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Free Theorems for Nested Types

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1 reverse FUNCTION FOR Ptree

Let Ptree be the type $\emptyset;\beta \vdash (\mu\phi.\lambda\alpha.\mathbb{1} + \alpha \times \phi(\alpha \times \alpha))\beta$. We will define a reverse function with type $\vdash \mathsf{reverse} : \mathsf{Nat}^{\beta}(\mathsf{Ptree}\,\beta)\,(\mathsf{Ptree}\,\beta)$.

To begin with, define the term swap as

$$\vdash L_\beta p.\,(\pi_2 p, \pi_1 p) : \mathsf{Nat}^{\beta}(\beta \times \beta)\,(\beta \times \beta)$$

and consider the map for Ptree

$$\vdash \mathsf{map}^{\beta\times\beta,\beta\times\beta}_{\mathsf{Ptree}\,\beta} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta \times \beta)\,(\beta \times \beta))\;(\mathsf{Nat}^{\beta}(\mathsf{Ptree}\,(\beta \times \beta))\,(\mathsf{Ptree}\,(\beta \times \beta)))$$

Then we get a term

$$\vdash \mathsf{map}^{\beta\times\beta,\beta\times\beta}_{\mathsf{Ptree}\,\beta}\,\mathsf{swap} : \mathsf{Nat}^{\beta}(\mathsf{Ptree}\,(\beta \times \beta))\;(\mathsf{Ptree}\,(\beta \times \beta))$$

Define the term s as

$$\begin{aligned}&\vdash L_\beta t.\mathsf{case}\; t \;\mathsf{of}\; \{u \mapsto \mathsf{inl}\,u;\; t' \mapsto \mathsf{inr}\,(\pi_1\, t', (\mathsf{map}^{\beta\times\beta,\beta\times\beta}_{\mathsf{Ptree}\,\beta}\,\mathsf{swap})_\beta\,(\pi_2\, t'))\}\\ &\quad : \mathsf{Nat}^{\beta}(\mathbb{1} + \beta \times \mathsf{Ptree}\,(\beta \times \beta))\;(\mathbb{1} + \beta \times \mathsf{Ptree}\,(\beta \times \beta))\end{aligned}$$

and consider the mediating morphism in of the initial algebra defining Ptree

$$\vdash \mathsf{in}_{\mathbb{1}+\alpha\times\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\beta}(\mathbb{1} + \beta \times \mathsf{Ptree}\,(\beta \times \beta))\;(\mathsf{Ptree}\,\beta)$$

Then we get a term

$$\vdash \mathsf{in}_{\mathbb{1}+\alpha\times\phi(\alpha\times\alpha)} \circ s : \mathsf{Nat}^{\beta}(\mathbb{1} + \beta \times \mathsf{Ptree}\,(\beta \times \beta))\;(\mathsf{Ptree}\,\beta)$$

Finally, consider the fold for Ptree

$$\vdash \mathsf{fold}^{\mathsf{Ptree}\,\beta}_{\mathbb{1}+\alpha\times\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\mathbb{1} + \beta \times \mathsf{Ptree}\,(\beta \times \beta))\;(\mathsf{Ptree}\,\beta))\;(\mathsf{Nat}^{\beta}(\mathsf{Ptree}\,\beta)\;(\mathsf{Ptree}\,\beta))$$

then we define the reverse function as

$$\vdash \mathsf{fold}^{\mathsf{Ptree}\,\beta}_{\mathbb{1}+\alpha\times\phi(\alpha\times\alpha)}(\mathsf{in}_{\mathbb{1}+\alpha\times\phi(\alpha\times\alpha)} \circ s) : \mathsf{Nat}^{\beta}(\mathsf{Ptree}\,\beta)\;(\mathsf{Ptree}\,\beta)$$

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