demand



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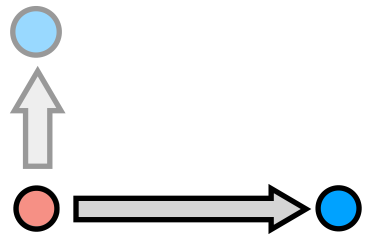
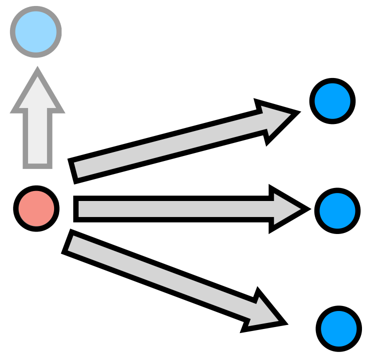


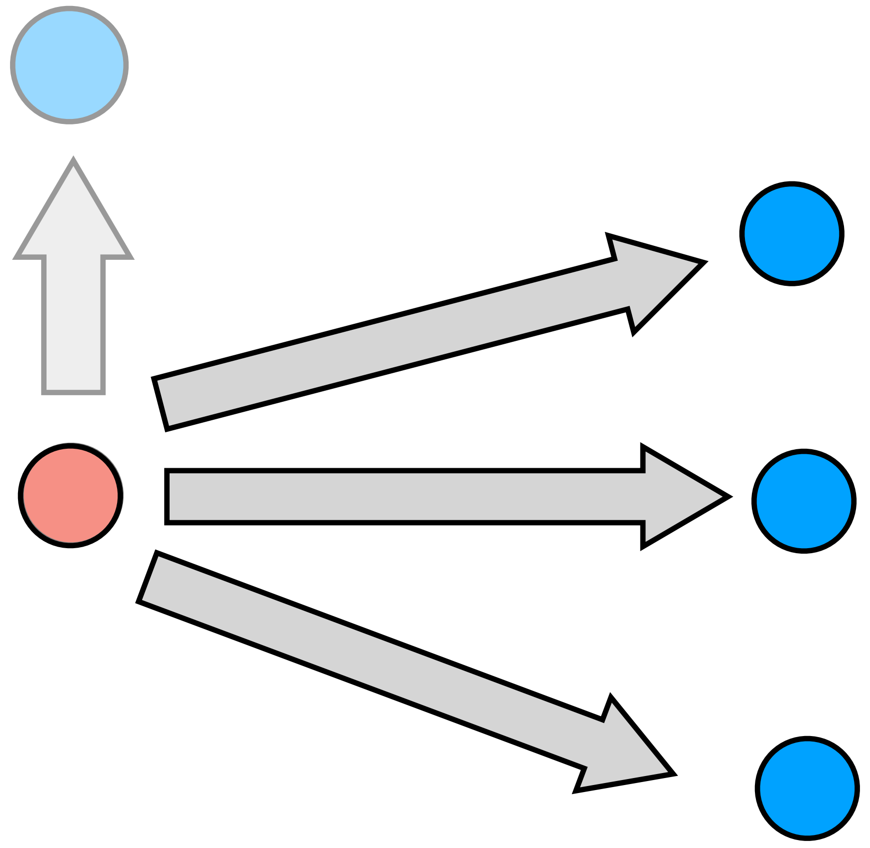
Cournot Market



Cournot Market

x : edge traffic

z : route traffic



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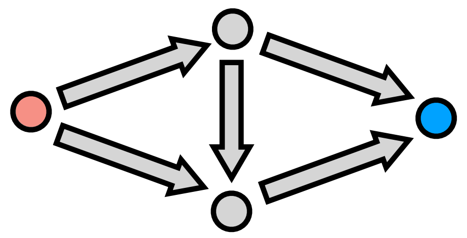
Potential Games

Routing Games



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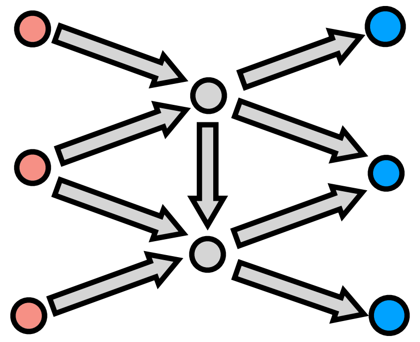
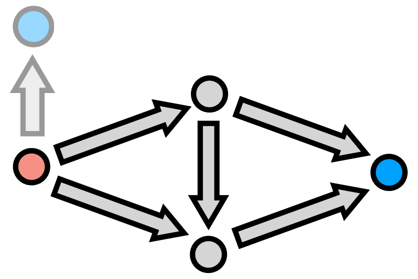
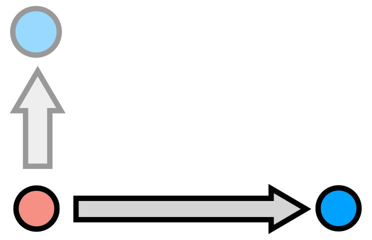


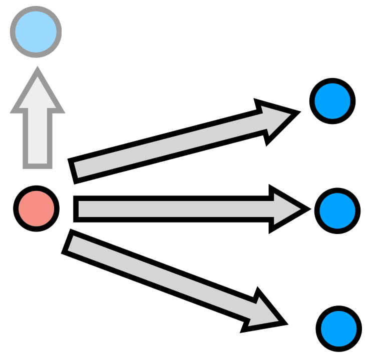
Variable Demand



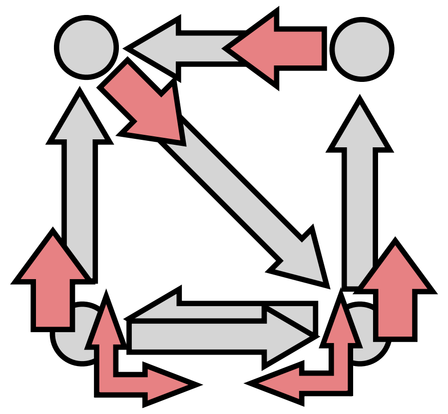
Supply & Demand



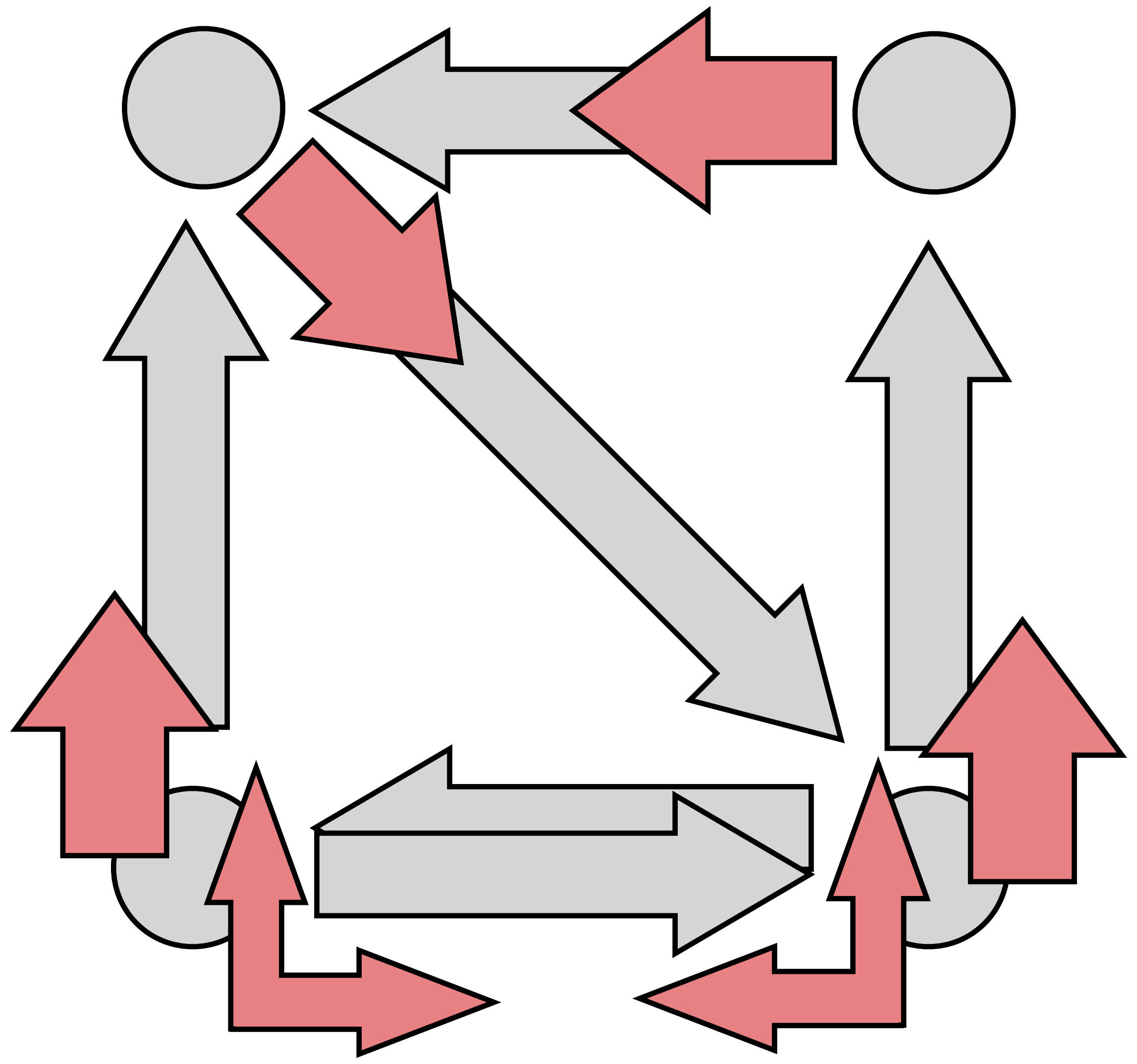
Cournot Market



MDP Congestion Game



Markov Decision Process Congestion Game



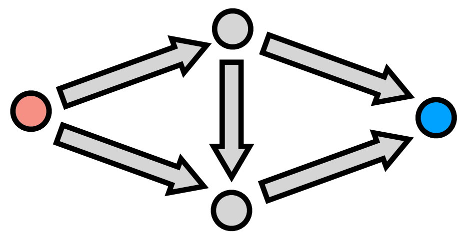
$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \;\; \lambda \qquad x \geq 0 \;\; \mu$$

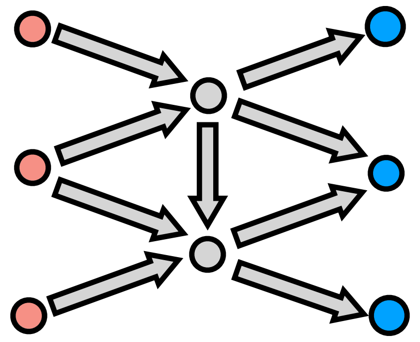
$$EWx = 0 \;\; v$$

Potential Games

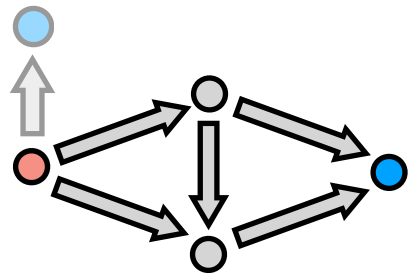
Routing Games



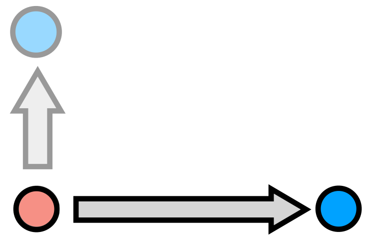
Multiple sources/ sinks



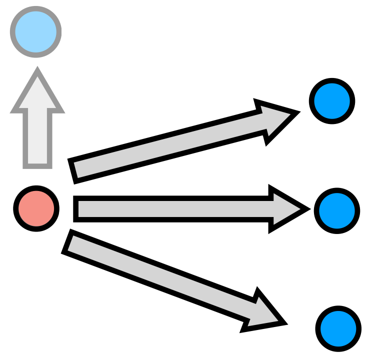
Variable Demand



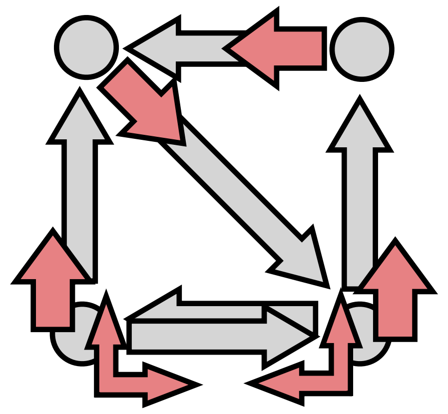
Supply & Demand



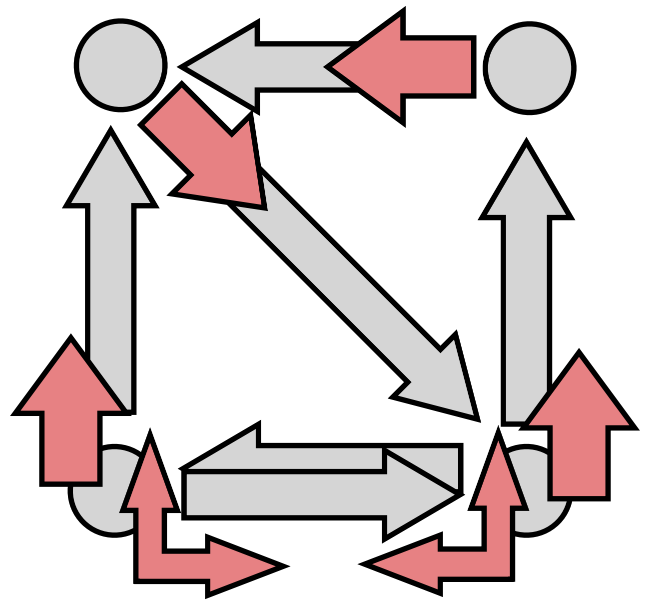
Cournot Market

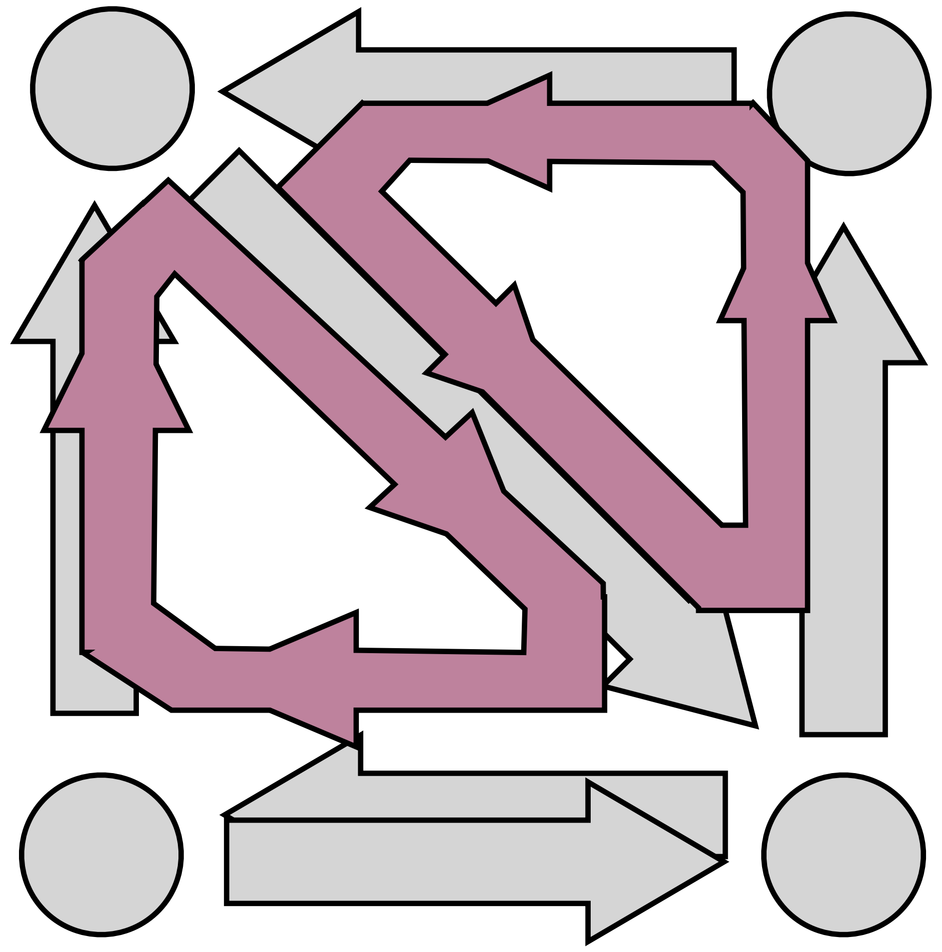


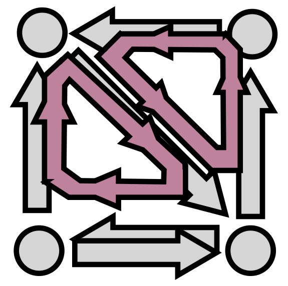
MDP Congestion Game



Markov Decision Process Congestion Game







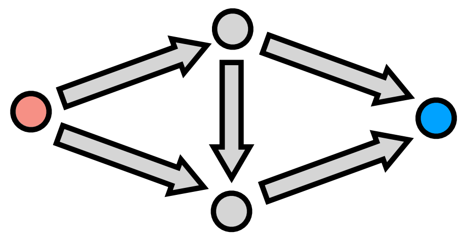
$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \quad \lambda \qquad x \geq 0 \quad \mu$$

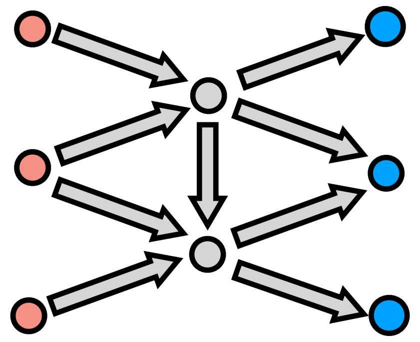
$$EWx = 0 \quad v$$

Potential Games

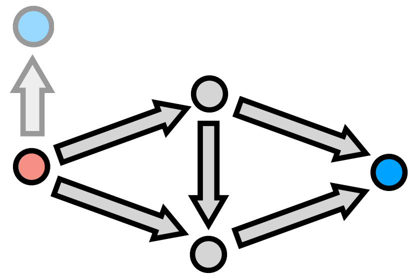
Routing Games



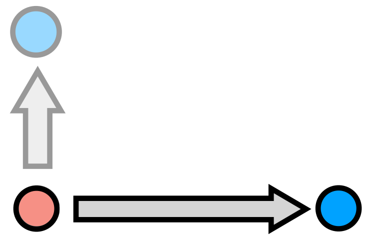
Multiple sources/ sinks



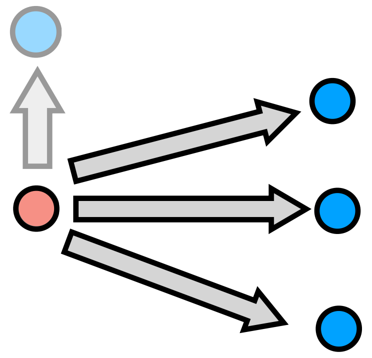
Variable Demand



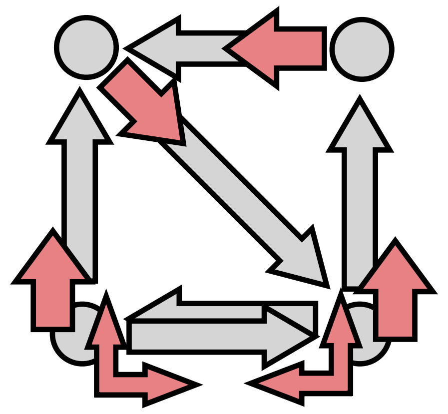
Supply & Demand



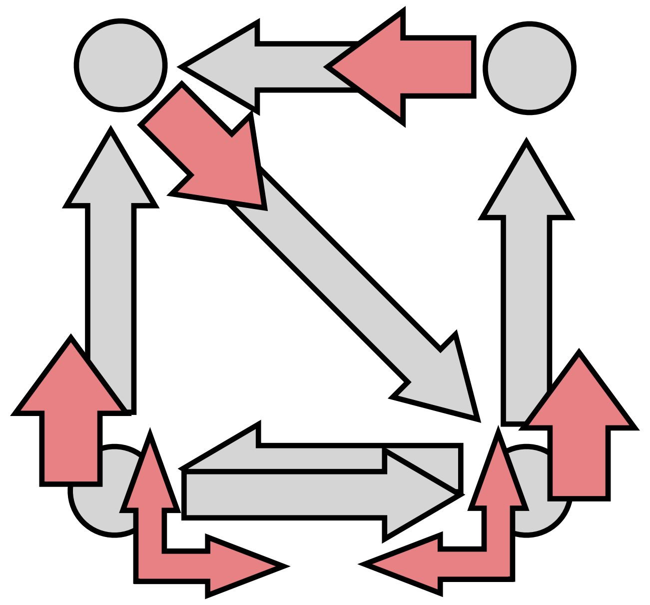
Cournot Market

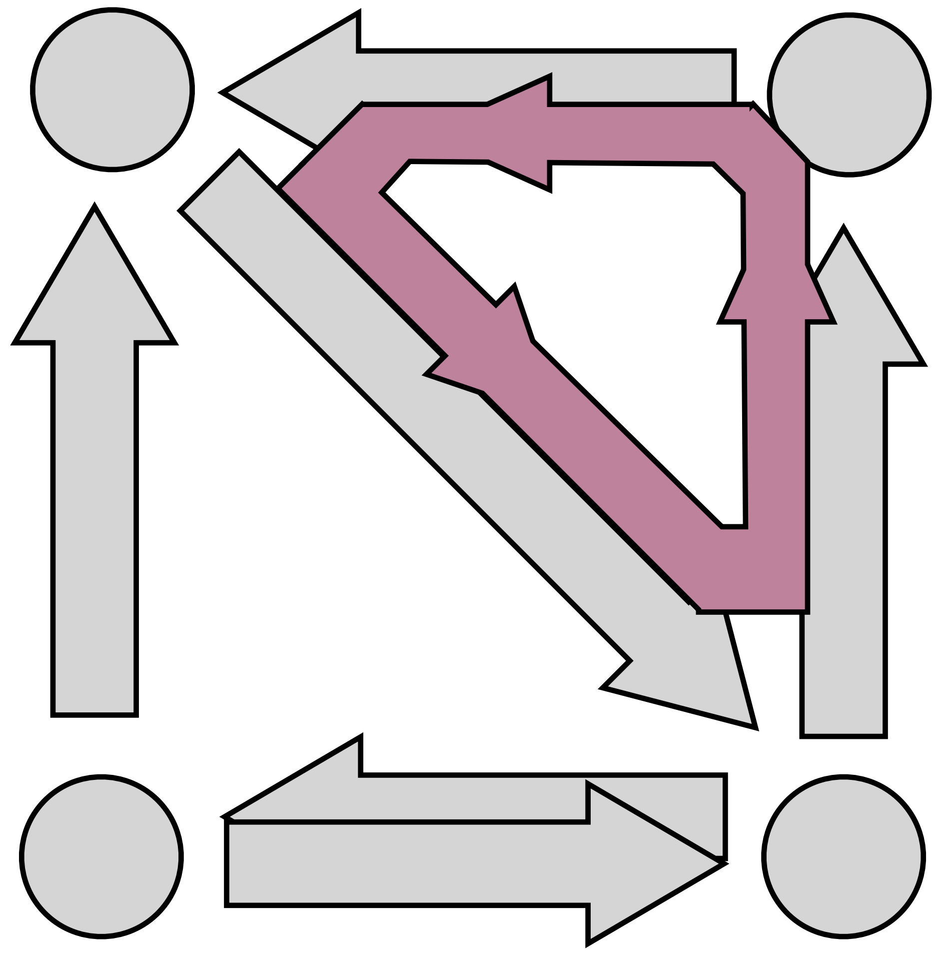


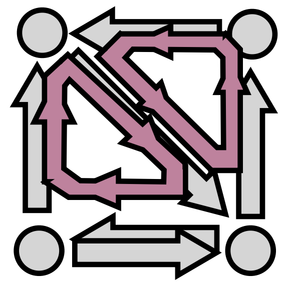
MDP Congestion Game

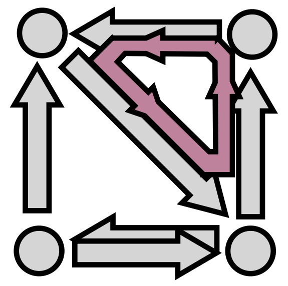


Markov Decision Process Congestion Game









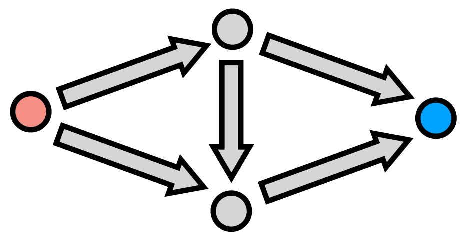
$$\min_{x} \quad F(x)$$

s.t. $\boxed{\mathbf{1}^T x = m \quad \lambda}$ $\boxed{x \geq 0 \quad \mu}$

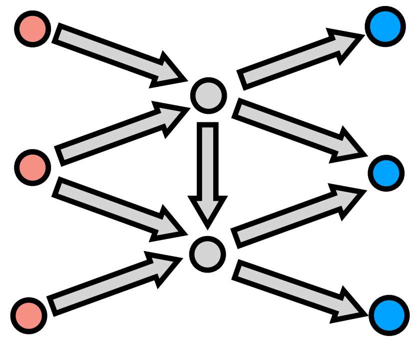
$\boxed{EWx = 0 \quad v}$

Potential Games

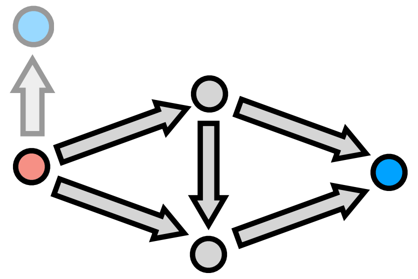
Routing Games



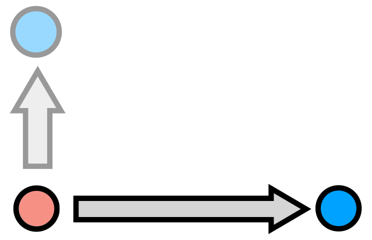
Multiple sources/ sinks



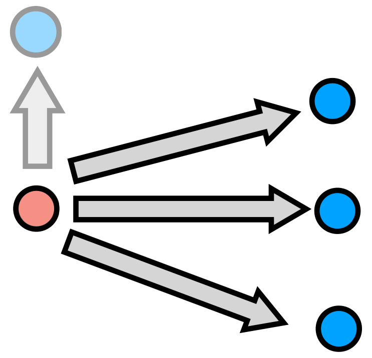
Variable Demand



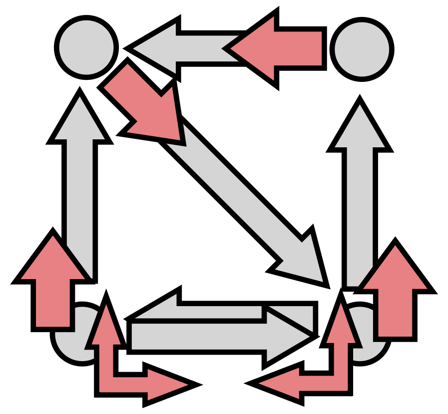
Supply & Demand



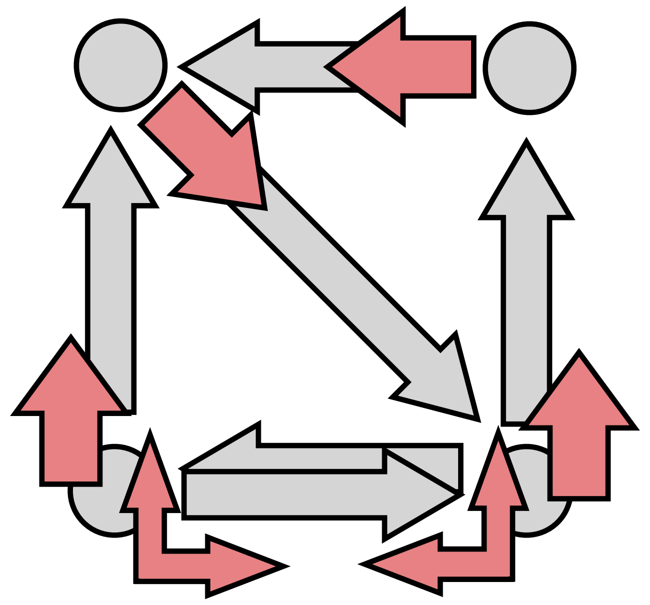
Cournot Market

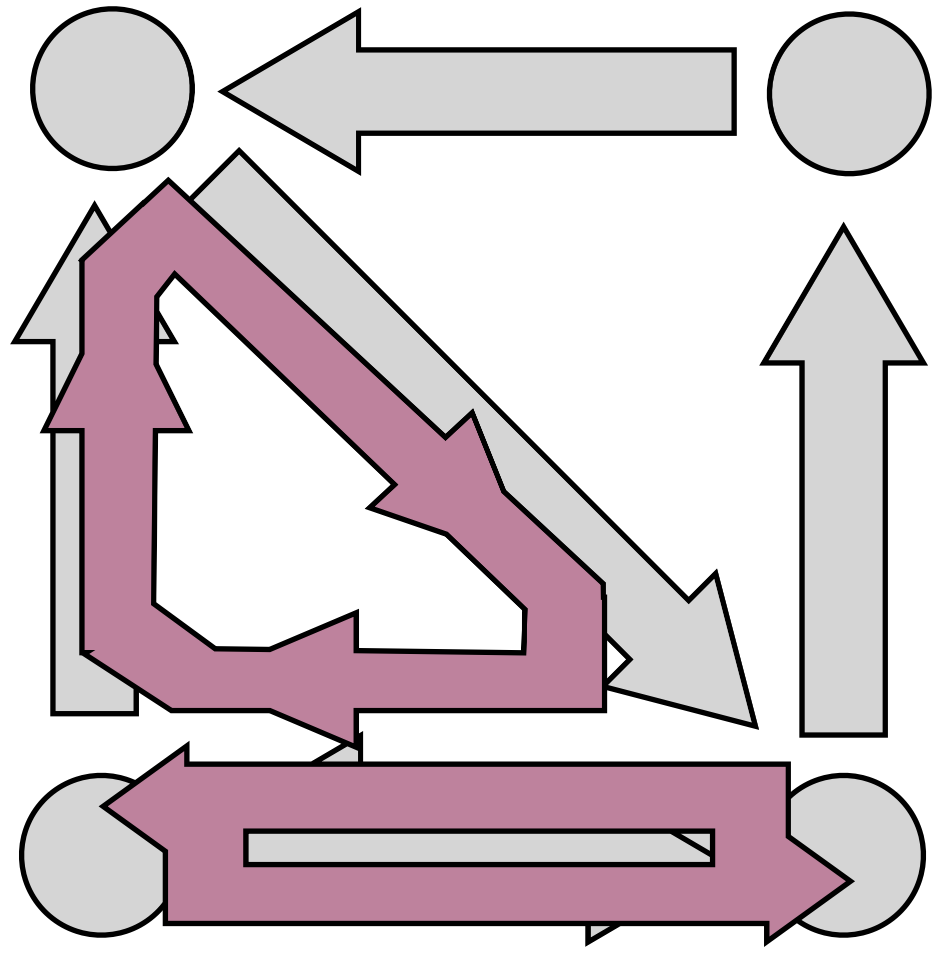


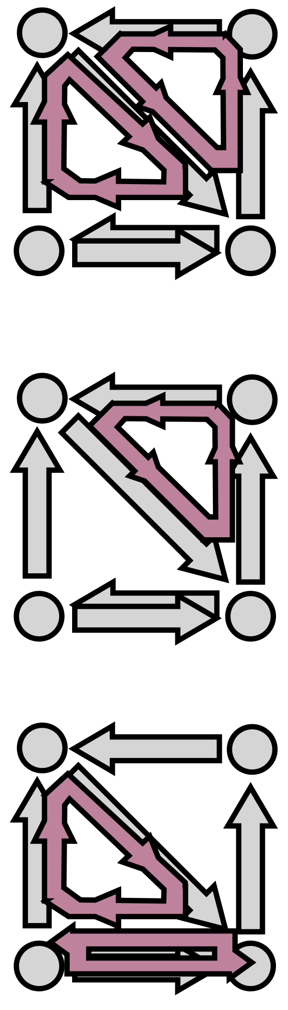
MDP Congestion Game



Markov Decision Process Congestion Game



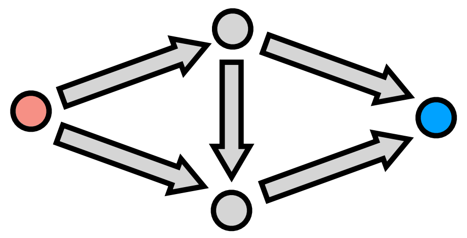




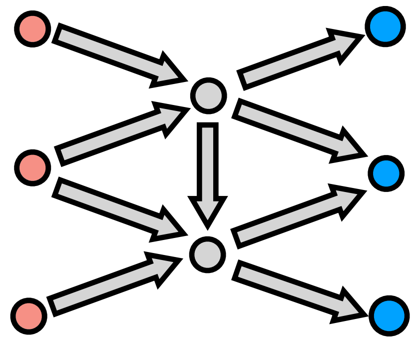
$$
\begin{aligned}
\min_{x} \quad & F(x) \\
\text{s.t.} \quad & \mathbf{1}^T x = m \quad \lambda \qquad x \geq 0 \quad \mu \\
& EWx = 0 \quad v
\end{aligned}
$$

Potential Games

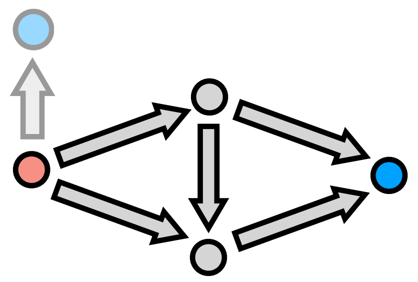
Routing Games



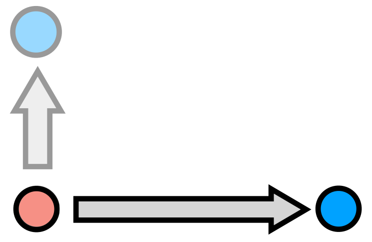
Multiple sources/ sinks



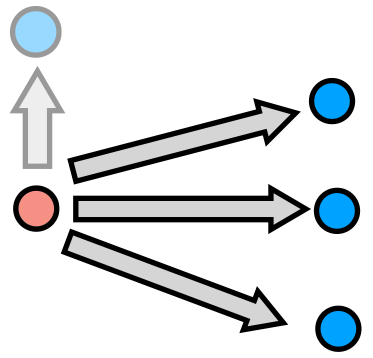
Variable Demand



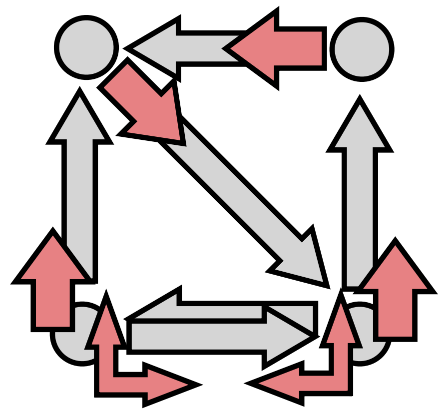
Supply & Demand



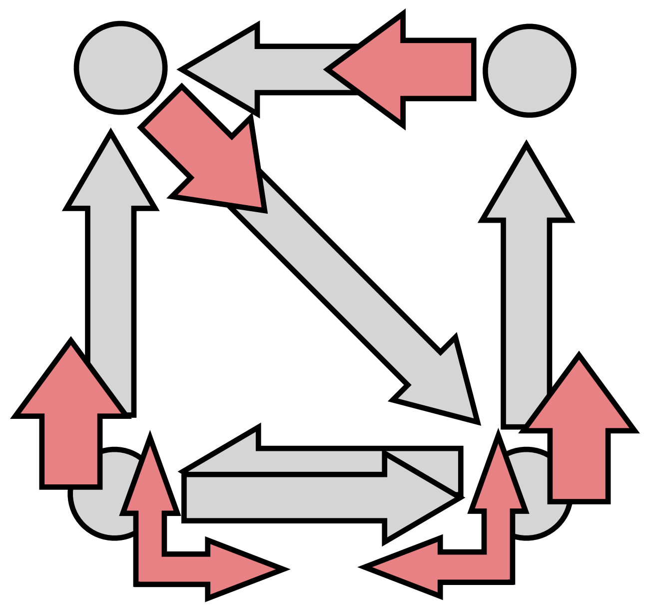
Cournot Market

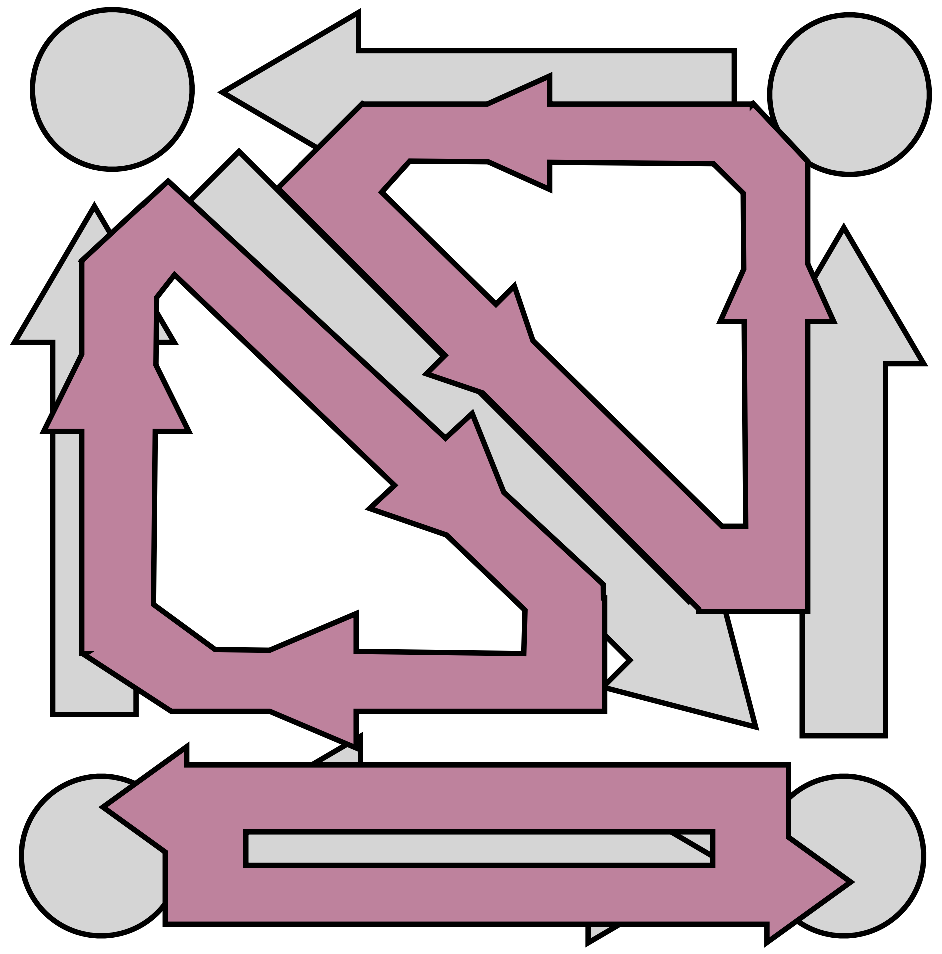


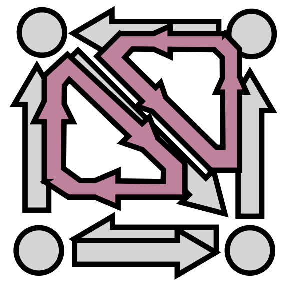
MDP Congestion Game

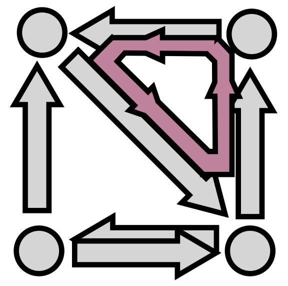


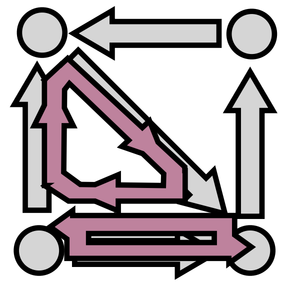
Markov Decision Process Congestion Game

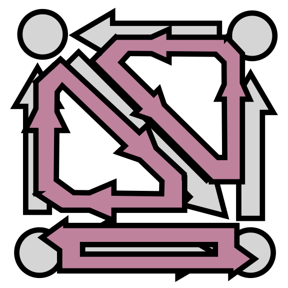












$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \;\; (\lambda) \qquad x \geq 0 \;\; (\mu)$$

$$EWx = 0 \;\; (v)$$

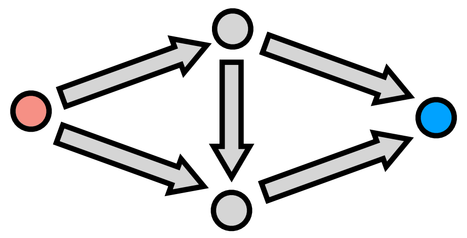
λ : average reward

μ : action inefficiency

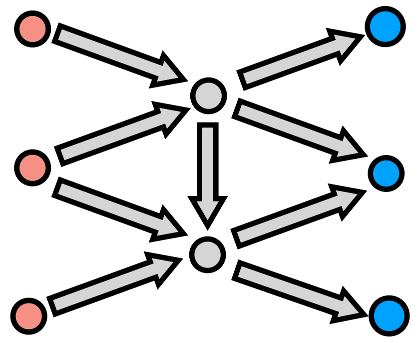
v : value function

Potential Games

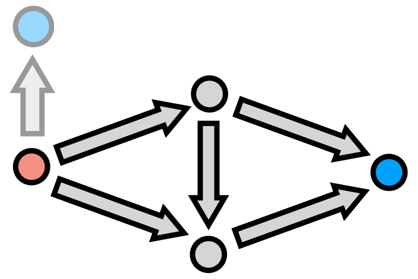
Routing Games



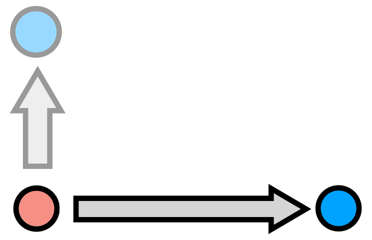
Multiple sources/ sinks



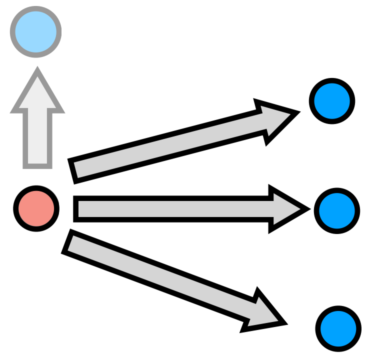
Variable Demand



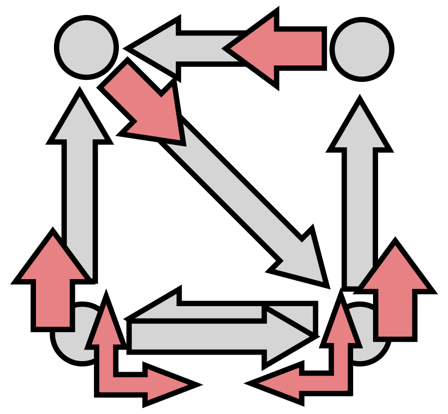
Supply & Demand



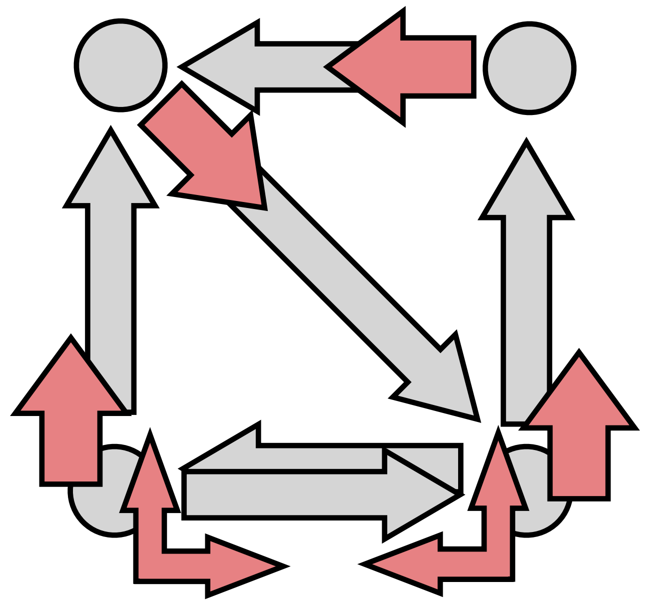
Cournot Market

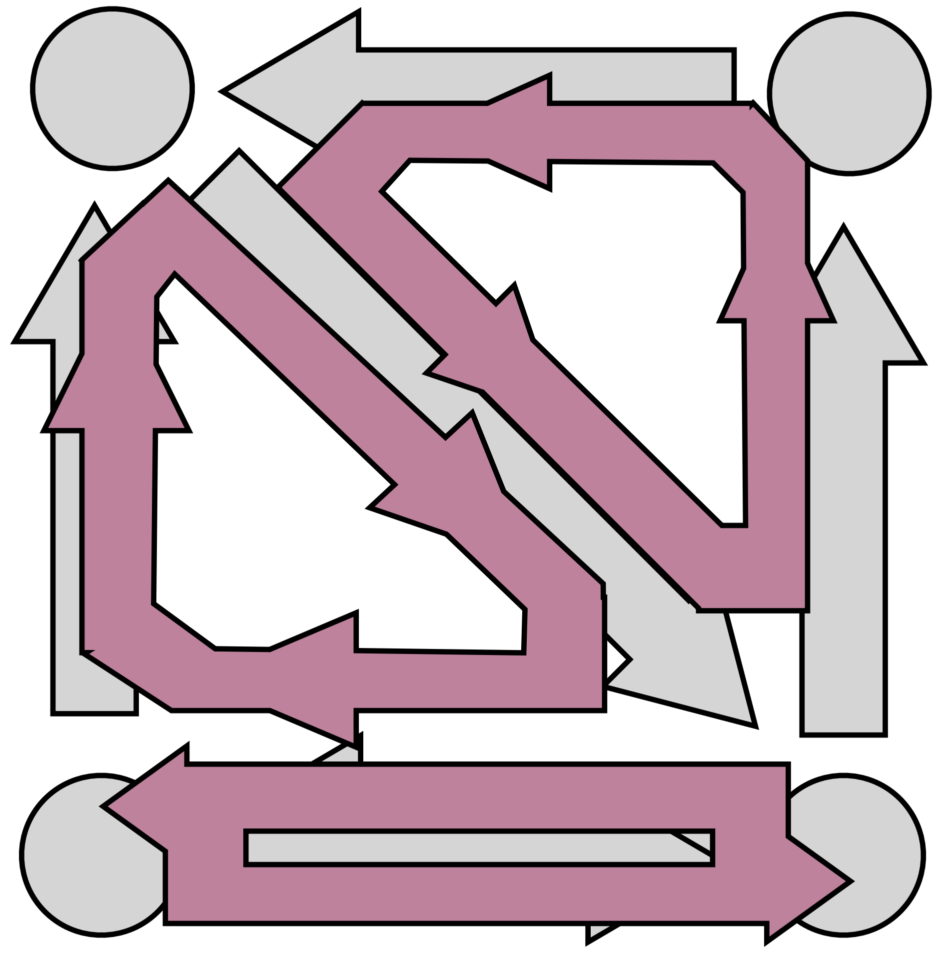


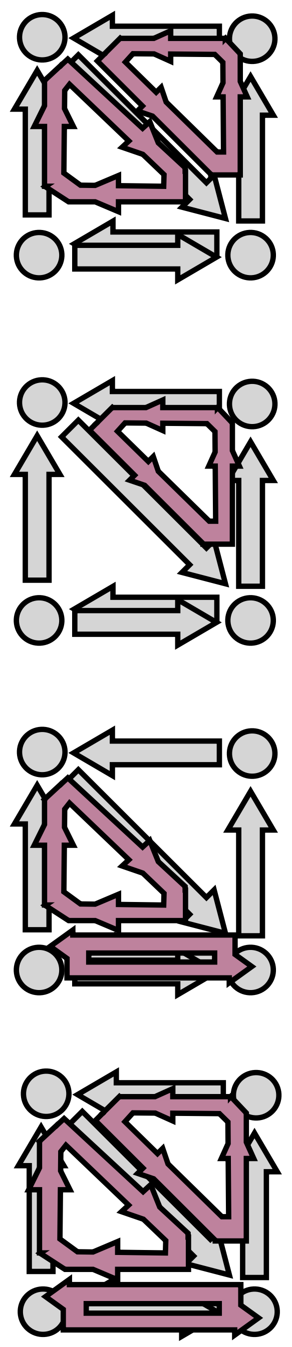
MDP Congestion Game



Markov Decision Process Congestion Game







APPLICATIONS

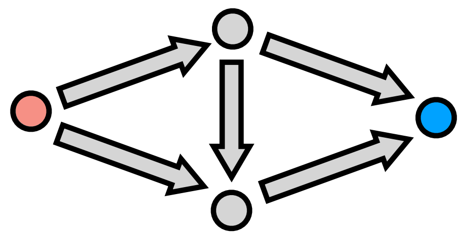
- Ride-sharing drivers planning routes
- Cars circling for street parking
- Air-traffic routing

PAPERS

- Markov decision process routing games **[Calderone, Sastry, 2017]**
- Infinite horizon average cost Markov decision process routing games **[Calderone, Sastry, 2017]**
- Adaptive constraint satisfaction for Markov decision process congestion games: Applications to transportation networks **[Li, Calderone, Ratliff, et al. 2021]**
- Variable demand and multi-commodity flow in Markovian network equilibrium **[Yu, Calderone, Ratliff, et al. 2021]**

Potential Games

Routing Games



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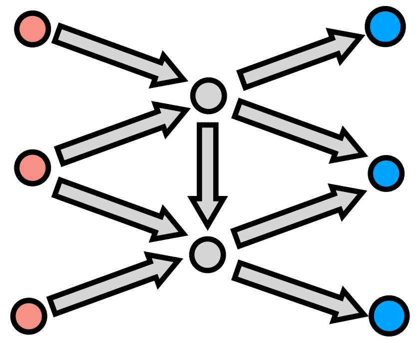
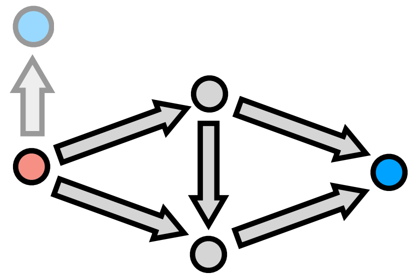
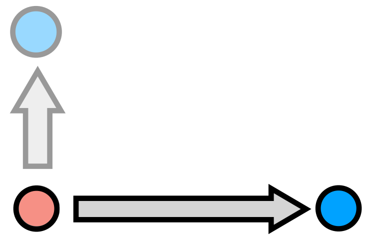


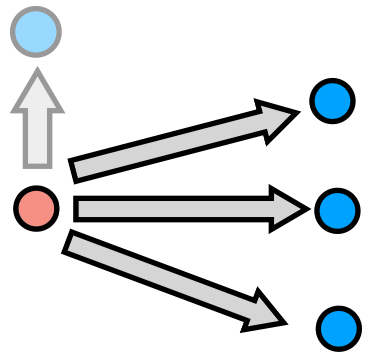
Variable Demand



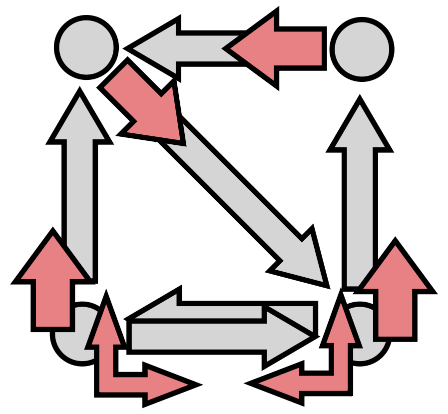
Supply & Demand



Cournot Market

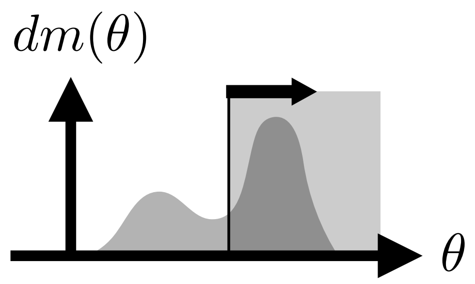


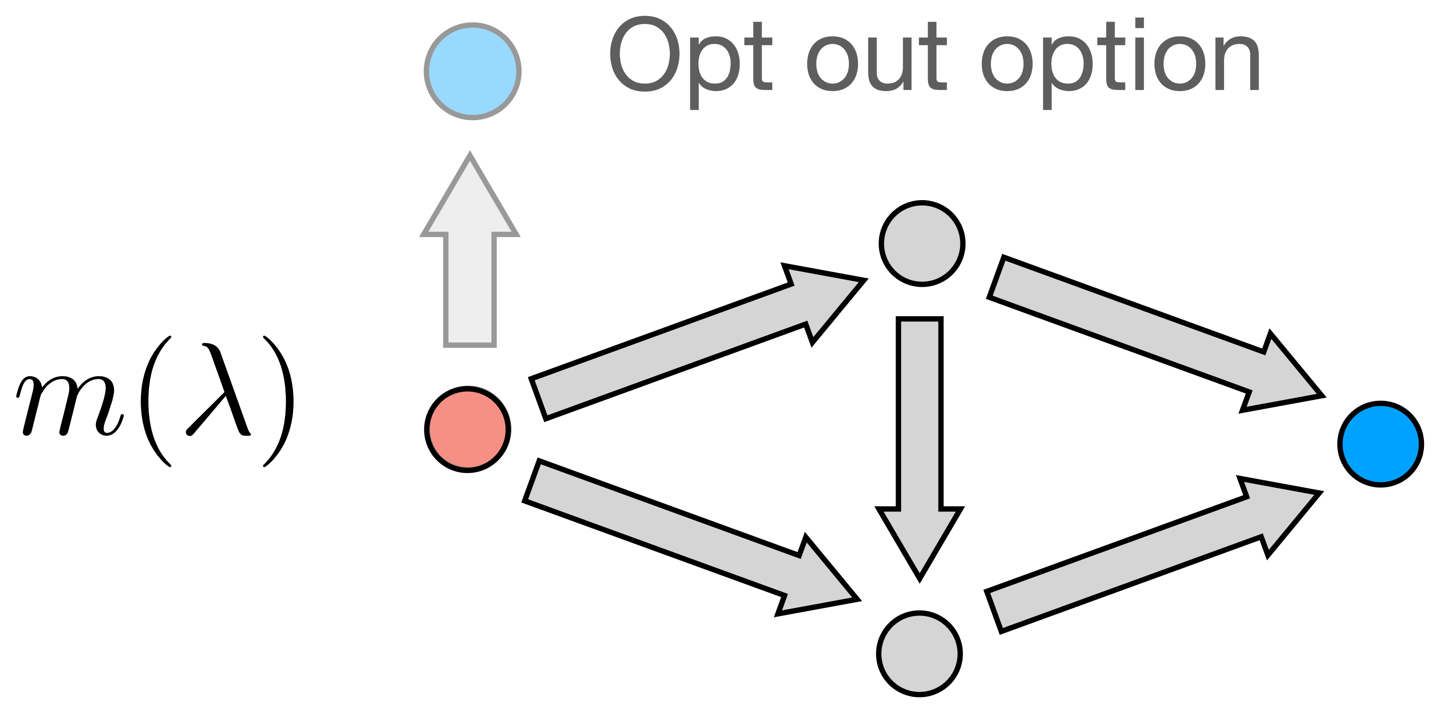
MDP Congestion Game

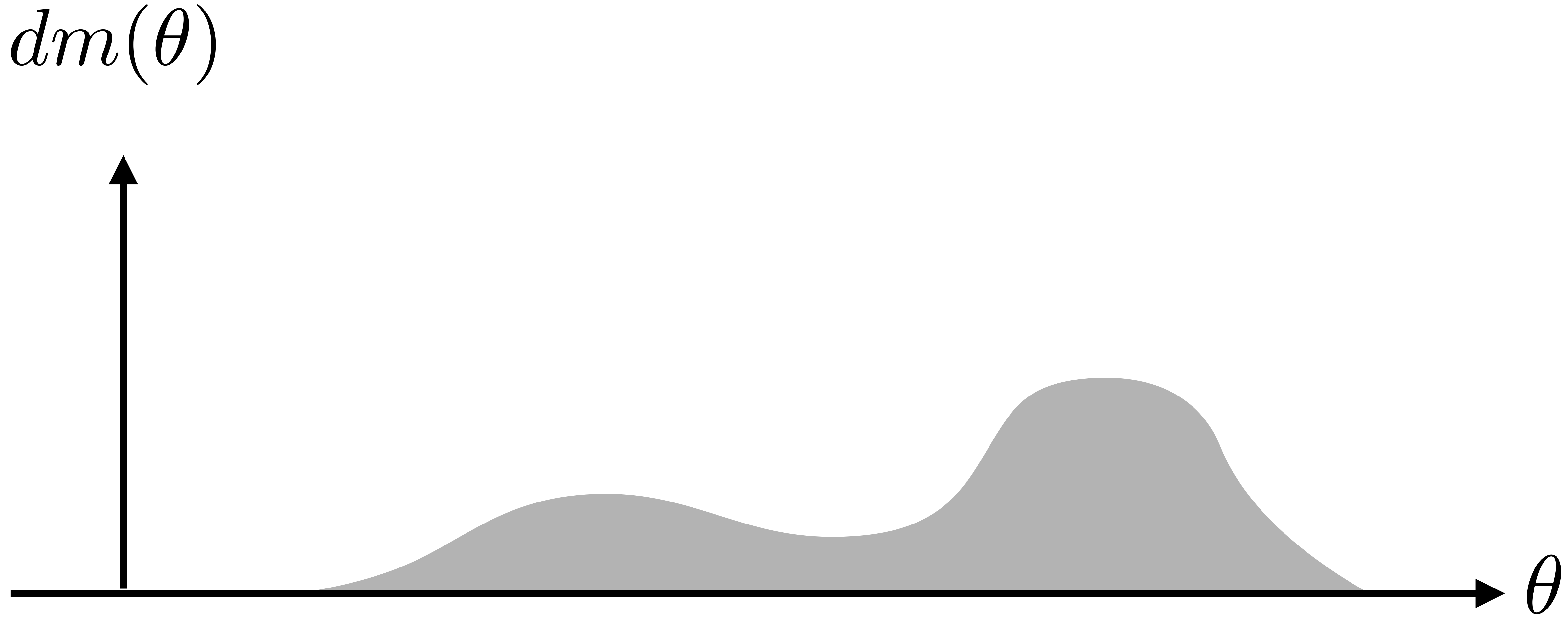


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences



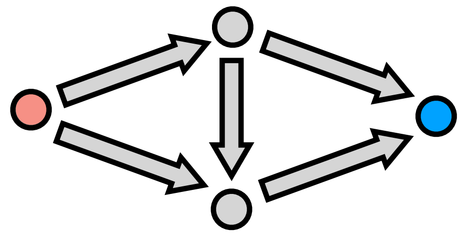




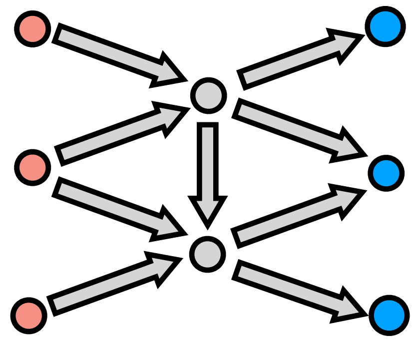
θ : Max cost $dm(\theta)$ will pay

Potential Games

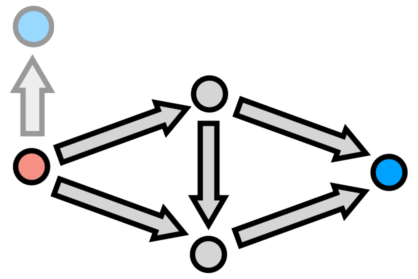
Routing Games



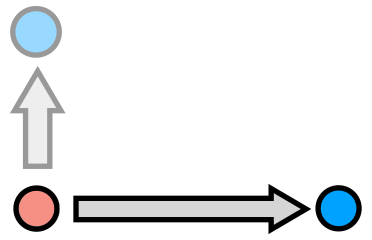
Multiple sources/ sinks



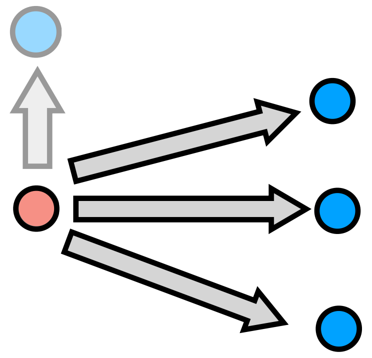
Variable Demand



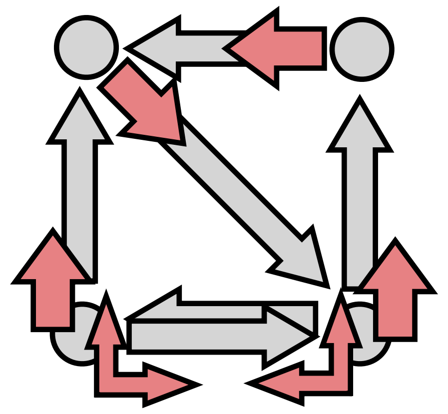
Supply & Demand



Cournot Market

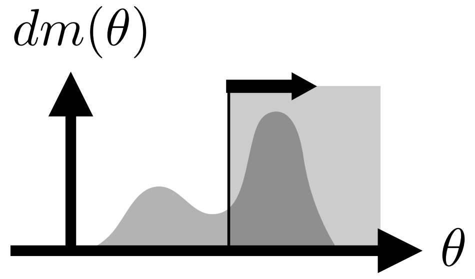


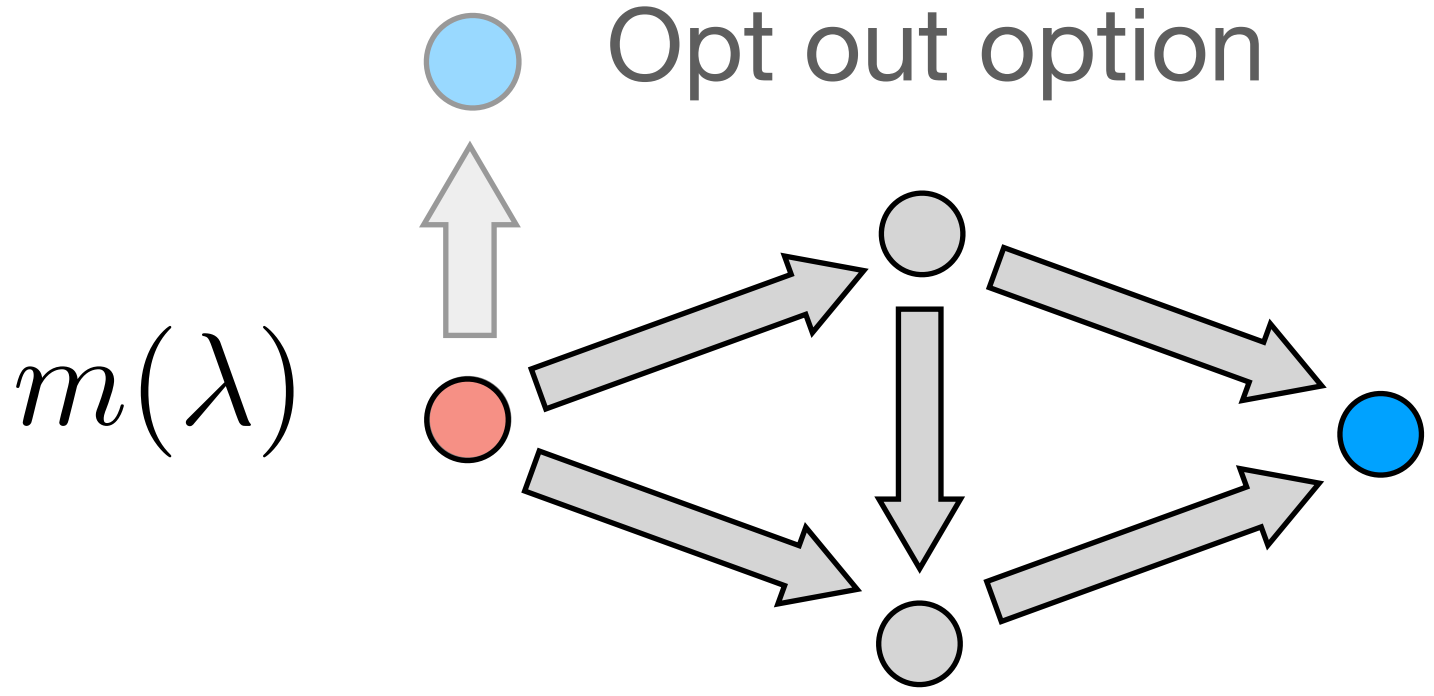
MDP Congestion Game

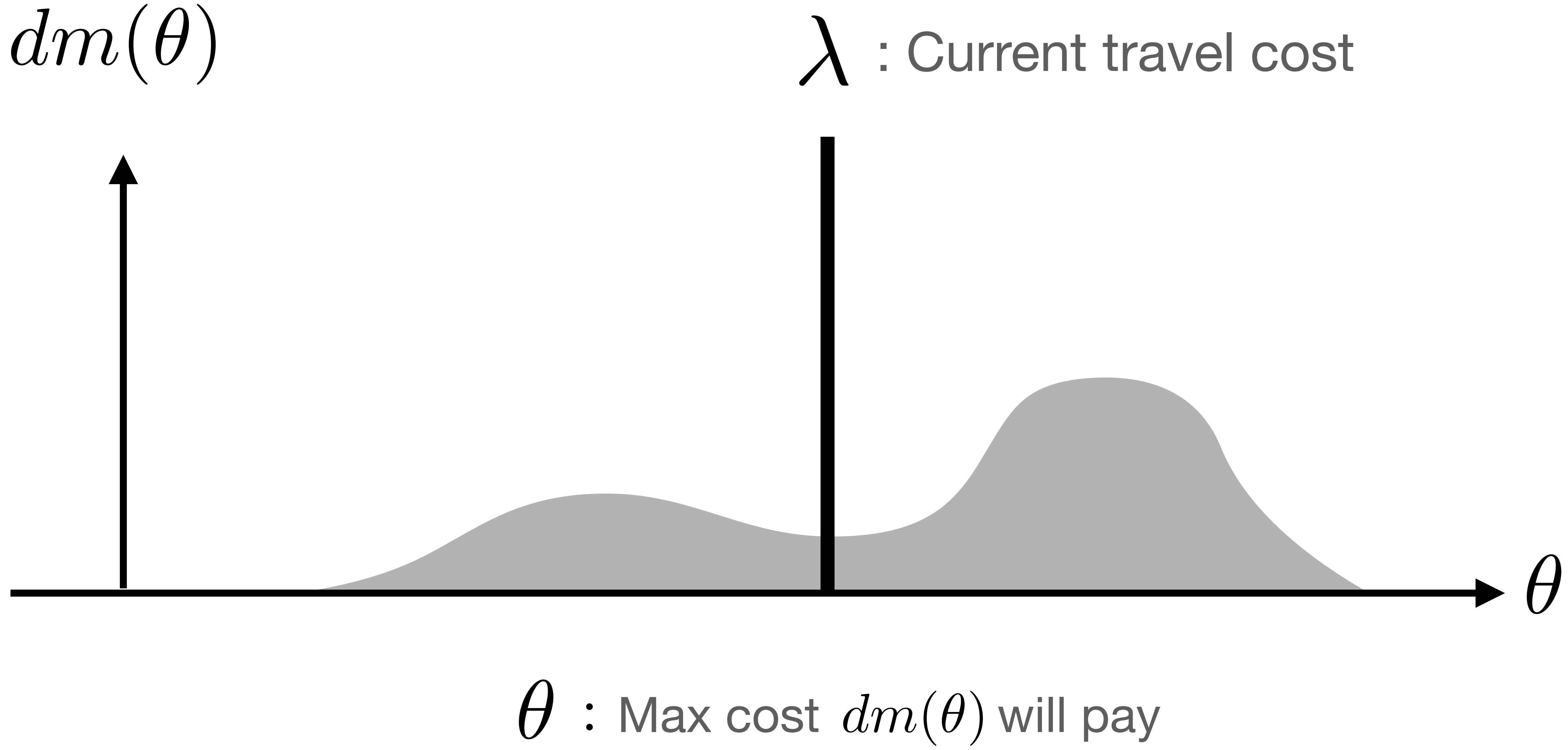


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences

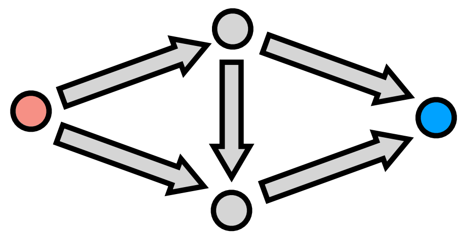




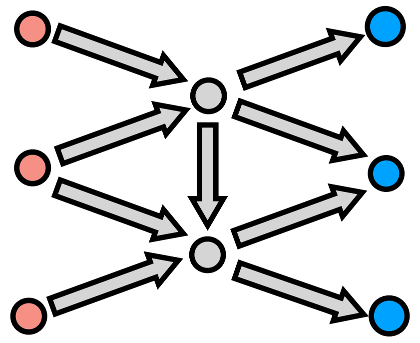


Potential Games

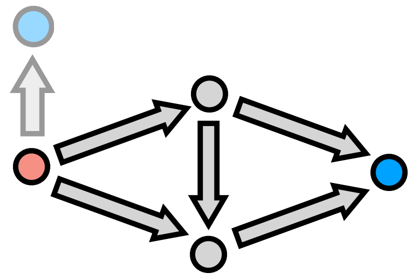
Routing Games



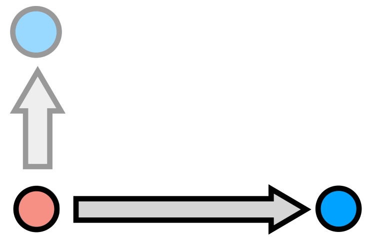
Multiple sources/ sinks



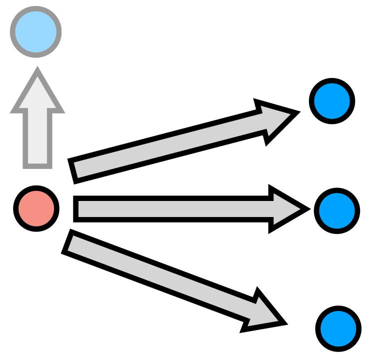
Variable Demand



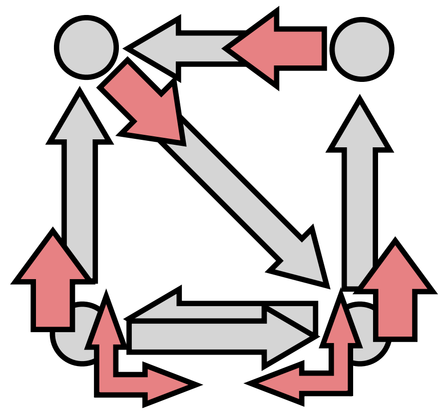
Supply & Demand



Cournot Market

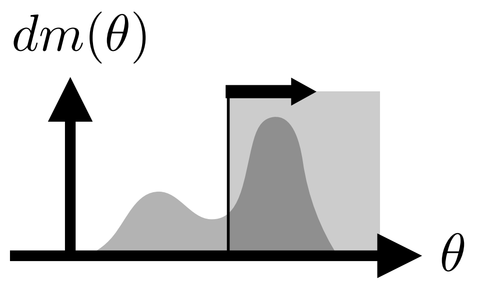


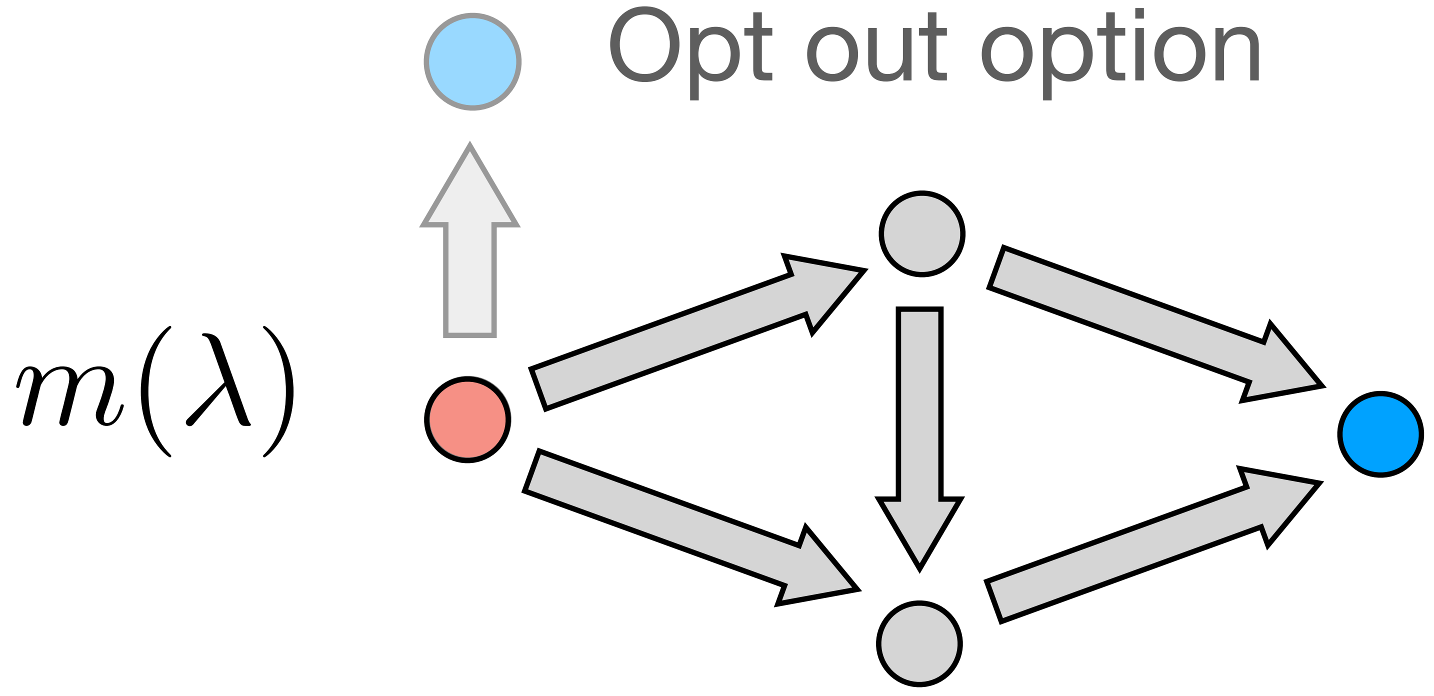
MDP Congestion Game

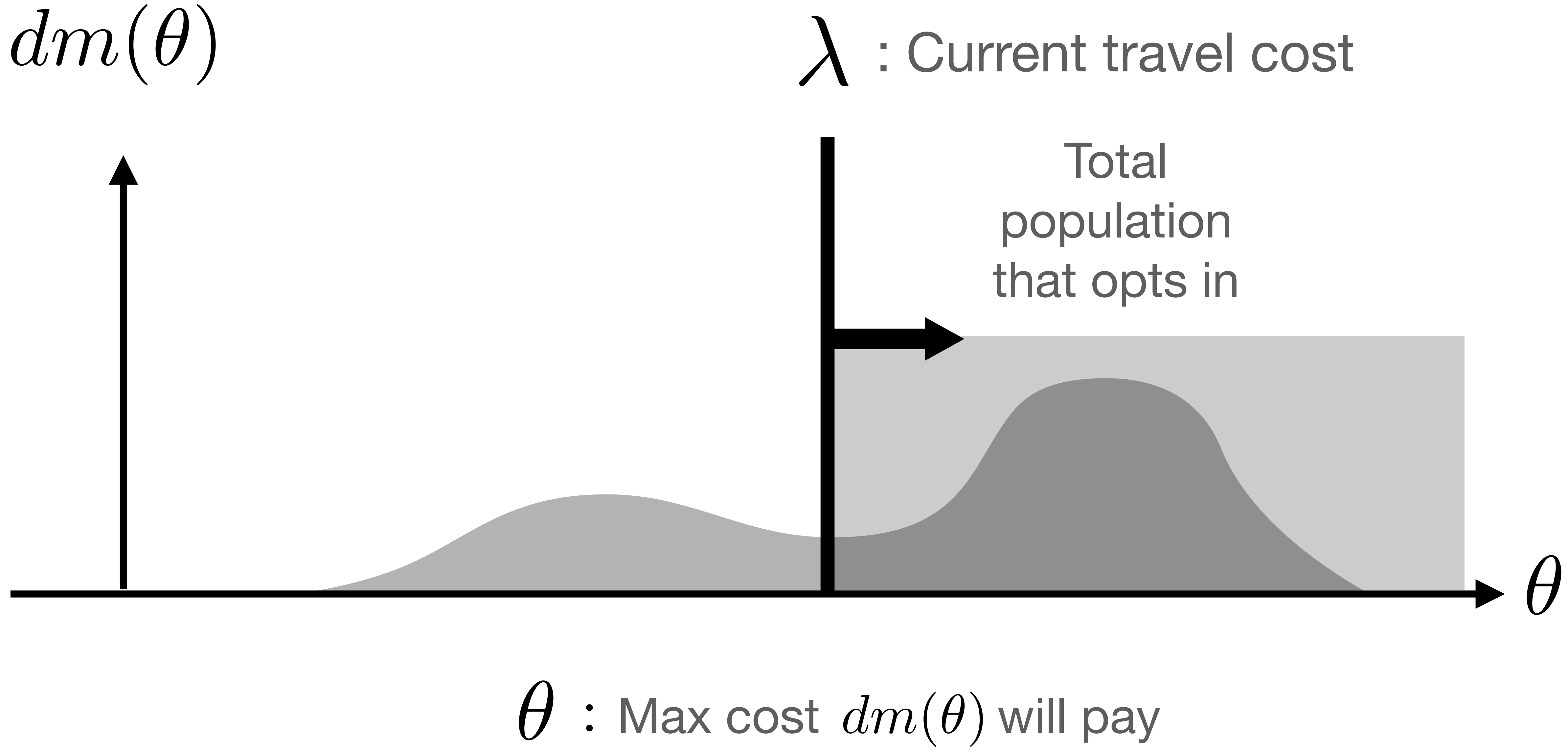


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences

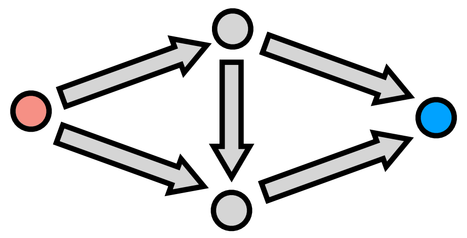




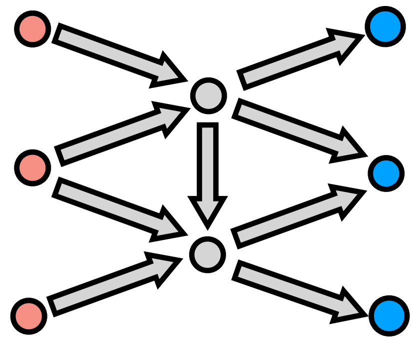


Potential Games

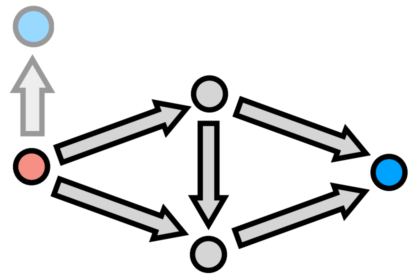
Routing Games



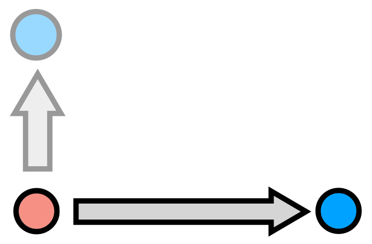
Multiple sources/ sinks



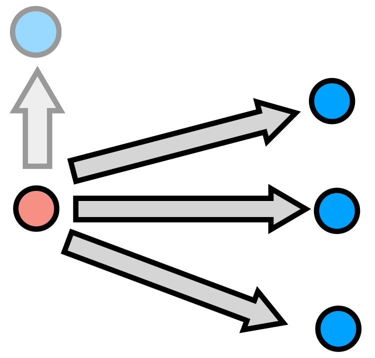
Variable Demand



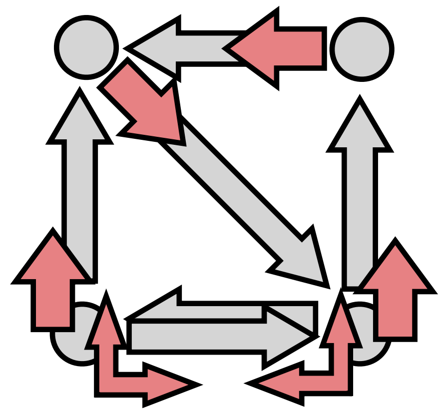
Supply & Demand



Cournot Market

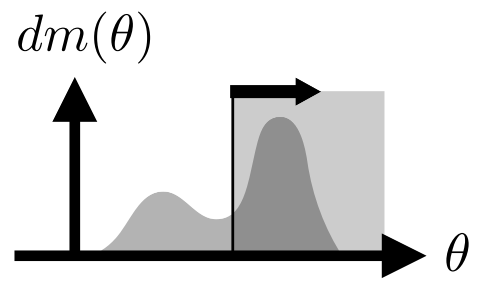


MDP Congestion Game

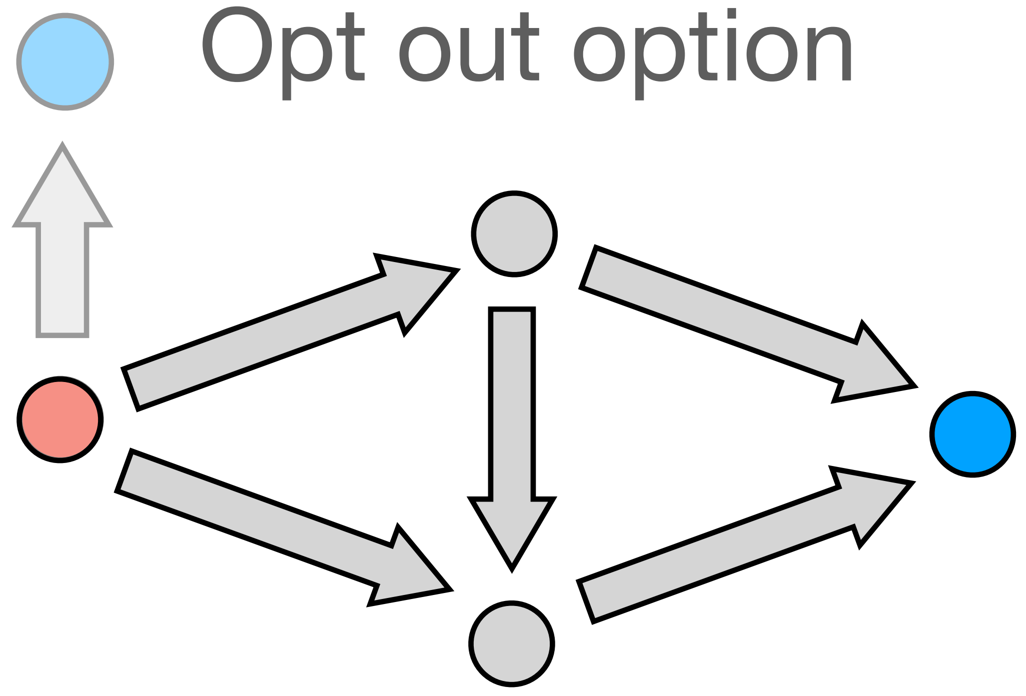


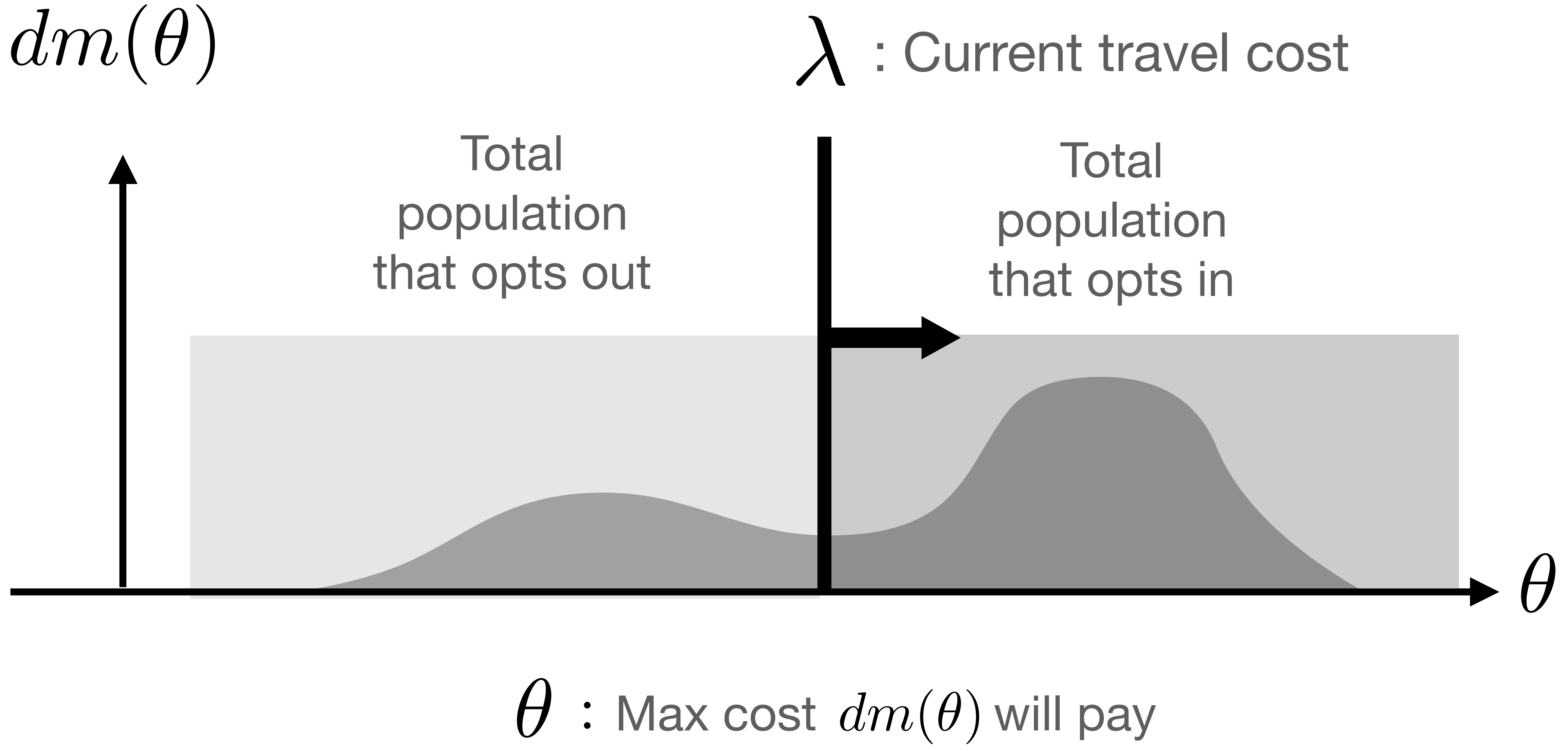
Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences



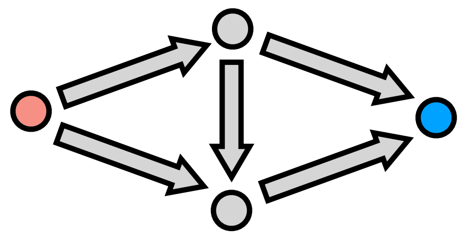
$m(\lambda)$



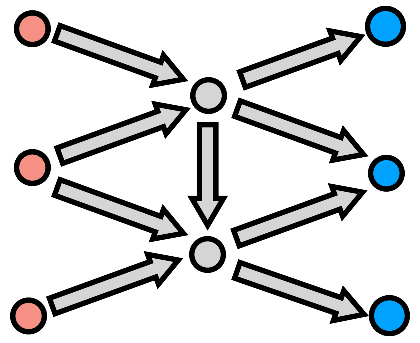


Potential Games

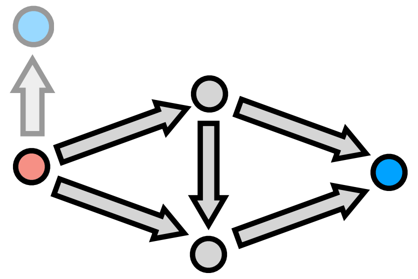
Routing Games



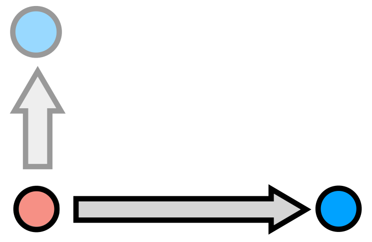
Multiple sources/ sinks



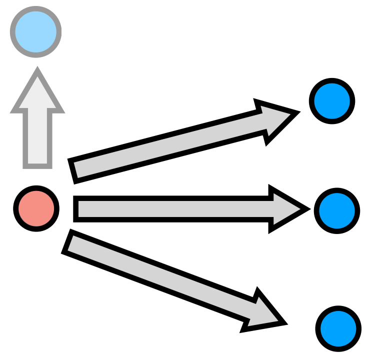
Variable Demand



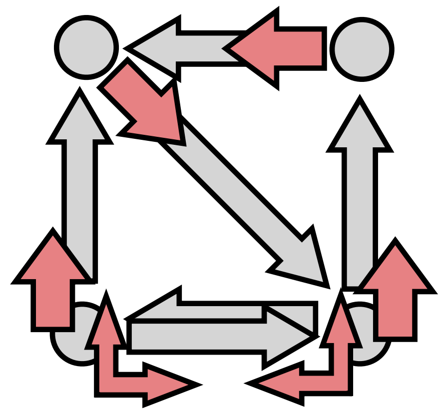
Supply & Demand



Cournot Market

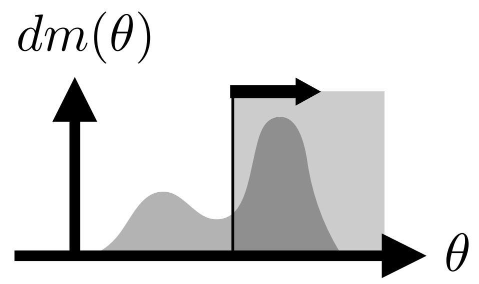


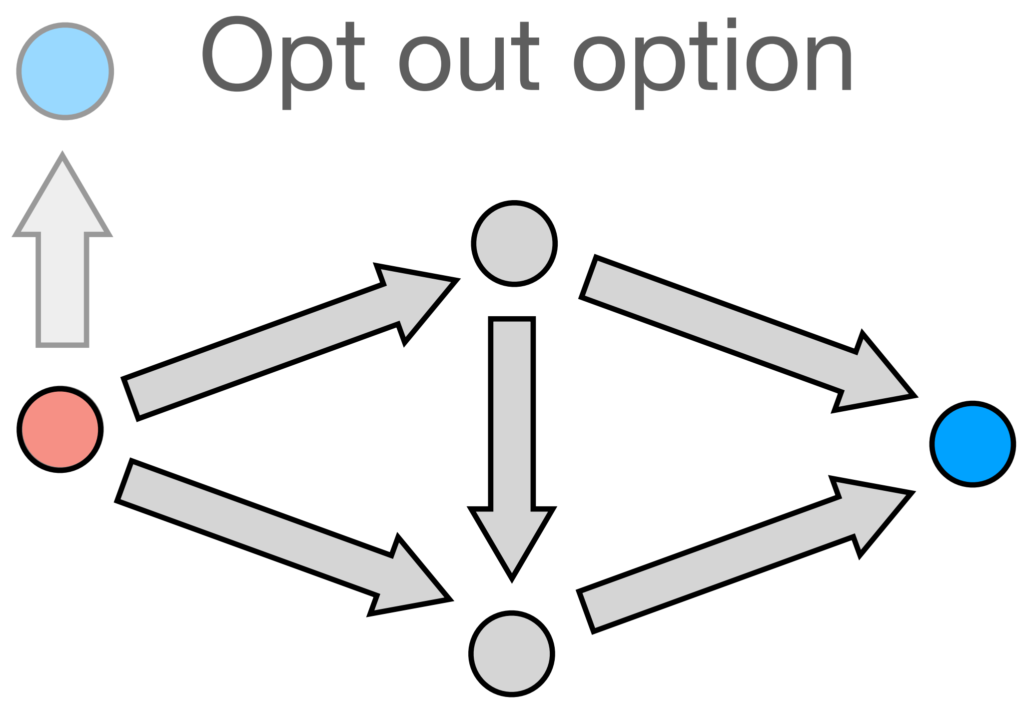
MDP Congestion Game



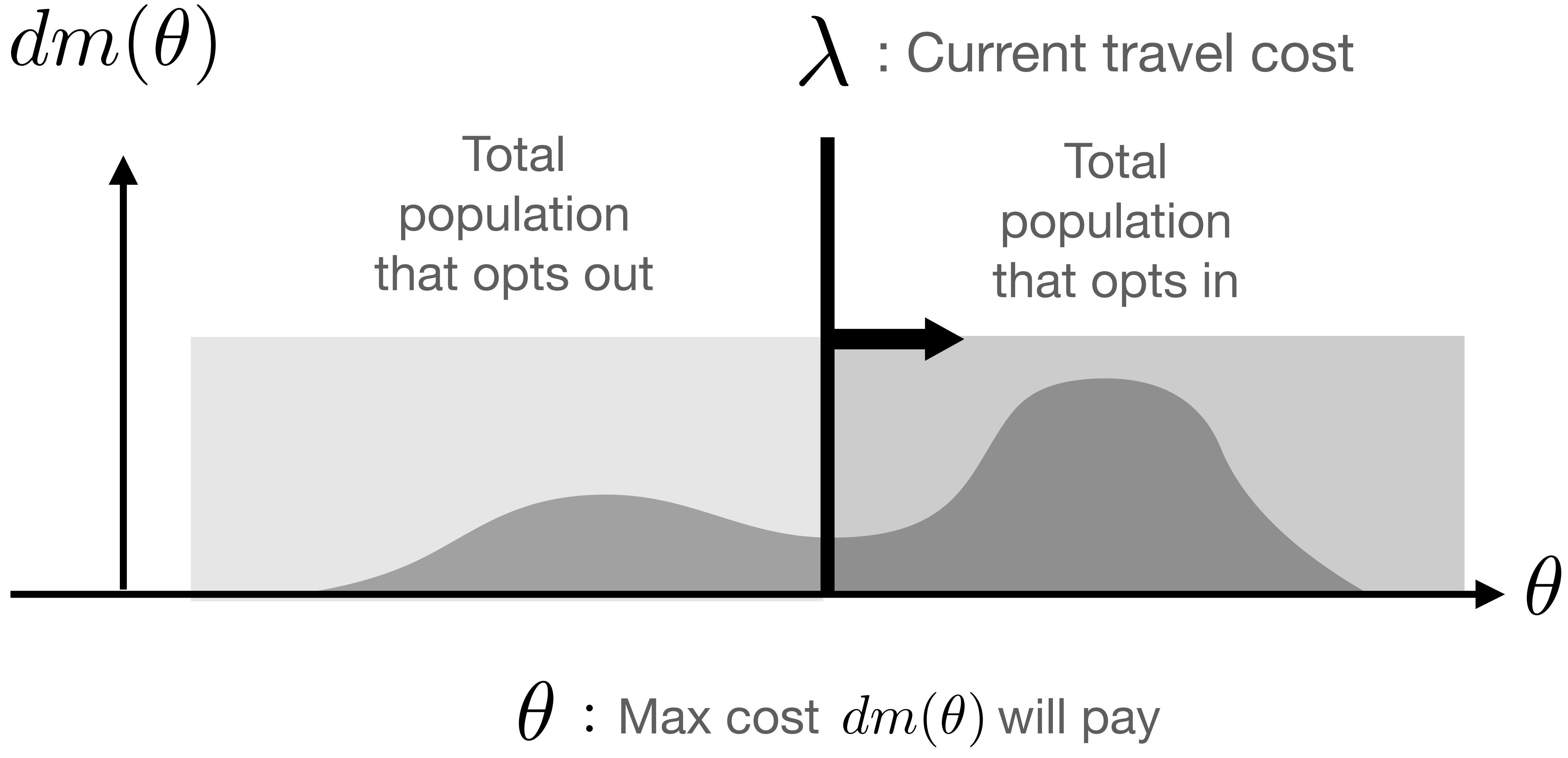
Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences



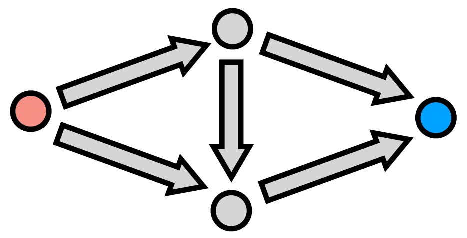


$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$

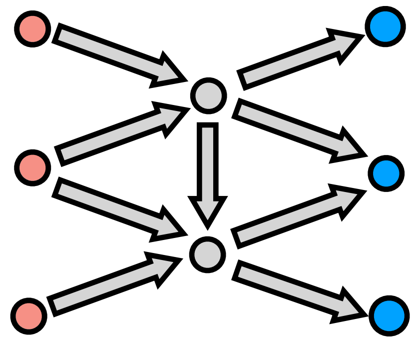


Potential Games

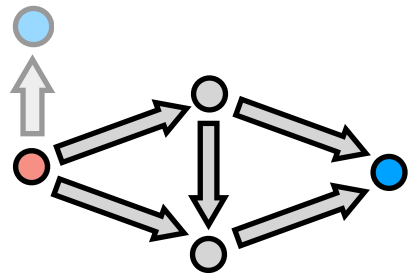
Routing Games



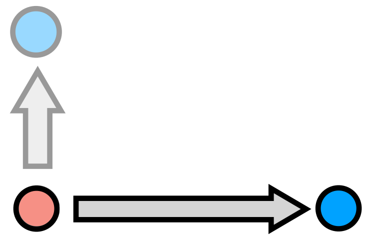
Multiple sources/ sinks



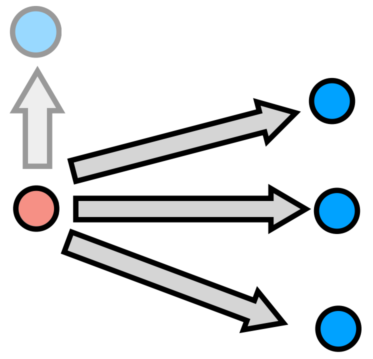
Variable Demand



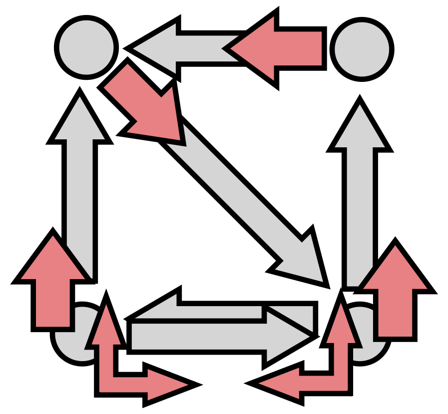
Supply & Demand



Cournot Market

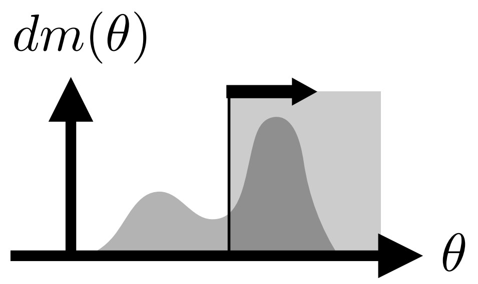


MDP Congestion Game



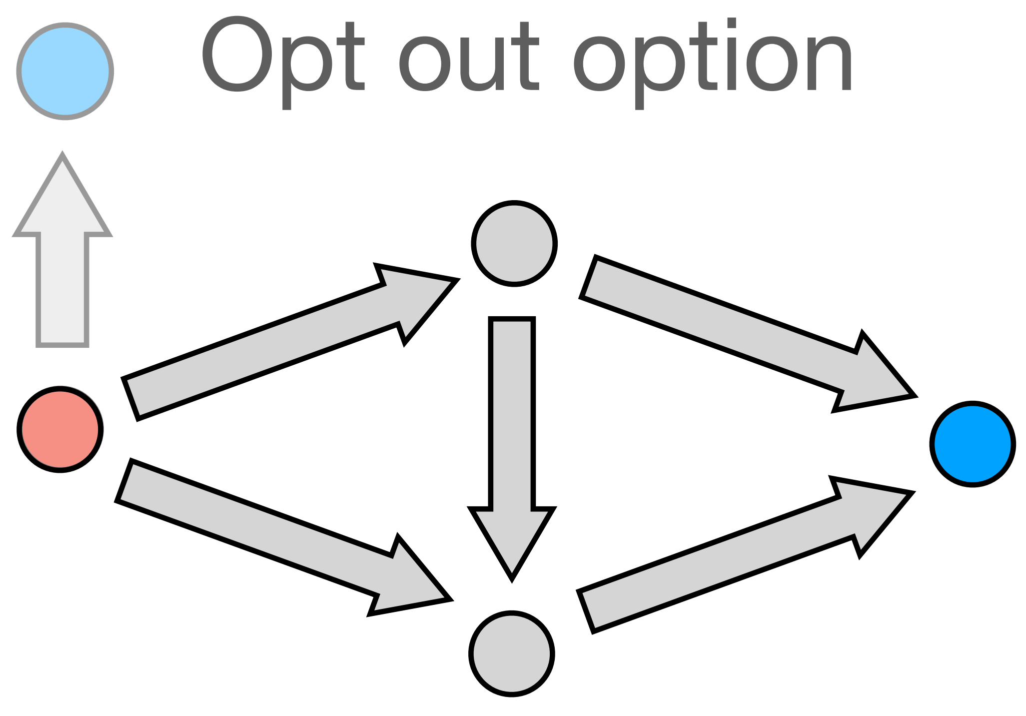
Variable Demand - Non-Homogeneous Preferences

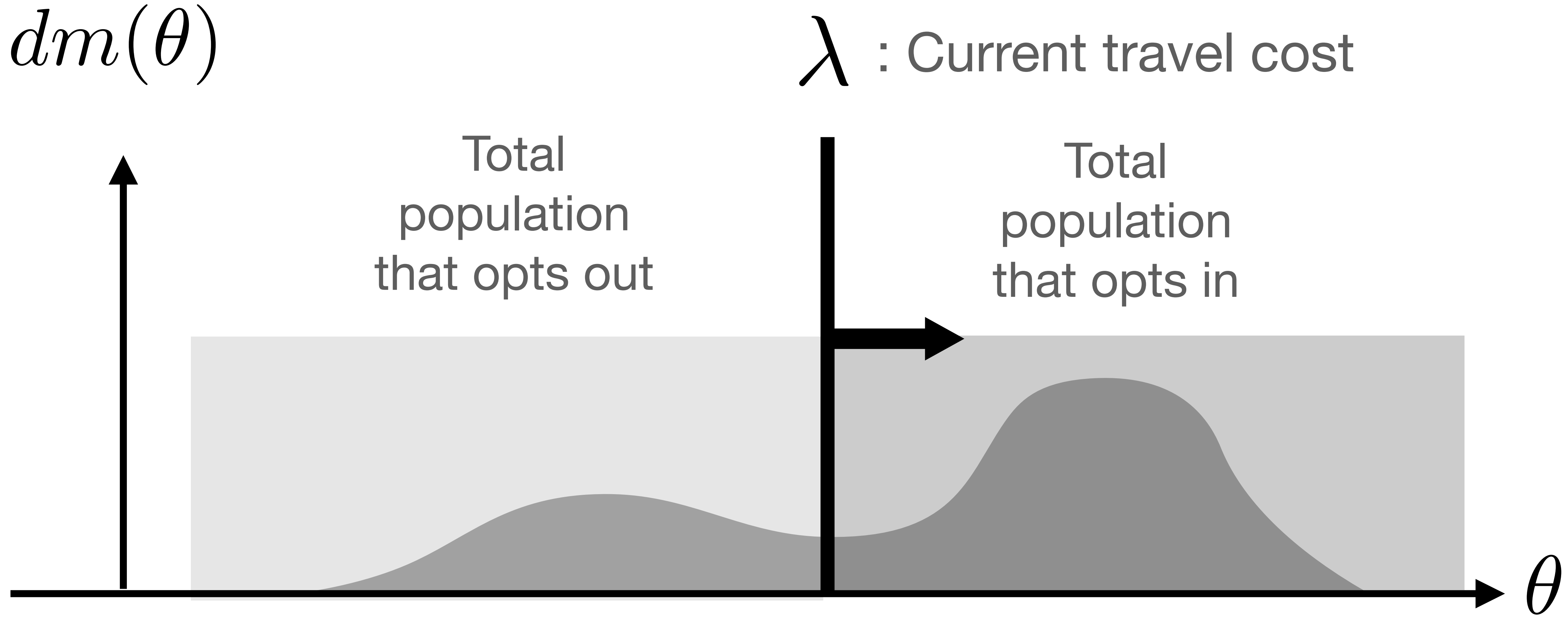
Non-homo-geneous prefences



Multi-Variate Preferences

$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$

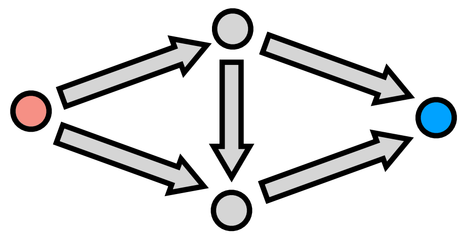




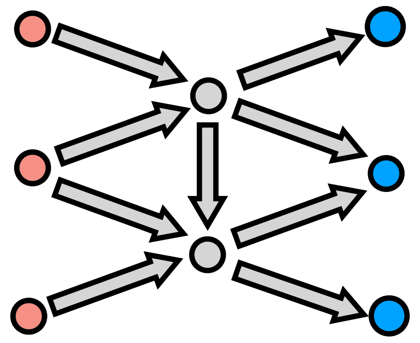
θ : Max cost $dm(\theta)$ will pay

Potential Games

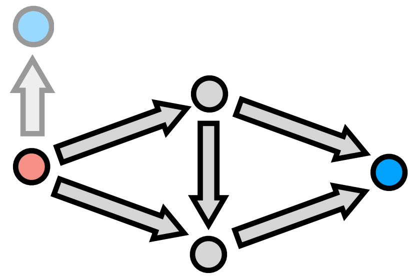
Routing Games



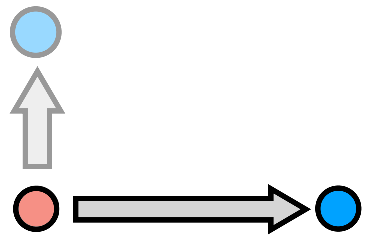
Multiple sources/ sinks



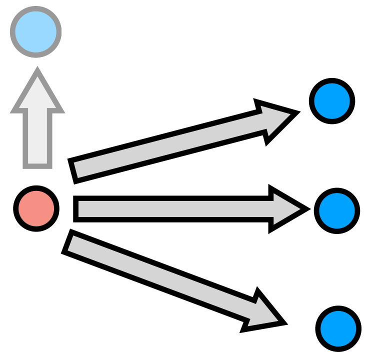
Variable Demand



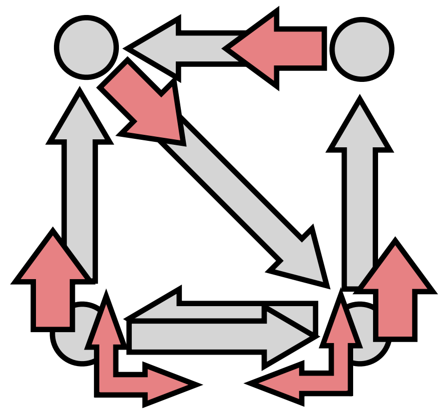
Supply & Demand



Cournot Market

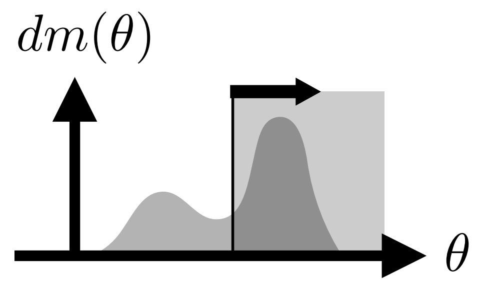


MDP Congestion Game

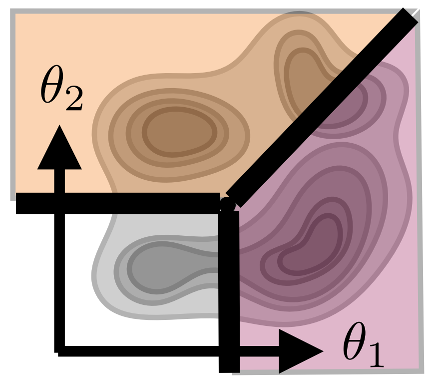


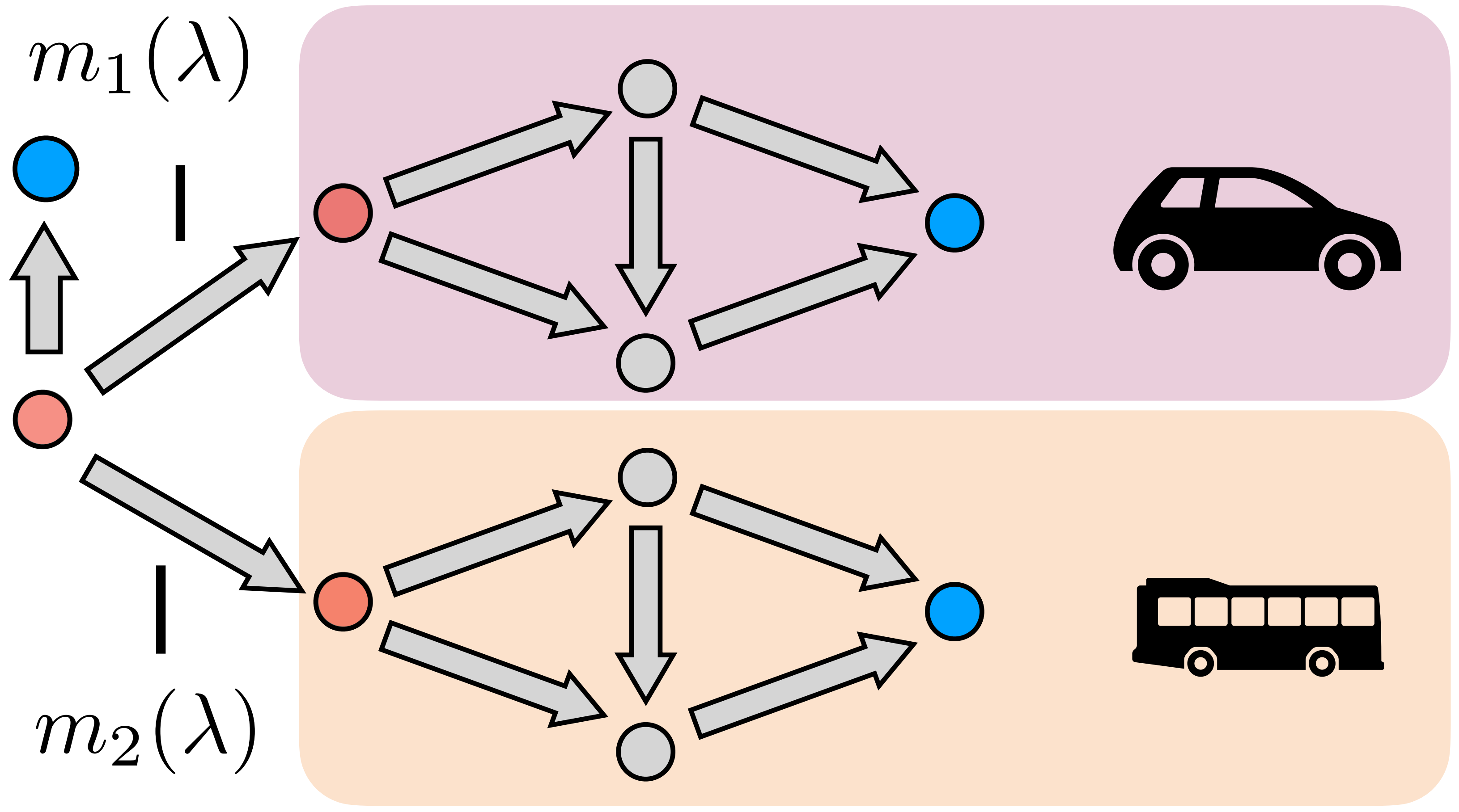
Variable Demand - Multi-Variate Non-Homogeneous Preferences

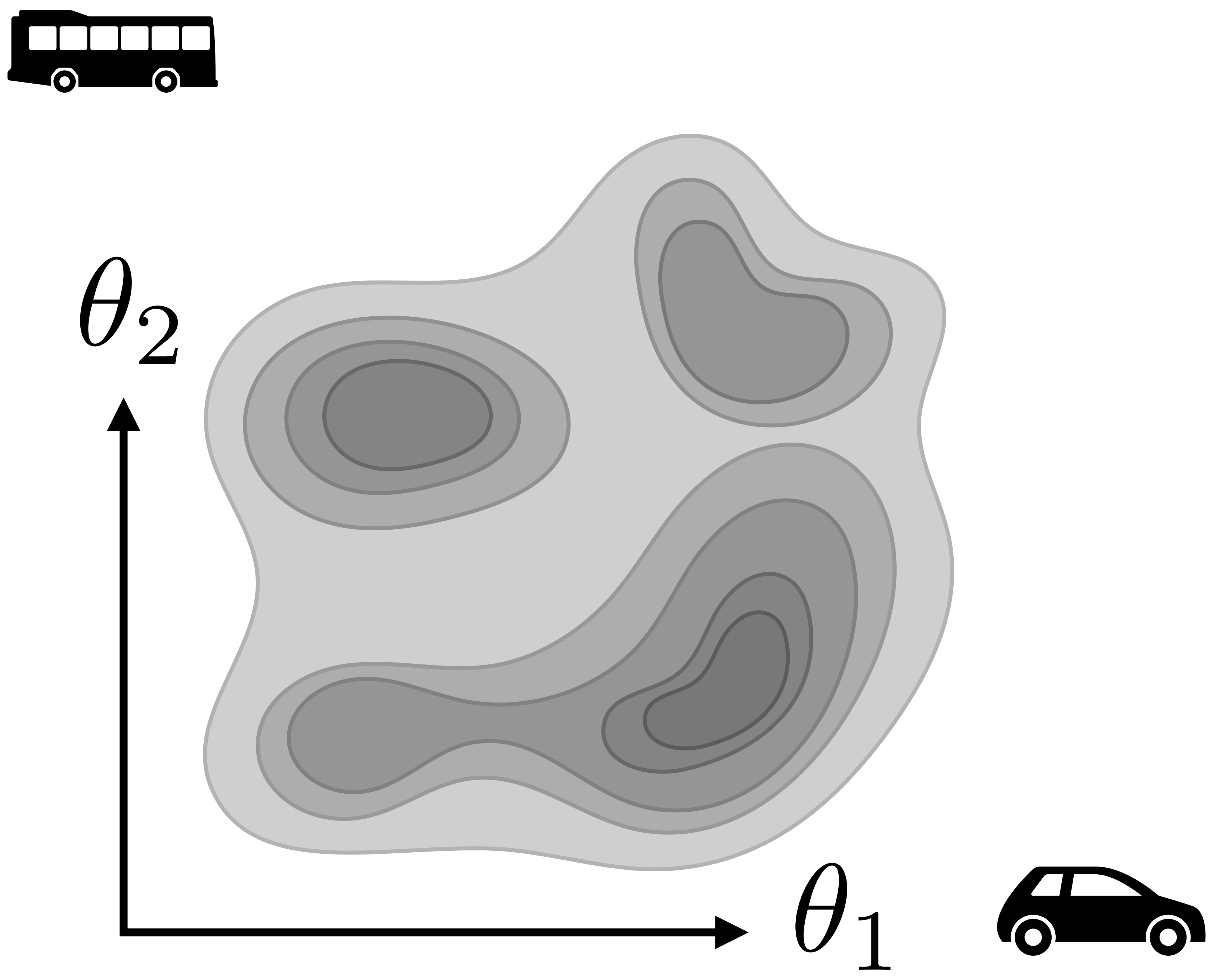
Non-homo-geneous prefences



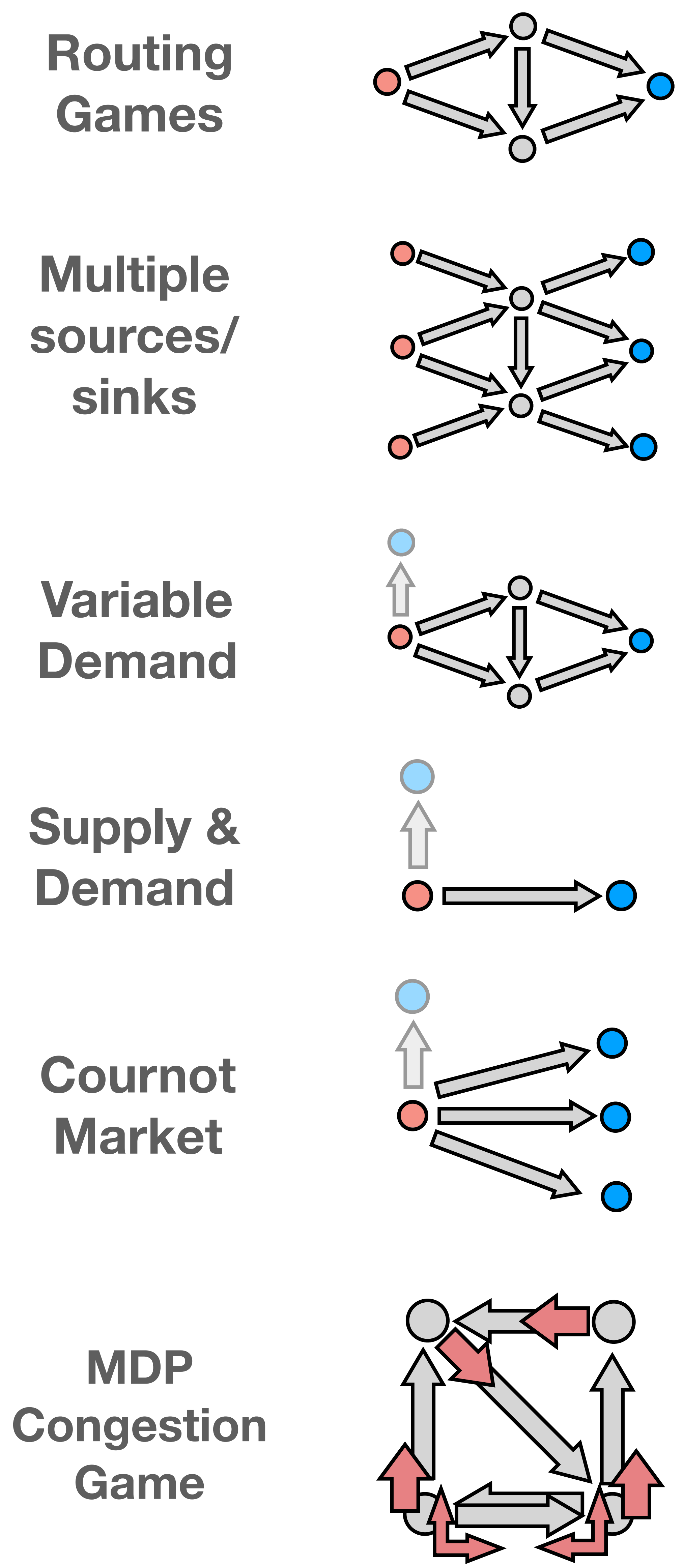
Multi-Variate Preferences



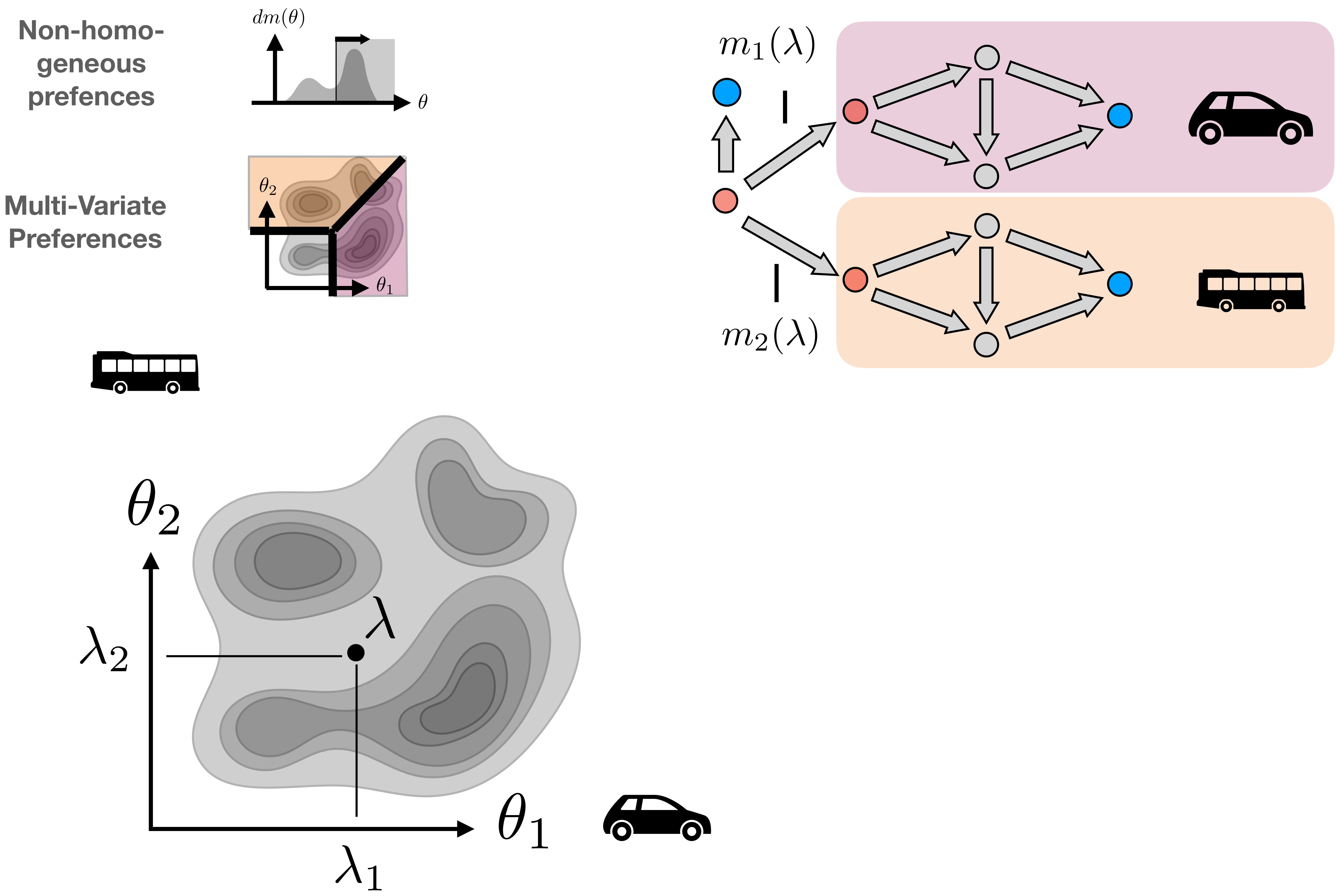




Potential Games



Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games

Routing Games

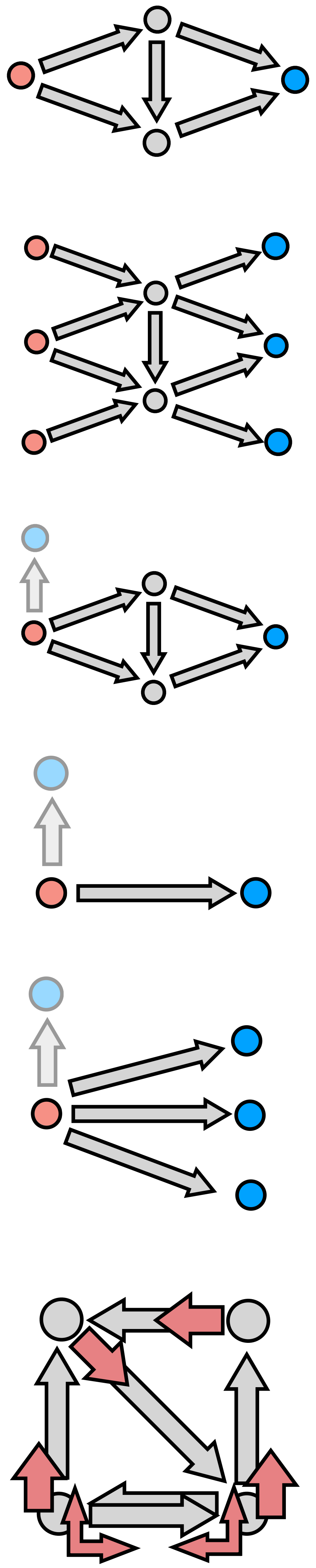
Multiple sources/ sinks

Variable Demand

Supply & Demand

Cournot Market

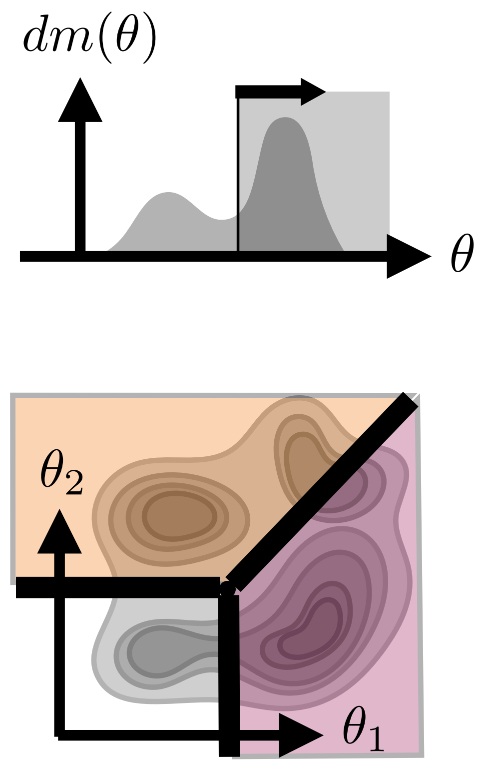
MDP Congestion Game

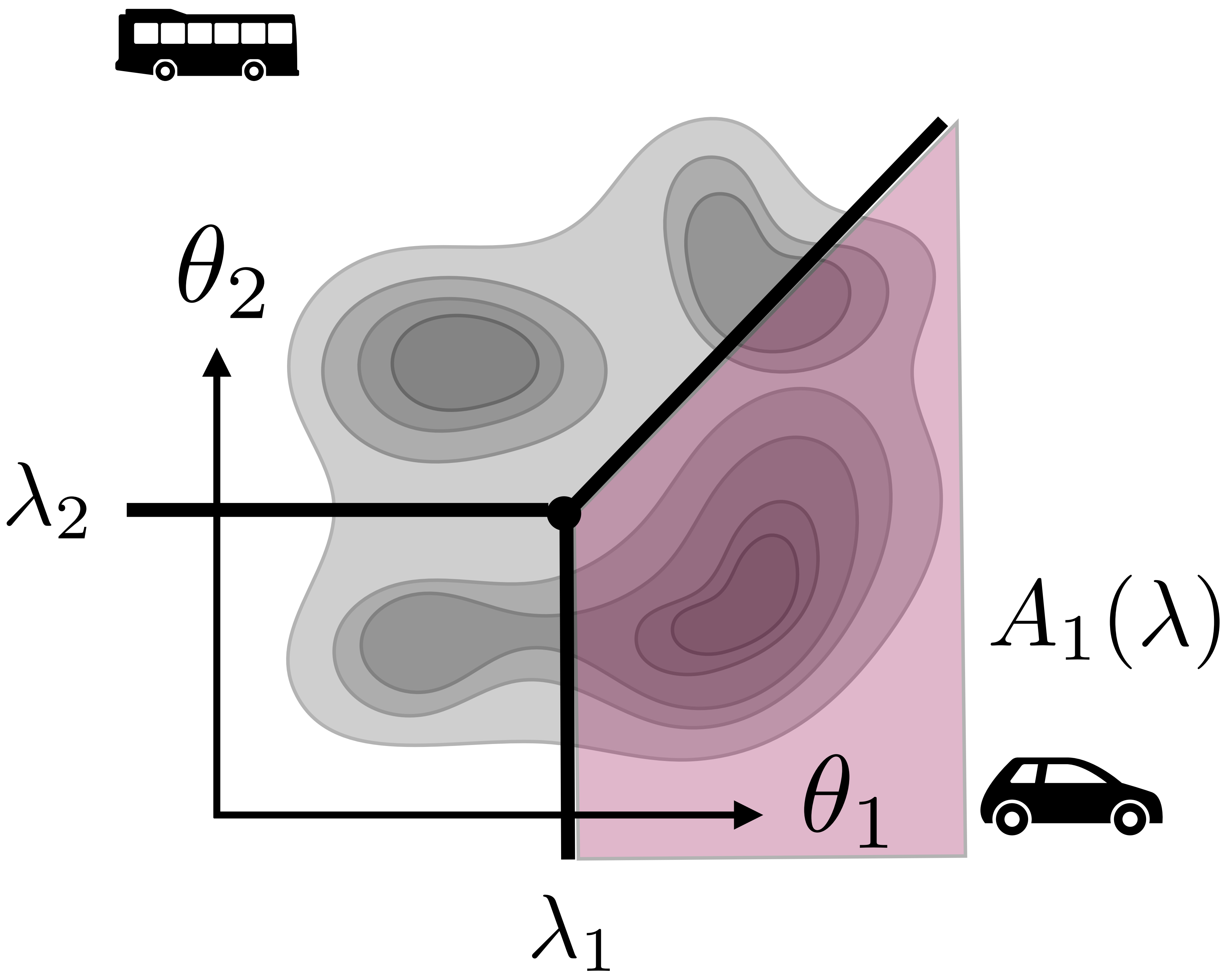


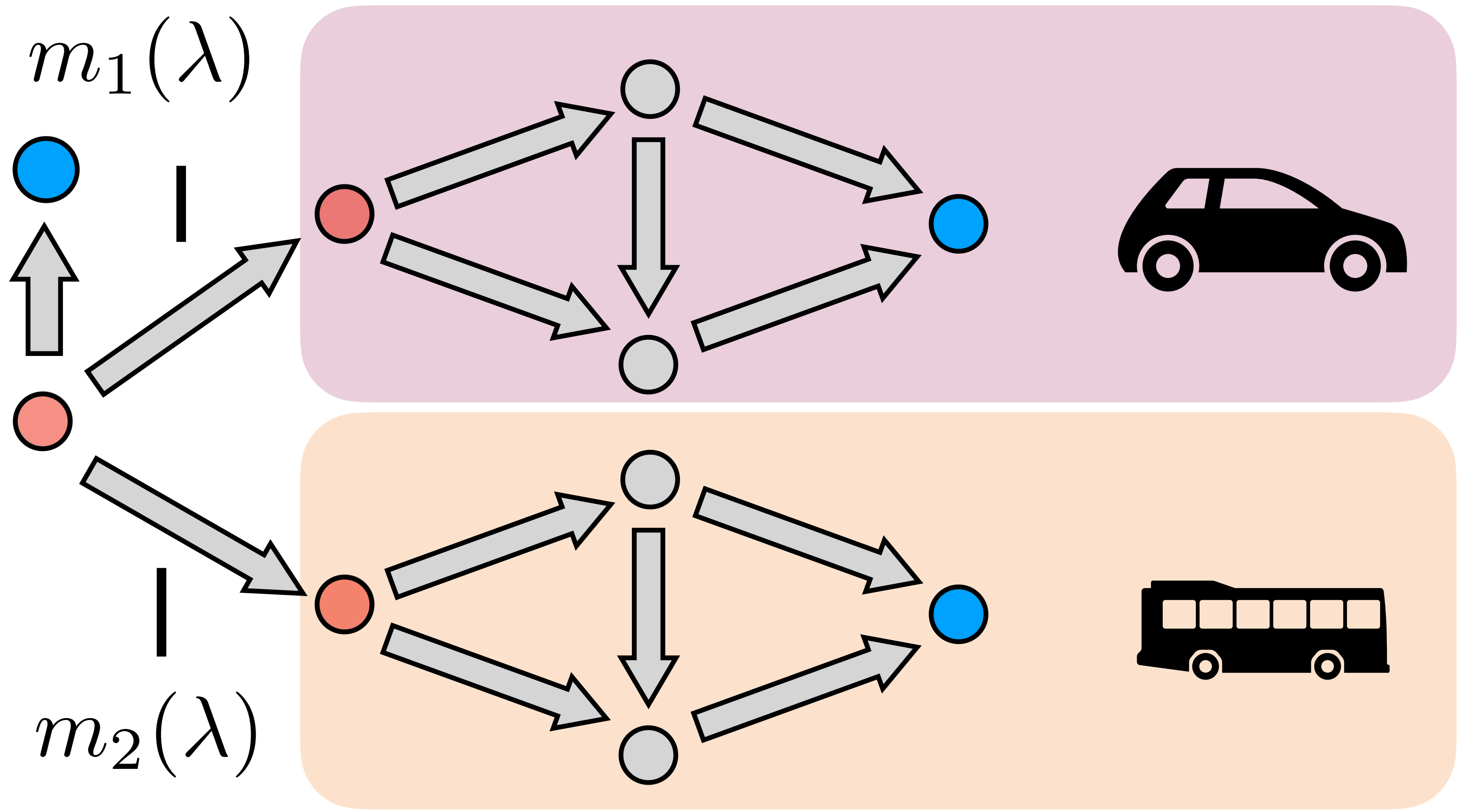
Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homogeneous prefences

Multi-Variate Preferences







$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

Potential Games

Routing Games

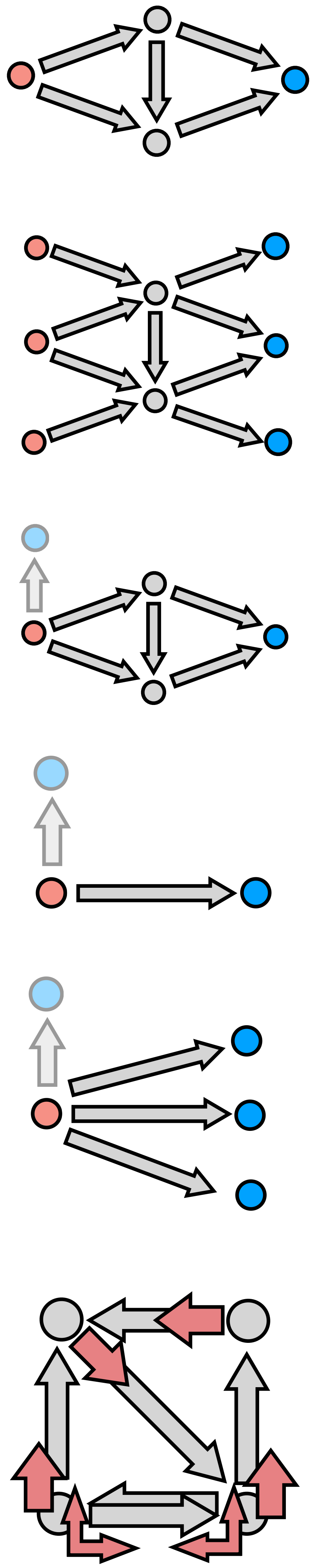
Multiple sources/ sinks

Variable Demand

Supply & Demand

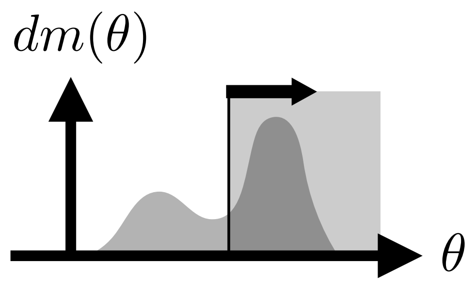
Cournot Market

MDP Congestion Game

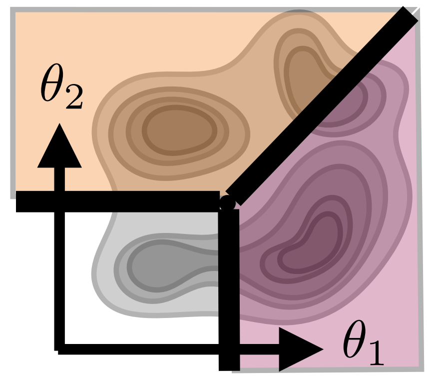


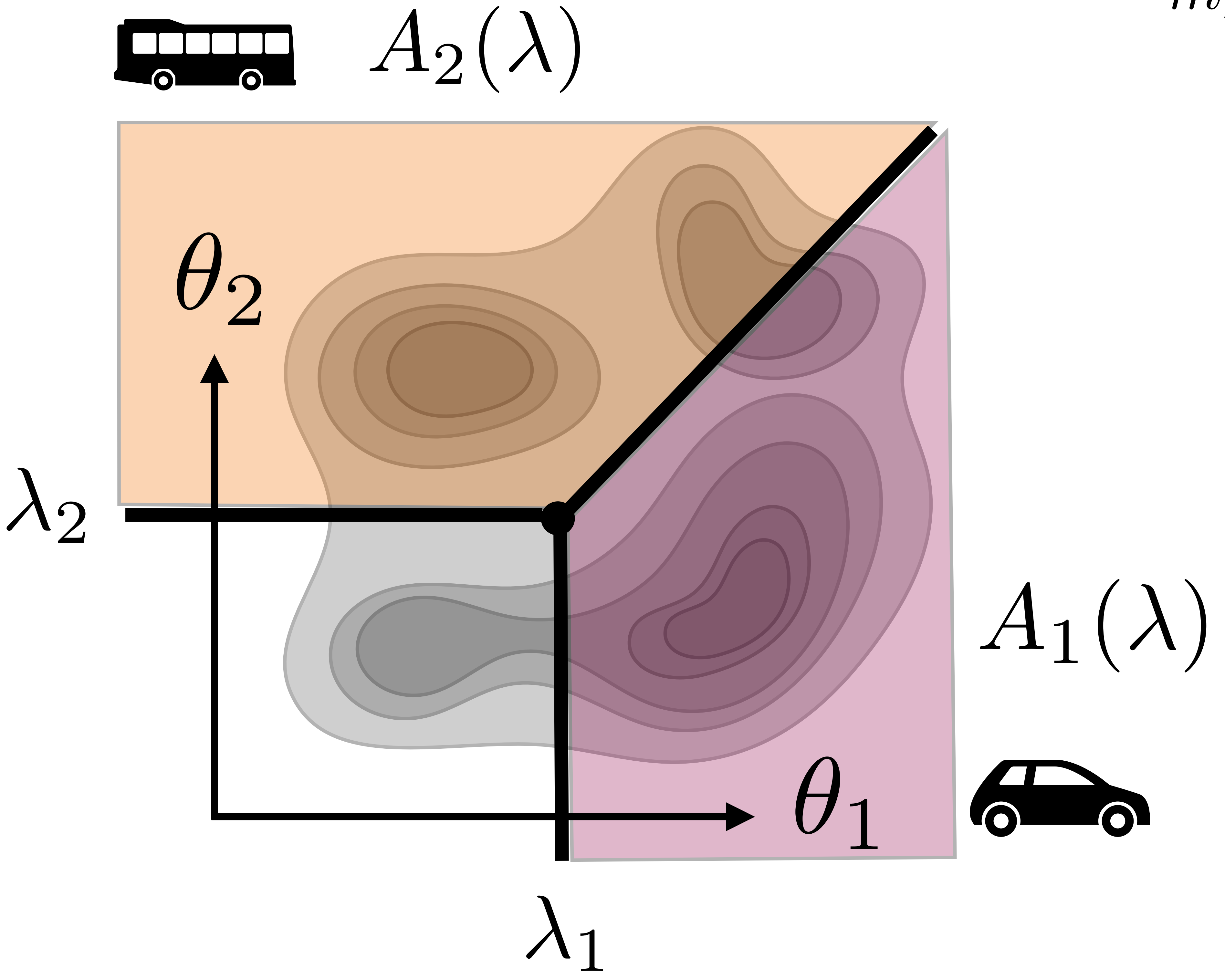
Variable Demand - Multi-Variate Non-Homogeneous Preferences

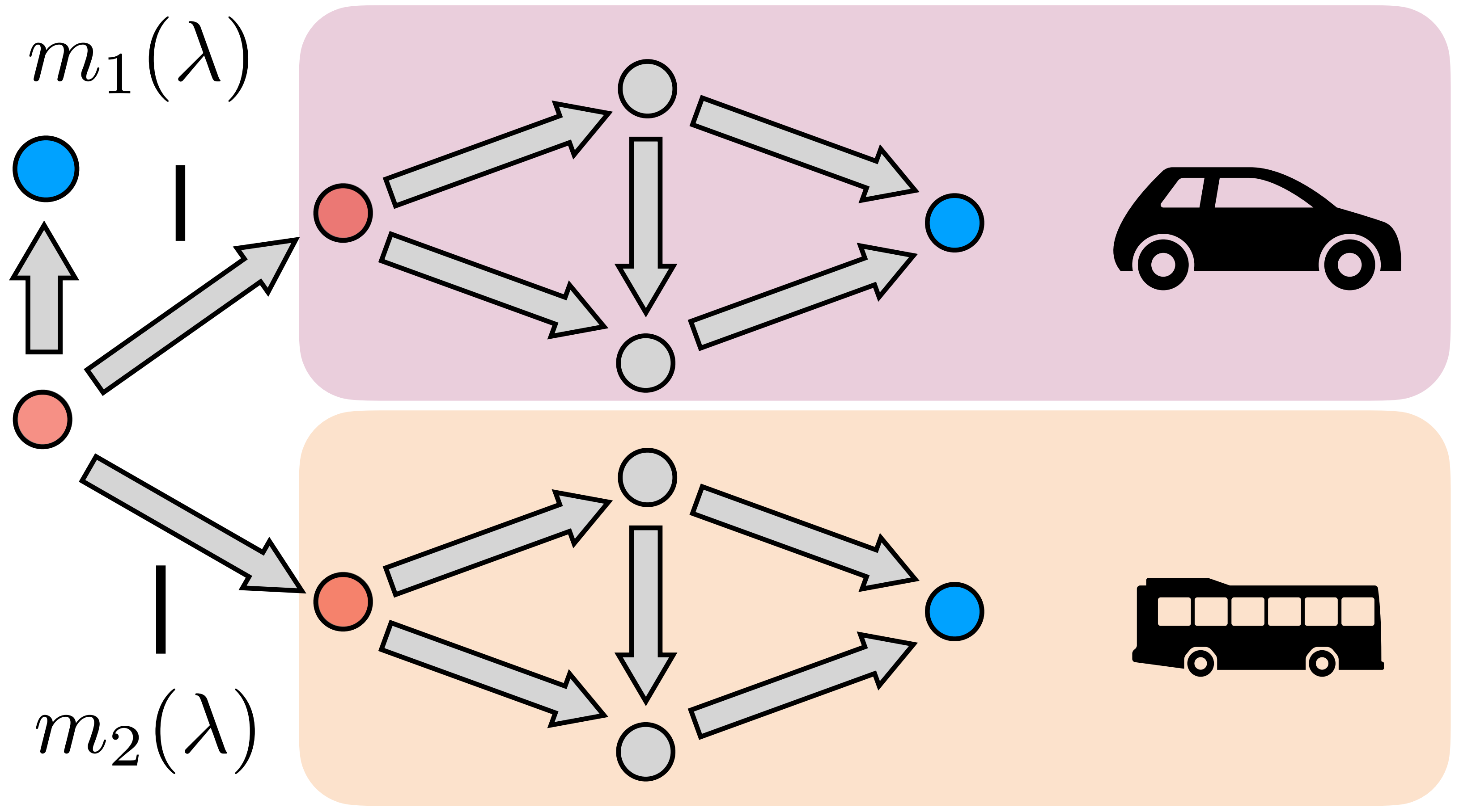
Non-homo-geneous prefences



Multi-Variate Preferences



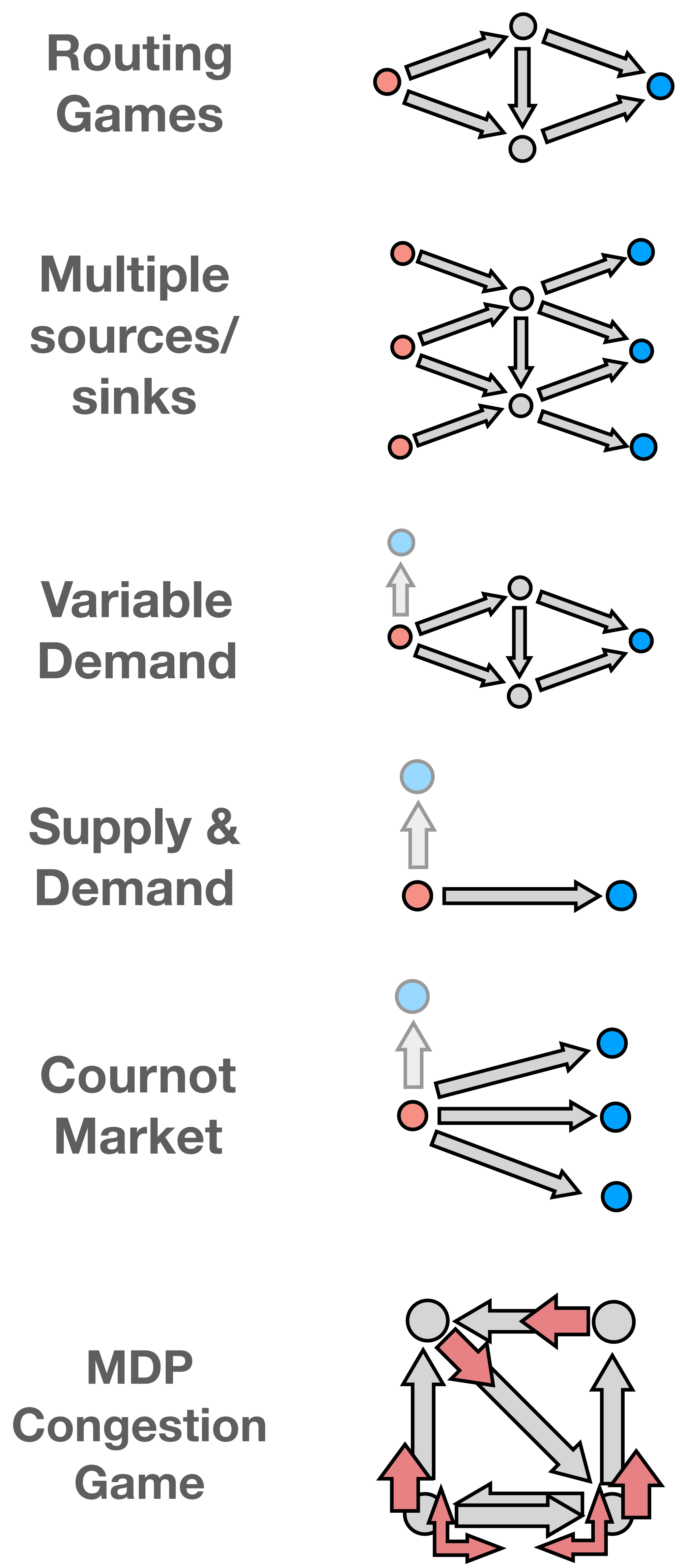




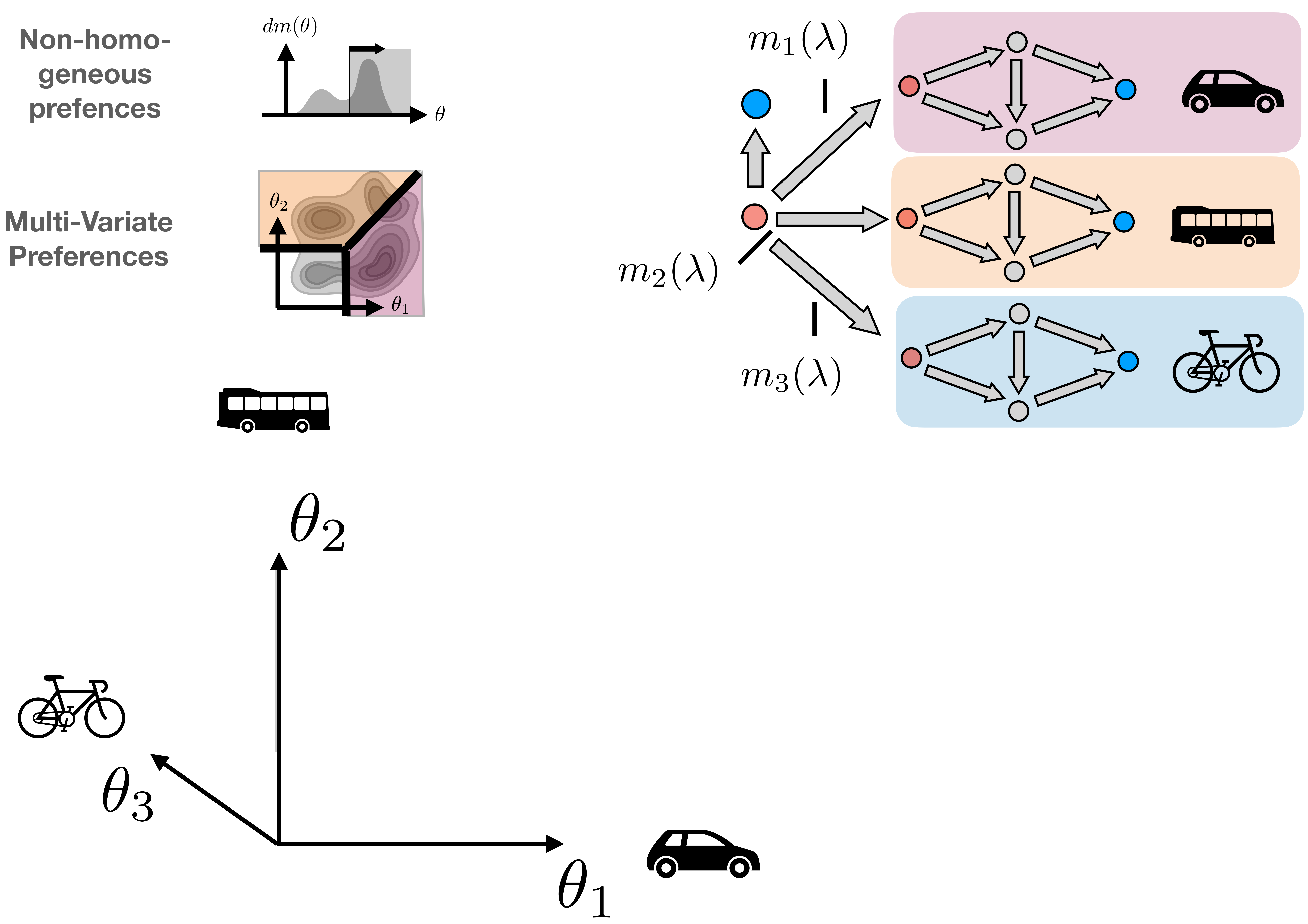
$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

$$m_2(\lambda) = \int_{A_2(\lambda)} dm(\theta)$$

Potential Games



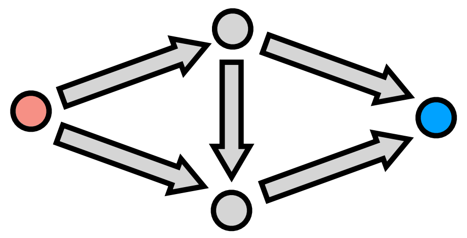
Variable Demand - Multi-Variate Non-Homogeneous Preferences



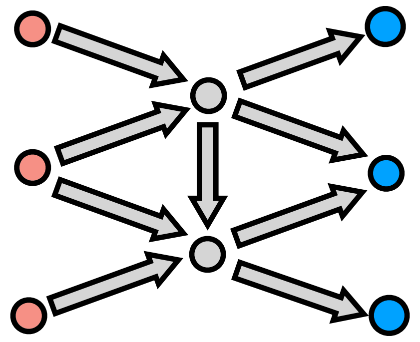
Potential Games

Variable Demand - Multi-Variate Non-Homogeneous Preferences

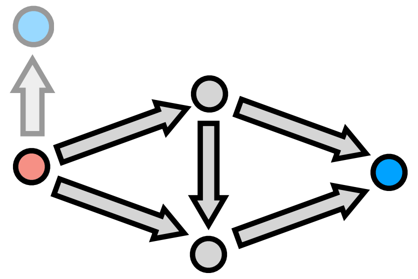
Routing Games



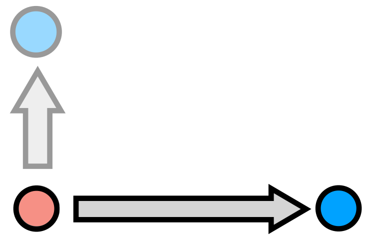
Multiple sources/ sinks



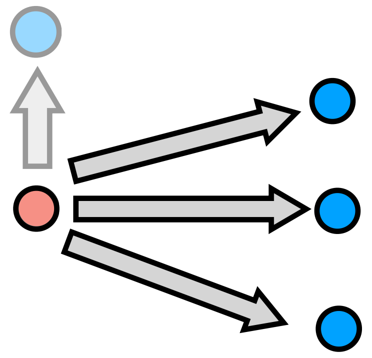
Variable Demand



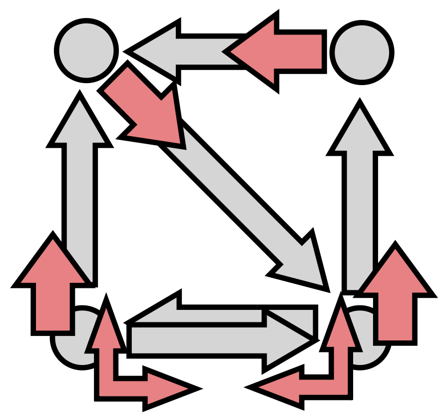
Supply & Demand



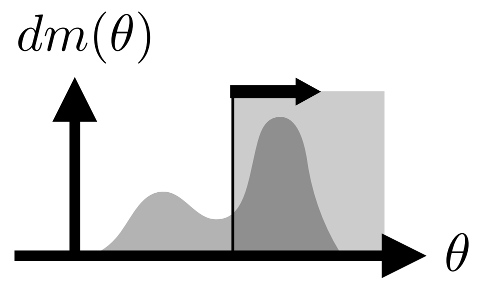
Cournot Market



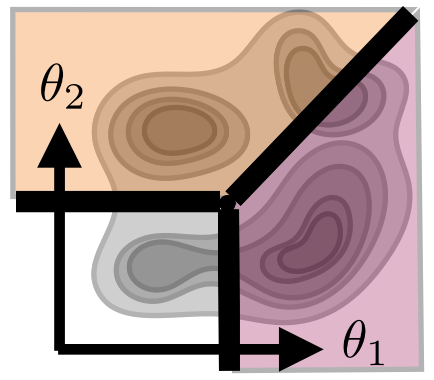
MDP Congestion Game

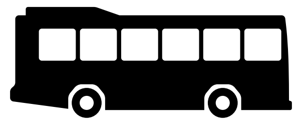


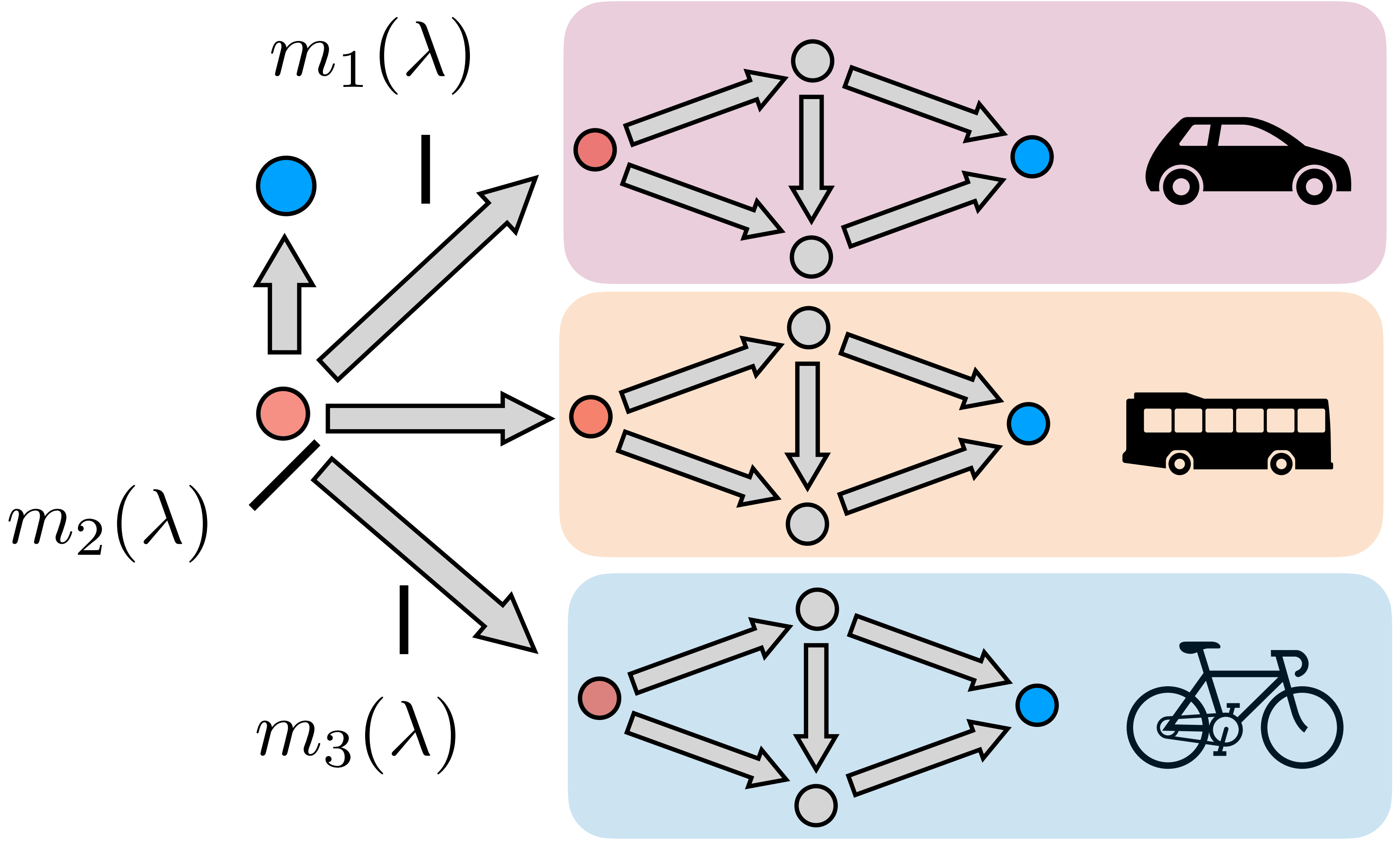
Non-homo-geneous prefences

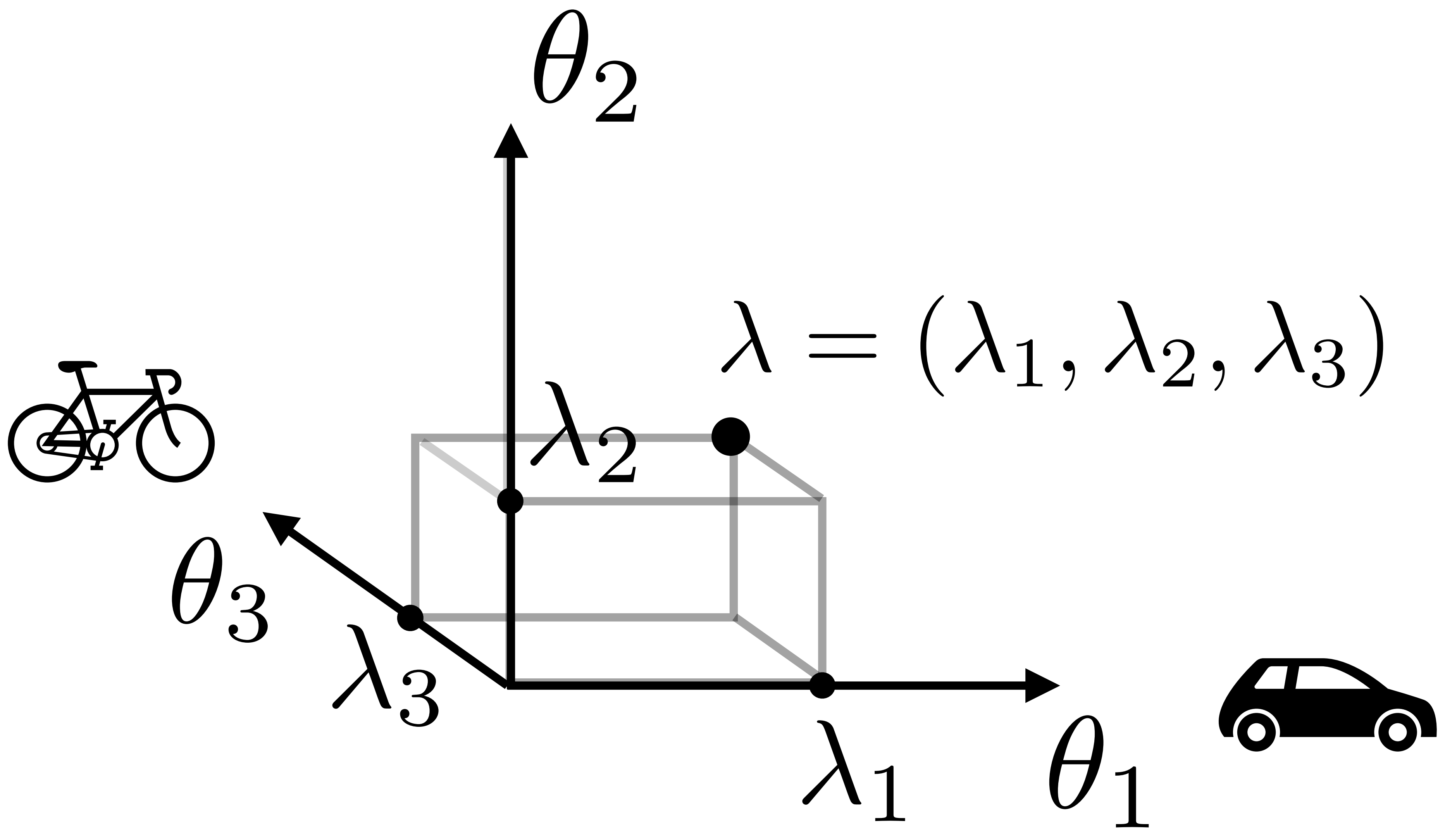


Multi-Variate Preferences









Potential Games

Variable Demand - Multi-Variate Non-Homogeneous Preferences

Routing Games

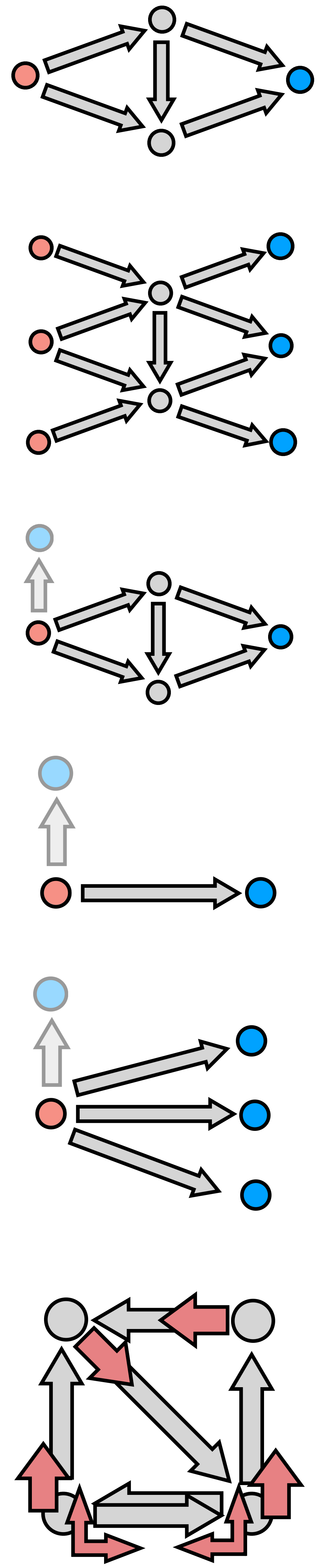
Multiple sources/ sinks

Variable Demand

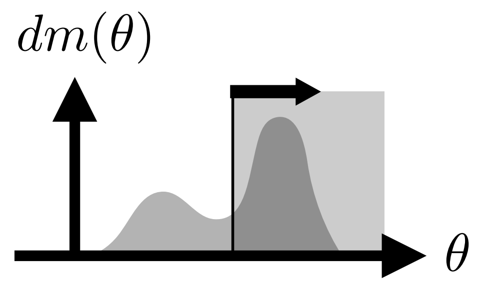
Supply & Demand

Cournot Market

MDP Congestion Game



Non-homo-geneous prefences



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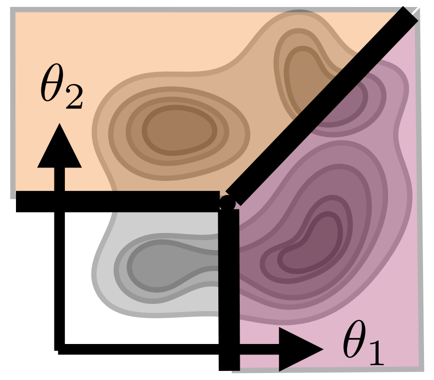

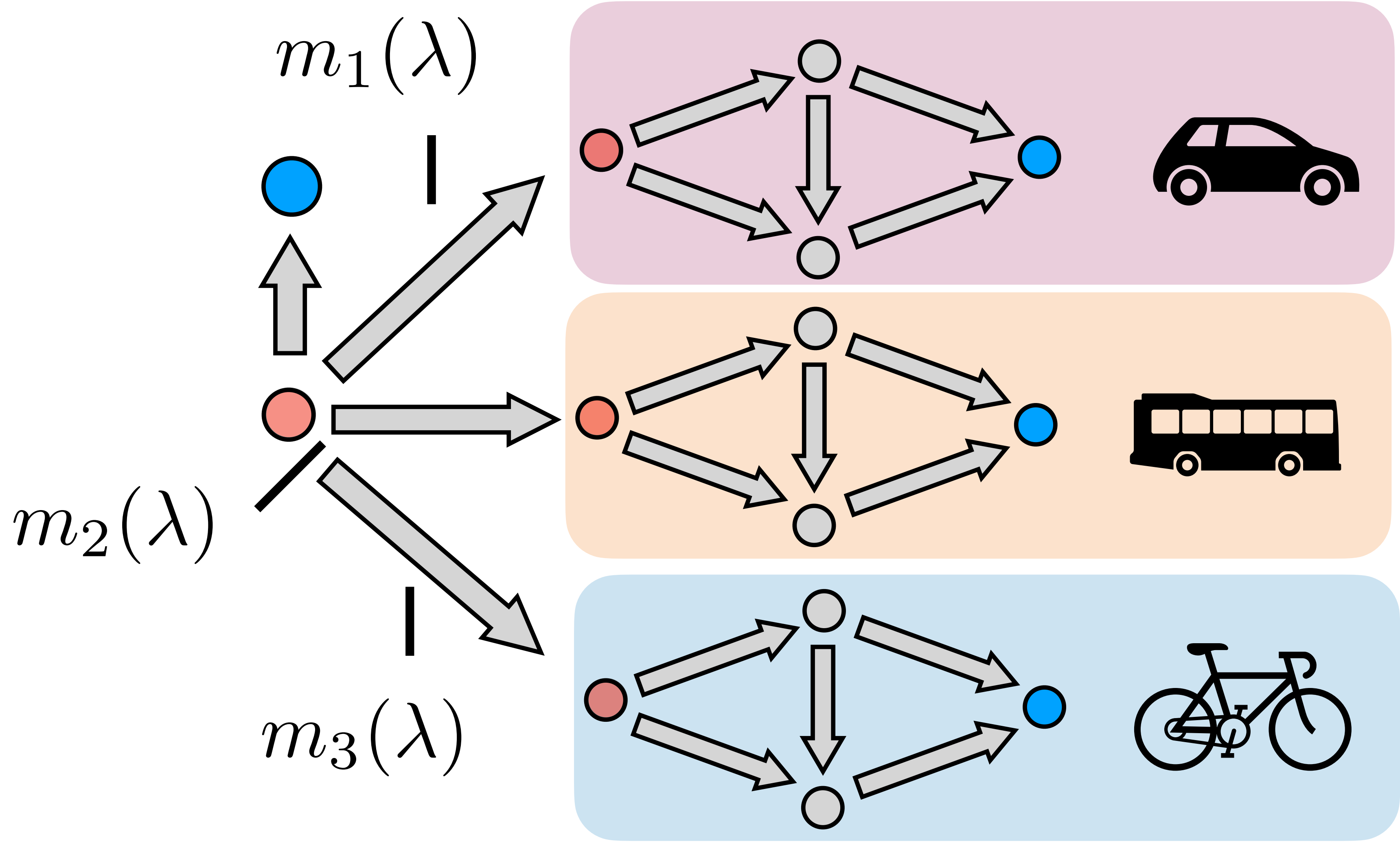

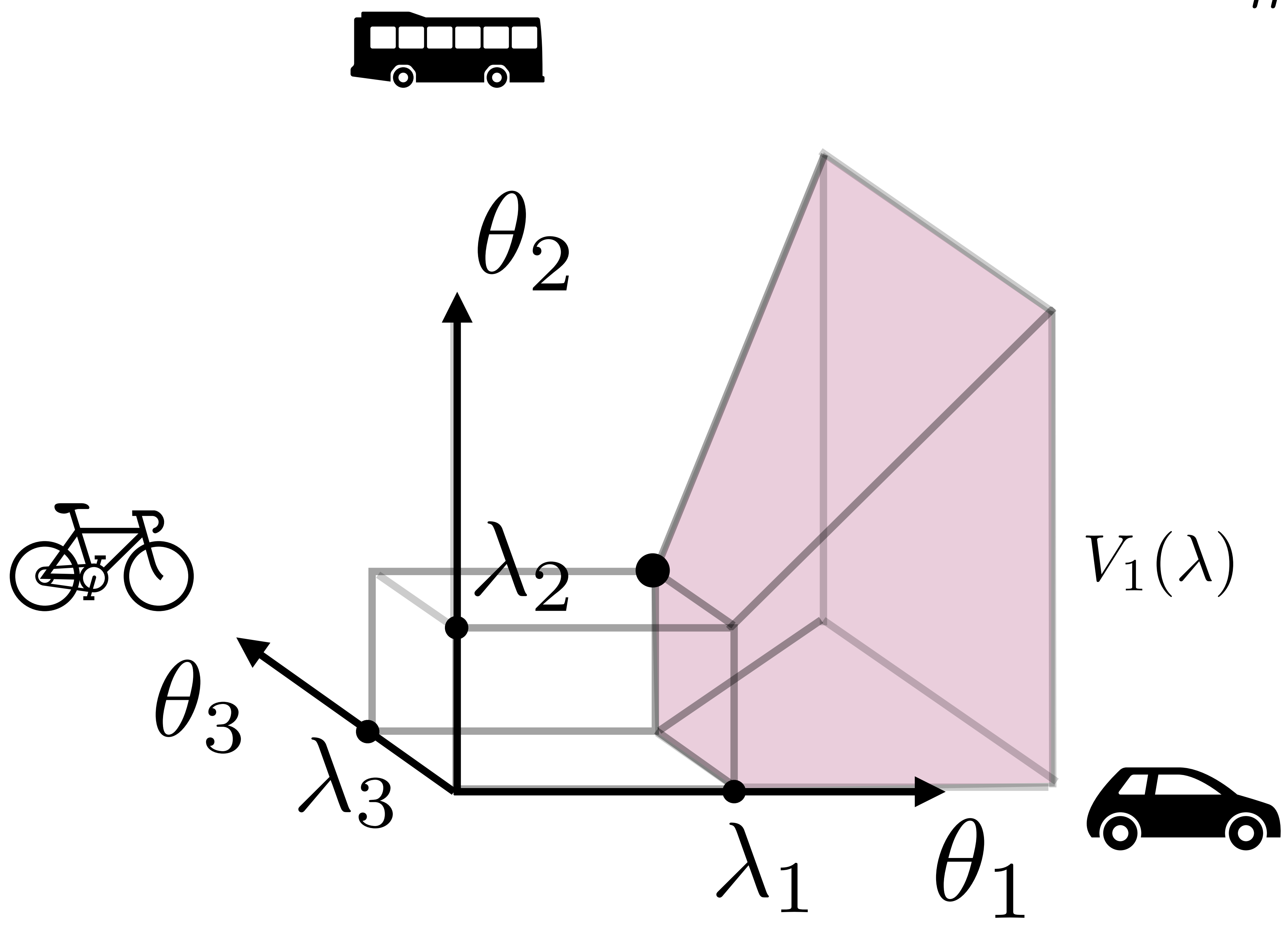

$$m_1(\lambda) = \int_{V_1(\lambda)} dm(\theta)$$

Potential Games

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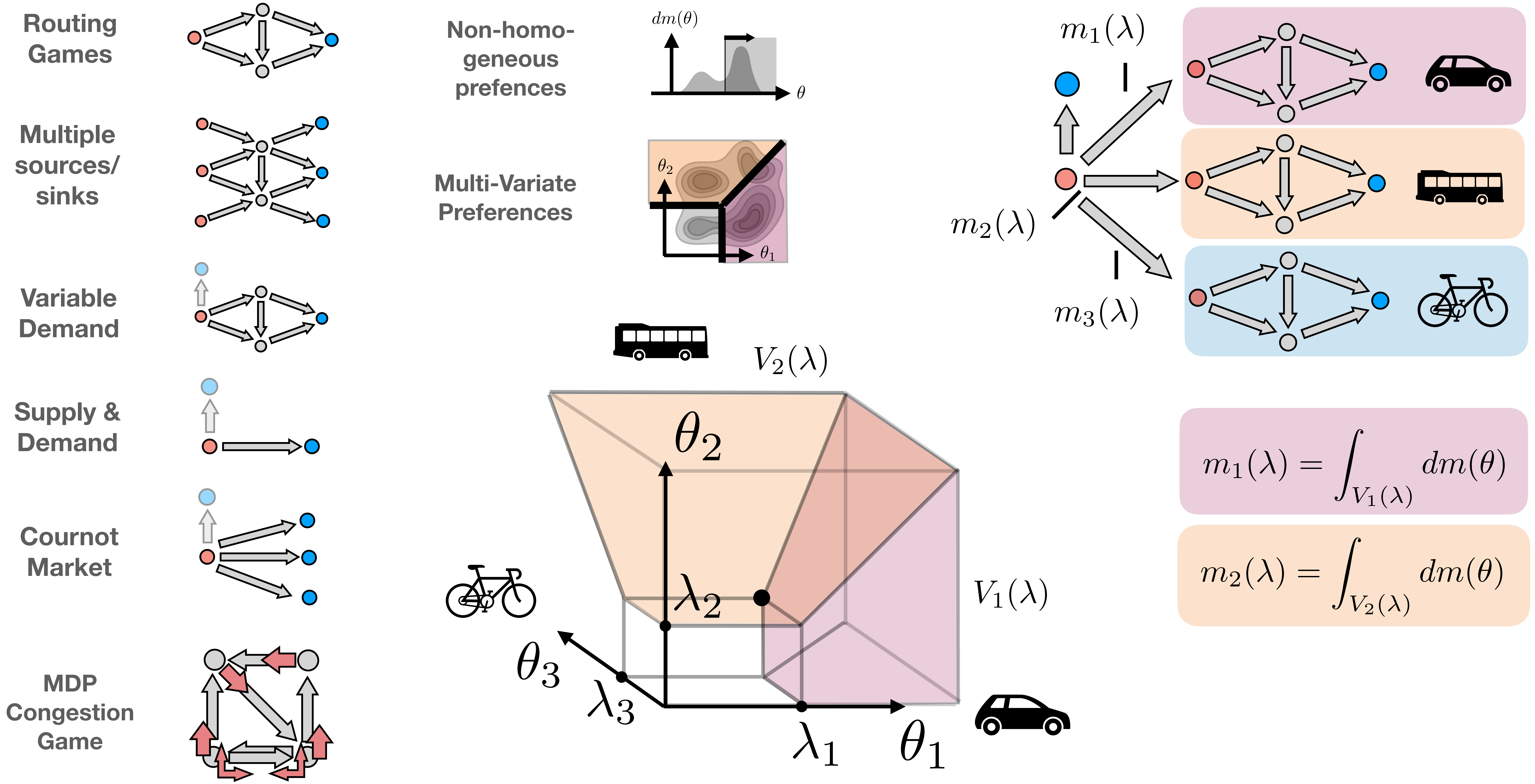
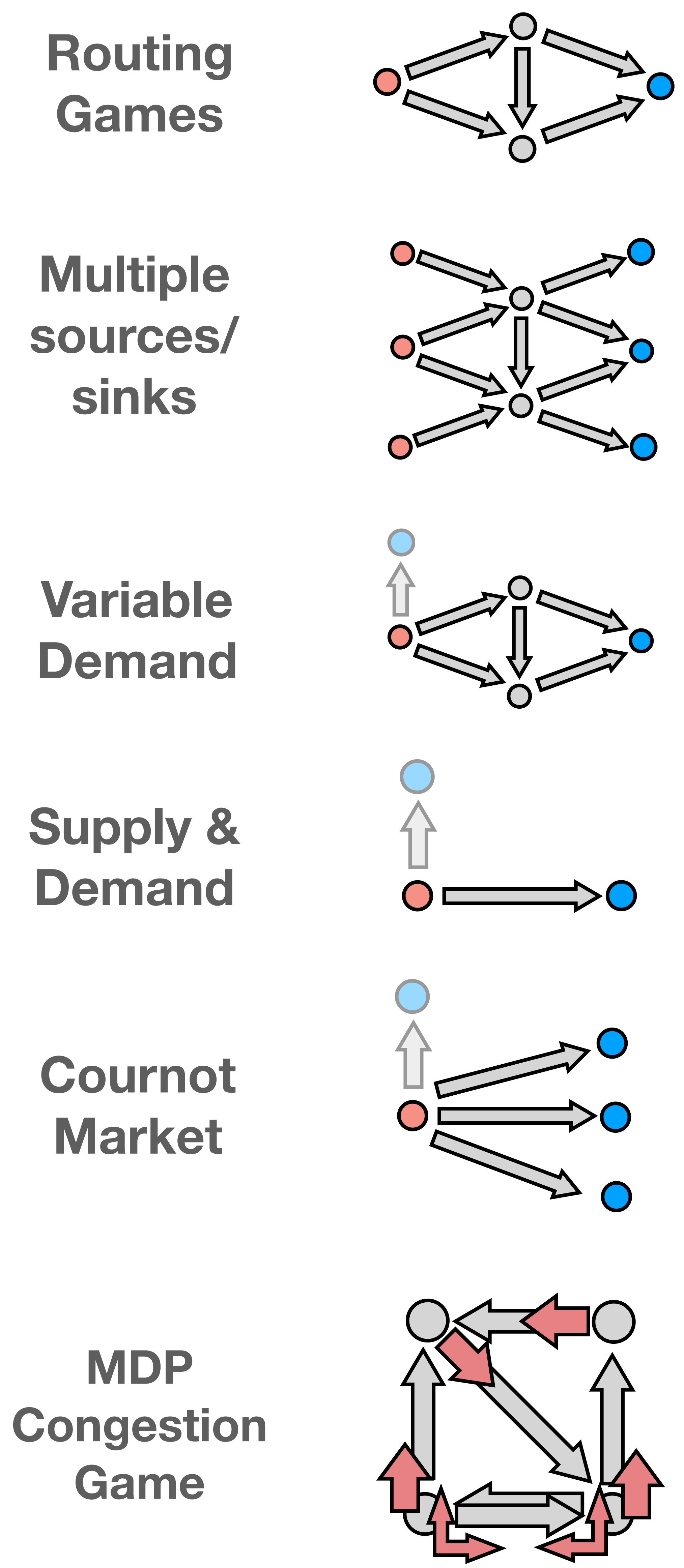


Potential Games



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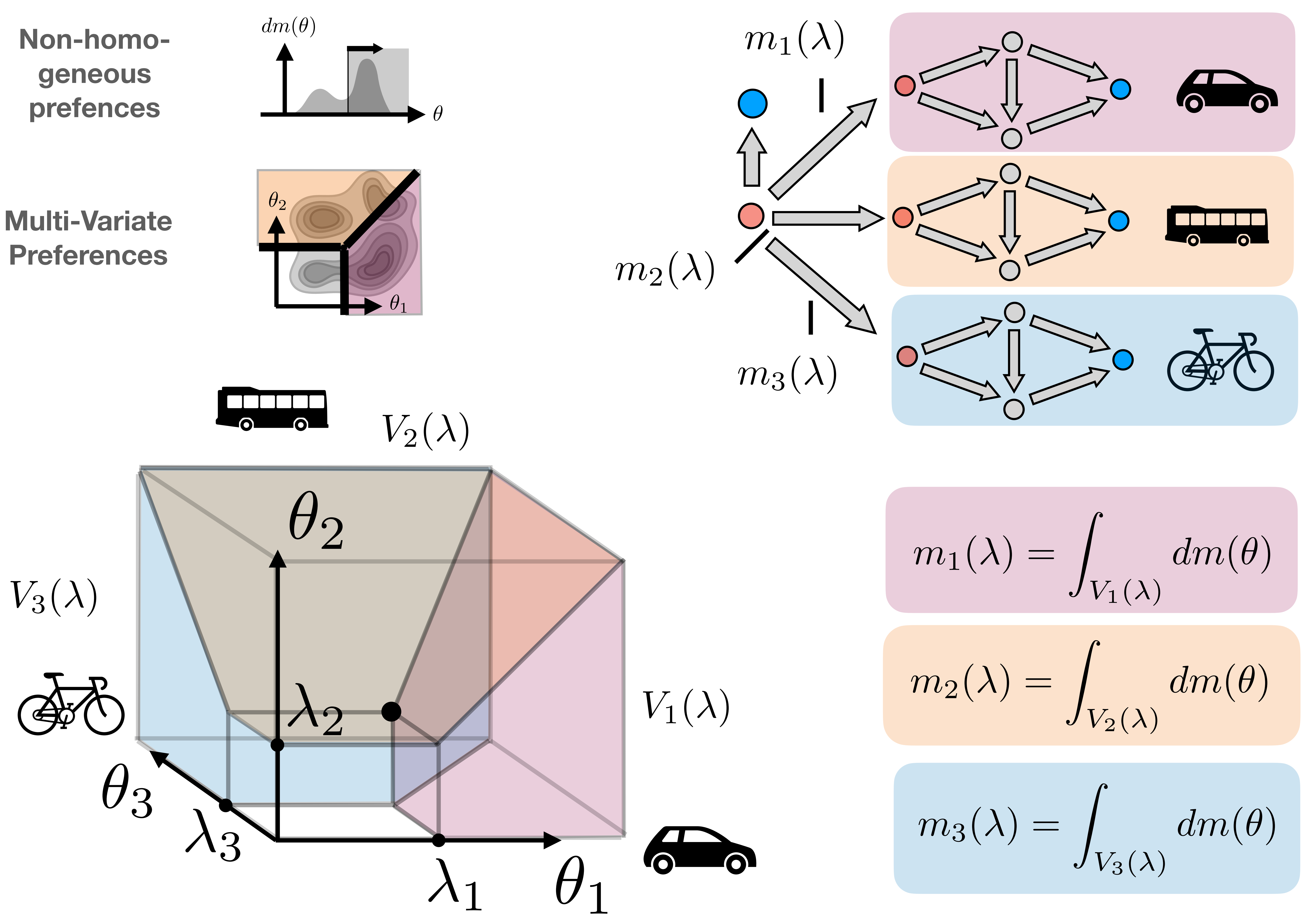
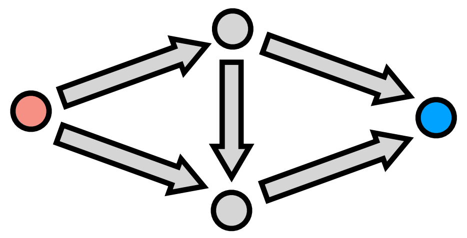


Potential Games

Routing Games



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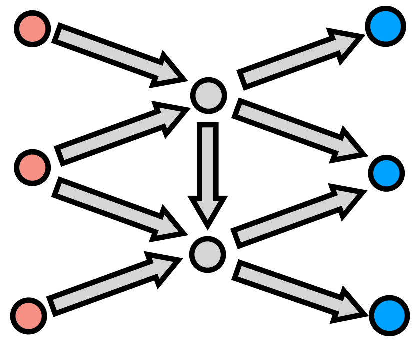

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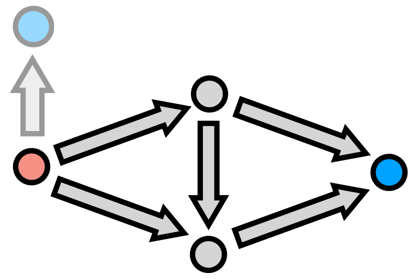

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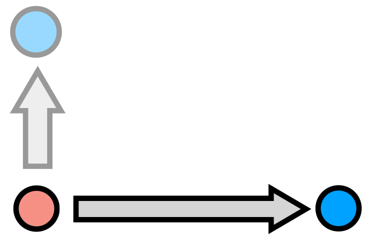
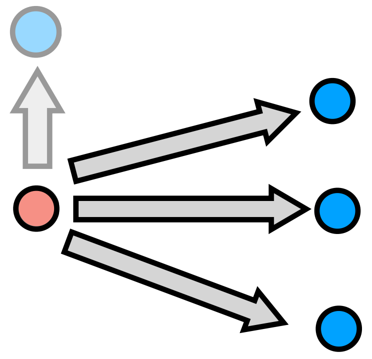


Cournot Market



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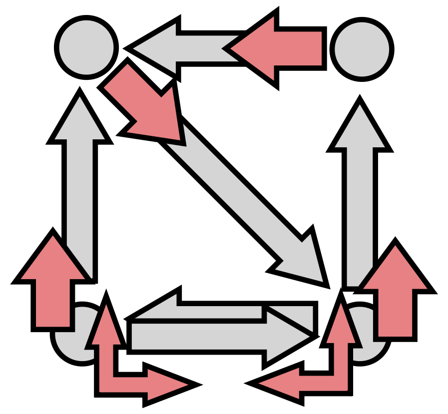

Variable Demand - Multi-Variate Non-Homogeneous Preferences

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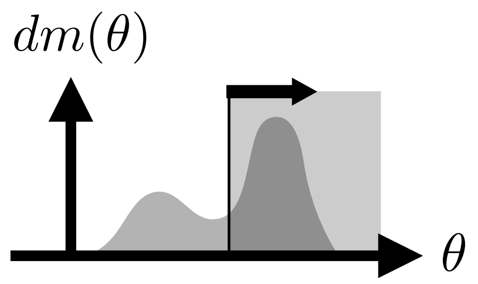

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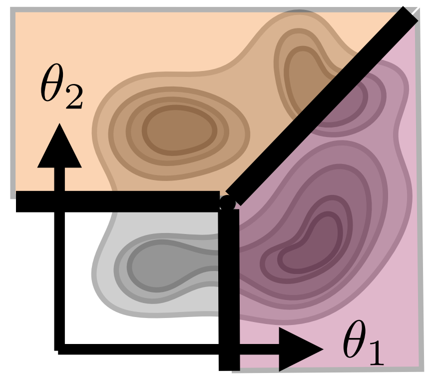
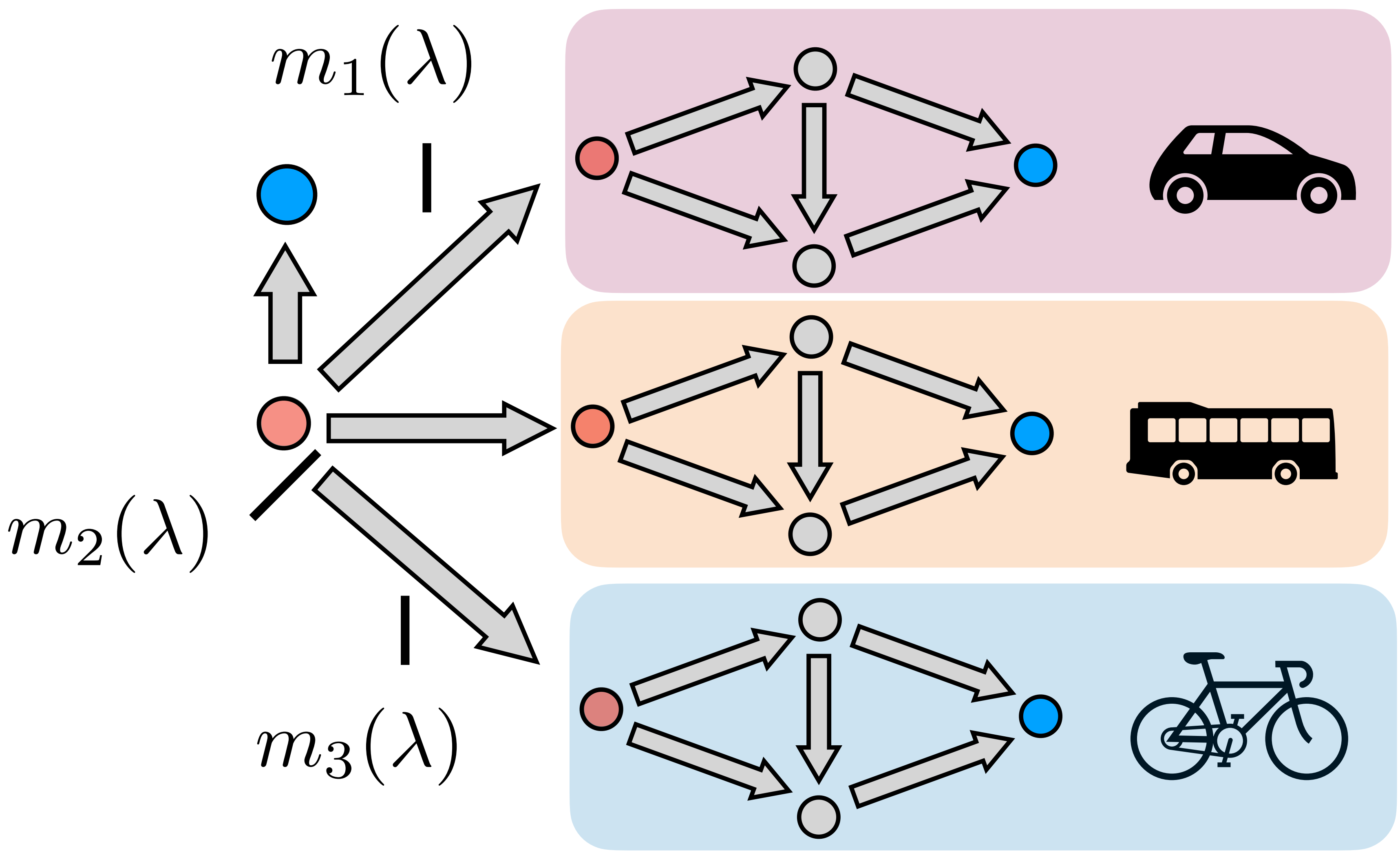




APPLICATIONS

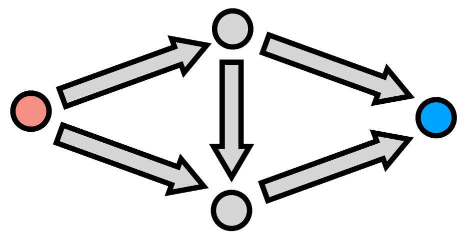
- Multi-modal transportation networks
- Non-homogeneous supply/demand

PAPERS

- External-cost continuous-type Wardrop equilibria in routing games
  [Calderone, Dong, Sastry, 2017]
- Multi-dimensional continuous type population potential games
  [Calderone, Ratliff, 2019]

Potential Games

Routing Games



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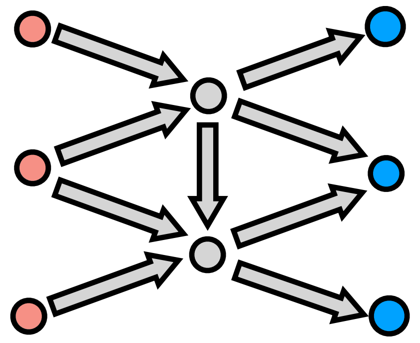

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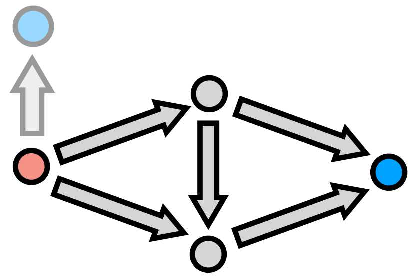

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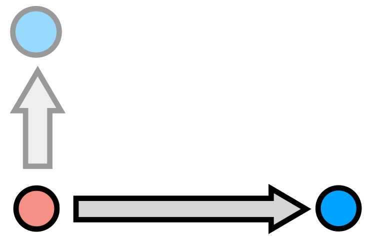

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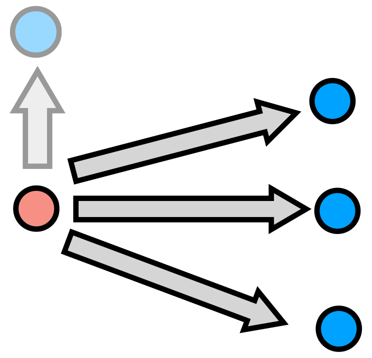

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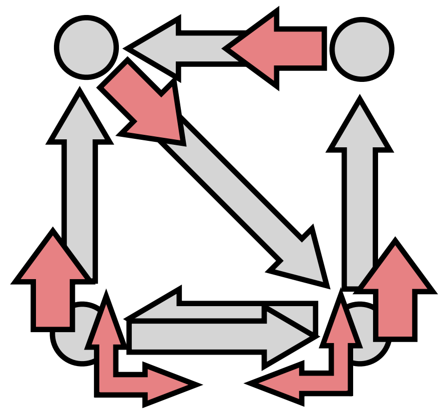

Braess Paradox

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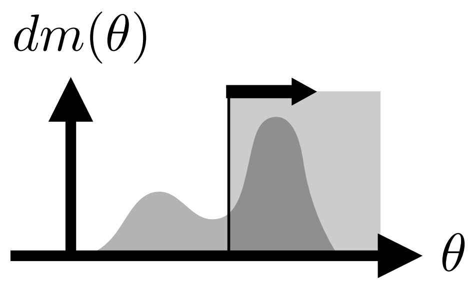

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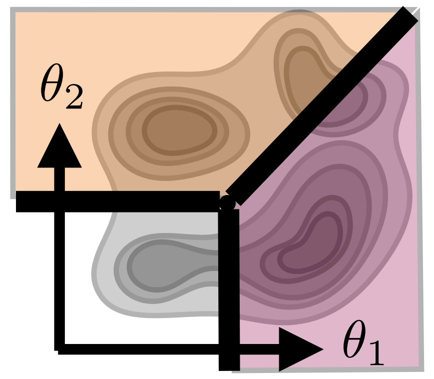

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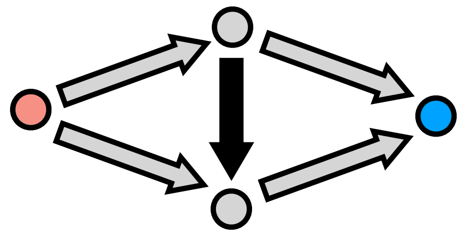

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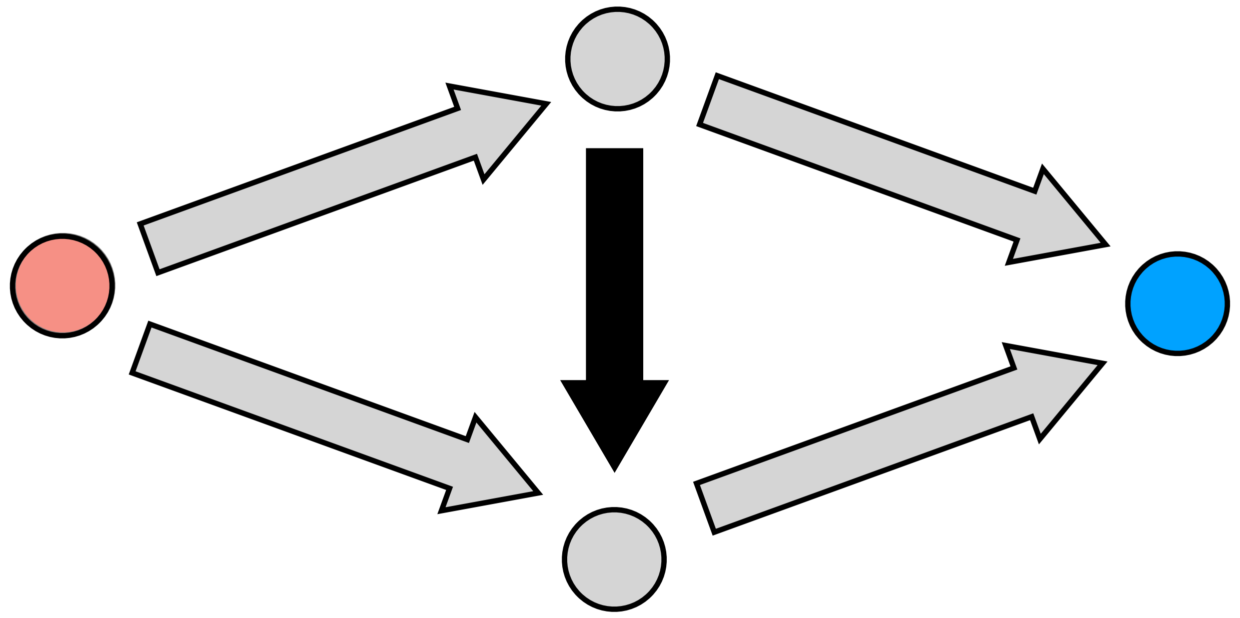
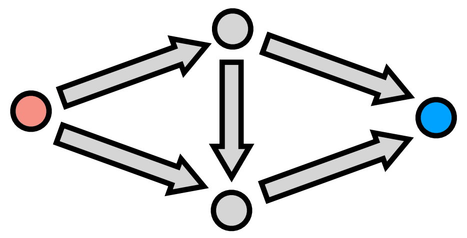


Potential Games

Routing Games



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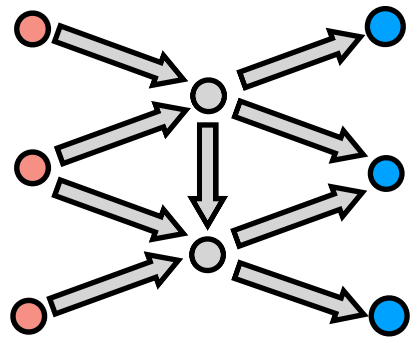

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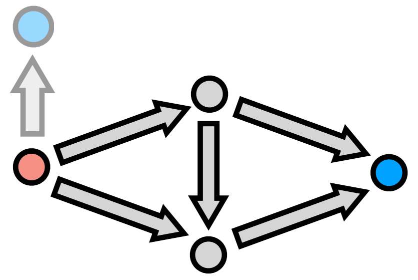

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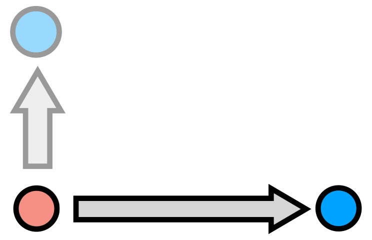

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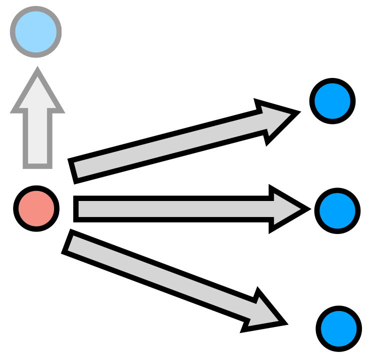

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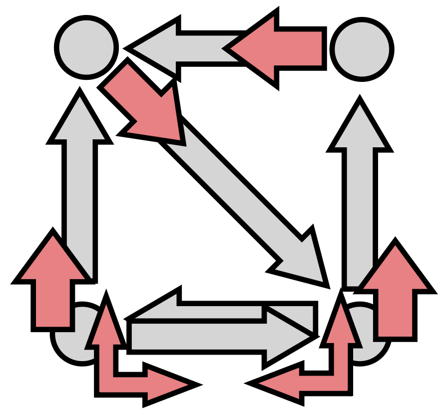

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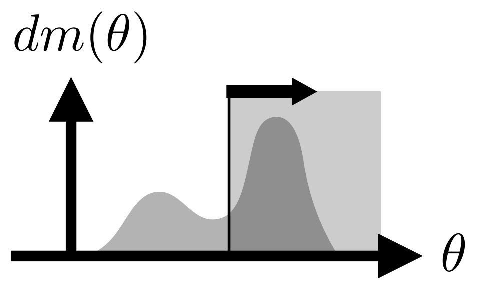

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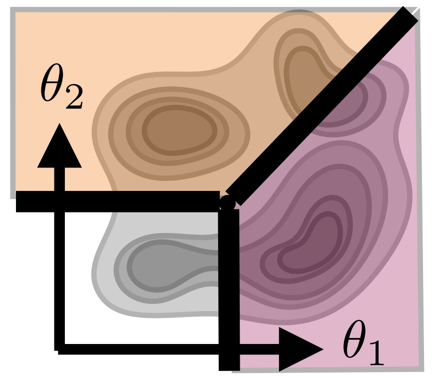

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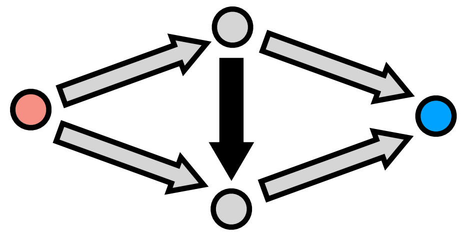

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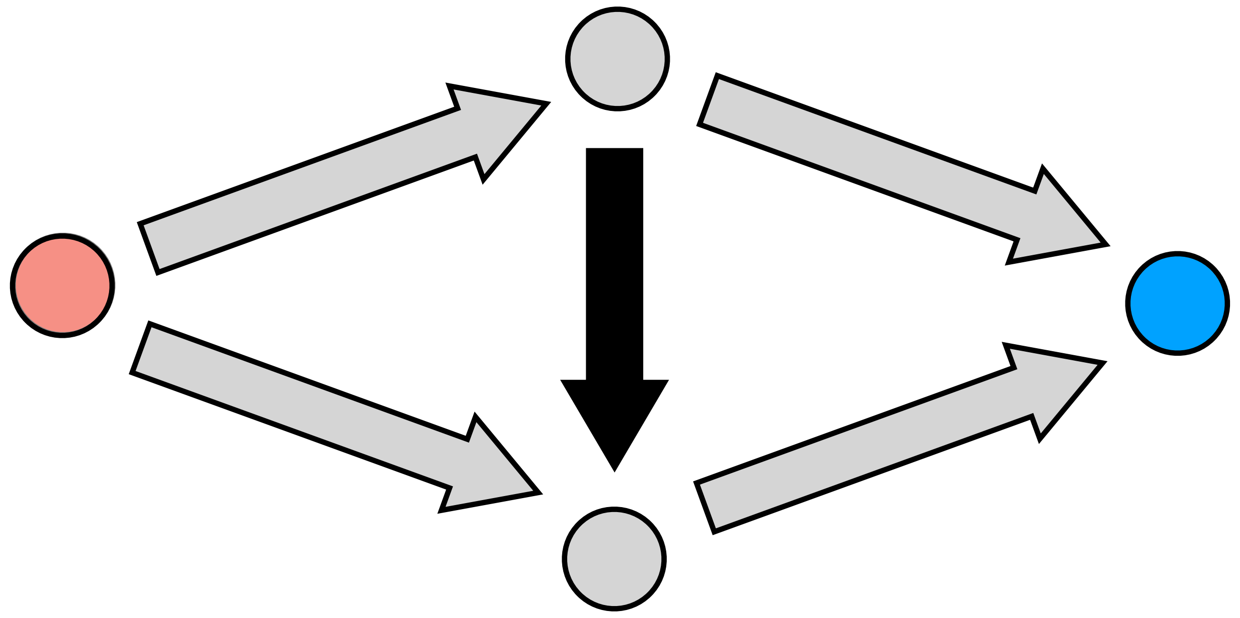
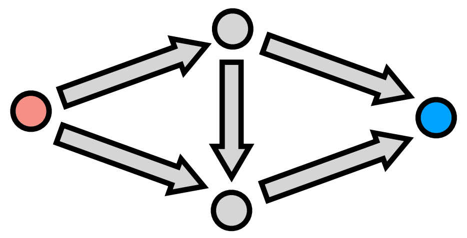


Adding center road can make traffic worse!

Potential Games

Routing Games



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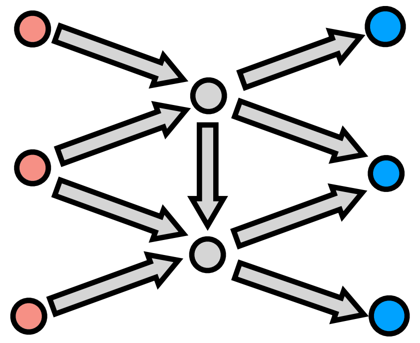

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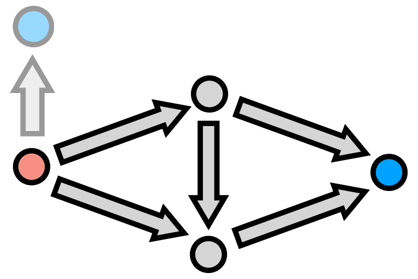

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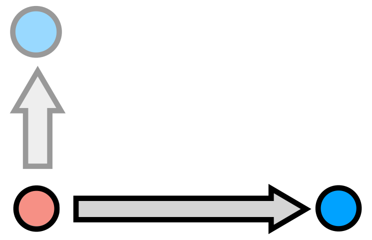

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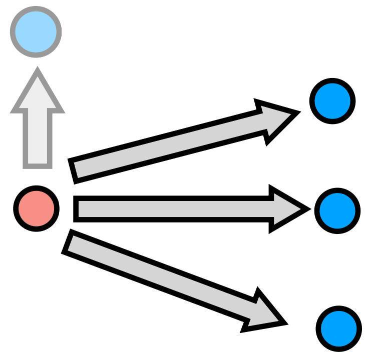

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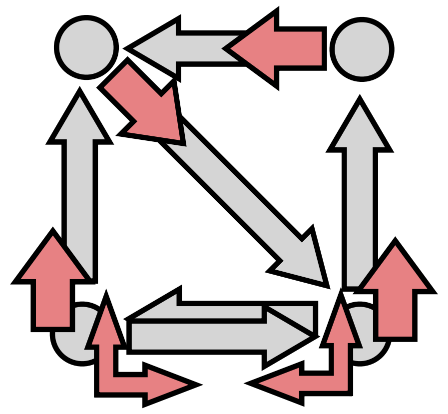

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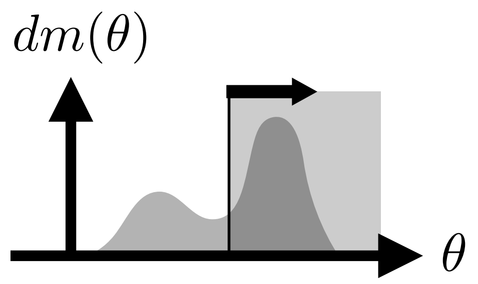

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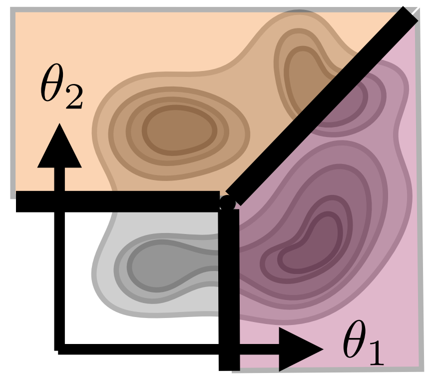

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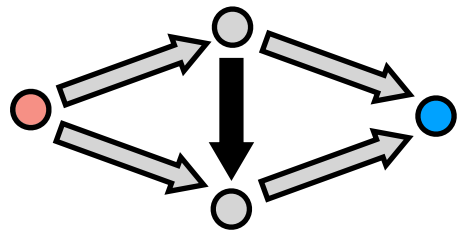

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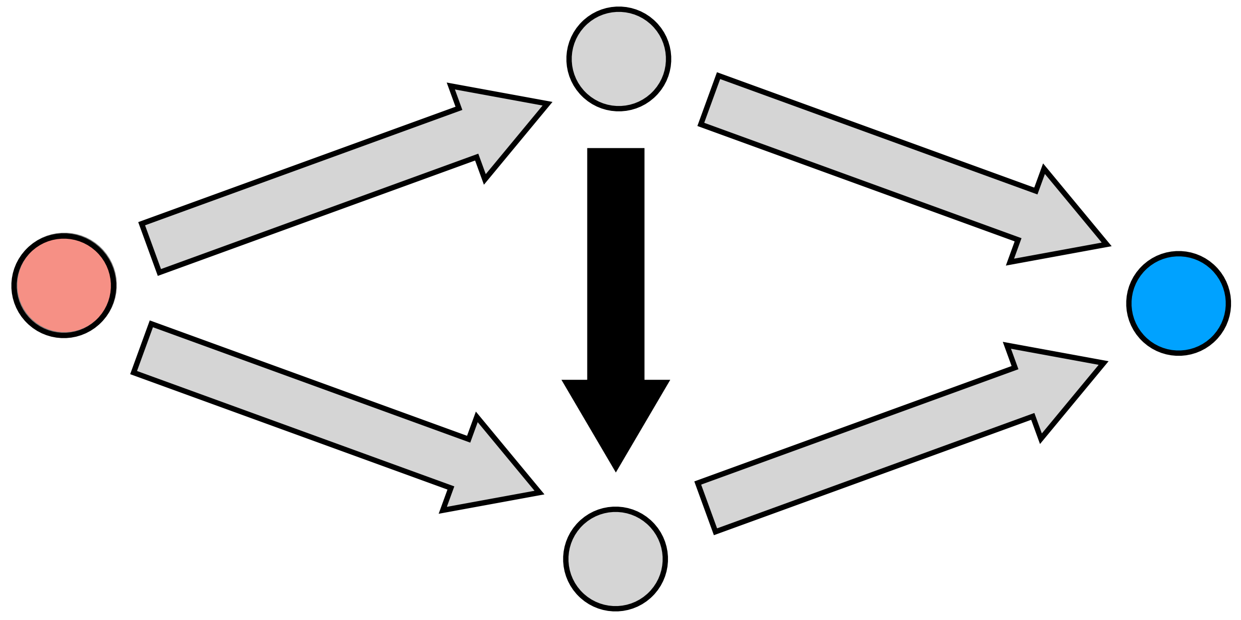
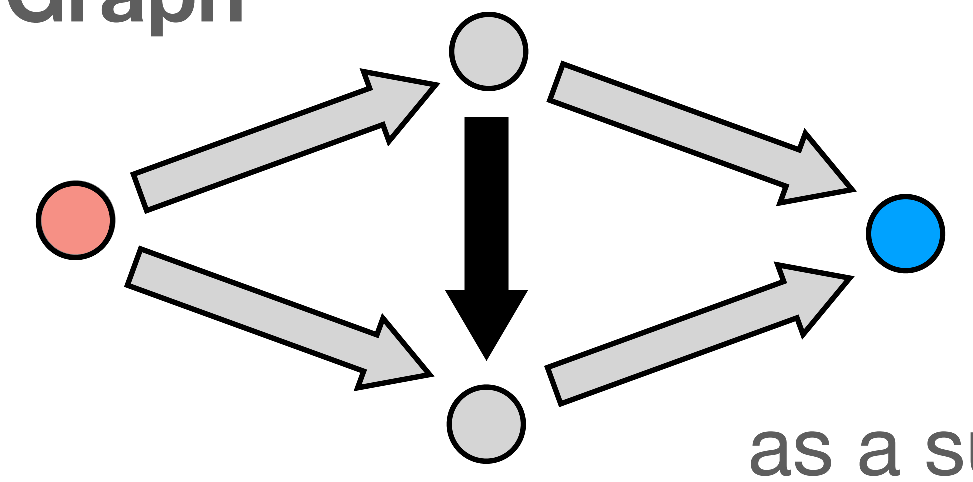


Adding center road can make traffic worse!

Network Characterization:

Every network has

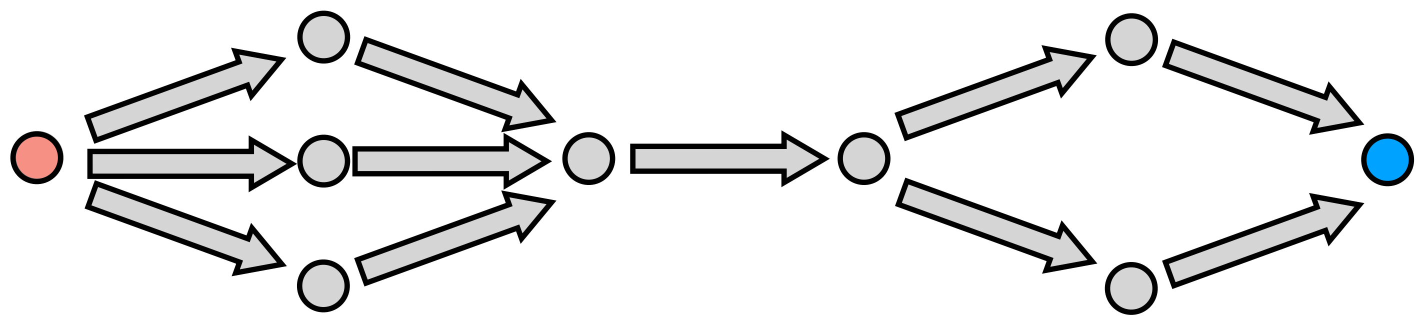
Braess Graph



as a subgraph

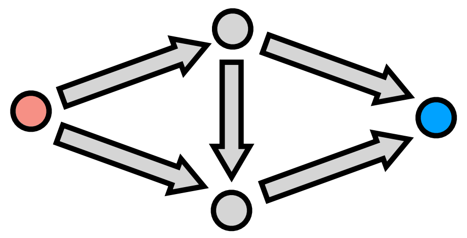
OR is

Series-Parallel Graph



Potential Games

Routing Games



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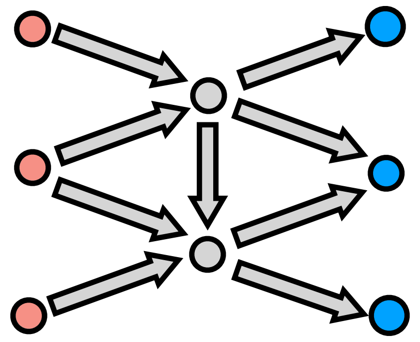

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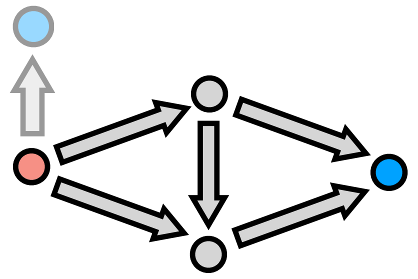

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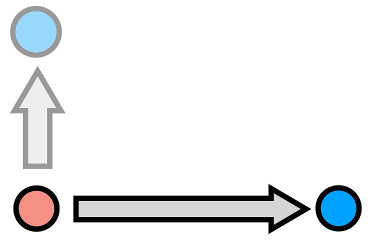

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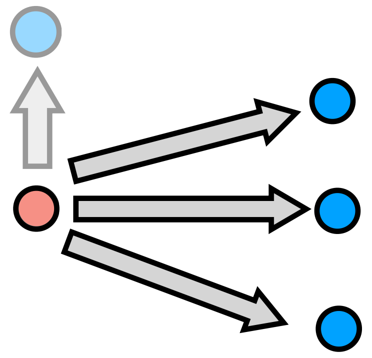

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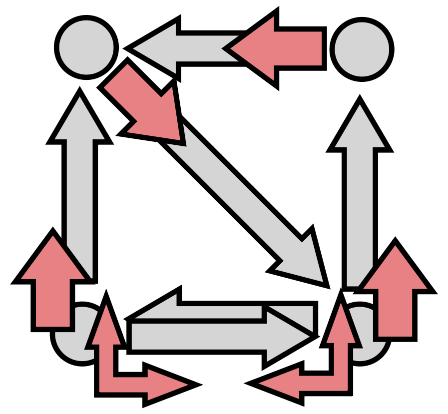
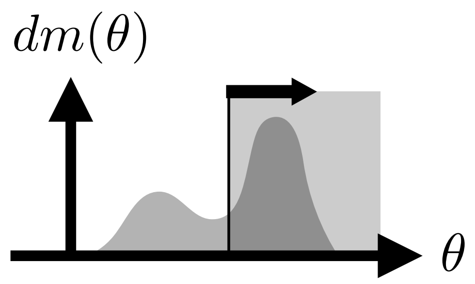


Braess Paradox

Non-homo-geneous prefences



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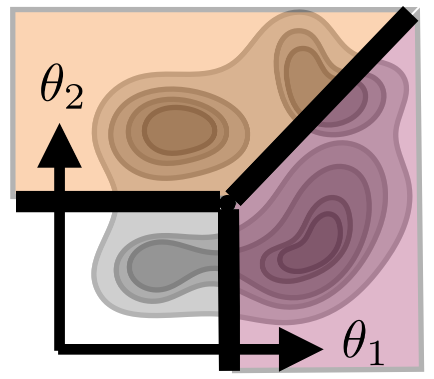

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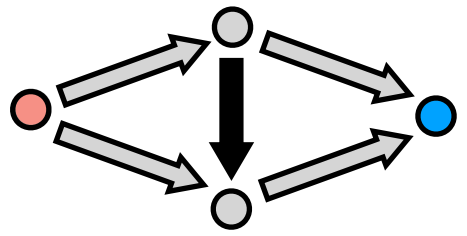

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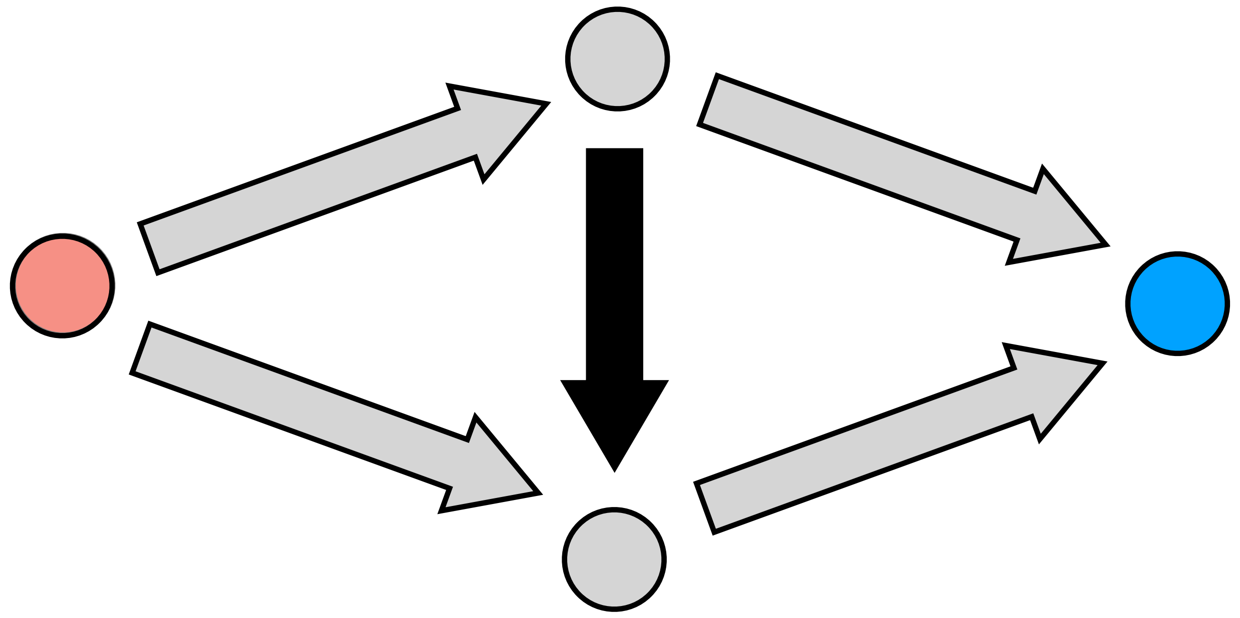

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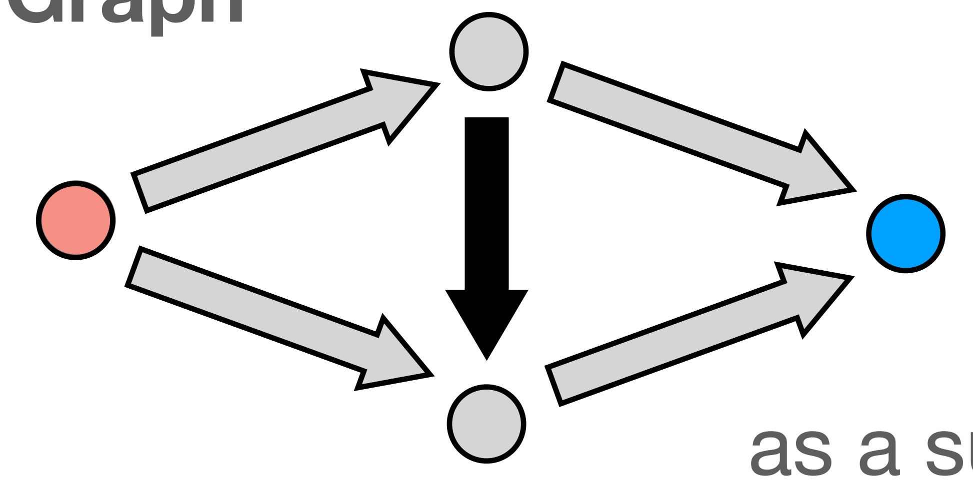

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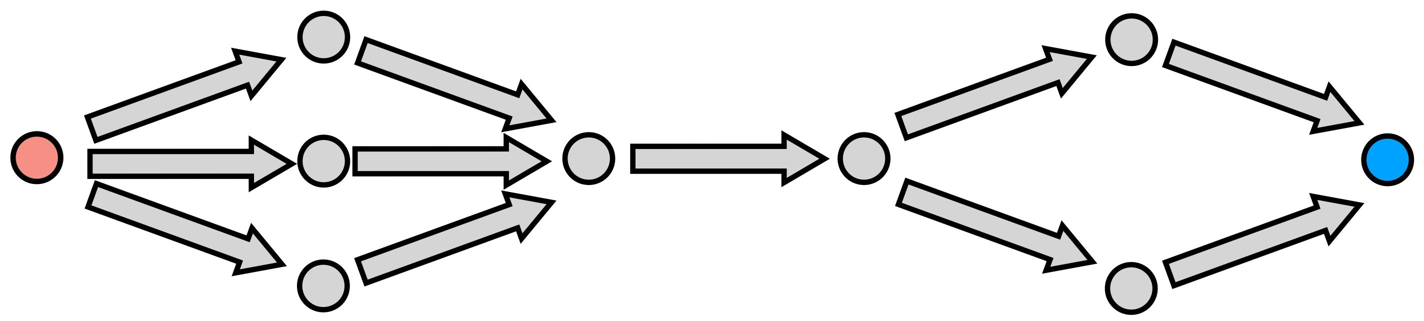

Series - Parallel graphs cannot suffer from Braess paradox ← **linear-algebraic characterization/proof**

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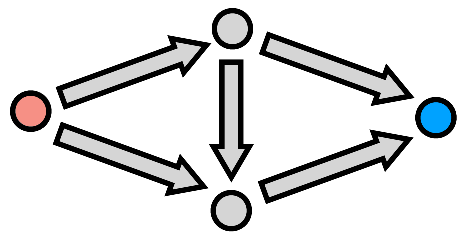
Routing Games



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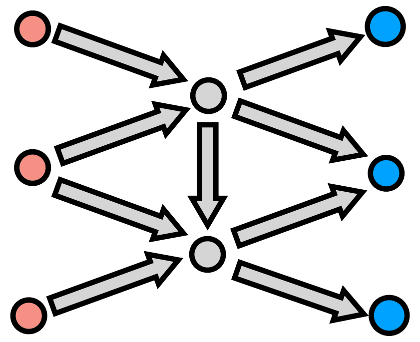

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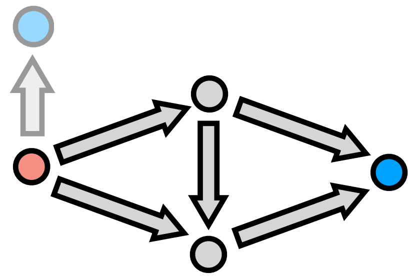

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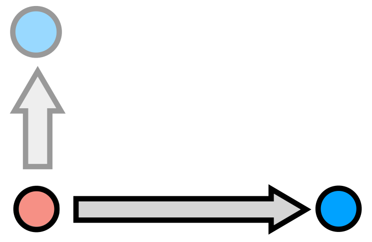

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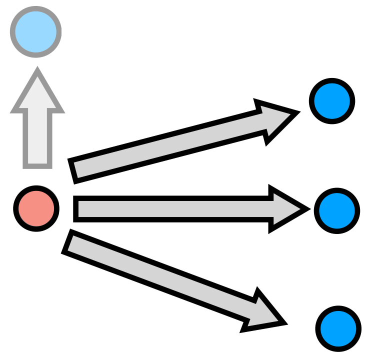

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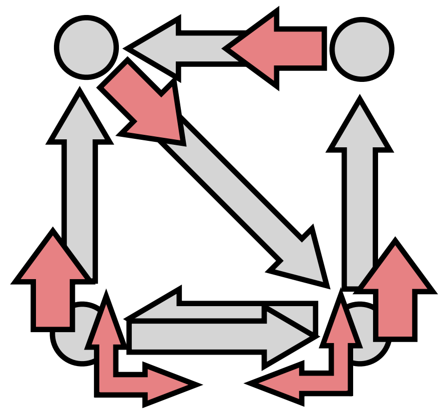
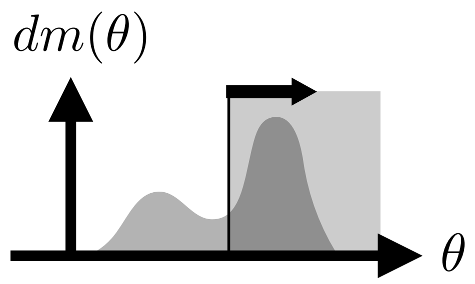
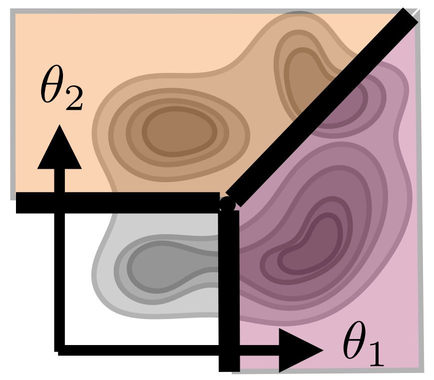


Braess Paradox

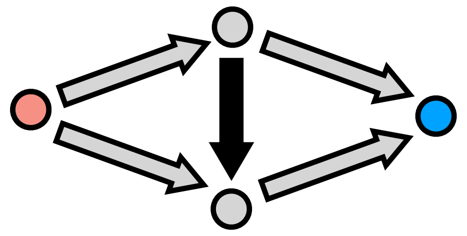
Non-homo-geneous prefences



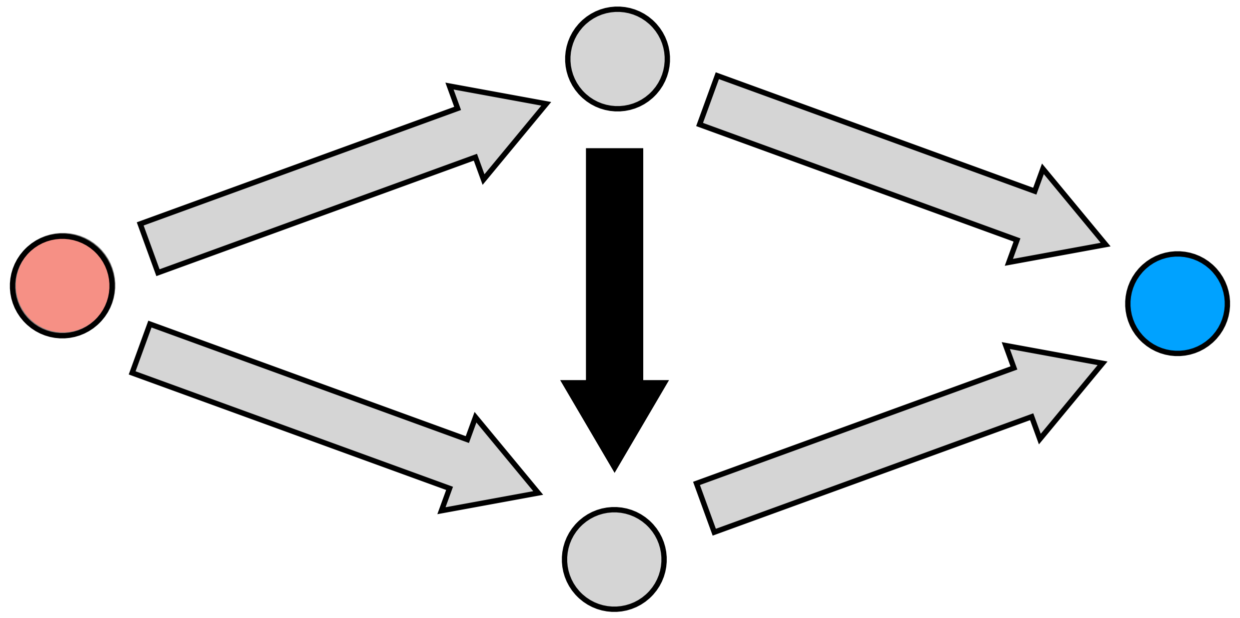
Multi-Variate Preferences



Braess Paradox



Braess Paradox



Adding center road can make traffic worse!

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- Topology of series-parallel networks **[Duffin, 1965]**
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- Sensitivity analysis for Markov decision process congestion games **[Li, Calderone, Ratliff, 2019]**
- Algebraic characterization of Braess paradox:
  Network efficiency in series-parallel and Braess networks **[Calderone, Ratliff, in prep]**