# Free Theorems for Nested Types

ANONYMOUS AUTHOR(S)

## 1 reverse FUNCTION FOR PLeaves

Let PLeaves be the type $\emptyset; \beta \vdash (\mu\phi.\lambda\alpha.\alpha + \phi(\alpha \times \alpha))\beta$. We will define a reverse function with type $\vdash \mathsf{reverse} : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\,(\mathsf{PLeaves}\,\beta)$.

To begin with, define the term swap as

$$\vdash L_{\beta}p.\,(\pi_2 p, \pi_1 p) : \mathsf{Nat}^{\beta}(\beta \times \beta)\,(\beta \times \beta)$$

and consider the map for PLeaves

$$\vdash \mathsf{map}^{\beta\times\beta,\beta\times\beta}_{\mathsf{PLeaves}\,\beta} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta \times \beta)\,(\beta \times \beta))\,\,(\mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,(\beta \times \beta))\,(\mathsf{PLeaves}\,(\beta \times \beta)))$$

Then we get a term

$$\vdash \mathsf{map}^{\beta\times\beta,\beta\times\beta}_{\mathsf{PLeaves}\,\beta}\,\mathsf{swap} : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,(\beta \times \beta))\,\,(\mathsf{PLeaves}\,(\beta \times \beta))$$

Define the term $s$ as

$$\begin{aligned}\vdash L_{\beta}t.\mathsf{case}\,t\,\mathsf{of}\,\{b \mapsto \mathsf{inl}\,b;\ t' \mapsto \mathsf{inr}\,((\mathsf{map}^{\beta\times\beta,\beta\times\beta}_{\mathsf{PLeaves}\,\beta}\,\mathsf{swap})_{\beta}\,t')\}\\ : \mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta \times \beta))\,\,(\beta + \mathsf{PLeaves}\,(\beta \times \beta))\end{aligned}$$

and consider the mediating morphism in of the initial algebra defining PLeaves

$$\vdash \mathsf{in}_{\alpha+\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta \times \beta))\,\,(\mathsf{PLeaves}\,\beta)$$

Then we get a term

$$\vdash \mathsf{in}_{\alpha+\phi(\alpha\times\alpha)} \circ s : \mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta \times \beta))\,\,(\mathsf{PLeaves}\,\beta)$$

Finally, consider the fold for PLeaves

$$\vdash \mathsf{fold}^{\mathsf{PLeaves}\,\beta}_{\alpha+\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta \times \beta))\,\,(\mathsf{PLeaves}\,\beta))\,\,(\mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\,\,(\mathsf{PLeaves}\,\beta))$$

then we define the reverse function as

$$\vdash \mathsf{fold}^{\mathsf{PLeaves}\,\beta}_{\alpha+\phi(\alpha\times\alpha)}(\mathsf{in}_{\alpha+\phi(\alpha\times\alpha)} \circ s) : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\,\,(\mathsf{PLeaves}\,\beta)$$

2020. 2475-1421/2020/1-ART1 $15.00
https://doi.org/