Free Theorems for Nested Types

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1 REPRESENTATION OF MAP

Let $\Gamma; \overline{\phi}, \overline{\gamma} \vdash H : \mathcal{F}$, $\overline{\Gamma; \overline{\beta}, \overline{\gamma} \vdash F : \mathcal{F}}$ and $\overline{\Gamma; \overline{\beta}, \overline{\gamma} \vdash G : \mathcal{F}}$. By definition of the semantic interpretation of map terms, we have

$$\begin{aligned} &[\![\Gamma; \emptyset \,|\, \emptyset \vdash \mathsf{map}_H^{\overline{F},\overline{G}} : \mathsf{Nat}^{\emptyset}\ (\overline{\mathsf{Nat}^{\overline{\beta},\overline{\gamma}}\, F\, G})\ (\mathsf{Nat}^{\overline{\gamma}}\, H[\overline{\phi :=_{\overline{\beta}} F}]\; H[\overline{\phi :=_{\overline{\beta}} G}])]\!]^{\mathsf{Set}} \rho \\ &\qquad = \lambda d\, \overline{\eta}\, \overline{B}.\ [\![\Gamma; \overline{\phi}, \overline{\gamma} \vdash H]\!]^{\mathsf{Set}} id_{\rho[\overline{\gamma := B}]} [\overline{\phi := \lambda \overline{A}.\eta_{\overline{A}\,\overline{B}}}] \end{aligned} \tag{1}$$

Then let $\Gamma; \overline{\alpha} \vdash F : \mathcal{F}$, $\overline{\Gamma; \emptyset \vdash \sigma : \mathcal{F}}$, $\overline{\Gamma; \emptyset \vdash \tau : \mathcal{F}}$ and $*$ be the unique element of $[\![\Gamma; \emptyset \vdash \emptyset]\!]^{\mathsf{Set}} \rho$. As a special case of the above definition, we have

$$\begin{array}{rl} & [\![\Gamma; \emptyset \,|\, \emptyset \vdash \mathsf{map}_F^{\overline{\sigma},\overline{\tau}} : \mathsf{Nat}^{\emptyset}\ (\overline{\mathsf{Nat}^{\emptyset}\, \sigma\, \tau})\ (\mathsf{Nat}^{\emptyset}\, F[\overline{\alpha := \sigma}]\; F[\overline{\alpha := \tau}])]\!]^{\mathsf{Set}} \rho\, * \\ = & \lambda \overline{f : [\![\Gamma; \emptyset \vdash \sigma]\!]^{\mathsf{Set}} \rho \to [\![\Gamma; \emptyset \vdash \tau]\!]^{\mathsf{Set}} \rho}.\ [\![\Gamma; \overline{\alpha} \vdash F]\!]^{\mathsf{Set}} id_\rho [\overline{\alpha := f}] \\ = & \lambda \overline{f : [\![\Gamma; \emptyset \vdash \sigma]\!]^{\mathsf{Set}} \rho \to [\![\Gamma; \emptyset \vdash \tau]\!]^{\mathsf{Set}} \rho}.\ map_{\lambda \overline{A}.\, [\![\Gamma; \overline{\alpha} \vdash F]\!]^{\mathsf{Set}} \rho [\overline{\alpha := A}]}\, \overline{f} \\ = & map_{\lambda \overline{A}.\, [\![\Gamma; \overline{\alpha} \vdash F]\!]^{\mathsf{Set}} \rho [\overline{\alpha := A}]} \end{array}$$

where the first equality is by Equation 1, the second equality is obtained by noting that $\lambda \overline{A}.\ [\![\Gamma; \overline{\alpha} \vdash F]\!]^{\mathsf{Set}} \rho [\overline{\alpha := A}$ is a functor in α, and map_G denotes the action of the functor G on morphisms.

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