Introduction

Dan Calderone djcal@uw.edu
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Post-doctoral scholar (Prof. Ratliff's group)
University of Washington

PhD: Berkeley, (under Shankar Sastry, 2017)
PostDoc: in AA & EE at UW (Ratliff, Ackimese, 2018-2019)
Lecturer: AA & EE at UW (2019-2022)

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Game theory & optimization
applied to transportation networks

Personal Interests: Math visualization

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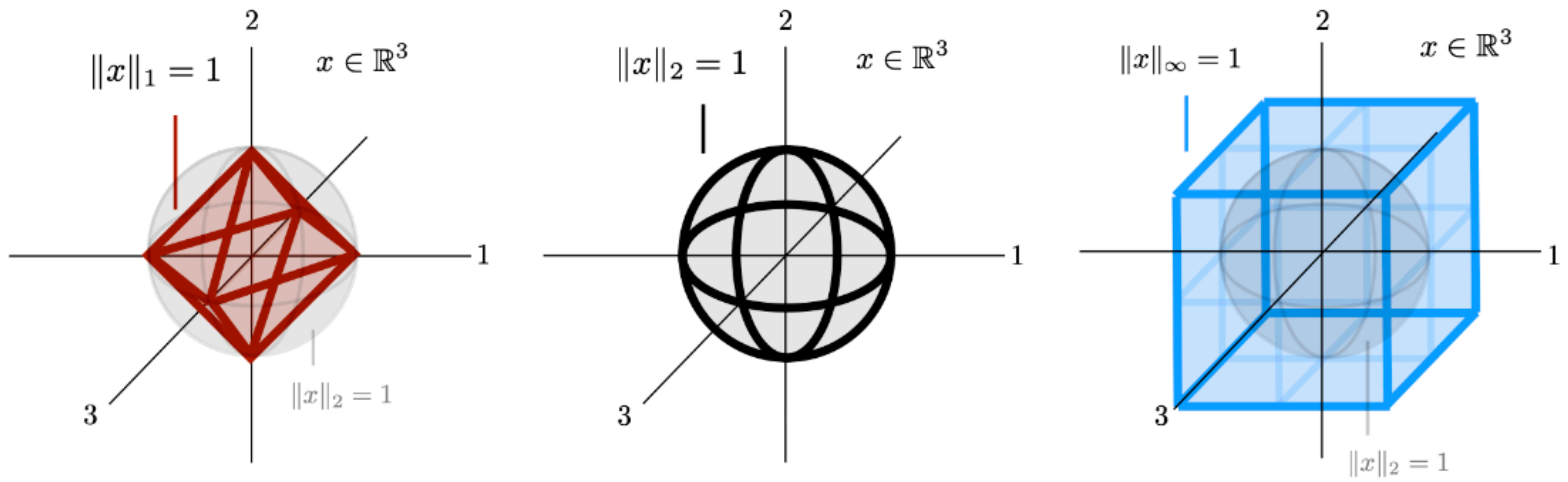

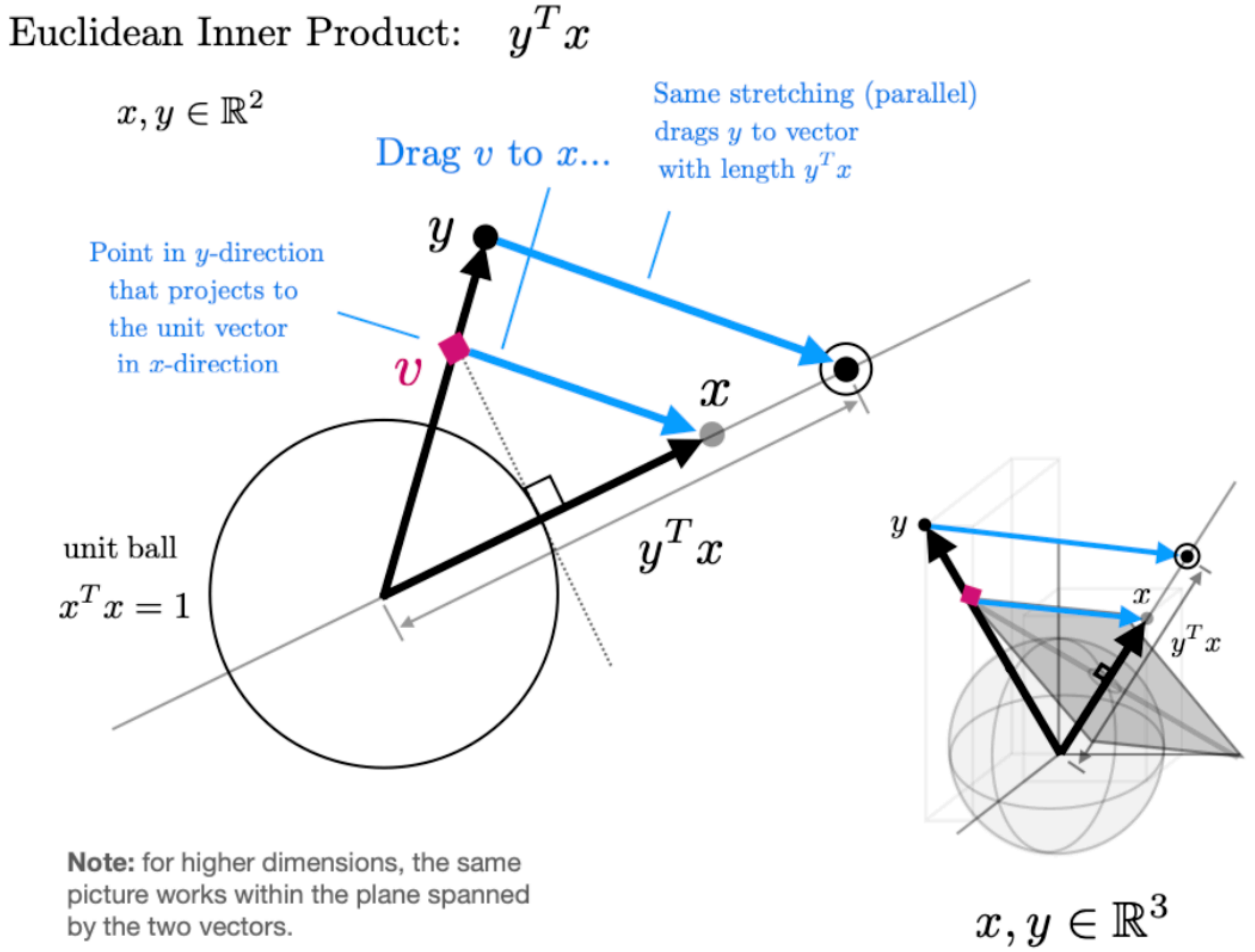

Note: for higher dimensions, the same picture works within the plane spanned by the two vectors.

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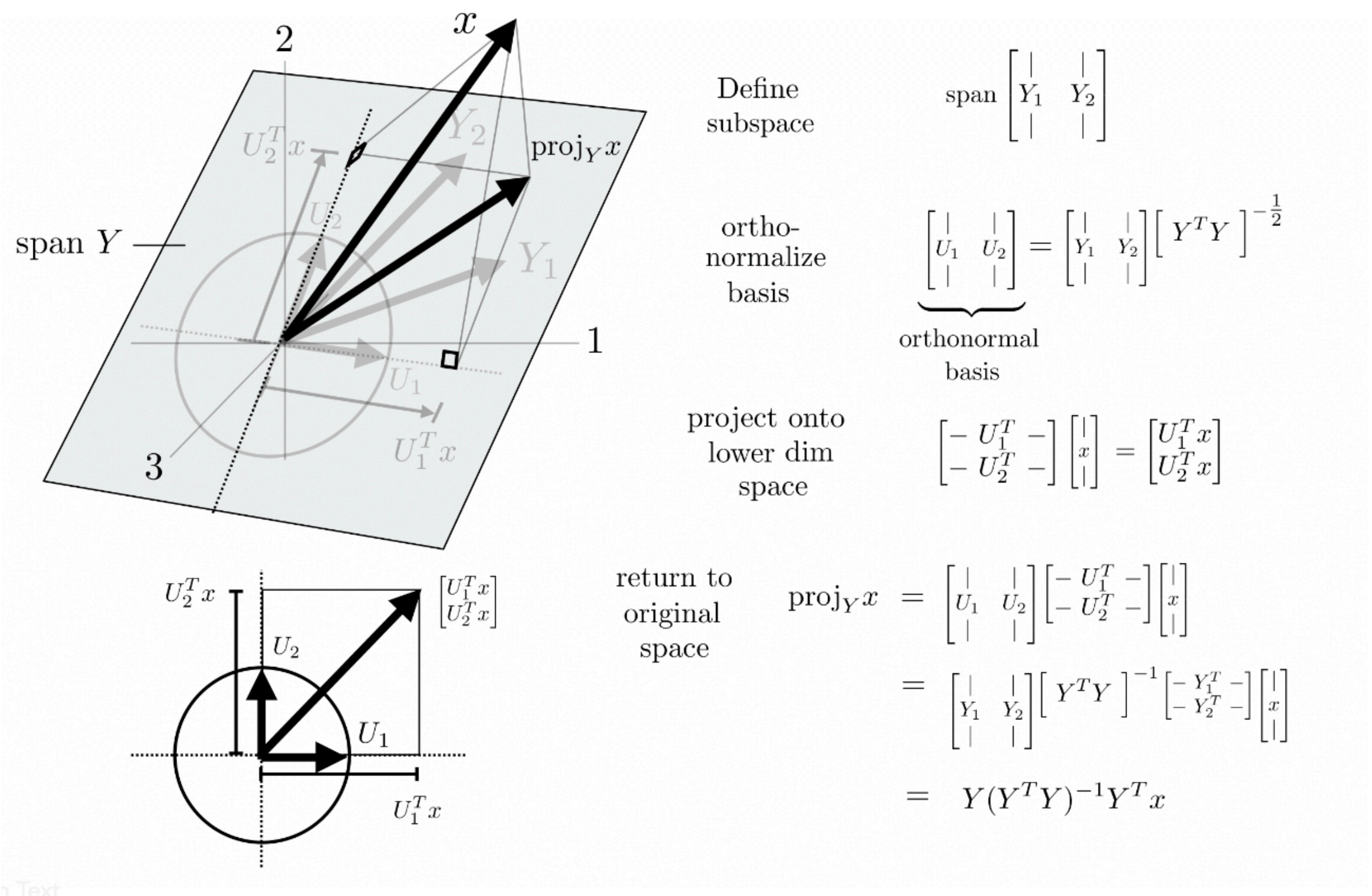

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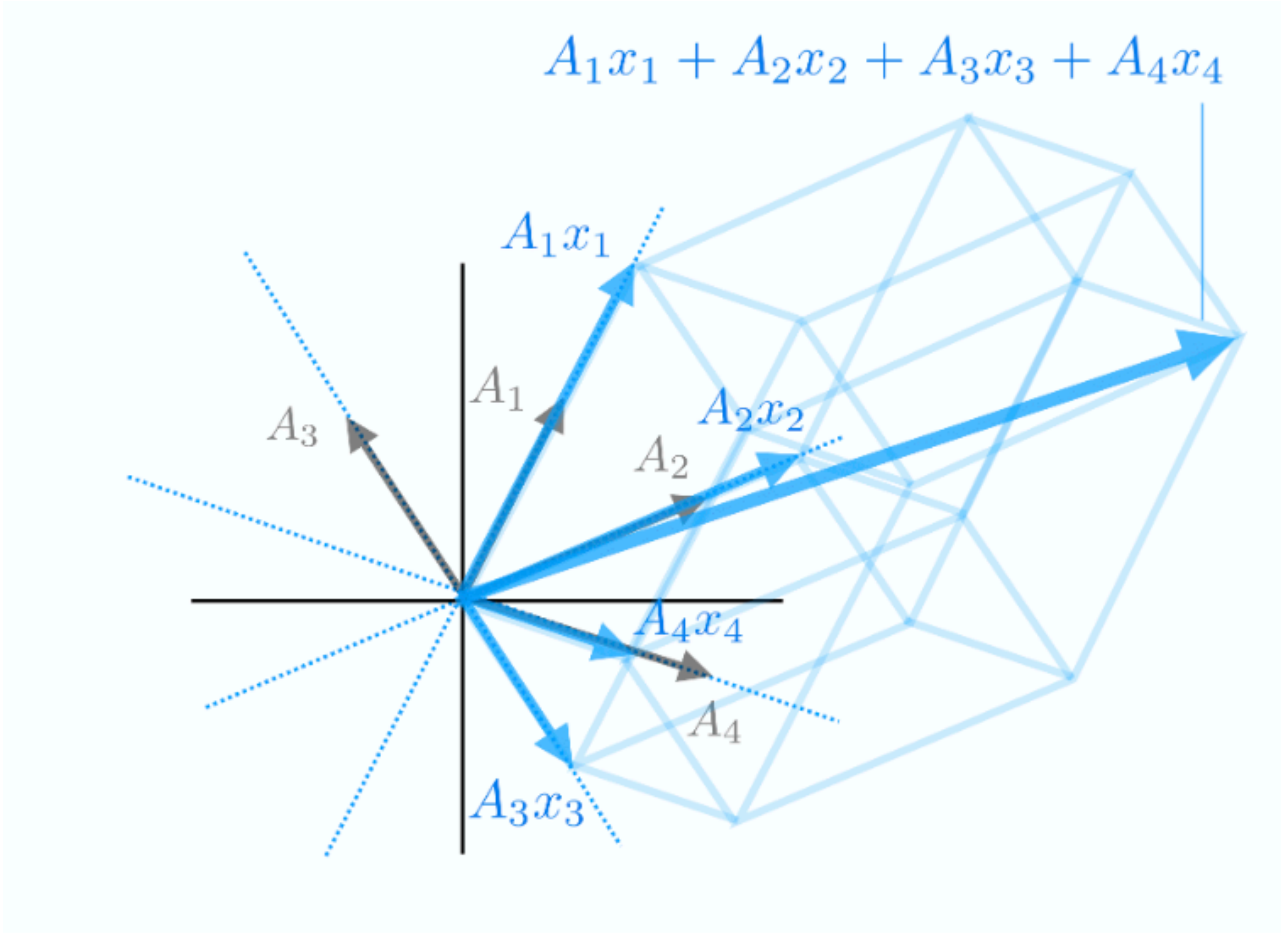

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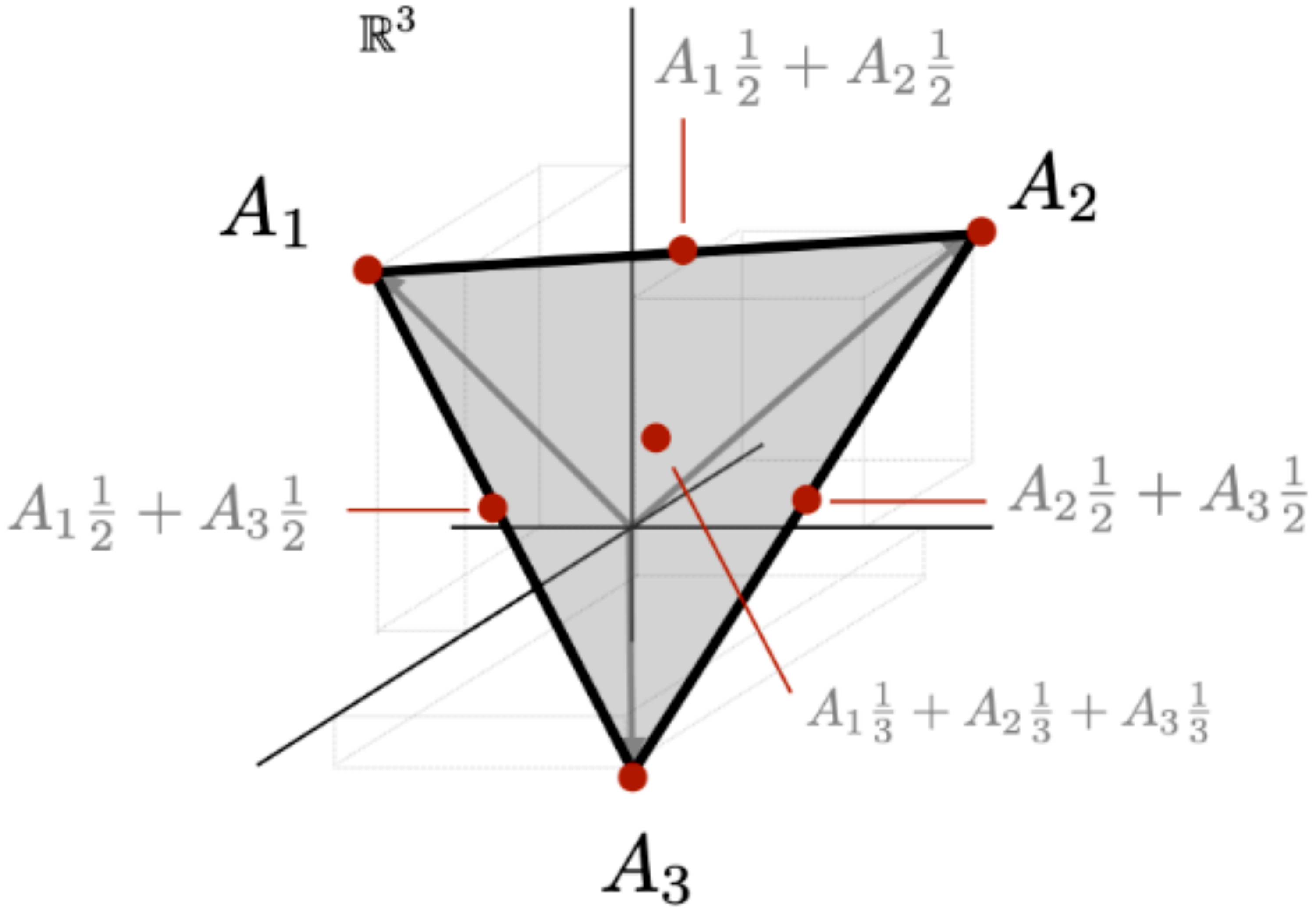

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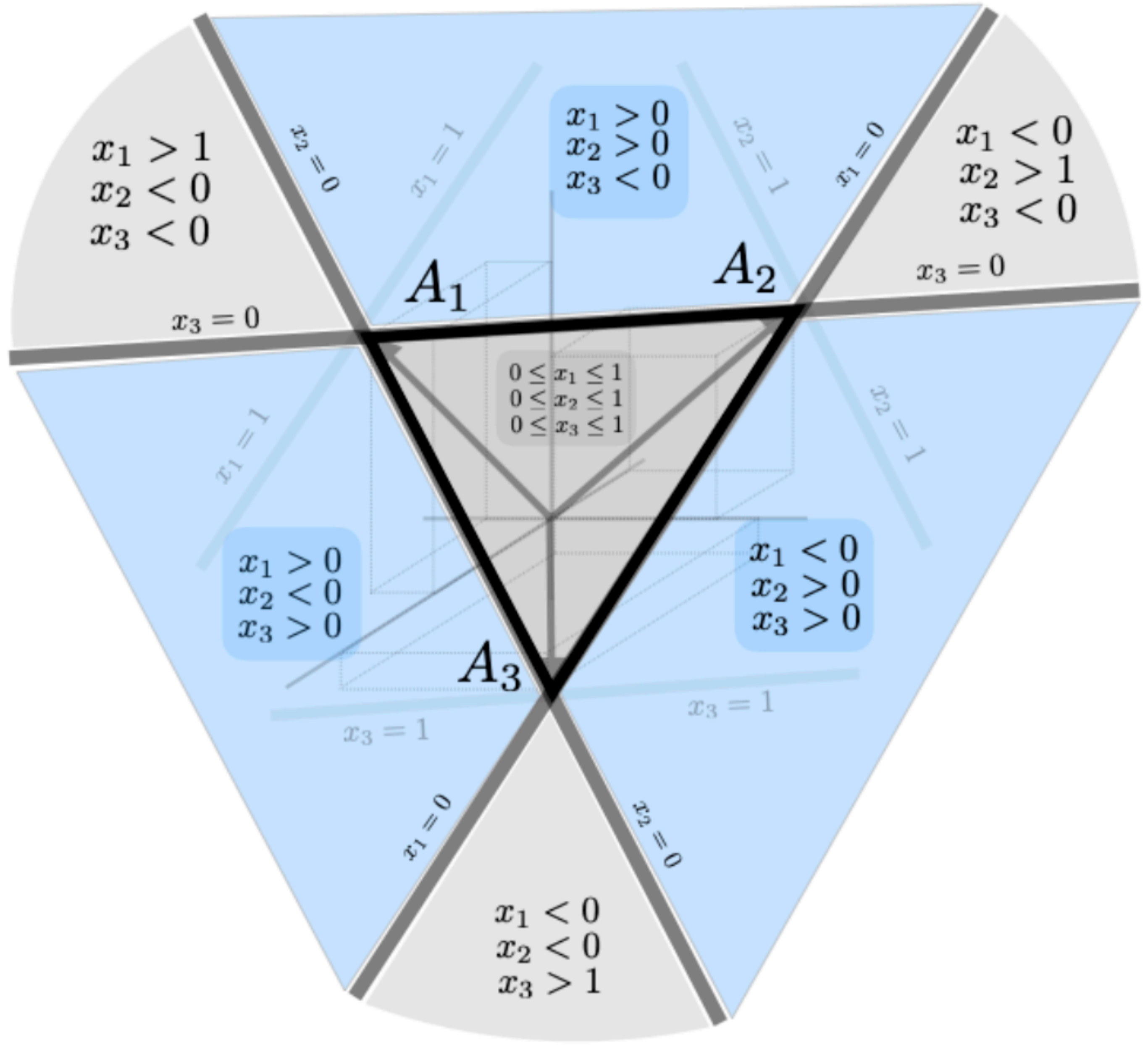

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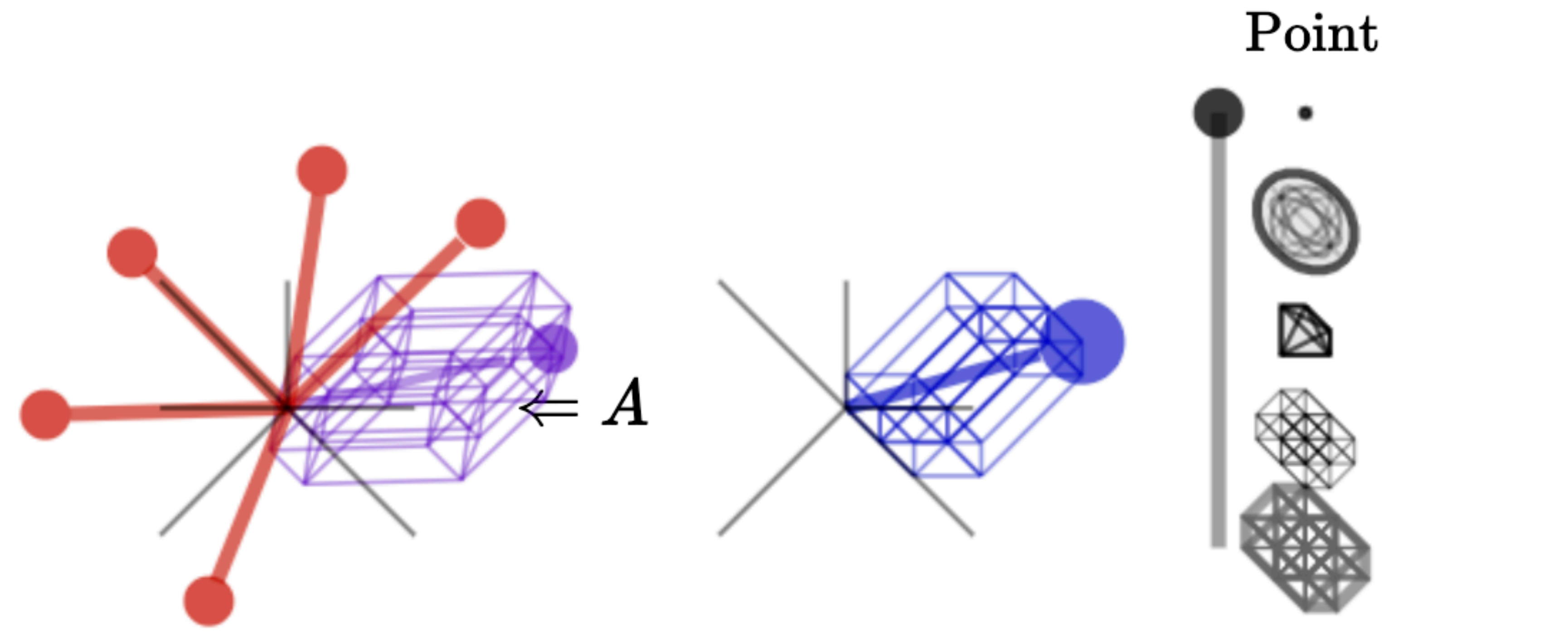

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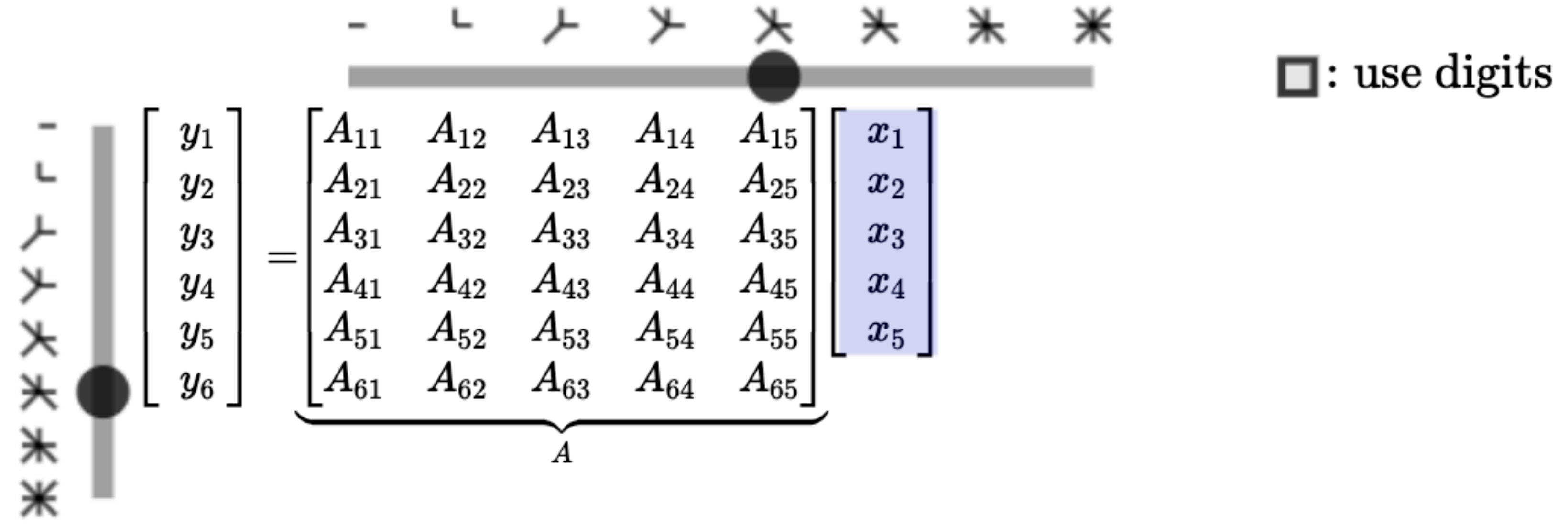

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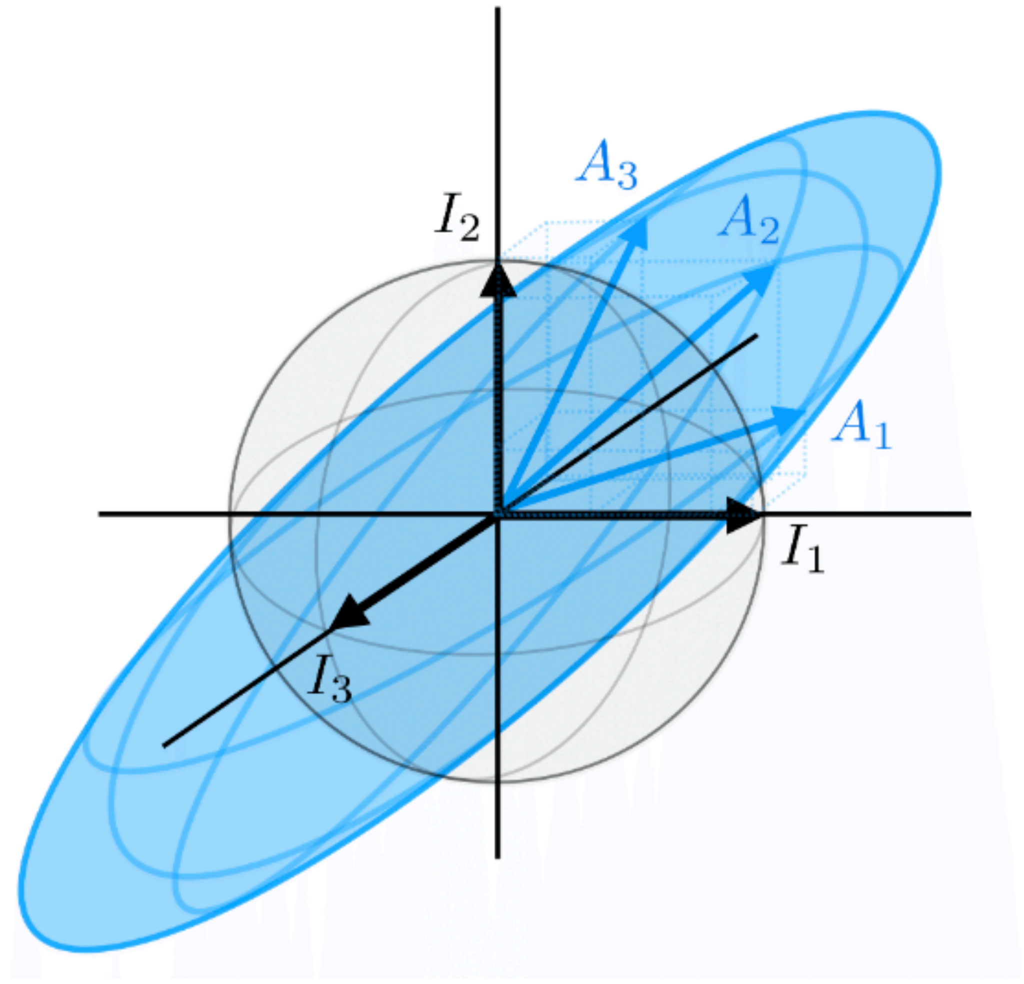

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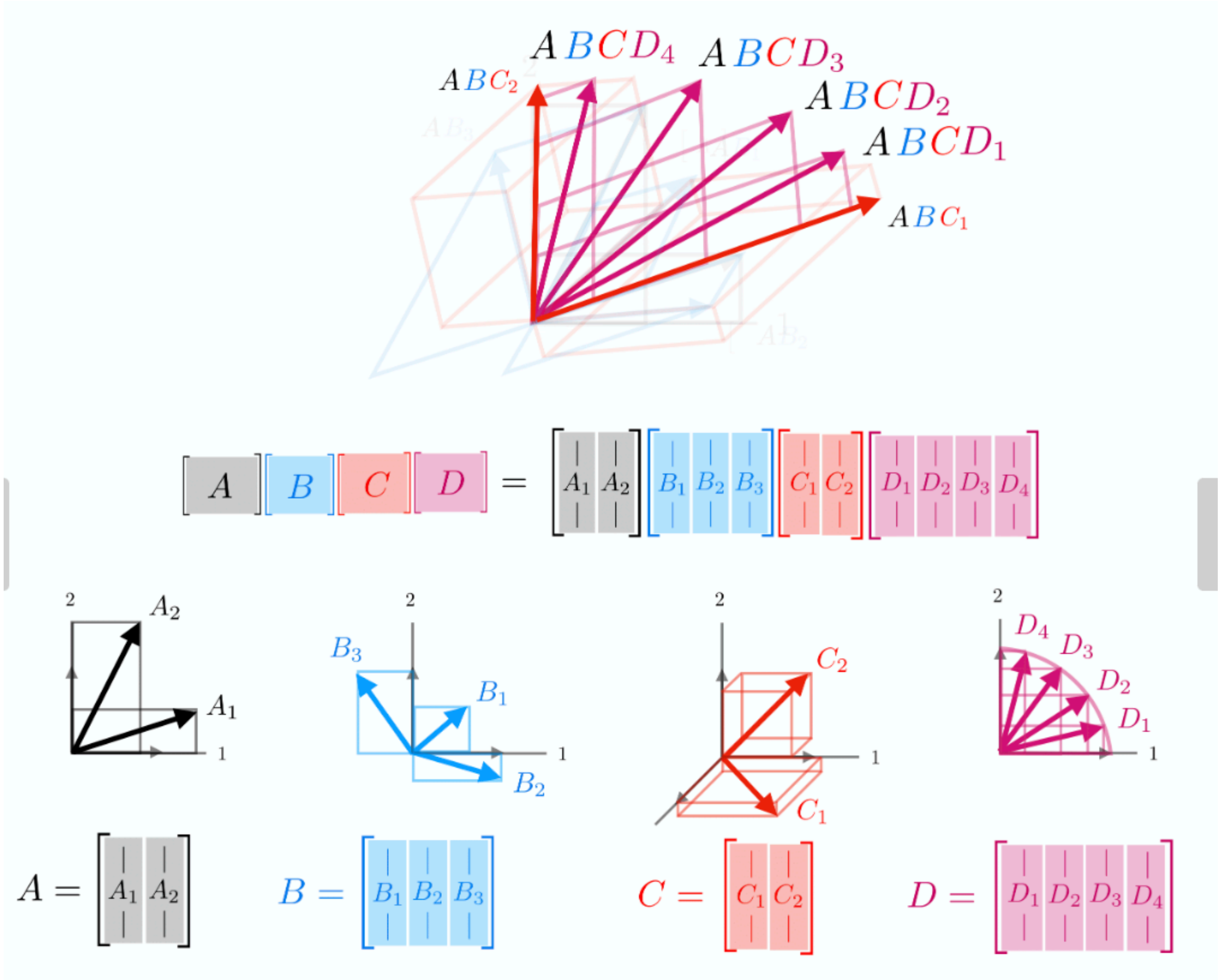

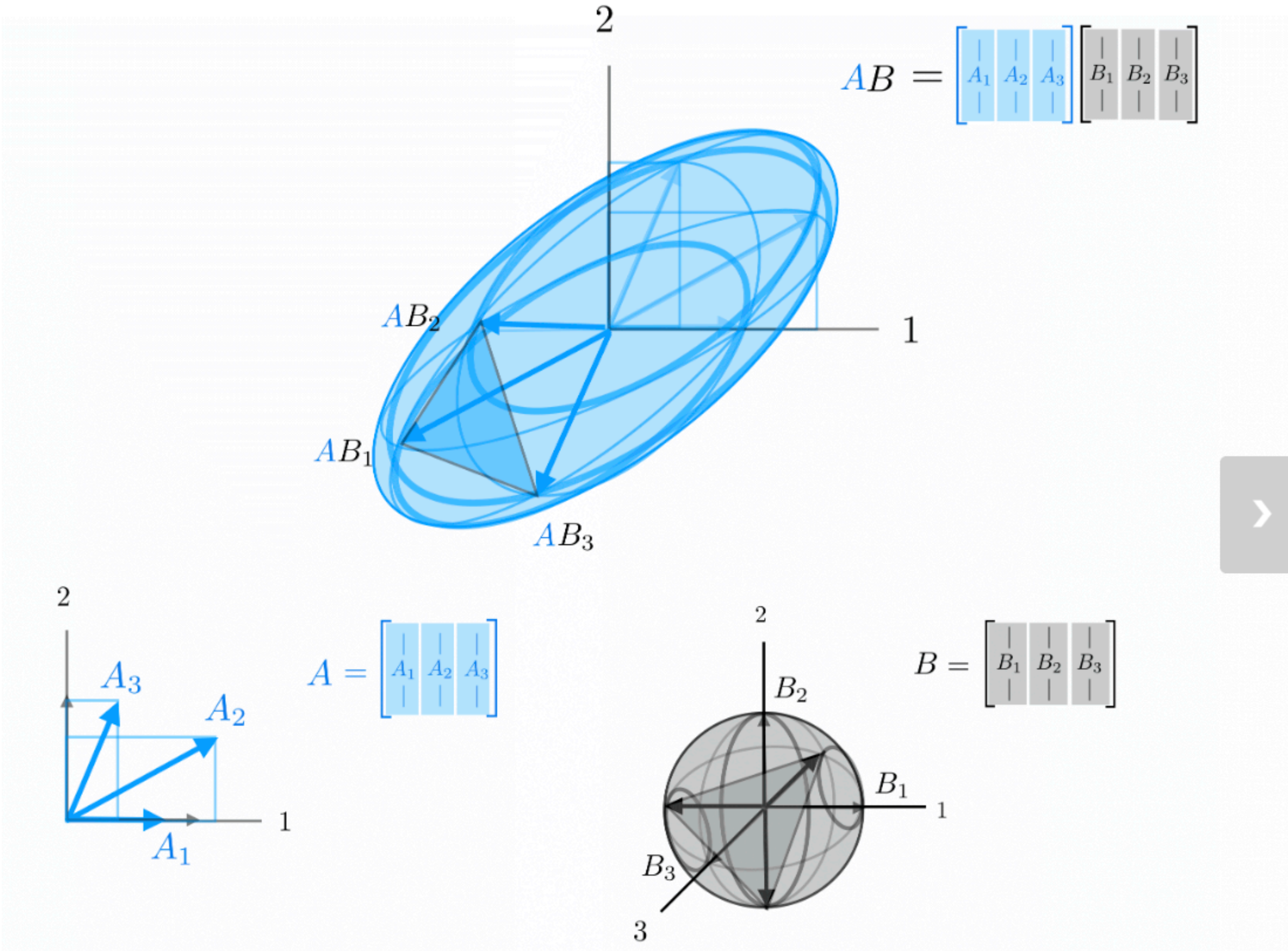

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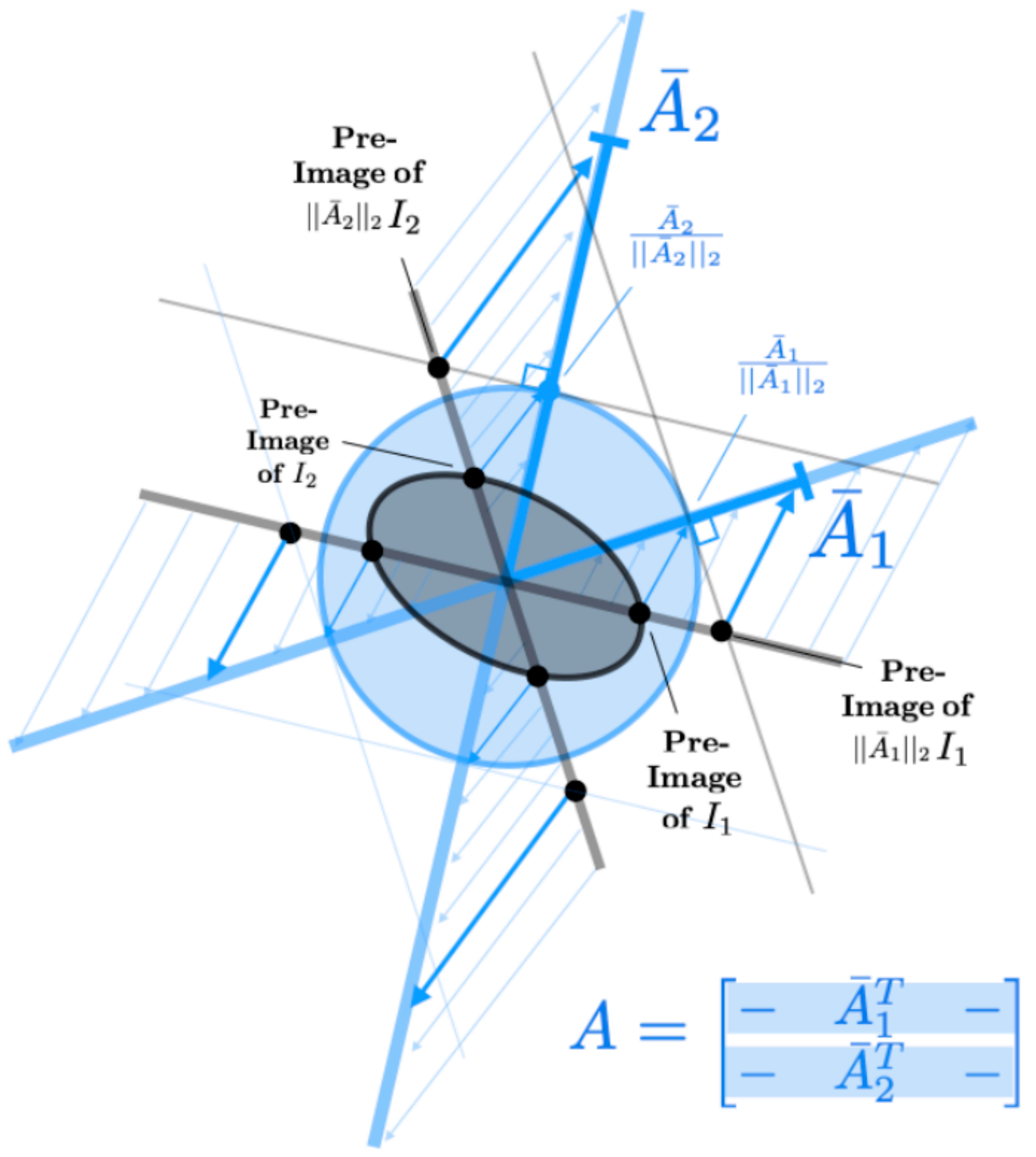

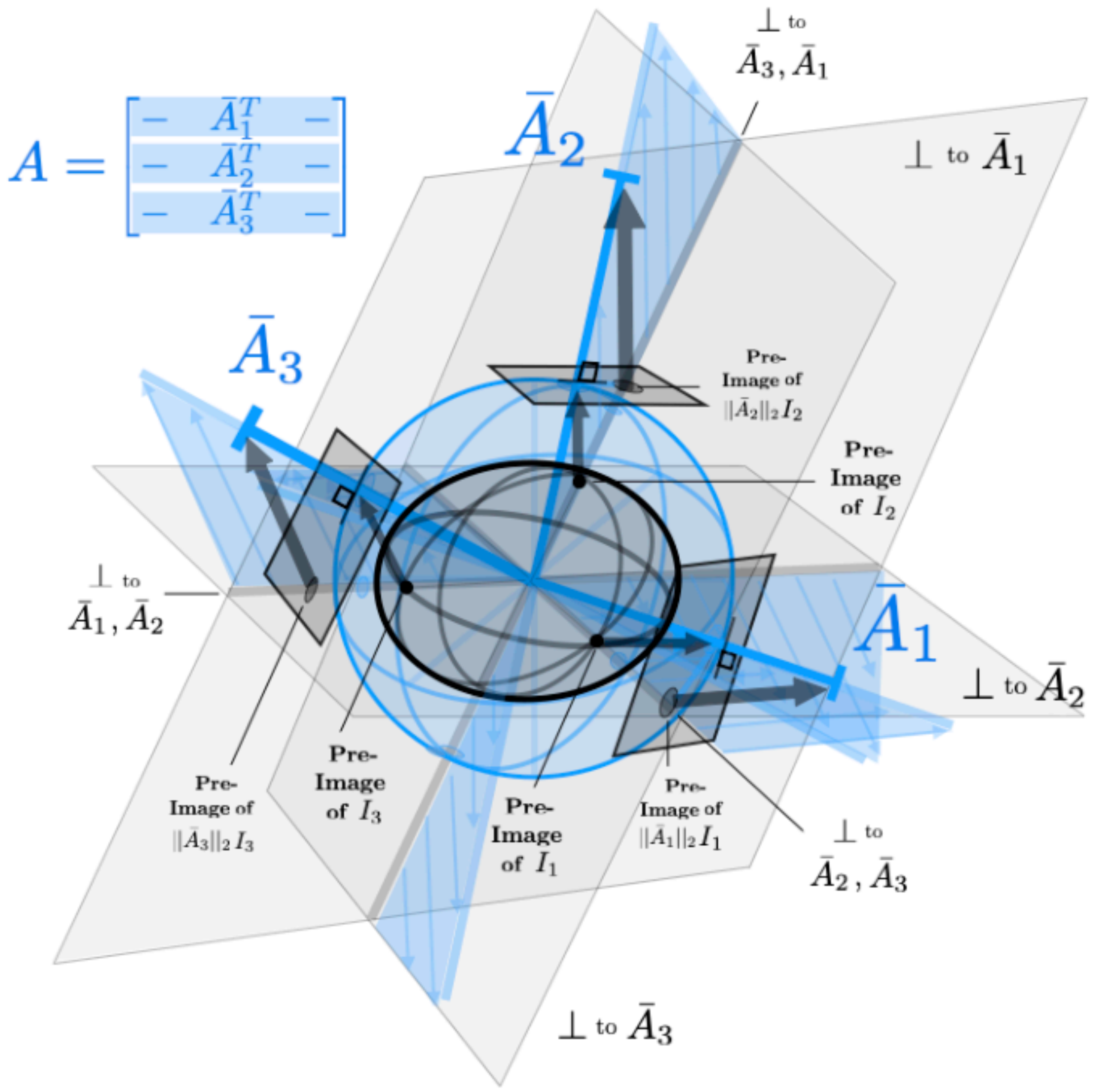

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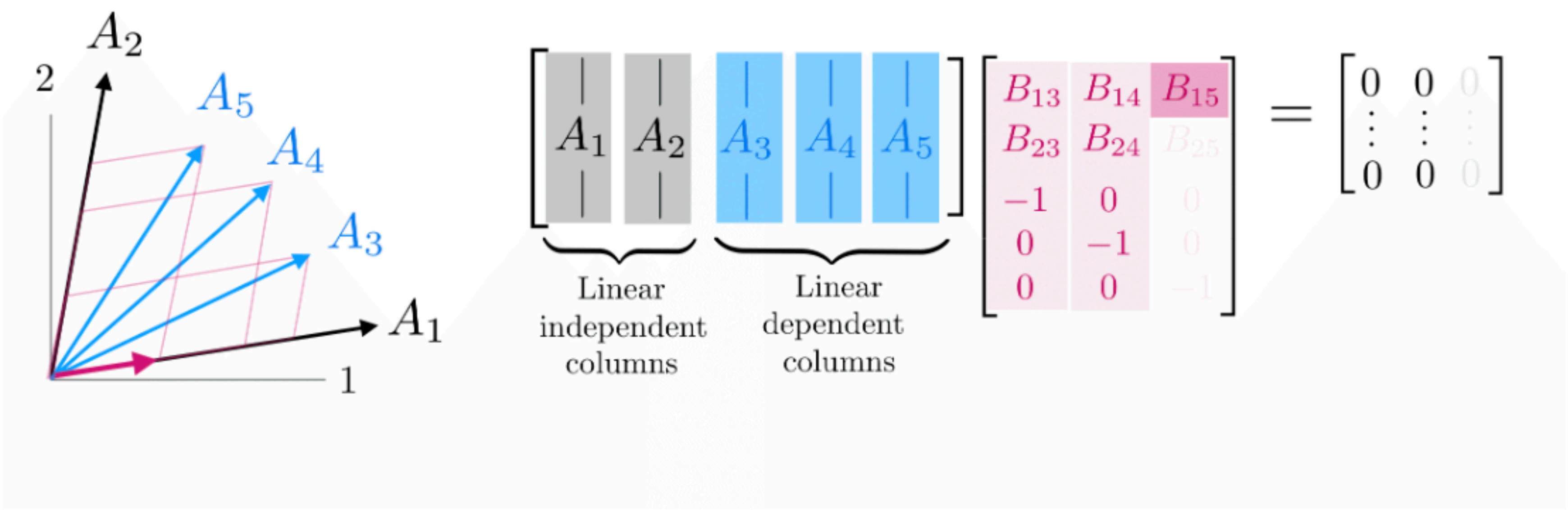

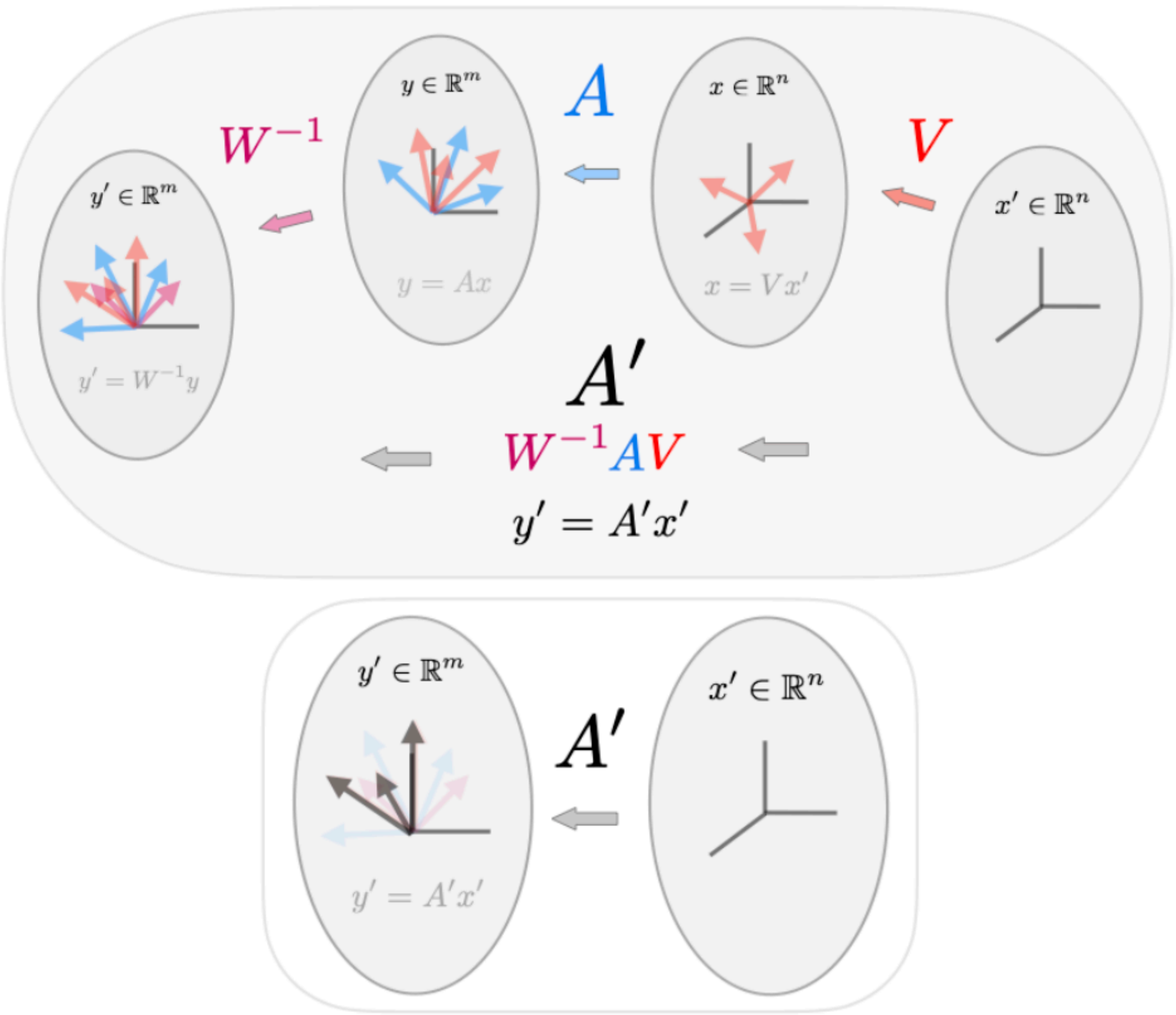

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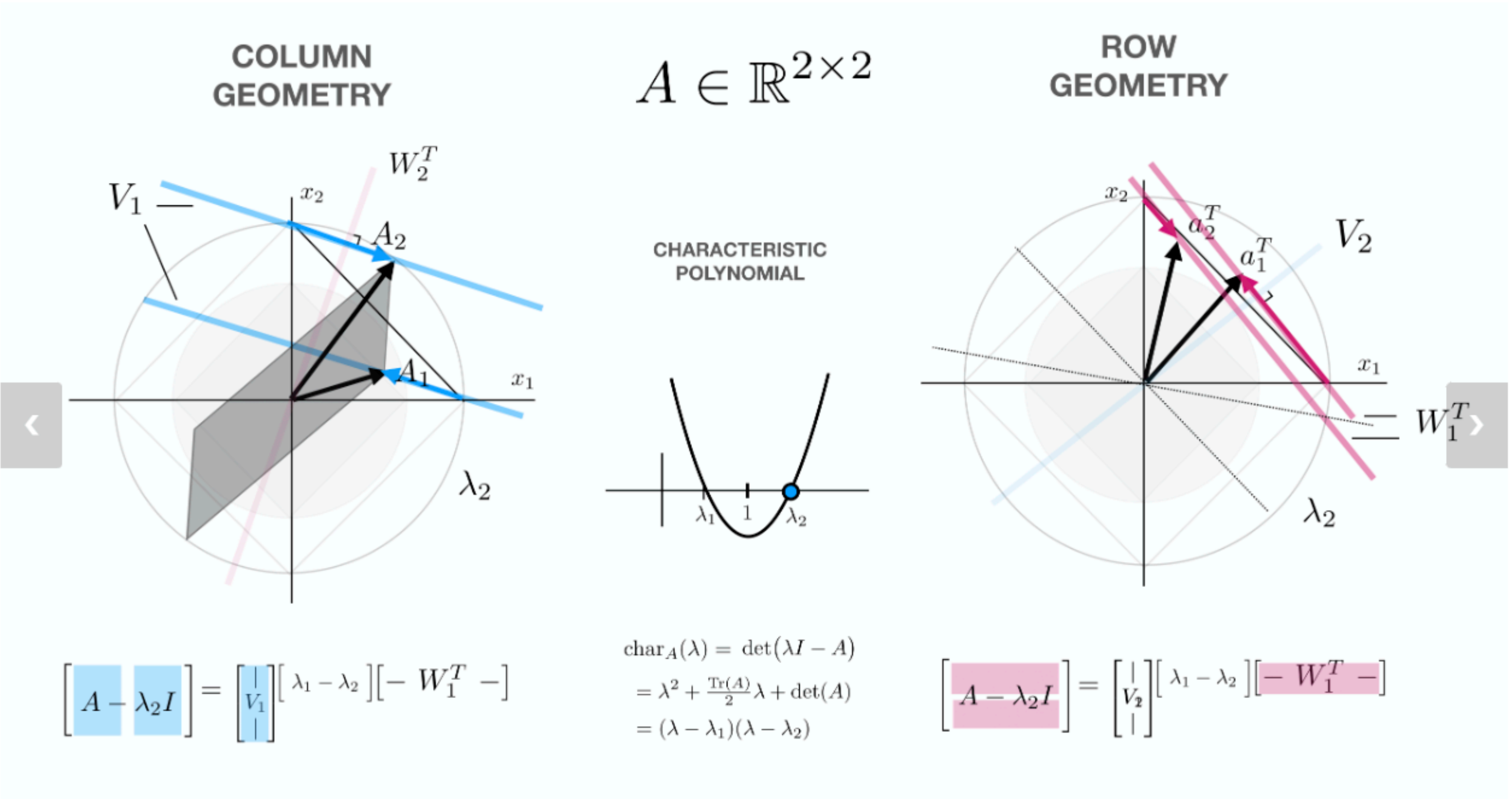

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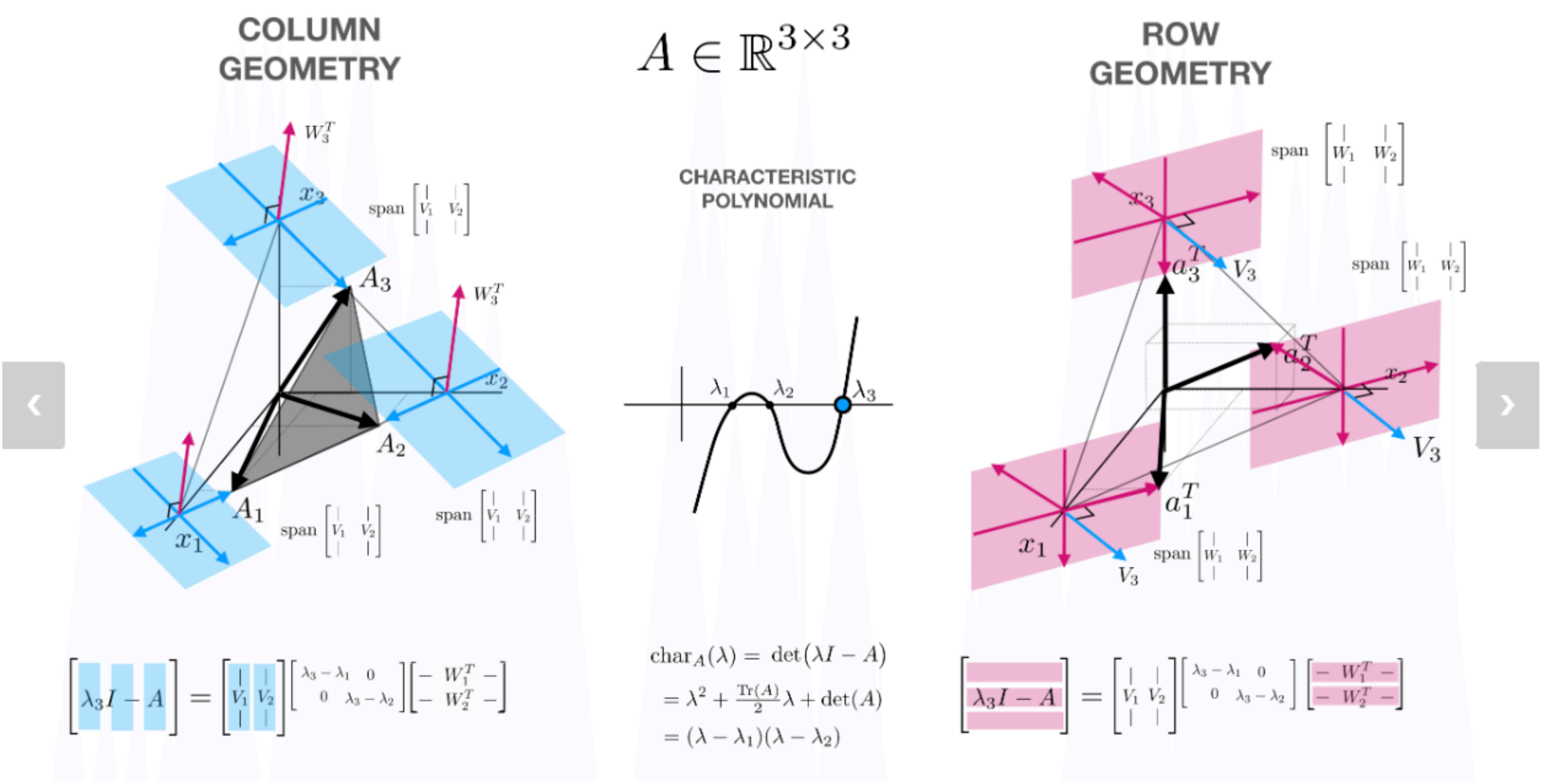

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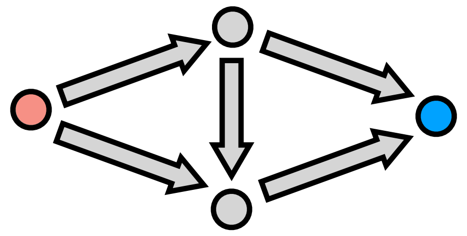
Potential Games

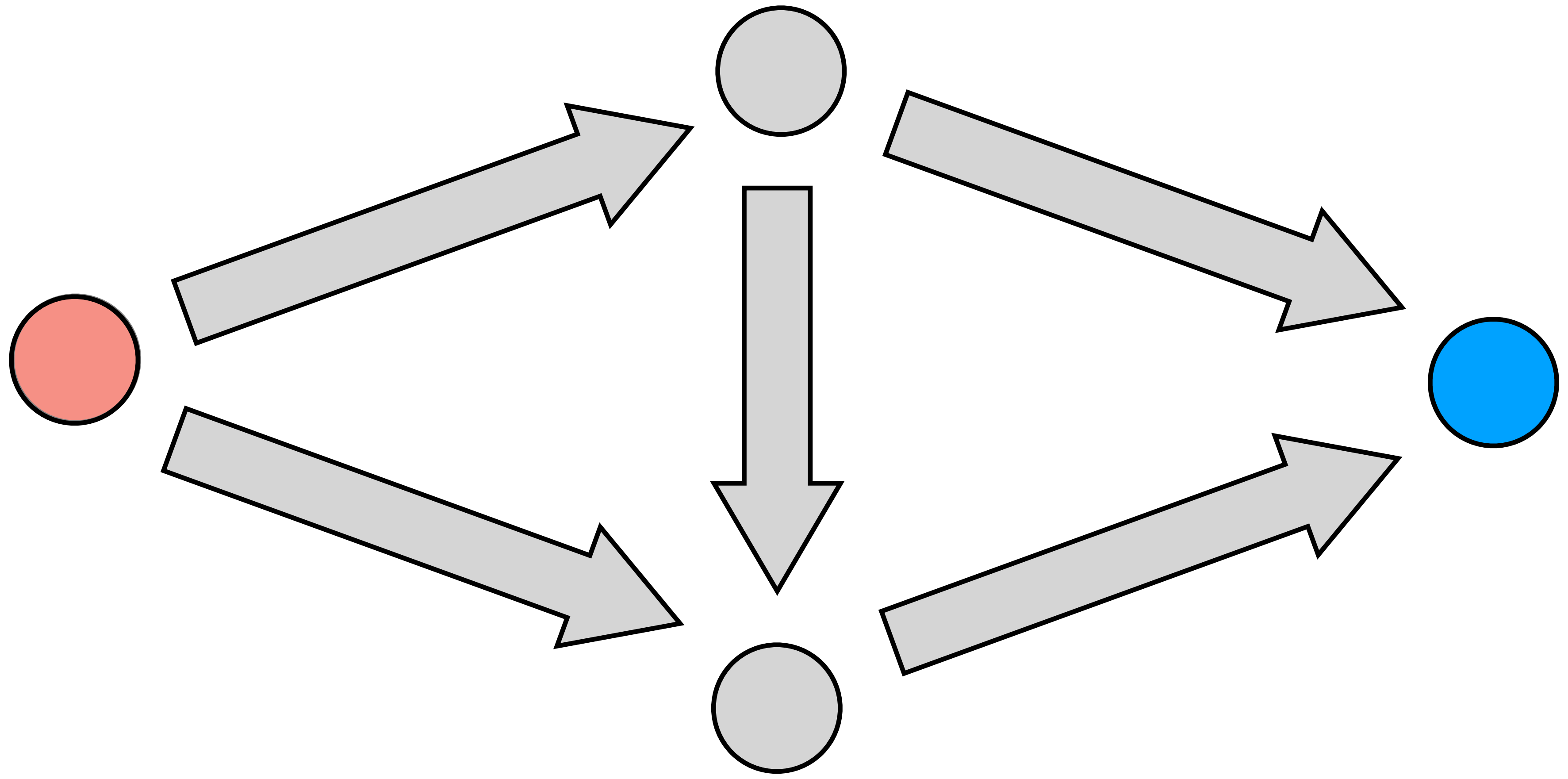
Routing Games



Routing Games

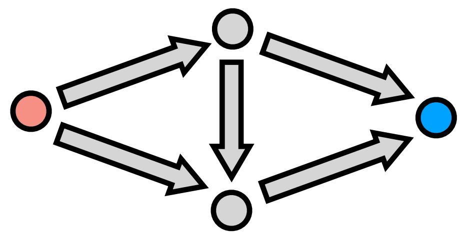
x : edge traffic

z : route traffic

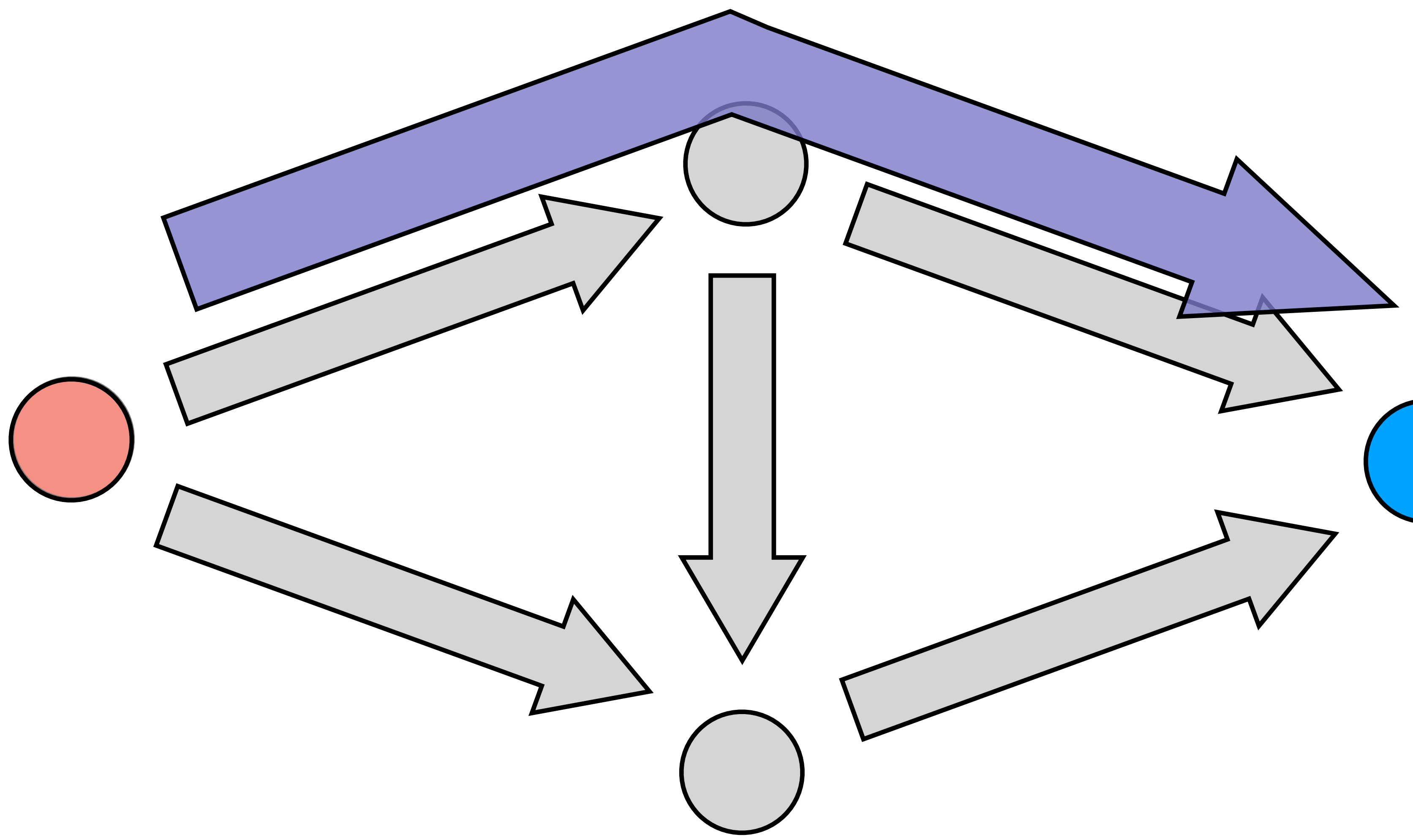


Potential Games

Routing Games



Routing Games



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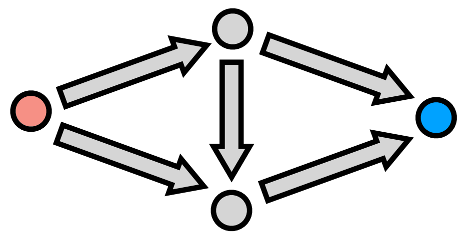
Potential Games

Routing Games



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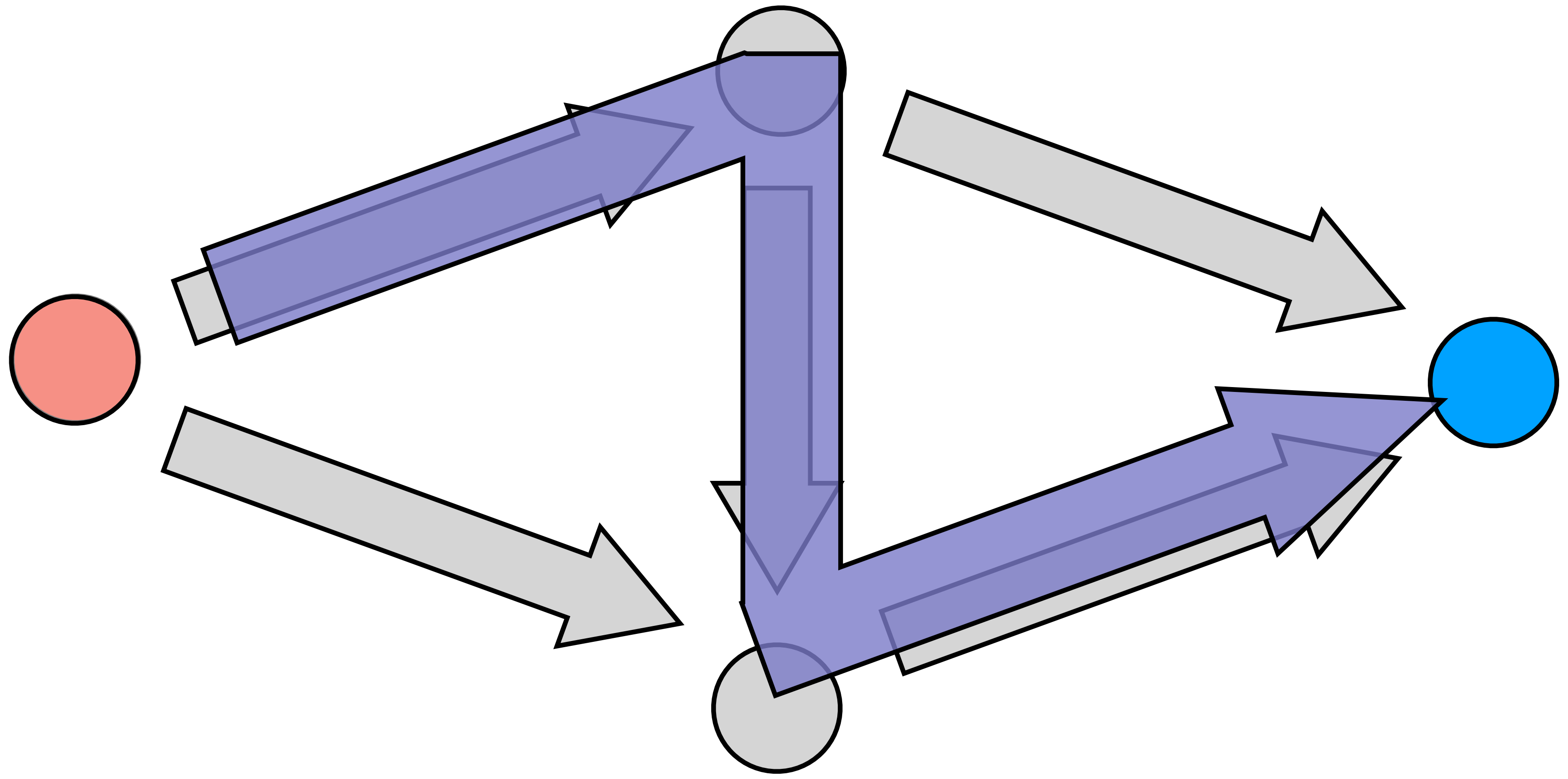
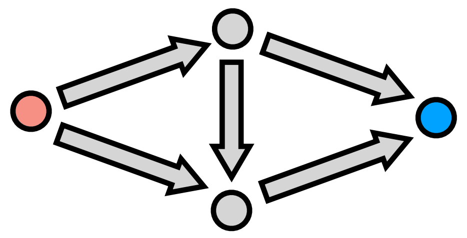


Potential Games

Routing Games



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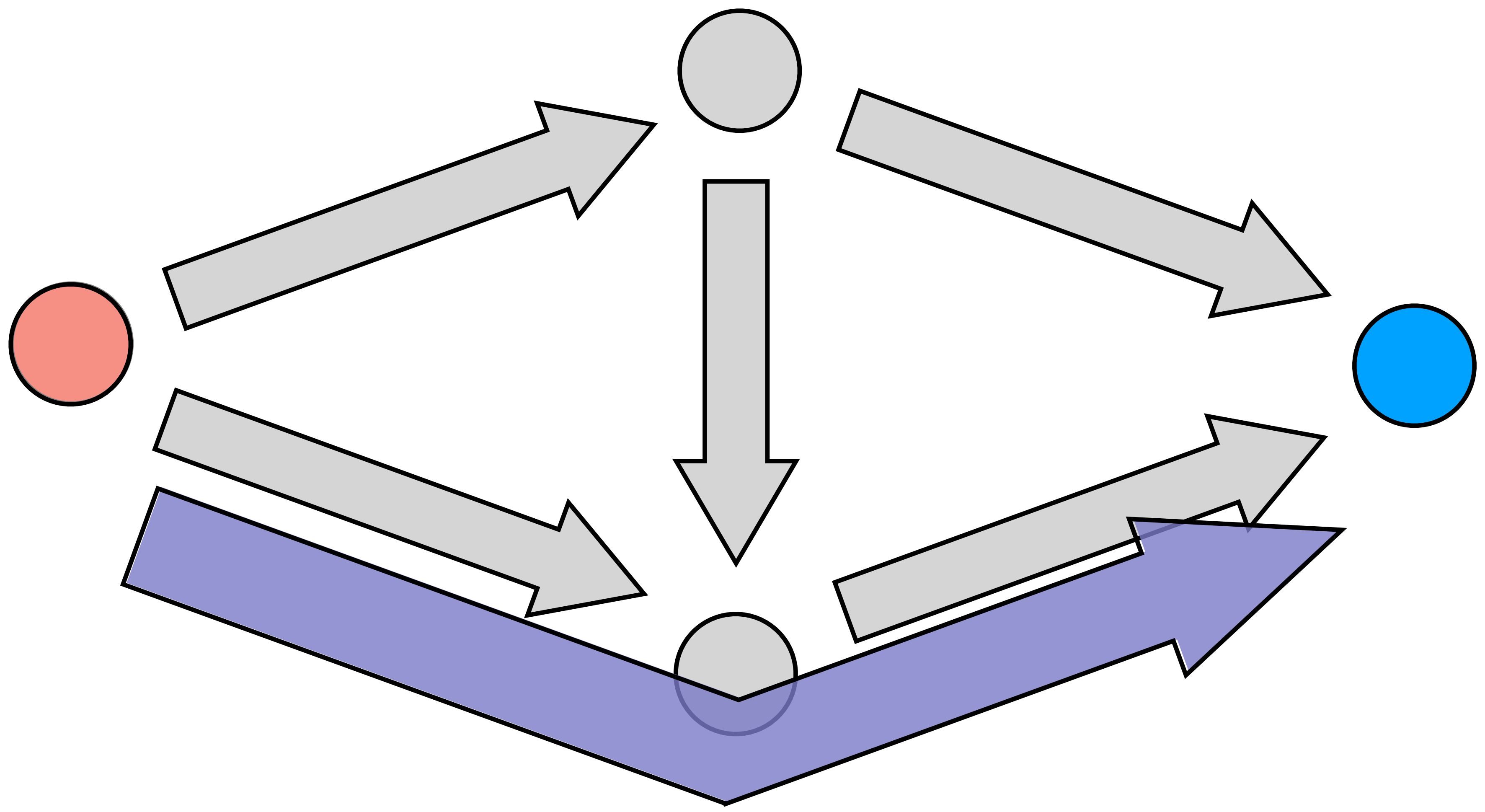

Potential Games

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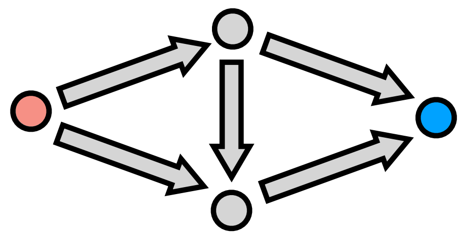

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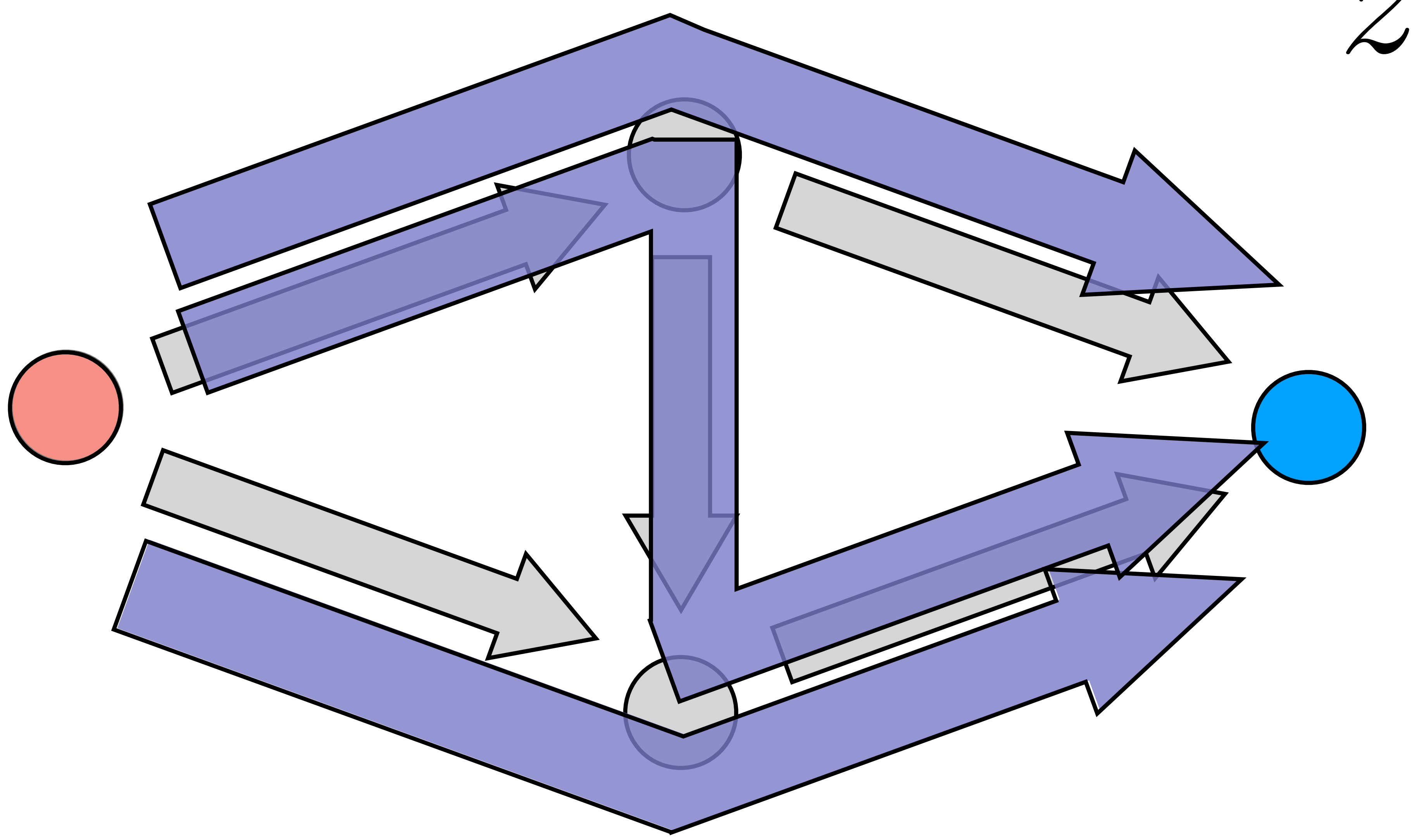
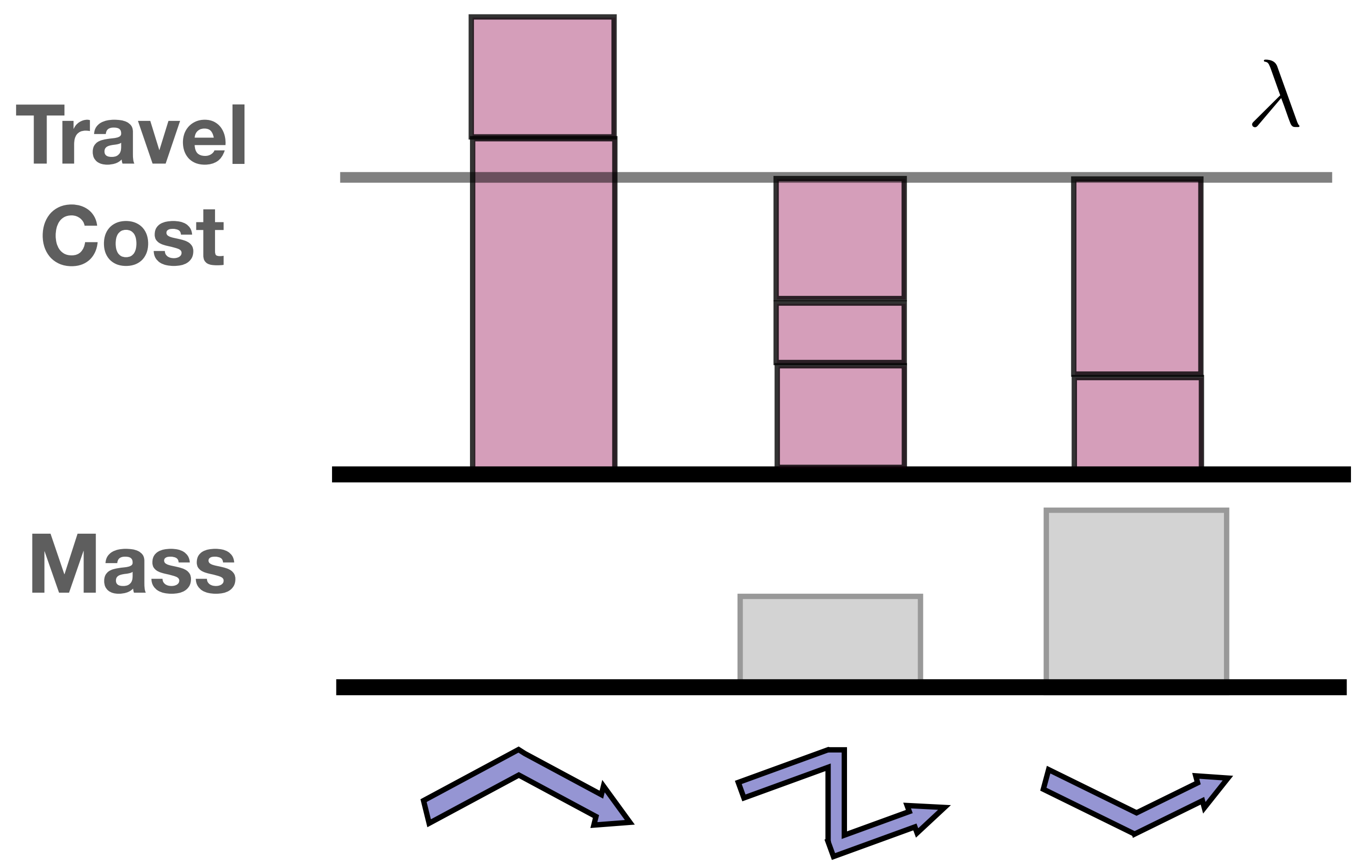
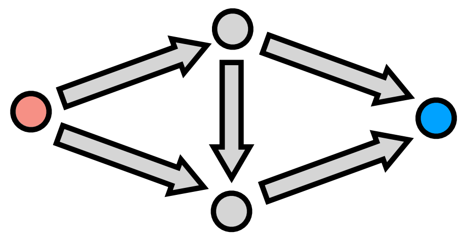


Wardrop Equilibrium



Potential Games

Routing Games



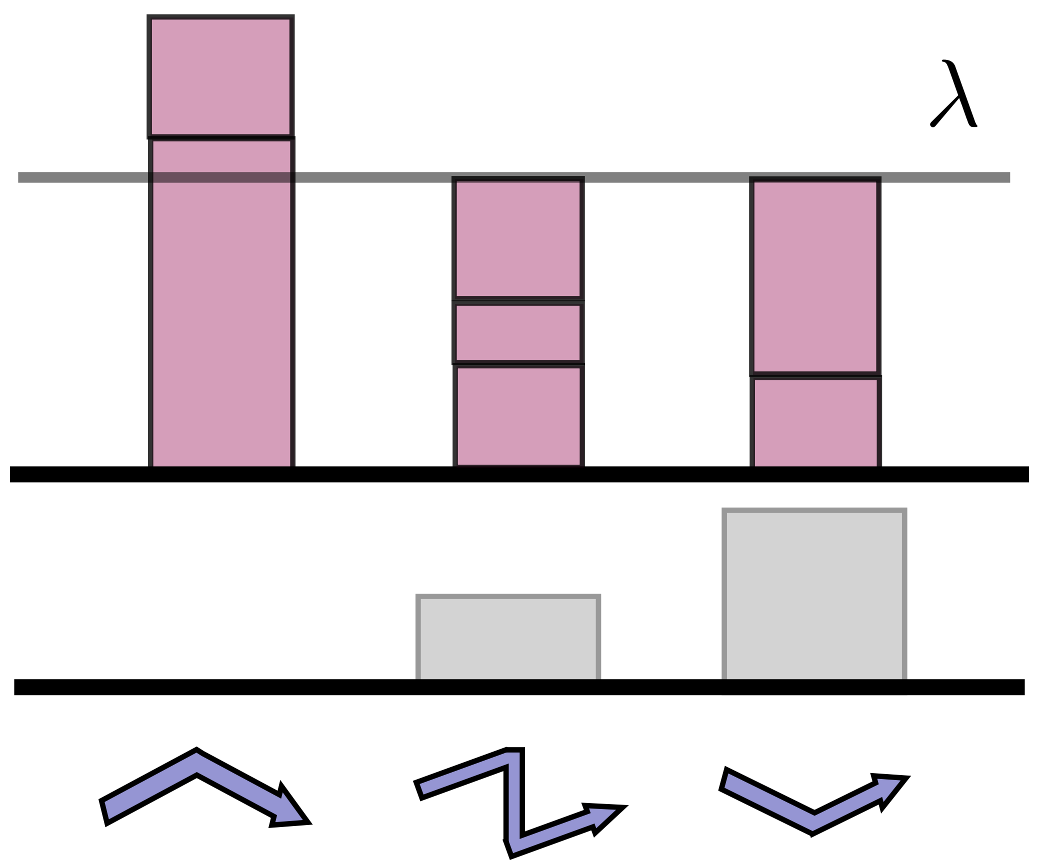
Potential Function

$$F(x)$$

Wardrop Equilibrium

Travel Cost

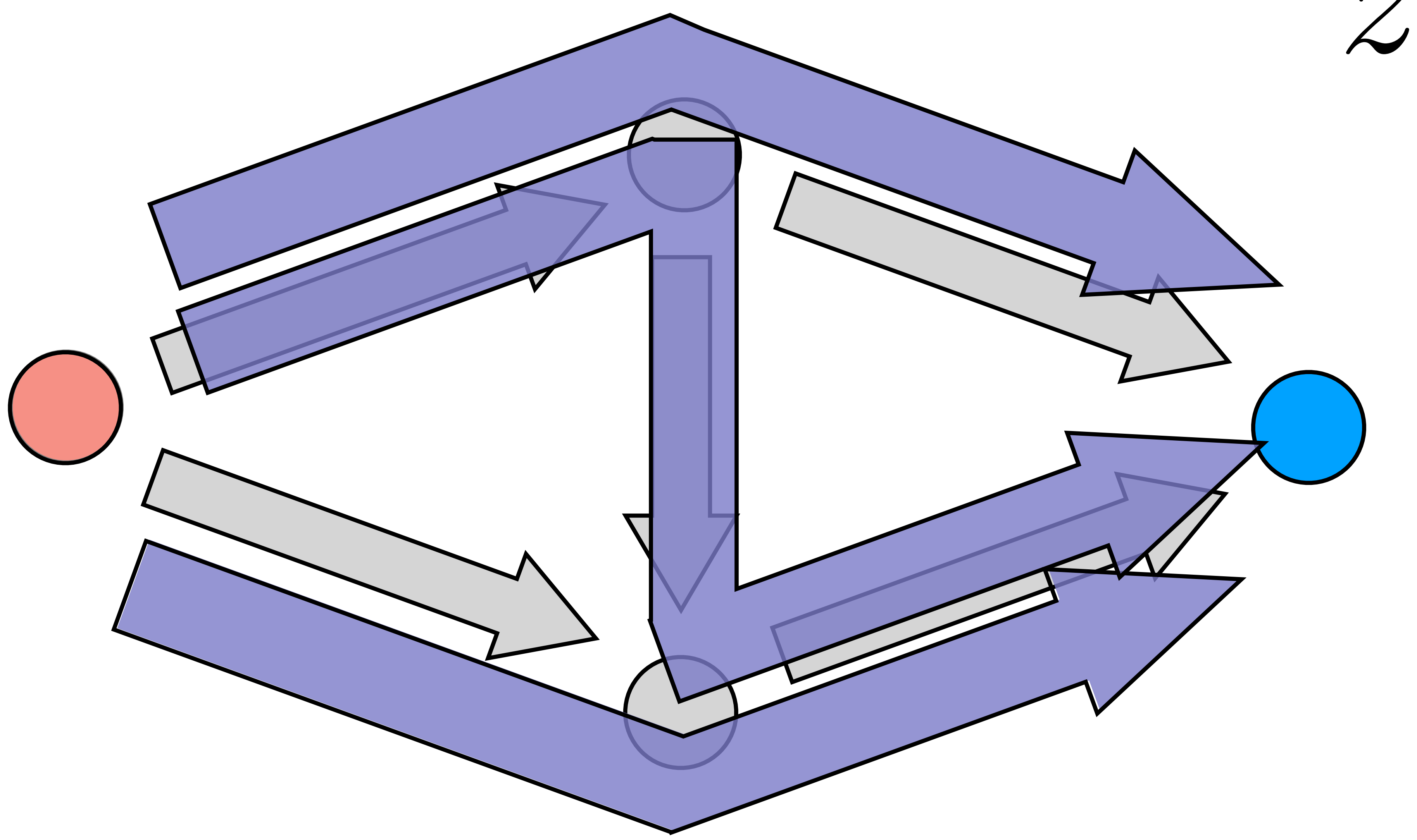
Mass



Routing Games

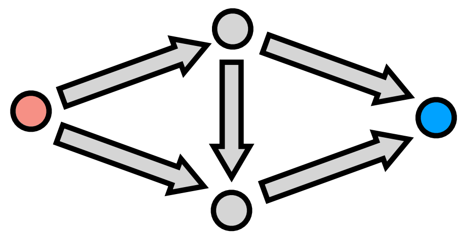
x : edge traffic

z : route traffic



Potential Games

Routing Games



Potential Function $F(x)$

Routing Games

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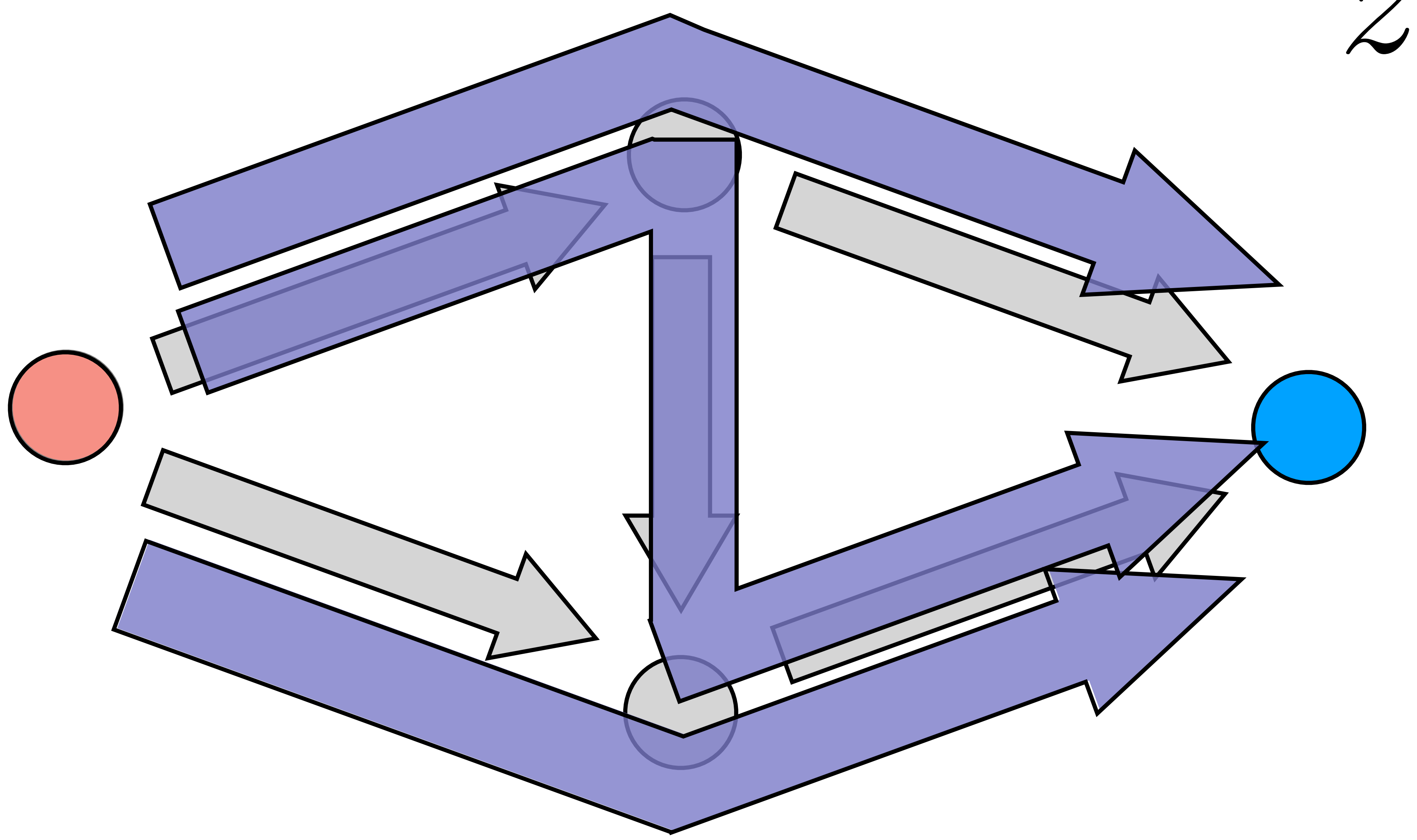
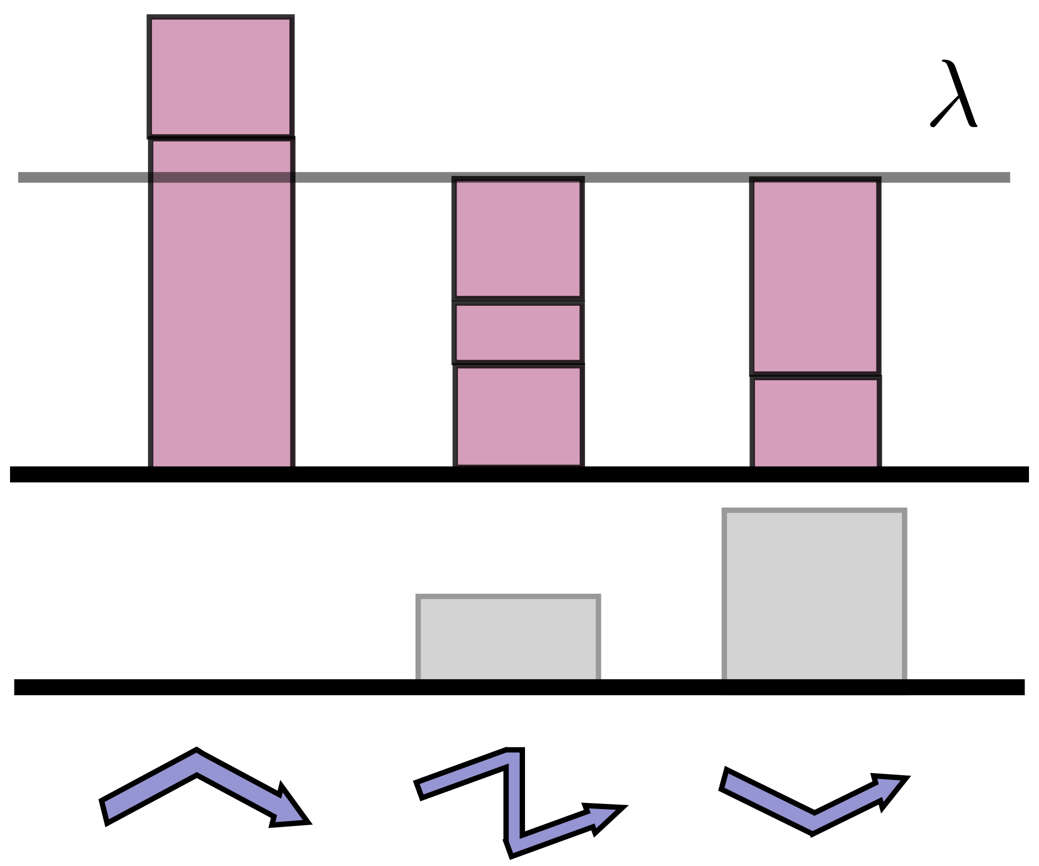


Wardrop Equilibrium

Travel Cost

Mass



$$\min_{x,z} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T z = m \qquad z \geq 0$$

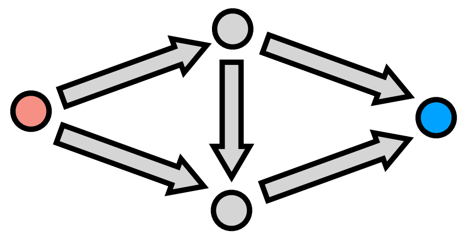
Mass conservation

$$x = \mathbf{R} z$$

Edges in routes

Potential Games

Routing Games

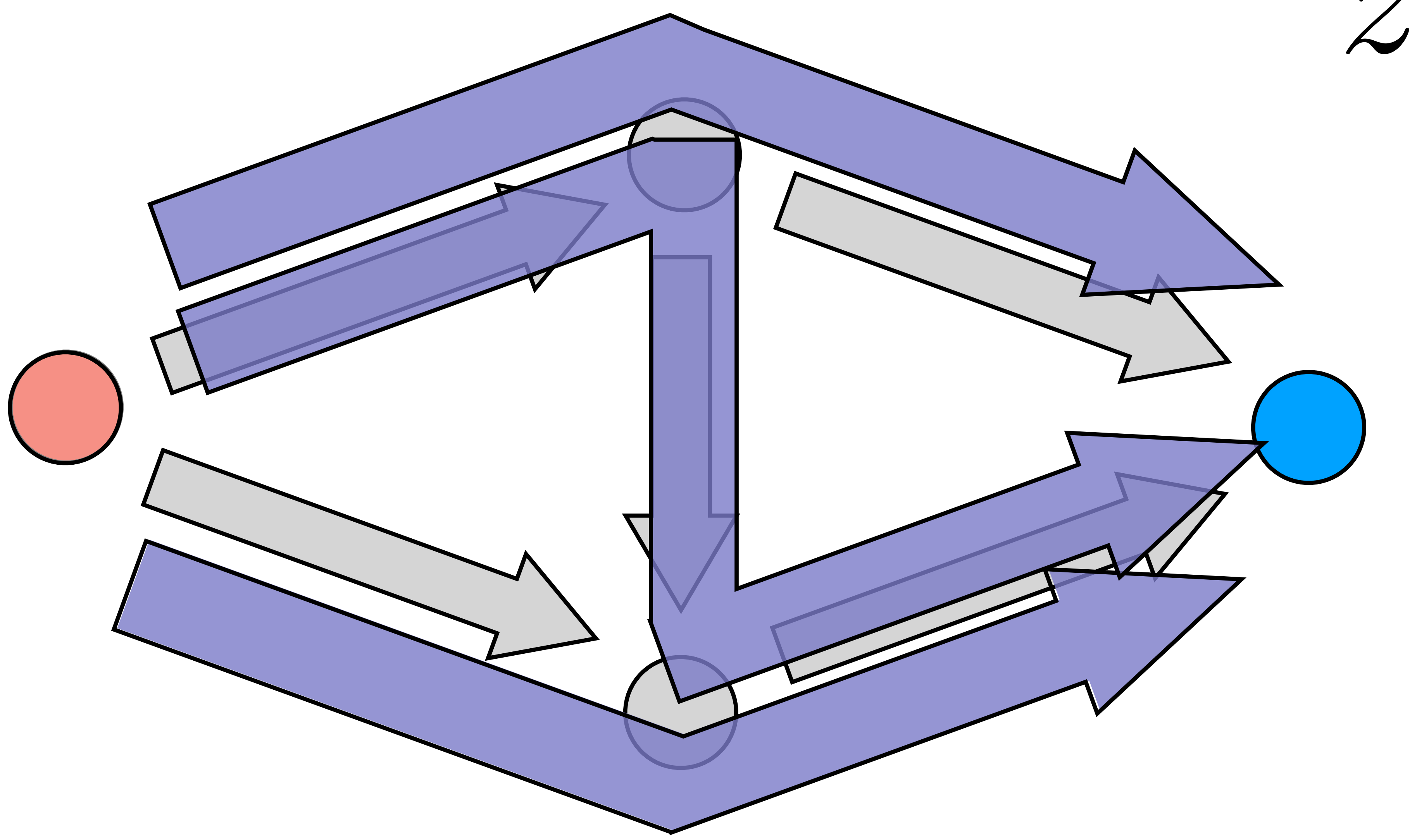


Potential Function $F(x)$

Routing Games

x : edge traffic

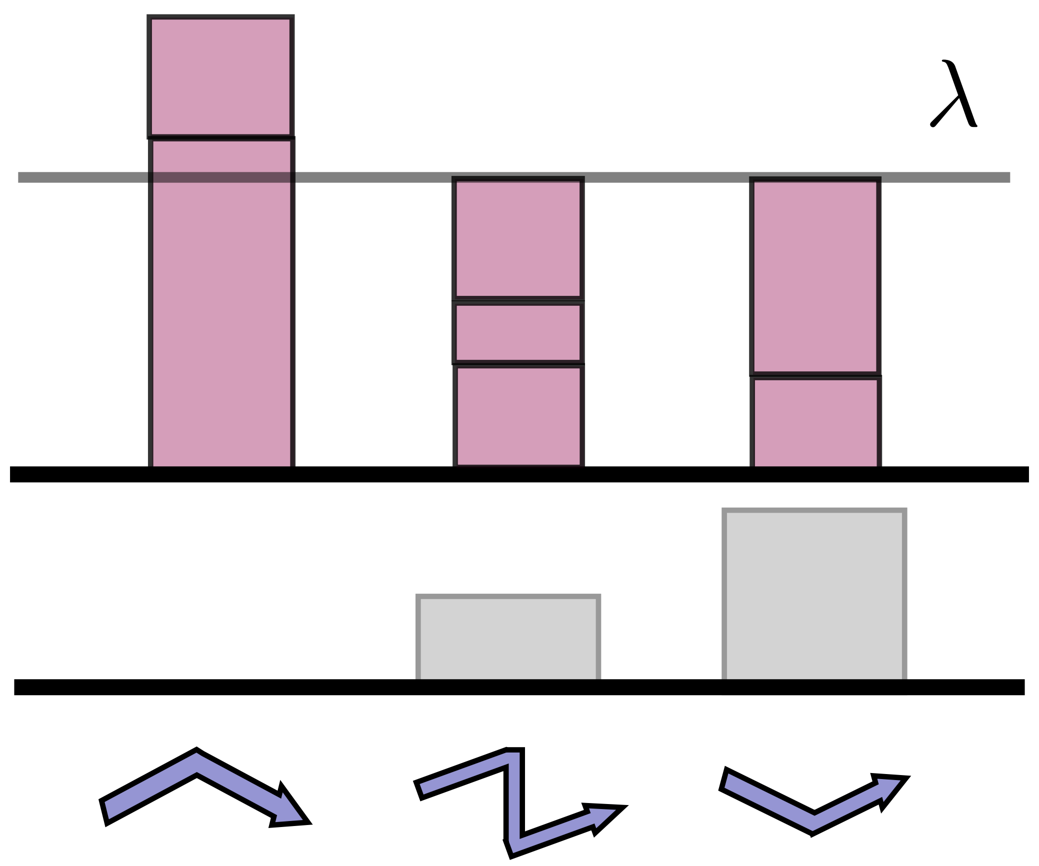
z : route traffic



Wardrop Equilibrium

Travel Cost

Mass



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Edges in routes

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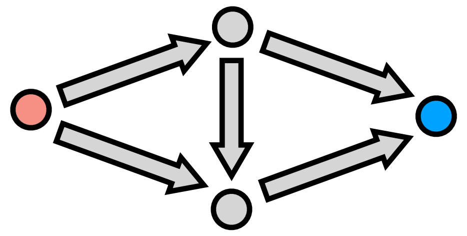
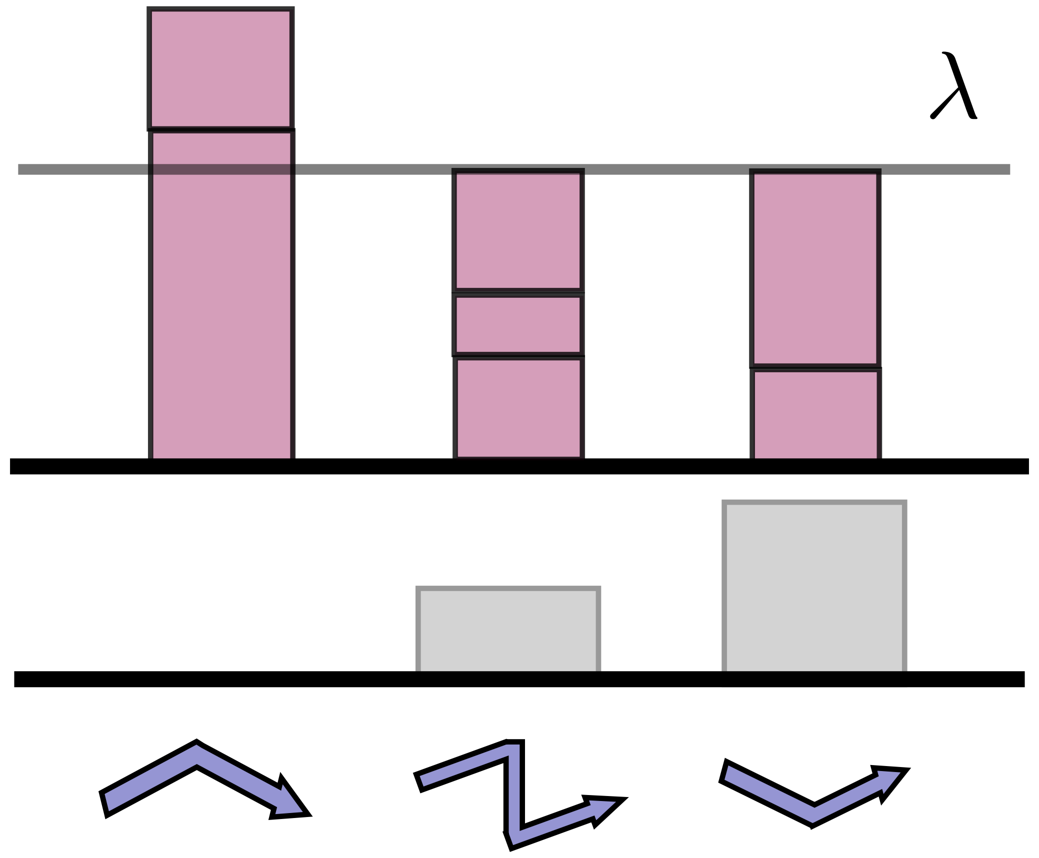


Potential Function $F(x)$

Wardrop Equilibrium



Routing Games

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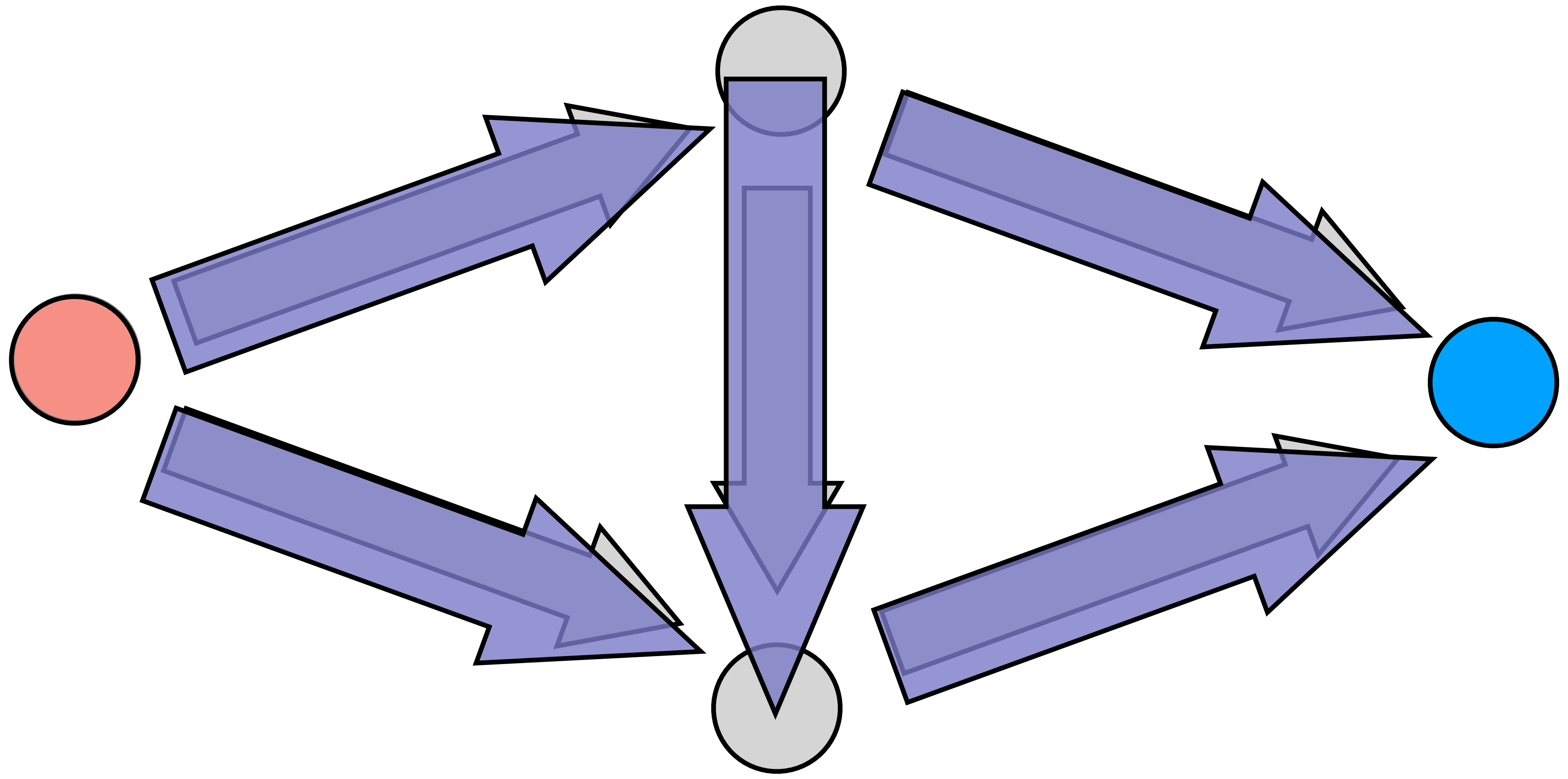
z : route traffic



$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad Ex = Sm, \quad x \geq 0$$

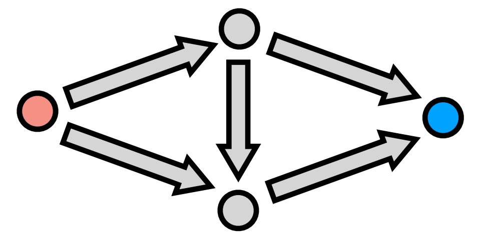
Mass conservation

Graph structure

Origin-destination

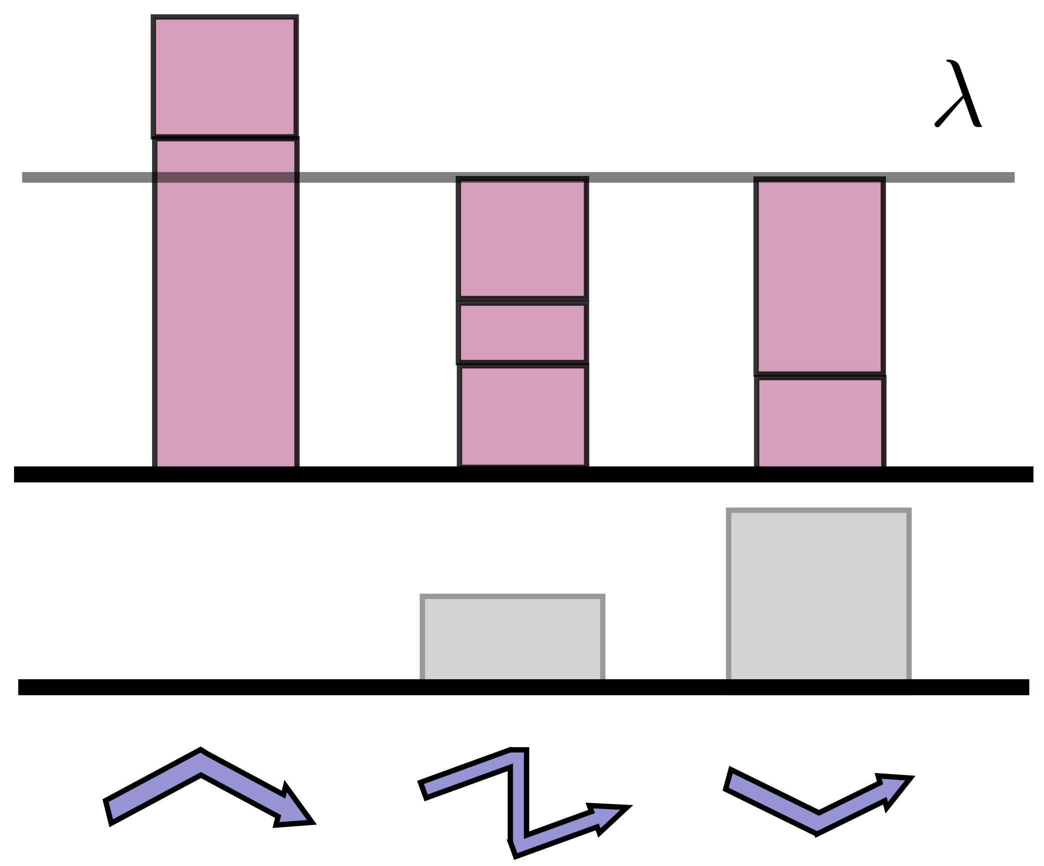
Potential Games

Routing Games



Potential Function $F(x)$

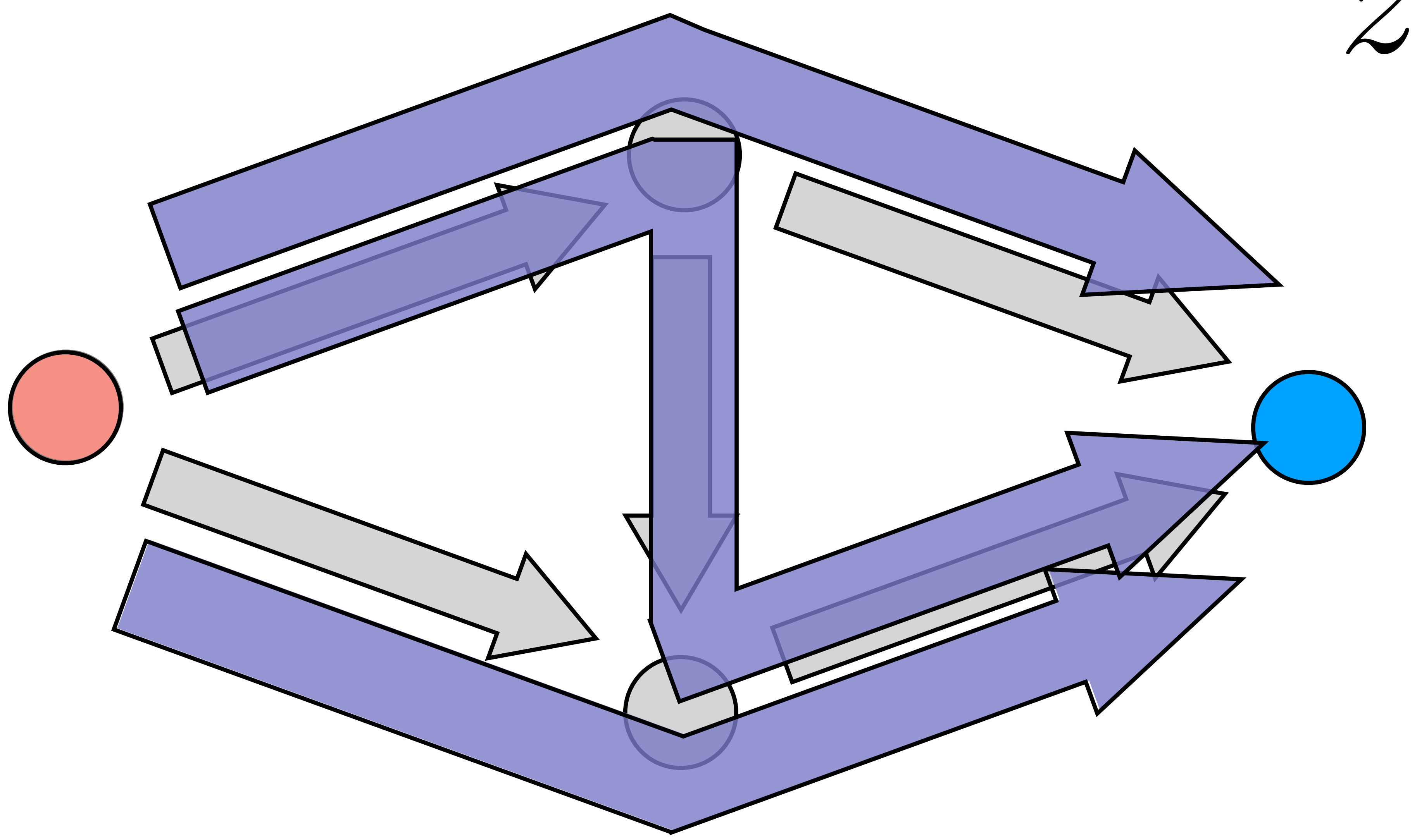
Wardrop Equilibrium



Routing Games

x : edge traffic

z : route traffic



$$\min_{x,z} \quad F(x)$$

s.t.

$\mathbf{1}^T z = m \quad (\lambda)$ $\qquad z \geq 0 \quad (\nu)$

$x = \mathbf{R}z \quad (w)$

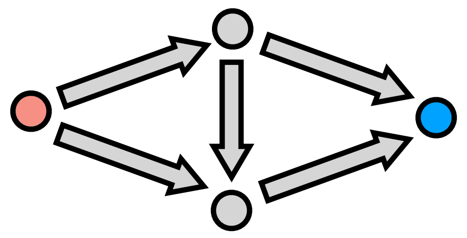
λ : travel cost

ν : route inefficiency

w : edge costs

Potential Games

Routing Games

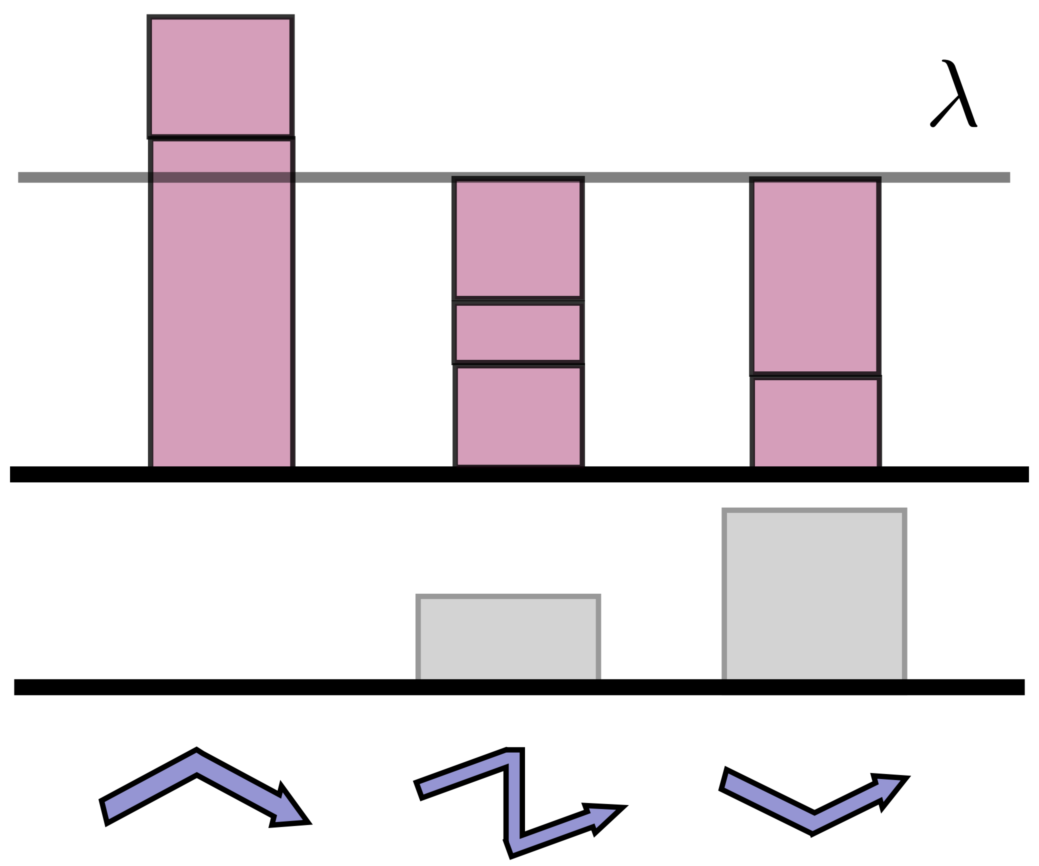


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

Mass



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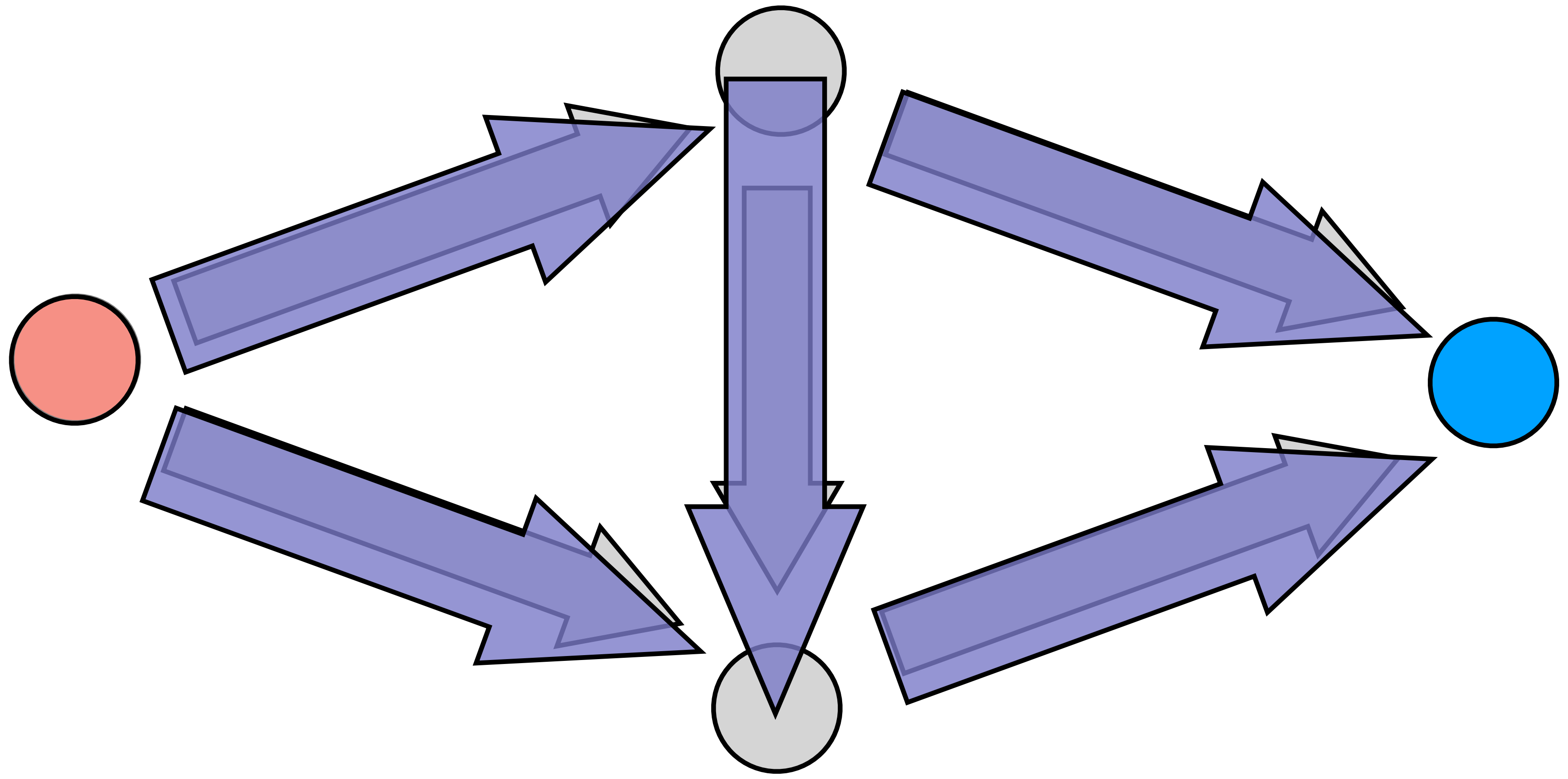


$$\min_{x} \quad F(x)$$

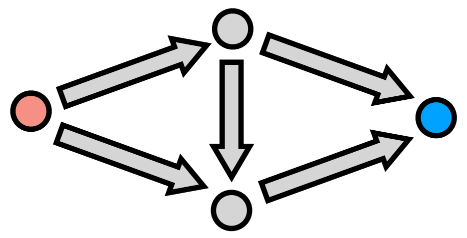
$$\text{s.t.} \quad \boxed{Ex = Sm, \ \boxed{v}} \quad \boxed{x \geq 0 \ \boxed{\mu}}$$

μ : edge inefficiency

v : value function

Potential Games

Routing Games

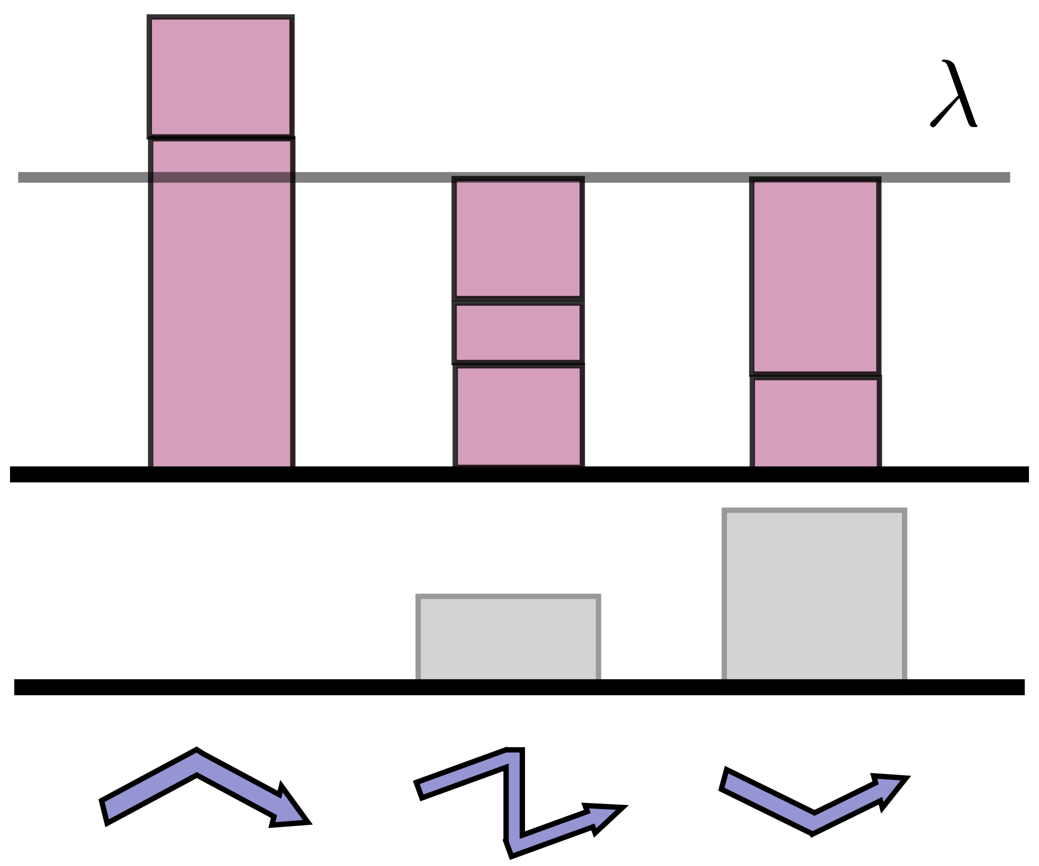


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

Mass



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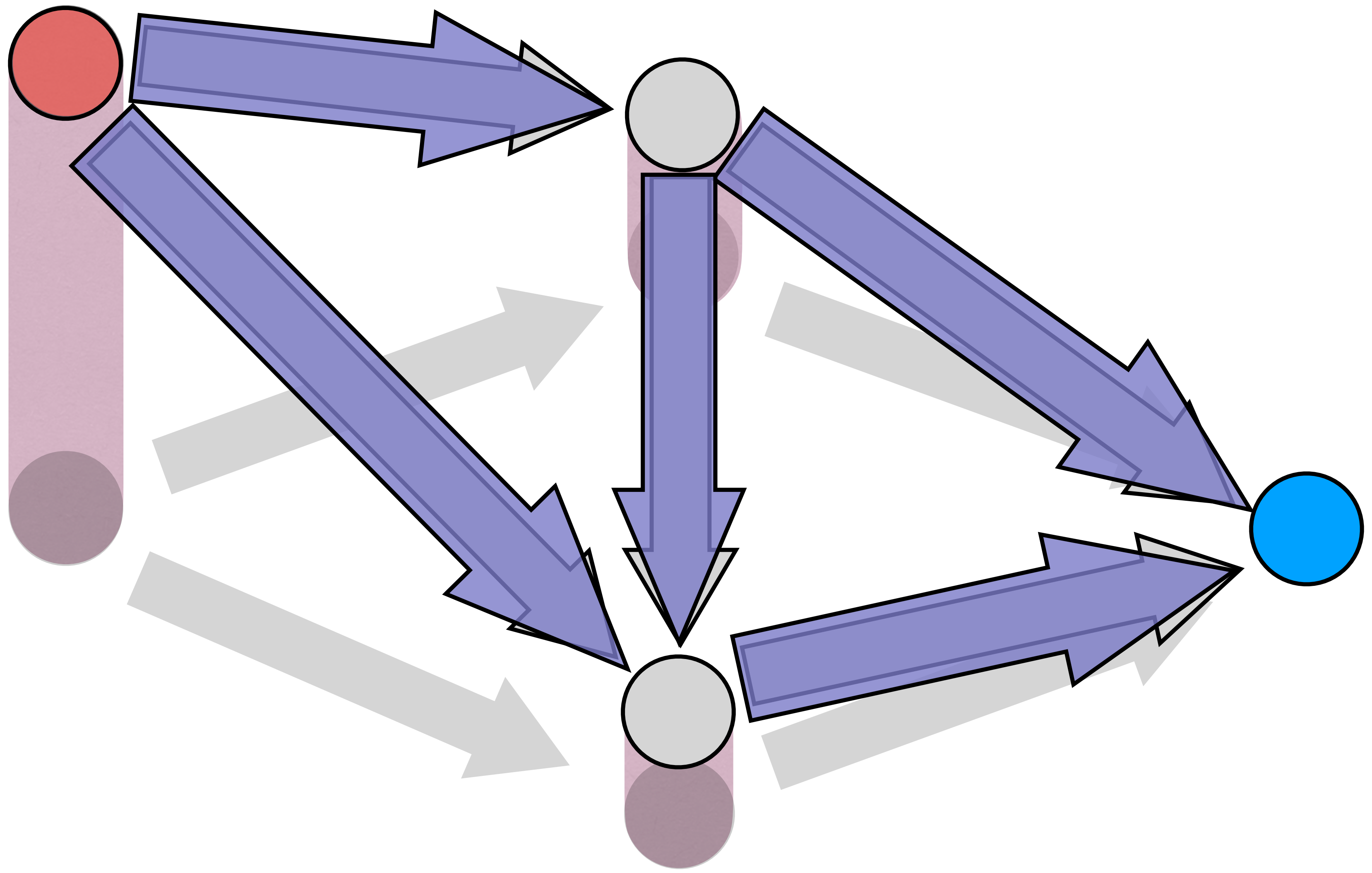

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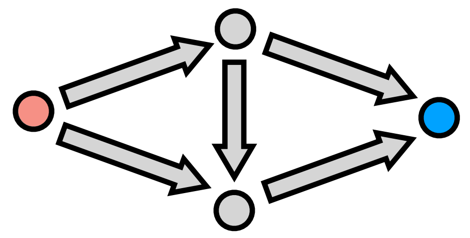
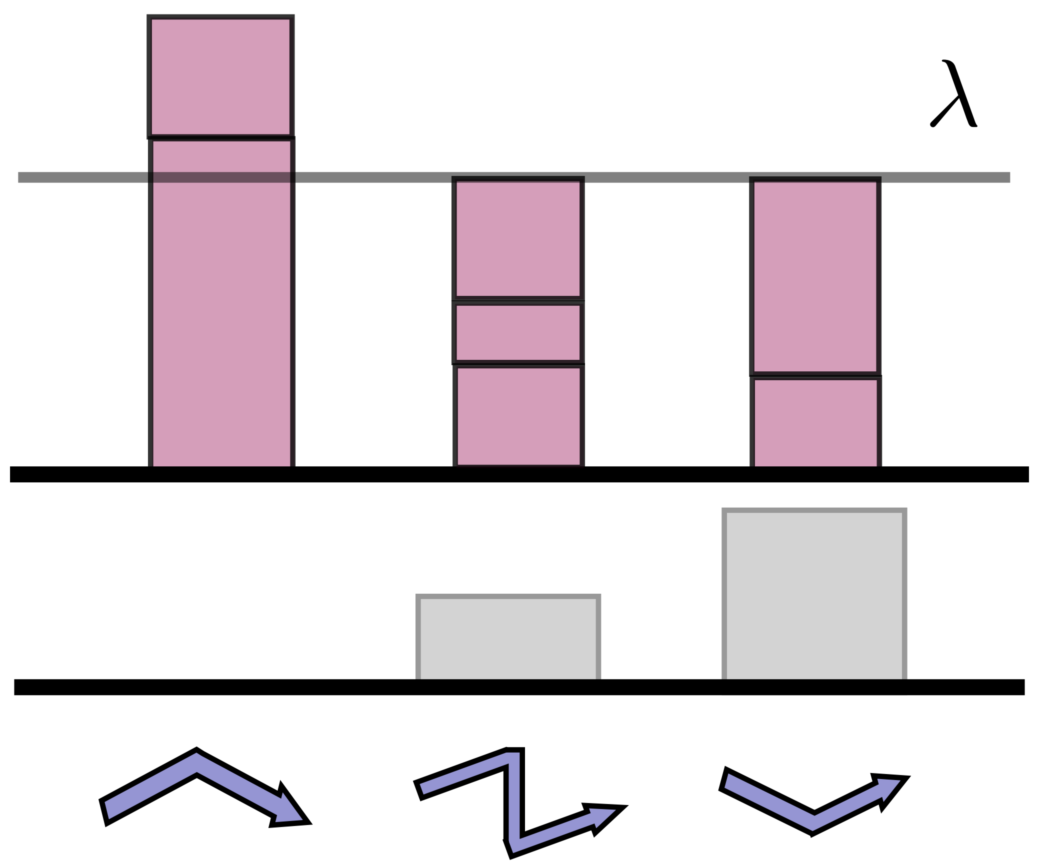
Routing Games



Potential Function $F(x)$

Wardrop Equilibrium



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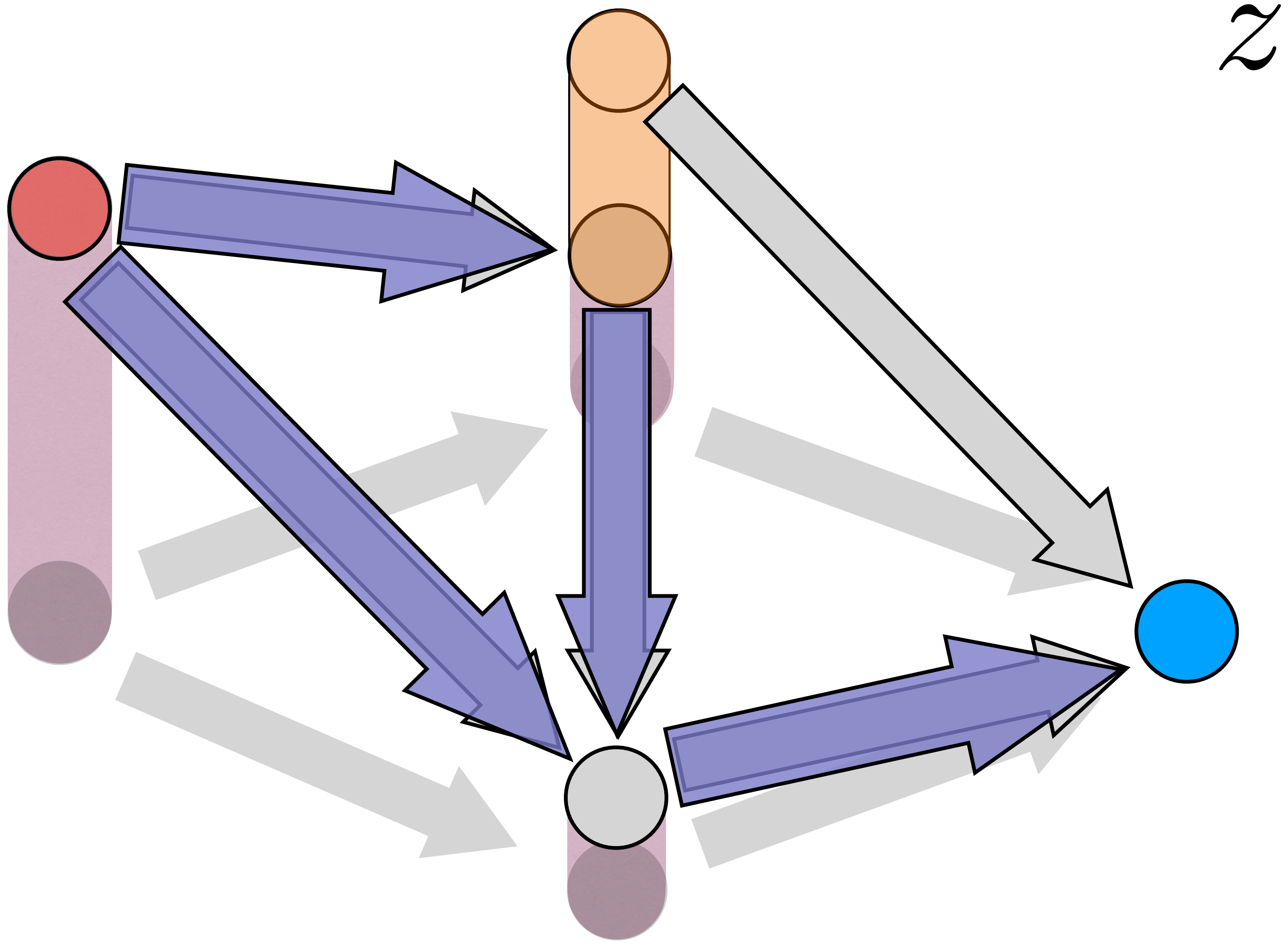

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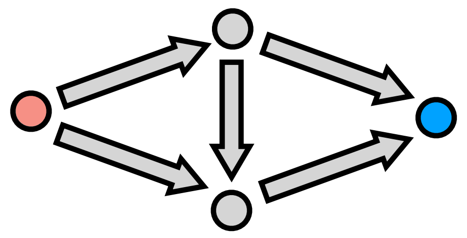
Routing Games

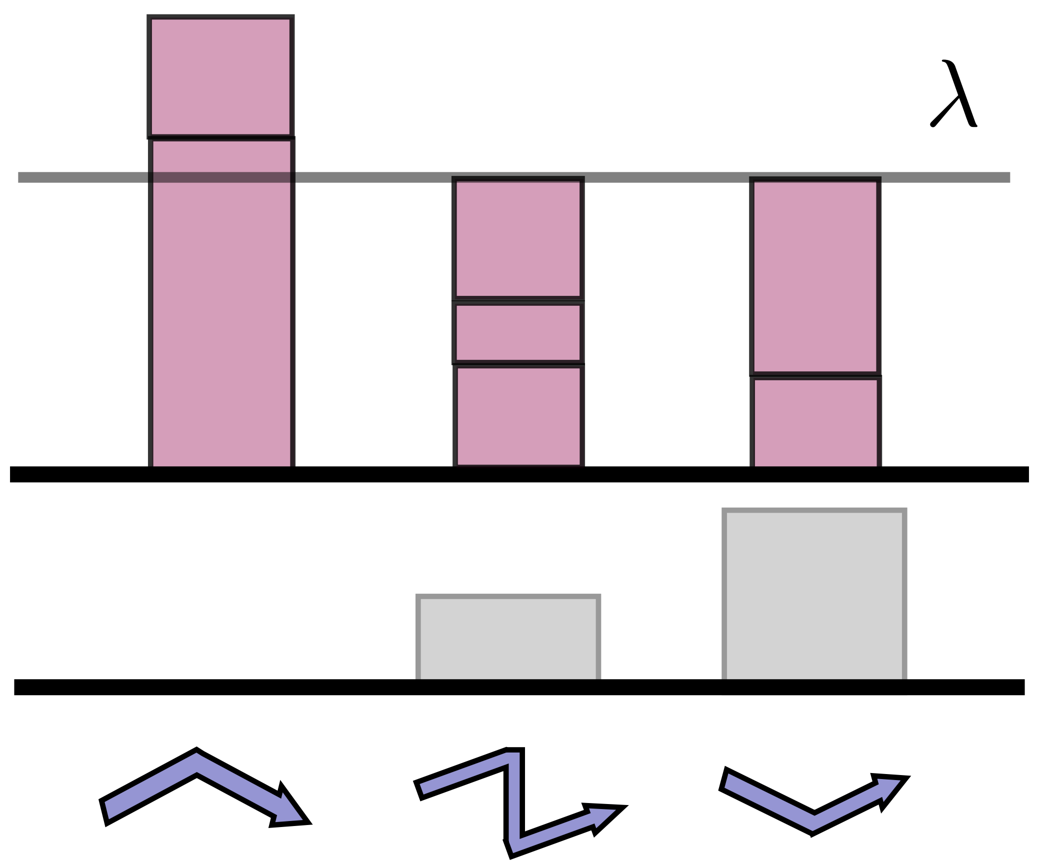


Potential Function $F(x)$

Wardrop Equilibrium

Travel Cost

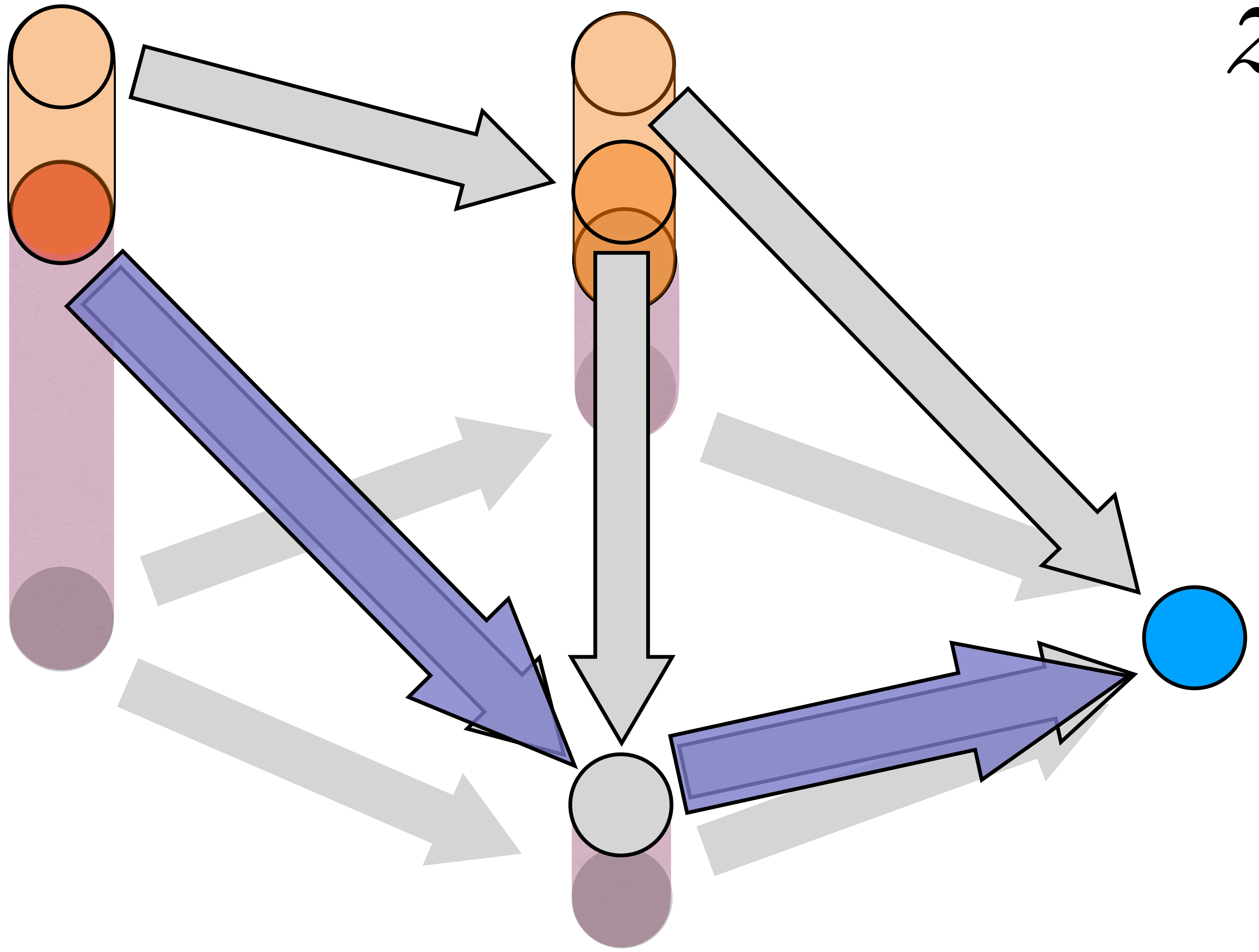
Mass



Routing Games

x : edge traffic

z : route traffic



$$\min_{x} \quad F(x)$$

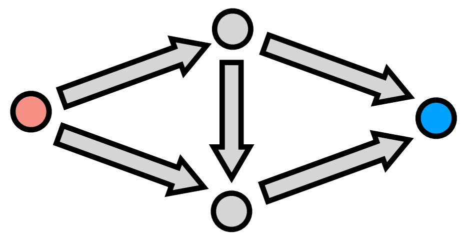
s.t. $\quad Ex = Sm, \; (v) \qquad x \geq 0 \; (\mu)$

μ : edge inefficiency

v : value function

Potential Games

Routing Games

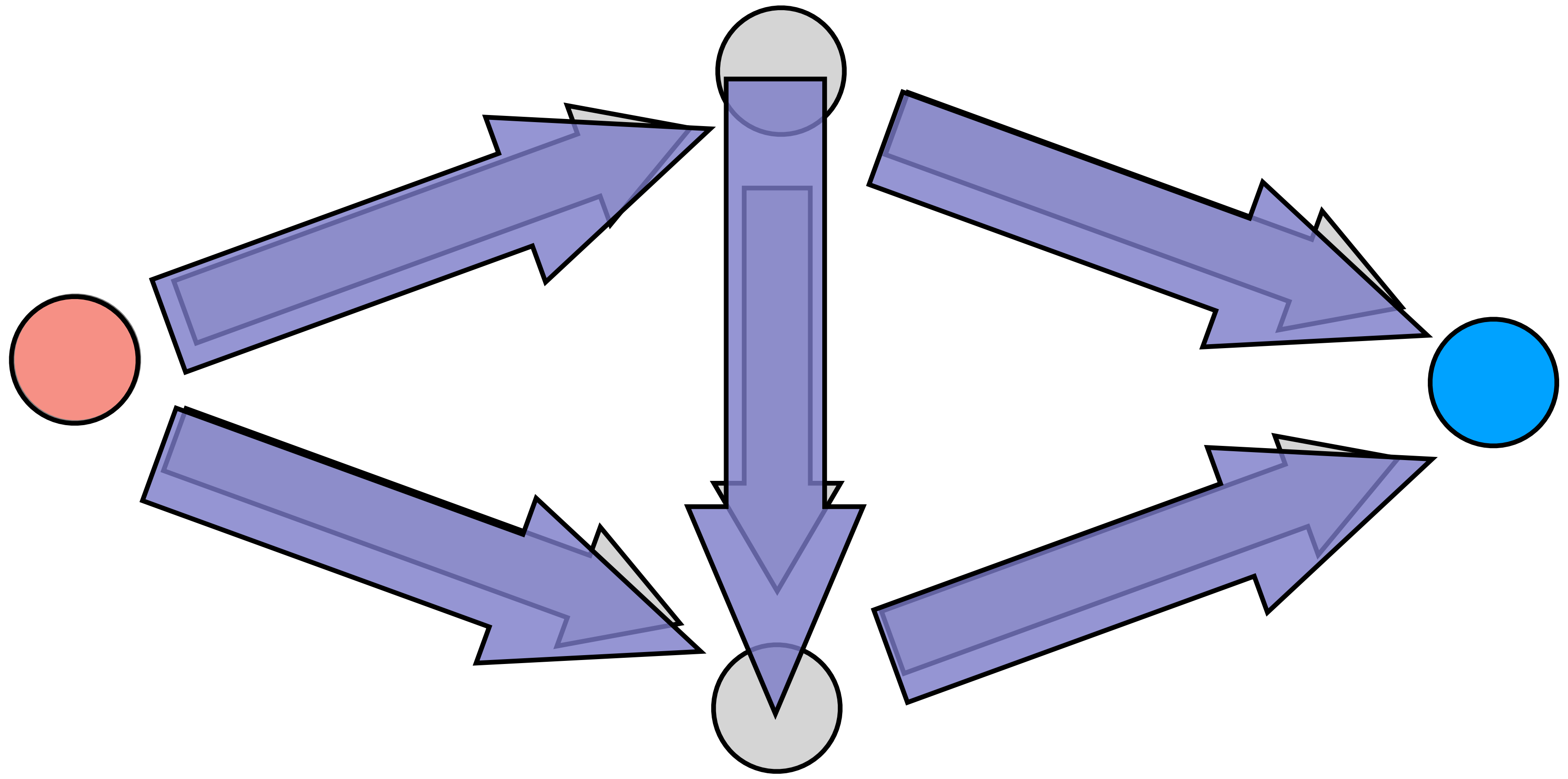


Potential Function $F(x)$

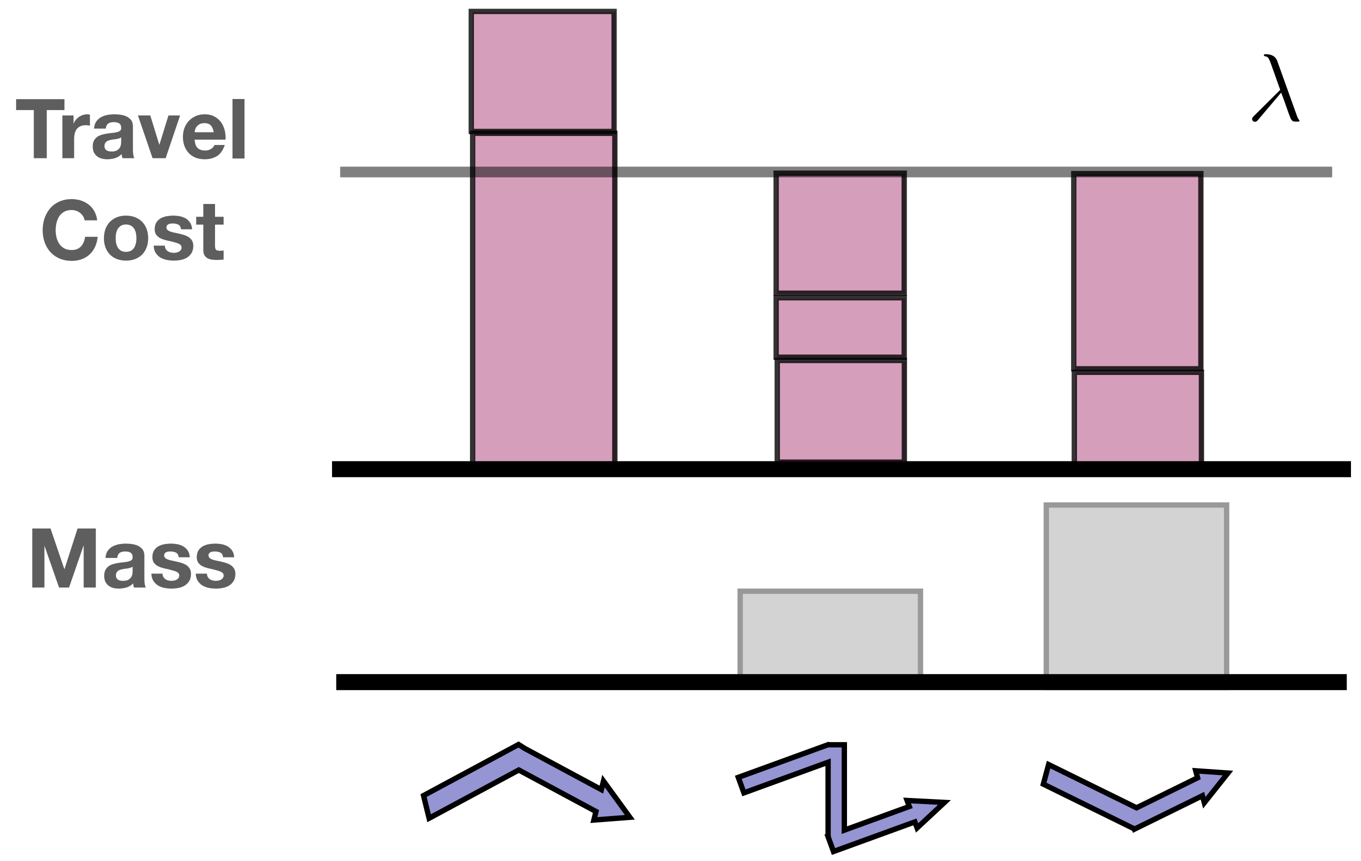
Routing Games

x : edge traffic

z : route traffic



Wardrop Equilibrium



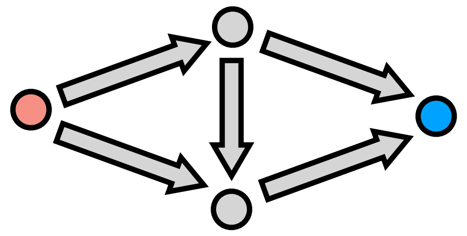
REFERENCES

- Some theoretical aspects of road traffic research **[Wardrop, 1952]**
- Studies in the economics of transportation **[Beckmann, McGuire, Winsten, 1956]**

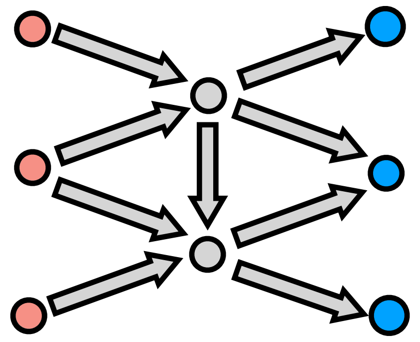
- The Traffic Assignment Problem: Models and Methods **[Patriksson, 2015]**

Potential Games

Routing Games



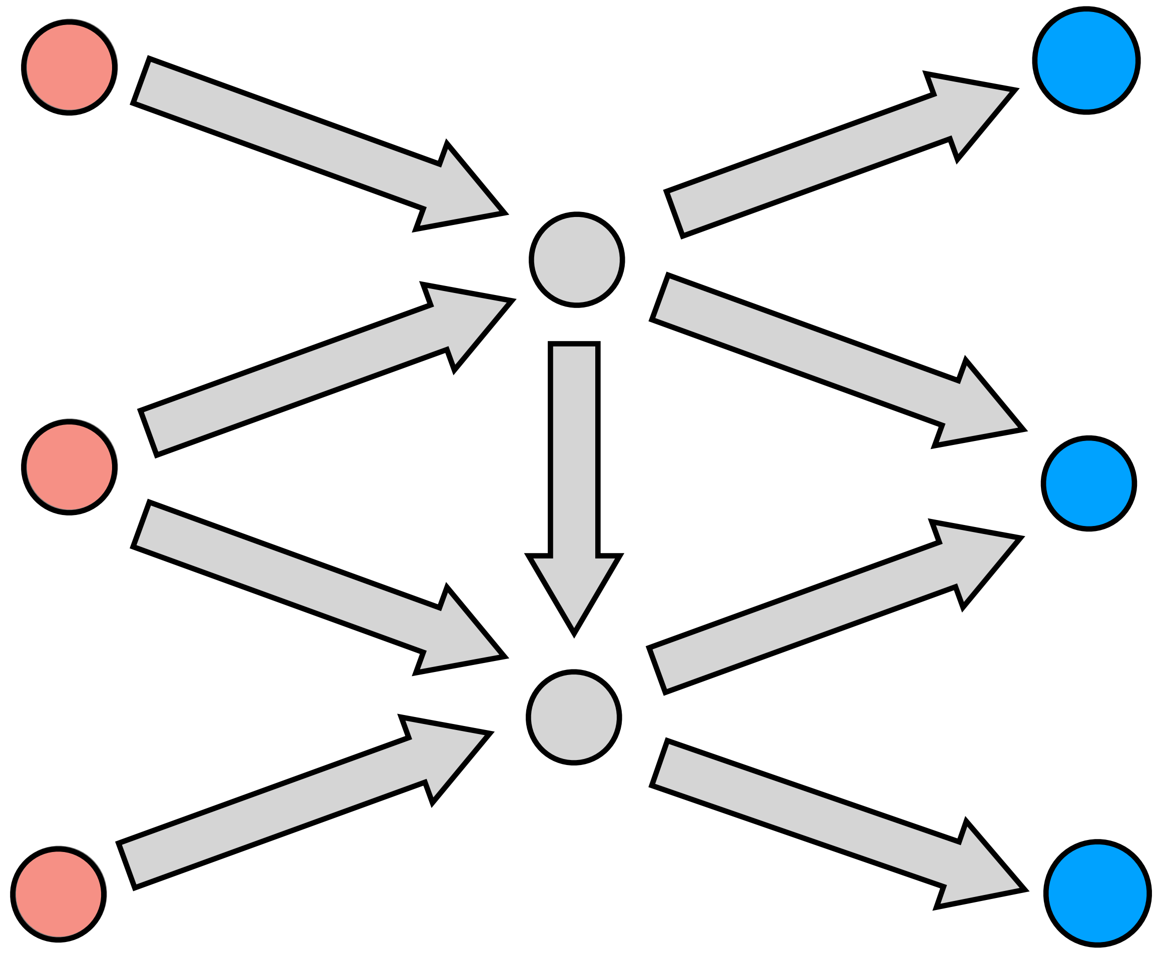
Multiple sources/ sinks



Multiple Source/Sinks

x : edge traffic

z : route traffic

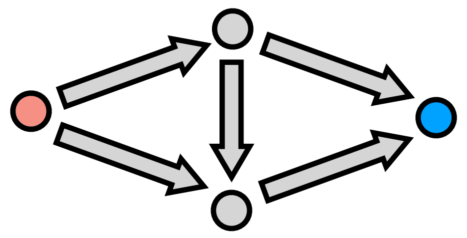


$$\min_x \quad F(x)$$

$$\text{s.t.} \quad E_i x_i = S_i m_i,\ \forall i \quad v_i \qquad x_i \geq 0,\ \forall i \quad \mu_i$$

Potential Games

Routing Games



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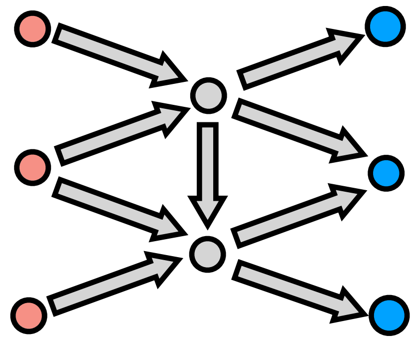
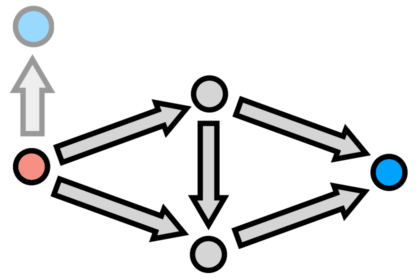


Variable Demand



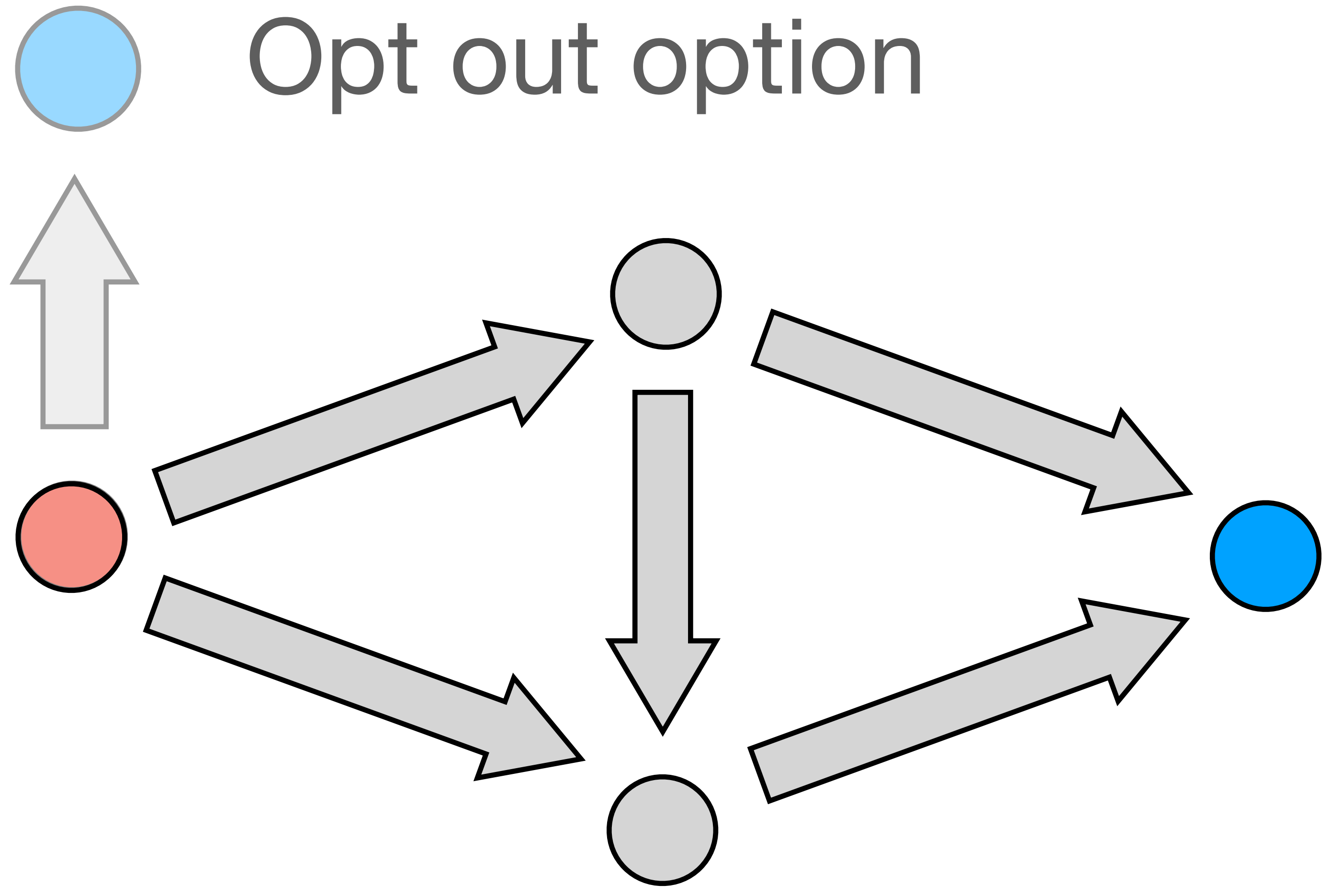
Variable Demand

x : edge traffic

z : route traffic

Opt out option

$m(\lambda)$

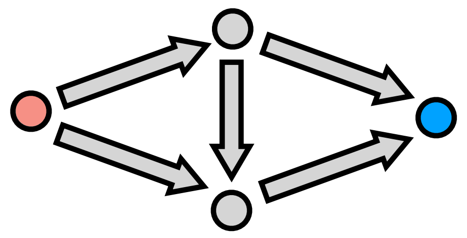


$$\min_{x} \quad F(x) + \int_0^m m^{-1}(u)\, du$$

$$\text{s.t.} \quad Ex = Sm, \quad v \qquad x \geq 0 \quad \mu$$

Potential Games

Routing Games



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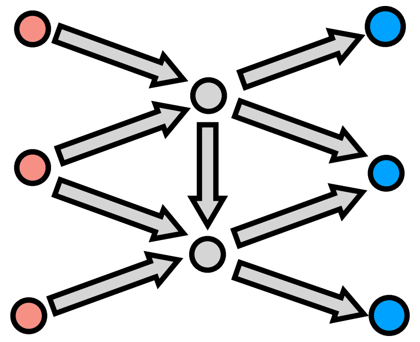
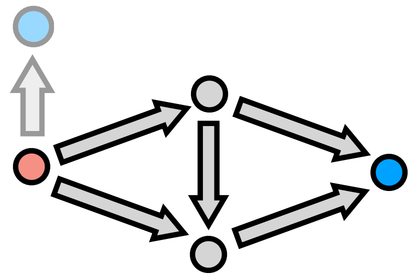
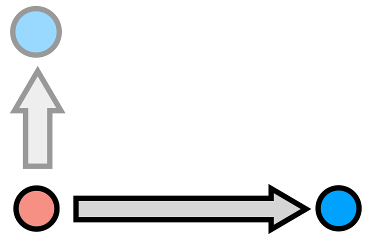


Variable Demand



Supply & Demand

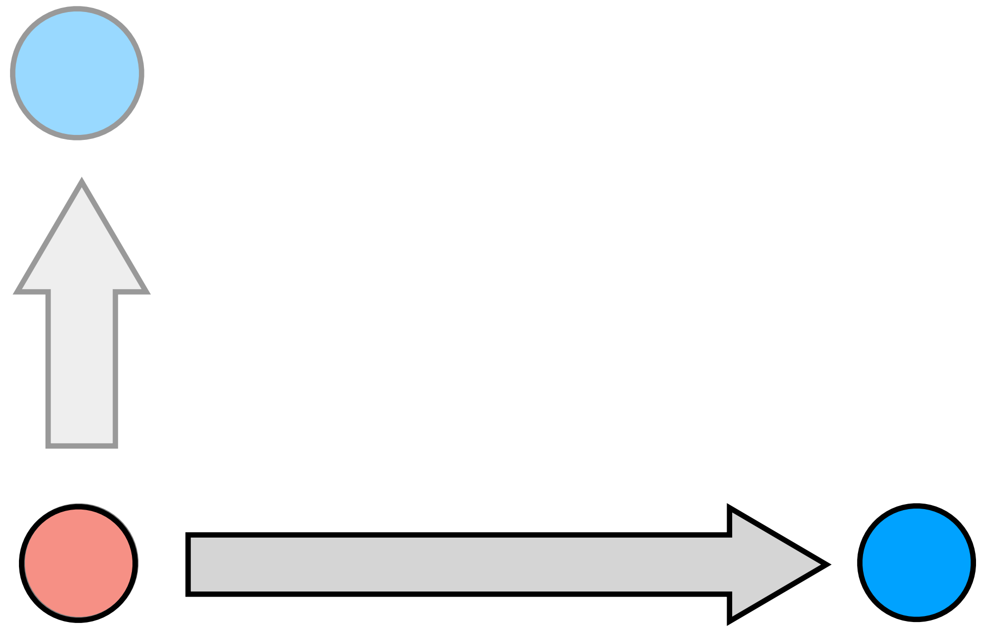


Supply & Demand

x : edge traffic

z : route traffic

$m(\lambda)$

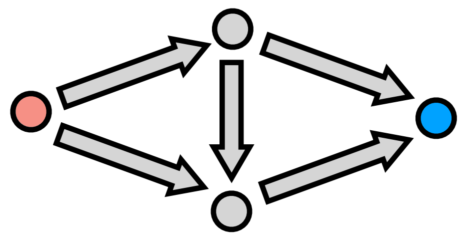


$$\min_x \quad F(x) + \int_0^m m^{-1}(u)\, du$$

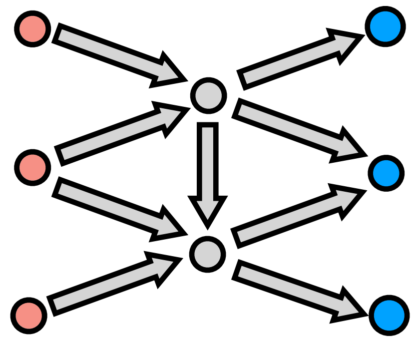
$$\text{s.t.} \quad Ex = Sm, \ \upsilon \qquad x \geq 0 \ \ \mu$$

Potential Games

Routing Games



Multiple sources/ sinks



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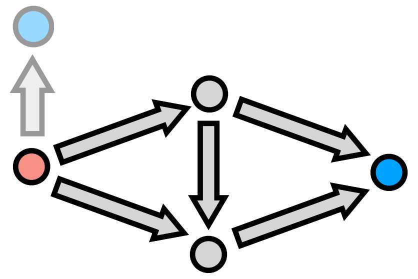

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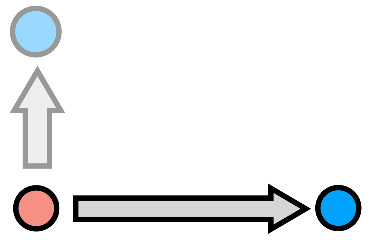
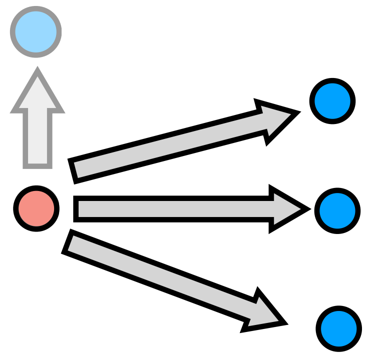


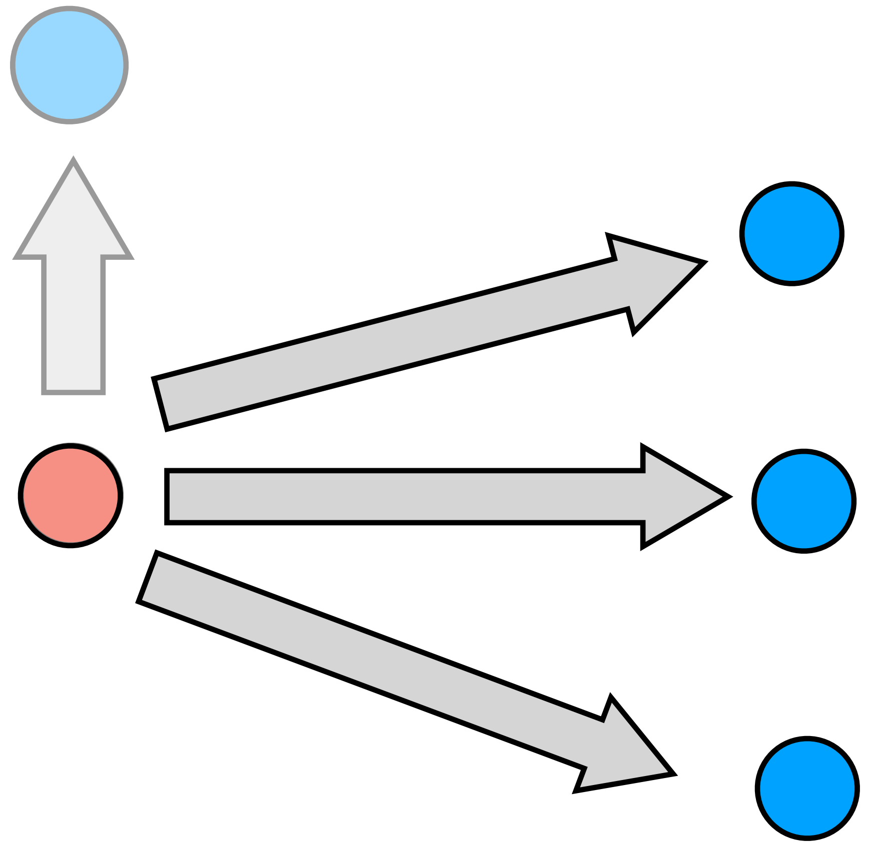
Cournot Market



Cournot Market

x : edge traffic

z : route traffic

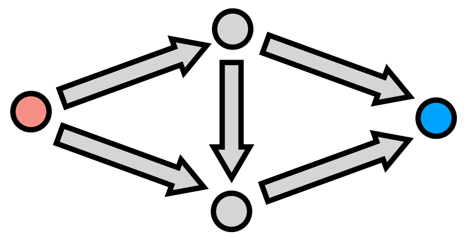


$$\min_{x} \quad F(x) + \int_0^m m^{-1}(u)\, du$$

$$\text{s.t.} \quad Ex = Sm, \; v \qquad x \geq 0 \; \mu$$

Potential Games

Routing Games



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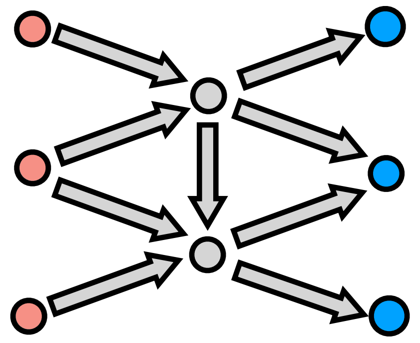
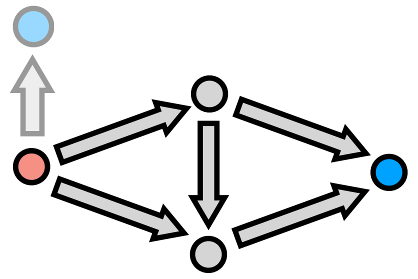


Variable Demand



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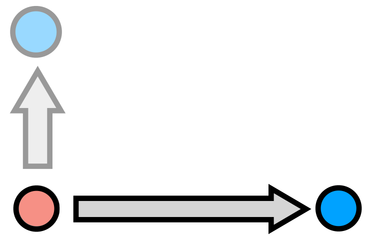
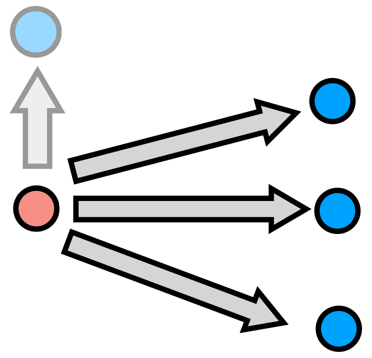
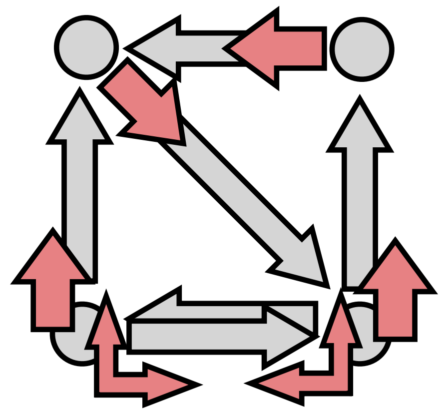


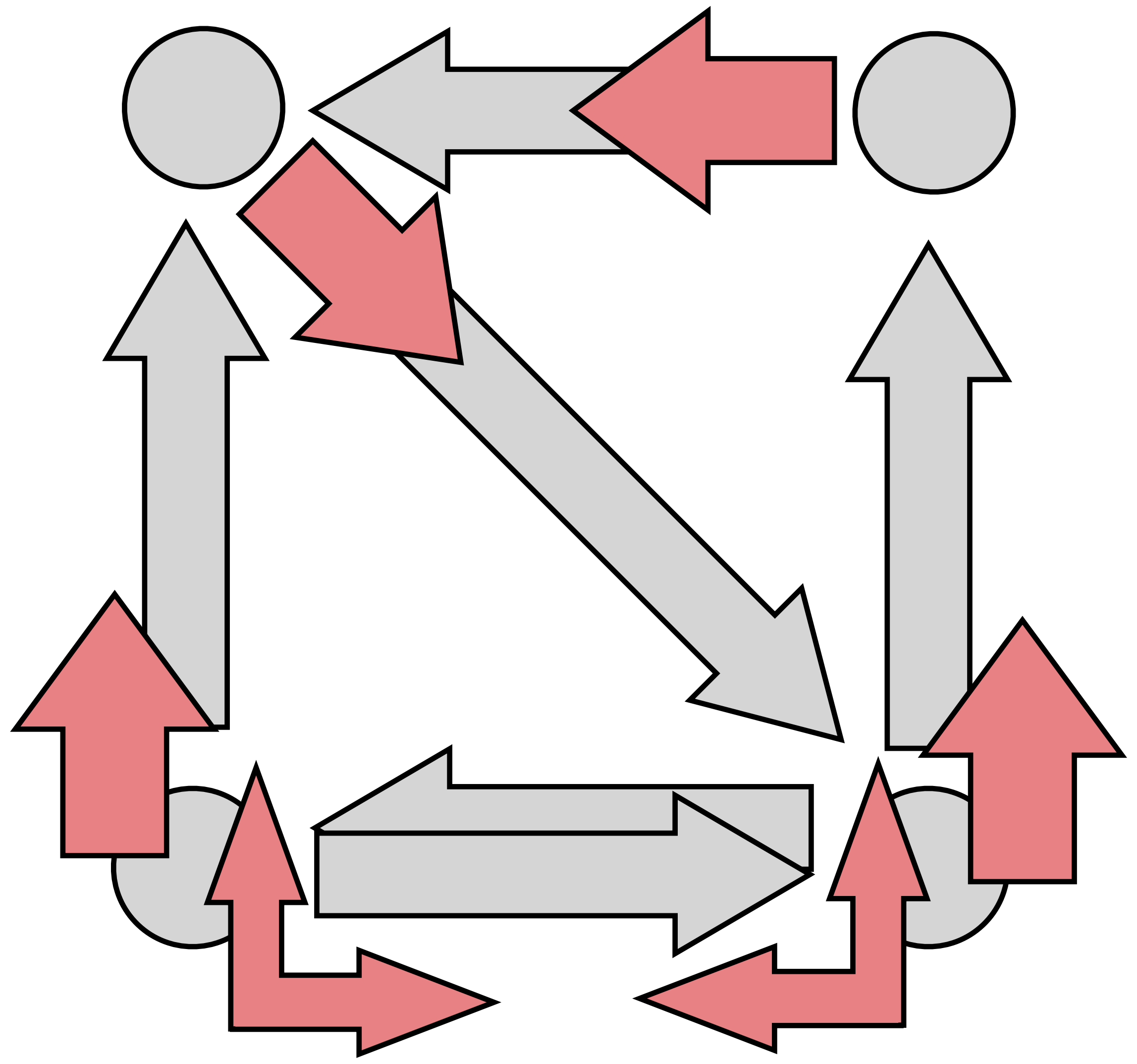
Cournot Market



MDP Congestion Game



Markov Decision Process Congestion Game



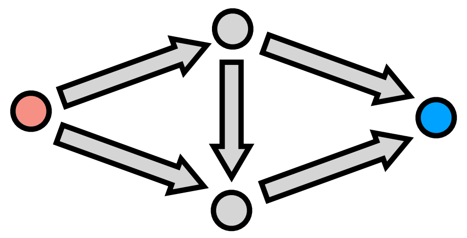
$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \;\; \lambda \qquad x \geq 0 \;\; \mu$$

$$EWx = 0 \;\; v$$

Potential Games

Routing Games



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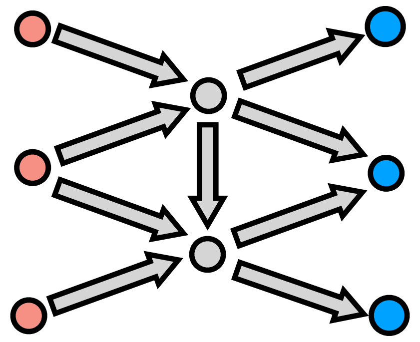
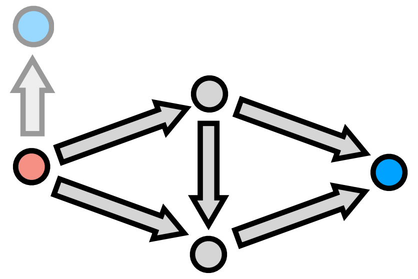
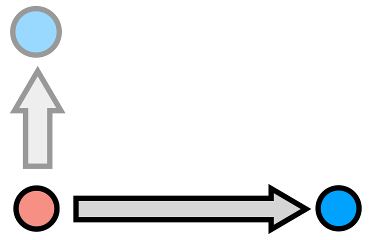


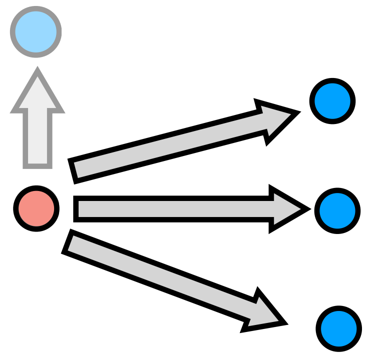
Variable Demand



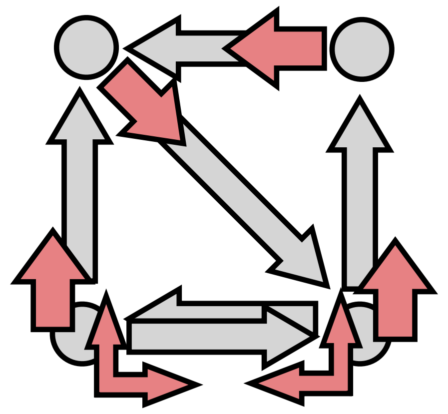
Supply & Demand



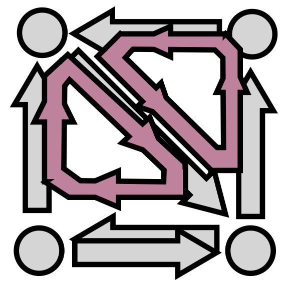
Cournot Market

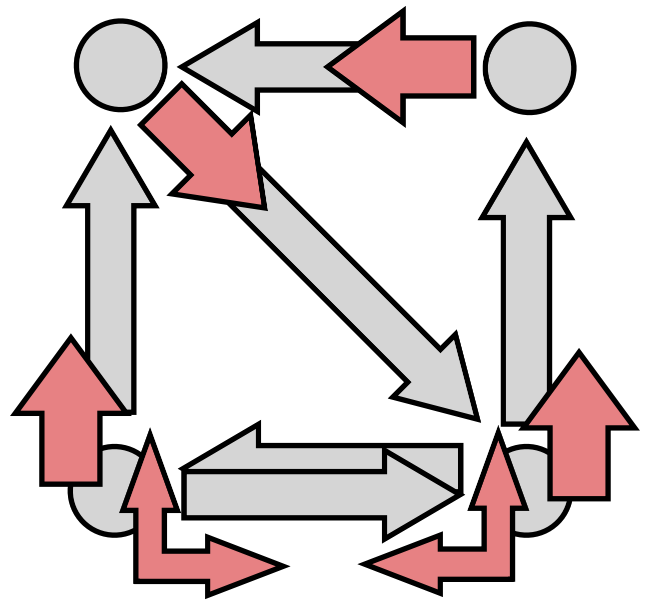


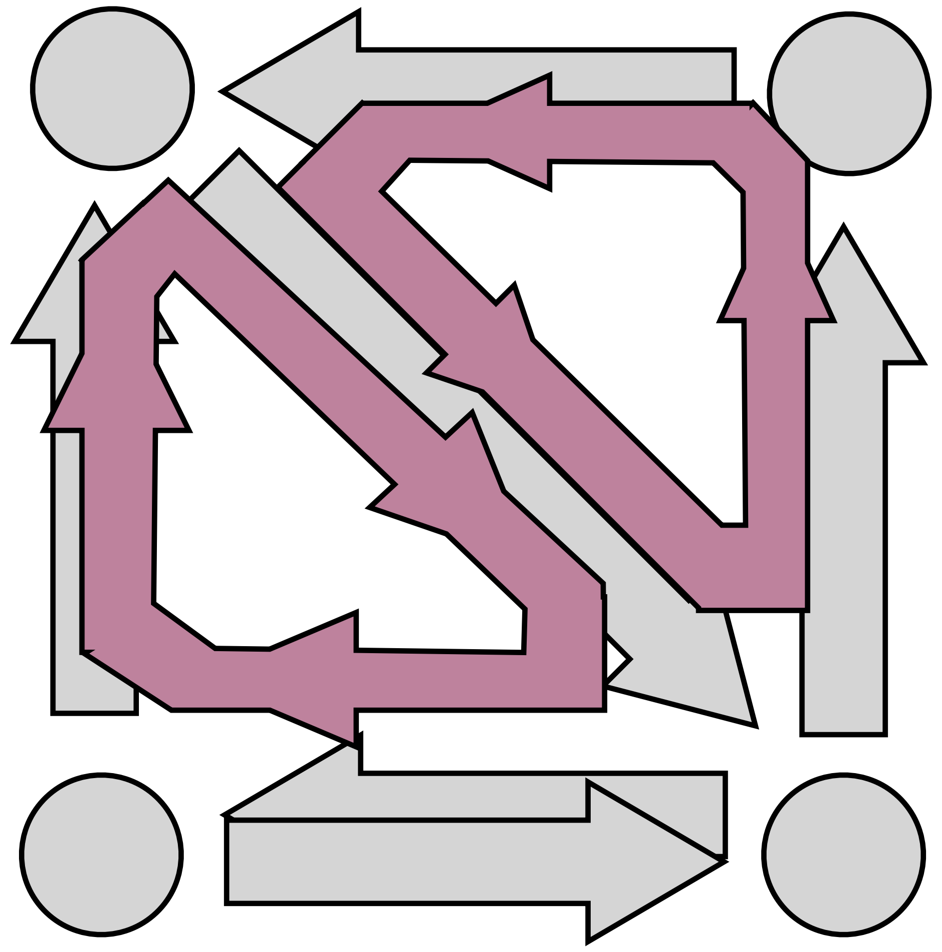
MDP Congestion Game



Markov Decision Process Congestion Game







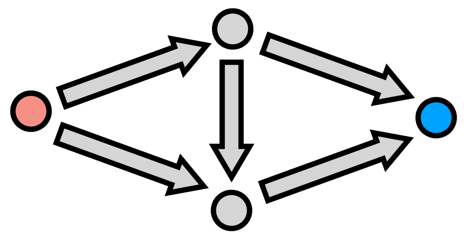
$$\min_{x} \quad F(x)$$

s.t. $\mathbf{1}^T x = m \quad \lambda$ $\qquad x \geq 0 \quad \mu$

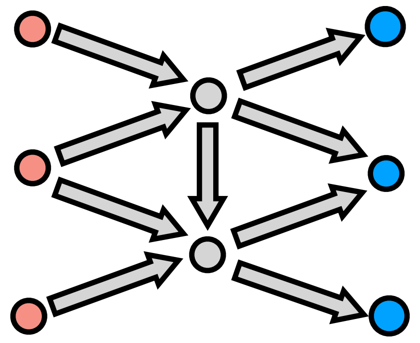
$EWx = 0 \quad v$

Potential Games

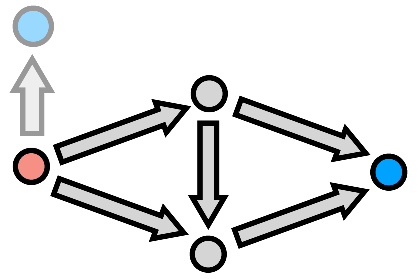
Routing Games



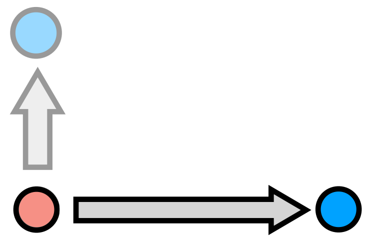
Multiple sources/ sinks



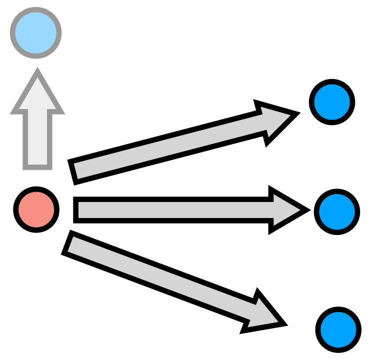
Variable Demand



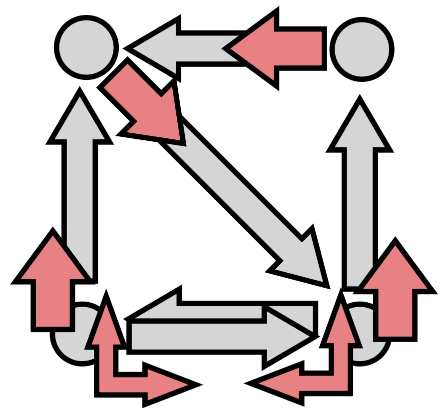
Supply & Demand



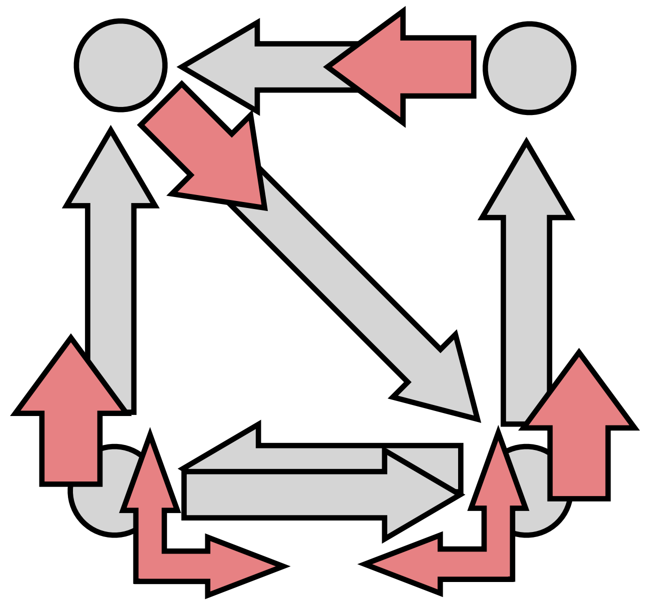
Cournot Market

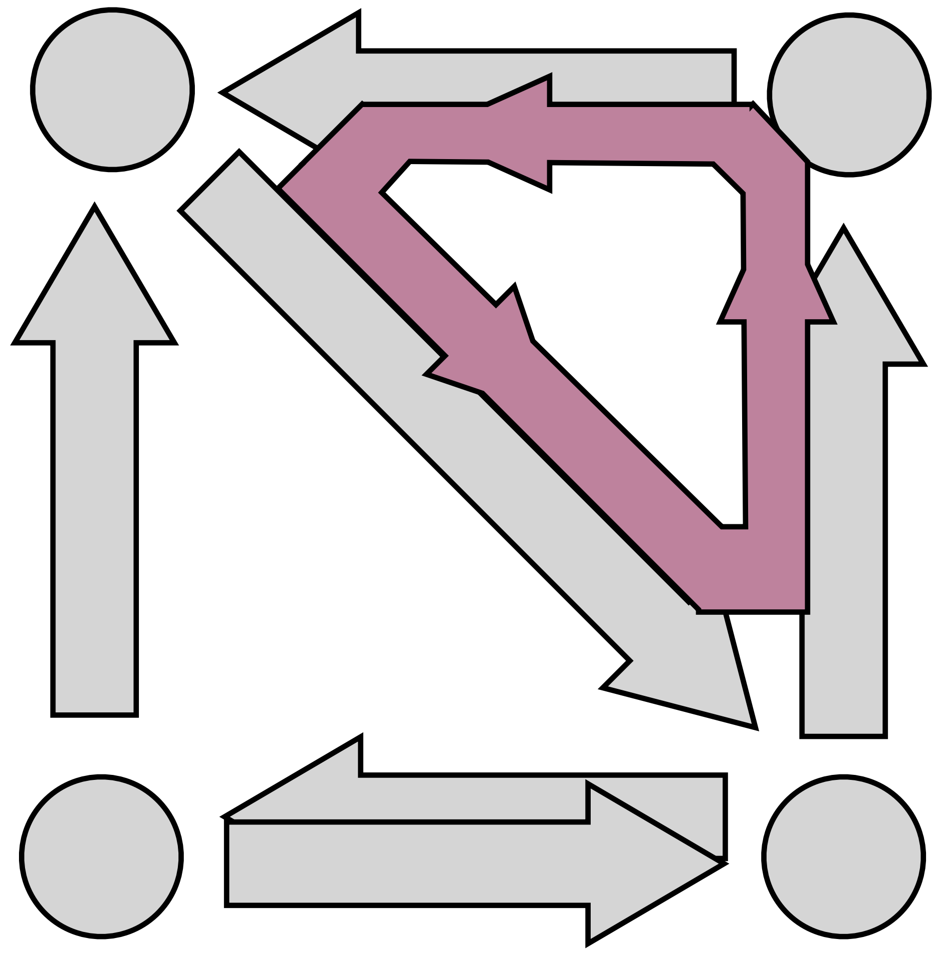


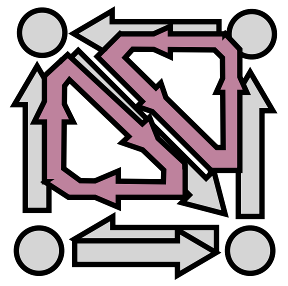
MDP Congestion Game

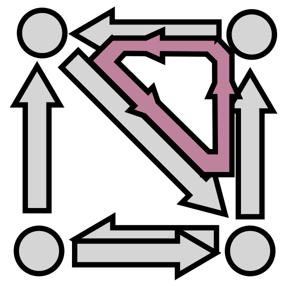


Markov Decision Process Congestion Game









$$\min_{x} \quad F(x)$$

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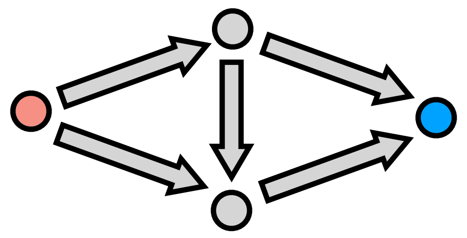
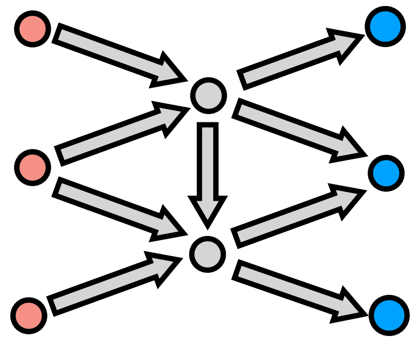
$$EWx = 0 \;\; v$$

Potential Games

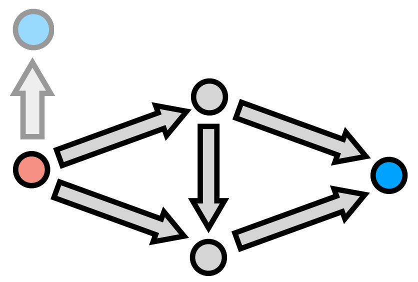
Routing Games



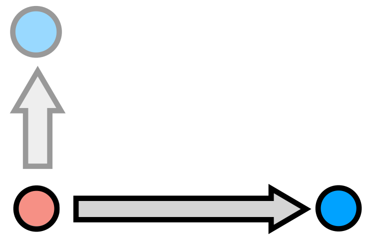
Multiple sources/ sinks



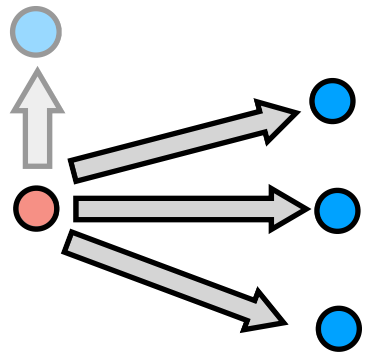
Variable Demand



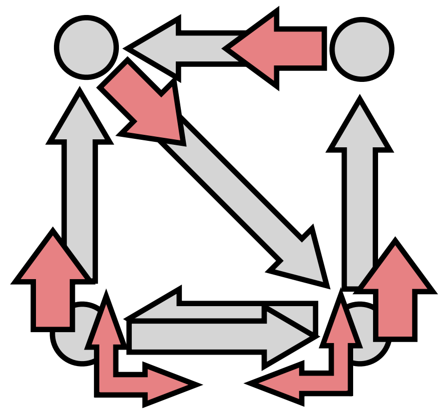
Supply & Demand



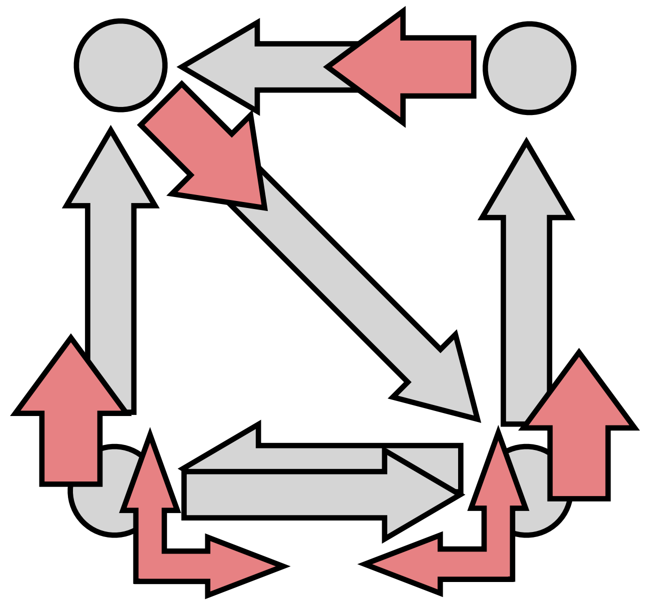
Cournot Market

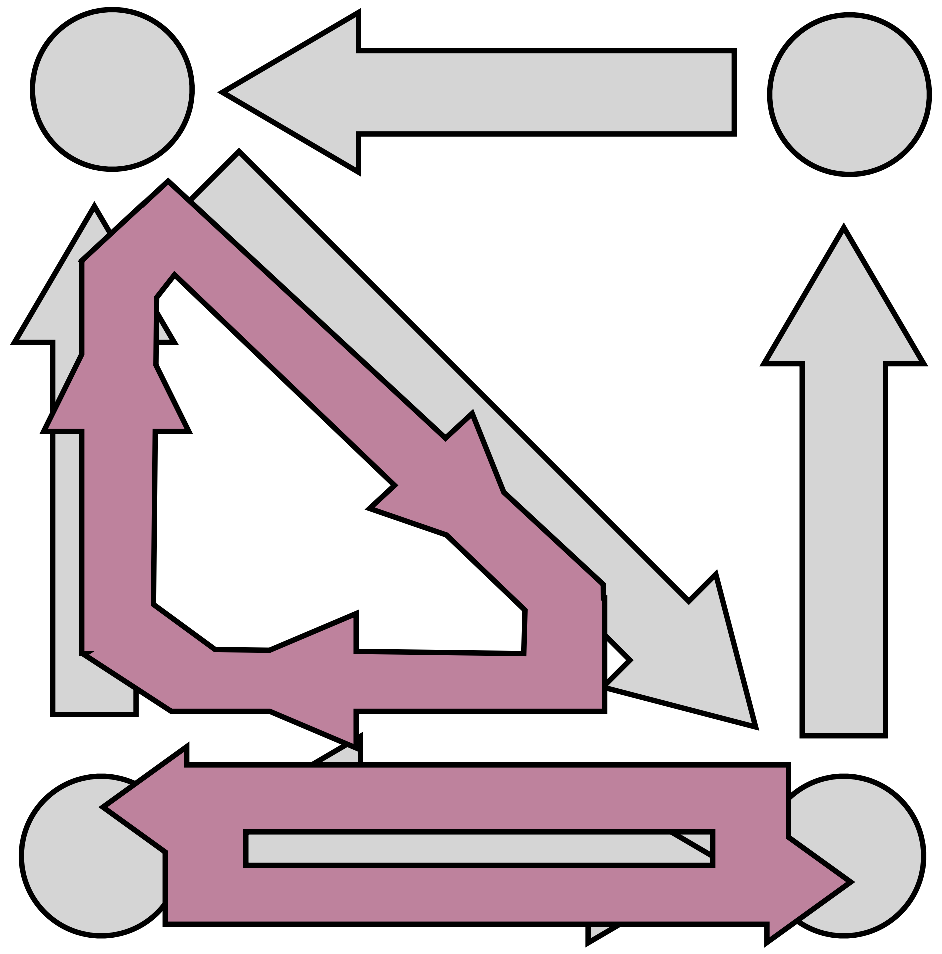


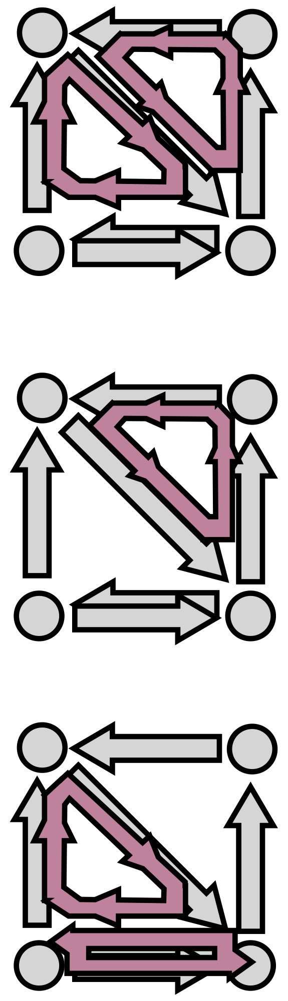
MDP Congestion Game



Markov Decision Process Congestion Game







$$\min_{x} \quad F(x)$$

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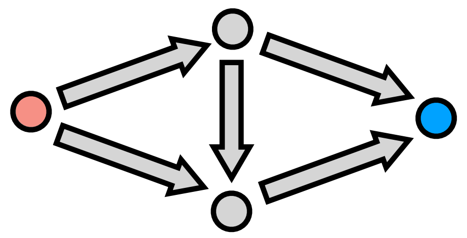
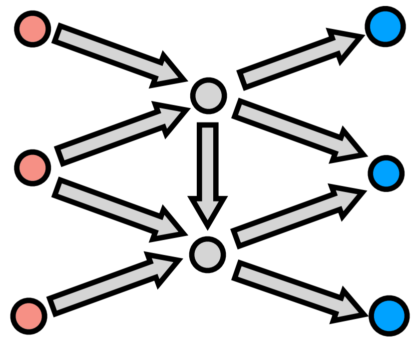
$$EWx = 0 \;\; v$$

Potential Games

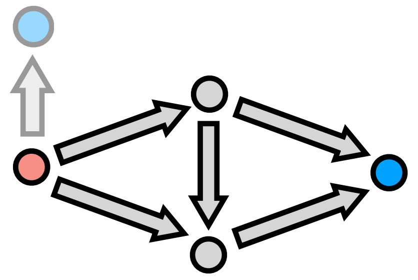
Routing Games



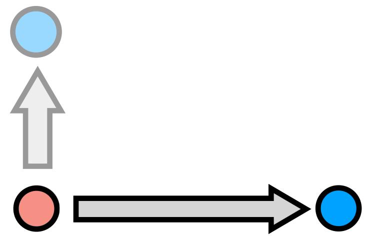
Multiple sources/ sinks



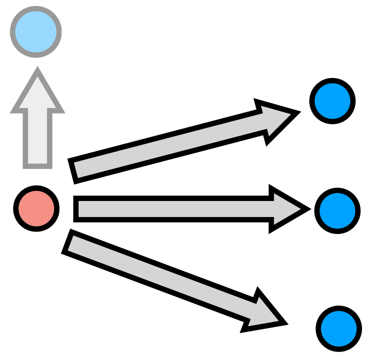
Variable Demand



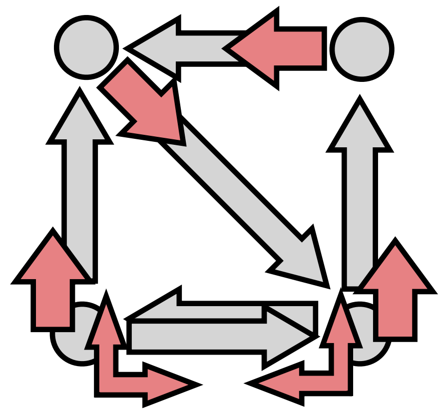
Supply & Demand



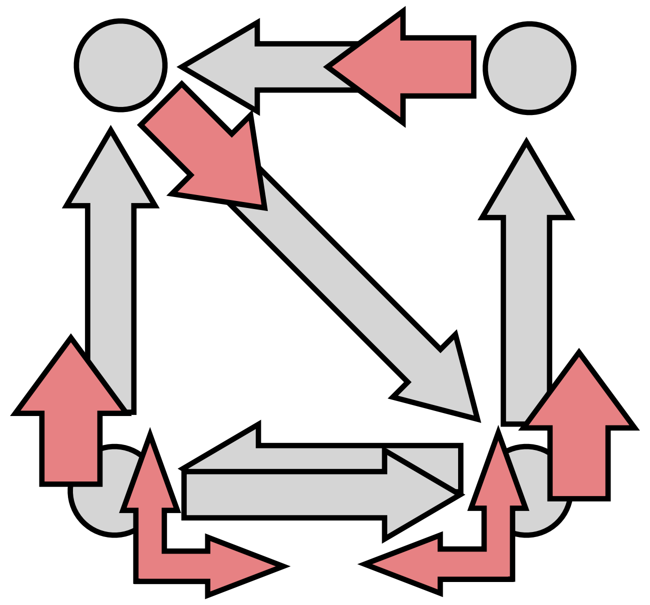
Cournot Market

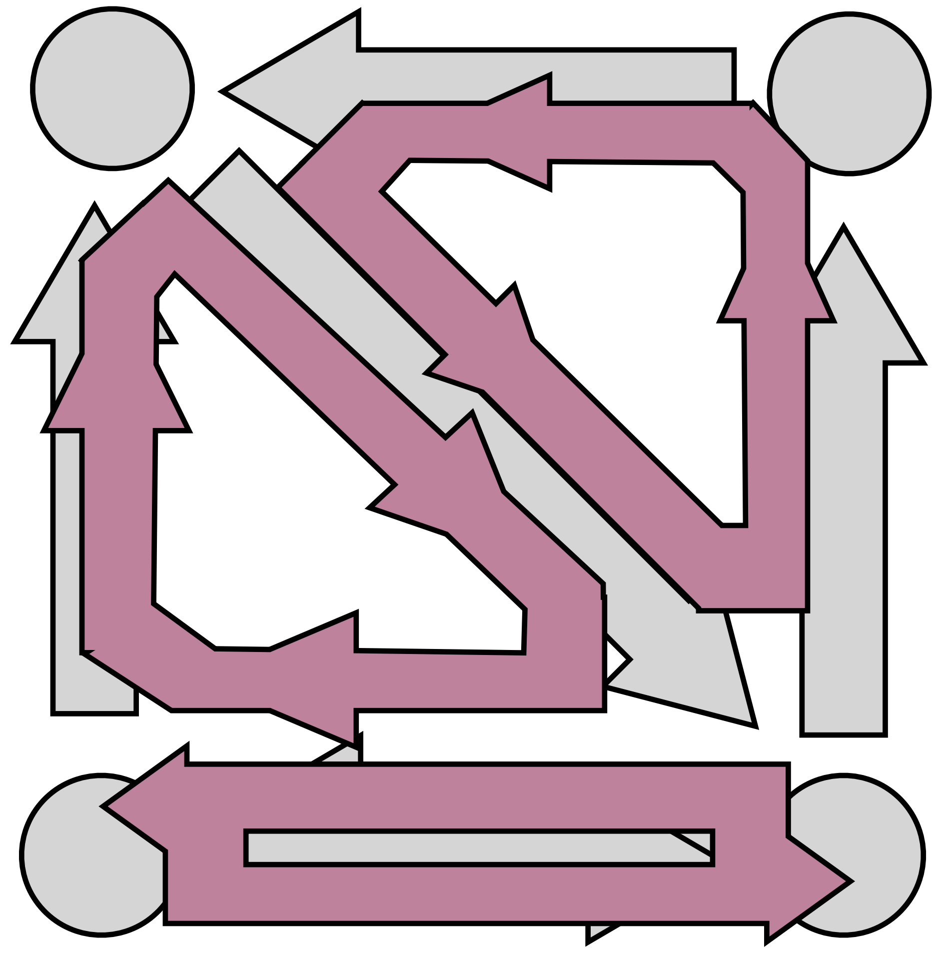


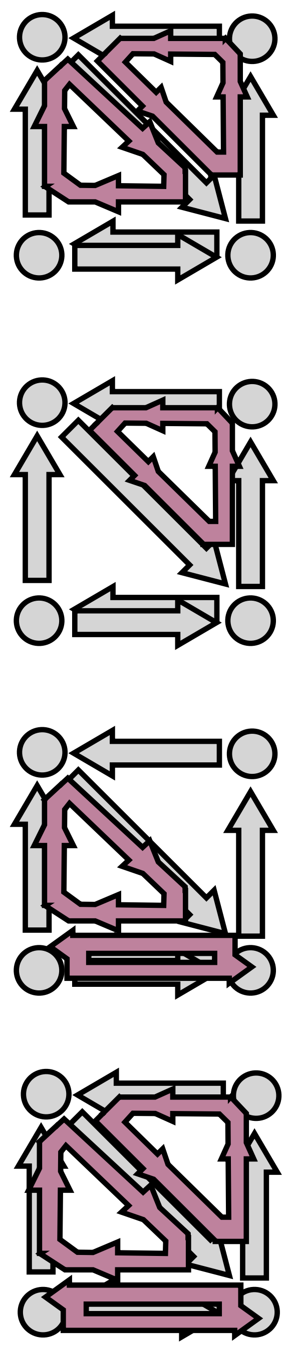
MDP Congestion Game



Markov Decision Process Congestion Game







$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \mathbf{1}^T x = m \quad (\lambda) \qquad x \geq 0 \quad (\mu)$$

$$EWx = 0 \quad (v)$$

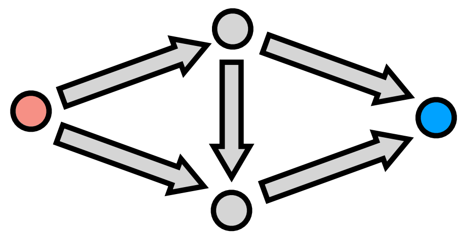
λ : average reward

μ : action inefficiency

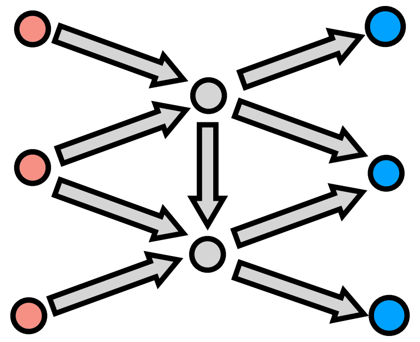
v : value function

Potential Games

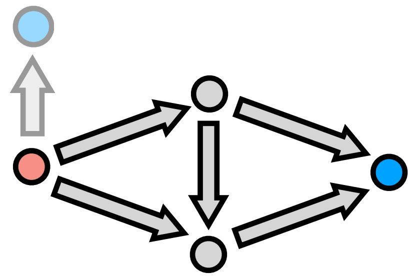
Routing Games



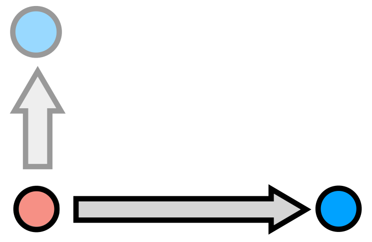
Multiple sources/ sinks



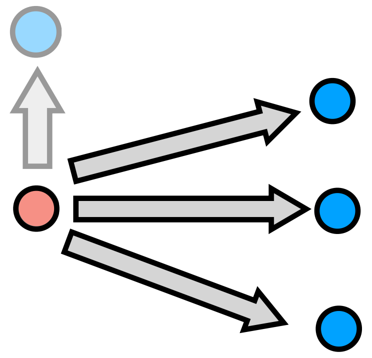
Variable Demand



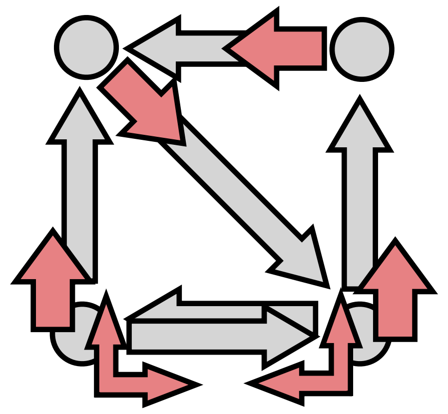
Supply & Demand



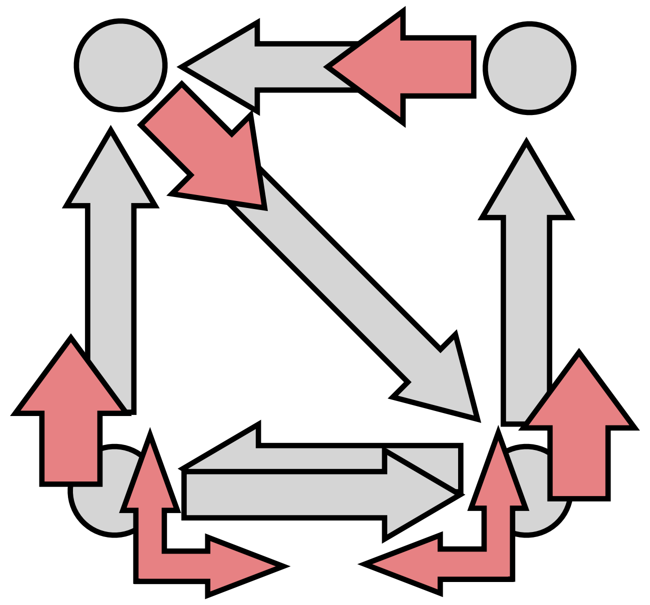
Cournot Market

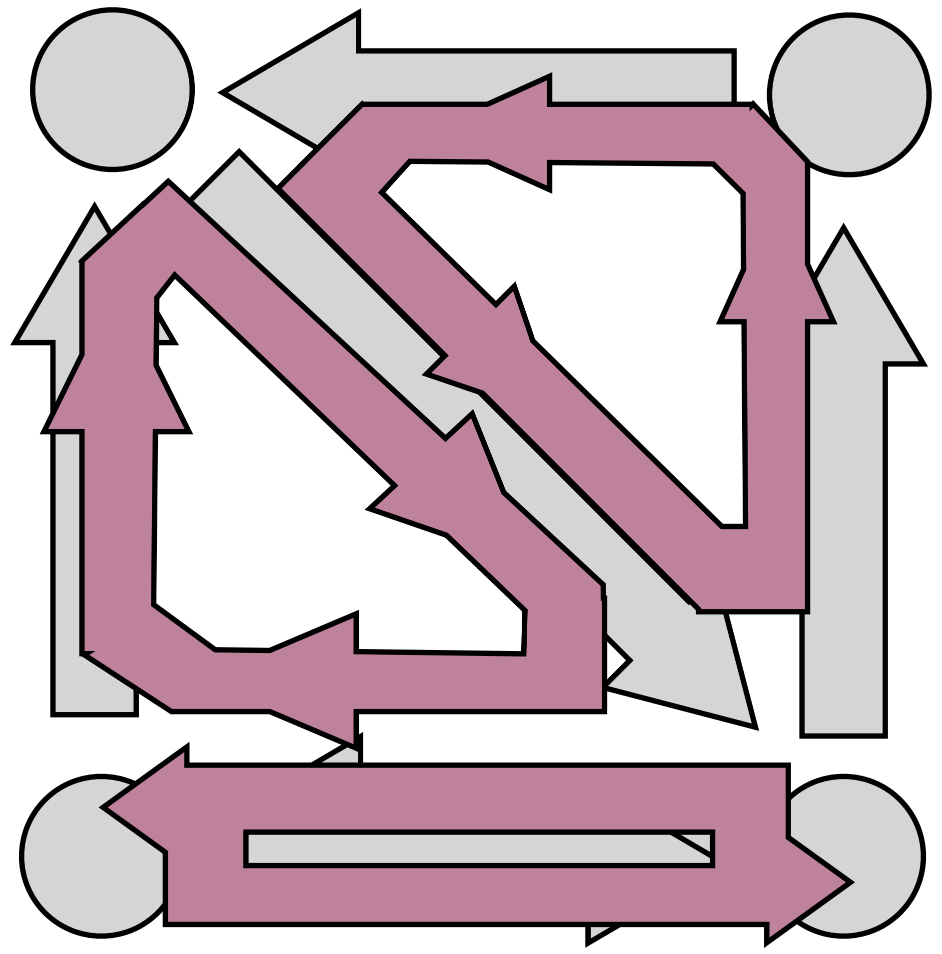


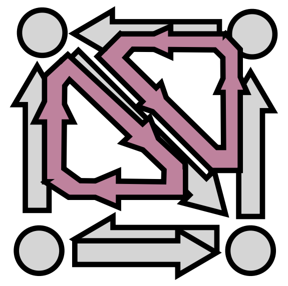
MDP Congestion Game

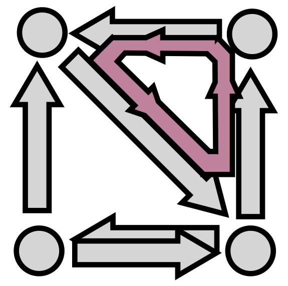


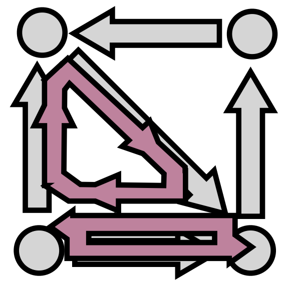
Markov Decision Process Congestion Game

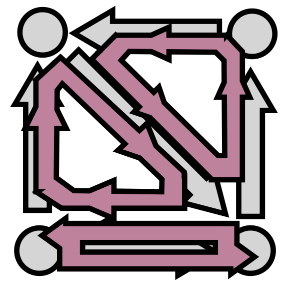












APPLICATIONS

- Ride-sharing drivers planning routes
- Cars circling for street parking
- Air-traffic routing

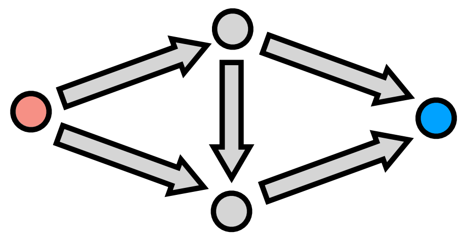
PAPERS

- Markov decision process routing games **[Calderone, Sastry, 2017]**
- Infinite horizon average cost Markov decision process routing games **[Calderone, Sastry, 2017]**
- Adaptive constraint satisfaction for Markov decision process congestion games: Applications to transportation networks **[Li, Calderone, Ratliff, et al. 2021]**
- Variable demand and multi-commodity flow in Markovian network equilibrium **[Yu, Calderone, Ratliff, et al. 2021]**

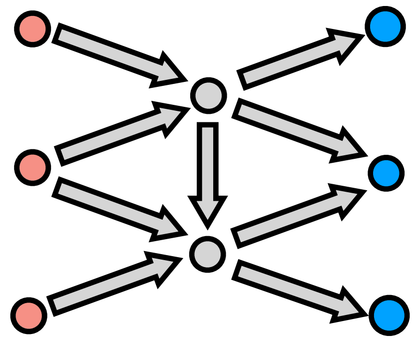
Potential Games

Variable Demand - Non-Homogeneous Preferences

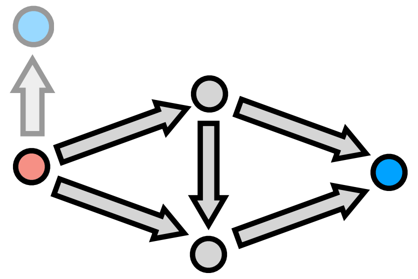
Routing Games



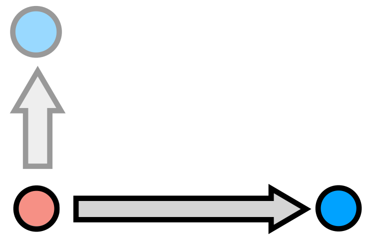
Multiple sources/ sinks



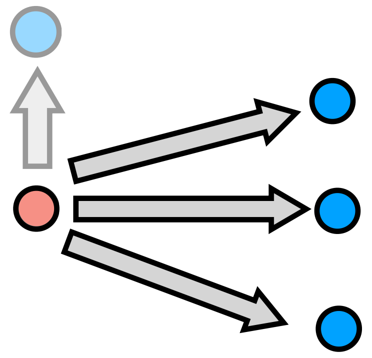
Variable Demand



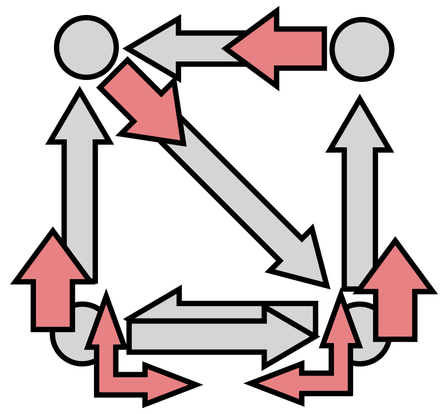
Supply & Demand



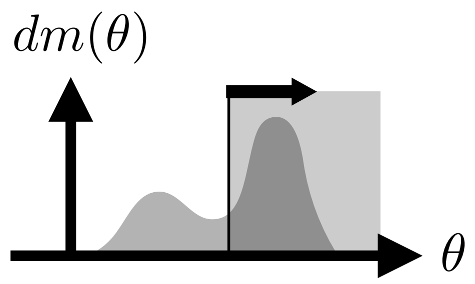
Cournot Market

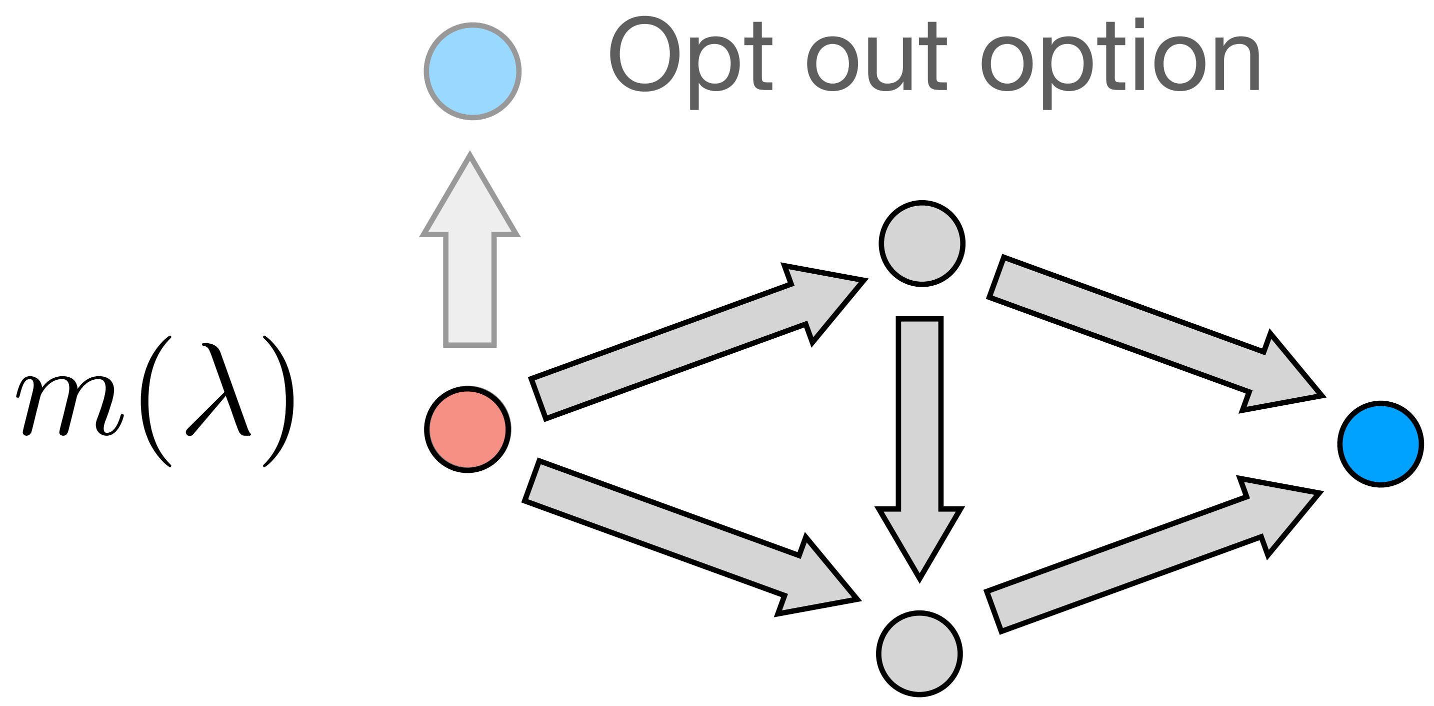


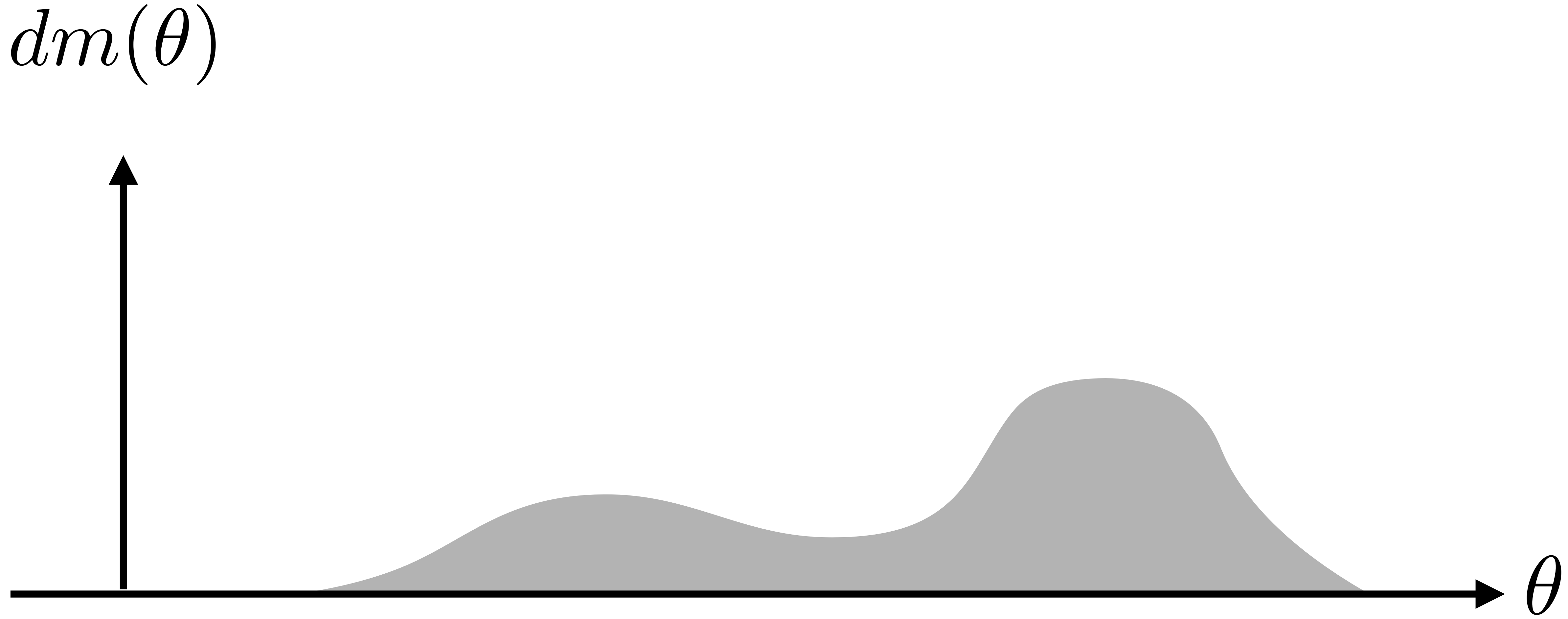
MDP Congestion Game



Non-homo-geneous prefences



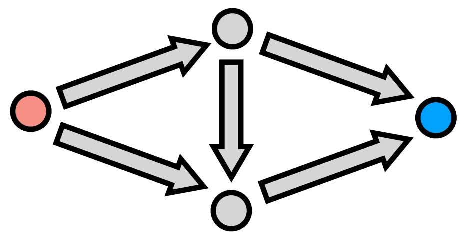




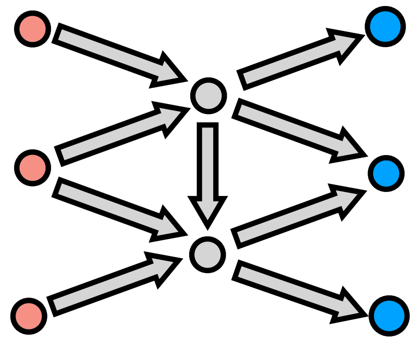
θ : Max cost $dm(\theta)$ will pay

Potential Games

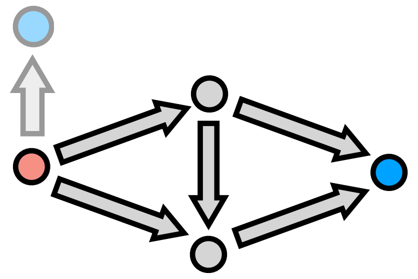
Routing Games



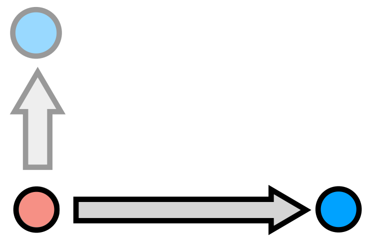
Multiple sources/ sinks



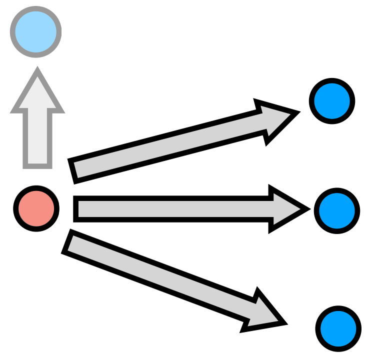
Variable Demand



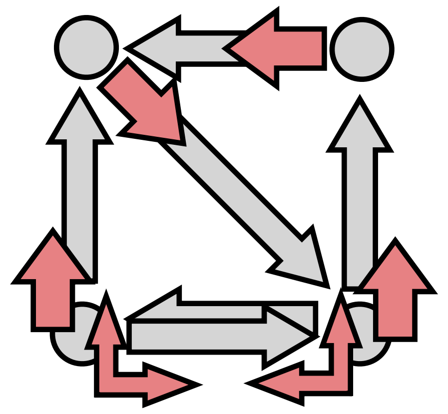
Supply & Demand



Cournot Market

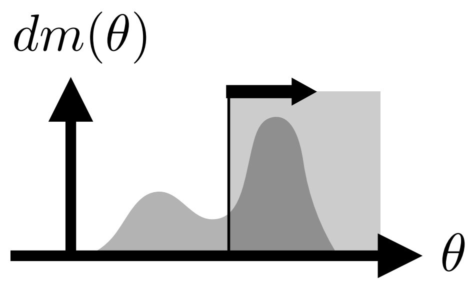


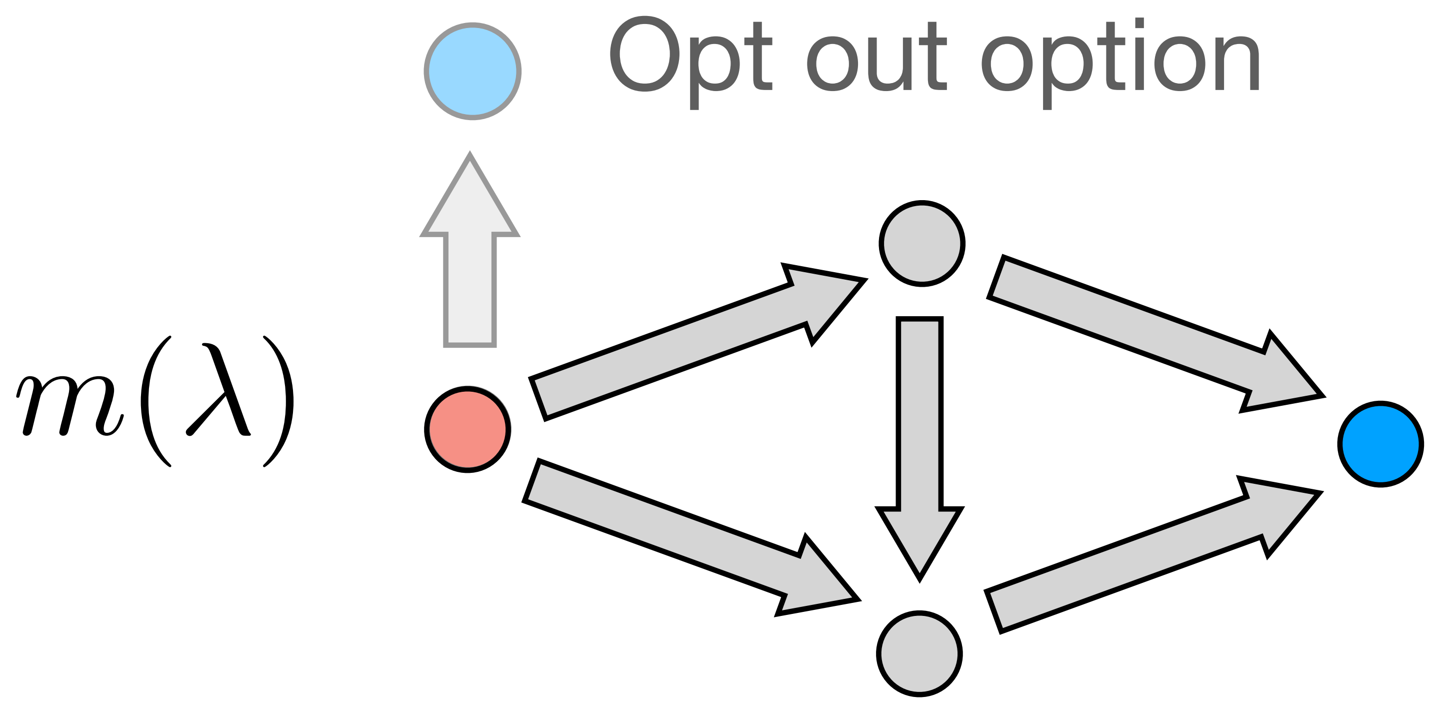
MDP Congestion Game

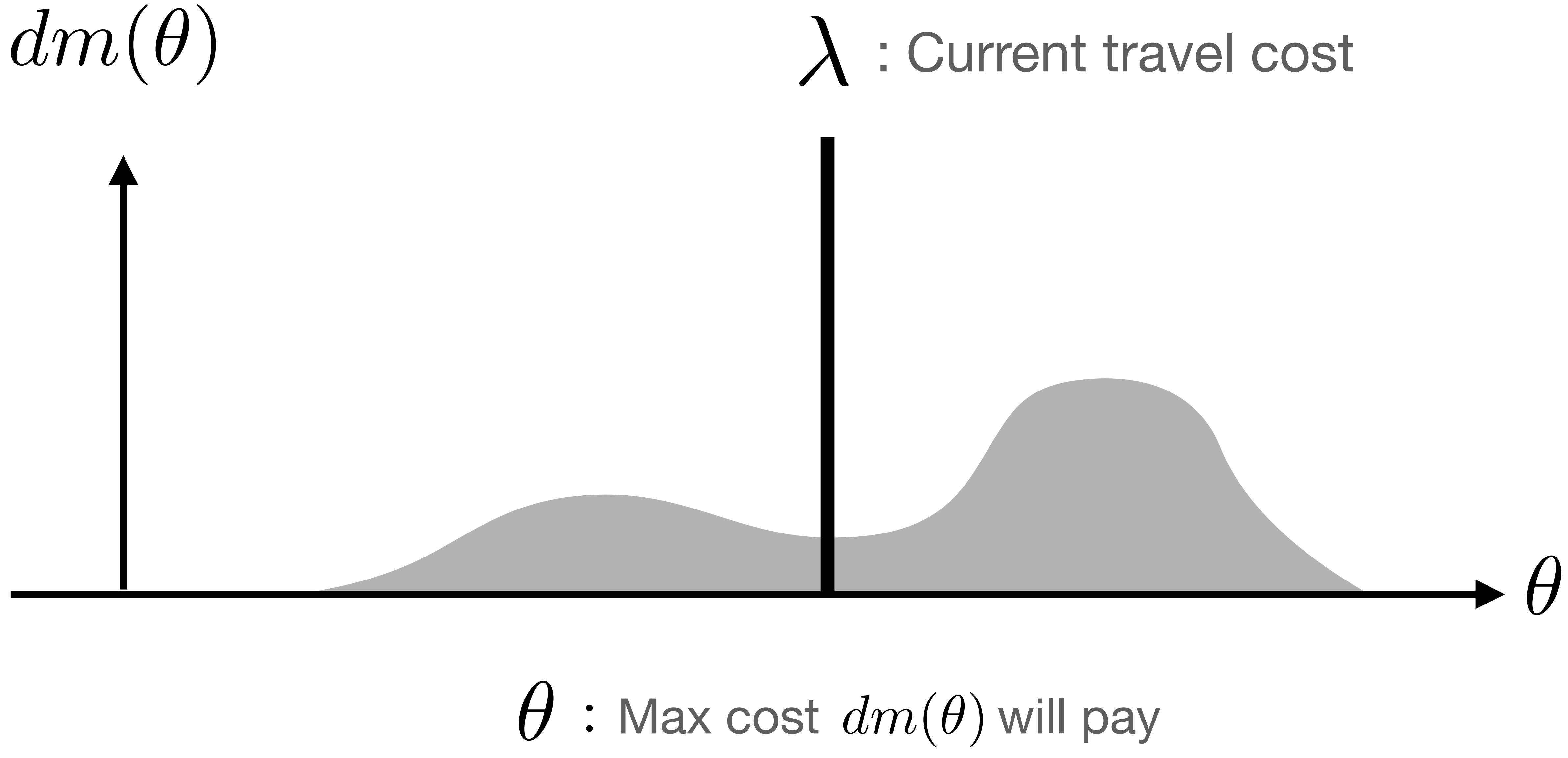


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences

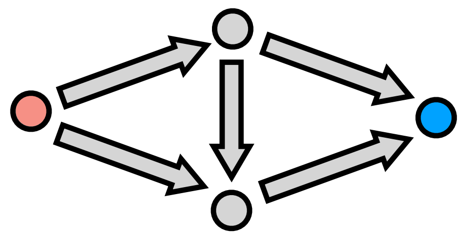




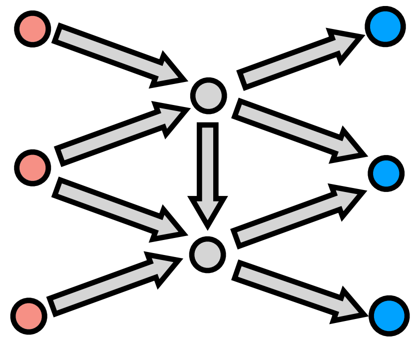


Potential Games

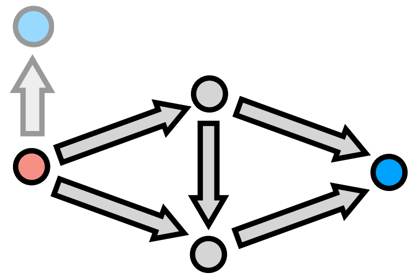
Routing Games



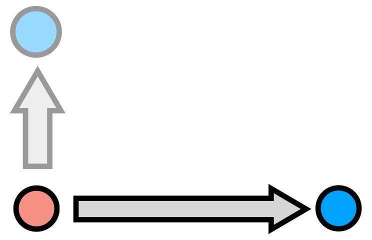
Multiple sources/ sinks



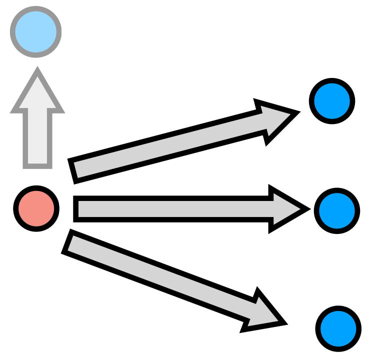
Variable Demand



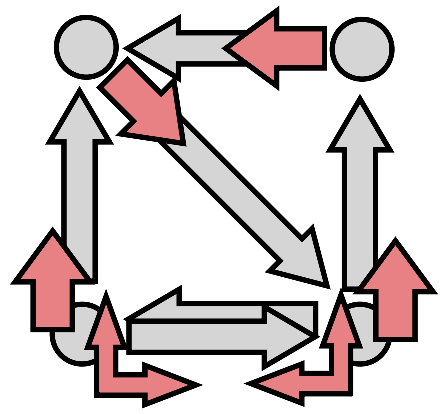
Supply & Demand



Cournot Market

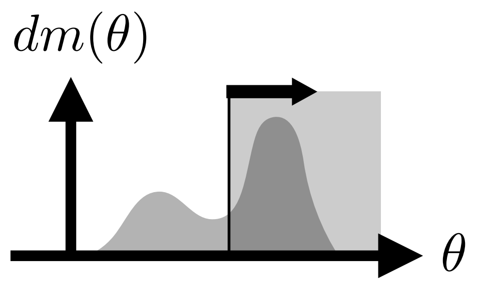


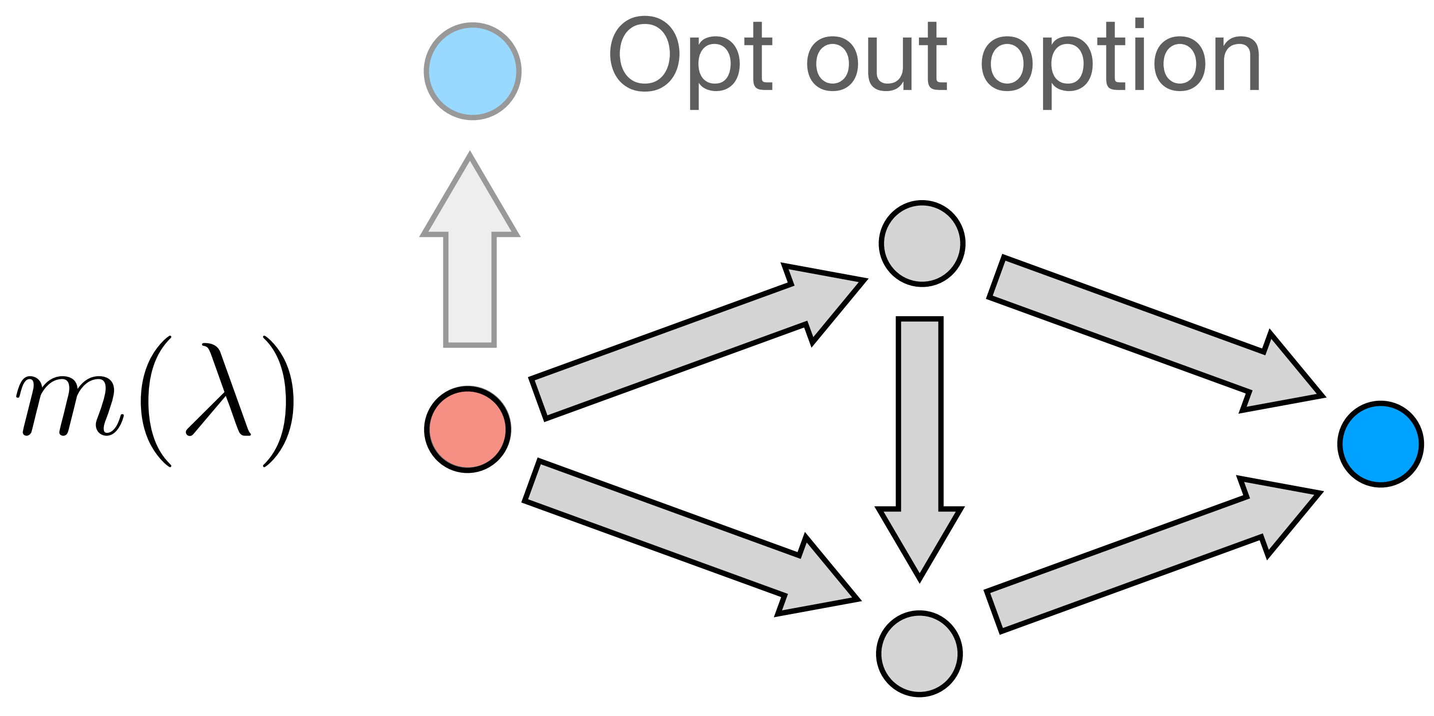
MDP Congestion Game

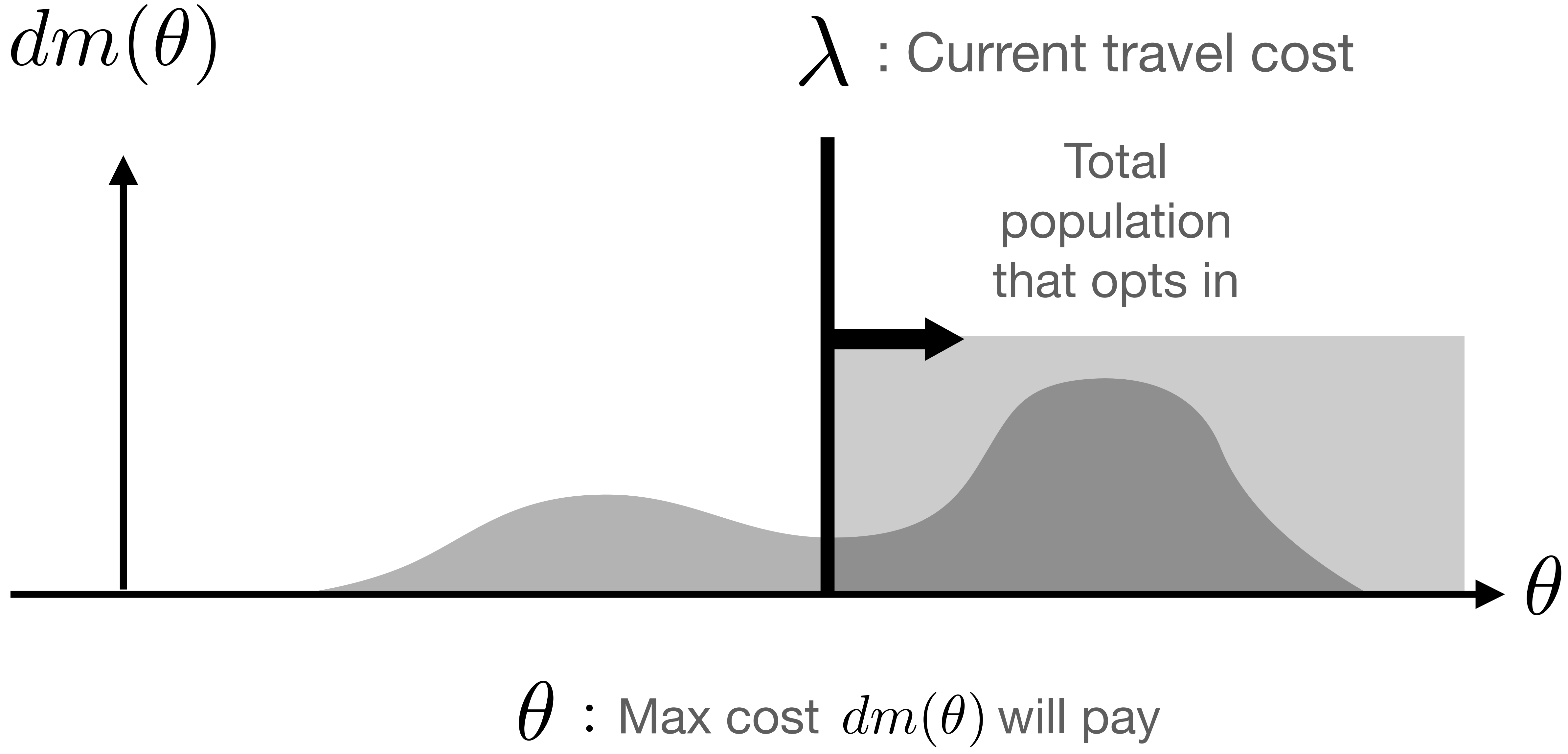


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences







Potential Games

Routing Games

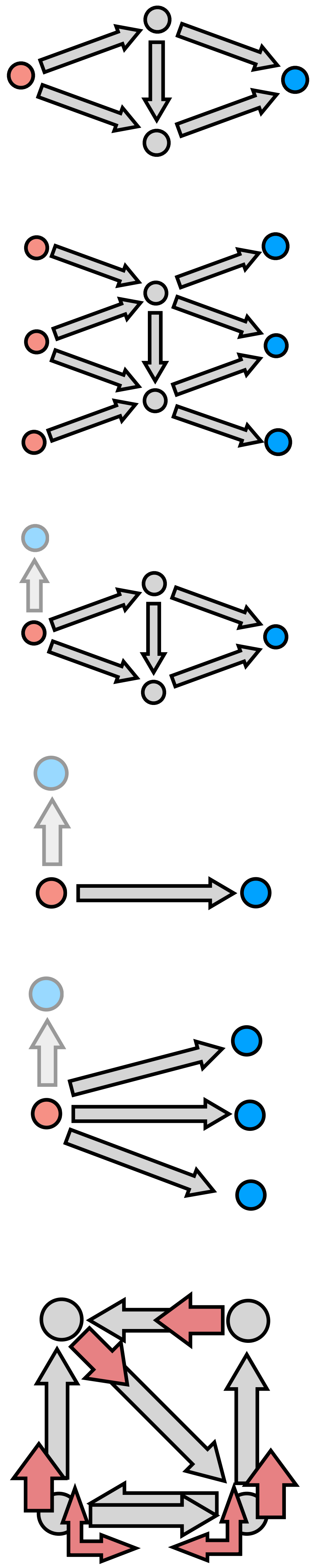
Multiple sources/ sinks

Variable Demand

Supply & Demand

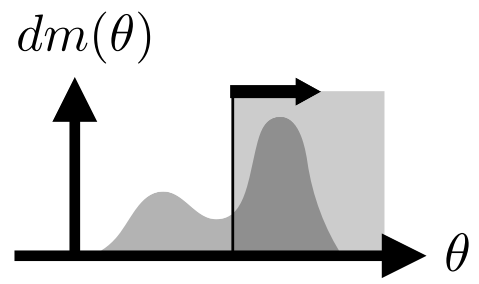
Cournot Market

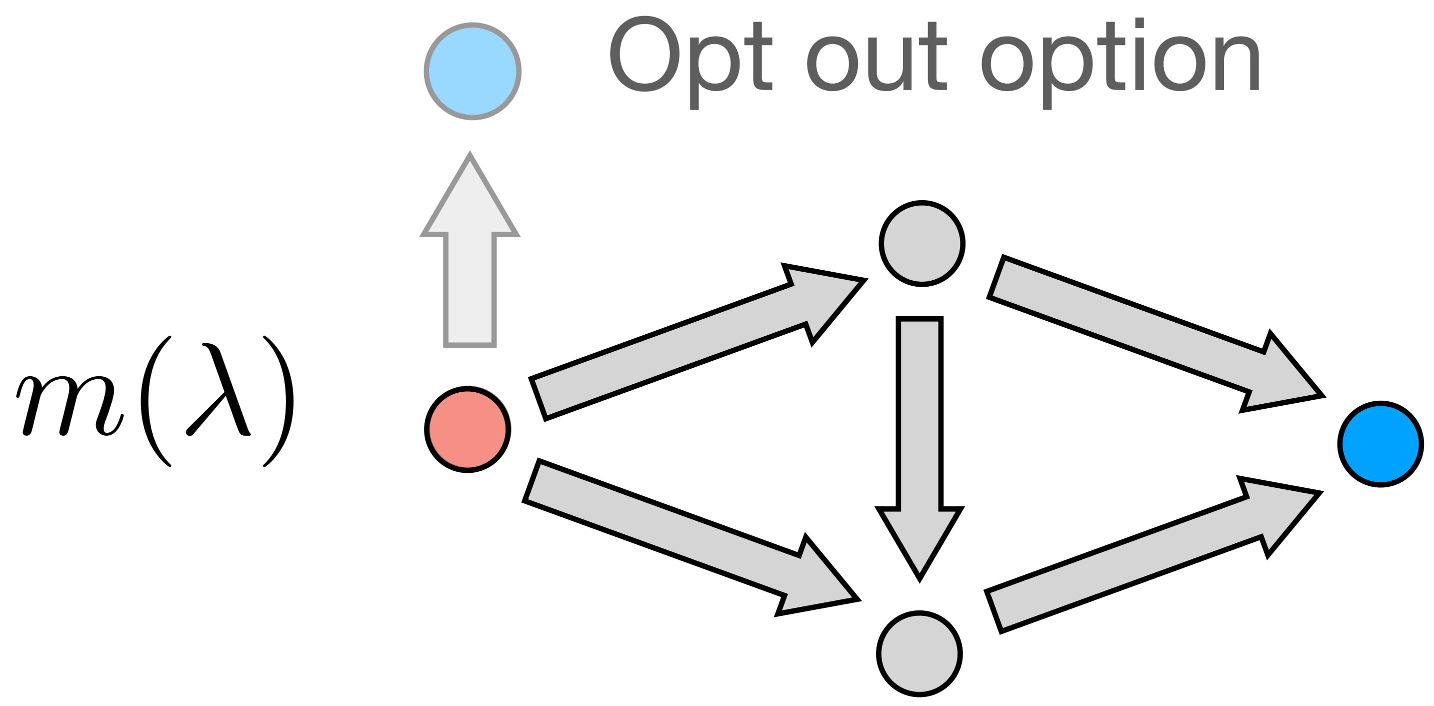
MDP Congestion Game

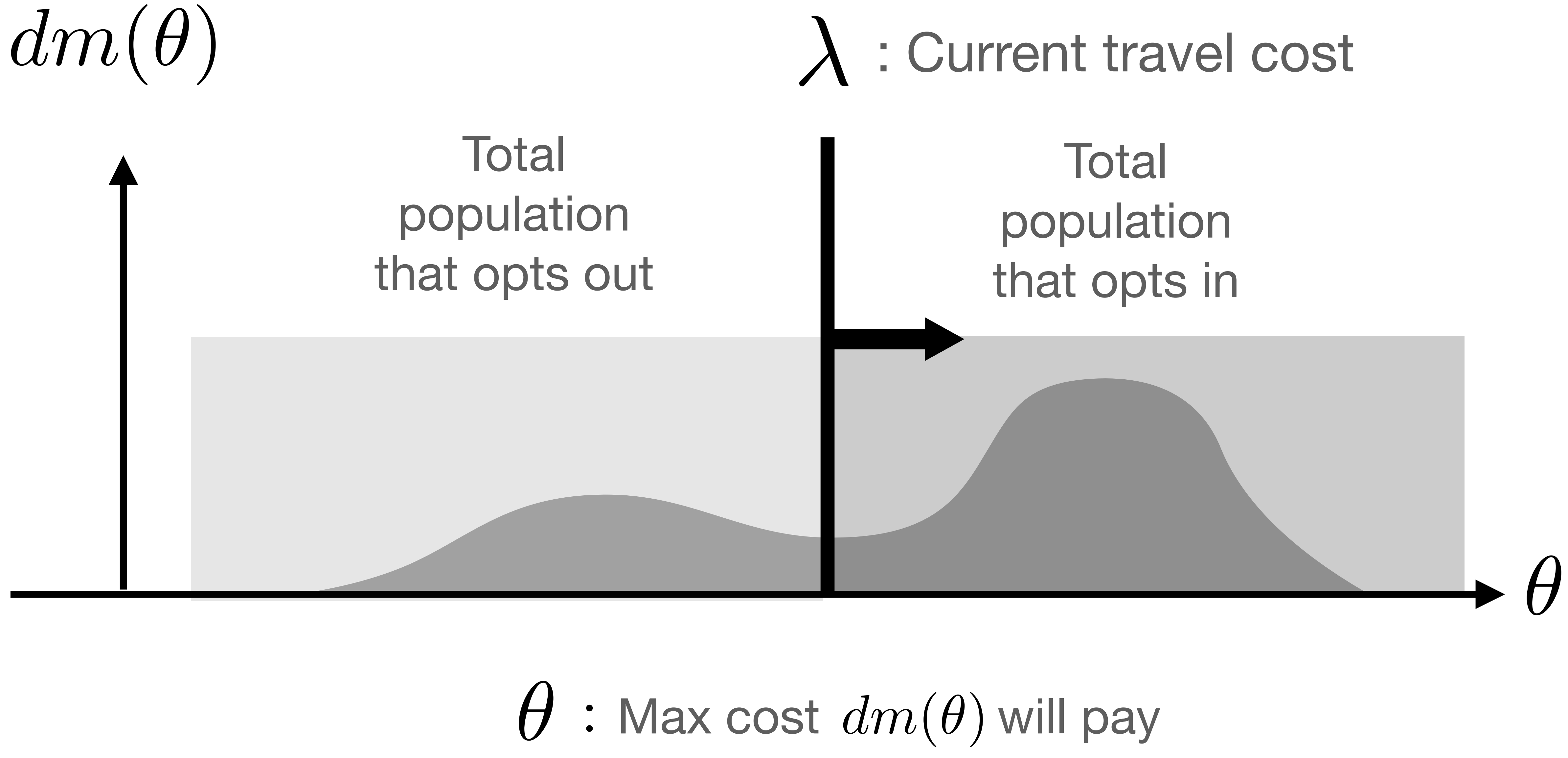


Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences

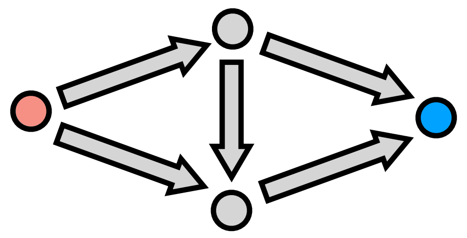






Potential Games

Routing Games



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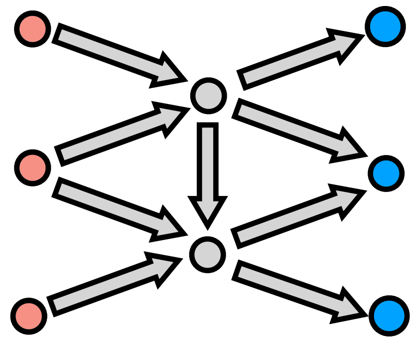
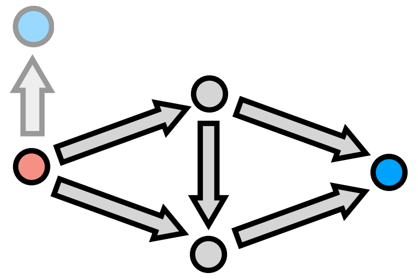
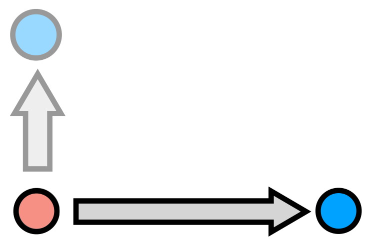


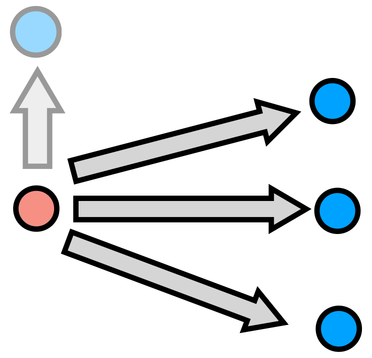
Variable Demand



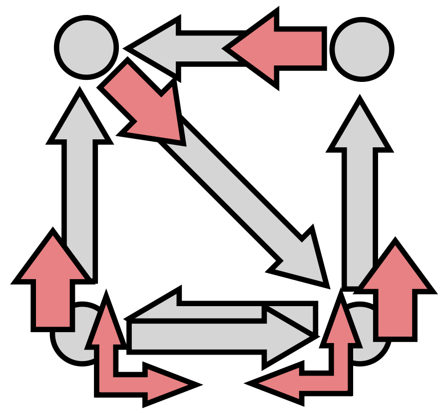
Supply & Demand



Cournot Market

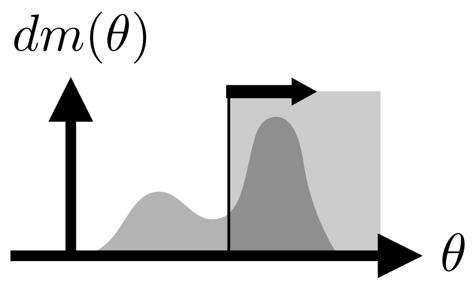


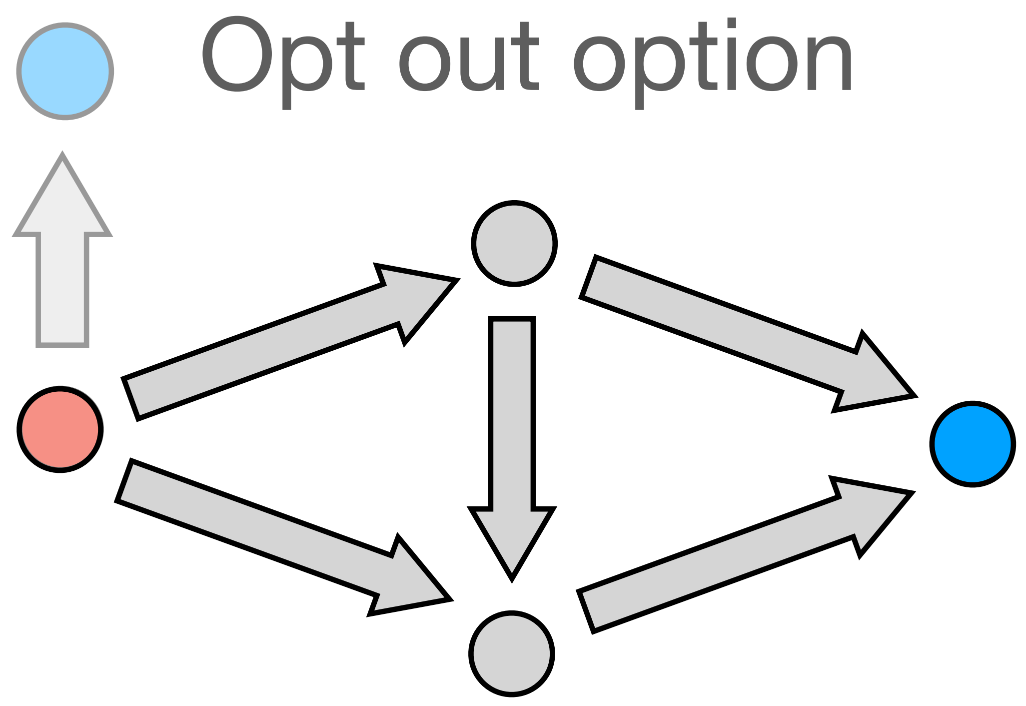
MDP Congestion Game



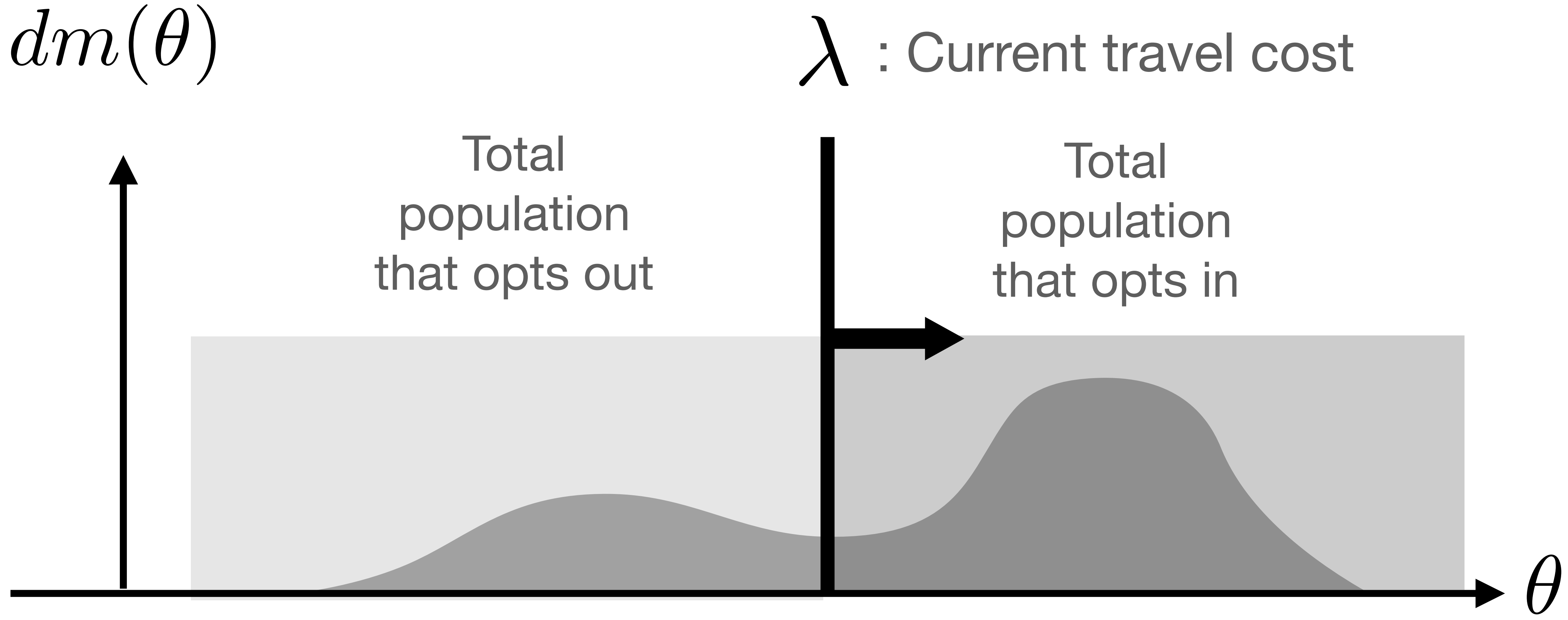
Variable Demand - Non-Homogeneous Preferences

Non-homo-geneous prefences





$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$



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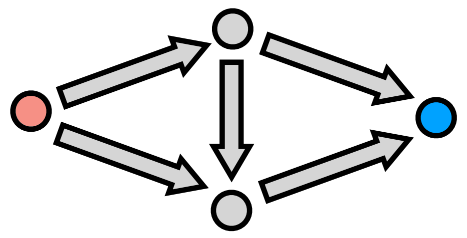
Potential Games

Routing Games



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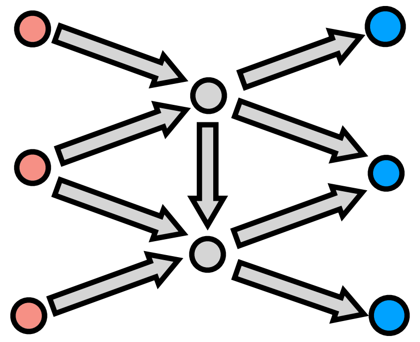
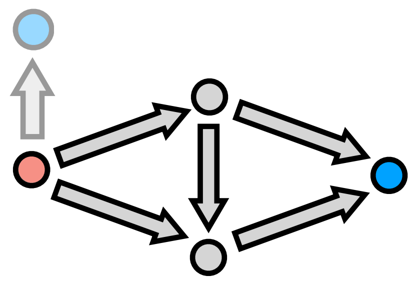
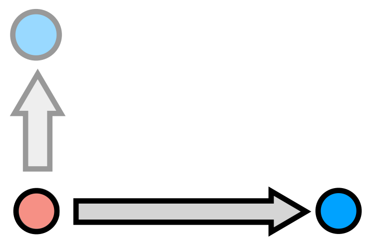


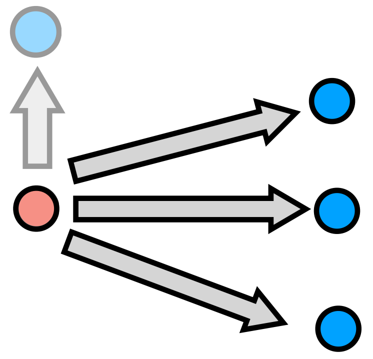
Variable Demand



Supply & Demand



Cournot Market



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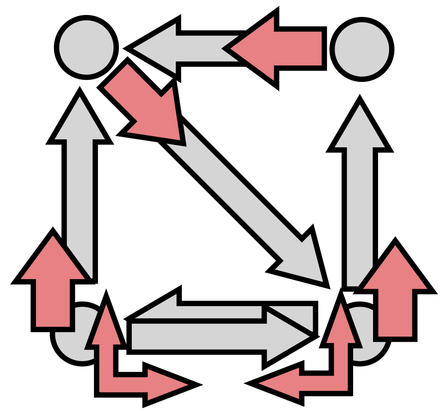
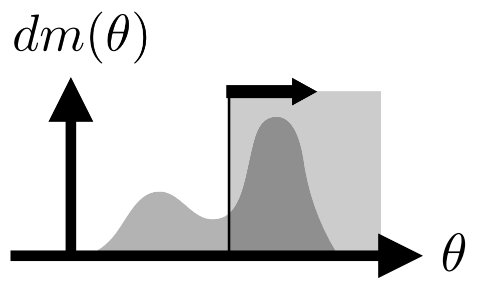


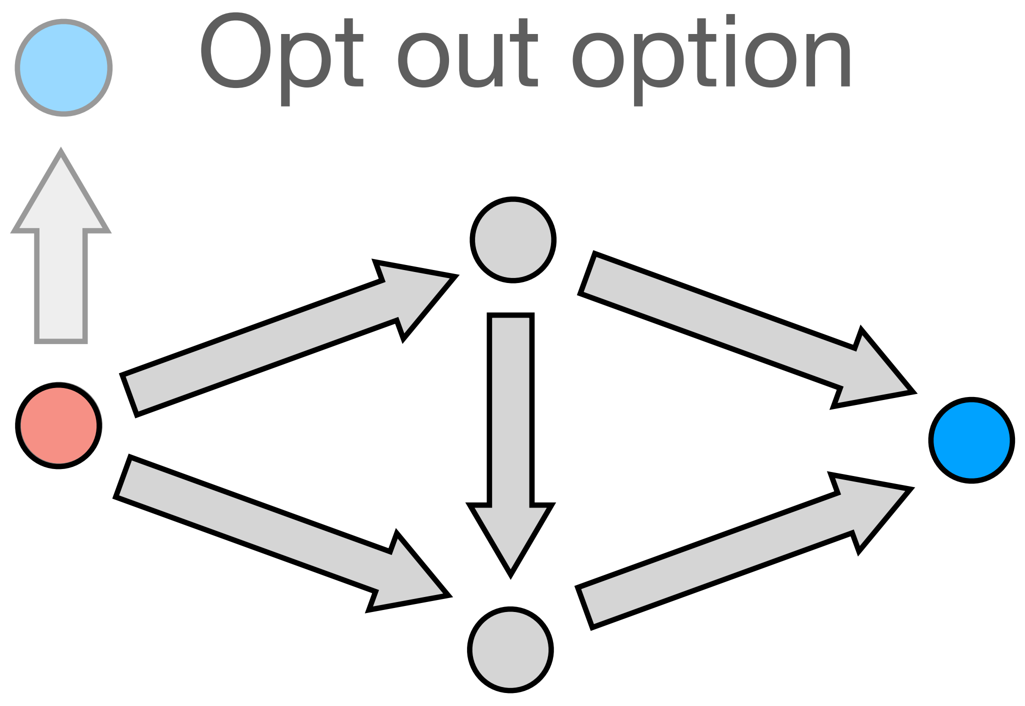
Variable Demand - Non-Homogeneous Preferences

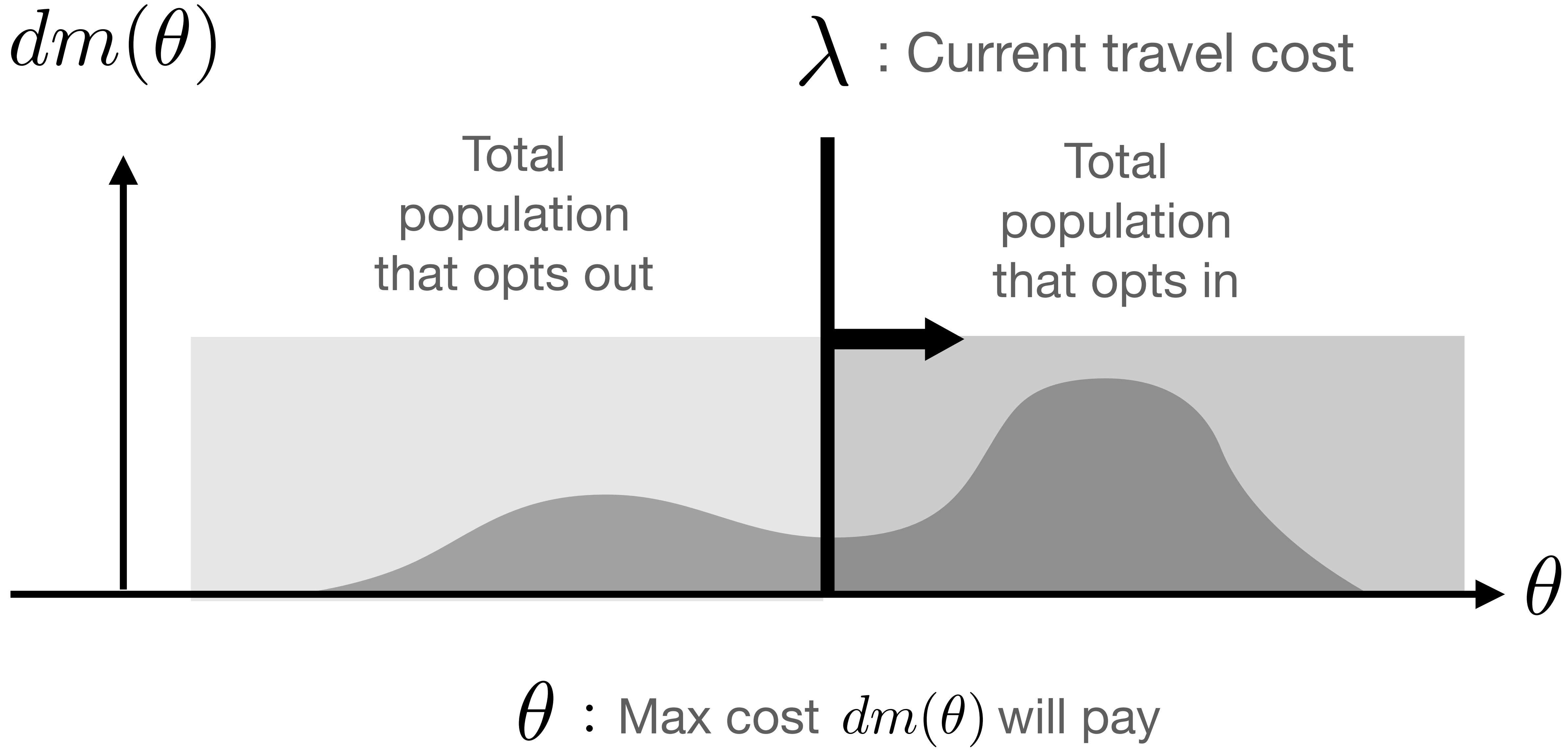
Non-homo-geneous prefences



Multi-Variate Preferences

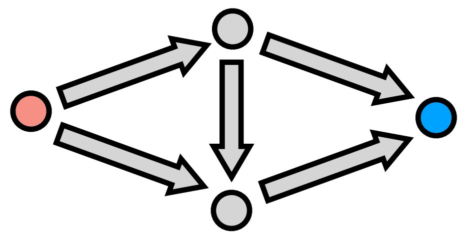
$$m(\lambda) = \int_{\lambda}^{\infty} dm(\theta)$$



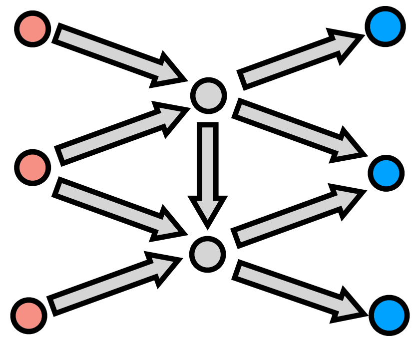


Potential Games

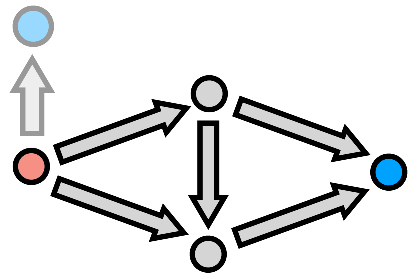
Routing Games



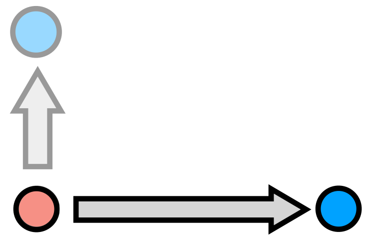
Multiple sources/ sinks



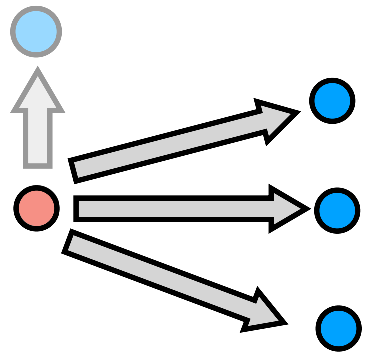
Variable Demand



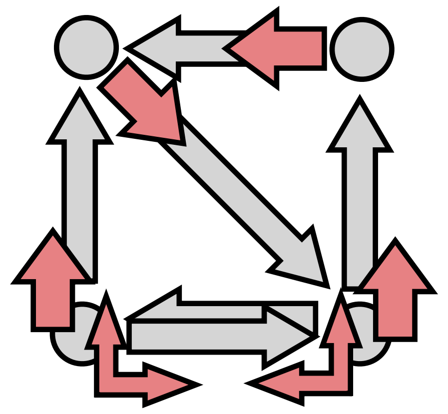
Supply & Demand



Cournot Market

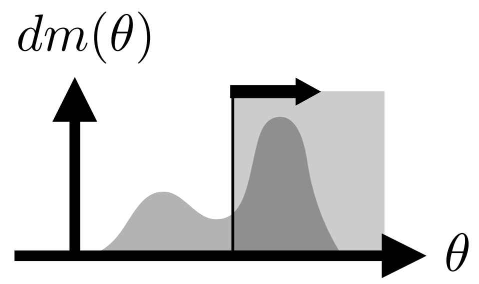


MDP Congestion Game

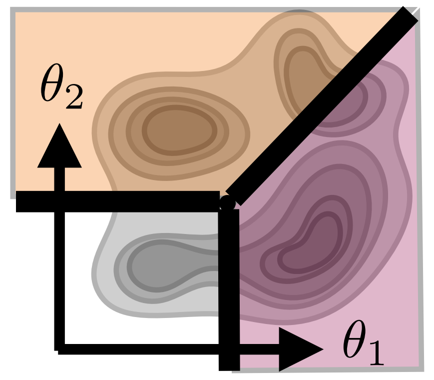


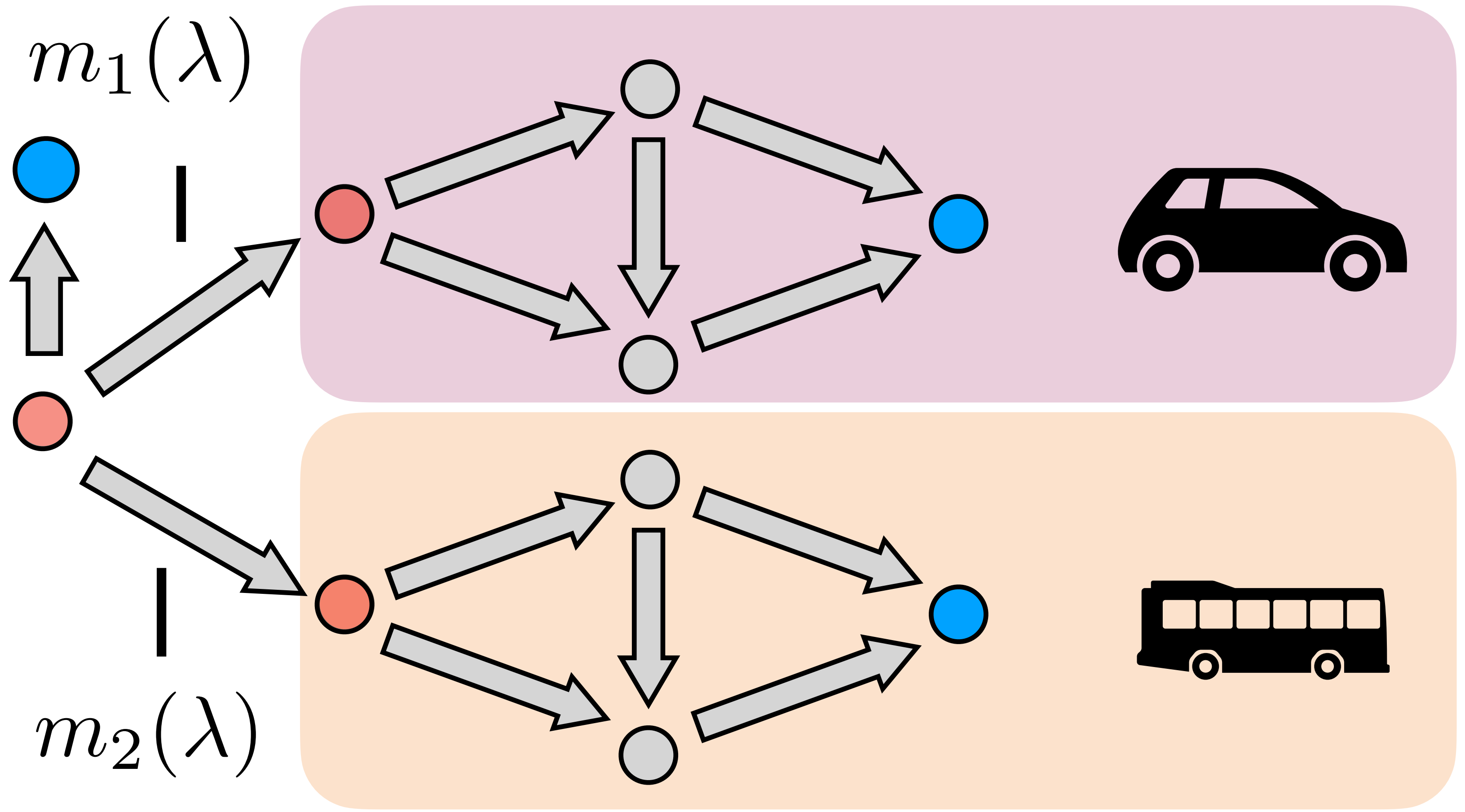
Variable Demand - Multi-Variate Non-Homogeneous Preferences

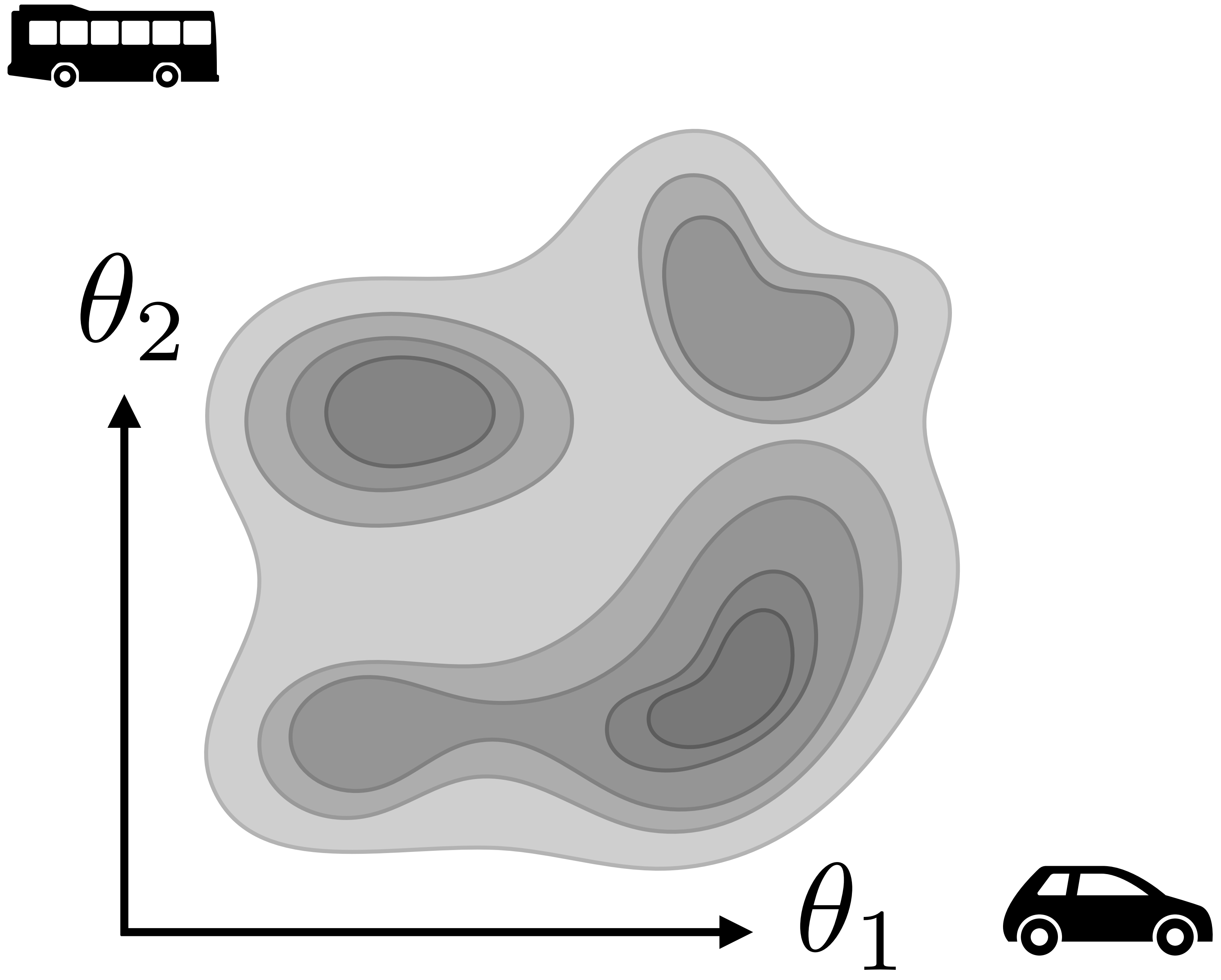
Non-homo-geneous prefences



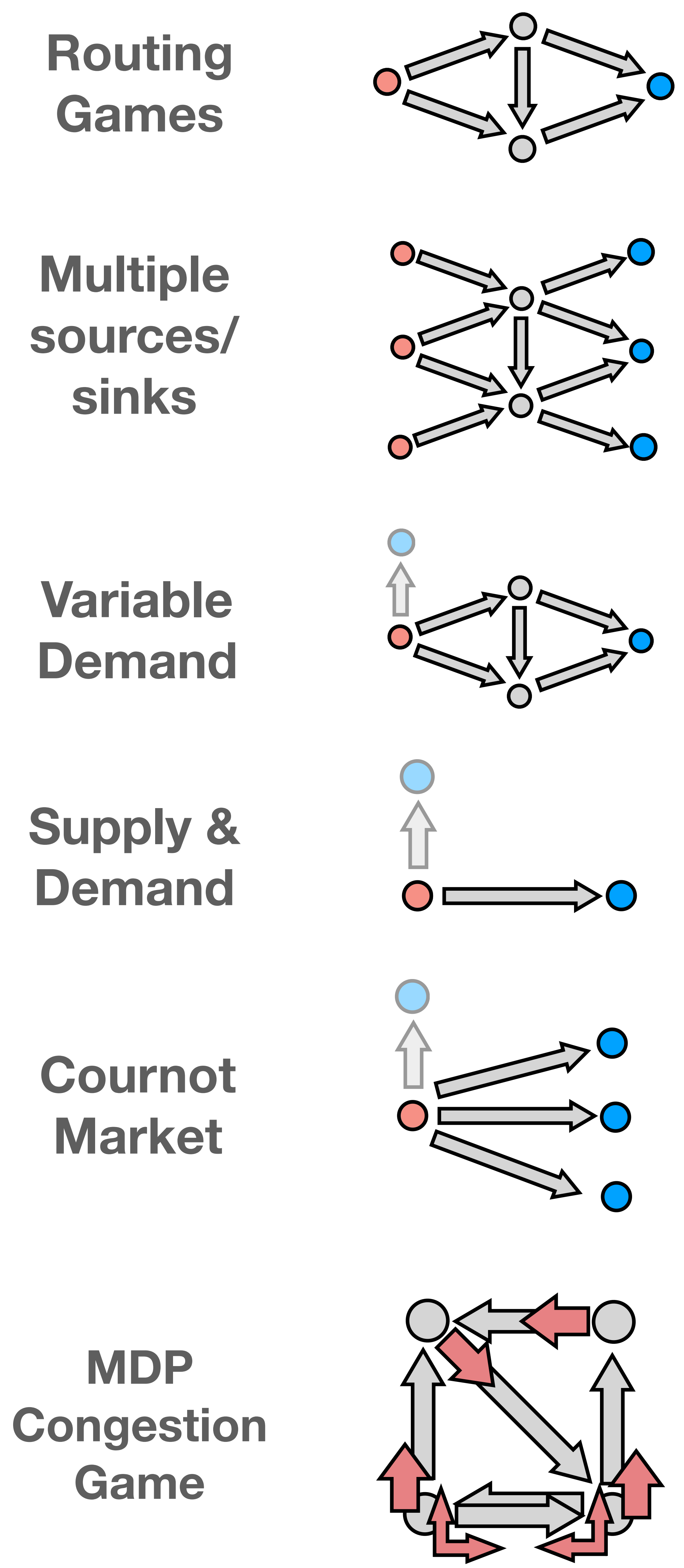
Multi-Variate Preferences



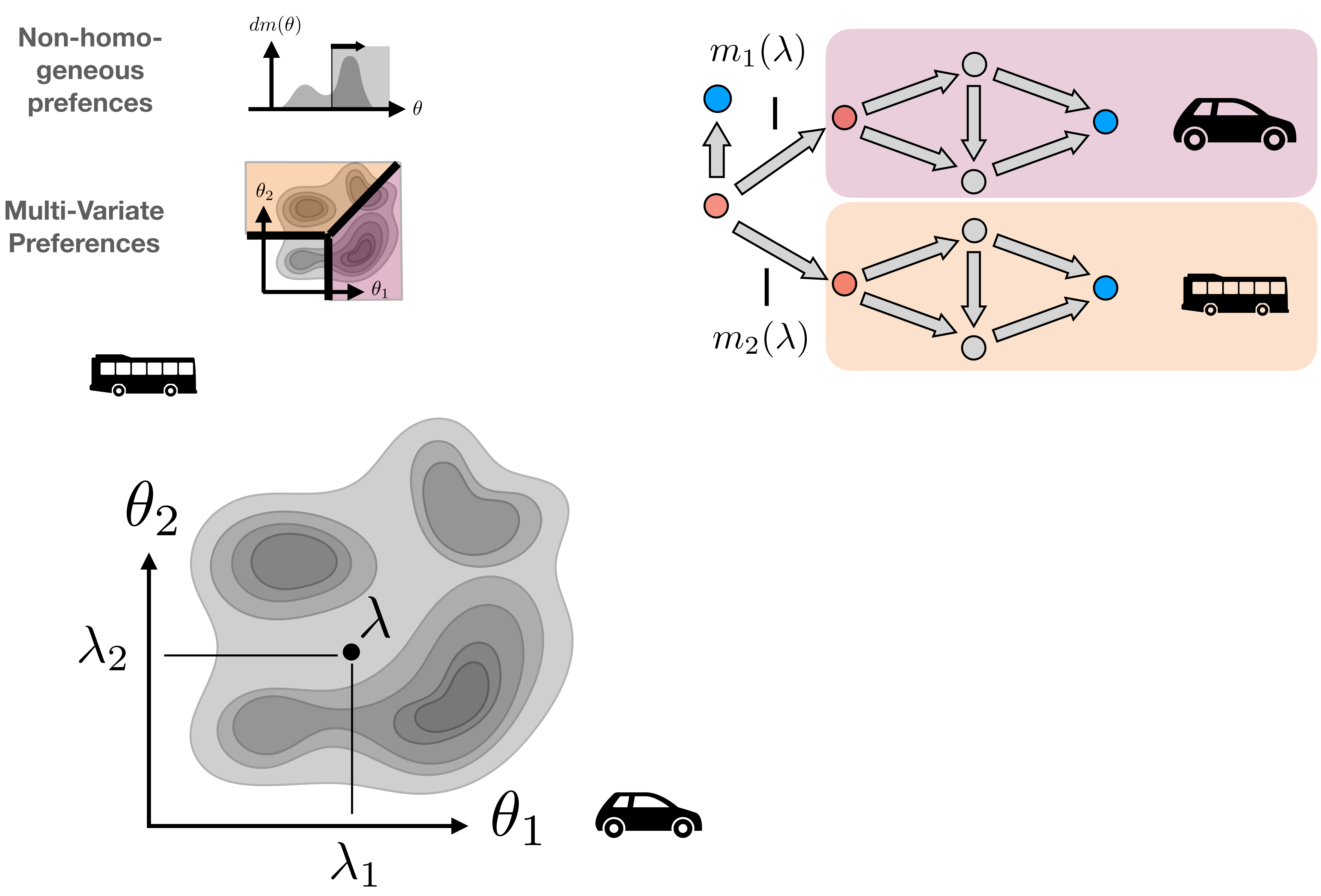




Potential Games



Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games

Routing Games

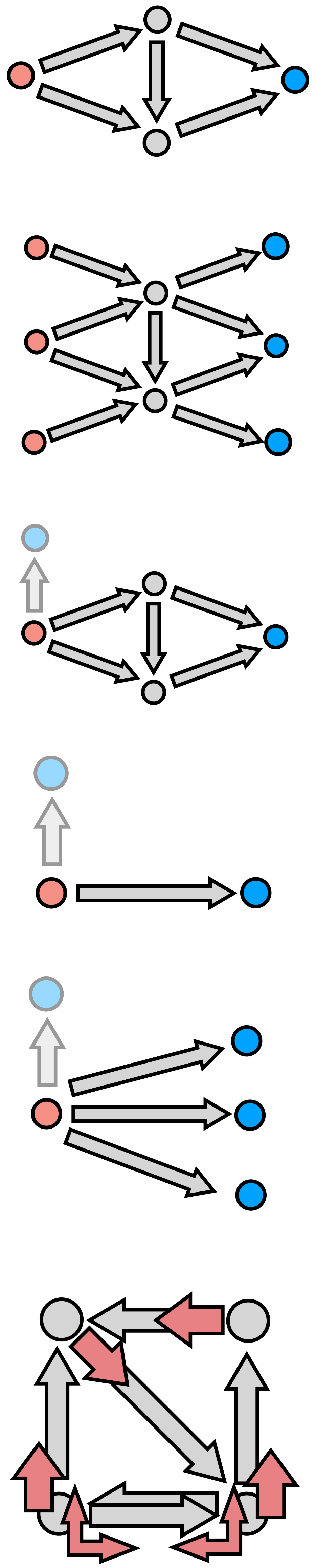
Multiple sources/ sinks

Variable Demand

Supply & Demand

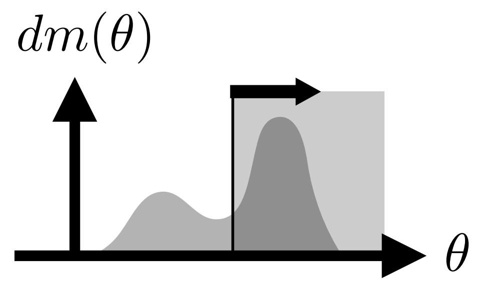
Cournot Market

MDP Congestion Game

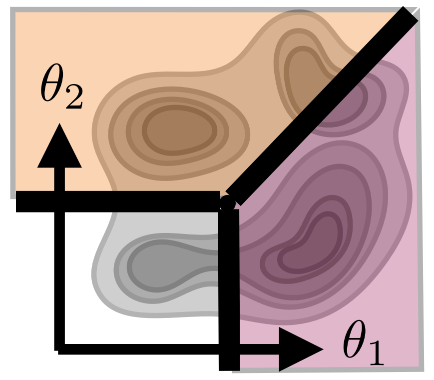


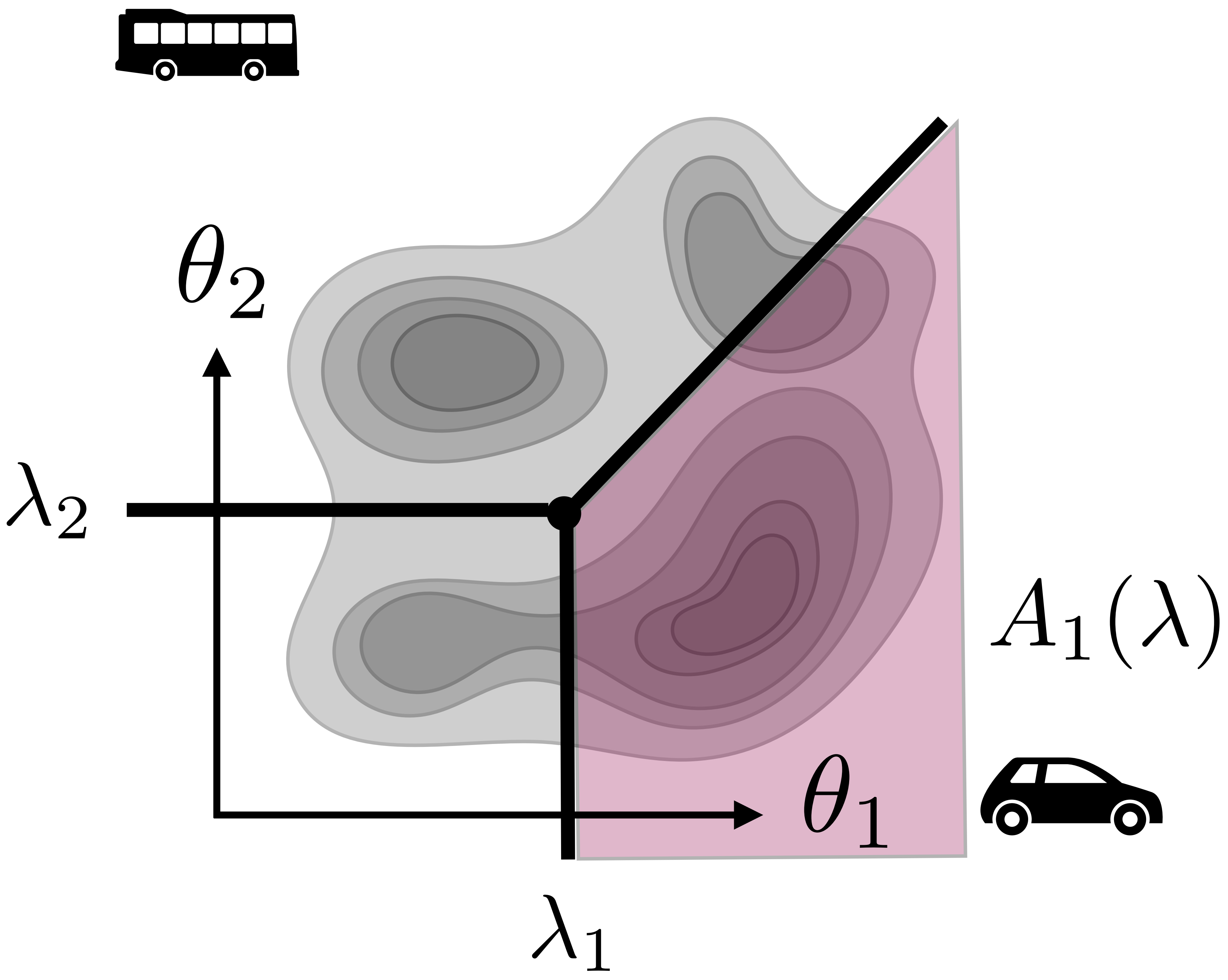
Variable Demand - Multi-Variate Non-Homogeneous Preferences

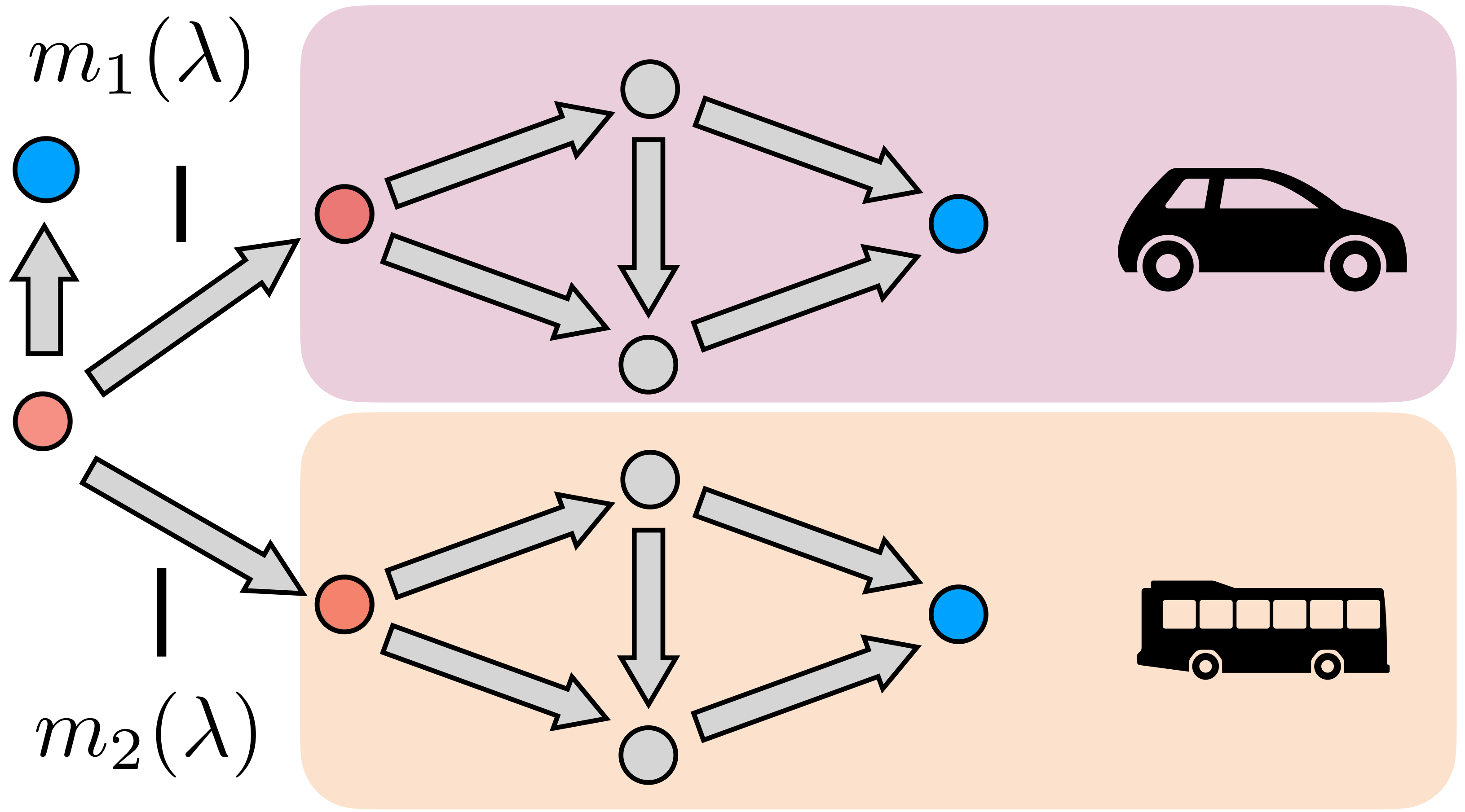
Non-homo-geneous prefences



Multi-Variate Preferences







$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

Potential Games

Routing Games

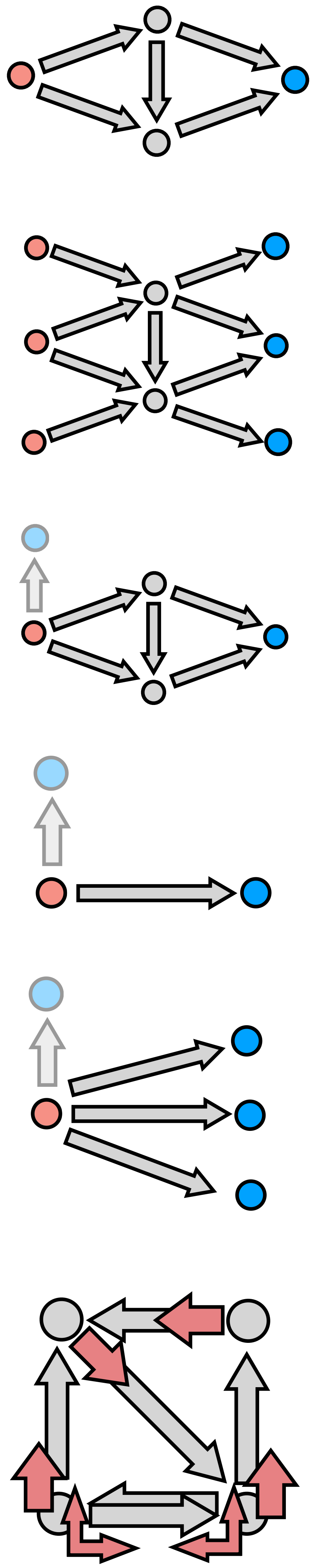
Multiple sources/ sinks

Variable Demand

Supply & Demand

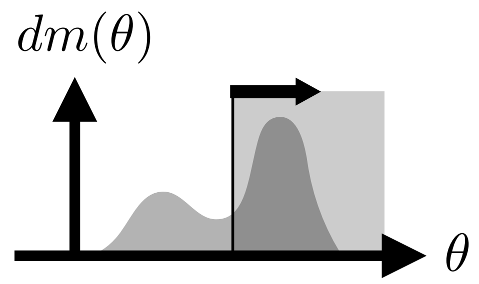
Cournot Market

MDP Congestion Game



Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous prefences



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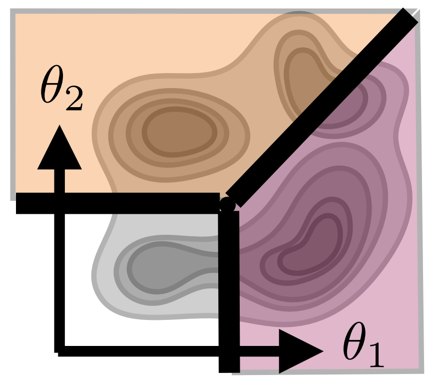
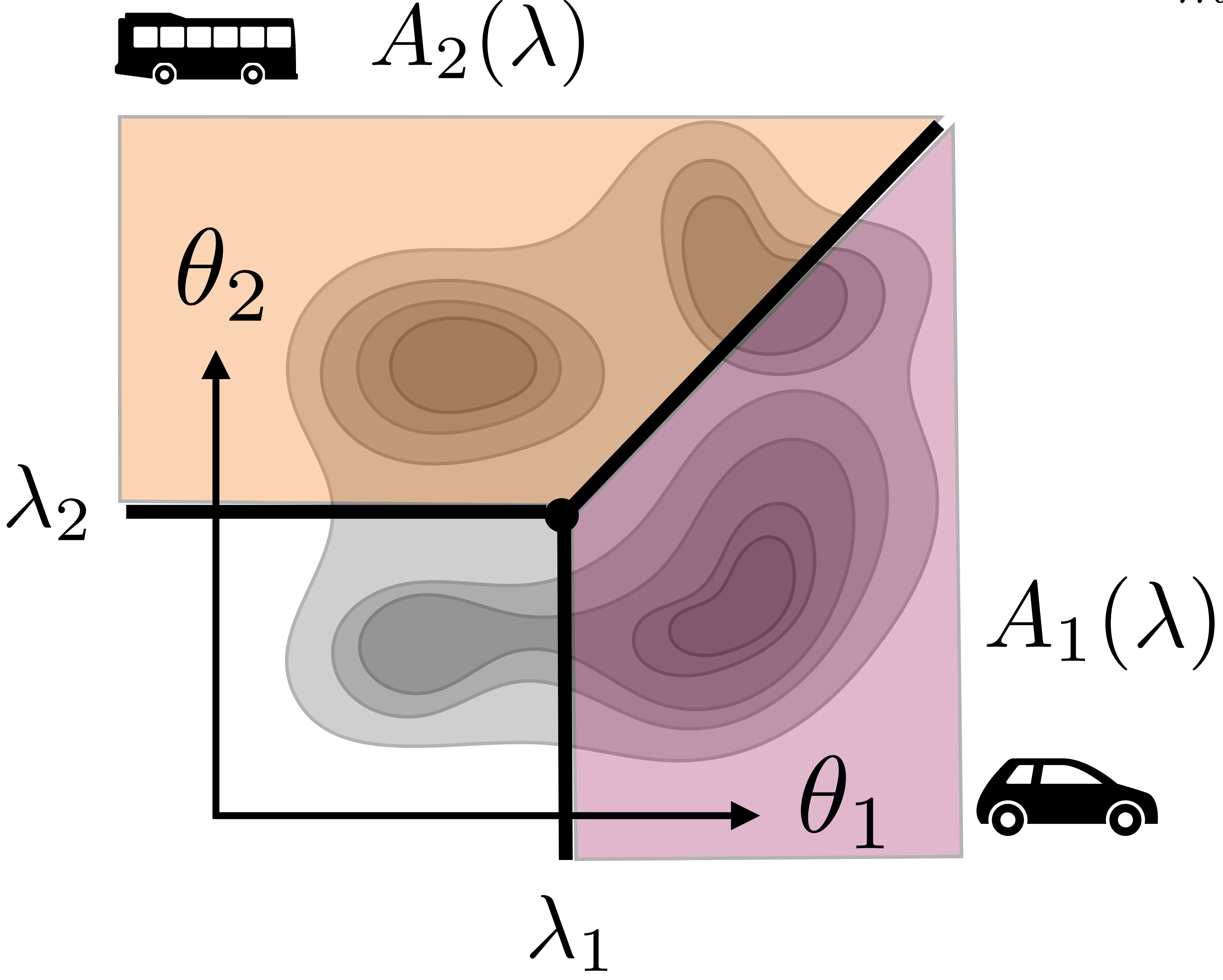
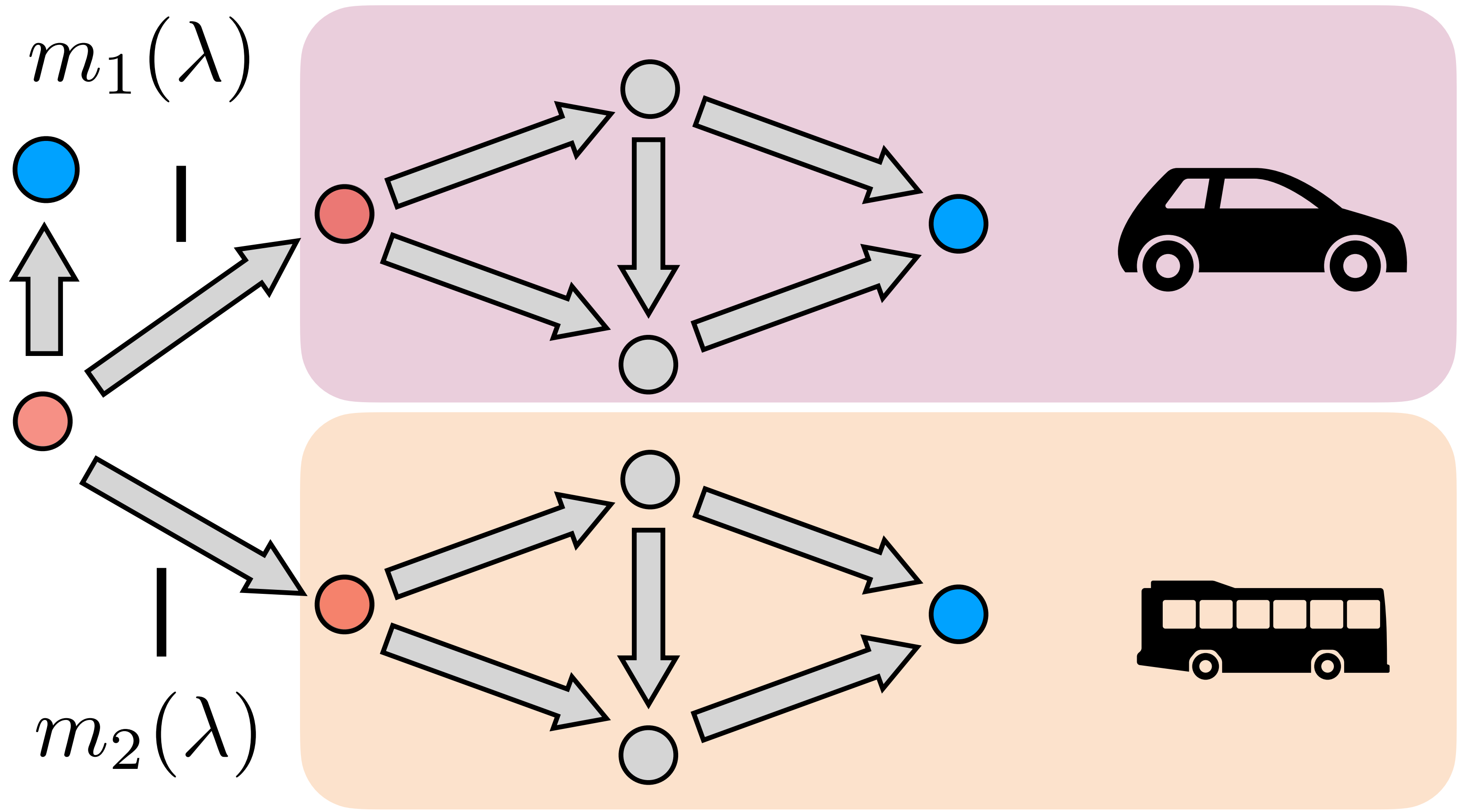




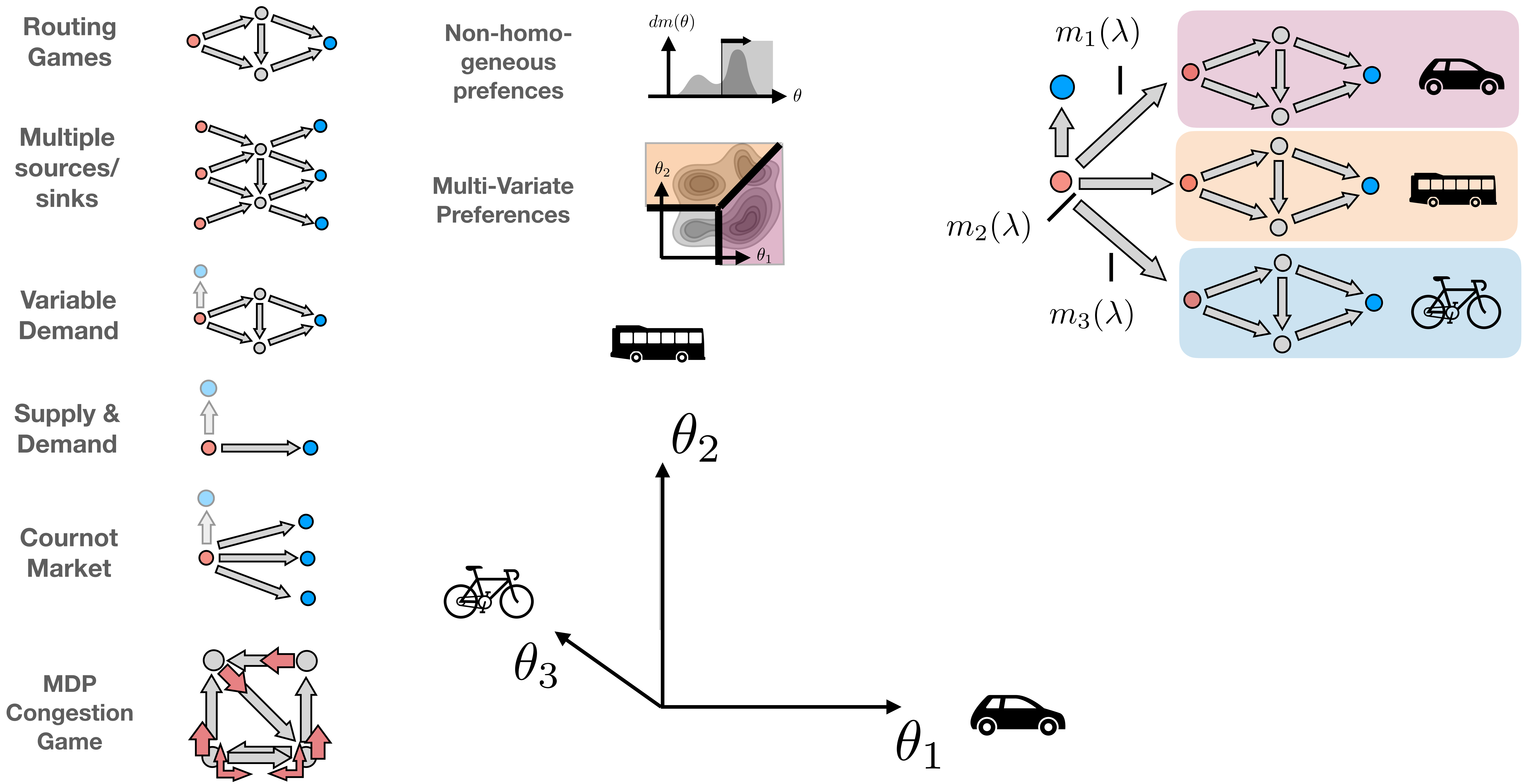


$$m_1(\lambda) = \int_{A_1(\lambda)} dm(\theta)$$

$$m_2(\lambda) = \int_{A_2(\lambda)} dm(\theta)$$

Potential Games

Variable Demand - Multi-Variate Non-Homogeneous Preferences



Potential Games

Routing Games

Multiple sources/ sinks

Variable Demand

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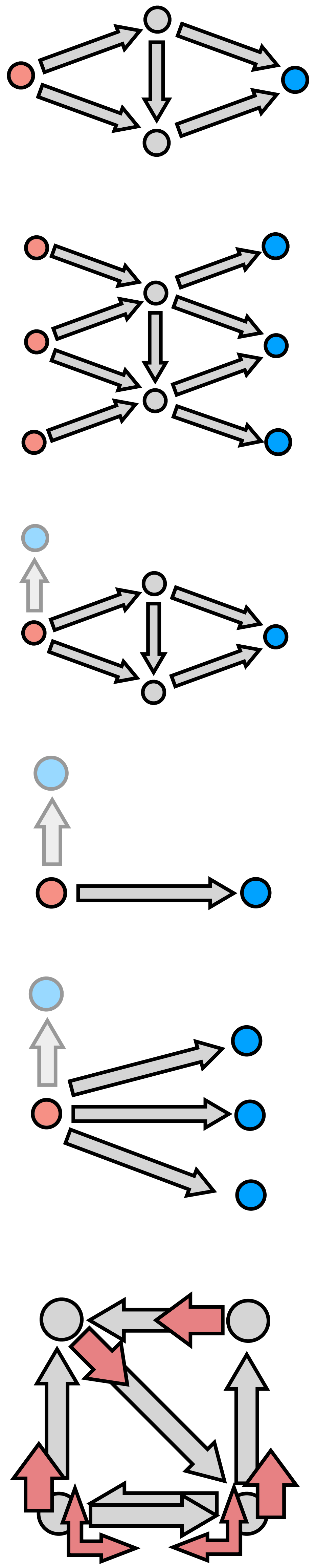

Variable Demand - Multi-Variate Non-Homogeneous Preferences

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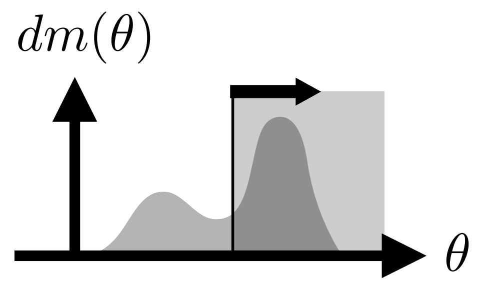

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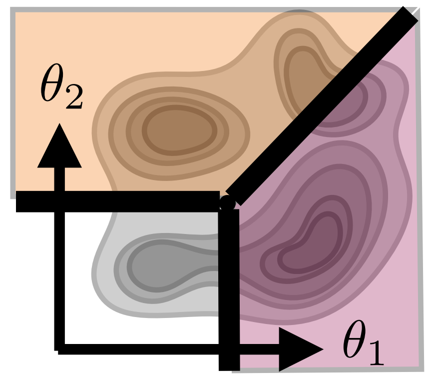
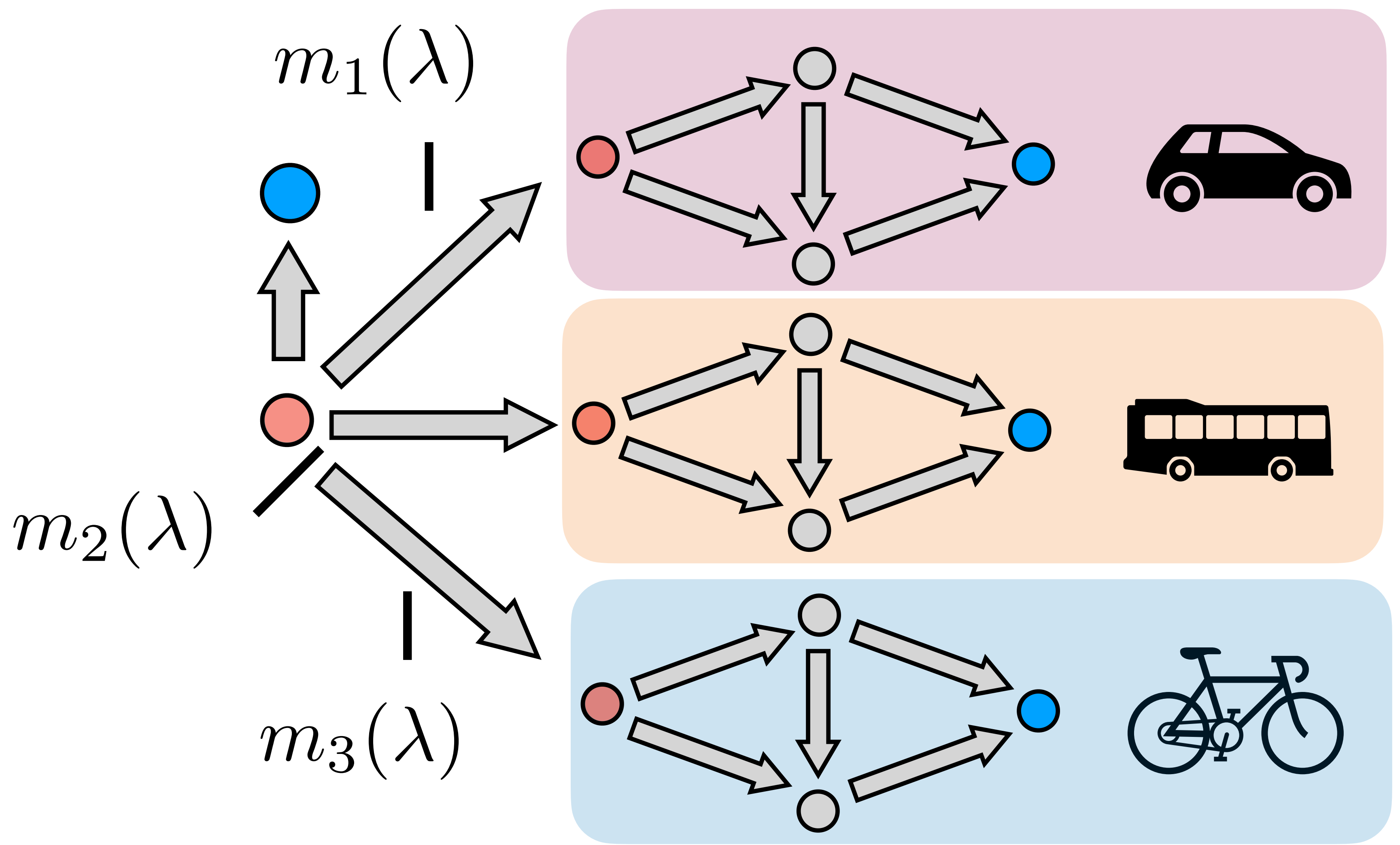
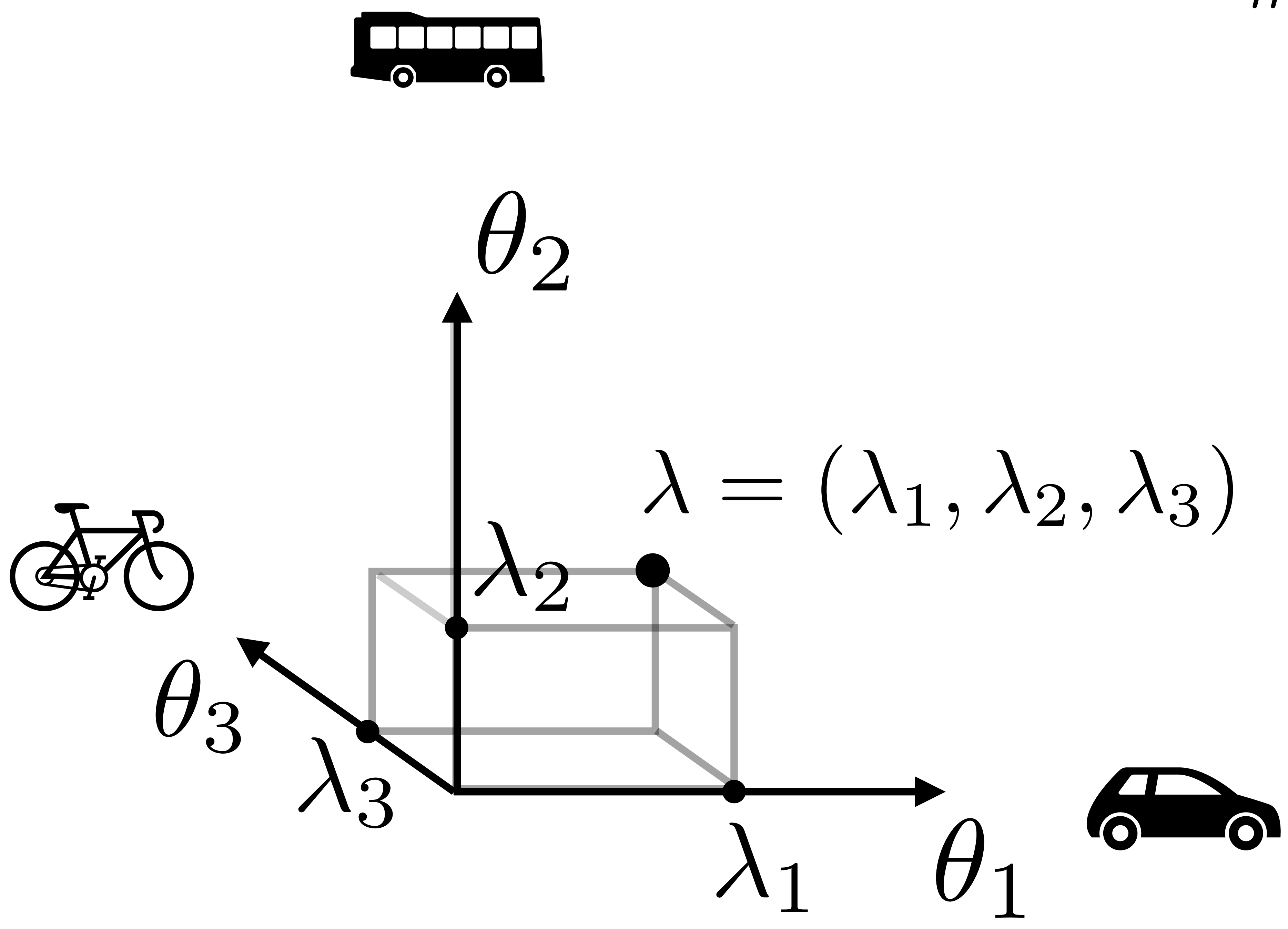
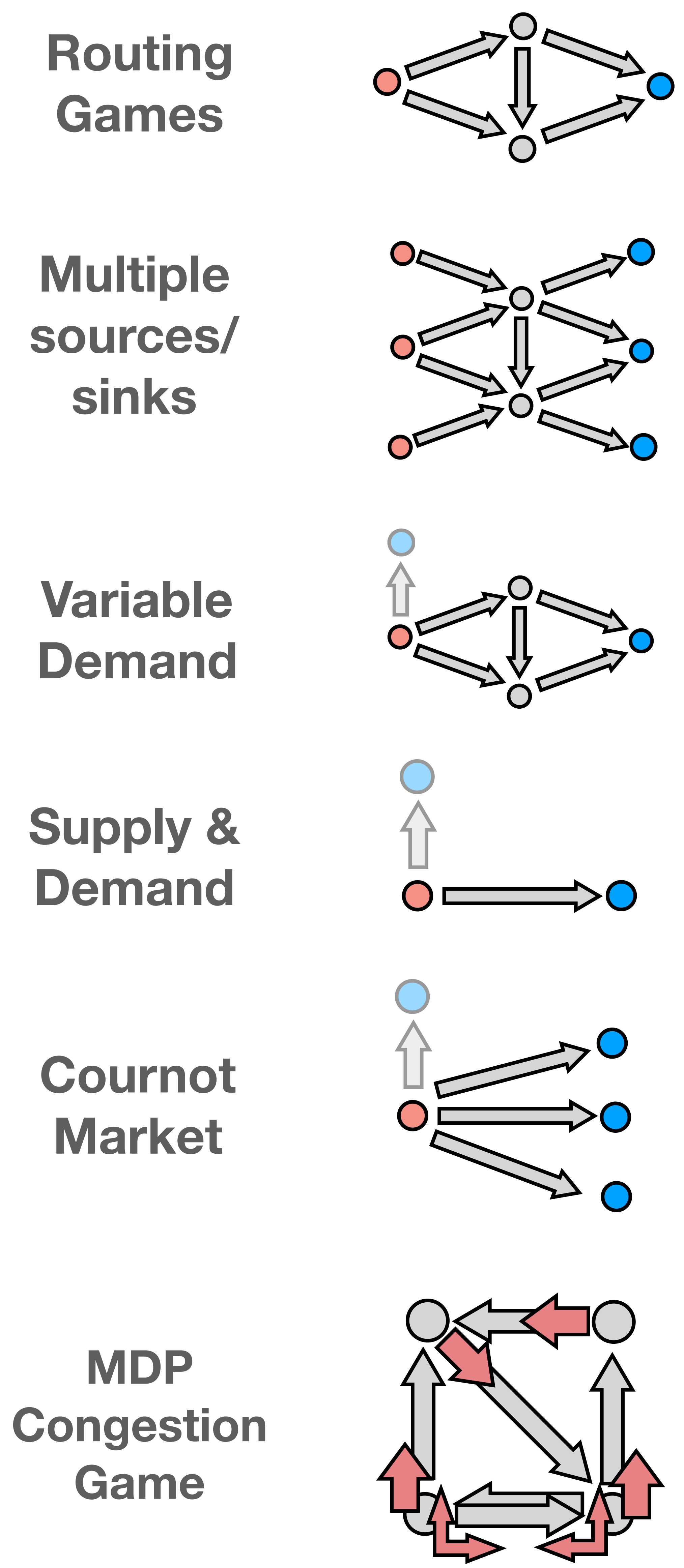






Potential Games



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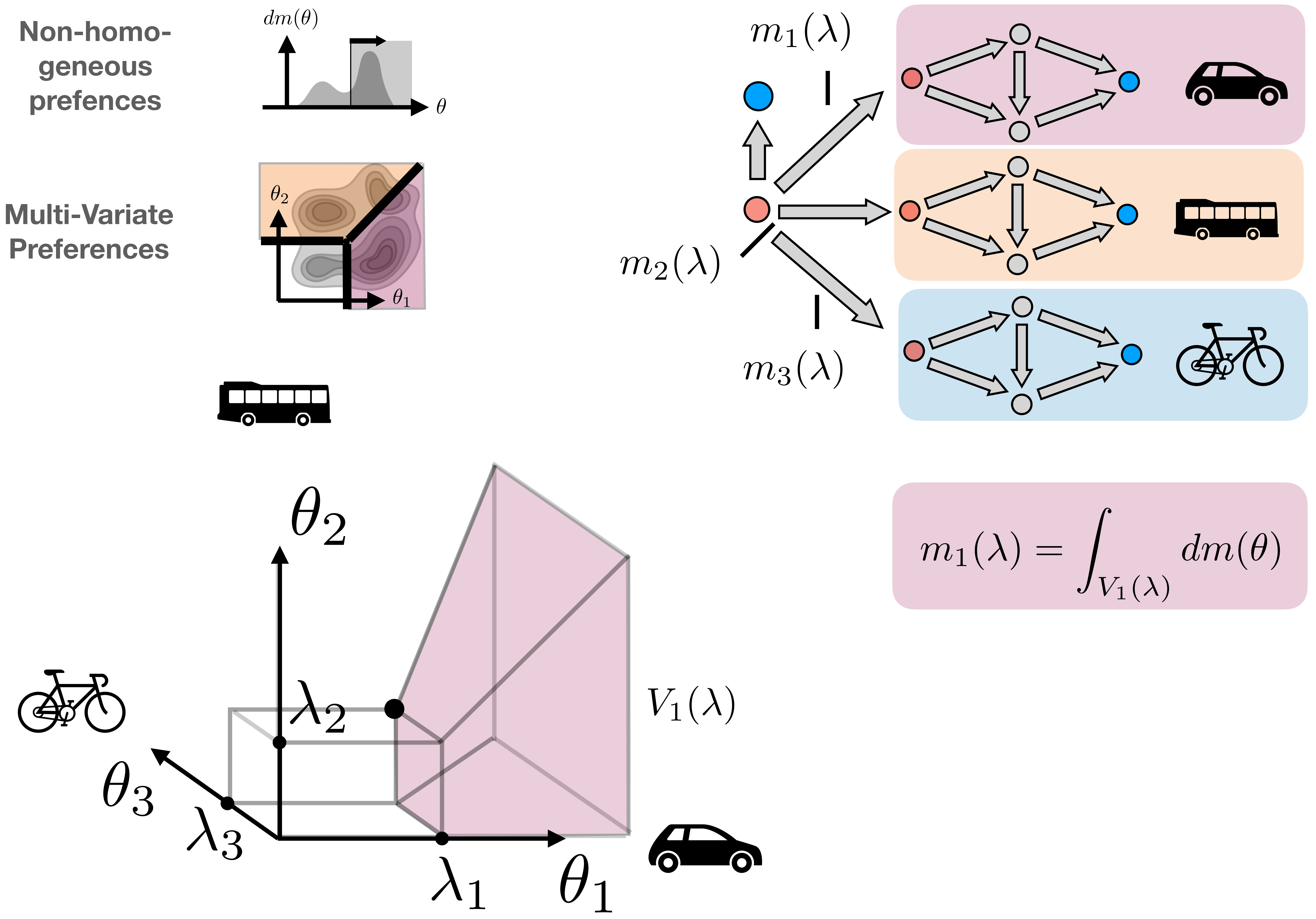
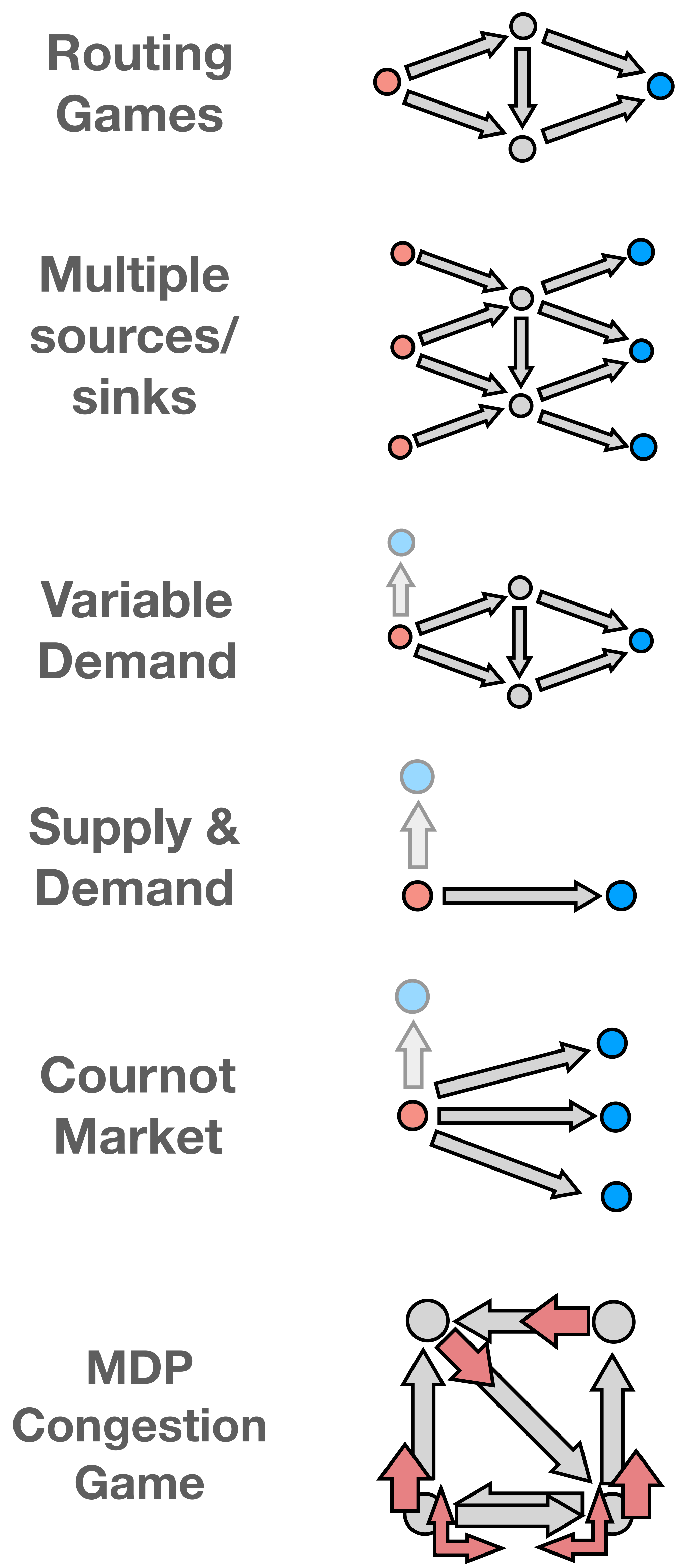


Potential Games



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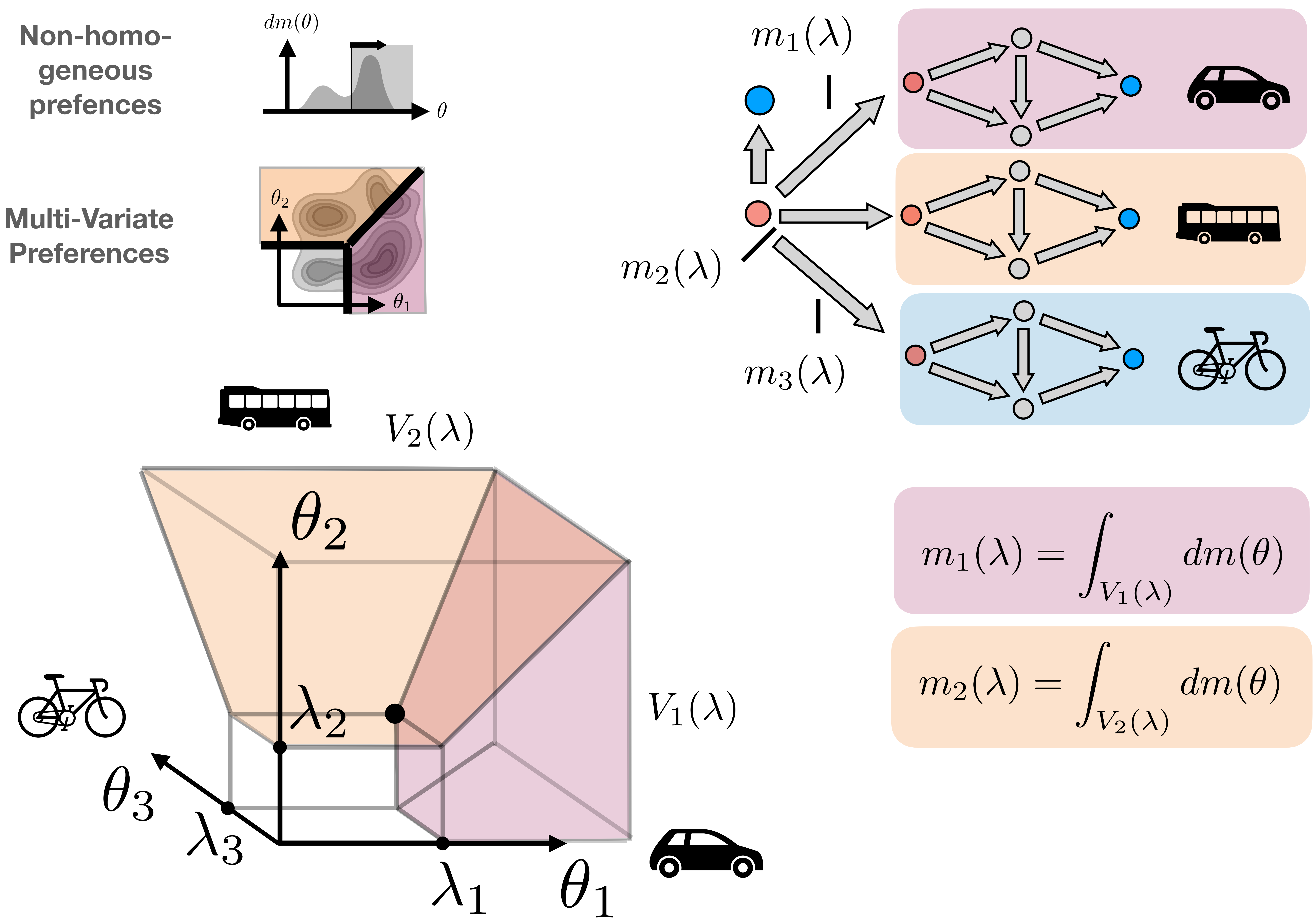

Potential Games

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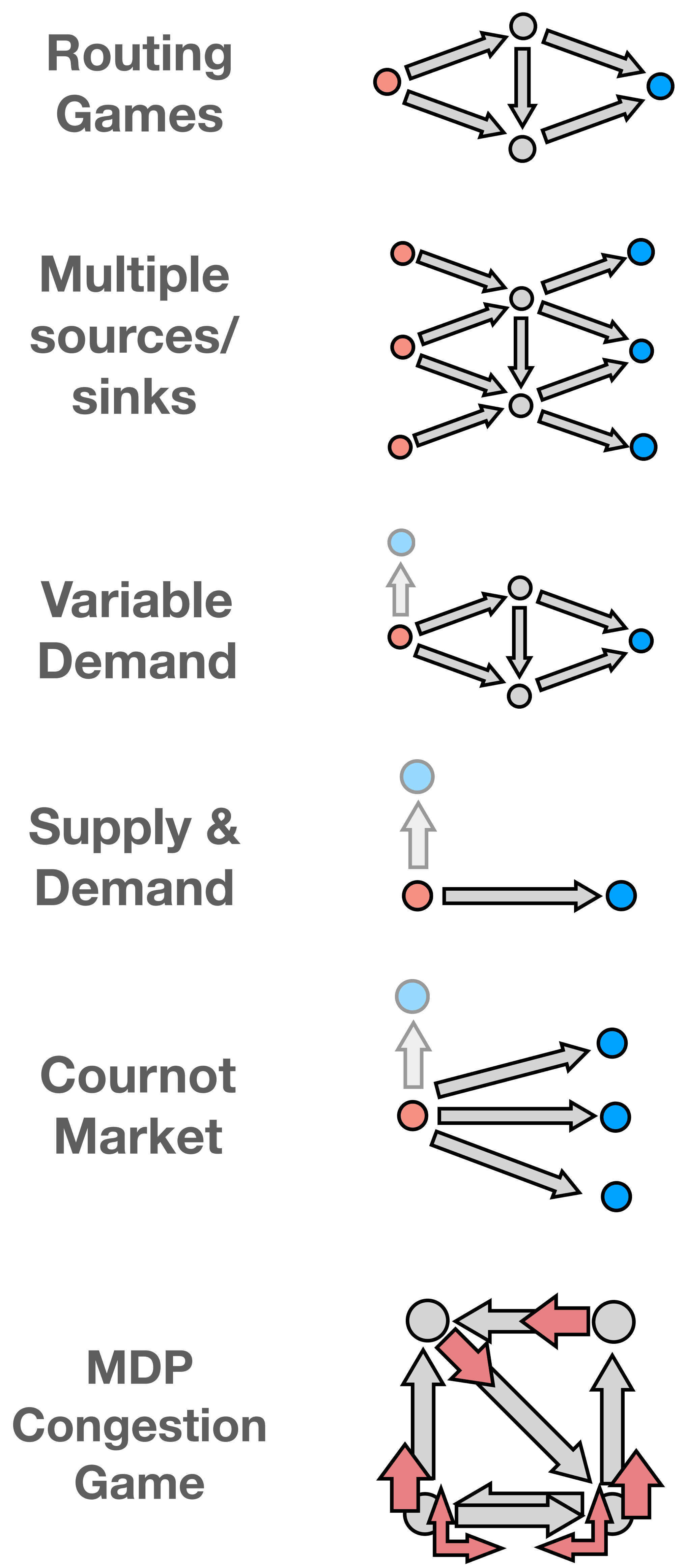
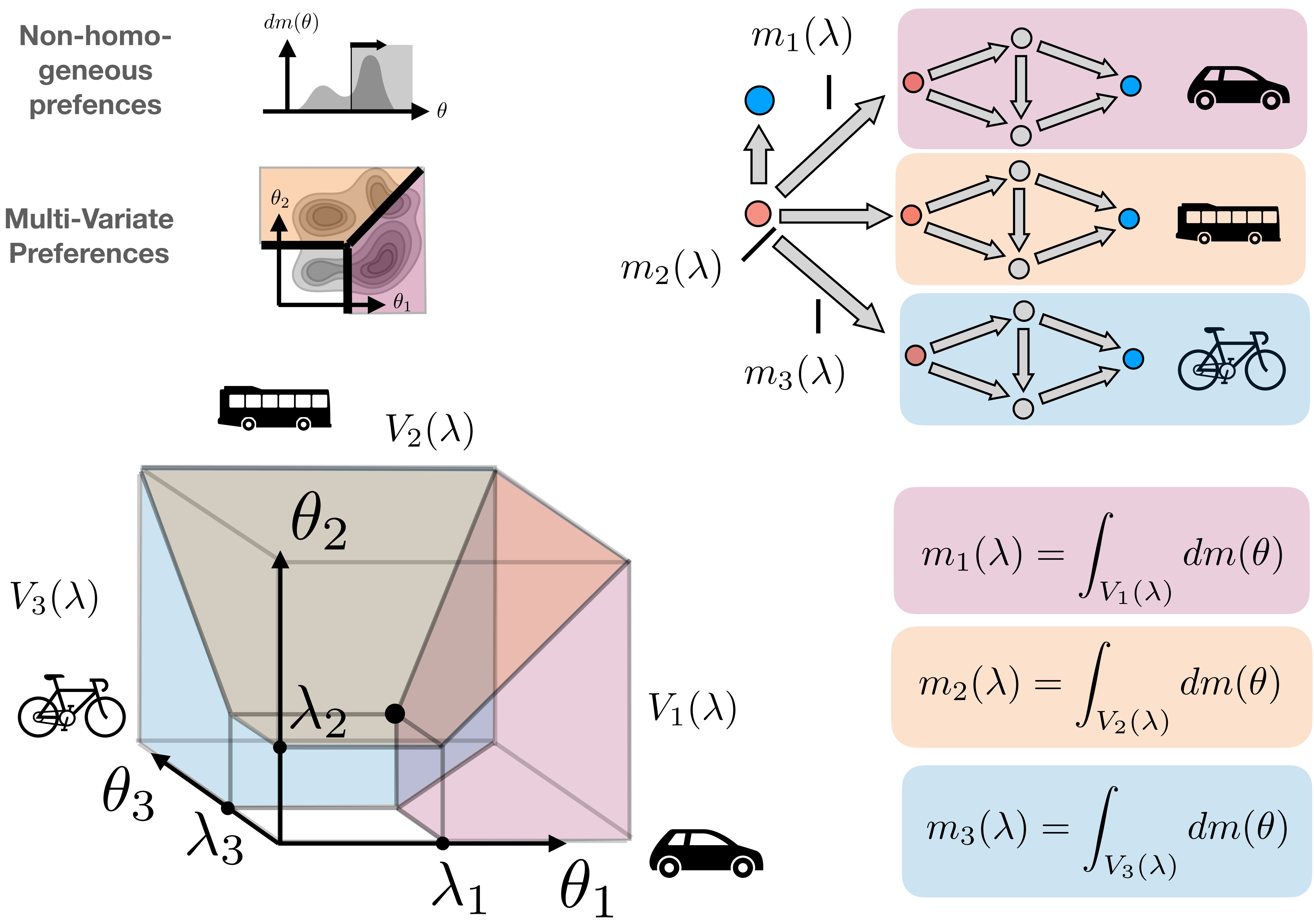
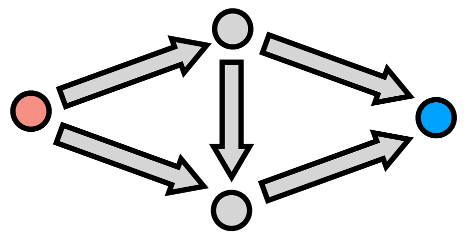


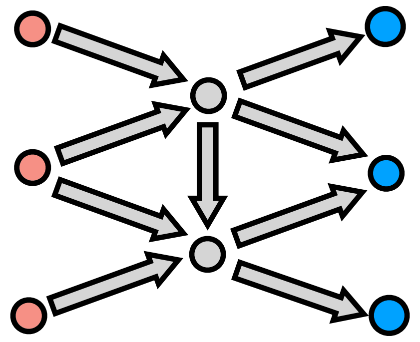


Potential Games

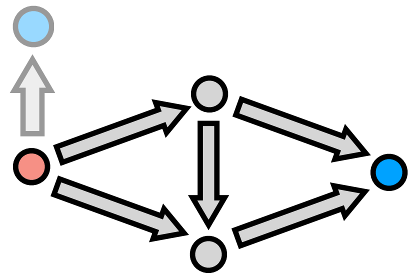
Routing Games



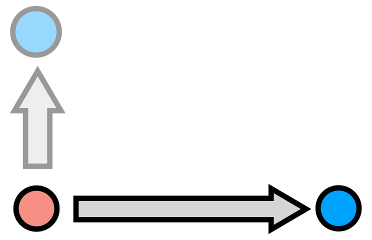
Multiple sources/ sinks



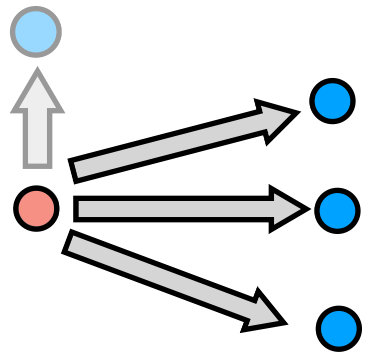
Variable Demand



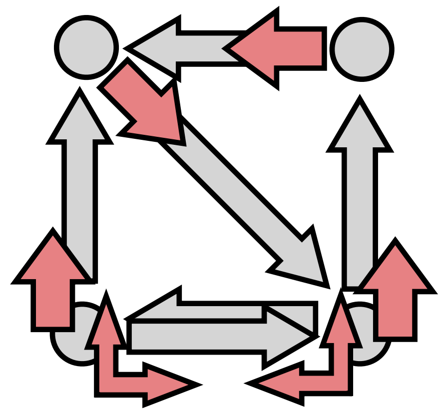
Supply & Demand



Cournot Market

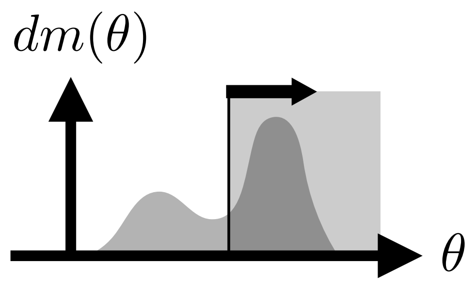


MDP Congestion Game



Variable Demand - Multi-Variate Non-Homogeneous Preferences

Non-homo-geneous prefences



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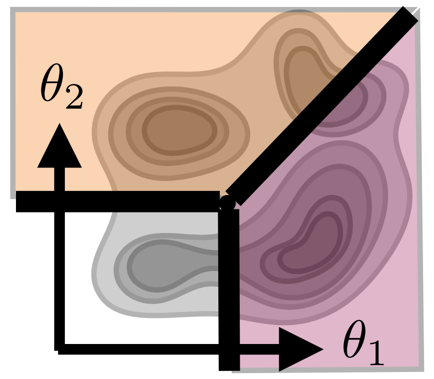
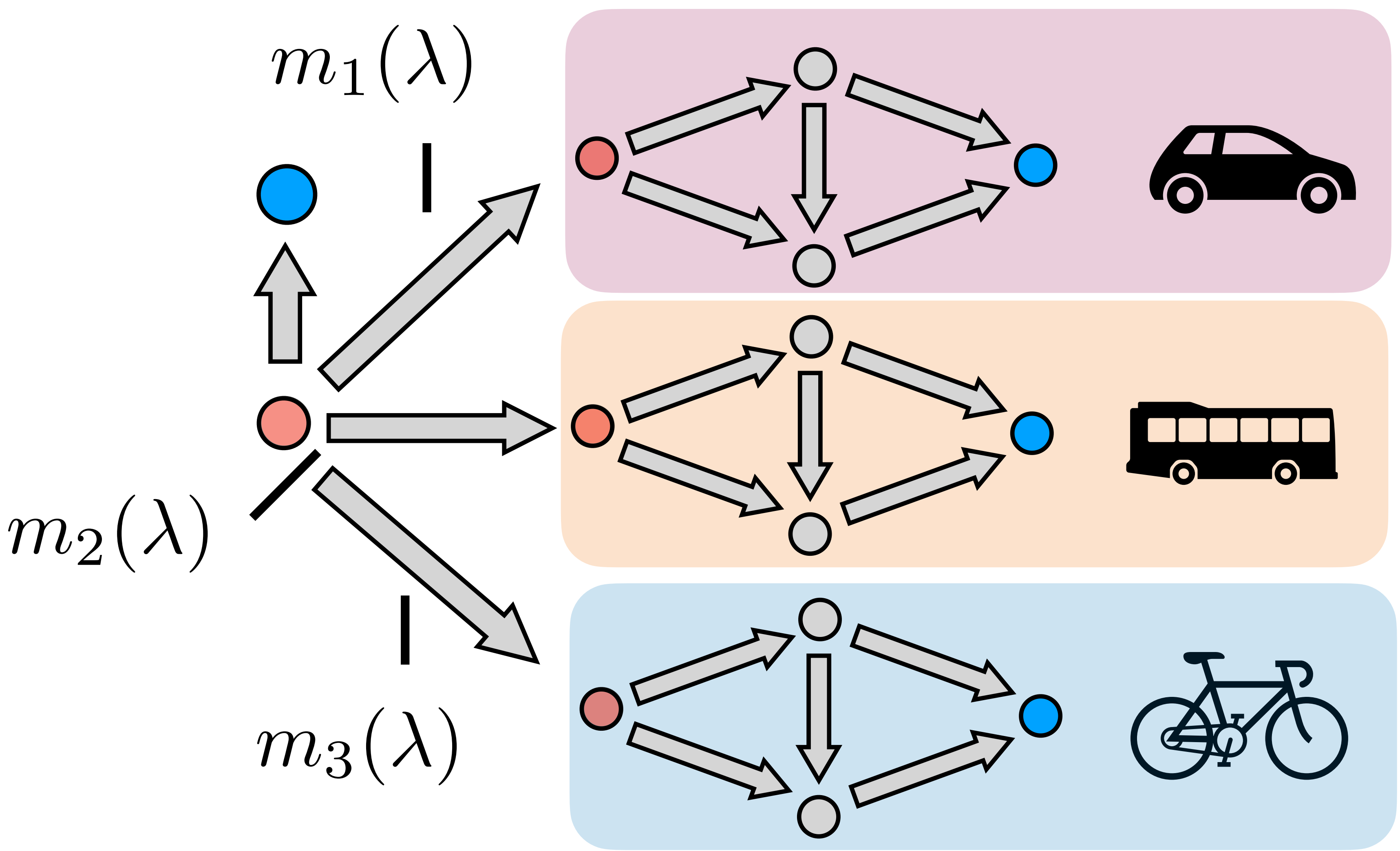




APPLICATIONS

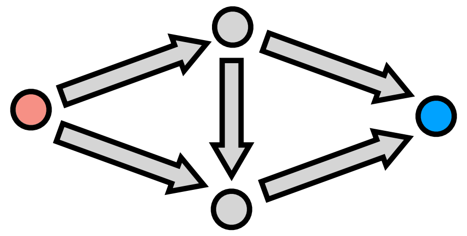
- Multi-modal transportation networks
- Non-homogeneous supply/demand

PAPERS

- External-cost continuous-type Wardrop equilibria in routing games
  [Calderone, Dong, Sastry, 2017]
- Multi-dimensional continuous type population potential games
  [Calderone, Ratliff, 2019]

Potential Games

Routing Games



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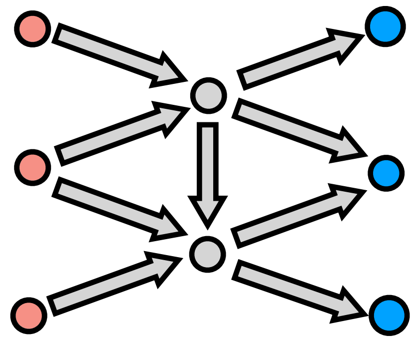
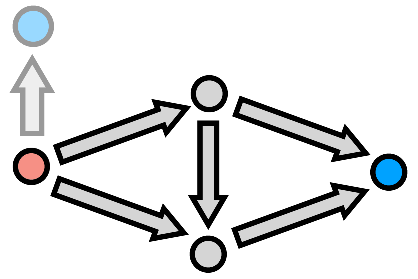


Variable Demand



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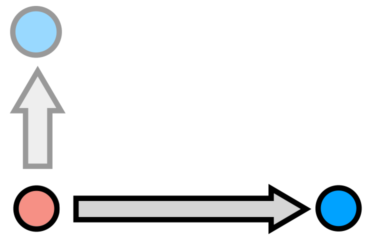
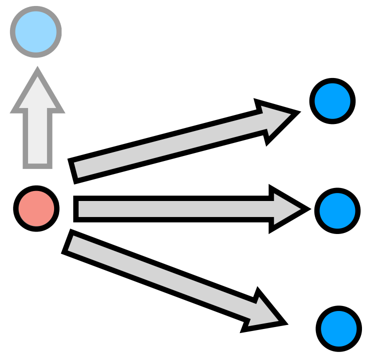
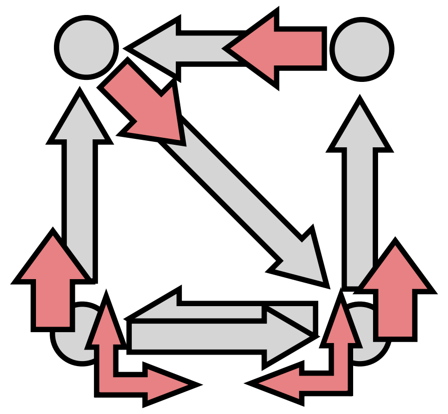


Cournot Market

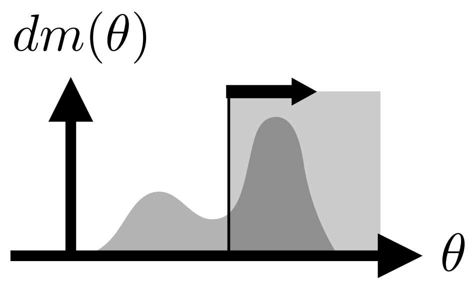


MDP Congestion Game

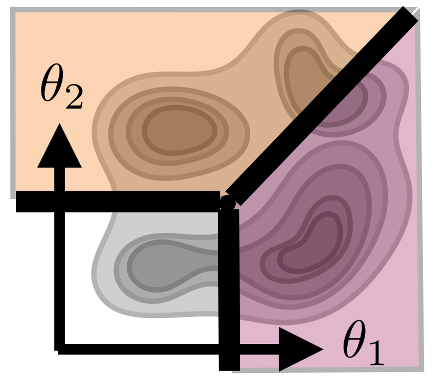


Braess Paradox

Non-homo-geneous prefences



Multi-Variate Preferences



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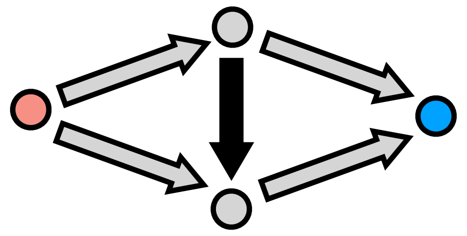

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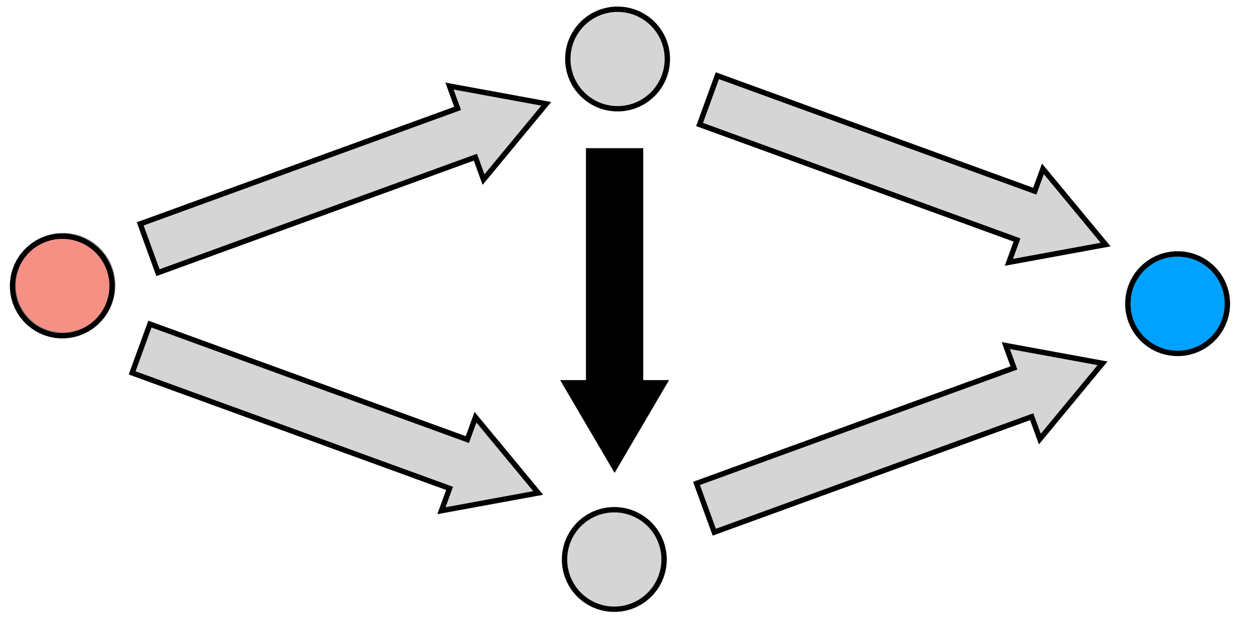
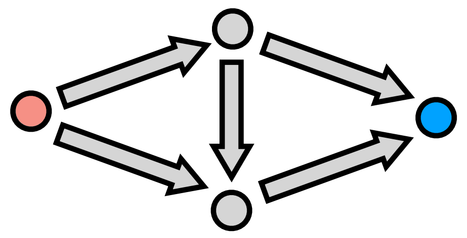


Potential Games

Routing Games



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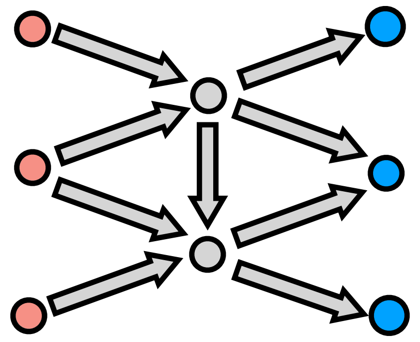
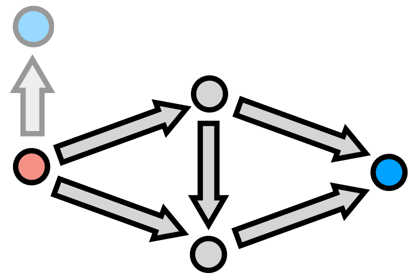


Variable Demand



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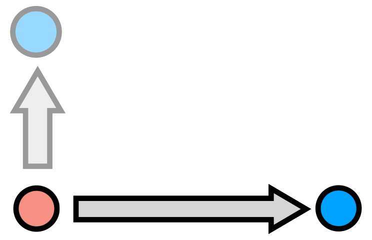
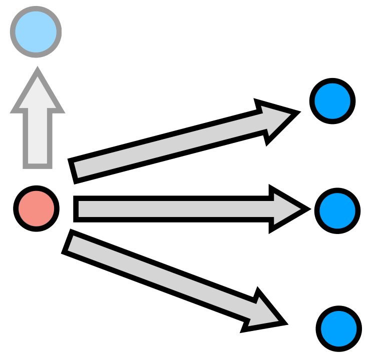
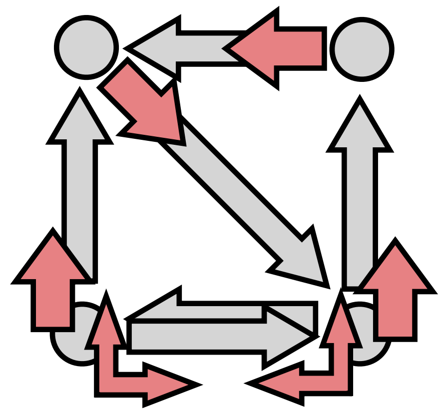


Cournot Market



MDP Congestion Game



Braess Paradox

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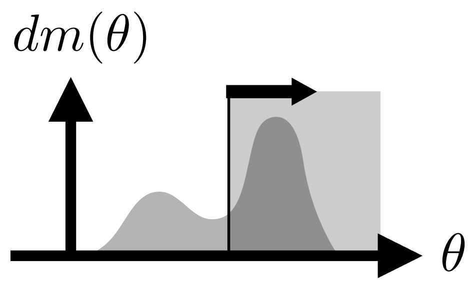
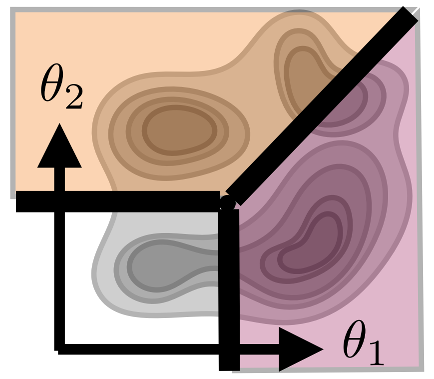


Multi-Variate Preferences



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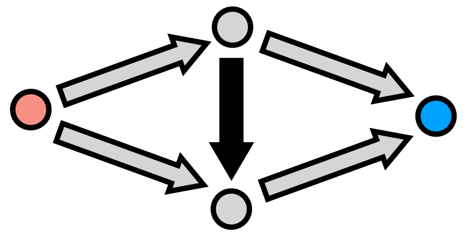

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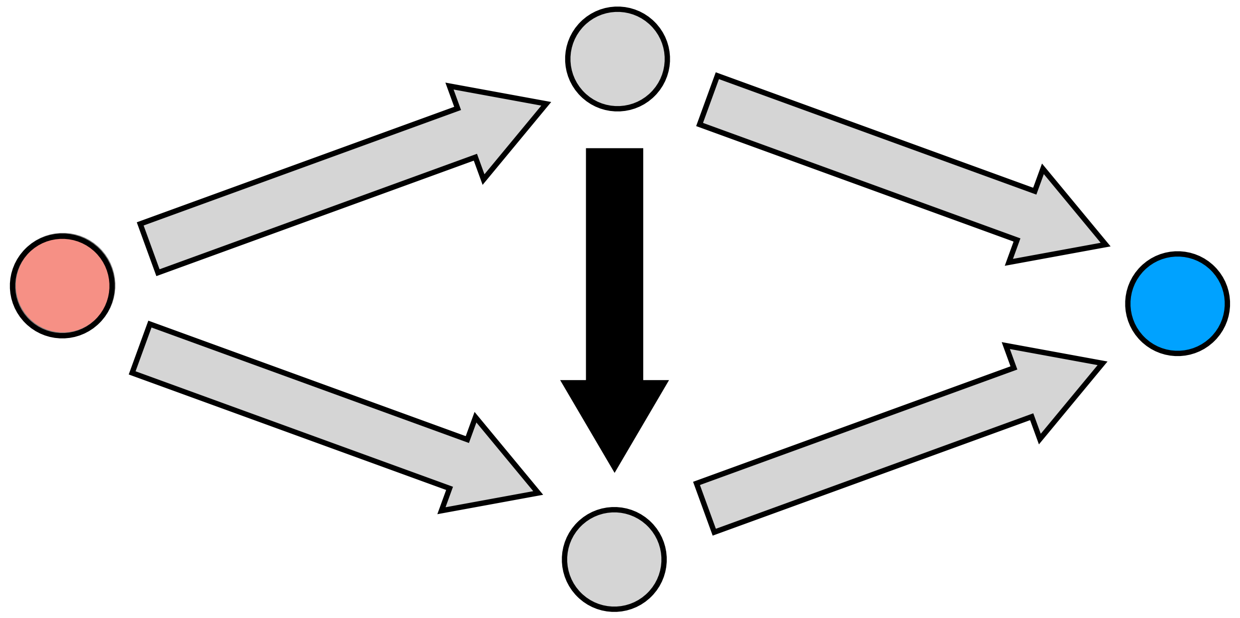
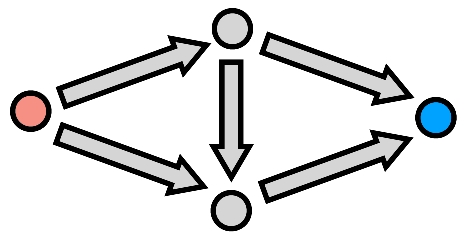


Adding center road can make traffic worse!

Potential Games

Routing Games



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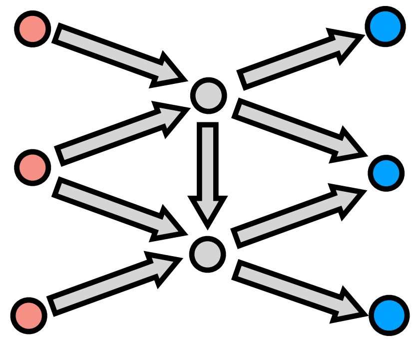
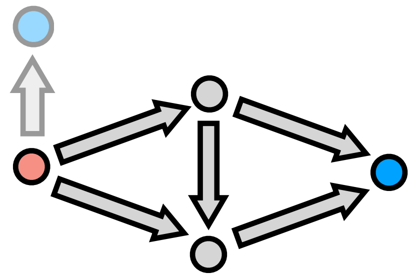
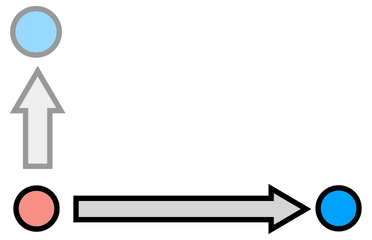


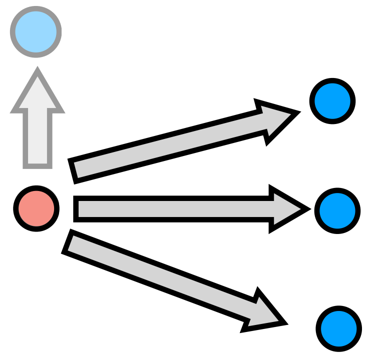
Variable Demand



Supply & Demand



Cournot Market



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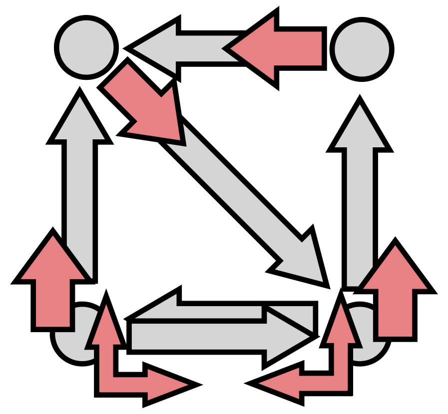

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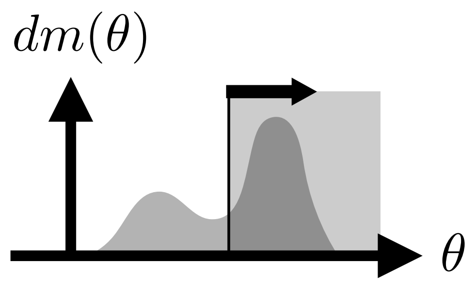

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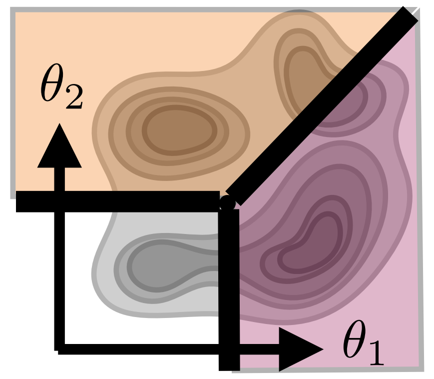

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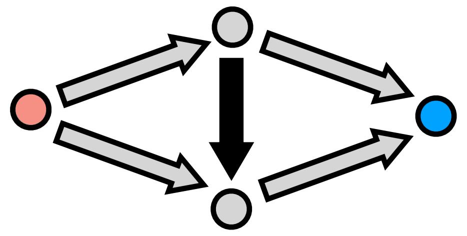

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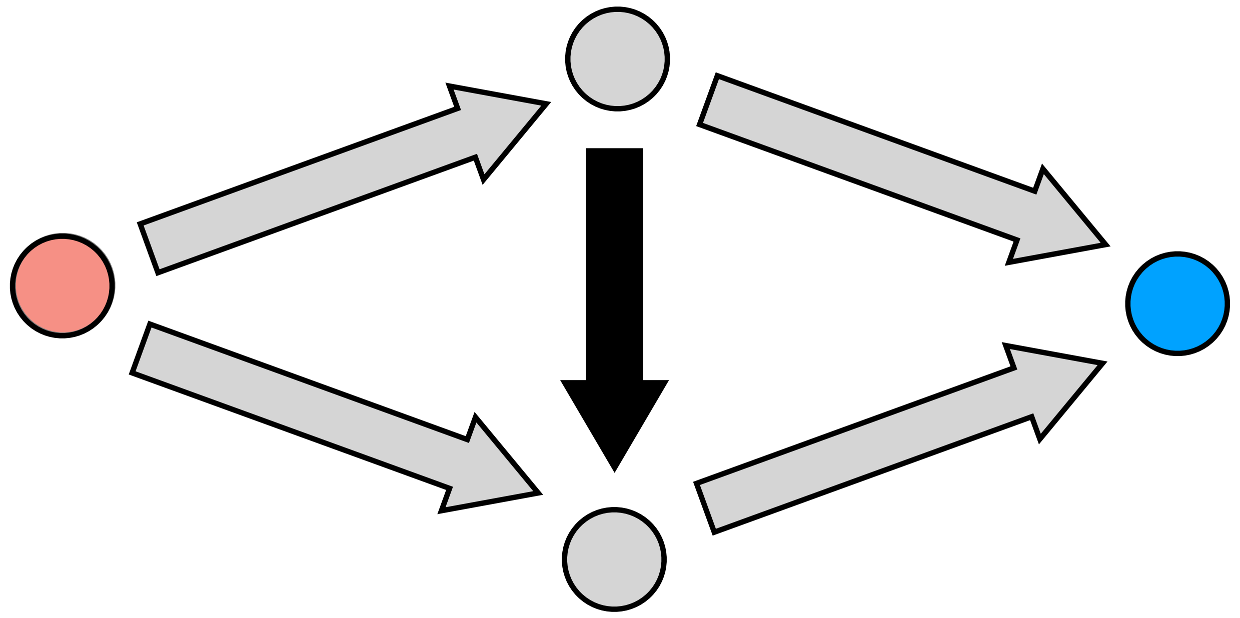
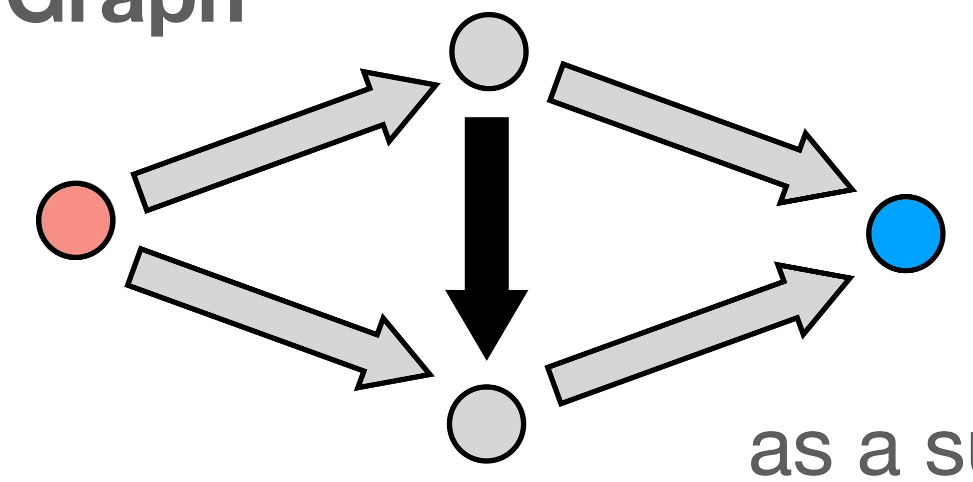


Adding center road can make traffic worse!

Network Characterization:

Every network has

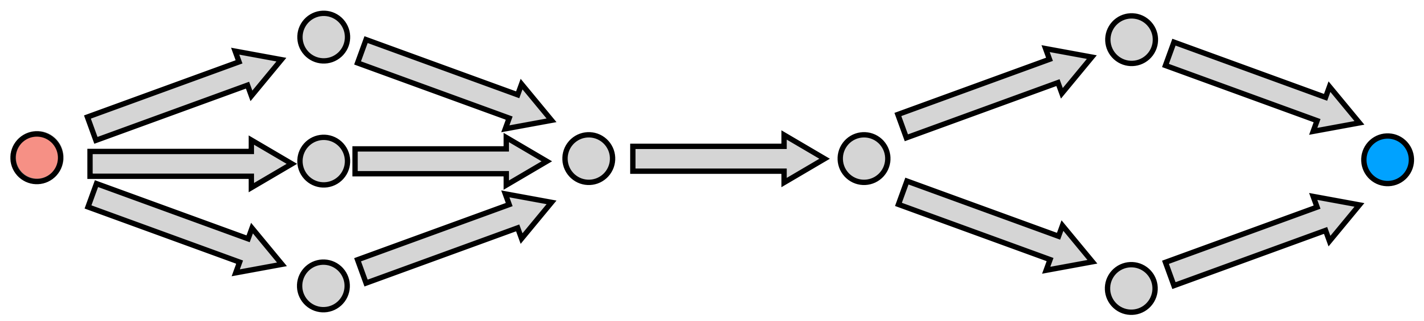
Braess Graph



as a subgraph

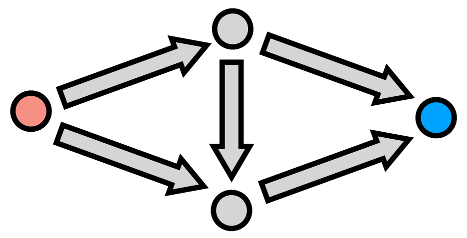
OR is

Series-Parallel Graph



Potential Games

Routing Games



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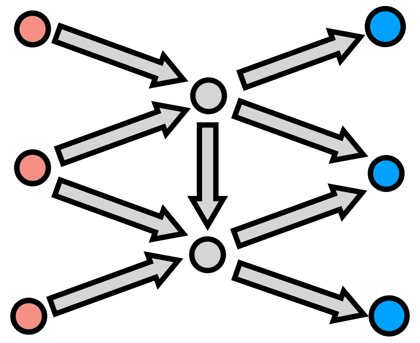
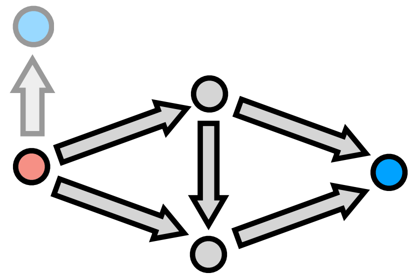
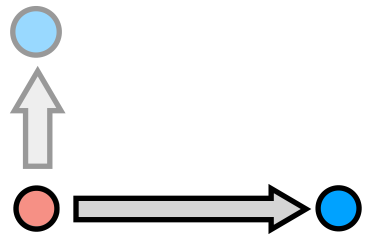


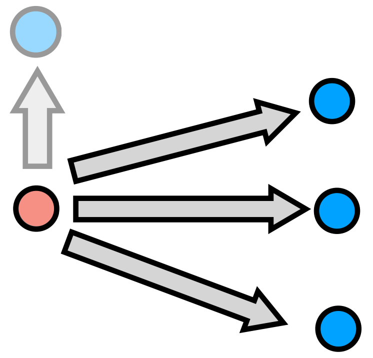
Variable Demand



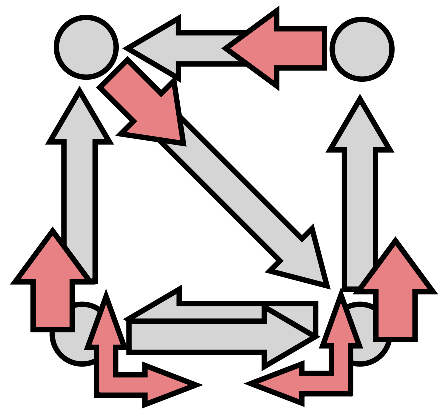
Supply & Demand



Cournot Market

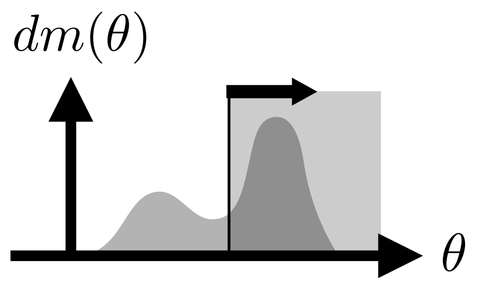


MDP Congestion Game

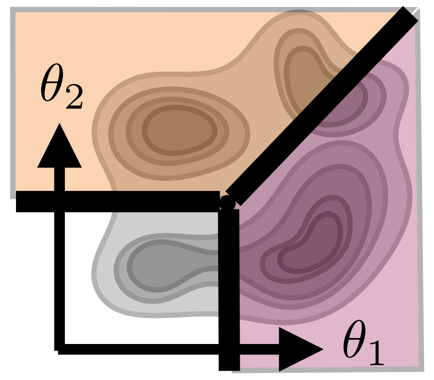


Braess Paradox

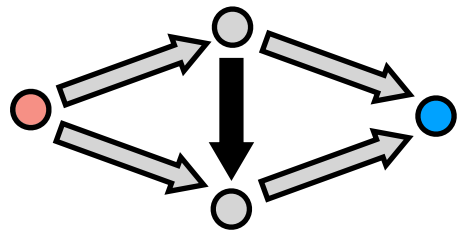
Non-homo-geneous prefences



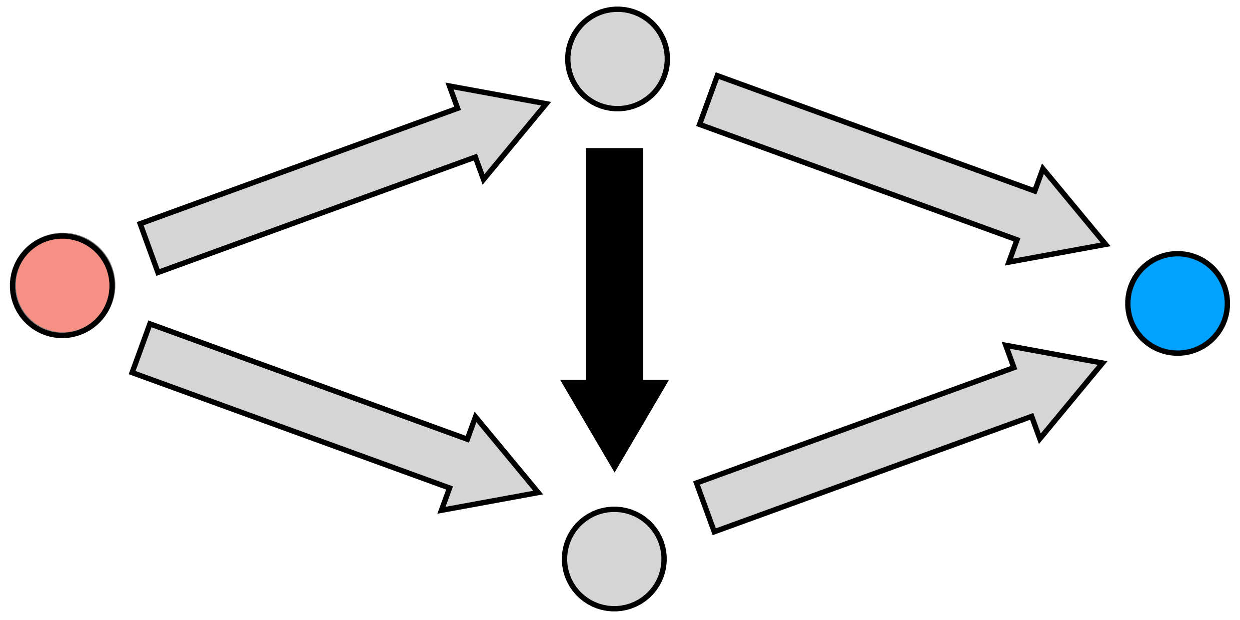
Multi-Variate Preferences



Braess Paradox



Braess Paradox

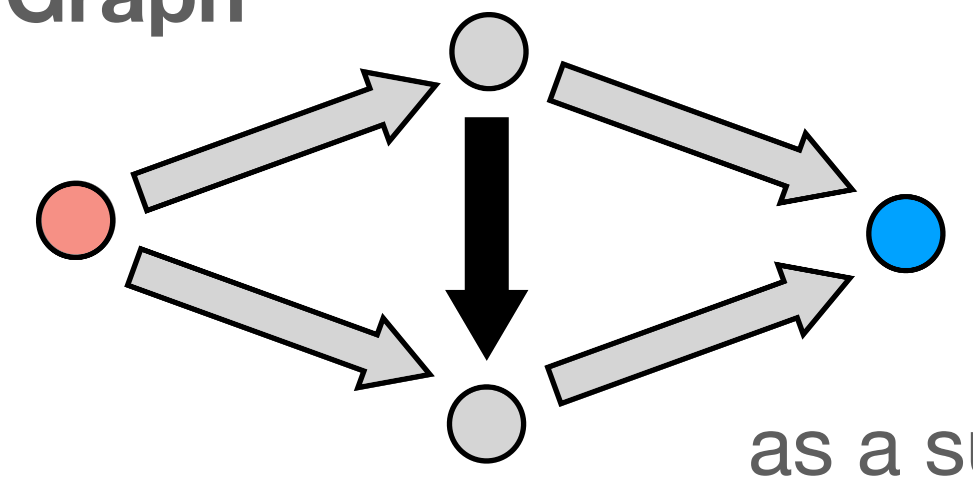


Adding center road can make traffic worse!

Network Characterization:

Every network has

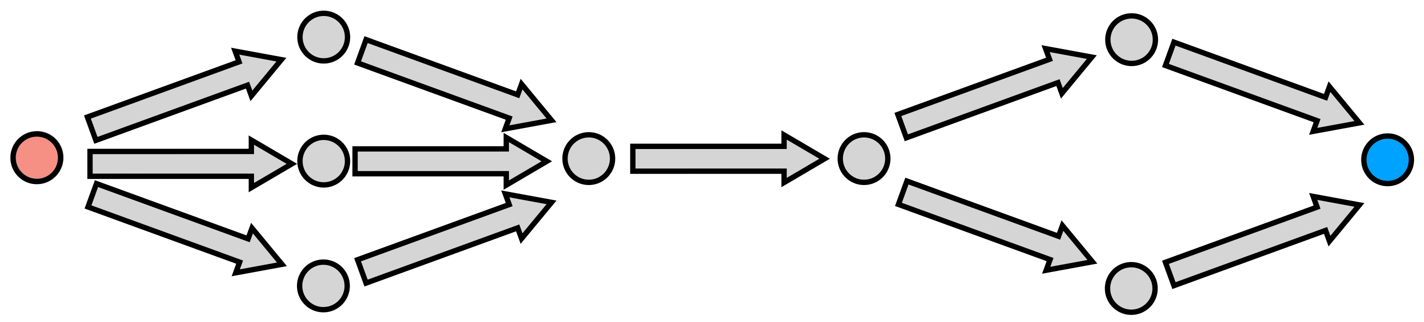
Braess Graph



as a subgraph

OR is

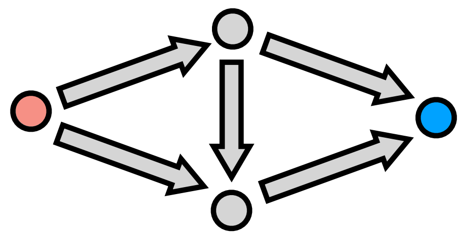
Series-Parallel Graph



Series - Parallel graphs cannot suffer from Braess paradox ← **linear-algebraic characterization/proof**

Potential Games

Routing Games



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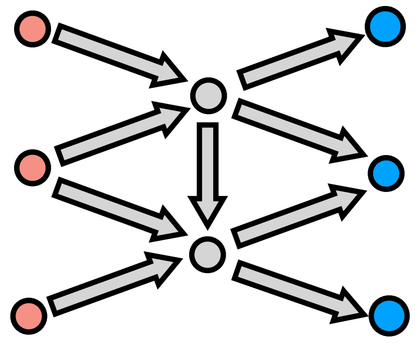
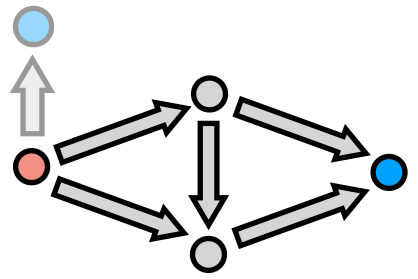
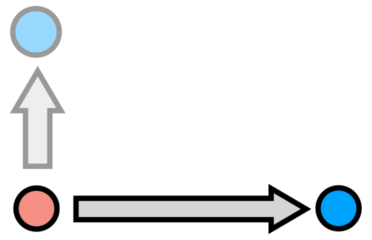


Variable Demand



Supply & Demand



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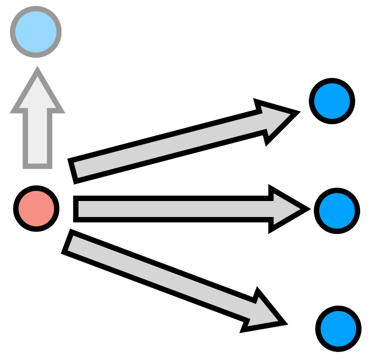

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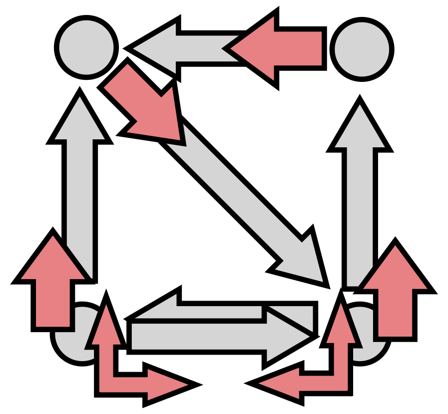

Braess Paradox

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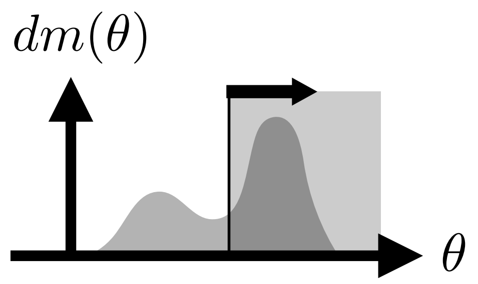

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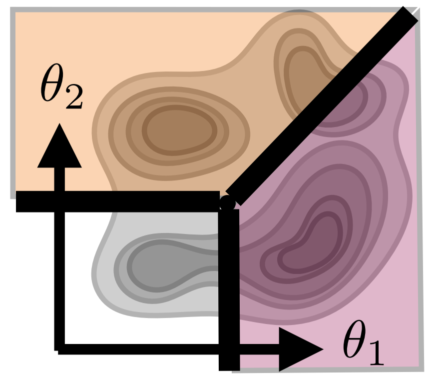
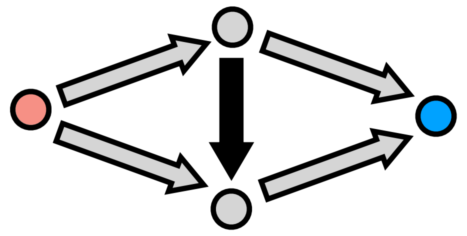
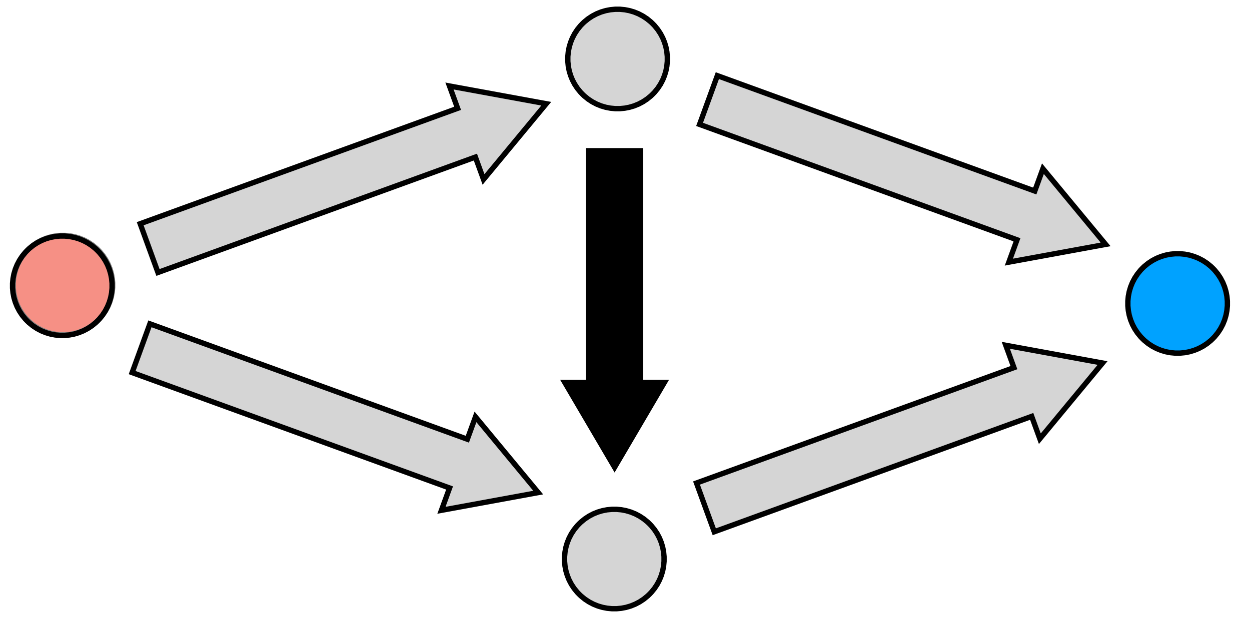


Braess Paradox



Braess Paradox



Adding center road can make traffic worse!

REFERENCES

- Uber ein Paradoxon der verkehrsplanung **[Braess, 1969]**
- Topology of series-parallel networks **[Duffin, 1965]**
- Network topology and the efficiency of equilibrium **[Milchtaich, 2006]**

PAPERS

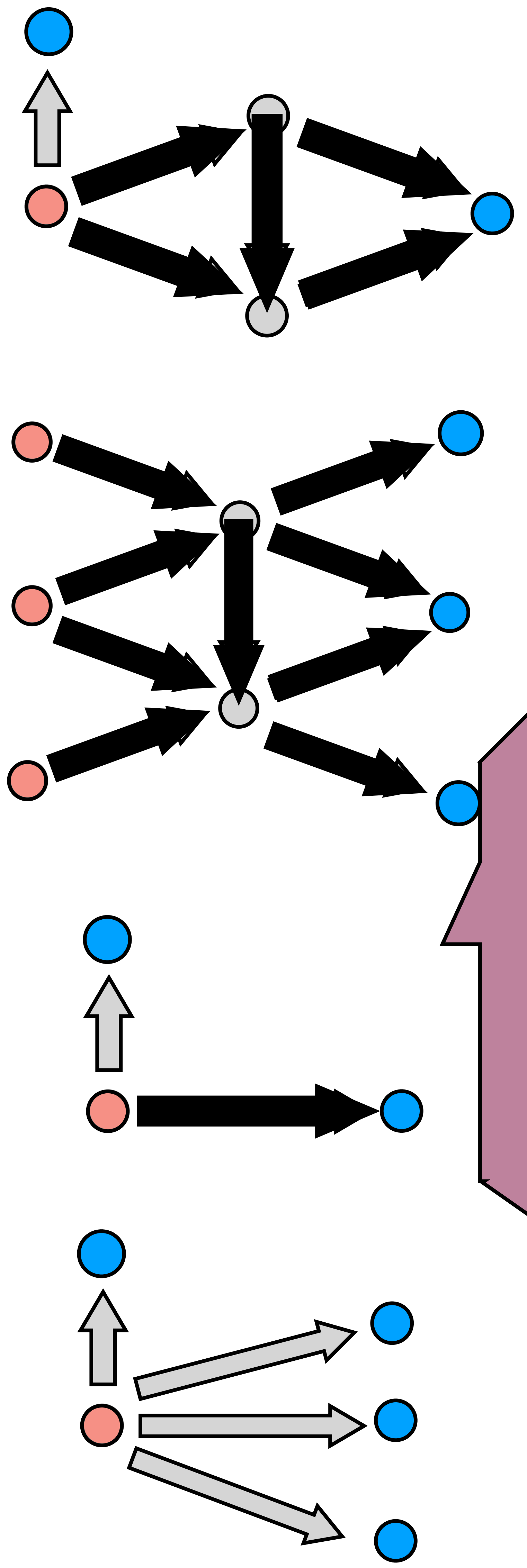
- Sensitivity analysis for Markov decision process congestion games **[Li, Calderone, Ratliff, 2019]**
- Algebraic characterization of Braess paradox: Network efficiency in series-parallel and Braess networks **[Calderone, Ratliff, in prep]**

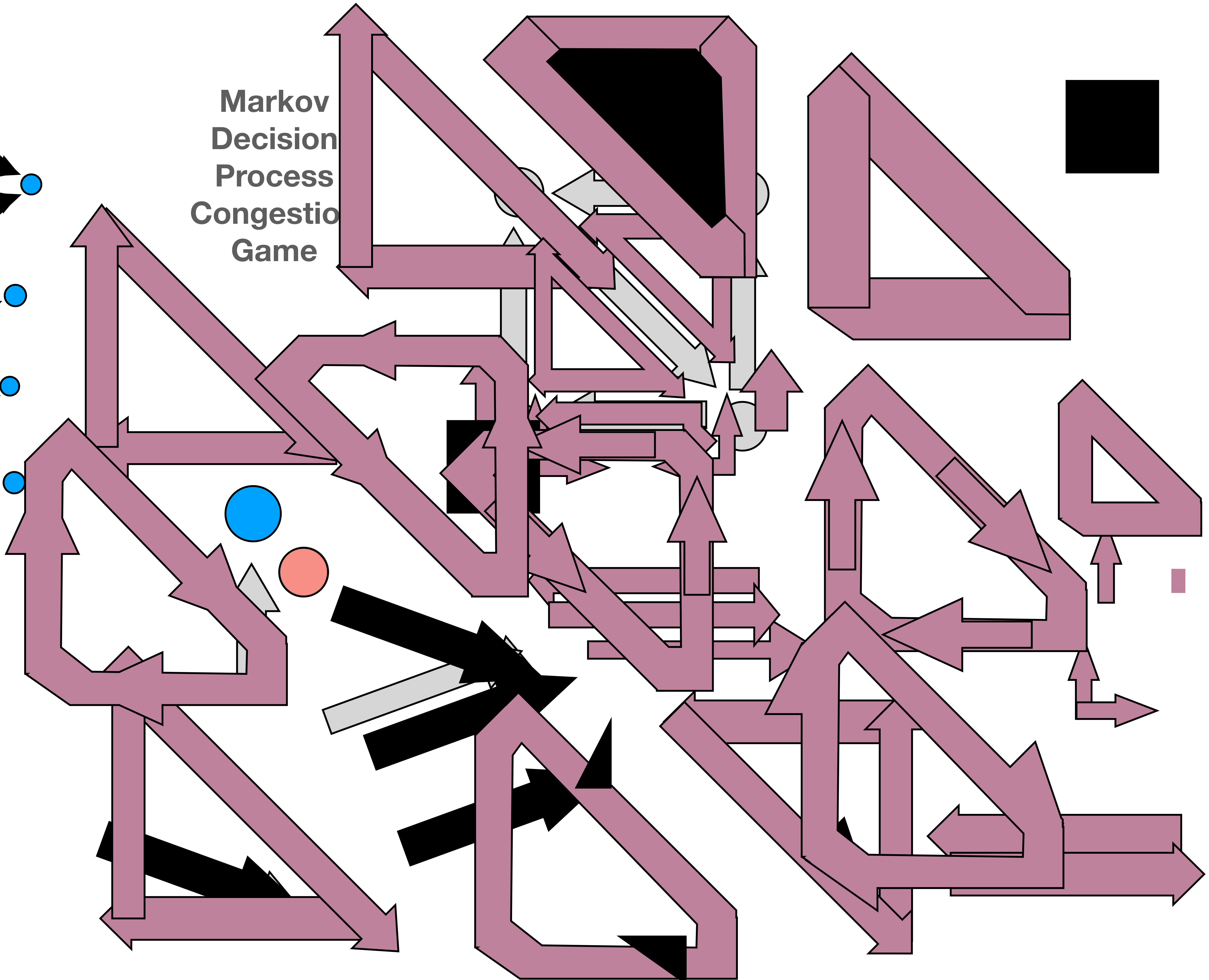
Variable Demand

Multiple sources/ sinks

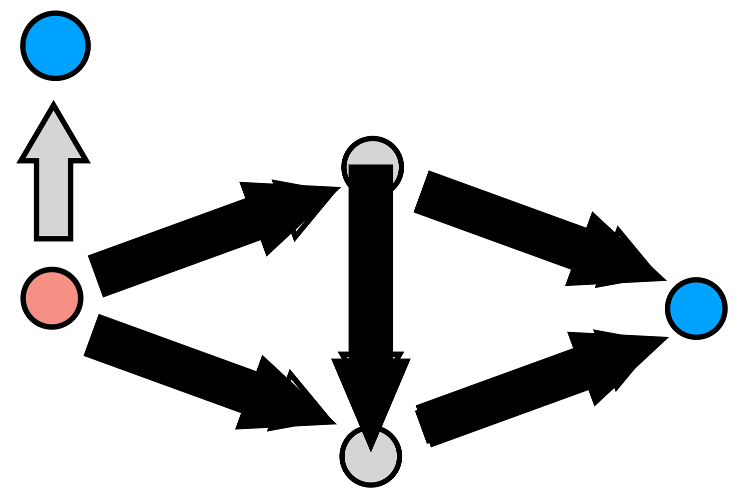
Supply & Demand

Cournot Competition

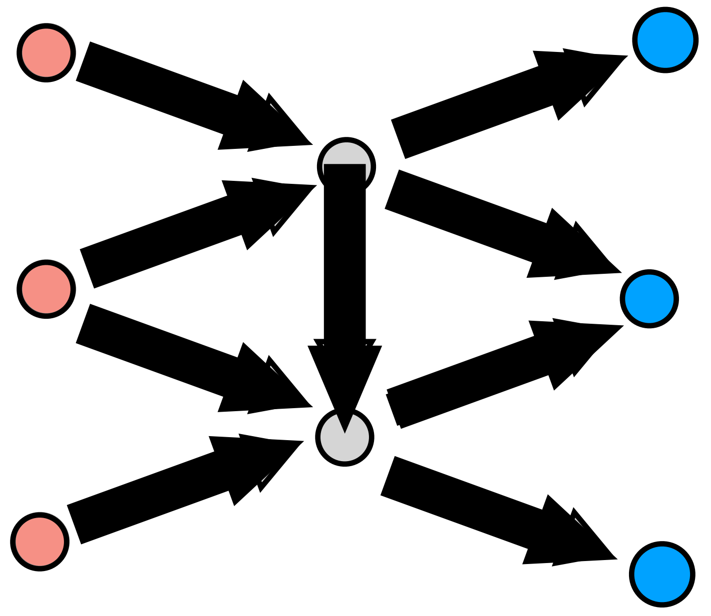




Variable Demand



Multiple sources/sinks



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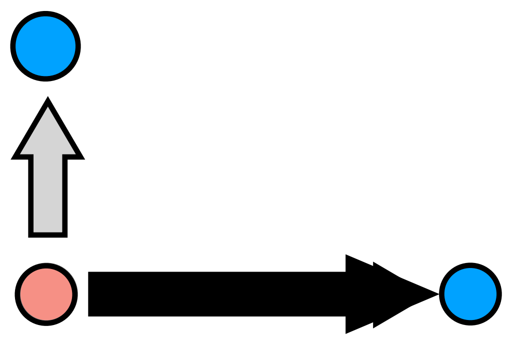

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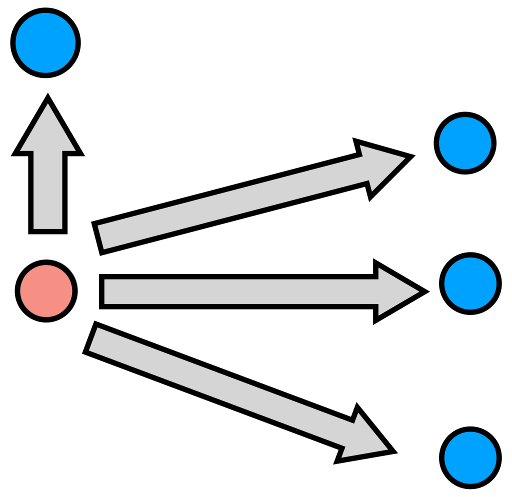
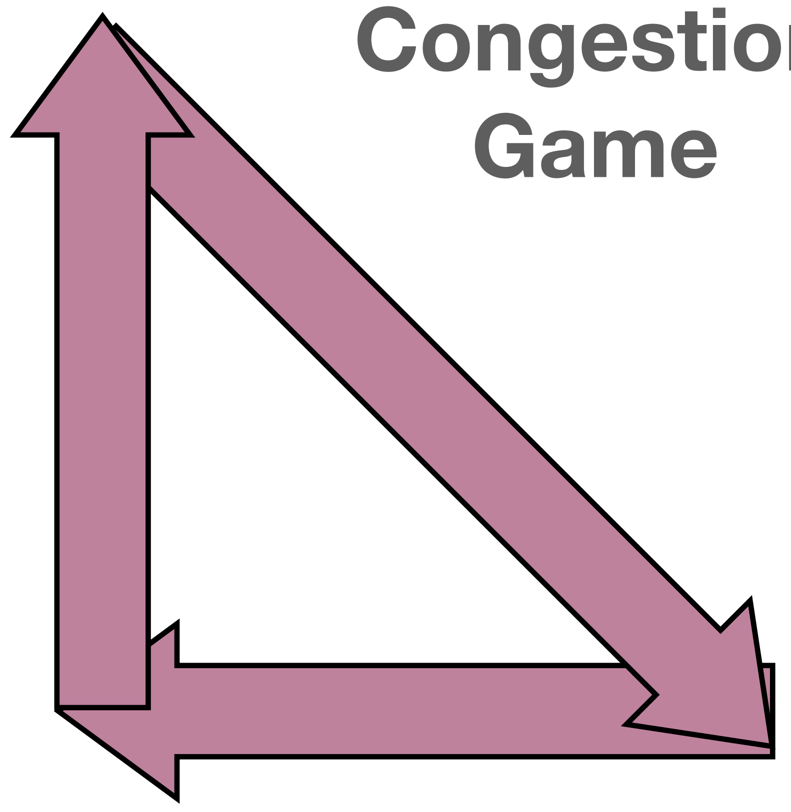
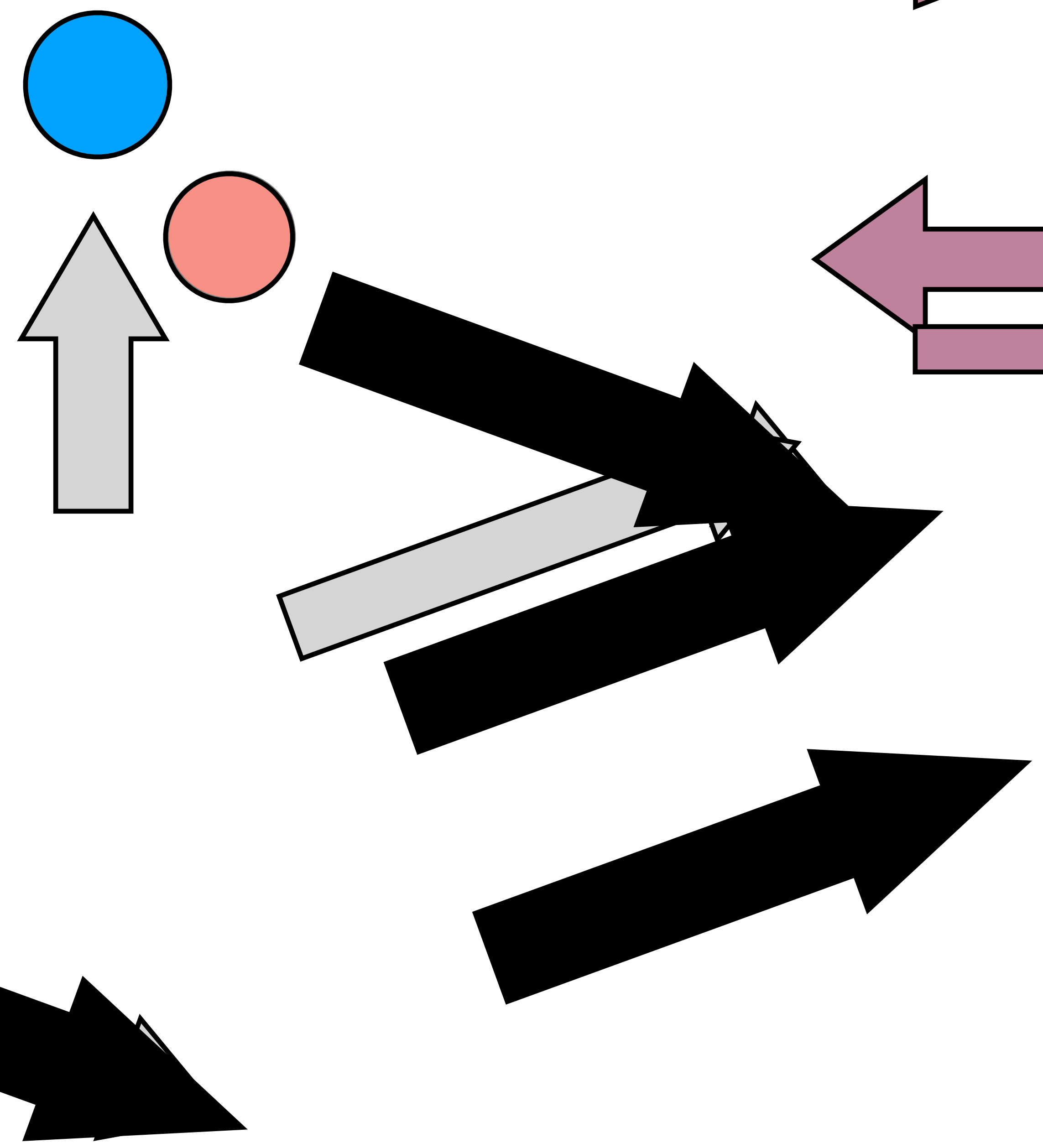
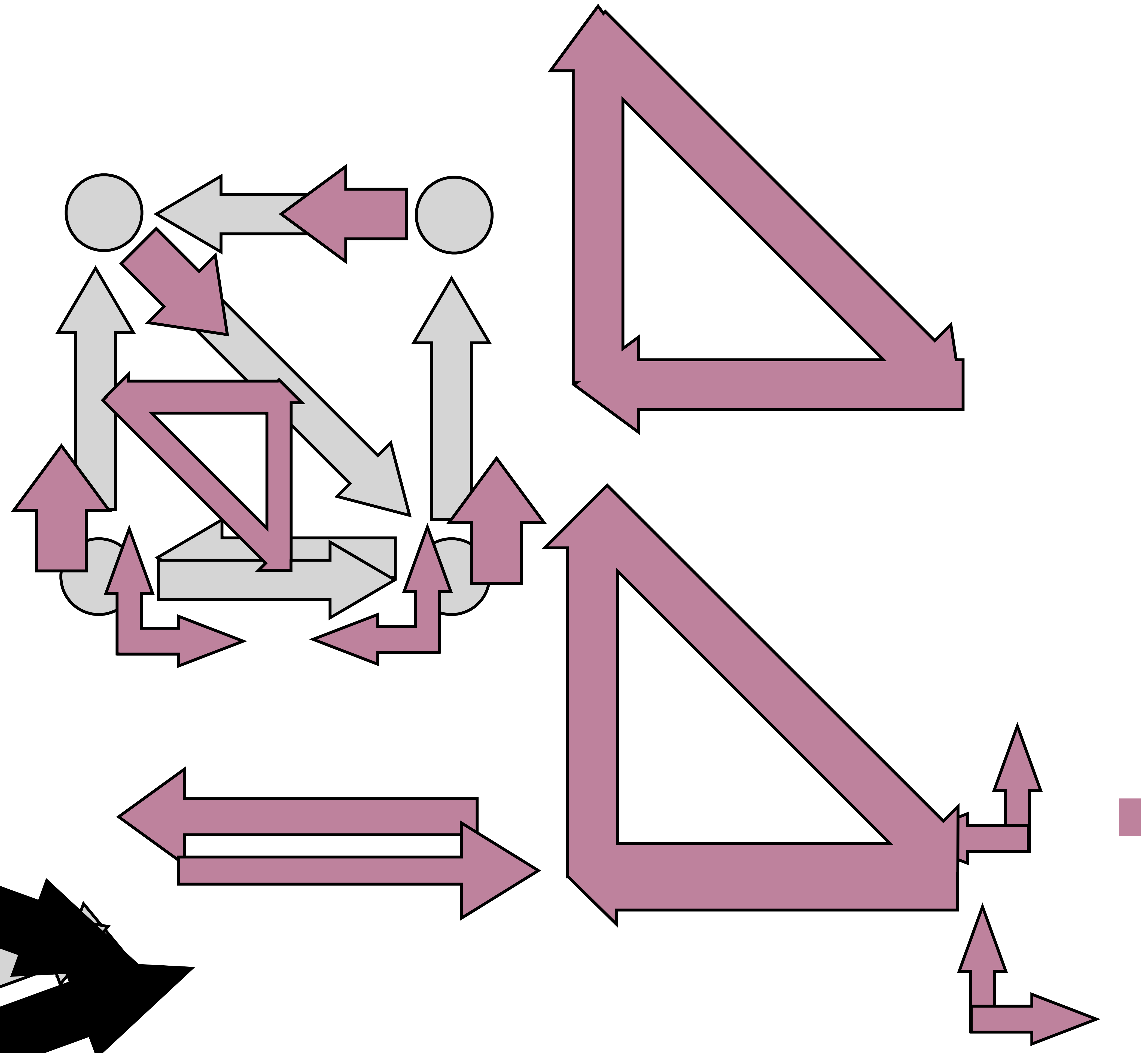
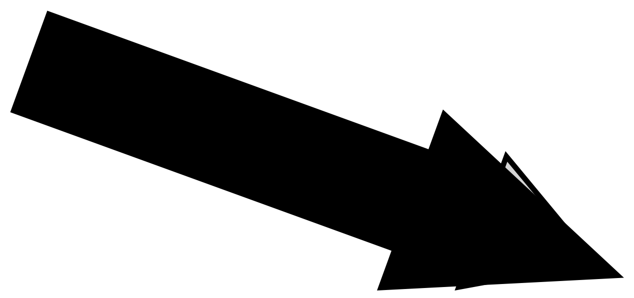


Markov Decision Process Congestion Game

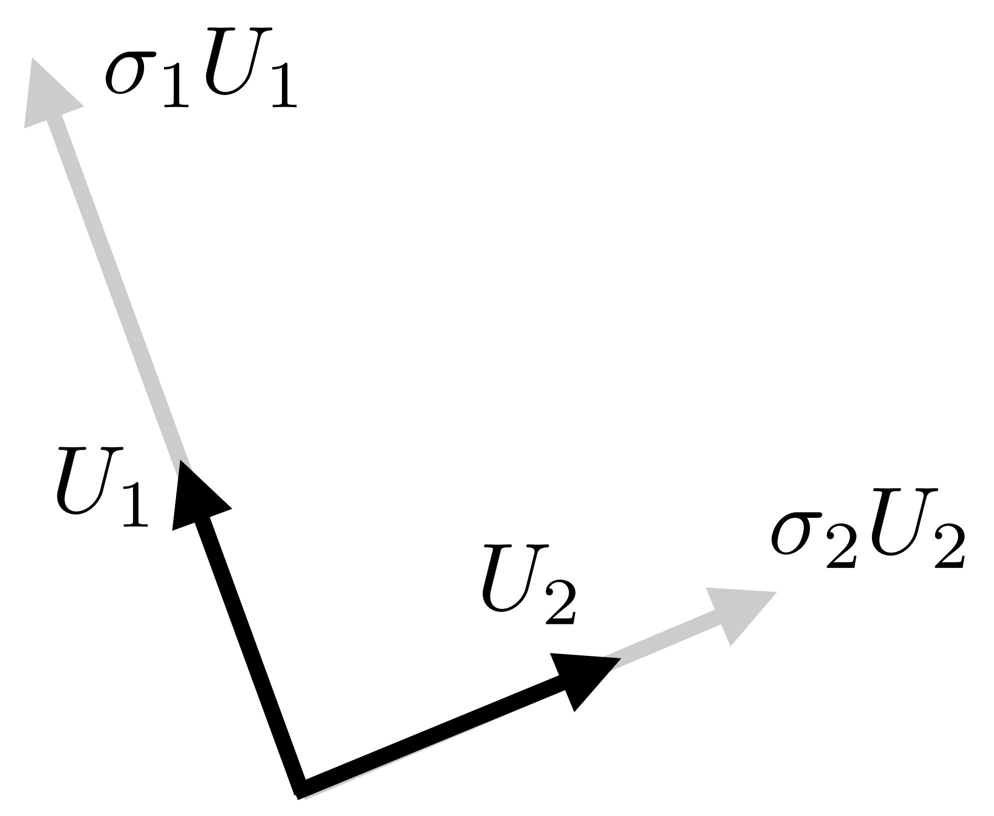








Velocity Space $\mathbb{R}^2$



directions of motion in velocity space

DECODER

$$D \in \mathbb{R}^{2\times 64}$$

$$D = U\,[\Sigma \quad 0]\,V^T$$

$$D = \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \\ & \vdots & \\ - & V_{64}^T & - \end{bmatrix}$$

Orthonormal basis for nullspace

$$D = \begin{bmatrix} | \\ U_1 \\ | \end{bmatrix} \sigma_1 \begin{bmatrix} - & V_1^T & - \end{bmatrix} + \begin{bmatrix} | \\ U_2 \\ | \end{bmatrix} \sigma_2 \begin{bmatrix} - & V_2^T & - \end{bmatrix}$$

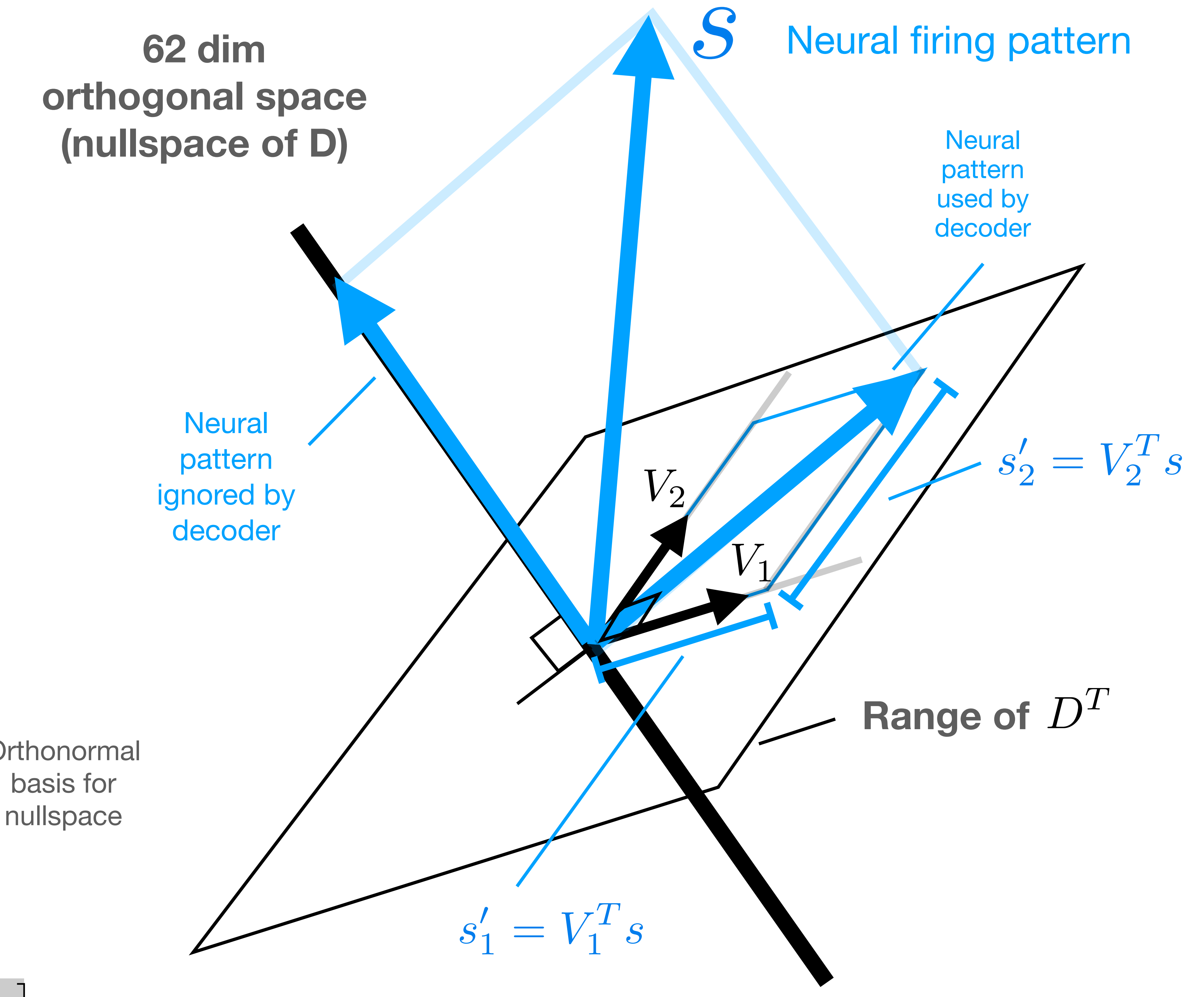
Correlations between neurons firing and motion in the U_1 direction

$$\begin{bmatrix} \sigma_1 (V_1)_1 & \cdots & \sigma_1 (V_1)_{64} \end{bmatrix}$$

... and the U_2 direction

$$\begin{bmatrix} \sigma_2 (V_2)_1 & \cdots & \sigma_2 (V_2)_{64} \end{bmatrix}$$

Neural Inputs: $\mathbb{R}^{64}$



Coordinate Transform: $s' = V^T s$

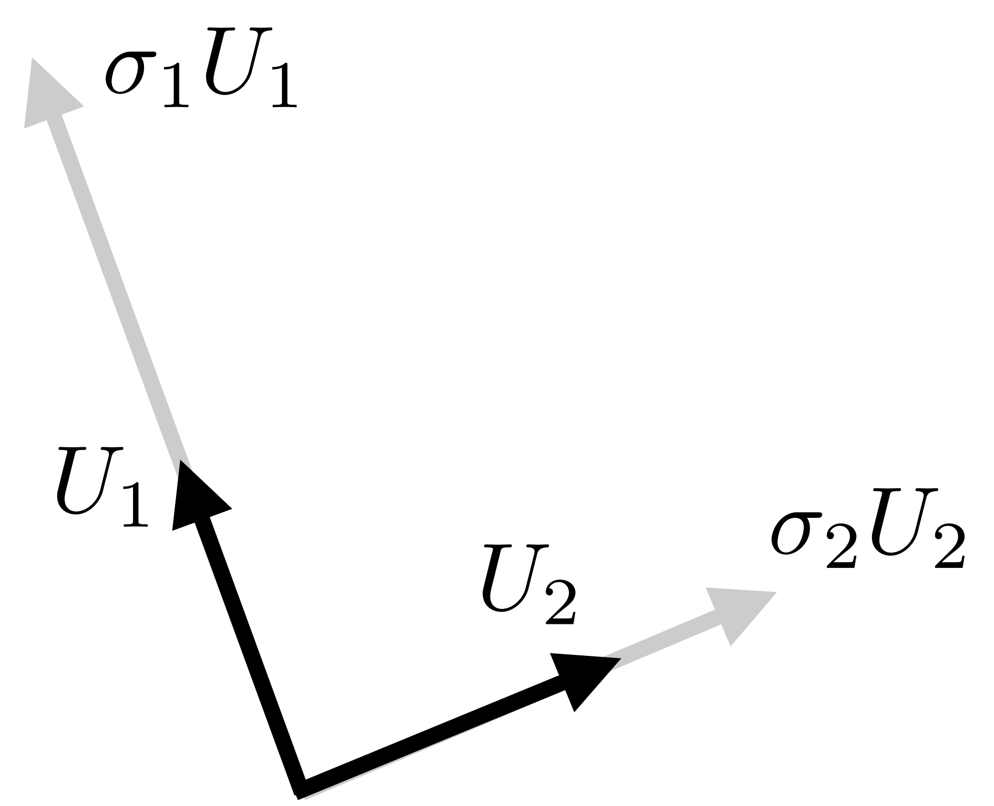
$$\begin{bmatrix} s_1' \\ s_2' \\ \vdots \end{bmatrix} = \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ & \vdots & \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix} = \begin{bmatrix} V_1^T s \\ V_2^T s \\ \vdots \end{bmatrix}$$

Velocity Space $\mathbb{R}^2$

DECODER

Neural Inputs: $\mathbb{R}^{64}$

If sigma1 = sigma2,
then U1 and U2 are arbitrary
and only span of V1 and V2 matters



directions of motion in velocity space

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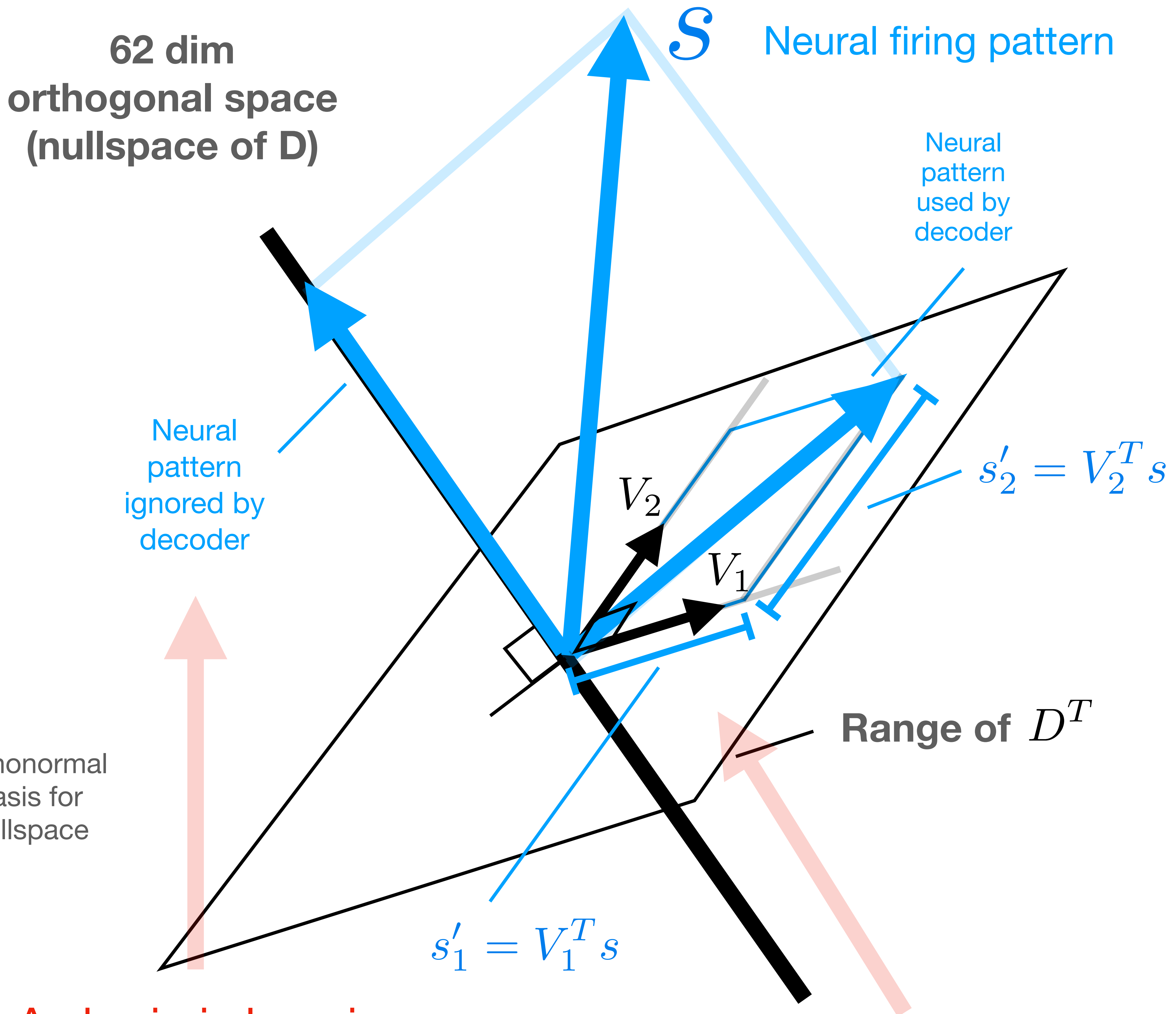
Orthonormal basis for nullspace

$$D = \begin{bmatrix} | \\ U_1 \\ | \end{bmatrix} \sigma_1 \begin{bmatrix} - & V_1^T & - \end{bmatrix} + \begin{bmatrix} | \\ U_2 \\ | \end{bmatrix} \sigma_2 \begin{bmatrix} - & V_2^T & - \end{bmatrix}$$



As brain is learning decoder, seems this component should shrink… (if not, brain is not halting firing of useless neurons.)

Would expect this subspace to converge and remain consistent over trials (for one person)

(span of V1 & V2 consistent)

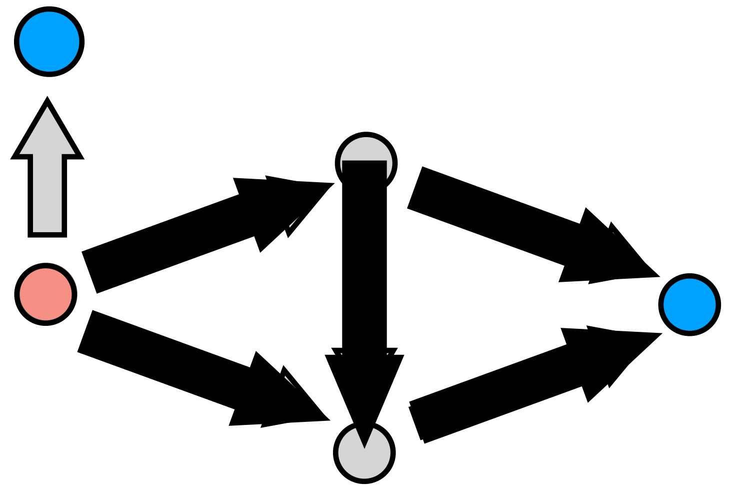
Correlations between neurons firing and motion in the U_1 direction

$$\begin{bmatrix} \sigma_1(V_1)_1 & \cdots & \sigma_1(V_1)_{64} \end{bmatrix}$$

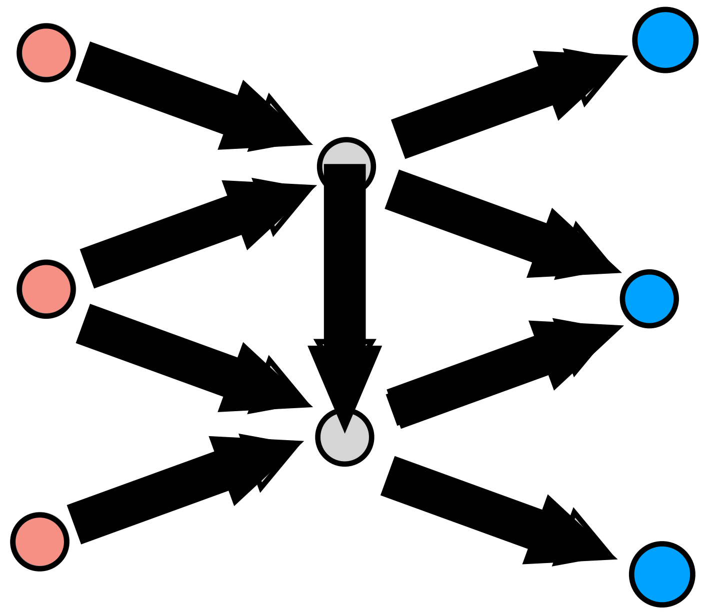
... and the U_2 direction

$$\begin{bmatrix} \sigma_2(V_2)_1 & \cdots & \sigma_2(V_2)_{64} \end{bmatrix}$$

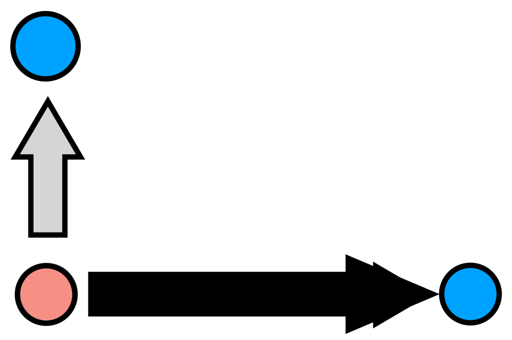
Variable Demand



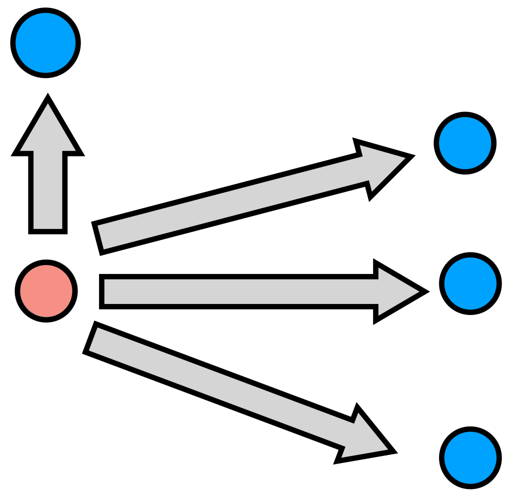
Multiple sources/ sinks



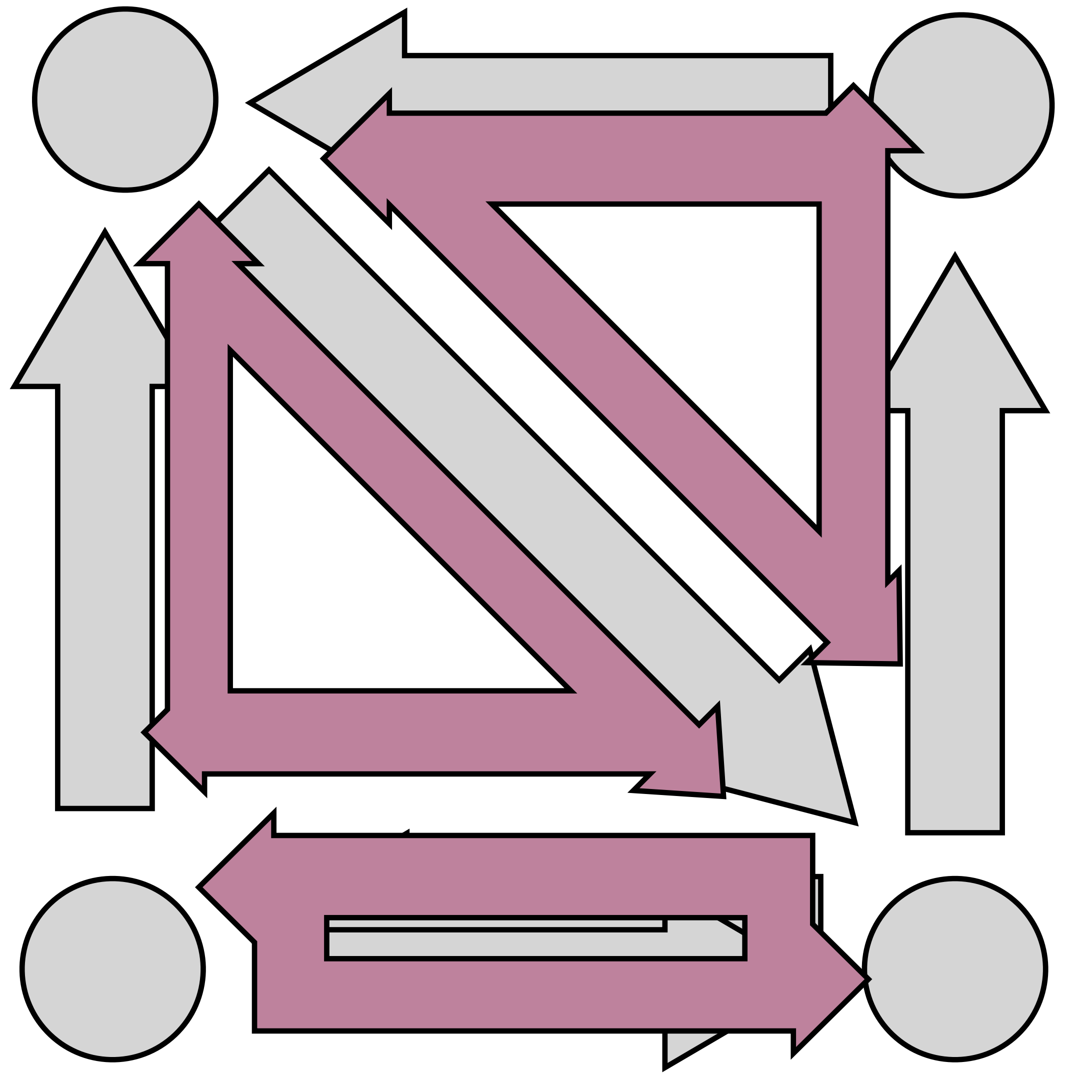
Supply & Demand

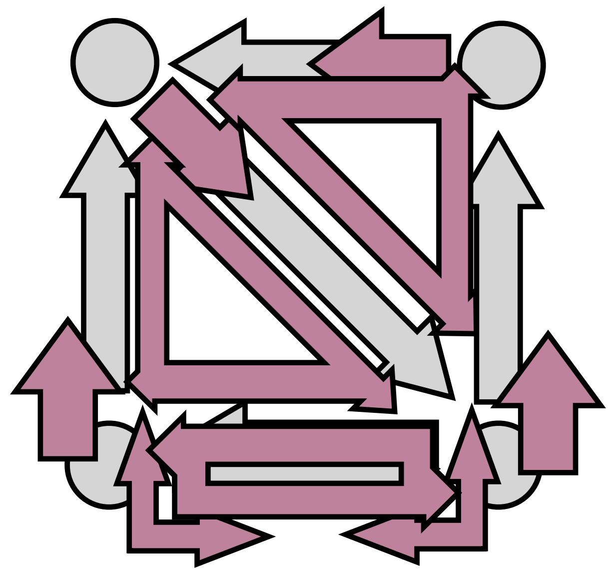


Cournot Competition



Markov Decision Process Congestion Game





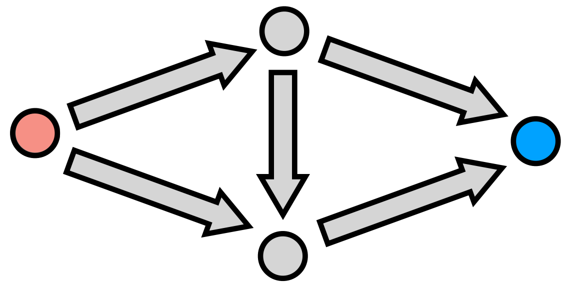
Finite or Infinite horizon

$$\begin{aligned} \min_x \quad & F(x) \\ \text{s.t.} \quad & EWx = 0 \qquad x \geq 0 \\ & \mathbf{1}^T x = m \end{aligned}$$

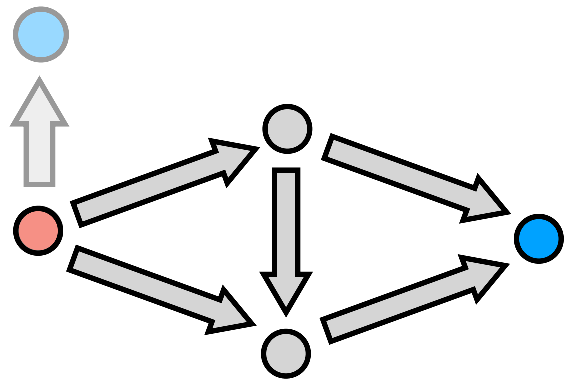
- Paper 1
- Paper 2
- Paper 3
- Paper 4

Potential Games

Routing Games



Variable Demand



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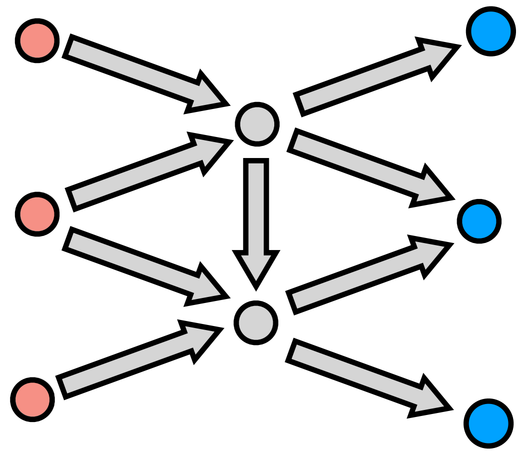

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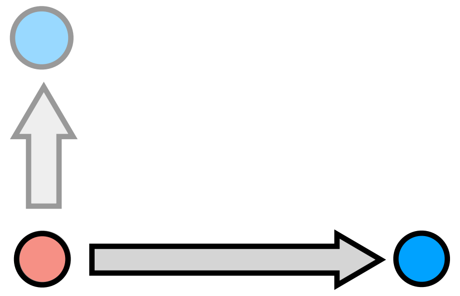
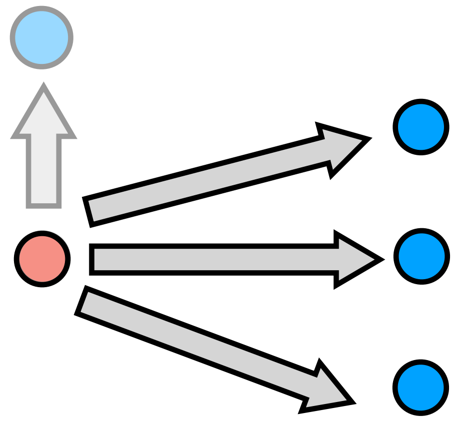
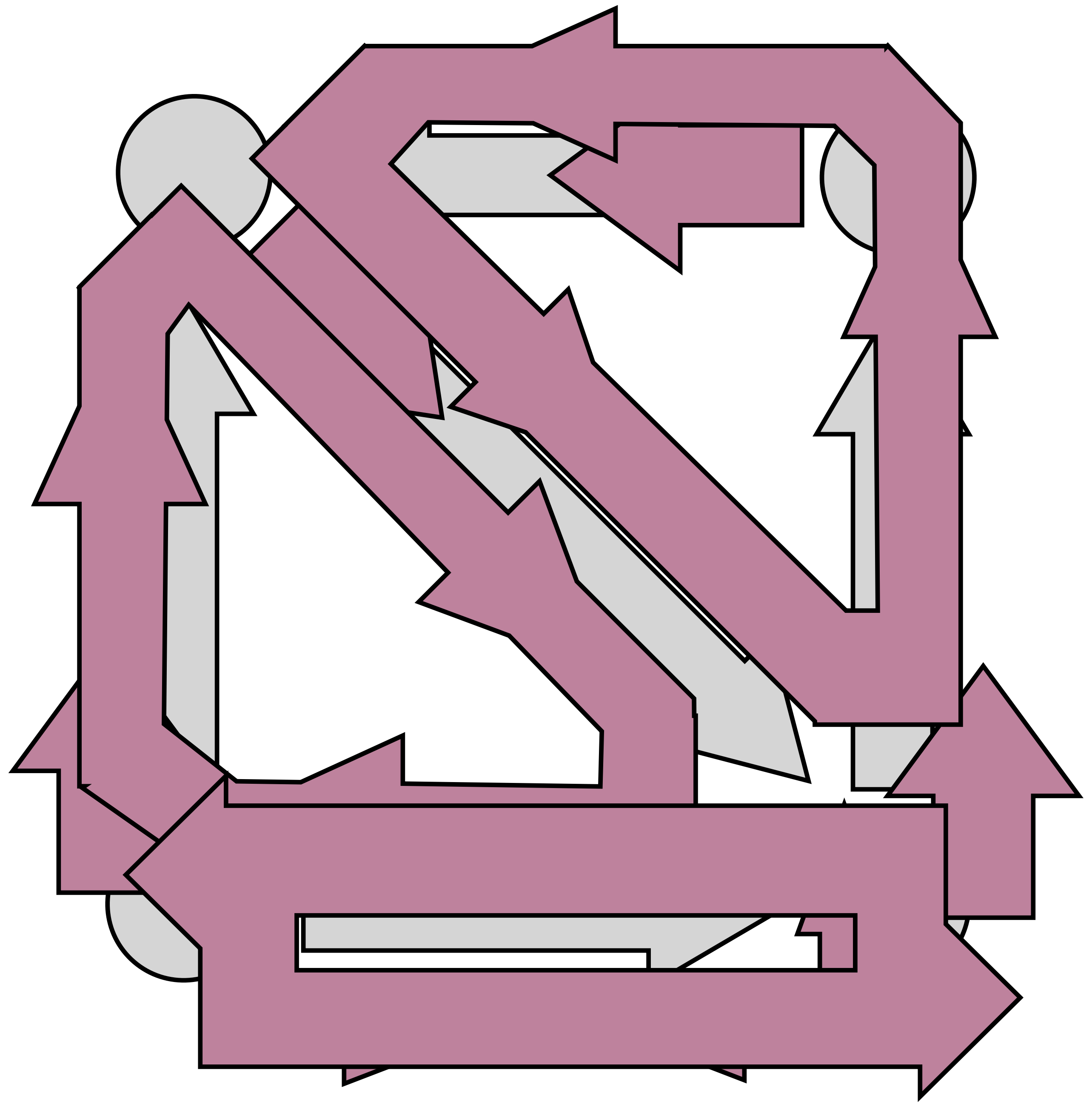


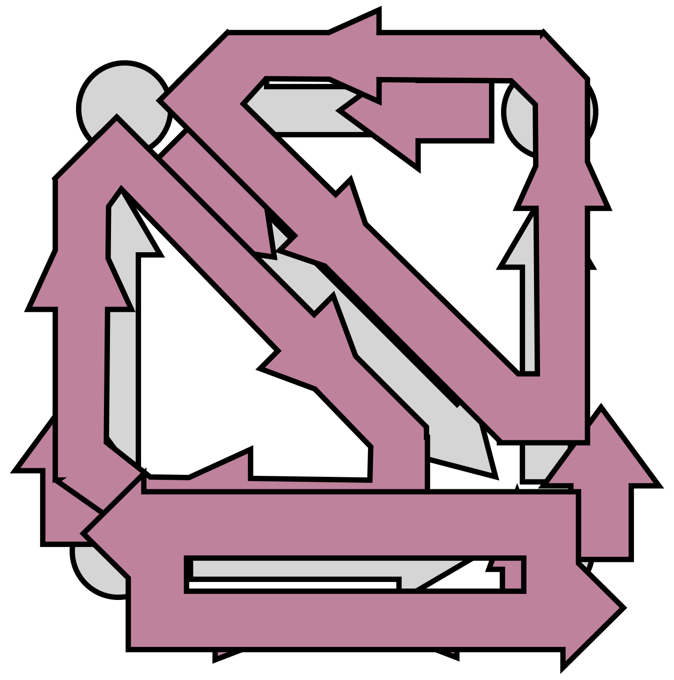
Market/ Cournot Competition



Markov Decision Process Congestion Game

Finite or Infinite Horizon





$$\min_x \quad F(x)$$

$$\text{s.t.} \quad EWx = 0 \qquad x \geq 0$$

$$\mathbf{1}^T x = m$$

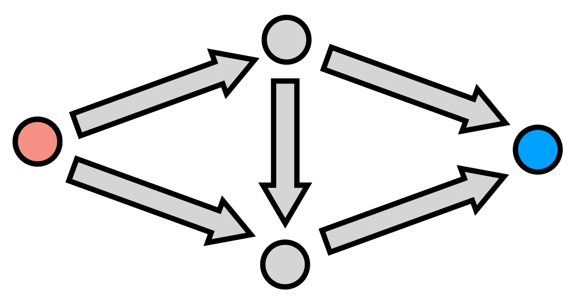
Potential Games

Routing Games

x : edge traffic

z : route traffic

Routing Games

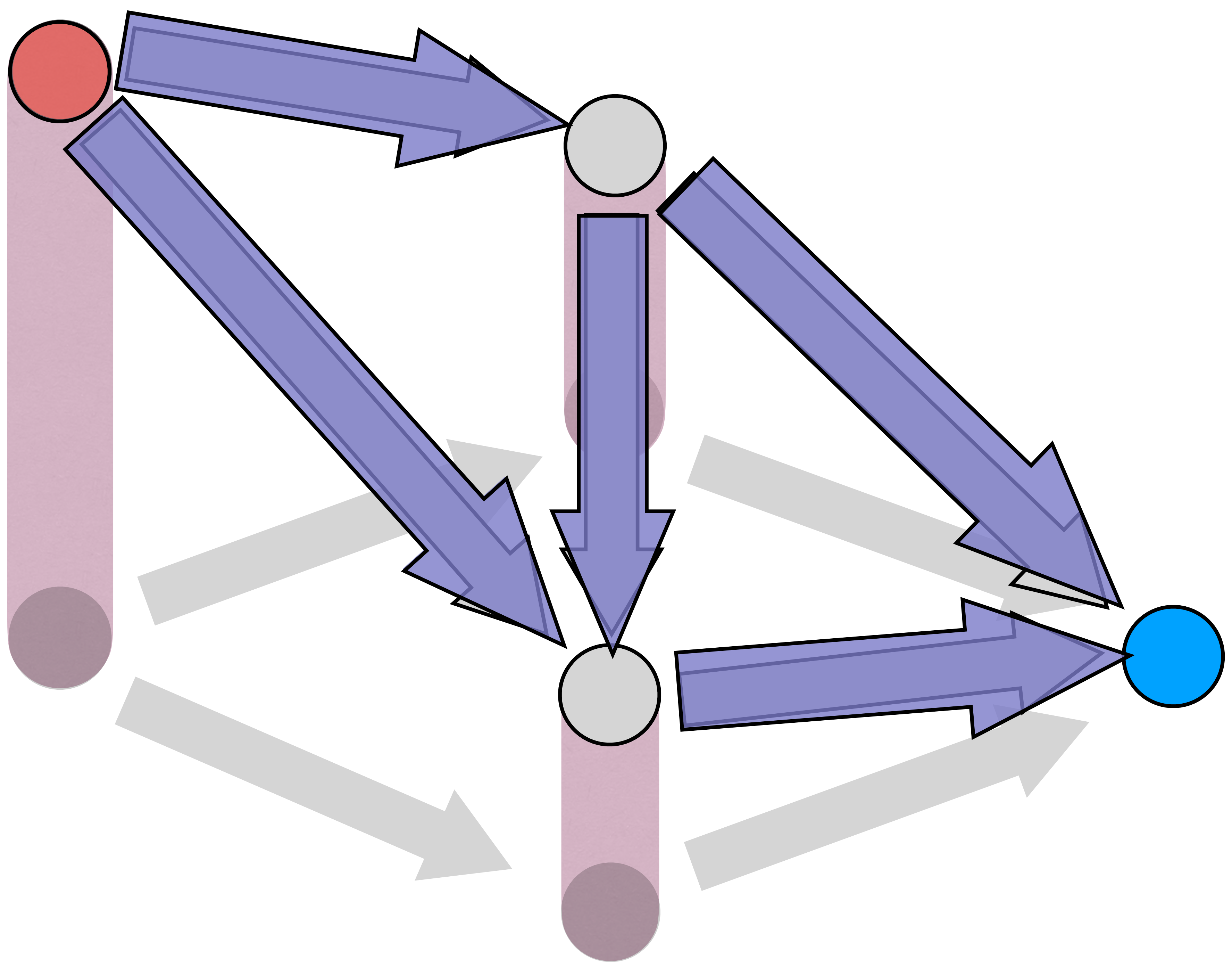


$F(x)$ **Potential Function**

$$\min_{x} \quad F(x)$$

$$\text{s.t.} \quad \boxed{Ex = Sm, \ \boxed{v}}$$

$$\boxed{x \geq 0 \ \boxed{\mu}}$$



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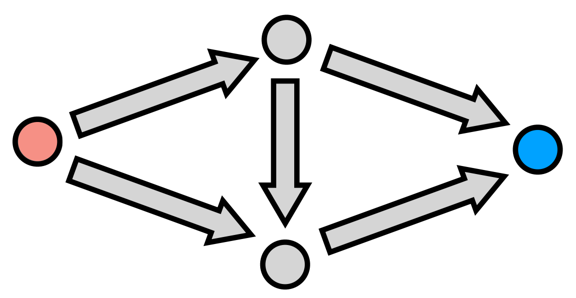

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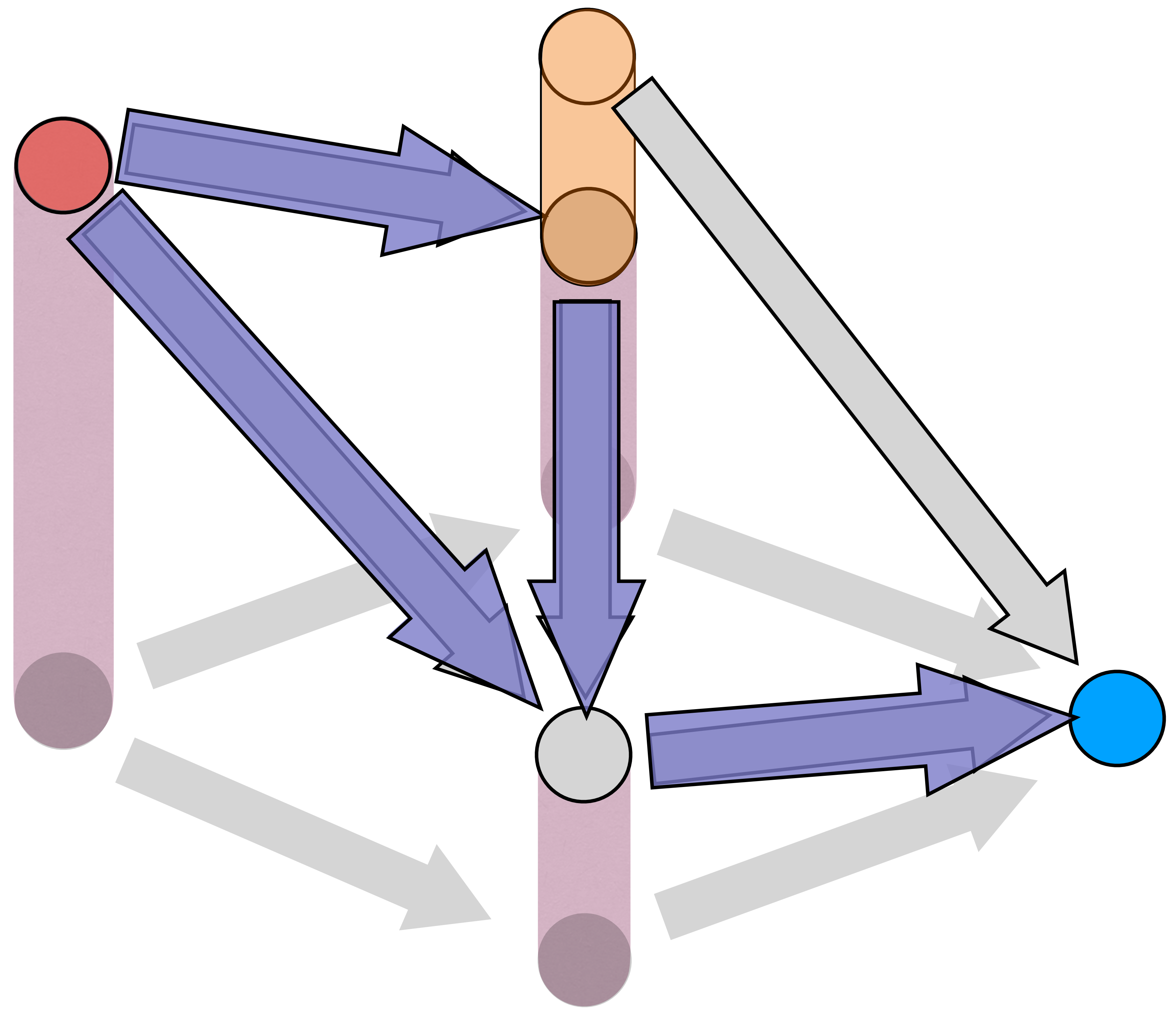

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$$\text{s.t.} \quad Ex = Sm, \quad v$$

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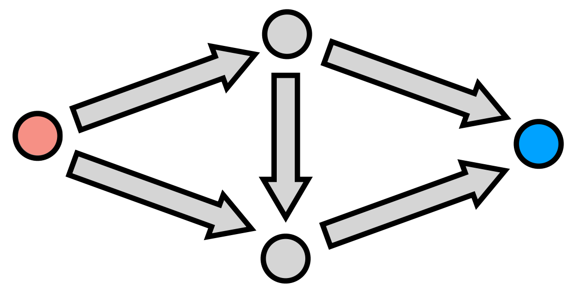
Potential Games

Routing Games



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