Free Theorems for Nested Types

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1 FREE THEOREM FOR TYPE OF POLYMORPHIC BOTTOM

Suppose $\vdash g : \mathsf{Nat}^{\alpha}\, \mathbb{1}\, \alpha$, let $G^{\mathsf{Set}} = [\![\vdash g : \mathsf{Nat}^{\alpha}\, \mathbb{1}\, \alpha]\!]^{\mathsf{Set}}$, and let $G^{\mathsf{Rel}} = [\![\vdash g : \mathsf{Nat}^{\alpha}\, \mathbb{1}\, \alpha]\!]^{\mathsf{Rel}}$. By Theorem **??**, $(G^{\mathsf{Set}}(\pi_1\rho), G^{\mathsf{Set}}(\pi_2\rho)) = G^{\mathsf{Rel}}\rho$. Thus, for all $\rho \in \mathsf{RelEnv}$ and any $(a, b) \in [\![\phi, \alpha; \emptyset \vdash \emptyset]\!]^{\mathsf{Rel}}\rho = 1$, eliding the only possible instantiations of a and b gives that

$$\begin{array}{rcl} (G^{\mathsf{Set}}, G^{\mathsf{Set}}) \;=\; (G^{\mathsf{Set}}(\pi_1\rho), G^{\mathsf{Set}}(\pi_2\rho)) & \in & [\![\vdash \mathsf{Nat}^{\alpha}\, \mathbb{1}\, \alpha]\!]^{\mathsf{Rel}}\rho \\ & = & \{\eta : K_1 \Rightarrow id\} \\ & = & \{(\eta_1 : K_1 \Rightarrow id, \eta_2 : K_1 \Rightarrow id)\} \end{array}$$

That is, G^{Set} is a natural transformation from the constantly 1-valued functor to the identity functor in Set. In particular, for every $S : \mathsf{Set}$, $G^{\mathsf{Set}}_S : 1 \to S$. Note, however, that if $S = \emptyset$, then there can be no such morphism, so no such natural transformation can exist in Set, and thus no term $\vdash g : \mathsf{Nat}^{\alpha} \mathbb{1}\, \alpha$ can exist in our calculus. That is, our calculus does not admit any terms with the closed type $\mathsf{Nat}^{\alpha} \mathbb{1}\, \alpha$ of the polymorphic bottom.

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