Free Theorems for Nested Types

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1 embed FUNCTION FOR PLeaves

Let PLeaves be the type $\emptyset;\beta \vdash (\mu\phi.\lambda\alpha.\alpha + \phi(\alpha\times\alpha))\beta$ and TLeaves be the type $\emptyset;\beta \vdash (\mu\phi.\lambda\alpha.\alpha + \phi\alpha \times \phi\alpha)\beta$. We will define a embed function with type $\vdash \mathsf{embed} : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\,(\mathsf{TLeaves}\,\beta)$.

Consider the map for TLeaves

$$\vdash \mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{TLeaves}\,\beta} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta\times\beta)\,\beta)\ (\mathsf{Nat}^{\beta}(\mathsf{TLeaves}\,(\beta\times\beta))\,(\mathsf{TLeaves}\,\beta))$$

which, applied to the projections $\beta\times\beta \to \beta$, yields terms

$$\vdash \mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{TLeaves}\,\beta}\,\pi_i : \mathsf{Nat}^{\beta}(\mathsf{TLeaves}\,(\beta\times\beta))\ (\mathsf{TLeaves}\,\beta)$$

for $i = 1, 2$. Then we define the term p as

$$\emptyset;\beta\,|\,t' : \mathsf{TLeaves}\,(\beta\times\beta) \vdash ((\mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{PLeaves}\,\beta}\,\pi_1)_\beta\,t', (\mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{PLeaves}\,\beta}\,\pi_2)_\beta\,t') : \mathsf{TLeaves}\,\beta\times\mathsf{TLeaves}\,\beta$$

Consider the mediating morphism in of the initial algebra defining TLeaves

$$\vdash \mathsf{in}_{\alpha+\phi\alpha\times\phi\alpha} : \mathsf{Nat}^{\beta}(\beta + \mathsf{TLeaves}\,\beta\times\mathsf{TLeaves}\,\beta))\ (\mathsf{TLeaves}\,\beta)$$

and define the term s as

$$\begin{aligned}\vdash L_\beta t.\mathsf{case}\ t\ \mathsf{of}\ \{b \mapsto (\mathsf{in}_{\alpha+\phi\alpha\times\phi\alpha})_\beta(\mathsf{inl}\,b);\ t' \mapsto (\mathsf{in}_{\alpha+\phi\alpha\times\phi\alpha})_\beta(\mathsf{inr}\,p)\}\\ : \mathsf{Nat}^{\beta}(\beta + \mathsf{TLeaves}\,(\beta\times\beta))\ (\mathsf{TLeaves}\,\beta)\end{aligned}$$

Finally, consider the fold for PLeaves

$$\vdash \mathsf{fold}^{\mathsf{TLeaves}\,\beta}_{\alpha+\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta + \mathsf{TLeaves}\,(\beta\times\beta))\ (\mathsf{TLeaves}\,\beta))\ (\mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\ (\mathsf{TLeaves}\,\beta))$$

and define the embed function as

$$\vdash \mathsf{fold}^{\mathsf{TLeaves}\,\beta}_{\alpha+\phi(\alpha\times\alpha)}\,s : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\ (\mathsf{TLeaves}\,\beta)$$

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