Free Theorems for Nested Types

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1 split FUNCTION FOR PLeaves

Let PLeaves be the type $\emptyset; \beta \vdash (\mu\phi.\lambda\alpha.\alpha + \phi(\alpha \times \alpha))\beta$. We will define a split function with type $\vdash \mathsf{split} : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta))\ (\mathsf{PLeaves}\,\beta \times \mathsf{PLeaves}\,\beta)$.

Consider the map for PLeaves

$$\vdash \mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{PLeaves}\,\beta} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta\times\beta)\,\beta)\ (\mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,(\beta\times\beta))\,(\mathsf{PLeaves}\,\beta))$$

Then we get a term

$$\vdash \mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{PLeaves}\,\beta}\,\pi_1 : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,(\beta\times\beta))\ (\mathsf{PLeaves}\,\beta)$$

Consider the mediating morphism in of the initial algebra defining PLeaves

$$\vdash \mathsf{in}_{\alpha+\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta\times\beta))\ (\mathsf{PLeaves}\,\beta)$$

and define the term s_1 as

$$\begin{aligned}\vdash L_\beta t.\mathsf{case}\ t\ \mathsf{of}\ \{b \mapsto (\mathsf{in}_{\alpha+\phi(\alpha\times\alpha)})_\beta(\mathsf{inl}\,b);\ t' \mapsto (\mathsf{map}^{\beta\times\beta,\beta}_{\mathsf{PLeaves}\,\beta}\,\pi_1)_\beta\,t'\}\\ : \mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta\times\beta))\ (\mathsf{PLeaves}\,\beta)\end{aligned}$$

Finally, consider the fold for PLeaves

$$\vdash \mathsf{fold}^{\mathsf{PLeaves}\,\beta}_{\alpha+\phi(\alpha\times\alpha)} : \mathsf{Nat}^{\emptyset}(\mathsf{Nat}^{\beta}(\beta + \mathsf{PLeaves}\,(\beta\times\beta))\ (\mathsf{PLeaves}\,\beta))\ (\mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\ (\mathsf{PLeaves}\,\beta))$$

then we define the split_1 function as

$$\vdash \mathsf{fold}^{\mathsf{PLeaves}\,\beta}_{\alpha+\phi(\alpha\times\alpha)}\,s_1 : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\ (\mathsf{PLeaves}\,\beta)$$

Analogously, we can define split_2 by using the second projection; then we can pair split_1 and split_2 to get split as

$$\vdash L_\beta x.((\mathsf{split}_1)_\beta x, (\mathsf{split}_2)_\beta x) : \mathsf{Nat}^{\beta}(\mathsf{PLeaves}\,\beta)\ (\mathsf{PLeaves}\,\beta \times \mathsf{PLeaves}\,\beta)$$

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