



$$\begin{aligned} \text{proj}_y^\perp x &= x - \text{proj}_y x \\ &= \underbrace{\left[ I - y(y^T y)^{-1} y^T \right]}_{\text{orthogonal projection matrix}} x \end{aligned}$$

$$\begin{aligned} \text{proj}_y x &= \frac{y}{\|y\|_2} \|x\|_2 \cos \theta \\ &= \frac{y}{\|y\|_2} \frac{y^T x}{\|y\|_2} \\ &= \underbrace{\left[ y(y^T y)^{-1} y^T \right]}_{\text{projection matrix}} x \end{aligned}$$