

Free Theorems for Nested Types

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1 reverse FUNCTION FOR PLeaves

Let PLeaves be the type \emptyset ; $\beta \vdash (\mu\phi.\lambda\alpha.\alpha + \phi(\alpha \times \alpha))\beta$. We will define a reverse function with type $\vdash \text{reverse} : \text{Nat}^\beta(\text{PLeaves } \beta) (\text{PLeaves } \beta)$.

To begin with, define the term swap as

$$\vdash L_\beta p. (\pi_2 p, \pi_1 p) : \text{Nat}^\beta(\beta \times \beta) (\beta \times \beta)$$

and consider the map for PLeaves

$$\vdash \text{map}_{\text{PLeaves } \beta}^{\beta \times \beta, \beta \times \beta} : \text{Nat}^\emptyset(\text{Nat}^\beta(\beta \times \beta) (\beta \times \beta)) (\text{Nat}^\beta(\text{PLeaves } (\beta \times \beta)) (\text{PLeaves } (\beta \times \beta)))$$

Then we get a term

$$\vdash \text{map}_{\text{PLeaves } \beta}^{\beta \times \beta, \beta \times \beta} \text{swap} : \text{Nat}^\beta(\text{PLeaves } (\beta \times \beta)) (\text{PLeaves } (\beta \times \beta))$$

Define the term s as

$$\begin{aligned} \vdash L_\beta t. \text{case } t \text{ of } \{b \mapsto \text{inl } b; t' \mapsto \text{inr } (\text{map}_{\text{PLeaves } \beta}^{\beta \times \beta, \beta \times \beta} \text{swap } t')\} \\ : \text{Nat}^\beta(\beta + \text{PLeaves } (\beta \times \beta)) (\beta + \text{PLeaves } (\beta \times \beta)) \end{aligned}$$

and consider the mediating morphism in of the initial algebra defining PLeaves

$$\vdash \text{in}_{\alpha + \phi(\alpha \times \alpha)} : \text{Nat}^\beta(\beta + \text{PLeaves } (\beta \times \beta)) (\text{PLeaves } \beta)$$

Then we get a term

$$\vdash \text{in}_{\alpha + \phi(\alpha \times \alpha)} \circ s : \text{Nat}^\beta(\beta + \text{PLeaves } (\beta \times \beta)) (\text{PLeaves } \beta)$$

Finally, consider the fold for PLeaves

$$\vdash \text{fold}_{\alpha + \phi(\alpha \times \alpha)}^{\text{PLeaves } \beta} : \text{Nat}^\emptyset(\text{Nat}^\beta(\beta + \text{PLeaves } (\beta \times \beta)) (\text{PLeaves } \beta)) (\text{Nat}^\beta(\text{PLeaves } \beta) (\text{PLeaves } \beta))$$

then we define the reverse function as

$$\vdash \text{fold}_{\alpha + \phi(\alpha \times \alpha)}^{\text{PLeaves } \beta} (\text{in}_{\alpha + \phi(\alpha \times \alpha)} \circ s) : \text{Nat}^\beta(\text{PLeaves } \beta) (\text{PLeaves } \beta)$$